QM: 1.1 - 1.5

1.1.)

$N = 14$

$N(14) = 1$

$N(15) = 1$

$N(16) = 3$

$N(22) = 2$

$N(24) = 2$

$N(25) = 5$

a.) $\langle j^2 \rangle = \sum_{j=0}^{\infty} j^2 P(j)$ $\quad P(j) = \frac{N(j)}{N}$

$$\langle j^2 \rangle = 14^2 \frac{1}{14} + 15^2 \frac{1}{14} + 16^2 \frac{3}{14} + 22^2 \frac{2}{14} + 24^2 \frac{2}{14} + 25^2 \frac{5}{14}$$

$$\Rightarrow \boxed{\langle j^2 \rangle = 459.57}$$

$$\langle j \rangle^2 = \left(\sum_{j=0}^{\infty} j \frac{N(j)}{N} \right)^2 \Rightarrow \left(1 + \frac{15}{14} + \frac{16 \cdot 3}{14} + \frac{22 \cdot 2}{14} + \frac{24 \cdot 2}{14} + \frac{25 \cdot 5}{14} \right)^2$$

$$\Rightarrow \boxed{\langle j \rangle^2 = 441} \qquad \langle j \rangle = 21$$

b.) $\Delta j = j - \langle j \rangle$

$\sigma^2 = \langle (\Delta j)^2 \rangle$ (1.11)

$$\sigma^2 = \langle (\Delta j)^2 \rangle = \sum_{j=0}^{\infty} (\Delta j)^2 \frac{N(j)}{N}$$

j	Δj	$(\Delta j)^2$
14	-7	49
15	-6	36
16	-5	25
22	1	1
24	3	9
25	4	16

c.)
using 1.12 $\sigma = \sqrt{\langle j^2 \rangle - \langle j \rangle^2}$

$$= \sqrt{459.57 - 441} \Rightarrow \boxed{\sigma = 4.31}$$

b.) cont'd
using 1.11

$$\sigma^2 = \frac{49(1) + 36(1) + 25(3) + 1(2) + 9(2) + 16(5)}{14} \Rightarrow \sigma^2 = 18.57$$

$$\boxed{\sigma = 4.31}$$

σ 1.12 eqn = σ 1.11 eqn

1.2

a.) $\sigma = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$ $\quad \langle x\rangle = \int_{-\infty}^{\infty} x\,P(x)\,dx$

$P(x) = \frac{1}{2\sqrt{hx}}\,dx$

$\langle x\rangle = \int_0^h x \frac{1}{2\sqrt{hx}}\,dx = \frac{h}{3} \Rightarrow \langle x\rangle^2 = \frac{h^2}{9}$

$\langle x^2\rangle = \int_0^h x^2 \frac{1}{2\sqrt{hx}}\,dx = \frac{1}{2\sqrt{h}}\int_0^h x^{3/2}\,dx = \frac{1}{5\sqrt{h}}\left(x^{5/2}\Big|_0^h\right)$

$\langle x^2\rangle = \frac{h^2}{5}$ $\quad \sigma = \sqrt{\frac{h^2}{5} - \frac{h^2}{9}} \Rightarrow h\sqrt{.2 - .111}$

$\boxed{\sigma = 0.298h}$

b.) $\langle x\rangle - \sigma = \frac{h}{3} - 0.298h$

$P = \int_0^{\langle x\rangle - \sigma} P(x)\,dx + \int_{\langle x\rangle + \sigma}^{h} P(x)\,dx$

$\hookrightarrow \int_0^{\frac{h}{3} - 0.298h} \frac{1}{2\sqrt{hx}}\,dx + \int_{\frac{h}{3}+0.298h}^{h} \frac{1}{2\sqrt{hx}}\,dx$ $\quad \frac{1}{2\sqrt{h}}\int x^{-1/2}\,dx = \frac{1}{2\sqrt{h}} \cdot \frac{1}{2} x^{1/2}$

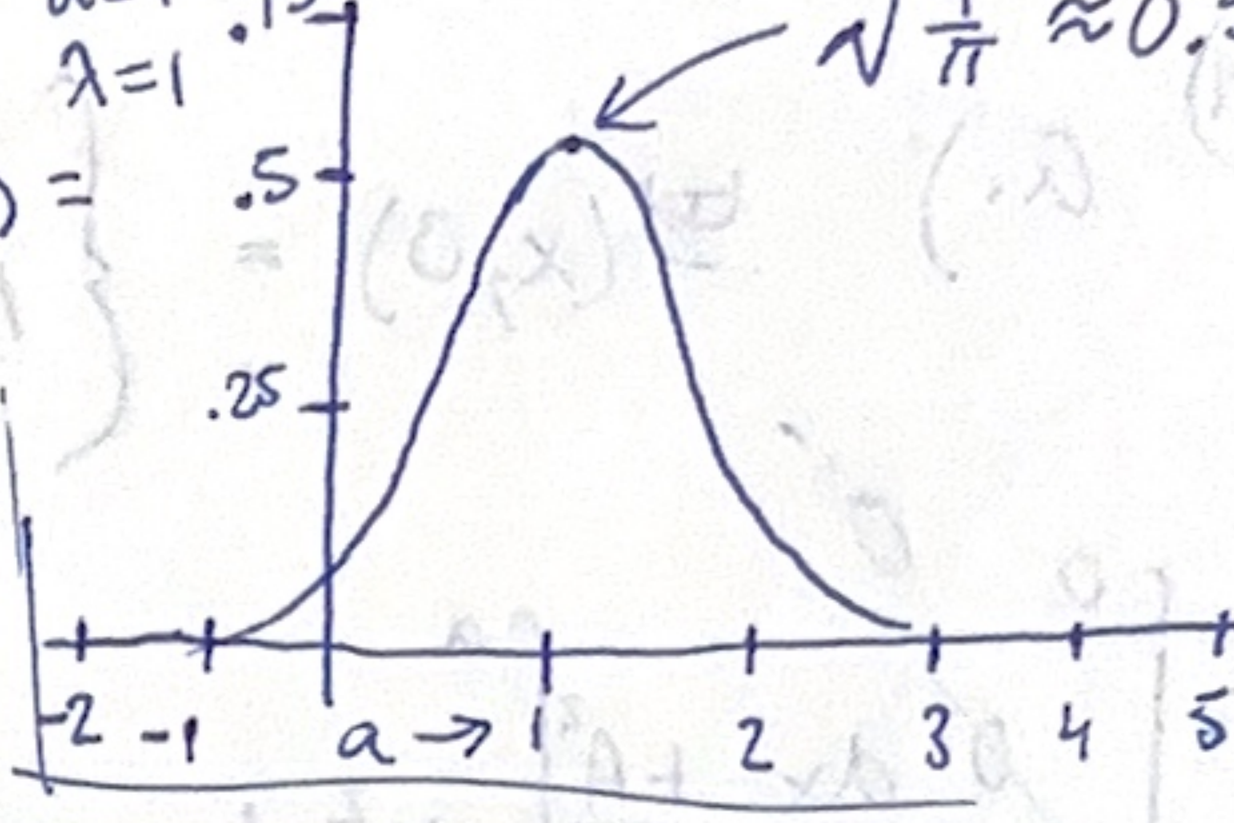
$\Rightarrow \frac{1}{\sqrt{h}}\left[x^{1/2}\Big|_0^{\frac{h}{3}-0.298h} + x^{1/2}\Big|_{\frac{h}{3}+0.298h}^{h}\right]$

$\Rightarrow \frac{1}{\sqrt{h}}\left[\sqrt{\frac{h}{3} - 0.298h} + \sqrt{h} - \sqrt{\frac{h}{3} + 0.298h}\right]$

$$\boxed{P = \frac{1}{\sqrt{h}}\left(\sqrt{\frac{h}{3} - 0.298h} + \sqrt{h} - \sqrt{\frac{h}{3} + 0.298h}\right)}$$

1.3 $\quad p(x) = Ae^{-\lambda(x-a)^2}$

c.) $A = \sqrt{\frac{\lambda}{\pi}}$ $a=1$ $\lambda=1$ $p(x) =$



$u = x-a$
$du = dx$

a.) $\displaystyle\int_{-\infty}^{\infty} p(x)\,dx = 1 \quad (1.16)$

$\hookrightarrow A\displaystyle\int_{-\infty}^{\infty} e^{-\lambda(x-a)^2}dx = 1 \Rightarrow \underbrace{2A\int_0^{\infty} e^{-\lambda u^2}du}$

$\frac{1}{a^2} = \lambda$ $\quad a = \frac{1}{\sqrt{\lambda}}$

$\sqrt{\pi}\,\frac{(2n)!}{n!}\left(\frac{a}{2}\right)^{2n+1}$

$\Rightarrow 2A\left(\sqrt{\pi}\cdot\frac{1}{2\sqrt{\lambda}}\right) = 1$

$\cancel{2}A\frac{\sqrt{\pi}}{\cancel{2}\sqrt{\lambda}} = 1 \Rightarrow \boxed{A = \sqrt{\frac{\lambda}{\pi}}} \quad A^2 = \frac{\lambda}{\pi}$

$u = x-a \quad x = u+a$
$du = dx$

b.) $\langle x\rangle = \displaystyle\int_{-\infty}^{\infty} x\,p(x)\,dx \Rightarrow A\int_{-\infty}^{\infty} x\,e^{-\lambda(x-a)^2} \Rightarrow A\int_{-\infty}^{\infty}(u+a)e^{-\lambda u^2}du$

$\Rightarrow \sqrt{\frac{\lambda}{\pi}}\left[\underbrace{\displaystyle\int_{-\infty}^{\infty} u\,e^{-\lambda u^2}du}_{0} + a\underbrace{\int_{-\infty}^{\infty} e^{-\lambda u^2}du}_{\sqrt{\frac{\pi}{\lambda}}}\right] \Rightarrow \sqrt{\frac{\lambda}{\pi}}\left[a\left(\sqrt{\pi/\lambda}\right)\right]$

$\boxed{\langle x\rangle = a}$

$u^2 + 2ua + a^2$

$\langle x^2\rangle = \displaystyle\int_{-\infty}^{\infty} x^2 p(x)\,dx \Rightarrow A\int_{-\infty}^{\infty} x^2 e^{-\lambda(x-a)^2}dx \Rightarrow A\int_{-\infty}^{\infty}(u+a)^2 e^{-\lambda u^2}du$

$\Rightarrow A\left[\displaystyle\int_{-\infty}^{\infty} u^2 e^{-\lambda u^2}du + \cancel{\int_{-\infty}^{\infty} u\,e^{-\lambda u^2}du}^{\,0} + a^2\underbrace{\int_{-\infty}^{\infty} e^{-\lambda u^2}du}_{\sqrt{\frac{\pi}{\lambda}}}\right] \Rightarrow 2\sqrt{\frac{\lambda}{\pi}}\underbrace{\int_0^{\infty} u^2 e^{-\lambda u}du}_{\sqrt{\pi}(2)\left(\frac{1}{2\sqrt{\lambda}}\right)^3} + a^2$

$A = \sqrt{\frac{\lambda}{\pi}}$

$\sqrt{\frac{\lambda}{\pi}}\,\frac{1}{2}\sqrt{\pi}\left(\frac{1}{\lambda^{3/2}}\right) + a^2 \Rightarrow \frac{1}{2}\sqrt{\frac{\lambda}{\pi}}\cdot\sqrt{\frac{\pi}{\lambda^3}} + a^2 \Rightarrow \boxed{\frac{1}{2\lambda} + a^2 = \langle x^2\rangle}$

$\sigma = \sqrt{\langle x^2\rangle - \langle x\rangle^2} = \sqrt{\frac{1}{2\lambda} + a^2 - a^2} \Rightarrow \boxed{\sigma = \frac{1}{\sqrt{2\lambda}}}$

1.4

a.) $\Psi(x,0) = \begin{cases} A(x/a) & 0 \le x \le a \\ A(b-x)/(b-a) & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$

$$\int_{-\infty}^{0} \cancel{0}\, dx + \frac{A^2}{a^2}\int_0^a x^2\, dx + \frac{A^2}{(b-a)^2}\int_a^b (b-x)^2\, dx + \int_b^{\infty} \cancel{0}\, dx = 1$$

$$\Rightarrow \frac{A^2}{a^2}\left(\frac{1}{3}\right)\left[x^3 \Big|_0^a\right] + \qquad \int_a^b b^2 - 2bx + x^2\, dx$$

a^3

$$\hookrightarrow b^2 x\Big|_a^b - 2b\left(\tfrac{1}{2}\right)x^2\Big|_a^b + \tfrac{1}{3}x^3\Big|_a^b$$

$$\hookrightarrow \frac{A^2}{3}a$$

$$\hookrightarrow \frac{A^2}{(b-a)^2}\left[b^2(b-a) - b(b-a)^2 + \tfrac{1}{3}(b-a)^3\right]$$

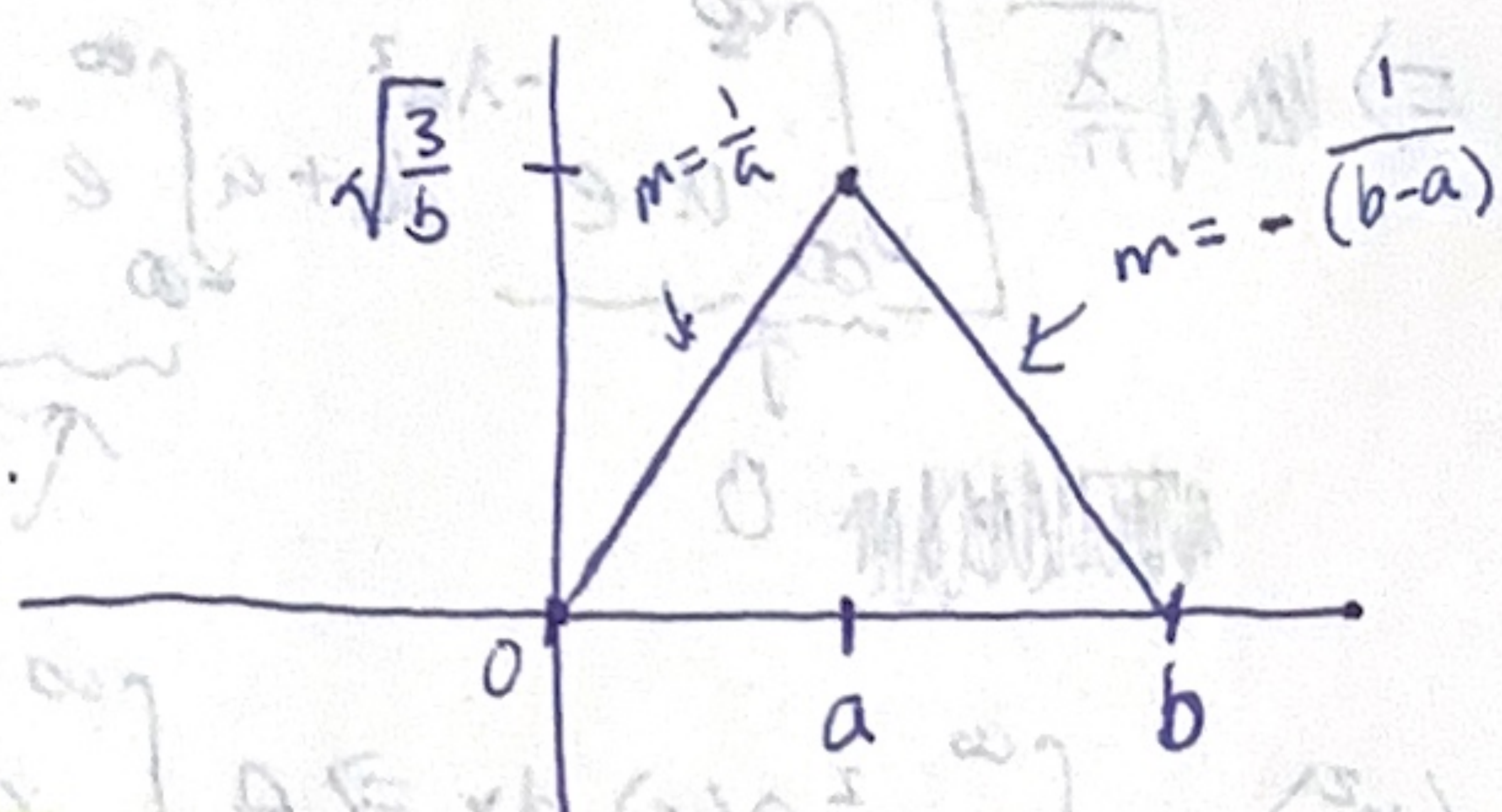
$$\frac{A^2}{3}a + \frac{A^2}{b-a}\overbrace{\left(b^2 - b(b-a) + \tfrac{1}{3}(b-a)^2\right)}^{(b-a)^2} = 1$$

$$\hookrightarrow \frac{A^2}{b-a}\left[b^2 - b(b-a) + \tfrac{1}{3}(b-a)^2\right]$$

$$\Rightarrow \frac{A^2}{3}(a + b - a) = 1$$

$$\hookrightarrow A^2 = \frac{3}{b} \Rightarrow \boxed{A = \sqrt{\frac{3}{b}}}$$

b.) $\Psi(x,0) = \begin{cases} \sqrt{\frac{3}{b}}\left(\frac{x}{a}\right) & 0 \le x \le a \\ \sqrt{\frac{3}{b}}\,\frac{(b-x)}{(b-a)} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$



c.) most likely at a because highest amplitude

d.) $P = \int_0^a |\Psi(x,0)|^2 dx \Rightarrow \int_0^a (A(x/a))^2 dx \Rightarrow \frac{3}{a^2 b}\int_0^a x^2 dx$

$$\Rightarrow \frac{3}{3a^2 b} x^3\Big|_0^a \Rightarrow \boxed{\frac{a}{b}}$$

If $b=a$ then $P=1$

If $b=2a$ then $P=\frac{1}{2}$

e.)

(1.4)

e.) $\langle x \rangle = \int_{-\infty}^{\infty} x\, p(x)\, dx \qquad A = \sqrt{\frac{3}{b}}$

$\Rightarrow \int_{-\infty}^{0} \cancel{0} \cdot x\, dx + \int_0^a \left(A\left(\frac{x}{a}\right)\right)^2 x\, dx + \int_a^b \left(A \frac{(b-x)}{(b-a)}\right)^2 x\, dx + \int_b^{\infty} \cancel{0} \cdot x\, dx$

$u = b - x \qquad x = b - u$

$du = -dx$

$\frac{3}{a^2 b}\int_0^a x^3\, dx + \frac{3}{b(b-a)^2}\int_a^b x(b-x)^2\, dx \Rightarrow -\int (b-u)\, u^2\, du \Rightarrow -\int bu^2 + \int u^3\, du$

$\Rightarrow -\frac{b}{3}u^3 + \frac{1}{4}u^4$

$\hookrightarrow \frac{1}{4}x^4 \Big|_0^a \qquad \Rightarrow -\frac{b}{3}(b-x)^3 \Big|_a^b + \frac{1}{4}(b-x)^4 \Big|_a^b$

$\frac{3a^2}{4b} + \frac{3}{b(b-a)^2} \cdot \qquad \frac{b(b-a)^3}{3} - \frac{(b-a)^4}{4} \qquad \Rightarrow \frac{b}{3}(b-a)^3 - \frac{1}{4}(b-a)^4$

$\hookrightarrow \frac{3a^2}{4b} + \frac{3}{b(b-a)^2} \cdot \frac{(b-a)^3(b+3a)}{4} \qquad \hookrightarrow \frac{4b(b-a)^3}{12} - \frac{3(b-a)^4}{12} = \frac{4b(b-a)^3 - 3(b-a)^4}{12}$

$\hookrightarrow \frac{3a^2}{4b} + \frac{(b-a)(b+3a)}{4b} \qquad \Rightarrow \frac{(b-a)^3\left[4b - 3b + 3a\right]}{12} = \frac{(b-a)^3(b+3a)}{12}$

$\hookrightarrow \frac{3a^2 + b^2 + 2ab - 3a^2}{4b} = \frac{b^2 + 2ab}{4b} = \boxed{\frac{b+2a}{4} = \langle x \rangle}$

(1.5) $\Psi(x,t) = Ae^{-\lambda|x|}e^{-i\omega t}$

a.) $\int_{-\infty}^{\infty} |\Psi(x,t)|^2\, dx = 1 \Rightarrow \int_{-\infty}^{\infty} Ae^{-\lambda|x|}e^{-i\omega t} \cdot Ae^{-\lambda|x|}e^{i\omega t}\, dx$

$\Rightarrow A^2 \int_{-\infty}^{\infty} e^{-2\lambda|x|}\, dx \Rightarrow A^2\left[\int_{-\infty}^{0} e^{2\lambda x}\, dx + \int_0^{\infty} e^{-2\lambda x}\, dx\right]$

$A^2\left[\frac{1}{2\lambda}\left(e^{2\lambda x}\Big|_{-\infty}^{0}\right) - \frac{1}{2\lambda}\left(e^{-2\lambda x}\Big|_0^{\infty}\right)\right] \Rightarrow A^2\left[\frac{1}{2\lambda}(1-0) - \frac{1}{2\lambda}(0-1)\right] = \frac{A^2}{\lambda} = 1$

$\boxed{A = \sqrt{\lambda}}$

1.5 cont'd

b.) $\Psi(x,t) = Ae^{-\lambda|x|}e^{-i\omega t}$ $\quad A = \sqrt{\lambda}$

$$\langle x\rangle = \int_{-\infty}^{\infty} x\,|\Psi(x,t)|^2\,dx \Rightarrow (Ae^{-\lambda|x|}e^{-i\omega t})\cdot(Ae^{-\lambda|x|}e^{i\omega t})$$

$$\Rightarrow A^2 e^{-2\lambda|x|} = \lambda e^{-2\lambda|x|}$$

$$\hookrightarrow \lambda\int_{-\infty}^{\infty} x e^{-2\lambda|x|}\,dx = 0 \qquad \boxed{\langle x\rangle = 0}$$

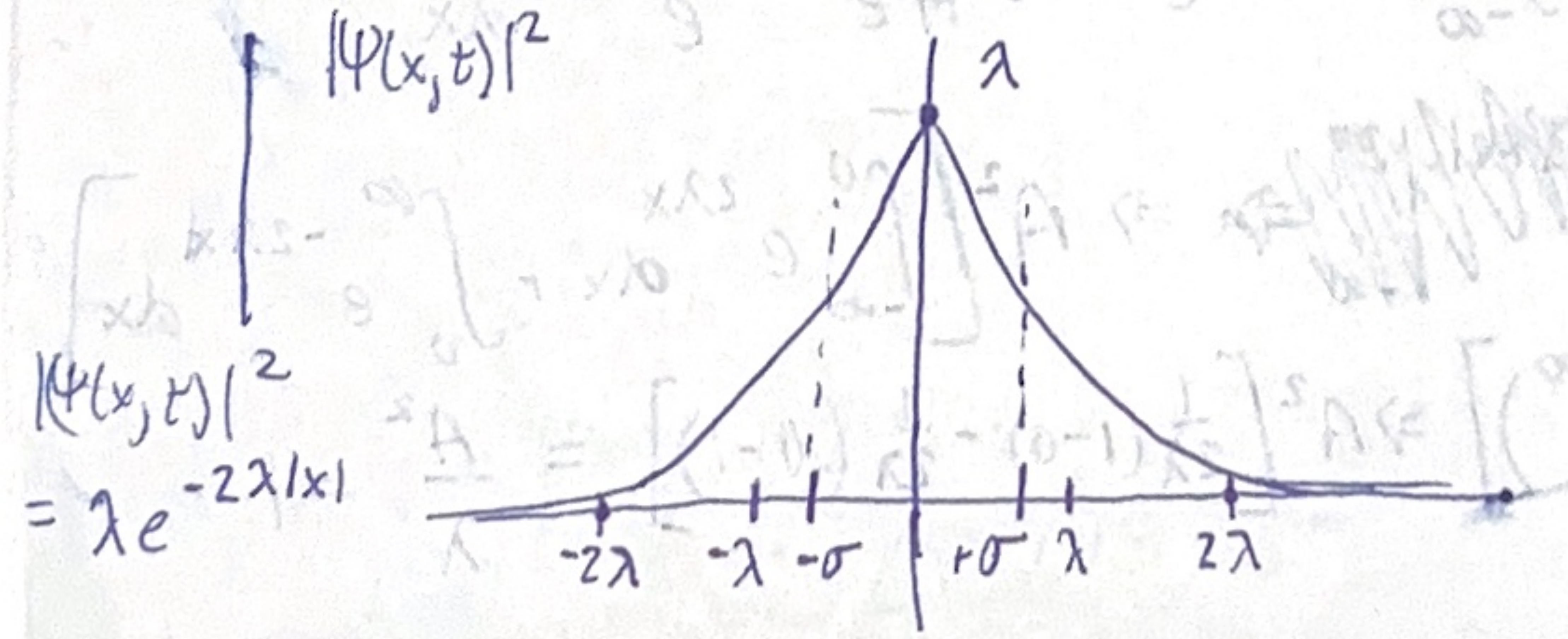
$$\langle x^2\rangle = \int_{-\infty}^{\infty} x^2|\Psi(x,t)|^2\,dx = \lambda\int_{-\infty}^{\infty} x^2 e^{-2\lambda|x|}\,dx$$

$$\Rightarrow 2\lambda\int_0^{\infty} x^2 e^{-2\lambda x}\,dx \qquad \frac{\partial}{\partial\lambda} = -2x \quad \frac{\partial^2}{\partial\lambda^2} = 4x^2 \Rightarrow x^2 = \frac{1}{4}\frac{\partial^2}{\partial\lambda^2}$$

$$\Rightarrow \frac{\lambda}{2}\frac{\partial^2}{\partial\lambda^2}\int_0^{\infty} e^{-2\lambda x}\,dx = \frac{\lambda}{2}\frac{\partial^2}{\partial\lambda^2}\left[-\frac{1}{2\lambda}e^{-2\lambda x}\Big|_0^{\infty}\right] \Rightarrow \frac{\lambda}{2}\frac{\partial^2}{\partial\lambda^2}\left(\frac{1}{2\lambda}\right)$$

$$\Rightarrow \frac{\lambda}{2}\left(\frac{1}{\lambda^3}\right) \Rightarrow \boxed{\langle x^2\rangle = \frac{1}{2\lambda^2}}$$

c.) $\sigma = \sqrt{\langle x^2\rangle - \langle x\rangle^2} = \sqrt{\frac{1}{2\lambda^2} - 0} \Rightarrow \boxed{\sigma = \frac{1}{\sqrt{2}\lambda}}$



(1.5) cont'd

c.) $$P = \int_{-\infty}^{\langle x\rangle - \sigma} |\Psi(x,t)|^2 dx + \int_{\langle x\rangle + \sigma}^{\infty} |\Psi(x,t)|^2 dx$$

$$\Rightarrow \lambda \int_{-\infty}^{\langle x\rangle - \sigma} e^{-2\lambda|x|} dx + \lambda \int_{\langle x\rangle + \sigma}^{\infty} e^{-2\lambda|x|} dx \Rightarrow \lambda \left[\int_{-\infty}^{-\frac{1}{\sqrt{2}\lambda}} e^{2\lambda x} dx + \int_{\frac{1}{\sqrt{2}\lambda}}^{\infty} e^{-2\lambda x} dx \right]$$

$$\lambda \left[\frac{1}{2\lambda} e^{2\lambda x} \Big|_{-\infty}^{-\frac{1}{\sqrt{2}\lambda}} + -\frac{1}{2\lambda} e^{-2\lambda x} \Big|_{\frac{1}{\sqrt{2}\lambda}}^{\infty} \right] \Rightarrow \frac{1}{2}\left[e^{\frac{-2}{\sqrt{2}}} - 0 - \left(0 - e^{\frac{-2}{\sqrt{2}}}\right) \right]$$

$= e^{\frac{-2}{\sqrt{2}}} \Rightarrow P$ of finding outside of $(\langle x\rangle - \sigma, \langle x\rangle + \sigma)$ is $e^{\frac{-2}{\sqrt{2}}} \approx 0.243$

(1.6) $$\frac{d\langle x\rangle}{dt} = \int x \frac{\partial}{\partial t} |\Psi|^2 dx = \frac{i\hbar}{2m} \int x \frac{\partial}{\partial x}\left(\Psi^* \frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x} \Psi \right) dx$$

integration by parts cannot be applied yet because the derivative is with respect to t but the integral is with respect to x.

(1.7) $$\frac{d\langle p\rangle}{dt} = \frac{d}{dt}\, m \frac{d\langle x\rangle}{dt} = \left(\frac{d}{dt}\right)(-i\hbar) \int \left(\Psi^* \frac{\partial \Psi}{\partial x} \right) dx$$

$$= -i\hbar \int_{-\infty}^{\infty} \frac{\partial}{\partial t}\left(\Psi^* \frac{\partial \Psi}{\partial x} \right) dx = -i\hbar \int_{-\infty}^{\infty} \left[\frac{\partial \Psi^*}{\partial t} \frac{\partial \Psi}{\partial x} + \Psi^* \frac{\partial}{\partial t}\left(\frac{\partial \Psi}{\partial x} \right) \right] dx$$

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi \Rightarrow \frac{\partial \Psi}{\partial t} = \frac{i\hbar}{2m} \frac{\partial^2 \Psi}{\partial x^2} - \frac{i}{\hbar} V\Psi$$

$$\frac{\partial \Psi^*}{\partial t} = -\frac{i\hbar}{2m} \frac{\partial^2 \Psi^*}{\partial x^2} + \frac{i}{\hbar} V\Psi^*$$

(1.7) cont'd

$$\frac{d\langle p\rangle}{dt} = -i\hbar\int_{-\infty}^{\infty}\left[\left(\frac{\partial \psi^*}{\partial t}\right)\frac{\partial \psi}{\partial x} + \psi^*\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial t}\right)\right]dx$$

$$\frac{\partial \Psi}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2\psi}{\partial x^2} - \frac{i}{\hbar}V\psi \qquad \frac{\partial \psi^*}{\partial t} = -\frac{i\hbar}{2m}\frac{\partial^2\psi^*}{\partial x^2} + \frac{i}{\hbar}V\psi^*$$

$$\frac{d\langle p\rangle}{dt} = -i\hbar\int_{-\infty}^{\infty}\left[\left(-\frac{i\hbar}{2m}\frac{\partial^2\psi^*}{\partial x^2} + \frac{i}{\hbar}V\psi^*\right)\frac{\partial\psi}{\partial x} + \psi^*\frac{\partial}{\partial x}\left(\frac{i\hbar}{2m}\frac{\partial^2\psi}{\partial x^2} - \frac{i}{\hbar}V\psi\right)\right]dx$$

$$= -i\hbar\int_{-\infty}^{\infty}\left[-\frac{i\hbar}{2m}\frac{\partial^2\psi^*}{\partial x^2}\frac{\partial\Psi}{\partial x} + \cancel{\frac{i}{\hbar}V\psi^*\frac{\partial\psi}{\partial x}} + \psi^*\frac{\partial}{\partial x}\frac{i\hbar}{2m}\frac{\partial^2\Psi}{\partial x^2} - \underbrace{\psi^*\frac{\partial}{\partial x}\left(\frac{i}{\hbar}V\psi\right)}_{-\frac{i}{\hbar}\frac{\partial V}{\partial x}\psi^*\psi}\right]dx$$

$$= -i\hbar\int_{-\infty}^{\infty}\left[-\frac{i\hbar}{2m}\frac{\partial^2\Psi^*}{\partial x^2}\frac{\partial\Psi}{\partial x} + \Psi^*\frac{\partial}{\partial x}\frac{i\hbar}{2m}\frac{\partial^2\Psi}{\partial x^2} - \frac{i}{\hbar}\frac{\partial V}{\partial x}\psi^*\psi\right]dx$$

$$\Rightarrow -i\hbar\int_{-\infty}^{\infty}\left[-\frac{i\hbar}{2m}\frac{\partial^2\Psi^*}{\partial x^2}\frac{\partial\Psi}{\partial x} + \frac{i\hbar}{2m}\Psi^*\frac{\partial^3\Psi}{\partial x^3} - \frac{i}{\hbar}\frac{\partial V}{\partial x}\Psi^*\Psi\right]dx$$

$$\Rightarrow -i\hbar\left[-\frac{i\hbar}{2m}\int_{-\infty}^{\infty}\frac{\partial^2\psi^*}{\partial x^2}\frac{\partial\psi}{\partial x}dx + \frac{i\hbar}{2m}\int_{-\infty}^{\infty}\psi^*\frac{\partial^3\Psi}{\partial x^3}dx - \frac{i}{\hbar}\int_{-\infty}^{\infty}\frac{\partial V}{\partial x}\psi^*\psi\, dx\right]$$

$$u = \frac{\partial\psi}{\partial x} \quad du = \frac{\partial^2\psi}{\partial x^2}dx \qquad v = \frac{\partial\psi^*}{\partial x} \quad dv = \frac{\partial^2\psi^*}{\partial x^2}dx$$

$$\Rightarrow -i\hbar\left[-\frac{i\hbar}{2m}\left(\overbrace{\cancel{\frac{\partial\psi}{\partial x}\frac{\partial\psi^*}{\partial x}\Big|_{-\infty}^{\infty}}}^{0} - \int_{-\infty}^{\infty}\frac{\partial\psi^*}{\partial x}\frac{\partial^2\psi}{\partial x^2}dx\right) + \frac{i\hbar}{2m}\int_{-\infty}^{\infty}\psi^*\frac{\partial^3\Psi}{\partial x^2}dx - \frac{i}{\hbar}\int_{-\infty}^{\infty}\frac{\partial V}{\partial x}\psi^*\psi\, dx\right]$$

$$\Rightarrow -i\hbar\left[\frac{i\hbar}{2m}\int_{-\infty}^{\infty}\frac{\partial\psi^*}{\partial x}\frac{\partial^2\psi}{\partial x^2}dx + \frac{i\hbar}{2m}\int_{-\infty}^{\infty}\psi^*\frac{\partial^3\psi}{\partial x^3}dx - \frac{i}{\hbar}\int_{-\infty}^{\infty}\frac{\partial V}{\partial x}\Psi^*\Psi\, dx\right]$$

$$u = \frac{\partial^2\psi}{\partial x^2} \quad du = \frac{\partial^3\psi}{\partial x^3}dx \qquad v = \psi^* \quad dv = \frac{\partial\psi^*}{\partial x}dx$$

$$\Rightarrow -i\hbar\left[\cancel{\frac{i\hbar}{2m}\left(\overbrace{\frac{\partial^2\Psi}{\partial x^2}\Psi^*\Big|_{-\infty}^{\infty}}^{0} - \int\Psi^*\frac{\partial^3\Psi}{\partial x^3}dx\right) + \frac{i\hbar}{2m}\int_{-\infty}^{\infty}\Psi^*\frac{\partial^3\Psi}{\partial x^3}dx} \;\ldots\right]$$

$$\Rightarrow -i\hbar\left(-\frac{i}{\hbar}\int_{-\infty}^{\infty}\frac{\partial V}{\partial x}\psi^*\Psi\, dx\right) = -\int_{-\infty}^{\infty}\Psi^*\frac{\partial V}{\partial x}\Psi\, dx = \left\langle -\frac{\partial V}{\partial x}\right\rangle$$

$$\boxed{\frac{d\langle p\rangle}{dt} = \left\langle -\frac{\partial V}{\partial x}\right\rangle} \quad \checkmark$$

(1.8) add V_0 const. show $e^{-iV_0t/\hbar}$

$$i\hbar \frac{\partial}{\partial t}\Psi = \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi + V'\Psi \qquad V' = V(x,t) + V_0$$

$$\Rightarrow \frac{\partial \Psi}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2 \Psi}{\partial x^2} \frac{i}{\hbar}(V(x,t) + V_0)\Psi$$

$$\frac{\partial \Psi}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2 \Psi}{\partial x^2} - \frac{i}{\hbar}V\Psi - \frac{i}{\hbar}V_0\Psi \qquad I = e^{\int^t \frac{iV_0}{\hbar}ds} = e^{iV_0t/\hbar}$$

$$e^{iV_0t/\hbar}\left(\frac{\partial \Psi}{\partial t} + \frac{i}{\hbar}V_0\Psi\right) = \frac{i\hbar}{2m}\frac{\partial^2 \Psi}{\partial x^2}e^{iV_0t/\hbar} - \frac{i}{\hbar}V\Psi e^{iV_0t/\hbar}$$

$$\frac{\partial}{\partial t}\left(e^{iV_0t/\hbar}\Psi\right) = \frac{i\hbar}{2m}\frac{\partial^2 \Psi}{\partial x^2}e^{iV_0t/\hbar} - \frac{i}{\hbar}V\Psi e^{iV_0t/\hbar}$$

$$\hookrightarrow \frac{\partial}{\partial t}\left(e^{iV_0t/\hbar}\Phi\right) = \frac{i\hbar}{2m}\frac{\partial^2}{\partial x^2}\left(e^{iV_0t/\hbar}\Phi\right) - \frac{i}{\hbar}V\left(e^{iV_0t/\hbar}\Phi\right)$$

$$\frac{\partial}{\partial t}(\Psi) = \frac{i\hbar}{2m}\frac{\partial^2}{\partial x^2}(\Psi) - \frac{i}{\hbar}V(\Psi)$$

$$\hookrightarrow \Psi = e^{iV_0t/\hbar}\Phi \Rightarrow \boxed{\Phi = e^{-iV_0t/\hbar}\Psi}$$

$$\langle z(x,p)\rangle = \int_{-\infty}^{\infty}\Phi^* z\left(x, -i\hbar\frac{\partial}{\partial x}\right)\Phi\, dx$$

$$\Rightarrow \int_{-\infty}^{\infty}\cancel{e^{iV_0t/\hbar}}\Psi^* z\left(x, -i\hbar\frac{\partial}{\partial x}\right)\cancel{e^{-iV_0t/\hbar}}\Psi\, dx$$

$$\Rightarrow \int_{-\infty}^{\infty}\Psi^* z\left(x, -i\hbar\frac{\partial}{\partial x}\right)\Psi\, dx$$

It does not effect the expectation value of a dynamic variable.