ShanghaiTech University

CS240 Algorithm Design and Analysis

Fall 2021

Problem Set 3

Due: 23:59, Nov.25, 2021

1. Submit your solutions to Gradescope (www.gradescope.com).
2. In "Account Settings" of Gradescope, set your FULL NAME to your Chinese name and enter your STUDENT ID correctly.
3. If you want to submit a handwritten version, scan it clearly. Camscanner is recommended.
4. When submitting your homework, match each of your solution to the corresponding problem number.

Note:

To show that any problem A is NP-Complete, we need to show four things:

(1) there is a non-deterministic polynomial-time algorithm that solves A, i.e., $A \in \mathrm{NP}$,

(2) any NP-Complete problem B can be reduced to A,

(3) the reduction of B to A works in polynomial time,

(4) the original problem A has a solution if and only if B has a solution.

Problem 1:

"Given numbers $s_1, s_2..s_n$, is there a subset that adds up to exactly $\frac{\sum_1^n s_i}{2}$?" Show that the problem is NP-Complete.

Problem 2:

Consider the $CLOSESAT$ problem, which is similar to the SAT problem except that you need to satisfy **n-1** clauses instead of **n** clauses, where n is the number of clauses. Notice that we don't set any limits on the number of variables in each clause. Show that the $CLOSESAT$ problem is NP-Complete.

Problem 3:

Suppose you are going to schedule courses in SIST and try to make the number of conflicts no more than K. You are given 3 sets of inputs: C = {...}, S = {...}, R = {{...}, {...}, ...}.

C is the set of distinct courses. S is the set of available time slots for all the courses. R is the set of requests from students, consisting of a number of subsets, each of which specifies the courses a student wants to take. A conflict occurs when two courses are scheduled at the same slot(same time) even though a student requests both of them. Prove this schedule problem is NP-complete.

Example:

K = 1; C = {a, b, c, d}, S = {1, 2, 3}, R = {{a, b, c}, {a, c}, {b, c, d}}

An acceptable schedule is:

a - 1; b - 2; c, d - 3;

Here only one conflict occurs.

Problem 4:

The binary quadratic programming problem can be stated as follows. Given a martix $A \in \mathbb{Z}^{m \times n}$ and a vector $b \in \mathbb{Z}^m$, is there an $x \in \{0,1\}^n$ such that $Ax \le b$? (Note: $x \in \{0,1\}^n$ means x is a vector with n elements and each element is either 0 or 1) Hint: Reduction from 3-SAT

Problem 5:

SIST allows students to work as TAs but would like to avoid TA cycles. A TA cycle is a list of TAs (A1, A2,...,Ak) such that A1 works as a TA for A2 in some course, A2 works as a TA for A3 in some course, $\cdots$, and finally Ak works as a TA for A1 in some course. We say a TA cycle is simple if it does not contain the same TA more than once. Given the TA arrangements of SIST, we want to find out whether there is a simple TA cycle containing at least K TAs. Prove this problem is NP-complete.

Problem 6:

Given a set E and m subsets of $E, S_1, S_2, ...S_m$, is there a way to select k of the m subsets such that the selected subsets are pairwise disjoint? Show that this problem is NP complete.

HINT: Reduction from Independent Set.