National Institute of Technology Meghalaya

Class Test-1, September 2023

Subject: Discrete Mathematics (CS205)

Branch: Computer Science and Engineering

Semester: 3rd

Time: 45 minutes. Marks:15

1. Given $f(x) = x^2 + 6$ and $g(x) = 2x - 1$, find 4
   a) $(f \circ g)(x)$
   b) $(g \circ f)(x)$
2. Show that if 11 positive numbers are chosen, two of them will have the same remainder when divided by ten. Use pigeonhole principle. 3

4. What is Skolemization process? Why is it needed? 3

   Convert the wffs into clausal form

$$\exists x \forall y \left(\left(\forall z P(f(x), y, z) \right) \cup \left(\exists u Q(x, u) \right) \right) \rightarrow \left(\exists v R(y, v) \right)$$

5. Find the inverse of the function. 2

   $f(x) = \frac{7+4x}{6-5x}$ and $f(x) = \frac{10}{\sqrt[5]{7-3x}}$

6. A local grocery stores in the outback newly opened. They were offering 1 free bottle milk to every 11th customer and 5 Kg of Rice for every 13th customer. If there were 1000 customers that visited them on opening day, how many customers walked away with free goodies? 3

	National Institute of Technology Meghalaya An Institute of National Importance		CLASS TEST 2 (SET A)
Branch	**Computer Science and Engineering**	Programme	**B.Tech**
Course Name	**Discrete Mathematics**	Semester	**III**
Course Code	**CS 205**	Year/Period	**2023/Autumn**
Time : 01 Hrs	**Answer All Questions**	Maximum Marks	**15**

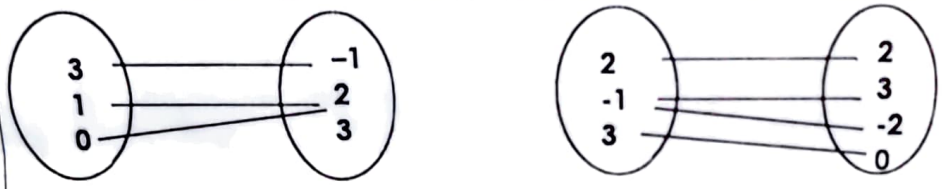
Knowledge Level (KL)			
	K1 : Remembering	**K3** : Applying	**K5** : Evaluating
	K2 : Understanding	**K4** : Analysing	**K6** : Creating

No.	Questions	Marks	COs	KL
1	Find out the generating function for the sequence <1, 4, 9, 16, 25, …>.	04	CO1, CO2	K1 and K2
2	How many ways can we fill a bag with n fruits with the following constraints? Find out the generating function. • The number of apples must be even. • The number of bananas must be a multiple of 5. • There can be at most four oranges. • There can be at most one pear.	04	CO3	K2
3	The set Zn* with the multiplication operator, G <Zn*, ×>, is also an abelian group. Prove it.	04	CO3	K3
4	Charles tosses two coins, what is the probability that he gets two heads and two tails.	01	CO2	K2
5	Find out the simplified polynomial representation of the bit string (10101001).	01	CO2, CO3	K3
6	Write all the Catalan numbers from 1-100	01	CO3	K4

Branch	**Computer Science and Engineering**	Programme	**B.Tech**
Course Name	**Discrete Mathematics**	Semester	**III**
Course Code	**CS 205**	Year/Period	**2023/Spring**
Time : 03 Hrs	**Answer All Questions in Q1** **Answer 4 Questions out of 6 from Q2 to Q7**	Maximum Marks	**100**

Knowledge Level (KL)	**K1** : Remembering	**K3** : Applying	**K5** : Evaluating
	K2 : Understanding	**K4** : Analysing	**K6** : Creating

Each Question Carry 02 Marks from Q1a to Q1j

Q.No	Questions	Marks	COs	KL
1a	$A = \{ x: x \in R \text{ and } x^2+7=0\}$. Find out the power set of A.	02	CO1	K2
1b	Define logical equivalence and the principle of duality.	02	CO2	K3
1c	Show that $\neg(p \leftrightarrow q)$ *and* $p \leftrightarrow \neg q$ are logically equivalent	02	CO3	K2
1d	What is the truth value of $\forall x P(x)$, where $P(x)$ is the statement "$x^2 < 10$", and the domain consists of the positive integers not exceeding 4?	02	CO2	K3
1e	Convert this into CNF form $\neg(\neg P \rightarrow (P \rightarrow Q))$	02	CO4	K2
1f	Which of the following relation mapping is a function? State the reason. 	02	CO3	K3
1g	Write the recurrence relation of "Tower of Hanoi" problem.	02	CO5	K2
1h	Prove that for every integer n, if n is odd, then n^2 is odd.	02	CO2	K2
1i	Charles tosses two coins, what is the probability that he gets two heads and two tails?	02	CO2	K3
1j	Define Galois field using example.	02	CO3	K3

Each Question Carry 20 Marks from Q2 to Q7

2a	i. State and explain in detail the wffs to clausal form conversion. ii. Convert the wffs to clausal form $\exists x ((\forall y P(f(x), y) \rightarrow (\exists z\, Q(x,z))) \rightarrow (\exists u\, R(x, u)))$	06	CO1	K2
2b	Prove that if n is an integer and $n^3 + 5$ is odd then n is even.(Use indirect proof)	04	CO1	K2, K3
2c	Determine the sequence generated by each of the following generating functions:- i. $f(x) = x^3/(1-x^2)$ ii. $f(x) = (2x-3)^3$	10	CO3	K3, K4
3a	Show that if 11 numbers are chosen from the set {1, 2, …20}, then one of them will be a multiple of another. (Use Pigeonhole Principle)	05	CO2	K2
3b	Find the recurrence relation with initial condition that uniquely determines the following geometric progressions 7, 14/5, 28/25, 56/125.	07	CO3	K3
3c	Find the cyclic subgroup of G $<Z_6, +>$. Justify your answer	08	CO4	K4
4a	Among 50 patients admitted to a hospital, 25 are diagnosed with pneumonia, 30 with bronchitis, and 10 with both pneumonia and bronchitis. Determine:	05	CO3	K2

	(a) The number of patients diagnosed with pneumonia or bronchitis (or both). (b) The number of patients not diagnosed with pneumonia or bronchitis. Use Inclusion-Exclusion Principle and show the respective Venn diagram.			
4b	Let A={1, 2, 3} and f_1, f_2, f_3 and f_4 be functions from A to A given by $f_1 = \{(1, 2), (2, 3), (3, 1)\}$ $f_2 = \{(1, 2), (2, 1), (3, 3)\}$ $f_3 = \{(1, 1), (2, 2), (3, 1)\}$ $f_4 = \{(1, 1), (2, 2), (3, 3)\}$ Compute $f_1 \circ f_2$; $f_2 \circ f_1$; $f_1 \circ f_3 \circ f_2$; $f_2 \circ f_4$; $f_4 \circ f_2$.	05	CO1	K3
4c	Find the coefficient of I. x^5 in $(1-2x)^{-7}$. II. x^8 in $1/(x-3)(x-2)^2$	10	CO3	K5
5a	Determine whether the following posets are lattices i. $(\{1, 3, 6, 9, 12\}, /)$ ii. $(Z, \geq)$ iii. $(P(S), \supseteq)$	05	CO5	K2
5b	Find the generating functions for the no. of integers solution to the equation $C_1 + C_2 + C_3 + C_4 = 20$ Where, $-3 \leq C_1$, $-3 \leq C_2$, $-5 \leq C_3 \leq 5$ and $0 \leq C_4$	07	CO4	K3
5c	How many bit strings of length 8 either start with a 1 or end with 00 can be constructed? Derive the process in details.	08	CO3	K4, K6
6a	Let A be a set with 10 distinct elements. i. How many different binary relations on A are there? ii. How many of them are reflexive? iii. How many of them are symmetric? iv. How many of them are reflexive and symmetric? v. How many of them are total ordering relations?	05	CO2	K2
6b	For n≥1 let S={1,2,3…..n} where n=0, S=Ø. And let a_n denote the number of subset of S that contain no consecutive integers. Find and solve the recurrence relation for a_n.	05	CO2	K3
6c	Solve the relation I. $a_n = n.a_{n-1}$ where, n≥1 and $a_0 = 1$. II. $a_n = 7.a_{n-1}$ where, n≥1 and $a_2 = 98$.	10	CO3	K5
7a	Find the result of $(x^5 + x^2 + x) \otimes (x^7 + x^4 + x^3 + x^2 + x)$ in GF(2^8) with irreducible polynomial $(x^8 + x^4 + x^3 + x + 1)$. $\otimes$: Represent the multiplication of two polynomials.	05	CO3	K2
7b	Find the unique solution for the recurrence relation $a_{n+1} - 1.5a_n = 0, n \geq 0$	07	CO4	K3
7c	Draw a Hasse diagram for (A,) (divisibility relation), Find out maximal and minimal elements. i. A = {1, 2, 3, 4, 5, 6}; ii. A = {1, 2, 3, 5, 7, 11, 13}; iii. A = {1,2,4,8,16,32}; iv. A = {1,2,3,6,12,24};)	08	CO5	K4

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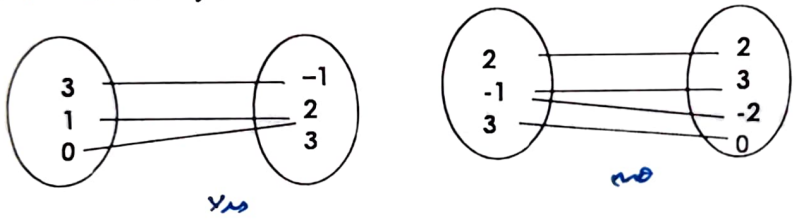
MID TERM

Branch	Computer Science and Engineering	Programme	B.Tech
Course Name	Discrete Mathematics	Semester	III
Course Code	CS 205	Year/Period	2023/ Autumn
Time : 02 Hrs		Maximum Marks	50

Knowledge Level (KL): **K1** : Remembering; **K2** : Understanding; **K3** : Applying; **K4** : Analysing; **K5** : Evaluating; **K6** : Creating

Answer All Questions

Each Question Carry 02 Marks from Q1a to Q1e

Q.No	Questions	Marks	COs	KL
1a	Convert the expression to CNF form $(P \rightarrow (Q \rightarrow R)) \rightarrow (P \rightarrow (R \rightarrow Q))$	02	CO1	K2
1b	What is Skolemization process. How it is helpful in getting the clausal forms from wffs.	02	CO3	K3
1c	Which of the following relation mapping is a function? State the reason. How do you determine if a relation is a function? 	02	CO2	K2
1d	What is the total number of anti-symmetric relations from Set A to itself which has 'n' elements? Justify with a suitable example.	02	CO1	K2
1e	Let f be the function from {a, b, c, d} to {1, 2, 3, 4} with f(a)=4, f(b)= 2, f(c)=1, f(d)= 3. Is f a bijection?	02	CO3	K3

Each Question Carry 10 Marks from Q2 to Q5

2a	What is equivalence relation? Explain with an example. Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. What is the composition of f and g? What is the composition of g and f?	05	CO3	K2, K3
2b	If the function f and g are defined as $f(x) = e^x$ and $g(x) = 3x - 2$ where f:R→R and g:R→R, then find the function $f \circ g$ and $g \circ f$. Also find $(fog)^{-1}$ and $(gof)^{-1}$.	5	CO2	K3, K4
3a	Prove the following identities, stating carefully which of the set laws you are using at each stage of the proof. (a) B ∪ (ø ∩ A) = B (b) (A ′ ∩ U) ′ = A (c) (C ∪ A) ∩ (B ∪ A) = A ∪ (B ∩ C)	05	CO2	K3
3b	Convert the wffs into clausal form $\exists x \forall y ((\forall z P(f(x), y, z)) \rightarrow (\exists u Q(x, u) \vee \exists v \forall w R(y, v, w)))$ What is a clause. Define horn clause, ground clause.	05	CO1	K4

4a	If the function f and g are defined as $f(x) = e^x$ and $g(x) = 3x - 2$ where f:R→R and g:R→R, then find the function $f \circ g$ *and* $g \circ f$. Also find $(fog)^{-1}$ *and* $(gof)^{-1}$.	05	CO2	K4, K5
4b	What is logical consequences. State with example. Give the proof of theorem "The proposition s is a logical consequences of $s_1, s_2, \cdots, s_n$ iff $s_1 \,\&\, s_2 \,\&\, \cdots \,\&\, s_n \,\&\, \neg s$ is inconsistent"	5	CO3	K5
5a	A local grocery store in the outback newly opened. They were offering 1 free bottle Marmite to every 11th customer and 1 free pound of meat for every 13th customer. If there were 1000 customers that visited them on opening day, how many customers walked away with free goodies? 160	5	CO4	K3, K4
5b	What is Generalized Pigeonhole Principle? Show that in a group of 10 people (where any two people are either friends or enemies), there are either three mutual friends or four mutual enemies, and there are either three mutual enemies or four mutual friends.	5	CO3	K5, K6

Good Luck