Maclaurin's Series

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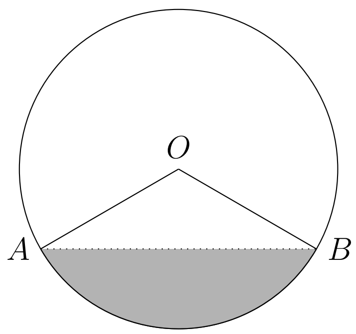
MACLAURIN'S SERIES [TBC Marks]

1 Find the first three non-zero terms in the expansion of $\mathrm{f}(x) = -\dfrac{x}{\sqrt{4-x^2}}$. [3]

2 Sector AOB of a circle centred at O with radius 4 cm is such that $\sphericalangle AOB = \frac{2\pi}{3} + \theta$ as shown in the diagram below:



Given that θ is sufficiently small for θ^4 and higher powers of θ to be neglected, find the approximate area of the shaded region in terms of powers of θ. [5]

3 It is given that $y = \sqrt[3]{(1+x^2)(1+4x^2)}$.

(a) State the Maclaurin's expansion of y in ascending powers of x up to and including the x^4 term. [4]

(b) Hence, show that $\frac{91487}{90000}$ is an accurate estimate for $\sqrt[3]{1.0504}$ up to 4 d.p.. [2]

4 Given that $y = 2^{\sin^{-1} x}$, where $\sin^{-1} x$ denotes the principal value;

(a) Show that:

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{1}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + \frac{x}{1-x^2}\cdot\frac{\mathrm{d}y}{\mathrm{d}x}.$$

[4]

(b) Hence, obtain the first three terms in the Maclaurin's series for y. [3]

5 The graph of $y = \mathrm{f}(x)$ is given by:

$$\mathrm{f}(x) = \begin{cases} \mathrm{e}^{1/x^2}, & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

(a) Show that $\mathrm{f}(x)$ is not equal to the Maclaurin Series of e^{1/x^2}. [3]

(b) Graph the function in **(a)**, labelling clearly the points at/around the origin. [3]

6 Given that $\tan 2x = a_0 + a_1x + a_2x^2 + a_3x^3 \cdots$, by using the fact that $\sin 2x = \cos 2x \cdot \tan 2x$, obtain the expansion of $\tan 2x$ in ascending powers of x, up to and including the term in x^3. [4]

7 It is given that $\mathrm{f}(x) = \dfrac{a-b}{(1-ax)(1-bx)}$, where $a, b > 0$.

(a) By expressing $\mathrm{f}(x)$ in terms of partial fractions and considering their Maclaurin expansions, show that:

$$\sum_{r=0}^{\infty}(a^{r+1} - b^{r+1})x^r$$

[4]

(b) Hence or otherwise, prove that, if x^3 and higher powers of x may be neglected, then $\dfrac{2}{(1-5x)(1-3x)} \approx 2 + 16x + 98x^2$. [2]

8 Find the Maclaurin's Series for e^{ix} up to and including the coefficient of x^7. [3]

(a) Hence, prove that $\mathrm{e}^{ix} = \cos x + i \sin x$. [3]