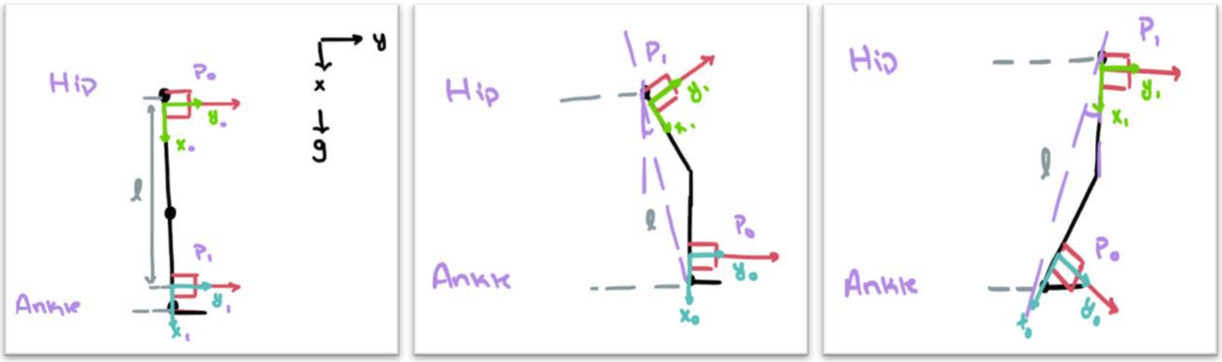

Kinematic relation expressing position of the ankle (P_0) with reference to the hip (P_1):

$$P_0 = P_1 + R_1^0 P_{10}^1 = P_1 - l x_1$$

where the rotation matrix is

$$R_1^0 = (x_1\ y_1\ z_1)$$

$$x_1 = [cos\theta_1, sin\theta_1, 0\ ]$$

$$y_1 = [-sin\theta_1, cos\theta_1, 0\ ]$$

$$z_1 = [0, 0, 1\ ]$$

and P_1 with reference to P_0 is

$$P_{10}^1 = [-l, 0, 0].$$

We want the equation in terms of the gyroscope input (radians/sec; $\dot{\theta}$) and the accelerometer input (meters/second^2; $\ddot{P}$). So,

$$\dot{x_1} = \dot{\theta}_1 y_1$$

$$\dot{y_1} = -\dot{\theta}_1 x_1.$$

and by taking the derivative (in terms of time) of the initial kinematic relation expressing position,

$$\dot{P}_0 = \dot{P}_1 - l\dot{x}_1$$

$$\dot{P}_0 = \dot{P}_1 - l\dot{\theta}_1 y_1$$

$$\ddot{P}_0 = \ddot{P}_1 + l\ddot{\theta}_1 y_1 - l\dot{\theta}_1{}^2 x_1$$

Since the accelerometer takes into consideration gravity, so a more accurate description of the equations are

$$(\ddot{P}_0 + g_x) = (\ddot{P}_1 + g_x) + l\ddot{\theta}_1 y_1 - l\dot{\theta}_1{}^2 x_1$$

$$[\ddot{P}_{0,x} + g_x, \ddot{P}_{0,y}] = [\ddot{P}_{1,x} + g_x, \ddot{P}_{1,y}] + l\ddot{\theta}_1 y_1 - l\dot{\theta}_1{}^2 x_1$$

$$(\ddot{P}_{0,x} + g_x)cos\theta_1 + \ddot{P}_{0,y}sin\theta_1 = \ddot{P}_{1,x} + l\dot{\theta}_1{}^2 x_1$$

$$-(\ddot{P}_{0,x} + g_x)sin\theta_1 + \ddot{P}_{0,y}cos\theta_1 = \ddot{P}_{1,y} - l\ddot{\theta}_1 y_1$$

or

$$(\ddot{P}_{0,x} + g_x)cos\theta_1 + \ddot{P}_{0,y}sin\theta_1 = \ddot{P}_{1,x} + l\dot{\theta}_1{}^2 x_1$$

$$\ddot{P}_{0,y}cos\theta_1 - (\ddot{P}_{0,x} + g_x)sin\theta_1 = \ddot{P}_{1,y} - l\ddot{\theta}_1 y_1$$

or

$$\begin{bmatrix} \ddot{P}_{0,x} + g_x & \ddot{P}_{0,y} \\ \ddot{P}_{0,y} & -(\ddot{P}_{0,x} + g_x) \end{bmatrix} \begin{bmatrix} cos\theta_1 \\ sin\theta_1 \end{bmatrix} = \begin{bmatrix} \ddot{P}_{1,x} + l\dot{\theta}_1{}^2 x_1 \\ \ddot{P}_{1,y} - l\ddot{\theta}_1 y_1 \end{bmatrix}.$$

Since we are solving for θ_1

$$\begin{bmatrix} \ddot{P}_{0,x} + g_x & \ddot{P}_{0,y} \\ \ddot{P}_{0,y} & -(\ddot{P}_{0,x} + g_x) \end{bmatrix}^{-1} \begin{bmatrix} \ddot{P}_{1,x} + l\dot{\theta}_1{}^2 x_1 \\ \ddot{P}_{1,y} - l\ddot{\theta}_1 y_1 \end{bmatrix} = \begin{bmatrix} cos\theta_1 \\ sin\theta_1 \end{bmatrix}.$$

All the terms in the matrices are found from the senor inputs or derived from the sensor inputs ($\ddot{\theta}_1$can be derived from two sets of data points with t_0 time separating the data points). From there, the $arcsin$ can be taken from the resulting (solution) matrix and the angle between the hip and ankle will be known.