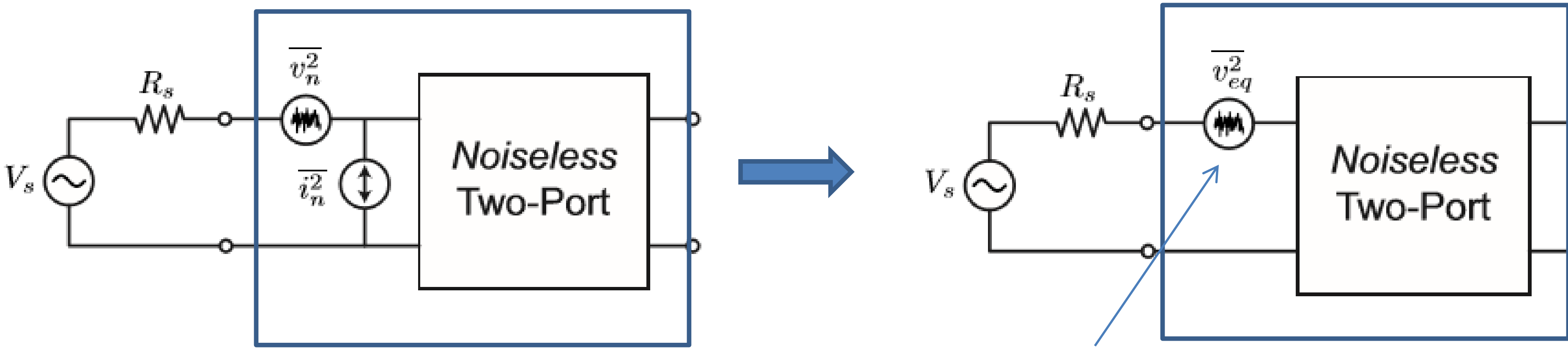
# Today's Agenda (Oct. 18)

- Hw7: Noise Modeling, NF Calculation
- Noise Circle and Ga Circle
- Review on Some Important Concepts in Two-port Theory (HW6)




$v_{eq} = i_n R_s + v_n$

$$\boxed{\overline{v_{eq}^2} = \overline{v_n^2} + \overline{i_n^2} R_s^2 + 2{*}R_s{*}\langle v_n{*}i_n \rangle}$$

**$\langle v_{eq}^2 \rangle$ depends on $R_s$**

$NF_{Rs} = 1 + \langle v_{eq}^2 \rangle / \langle v_{n,s}^2 \rangle = 1 + \langle v_{eq}^2 \rangle / 4kTR_s$ => $\mathbf{\langle v_{eq,Rs}^2 \rangle = 4kT{*}R_s{*}(NF_{Rs} - 1)}$

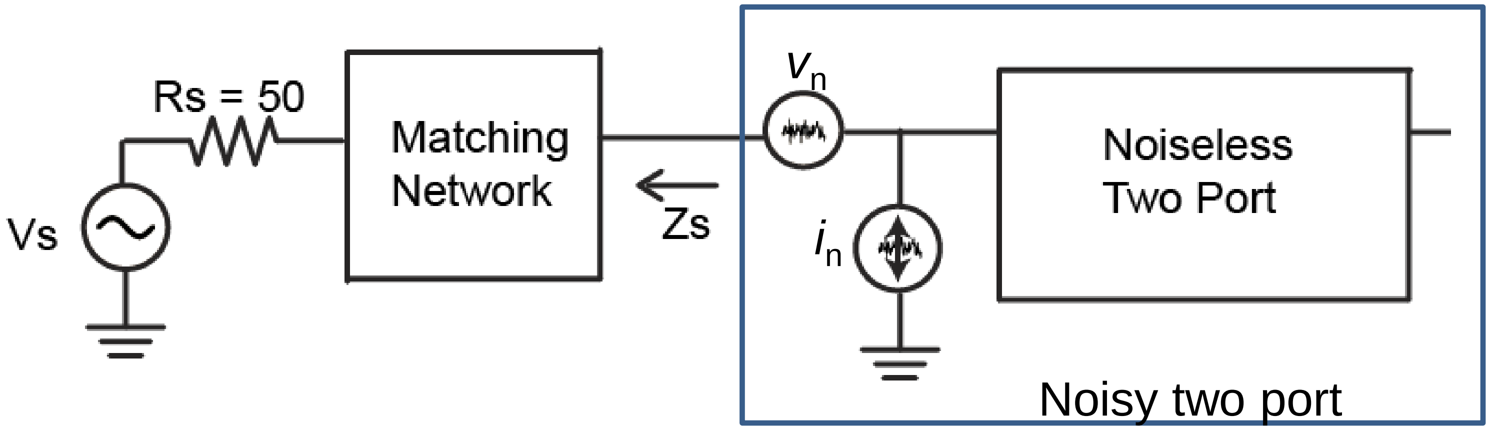
$\langle v_{eq,Rs}^2 \rangle$ can be calculated if $NF_{Rs}$ is given

$\langle v_{eq,Rs2}^2 \rangle$ is unknown without knowing $NF_{Rs2}$

Ex. A circuit has NF = 10 dB with 50-Ω Rs. What is the new NF if the source resistance becomes 100 Ω ? I don't know

# Source Impedance For The Best NF

- $R_s$ can be matched to a complex $Z_s = R_{Zs}+jX_{Zs}$
- Assume $i_n = i_c + i_u$ $\quad\quad$ $i_c = Y_c v_n$ and $<i_u v_n> = 0$

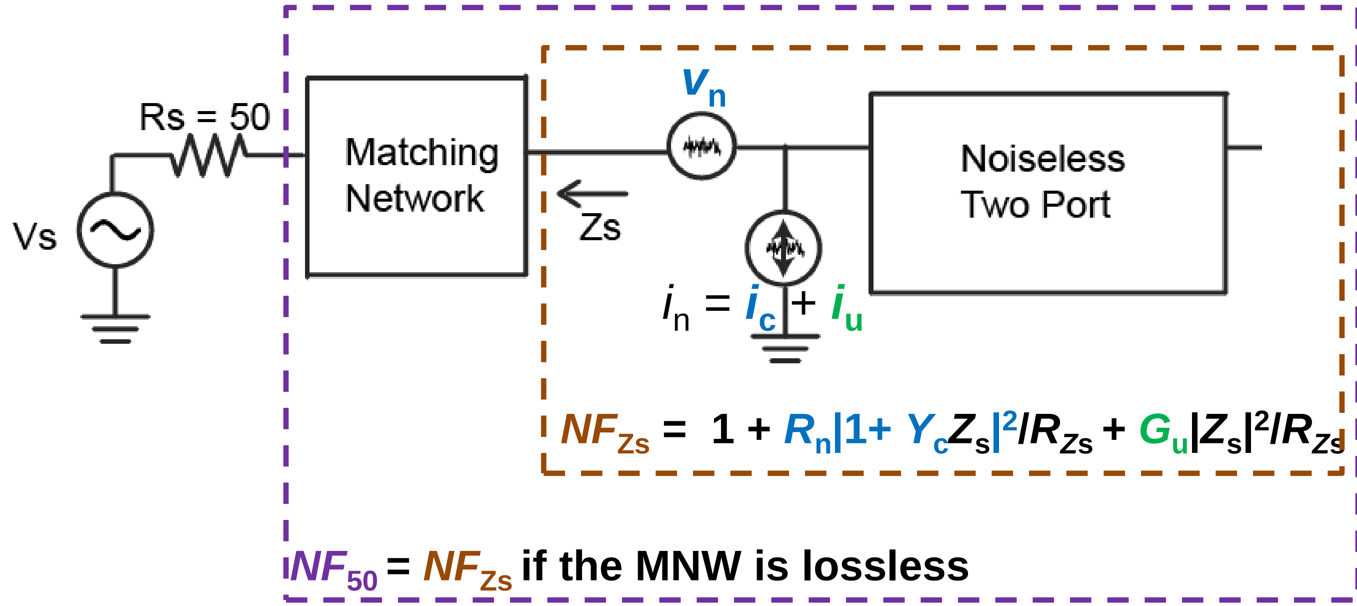



$v_{eq} = v_n(1+ Y_c Z_s) + Z_s i_u$

$$NF_{Rs} = 1 + <v_{eq}^2>/4kTR_{Zs} = \mathbf{1} + \mathbf{<v_n^2>|1+ Y_c Z_s|^2/4kTR_{Zs}} + \mathbf{<i_u^2>|Z_s|^2/4kTR_{Zs}}$$

$$= \mathbf{1} + \mathbf{R_n|1+ Y_c Z_s|^2/R_{Zs}} + \mathbf{G_u|Z_s|^2/R_{Zs}}$$

$<v_n^2> = 4kTR_n$ and $<i_u^2> = 4kTG_u$




Assume $\langle v_n^2 \rangle = 4kTR_n$ and $\langle i_u^2 \rangle = 4kTG_u$

Simplified case with $Y_c = 0$ (noise voltage and source are uncorrelated)

$$NF_{Zs} = 1 + R_n/R_{Zs} + G_u|Z_s|^2/R_{Zs} \text{ ,}$$

$$NF_{min} = 1 + 2(R_n G_u)^{0.5} \quad \text{with} \quad Z_s = (R_n/G_u)^{0.5} + 0j$$

No simplification

$$\boldsymbol{NF_{Zs} = 1 + R_n|1 + Y_c Z_s|^2/R_{Zs} + G_u|Z_s|^2/R_{Zs}}$$

$$Y_c = G_c + jB_c, \qquad Z_s = (G_s + jB_s)^{-1}$$

$$B_{opt} = B_s = -B_c$$

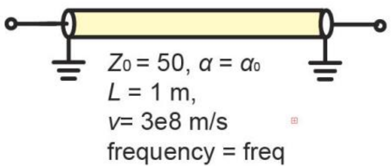
$$G_{opt} = G_s = \sqrt{\frac{G_u}{R_n} + G_c^2}$$

- The minimum acheivable noise figure is

$$F_{min} = 1 + 2G_c R_n + 2\sqrt{R_n G_u + G_c^2 R_n^2}$$

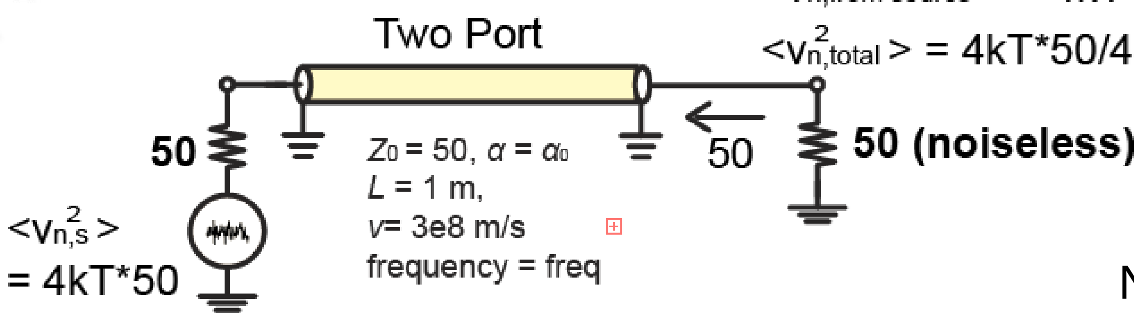
$$F = F_{min} + \frac{R_n}{G_s}|Y_s - Y_{opt}|^2$$

# NF of a Lossy Transmission Line (1/3)



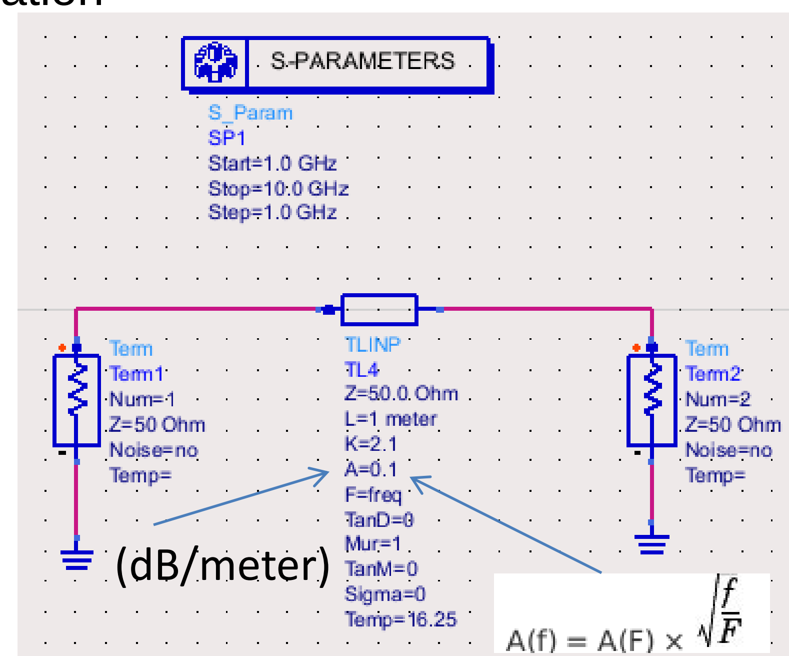

$<v_{n,\text{from source}}^2> = 4kT*50/4*\exp(-2\alpha_0 L)$

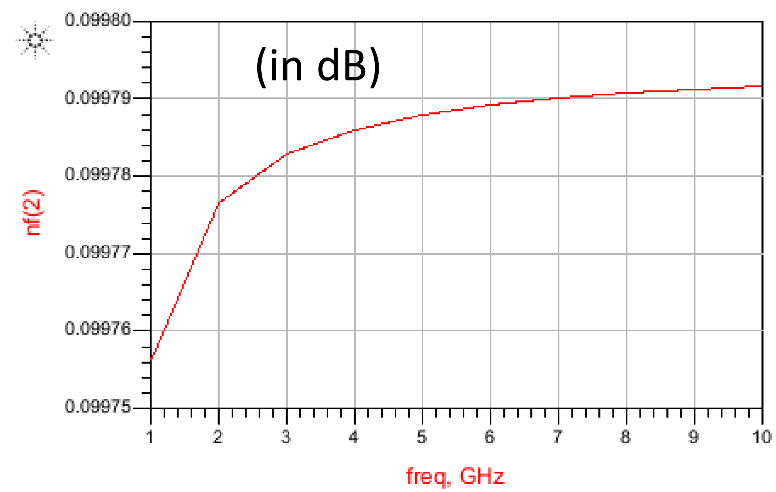
$<v_{n,\text{total}}^2> = 4kT*50/4$




NF = $\exp(2\alpha_0 L)$

Simulation




$dB[\exp(-\alpha_0)] = A = -0.1$

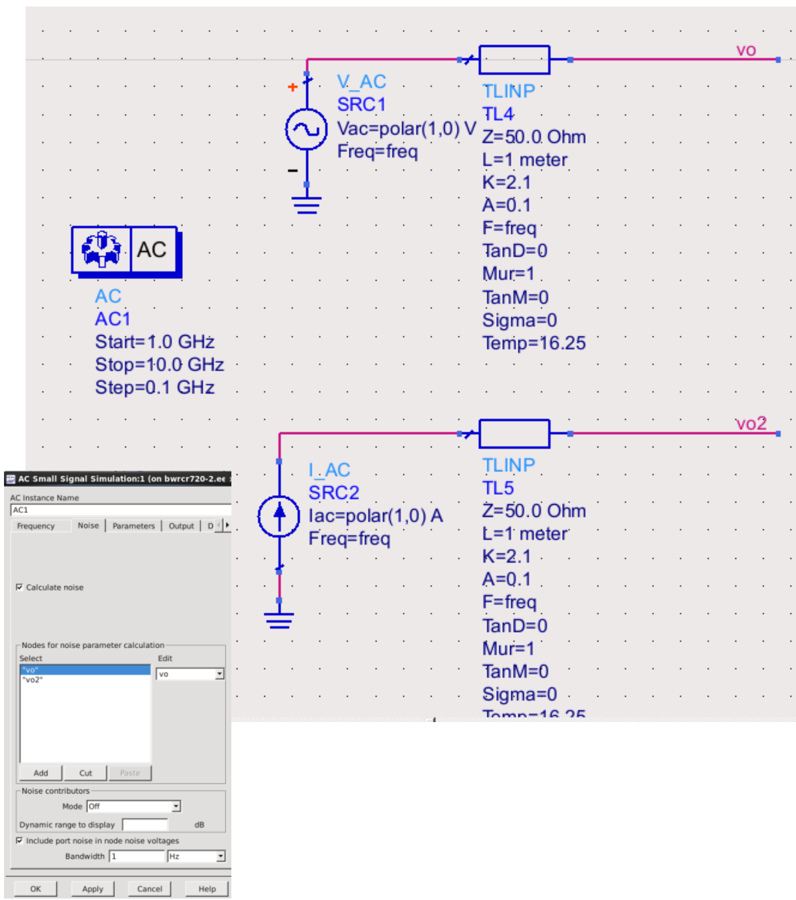
$\alpha_0 = 0.0115$

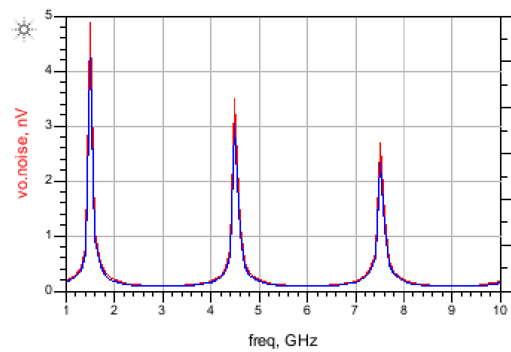
$\exp(2\alpha_0 L) = 1.023$

NF = 0.099 dB

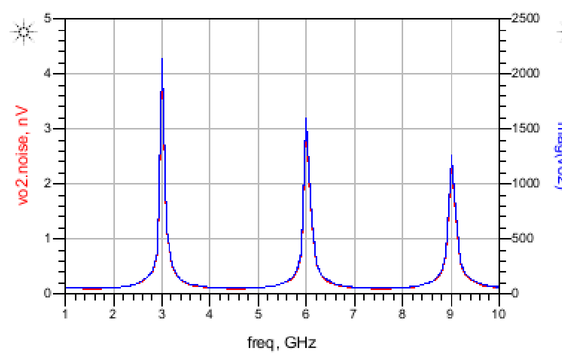
# NF of a Lossy Transmission Line (2/3)

Calculate NF by input referred voltage and noise sources (very difficult)




| freq | vo.noise/vo |
|---|---|
| 1.000 GHz | 9.523E-11 / -1.105 |
| 1.100 GHz | 9.537E-11 / 178.576 |
| 1.200 GHz | 9.552E-11 / -1.929 |
| 1.300 GHz | 9.566E-11 / 177.118 |
| 1.400 GHz | 9.579E-11 / -5.436 |
| 1.500 GHz | 9.591E-11 / 144.159 |
| 1.600 GHz | 9.601E-11 / -172.609 |
| 1.700 GHz | 9.610E-11 / 3.372 |
| 1.800 GHz | 9.617E-11 / -177.849 |
| 1.900 GHz | 9.622E-11 / 1.552 |
| 2.000 GHz | 9.626E-11 / -178.811 |
| 2.100 GHz | 9.627E-11 / 0.942 |
| 2.200 GHz | 9.628E-11 / -179.241 |
| 2.300 GHz | 9.626E-11 / 0.616 |
| 2.400 GHz | 9.624E-11 / -179.502 |




| freq | vo2.noise/vo2 |
|---|---|
| 1.000 GHz | 1.932E-12 / -89.585 |
| 1.100 GHz | 1.929E-12 / 90.325 |
| 1.200 GHz | 1.926E-12 / -89.757 |
| 1.300 GHz | 1.923E-12 / 90.167 |
| 1.400 GHz | 1.921E-12 / -89.905 |
| 1.500 GHz | 1.918E-12 / 90.025 |
| 1.600 GHz | 1.916E-12 / -90.045 |
| 1.700 GHz | 1.915E-12 / 89.884 |
| 1.800 GHz | 1.913E-12 / -90.190 |
| 1.900 GHz | 1.912E-12 / 89.731 |
| 2.000 GHz | 1.911E-12 / -90.355 |
| 2.100 GHz | 1.911E-12 / 89.549 |
| 2.200 GHz | 1.911E-12 / -90.563 |
| 2.300 GHz | 1.911E-12 / 89.303 |
| 2.400 GHz | 1.912E-12 / -90.864 |

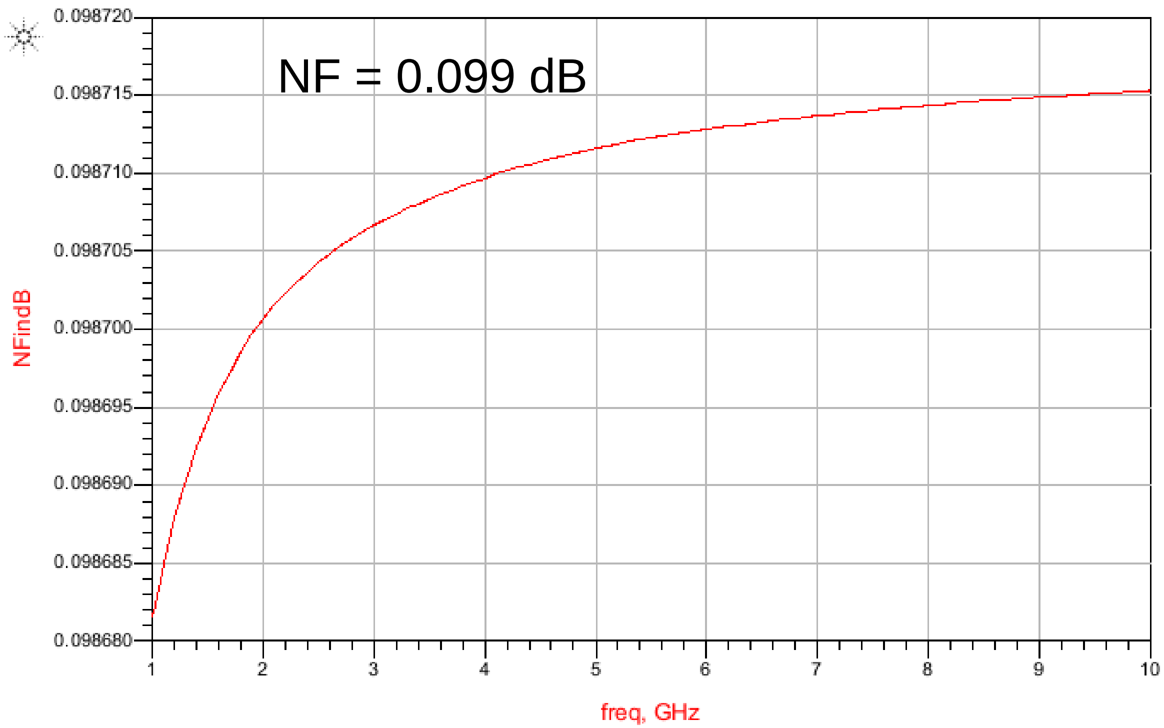
# NF of a Lossy Transmission Line (3/3)

Assume the input referred noise current and noise voltage are uncorrelated

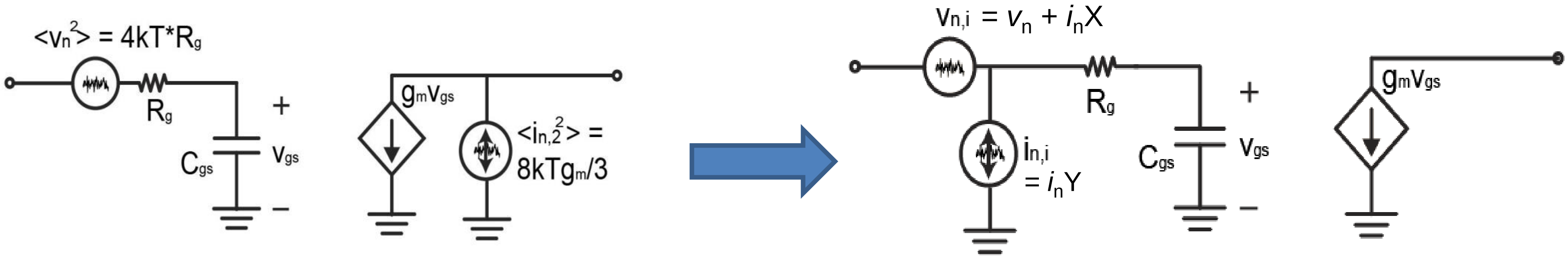
$$\overline{v_{eq}^2} = \overline{v_n^2} + \overline{i_n^2} R_s^2 \qquad NF_{Rs} = 1 + \langle v_{eq}^2 \rangle / \langle v_{n,s}^2 \rangle$$

Eqn NF = 1 + (mag(vo2.noise/vo2*50)) *(mag(vo2.noise/vo2*50))/4/1.38e-23/290/50+(mag(vo.noise/vo)) *(mag(vo.noise/vo))/4/1.38e-23/290/50

Eqn NFindB=10*log(NF)



# Noise Modeling Example



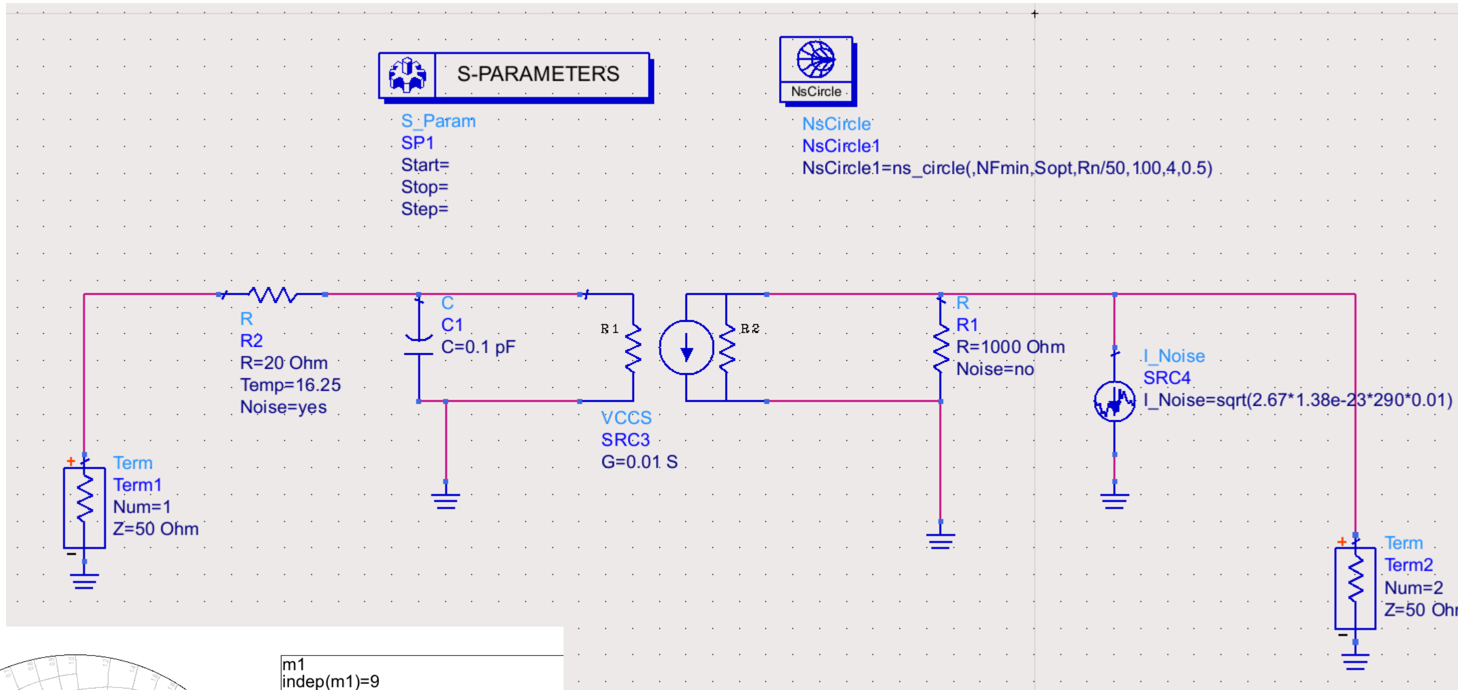

- Straightforwardly, $v_{n,i}$ and $i_{n,i}$ are partially correlated
- Test of concept: what is the $v_{gs}$ noise when the input is short-circuited?
- NF is determined only by Zs
- Zs for the best NF does not achieve the best (transducer) power gain
- Recall: (transducer) power gain is determined by both $Z_s$ and $Z_L$

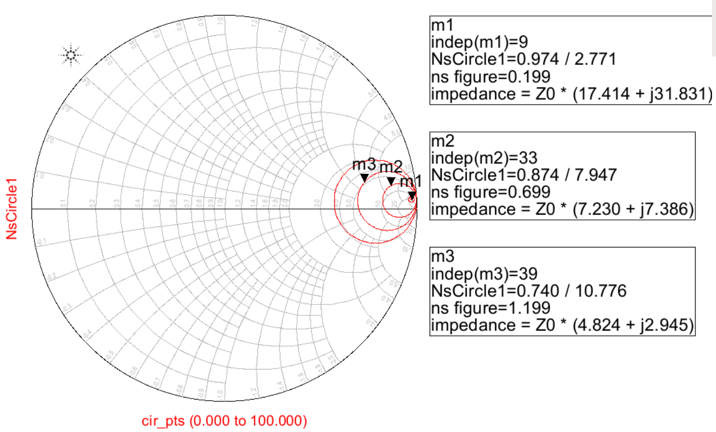
  (What is the maximum gain of this circuit?)

# Noise Circle Simulation

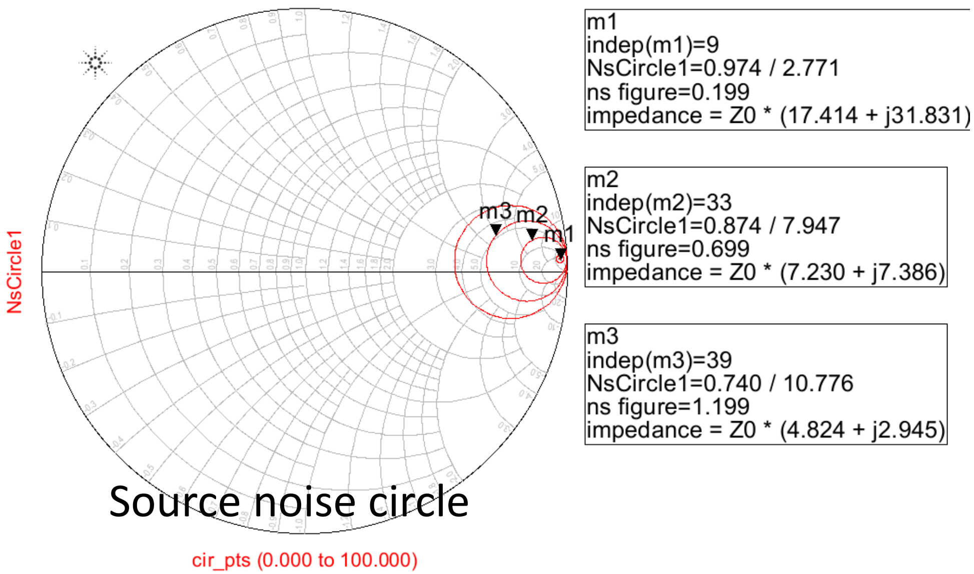
We know that different source impedances (Zs) correspond to different NF

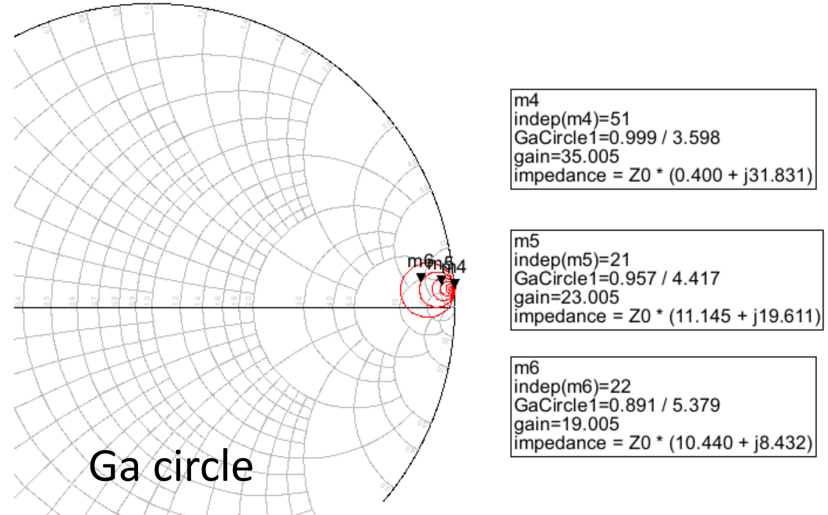
The NF contour can be plotted versus the source impedance on the Smith Chart



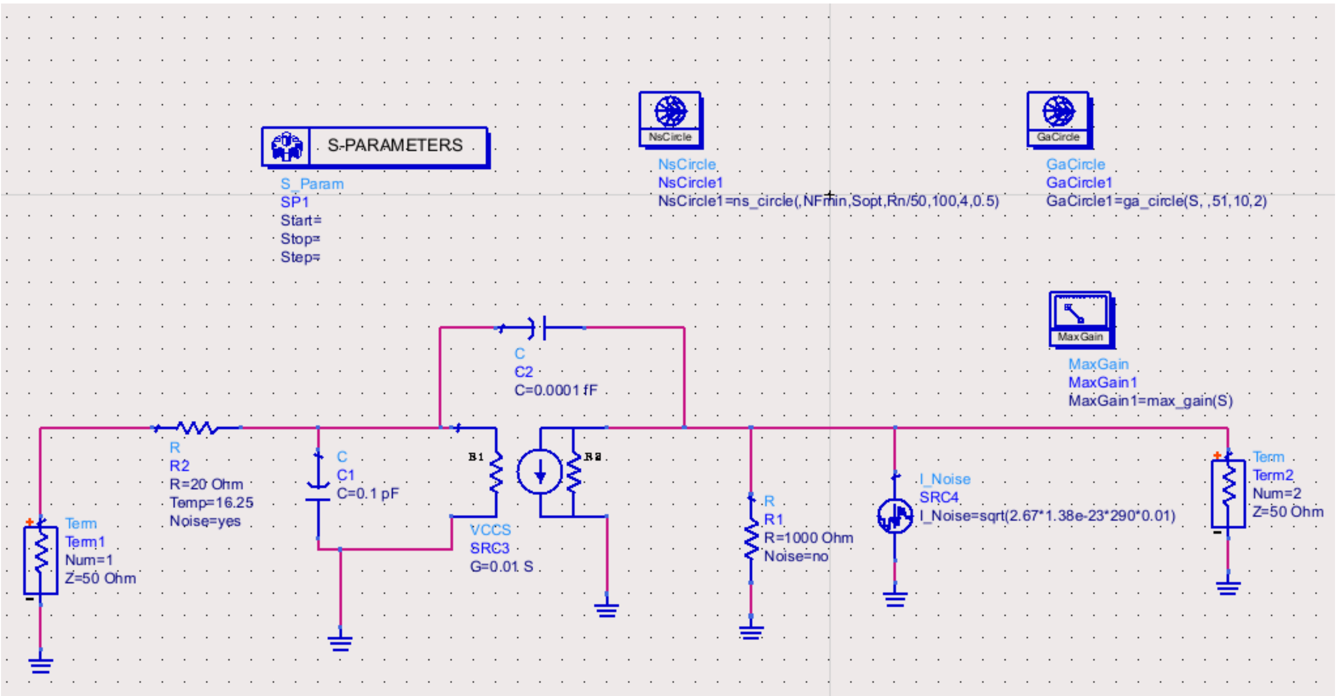




| freq | NFmin | Rn | Sopt |
|---|---|---|---|
| 1.000 GHz | 0.199 | 86.687 | 0.974 / 2.771 |



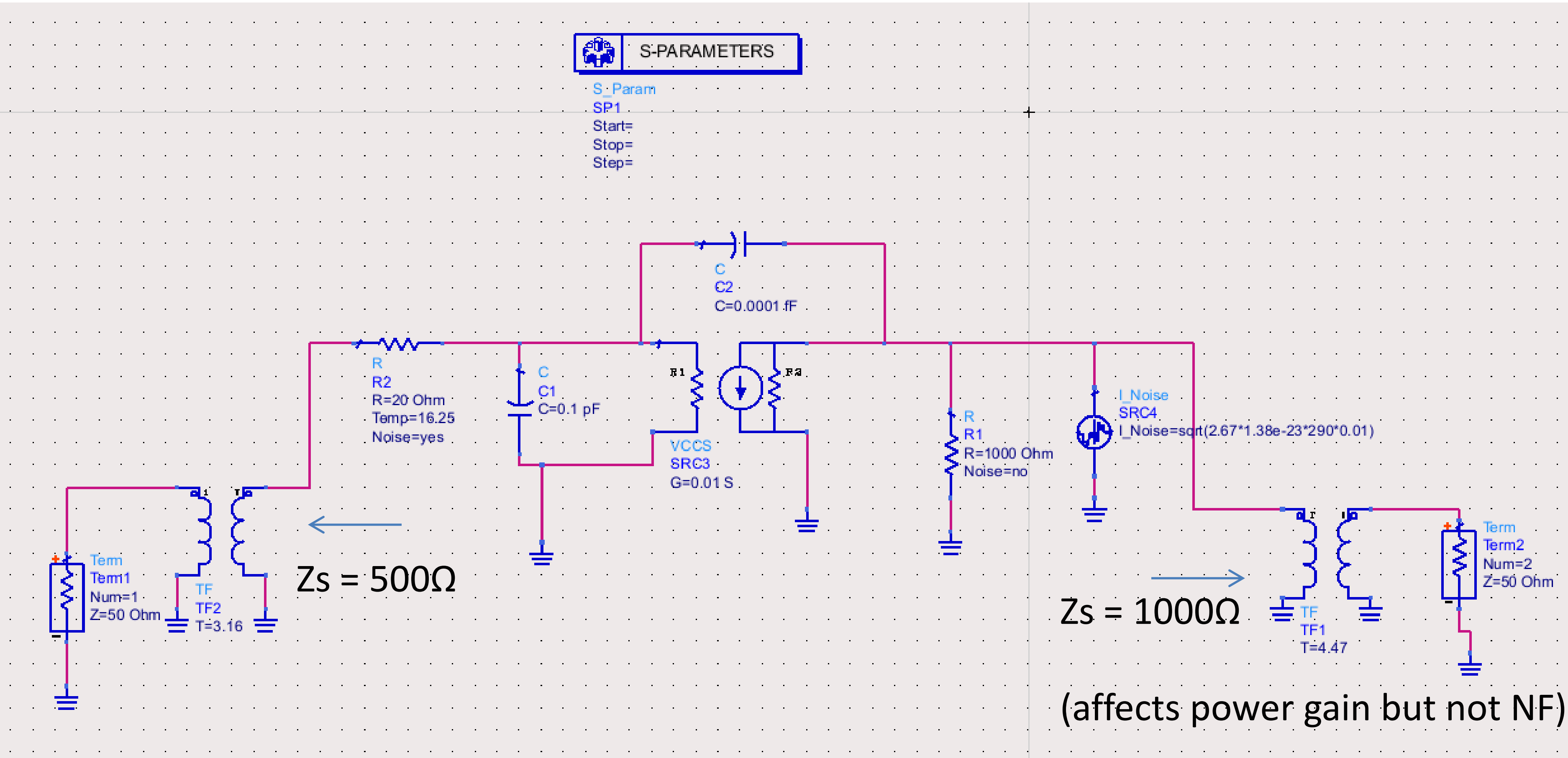

| freq | MaxGain1 |
|---|---|
| 1.000 GHz | 35.005 |




From the Noise and Ga circles:
Design with Zs = 500

Can achieve :

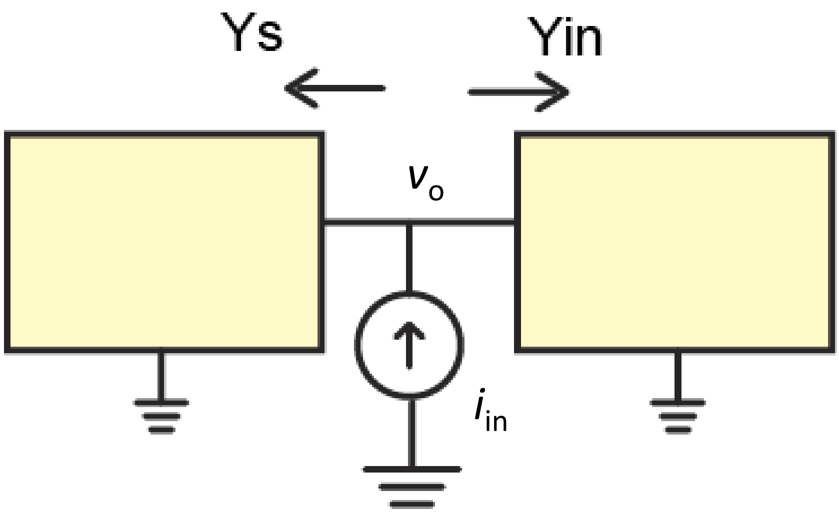
0.7dB NF (not the best)
17 dB transducer power gain (not the best)

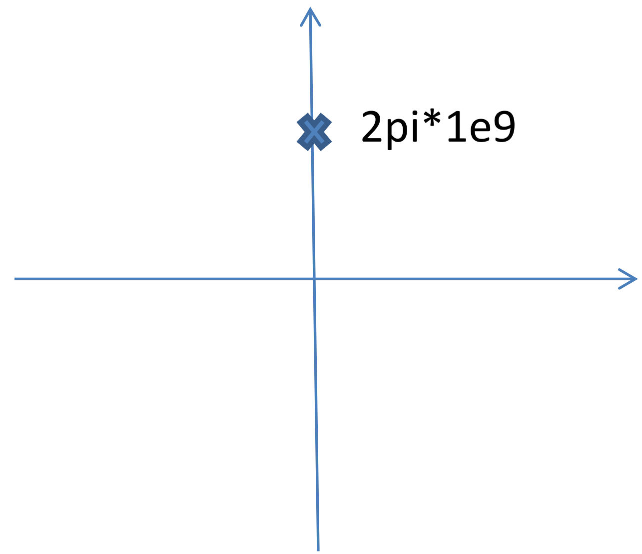



| freq | nf(2) | dB(S(2,1)) | dB(S(1,1)) | dB(S(2,2)) |
|---|---|---|---|---|
| 1.000 GHz | 0.748 | 16.544 | -0.062 | -66.425 |

very bad input matching

# HW6: System Pole Location

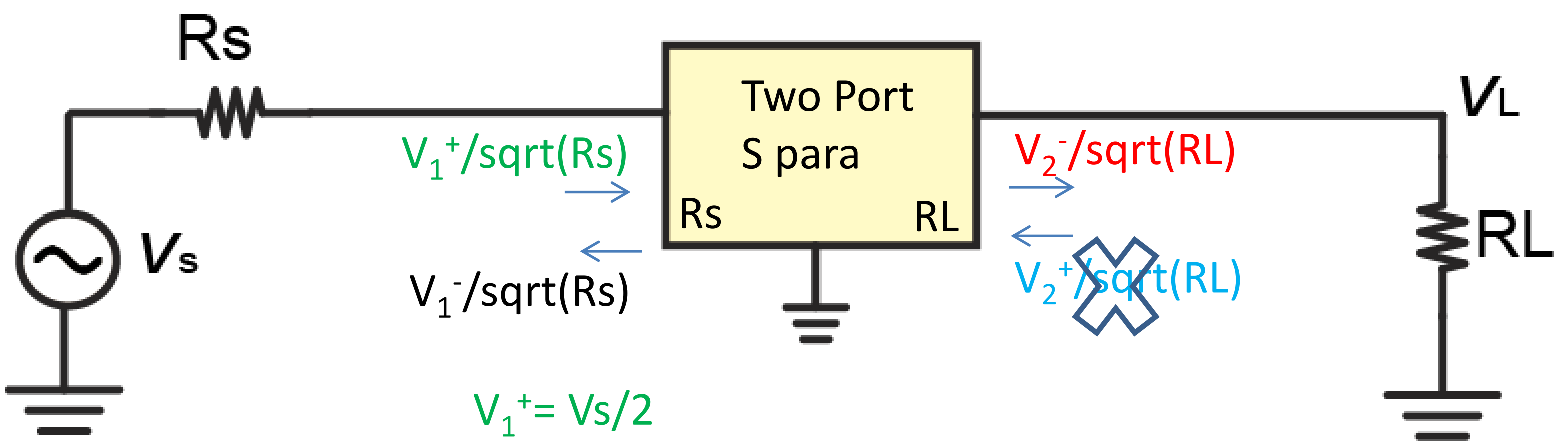



System pole at $s = j2pi*1\ GHz$ means $v_o/i_{in}$ is infinite if the current is at 1 GHz.

$v_o/i_{in}$ at 1 GHz = $(1/Y_{in}) + (1/Y_s)$ = infinite => $Y_{in} + Y_s = 0$ at 1 GHz

# Voltage Gain and Transducer Power Gain




Voltage Gain
$= V_L/V_s$

(Transducer) Power Gain
$= 2P_L \ / \ 2P_{avs}$

$= (|V_L|^2/2R_L) \ / \ (|V_s|^2/8R_s)$

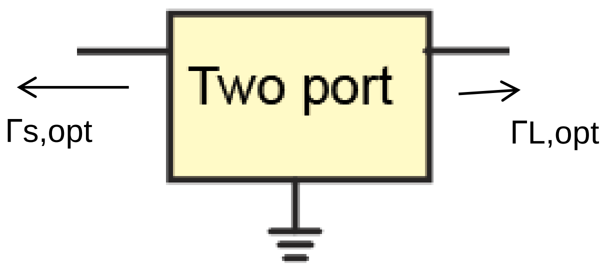
$= \boldsymbol{|V_L/V_s|^2} \times (4R_s/R_L)$

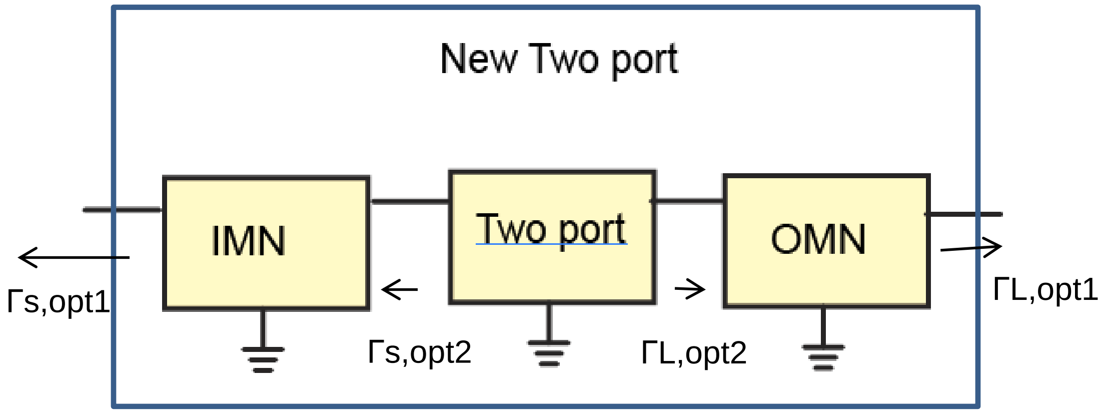
$= |S_{21}|^2$ (source reference = Rs load reference = RL)

=> Voltage gain is not $|S_{21}|$

# Maximum Transducer Power Gain and Stability

When you add lossless input and output matching networks to a two port, the **maximum gain** and ***k*** of the new two port do not change!




***k* does not change**

$$G_{max} = \frac{Y_{21}}{Y_{12}}\left(K - \sqrt{K^2 - 1}\right)$$

Need to prove Y21/Y12 of a two port does not change when adding series or parallel inductor/capacitor (easy)