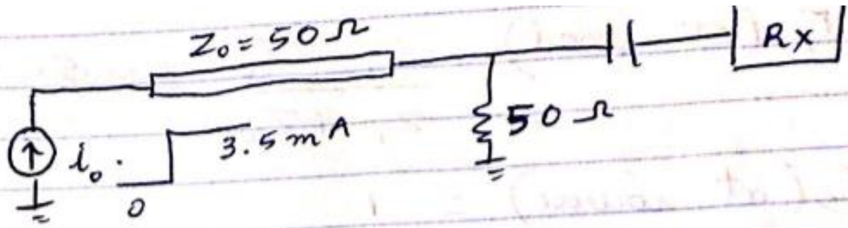
Prob. 1

(a) Contributed by Sashank

1. a)



$L = 0.15\,m$

$v = 1.5 \times 10^8\ m/s$

t_d of T-line $= 1\ ns$

Let 50Ω termination be ℓ, source 0 in space

@ $t = 0$

$i^+(0) = i_o$

$v^+(0) = Z_0 i^+(0) = Z_0 i_o = 175\ mV$

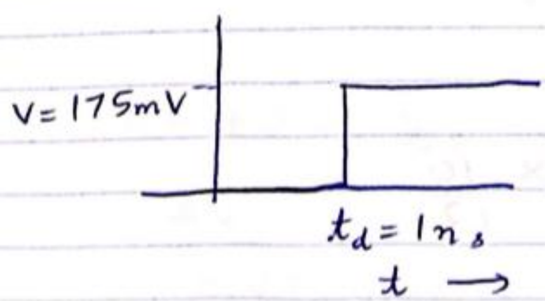
@ $t = t_d$

$v^+(\ell) = Z_0 i_o = 175\ mV$

$\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} = 0$

$v^-(\ell) = 0$

No further reflections.



(b) At steady state, the line doesn't matter. Only the load matters, so the load voltage is again 175 mV.

(c) Contributed by Sen Lin

(c) $\rho_L = \dfrac{Z_L - Z_0}{Z_L + Z_0}$, at source. $\rho_s = 1$.

$$V_{final} = V_0 + \rho_L V_0 + \rho_L V_0 + \rho_L^2 V_0 + \rho_L^2 V_0 + \cdots$$

$$V_{final} = V_0 \cdot \left(1 + 2\sum_{i=1}^{\infty} \rho_L^{i}\right)$$

When $Z_0 < 50\Omega$, $\rho_L > 0$. $V_{final} = V_{max} = 175\,mV$.

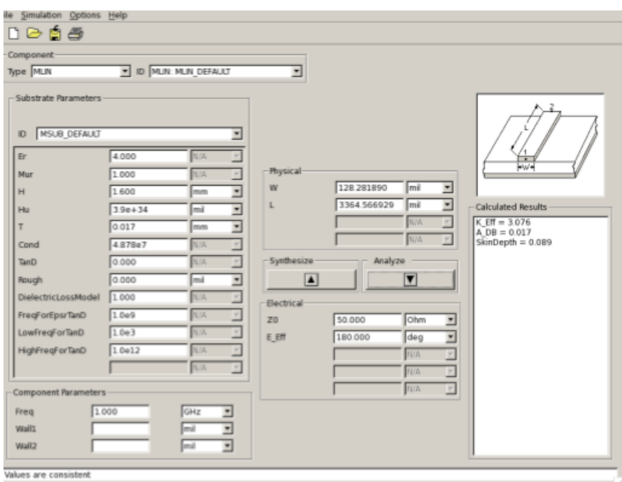
When $Z_0 > 50\Omega$. $\rho_L < 0$,

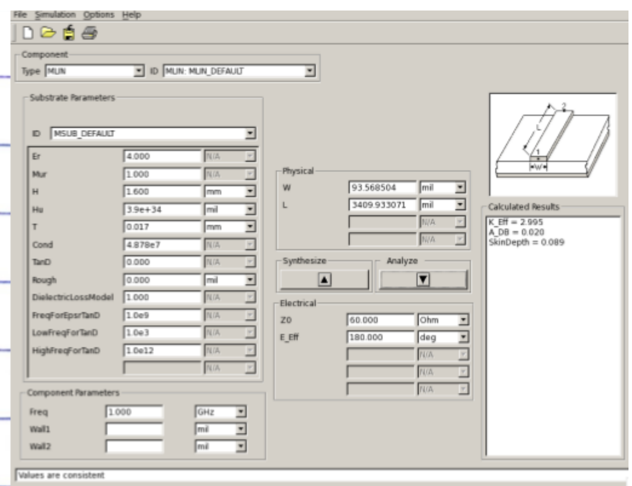
$$V_{max} = (1+\rho_L)\cdot V_0 = 3.5m \times Z_0 \times \left(1 + \frac{50 - Z_0}{50 + Z_0}\right)$$

$$V_{max} \leq 200\ mV.$$

$$\Rightarrow Z_0 \leq 66.7\Omega.$$

(d) Contributed by Luya Zhang





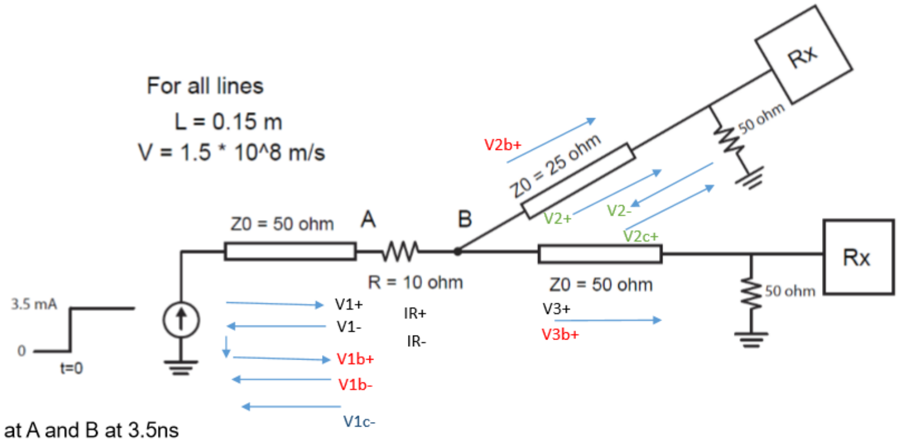
(1) Z0 = 50 ohm, W = 128.28 mil = 3.26 mm

(2) Z0 = 60 ohm, W = 93.568 mil = 2.38 mm

(e) It is almost impossible. Typical tolerance is like ±1 mil. You can have refund if the tolerance is not satisfied.

Prob. 2

HW2.2



V $^+$ and **V** $^-$ at A and B at 3.5ns

At 1 ns:
V1 $^+$ = 175mV, **V1** $^-$ = 175*(26.7-50)/(26.7+50)=175*-0.30 = -53 mV
IR+ = 175/50+53/50 = 4.56 mA
V2 $^+$ = **V3** $^+$ = **V1** $^+$ + **V1** $^-$ **- 10* IR** $^+$ = 77.4 mV

(a)

At 1.5 ns

At point A: V+ = 175mV and V- = -53mV

At point B: V+ = 77.4mV and V- = 0 mV

(b)

At 3.5 ns:
V1 $^-$ has created a new forward traveling wave : **V1b** $^+$ = **V1** $^-$ = -53 mV
V1b $^+$ has created **V2b** $^+$ and **V3b** $^+$ as **V1** $^+$ creating **V2** $^+$ and **V3** $^+$
=> **V2b** $^+$ = -23.4
V1b $^-$ = 16 mV

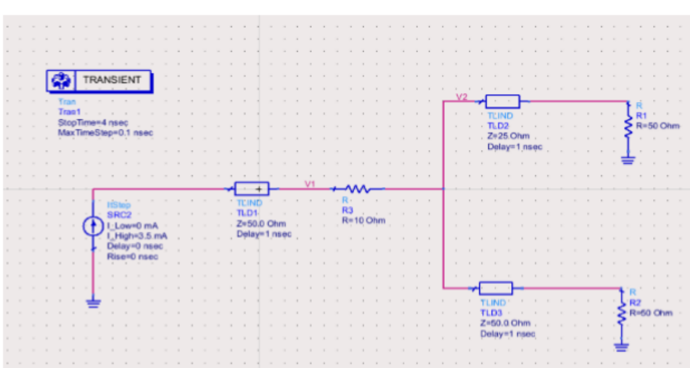
At 3.5 ns:
V2 $^+$ has created a new backward traveling wave : **V2** $^-$ = 25.8 mV
V2 $^-$ has created **V2c** $^+$ **: V2c** $^+$ **=** 25.8* (60//50 - 25)/ (60//50 + 25) = 1 mV
IR- = (25.8 -1)/25 *(50/110) = 0.48mA
V1c $^-$ **= V2** $^-$ + **V2c** $^+$ - 10* **IR** $^-$ = 22 mV

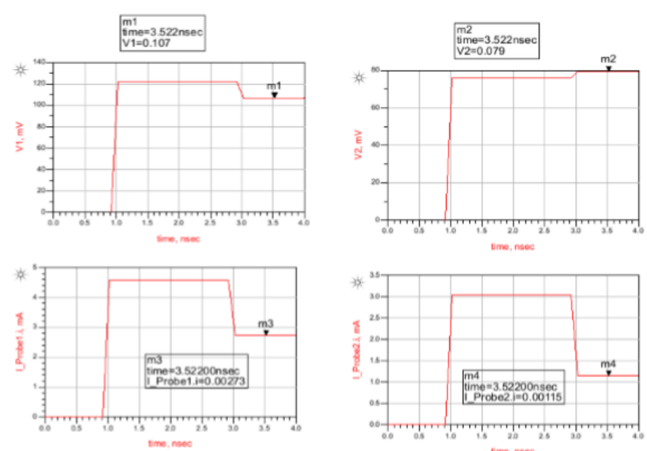
At Point A: **V** $^+$ = 175 + -53 = 122 mV , **V** $^-$ = -53+ 16 + 22= -15 mV

At Point B: **V** $^+$ = 77.4 + -23.4+ -1 = 53 mV, **V** $^-$ = 25.8 mV

HW2.2

Check it in simulation





Only **V** and **I** can be measured

$V^+ + V^- = \mathbf{V}$

$V^+/Z_0 - V^-/Z_0 = \mathbf{I}$

(c)

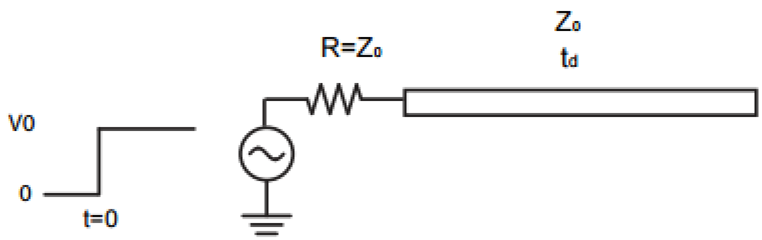
At t= infinite, neglecting the T-lines yield VA= 0.122 V, IA= 3.5 mA, VB= 87.5 mV, and IB= 1.75 mA.

According to $(V^+ + V^-) = V$ and $(V^+ - V_-)/Z_0 = I$, we have

At point A: V+ = 149 mV and V- = -27 mV

At point B: V+ = 65.7 mV and V- = 21.9 mV

Prob. 3



3. A step voltage source is connected to a transmission line, as shown above.

   (a) Draw the time-domain response of the current flowing through the voltage source.

   (b) Using the time-domain response to calculate the total energy delivered by the voltage source and the total energy consumed on the resistor. What is the energy stored on the transmission line at $t = \infty$s?

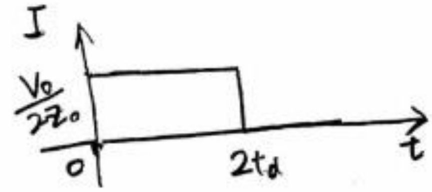
   (c) Express the total capacitance of the transmission line by the line parameters Z_0 and t_d.

   (d) Repeat parts (a)-(b) but with the source resistance changed to $2Z_0$.

   (e) Calculate the energy stored on the transmission line at $t = \infty$s. Is there a way to bypass the tedious sum of infinite geometric series?

(3) Contributed by Sen Lin

3. (a)



(b) $E_0 = V_0 \cdot \frac{V_0}{2Z_0} \cdot 2t_d = \frac{V_0^2}{Z_0} \cdot t_d$.

$E_{stored} = E_0 - I^2 \cdot Rt = -\left(\frac{V_0^2}{2Z_0}\right)^2 \cdot R \cdot 2t_d + \frac{V_0^2}{Z_0} \cdot t_d = \frac{V_0^2}{2Z_0} t_d$.

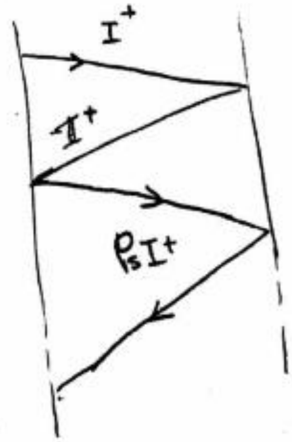
(c) $Z_0 = \sqrt{\frac{L'}{C'}}$, $\quad c' = \frac{L'}{Z_0^2}$, $\quad v = \frac{1}{\sqrt{L'C'}}$ $\quad c' = \frac{1}{L'v^2}$

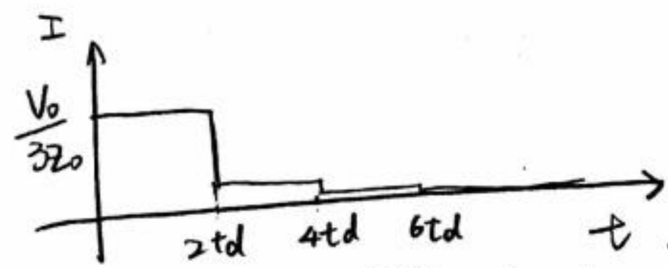
$\Rightarrow \quad c' = \frac{1}{Z_0 \cdot v} \Rightarrow$ total cap $= c' \cdot L = \frac{t_d}{Z_0}$.

(d) $t = 0$, $I = I^+ = \frac{V_0}{3Z_0}$

$t = 2t_d$, $I = \rho_s I^+ = \left(\frac{1}{3}\right)^2 \cdot \frac{V_0}{Z_0}$

$\cdots$ $t = 2nt_d$, $I = \left(\frac{1}{3}\right)^{n+1} \cdot \frac{V_0}{Z_0}$





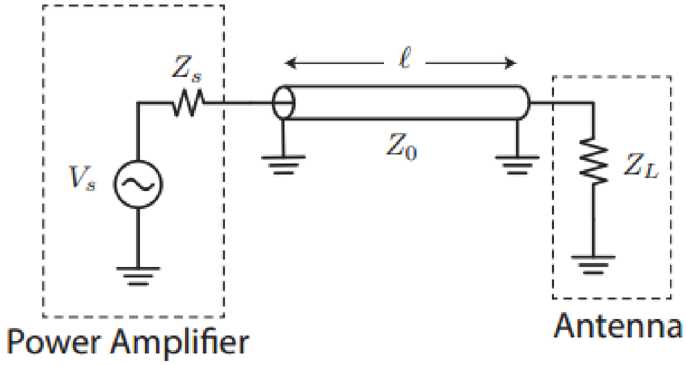
Energy from source: $E_0 = V_0 I \cdot 2t_d = V_0 \cdot 2t_d \cdot \sum_{i=1}^{\infty} \left(\frac{1}{3}\right)^i \frac{V_0}{Z_0} = \frac{V_0^2}{Z_0} \cdot t_d$

Energy consumed by resistor: $E_R = V_0 \frac{I^2}{2} \cdot 2t_d = \frac{V_0^2}{2Z_0} \cdot t_d$

Energy stored $E_{stored} = \frac{V_0^2}{2Z_0} \cdot t_d$ (3)

3. (e) $E_{stored} = \frac{1}{2} C V^2 = \frac{1}{2} \cdot \frac{t_d}{Z_0} \cdot V_0^2 = \frac{V_0^2 t_d}{2Z_0}$.

Prob. 4



4. Consider a power amplifier (modeled by an equivalent source) is driving and antenna load through a section of transmission line $Z_0 = 50\Omega$ ($\epsilon_r = 4$) of length 21.875cm. The antenna is designed nominally to match to 50Ω but due to its placement on a metallic surface, the actual impedance is unknown. Fortunately, as a former student of 142/242, you have the right tools to solve this problem. Unfortunately you are alone in the lab and the network analyzer and oscilliscope are locked up. To meet a difficult deadline, you decide to work with the tools at hand.

   Suppose that we use a capacitive probe and scan it along the transmission line. We find that as we move away from the load, the magnitude of the voltage on the line drops and hits a minimum at a distance of 4.75mm from the load. Also, if we move to a distance of 36mm from the load, the voltage is maximum with a $SWR = 3$.

   (a) What is the operating frequency of the power amplifier?
   (b) What is the load (antenna) impedance at the frequency?
   (c) In sinusoidal steady state, what are the boundary conditions at the load?
   (d) Similarly, what are the boundary conditions at the source? Assume $Z_s = 5\Omega$ but keep your answer in symbolic form.
   (e) Using the above boundary conditions, find the voltage waveform along the transmission line and plot its magnitude. Verify the results of the probe experiment you did in part (a). Use Mathematica or a similar package to do the calculation and plots.
   (f) Suppose that the voltage source (amplifier) has a swing of 10V. How much power is delivered by the source and how much power reaches the load (antenna)? What is the efficiency? How much better/worse is this than an antenna matched to 50Ω?
   (g) Suppose the transmission line is low loss so that all of your above calculations are approximately valid, except your power calculation. If the line loss is .1 dB per centimeter, redo the power calculation. You can use Mathematica or a similar tool.

(a) At load, the voltage in phasor expression can be expressed by $V(0) = V^+ + V^-$. At $x=-l$, the voltage can be expressed by $V(-l) = V^+\exp(j2\pi l/\lambda) + V^-\exp(-j2\pi l/\lambda) = V^+\exp(j2\pi l/\lambda) * [1 + V^-/V^+ \exp(-j4\pi l/\lambda)]$. For the maximum voltage to happen, $V^-/V^+ \exp(-j4\pi l/\lambda)$ is a real positive number, and for the minimum voltage to happen, $V^-/V^+ \exp(-j4\pi l/\lambda)$ is a real negative number. Therefore, $4\pi(\Delta l)/\lambda = \pi$ or $\Delta l = \boldsymbol{\lambda/4}$. Since $\Delta l = (0.036-0.00475)$, $\boldsymbol{\lambda = 0.125}$ and **frequency = 1.2 GHz** with er = 4 (speed = 1.5e8m/s).

(b) The minimum voltage is at $x = -0.00475$ *or* $l = 0.00475$, *which means* $V^-/V^+ \exp(-j4\pi*0.00475 / \mathbf{0.125}) = -|V^-/V^+|$ or

$$V^-/V^+ = -|V^-/V^+| \exp(j4\pi*0.00475 / \mathbf{0.125})$$

On the other hand, SWR = 3 means: $3 = [1+ | V^-/V^+|] / [1- | V^-/V^+ |]$ or $|V^-/V^+|=0.5$

- ⇨ $V^-/V^+ = rhoL = -0.444-0.230j$
- ⇨ $Z_L=Z0*(1+rhoL)/(1-rhoL)=17.5-10.7j$

(c) The boundary conditions at load are:

$V_L= V^+ + V^-$
$I_L = V^+/50 – V^-/50$
$Z_L = V_L/ I_L$

(d) The boundary conditions at source are:

$V_s – V^+\exp(j\, 2\, \pi*length / \boldsymbol{\lambda}) * [1 + V^-/V^+ \exp(-j4\pi* length / \boldsymbol{\lambda}))] = I_s *Zs$

$I_s = 1/Z0 * V^+\exp(j\, 2\, \pi*length / \boldsymbol{\lambda}) * [1 - V^-/V^+ \exp(-j4\pi\, length / \boldsymbol{\lambda}))]$

Note that V^-/V^+, $\boldsymbol{\lambda}$, Zs, Z0, and *line length* are all known.

(e) From (d), we have

$V_s =$

$V^+\exp(j\, 2\, \pi*length / \boldsymbol{\lambda}) * \{ 1 + V^-/V^+ \exp(-j4\pi\, length / \boldsymbol{\lambda})) + Zs/Z0* [1 - V^-/V^+ \exp(-j4\pi\, length / \boldsymbol{\lambda})] \}$

$= V^+\exp(j\, 2\, \pi*length / \boldsymbol{\lambda}) * \{ 1 + rhoL *\exp(-j4\pi\, length / \boldsymbol{\lambda})) + Zs/Z0* [1 - rhoL *\exp(-j4\pi\, length / \boldsymbol{\lambda})] \}$

or

V^+

$= V_s \exp(- j\, 2\, \pi*length / \boldsymbol{\lambda}) / \{ 1 + rhoL *\exp(-j4\pi* length / \boldsymbol{\lambda})) + Zs/Z0* [1 - rhoL *\exp(-j4\pi* length / \boldsymbol{\lambda})] \}$

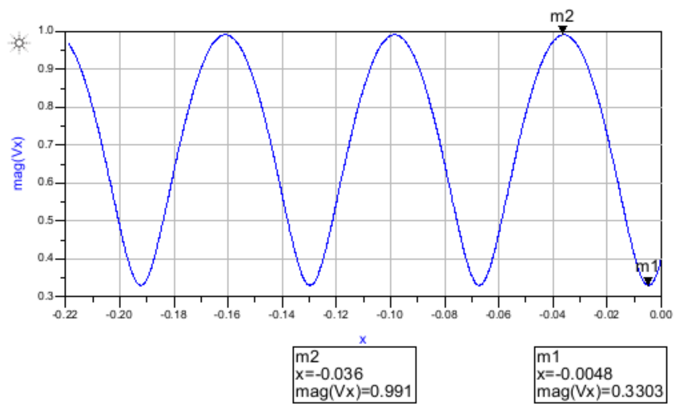
Recall again that

$V^- = rhoL* V^+$

Substitute V^- and V^+ into $V(x) = V^+\exp(-j2\pi x/\boldsymbol{\lambda}) + V^- \exp(j2\pi x/\boldsymbol{\lambda})$, the voltage at any point can be calculated

Calculation:

```
Eqn Rs=5
Eqn Vm=Vp*rhoL
Eqn Z0=50
Eqn rhoL=-0.5*exp(j*4*pi*0.00475/0.125)
Eqn Length=0.21875
Eqn Vs=1
Eqn Vp=Vs*exp(-j*2*pi*Length/0.125)/ [  1+rhoL*exp(-j*4*pi*Length/0.125)   + Rs/Z0* (1-rhoL*exp(-j*4*pi*Length/0.125))]
Eqn Vx=Vp*exp(-j*2*pi*x/0.125)+Vm*exp(j*2*pi*x/0.125)
```



(f)

The input impedance of the network is

Zin = Z0* [1+rhoL* exp(-j4π* *length* / $\boldsymbol{\lambda}$)]/ [1-rhoL* exp(-j4π* *length* / $\boldsymbol{\lambda}$)]

= 103.6+63*j

Power from the source = 0.5* $(Vs)^2$/ $|Zs+ Zin|^2$ * Re(Zs + Z_{in}) = 0.343 W

Efficiency is 103.6/(5+103.6) = 95.4%

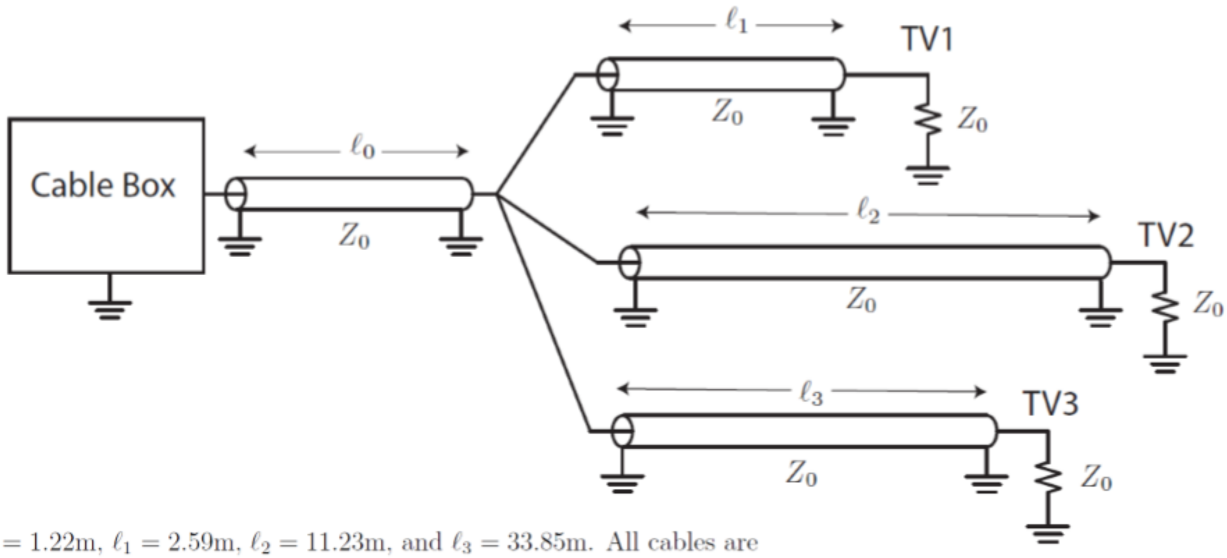
With a matched antenna, we have a higher power of 0.5* $(Vs)^2$/ $|Zs+ 50|^2$ * Re(Zs + 50) = 0.909 W, but the efficiency is lower.

Efficiency is 50/(5+50) = 90.9%

(g)

Assume the line loss is 0.1 dB/cm. The power into the network changes little, but the power into the antenna and efficiency both degrade by 2.1875 dB (60% of the old value)

Prob. 5



(a) Suppose $\ell_0 = 1.22$m, $\ell_1 = 2.59$m, $\ell_2 = 11.23$m, and $\ell_3 = 33.85$m. All cables are 75Ω and have a velocity of propagation of 1×10^8m/s. If the TV is tuned to channel 14, approximately at 473 MHz, what is the impedance seen by the cable box?

Lambda = v/f = 10^8/473 MHz = 0.21142m

l_0 =1.22m = 5.77 lambda

ZL = Z0//Z0//Z0 = Z0/3

rhoL = (Z0/3- Z0) / (Z0/3 +Z0) = -0.5

Impedance seen by the cable box is: Z0* (1 + rhoL *exp (-*j 2**2p*i* * 5.77) / (1 - rhoL *exp (-*j 2**2p*i* * 5.77)

= 200-66j

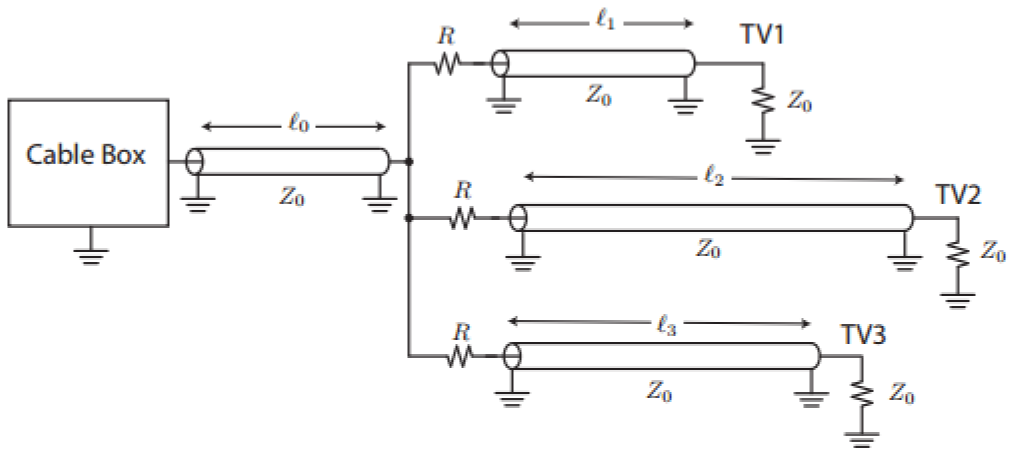
(b) Assume the Cable Box puts out a signal at -30 dBm. How much power reaches each television set?

= -35 dBm (1/3 of the delivered power)

(c) It is discovered that the system performance gets much worse when the first TV is unplugged, as shown. Explain what is happening. How much power reaches TV2 and TV3?

Extreme low power! Notice l1 is about 12.25 wavelength

(d) After explaining the concept of impedance matching, your friends come up with the following solution. Does it work? What is the impedance seen by the Cable Box? How much power reaches each TV set?



It works. Let's work in terms of R.

The impedance seen by the box is

Z0* (1 + rhoL *exp (-*j 2**2*p*i * 5.77) / (1 - rhoL *exp (-*j 2**2*p*i * 5.77),

where rhoL = (Z0/3 + R - Z0) / (Z0/3 +R + Z0) = -0.5. For example, if R = 150 ohm, then the impedance seen by the box is 75 ohm.

The power into each branch is -35 dBm, but R/(R+75) of power is consumed on the series resistor, so the power into each TV is -35 dBm + 10*log(R/(R+75)). For example, if R = 150 ohm, then the power is **-38.5 dBm.**