Today’s Agenda (Nov. 8)

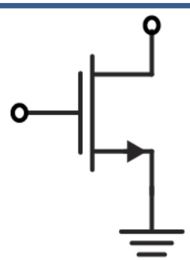
- Reviews and Quick Questions
- Distortion with Feedback
- Sample Distortion Problems

Important Distortion Matrices

i_{out} (or v_{out}) $= a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 + \ldots$

Very Important: v_{in} and i_{out} are small-signal !!!

Extract the correct power series for distortion analysis



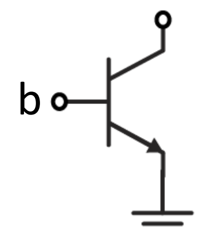
Long channel device

$I_d = 0.5\mu_n C_{ox} W/L*(V_{gs}-V_{th})^2$
$= 0.5\mu_n C_{ox} W/L*V_{th}^2 - \mu_n C_{ox} W/L*V_{th}\boldsymbol{V_{gs}} + 0.5\mu_n C_{ox} W/L*\boldsymbol{V_{gs}^2}$

$i_d = 0.5\mu_n C_{ox} W/L*(V_{gs0}-V_{th} + v_{in})^2 =$

$0.5\mu_n C_{ox} W/L*(V_{gs0}-V_{th})^2 + \mu_n C_{ox} W/L*(V_{gs0}-V_{th})\, v_{in} + 0.5\mu_n C_{ox} W/L*v_{in}^2$

$\Rightarrow\; i_{out} = \mu_n C_{ox} W/L*(V_{gs0}-V_{th})\, v_{in} + 0.5\mu_n C_{ox} W/L*v_{in}^{2}$



BJT

$I_c \approx I_{s0} exp(V_b/V_T)$
$= I_{s0} + I_{s0}/V_T*V_b + I_{s0}/2V_T^2*V_b^2 + I_{s0}/6V_T^3*V_b^3 + \ldots$

$i_c \approx I_{s0} exp[(V_b + v_{in})/V_T] = \boldsymbol{I_{c0}} exp(v_{in}/V_T)$
$= I_{c0}/V_T*v_{in} + I_{c0}/(2V_T^2)*v_{in}^2 + I_{c0}/(6V_T^3)*v_{in}^3 + \ldots$

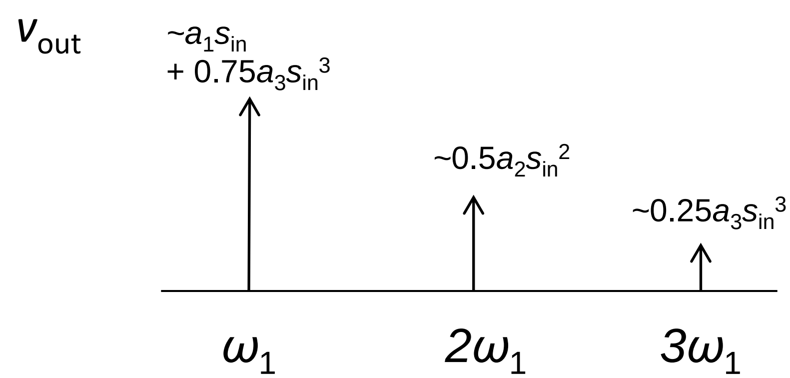
$I_{c0} = I_{s0} exp(V_b/V_T)$

Important Distortion Matrices

i_{out} (or v_{out}) = $a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 + \ldots$

Single-Tone Excitation: $v_{in} = s_{in}\cos(\omega_1 t)$

- Gain = $a_1 + (3a_3/4) \times s_{in}^2$
- HD2 = $a_2/(2a_1) \times s_{in}$
- HD3 = $a_3/(4a_1) \times s_{in}^2$
- IP_{1dB}: $s_{in} = \mathrm{sqrt}(|4a_1|/|3a_3| \times 0.11)$

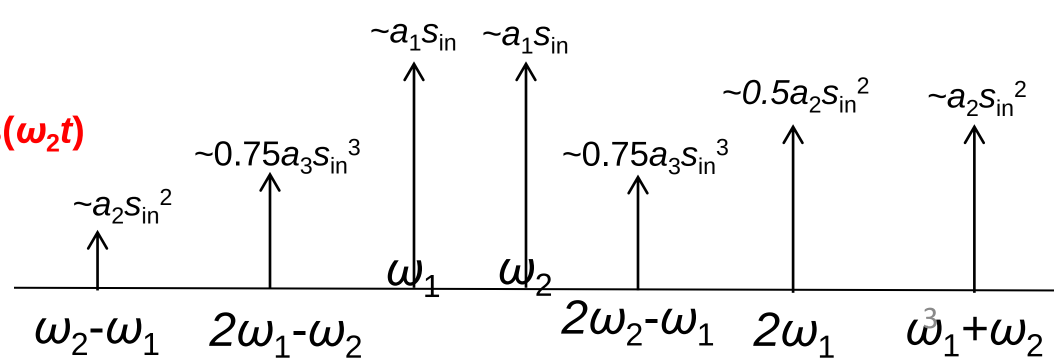


Two-Tone Characterization: $v_{in} = s_{in}\cos(\omega_1 t) + s_{in}\cos(\omega_2 t)$

- IM2 = $a_2/a_1 \times s_{in}$
- IIP2 $s_{in} = a_1/a_2$
- IM3 = $3a_3/(4a_1) \times s_{in}^2$
- IIP3 $s_{in} = \mathrm{sqrt}(|4a_1/3a_3|)$

Small sig. with blocker: $v_{in} = s_{in}\cos(\omega_1 t) + s_b\cos(\omega_2 t)$

- Gain = $a_1 (1 + (3a_3/2a_1) s_b^2)$



Quick Questions

IIP2 = 5 dBm, Input@ω_1 = 0 dBm, Input@ω_2 =-10 dBm, What is the output IM2?

Assume x = OIP2

(Output 2nd intermodulation)/(output ω1) = (x-5-15) - (x-5) = -15 dBc

(Output 2nd intermodulation)/(output ω2) = (x-5-15) - (x-15) = -5 dBc

Is the upper-band IIP2 always the same to the lower-band IIP2?

No! Maybe there is a output filter!

IIP3 = 5 dBm, Input@ω_1 = 4 dBm, Input@ω_2 = 0 dBm, What is the output IM3?

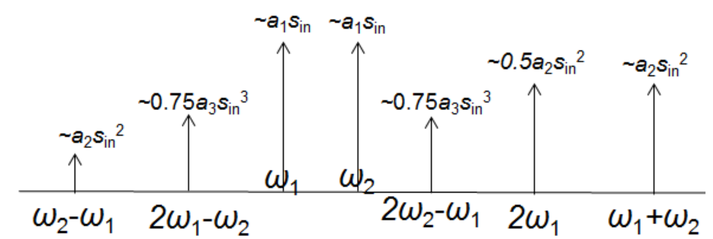
Assume x = OIP3

(Output $2\omega_2$-ω_1)/(output ω_1) = (x-5*2-1) - (x-1) = -10 dBc

(Output $2\omega_2$-ω_1)/(output ω_2) = (x-5*2-1) - (x-5) = -6 dBc

(Output $2\omega_1$-ω_2)/(output ω_1) = (x-1*2-5) - (x-1) = -6 dBc

(Output $2\omega_1$-ω_2)/(output ω_2) = (x-1*2-5) - (x-5) = -2 dBc



With $\omega_1 \approx \omega_2$, IIP3 = 10 dBm. (i) What is the IP_{1dB}?
(ii) What is the HD3 with input@ω_1 of 5 dBm and 20-dB filter loss to $3\omega_1$?

(i) IIP3 = sqrt($|4a_1/3a_3|$), $s_{in,P1dB}$ = sqrt($|4a_1|/|3a_3|\times 0.11$) => IIP3 = IP_{1dB} + 9.5 dB

(ii) IIP3 = sqrt($|4a_1/3a_3|$) in power is 10 dBm => 0.01(W)= $|4a_1/3a_3|/(2\times 50)$ => $|4a_1/3a_3|$ = 1

Input power is 5 dBm (3.2 mW) => $s_{in}^2 = 0.32$

HD3 before filter = $a_3/(4a_1)\times s_{in}^2$ =0.33*0.32 = 0.106 = **-19.5 dB**

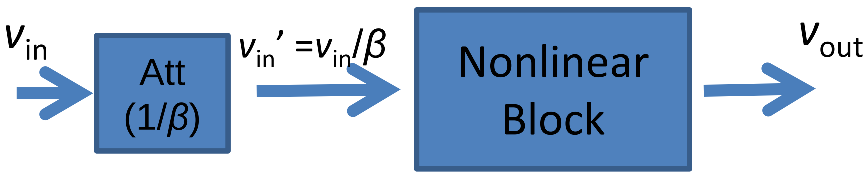
HD3 = -19.5 dB - 20 dB (filter) = -39.5 dB

Distortion With Attenuator

Case A

$$v_{out} = a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 + \ldots$$

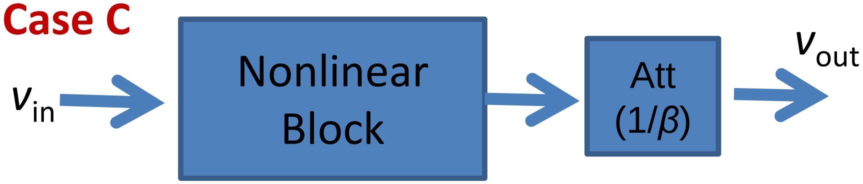
Case B



$$v_{out} = a_1 v_{in}' + a_2 v_{in}'^2 + a_3 v_{in}'^3 + \ldots$$

$$v_{out} = (a_1/\beta) v_{in} + (a_2/\beta^2) v_{in}^2 + (a_3/\beta^3) v_{in}^3 + \ldots$$

Case C

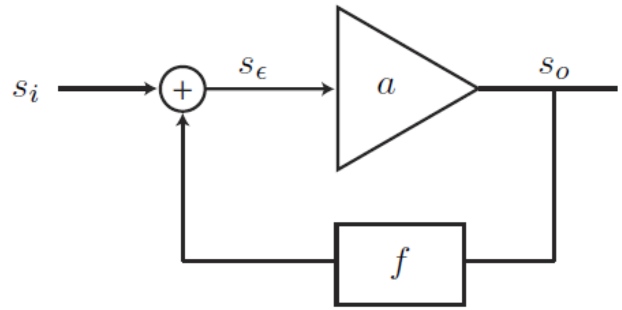


$$v_{out} = (a_1/\beta) v_{in} + (a_2/\beta) v_{in}^2 + (a_3/\beta) v_{in}^3 + \ldots$$

$$\boldsymbol{v_{in} = s_{in}\cos(\omega_1 t)}$$

	Case A	Case B	Case C
Same Input Level			
Output Sig.	$a_1 s_{in}$	$(a_1/\beta)\times s_{in}$	$(a_1/\beta)\times s_{in}$
Output 2nd	$(a_2/2)\times s_{in}^2$	$(a_2/2\beta^2)\times s_{in}^2$	$(a_2/2\beta)\times s_{in}^2$
HD2	$a_2/(2a_1)\times s_{in}$	$\boldsymbol{a_2/(2a_1\beta)\times s_{in}}$	$a_2/(2a_1)\times s_{in}$
Generate the Same Output Level			
Output Sig.	$a_1 s_{in}$	$(a_1/\beta)\times\beta s_{in}$	$(a_1/\beta)\times\beta s_{in}$
Output 2nd	$(a_2/2)\times s_{in}^2$	$(a_2/2\beta^2)\times\beta^2 s_{in}^2$	$(a_2/2\beta)\times\beta^2 s_{in}^2$
HD2	$a_2/(2a_1)\times s_{in}$	$a_2/(2a_1)\times s_{in}$	$\boldsymbol{a_2/(2a_1)\times\beta s_{in}}$

Put input attenuator => linearity does not improve
Put output attenuator => linearity degrades



- We usually implement the feedback with a passive network
- Assume that the only distortion is in the forward path a

$$s_o = a_1 s_\epsilon + a_2 s_\epsilon^2 + a_3 s_\epsilon^3 + \cdots$$

$$s_\epsilon = s_i - f s_o$$

$$s_o = a_1(s_i - f s_o) + a_2(s_i - f s_o)^2 + a_3(s_i - f s_o)^3 + \cdots$$

$$s_o = b_1 s_i + b_2 s_i^2 + b_3 s_i^3 + \cdots$$

$$\boxed{b_1 = \frac{a_1}{1 + a_1 f} = \frac{a_1}{1 + T}} \quad \boxed{b_2 = \frac{a_2}{(1+T)^3}} \quad \boxed{b_3 = \frac{a_3(1 + a_1 f) - 2a_2^2 f}{(1 + a_1 f)^5}}$$

$$b_1 = \frac{a_1}{1 + a_1 f} = \frac{a_1}{1 + T}$$

$$b_2 = \frac{a_2}{(1 + T)^3}$$

$$b_3 = \frac{a_3(1 + a_1 f) - 2a_2^2 f}{(1 + a_1 f)^5}$$

Single-Tone Excitation: $v_{in} = s_{in}\cos(\omega_1 t)$

- Gain = a_1
- HD2 = $a_2/(2a_1)\times s_{in}$
- HD3 = $a_3/(4a_1)\times s_{in}^2$
- IP_{1dB}: s_{in} = sqrt($|4a_1|/|3a_3|\times 0.11$)

$v_{out} = a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 + \ldots$

Two-Tone Excitation: $v_{in} = s_{in}\cos(\omega_1 t) + s_{in}\cos(\omega_2 t)$

- IM2 = $a_2/a_1\times s_{in}$
- IIP2 $s_{in} = a_1/a_2$
- IM3 = $3a_3/(4a_1)\times s_{in}^2$
- IIP3 s_{in} = sqrt($|4a_1|/|3a_3|$)

A small sig. with blocker: $v_{in} = s_{in}\cos(\omega_1 t) + s_b\cos(\omega_2 t)$

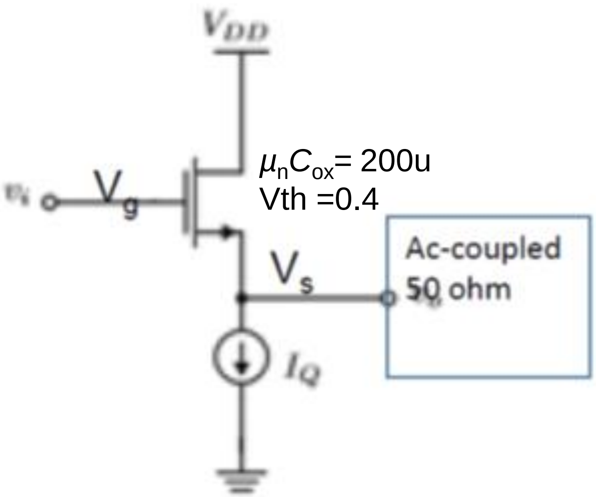
Gain = $a_1 (1 + (3a_3/2a_1) s_b^2)$

First notice the Gain degrades by a factor of (1+T)

With FB	Same input to the no-feedback case (input = s_{in})	Same output to the no-feedback case (input = $(1+T)s_{in}$)
HD2	$b_2/(2b_1)\times s_{in} = (a_2/2a_1)\times s_{in}/(1+T)^2$	$(a_2/2a_1)\times s_{in}/(1+T)$
IM2	$a_2/a_1\times s_{in}/(1+T)^2$	$(a_2/a_1)\times s_{in}/(1+T)$
HD3	$b_3/(4b_1)\times s_{in}^2 =$ $[\, (a_3/4a_1)/(1+T)^3 - (a_2^2 f)/(2a_1(1+T)^4) \,]\times s_{in}^2$	$[\, (a_3/4a_1)/(1+T) - (a_2^2 f)/(2a_1(1+T)^2) \,]\times s_{in}^2$
IM3	$3b_3/(4b_1)\times s_{in}^2$	$3b_3/(4b_1)\times s_{in}^2(1+T)^2$

- In most cases, linearity can be truly improved!
- Feedback might be used to make $b_3 = 0$
- **If $a_3 = 0$ and $a_2 \neq 0$ (e.g. long-channel NMOS), then feedback creates $b_3 \neq 0$**

(i) Find the output referred OIP_3 of the amplifier assuming that it drives a load $R_L = 50\Omega$ (AC coupled). Bias the amplifier in order to drive a voltage swing of 100mV onto the load with an IM_3 better than -50 dBc. when input is 0.12V



IM3 = -50 dBc @ Vout = 0.1V

=> OIP3 = 0.1V+25 dB = 1.78V

Without feedback (use the correct power series in page 2)

$$i_{out} = \mu_n C_{ox} W/L*(V_{gs0}-V_{th})\, v_{in} + 0.5\mu_n C_{ox} W/L*v_{in}^2$$

Replace v_{in} by $v_{in} - i_{out}50$ in the above power series

New power series with FB

Use $I_{dc} = 0.5\mu_n C_{ox} W/L*(V_{gs0}-V_{th})^2$

$$i_{out} = \underbrace{\frac{\sqrt{\frac{2I_{dc}\,\mu_n C_{ox} W}{L}}}{1+50\sqrt{\frac{2I_{dc}\mu_n C_{ox} W}{L}}}}_{b_1} v_{in,new} + ?\, v_{in,new}^2 + \underbrace{\frac{-2(0.5\times\mu_n C_{ox} W/L)^2\times 50}{\left[1+50\sqrt{\frac{2I_{dc}\mu_n C_{ox} W}{L}}\right]^5}}_{b_3} * v_{in,new}^3$$

With a two-tone input

$b_1 s_{in} = 0.1$

$3b_3/(4b_1)\times s_{in}^2 = 0.1*0.0032$

$\xrightarrow{s_{in} = 0.12}$

=> I_{dc} = 4.2 mA, W/L = 6150

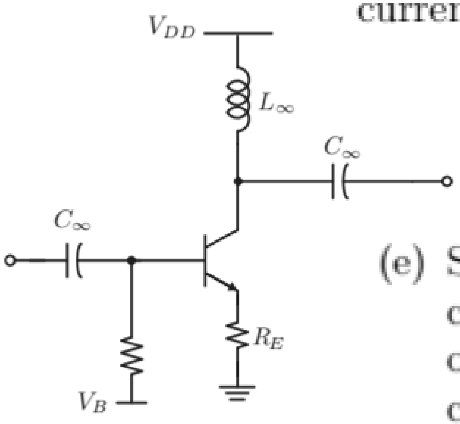
BJT Emitter Degeneration

16'Spring Final

2. (25 points) Consider the following power amplifier. The power transistor has a $V_{CE,sat} = 0.3$V and a degeneration resistance of $R_E = 3\Omega$ and is biased at a quiescent current $I_Q = 1$A. The power supply voltage is 20V.

Feedback resistor

Use to find the correct power series



(e) Suppose the signal is a multi-carrier OFDM signal. We wish to limit the largest component of the distortion to be 10 dB down from the sub-carrier signal (think of the signal as a multi-tone input and consider the strongest distortion component only). Specify the required back-off to meet this specification.

Assume IM3 is the strongest distortion: IM3 = -10 dBc

Without feedback and assume I_Q = 1A
(use the correct power series in page 2)

$i_{out} = I_{c0}/V_T * v_{in} + I_{c0}/(2V_T^2) * v_{in}^2 + I_{c0}/(6V_T^3) * v_{in}^3 + \ldots$

$v_{in,new} = v_{in} - i_{out} R_E$

$(1+T) = (1+ R_E I_{c0}/V_T)$; $f = R_E = 3$; I_{c0} = 1A ; I_{c0}/V_T = 40 ; V_T = 0.025

$b_1 = (I_{c0}/V_T)/(1+ R_E I_{c0}/V_T) = 0.33$

$b_3 = (I_{c0}/6V_T^3)/(1+ R_E I_{c0}/V_T)^4 - 2[I_{c0}^2/(4V_T^4)] \times R_E/(1+R_E I_{c0}/V_T)^5$ = 5e-5 - 14.8e-5 = -9.8e-5

IIP3 = sqrt(|4b_1|/|3b_3|) = 67V; Input for -10-dB IM3 => 5-dB back off => Input = 21V