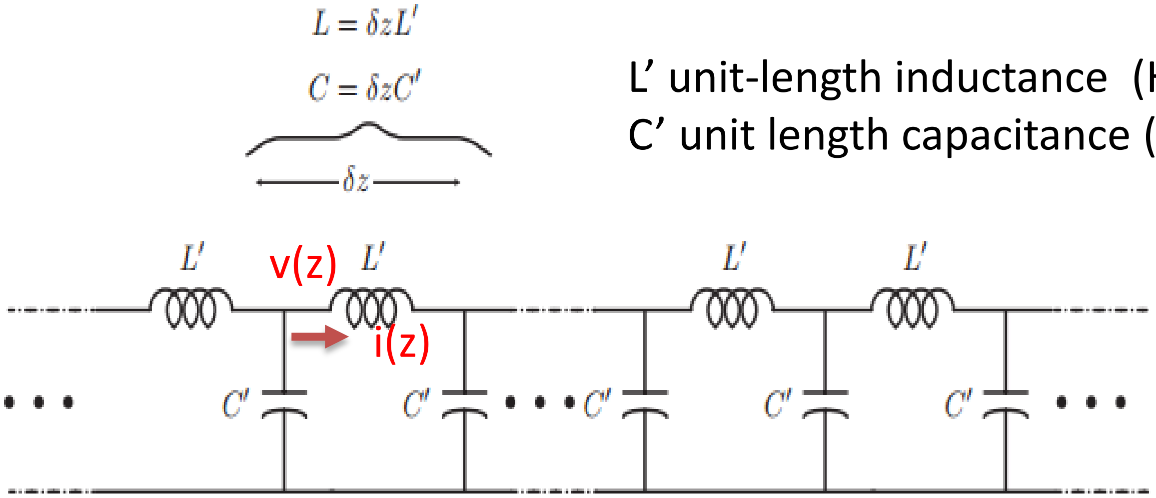
Today's plan

- Review on transmission line in frequency domain
- Review on VSWR method to find a unknown load impedance
- Discussion on HW2.5 and 2.6

Transmission Line in Time Domain

Last week



L’ unit-length inductance (H/m)
C’ unit length capacitance (F/m)

Solve KCL and KVL

$$-\frac{\partial i}{\partial z} = C'\frac{\partial v}{\partial t}$$

$$-\frac{\partial v}{\partial z} = L'\frac{\partial i}{\partial t}$$

$$\Rightarrow$$

$$\frac{\partial^2 v}{\partial z^2} = L'C'\frac{\partial^2 v}{\partial t^2}$$

$$\frac{\partial^2 i}{\partial z^2} = L'C'\frac{\partial^2 i}{\partial t^2}$$

$$\Rightarrow$$

$$v(z,t) = \overset{v^+}{\boxed{f^+(z-vt)}} + \overset{v^-}{\boxed{f^-(z+vt)}}$$

i^+ $\quad \frac{f^+}{Z_0}$ $\qquad$ i^- $\quad -\frac{f^-}{Z_0}$

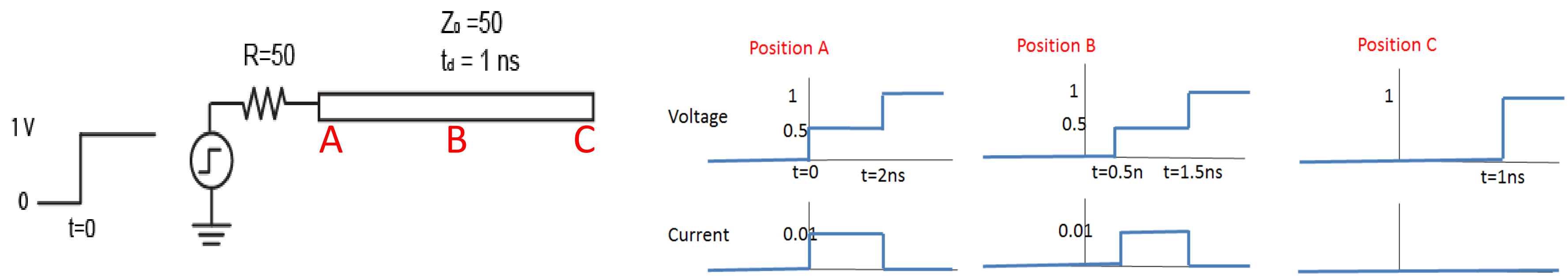
$$v = \sqrt{\frac{1}{L'C'}}$$

$$Z_0 = \sqrt{\frac{L'}{C'}}$$

- Is $V(z,t)/I(z,t) = Z_0$?
- The discussion last week focus on “transient response” of “dc excitation”
- Today we will focus on “steady-state response” of “ac excitation”

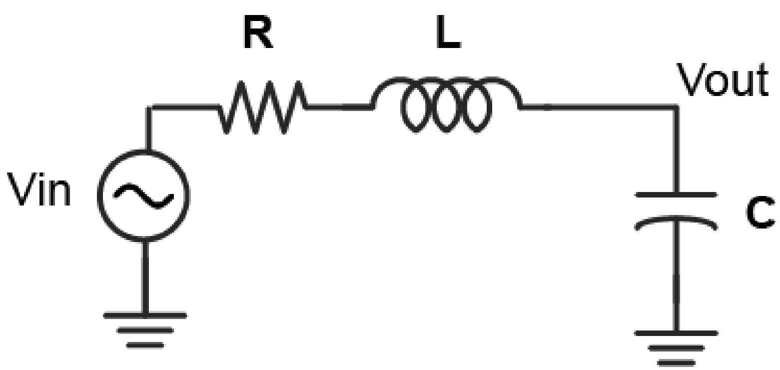
Old Example: Time-Domain Diagram

Last week



- This is a typical "transient response" of "dc excitation"
- "Transient response" of "ac excitation" can be studied with the same method (time-domain diagram)
- Transient response" is more difficult to analyze if the load has memory (e.g. inductor)
- "Steady-state response" for dc excitation is usually super easy
  To make the example complicated: $(\sum V^+/50) - (\sum V^-/50) = 0$; $1 = (\sum V^+) + (\sum V^-) + 50*[(\sum V^+/50) - (\sum V^-/50)]$
- **"Steady-state response" of "ac excitation" is also very important** (Transient response is short, RF circuit is narrowband…)
- Great tools for analyzing **steady-state ac response**: phasor expression

Phasor Expression



$$A(\omega) = \frac{Vout(\omega)}{Vin(\omega)} = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}$$

A complex number

$$Vin(t) = V_0 * \cos(\omega t + \Omega)$$

into phaser expression

$$Vin(\omega) = V_0 * \exp(j\Omega)$$

Play in the frequency domain

$$Vout(\omega) = A(\omega) * Vin(\omega)$$

Translate back into time domain

$$Vout(t) = Re[Vout(\omega) * \exp(j\omega t)] = V_0 * |A(\omega)| * \cos[\omega t + \Omega + angle(A(\omega))]$$

Transmission Line in Frequency Domain

Steady-state time-domain formula

Forward traveling voltage waves lumped together

Backward traveling voltage waves lumped together

$$v(z,t) = f^{+}(z - vt) + f^{-}(z + vt)$$

$$\frac{f^{+}}{Z_0} \qquad -\frac{f^{-}}{Z_0}$$

Forward current wave

Backward current wave

Only one frequency in the system

Steady-state (ac) phasor expression

$$v(z) = V^{+}e^{-\gamma z} + V^{-}e^{\gamma z}$$

$$i(z) = \frac{V^{+}}{Z_0}e^{-\gamma z} - \frac{V^{-}}{Z_0}e^{\gamma z}$$

f^{+} = A*exp(-αz)*cos(βz-ωt+θ)

In phasor

=> f^{+} = A*exp(-αz)*exp(-$j\beta z$-$j\theta$)

=[A*exp(-$j\theta$)]* exp[-(α+$j\beta$)z]

$$\gamma = j\beta \; (\alpha = 0)$$

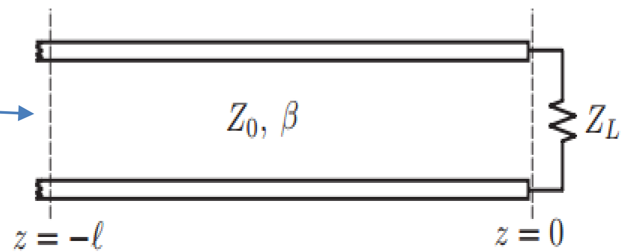
β = 2π/λ

No loss

Impedance Transform By Transmission Line

We are in ac or phasor domain now !!!

$Z_{in}(\omega)$



Steady-state solution in phasor

$$v(z) = V^{+}e^{-\gamma z} + V^{-}e^{\gamma z}$$

$$i(z) = \frac{V^{+}}{Z_0}e^{-\gamma z} - \frac{V^{-}}{Z_0}e^{\gamma z}$$

$$\gamma = j\beta = j2\pi/\lambda$$

Use boundary condition at the $Z_L(\omega)$ load (z=0):

$V^{-} = V^{+}\rho_L = V^{+}(Z_L - Z_0)/(Z_L + Z_0)$

$\rho_L = 1$ *when* $Z_L = inf$

$\rho_L = -1$ *when* $Z_L = 0$

$$\boldsymbol{\rho_L = (Z_L - Z_0)/(Z_L + Z_0)}$$

$(V^{-} + V^{+}) / (V^{+}/Z_0 - V^{-}/Z_0) = Z_L$

$$v(z) = V^{+}\left(e^{-j\beta z} + \rho_L e^{j\beta z}\right)$$

$$i(z) = \frac{V^{+}}{Z_0}\left(e^{-j\beta z} - \rho_L e^{j\beta z}\right)$$

$Z_{in} = v(-l)/i(-l) =$

$Z_0\,(1 + \rho_L e^{-j2\beta l})/(1 - \rho_L e^{-j2\beta l})$

Discuss the cases with

i) $l = 0$

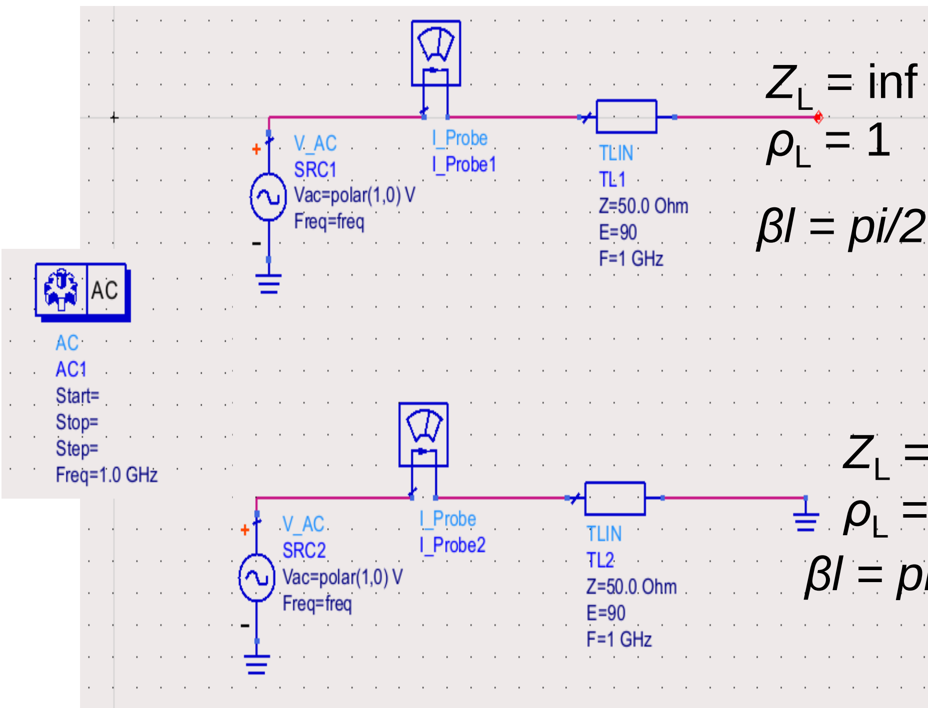
ii) $\rho_L = 0$

iii) $\rho_L \neq 0$

iv) $\rho_L = -1 \Rightarrow Z_{in} = jZ_0 \tan(\beta l) = jZ_0 \tan(\omega l/v)$

ADS Simulation: It is an ac simulation

$$Z_{in} = v(-l)/i(-l) = Z_0 * (1 + \rho_L e^{-j2\beta l})/(1 - \rho_L e^{-j2\beta l})$$



Z_L = inf
ρ_L = 1
$\beta l = pi/2 = 90°$

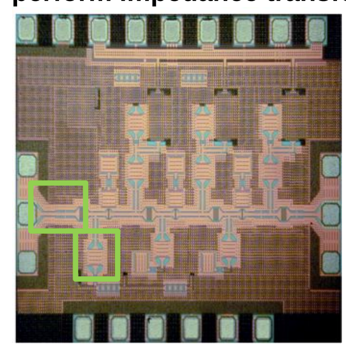
Z_L = 0
ρ_L = -1
$\beta l = pi/2 = 90°$

Measure the current

freq	mag(I_Probe1.i)	mag(I_Probe2.i)
1.000 GHz	4.000E8	1.225E-18
	Zin = 0	Zin = inf

Real Work Example

Use transmission line to perform impedance transform

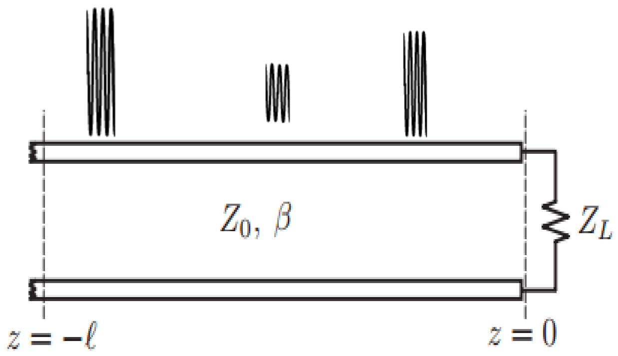


World's First 60 GHz CMOS Amplifier, ISSCC 2004, **Technology Directions Award**

C. H. Doan, S. Emami, A. M. Niknejad, and R. W. Brodersen

Use VSWR Measurement to Find the Unknown Load Impedance

In old days, people measured the maximum and minimum voltage magnitude on a T-line to find the load impedance (at a specific frequency)



$$v(z) = V^+\left(e^{-j\beta z} + \rho_L e^{j\beta z}\right)$$

$$|v(z)| = \left|V^+\left(e^{-j\beta z} + \rho_L e^{j\beta z}\right)\right| = |V^+|\left|e^{-j\beta z}\right|\left|1 + \rho_L e^{2j\beta z}\right|$$

$$= |V^+|\left|1 + \rho_L e^{2j\beta z}\right|$$

$\rho_L = (Z_L - Z_0)/(Z_L + Z_0)$: If we find ρ_L then we know Z_L

What is the maximum voltage swing on the line? $|V^+|*(1+|\rho_L|)$

What is the minimum voltage swing on the line? $|V^+|*(1-|\rho_L|)$

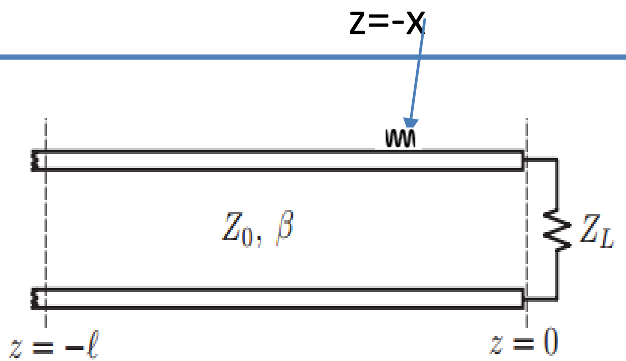
$|\rho_L|$ can be solved

VSWR = maximum swing/minimum swing = $(1+|\rho_L|) / (1-|\rho_L|)$

We know roughly if Z_L is close to Z_0 or not, but more information needed to solve ρ_L or Z_L

Use VSWR Measurement to Find the Unknown Load Impedance

Assume we got the
minimum voltage swing at z =-x



$$v(z) = V^+\left(e^{-j\beta z} + \rho_L e^{j\beta z}\right)$$

$$\left|v(z)\right| = \left|V^+\left(e^{-j\beta z} + \rho_L e^{j\beta z}\right)\right| = \left|V^+\right|\left|e^{-j\beta z}\right|\left|1 + \rho_L e^{2j\beta z}\right|$$

$$= \left|V^+\right|\left|1 + \rho_L e^{2j\beta z}\right|$$

If the minimum voltage swing happens at z=-x

➡ $|V^+|{*}|\ 1 + \rho_L e^{j2\beta z}|$ has a minimum at z=-x

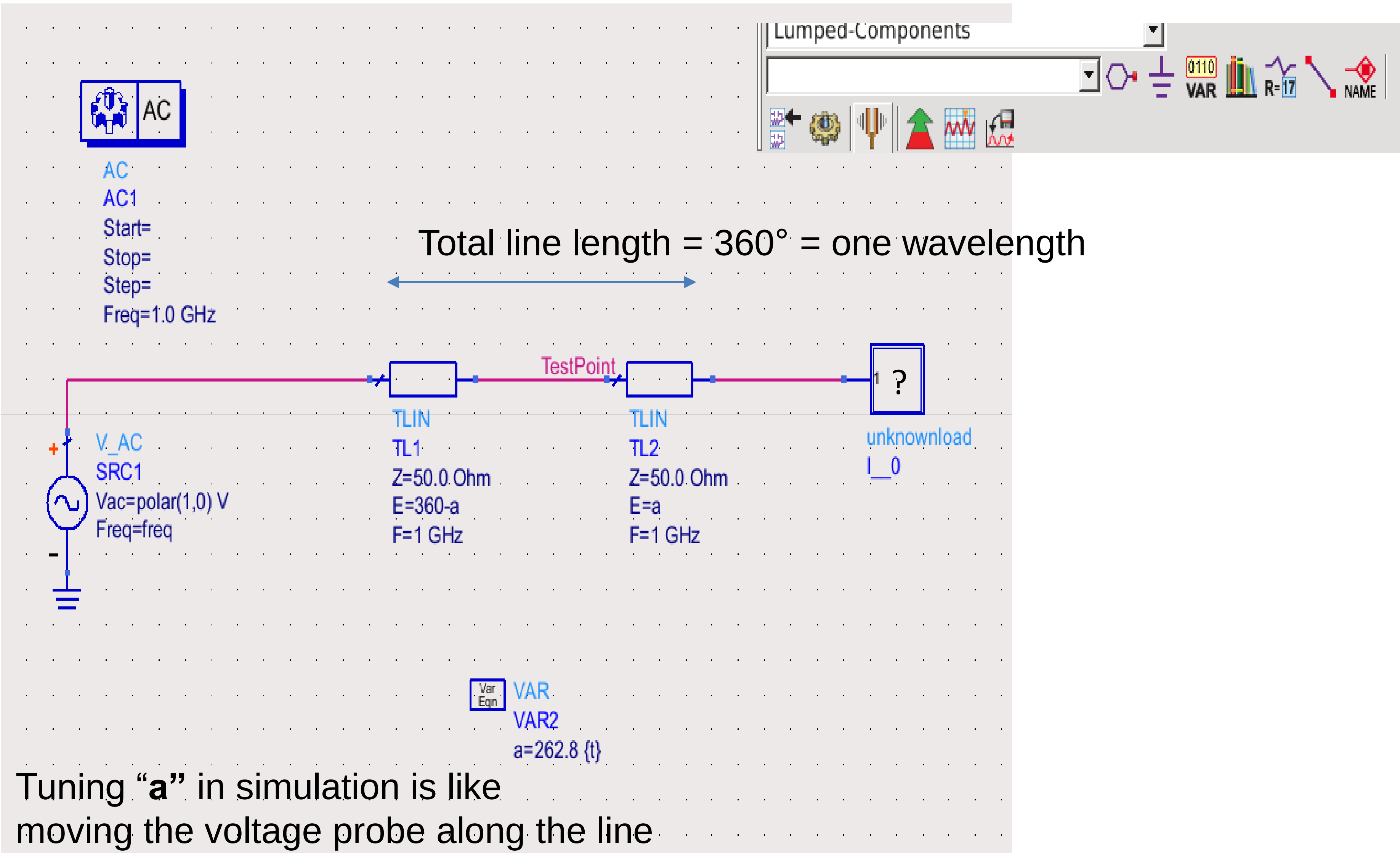
➡ $\rho_L e^{-j2\beta x} = -|\rho_L|$

Recall that $|\rho_L|$ has already been solved

➡ ρ_L can be solved by $\boldsymbol{\rho_L = -|\rho_L| e^{j2\beta x}}$

➡ Z_L can be solved

Conducted in ADS Simulation

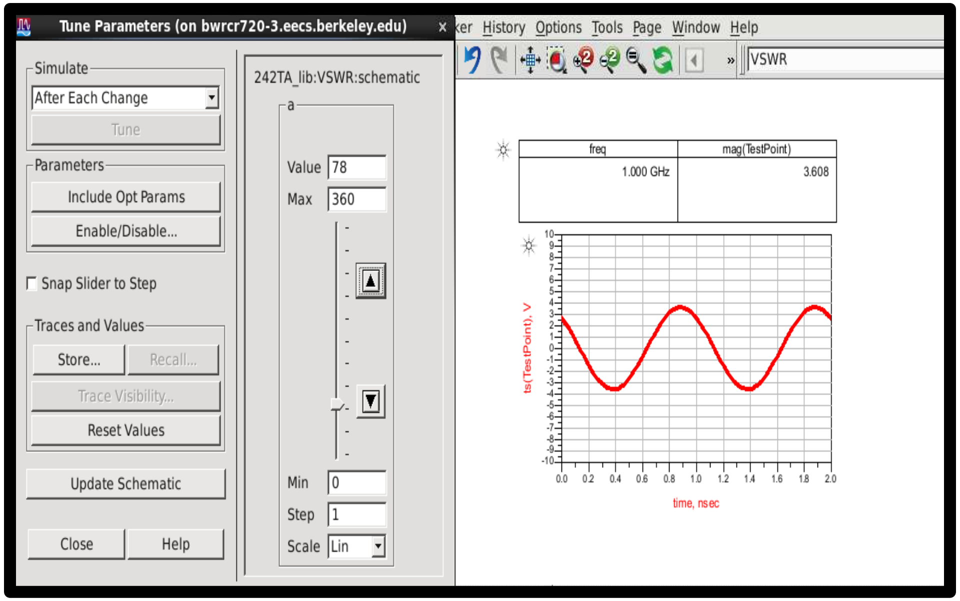


Total line length = 360° = one wavelength

Tuning “**a**” in simulation is like
moving the voltage probe along the line

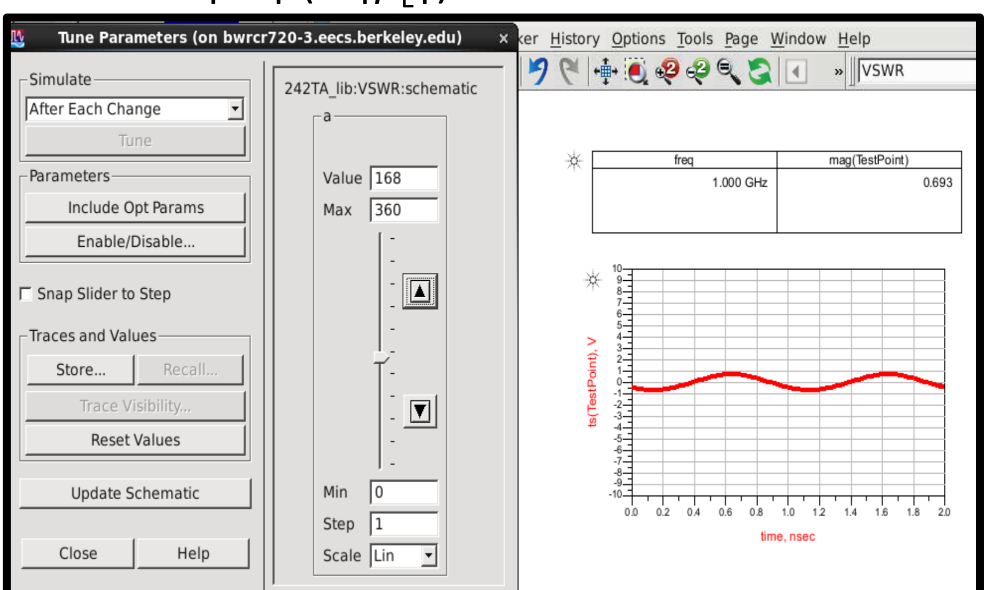
Found the maximum voltage swing at **a=78°**

$|V^+|\ (1+|\rho_L|) = 3.608$



Found the minimum voltage swing at **a=168°**

$|V^+|\ (1-|\rho_L|) = 0.693$



VSWR =
maximum swing/minimum swing =
$\mathbf{(1+|\rho_L|)/(1-|\rho_L|)}$

$3.608 / 0.693 = 5.21 = \mathbf{(1+|\rho_L|) / (1-|\rho_L|)}$

$\Rightarrow \quad |\rho_L| = 0.678$

The minimum voltage swing was found at **a**=168°

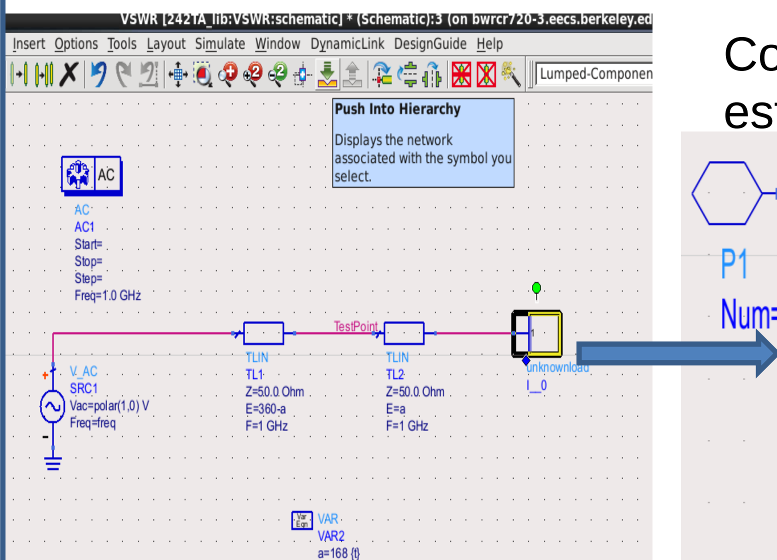
a=168° => $x = (\lambda/360°)168°$ => $\beta x = 2\pi/\lambda * x = 2.932$

$$\rho_L = -|\rho_L| e^{j2\beta x} = -0.678\, e^{j2*2.932} = -0.619 + j0.276$$

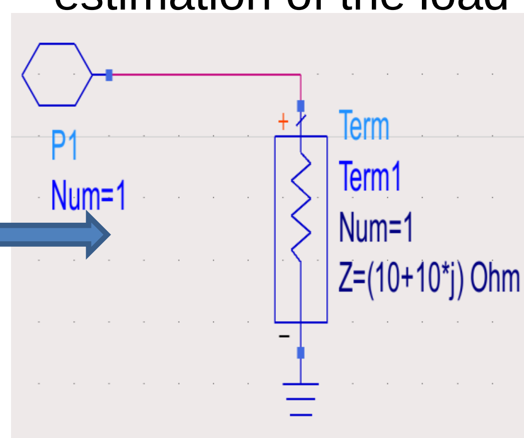
$\rho_L = (Z_L - Z_0)/(Z_L + Z_0)$: If we find ρ_L then we know Z_L by $Z_L = Z_0(1+\rho_L)/(1-\rho_L)$

$$Z_L = 50 * [1 + (-0.619 + j0.276)] / [1 - (-0.619 + j0.276)] = 10.0 + 10.2j \text{ ohm}$$

Click on the unknown load block and click on "push into hierarchy"



Cool, we have a good estimation of the load

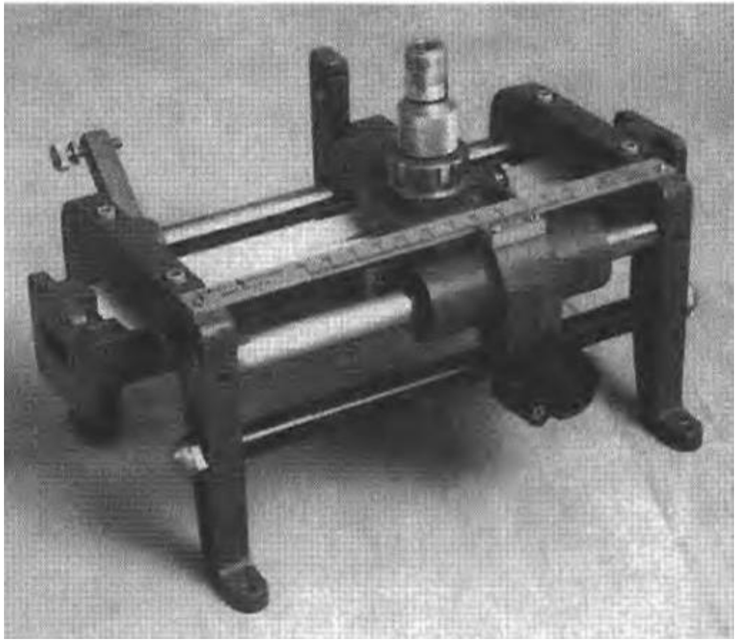


Practice

- Change the load impedance and run the procedure again
- How about using the maximum voltage swing to find ρ_L
- How to modify the procedure if the T-line has $Z_0 = 60$?

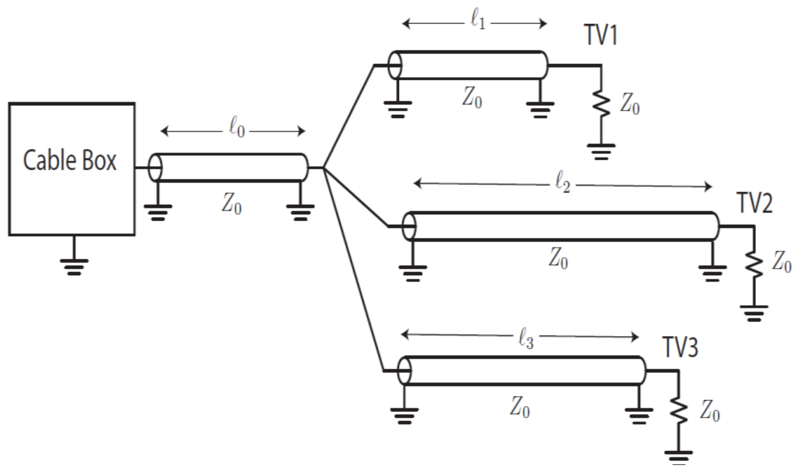
Advantages of the VSWR Method

- (1) Do not have to measure the absolute magnitude accurately. Only the ratio is need!
- (2) The location measurement in the second step can be very accurate



HW2.5 is on using the VSWR method to find a unknown impedance

HW2.6: Transmission line impedance transformation



(a) Suppose $\ell_0 = 1.22$m, $\ell_1 = 2.59$m, $\ell_2 = 11.23$m, and $\ell_3 = 33.85$m. All cables are 75Ω and have a velocity of propagation of 1×10^8m/s. If the TV is tuned to channel 14, approximately at 473 MHz, what is the impedance seen by the cable box?

λ = v/f = 10^8/473 MHz = 0.21142m

l_0 =1.22m = 5.77 λ

$\boldsymbol{\rho_L}$ for the trace l_0 is -0.5 (Why ?)

$Z_{in} = Z_0 * (1 + \rho_L e^{-j2\beta l}) / (1 - \rho_L e^{-j2\beta l})$

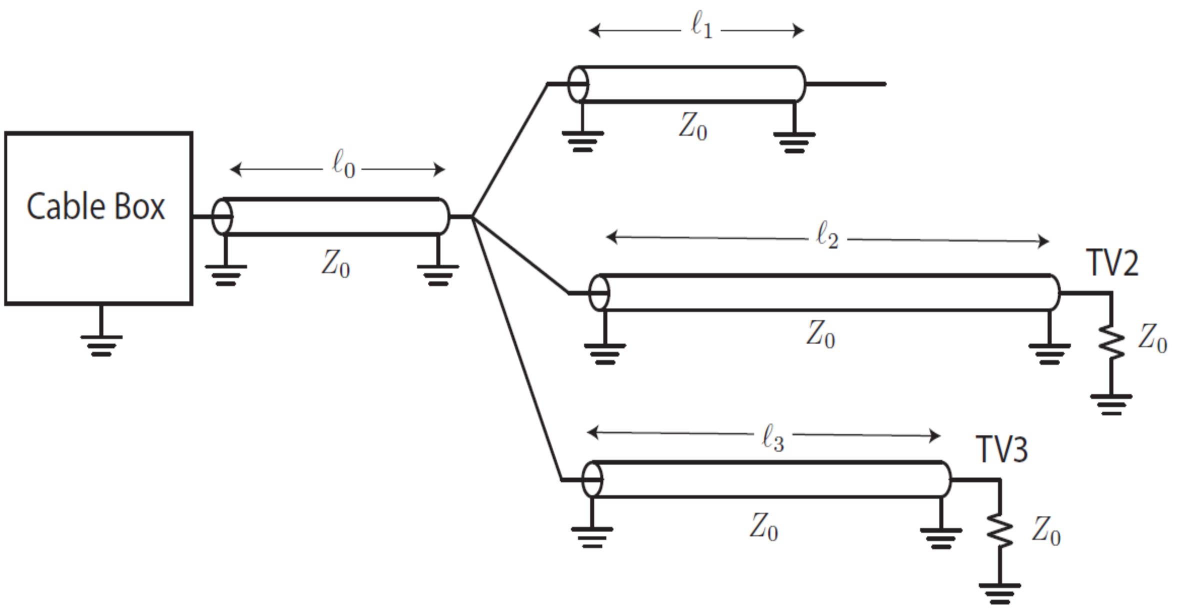
$\beta l = 2\pi/\lambda * (5.77\lambda)$

(b) Assume the Cable Box puts out a signal at -30 dBm. How much power reaches each television set?

-35 dBm (each TV gets one-third of power)

What is the unit dBm?

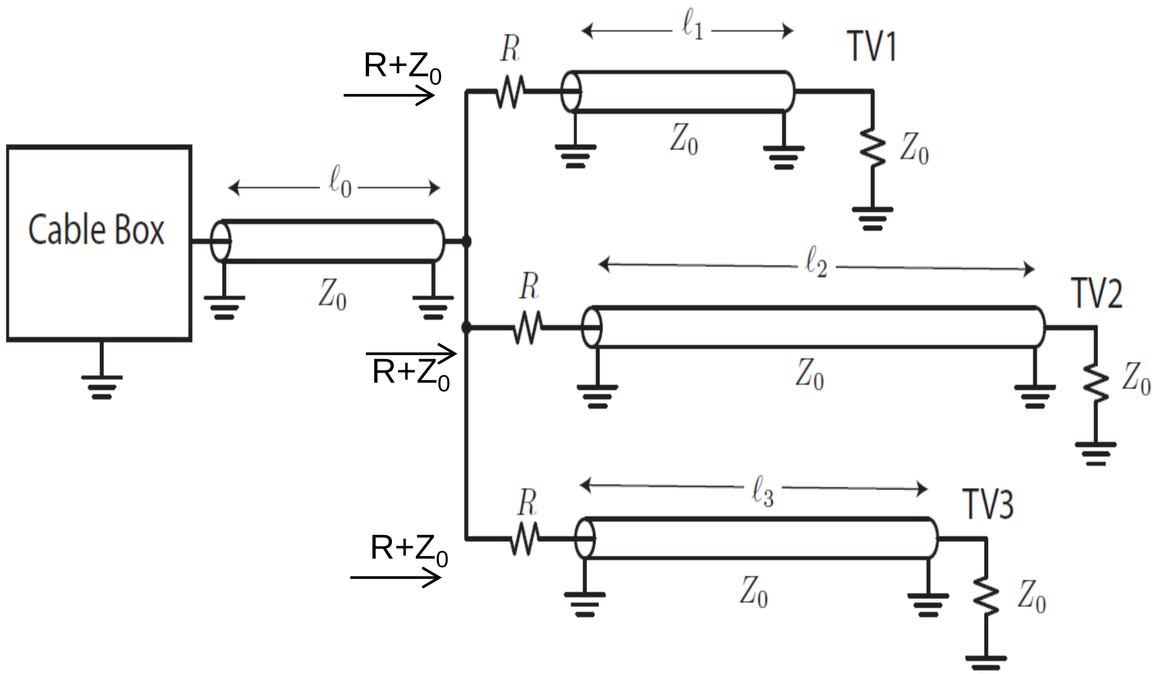
(c) It is discovered that the system performance gets much worse when the first TV is unplugged, as shown. Explain what is happening. How much power reaches TV2 and TV3?



λ = v/f = 10^8/473 MHz = 0.21142m

l_1 =2.59m = 12.25λ, so βl_1= 2π * 12.25 (an odd number × a quarter wavelength)

(d) After explaining the concept of impedance matching, your friends come up with the following solution. Does it work? What is the impedance seen by the Cable Box? How much power reaches each TV set?



Each TV branch gets -35 dBm, if the cable box still outputs -30 dBm

Actual powers into the TVs are lower than -35 dBm. Need to scale by $[Z_0/(Z_0+R)]$

With TV1 disconnected, TV2 and TV3 both get a portion of the power outputted by the cable box

$$[(Z_0+R)R] \,/\, [(Z_0+R)R + (Z_0+R)R + (Z_0+R)(Z_0+R)]$$