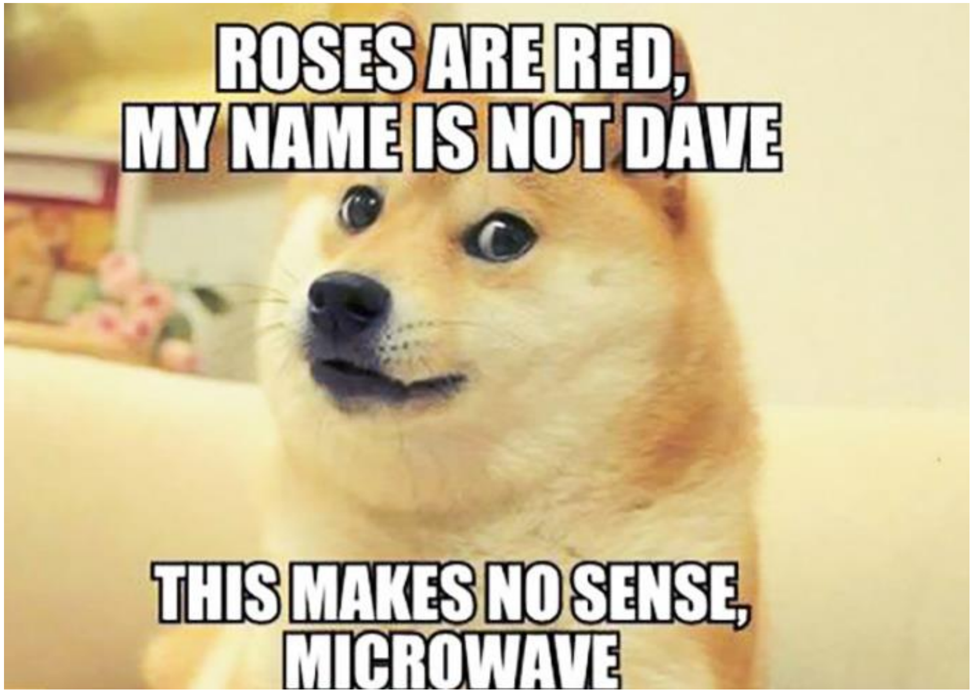
Today's Agenda (09272017)

- HW4#1. More *S*-parameter Questions
- HW4#2,3. Simple Smith Chart Manipulation
- HW4#4(d)(e) Smith Chart Matching Examples



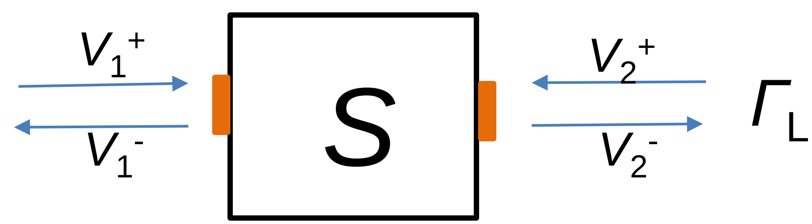
1. Calculate the scattering parameters of the following circuits:

(a) Find the input S_{11} for a general two-port terminated at port 2 with a load reflection coefficient of Γ_L.

Assume the two-port is described by a *S*-parameter with reference Z_0
Assume the required input S_{11} is with reference Z_0
Assume the load reflection Γ_L is with reference Z_0

$$\begin{pmatrix} \frac{V_1^-}{\sqrt{Z_1}} \\ \frac{V_2^-}{\sqrt{Z_2}} \\ \frac{V_3^-}{\sqrt{Z_3}} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \begin{pmatrix} \frac{V_1^+}{\sqrt{Z_1}} \\ \frac{V_2^+}{\sqrt{Z_2}} \\ \frac{V_3^+}{\sqrt{Z_3}} \end{pmatrix}$$

$$\begin{pmatrix} V_1^- \\ V_2^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \end{pmatrix}$$



$$V_2^+ = V_2^- \Gamma_L$$

$$V_1^+ = V_2^- (1-S_{22}\Gamma_L)/S_{21}$$

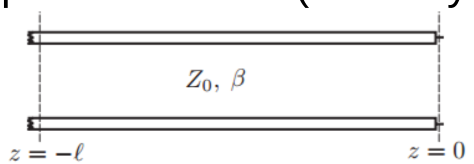
$$V_1^- = V_2^- \{(1-S_{22}\Gamma_L)S_{11}/S_{21} + S_{12}\Gamma_L\}$$

$$V_1^-/V_1^+ = \boldsymbol{S_{11} + S_{12}S_{21}\Gamma_L/(1-S_{22}\Gamma_L)}$$

Recall how did we calculate S11 of a two port

S-Parameter and Power

In phasor domain (ac analysis)



Recall what can exist on a cable

$$v(z) = V^+ e^{-\gamma z} + V^- e^{\gamma z}$$

$$i(z) = \frac{V^+}{Z_0} e^{-\gamma z} - \frac{V^-}{Z_0} e^{\gamma z}$$

$$\gamma = j\beta = j2\pi/\lambda$$

Power sent:

P_{in} = 0.5*Real{ $v(z)$ $i(z)^*$}

= 0.5*Real{
$|V^+|^2/Z_0$ - $|V^-|^2/Z_0$ + $V^-(V^{+*})\, e^{2\gamma z}/Z_0$ - $V^+(V^{-*})\, e^{-2\gamma z}/Z_0$
}

= $\mathbf{|V^+|^2/2Z_0}$ **-** $\mathbf{|V^-|^2/2Z_0}$

Power Sent = Forward-wave power - Backward-wave power

$P_{in} = [|V_1^+|^2 - |V_1^-|^2]\ \boldsymbol{/2Z_0}$

$P_{load} = [|V_2^-|^2 - |V_2^+|^2]\ \boldsymbol{/2Z_0} = |V_2^-|^2 [\ 1 - |\Gamma_L|^2\ ]\ \boldsymbol{/2Z_0}$

$\textit{Power Gain} = P_{load}/P_{in} = |S_{21}/(1-S_{22}\Gamma_L)|^2 \times [1 - |\Gamma_L|^2\,] \;/\; [1 - |S_{11} + S_{12}S_{21}\Gamma_L/(1-S_{22}\Gamma_L)|^2\,]$

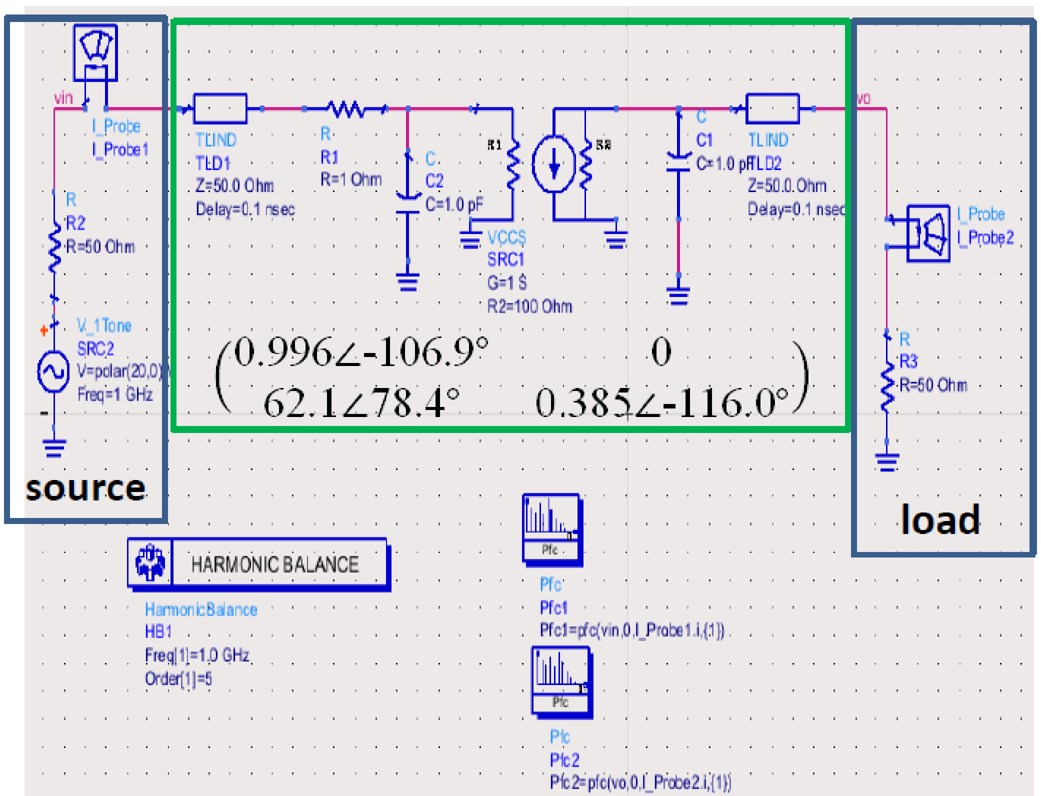
Remarks:

1. P_{avs} of a source with $Z_s = Z_0$ is $|V_1^+|^2/2Z_0$ (explained later)
2. If $\Gamma_L = 0$, $P_{load}/P_{in} = |S_{21}|^2 / (1 - |S_{11}|^2)$ (power gain)
3. If $\Gamma_L = 0$, $P_{load} = |V_2^-|^2 / 2Z_0$
4. If $Z_s = Z_0$ and $\Gamma_L = 0$, $P_{load} = P_{avs}|S_{21}|^2$ (using 1 and 3)
5. If $Z_s = Z_0$ and $\Gamma_L = 0$, $P_{in} = P_{avs}(1 - |S_{11}|^2)$ (using 1 and 4)

Old Slides

Why S-Parameters ? (B)

S-parameter has a straightforward connection to the power flow in a microwave system



- In microwave systems, load and source impedances are usually 50Ω (why?)
- In this case, the source can provide maximum power of 1 W

  P_{avs}

  Power to the load: P_{load}

  1*$|S_{21}|^2$= 3856
- Power into the circuit: P_{load}

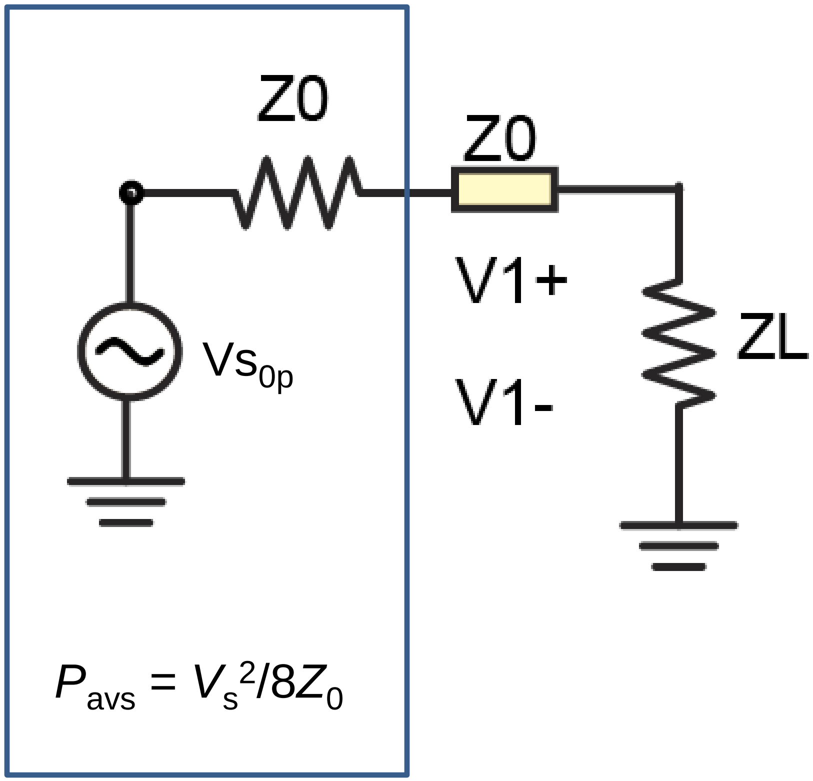
  1*(1-$|S_{11}|^2$)=1 - 0.9964^2 =0.0072

(S_{11} = 0.996 indicates most power from the source is reflected)

- **Matching:** Create a 50-Ω load impedance for the source so all the 1-W power can be delivered into the circuit

S-parameter and Power

- ***P***



$P_{avs} = V_s^2/8Z_0$

$V_1^- = V_1^+ \Gamma_L$

$V_1^+ + V_1^- = V_s - Z_0(V_1^+/Z_0 - V_1^-/Z_0)$

$\Rightarrow V_1^+ = V_s/2$

$\boldsymbol{\Rightarrow P_{avs} = |V_1^+|^2/2Z_0}$

$$\boldsymbol{P_{in} = |V_1^+|^2/2Z_0 - |V_1^-|^2/2Z_0}$$
$$\boldsymbol{= P_{avs}(1 - |V_1^- / V_1^+|^2)}$$
$$\boldsymbol{= P_{avs}(1 - |\Gamma_L|^2)}$$

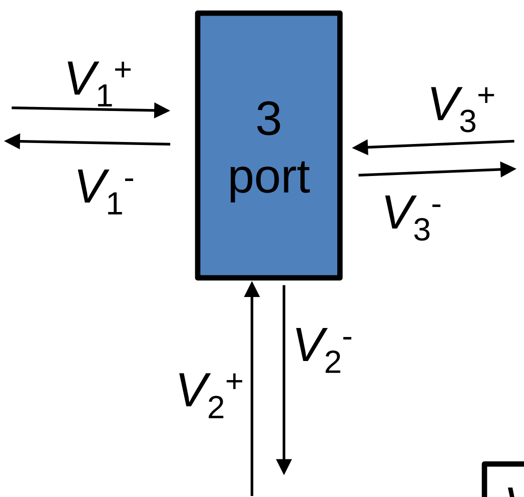
1. Calculate the scattering parameters of the following circuits:

   (a) Find the input S_{11} for a general two-port terminated at port 2 with a load reflection coefficient of Γ_L. (see page 2)

   (b) In the previous problem, what is the power that reaches the load in terms of the two-port scattering parameters and Γ_L? Suppose the input is driven with a matched source. ($P_{avs} = P_{in}$)

   (c) Derive the two-port scattering parameters of a three-port where port 3 is terminated in a load with reflection coefficient Γ_L.

Need to find the reduced two-port *S*-parameter



$$\boxed{\begin{aligned} V_1^- &= S_{11}V_1^+ + S_{12}V_2^+ + S_{13}V_3^+ \\ V_2^- &= S_{21}V_1^+ + S_{22}V_2^+ + S_{23}V_3^+ \\ V_3^- &= S_{31}V_1^+ + S_{32}V_2^+ + S_{33}V_3^+ \end{aligned}}$$

$$V_3^+ = V_3^- \Gamma_L$$

$$\boxed{\begin{aligned} V_1^- &= S_{11}V_1^+ + S_{12}V_2^+ + S_{13}V_3^+ \\ V_2^- &= S_{21}V_1^+ + S_{22}V_2^+ + S_{23}V_3^+ \\ 0 &= S_{31}V_1^+ + S_{32}V_2^+ + (S_{33} - 1/\Gamma_L)V_3^+ \end{aligned}}$$

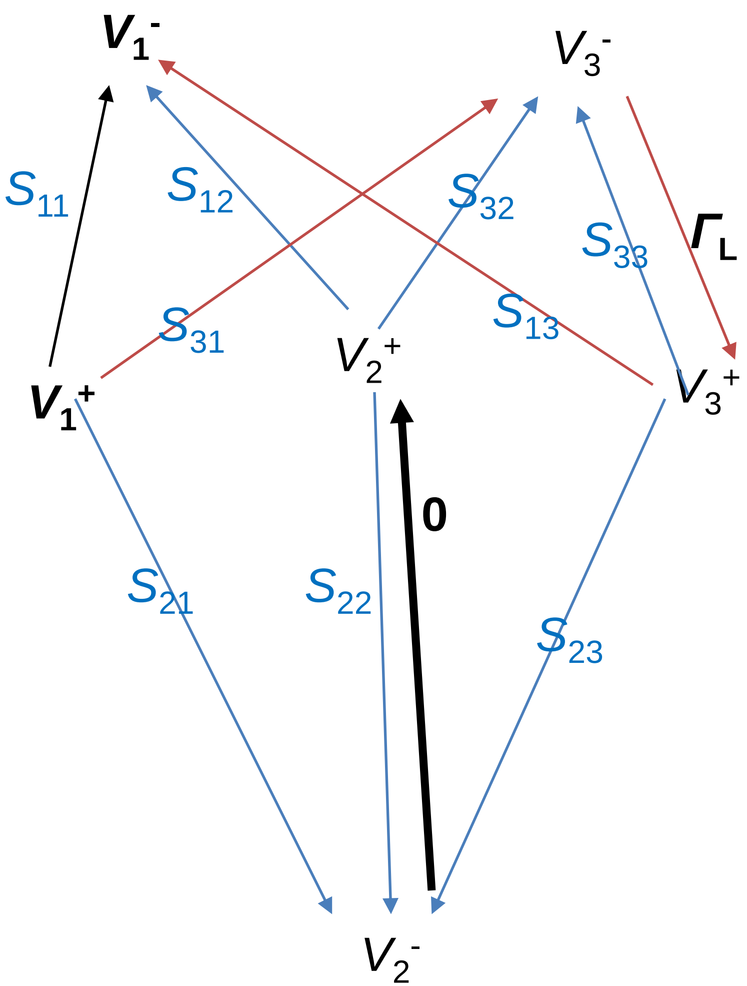
$$\boxed{\begin{aligned} V_1^- &= S_{11}V_1^+ + S_{12}V_2^+ + S_{13}(S_{31}V_1^+ + S_{32}V_2^+)/(1/\Gamma_L - S_{33}) \\ V_2^- &= S_{21}V_1^+ + S_{22}V_2^+ + S_{23}(S_{31}V_1^+ + S_{32}V_2^+)/(1/\Gamma_L - S_{33}) \end{aligned}}$$

$$S_{11\_NEW} = S_{11} + S_{13}S_{31}/(\Gamma_L^{-1} - S_{33})$$

Can $\Gamma_L^{-1} = S_{33}$?

Signal Flow Approach

A systematic method to calculate



$$\boxed{\begin{aligned} V_1^- &= S_{11}V_1^+ + S_{12}V_2^+ + S_{13}V_3^+ \\ V_2^- &= S_{21}V_1^+ + S_{22}V_2^+ + S_{23}V_3^+ \\ V_3^- &= S_{31}V_1^+ + S_{32}V_2^+ + S_{33}V_3^+ \end{aligned}}$$

$$V_3^- = V_3^+\Gamma_L$$

- Add the arrow from b3 to a3 with RL
- Since $S_{11\_new} = V_1^-/V_1^+$ with $V_2^+ = 0$, add an arrow from V_2^- to V_2^+ with 0
- Find the close loops

$$S_{11\_new} = V_1^-/V_1^+ = N/D$$

$$N = S_{11}(1- S_{33}\Gamma_L) + S_{31}\Gamma_L S_{13}(1)$$

$$D = (1- S_{33}\Gamma_L)$$

$$\Rightarrow \boldsymbol{S_{11\_new}} = S_{11} + S_{31}\Gamma_L S_{13}/(1- S_{33}\Gamma_L)$$

Mason's gain formula (see Wiki)

Signal Flow Approach

Formula [edit]

The gain formula is as follows:

$$G = \frac{y_{\text{out}}}{y_{\text{in}}} = \frac{\sum_{k=1}^{N} G_k \Delta_k}{\Delta}$$

$$\Delta = 1 - \sum L_i + \sum L_i L_j - \sum L_i L_j L_k + \cdots + (-1)^m \sum \cdots + \cdots$$

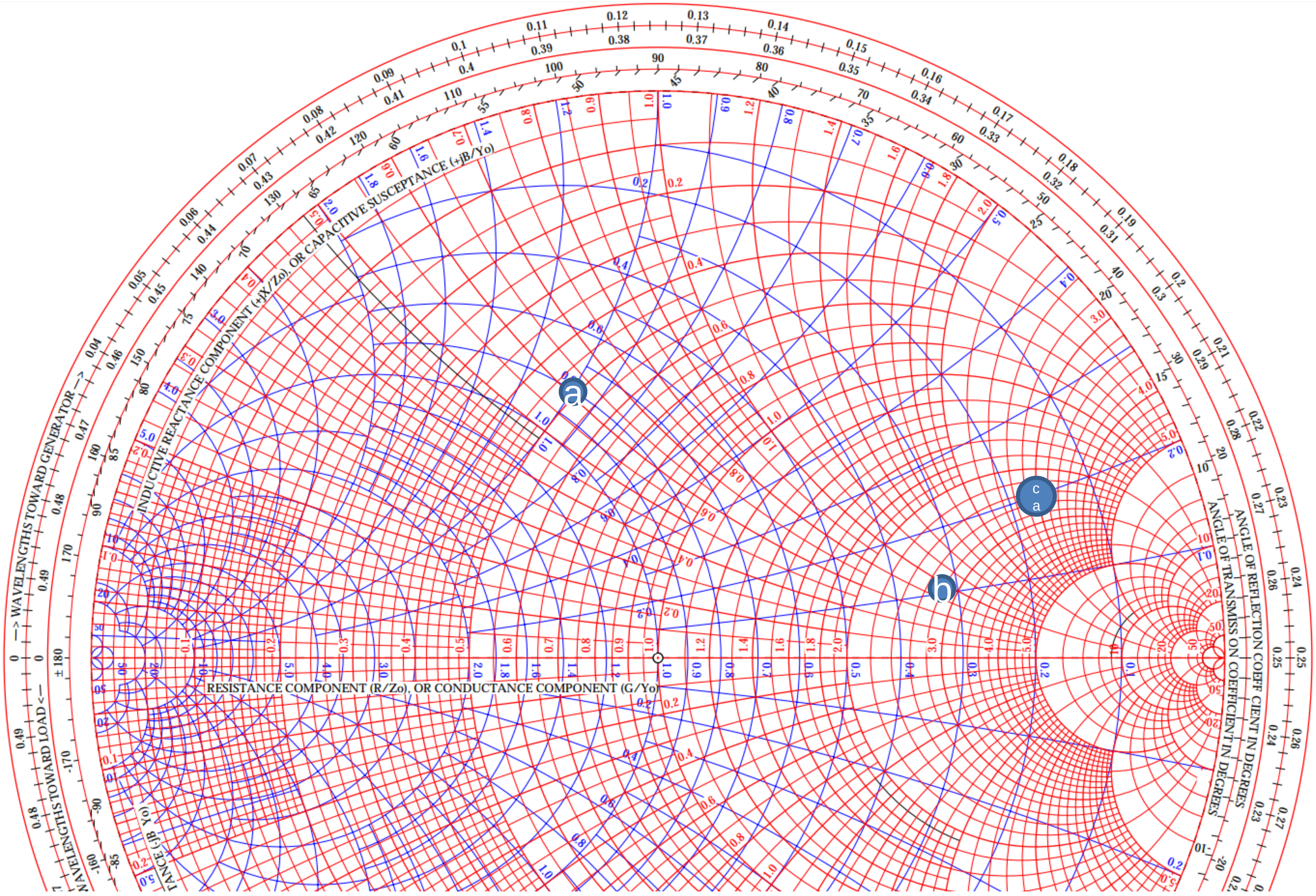
where:

- Δ = the determinant of the graph.
- y_{in} = input-node variable
- y_{out} = output-node variable
- G = complete gain between y_{in} and y_{out}
- N = total number of forward paths between y_{in} and y_{out}
- G_k = path gain of the kth forward path between y_{in} and y_{out}
- L_i = loop gain of each closed loop in the system
- L_iL_j = product of the loop gains of any two non-touching loops (no common nodes)
- $L_iL_jL_k$ = product of the loop gains of any three pairwise nontouching loops
- Δ_k = the cofactor value of Δ for the k^{th} forward path, with the loops touching the k^{th} forward path removed. *

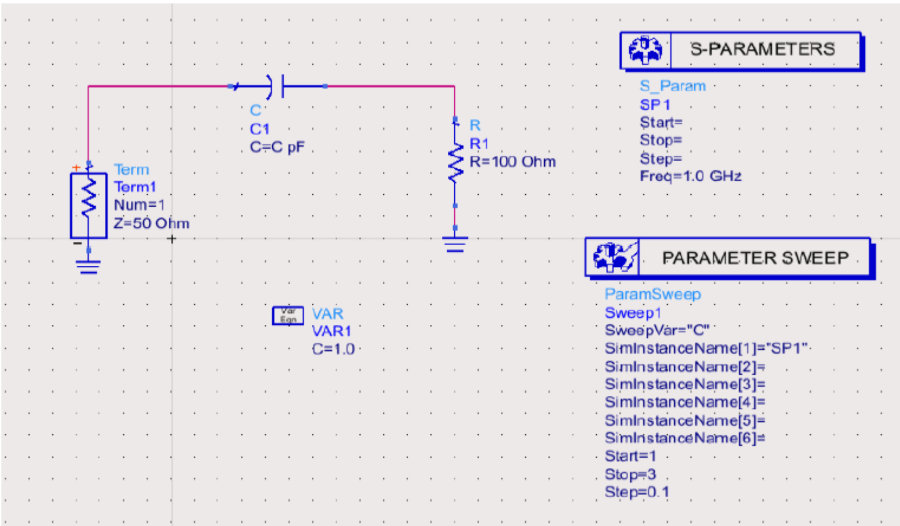
2. (a) Assume $Z_0 = 50\Omega$. Using Smith Chart to find ρ_L for a load impedance $Z_L = 25 + 30j\Omega$. Use Z chart: Z'=0.5+0.6j

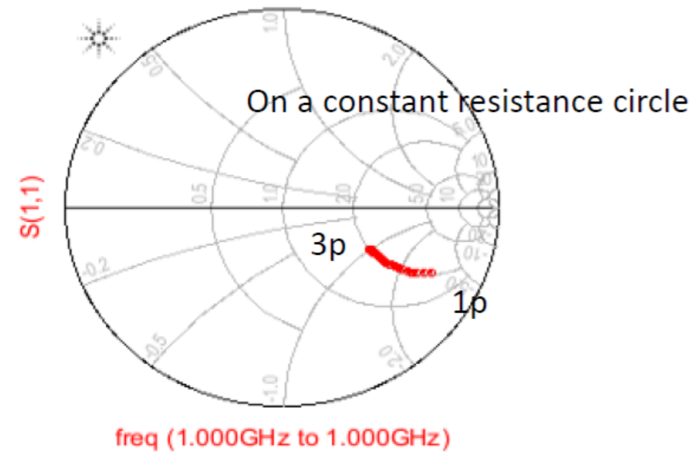
(b) Assume $Z_0 = 50\Omega$. Using Smith Chart to find the load impedance for $\rho_L = 0.5 + 0.1j$.

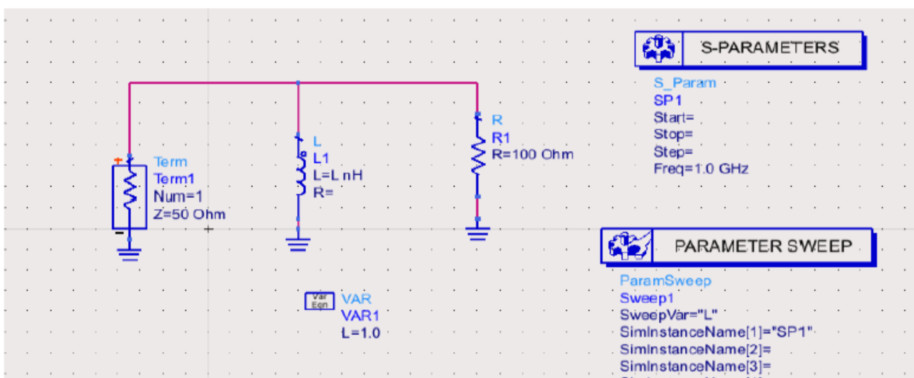
(c) Repeat (a) and (b) with Z_0 changed to 10Ω.

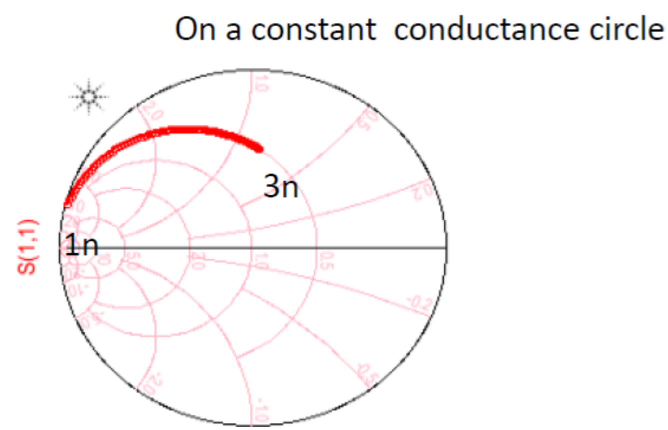


(d) For the following two circuits, trace ρ_L on a Smith Chart with $Z_0 = 50\Omega$.







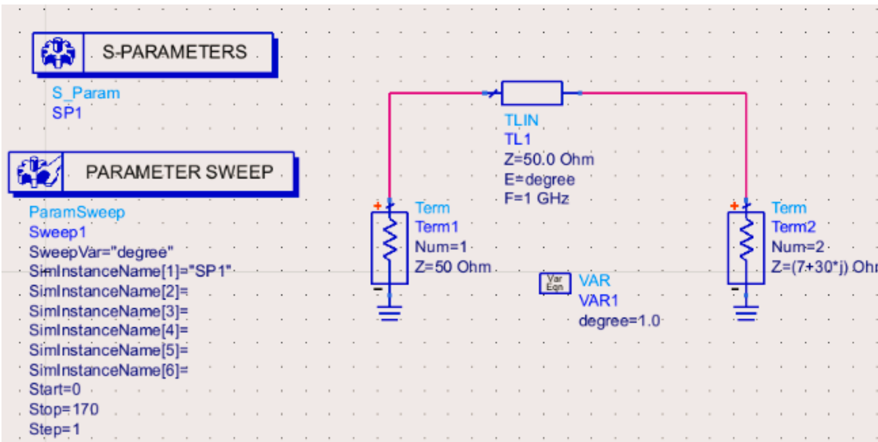


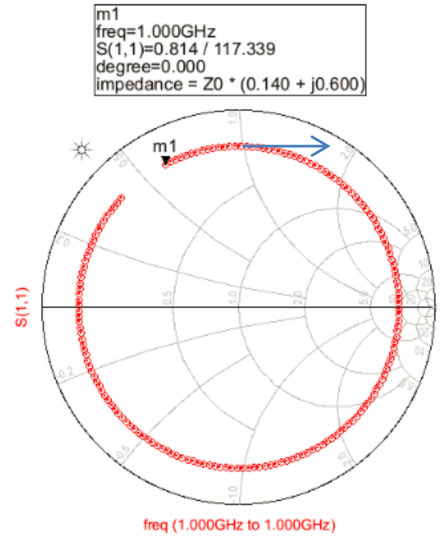
- R=100 circle and G=0.01 circle have one intersection
- All constant resistance circles pass 1 and all constant conductance circles pass -1

Series and Parallel Transmission line

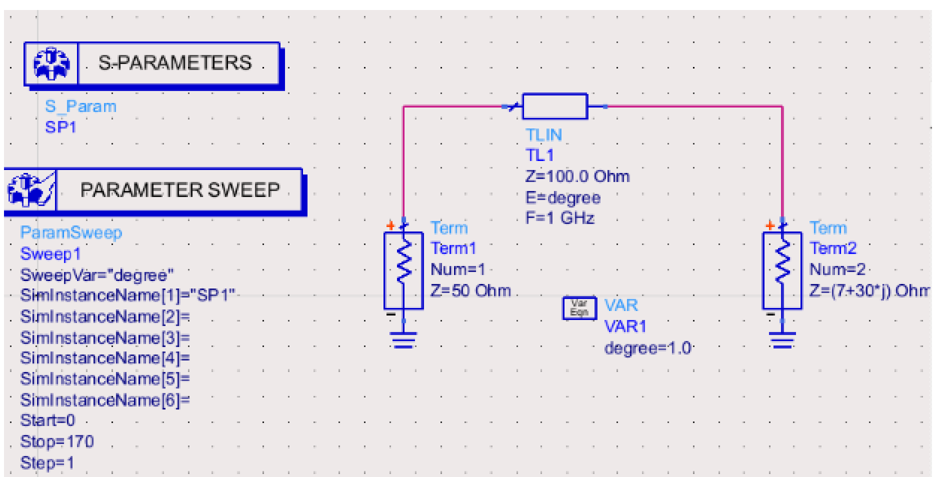
T-line char. impedance = Z_0

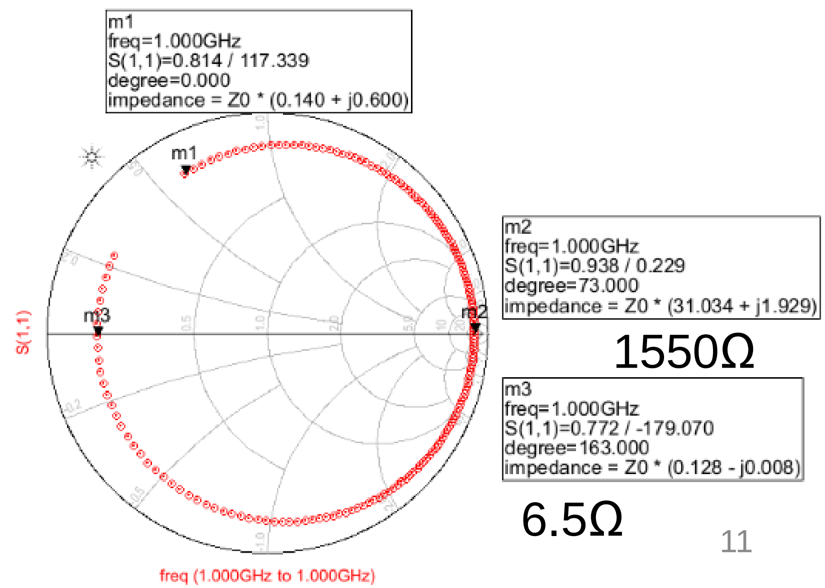
$$\Gamma_{in} = \Gamma_{L,Z0} \times \exp(-2j\beta l)$$





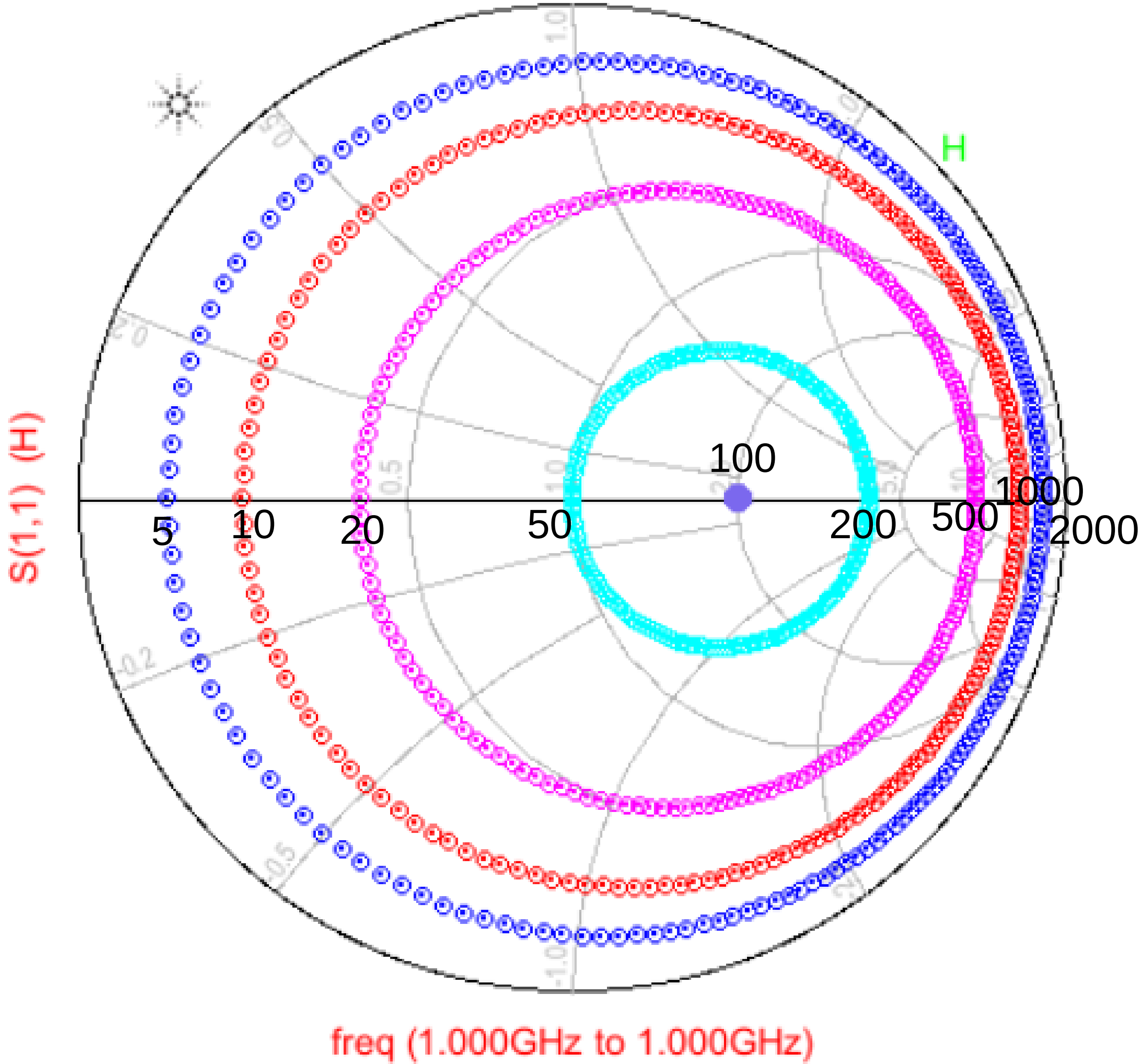
Same magnitude clockwise





1550Ω

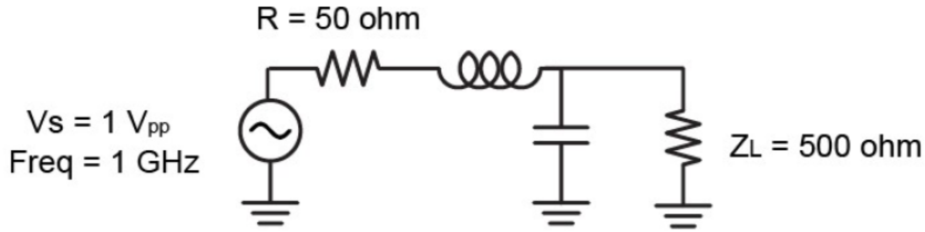
6.5Ω



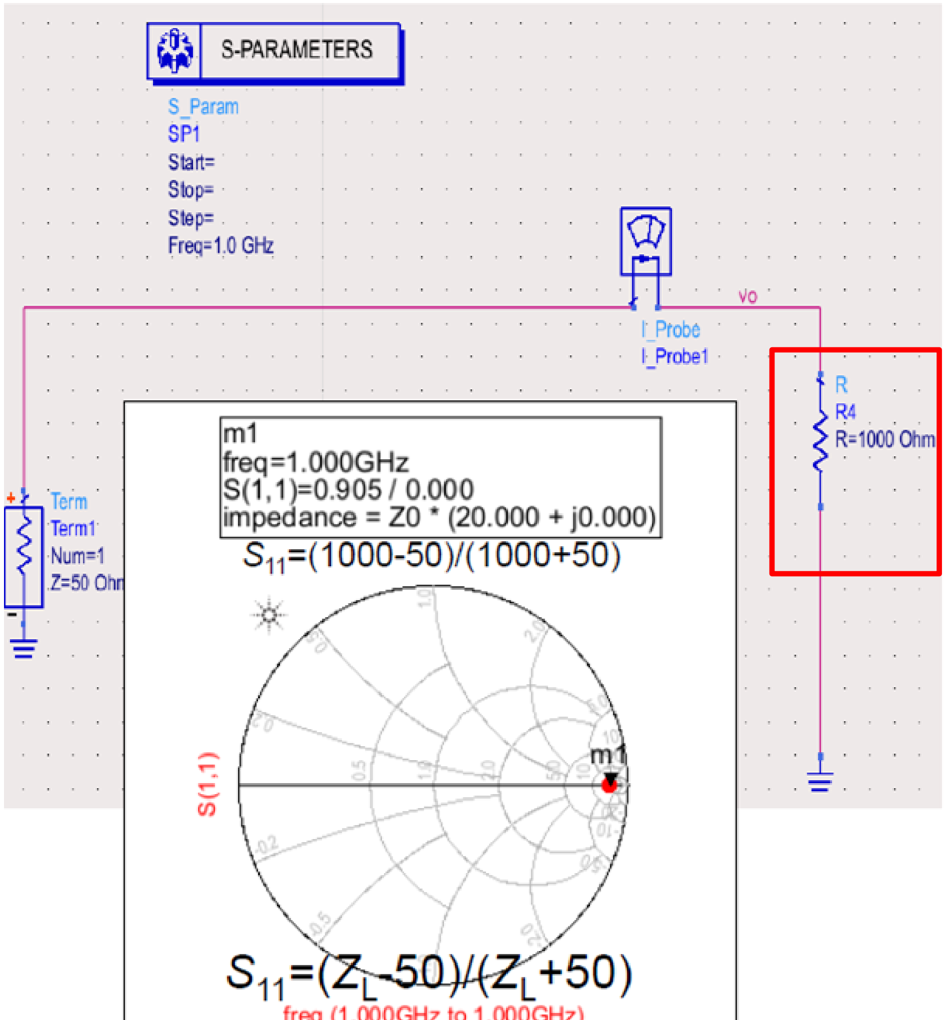
50Ω Smith Chart
100Ω series T-line

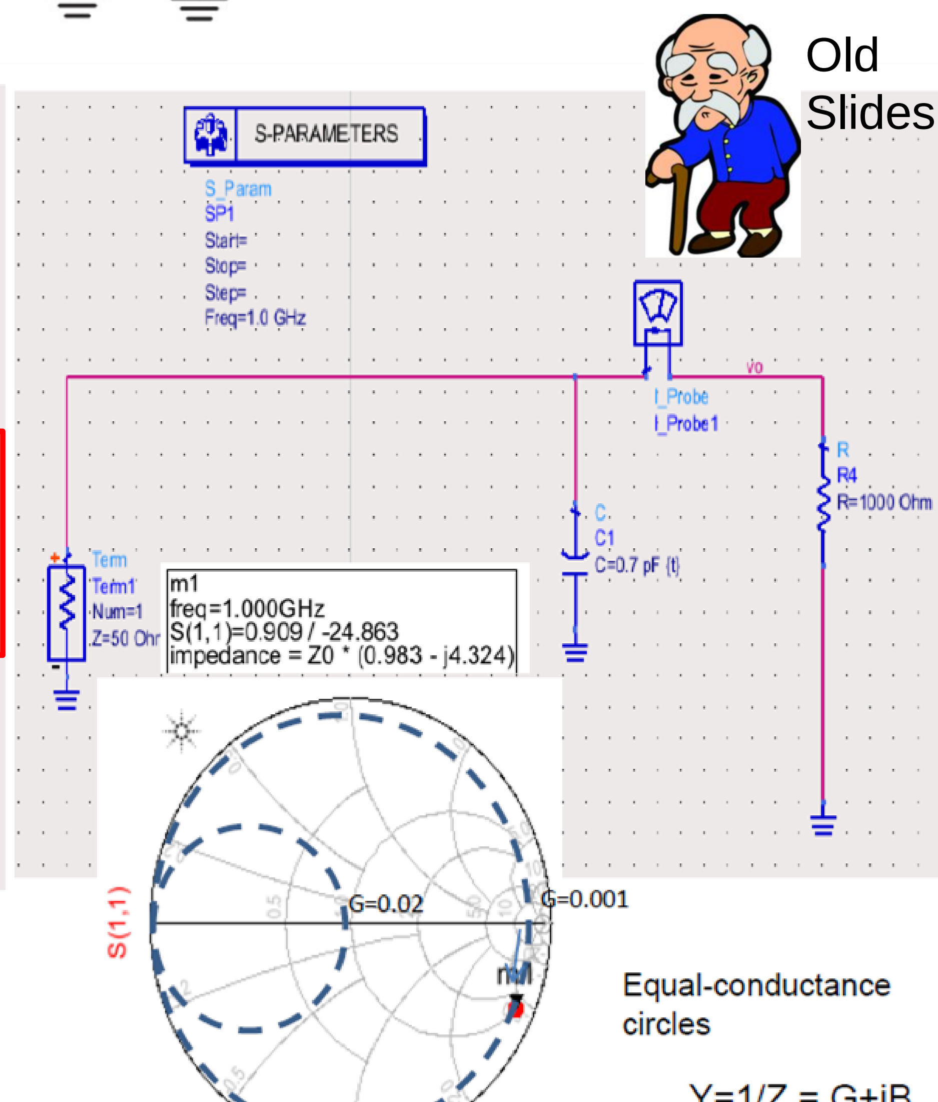
- The S11 trajectories are also circles
- The accurate position is difficult to find using Smith Chart

HW4.4(d)



Old Slides

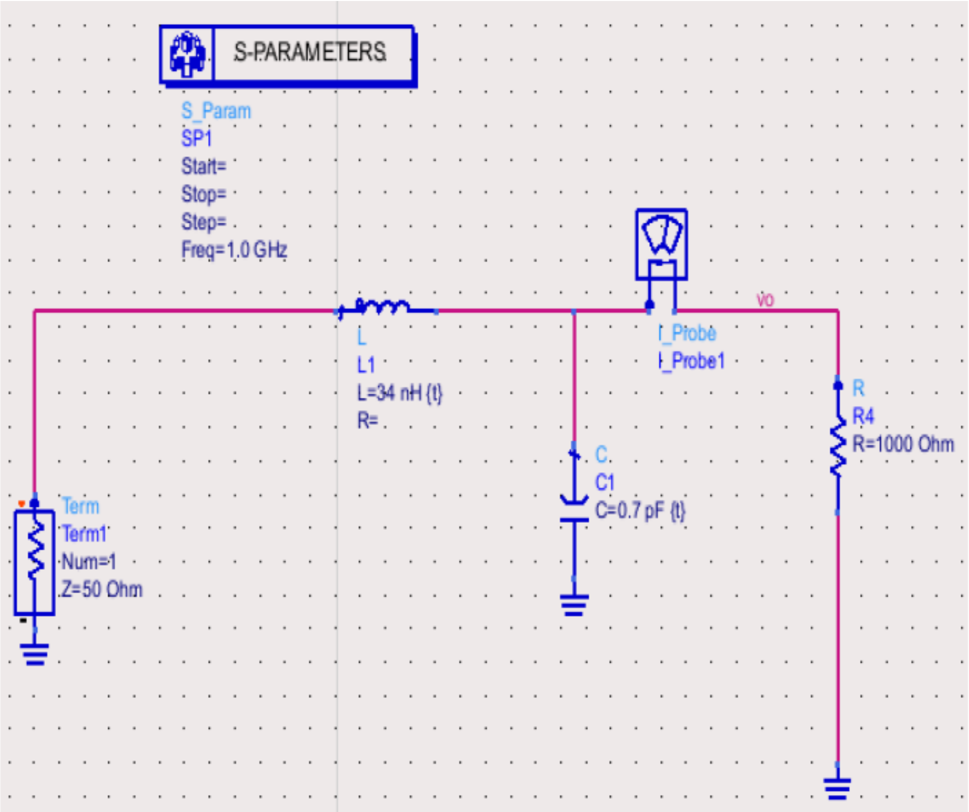


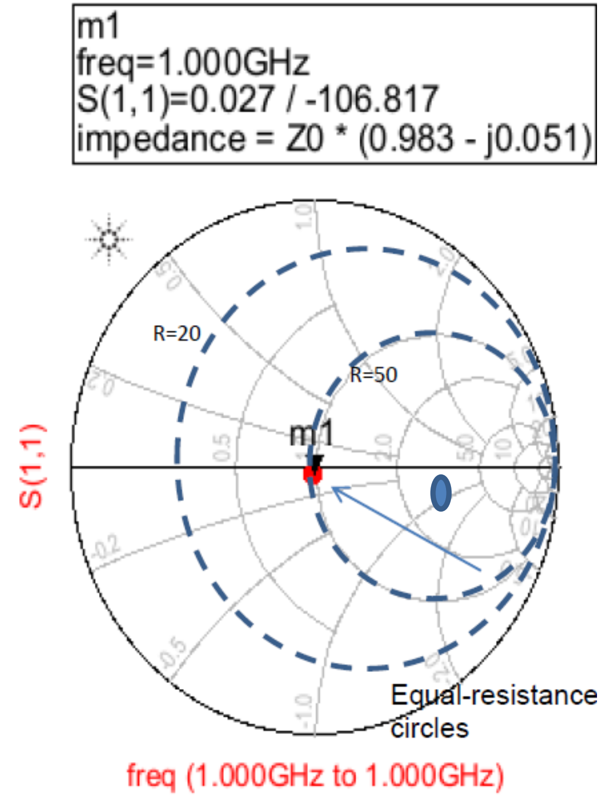


Equal-conductance circles

Y=1/Z = G+jB

Old Slides

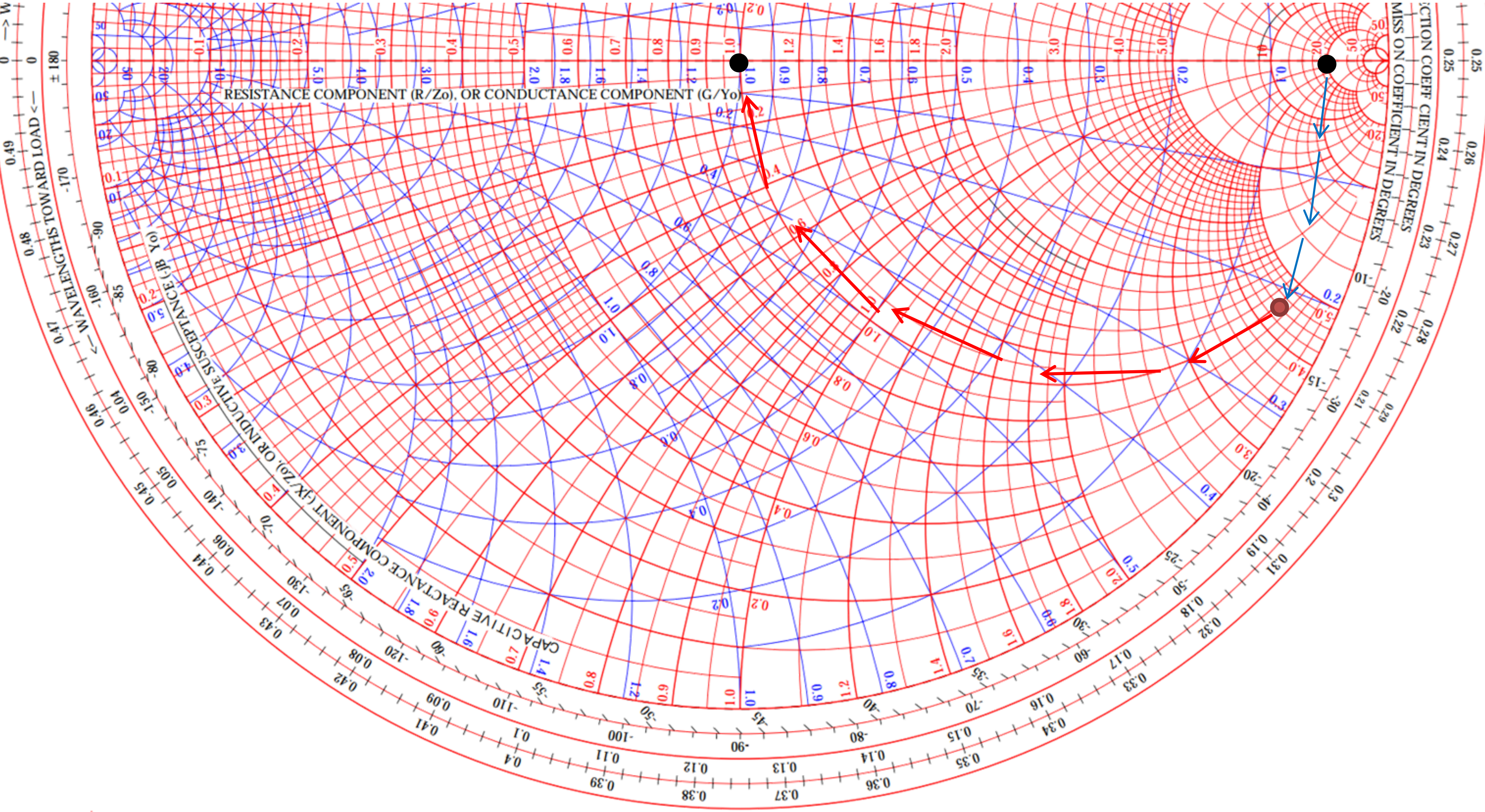




- Shunt-series matching
- First step: Move S11 on Smith Chart (along the equal-conductance circle) to a point on the equal-resistance circle with R = 50
- Second step: Move S11 on Smith Chart (along the equal-resistance circle) to S11 =0
- The shunt component can be an inductor, but then the series one must be a capacitor

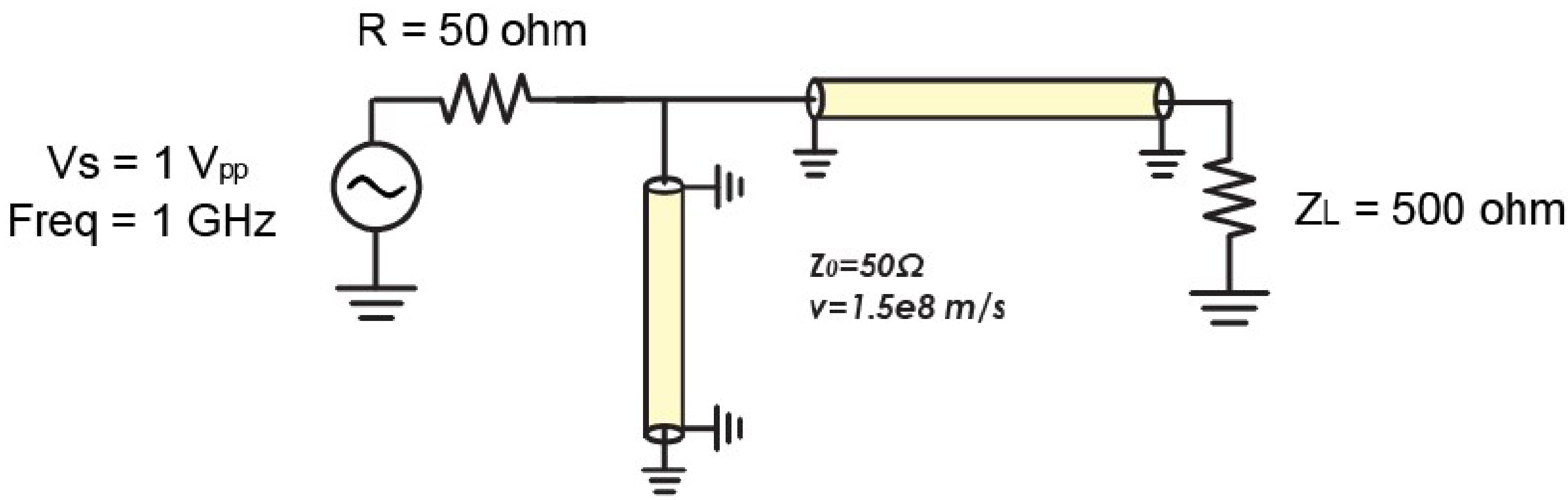
Can we put a series component first ???

Determine the Component Values



First Step: Add $B' \approx 0.22j$ => $B = 0.22j{*}Y_0 = 0.0044j$ S => $2\pi fC = 0.0044$ => $C = 0.70$ pF

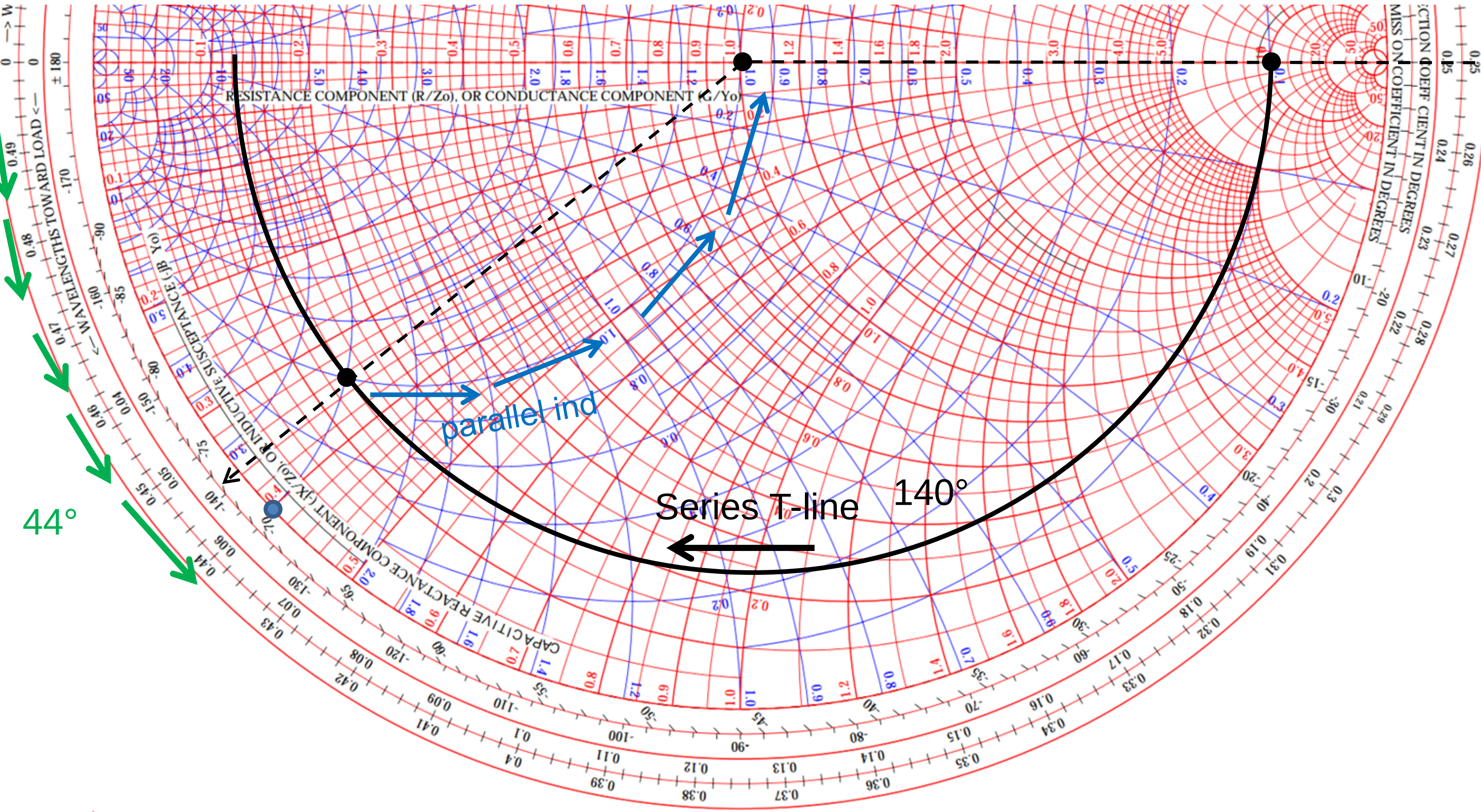
Second Step: $X' \approx -4.5j$ => $X = -4.5j{*}Z_0 = -225j\ \Omega$ => $2\pi fL = 225$ => $L = 35$ nH



(f) (*242A only*) Alternatively, the impedance matching network can be designed by using transmission lines, as illustrated above. What are the length of the two lines?

(g) (*242A only*) Repeat (f) with Z_0 of the transmission lines changed to 100Ω.

Suggestion: Use Smith Chart with Z_0 = 100 is more convenient (Z_L' = 5 to Z_{in}' = 0.5)



First Step: Add series T-line; travel 140° on Smith Chart => line length ≈ 140/180 * 0.25 wavelength

Second Step: *B'* ≈ 2.5*j* so we have to add a parallel inductor with *B'* ≈ -2.5*j* *or X' = 0.4j*

=> short-circuited line length ≈ 44/180 * 0.25 wavelength