EE242-Problem Set 5

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In this homework, reflection coefficient is expressed as ρ or Γ.

1

z_L and y are symmetric about the origin. So we can find the z_L on the Smith chart first and then find the symmetric point of it.

1.a

$z_L = 1.4 + 2j$ is shown in Figure. 1.

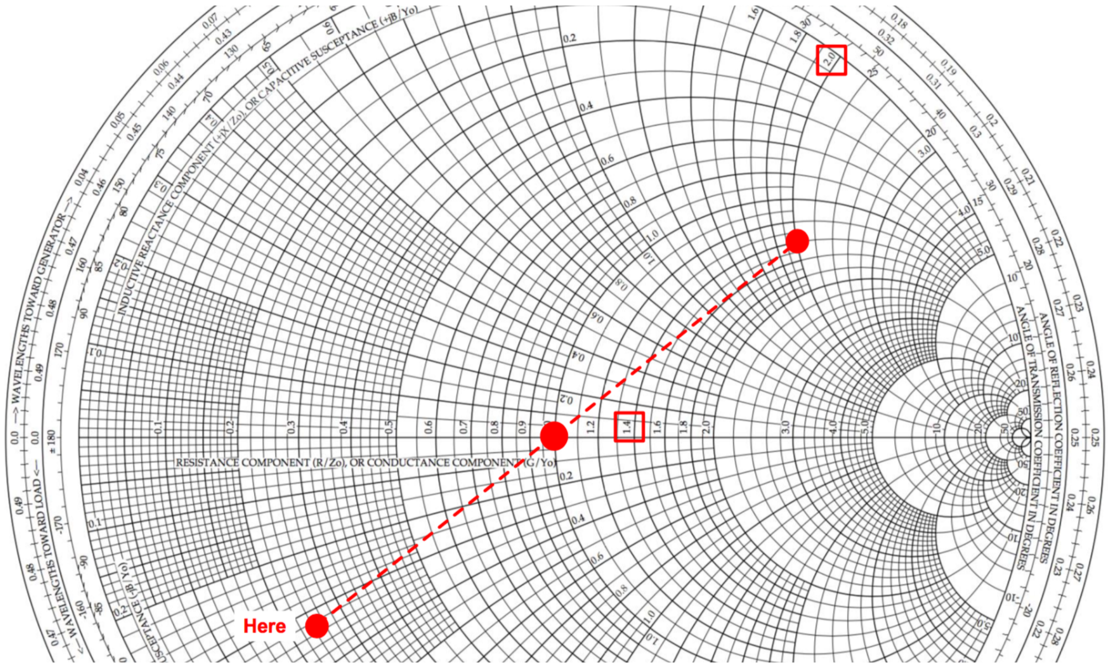

Figure 1: Smith chart

1.b

$z_L = 0.5 + 0.9j$ is shown in Figure. 2.

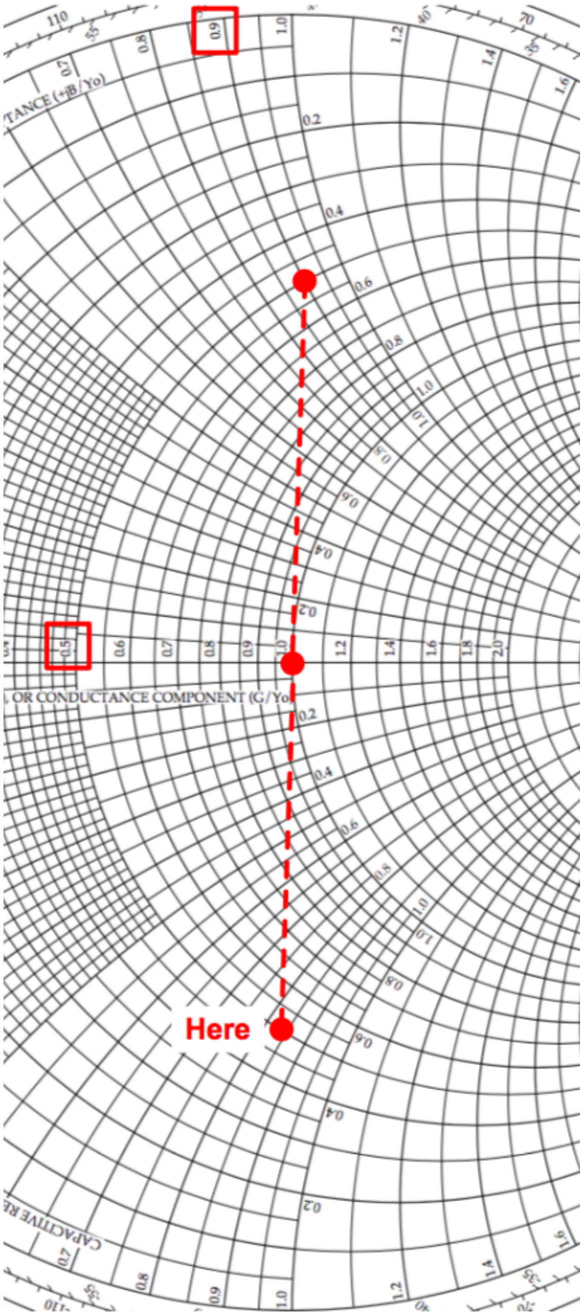

Figure 2: Smith chart

1.c

$z_L = 1.6 - 0.3j$ is shown in Figure. 3.

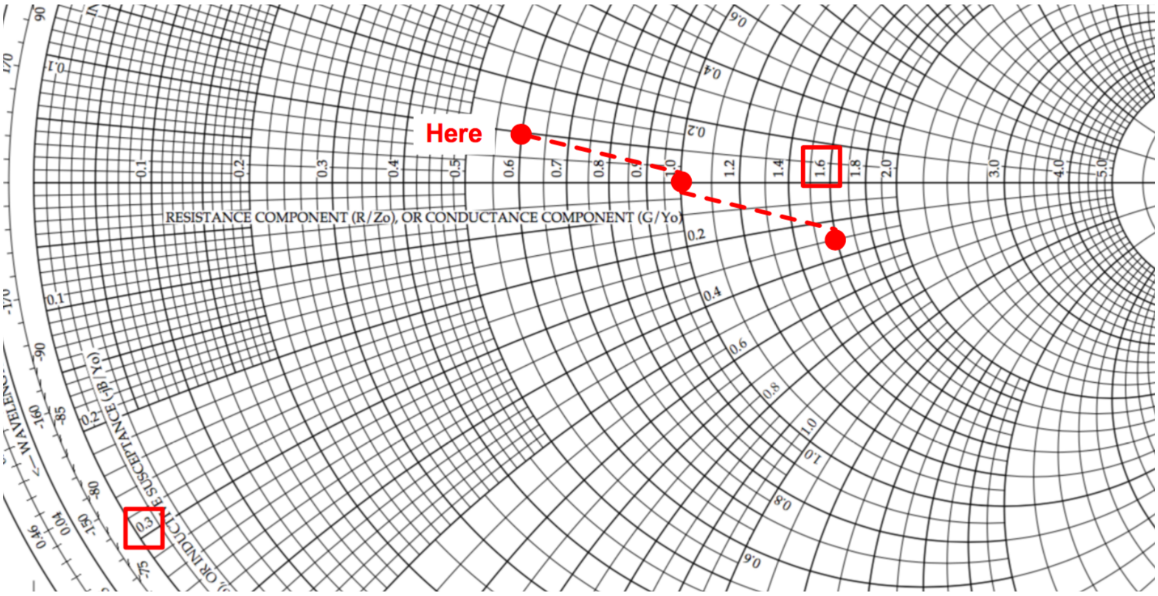

Figure 3: Smith chart

2

Assumptions:

1. In this question, when calculating the inductance or capacitance of lumped components, I assume the target frequency is $f = 1(GHz)$.

2. The Z_0 of transmission line is equal to the matching target impedance in each part.

2.a

For part a, I'll use equation method to do the matching.

First of all, we convert the Z_L into parallel structure as shown in Figure. 4. The new R and X can be calculated as:

$R_1 = [1 + (\frac{100}{70})^2] \times 70 = 212.8571(\Omega)$

$jX_1 = j[1 + (\frac{70}{100})^2] \times 100 = 149j(\Omega)$

$\because 212.8571(\Omega) > 50(\Omega)$

$\therefore$ We need to add a parallel component first to convert $212.8571(\Omega)$ into $50(\Omega)$. And then we add a serial component to make the impedance a real number. Assume the impedance of the parallel component is jX_p and that of the serial component is jX_s, as shown in Figure. 4.

With jX_p, we have $Q = \frac{R_1}{\frac{X_1 X_p}{X_1 + X_p}} = (\frac{1}{X_1} + \frac{1}{X_p})R_1$.

$\because R_2 = 50(\Omega)$

$\therefore \frac{R_1}{1+Q^2} = 50$

Plug in the values of R_1 and X_1, we get:

$jX_p = j565.8322(\Omega)$ and $Q = 1.8048$

$\therefore$ To convert it into serial structure as shown in Figure. 4, we get:

$R_2 = 50(\Omega)$

$jX_2 = j\frac{\frac{X_1 X_p}{X_1+X_p}}{1+Q^{-2}} = j90.2378(\Omega)$

$\therefore$ To make the impedance a real number, jX_s needs to be equal to $-jX_2$.

$\therefore jX_s = -jX_2 = -j90.2378(\Omega)$

$\therefore jX_p = j565.8322(\Omega) \quad jX_s = -j90.2378(\Omega)$

To implement X_p and X_s with lumped components, we can use inductor and capacitor respectively as shown in Figure. 5.

$\therefore jX_p = j2\pi fL \quad \Rightarrow \quad L = 90.06(nH)$

$\therefore jX_s = \frac{1}{j2\pi fC} \quad \Rightarrow \quad C = 1.76(pF)$

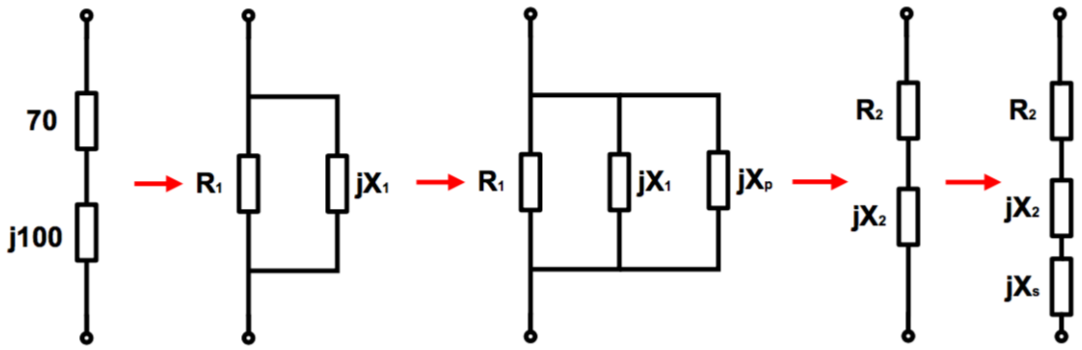

Figure 4: Impedandce matching.

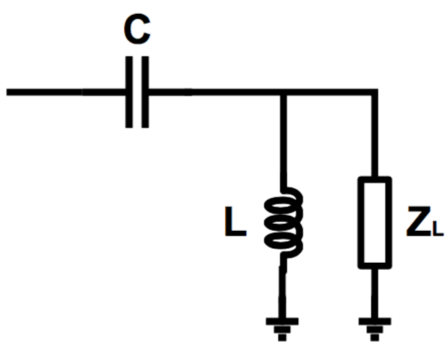

Figure 5: Impedandce matching.

2.b

For part b, I will use equation method.

As shown in Figure. 6, the transmission lines are labeled 1 and 2. Assume the impedance seen from the left end of T-line 1 is $a + bj(\Omega)$. The first step is to convert the $a + bj(\Omega)$ into parallel structure as shown in Figure. 6. The second step is to use T-line 2 to cancel out jX_1.

$\because R_1 = 50(\Omega)$

$\therefore (1 + Q^2)a = 50$

$(1 + \frac{b^2}{a^2})a = 50$

$\therefore a^2 + b^2 = 50a$ (equation 1)

With transmission line 1, Z_L is converted into $a + bj(\Omega)$. Their relation is:

$a + bj = Z_0 \frac{Z_L + jZ_0 tan(kl_1)}{Z_0 + jZ_L tan(kl_1)}$

Assume $tan(kl_1) = A$, we have:

$a + bj = 50 \times \frac{70+100j+50Aj}{50-100A+70Aj} = 50 \times \frac{35A^2+35+(50-50A^2-124A)j}{(5-10A)^2+49A^2}$

$\therefore a = 50 \times \frac{35A^2+35}{(5-10A)^2+49A^2}$

$b = 50 \times \frac{50-50A^2-124A}{(5-10A)^2+49A^2}$

Plug a and b into equation 1, we can get the value of A:

$A_1 = -1.0332$

$A_2 = 11.0332$

I'll use the value of A_2, then $tan(kl_1) = 11.0332 \quad \Rightarrow \quad l_1 = 0.2356\lambda$

With the value of A_2, we have:

$a = 12.5899(\Omega)$

$jb = -21.7023j(\Omega)$

$R_1 = 50(\Omega)$

$jX_1 = -29.006j(\Omega)$

To cancel out the jX_1, the impedance of T-line 2 should be $-jX_1 = 29.006j(\Omega)$.

Transmission line 2 is open-loaded, so its impedance is:

$jZ_0 tan(kl_2) = 29.006j \quad \Rightarrow \quad l_2 = 0.0837\lambda$

In conclusion, $l_1 = 0.2356\lambda \quad l_2 = 0.0837\lambda$

2.c

For part c, I'll use Smith chart to match the impedance.

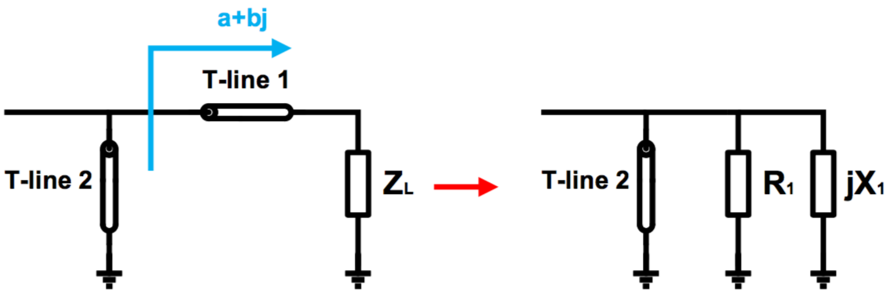

Figure 6: Impedandce matching.

First of all, $z = \frac{Z_L}{100} = 1.6 - 0.3j$. As shown in Figure. 7, z can be found on Smith chart as point A. So first step is to move the point on equal-conductance circle, which means adding a parallel component. Point A's susceptance is about $0.1j$. Point A's equal-conductance circle is very close to equal-conductance circle of 0.6, so we just move the point on equal-conductance of 0.6 until it hits the equal-resistance circle of 1, which is point B. Point B's susceptance is about $0.5j$. Thus, we need to add a parallel component whose impedance is $jX_p = \frac{1}{\frac{0.5j - 0.1j}{100}} = -j250(\Omega)$. So the capacitor we will use as parallel component is $(0.5j - 0.1j) \times \frac{1}{100} = j2\pi fC \Rightarrow C = 636.62(fF)$. And then, point B's reactance is about $-0.8j$. So we need a serial impedance of $jX_s = 0.8j \times 100 = 80j(\Omega)$.

In conclusion, firstly we need to add a parallel component of $jX_p = -j250(\Omega)$ and then add a serial component of $jX_s = 80j(\Omega)$. If we implement the impedance with inductor, its inductance can be calculated as $0.8j \times 100 = j2\pi fL \Rightarrow L = 12.73(nH)$. The matching network is shown in Figure. 8.($L = 12.73(nH) \quad C = 636.62(fF)$)

This process can be verified in ADS as shown in Figure. 9. $L = 12.89(nH)$ and $C = 595.09(fF)$. As we can see here, the inductance and capacitance from Smith chart method are not very accurate, because it's hard to read out accurate values from Smith chart.

2.d

I'll do part d with equation method.

As shown in Figure. 10, the transmission lines are labeled 1 and 2. Assume the impedance seen from the left end of T-line 1 is $a + bj(\Omega)$. The first step is to convert the $a + bj(\Omega)$ into parallel structure as shown in Figure. 10. The second step is to use T-line 2 to cancel out jX_1.

$\because R_1 = 100(\Omega)$

$\therefore (1 + Q^2)a = 100$

$(1 + \frac{b^2}{a^2})a = 100$

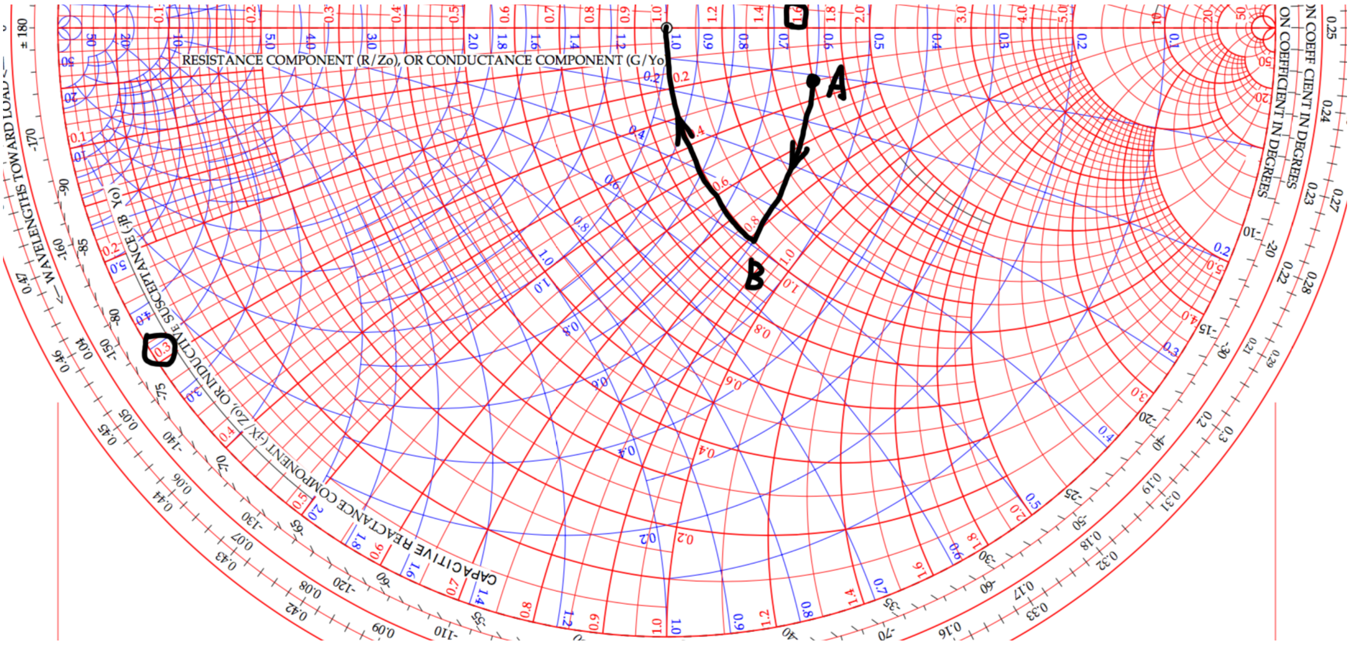

Figure 7: Impedandce matching.

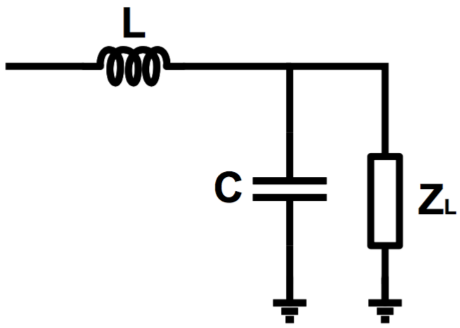

Figure 8: Impedandce matching.

$\therefore a^2 + b^2 = 100a$ (equation 1)

With transmission line 1, Z_L is converted into $a + bj(\Omega)$. Their relation is:

$a + bj = Z_0 \frac{Z_L + jZ_0 tan(kl_1)}{Z_0 + jZ_L tan(kl_1)}$

Assume $tan(kl_1) = A$, we have:

$a + bj = 100 \times \frac{160-30j+100Aj}{100+(160-30j)Aj} = 100 \times \frac{160+160A^2+j(130A^2-265A-30)}{(10+3A)^2+256A^2}$

$\therefore a = 100 \times \frac{160+160A^2}{(10+3A)^2+256A^2}$

$b = 100 \times \frac{130A^2-265A-30}{(10+3A)^2+256A^2}$

Plug a and b into equation 1, we can get the value of A:

$A = 0.941$

Then $tan(kl_1) = 0.941 \quad \Rightarrow \quad l_1 = 0.12\lambda$

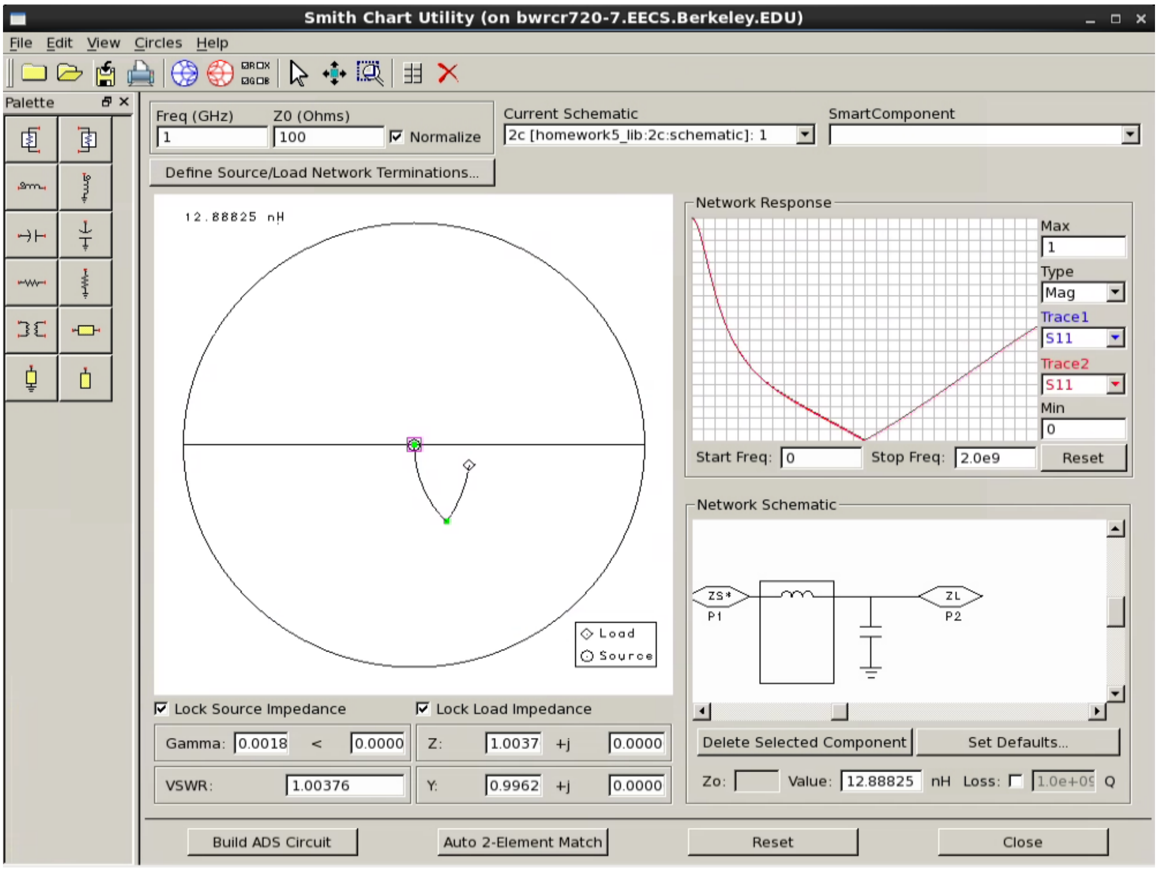

Figure 9: Impedandce matching.

With the value of A, we have:

$a = 146.199(\Omega)$

$jb = -53.981j(\Omega)$

$R_1 = 100(\Omega)$

$jX_1 = -183.66j(\Omega)$

To cancel out the jX_1, the impedance of T-line 2 should be $-jX_1 = 183.66j(\Omega)$.

Transmission line 2 is open-loaded, so its impedance is:

$jZ_0 tan(kl_2) = 183.66j \quad \Rightarrow \quad l_2 = 0.171\lambda$

In conclusion, $l_1 = 0.12\lambda \quad l_2 = 0.171\lambda$

2.e

For part e, I will use equation method.

$\because 25(\Omega) < 50(\Omega)$

$\therefore$ We need to add a serial component and then add a parallel component to cancel out the imaginary part.

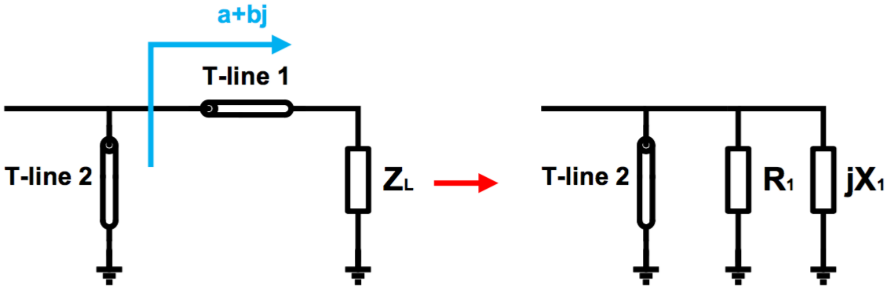

Figure 10: Impedandce matching.

Assume the impedance of serial component is jX_s and that of parallel component is jX_p as shown in Figure. 11.

After adding the serial component, the Q factor becomes $Q = \frac{90+X_s}{25}$.

$\because R_1 = 50(\Omega)$

$\therefore (1 + Q^2) \times 25 = 50 \quad \Rightarrow \quad Q = 1 \quad jX_s = -j65(\Omega)$

$jX_1 = j(90 + X_s)(1 + Q^{-2}) = j50(\Omega)$

$\therefore$ To cancel out the imaginary part of the impedance, $jX_p = -jX_1 = -j50(\Omega)$

$\therefore$ In conclusion, $jX_s = -j65(\Omega) \quad jX_p = -j50(\Omega)$.

How to convert X_s and X_p into actual inductors and capacitors has been shown in part a. So in this part, I just skip it.

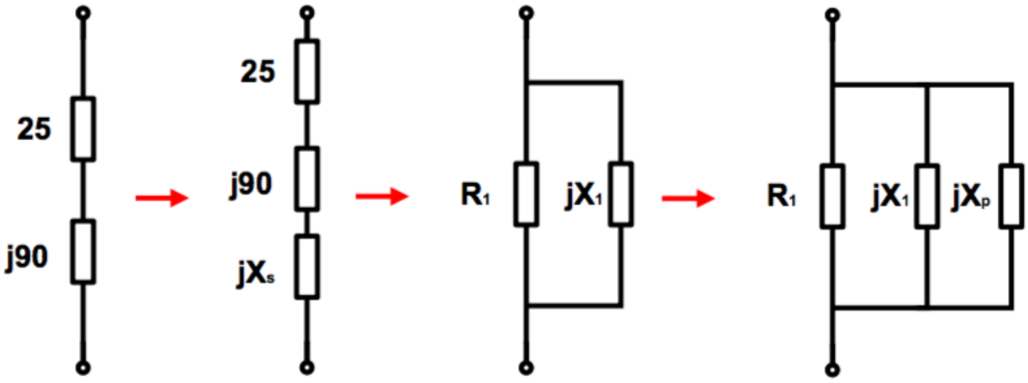

Figure 11: Impedandce matching.

2.f

I'll use Smith chart to do part f.

It’s really hard to draw accurate circles in Photoshop on Smith chart by hand. So I’ll just use a tool called ”Smith chart” in ADS to do the matching. As shown in Figure. 12, we add a T-line in series with Z_L and then add a parallel T-line to cancel out the imaginary part. $l_1 = 0.275\lambda \quad l_2 = 0.0575\lambda$.

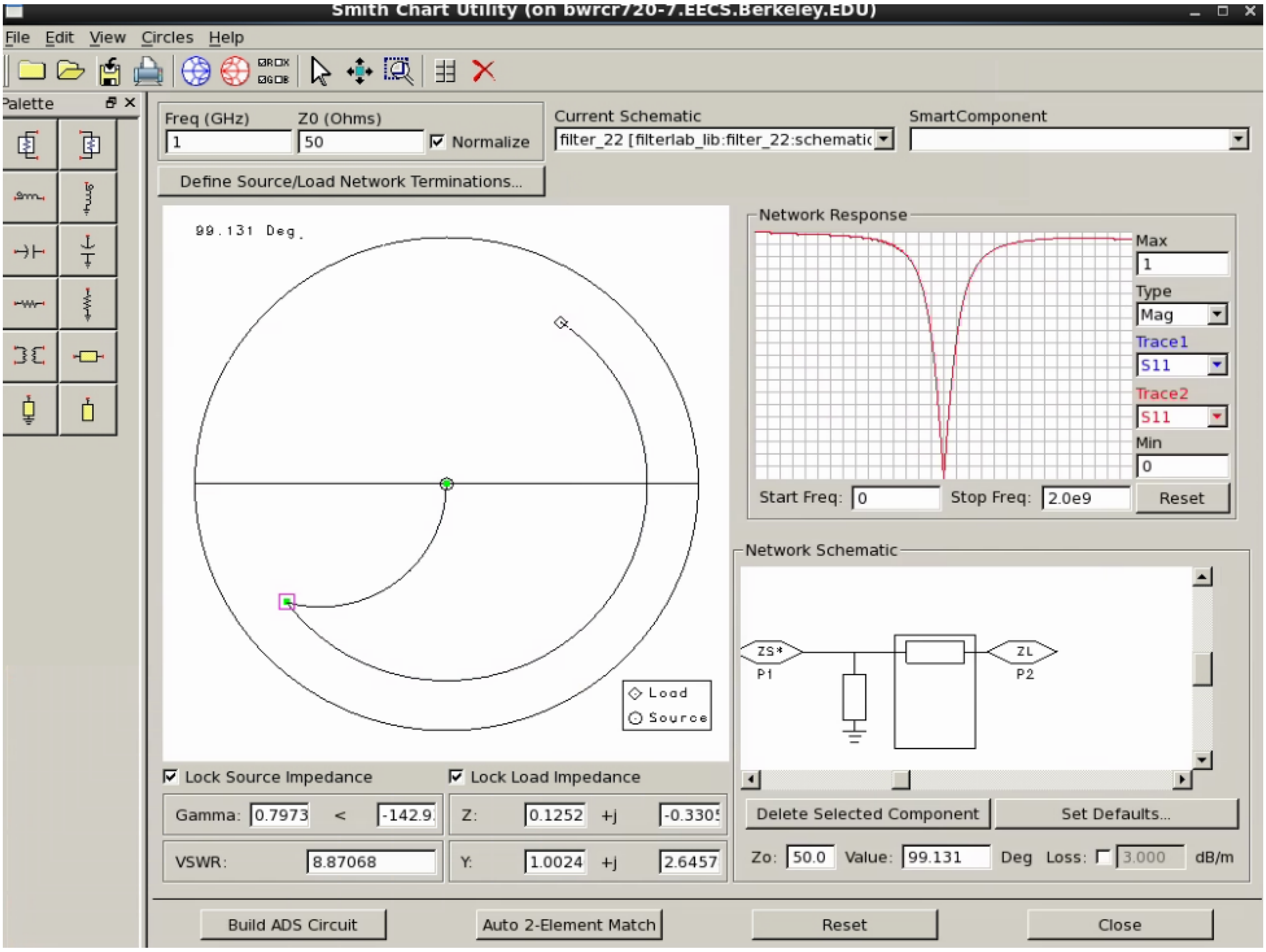

Figure 12: Impedandce matching.

3

3.a

To make bandwidth less than 5%:

$$BW \approx \frac{1}{3Q_{total}} < 5\% \quad \Rightarrow \quad Q > \frac{20}{3}$$

The $\pi-$matching network is shown in Figure. 13. Assume the intermediate resistance is R_i, then we have:

$Q_{total} = \frac{Q_1+Q_2}{2} = \frac{\sqrt{\frac{1000}{R_i}-1}+\sqrt{\frac{50}{R_i}-1}}{2} > \frac{20}{3} \quad \Rightarrow \quad R_i < 8(\Omega)$

$\therefore$ Set R_i to be $R_i = 5(\Omega)$

$\therefore Q_1 = \sqrt{\frac{1000}{5} - 1} = 14.1 \quad Q_2 = \sqrt{\frac{50}{5} - 1} = 3$

With the same method used in previous questions. Values of jX_1, jX_2, jX_3, and jX_4 can be calculated:

$jX_1 = j16.67(\Omega) \quad \Rightarrow \quad L_1 = 2.65(nH)$

$jX_2 = -j15(\Omega) \quad \Rightarrow \quad C_2 = 10.61(pF)$

$jX_3 = -j70.54(\Omega) \quad \Rightarrow \quad C_3 = 2.26(pF)$

$jX_4 = j70.89(\Omega) \quad \Rightarrow \quad L_4 = 11.28(nH)$

$IL = \frac{1}{1+\frac{Q_1}{20}+\frac{Q_1}{20}+\frac{Q_2}{20}+\frac{Q_2}{20}} = 0.3689$ With ideal components, the simulation setup is shown in Figure. 14 and the results are shown in Figure. 15. From the results, we know the bandwidth is about 3.95%, which meets the spec. Then I add parasitics into the simulation, and its setup is shown in Figure. 16. Results are shown in Figure. 17. As we can see, there are totally no $-10dB$ point any more. So I just measure the $+3dB$ bandwidth, which is 13.27%, much larger than expected.

For the value of $|S_{11}|^2 + |S_{21}|^2$, because the network is lossy, the value is not 1 anymore. This is reasonable because part of the input power doesn't get into the network (S_{11}^2) and part of the input power arrives the output (S_{21}^2) and the rest is dissipated in the network (loss).

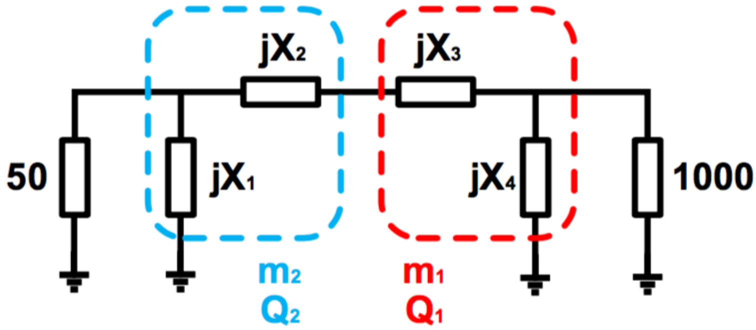

Figure 13: Impedandce matching.

3.b

As shown in Figure. 18, assume we have N stages of L-matching network and all of them have equal Q and m, which is Q_0 and m_0 respectively.

$\because m_0^N = \frac{1000}{50} = 20 \quad \Rightarrow \quad m_0 = 20^{\frac{1}{N}}$

$\therefore Q_0 = \sqrt{m_0 - 1} = \sqrt{20^{\frac{1}{N}} - 1}$

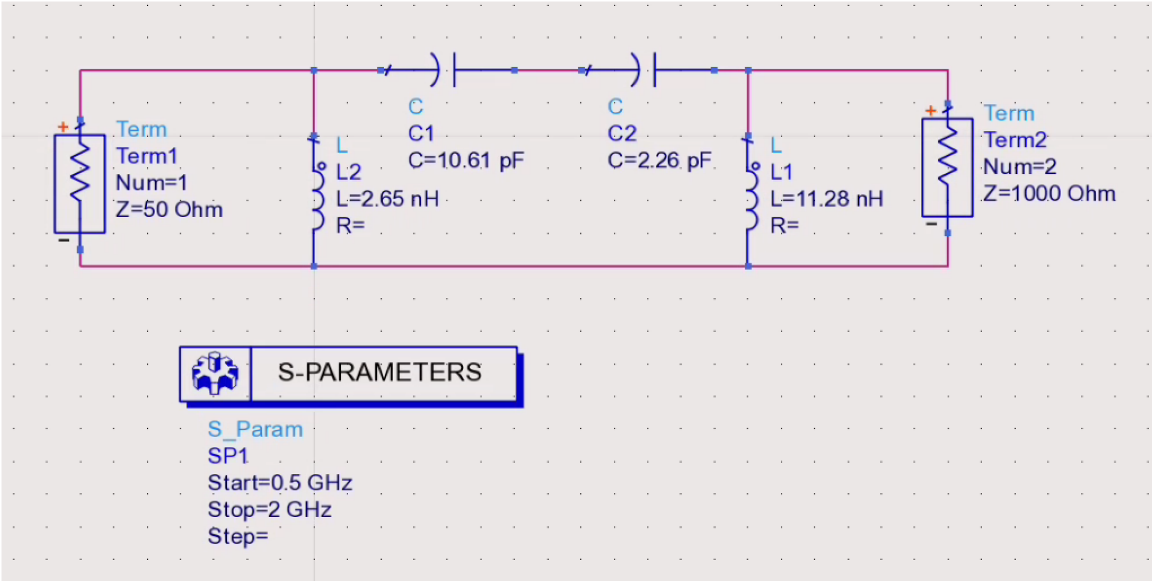

Figure 14: ADS simulation setup with ideal components.

$\therefore IL = \frac{1}{1+N(\frac{Q_0}{20}+\frac{Q_0}{20})} = \frac{1}{1+\frac{N}{10}\cdot\sqrt{20^{\frac{1}{N}}-1}}$

With the help of MATLAB, when $N = 2$, IL is lowest as shown in Figure. 19.

When $N = 2$,

$m_0 = 4.47$

$Q_0 = 1.86$

$IL = 0.7285$

With the same method used in previous questions, the inductance and capacitance can be calculated:

$L_1 = 85.41(nH)$

$L_2 = 19.10(nH)$

$C_1 = 0.382(pF)$

$C_2 = 1.71(pF)$

Simulation setup and result are shown in Figure. 20 and Figure. 21. As we can see here, at $1(GHz)$, $|S_{21}| = 0.833$, so the power delivered to load is $|S_{21}|^2 = 0.6939$, which is very close to the calculation result of IL.

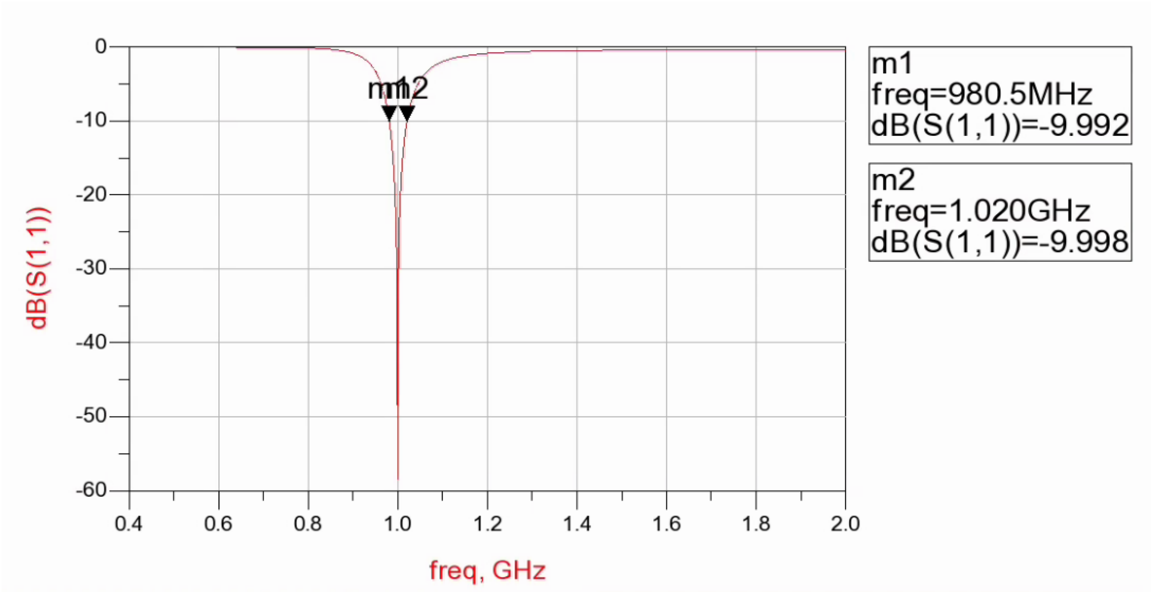

Figure 15: ADS simulation results.

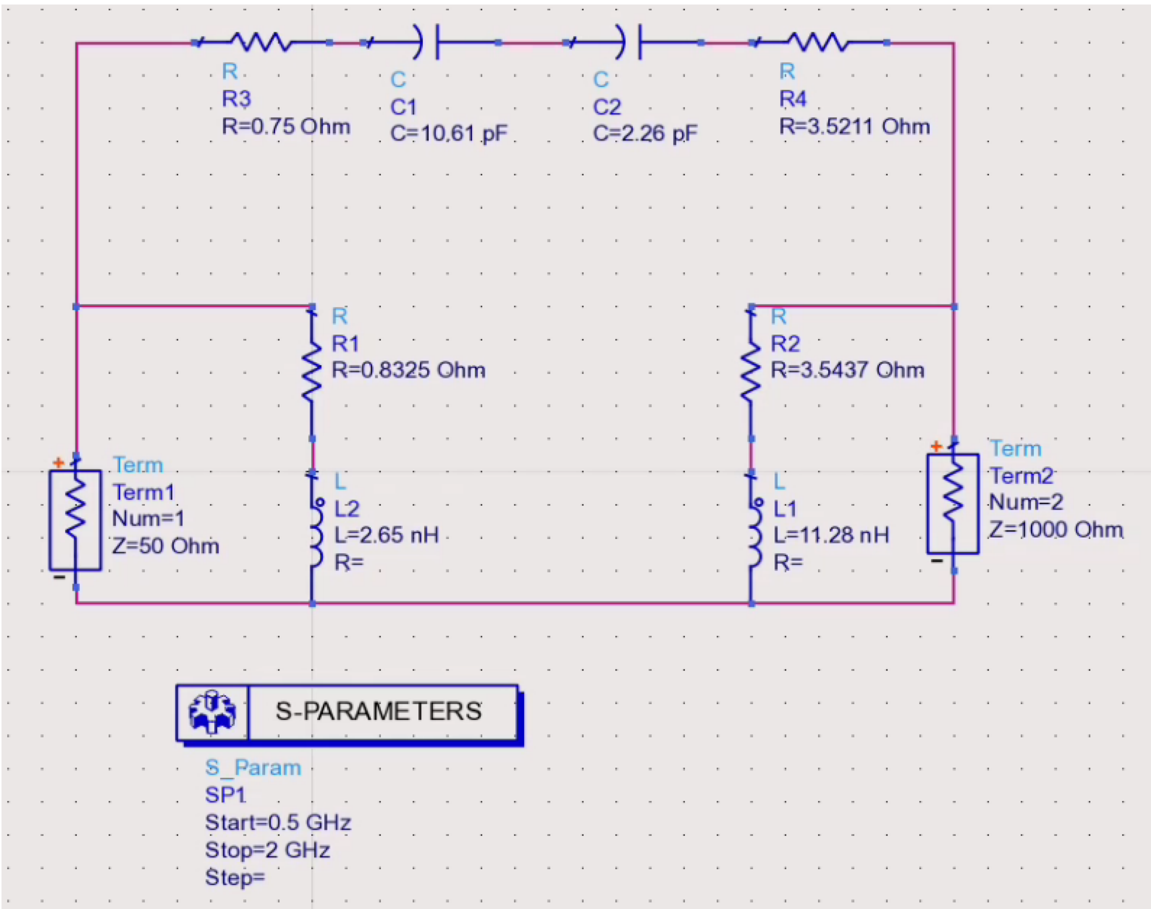

Figure 16: ADS simulation setup with ideal components.

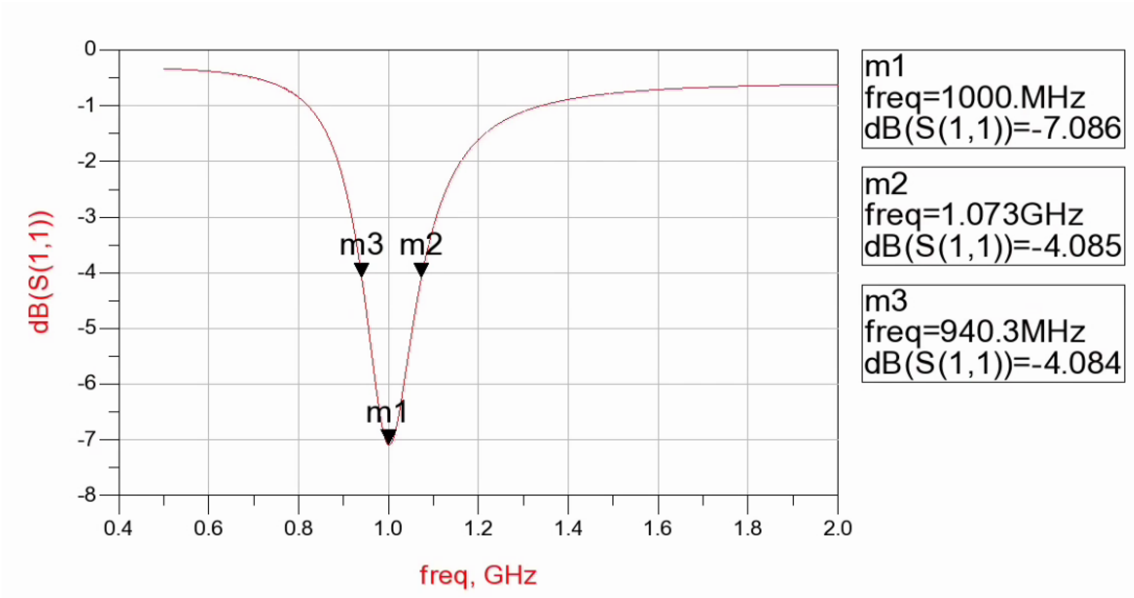

Figure 17: ADS simulation results.

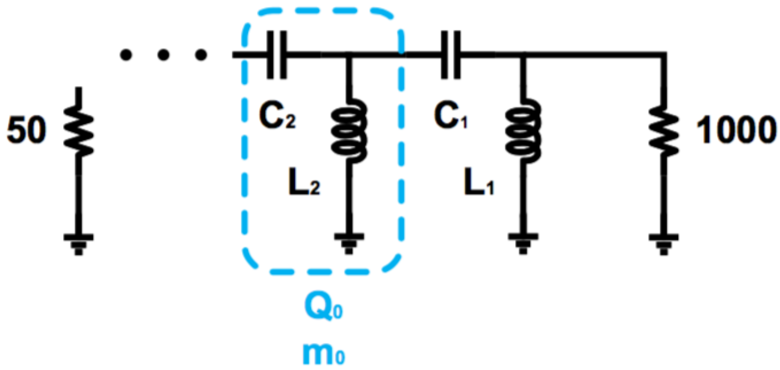

Figure 18: Matching network.

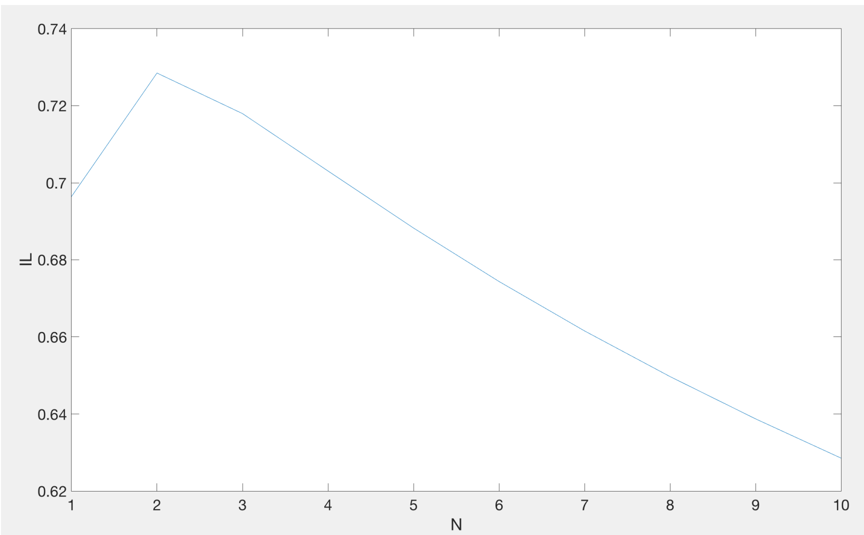

Figure 19: Optimal N.

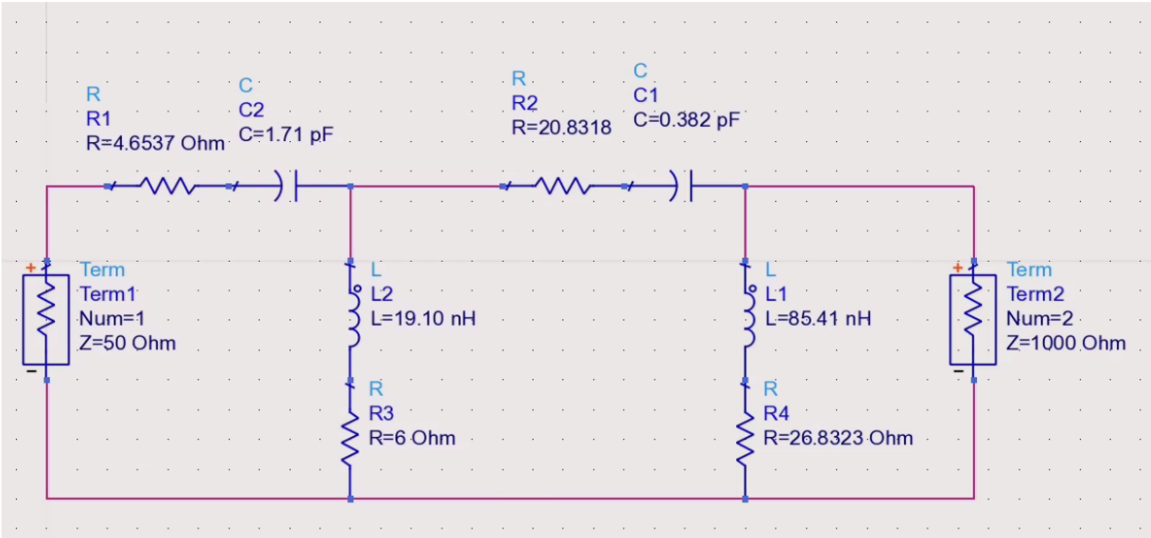

Figure 20: Simulation setup.

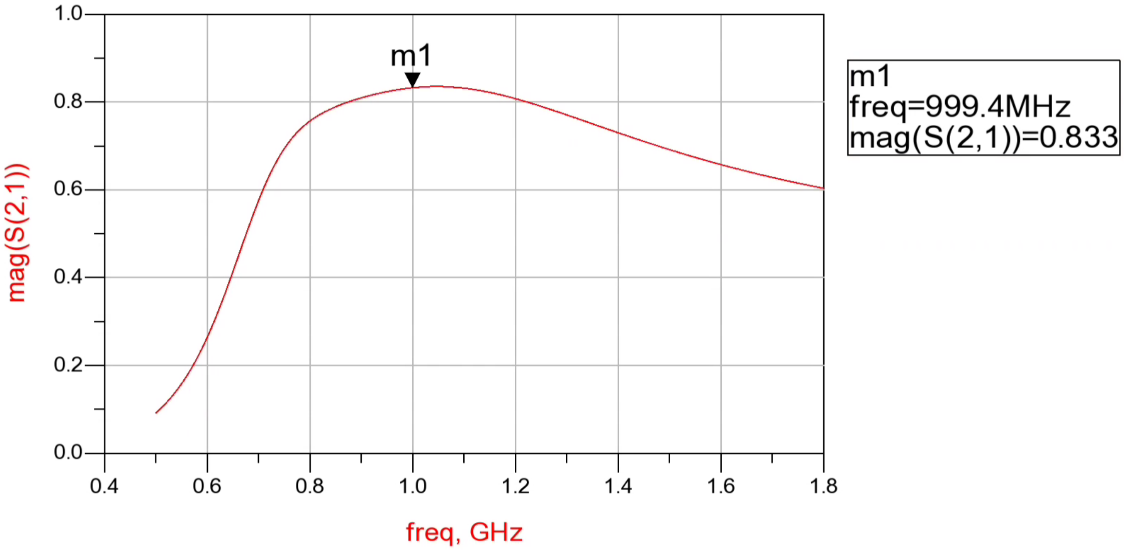

Figure 21: Simulation results.