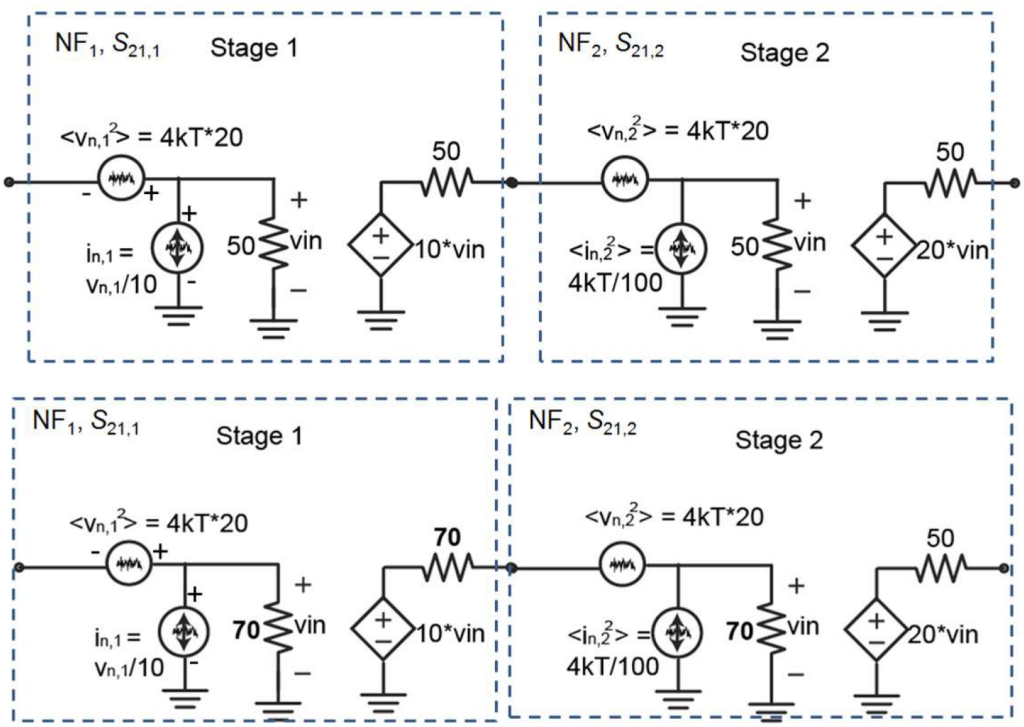
EE142 Problem Set 7

Vighnesh Iyer

October 21, 2017

1 Noise Figure of Cascade Blocks and Lossy Transmission Line



(a) For the above two cascade circuits, calculate the power gains and noise figures for each stage (i.e. $S_{21,1}$, $S_{21,2}$, NF_1, NF_2) and the two stage circuits ($S_{21,total}$, NF_{total}). The resistors are assumed to be noiseless.

1.a Cascade 1

For the first cascase's stage 1, we begin by input referring the noise sources and collapsing the voltage and current noise into $\overline{v_{eq}^2}$. From lecture:

$$\overline{v_{eq}^2} = \overline{v_n^2} + \overline{i_n^2} R_s^2 + 2 \cdot R_s \cdot \overline{v_n i_n}$$

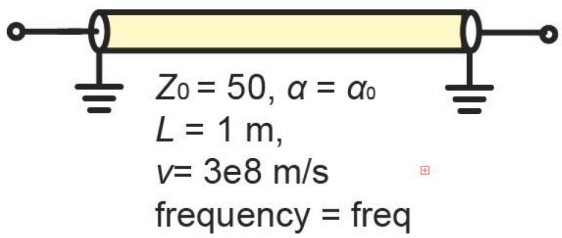
$$F = 1 + \frac{N_{amp,i}}{N_s} = 1 + \frac{\overline{v_{eq}^2}}{\overline{v_s^2}}$$

Assume we are calculating noise figure in a 50Ω environment, $R_s = 50\Omega$ and $\overline{v_s^2} = 4kTR_s$. We assume that all noise sources represented are defined as *spot noise*. Furthermore, we *don't* assume that $\overline{v_n^2}$ and $\overline{i_n^2}$ are uncorrelated, and include covariance terms when needed.

$$
\begin{aligned}
i_{n,1} &= v_{n,1}/10 \rightarrow \overline{i_{n,1}^2} = \overline{v_{n,1}^2}/100 \\
F_1 &= 1 + \frac{\overline{v_{eq,1}^2}}{\overline{v_s^2}} = \frac{4kT \cdot 20 + 4kT \cdot 0.2 \cdot 50^2}{4kT \cdot R_s} = 10.4 \\
NF_1 &= 10 \cdot \log(F_1) = 23.42 \text{ dB}
\end{aligned}
$$

(b) Is the formula $NF_{total} = NF_1 + \frac{NF_2 - 1}{|S_{21,1}^2|}$ applicable?

(c) For a lossy transmission line illustrated below, derive its noise figure.



Starting with the definition of F:

$$
\begin{aligned}
F &= \frac{SNR_i}{SNR_o} = \frac{P_{sig}/\overline{v_{n,s}^2}}{P_{sig} \cdot \text{ loss}/\overline{v_{n,s}^2} \cdot \text{ loss}^2} \\
F &= \frac{1}{\text{loss}}
\end{aligned}
$$

where 'loss' represents the power loss of the transmission line in steady state from input to output. This is simple to compute, assuming that the line is driven with a source impedance of Z_0 with a mean squared noise voltage of $4kTZ_0$ and terminated with a noiseless load of Z_0.

$$
\begin{aligned}
\text{Power @ Load} &= V_+ e^{-\gamma l} \cdot \frac{1}{Z_0} e^{-\gamma l} V_+ |_{l=0} = \frac{V_+^2}{Z_0} \\
\text{Power @ Source} &= V_+ e^{-\gamma l} \cdot \frac{1}{Z_0} e^{-\gamma l} V_+ |_{l=-l} = \frac{V_+^2 e^{2\gamma l}}{Z_0} \\
\text{Power @ Load / Power @ Source} &= e^{-2\gamma l} \\
F &= \frac{1}{e^{-2\gamma l}} = e^{2\gamma l}
\end{aligned}
$$

We know the propagation constant $\gamma = \alpha + j\beta$. Because the velocity of the line is at the speed of light, the imaginary term goes to zero and γ is dominated by α.

$$F = e^{2\alpha_0 L}$$

The noise figure seems frequency independent, but any real line has $\beta = \frac{2\pi}{\lambda}$.

(d) If the tline is used to connect the above two cascade circuits to the 50Ω soucrce (e.g. antenna), what will be the new total noise figures

2 Matching for Low Noise Versus Matching for High Gain

In this problem, your answers should be functions of frequency.

(a) For a simplified common-source model shown below (with noise sources drawn) derive the input referred noise voltage and noise current.