EE 142 Problem Set 3

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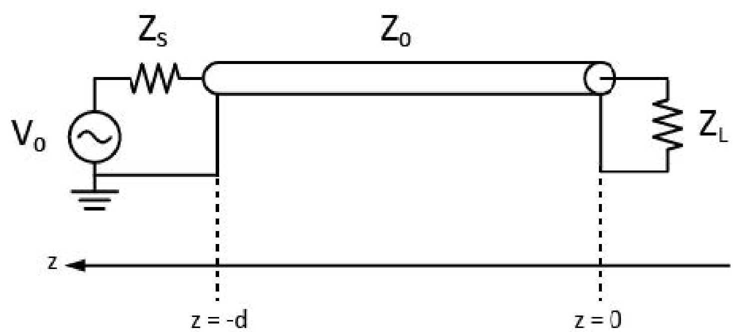
1 T-Lines at Steady State



Voltage source generates 10 Ghz sine with 10V amplitude.

Tline terminated with $Z_L = 80 - 40j\Omega$, and $Z_0 = 100\Omega$. $\epsilon_{eff} = 4$ and $d = 22.5$ mm.

1. Find the reflection coefficient at the load (z = 0) and at the source (z = -d)

   At the load:

$$\rho_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$
$$\rho_L = -0.0588 - 0.23539j$$
$$|\rho_L| = 0.242$$

   At any point on the line, we can derive an effective generalized $\rho(z)$ which represents the ratio of the backwards and forward traveling waves at a given point on the tline.

$$V(z) = V_0^+(e^{-j\beta z} + \rho_L e^{-j\beta z})$$
$$\rho(z) = \frac{V_0^+ \rho_L e^{j\beta z}}{V_0^+ e^{-j\beta z}}$$
$$\rho(z) = \rho_L e^{2j\beta z}$$

Notice that since $\beta = 2\pi/\lambda$, $\rho(z)$ repeats every $\lambda/2$ traversed along the tline back to the generator. We can find c_p and λ for this tline and frequency.

$$c_p = \frac{c_0}{\sqrt{\epsilon_{eff}}} \approx 1.5e8 \text{ m/s}$$
$$\lambda = \frac{c_p}{f} = 0.015 \text{ m}$$
$$d/\lambda = 1.5 = 3 \cdot \frac{1}{2}\lambda$$

So, $\rho(z)$ at $z = -d$ is $rho_L = 0.242$.

2. Find the input impedance at the source (z = -d) and at z = 18.75mm.

   The general form is:

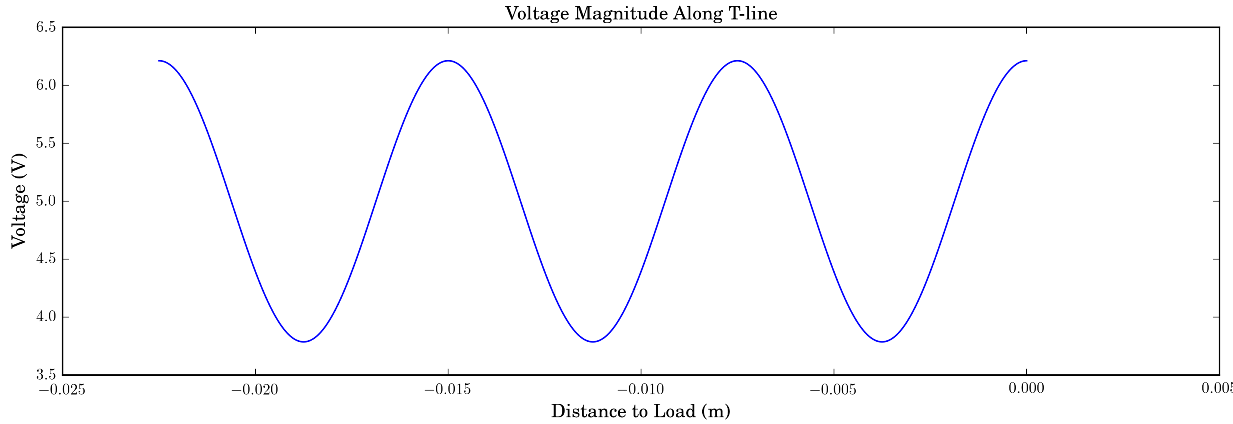
$$Z_{in}(-l) = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$
$$\beta = \frac{2\pi}{\lambda} = 418.879$$
$$Z_{in}(0) = Z_L = 100\Omega$$
$$Z_{in}(-18.75 \text{ mm}) = Z_{in}(\lambda + \lambda/4) = \frac{Z_0^2}{Z_L} = 100 + 50j$$

3. Plot the magnitude of the voltage along the line. Find voltage maximum, minimum, and SWR.

   We assume that $Z_S = Z_0$:

$$SWR = \frac{V_{max}}{V_{min}} = \frac{1 + |\rho_L|}{1 - |\rho_L|}$$
$$SWR = 1.64$$
$$V^+ = \frac{Z_0}{Z_0 + Z_S} = 5 \text{ V}$$
$$V_{max} = |V^+|(1 + |\rho_L|) = 6.2 \text{ V}$$
$$V_{min} = |V^+|(1 - |\rho_L|) = 3.8 \text{ V}$$

   Plot of voltage magnitude along line:



2 T-Line Modeling

We will derive an equivalent two-port circuit model for a short section of transmission line ($l << \lambda$) including loss.

1. For a "pi" equivalent circuit shown below, find the two-port Z matrix.