EE142 Problem Set 5

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Problem 1

Find y for the following normalized impedance on Smith Chart.

The straightforward procedure is to plot z_L on the impedance Smith Chart, and then look at what constant admittance and constant suspectance curves cross over the point in the admittance smith chart.

But, we can also plot z_L on the impedance smith chart, then rotate the point by π degrees along the constant SWR circle, and then read off the admittance by looking at the constant resistance and reactance curves.

I'm going to use the second technique; annotated charts aren't included in this document, but I'll compare the chart result I get to the exact calculation.

(a) $z_L = 1.4 + 2j$

$$y_L = \frac{1}{z_L} = \frac{1}{\alpha + \beta j} = \frac{\alpha - \beta j}{\alpha^2 + \beta^2} = 0.234899 - 0.33557j$$
$$y_{L,chart} = 0.22 - 0.32j$$

(b) $z_L = 0.5 + 0.9j$

$$y_L = 0.471698 - 0.849j$$
$$y_{L,chart} = 0.45 - 0.85j$$

(c) $z_L = 1.6 - 0.3j$

$$y_L = 0.60377 + 0.1132j$$
$$y_{L,chart} = 0.6 + 0.12j$$

Problem 2

Use the Smith Chart. Also use equations for lumped component matching to check.

(a) Match $Z_L = 70 + 100j\Omega$ to 50 Ohm with lumped components.

Let's clear up some things:

$$Z_C = \frac{1}{j\omega C} \qquad X_C = \Im Z_C = -\frac{1}{\omega C}$$

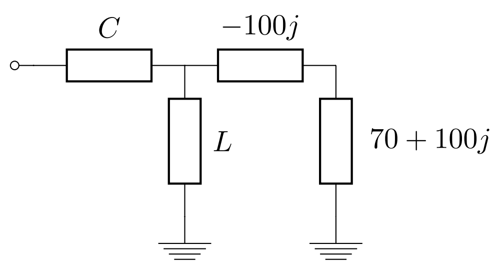
$$Z_L = j\omega L \qquad X_L = \Im Z_L = \omega L$$

$$\text{To find } C = \frac{1}{\omega X_c}$$

$$\text{To find } L = \frac{X_L}{\omega}$$

$$\text{where: } \omega = 2\pi f$$

The load is complex, so we have to resonant out the load's complex impedance so only a real part is seen before solving using the L network method.



Now, the L-network will see a purely real 70Ω impedance with which we can use the regular matching equations.

$$R_S = 50$$

$$R_L = 70$$

$$R_{hi} = max(R_S, R_L) = 70$$

$$R_{lo} = min(R_S, R_L) = 50$$

$$\text{Boosting factor: } m = \frac{R_{hi}}{R_{lo}} = 1.4$$

$$Q = \sqrt{m-1} = 0.632$$

$$\text{Dropping resistance so, } X_p = \frac{R_L}{Q} = 110.76$$

$$X_p' = \frac{X_p}{1 + Q^{-2}} = 31.613$$

$$X_s = -X_p' = -31.613$$

We arrive at the capacitor reactance of $-79.15j$ and the inductor reactance of $110.76j$. The circuit is simulated in ADS to match at 1 Ghz with component values $C = 5.0344$ pF, $L = 17.6$ nH, and $C_{res} = 1.59$ pF. S-parameter simulation verifies that the source and load are perfectly matched at 1 Ghz with $S_{21} = 0dB$.

The same calculation can be performed using the smith chart.

$$Z_{L,norm} = 1.4 + 2j$$
$$Z_{L,real} = 1.4$$
$$X_p = (1/0.45)j \cdot 50 = 111.1j$$
$$X_s = -0.62 \cdot 50 = -31j$$

The values calculated using the Smith Chart are very close to the values from the equations.

(b) Match $Z_L = 70 + 100j\Omega$ to 50 Ohm using transmission lines.

(c) Match $Z_L = 160 - 30j\Omega$ to 100 Ohm using lumped circuits.

Assume a resonating inductor with reactance $30j$ to make the load purely real.

$$R_S = 100$$
$$R_L = 160$$
$$R_{hi} = max(R_S, R_L) = 160$$
$$R_{lo} = min(R_S, R_L) = 100$$
$$\text{Boosting factor: } m = \frac{R_{hi}}{R_{lo}} = 1.6$$
$$Q = \sqrt{m-1} = 0.775$$
$$\text{Dropping resistance so, } X_p = \frac{R_L}{Q} = 206.452$$
$$X_p' = \frac{X_p}{1+Q^{-2}} = 77.47$$
$$X_s = -X_p' = -77.47$$

We simulate in ADS with $L_{res} = 4.77$ nH, $L = 32.858$ nH, $C = 2.05$ pF. The simulation shows that these values give a perfect match at 1 Ghz. This match appears more broadband than the one in part a). The Smith Chart again gives very similar values.

(d) Match $Z_L = 160 - 30j\Omega$ to 100 Ohm using transmission lines.

(e) Match $Z_L = 25 + 90j\Omega$ to 50 Ohm using lumped circuits. Assume a resonating capacitor with reactance $-90j$ to make the load purely real.

$$R_S = 50$$
$$R_L = 25$$
$$R_{hi} = max(R_S, R_L) = 50$$
$$R_{lo} = min(R_S, R_L) = 25$$
$$\text{Boosting factor: } m = \frac{R_{hi}}{R_{lo}} = 2.0$$
$$Q = \sqrt{m-1} = 1.0$$
$$\text{Boosting resistance so, } X_s = Q \cdot R_L = 25$$
$$X_s' = X_s(1+Q^{-2}) = 50$$
$$X_p = -X_s' = -50$$

We simulate in ADS with $C_{res} = 1.768$ pF, $C = 3.183$ pF, $L = 3.98$ nH. The results confirm a perfect match at 1 Ghz.

(f) Match $Z_L = 25 + 90j\Omega$ to 50 Ohm using transmission lines.

Problem 3

(a) Design a Π matching network between a 1000Ω load impedance and a 50Ω source impedance at 1 Ghz. The inductor and capacitor quality factors are 20. The target bandwidth for $|S_{11}| < -10$ dB is 5%. Calculate the insertion loss and verify your design using ADS. Check if $|S_{11}|^2 + |S_{21}|^2 = 1$ holds.

Let's first analyze an L-network to see if it can fit our design requirements.

$$Q_{cap} = Q_{ind} = 20$$
$$m = \frac{R_{hi}}{R_{lo}} = 20 \rightarrow Q = \sqrt{m-1} \approx 4.359$$
$$Q_{total,L-network} = \frac{Q}{2} \approx 2.2$$
$$S_{11} \text{ -10 dB BW} \approx \frac{1}{3 \cdot Q_{total}} = 15\%$$
$$\text{Insertion Loss} = \frac{1}{1 + \frac{Q}{Q_{cap}} + \frac{Q}{Q_{ind}}} = 0.694$$

The bandwidth of the L-network is too high and isn't selective enough for our requirements. The bandwidth is set (approximately) by the circuit Q and so we need to use a Π network so Q doesn't depend on m.

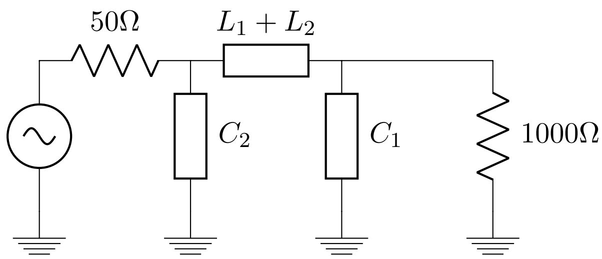
$$\frac{1}{3 \cdot Q_{tot}} \approx 0.05 \rightarrow Q_{tot} \geq 6$$
$$Q_1 = \sqrt{\frac{R_L}{R_i} - 1}$$
$$Q_2 = \sqrt{\frac{R_S}{R_i} - 1}$$
$$Q_{tot} = \frac{Q_1 + Q_2}{2}$$
$$R_i \leq 8.108$$

We find that the intermediate resistance should be less than 8.108 Ω to keep the bandwidth below 5%. We will design for $R_i = 5\Omega$.

Here are all the derived parameters:

	L-network 1	L-network 2
m	200	10
Q	14.11	3
X_p	70.888	16.666
X_s	70.534	15.0
C	2.245 pF	9.549 pF
L	11.2 nH	1.39 nH

assuming capacitors are placed in parallel and inductors in series. Q_{tot} is around 8.5.



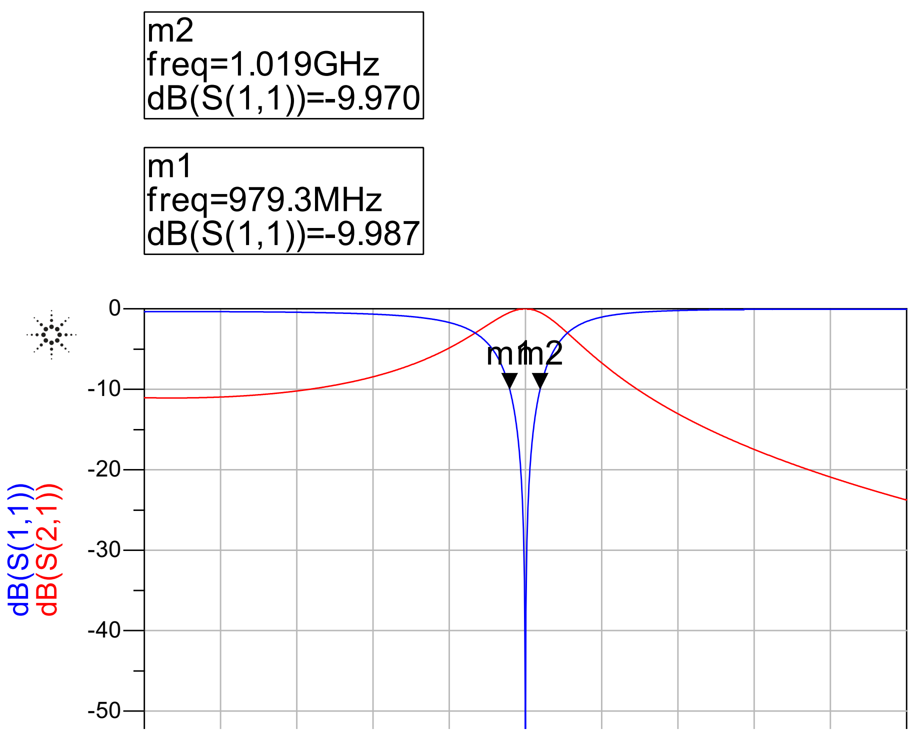


Figure 1: Simulation with lossless components

We run a simulation and find that for lossless components, the bandwidth of S_{11} down to -10 dB is 39.7 Mhz, which is within our 50 Mhz design spec.

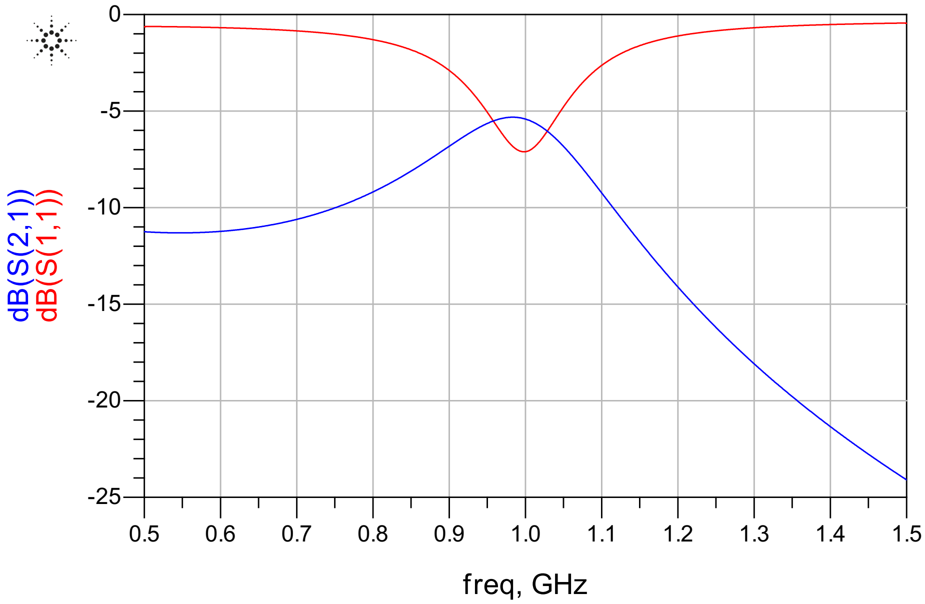
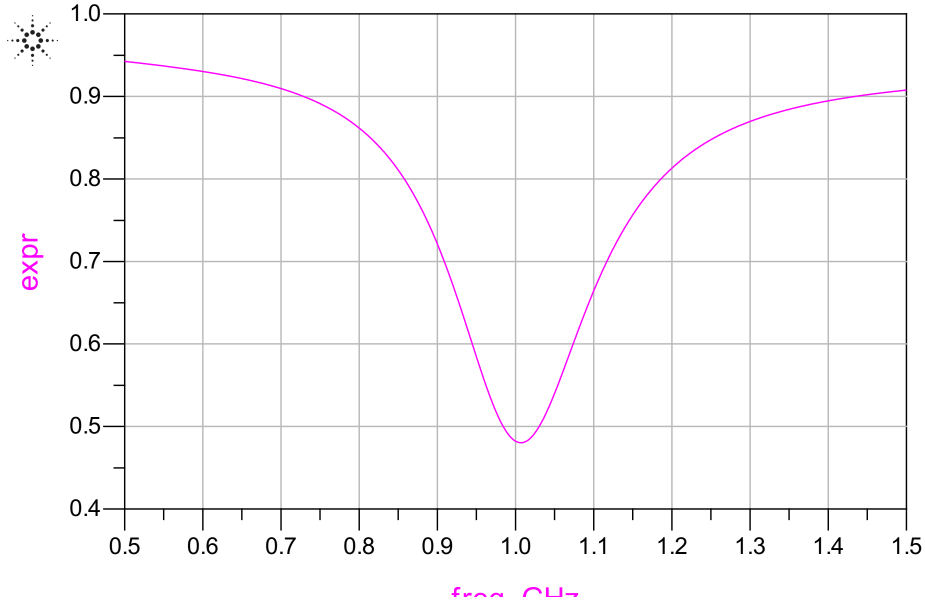


Figure 2: Simulation with finite Q

When adding finite Q components, the bandwidth isn't very different, but the insertion loss at 1 Ghz becomes significant. However the S_{11} -10dB bandwidth becomes infinite/zero since S_{11} never dips below -10dB. It is possible that the Q of these components isn't sufficient to achieve -10dB input selectivity. It's also possible that my design isn't optimized sufficiently.

Eqn expr = abs(S(1,1))**2 + abs(S(2,1))**2

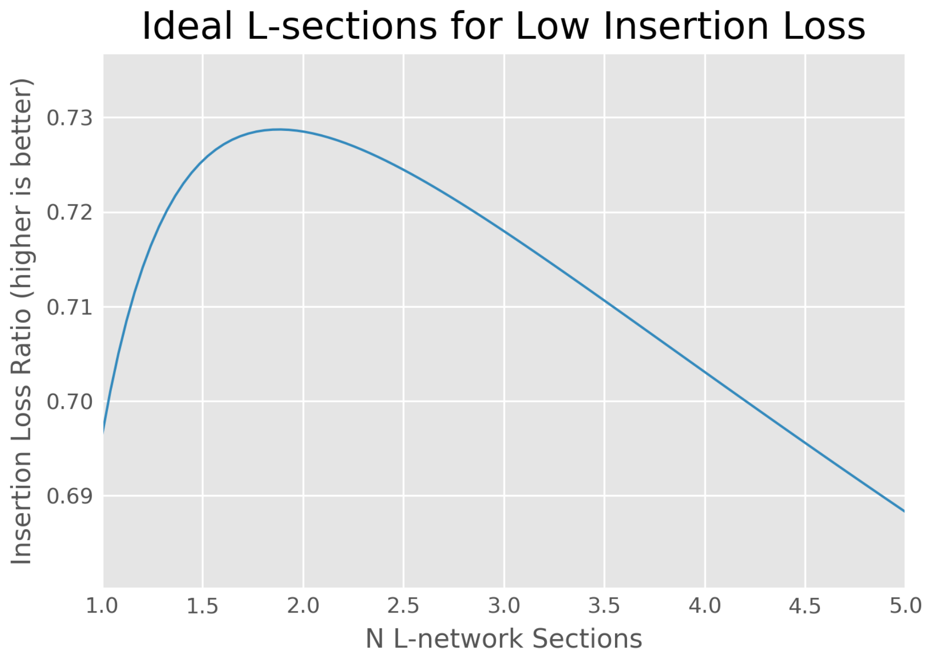


In the finite Q simulation, the relationship $|S_{11}|^2 + |S_{21}|^2 = 1$ doesn't hold near the center frequency. This is due to the internal loss of the finite Q components.

(b) Design a matching network between a 1000Ω load impedance and a 50Ω source impedance at 1 Ghz. The inductor and capacitor quality factors are 20. The design goal is to achieve the lowest insertion loss. Calculate the insertion loss and verify your design using ADS.

$$\mathrm{IL} = \frac{1}{1 + \frac{N}{Q_u}\sqrt{(\frac{R_{hi}}{R_{lo}})^{1/N} - 1}}$$

We use this equation with our design variables to find the ideal value of N. We let $Q_u = Q_{cap}||Q_{ind} = 10$.



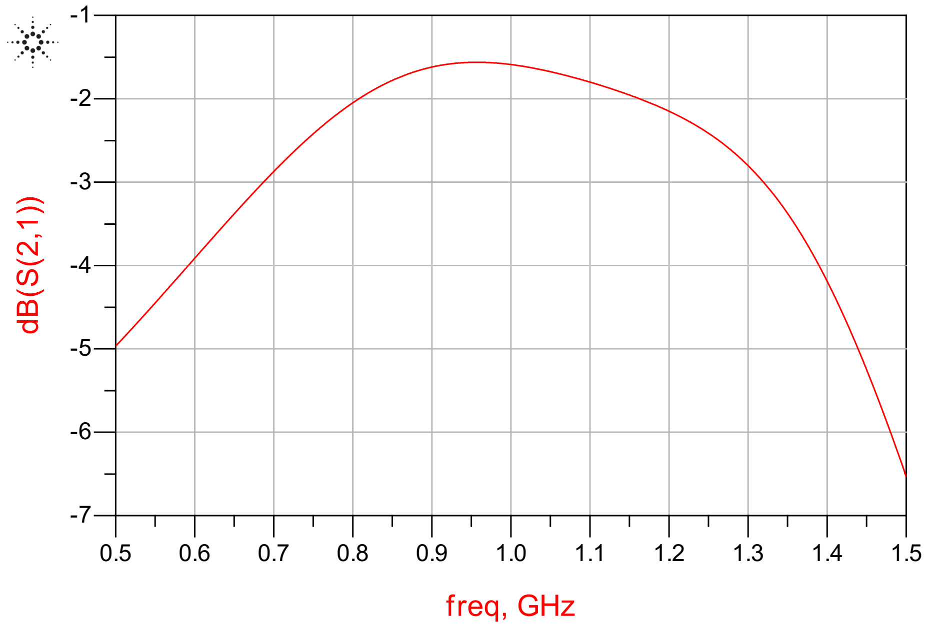
Insertion loss is minimized with 2 L-network stages. $IL_{max} = 0.729$.

$$R_{i,opt} = \sqrt{R_L R_S} = 223.6$$
$$Q_{i,opt} = \sqrt{(\frac{R_{hi}}{R_{lo}})^{1/N} - 1} = 1.863$$

Now we can again go through the process of calculating actual component values.

	L-network 1	L-network 2
X_p	536.66	120.0
X_s	416.66	93.168
C	0.296 pF	1.326 pF
L	66.3 nH	14.8 nH

Again assuming that capacitors are in shunt and inductors in series. Use these values and run a finite Q simulation.



The simulation confirms a low insertion loss of around -1.5 dB. This is about 4dB better than the Π network.