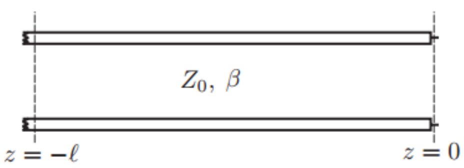
Today's Plan (9/20/2017)

- S-parameter Review
- S-parameter Calculation (Examples)
- Why *S*-parameter?
- Matching Network Design on Smith Chart
- Hints on hw3.4

S-Parameter Basics

In phasor domain (ac analysis)

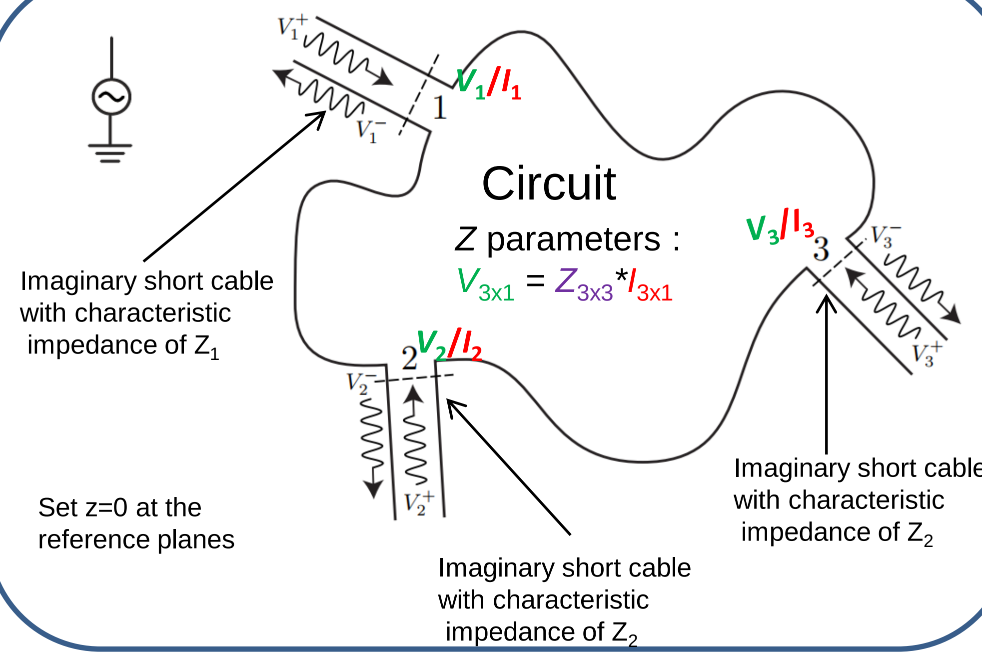


Recall what can exist on a cable

$$v(z) = V^+ e^{-\gamma z} + V^- e^{\gamma z}$$

$$i(z) = \frac{V^+}{Z_0} e^{-\gamma z} - \frac{V^-}{Z_0} e^{\gamma z}$$

$$\gamma = j\beta = j2\pi/\lambda$$



$$\begin{pmatrix} \frac{V_1^-}{\sqrt{Z_1}} \\ \frac{V_2^-}{\sqrt{Z_2}} \\ \frac{V_3^-}{\sqrt{Z_3}} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \begin{pmatrix} \frac{V_1^+}{\sqrt{Z_1}} \\ \frac{V_2^+}{\sqrt{Z_2}} \\ \frac{V_2^+}{\sqrt{Z_3}} \end{pmatrix}$$

Backward Terms Forward Terms

- *S* parameters: $[V_i^-/(Z_i^{0.5})]_{3x1} = S_{3x3} * [V_i^+/(Z_i^{0.5})]_{3x1}$
- $V_i = V_i^+ + V_i^-$; $I_i = V_i^+/Z_i - V_i^- /Z_i$

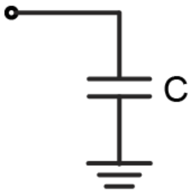
  => $V_i^+ = (V_i + I_i Z_i)/2$; $V_i^- = (V_i - I_i Z_i)/2$

  => $V_i^+/(Z_i^{0.5}) = [V_i/(Z_i^{0.5}) + I_i(Z_i^{0.5})]/2$; $V_i^- /(Z_i^{0.5}) = [V_i/(Z_i^{0.5}) - I_i(Z_i^{0.5})]/2$

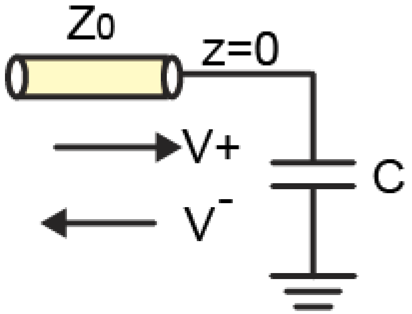
  => $S_{3x3} = (Z_{3x3} - [Z_i]_{3x3})(Z_{3x3} + [Z_i]_{3x3})^{-1}$
- ***S* parameters depend on the used reference impedance**

Simple One-Port Example

What is the S-parameter of a capacitor under reference impedance of Z_0 ?



Method 1:
From definition



$$v(z) = V^{+}e^{-\gamma z} + V^{-}e^{\gamma z}$$

$$i(z) = \frac{V^{+}}{Z_0}e^{-\gamma z} - \frac{V^{-}}{Z_0}e^{\gamma z}$$

$V^+ + V^- = (V^+/Z_0 - V^-/Z_0)*(1/j\omega C)$

$\Rightarrow S_{11} = V^-/V^+ = [(j\omega C)^{-1} - Z_0]/[(j\omega C)^{-1} + Z_0]$

One port $S_{11} = (Z_L - Z_0)/(Z_L + Z_0)$ depends on the selected Z_0

Note: if z = a ≠ 0,
use $\underline{V^+e^{-ra}}$ as the new V^+ and $\underline{V^-e^{ra}}$ as the new V^-
the calculated S_{11} does not change

Method 2:
Converted from Z-parameter

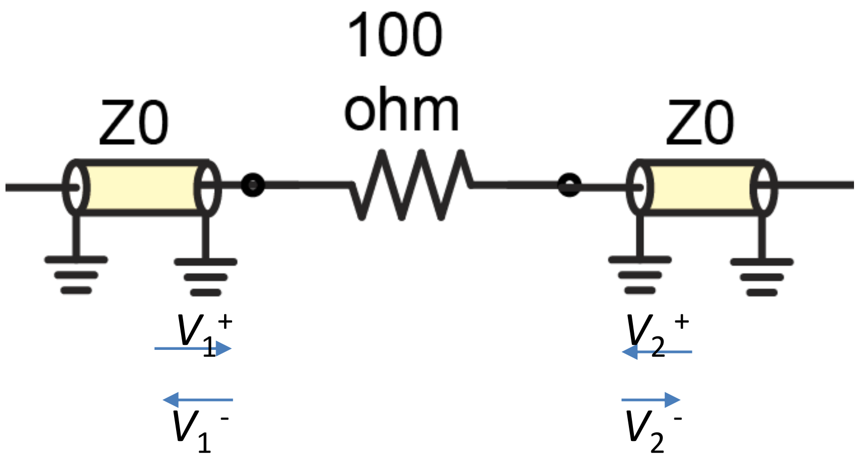
$$S_{3x3} = (Z_{3x3} - [Z_i]_{3x3})(Z_{3x3} + [Z_i]_{3x3})^{-1}$$

$$S_{1x1} = (Z_{1x1} - [Z_i]_{1x1})(Z_{1x1} + [Z_i]_{1x1})^{-1} = (Z_{in} - Z_0)/(Z_{in} + Z_0)$$

Simple Two-Port Example

What is the S-parameter of a series resistor
under reference impedance of Z_0 ?

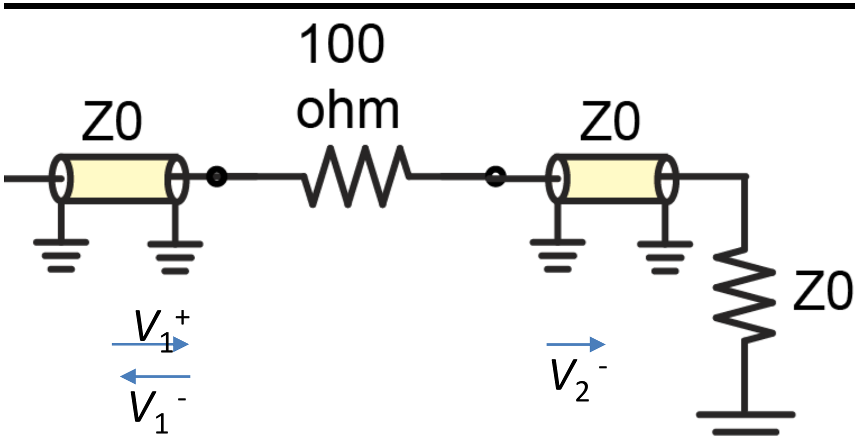
Method 1:
From definition



$$\begin{pmatrix} V_1^- \\ V_2^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \end{pmatrix}$$

$S_{11} = V_1^-/V_1^+$ if $V_2^+ = 0$

$S_{21} = V_2^-/V_1^+$ if $V_2^+ = 0$



$(V_1^+ + V_1^-) - 100(V_1^+/Z_0 \;-\; V_1^-/Z_0\,) = V_2^-$

$(V_1^+/Z_0 \;-\; V_1^-/Z_0\,) = V_2^-/Z_0$

(2 eqs, 3 variables) => $S_{11}=50/(Z_0+50)$; $S_{21}=Z_0/(Z_0+50)$

S_{11} can be calculated rapidly by seeing $Z_L = 100+Z_0$

Simple Two-Port Example

Method 2:
Converted from Z-parameter

S = $(Z - [Z_i])(Z + [Z_i])^{-1}$

$$Z = \begin{pmatrix} \infty & \infty \\ \infty & \infty \end{pmatrix} \quad Z_i = \begin{pmatrix} Z_0 & 0 \\ 0 & Z_0 \end{pmatrix}$$

S = $([Y_i] - Y) * ([Y_i] + Y)^{-1}$

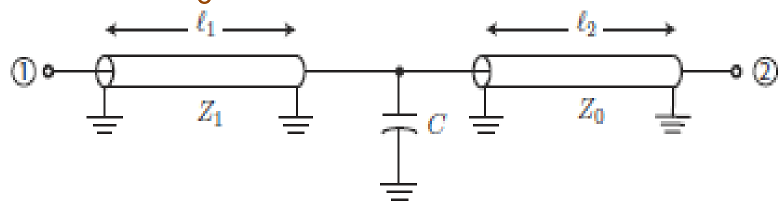
$$Y = \begin{pmatrix} 0.01 & -0.01 \\ -0.01 & 0.01 \end{pmatrix}$$

$$Y_i = \begin{pmatrix} 1/Z_0 & 0 \\ 0 & 1/Z_0 \end{pmatrix}$$

=> $S_{11}=50/(Z_0+50)$; $S_{21}=Z_0/(Z_0+50)$

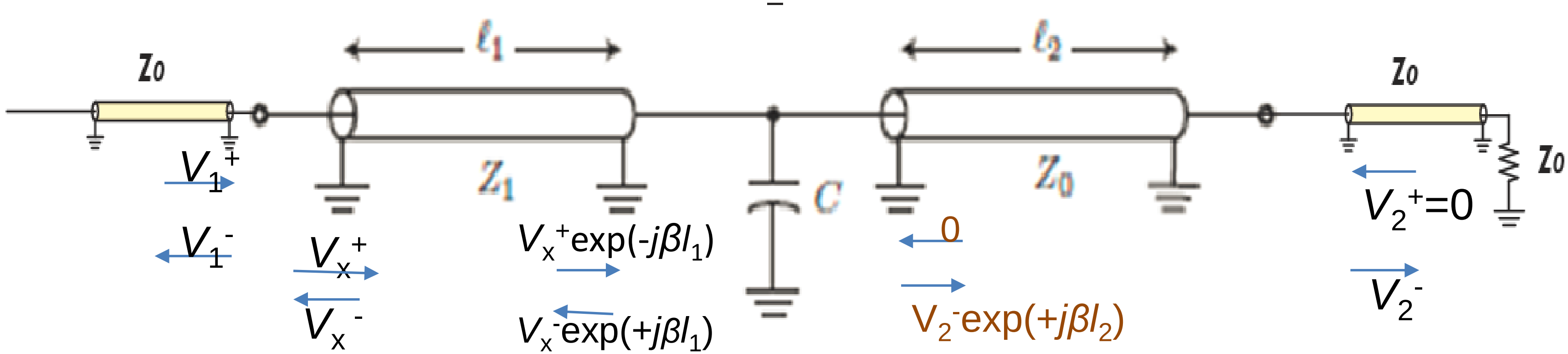
Midterm-Level Two-Port Example

What is the S-parameter of the beloe two-port circuit
under reference impedance of Z_0 ?



$S_{11} = V_1^- / V_1^+$ if $V_2^+ = 0$

$S_{21} = V_2^- / V_1^+$ if $V_2^+ = 0$



1. $V_1^+ + V_1^- = V_x^+ + V_x^-$
2. $V_1^+ / Z_0 - V_1^- / Z_0 = V_x^+ / Z_1 - V_x^- / Z_1$
3. $V_x^+ \exp(-j\beta l_1) = V_x^+ \exp(-j\beta l_1) * \rho_L$, where $\rho_L = (Z_0 // (j\omega C)^{-1} - Z_1) / (Z_0 // (j\omega C)^{-1} + Z_1)$
4. $V_x^- \exp(+j\beta l_1) + V_x^+ \exp(-j\beta l_1) = V_2^- \exp(+j\beta l_1)$

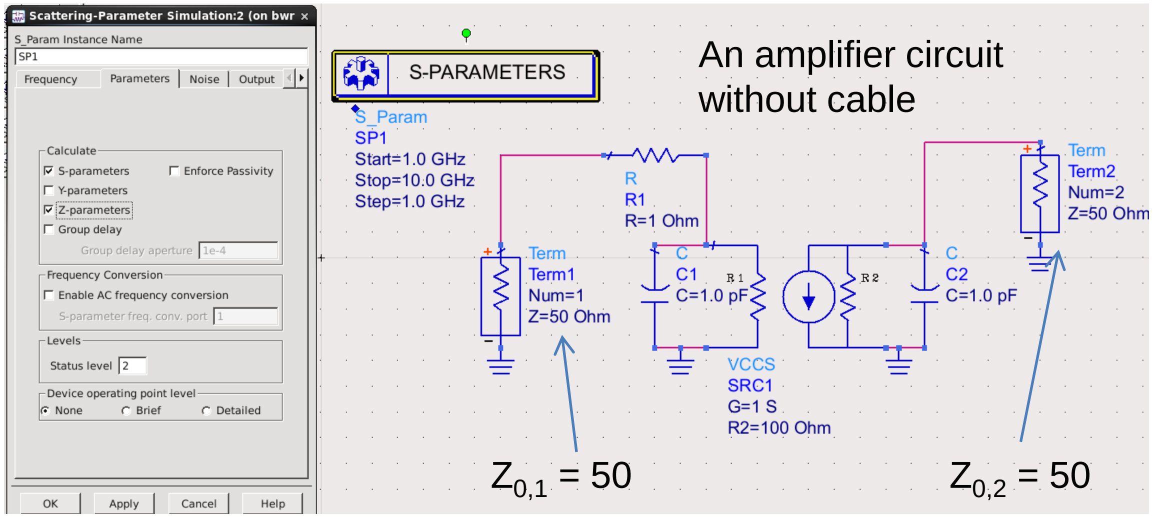
(4 eqs, 5 variables: V_1^+, V_1^-, V_x^+, V_x^-, V_2^-) => S_{11} and S_{21} can be obtained

Why S-Parameters ?

- *A. S*-parameter has a higher immunity when a cable is connected, if $Z_{0,Spara} = Z_{0,Cable}$

- *B. S*-parameter has a straightforward connection to the power flow in a microwave system

Why S-Parameters (A)



$$\begin{pmatrix} V_1^- \\ V_2^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \end{pmatrix}$$

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

Z parameter

freq	Z			
	Z(1,1)	Z(1,2)	Z(2,1)	Z(2,2)
1.000 GHz	159.158 / -89.640	1.278E-16 / 56.162	1.348E4 / 57.858	84.673 / -32.142

S parameter
Z_0 = 50

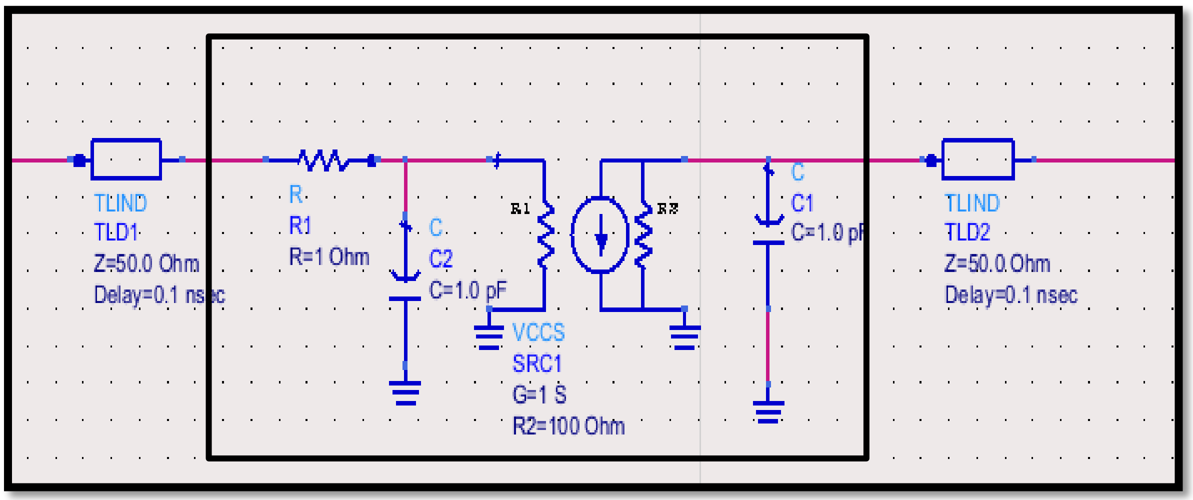
freq	var("S")			
	(1,1)	(1,2)	(2,1)	(2,2)
1.000 GHz	0.996 / -34.880	0.000 / 0.000	62.139 / 150.403	0.385 / -43.971

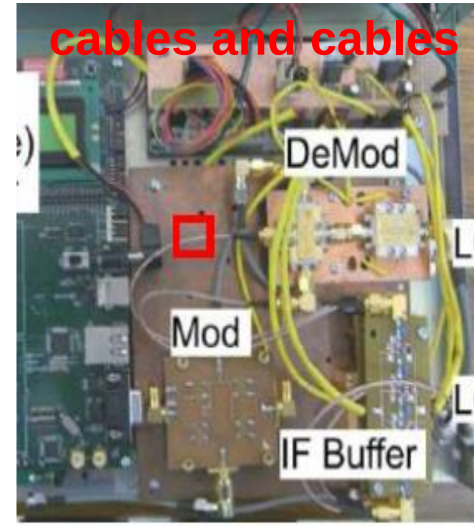
S parameter
Z_0 = 60

freq	var("S")			
	(1,1)	(1,2)	(2,1)	(2,2)
1.000 GHz	0.996 / -41.311	0.000 / 0.000	68.166 / 145.7...	0.334 / -56.562

- Both Z-parameters and S-parameters are complex numbers and are functions of frequency
- Only one Z-matrix for a given circuits at a frequency
- S parameters depend on the used reference impedance

An amplifier circuit with cable

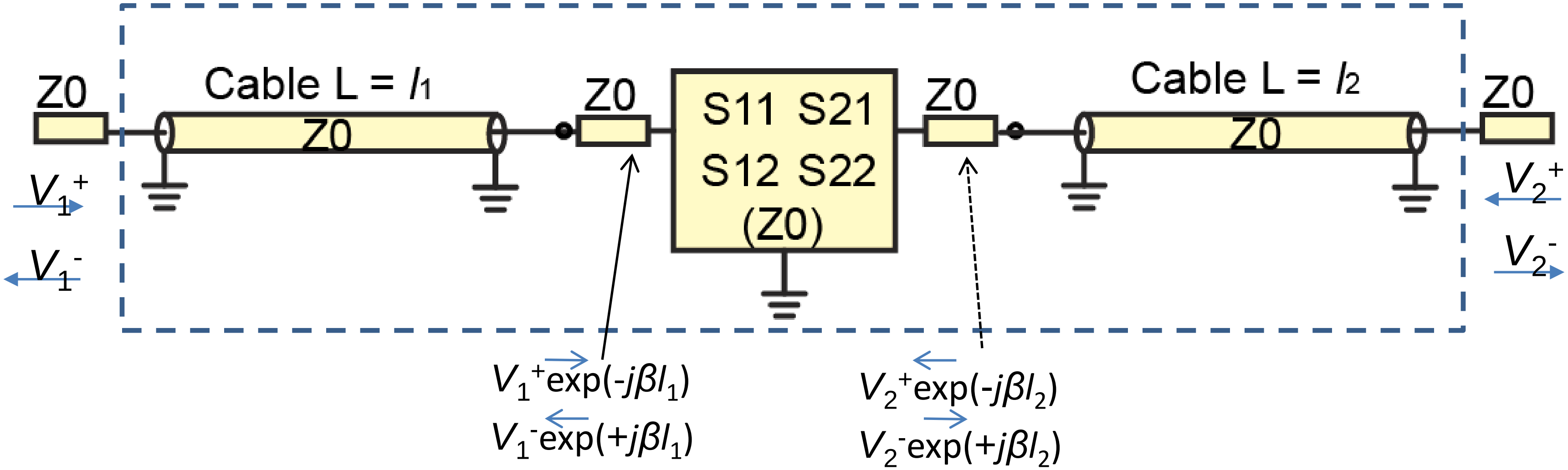




	No cable		Add the two cable with Z_0 of 50
***Z*-parameter**	$\begin{pmatrix} 159.2\angle -89.6^\circ & 0 \\ 13480\angle 58^\circ & 84.7\angle -32^\circ \end{pmatrix}$	→ Changes a lot	$\begin{pmatrix} 37.1\angle -89.8^\circ & 0 \\ 3179\angle 25.4^\circ & 36.9\angle -39.1^\circ \end{pmatrix}$
***S*-parameter $Z_0 = 50$**	$\begin{pmatrix} 0.996\angle -34.9^\circ & 0 \\ 62.1\angle 150.4^\circ & 0.385\angle -43.9^\circ \end{pmatrix}$	→ Looks very similar	$\begin{pmatrix} 0.996\angle -106.9^\circ & 0 \\ 62.1\angle 78.4^\circ & 0.385\angle -116.0^\circ \end{pmatrix}$
***S*-parameter $Z_0 = 60$**	$\begin{pmatrix} 0.996\angle -41.3^\circ & 0 \\ 68.2\angle 145.7^\circ & 0.334\angle -56.5^\circ \end{pmatrix}$	→ Changes	$\begin{pmatrix} 0.997\angle -116.6^\circ & 0 \\ 58.9\angle 71.8^\circ & 0.426\angle -128.6^\circ \end{pmatrix}$

Simple Proof

S-parameter magnitude do not change when cables are connected, if $Z_{0,Spara} = Z_{0,Cable}$

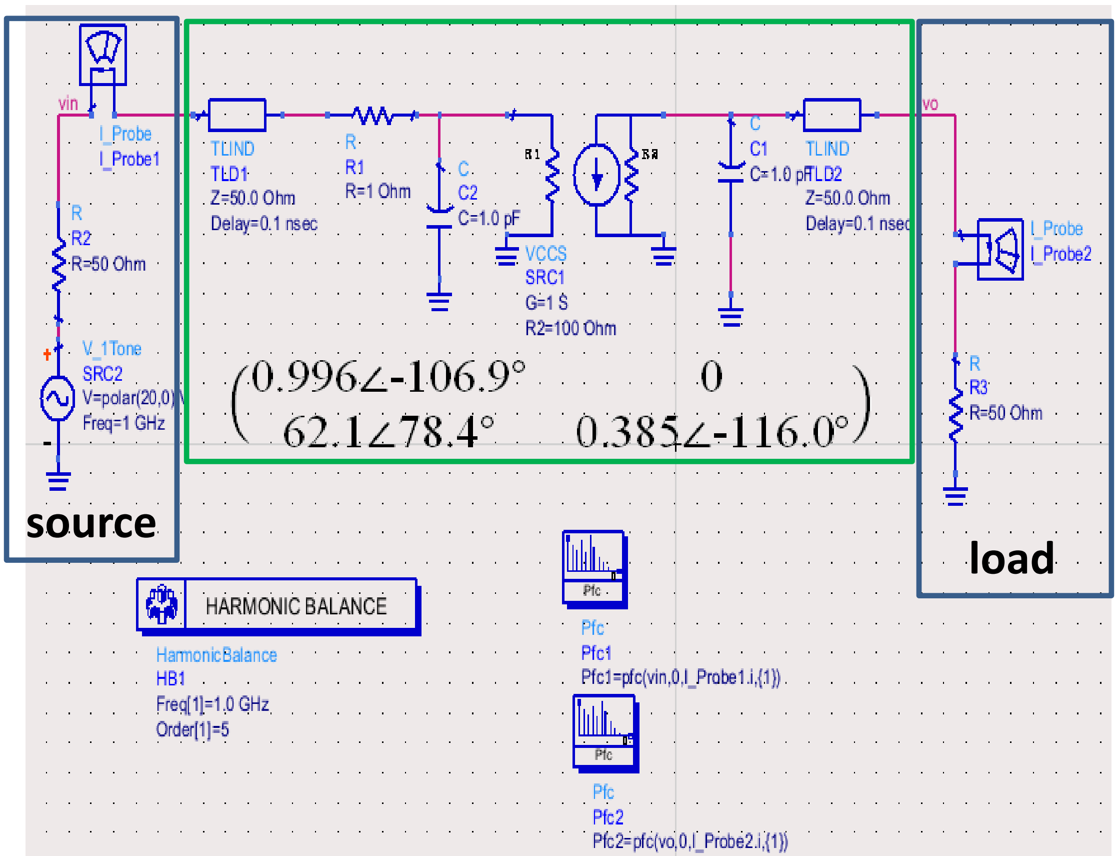


$$\begin{pmatrix} V_1^-\exp(+j\beta l_1) \\ V_2^-\exp(+j\beta l_2) \end{pmatrix} = \left(S_{\text{no cable}}\right) \begin{pmatrix} V_1^+\exp(-j\beta l_1) \\ V_2^+\exp(-j\beta l_2) \end{pmatrix} \qquad \begin{pmatrix} V_1^- \\ V_2^- \end{pmatrix} = \left(S_{\text{with cable}}\right) \begin{pmatrix} V_1^+ \\ V_2^+ \end{pmatrix}$$

$$\left(S_{\text{with cable}}\right) = \begin{pmatrix} \exp(-j\beta l_1) & 0 \\ 0 & \exp(-j\beta l_2) \end{pmatrix} \left(S_{\text{no cable}}\right) \begin{pmatrix} \exp(-j\beta l_1) & 0 \\ 0 & \exp(-j\beta l_2) \end{pmatrix} = \begin{pmatrix} S_{11}\exp[-j\beta(2l_1)] & S_{12}\exp[-j\beta(l_1+l_2)] \\ S_{21}\exp[-j\beta(l_1+l_2)] & S_{22}\exp[-j\beta(2l_2)] \end{pmatrix}$$

Why S-Parameters ? (B)

S-parameter has a straightforward connection to the power flow in a microwave system



- In microwave systems, load and source impedances are usually 50Ω (why?)
- In this case, the source can provide maximum power of 1 W

  Power to the load:
  $1 * |S_{21}|^2 = 3856$

- Power into the circuit:
  $1 * (1 - |S_{11}|^2) = 1 - 0.9964^2 = 0.0072$

(S_{11} = 0.996 indicates most power from the source is reflected)

- **Matching:** Create a 50-Ω load impedance for the source so all the 1-W power can be delivered into the circuit

Typical Commercial Amplifier S-parameters

ZX60-183-S+

BROADBAND AMPL / SMA / RoHS

Connector Type: SMA

Connector types may vary. Please refer to datasheet for details.

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Quantity	Unit Price
1 - 4	$239.95
5 - 9	$219.95
10 or more	$209.95

will learn in later lectures

GAIN: $10\log(|S_{21}|)$ DIRECTIVITY: $10\log(|S_{21}|/|S_{12}|)$ VSWR IN: $(1+|S_{11}|)/(1-|S_{11}|)$ VSWR OUT: $(1+|S_{22}|)/(1-|S_{22}|)$

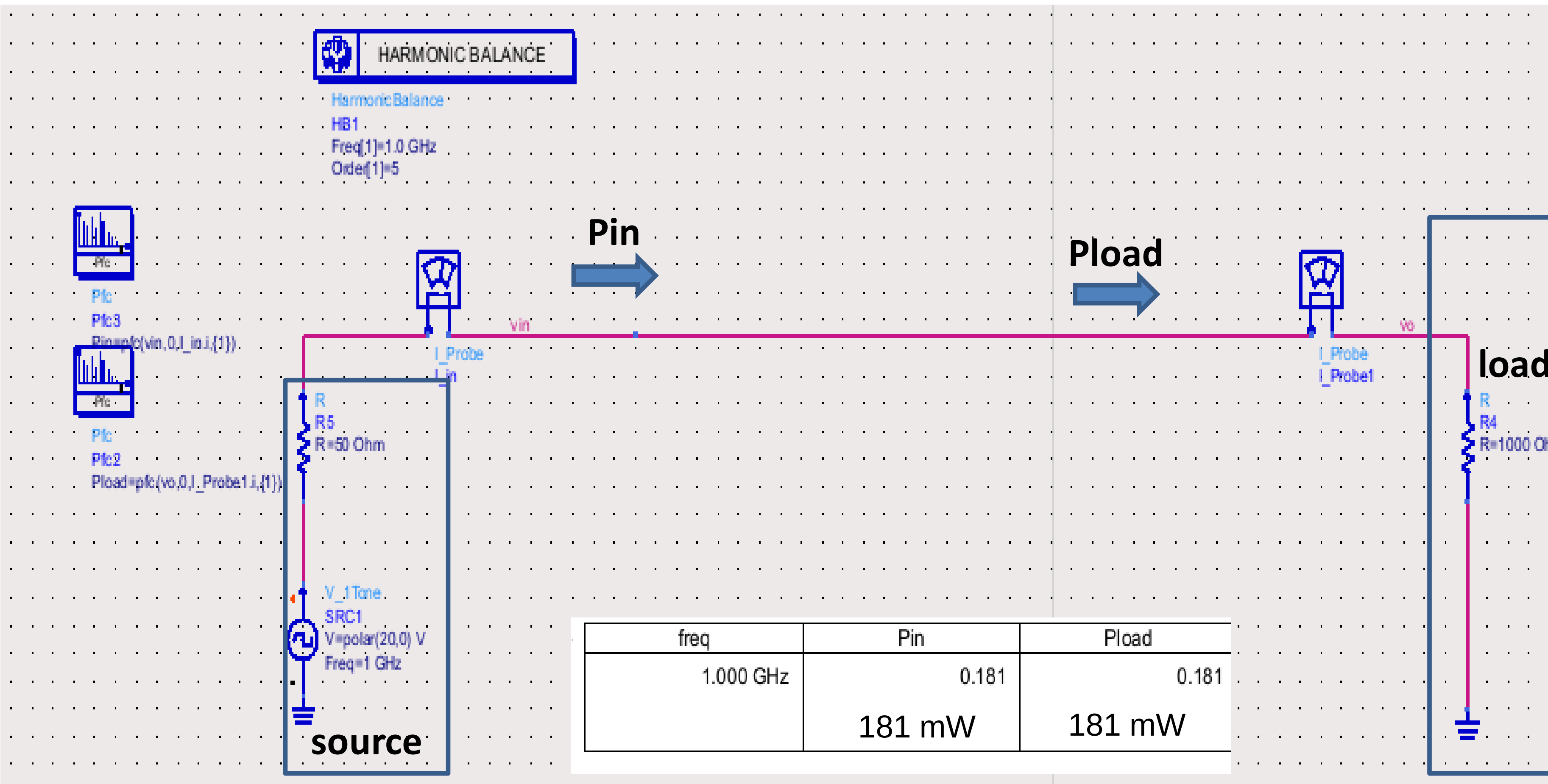
FREQUENCY (MHz)	GAIN (dB)	DIRECTIVITY	VSWR IN (:1)	VSWR OUT (:1)	POWER OUT @1 dB COMPR. (dBm)	IP3 (dBm)	NF (dB)
6000.00	24.24	43.74	1.13	1.23	17.80	27.78	7.40
7000.00	24.13	41.84	1.06	1.31	18.44	27.98	7.03
8000.00	24.19	38.28	1.26	1.47	18.49	27.72	6.90
9000.00	23.91	36.50	1.57	1.62	18.73	27.47	7.08

- Notice that the amplifier module is designed with $|S_{11}|$ and $|S_{22}|$ close to zero
- $|S_{11}|\sim 0$ means the module has a input impedance of 50Ω **when the load is 50Ω**
- $|S_{22}|\sim 0$ means the module has a output impedance of 50Ω **when the source is 50Ω**

Recall: $S_{11} = V_1^-/V_1^+$ if $V_2^+ = 0$; $S_{21} = V_2^-/V_1^+$ if $V_2^+ = 0$

Impedance Matching Using Smith Chart

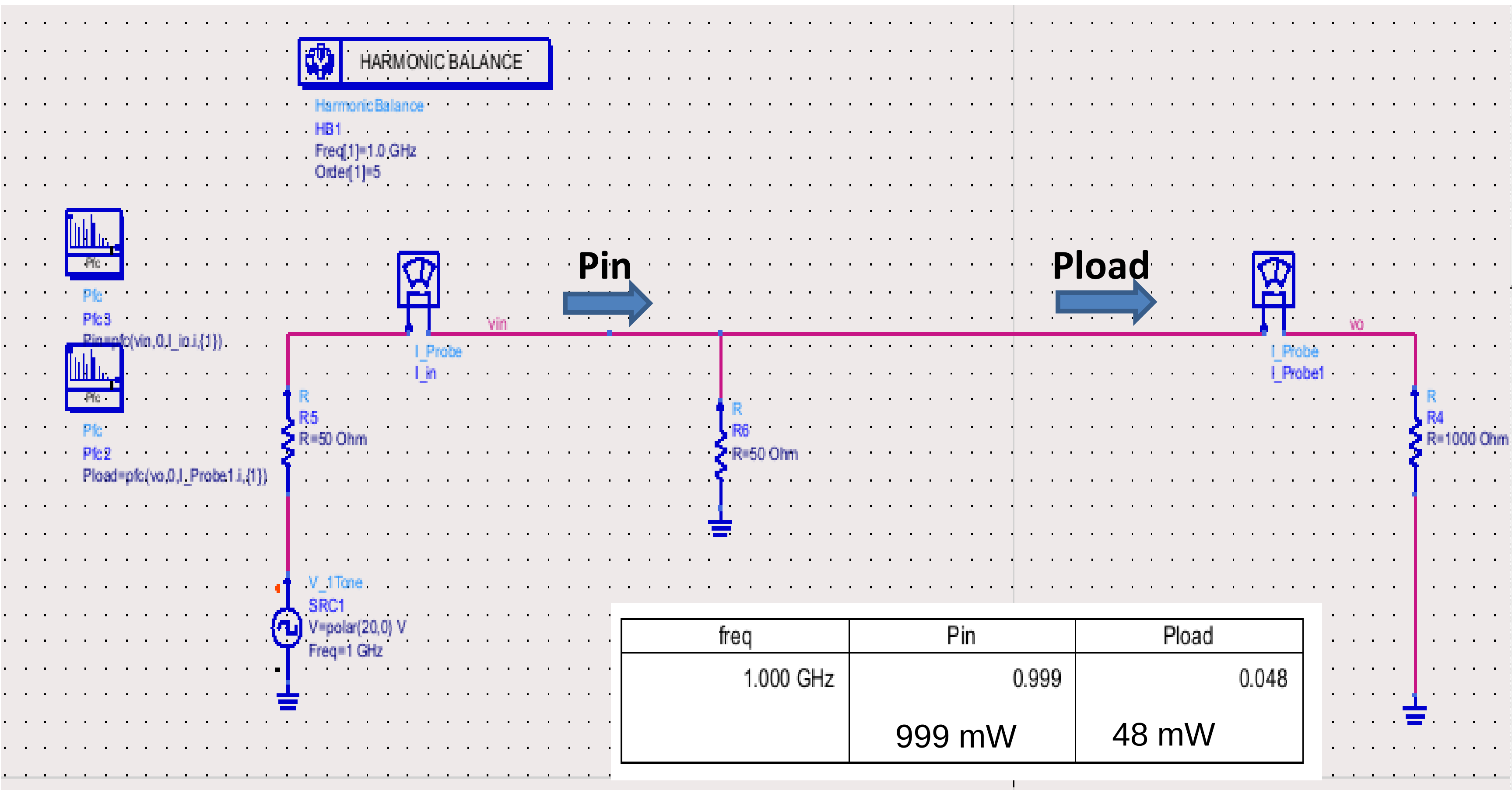
How can I extract the maximum available power (1 W) from the source ?
*****The source can deliver a maximum power of 1 W*****



freq	Pin	Pload
1.000 GHz	0.181	0.181
	181 mW	181 mW

Impedance Matching Using Smith Chart

Put a 50 ohm shunt resistor so the source see a ~50 ohm load

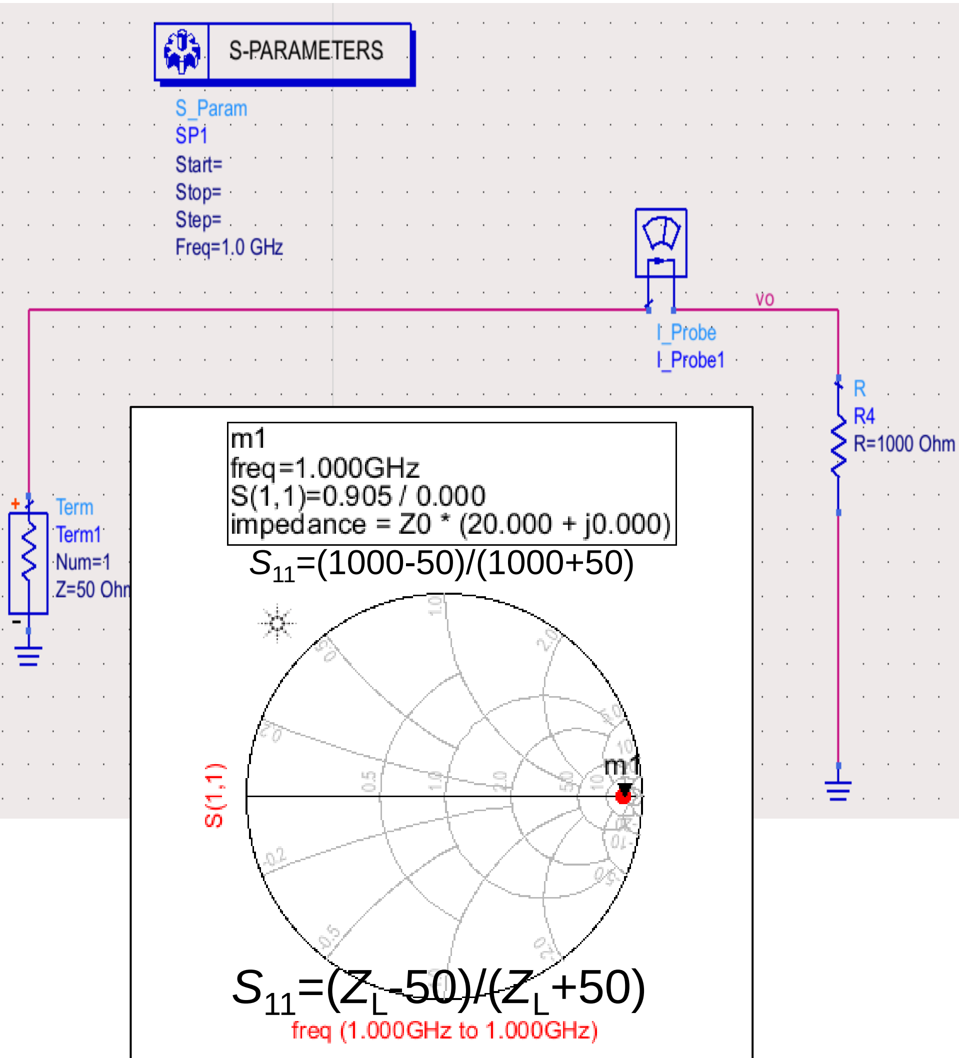


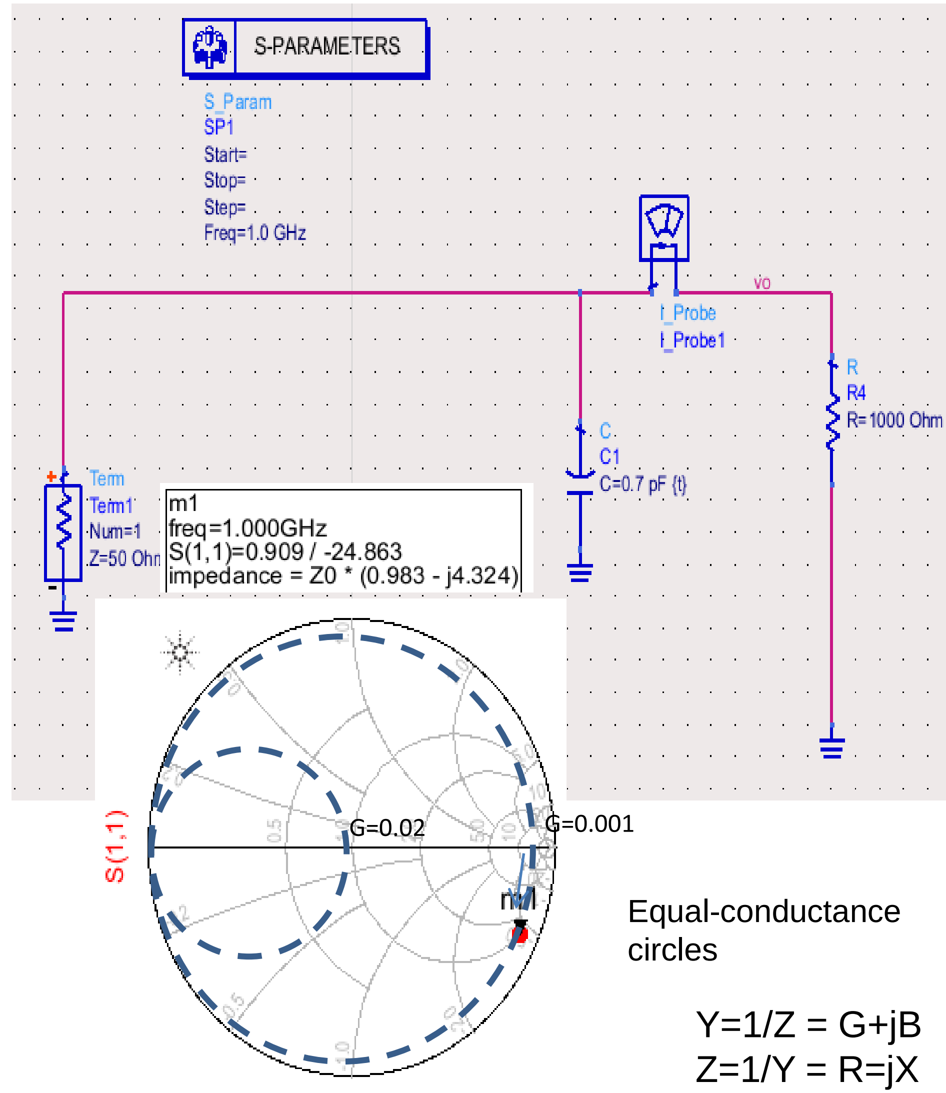
freq	Pin	Pload
1.000 GHz	0.999	0.048
	999 mW	48 mW

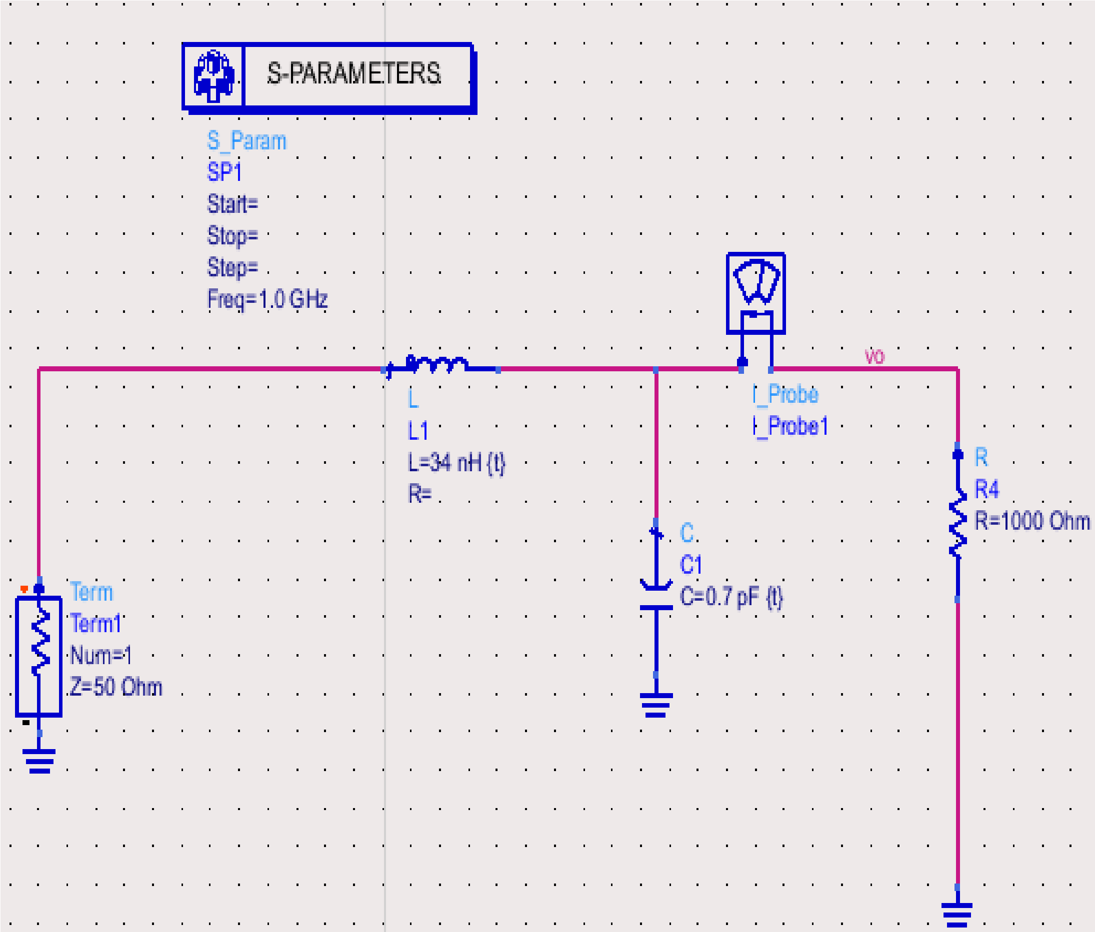
More power is extracted from the source (999 mW)…. But…

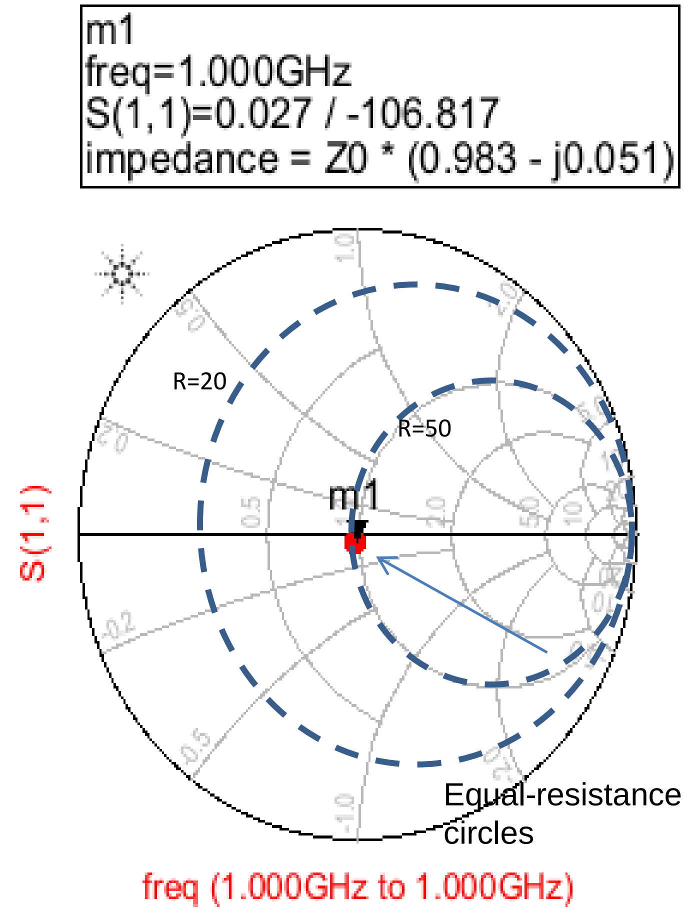
Impedance Matching Using Smith Chart

- Present a 50-Ω impedance to source to extract the highest power using (low-loss) matching
- The 50-Ω input impedance does not vary when a cable is introduced between circuit and source

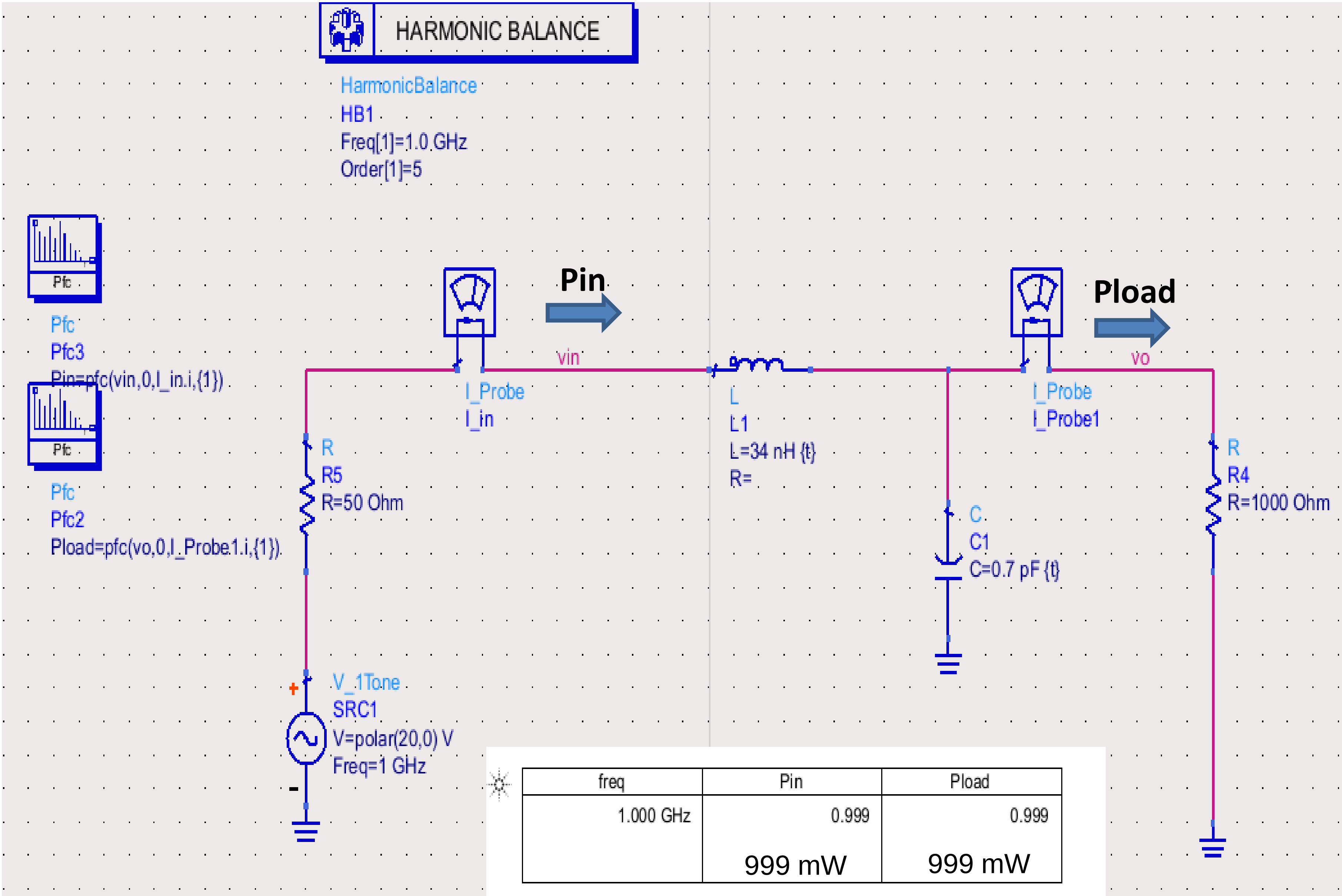








- Shunt-series matching
- First step: Move S11 on Smith Chart (along the equal-conductance circle) to a point on the equal-resistance circle with R = 50
- Second step: Move S11 on Smith Chart (along the equal-resistance circle) to S11 =0
- The shunt component can be an inductor, but then the series one must be a capacitor



freq	Pin	Pload
1.000 GHz	0.999 999 mW	0.999 999 mW

Great! Now the maximum power (1 W) is delivered to the load

Hints for HW3.4

- Refer to

EECS 117 *Lecture 6: Lossy Transmission Lines and the Smith Chart*

http://rfic.eecs.berkeley.edu/~niknejad/ee117/pdf/lecture6.pdf

I suggest using microstrip line for the transmission line and use ADS linecal

T-line quarter-wave resonator: $Q = \beta/2\alpha = \pi/(\lambda\alpha)$

α *(without skin effect)* = R'/Z_0,

$R' = 0.025/\text{width}$ and Z_0 is a function of line width and substrate thickness (10 um)

Maybe there is an optimal width and Z_0 for the best Q

If you want to use meander T-line, assume the distance between two parallel lines is 40 um

LC tank resonator: $Q = \omega L/R_L$, where R_L is the series resistance of the inductor

Loop inductor:

$$L_{loop} \approx \mu_o \mu_r \left(\frac{D}{2}\right) \cdot \left(\ln\left(\frac{8 \cdot D}{d}\right) - 2\right)$$