1. (a). $\varepsilon_r = 4$

$$v = \frac{1}{\sqrt{\varepsilon_r}} \cdot c = \frac{1}{\sqrt{4}} \cdot 3\times10^8 m/s = 1.5\times10^8 m/s$$

$\lambda f = v$

$$\lambda = \frac{v}{f} = \frac{1.5\times10^8 m/s}{1\times10^{10} m/s} = 15mm$$

$$\frac{d}{\lambda} = 1.5$$

$d = 1.5\lambda$

① Use Smith chart. normalize z_L

$$z_L = \frac{Z_L}{Z_0} = \frac{80-j40}{100} = 0.8-0.4j$$

So $r=0.8$, $x=-0.4$ And find the corresponding point on the chart.

So $|\rho_L| \simeq 0.25$, $\measuredangle \rho_L \simeq 250°$

②. Use calculation

$$\rho_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{80-40j-100}{80-40j+100} = 0.2425\, e^{j256°}$$

At the source, since $d = 1.5\lambda$

$\therefore \rho_L(\text{at the source}) = \rho_L(\text{at the load}) = 0.2425\, e^{j256°}$.

(b). ① Use Smith chart

$z = 18.75mm$

$$\frac{z}{\lambda} = 1.25 \quad \text{quarter wavelength.}$$

draw a circle, and use the opposite point on the circle.

②. read its r and x from the chart.

about $100+50j$.

②. Use calculation

Since it is quarter wavelength

$$z_{in} = \frac{z_0^2}{z_L} = \frac{100^2}{80-40j} = 100+50j$$

$\therefore$ input impedance at the source is $80-40j$ (half wave length, so same as Z_L)

input impedance at $z=-18.75mm$ is $100+50j$.

1. (c). $Z_S = 100\Omega$

$$V_{in} = \frac{Z_{in}}{Z_{in}+Z_S}\cdot V_S = V^+(1+\rho_{LS})$$

$$\rho_{LS} = \rho_{LL} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-2-4j}{18-4j} = 0.2425\, e^{j\,256°}$$

$Z_{in} = Z_L$

$Z_{in} = 80 - 40j$

$$\frac{80-40j}{80-40j+100}\cdot 10 = V^+\left(1+\frac{80-40j-100}{80-40j+100}\right)$$

$$= V^+\left(\frac{80-40j+100+80-40j-100}{80-40j+100}\right)$$

$$= V^+\left(\frac{160-80j}{80-40j+100}\right)$$

$\therefore V^+ = 5V$

$|V^+| = 5V$

① For SWR, just find from the Smith Chart.

SWR $\simeq 1.65$

②. For SWR, Calculation.

$$SWR = \frac{1+|\rho_L|}{1-|\rho_L|} = \frac{1+0.2425}{1-0.2425} = 1.64$$

$$SWR = \frac{|V_{max}|}{|V_{min}|} = 1.64$$

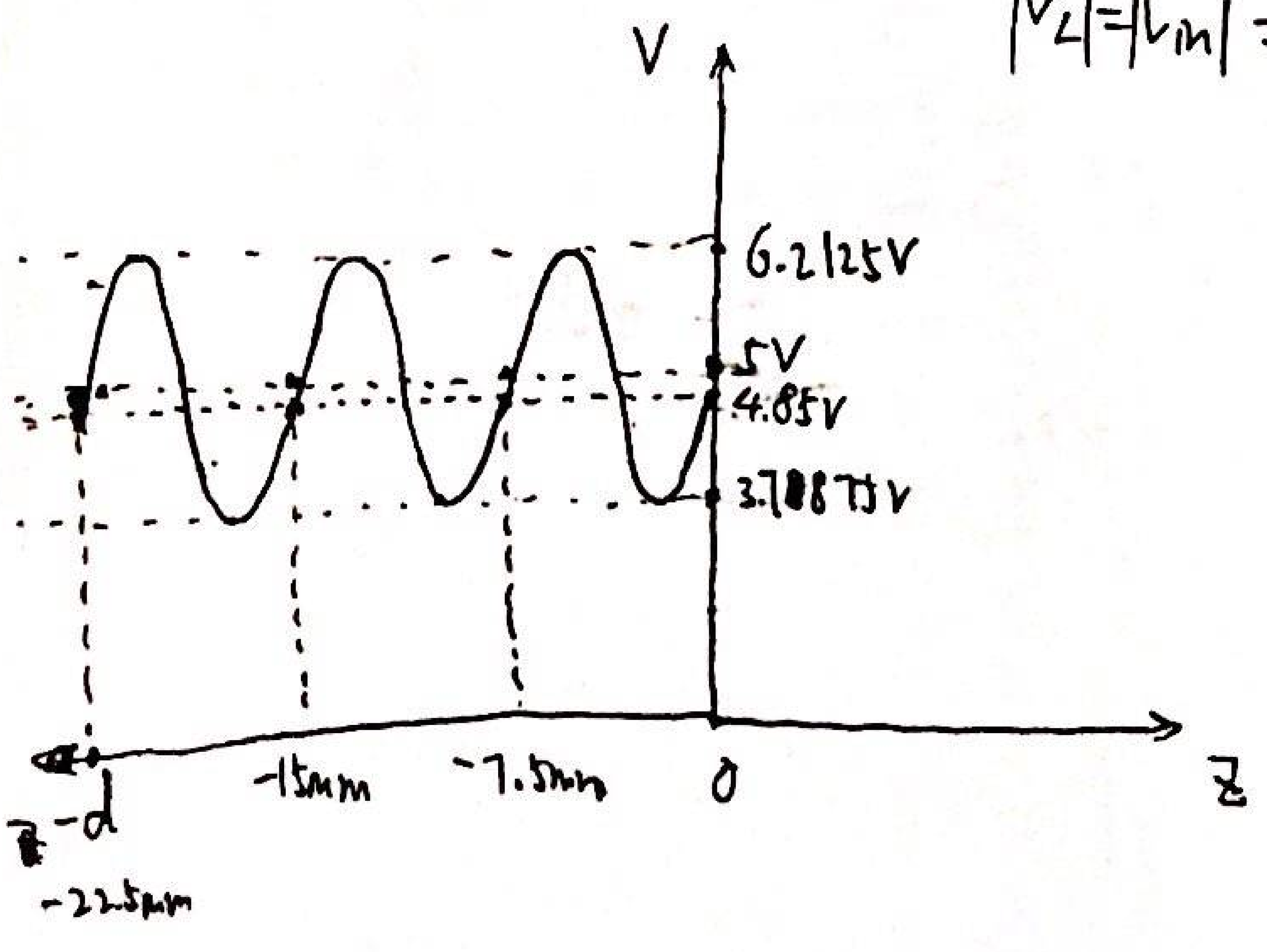
$$|V_{max}| = (1+|\rho_L|)\cdot|V^+| = 1.2425\cdot 5 = 6.2125V$$

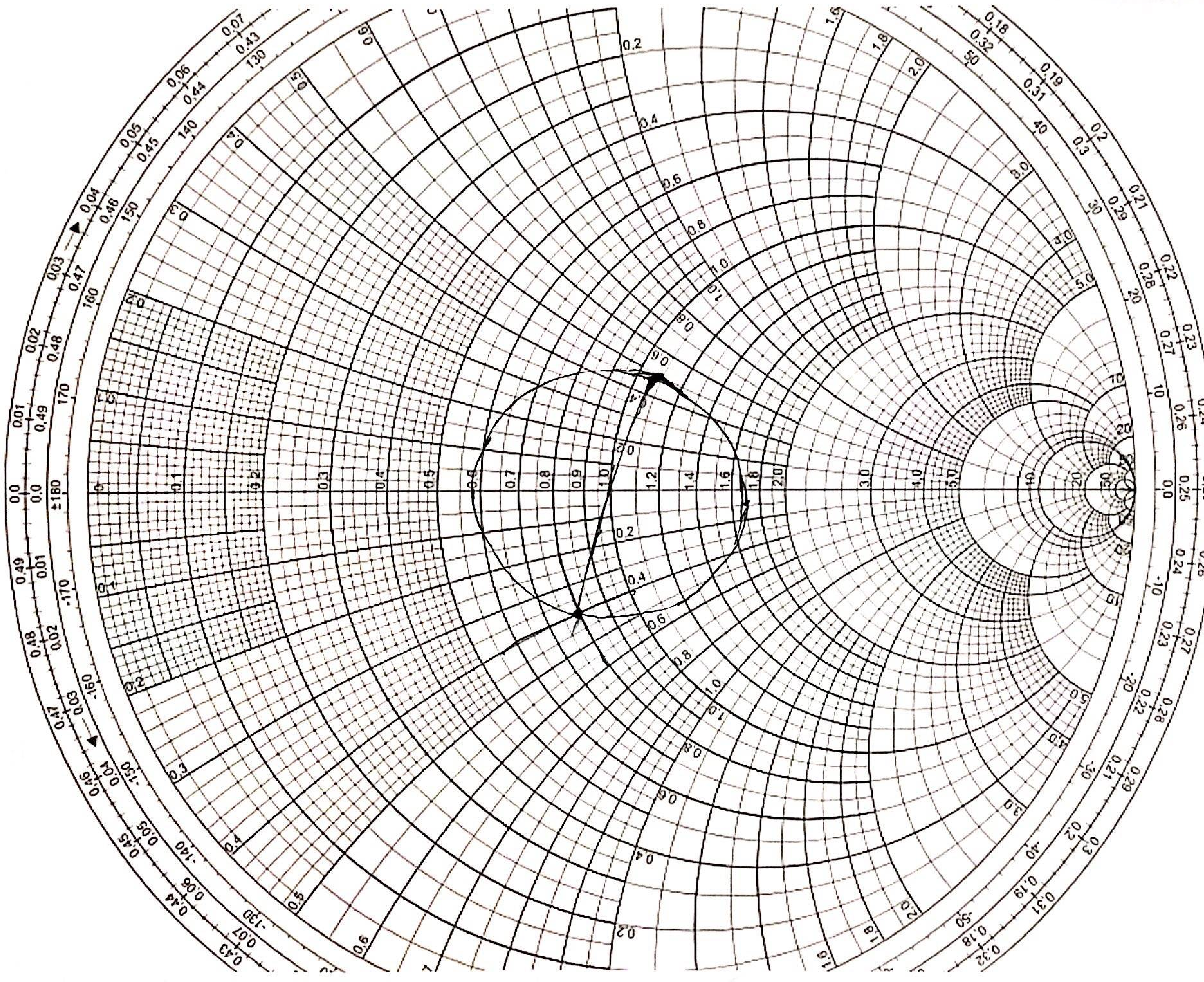
$$|V_{min}| = (1-|\rho_L|)\cdot|V^+| = 3.7875V$$

$$|V_L| = |V_{in}| = |V^+\cdot(1+\rho_L)| = 5\cdot\left(1+\frac{80-40j-100}{80-40j+100}\right) \simeq |5\cdot 0.97\, e^{-14° j}|$$

$$= |5\cdot 0.97|$$

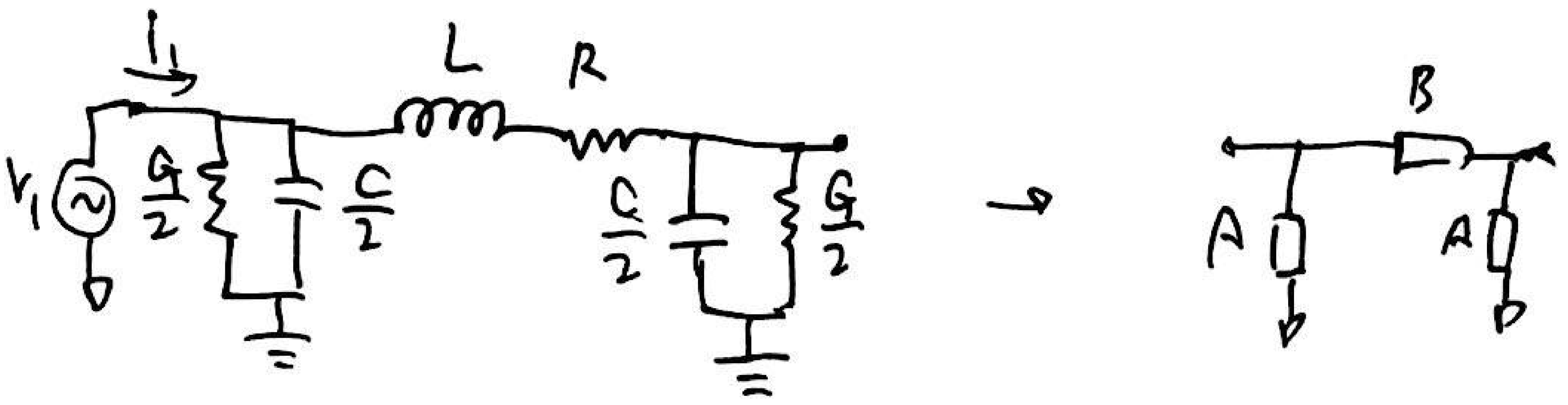
$$= 4.85V$$




0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.2
1.4
1.6
1.8
2.0
3.0
4.0
5.0
10
20
50

2. (a).

$$Z_{11} = \left.\frac{V_1}{i_1}\right|_{i_2=0}$$

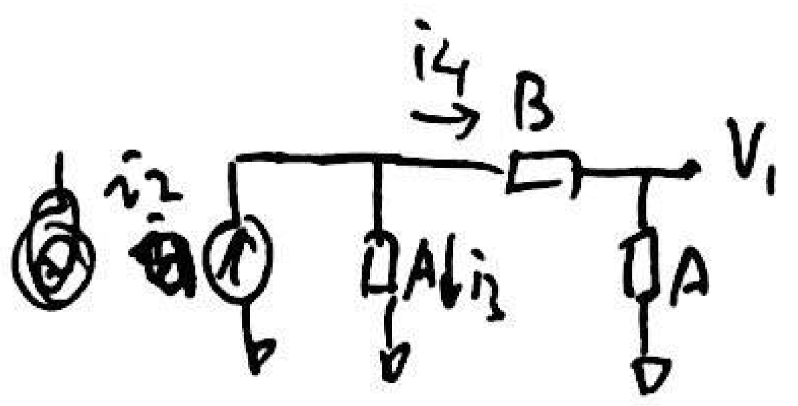


$$Z_{11} = \left(\frac{G}{2} \parallel \frac{C}{2}\right) \parallel \left[L+R+\left(\frac{C}{2}\parallel\frac{G}{2}\right)\right]$$ ("+" means in series)

$$= \left(\frac{2}{G} \parallel \frac{2}{j\omega C}\right) \parallel \left[j\omega L+R+\left(\frac{2}{j\omega C}\parallel\frac{2}{G}\right)\right] = A \parallel (A+B)$$

$$Z_{22} = Z_{11}$$

$$Z_{12} = Z_{21} = \left.\frac{V_1}{i_2}\right|_{i_1=0}.$$



$$\begin{cases} V_1 = i_4 \cdot A \\ i_2 = i_3 + i_4 \\ i_3 A = i_4(A+B) \end{cases}$$

$$\therefore\ V_1 = i_4 \cdot A = \frac{A}{2A+B}\cdot i_2 \cdot A$$

$$Z_{12} = Z_{21} = \frac{A^2}{2A+B}$$ $$A = \frac{2}{G} \parallel \frac{2}{j\omega C} = \frac{2}{j\omega C + G}$$

$$B = j\omega L + R$$

$$= \frac{\left(\frac{2}{G}\parallel\frac{2}{j\omega C}\right)^2}{2\cdot\left(\frac{2}{G}\parallel\frac{2}{j\omega C}\right)+j\omega L+R}$$

$$\therefore Z_{matrix} = \begin{bmatrix} \left(\frac{2}{G}\parallel\frac{2}{j\omega C}\right)\parallel\left[j\omega L+R+\left(\frac{2}{j\omega C}\parallel\frac{2}{G}\right)\right] & \frac{\left(\frac{2}{G}\parallel\frac{2}{j\omega C}\right)^2}{2\cdot\left(\frac{2}{G}\parallel\frac{2}{j\omega C}\right)+j\omega L+R} \\ \frac{\left(\frac{2}{G}\parallel\frac{2}{j\omega C}\right)^2}{2\left(\frac{2}{G}\parallel\frac{2}{j\omega C}\right)+j\omega L+R} & \left(\frac{2}{G}\parallel\frac{2}{j\omega C}\right)\parallel\left[j\omega L+R+\left(\frac{2}{j\omega C}\parallel\frac{2}{G}\right)\right] \end{bmatrix}$$

$$= \begin{bmatrix} A\parallel(A+B) & \frac{A^2}{2A+B} \\ \frac{A^2}{2A+B} & A\parallel(A+B) \end{bmatrix}$$

(More detailed organized form is in (c).)

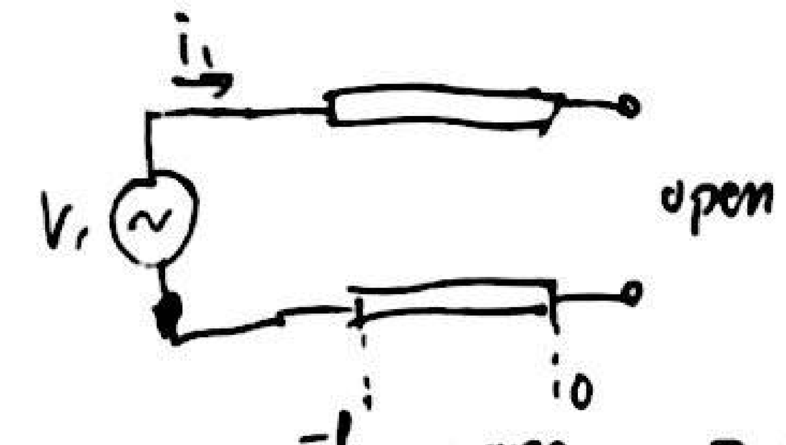
2. (b). $\gamma = \alpha + j\beta = \sqrt{(R' + j\omega L')(G' + j\omega C')}$

$$V(z) = V^+ e^{-\gamma z} + V^- e^{\gamma z} = V^+ \left(e^{-\gamma z} + \rho_L e^{\gamma z}\right)$$

$$i(z) = \frac{V^+}{Z_0} e^{-\gamma z} - \frac{V^-}{Z_0} e^{\gamma z} = \frac{V^+}{Z_0}\left(e^{-\gamma z} - \rho_L e^{\gamma z}\right)$$

$$Z_{in}(z) = \frac{V(z)}{i(z)} = Z_0 \cdot \frac{e^{-\gamma z} + \rho_L e^{\gamma z}}{e^{-\gamma z} - \rho_L e^{\gamma z}} = Z_0 \cdot \frac{1 + \rho_L e^{2\gamma z}}{1 - \rho_L e^{2\gamma z}}, \quad \rho_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_{11} = Z_{in}(-l) = Z_0 \cdot \frac{1 + \rho_L e^{2\gamma z}}{1 - \rho_L e^{2\gamma z}}$$

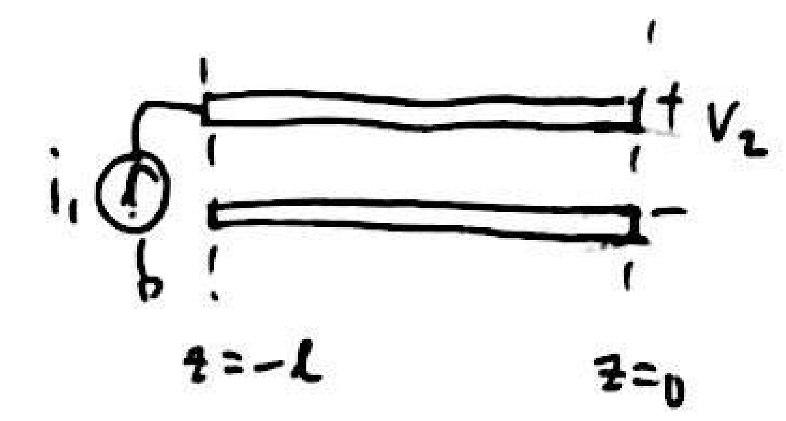


$-l$. ~~open~~ . $\therefore Z_{11} = \frac{V_1}{i_1}\Big|_{i_2=0}$ open

$\therefore \rho_L = 1$

$$\therefore Z_{22} = Z_{11} = Z_0 \frac{1 + e^{-2\gamma l}}{1 - e^{-2\gamma l}}, \quad Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}, \quad \gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

$$Z_{12} = Z_{21} = \frac{V_2}{i_1}\Big|_{i_2=0}$$



$$i_2 = i(0) = \frac{V^+}{Z_0}(1 - \rho_L) = 0 \Rightarrow \rho_L = 1$$

$$V_1 = V(-l) = V^+\left(e^{\gamma l} + \rho_L e^{-\gamma l}\right)$$
$$= V^+\left(e^{\gamma l} + e^{-\gamma l}\right)$$

$$i_1 = i(-l) = \frac{V^+}{Z_0}\left(e^{\gamma l} - \rho_L e^{-\gamma l}\right)$$
$$= \frac{V^+}{Z_0}\left(e^{\gamma l} - e^{-\gamma l}\right)$$

$$V_2 = V(0) = V^+(1 + \rho_L) = 2V^+$$

$$\therefore Z_{12} = Z_{21} = \frac{V_2}{i_1}\Big|_{i_2=0}$$
$$= \frac{2Z_0}{e^{\gamma l} - e^{-\gamma l}}, \quad Z_0 = \sqrt{\frac{L'}{C'}}, \quad \gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

$$Z_{matrix} = \begin{bmatrix} \frac{1 + e^{-2\gamma l}}{1 - e^{-2\gamma l}} & \frac{2}{e^{\gamma l} - e^{-\gamma l}} \\ \frac{2}{e^{\gamma l} - e^{-\gamma l}} & \frac{1 + e^{-2\gamma l}}{1 - e^{-2\gamma l}} \end{bmatrix} \cdot Z_0 = \begin{bmatrix} \frac{e^{\gamma l} + e^{-\gamma l}}{e^{\gamma l} - e^{-\gamma l}} & \frac{2}{e^{\gamma l} - e^{-\gamma l}} \\ \frac{2}{e^{\gamma l} - e^{-\gamma l}} & \frac{e^{\gamma l} + e^{-\gamma l}}{e^{\gamma l} - e^{-\gamma l}} \end{bmatrix} \cdot Z_0$$

2.(b) continued

$$A \parallel (A+B) = \frac{\frac{2}{j\omega C+G}\cdot\left(j\omega L+R+\frac{2}{j\omega C+G}\right)}{\frac{4}{j\omega C+G}+j\omega L+R}$$

$$= \frac{2\cdot\left[(j\omega L+R)(j\omega C+G)+2\right]}{\cancel{4j\omega C+4G}\ 4(j\omega C+G)+(j\omega L+R)(j\omega C+G)^2}$$

$$\frac{A^2}{2A+B} = \frac{\left(\frac{2}{j\omega C+G}\right)^2}{\frac{4}{j\omega C+G}+j\omega L+R} = \frac{4}{4(j\omega C+G)+(j\omega L+R)(j\omega C+G)^2}$$

$$Z_{matrix} = Z_0\cdot\begin{bmatrix} \frac{e^{\gamma l}+e^{-\gamma l}}{e^{\gamma l}-e^{-\gamma l}} & \frac{2}{e^{\gamma l}-e^{-\gamma l}} \\ \frac{2}{e^{\gamma l}-e^{-\gamma l}} & \frac{e^{\gamma l}+e^{-\gamma l}}{e^{\gamma l}-e^{-\gamma l}} \end{bmatrix}$$

$$\therefore \begin{cases} e^{\gamma l}+e^{-\gamma l} = \left[(j\omega L+R)(j\omega C+G)+2\right] \\ \frac{1}{Z_0}(e^{\gamma l}-e^{-\gamma l}) = \frac{1}{2}\left[4(j\omega C+G)+(j\omega L+R)(j\omega C+G)^2\right] = 2(j\omega C+G)+\frac{1}{2}(j\omega L+R)(j\omega C+G)^2 \end{cases}$$

(c)

$$\cancel{e^{\gamma l}+e^{-\gamma l}}$$

$$e^{\gamma l} \simeq 1+\gamma l+\frac{1}{2}\gamma^2 l^2+\frac{1}{6}\gamma^3 l^3$$

$$e^{-\gamma l} \simeq 1-\gamma l+\frac{1}{2}\gamma^2 l^2-\frac{1}{6}\gamma^3 l^3$$

$$e^{\gamma l}+e^{-\gamma l} = 2+\gamma^2 l^2 \cancel{= 2+(\alpha+j\beta)^2 l^2}$$

$$e^{\gamma l}-e^{-\gamma l} = 2\gamma l+\frac{1}{3}\gamma^3 l^3$$

$$\cancel{\therefore \begin{cases} 2+\gamma^2 l^2 = 2+(j\omega L+R)(j\omega C+G) \\ (2\gamma l+\frac{1}{3}\gamma^3 l^3)\frac{1}{Z_0} = \frac{1}{Z_0}\gamma l(2+\frac{1}{3}\gamma^2 l^2) = 2(j\omega C+G)+\frac{1}{2}(j\omega L+R)(j\omega C+G)^3 \end{cases}}$$

~~To match each equation.~~

2 (c) continued.

$$Z_{matrix} = Z_0 \begin{bmatrix} \frac{2+\gamma^2 l^2}{2\gamma l + \frac{1}{3}\gamma^3 l^3} & \frac{2}{2\gamma l + \frac{1}{3}\gamma^3 l^3} \\ \frac{2}{2\gamma l + \frac{1}{3}\gamma^3 l^3} & \frac{2+\gamma^2 l^2}{2\gamma l + \frac{1}{3}\gamma^3 l^3} \end{bmatrix}$$

$$Z_0 = \sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}} \; . \quad \gamma = \sqrt{(R'+j\omega L')(G'+j\omega C')}$$

compare to simple mode

$$\begin{bmatrix} \frac{2+(j\omega L+R)(j\omega C+G)}{2(j\omega C+G)+\frac{1}{2}(j\omega L+R)(j\omega C+G)^2} & \frac{2}{2(j\omega C+G)+\frac{1}{2}(j\omega L+R)(j\omega C+G)^2} \\ \frac{2}{2(j\omega C+G)+\frac{1}{2}(j\omega L+R)(j\omega C+G)^2} & \frac{2+(j\omega L+R)(j\omega C+G)}{2(j\omega C+G)+\frac{1}{2}(j\omega L+R)(j\omega C+G)^2} \end{bmatrix}$$

(d). From observation

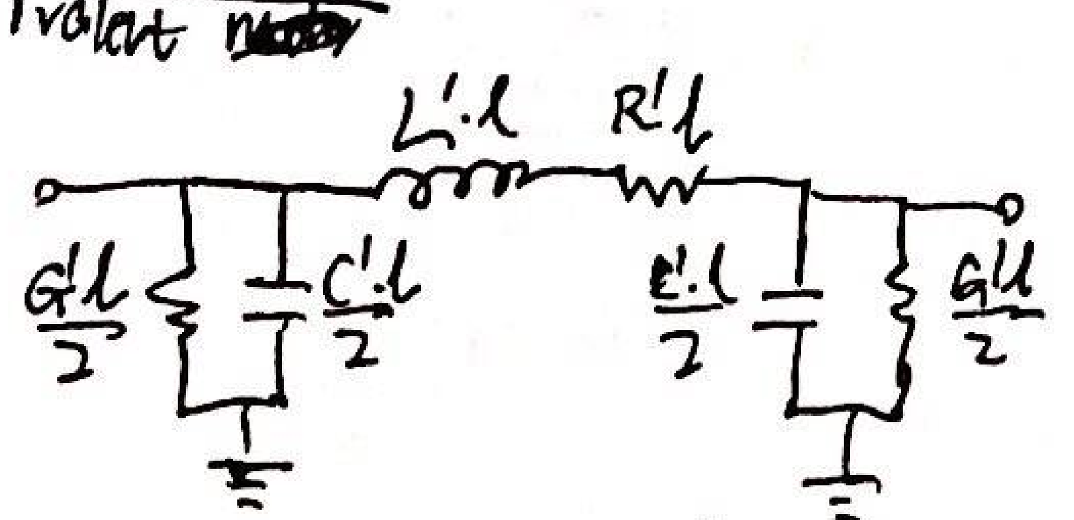
$$\begin{cases} 2+\gamma^2 l^2 = 2+(j\omega L+R)(j\omega C+G) \\ \frac{1}{Z_0}\cdot \gamma l\left(2+\frac{1}{3}\gamma^2 l^2\right) = (j\omega C+G)\cdot\left[2+\frac{1}{2}(j\omega L+R)(j\omega C+G)\right] \end{cases}$$

$$\therefore \begin{cases} \gamma l = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{(R'+j\omega L')(G'+j\omega C')}\cdot l \\ \frac{\gamma l}{Z_0} = j\omega C+G = (j\omega C'+G')l \end{cases}$$

$$\therefore \begin{cases} R = R' l \\ G = G' l \\ C = C' l \\ L = L' l \end{cases}$$

(The 3rd order has coefficient mismatch, but 0, 1, 2 order match, so the approximation is very close)

(e) So the model is ~~equivalent model~~



so the pi model is a good approximation.

3. (a) Load impedance should be $50\,\Omega$.

$$P_{max} = \frac{1}{2}\left(\frac{V_{PP}}{2\cdot 2R}\right)^2 \cdot R = \frac{V_{PP}^2}{32}\cdot\frac{1}{50\,\Omega} = \frac{1}{32}\cdot\frac{1}{50} = 0.625\,mW$$

(b).

$$P_L = \frac{1}{8}\cdot\left(\frac{V_{PP}}{R+Z_L}\right)^2\cdot Z_L = \frac{1}{8}\left(\frac{1}{50+500}\right)^2\cdot 500 = 0.2066\,mW$$

$$V_L = \frac{Z_L}{Z_L+R}\cdot V_S = 0.909\,V_{PP}$$

(c).

$$Z_0 = \sqrt{R\cdot Z_L} = \sqrt{50\cdot 500} \approx 158.1\,\Omega$$

$$V_L = \frac{Z}{Z+R}\cdot V_S = 0.905\,V_{PP}$$

$$P_L = \frac{1}{8}\cdot\left(\frac{V_{PP}}{2R}\right)^2\cdot R = 0.625\,mW$$

(d).

$$\rho_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{500-158.1}{500+158.1} = 0.52,$$

Let Δf be the diviation.

$$P_{new} = \frac{1}{8}\frac{V_{PP}^2}{|Z_{in}+R|^2}\cdot Re\{Z_{in}\} = \frac{1}{2}P_{old} = \frac{1}{2}\cdot 0.625\,mW$$

$$Z_{in} = Z_0\cdot\frac{1+\rho_L e^{-2jk(\frac{\lambda}{4}+x)}}{1-\rho_L e^{-2jk(\frac{\lambda}{4}+x)}} = 50\cdot\frac{1-0.52\cdot e^{-2jkx}}{1+0.52\,e^{-2jkx}}$$

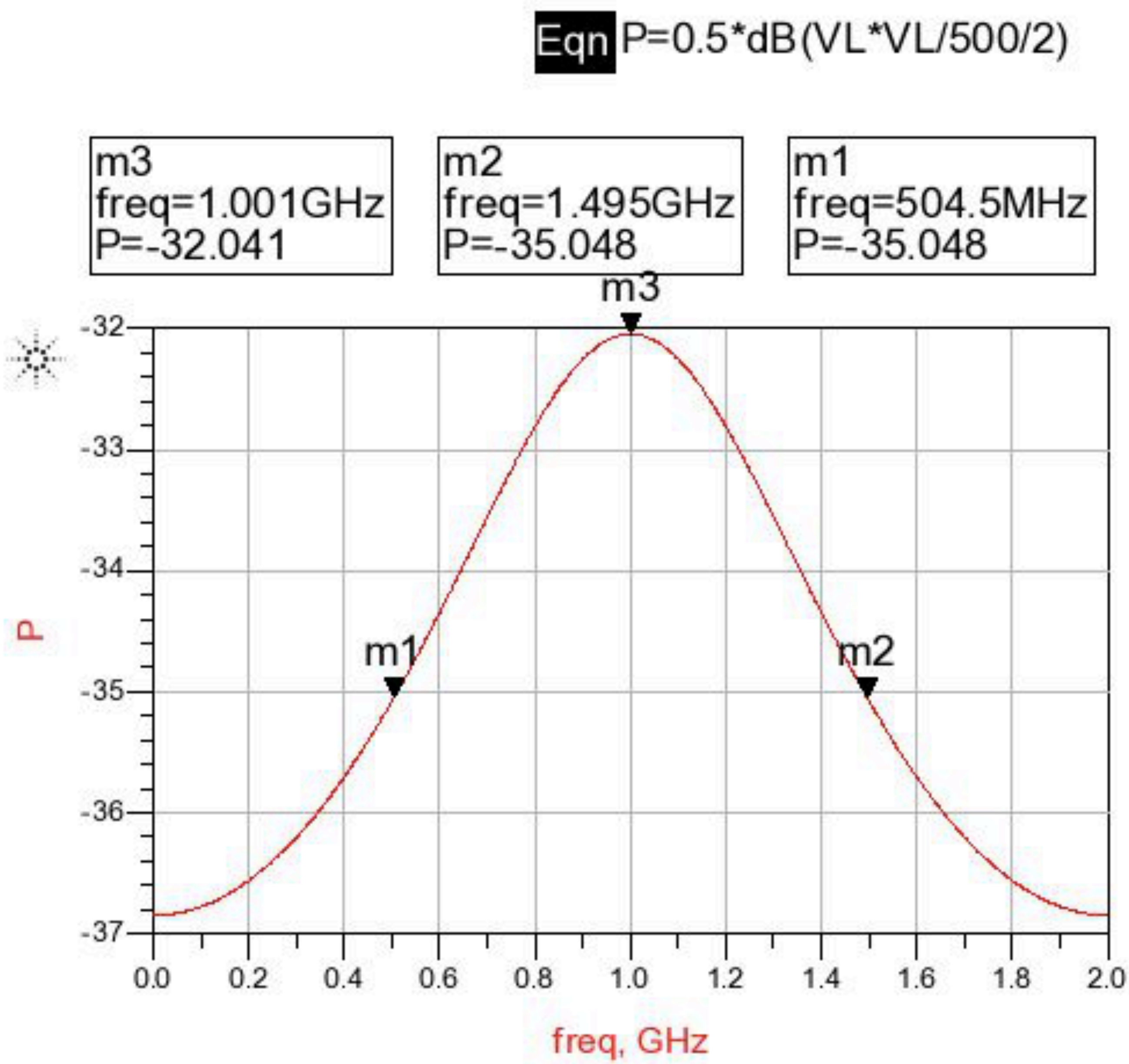
Solve the equation, $\frac{1}{|Z_{in}+R|^2}Re\{Z_{in}\} = 0.3125\,mW$

we get $x \approx \pm 11.13\,mm$

$\Delta f \simeq \pm\,495\,MHz$

$\therefore$ frequency range from $505\,MHz \sim 1.495\,GHz$.

The simulation yields same results.


Eqn P=0.5*dB(VL*VL/500/2)
m3
freq=1.001GHz
P=-32.041
m2
freq=1.495GHz
P=-35.048
m1
freq=504.5MHz
P=-35.048
m3
m1
m2
-32
-33
-34
-35
-36
-37
P
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
freq, GHz