HW4 SOL

1. Calculate the scattering parameters of the following circuits:

   (a) Find the input S_{11} for a general two-port terminated at port 2 with a load reflection coefficient of Γ_L.

   (b) In the previous problem, what is the power that reaches the load in terms of the two-port scattering parameters and Γ_L? Suppose the input is driven with a matched source.

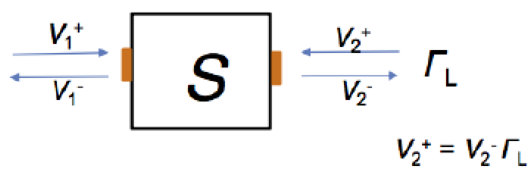
   (c) Derive the two-port scattering parameters of a three-port where port 3 is terminated in a load with reflection coefficient Γ_L.

1.a

As shown in Figure. 1, the directions of V_2^+ and V_2^- are different from those in transmission line. Thus, we have: $V_2^+ = \Gamma_L V_2^-$. Definition of S-parameter is:

$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+$$

$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+$$



From the equations above, we get:

$$V_1^+ = V_2^- \frac{(1-S_{22}\Gamma_L)}{S_{21}}$$

$$V_1^- = V_2^-[(1-S_{22}\Gamma_L)\frac{S_{11}}{S_{21}} + S_{12}\Gamma_L]$$

$$\therefore \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$$

1b

$$\therefore \frac{P_{load}}{P_{in}} = \frac{|v_2^-|^2(1-|\Gamma_L|^2)}{|v_1^+|^2 - |v_1^-|^2} = \frac{(1-|\Gamma_L|^2)}{\left|\frac{1-\Gamma_L S_{22}}{S_{21}}\right|^2 - \left|(S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L})\right|^2 \left|\frac{1-\Gamma_L S_{22}}{S_{21}}\right|^2}$$

$$= \frac{\left|\frac{S_{21}}{1-\Gamma_L S_{22}}\right|^2 (1-|\Gamma_L|^2)}{1 - \left|(S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L})\right|^2}$$

1.c

With port 3, we have:

$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+ + S_{13}V_3^+$$
$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+ + S_{23}V_3^+$$
$$V_3^- = S_{31}V_1^+ + S_{32}V_2^+ + S_{33}V_3^+$$
$$\because V_3^+ = \Gamma_L V_3^-$$

$\therefore$ Equations above can be written as:

$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+ + S_{13}V_3^+$$
$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+ + S_{23}V_3^+$$
$$0 = S_{31}V_1^+ + S_{32}V_2^+ + (S_{33} - \tfrac{1}{\Gamma_L})V_3^+$$

By solving equations above, we have:

$$V_1^- = (S_{11} + \tfrac{S_{13}S_{31}}{\frac{1}{\Gamma_L} - S_{33}})V_1^+ + (S_{12} + \tfrac{S_{13}S_{32}}{\frac{1}{\Gamma_L} - S_{33}})V_2^+$$
$$V_2^- = (S_{21} + \tfrac{S_{23}S_{31}}{\frac{1}{\Gamma_L} - S_{33}})V_1^+ + (S_{22} + \tfrac{S_{23}S_{32}}{\frac{1}{\Gamma_L} - S_{33}})V_2^+$$

$\therefore$ New S-parameter is:

$$\begin{bmatrix} S_{11} + \frac{S_{13}S_{31}}{\frac{1}{\Gamma_L} - S_{33}} & S_{12} + \frac{S_{13}S_{32}}{\frac{1}{\Gamma_L} - S_{33}} \\ S_{21} + \frac{S_{23}S_{31}}{\frac{1}{\Gamma_L} - S_{33}} & S_{22} + \frac{S_{23}S_{32}}{\frac{1}{\Gamma_L} - S_{33}} \end{bmatrix}$$

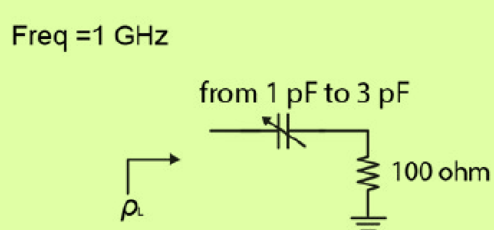
2. The Smith Chart is a graphical tool for convert between impedance and reflection coefficient. A blank Smith chart can be downloaded from `http://www.acs.psu.edu/drussell/Demos/SWR/SmithChart.pdf`

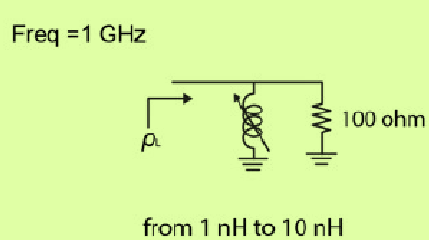
   (a) Assume $Z_0 = 50\Omega$. Using Smith Chart to find ρ_L for a load impedance $Z_L = 25 + 30j\Omega$.

   (b) Assume $Z_0 = 50\Omega$. Using Smith Chart to find the load impedance for $\rho_L = 0.5 + 0.1j$.

   (c) Repeat (a) and (b) with Z_0 changed to 10Ω.

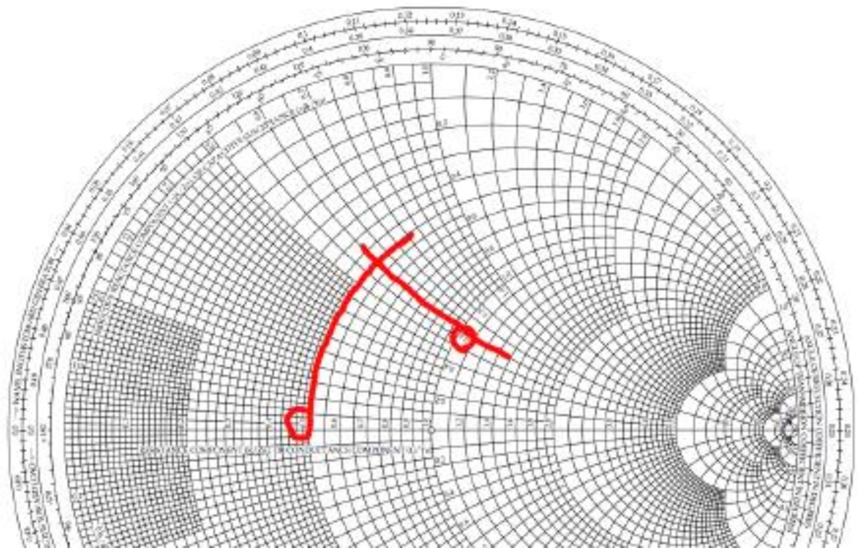
   (d) For the following two circuits, trace ρ_L on a Smith Chart with $Z_0 = 50\Omega$.





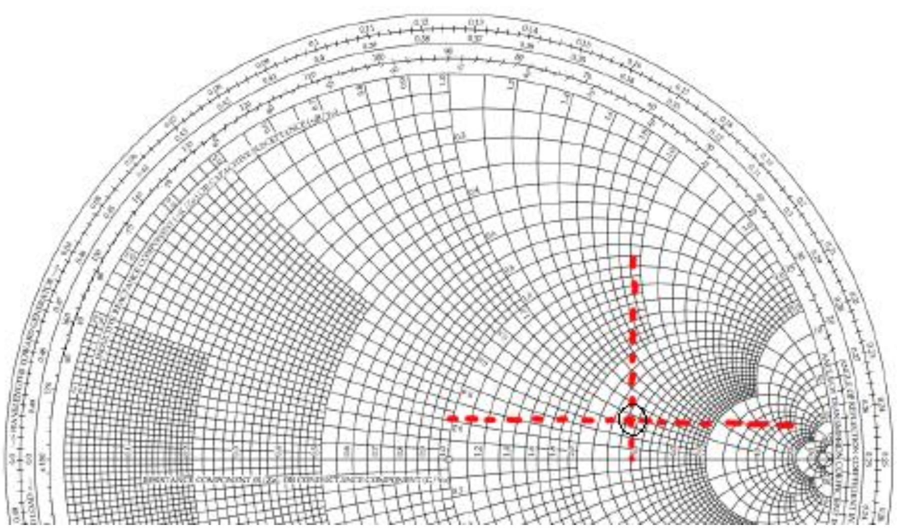
2.a

First of all, we need to calculate $z = \frac{Z_L}{Z_0} = 0.5 + 0.6j$. So what we need to do is to find 0.5 circle and $0.6j$ circle and their crosspoint is ρ_L. As shown in Figure. 2, the crosspoint is drawn. But to be honest, it's really hard to read out the accurate values of real and imaginary parts of ρ_L, so I still need to do calculation: $\rho_L = \frac{Z_L - Z_0}{Z_L + Z_0} = -0.1494 + 0.4598j$.



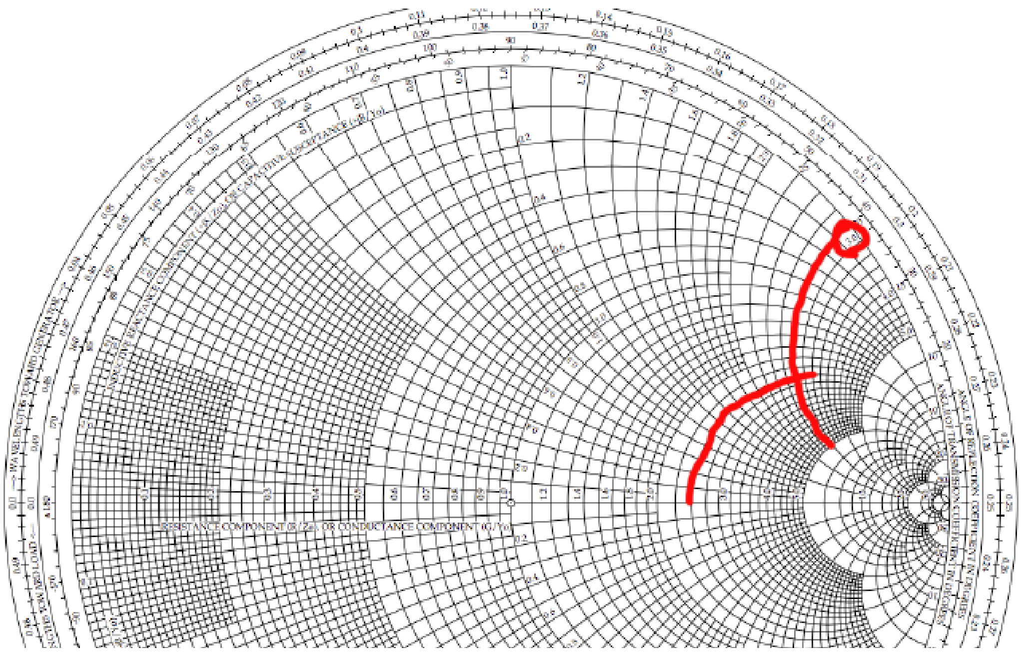
2.b

First, we need to find ρ_L on Smith chart as shown in Figure. 3. The crosspoint shown in Figure. 3 is the ρ_L. And then we need to read out the resistance and capacitance according to which circles the point is on. $z = 2.8462 + 0.7692j$. (Actually I calculate that, because it's really hard to read the chart.) So $Z_L = Z_0 \cdot z = 142.3 + 38.5j(\Omega)$.



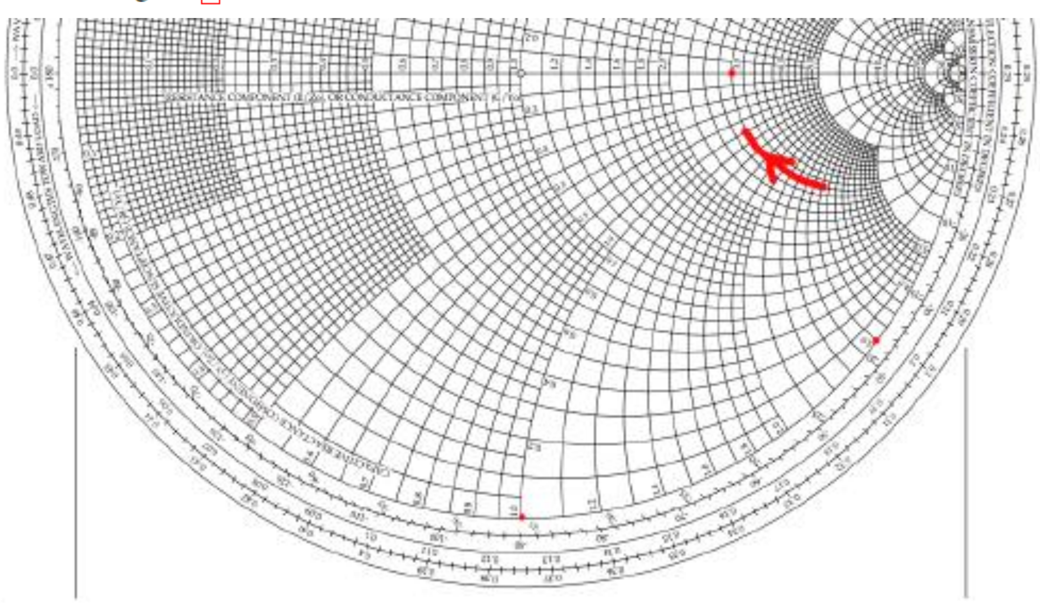
2.c

For $Z_L = 25 + 30j(\Omega)$, $z = 2.5 + 3j$. Smith chart is shown in Figure. 4. And corresponding ρ_L is $0.67 + 0.28j$. For $\rho_L = 0.5 + 0.1j$, we can get the same z as part b, which is $z = 2.8462 + 0.7692j$. But Z_L is different, which is $Z_L = Z_0 \cdot z = 28.462 + 7.692j(\Omega)$.

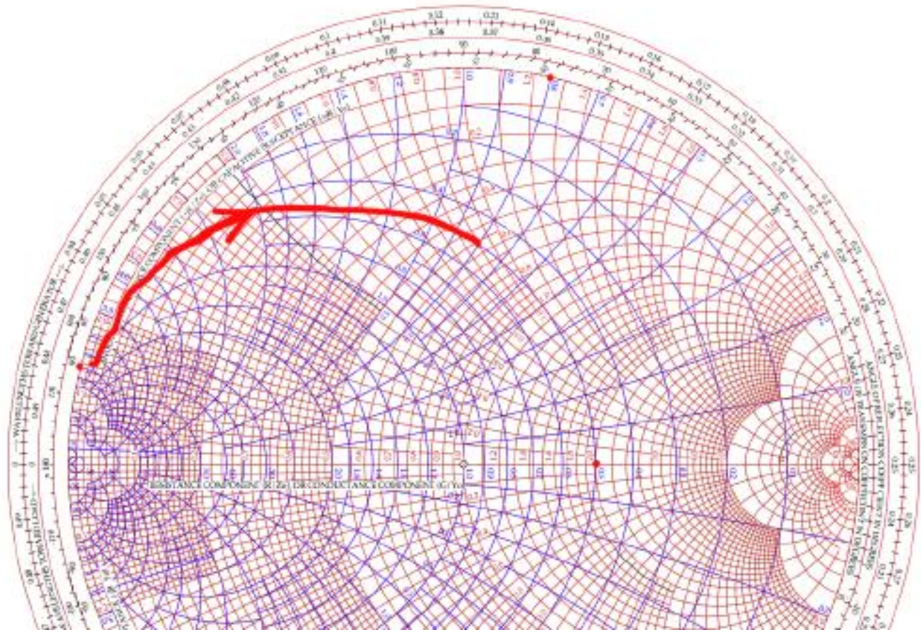


2.d

For capacitor one, the impedance changes between $z = 2 - 3.18j$ and $z = 2 - 1.06j$. The trace of ρ_L is shown in Figure. 5.



For inductor one, the impedance changes between $\frac{1}{z} = 0.5 - 7.96j$ and $\frac{1}{z} = 0.5 - 0.796j$.

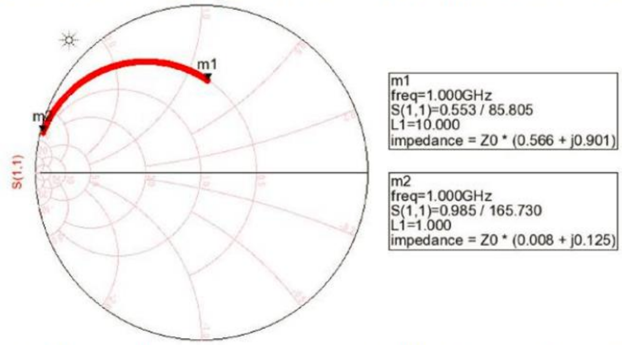


3. The Smith Chart you downloaded is called an Impedance Smith Chart. As we learned in class, we can also do an Admittance Smith Chart. A combined Impednace/Admittance chart can be found at `http://www.eecircle.com/applets/006/imped_admit_smithchart.pdf`.

   (a) Assume $Y_0 = 0.02S$. Using the Admittance Smith Chart to find the ρ_L for a load impedance $Z_L = 25 + 30j\Omega$

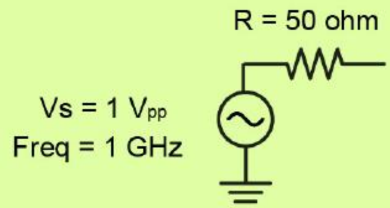
   (b) Trace ρ_L for the second circuit shown above with the parallel inductor, but on the Admittance Smith Chart with $Y_0 = 0.02S$. Is using the Admittance Smith Chart helpful?

3. Admittance Smith chart:
   a) $y_L = 0.82 - j0.98$. Using Smith Chart, $\rho_L \approx 0.48\angle 108^o$.
   b) The plot of ρ_L for the variable inductor in parallel with the resistor is given below:



In this case, since the conductance remains constant while the susceptance changes, using the admittance Smith chart is easier.

4. Impedance matching is an important technique to extract the highest power from the source.



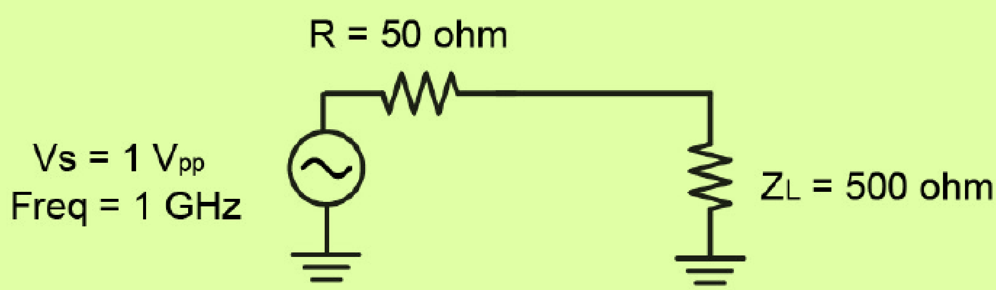
(a) What is the maximum power that can be extracted from the source shown above? What is the load impedance for the maximum power delivery to happen?

4.a

First of all, the rms-value of the source is $V_{rms} = \frac{V_{pp}}{2\sqrt{2}} = \frac{1}{2\sqrt{2}}(V)$

Maximum power delivery happens when $Z_L = R = 50\Omega$

$P_{avs} = \frac{1}{2}\frac{V_{rms}^2}{2R} = 0.625(mW)$



(b) Now we want to use this source to drive a 500Ω load and we connect it to the source directly, as illustrated above. What is the power delivered to the load and the load voltage?

The rms-value of load voltage is:

$V_{L,rms} = V_{rms}\frac{Z_L}{Z_L+R} = \frac{1}{2\sqrt{2}}\frac{10}{11}(V) \approx 0.3214(V)$ (Corresponding $V_{pp} = \frac{10}{11} = 0.9091(V)$)

$$P_L = \frac{V_{L,rms}^2}{Z_L} = 0.20661(mW)$$

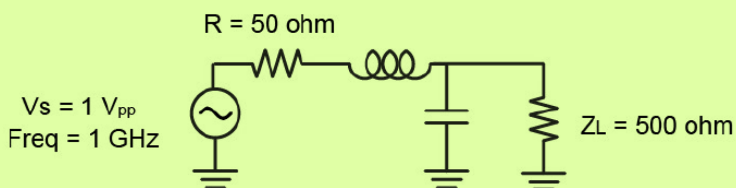
(c) Let's try to achieve impedance matching by putting a resistor in parallel with our load. What should be the resistor value in order to extract the maximum power from the source? What is the actual power delivered to the load? Comment on this approach.

To match the impedance,

$R = Z_L // R_{match} \quad \Rightarrow \quad R_{match} = 55.56\Omega$

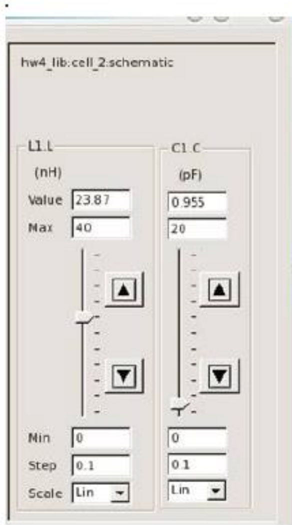
$P_L = \frac{(\frac{1}{2} V_{rms})^2}{Z_L} = 62.5(\mu W)$

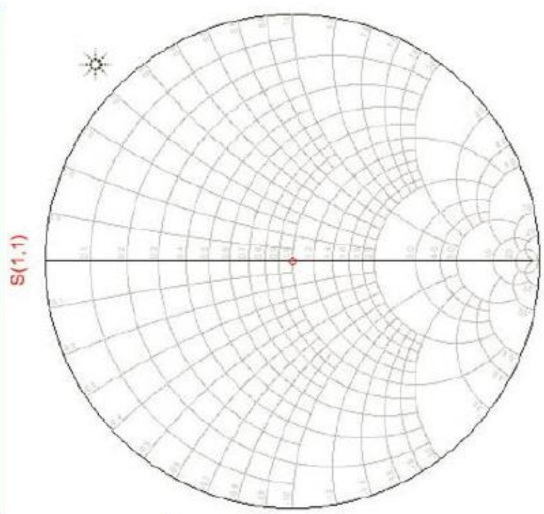
This approach is stupid because it's worse than performance of circuit without matching network. Most of the power is consumed on the matching network instead of the load.



(d) You can see that the impedance matching network should not eat the precious power delivered from the source. Please use Smith Chart to design an impedance matching network using an ideal shunt capacitor and an ideal series inductor, as shown above. What are the capacitor and the inductor values?

d) A matching network is made using series inductor and parallel capacitor. The shunt capacitor will change the admittance of the load along the constant conductance circle till r=1; then the series inductor will nullify the remaining reactance to make it matched. Hence the impedance Smith chart is useful for this scenario. With the help of Smith chart, $\boldsymbol{L = 23.87nH, C = 0.955pF}$.





(e) With the help of the matching network you designed in (d), what is the power delivered to the load? Calculate the load voltage and power by using KCL.

4.e

The impedance of inductor is $j2\pi fL = 150j(\Omega)$ and impedance of capacitor is $\frac{1}{j2\pi fC} = -166.67j(\Omega)$.

$\therefore Z_L//Z_C = 50 - 150j(\Omega)$

$\therefore$ $150j$ from inductor and $-150j$ from capacitor are cancelled away $\therefore$ The current flowing through the inductor is $I_{rms,inductor} = \frac{V_{rms}}{50+50} = \frac{1}{200\sqrt{2}}(A)$

$\therefore$ The current flowing through the load is $I_{rms,load} = I_{rms,inductor}\frac{-166.67j}{-166.67j+500} = \frac{1}{200\sqrt{2}} \times 0.3163e^{-1.25j}(A)$

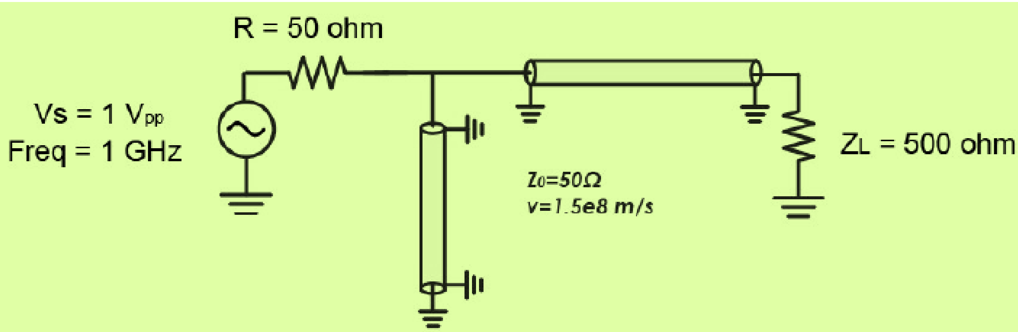
$\because$ The load is real

$\therefore$ Its voltage and current don't have phase difference

$\therefore$ We can ignore the phase part

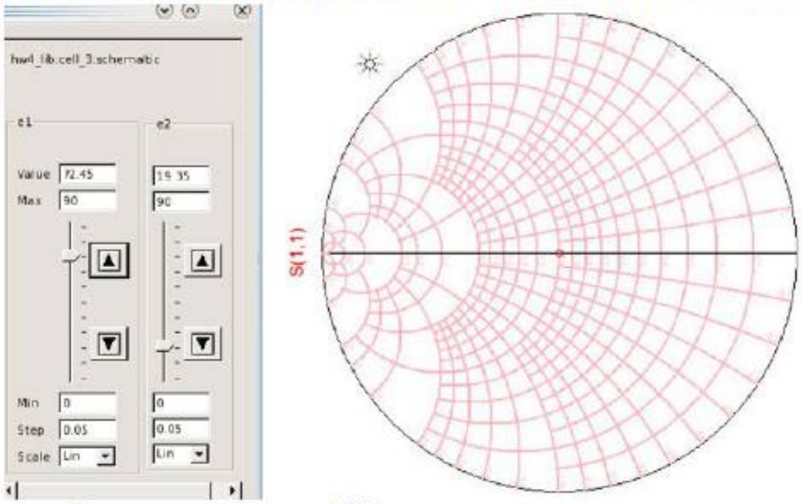
$\therefore P_L = |I_{rms,load}|^2 Z_L = (\frac{1}{200\sqrt{2}}0.3163)^2 \times 500 = 0.625(mW) = P_{avs}$

$|V_{rms,load}| = |I_{rms,load}| \times Z_L = 0.559(V)$ (Corresponding $V_{pp} = 1.581(V)$)



(f) (*242A only*) Alternatively, the impedance matching network can be designed by using transmission lines, as illustrated above. What are the length of the two lines?

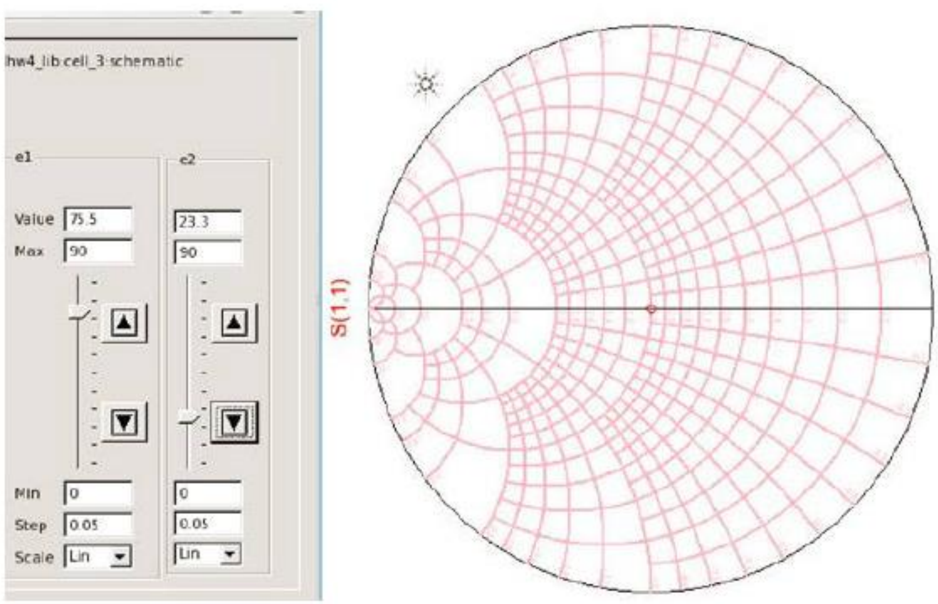
f) A matching network is made using a transmission line of length l_1 (electrical length e_1) in series with the load to change its impedance along the constant resistance circle till g=1, and then a transmission line of length l_2 (electrical length e_2) nullifies the remaining susceptance to make it matched. Hence the admittance Smith chart is useful in this scenario.



Since $e_1 = \beta l_1 = \frac{2\pi}{\lambda} l_1 = 72.45^o$, $\boldsymbol{l_1} = \frac{72.45}{2\times180}\lambda \approx 0.201\lambda \approx \mathbf{3.02}\boldsymbol{cm}$.

Similarly, $e_2 = \beta l_2 = \frac{2\pi}{\lambda} l_2 = 19.35^o$, $\boldsymbol{l_2} = \frac{19.35}{2\times180}\lambda \approx 0.054\lambda \approx \mathbf{8.07}\boldsymbol{mm}$.

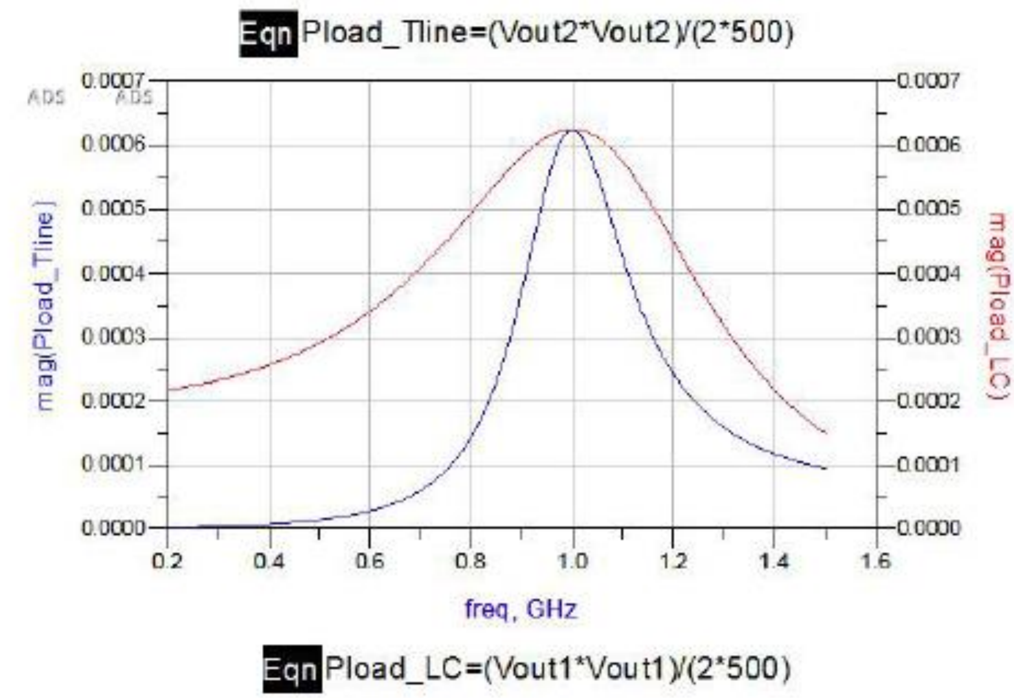
(g) (*242A only*) Repeat (f) with Z_0 of the transmission lines changed to 100Ω.



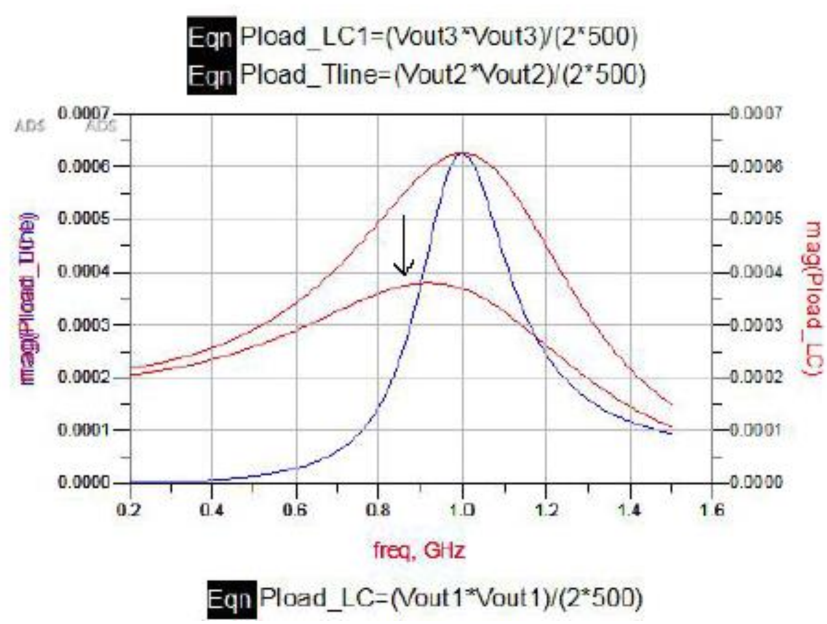
Since $e_1 = \beta l_1 = \frac{2\pi}{\lambda} l_1 = 75.5^o$, $\boldsymbol{l_1} = \frac{75.5}{2\times180}\lambda \approx 0.210\lambda \approx \mathbf{3.15}\boldsymbol{cm}$.

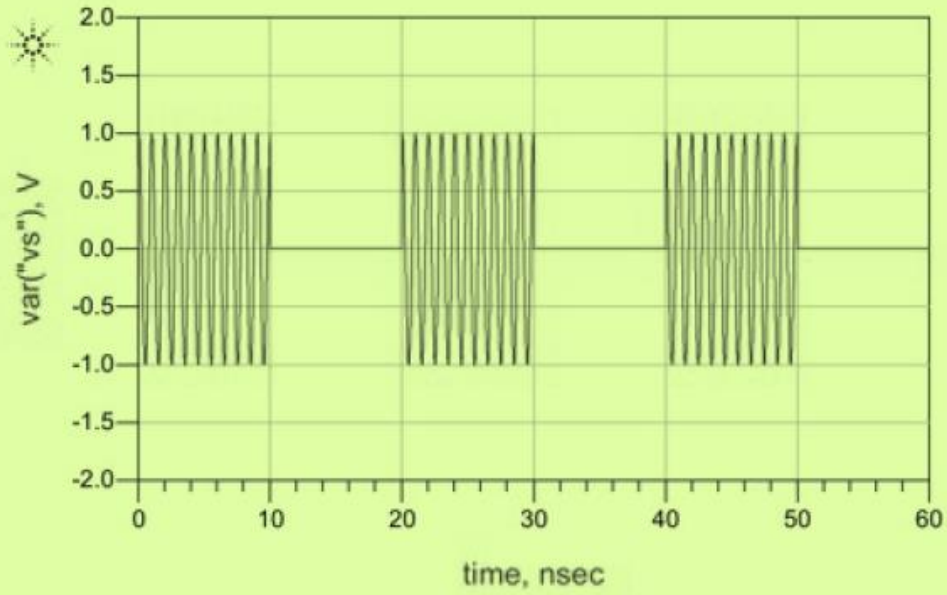
Similarly, $e_2 = \beta l_2 = \frac{2\pi}{\lambda} l_2 = 23.3^o$, $\boldsymbol{l_2} = \frac{23.3}{2\times180}\lambda \approx 0.065\lambda \approx \mathbf{9.70}\boldsymbol{mm}$.

(h) (*242A only*) Simulate the frequency response of the circuit you designed in (d) and (f). Plot the power delivered to the load versus the source frequency from 0.5 to 1.5 GHz.

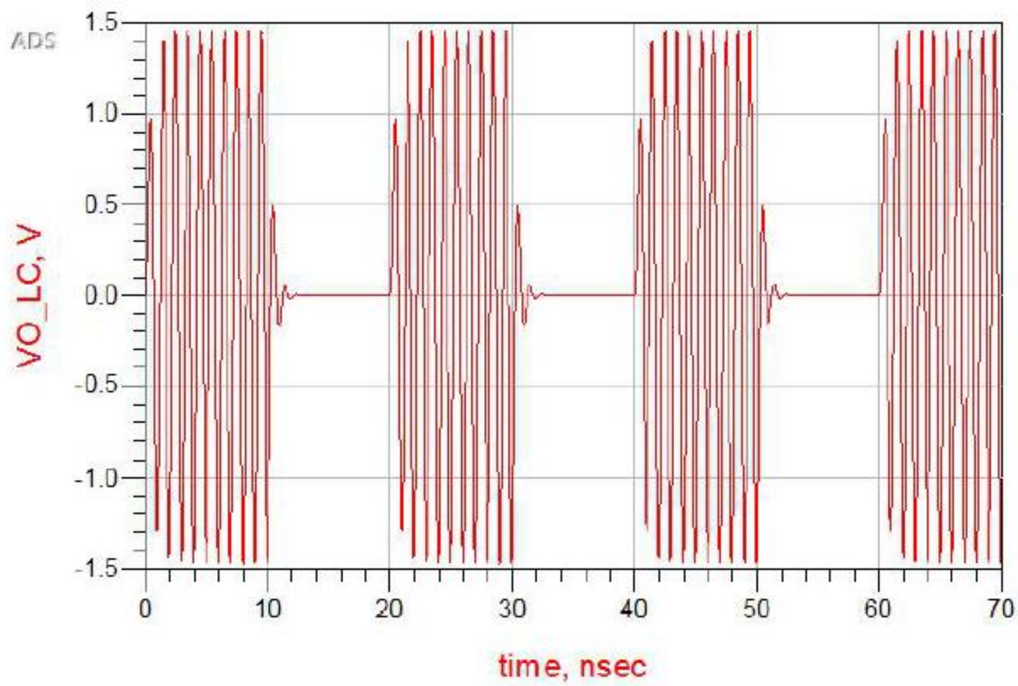


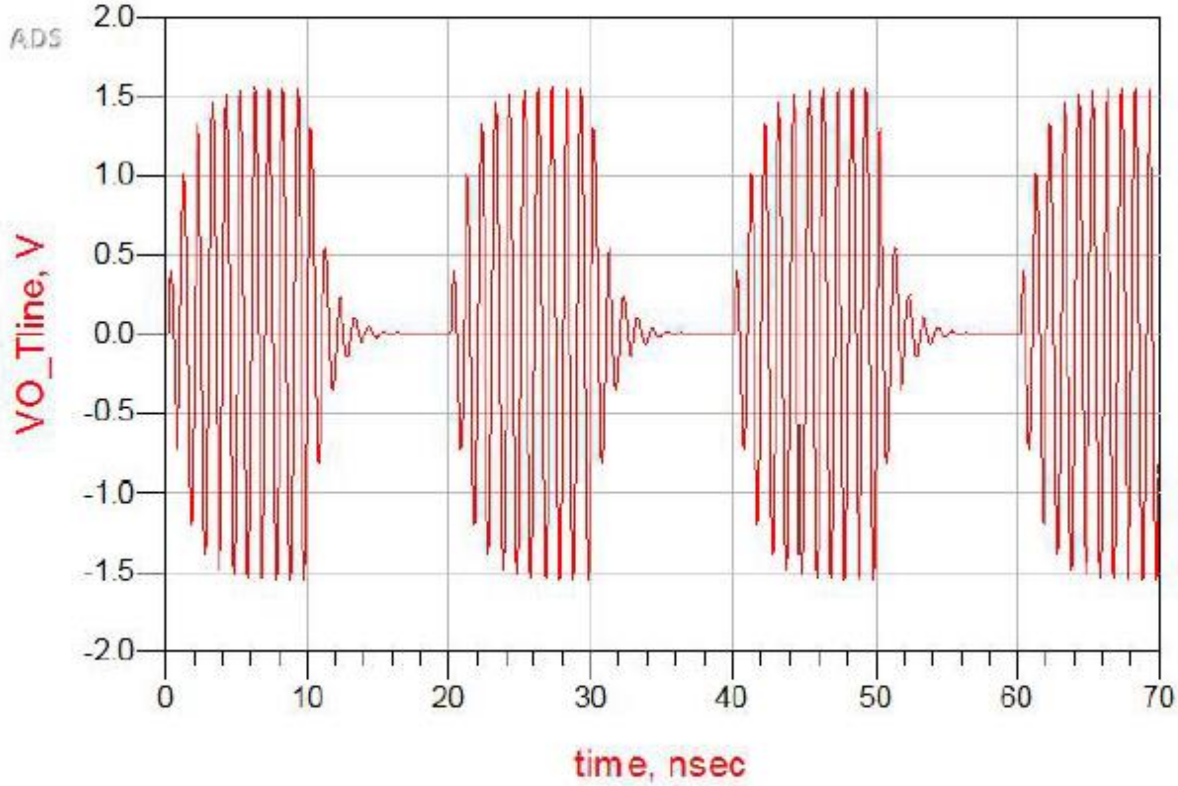
(i) (*242A only*) Re-simulate (h) but now both the capacitor and the inductor have finite quality factor of 10 at 1 GHz.





(j) (*242A only*) Assume the input source is a RF pulse with RF carrier frequency of 1 GHz, pulse repetition period of 20 ns, and pulse width of 10 ns, as shown above. Simulate the load voltage with your designed impedance matching networks in (d) and (f) and the one without any matching network. Compare the three results.




ADS
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
VO_Tline, V
0
10
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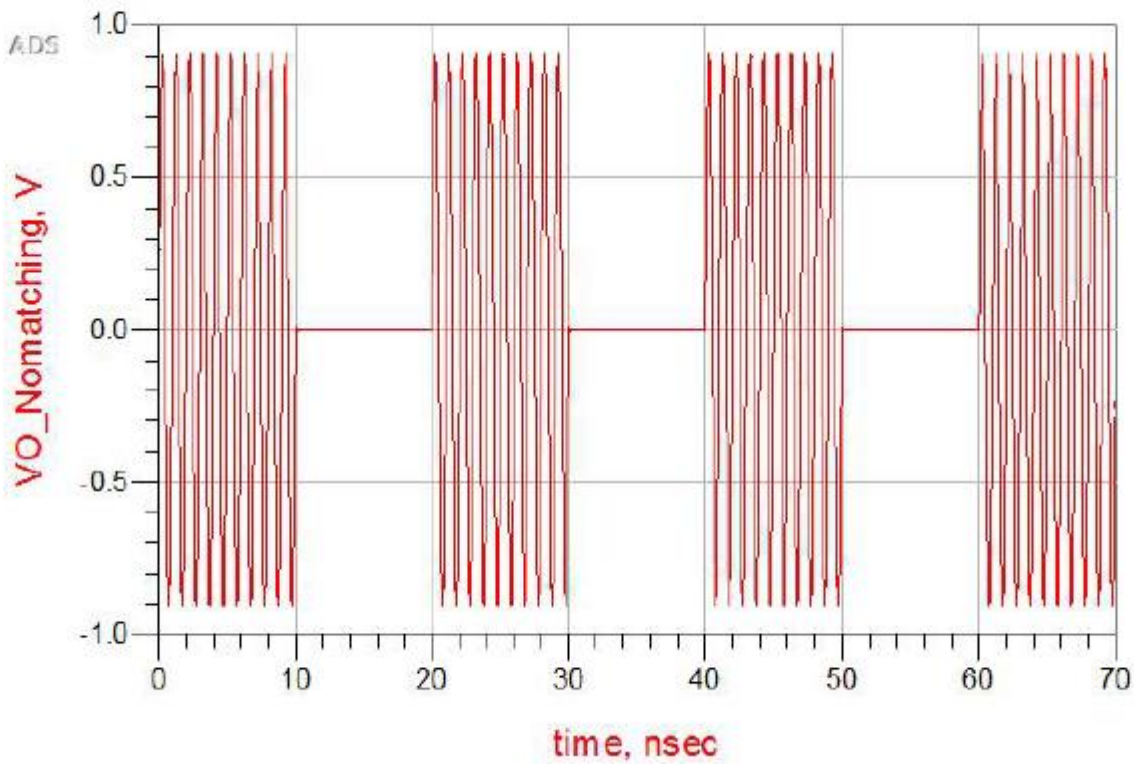
30
40
50
60
70
time, nsec


ADS
1.0
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VO_Nomatching, V
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time, nsec

(k) (*242A only*) Here are some follow-up questions for (j). Some of them might need to be answered by the aid of simulation tools.

i. What's the maximum power that can be extracted from the source?
ii. How much power is delivered to the load?
iii. Why is not all the power delivered to the load?

I. The maximum power that can be extracted from the source is

$$\frac{1}{2}*\frac{1}{2}*\frac{(1/2)^2}{50}=1.25mW$$

Ii

1) without matching network, the power delivered to the load is ~0.4132mW
2) With LC matching network, , the power delivered to the load is ~1.19996mW
3) With T-line, the power delivered to the load is ~1.1mW

The simulation results are shown in the following figures.

iii

Now there are other frequencies except 1GHz in the input signal and the matching works cannot achieve maximum power delivery at all these frequencies.