Integrated Circuits for Communication

Berkeley

Effect of Feedback on Distortion

Prof. Ali M. Niknejad

U.C. Berkeley

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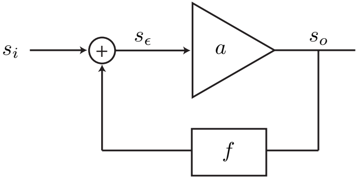
Effect of Feedback on Disto



- We usually implement the feedback with a passive network
- Assume that the only distortion is in the forward path a

$$s_o = a_1 s_\epsilon + a_2 s_\epsilon^2 + a_3 s_\epsilon^3 + \cdots$$

$$s_\epsilon = s_i - f s_o$$

$$s_o = a_1(s_i - f s_o) + a_2(s_i - f s_o)^2 + a_3(s_i - f s_o)^3 + \cdots$$

Feedback and Disto (cont)

- We'd like to ultimately derive an equation as follows

$$s_o = b_1 s_i + b_2 s_i^2 + b_3 s_i^3 + \cdots$$

- Substitute this solution into the equation to obtain

$$\begin{aligned} b_1 s_i + b_2 s_i^2 + b_3 s_i^3 + \cdots = {} & a_1(s_i - f b_1 s_i - f b_2 s_i^2 - f b_3 s_i^3 + \cdots) \\ & + a_2(s_i - f b_1 s_i - f b_2 s_i^2 - f b_3 s_i^3 + \cdots)^2 \\ & + a_3(s_i - f b_1 s_i - f b_2 s_i^2 - f b_3 s_i^3 + \cdots)^3 + \cdots \end{aligned}$$

- Solve for the first order terms

$$b_1 s_i = a_1(s_i - f b_1 s_i)$$

$$b_1 = \frac{a_1}{1 + a_1 f} = \frac{a_1}{1 + T}$$

Feedback and Disto (square)

- The above equation is the same as linear analysis (loop gain $T = a_1 f$)
- Now let's equate second order terms

$$b_2 s_i^2 = -a_1 f b_2 s_i^2 + a_2 (s_i - f b_1 s_i)^2$$

$$b_2 (a + a_1 f) = a_2 \left(1 - \frac{f a_1}{1 + T}\right)^2$$

$$b_2 (1 + T)^3 = a_2 (1 + T - T)^2 = a_2$$

$$b_2 = \frac{a_2}{(1 + T)^3}$$

- Same equation holds at high frequency if we replace with $T(j\omega)$

Feedback and Disto (cube)

- Equating third-order terms

$$b_3 s_i^3 = a_1(-f b_3 s_i^3) + a_2(-f b_2 2 s_i^3) + a_3(s_i - f b_1 s_i)^3 + \cdots$$

$$b_3(1 + a_1 f) = -2 a_2 b_2 f \frac{1}{1+T} + \frac{a_3}{(1+T)^3}$$

$$b_3(1 + T) = \frac{-2 a_2 f}{1+T} \frac{a_2}{(1+T)^3} + \frac{a_3}{(1+T)^3}$$

$$b_3 = \frac{a_3(1 + a_1 f) - 2 a_2^2 f}{(1 + a_1 f)^5}$$

Second Order Interaction

- The term $2a_2^2 f$ is the second order interaction
- Second order disto in fwd path is fed back and combined with the input linear terms to generate third order disto
- Can get a third order null if

$$a_3(1 + a_1 f) = 2a_2^2 f$$

HD_2 in Feedback Amp

$$HD_2 = \frac{1}{2}\frac{b_2}{b_1^2}s_{om}$$

$$= \frac{1}{2}\frac{a_2}{(1+T)^3}\frac{(1+T)^2}{a_1^2}s_{om}$$

$$= \frac{1}{2}\frac{a_2}{a_1^2}\frac{s_{om}}{1+T}$$

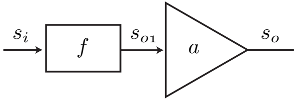
- Without feedback $HD_2 = \frac{1}{2}\frac{a_2}{a_1^2}s_{om}$
- For a *given output* signal, the negative feedback reduces the second order distortion by $\frac{1}{1+T}$

HD_3 in Feedback Amp

$$HD_3 = \frac{1}{4}\frac{b_3}{b_1^3}s_{om}^2$$

$$= \frac{1}{4}\frac{a_3(1+T) - 2a_2^2 f}{(1+T)^5}\frac{(1+T)^3}{a_1^3}s_{om}^2$$

$$= \underbrace{\frac{1}{4}\frac{a_3}{a_1^3}s_{om}^2}_{\text{disto with no fb}} \quad \frac{1}{(1+T)}\left[1 - \frac{2a_2^2 f}{a_3(1+T)}\right]$$



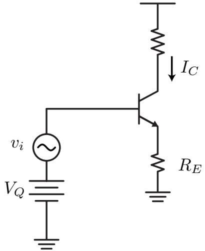
- Notice that the distortion is improved for a given output signal level. Otherwise we can see that simply decreasing the input signal level improves the distortion.
- Say $s_{o1} = fs_i$ with $f < 1$. Then

$$s_o = a_1 s_{o1} + a_2 s_{o1}^2 + a_3 s_{o1}^3 + \cdots = \underbrace{a_1 f}_{b_1} s_i + \underbrace{a_2 f^2}_{b_2} s_i^2 + \underbrace{a_3 f^3}_{b_3} s_i^3 + \cdots$$

- But the distortion is unchanged for a given output signal

$$HD_2 = \frac{1}{2} \frac{b_2}{b_1^2} s_{om} = \frac{1}{2} \frac{a_2}{a_1^2} s_{om}$$

BJT With Emitter Degeneration



The total input signal applied to the base of the amplifier is

$$v_i + V_Q = V_{BE} + I_E R_E$$

- The V_{BE} and I_E terms can be split into DC and AC currents (assume $\alpha \approx 1$)

$$v_i + V_Q = V_{BE,Q} + v_{be} + (I_Q + i_c)R_E$$

- Subtracting bias terms we have a separate AC and DC equation

$$V_Q = V_{BE,Q} + I_Q R_E$$

$$v_i = v_{be} + i_C R_E$$

Feedback Interpretation

- The AC equation can be put into the following form

$$v_{be} = v_i - i_c R_E$$

- Compare this to our feedback equation

$$s_\epsilon = s_i - f s_o$$

- The equations have the same form with the following substitutions

$$s_\epsilon = v_{be}$$

$$s_o = i_c$$

$$s_i = v_i$$

$$f = R_E$$

BJT with Emitter Degen (II)

- Now we know that

$$i_c = a_1 v_{be} + a_2 v_{be}^2 + a_3 v_{be}^3 + \cdots$$

- where the coefficients $a_{1,2,3,\ldots}$ come from expanding the exponential into a Taylor series

$$a_1 = g_m \quad a_2 = \frac{1}{2}\frac{I_Q}{V_t^2} \quad \cdots$$

- With feedback we have

$$i_c = b_1 v_i + b_2 v_i^2 + b_3 v_i^3 + \cdots$$

Emitter Degeneration (cont)

- The loop gain $T = a_1 f = g_m R_E$

$$b_1 = \frac{g_m}{1 + g_m R_E}$$

$$b_2 = \frac{\frac{1}{2}\left(\frac{q}{kT}\right)^2 I_Q}{(1 + g_m R_E)^3}$$

$$b_3 = \frac{1}{4 \cdot 6} \frac{\left(\frac{q}{kT}\right)^3 I_Q}{(1 + g_m R_E)^4} \left[1 - \frac{2\left(\frac{1}{2}\left(\frac{q}{kT}\right)^2 I_Q\right)^2 R_E}{\frac{1}{6}\left(\frac{q}{kT}\right)^3 I_Q (1 + g_m R_E)}\right]$$

- For large loop gain $g_m R_E \to \infty$

$$b_3 = \frac{-1}{12} \frac{\left(\frac{q}{kT}\right)^3 I_Q}{(1 + g_m R_E)^4}$$

- Using our previously derived formulas we have

$$HD_2 = \frac{1}{2}\frac{b_2}{b_1^2}s_{om}$$

$$= \frac{1}{4}\frac{\hat{i}_c}{I_Q}\frac{1}{1+g_mR_E}$$

$$HD_3 = \frac{1}{4}\frac{b_3}{b_1^3}s_{om}^2$$

$$= \frac{1}{24}\left(\frac{\hat{i}_c}{I_Q}\right)^2\frac{1-\frac{3g_mR_E}{1+g_mR_E}}{1+g_mR_E}$$

Harmonic Distortion Null

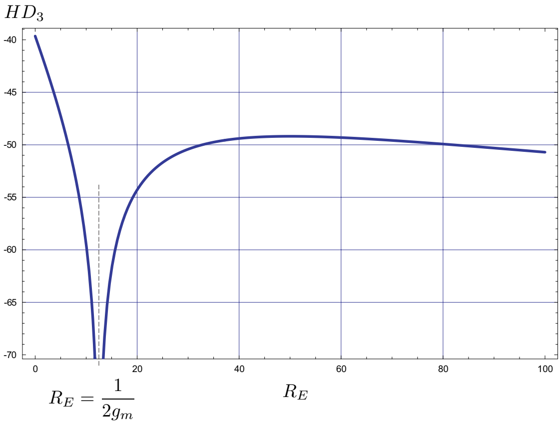
- We can adjust the feedback to obtain a null in HD_3
- $HD_3 = 0$ can be achieved with

$$\frac{3g_m R_E}{1 + g_m R_E} = 1$$

or

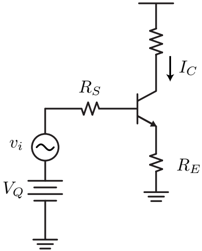
$$R_E = \frac{1}{2g_m}$$

HD_3 Null Example



- Example: For $I_Q = 1\mathrm{mA}$, $R_E = 13\Omega$

BJT with Finite Source Resistance



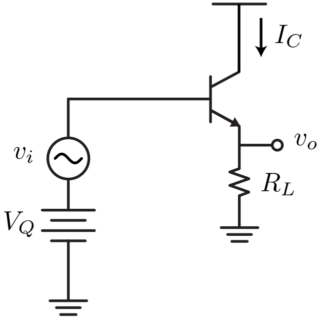
$$v_i + V_Q - I_B R_B = V_{BE} + I_E R_E$$

- Assuming that $\alpha \approx 1$, $\beta = \beta_0$ (constant). Let $R_B = R_S + r_b$ represent the total resistance at the base.

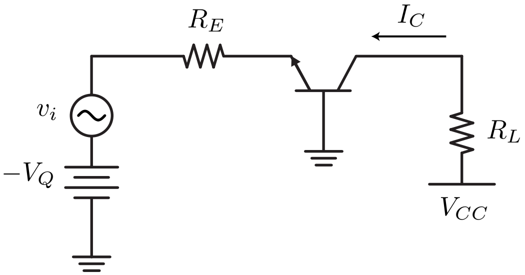
$$v_i + V_Q = V_{BE} + I_C \left(R_E + \frac{R_B}{\beta_0} \right)$$

- The formula is the same as the case of a BJT with emitter degeneration with $R'_E = R_E + R_B/\beta_0$

Emitter Follower



- The same equations as before with $R_E = R_L$



- Same equation as CE with R_E feedback

$$v_i - V_Q + I_C R_E = -V_{BE}$$

Calculation Tools: Multi-Tone Excitation

N Tones in One Shot

- Consider the effect of an m'th order non-linearity on an input of N tones

$$y_m = \left(\sum_{n=1}^{N} A_n \cos \omega_n t \right)^m$$

$$y_m = \left(\sum_{n=1}^{N} \frac{A_n}{2} \left(e^{\omega_n t} + e^{-\omega_n t} \right) \right)^m$$

$$y_m = \left(\sum_{n=-N}^{N} \frac{A_n}{2} e^{\omega_n t} \right)^m$$

- where we assumed that $A_0 \equiv 0$ and $\omega_{-k} = -\omega_k$.
-

Product of sums...

- The product of sums can be written as lots of sums...

$$= \underbrace{\sum () \times \sum () \times \sum () \cdots \times \sum ()}_{m-\text{times}}$$

$$= \sum_{k_1=-N}^{N} \sum_{k_2=-N}^{N} \cdots \sum_{k_m=-N}^{N} \frac{A_{k_1} A_{k_2} \cdots A_{k_m}}{2^m} \times e^{j(\omega_{k_1}+\omega_{k_2}+\cdots+\omega_{k_m})t}$$

- Notice that we generate frequency component $\omega_{k_1} + \omega_{k_2} + \cdots + \omega_{k_m}$, sums and differences between m non-distinct frequencies.
- There are a total of $(2N)^m$ terms.

Example

- Let's take a simple example of $m = 3$, $N = 2$. We already know that this cubic non-linearity will generate harmonic distortion and IM products.
- We have $(2N)^m = 4^3 = 64$ combinations of complex frequencies. $\omega \in \{-\omega_2, -\omega_1, \omega_1, \omega_2\}$. There are 64 terms that looks like this (HD_3)

$$\omega_1 + \omega_1 + \omega_1 = 3\omega_1$$

$$\omega_1 + \omega_1 + \omega_2 = 2\omega_1 + \omega_2$$

$(IM3)$

$$\omega_1 + \omega_1 - \omega_2 = 2\omega_1 - \omega_2$$

(Gain compression or expansion)

$$\omega_1 + \omega_1 - \omega_1 = \omega_1$$

Frequency Mix Vector

- Let the vector $\vec{k} = (k_{-N}, \cdots, k_{-1}, k_1, \cdots, k_N)$ be a $2N$-vector where element k_j denotes the number of times a particular frequency appears in a given term.
- As an example, consider the frequency terms

$$\left.\begin{array}{l} \omega_2 + \omega_1 + \omega_2 \\ \omega_1 + \omega_2 + \omega_2 \\ \omega_2 + \omega_2 + \omega_1 \end{array}\right\} \vec{k} = (0, 0, 1, 2)$$

Properties of $\vec{k}$

- First it's clear that the sum of the k_j must equal m

$$\sum_{j=-N}^{N} k_j = k_{-N} + \cdots + k_{-1} + k_1 + \cdots + k_N = m$$

- For a fixed vector $\vec{k_0}$, how many different sum vectors are there?
- We can sum m frequencies $m!$ ways. But the order of the sum is irrelevant. Since each k_j coefficient can be ordered $k_j!$ ways, the number of ways to form a given frequency product is given by

$$(m; \vec{k}) = \frac{m!}{(k_{-N})! \cdots (k_{-1})!(k_1)! \cdots (k_N)!}$$

Extraction of Real Signal

- Since our signal is real, each term has a complex conjugate present. Hence there is another vector $\vec{k_0'}$ given by

$$\vec{k_0'} = (k_N, \cdots, k_1, k_{-1}, \cdots, k_{-N})$$

- Notice that the components are in reverse order since $\omega_{-j} = -\omega_j$. If we take the sum of these two terms we have

$$2\Re\left\{e^{j(\omega_{k_1}+\omega_{k_2}+\cdots+\omega_{k_m})t}\right\} = 2\cos(\omega_{k_1}+\omega_{k_2}+\cdots+\omega_{k_m})t$$

- The amplitude of a frequency product is thus given by

$$\frac{2\times(m;\vec{k})}{2^m} = \frac{(m;\vec{k})}{2^{m-1}}$$

Example: IM_3 Again

- Using this new tool, let's derive an equation for the IM_3 product more directly.
- Since we have two tones, $N = 2$. IM_3 is generated by a $m = 3$ non-linear term.
- A particular IM_3 product, such as $(2\omega_1 - \omega_2)$, is generated by the frequency mix vector $\vec{k} = (1, 0, 2, 0)$.

$$(m; \vec{k}) = \frac{3!}{1! \cdot 2!} = 3 \qquad\qquad 2^{m-1} = 2^2 = 4$$

- So the amplitude of the IM_3 product is $3/4 a_3 s_i^3$. Relative to the fundamental

$$IM_3 = \frac{3}{4} \frac{a_3 s_i^3}{a_1 s_i} = \frac{3}{4} \frac{a_3}{a_1} s_i^2$$

Harder Example: Pentic Non-Linearity

- Let's calculate the gain expansion/compression due to the 5th order non-linearity. For a one tone, we have $N = 1$ and $m = 5$.
- A pentic term generates fundamental as follows

$$\omega_1 + \omega_1 + \omega_1 - \omega_1 - \omega_1 = \omega_1$$

- In terms of the $\vec{k}$ vector, this is captured by $\vec{k} = (2, 3)$. The amplitude of this term is given by

$$(m; \vec{k}) = \frac{5!}{2! \cdot 3!} = \frac{5 \cdot 4}{2} = 10 \qquad 2^{m-1} = 2^4 = 16$$

- So the fundamental amplitude generated is $a_5 \frac{10}{16} S_i^5$.

Apparent Gain Due to Pentic

- The apparent gain of the system, including the 3rd and 5th, is thus given by

$$\mathrm{AppGain} = a_1 + \frac{3}{4} a_3 S_i^2 + \frac{10}{16} a_5 S_i^4$$

- At what signal level is the 5th order term as large as the 3rd order term?

$$\frac{3}{4} a_3 S_i^2 = \frac{10}{16} a_5 S_i^4 \qquad S_i = \sqrt{1.2 \frac{a_3}{a_5}}$$

- For a bipolar amplifier, we found that $a_3 = 1/(3! V_t^3)$ and $a_5 = 1/(5! V_t^5)$. Solving for S_i, we have

$$S_i = V_t \sqrt{1.2 \times 5 \times 4} \approx 127\,\mathrm{mV}$$