EE142 Problem Set 4

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Problem 1

Calculate the scattering parameters of the following circuits:

(a) Find the input S_{11} for a general two-port terminated at port 2 with a load reflection coefficient of Γ_L.

We will call the wave going into port 1 V_1^+, the wave coming out of port 1 V_1^-, the wave *into* port 2 V_2^+ and the wave out of port 2 V_2^-.

We can then write the voltage waves in terms of the *two-port* S parameters.

$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+$$
$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+$$

Now, if port two is terminated by a load which results in reflection coefficient Γ_L, then the network effectively becomes a 1 port network. We can write the one-port S_{11} in terms of the two-port S parameters.

Throughout this problem we will assume that the two-port S parameters, the one port S_{11}, and Γ_L are with respect to a reference of Z_0.

$$V_2^+ = V_2^-\Gamma_L$$
$$V_1^- = S_{11}V_1^+ + S_{12}V_2^-\Gamma_L \quad (1)$$
$$V_2^- = S_{21}V_1^+ + S_{22}V_2^-\Gamma_L \quad (2)$$

$$\text{Rewriting equ 2: } V_2^- = \frac{S_{21}V_1^+}{1 - S_{22}\Gamma_L}$$

$$\text{Plug into equ 1: } V_1^- = S_{11}V_1^+ + S_{12}\Gamma_L\frac{S_{21}V_1^+}{1 - S_{22}\Gamma_L}$$

$$\text{Finally: } \boxed{\frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}}$$

$$\text{Notice: } \frac{V_1^-}{V_1^+} = S_{11,one-port}$$

(b) In the previous problem, what is the power that reaches the load in terms of the two-port scattering parameters and Γ_L? Suppose the input is driven with a matched source.

We derived the available power from the source for a 1-port network to be:

$$P_{avs} = \frac{|V_s|^2}{8Z_0}$$

where V_s is the amplitude at the generator. Assuming a perfect input match:

$$V_1^+ = V_s/2$$

The power seen by the load can be found as:

$$P_L = \frac{|V_2^-|^2 - |V_2^+|^2}{2Z_0} = \frac{|V_2^-|^2(1 - |\Gamma_L|^2)}{2Z_0}$$

Recall from the previous part that V_2^- can be written in terms of S parameters:

$$V_2^- = \frac{S_{21}v_1^+}{1 - S_{22}\gamma_L}$$

Then, P_L can be written as such:

$$P_L = \frac{|S_{21}|^2|v_1^+|^2}{|1 - S_{22}\Gamma_L|^2} \cdot \frac{1 - |\Gamma_L|^2}{2Z_0}$$

Substituting for V_1^+:

$$\boxed{P_L = \frac{|S_{21}|^2|v_s|^2}{4|1 - S_{22}\Gamma_L|^2} \cdot \frac{1 - |\Gamma_L|^2}{2Z_0}}$$

(c) Derive the two-port scattering parameters of a three-port where port 3 is terminated in a load with reflection coefficient Γ_L.