Today's Agenda (Oct.4 2017)

Review Homework Problems

- HW#5.2.(f) Single-stage T-line Matching Network
- HW#5.3 Multi-Stage Matching Network

Review Methods to Calculate Transfer Function

- Two-port method
- Feedback factor approximation
- Return ratio method (maybe next time)

Important Concepts on Two-port Stability

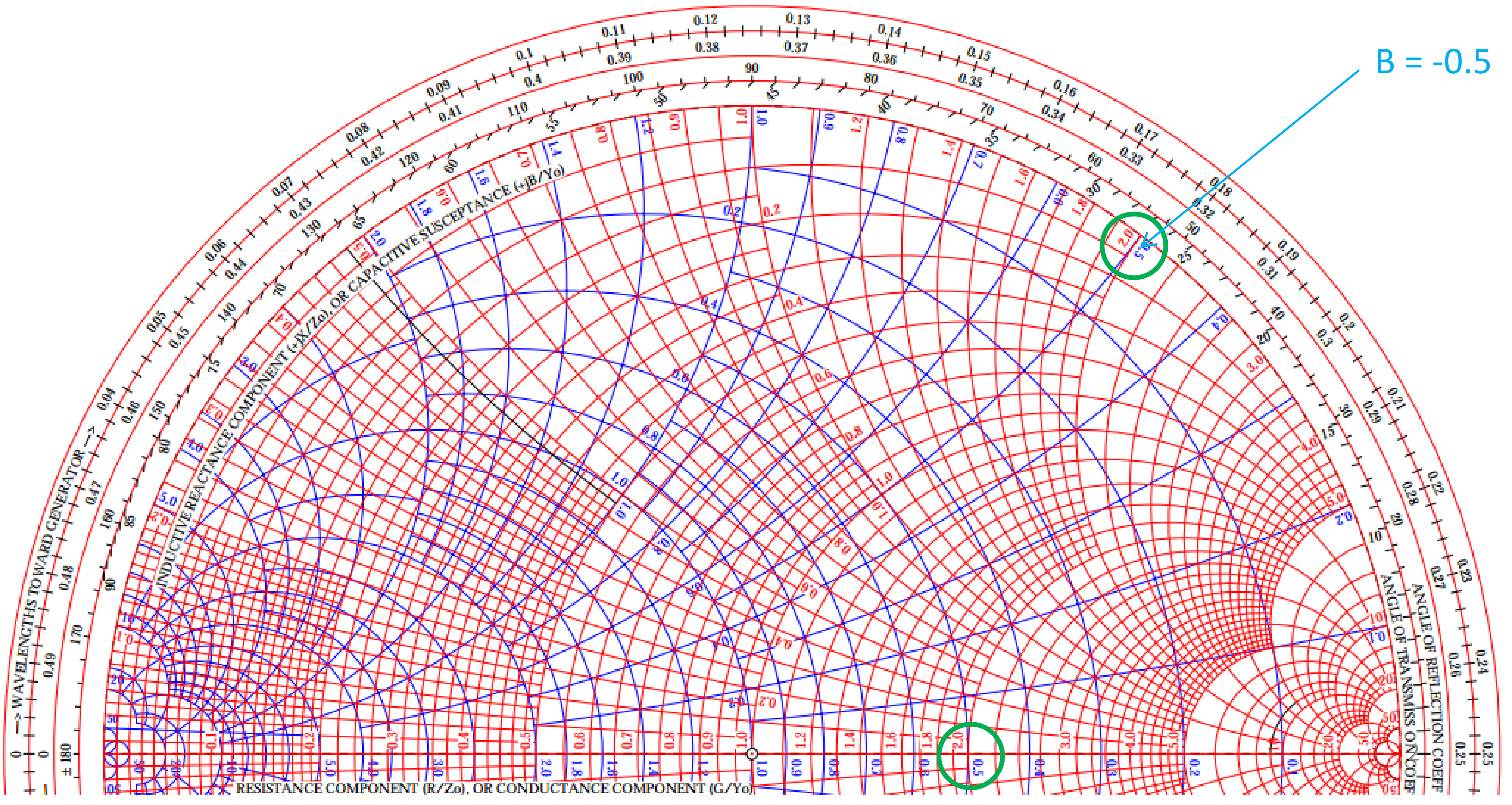
Final Remarks on Smith Chart:

Y chart and Z chart must tell the same story

(e.g. Upper plane => Z chart tells *X* is positive; Y chart tells *B* is negative

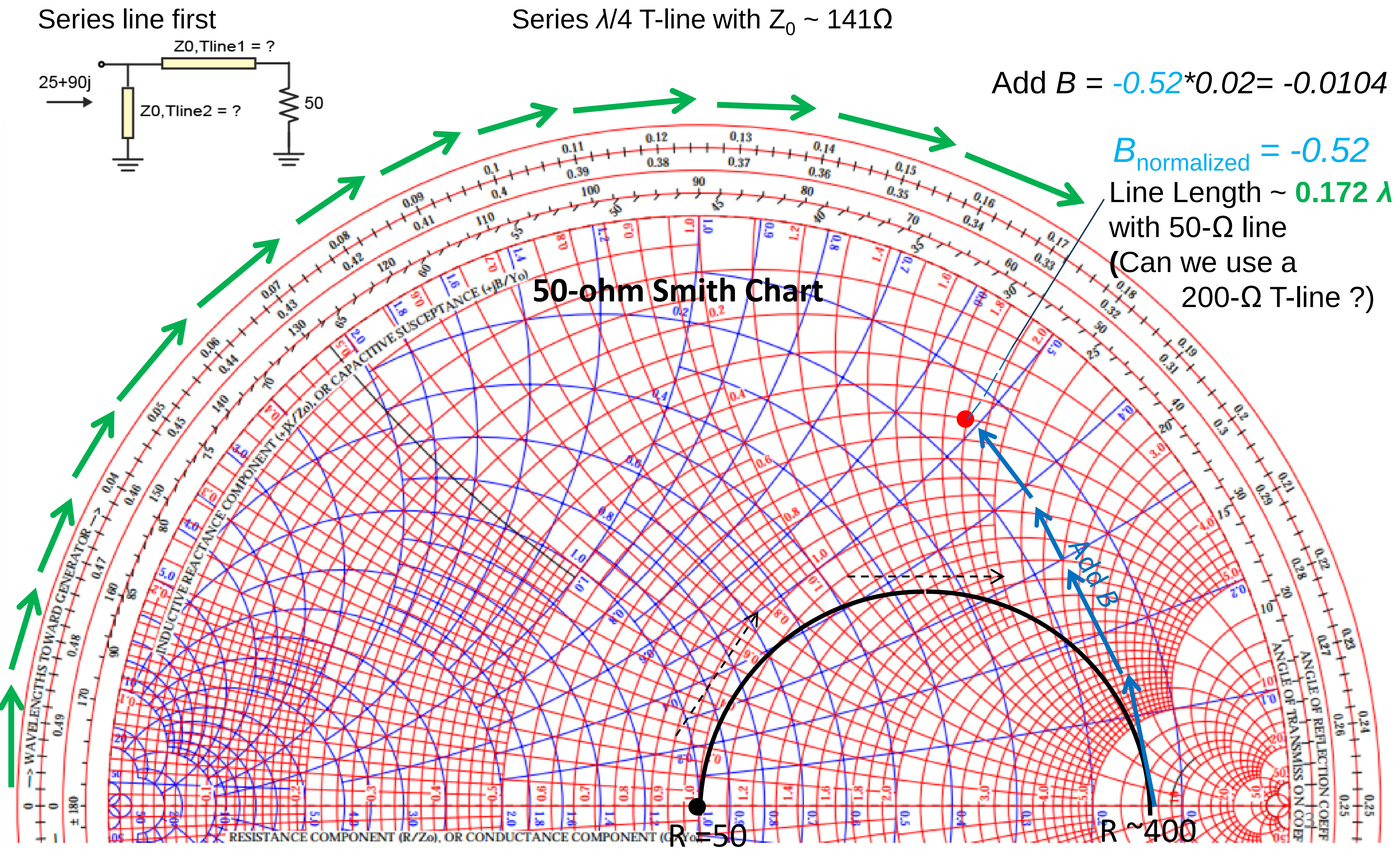
(e.g. R+2j = 1/(G-0.5j) => R = 0 and G = 0

(e.g. 2+*X*j = 1/(0.5+*B*j) => X = 0 and *B* = 0

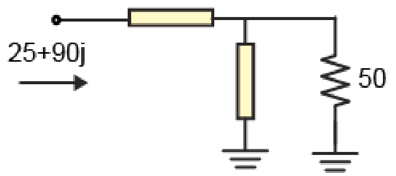


2. For the following problems, you may use the Smith Chart. For lumped component calculations, use equations and compare to the accuracy of using the Smith Chart.

   (f) Design a matching network to match 50 ohm to $Z_L = 25 + j90$ ohm using transmission lines.



Parallel line first

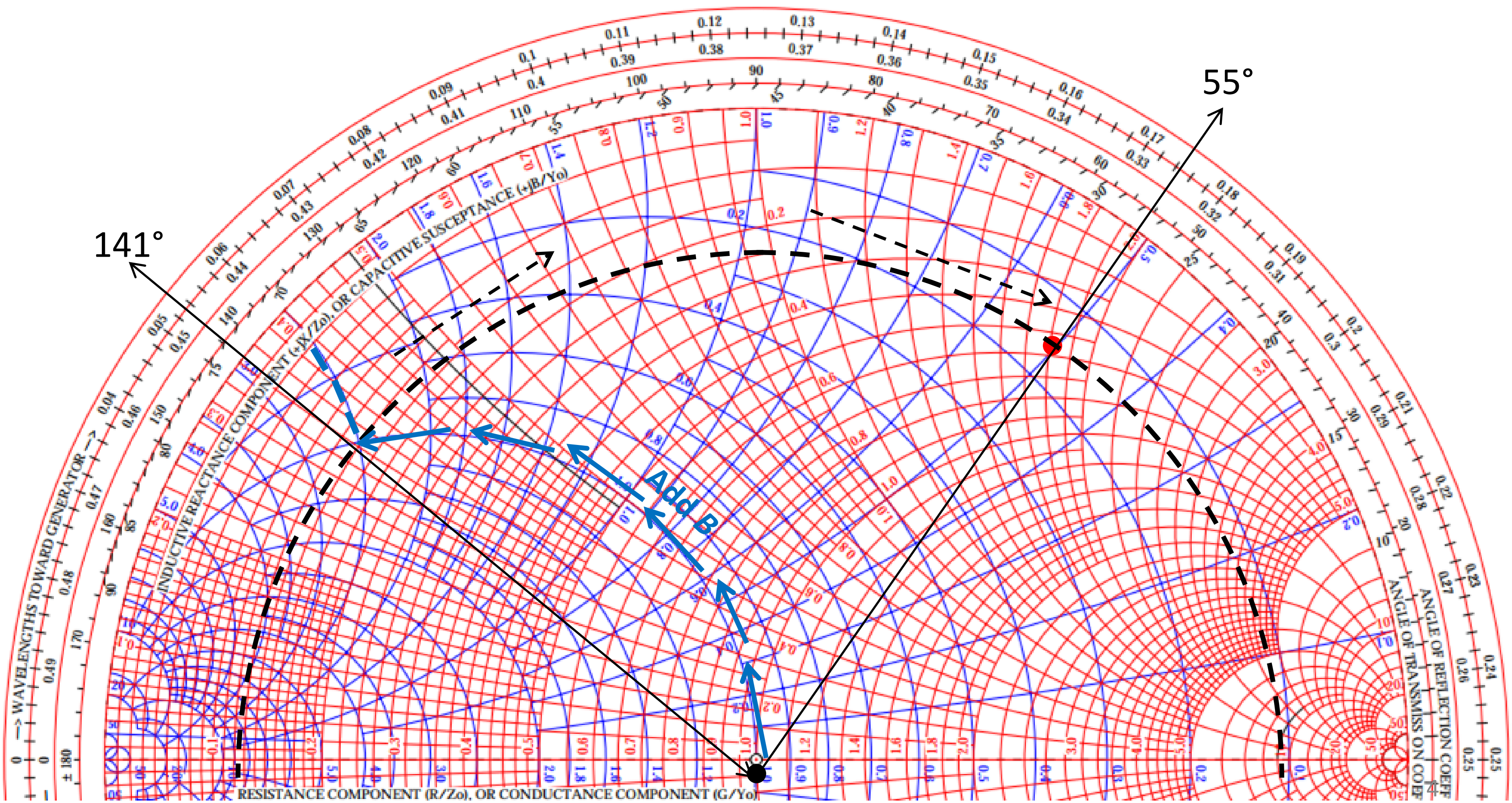


$B_{normalized}$ = -2.5

Parallel line length ~ **0.054 λ** with 50-Ω line

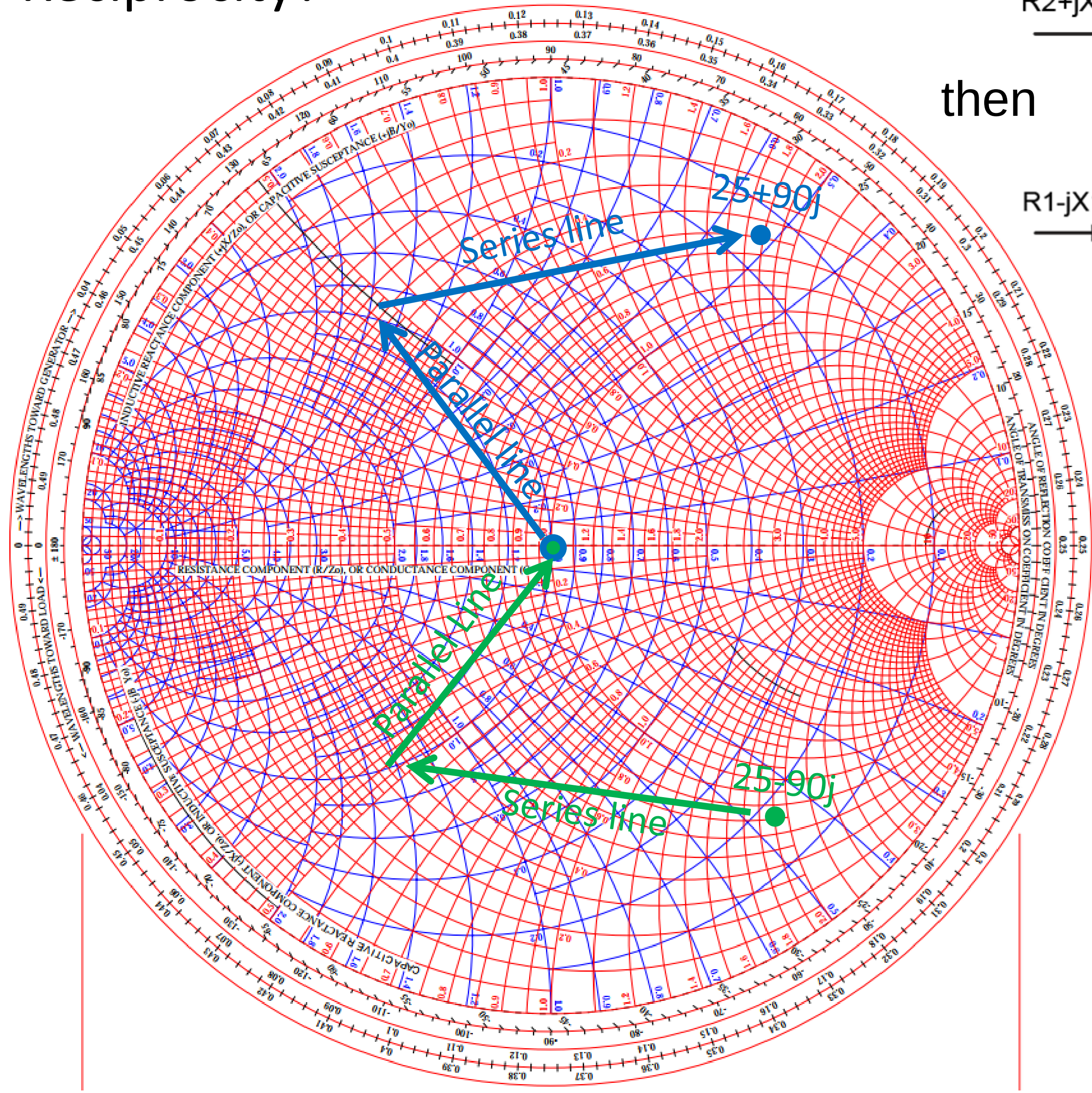
(can the line be open-circuited ?)

Series line length of (141°-55°)/180° *0.25λ **~ *0.12* λ**

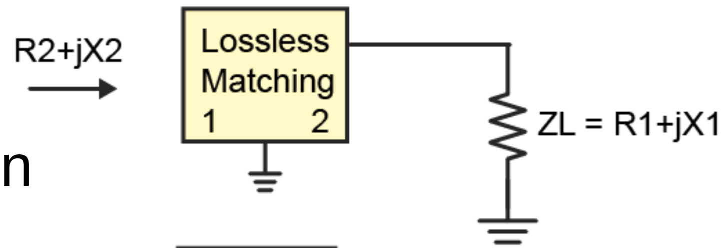


50-ohm Smith Chart

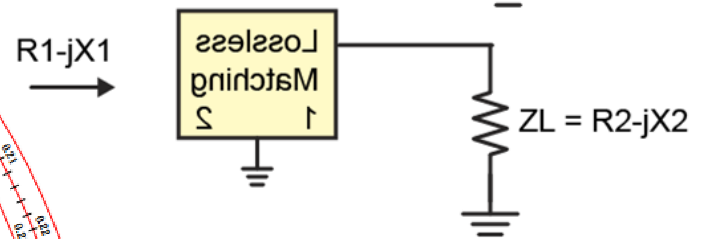
Have you already noticed the Reciprocity?

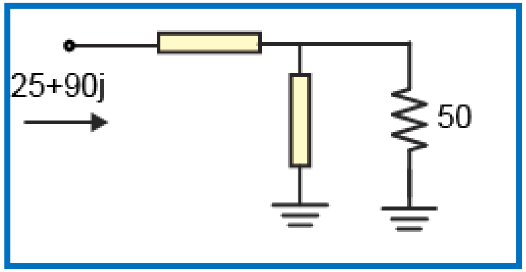


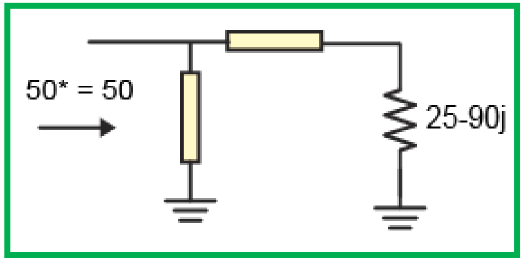
If



then





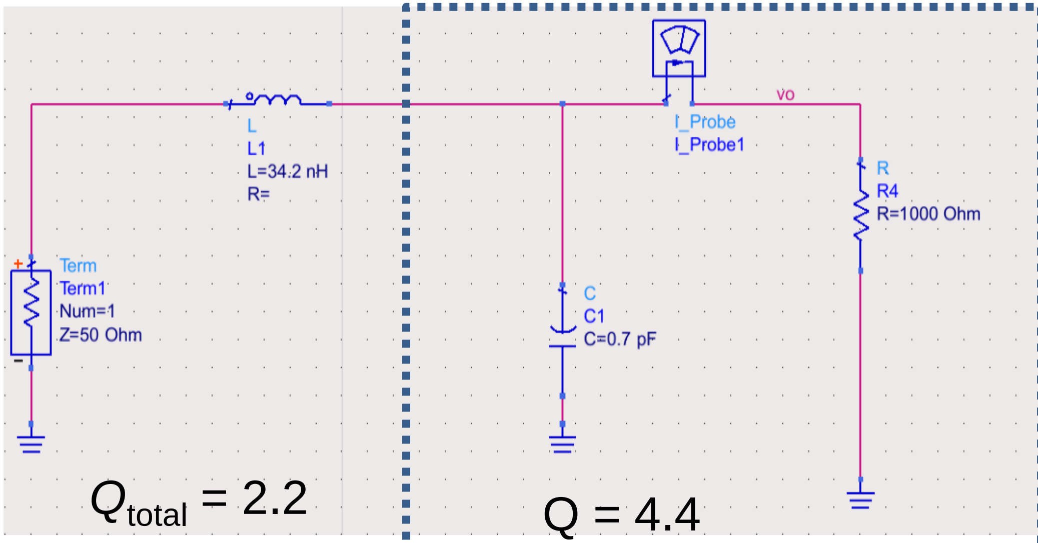


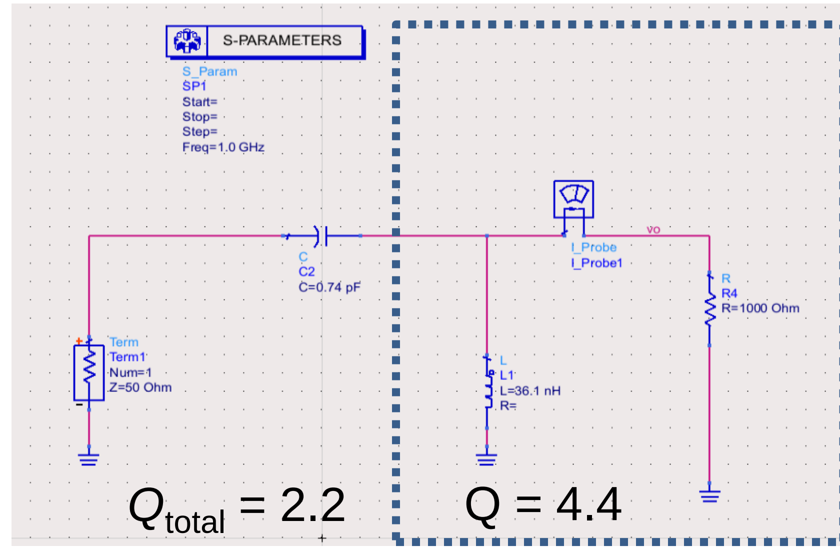
3. (a) Design a π matching network between a 1000Ω load impedance and a 50Ω source impedance at 1 GHz. The inductor and capacitor quality factors are 20. The target bandwidth for $|S_{11}| <$ -10 dB is 5%. Calculate the insertion loss and verify your design using ADS or Cadence. Check in ADS if the relation $|S_{11}|^2 + |S_{21}|^2 = 1$ holds.

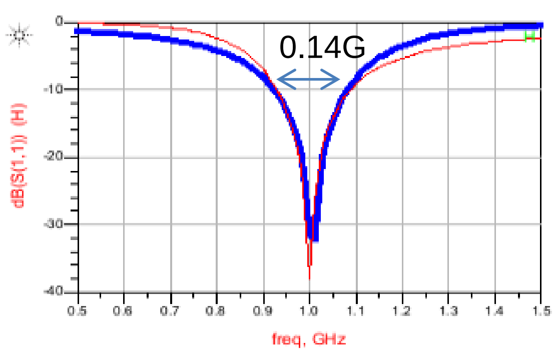
Can a 1-stage matching work? No!c

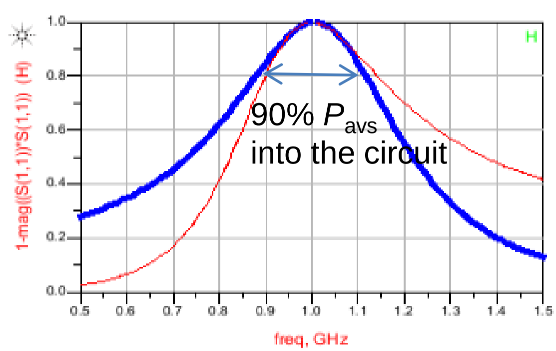
m = 1000/50=20; ***Q*** = sqrt(m-1) = 4.4
Shunt C: X = 1000/4.4 = 227 => C = 0.7 pF
Series L: $X = 227/(1+1/4.4^2)$ = 215 => L = 34.2 nH

Parallel L: X = 1000/4.4 = 227 => L = 36.1 nH
Series C: $X= 227/(1+1/4.4^2)$ = 215 => C= 0.74 pF





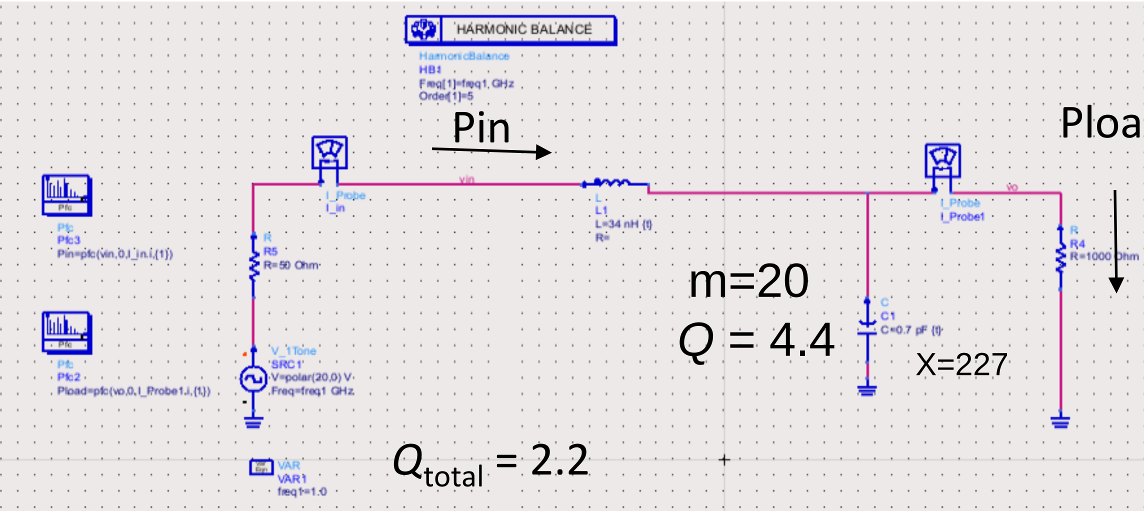


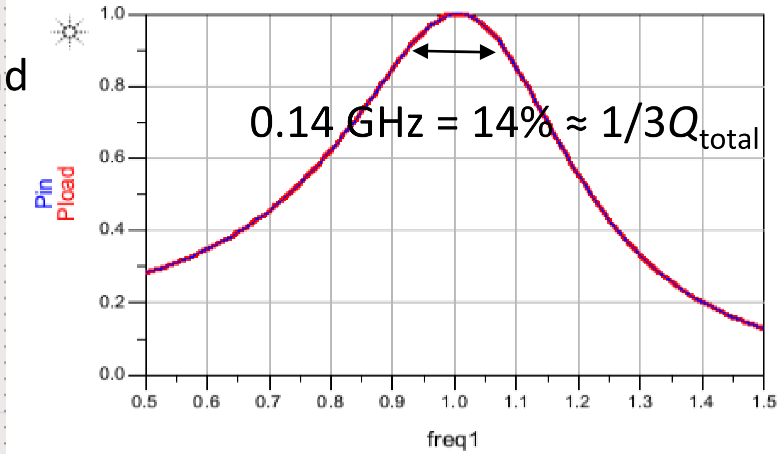


$|S_{11}|$<-10 dB Bandwidth =
0.14 GHz/1 GHz = 0.14 ≈ **1/3Q_{total}**

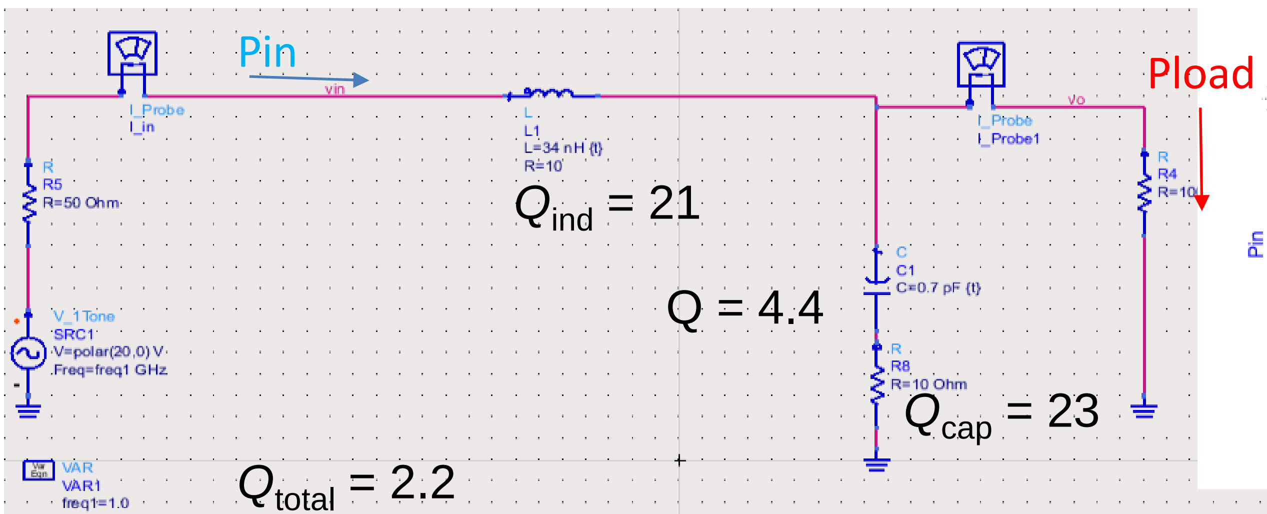
With lossy components: $IL = 1/(1+Q/Q_{cap}+Q/Q_{ind})$

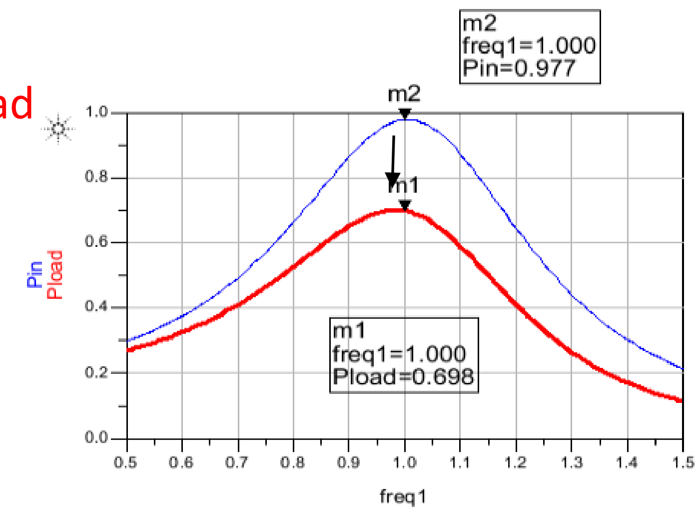
This is the L-matching shown earlier





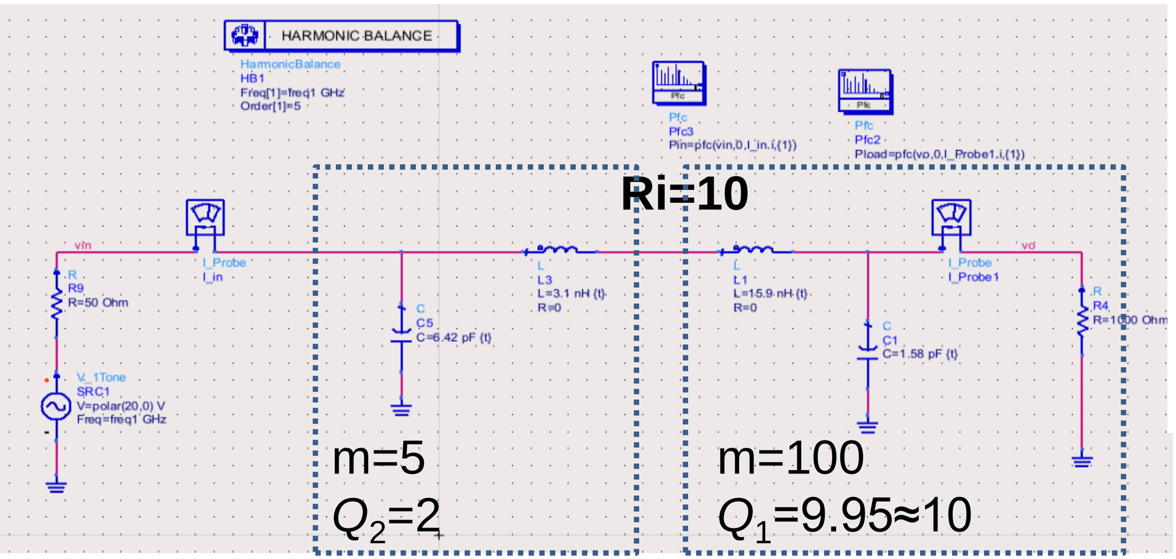
Introduce finite-Q inductor and capacitor

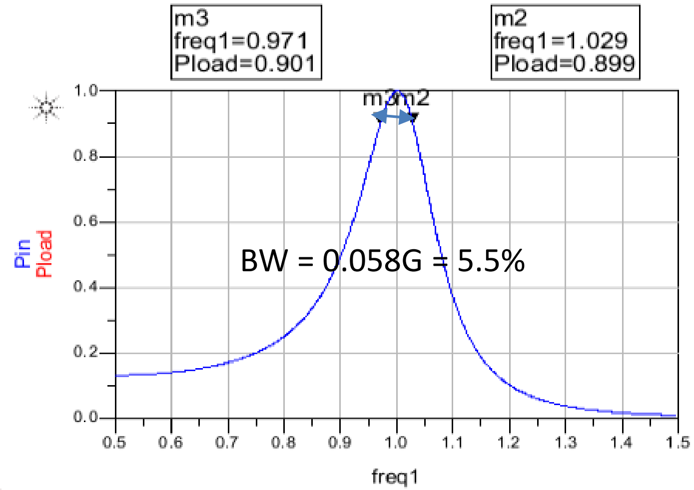




$IL \sim P_{load}/P_{in} = 1/(1+4.4/21+4.4/23) = 0.71$

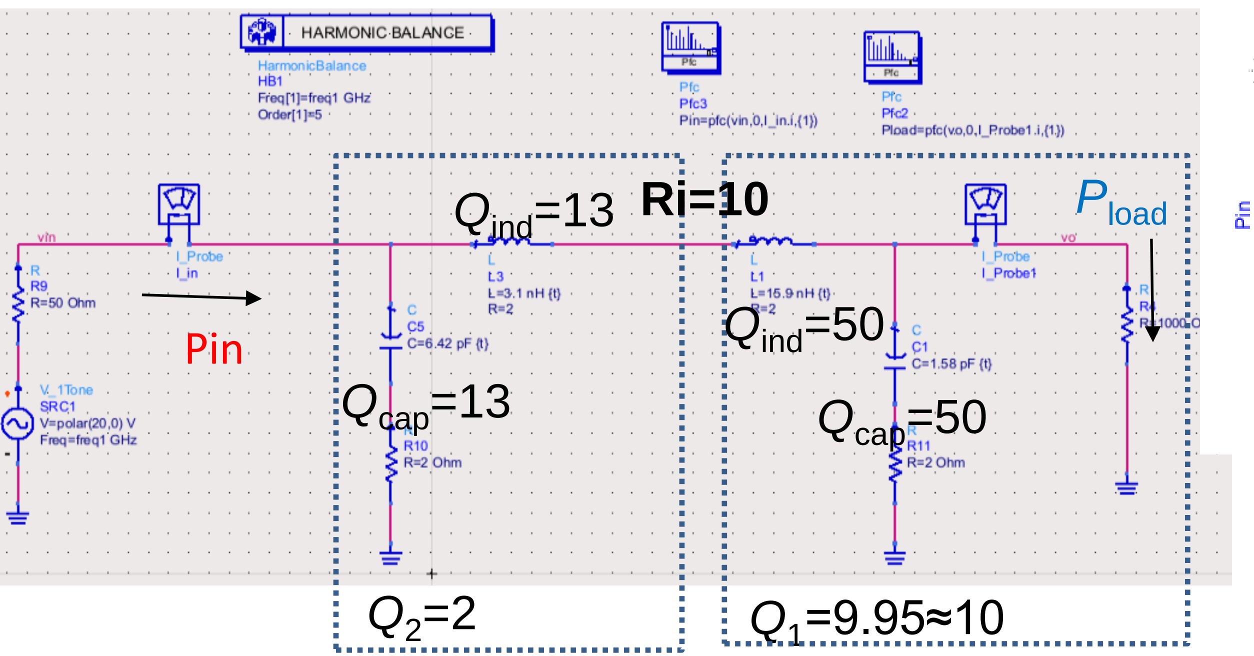
Pi-matching

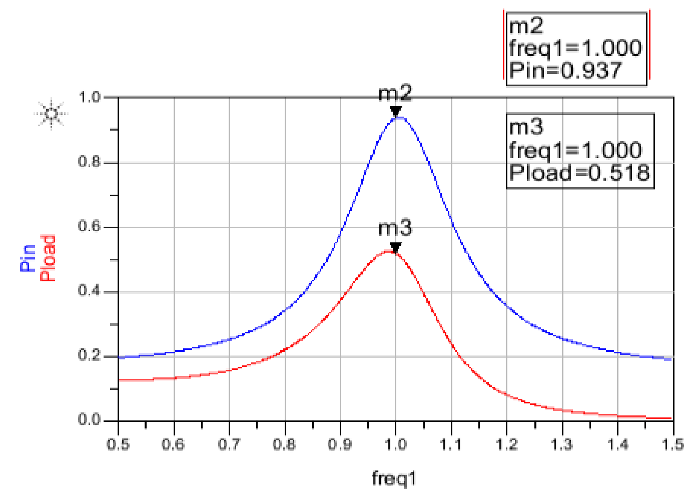




$$BW \approx 1/3Q_{total} = 5.5\%$$

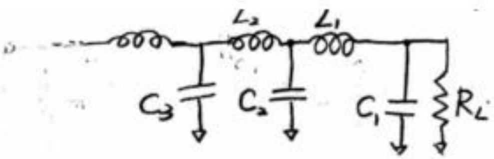
$$Q_{total} = (2+10)/2 = 6$$





$$IL = P_{load}/P_{in}$$
$$= 1/(1+\mathbf{10}/50+\mathbf{10}/50+\mathbf{2}/13+\mathbf{2}/13)$$
$$= 0.59$$

(b) Design a matching network between a 1000Ω load impedance and a 50Ω source impedance at 1 GHz. The inductor and capacitor quality factors are 20. The design goal is to achieve the lowest insertion loss. Calculate the insertion loss and verify your design using ADS or Cadence.



$$IL = \frac{1}{1+\frac{N}{Q_u}\sqrt{\left(\frac{R_{hi}}{R_{lo}}\right)^{\frac{1}{N}}-1}}$$

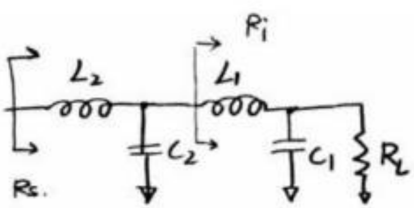
$Q_u = Q_{cap} \parallel Q_{ind} = 10$. $R_{hi}=1000$, $R_{lo}=50$.

$$IL = \frac{1}{1+\frac{N}{10}\sqrt{20^{\frac{1}{N}}-1}}$$.

When $N=2$, IL reaches the max value

The matching network should be two-stage.

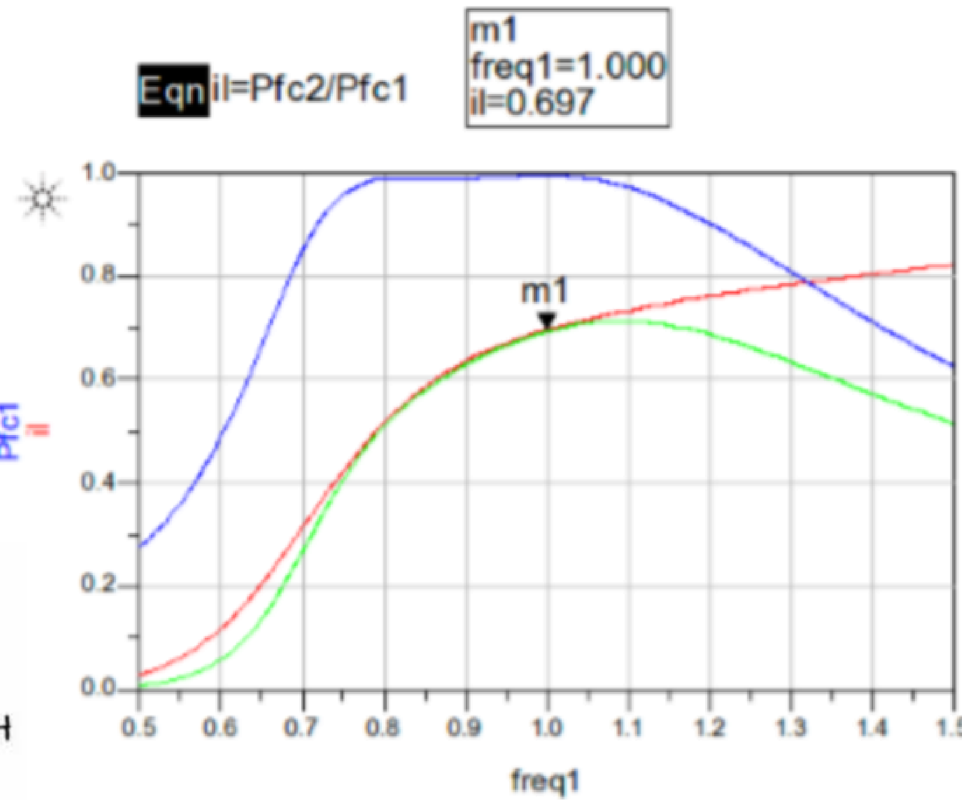
$$Q = \sqrt{\sqrt{20}-1} = 1.863, \quad IL_{max} = \frac{1}{1+\frac{2}{10}\sqrt{\sqrt{20}-1}} = 0.729$$



$$X_{p1} = \frac{R_L}{Q}, \quad X_{s1} = \frac{X_{p1}}{1+Q^{-2}} \quad \begin{cases} C_1 = 296\text{fF} \\ L_1 = 66.4\text{nH} \end{cases}$$

$$X_{p2} = \frac{R_i}{Q}, \quad X_{s2} = \frac{X_{p2}}{1+Q^{-2}} \quad \begin{cases} C_2 = 1.32\text{pF} \\ L_2 = 14.8\text{nH} \end{cases}$$

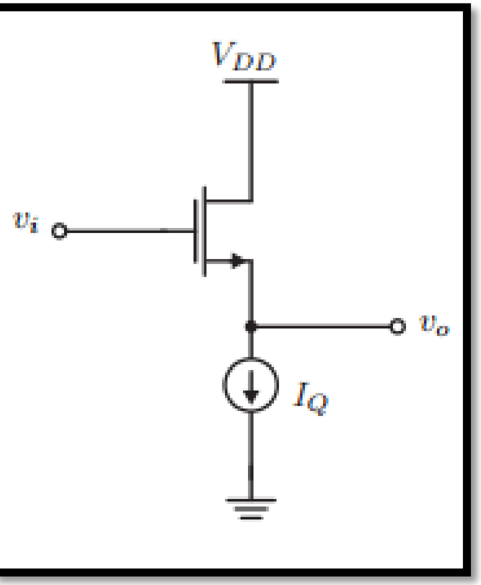
Designs are verified in ADS.
(see attachments)



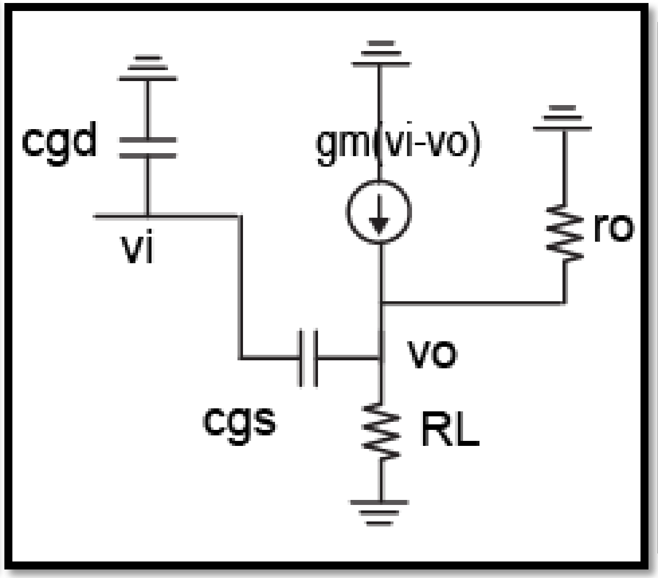
Review methods to calculate circuit close-loop transfer function

- KCL
- Two-port method
- Feedback factor approximation
- Return ratio method (maybe next time)

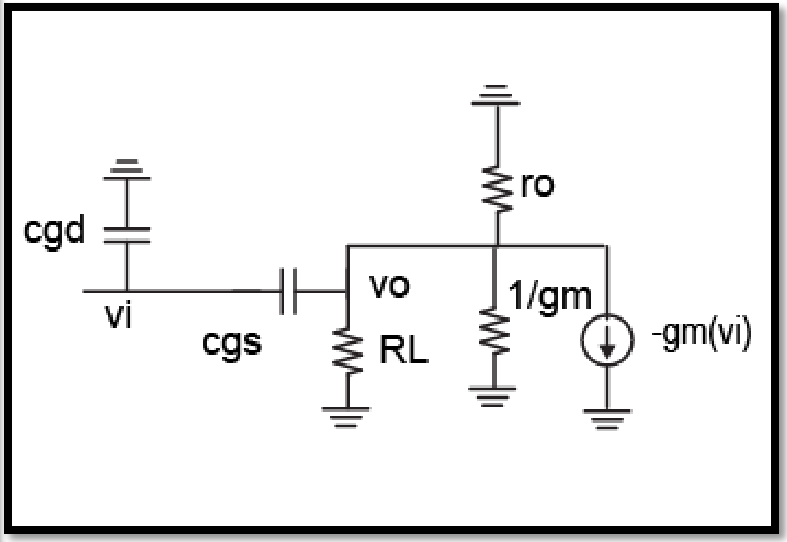
Calculate Vo/Vi



Draw the small-signal schematic

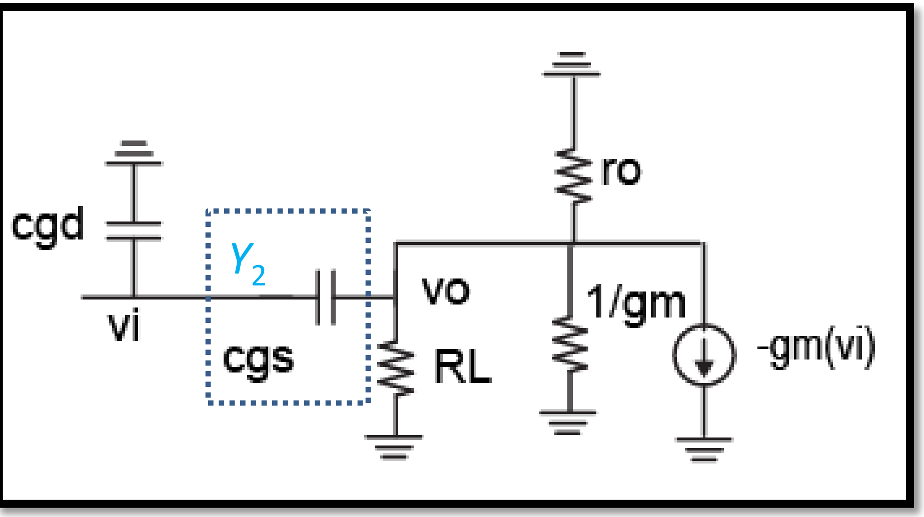


Use some imagination



- You can always apply KCL to solve V_o/V_i
- It looks appropriate to describe the circuit with two-port Y matrix!

Two-port Method: *Y*-Matrix Approach

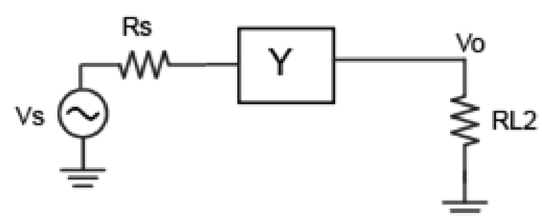


$$Y_1 = \begin{bmatrix} j\omega C_{gd} & 0 \\ -gm & gm+(1/RL)+(1/ro) \end{bmatrix}$$

$$Y_2 = \begin{bmatrix} j\omega C_{gs} & -j\omega C_{gs} \\ -j\omega C_{gs} & j\omega C_{gs} \end{bmatrix}$$

$$Y = \begin{bmatrix} j\omega C_{gd}+j\omega C_{gs} & -j\omega C_{gs} \\ -gm-j\omega C_{gs} & gm+(1/RL)+(1/ro)+j\omega C_{gs} \end{bmatrix}$$

Can you get the G,H or Z matrix easier than Y Matrix?



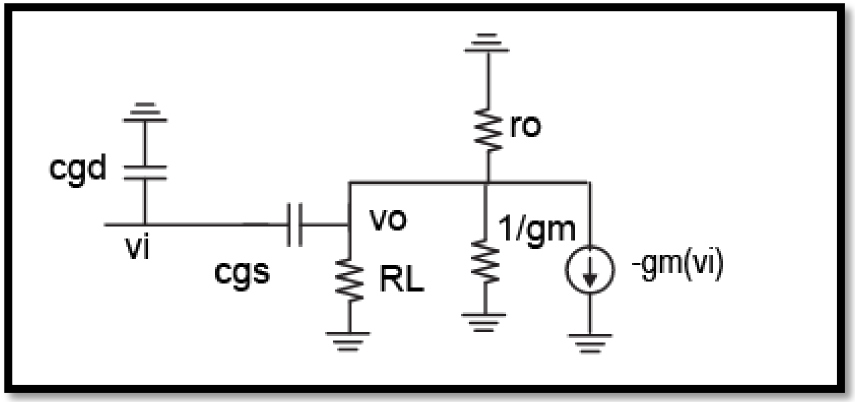
$$\frac{Vo}{Vs} = \frac{-Y_{21}*Ys}{(Y_s+Y_{11})(Y_{L2}+Y_{22}) - Y_{12}Y_{21}}$$

Substitute $Y_s = \infty$, $Y_L = 0$, and *Y*-parameters into the formula

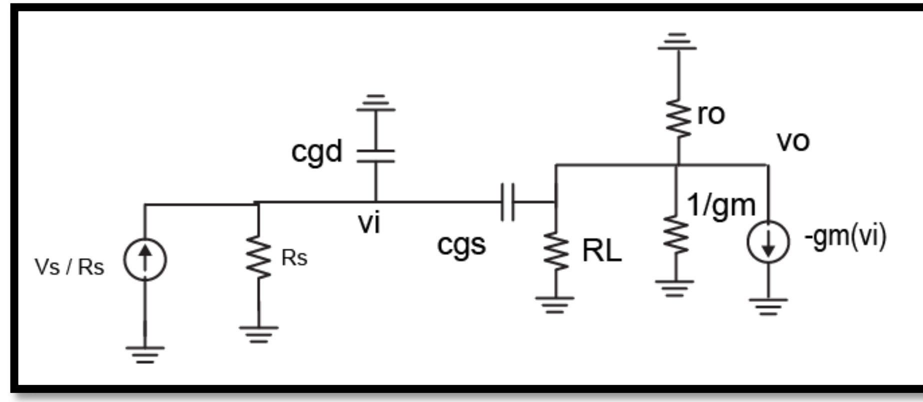
$\Rightarrow V_o/V_s = -Y_{21}/Y_{22} = (g_m + j\omega C_{gs}) / (g_m+1/R_L+1/r_o+ j\omega C_{gs})$

Feedback Factor Method:
Approximation introduced in many textbooks (e.g., Smith, G&M, Ravazi)

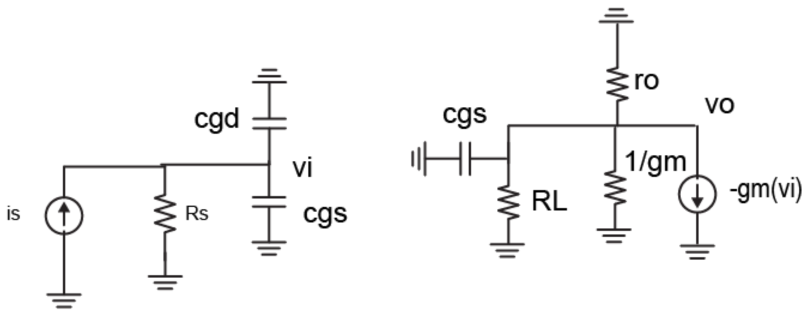
(1) Distinguish the circuit as a shunt-shunt feedback



(2) It is a current-voltage amp



(3) Open loop gain calculation



$[v_o/i_s]_{open}$ = vi/is * vo/vi

vi/is = $(1/R_s + j\omega C_{gs} + j\omega Cgd)^{-1}$

vo/vi = $gm/\,(1/r_o + gm + 1/R_L + j\omega C_{gs})$

(4) Close-loop gain calculation

Feedback factor = $j\omega C_{gs}$

$$[v_o/i_s]_{close} = [v_o/i_s]_{open} \,/\, \{1 + j\omega C_{gs}\,[v_o/i_s]_{open}\,\}$$

$$v_s = i_s R_s$$

$$[v_o/v_s]_{close} = 1/R_s{*}[v_o/i_s]_{open} \,/\, \{1+j\omega C_{gs}[v_o/i_s]_{open}\,\}$$

$$R_s = 0$$

$$[v_o/v_s]_{close} = gm \,/\, (gm+1/R_L+1/r_o+ j\omega C_{gs})$$

Comparison (1/2)

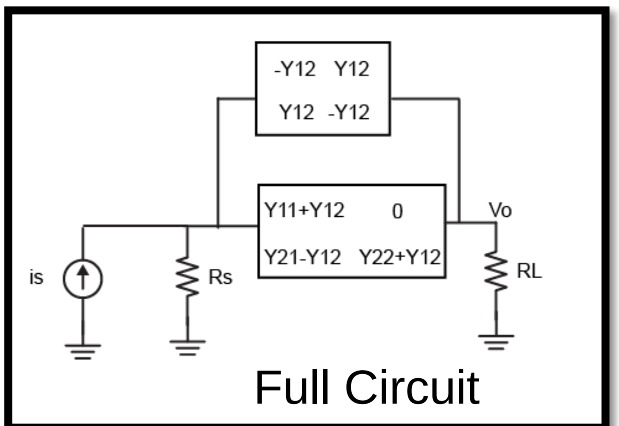
True (2-port approach)

$$v_o/v_s = (gm + j\omega C_{gs}) / (gm+1/R_L+1/r_o+ j\omega C_{gs})$$

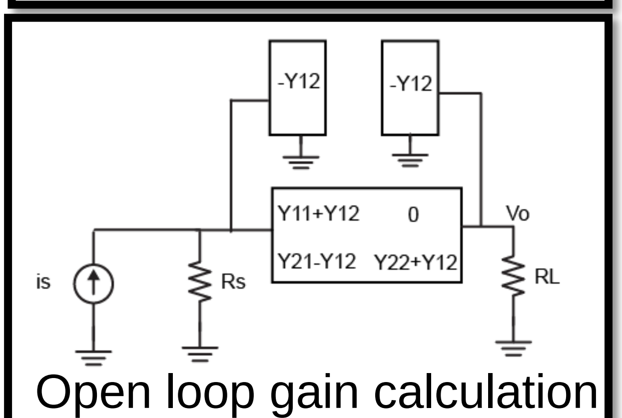
Approximated (feedback factor approach)

$$v_o/v_s = gm / (gm+1/R_L+1/r_o+ j\omega C_{gs})$$

- The two methods do not match
- Use the feedback factor approach on a *Y*-matrix to see what is neglected (no loss of generality)



Full Circuit



Open loop gain calculation

$$[v_o/i_s]_{open} = -(Y_{21}-Y_{12}) / [(Y_s+Y_{11})(Y_{22}+Y_L)]$$

Close-loop gain calculation

Feedback factor = $-Y_{12}$

$$[v_o/i_s]_{close} = [v_o/i_s]_{open} / \{1 + - Y_{12}[v_o/i_s]_{open} \}$$

$v_s = i_s R_s$

$$[v_o/v_s]_{close} = 1/R_s * [v_o/i_s]_{open} / \{1 + - Y_{12} [v_o/i_s]_{open} \}$$

$R_s = 0$

$$[v_o/v_s]_{close} = -Y_s(Y_{21}-Y_{12}) / [(Y_s+Y_{11})(Y_L+Y_{22})+Y_{12}(Y_{21}-Y_{12})]$$

Comparison (2/2)

True

$$\frac{Vo}{Vs} = \frac{-Y_{21}*Ys}{(Y_s+Y_{11})(Y_{L2}+Y_{22}) - Y_{12}Y_{21}}$$

Approximated

$$v_o/v_s = -Y_s(Y_{21}-Y_{12}) \,/\, [(Y_s+Y_{11})(Y_L+Y_{22})+Y_{12}(Y_{21}-Y_{12})\,]$$

In the approximation (feedback factor approach), we replace Y_{21} by Y_{21}-Y_{12}, which is usually acceptable

Similarly, applying feedback factor approach to a series-series feedback assumes $Z_{21} - Z_{12} \sim Z_{21}$

Important Concept on Two-port Stability

- The two-port network is unstable if it supports non-zero currents/voltages with passive terminations

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

- Since $i_1 = -v_1 Y_S$ and $i_2 = -v_2 Y_L$

$$\begin{pmatrix} y_{11} + Y_S & y_{12} \\ y_{21} & y_{22} + Y_L \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$$

- The only way to have a non-trial solution is for the determinant of the matrix to be zero at a particular frequency

$$Y_S + Y_{in} = 0$$

- Or equivalently

$$Y_L + Y_{out} = 0$$

Or equivalently,
a system pole is on the imaginary axis
(not stable and could oscillate)

Two-port Unconditionally Stable
For all passive Y_L and Y_s

$$\Re(Y_{in}) = \Re\left(y_{11} - \frac{y_{12}y_{21}}{Y_L + y_{22}}\right) > 0$$

$$\Re(Y_{out}) = \Re\left(y_{22} - \frac{y_{12}y_{21}}{Y_S + y_{11}}\right) > 0$$

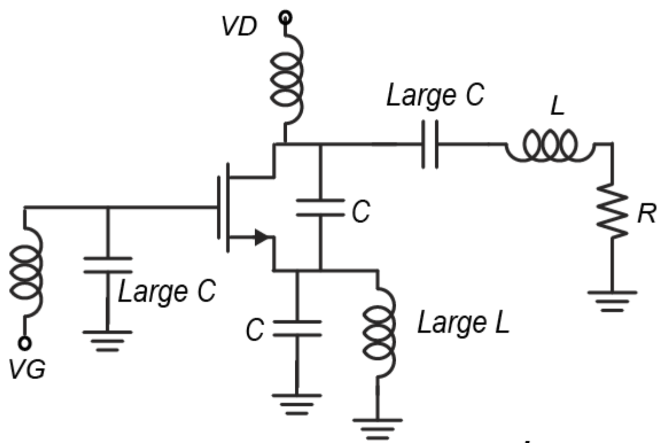
Impossible to be unstable

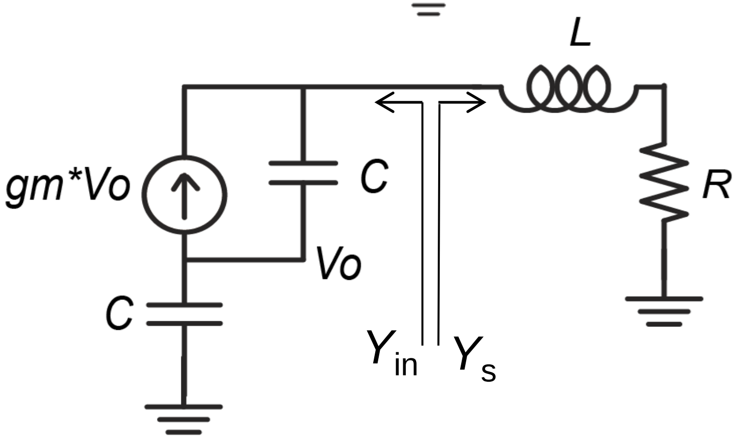
In RF circuits, we usually only control the impedances (e.g. Y_L) at the in-band frequency

Important Concept on Two-port Stability

If a passive (Y_L, Y_S) creates Re($Y_S + Y_{IN}$) < 0, is this circuit stable?

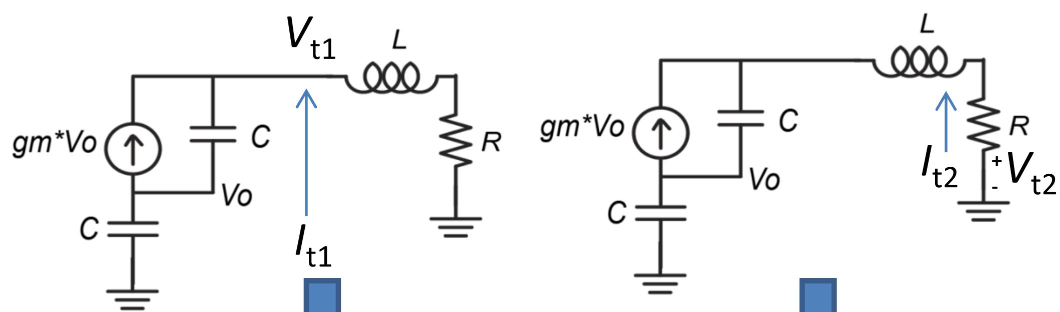
"I don't know" because I do not know the system pole locations according to the give condition





$Y_{in} = s^2C^2/(2sC + gm)$; $Y_s = 1/(R+sL)$

System Pole = pole of V_{t1}/I_{t1} or pole of I_{t2}/V_{t2}



System Pole at roots of

$s^3LC^2+s^2RC^2+s2C+gm = 0$

gm = 0.01

L=1e-9

C=1e-12

R=5

Three system poles at:

1.0e+10 *

-2e-16 + 4.4721i

-2e-16 - 4.4721i

-0.5000

Still stable