

# EE142 Problem Set 5

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## Problem 1

Find  $y$  for the following normalized impedance on Smith Chart.

The straightforward procedure is to plot  $z_L$  on the impedance Smith Chart, and then look at what constant admittance and constant susceptance curves cross over the point in the admittance smith chart.

But, we can also plot  $z_L$  on the impedance smith chart, then rotate the point by  $\pi$  degrees along the constant SWR circle, and then read off the admittance by looking at the constant resistance and reactance curves.

I'm going to use the second technique; annotated charts aren't included in this document, but I'll compare the chart result I get to the exact calculation.

(a)  $z_L = 1.4 + 2j$

$$y_L = \frac{1}{z_L} = \frac{1}{\alpha + \beta j} = \frac{\alpha - \beta j}{\alpha^2 + \beta^2} = 0.234899 - 0.33557j$$
$$y_{L,chart} = 0.22 - 0.32j$$

(b)  $z_L = 0.5 + 0.9j$

$$y_L = 0.471698 - 0.849j$$
$$y_{L,chart} = 0.45 - 0.85j$$

(c)  $z_L = 1.6 - 0.3j$

$$y_L = 0.60377 + 0.1132j$$
$$y_{L,chart} = 0.6 + 0.12j$$

## Problem 2

Use the Smith Chart. Also use equations for lumped component matching to check.

(a) Match  $Z_L = 70 + 100j\Omega$  to 50 Ohm with lumped components.

Let's clear up some things:

$$Z_C = \frac{1}{j\omega C}$$

$$X_C = \Im Z_C = -\frac{1}{\omega C}$$

$$Z_L = j\omega L$$

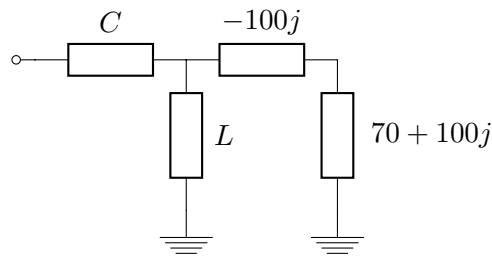
$$X_L = \Im Z_L = \omega L$$

$$\text{To find } C = \frac{1}{\omega X_c}$$

$$\text{To find } L = \frac{X_L}{\omega}$$

$$\text{where: } \omega = 2\pi f$$

The load is complex, so we have to resonant out the load's complex impedance so only a real part is seen before solving using the L network method.



Now, the L-network will see a purely real  $70\Omega$  impedance with which we can use the regular matching equations.

$$R_S = 50$$

$$R_L = 70$$

$$R_{hi} = \max(R_S, R_L) = 70$$

$$R_{lo} = \min(R_S, R_L) = 50$$

$$\text{Boosting factor: } m = \frac{R_{hi}}{R_{lo}} = 1.4$$

$$Q = \sqrt{m^2 - 1} = 0.632$$

$$\text{Dropping resistance so, } X_p = \frac{R_L}{Q} = 110.76$$

$$X'_p = \frac{X_p}{1 + Q^{-2}} = 31.613$$

$$X_s = -X'_p = -31.613$$

We arrive at the capacitor reactance of  $-79.15j$  and the inductor reactance of  $110.76j$ . The circuit is simulated in ADS to match at 1 GHz with component values  $C = 5.0344$  pF,  $L = 17.6$  nH, and  $C_{res} = 1.59$  pF. S-parameter simulation verifies that the source and load are perfectly matched at 1 GHz with  $S_{21} = 0dB$ .

The same calculation can be performed using the smith chart.

$$Z_{L,norm} = 1.4 + 2j$$

$$Z_{L,real} = 1.4$$

$$X_p = (1/0.45)j \cdot 50 = 111.1j$$

$$X_s = -0.62 \cdot 50 = -31j$$

The values calculated using the Smith Chart are very close to the values from the equations.

- (b) Match  $Z_L = 70 + 100j\Omega$  to 50 Ohm using transmission lines.

This is the general procedure for parallel stub matching using a Smith Chart.

- Find the load impedance and normalize it to the transmission line  $Z_0$  (nominally 50 $\Omega$ )
- Plot the load impedance on the Smith Chart
- Reflect the point across the origin so we have a new point representing the load admittance. **The impedance Smith Chart is now an admittance chart.**
- Draw the constant SWR circle and move towards the generator until we intersect with the admittance = 1 circle
- Note the angle in  $\lambda$  that we had to move across to get from the load admittance to the  $G=1$  circle. This is the distance to move from the load towards the generator along the main line before inserting the parallel stub.
- Now, we are at a point where the normalized admittance can be written in the form  $1 + jB$ . We need to insert a short or open stub with an admittance in THIS location of  $0 - jB$ , so mark the point  $-jB$  on the Smith Chart.
- To figure out the length of the stub, note that on this **admittance** chart, short is at the very right and open is at the very left. Now, move towards the short or open load and read off the angle swept in  $\lambda$ .

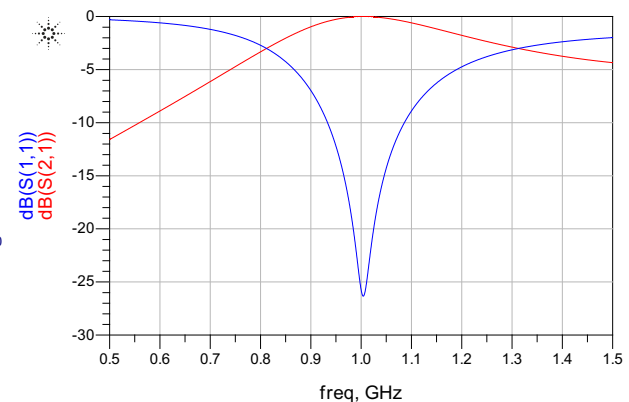
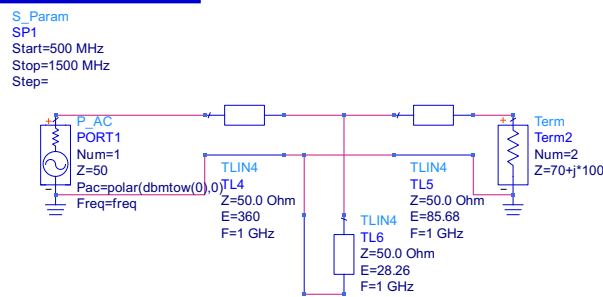
We follow the procedure:

$$l_{from,load,on-main-tline} = (0.5 - 0.4465) + 0.1845 = 0.238\lambda$$

$$l_{to,short,on-stub-tline} = 0.25 - 0.1715 = 0.0785\lambda$$

Confirm with ADS simulation (at 1 GHz):

#### S-PARAMETERS



- (c) Match  $Z_L = 160 - 30j\Omega$  to 100 Ohm using lumped circuits.

Assume a resonating inductor with reactance  $30j$  to make the load purely real.

$$R_S = 100$$

$$R_L = 160$$

$$R_{hi} = \max(R_S, R_L) = 160$$

$$R_{lo} = \min(R_S, R_L) = 100$$

$$\text{Boosting factor: } m = \frac{R_{hi}}{R_{lo}} = 1.6$$

$$Q = \sqrt{m - 1} = 0.775$$

$$\text{Dropping resistance so, } X_p = \frac{R_L}{Q} = 206.452$$

$$X'_p = \frac{X_p}{1 + Q^{-2}} = 77.47$$

$$X_s = -X'_p = -77.47$$

We simulate in ADS with  $L_{res} = 4.77$  nH,  $L = 32.858$  nH,  $C = 2.05$  pF. The simulation shows that these values give a perfect match at 1 Ghz. This match appears more broadband than the one in part a). The Smith Chart again gives very similar values.

- (d) Match  $Z_L = 160 - 30j\Omega$  to 100 Ohm using transmission lines.

- (e) Match  $Z_L = 25 + 90j\Omega$  to 50 Ohm using lumped circuits. Assume a resonating capacitor with reactance  $-90j$  to make the load purely real.

$$R_S = 50$$

$$R_L = 25$$

$$R_{hi} = \max(R_S, R_L) = 50$$

$$R_{lo} = \min(R_S, R_L) = 25$$

$$\text{Boosting factor: } m = \frac{R_{hi}}{R_{lo}} = 2.0$$

$$Q = \sqrt{m - 1} = 1.0$$

$$\text{Boosting resistance so, } X_s = Q \cdot R_L = 25$$

$$X'_s = X_s(1 + Q^{-2}) = 50$$

$$X_p = -X'_s = -50$$

We simulate in ADS with  $C_{res} = 1.768$  pF,  $C = 3.183$  pF,  $L = 3.98$  nH. The results confirm a perfect match at 1 Ghz.

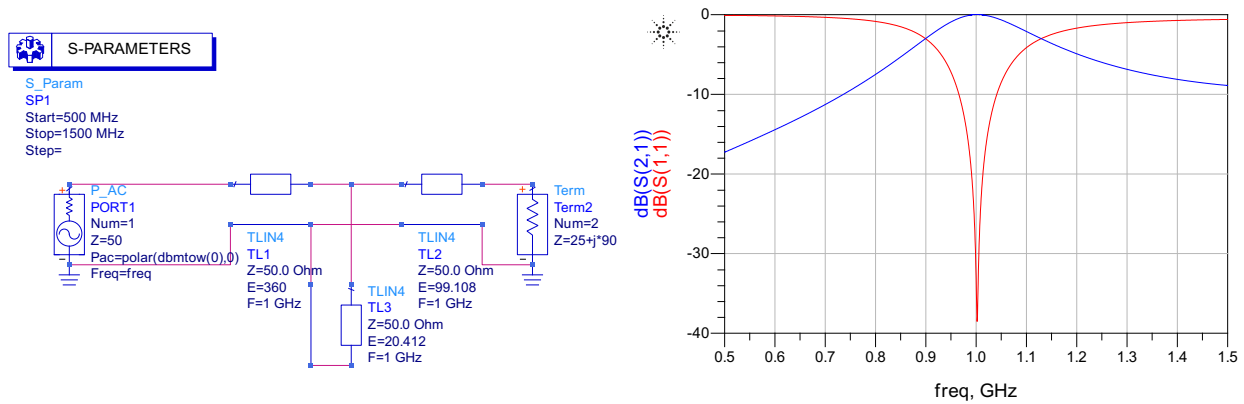
- (f) Match  $Z_L = 25 + 90j\Omega$  to 50 Ohm using transmission lines.

Following same procedure:

$$Z_{L,norm} = 0.5 + 1.8j$$

$$l_{from,load,on-main-tline} = (0.5 - 0.4237) + 0.199 = 0.2753\lambda$$

$$l_{to,short,on-stub-tline} = (0.25 - 0.1933) = 0.0567\lambda$$



### Problem 3

- (a) Design a  $\Pi$  matching network between a  $1000\Omega$  load impedance and a  $50\Omega$  source impedance at 1 GHz. The inductor and capacitor quality factors are 20. The target bandwidth for  $|S_{11}| < -10$  dB is 5%. Calculate the insertion loss and verify your design using ADS. Check if  $|S_{11}|^2 + |S_{21}|^2 = 1$  holds.

Let's first analyze an L-network to see if it can fit our design requirements.

$$\begin{aligned}
 Q_{cap} &= Q_{ind} = 20 \\
 m &= \frac{R_{hi}}{R_{lo}} = 20 \rightarrow Q = \sqrt{m - 1} \approx 4.359 \\
 Q_{total, L-network} &= \frac{Q}{2} \approx 2.2 \\
 S_{11} \text{ -10 dB BW} &\approx \frac{1}{3 \cdot Q_{total}} = 15\% \\
 \text{Insertion Loss} &= \frac{1}{1 + \frac{Q}{Q_{cap}} + \frac{Q}{Q_{ind}}} = 0.694
 \end{aligned}$$

The bandwidth of the L-network is too high and isn't selective enough for our requirements. The bandwidth is set (approximately) by the circuit Q and so we need to use a  $\Pi$  network so Q doesn't depend on  $m$ .

$$\frac{1}{3 \cdot Q_{tot}} \approx 0.05 \rightarrow Q_{tot} \geq 6$$

$$Q_1 = \sqrt{\frac{R_L}{R_i} - 1}$$

$$Q_2 = \sqrt{\frac{R_S}{R_i} - 1}$$

$$Q_{tot} = \frac{Q_1 + Q_2}{2}$$

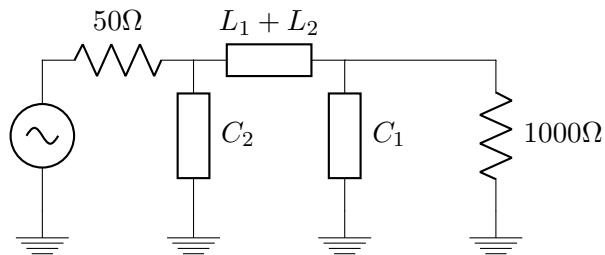
$$R_i \leq 8.108$$

We find that the intermediate resistance should be less than  $8.108 \Omega$  to keep the bandwidth below 5%. We will design for  $R_i = 5\Omega$ .

Here are all the derived parameters:

	L-network 1	L-network 2
m	200	10
Q	14.11	3
$X_p$	70.888	16.666
$X_s$	70.534	15.0
$C$	2.245 pF	9.549 pF
$L$	11.2 nH	1.39 nH

assuming capacitors are placed in parallel and inductors in series.  $Q_{tot}$  is around 8.5.



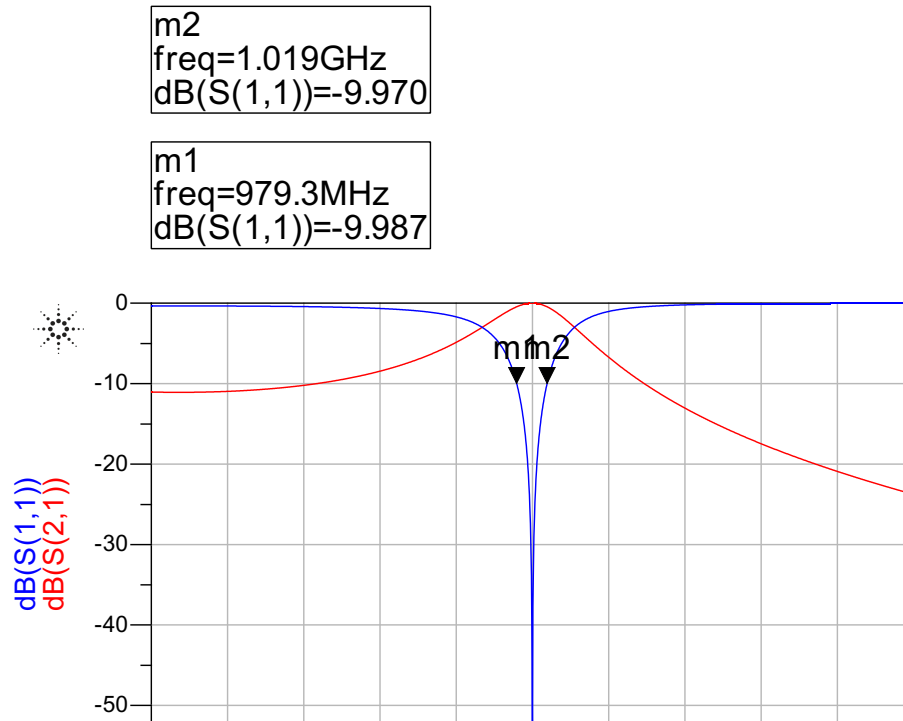


Figure 1: Simulation with lossless components

We run a simulation and find that for lossless components, the bandwidth of  $S_{11}$  down to -10 dB is 39.7 Mhz, which is within our 50 Mhz design spec.

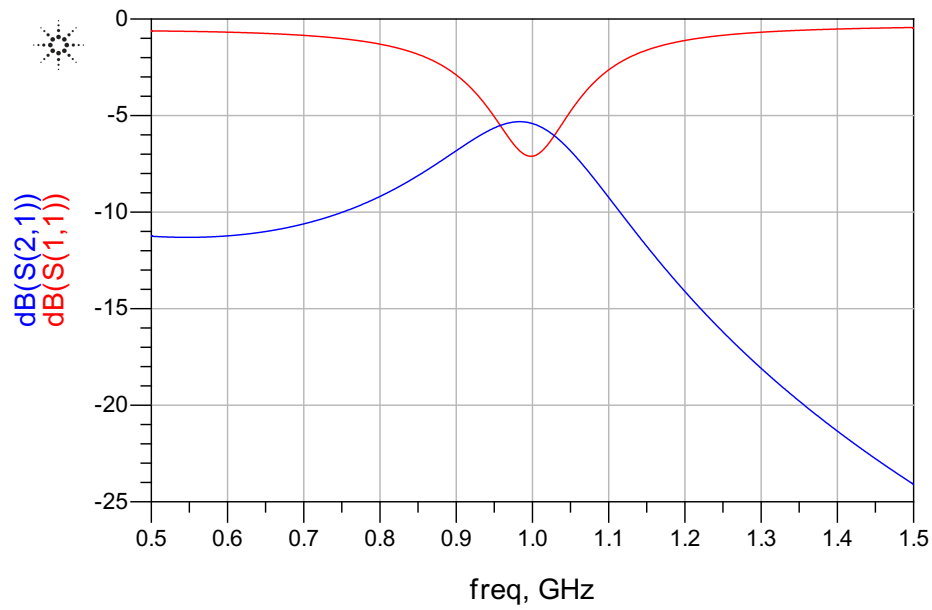
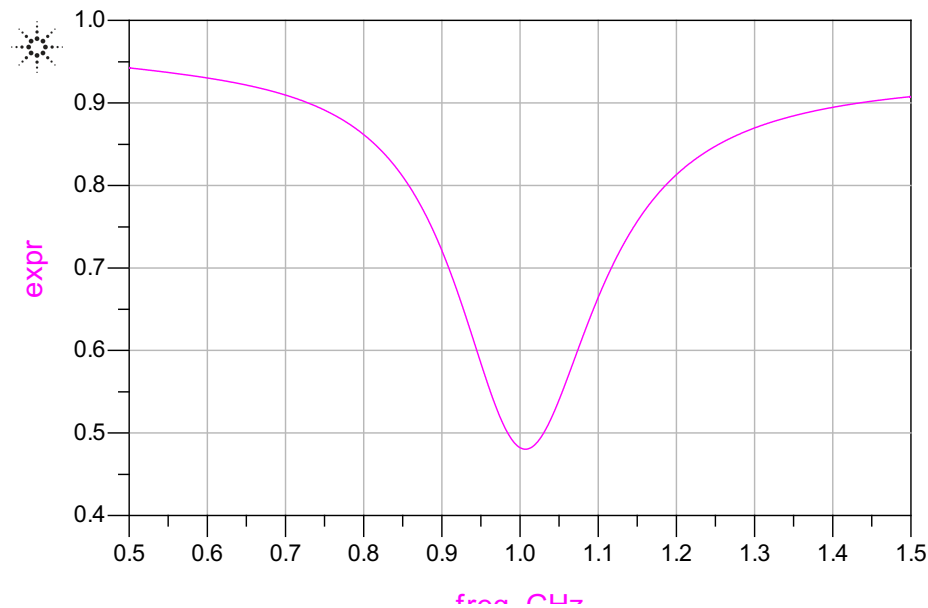


Figure 2: Simulation with finite Q

When adding finite Q components, the bandwidth isn't very different, but the insertion loss at 1 GHz becomes significant. However the  $S_{11}$  -10dB bandwidth becomes infinite/zero since  $S_{11}$  never dips below -10dB. It is possible that the Q of these components isn't sufficient to achieve -10dB input selectivity. It's also possible that my design isn't optimized sufficiently.



$$\text{Eqn } \text{expr} = \text{abs}(S(1,1))^{**2} + \text{abs}(S(2,1))^{**2}$$

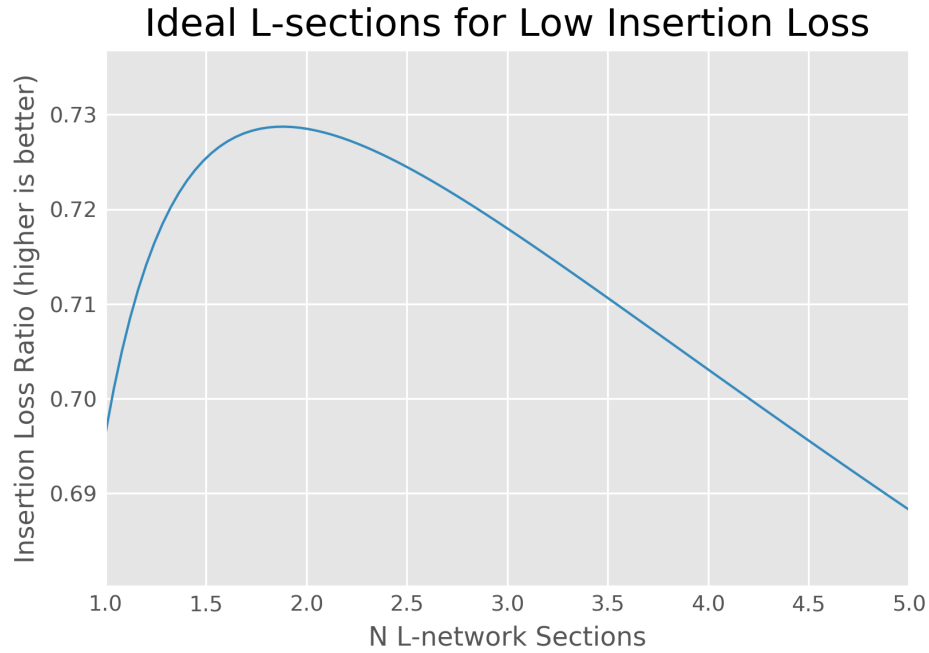


In the finite Q simulation, the relationship  $|S_{11}|^2 + |S_{21}|^2 = 1$  doesn't hold near the center frequency. This is due to the internal loss of the finite Q components.

- (b) Design a matching network between a  $1000\Omega$  load impedance and a  $50\Omega$  source impedance at 1 GHz. The inductor and capacitor quality factors are 20. The design goal is to achieve the lowest insertion loss. Calculate the insertion loss and verify your design using ADS.

$$IL = \frac{1}{1 + \frac{N}{Q_u} \sqrt{\left(\frac{R_{hi}}{R_{lo}}\right)^{1/N} - 1}}$$

We use this equation with our design variables to find the ideal value of  $N$ . We let  $Q_u = Q_{cap} || Q_{ind} = 10$ .



Insertion loss is minimized with 2 L-network stages.  $IL_{max} = 0.729$ .

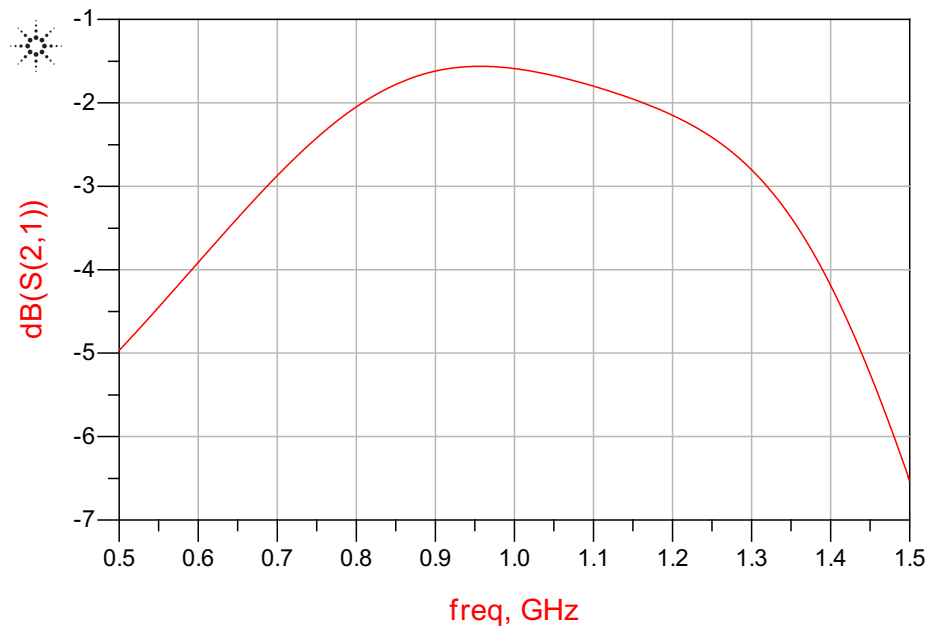
$$R_{i,opt} = \sqrt{R_L R_S} = 223.6$$

$$Q_{i,opt} = \sqrt{\left(\frac{R_{hi}}{R_{lo}}\right)^{1/N} - 1} = 1.863$$

Now we can again go through the process of calculating actual component values.

	L-network 1	L-network 2
$X_p$	536.66	120.0
$X_s$	416.66	93.168
$C$	0.296 pF	1.326 pF
$L$	66.3 nH	14.8 nH

Again assuming that capacitors are in shunt and inductors in series. Use these values and run a finite Q simulation.



The simulation confirms a low insertion loss of around -1.5 dB. This is about 4dB better than the  $\Pi$  network.