

Midterm Exam (closed book/notes)
Thursday, October 30, 2012

Guidelines: Closed book. You may use a calculator. Do not unstaple the exam. In order to maximize your score, write clearly and indicate each step of your calculations. We cannot give you partial credit if we do not understand your reasoning. Feel free to use scratch paper.

Common two-port equation:

$$Y_{in} = Y_{11} - \frac{Y_{12}Y_{21}}{Y_L + Y_{22}}$$
$$Y_{out} = Y_{22} - \frac{Y_{12}Y_{21}}{Y_S + Y_{11}}$$

The voltage gain of a two-port can be written as

$$A'_v = \frac{-Y_S y_{21}}{(Y_S + y_{11})(Y_L + y_{22}) - y_{12}y_{21}}$$

or as

$$A'_v = \frac{A_{vu}}{1 + T}$$

where T is identified as the loop gain

$$T = A_{vu}f = \frac{-y_{12}y_{21}}{(Y_S + y_{11})(Y_L + y_{22})}$$

and

$$A_{vu} = A'_v|_{y_{12}=0} = \frac{-Y_S y_{21}}{(Y_S + y_{11})(Y_L + y_{22})}$$

The power gain of a two-port is given by

$$G_p = \frac{P_L}{P_{in}} = \frac{|Y_{21}|^2}{|Y_L + Y_{22}|^2} \frac{\Re(Y_L)}{\Re(Y_{in})}$$

The maximum gain is given by

$$G_{max} = \frac{Y_{21}}{Y_{12}} (K - \sqrt{K^2 - 1})$$

where K is the stability factor

$$K = \frac{2\Re(Y_{11})\Re(Y_{22}) - \Re(Y_{12}Y_{21})}{|Y_{12}Y_{21}|}$$

Simple trigonometric identify:

$$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(x+y) = \cos(y)\sin(x) + \cos(x)\sin(y)$$

$$2\cos(x)\cos(y) = \cos(x+y) + \cos(x-y)$$

Distortion equations: $s_o = a_1 s_i + a_2 s_i^2 + a_3 s_i^3 + \dots$

$$IM_2 = 2HD_2 = \frac{a_2}{a_1} S_i$$

$$IM_3 = 3HD_3 = \frac{3}{4} \frac{a_3}{a_1} S_i^2$$

Series inversion: If, $s_i = a_1 s_o + a_2 s_o^2 + a_3 s_o^3 + \dots$, then $s_o = b_1 s_i + b_2 s_i^2 + b_3 s_i^3 + \dots$ where

$$b_1 = \frac{1}{a_1}$$

$$b_2 = -\frac{a_2}{a_1^3}$$

$$b_3 = \frac{2a_2^2}{a_1^5} - \frac{a_3}{a_1^4}$$

Cascade of two power series:

$$c_1 = a_1 b_1$$

$$c_2 = b_1 a_2 + b_2 a_1^2$$

$$c_3 = b_1 a_3 + 2b_2 a_1 a_2 + b_3 a_1^3$$

Effect of feedback on distortion

$$b_1 = \frac{a_1}{1 + a_1 f} = \frac{a_1}{1 + T}$$

$$b_2 = \frac{a_2}{(1 + T)^3}$$

$$b_3 = \frac{a_3(1 + T) - 2a_2^2 f}{(1 + T)^5}$$

The intercept points for a cascade of two block is given by (where the second-order interaction has been neglected):

$$\frac{1}{IIP2} = \frac{1}{IIP2^A} + \frac{a_1}{IIP2^B}$$

$$\frac{1}{IIP3^2} = \frac{1}{IIP3_A^2} + \frac{a_1^2}{IIP3_B^2}$$

Taylor Series Expansion about $x = 0$

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

Binomial Theorem

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$$

Newton's Generalized Binomial Theorem (for any r) is expressed as follows. Note that this is a finite sum only for positive integer values of r and an infinite series otherwise:

$$(x + y)^r = x^r + rx^{r-1}y + \frac{r(r-1)}{2!}x^{r-2}y^2 + \frac{r(r-1)(r-2)}{3!}x^{r-3}y^3 + \dots$$

Some useful Taylor Series expansions:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\frac{1}{1 + e^x} = \frac{1}{2} - \frac{1}{4}x + \frac{1}{48}x^3 + \dots$$

$$\frac{e^x}{1 + e^x} = \frac{1}{2} + \frac{1}{4}x - \frac{1}{48}x^3 + \dots$$

$$\tanh(x) = x - \frac{1}{3}x^3 + \dots$$

Bipolar Device I-C relation (Forward Active)

$$I_C = I_S e^{\left(\frac{qV_{BE}}{kT}\right)}$$

MOS Square Law Device Physics (Saturation)

$$I_{DS} = \mu C_{ox} \frac{W}{L} \frac{1}{2} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

$$C_{GS} = \frac{2}{3} W \cdot L C_{ox}$$

$$\omega_T = \frac{g_m}{C_{gs}} = \frac{3}{2} \frac{\mu (V_{GS} - V_T)}{L^2}$$

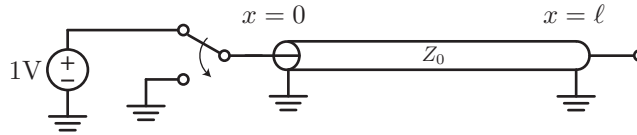
The thermal noise limit kT has a noise power of -174 dBm/ \sqrt{Hz} . Noise figure is defined as the ratio of SNR at the input relative to the output

$$F = \frac{SNR_i}{SNR_o}$$

When two matched blocks are in cascade, the overall noise figure is given by

$$F = 1 + (F_1 - 1) + \frac{F_2 - 1}{G_1} = F_1 + \frac{F_2 - 1}{G_1}$$

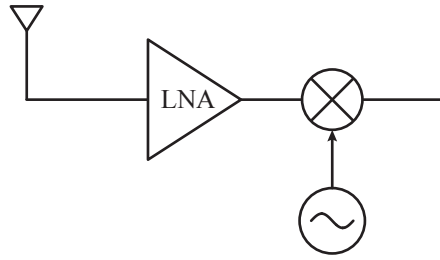
1. (25 points) The following transmission line, initially charged to 1V, is discharged by the switch at time $t = 0$ s. The transmission line has a length of 10cm, a distributed capacitance of $C' = 0.1\text{pF/mm}$ and distributed inductance of $L' = 1\text{nH/mm}$.



- (a) (5 points) What is the velocity a traveling wave on the line.
- (b) (10 points) Plot the **current** waveform as a function of time at the input of the line. Explain your plot with a few lines of derivation or by sketching a bounce diagram. Label the plot x and y axis scales appropriately with numerical values.

(c) (*5 points*) What is the initial energy on the line? What is the final energy on the line? Explain.

(d) (*5 points*) Qualitatively sketch the waveform if the transmission line were lossy.



2. (25 points) Consider the following simple receiver consisting of a low-noise amplifier followed by a mixer. The performance of each block has been characterized in a 50Ω environment:

	LNA	Mixer
NF	3dB	10dB
Power Gain	16 dB	0 dB
Input Intercept IIP_3	-10 dBm	+6 dBm

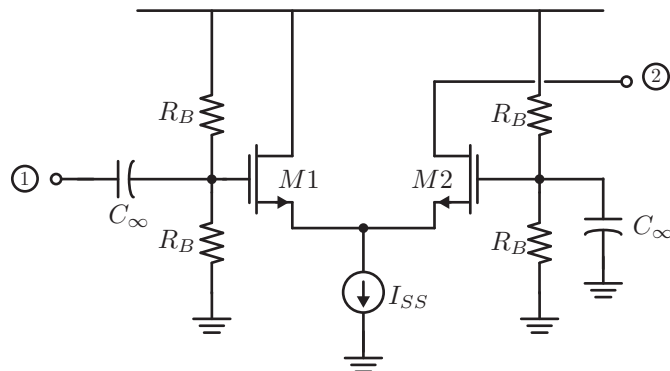
- (a) (6 points) Calculate the overall noise figure of the cascade. Which block dominates? Why?

- (b) (*6 points*) Calculate the overall input third intercept point (dBm) for the receiver. Which block dominates? Why?

- (c) (*6 points*) What is the minimum detectable signal if the required $\text{SNR} = 10$ dB.
The channel bandwidth is 100 MHz.

- (d) (*7 points*) While the receiver is tuned to 1 GHz, suppose two interfering signals appear at 1.1 GHz and 1.2 GHz, each of equal power P_B . Calculate the total allowable power P_B that the receiver can tolerate such that the in-band distortion products produced by these blockers does not have power more than the minimum detectable signal level.

3. (25 points) Consider the following two-stage amplifier as a two-port element. For each transistor, $f_T = 25\text{GHz}$ and $C_{gd} = C_{db} = 0.2C_{gs}$.

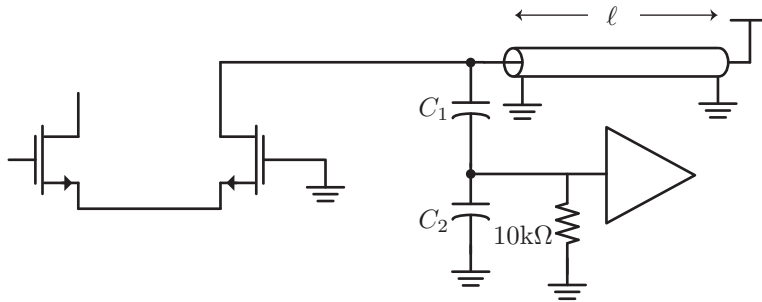


- (a) (7 points) Find the y-parameters assuming the two-port is unilateral.

(b) (6 points) Calculate the maximum possible gain that you can attain from this two-port (*Hint*: Use the fact that it is a unilateral two-port.).

(c) (6 points) Design an input matching network to obtain the maximum gain. Size R_B for a network Q of 5. The center frequency is 5 GHz, the bias current is $I_{EE} = 2\text{mA}$, the overdrive voltage $V_{dsat} = 200\text{mV}$, and for the selected bias point $r_o = 50\text{k}\Omega$.

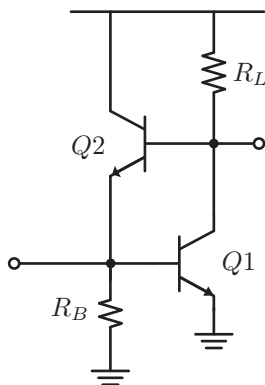
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- (d) (6 points) Now consider the interface to the second stage, shown above. The supply feed comes in on a transmission line ($Z_0 = 50\Omega$, $v = c$), and C_1 and C_2 are sized to provide the maximum gain from the input load of $10\text{k}\Omega$ presented by the second amplifier. Calculate the values of C_1 , C_2 , and ℓ .

(cont.)

4. (25 points) Consider the following amplifier. Assume ideal BJTs (neglect r_π and r_o).



- (a) (8 points) Calculate the two-port parameters of the circuit. Hint, you can break it up into two simpler components.

(b) (7 points) Find the loop gain using two-port theory. If you cannot solve the above part, use return-ratio analysis.

(c) (10 points) Assume that Q1 dominates the distortion and calculate the HD_2 for the amplifier.