EE240B HW2

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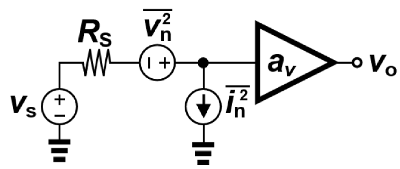
1 Electronic Noise



Noise source	Amplifier A	Amplifier B
$\sqrt{\frac{\overline{v_n^2}}{\Delta f}}$	$1 \frac{\text{nV}}{\sqrt{\text{Hz}}}$	$10 \frac{\text{nV}}{\sqrt{\text{Hz}}}$
$\sqrt{\frac{\overline{i_n^2}}{\Delta f}}$	$1 \frac{\text{pA}}{\sqrt{\text{Hz}}}$	$1 \frac{\text{fA}}{\sqrt{\text{Hz}}}$

(a) Calculate the input referred noise in $V/\sqrt{\text{Hz}}$ achieved with the two amplifiers for $R_S = 50\Omega, 5\text{M}\Omega$. Only consider the amplifier noise. Assume the amplifier voltage and current sources are uncorrelated.

R_s	Amplifier A	Amplifier B
50 Ω	$1.05 \frac{\text{nV}}{\sqrt{\text{Hz}}}$	$10 \frac{\text{nV}}{\sqrt{\text{Hz}}}$
5 MΩ	$5000 \frac{\text{nV}}{\sqrt{\text{Hz}}}$	$15 \frac{\text{nV}}{\sqrt{\text{Hz}}}$

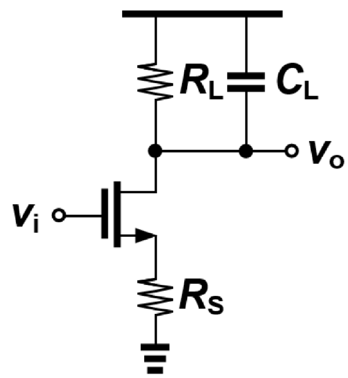
(b) Significance of the result?

If the input voltage source's resistance is low, an amplifier with low voltage noise should be chosen (since the amplifier's current noise won't see a large resistance). Use a BJT op-amp.

If the input voltage source's resistance is high, an amplifier with low current noise should be chosen. This is usually a FET op-amp.

2 Amplifier Noise

Derive analytical expressions as a function of $g_m, \gamma, f_T, R_L, R_S$, and C_L. Consider only thermal noise from R_S, R_L, and transistor shot noise. Ignore flicker noise, r_o, and body effect. C_L is fixed and C_{gs} is expressed in terms of f_T.



(a) The voltage gain $a_v(s) = v_o/v_i$

This is a standard source-degenerated common-source amplifier with a complex load. We need to consider C_{gs} which will give a 2nd-order transfer function and makes the algebra complicated. I'm using the sympy CAS to do the algebra. I'll provide the source and the derived solution. Check this IPython notebook for all the source code.

```
from sympy import *
def ll(a, b): # Put impedances a and b in parallel symbolically
    return (a*b) / (a + b)
s, RL, CL, ZL, Cgs, Rs, gm = symbols("s R_L C_L Z_L C_{gs} R_s g_m")
igate, isource, idrain = symbols("i_g i_s i_d")
vs, vd, vi = symbols("v_s v_d v_i")
sol = nonlinsolve([
    igate - ((vi - vs) / (1 / (s*Cgs))),
    isource - (vs / Rs),
    idrain - gm*(vi - vs),
    -vd - idrain*ZL,
    isource - igate - idrain
], [igate, isource, idrain, vd, vs])
vd_sol = sol.args[0][3]
gain = vd_sol / vi
ZL_sol = ll(CL, RL)
ZL_sol = ZL_sol.subs(CL, (1 / (s*CL))).simplify()
display(gain.subs(ZL, ZL_sol).simplify())
```

$$a_v(s) = -\frac{R_L g_m}{(C_L R_L s + 1)\,(C_{gs} R_s s + R_s g_m + 1)}$$

(b) The PSD of the noise in $\frac{\mathrm{V}}{\sqrt{\mathrm{Hz}}}$ at node v_o

I derived the noise due to R_S, R_L, and the transistor using the hybrid-pi model and a CAS.

$$
\begin{aligned}
V_{n,out,R_s}(s) &= \frac{R_L g_m v_{nS}}{(C_L R_L s + 1)(C_{gs} R_s s + R_s g_m + 1)} \\
V_{n,out,R_L}(s) &= \frac{R_L i_{nL}}{C_L R_L s + 1} \\
V_{n,out,transistor}(s) &= -\frac{R_L i_{nT}(C_{gs} R_s s + 1)}{(C_L R_L s + 1)(C_{gs} R_s s + R_s g_m + 1)}
\end{aligned}
$$

We can easily substitute for v_{ns}, i_{nL}, i_{nT} with the actual noise voltage and current of R_S, R_L and the transistor respectively. To find output noise power, these quantities need to be squared and added together.

(c) The total noise integrated over all frequencies at node v_o

Take the total output noise power and integrate from $s = 0 \to \infty j$. See the lecture slides for the standard integrals.

(d) The dynamic range of the circuit as a function of the peak-to-peak voltage range $V_{o,pp}$ at node v_o

$$
\text{Output Dynamic Range } = dB_{power}\left(\frac{V_{o,pp}^2 + \overline{V_{n,out,total}^2}}{\overline{V_{n,out,total}^2}}\right)
$$

(e) The minimum detectable signal in $\frac{\mathrm{nV}}{\sqrt{\mathrm{Hz}}}$ at low frequency and μV_{rms} based on the total noise at the output of the amplifier

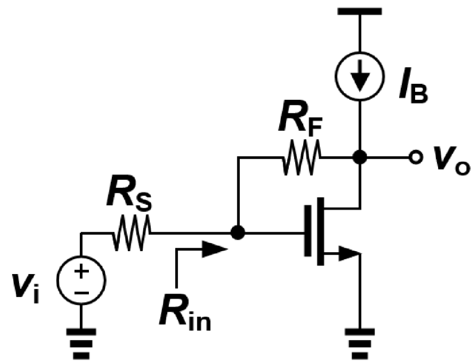
Find the transfer function of the noise sources, but this time treat C_{gs} and C_L as opens at low frequency (set them to 0).

$$
\overline{V_{n,out,lowfreq}} = \frac{2R_L\left(I_d R_L q + 2KR_L R_s T g_m^2 + 2KT(R_s g_m + 1)^2\right)}{(R_s g_m + 1)^2}
$$

(f) Comment on the effect of R_S on the dynamic range and minimum detectable signal of the circuit

R_S suppresses the shot noise of the transistor which improves the minimum detectable signal, but it reduces the maximum output swing by a larger margin which degrades the dynamic range. There is probably a specification-specific optimal choice of R_S if output swing isn't critical.

3 Noise Cancelling Amplifier



(a) Find the input impedance R_{in}

Neglecting r_o, the input impedance $R_{in} = \frac{1}{g_m}$

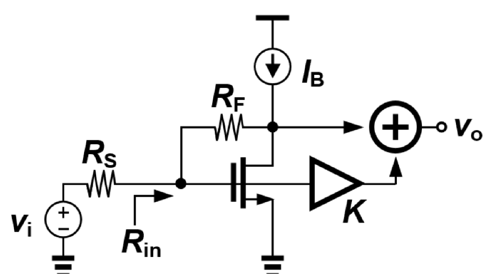
(b) What is the output noise under the constraint of impedance matching ($R_{in} = R_S$)? Ignore noise from current source I_B. First find the noise due to R_S:

$$V_g = \frac{v_{n,R_S}}{1 + g_m R_S}$$
$$V_o = \frac{v_{n,R_S}(1 - g_m R_F)}{1 + g_m R_S}$$
$$\overline{V_o^2} = kT\frac{1}{g_m}(1 - g_m R_F)^2$$

Then the noise due to R_F:

$$V_g = 0$$
$$V_o = V_g + V_{n,R_F} + g_m V_g R_F = V_{n,R_F}$$
$$\overline{V_o^2} = \overline{V_{n,R_F}^2} = 4KTR_F$$

(c) What is the output noise with the modified topology below, under a matching condition? How would you pick the value of K? Is K positive or negative?



$$\overline{V_o^2} = kT\frac{1}{g_m}K^2 \pm (kT\frac{1}{g_m}(1 - g_m R_F)^2 + 4kTR_F)$$

(d) Replace the functional block above with a real circuit, and re-calculate the output noise.