EE 240B – Fall 2019

Advanced Analog Integrated Circuits

Lecture 7: Noise- and SNR-Limited Amplifier Design Methodology

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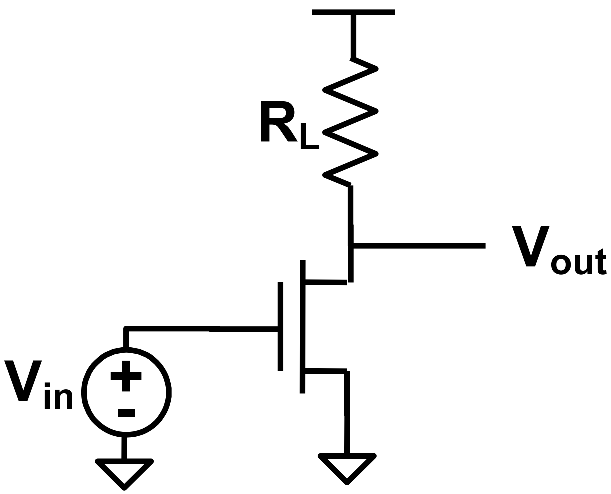
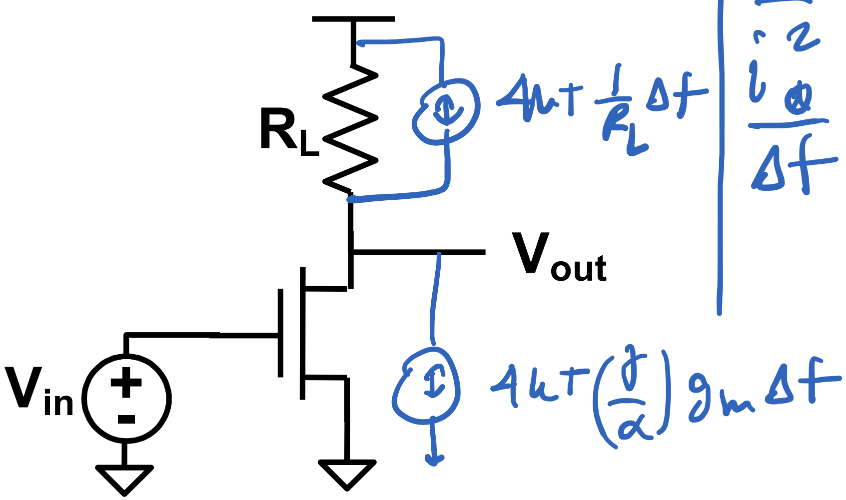
Noise Density Limited Amplifier Design Methodology



- **Input specifications:**
  - Minimum small signal gain A_v
  - Supply voltage V_{dd}
  - Fixed V*
  - Maximum input-referred noise spectral density $v_{i,n}^2/\Delta f$
- **Goal: minimize power**

Small Signal Model and Noise Analysis



$$\frac{\overline{i_o^2}}{\Delta f} = 4kT\left(\frac{1}{R_L} + \frac{\gamma}{\alpha} g_m\right)$$

$$\frac{\overline{v_i^2}}{\Delta f} = \frac{\overline{i_o^2}/\Delta f}{g_m^2} = 4kT\frac{1}{g_m}\left(\frac{1}{g_m R_L} + \frac{\gamma}{\alpha}\right)$$

$$\frac{\overline{v_{i,max}^2}}{\Delta f} < \overline{v_i^2} = 4kT\frac{1}{g_m}\left(\frac{1}{A_{v0}} + \frac{\gamma}{\alpha}\right)$$

excess noise

Resulting Design

$$g_m \geq \frac{4kT\left(\frac{1}{A_{vo}} + \frac{\gamma}{\alpha}\right)}{\left(\overline{v_{i,max}^2}/\Delta f\right)}$$

$$I_D = \frac{g_m V^*}{2} = \frac{4kT}{2}\left(\frac{1}{A_{vo}} + \frac{\gamma}{\alpha}\right) \cdot \frac{V^*}{\left(\overline{v_{i,max}^2}/\Delta f\right)}$$

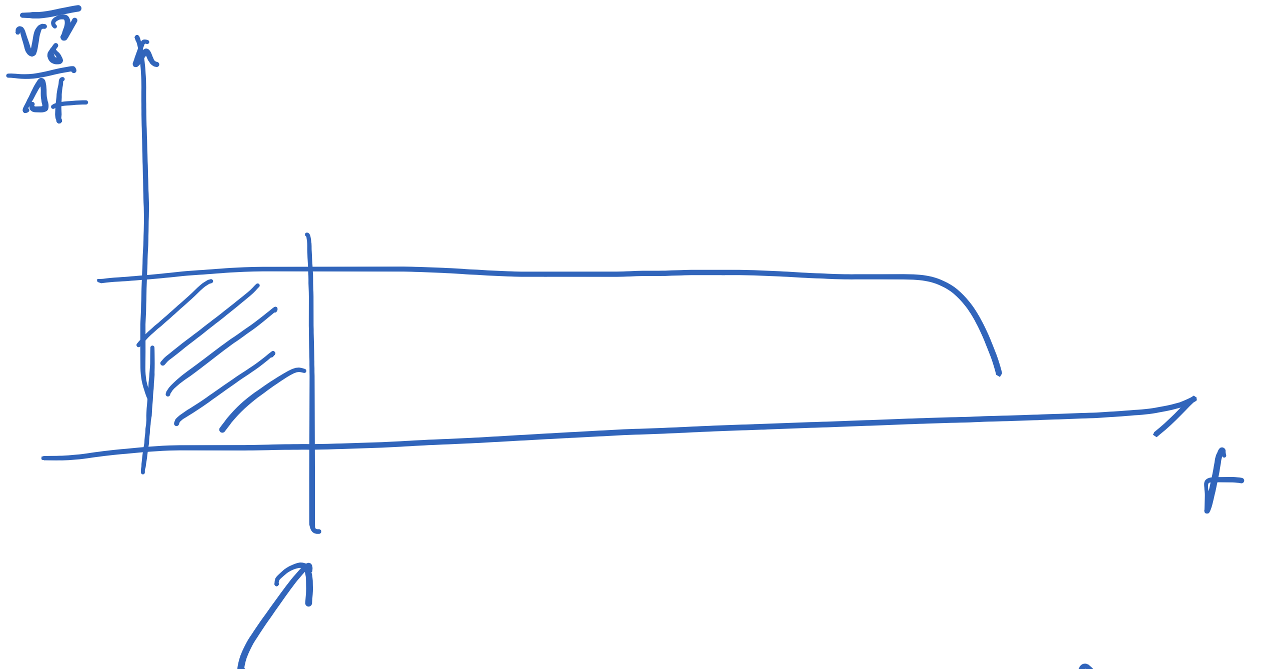
$$A_v = g_m (R_L \| r_o) = \frac{g_m}{\frac{1}{R_L} + \frac{1}{r_o}} = \frac{g_m R_L}{1 + \frac{g_m R_L}{a_{vo}}} = \frac{A_{vo}}{1 + \frac{A_{vo}}{a_{vo}}}$$

$$A_{vo} = A_v\left(1 + \frac{A_{vo}}{a_{vo}}\right) \qquad A_{vo}\left(1 - \frac{A_v}{a_{vo}}\right) = A_v$$

$$\boxed{A_{vo} = \frac{A_v}{1 - \frac{A_v}{a_{vo}}}} = g_m R_L$$

Discussion (1)

- **Why did we not even specify the capacitive load?**



Explicitly set BW for noise (digital or analog domain)

Discussion (2)

- **If you could exactly set a_{v0}, what value would you pick?**

A_v – gain

a_{vo} – intrinsic gain

A_{vo} – gain if $r_o = \infty\Omega$

$$A_{vo} = \frac{A_v}{1 - \frac{A_v}{a_{vo}}}$$

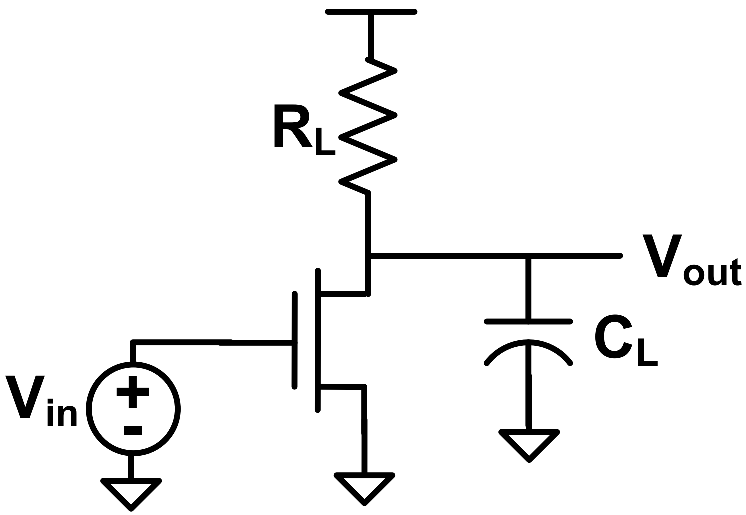
$$g_m = \frac{4kT\left(1 + \frac{\gamma}{\alpha} A_{vo}\right)}{A_{vo}\left(\overline{v_{in}^2}/\Delta f\right)}$$

$$= \frac{4kT\left(\frac{\gamma}{\alpha}\right)}{\left(\overline{v_{in}^2}/\Delta f\right)}$$

$(a_{vo} = A_v)$

$R_L = \infty\Omega \Rightarrow$ Supply Limited

Integrated Noise-Limited Amplifier



- **Input specifications:**
  - Minimum small signal gain A_v
  - Minimum 3dB bandwidth ω_{bw}
  - Supply voltage V_{dd}
  - Fixed V*
  - Maximum noise variance $v_{o,n}^2$
- **Goal: minimize power**

C_L is a design parameter

Required C_L, g_m, and I_D

$$\overline{V_{o,T}^2} = \frac{kT}{C_L}\left(1 + \frac{\gamma}{\alpha} A_{vo}\right)$$

$$A_{vo} \triangleq g_m R_L$$

$$\boxed{C_L \geq \frac{kT\left(1 + \frac{\gamma}{\alpha} A_{vo}\right)}{\overline{V_{o,T}^2}}}$$

$$GBW = \frac{g_m}{C_L} = A_v\, \omega_{BW}$$

$$g_m = \frac{A_v\, \omega_{BW} \cdot kT\left(1 + \frac{\gamma}{\alpha} A_{vo}\right)}{\overline{V_{o,T}^2}}$$

$$I_D = \frac{g_m V^*}{2} \longrightarrow$$

R_L SET AGAIN BY GAIN

Discussion (1)

- **For both noise-density and integrated noise-limited amplifiers, what V* should you pick?**

$V^* \leftrightarrow J$ (current density)

– Input noise density $\rightarrow$ V^* large enough to meet GBW req

– For integrated noise, adding C_L to meet GBW. $C_{tot} > C_L$, $\uparrow V^*$

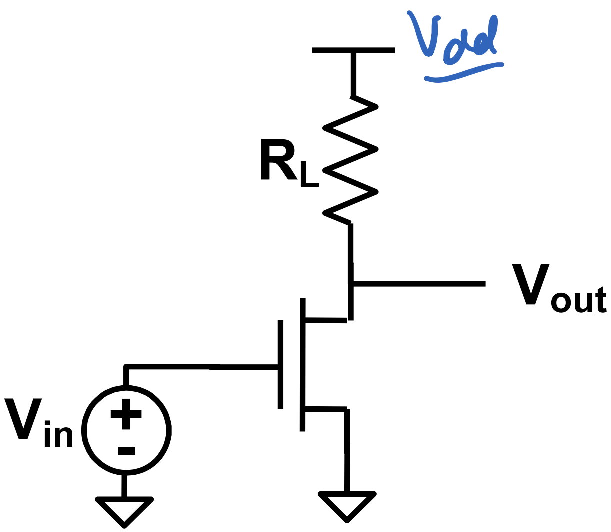
Discussion (2)

- **How would one know the $v_{i,n}^2/\Delta f$ or $v_{o,n}^2$ spec?**

- Usually we care about SNR, not just noise

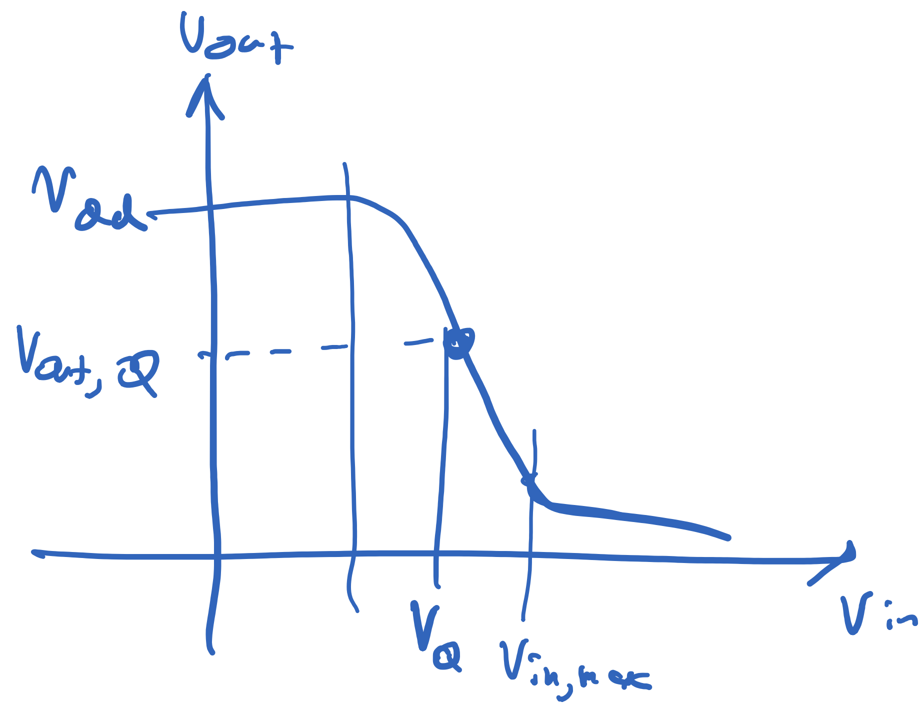
- What is limiting out signals (e.g. V_{DD})

Signal Swing Limitations



$$V_{out,Q} \simeq \frac{V_{dd}}{2} + \delta V$$

$$V_{out,min} \simeq V^*$$



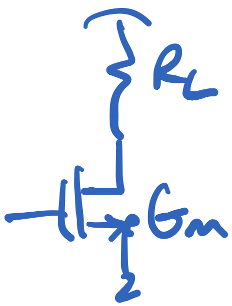
LINEARITY DETERMINES SWING

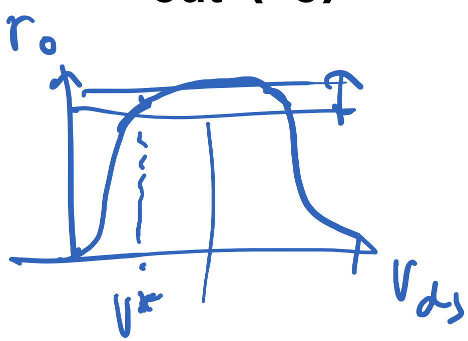
Why Linearity Matters

- **Option 1: Retaining the original shape of the input inherently matters**
  - E.g., oscilloscope, spectrum analyzer
  - (Actually also often matters in communication systems)
- **Option 2: Need to be able to discern a (small) signal out of the combination of many others**
  - E.g., RF, neural front-ends
  - “Other” signals could
- **Precise linearity metric depends on usage scenario**
  - More next time – will use simplified metric for now

Sources of Non-Linearity

- **Output limited: Non-linear Z_{out} (r_o)**





– Want $r_o > k \cdot R_L$

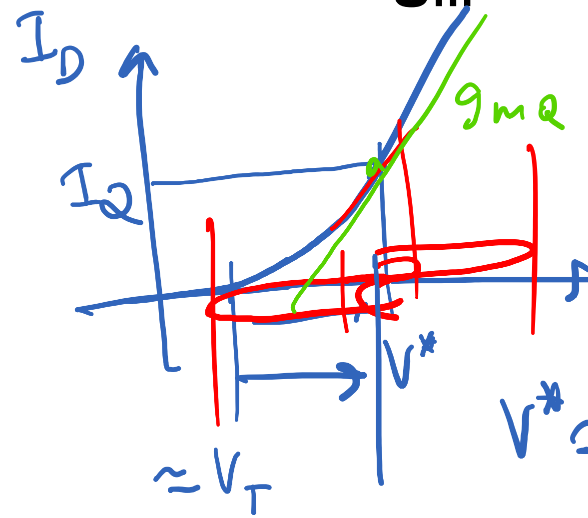
$$a_{vo} = g_m r_o \geq k \underbrace{R_L \cdot g_m}_{A_{vo}}$$

$$a_{vo} \geq k \frac{A_v}{1 - A_v/a_{vo}}$$

$$a_{vo} - A_v \geq k A_v$$

$$a_{vo} \geq (k+1) A_v$$

- **Input limited: Non-linear g_m**

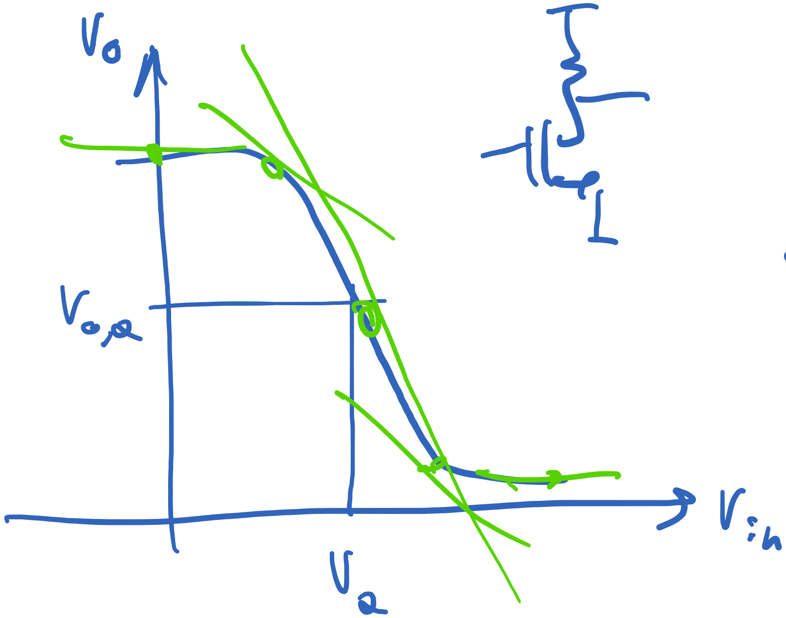


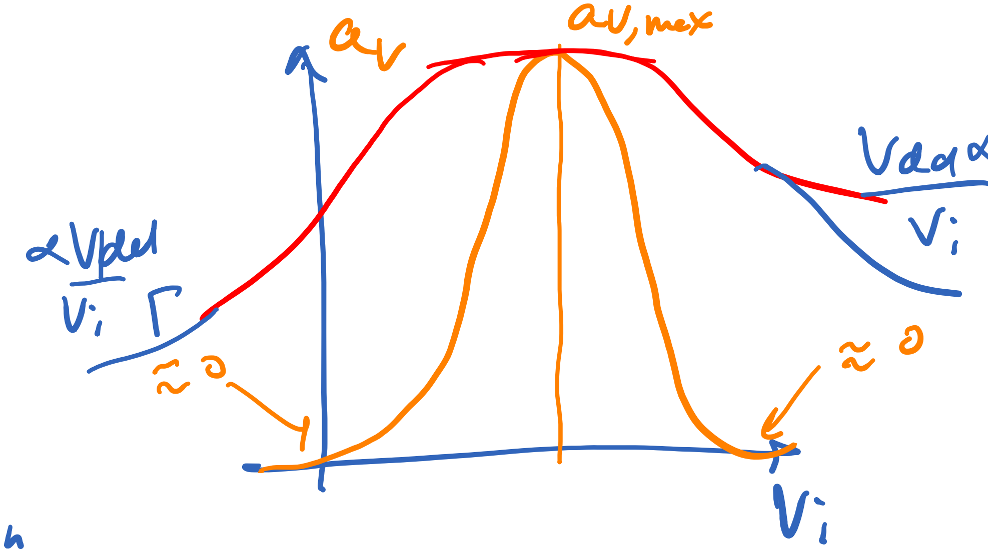
$V^* \simeq V_{od}$

input swing $< V^*$

$\Rightarrow I_d$ "clips"

Linearity: Small vs. Large-Signal Gain





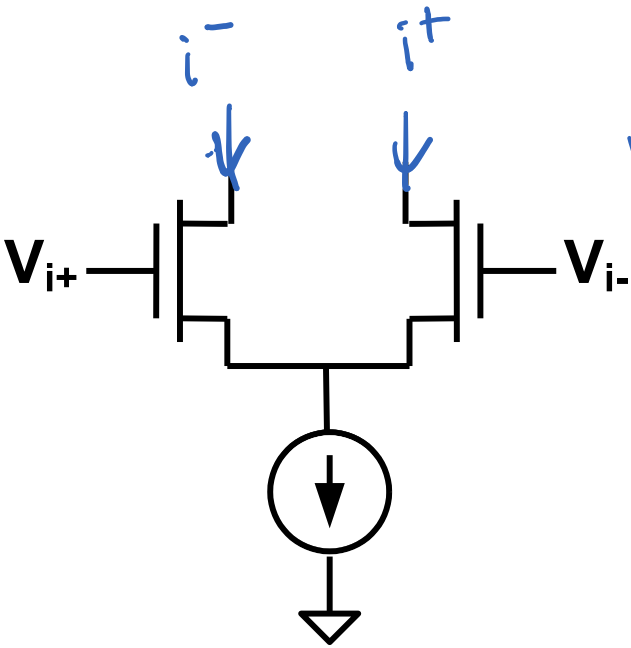
- **Small Signal:**

$$a_{vo} = \frac{dV_{out}}{dV_{in}}$$

- **Large Signal:**

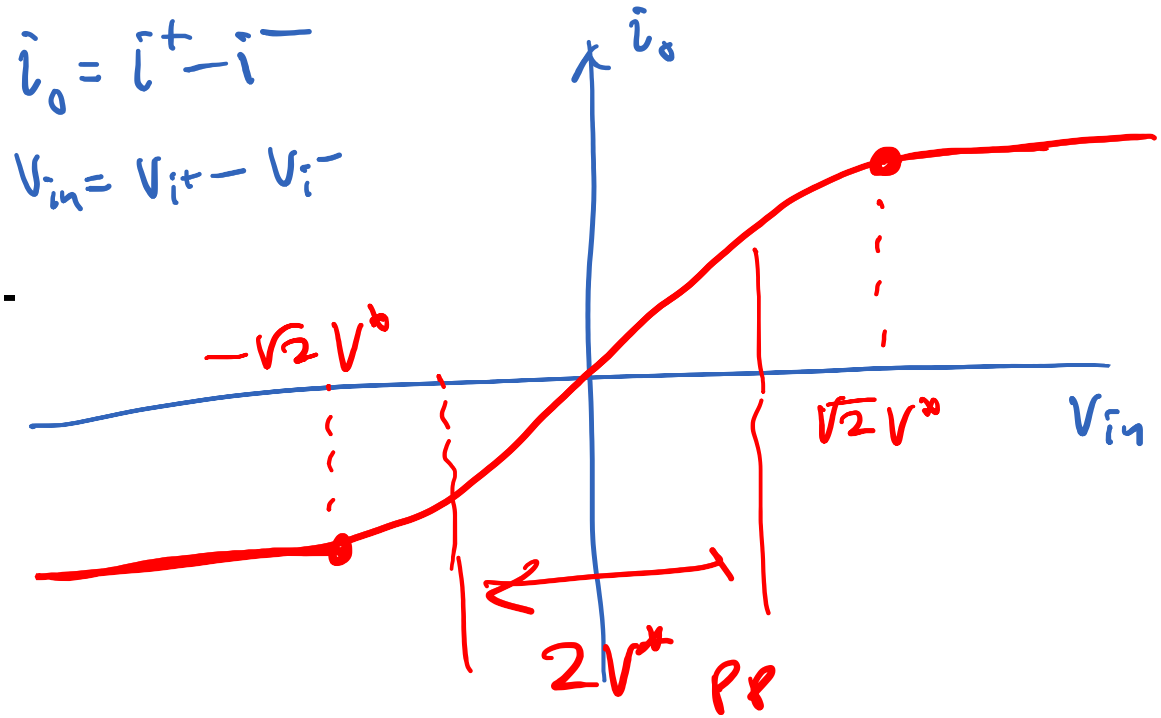
$$A_{vo} = \frac{V_{out} - V_{out\_o}}{V_{in} - V_{in\_o}}$$

Input Non-Linearity with a Diff. Pair



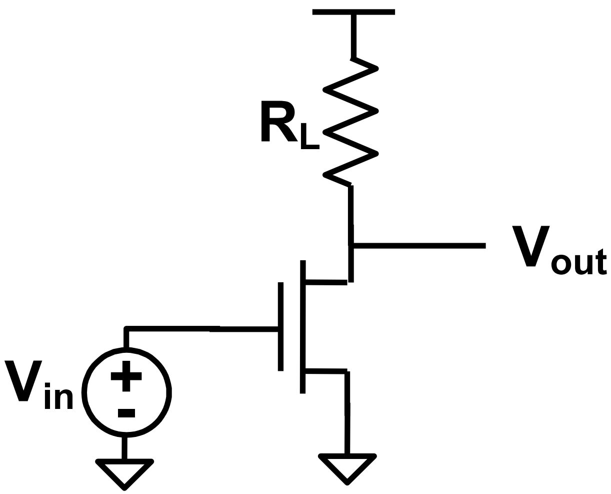
$i_o = i^+ - i^-$

$V_{in} = V_{i^+} - V_{i^-}$



≈ 2X Better than single FET CS amp

Full Circle: SNR-Limited Design (noise density)



- **Input specifications:**
  - Minimum small signal gain A_v
  - Supply voltage V_{dd}
  - Input-referred maximum linear amplitude $V_{i,max}$ ($\approx V^*$)
  - Signal shape (usually sinusoid) and amplitude V_{sig}
  - Externally determined bandwidth f_{bw}
  - Minimum signal-to-noise ratio SNR_{min}
- **Goal: minimize power**

Required $v_{i,n}/\Delta f$

- Assume sinusoidal signal drive
- Signal power $= \frac{1}{2} V_{sig}^2$
- Input refer noise power =

$$4kT\left(\frac{\gamma}{\alpha}+\frac{1}{A_{v0}}\right) f_{bw}\cdot\frac{1}{g_m}$$

$$SNR_{min} = \frac{\cancel{\frac{1}{2}}\, V_{sig}^2 \cdot g_m}{\cancel{4}8kT\cdot\left(\frac{\gamma}{\alpha}+\frac{1}{A_{v0}}\right) f_{bw}} \Rightarrow \text{Solve for } g_m$$

Required V*

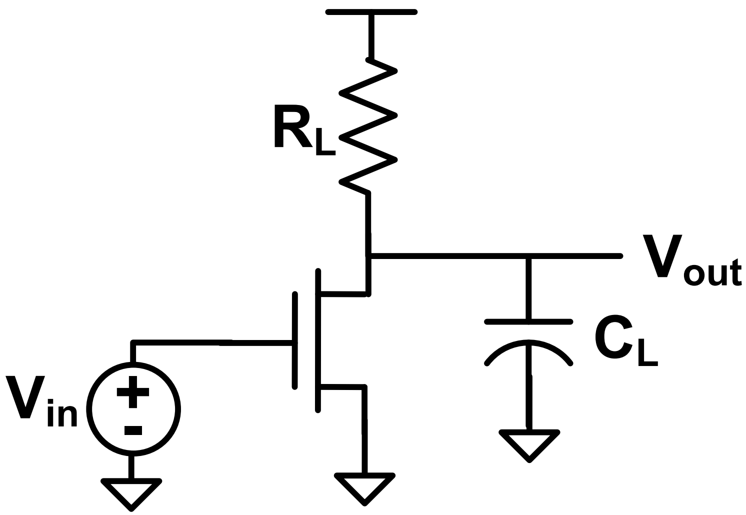
$$g_m > \frac{8kT\left(\frac{\gamma}{\alpha} + \frac{1}{A_{vo}}\right) f_{bw} \cdot SNR_{min}}{V_{sig}^2}$$

$$V_{i,max} < \frac{V^*}{2} \qquad V^* = 2V_{i,max}$$

$$\frac{\cancel{2} I_D}{\cancel{2} V_{i,max}} = \frac{8kT\left(\frac{\gamma}{\alpha} + \frac{1}{A_{vo}}\right) f_{bw} \cdot SNR_{min}}{V_{sig}^2}$$

$$I_D = \frac{8kT\left(\frac{\gamma}{\alpha} + \frac{1}{A_{vo}}\right) f_{bw} \cdot SNR}{V_{sig}} \left(\frac{V_{i,max}}{V_{sig}}\right)$$

SNR-Limited Design (total noise)



- **Input specifications:**
  - Minimum small signal gain A_v
  - Minimum 3dB bandwidth ω_{bw}
  - Supply voltage V_{dd}
  - Input-referred maximum linear amplitude $V_{i,max}$
  - Signal shape (usually sinusoid) and amplitude V_{sig}
  - Minimum signal-to-noise ratio SNR_{min}

- **Goal: minimize power**

Methodology

① • $\frac{V^*}{2} = V_{i,max}$

③ • $g_m = A_v \cdot \omega_{bw} \cdot C_L$

④ • $I_d = \frac{g_m V^*}{2}$

② • $SNR_{min} = \frac{\frac{1}{2} A_v^2 V_{sig}^2}{\frac{kT}{C_L}\left(1 + \frac{\gamma}{\alpha} A_{v0}\right)}$

$$C_L = \frac{2kT\, SNR_{min}\left(1 + \frac{\gamma}{\alpha} A_{v0}\right)}{A_v^2 V_{sig}^2}$$

Discussion