

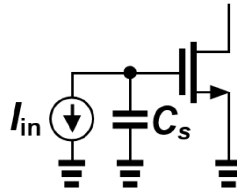
EE240B HW3

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1 Amplifier Noise

For the circuit below calculate the transconductance g_m that minimizes the minimum detectable signal (when I_{in} equals to input-referred current noise) as a function of γ , ω_T , and C_s . In the plot of MDS vs g_m , comment briefly on why there is a minimum and how the slope relates to g_m . Consider C_{gs} and ignore C_{gd} .



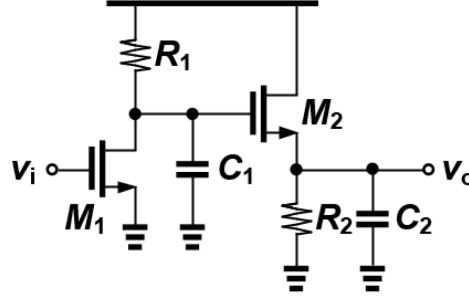
I think the answer is just that, to first order, the MDS is independent of g_m ; this is because $\omega_T = \frac{g_m}{C_s}$ and ω_T is a process and bias constant. Therefore any increase in C_s would also necessitate a proportional increase in g_m . Notice that the output noise caused by a current noise at the input is $\overline{i_s^2} \left(\frac{\omega_T}{\omega} \right)^2$ which is a constant!

The solution doesn't make sense. Firstly the 2 expressions given for $\overline{i_{out}^2}$ aren't equal; they are off by a factor of 4.

But that shouldn't change the optimal g_m anyways. Indeed if you differentiate and set to 0 the expression in the solution, the solution $g_{m,opt} = C_s \omega_T$ is what comes out, but that just equals g_m again! How is it legitimate to differentiate the expression in the solution wrt g_m if ω_T itself can be written in terms of g_m ?

2 Amplifier Design

- (a) What is the total noise at the output of the common-source-common-drain cascade shown below? Ignore flicker noise, r_o , and capacitor except those drawn in the diagram. You should provide your answer in terms of kT , C_1 , C_2 , γ , A_{v1} , A_{v2} , ω_{p1} , ω_{p2} . $A_{v1,v2}$ and $\omega_{p1,p2}$ are the low-frequency voltage gains and dominant poles of the two stages.



$$\begin{aligned}
 \overline{v_{n,o}^2} &= \underbrace{4kT \left(g_{m2}\gamma + \frac{1}{R_2} \right)}_{\text{Current noise from } M_2 \text{ and } R_2} \cdot \overbrace{\left(R_2 \parallel \frac{1}{g_{m2}} \right)^2}^{\text{Converted to voltage by } R_{out}} \cdot \underbrace{\int_0^\infty \left| \frac{1}{1 + s/\omega_{p2}} \right|^2 df}_{\text{Shaped by output pole}} + \\
 &\quad \underbrace{4kT \left(g_{m1}\gamma + \frac{1}{R_1} \right) \cdot R_1^2}_{\text{Current noise from } M_1 \text{ and } R_1} \int_0^\infty \left| \frac{1}{1 + s/\omega_{p1}} \right|^2 df \cdot A_{v2}^2 \cdot \int_0^\infty \left| \frac{1}{1 + s/\omega_{p2}} \right|^2 df \\
 &= 4kT \left(g_{m2}\gamma + \frac{1}{R_2} \right) \left(\frac{R_2}{1 + R_2 g_{m2}} \right)^2 \frac{\omega_{p2}}{4} + 4kT \left(g_{m1}\gamma + \frac{1}{R_1} \right) R_1^2 A_{v2}^2 \frac{\omega_{p1} \omega_{p2}}{16}
 \end{aligned}$$

Sub $R_1 = \frac{1}{\omega_{p1} C_1}$ and R_2 from solving $\omega_{p2} = \frac{1}{(R_2 \parallel 1/g_{m2}) C_2}$. Just some painful algebra.

- (b) We have a fixed power budget for the two stages of amplifier and we would like to minimize the total noise at the output. To simplify the analysis, we can assume the V^* 's and GBW of the two stages are identical, $R_1 C_1 = R_2 C_2$, and $g_{m1} R_1, g_{m2} R_2 \gg 1$. What portion of power would you allocate to the M_1 in order to minimize the total noise at the output? (Hint: In order to maintain fixed GBW, the power consumption of each stage is linearly proportional to its load capacitance)

With the simplifying assumptions:

$$\begin{aligned}
 A_{v1} &= -g_{m1} R_1 \\
 A_{v2} &= 1 \\
 \omega_{p1} &= \frac{1}{R_1 C_1} \\
 \omega_{p2} &= \frac{g_{m2}}{C_2} = \frac{2I_{d2}}{V^* C_2}
 \end{aligned}$$

Setting GBW equal:

$$\begin{aligned}
 A_{v1} \omega_{p1} &= A_{v2} \omega_{p2} \\
 -\frac{g_{m1}}{C_1} &= \frac{g_{m2}}{C_2}
 \end{aligned}$$

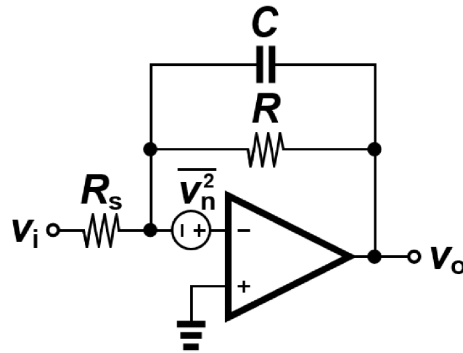
This proves the hint which is that power is proportional to input capacitance for equal GBW.

I substituted the values above to the noise equation, and then performed the substitution $g_{m1} = I_{d1}$ and $g_{m2} = KI_{d1}$. After differentiating wrt I_{d1} , setting to zero, and solving for K , I get something on the order of:

$$K = \frac{\sqrt{1/I_{d1}}}{I_{d1}}$$

3 Filter Noise

You are given an active RC low pass filter with noisy resistors and noisy op-amp shown below.



- (a) Assume the op-amp has infinite DC gain and infinite GBW (ω_u). Calculate v_o/v_i and total output noise. What issue did you find?

$$\begin{aligned} \frac{v_o}{v_i} &= \frac{R \parallel C}{R_s} = \frac{\frac{R}{1+sRC}}{R_s} = \frac{R}{R_s} \frac{1}{1+sRC} \\ \overline{v_{n,out,R_s}^2} &= \int_0^\infty \overline{v_{n,R_s}^2} \left(\frac{v_o}{v_i} \right)^2 df \\ &= 4kTR_s \left(\frac{R}{R_s} \right)^2 \int_0^\infty \left| \frac{1}{1+sRC} \right|^2 df \\ &= 4kTR_s \left(\frac{R}{R_s} \right)^2 \frac{1}{4RC} \end{aligned}$$

- (b) Assume the op-amp has infinite DC gain and finite GBW (i.e. You can model the op-amp gain $A(s) = -\omega_u/s$ where $\omega_u \gg 1/RC$). Calculate v_o/v_i and total output noise. How does the noise relate to ω_u ?