HW3

1)

Output current noise:

$$i2_{on} := 4kT \cdot g_m \cdot \gamma$$

$$i2_{out} := {i_s}^2 \cdot \left[\frac{g_m}{\omega \cdot \left(C_s + \frac{g_m}{\omega_T}\right)}\right]^2 \rightarrow \qquad i2_{out} := {i_s}^2 \cdot \left(\frac{\omega_T}{\omega}\right)^2$$

$$i2_{s\_min} := 4kT \cdot \gamma \cdot \omega^2 \cdot \frac{1}{g_m} \cdot \left(C_s + \frac{g_m}{\omega_T}\right)^2$$

$$\frac{di2_{s\_min}}{dg_m} = 0 \quad \text{for} \quad g_{min} = \omega_T \cdot C_s$$

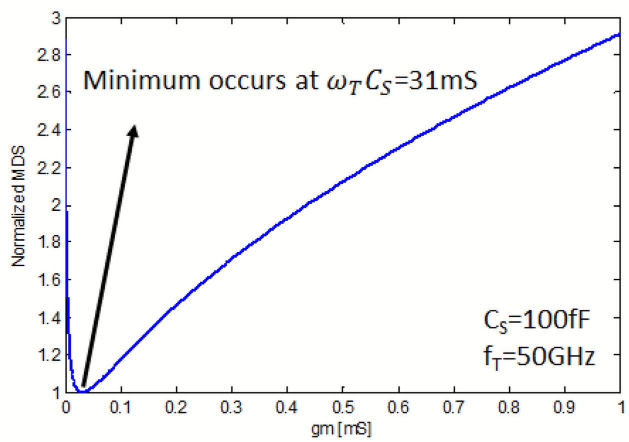
For CGS<CS:
Increasing gm --> Noise power increases with gm --> Signal power increases gm^2
So, MDS decreases with sqrt(gm)

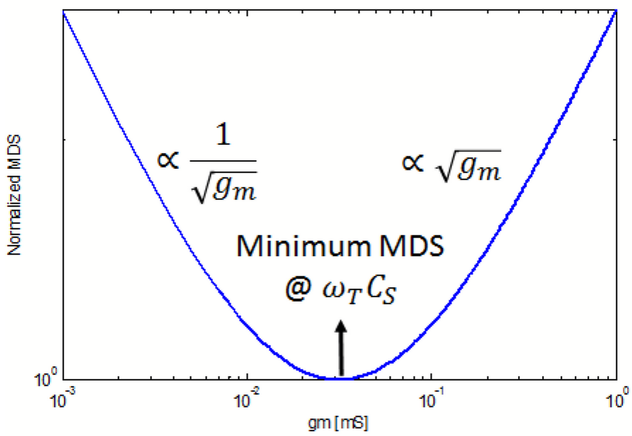
*But increasing gm also increases CGS. After it becomes larger than CS, signal power stop increasing.

For CGS>CS:
Increasing gm --> Noise power increases with gm --> Signal power remains same
So, MDS increases with sqrt(gm)

As a result, the point where CS=CGS is local minima for MDS. Therefore, gm=wTxCGS=wTxCS

The gm dependent part is plotted (both in linear and log scales) for Cs=100fF and wT=50GHz. As plot indicates, the minimum is very steep.





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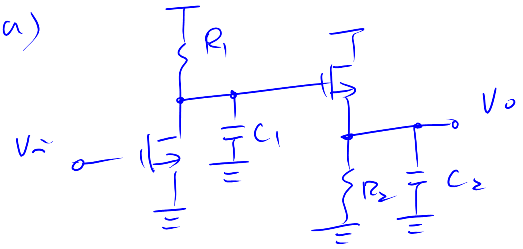
(a)



$$\Rightarrow \quad A_{v1} = -g_{m1}R_1 \quad , \quad \omega_{p1} = \frac{1}{R_1C_1}$$

$$A_{v2} = \frac{g_{m2}R_2}{1+g_{m2}R_2} \quad , \quad \omega_{p2} = \frac{1}{\left(R_2 // \frac{1}{g_{m2}}\right)C_2}$$

$$\therefore \quad \overline{v_{n,o}^2} = 4kT\left(\gamma g_{m2} + \frac{1}{R_2}\right)\int_0^\infty \left|\frac{R_2 // \frac{1}{g_{m2}}}{1+s/\omega_{p2}}\right|^2 df \; +$$

$$4kT\left(\gamma g_{m1} + \frac{1}{R_1}\right)\int_0^\infty \left|\frac{R_1}{1+s/\omega_{p1}}\right|^2 \cdot \left|\frac{A_{v2}}{1+s/\omega_{p2}}\right|^2 df$$

$$= 4kT\left(\gamma g_{m2} + \frac{1}{R_2}\right)\left(\frac{R_2}{1+g_{m2}R_2}\right)^2 \frac{\omega_{p2}}{4} \; +$$

$$4kT\left(\gamma g_{m1} + \frac{1}{R_1}\right) R_1^2 A_{v2}^2 \; \frac{1}{4}\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)$$

$$= \frac{kT}{C_2}\left(\gamma A_{v2} + 1 - A_{v2}\right) + \frac{kT}{C_1}\left(\gamma A_{v1} + 1\right) A_{v2}^2 \; \frac{1}{1+\omega_{p1}/\omega_{p2}}$$

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(b) Since $g_{m1}R_1 \gg 1$, $g_{m2}R_2 \gg 1$ $\Rightarrow$ $A_{v2} \to 1$, $\omega_{p2} \to \infty$

$$\therefore \quad v_{n,o}^2 \cong \frac{kT}{C_2}\gamma + \frac{kT}{C_1}\gamma A_{v1}$$

$\because$ GBW is fixed $\therefore$ $I \propto g_m = GBW \cdot C_L \propto C_L$

$$P_{stg1} : P_{stg2} = C_1' : C_2'$$

$$= k(C_1+C_2) : (1-k)(C_1+C_2)$$

$$v_{n,o}^2 = \frac{kT\gamma}{C_1+C_2}\left(\frac{1}{1-k} + \frac{A_{v1}}{k}\right) \Rightarrow \frac{\partial}{\partial k} = 0$$

$$\Rightarrow -(1-k)^2(-1) - A_{v1}\cdot k^{-2} = 0$$

$$\Rightarrow k = \frac{\sqrt{A_{v1}}}{\sqrt{A_{v1}}+1}$$

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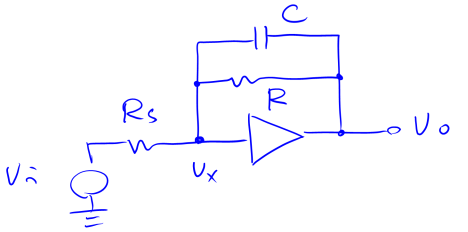
3



(a) $\boxed{A=\infty}$ $\Rightarrow V_x = 0$

$$\frac{V_o}{V_i} = -\frac{Z}{R_s} = \frac{\frac{R}{1+sRC}}{R_s} = -\underline{\frac{R}{R_s}\frac{1}{1+s/\omega_o}} \#$$

$$V_{n,o}^2 = \int_0^\infty \left[\frac{V_{n,Rs}^2}{\Delta f}\left(-\frac{R}{R_s}\frac{1}{1+s/\omega_o}\right)^2 + \frac{i_{n,R}^2}{\Delta f}\left(\frac{R}{1+sRC}\right)^2 + \frac{V_{n,A}^2}{\Delta f}\left(1+\frac{\frac{R}{1+sRC}}{R_s}\right)^2\right]df$$

$$= \int_0^\infty \left[4KTR_s\left(\frac{R}{R_s}\right)^2\left|\frac{1}{1+s/\omega_o}\right|^2 + 4KTR\left|\frac{1}{1+s/\omega_o}\right|^2 + \underbrace{\frac{V_{n,A}^2}{\Delta f}\left|1+\frac{R}{R_s}\frac{R}{1+sRC}\right|^2}_{=\infty}\right]df$$

$\Rightarrow \overline{V_{n,o}^2} = \infty$ (Due to we assume GBW $=\infty$) #

(b) $\boxed{A = -\frac{\omega_u}{s}}$ $\Rightarrow V_x = \frac{V_o}{A} = -\frac{s}{\omega_u}V_o$

$$-\frac{s}{\omega_u}V_o + \frac{-\frac{s}{\omega_u}V_o - V_i}{R_s} \times \frac{R}{1+sRC} = V_o$$

$$V_o\left[-\frac{s}{\omega_u} - \frac{R}{R_s}\frac{s}{\omega_u}\frac{1}{1+sRC} - 1\right] = V_i\frac{R}{R_s}\frac{1}{1+sRC}$$

$$\frac{V_o}{V_i} = -\frac{R}{R_s}\cdot\frac{1}{1+s/\omega_o}\cdot\frac{1}{\frac{s}{\omega_u}+\frac{R}{R_s}\frac{s}{\omega_u}\frac{1}{1+s/\omega_o}+1}$$

$$= -\frac{R}{R_s}\frac{1}{\left(1+\frac{s}{\omega_o}\right)\frac{s}{\omega_u}+\frac{R}{R_s}\frac{s}{\omega_u}+\left(1+\frac{s}{\omega_o}\right)}$$

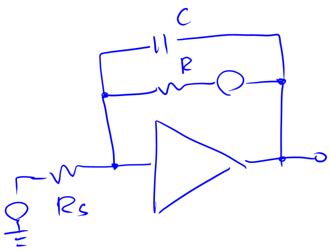
$$= \underline{-\frac{R}{R_s}\frac{1}{\frac{s^2}{\omega_o\omega_u}+\left(\frac{R+R_s}{R_s}\frac{1}{\omega_u}+\frac{1}{\omega_o}\right)s+1}} \#$$

$$= -\frac{R}{R_s}\frac{1}{\frac{s^2}{\omega_n^2}+\frac{s}{\omega_n Q}+1}, \text{ where } \begin{cases}\omega_n^2 = \omega_o\omega_u \\ \omega_n Q = \left(\frac{R+R_s}{R_s}\frac{1}{\omega_u}+\frac{1}{\omega_o}\right)^{-1}\end{cases}$$

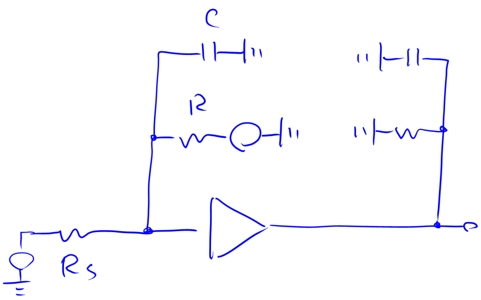
noise by R_s

$$v_{n,in} = v_{n,R}$$

noise by R



$\Rightarrow$



$$\begin{cases} \left.\dfrac{V_o}{V_i}\right|_{OL} = \dfrac{\frac{R}{1+sRC}}{R_s + \frac{R}{1+sRC}} \cdot A \\[2ex] \left.v_{n,o}\right|_{OL} = \dfrac{\frac{R_s}{1+sR_sC}}{R + \frac{R_s}{1+sR_sC}} \cdot A \cdot v_{n,R} \end{cases} \qquad \therefore \left.v_{n,in}\right|_{OL} = \frac{R_s}{R} v_{n,R}$$

noise by A

$$\left.v_{n,o}\right|_{OL} = v_{n,A} \cdot A \qquad \therefore \left.v_{n,in}\right|_{OL} = \frac{v_{n,A} \cdot A}{\left.\frac{V_o}{V_i}\right|_{OL}} = v_{n,A} \frac{R_s + R + sRR_sC}{R}$$

$$= v_{n,A} \frac{R_s}{R_x}\left(1 + \frac{s}{\omega_x}\right)$$

$$, \text{ where } R_x = R /\!/ R_s, \ \omega_x = \frac{1}{R_x C}$$

$\therefore$ Total noise

$$v_{n,o}^2 = \left.v_{n,in}^2\right|_{CL} \times \left(\frac{V_o}{V_i}\right)^2 = \left.v_{n,in}\right|_{OL} \times \left(\frac{V_o}{V_i}\right)^2$$

$$= \int_0^\infty 4kTR_s \left| -\frac{R}{R_s} \frac{1}{\frac{s^2}{\omega_n^2} + \frac{s}{\omega_n Q} + 1} \right|^2 df \ +$$

$$\int_0^\infty 4kTR \left(\cancel{\frac{R_s}{R}}\right)^2 \left| \cancel{-\frac{R}{R_s}} \frac{1}{\frac{s^2}{\omega_n^2} + \frac{s}{\omega_n Q} + 1} \right|^2 df \ +$$

$$\int_0^\infty v_{n,A}^2 \left(\frac{\cancel{R_s}}{R_x}\right)^2 \left(1 + \frac{s}{\omega_x}\right)^2 \left| -\frac{R}{\cancel{R_s}} \frac{1}{\frac{s^2}{\omega_n^2} + \frac{s}{\omega_n Q} + 1} \right|^2 df$$

$$= kT\left(\frac{R^2}{R_s} + R\right) \cancelto{\sim\omega_o}{\omega_n Q} + v_{n,A}^2 \left(\frac{R}{R_x}\right)^2 \frac{\cancelto{\sim\omega_o}{\omega_n Q}}{4} \left(1 + \frac{\omega_u \omega_o}{\omega_x^2}\right)$$

$$\simeq \frac{kT}{C}\left(\frac{R}{R_s} + 1\right) + v_{n,A}^2 \left(\cancel{\frac{R}{R_x}}\right)^2 \frac{\omega_o}{4} \left[\cancelto{0}{1+} \frac{\omega_u}{\omega_o} \left(\cancel{\frac{R_x}{R}}\right)^2\right]$$

$$\cong \frac{KT}{C}\left(\frac{R}{R_S}+1\right) + v_{n,A}^2 \frac{\omega_u}{4}$$

if we let $\overline{v_{n,A}^2} = 4KTR_{eq}$

$$= \underbrace{\frac{KT}{C}\left(\frac{R}{R_S}+1+\frac{R_{eq}}{R}\frac{\omega_u}{\omega_0}\right)}$$ #.