EE 240B – Spring 2019

Advanced Analog Integrated Circuits
Lecture 5: Noise and SNR Analysis

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General Noise Analysis

- **Method:**
  1) **Create small-signal model**
  2) **All inputs = 0 (linear superposition)**
  3) **Pick output v_o or i_o**
  4) **For each noise source v_x, i_x** ← N noise source
     Calculate $H_x(s) = v_o(s) / v_x(s)$ (… i_o, i_x)
  5) **Total noise at output is:**

$$\overline{v_{on,T}^{2}(f)} = \sum_{x}^{N} \left|H_x(s)\right|^{2}_{s=2\pi jf} \overline{v_x^2(f)}$$

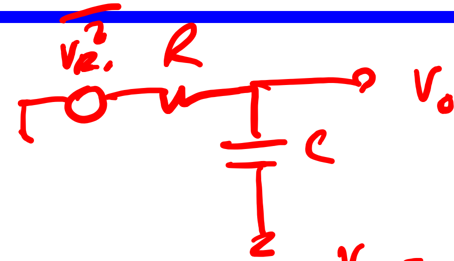
Sum over noise sources

$$\overline{v_{on,T}^{2}} = \int_{0}^{\infty} \overline{v_{on,T}^{2}(f)}\,df$$

Tedious but simple …

Important Integrals

$$\int_0^\infty \left| \frac{1}{1+\frac{s}{\omega_o}} \right|^2 df = \frac{\omega_o}{4}$$



$$V_o = \frac{1}{1+j\omega RC} V_R$$

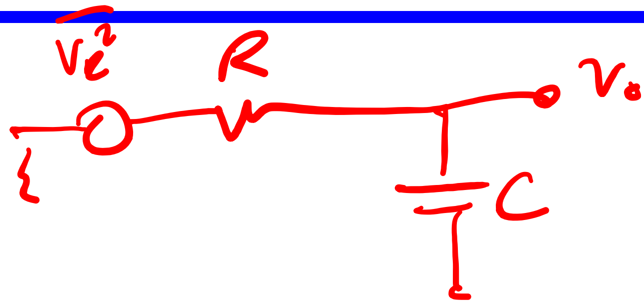
$$\int_0^\infty \left| \frac{1}{1+\frac{s}{\omega_o Q}+\frac{s^2}{\omega_o^2}} \right|^2 df = \int_0^\infty \left| \frac{\frac{s}{\omega_o}}{1+\frac{s}{\omega_o Q}+\frac{s^2}{\omega_o^2}} \right|^2 df = \frac{\omega_o Q}{4}$$

$$\overline{V_o^2} = \left| \frac{1}{1+j\omega RC} \right|^2 \overline{V_R^2}$$

$$= \left(\frac{1}{1+\omega^2 R^2 C^2} \right) \overline{V_R^2}$$

$$\int_0^\infty \left| \frac{\frac{s}{\omega_z}+1}{1+\frac{s}{\omega_o Q}+\frac{s^2}{\omega_o^2}} \right|^2 df = \frac{\omega_o Q}{4}\left(\frac{\omega_o^2}{\omega_z^2}+1 \right)$$

Noise in a Real Circuit: RC



$\omega_o = \frac{1}{RC}$

- **Noise on the capacitor:**

$$\overline{v_{on}^2(f)} = \int \overbrace{4k_B TR}^{\overline{v_R^2}} \left| \frac{1}{1+\underset{2\pi jf}{s}RC} \right|^2 df$$

$$\rightarrow \overline{v_{oT}^2} = 4k_B TR \left(\frac{1}{4RC}\right) = \frac{k_B T}{C}$$

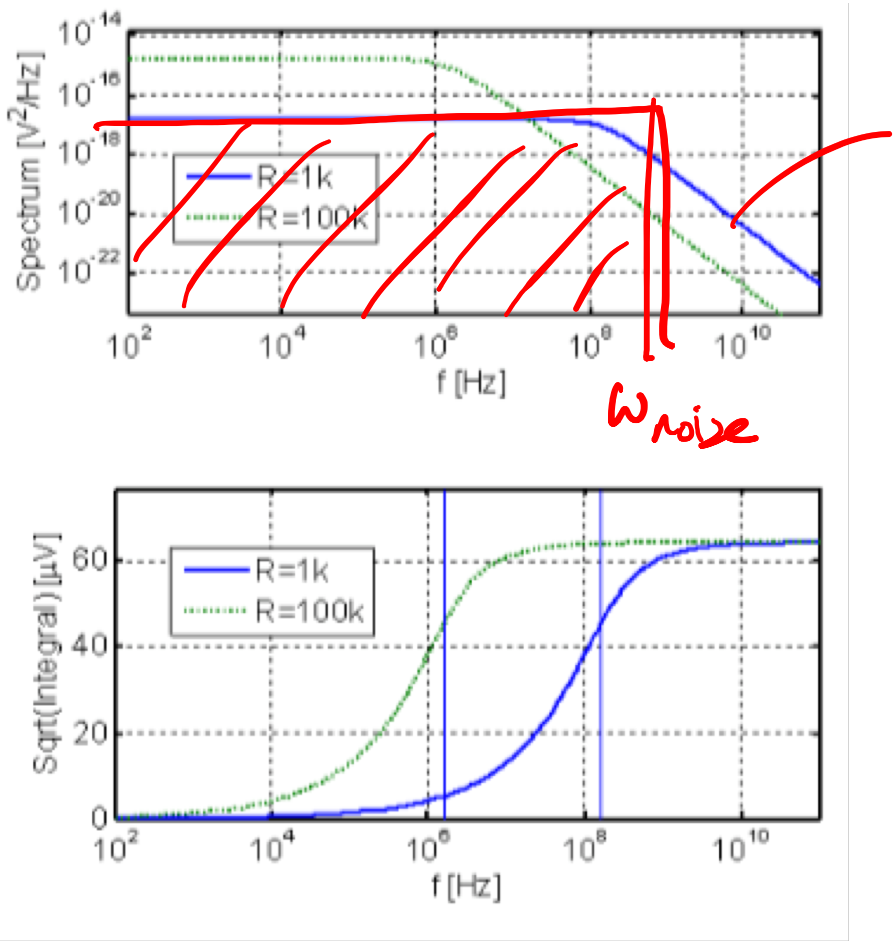
$$\left(\frac{\omega_o}{4}\right)$$

- **Note that effective bandwidth is:**

$$\Delta f = \frac{1}{4RC} = \frac{\omega_o}{4} = \frac{\pi}{2} f_o$$

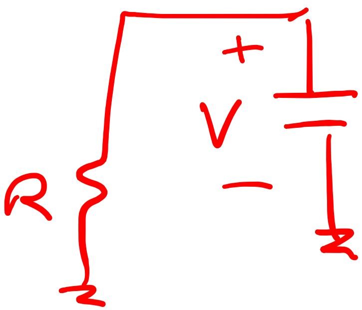
NOISE BANDWIDTH

Noise PSD



Spectrum [V²/Hz]
R=1k
R=100k
f [Hz]
ω noise
Sqrt(Integral) [µV]
R=1k
R=100k
f [Hz]

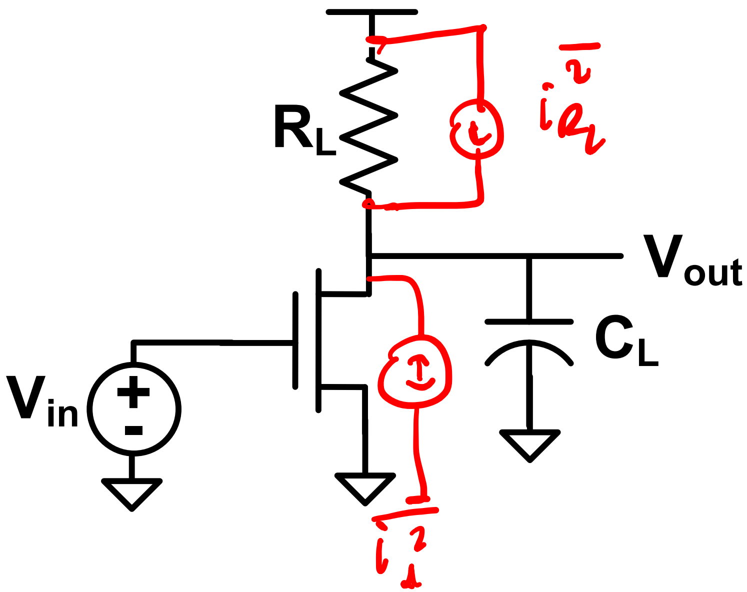
Equipartition Theorem



$$\frac{1}{2} C V^2 = \frac{kT}{2}$$

$$V^2 = \frac{kT}{C}$$

CS Amplifier Noise



$$\overline{v_{on}^2(f)} = 4k_BT\left(\frac{1}{R_L}+\gamma g_m\right)\left|\frac{R_L}{1+sR_LC_L}\right|^2$$

$$\overline{v_{oT}^2} = 4k_BT\left(\frac{1}{R_L}+\gamma g_m\right)R_L^2\int_0^\infty\left|\frac{1}{1+sR_LC_L}\right|^2 df$$

$$= 4k_BT\left(\frac{1}{R_L}+\gamma g_m\right)R_L^2\frac{1}{4R_LC_L}$$

$$= \frac{k_BT}{C_L}\left(1+\gamma g_m R_L\right)$$

$$= \frac{k_BT}{C_L}\left(1+\gamma\left|A_{vo}\right|\right)$$

$$= \frac{k_BT}{C_L}n_F$$

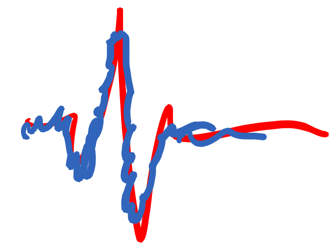
$$v_o = \left(i_d + i_{R_L}\right)\frac{R_L}{1+j\omega R_LC_L}$$

$$\overline{v_o^2}(f) = \underbrace{\left(\overline{i_d^2}+\overline{i_{R_L}^2}\right)}_{=4KT\left(g_m\cdot\gamma+\frac{1}{R_L}\right)}\left(\frac{R_L}{1+\omega^2R_LC_L}\right)^2\Delta f$$

Signal-To-Noise Ratio

- **SNR:**

$$SNR = \frac{P_{sig}}{P_{noise}}$$



- **Signal Power (sinusoidal source):**

$$P_{sig} = \tfrac{1}{2} V^2_{zero-peak} \quad \propto \quad V_{DD}^2$$

- **Noise Power (assuming thermal noise dominates):**

$$P_{noise} = \frac{k_B T}{C} n_f$$ at output

- **So:**

$$\boxed{SNR = \frac{\tfrac{1}{2} C V^2_{zero-peak}}{n_f k_B T}}$$

$$SNR \quad \uparrow +6dB$$

$$\downarrow$$

$$C \quad \uparrow \times 4$$

SNR versus Bits

- **Quantization "noise"**
  - Quantizer step size: Δ
  - Box-car pdf variance: $S_Q = \dfrac{\Delta^2}{12}$

N	dB
8	50
16	98
24	146

- **SNR of N-Bit sinusoidal signal**
  - Signal power $$P_{sig} = \frac{1}{2}\left(\frac{1}{2}2^{N_{bits}}\Delta\right)^2$$
  - SNR $$SNR = \frac{P_{sig}}{S_Q} = 1.5 \times 2^{2N_{bits}}$$
  - 6.02 dB per Bit $$= \left[1.76 + 6.02 N_{bits}\right] \quad \text{dB}$$

SNR versus C_L

- **For a 1V sinusoidal signal at 100°C:**

Bits	SNR [dB]	C
3.0	20	4.1 aF
6.3	40	412 aF
9.7	60	41 fF
13.0	80	4.1 pF
16.3	100	412 pF
19.6	120	41 nF
23.0	140	4.1 μF

SNR versus Power

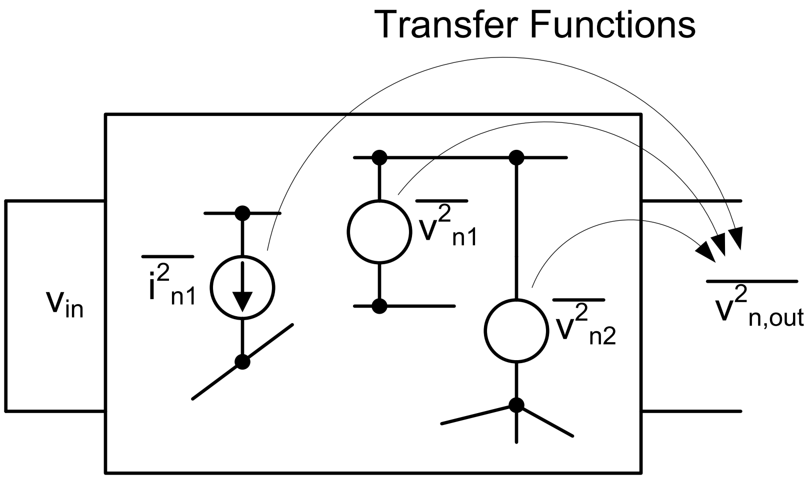
- 1 Bit → 6dB → 4x SNR
- 4x SNR → 4x C
- Circuit bandwidth ~g_m/C → 4x g_m
- Keeping V* constant → 4x I_D, 4x W

- Thermal noise limited circuit:
  - Each bit QUADRUPLES power!

- Comparison vs. digital circuits…

Input and Output Referred Noise

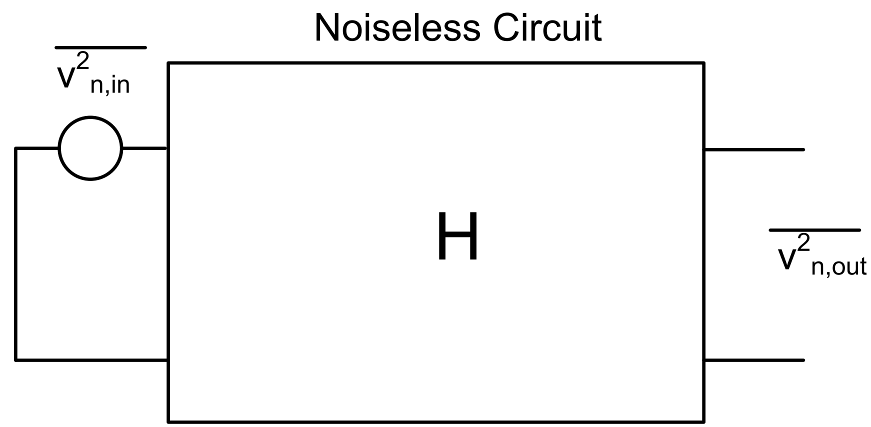
Output



$$v_{out} = H_i \cdot s_{n,i}$$

$$\overline{v^2_{n,out}} = \sum_i |H_i|^2 \overline{s^2_{n,i}}$$

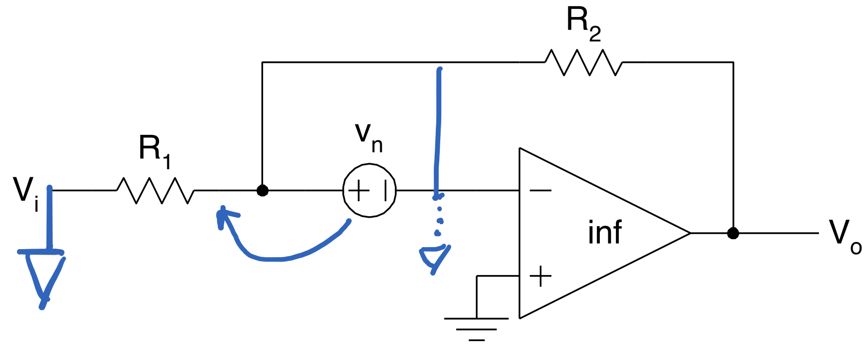
Input



$$v_{out} = H \cdot v_{in}$$

$$\overline{v^2_{n,in}(\omega)} = \frac{\overline{v^2_{n,out}(\omega)}}{|H|^2}$$

Noise Example



- **Ignoring noise from R_1, R_2:**

$$v_o = -v_i \underbrace{\frac{R_2}{R_1}}_{-A_{v0}} + v_n\left(1+\frac{R_2}{R_1}\right) = -v_i \underbrace{\frac{R_2}{R_1}}_{-A_{v0}} + v_n \frac{R_1+R_2}{R_1}$$

$$\overline{v_{ieq}^2} = \overline{v_n^2}\left(\underbrace{\frac{R_1+R_2}{R_1}}_{\text{Noise gain}}\underbrace{\frac{R_1}{R_2}}_{\frac{1}{\text{Gain}}}\right)^2 = \overline{v_n^2}\left(\frac{R_1+R_2}{R_2}\right)^2 = \overline{v_n^2}\left(1+\frac{1}{|A_{v0}|}\right)^2$$

Complete Op-Amp Noise Model

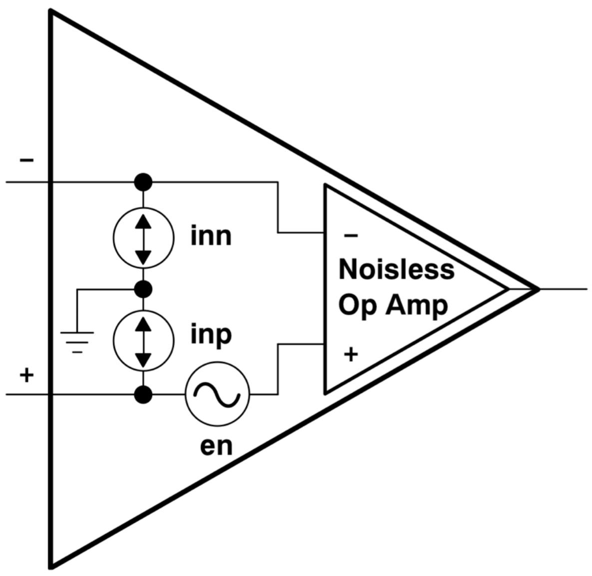
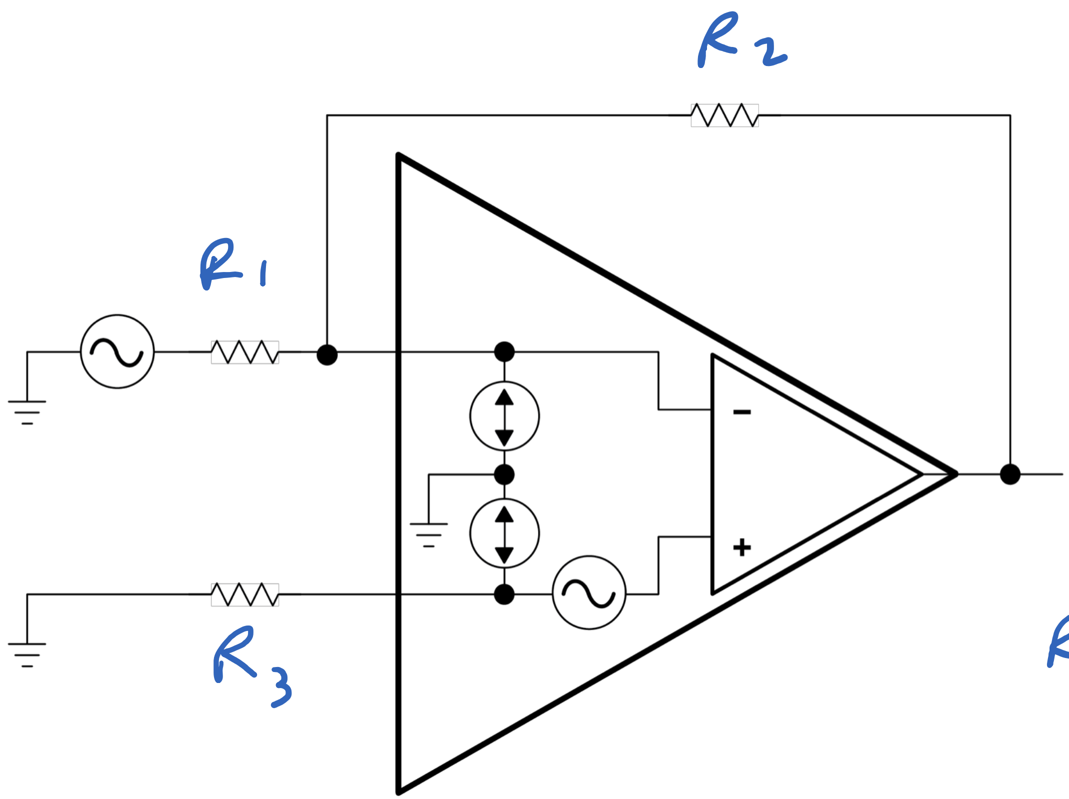


Figure 9. Op Amp Noise Model

http://www.ti.com/lit/an/slva043b/slva043b.pdf

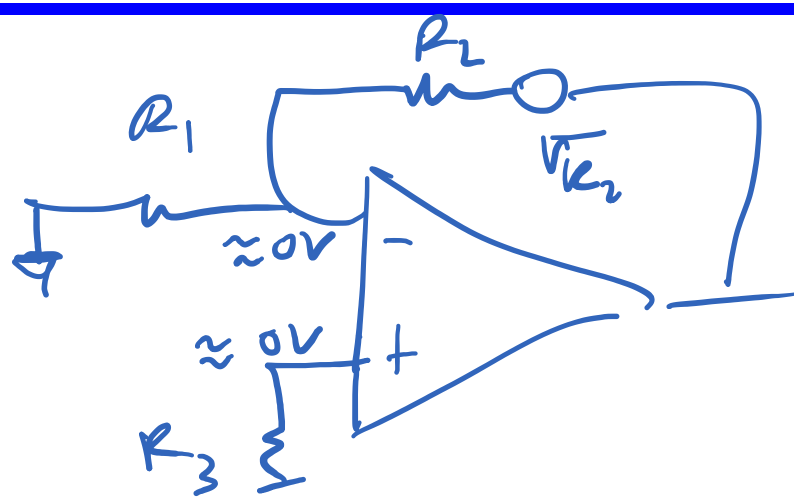
Complete Amplifier



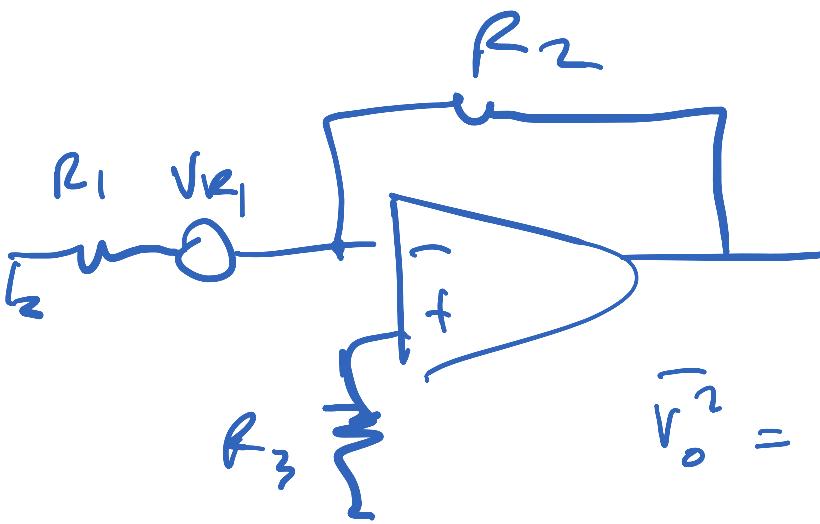
$R_3 = R_1 || R_2$

- **Include R3 for bipolar op-amps (offset cancellation)**

Noise Due to Feedback (R1 and R2)



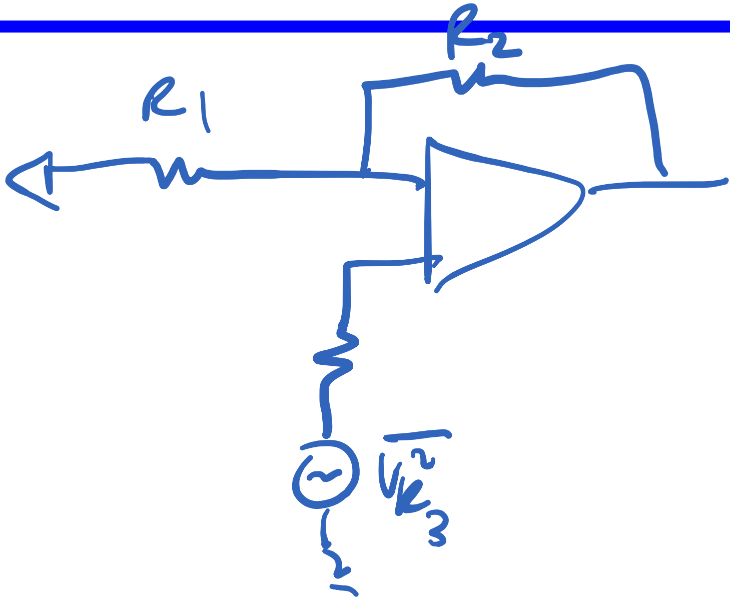
$$\overline{v_{n1}^2} = \overline{v_{R_2}^2}$$
$$= 4kT R_2 \Delta f$$



$$\overline{v_{n2}^2} = \left(-\frac{R_2}{R_1}\right)^2 \overline{v_{R_1}^2}$$

$$\overline{v_o^2} = 4kT R_2 \left[1 + \frac{R_2^{\cancel{2}}}{R_1^{\cancel{2}}} \cdot \cancel{R_1}\right] \Delta f$$
$$= 4kT R_2 (1 + A_v) \Delta f$$

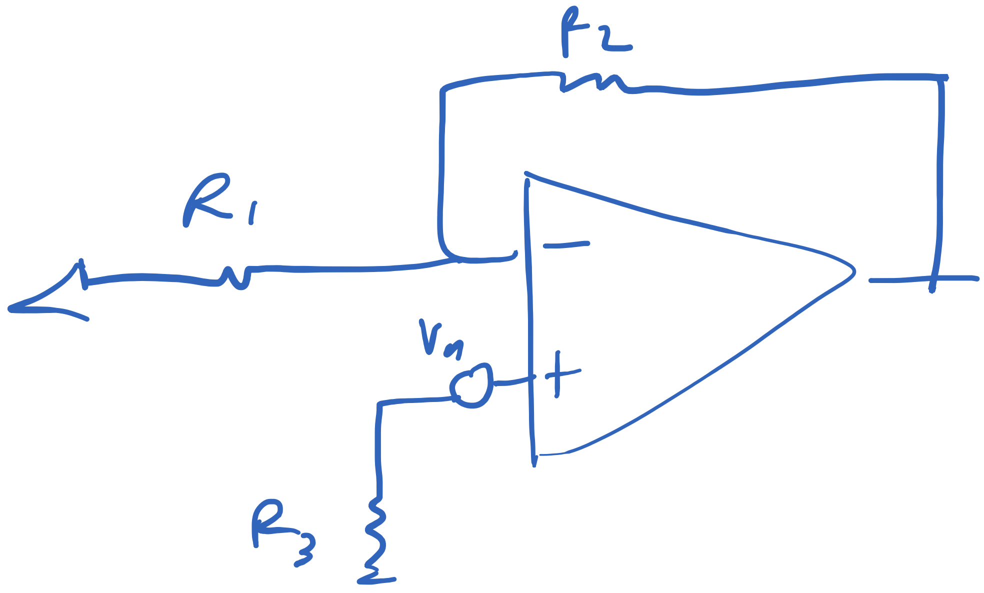
Noise Due to R3



$$\overline{v^2_{oR_3}} = \left(1 + \frac{R_2}{R_1}\right)^2 \cdot 4kT R_3 \Delta f$$

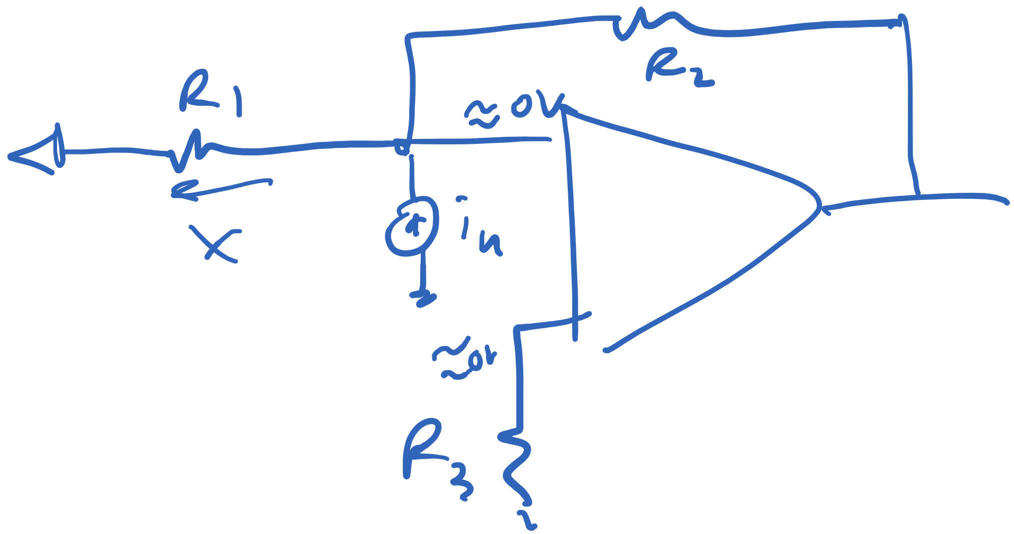
$R_3 = R_1 \| R_2$ for offset

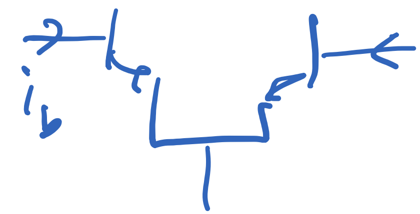
Noise Due to Op-Amp Voltage Noise



$$\overline{v_o^2} = \left(1 + \frac{R_2}{R_1}\right)^2 \cdot \overline{v_n^2}$$

Noise Due to Op-Amp Current Noise





$$v_o = (R_2)^2 \cdot \overline{i_n^2}$$

Total Noise

$$\overline{v_{i,eq}^2} = \overline{v_{R_1,n}^2}\left(1+\frac{1}{a_v}\right) + \overline{v_{v,n}^2}\left(1+\frac{1}{a_v}\right)^2 + \overline{i_{i,n}^2}R_1^2$$

Source and feedback resistor

Amplifier voltage noise

Amplifier current noise

$\overline{v_{v,n}^2} = 4\frac{\text{nV}}{\sqrt{Hz}}$ $\overline{i_{i,n}^2} = 1.2\frac{\text{pA}}{\sqrt{Hz}}$ (uncorrelated) a_v large

$\overline{v_{v,n}^2}$ dominates over $\overline{i_{i,n}^2}$, correlation no concern

$R_1 = 50\Omega$

$$\frac{\overline{v_{i,eq}^2}}{\Delta f} = \sqrt{\left(0.9\frac{\text{nV}}{\sqrt{Hz}}\right)^2 + \left(4\frac{\text{nV}}{\sqrt{Hz}}\right)^2 + \left(0.06\frac{\text{nV}}{\sqrt{Hz}}\right)^2}$$

Low source resistance:
Voltage noise dominates
Use BJT

$R_1 = 1\text{M}\Omega$

$$\frac{\overline{v_{i,eq}^2}}{\Delta f} = \sqrt{\left(126\frac{\text{nV}}{\sqrt{Hz}}\right)^2 + \left(4\frac{\text{nV}}{\sqrt{Hz}}\right)^2 + \left(1200\frac{\text{nV}}{\sqrt{Hz}}\right)^2}$$

High source resistance:
Current noise dominates
Use MOS

Other Noise Topics

- Later in EE 240B
  – Noise in sampled data systems – (Low) noise amplifier design ...
- RF noise metrics (EE 242A) – Noise figure
  – Receiver sensitivity
  – Phase noise in oscillators
- Cyclostationary noise
  - – Noise in circuits with high signal amplitude which modulates the noise power spectral densities
  - – E.g. oscillators, mixers, comparators