EE 240B – Spring 2019

Advanced Analog Integrated Circuits
Lecture 5: Electronic Noise

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Electronic Noise

- **Why is noise important?**
  - Sets minimum signals we can deal with
  - Ensuring sufficiently low noise will be another "active constraint" that we will develop a design methodology around

- **Most often care about "signal-to-noise ratio"**
  - How do you make the signal larger?
  - Will see today and next time how to make noise smaller, but can you guess what it takes?

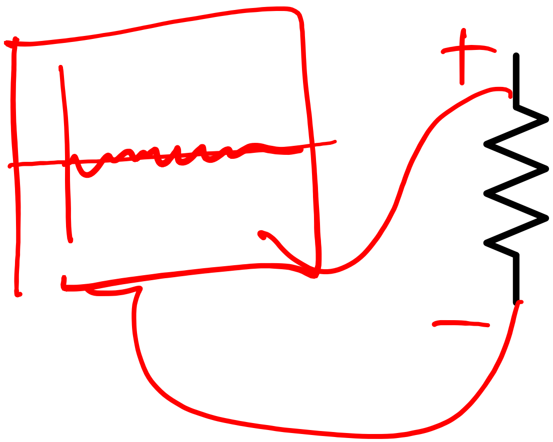
$$SNR \triangleq \frac{P_{sig}}{P_{noise}}$$

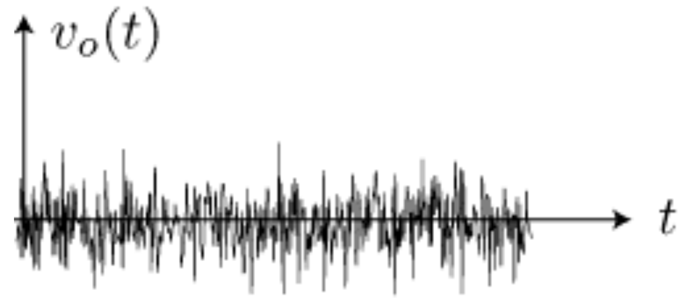
Types of “Noise”

- **Interference**
  - Not actually “noise” – deterministic
  - Signal coupling
    - Capacitive, inductive, substrate, etc.
  - Supply variations

- **Device noise**
  - Caused by discreteness of charge – shot noise
  - “Fundamental physics” related – thermal noise
  - “Technology” related – flicker noise

Noise in Electrical Circuits

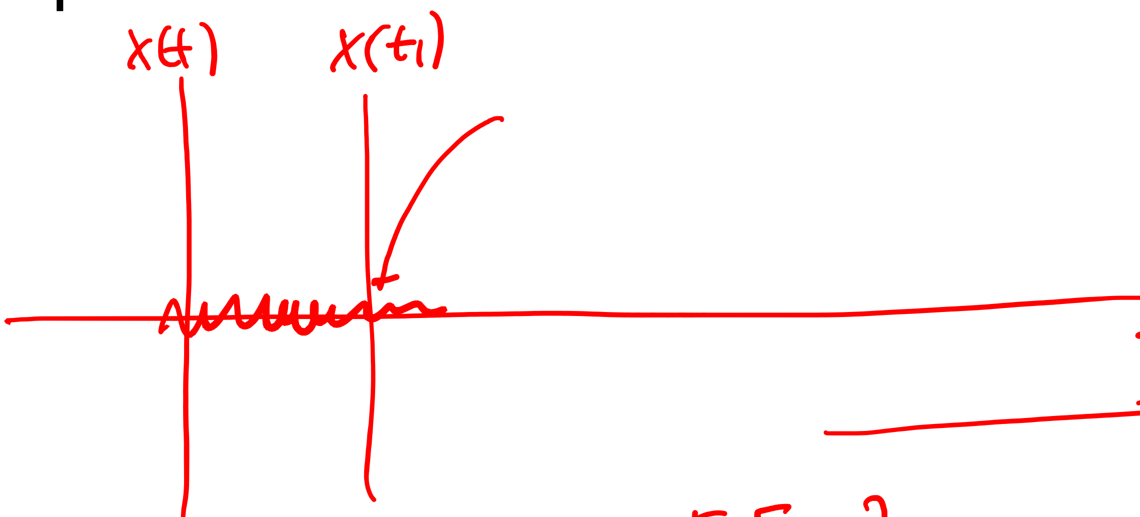


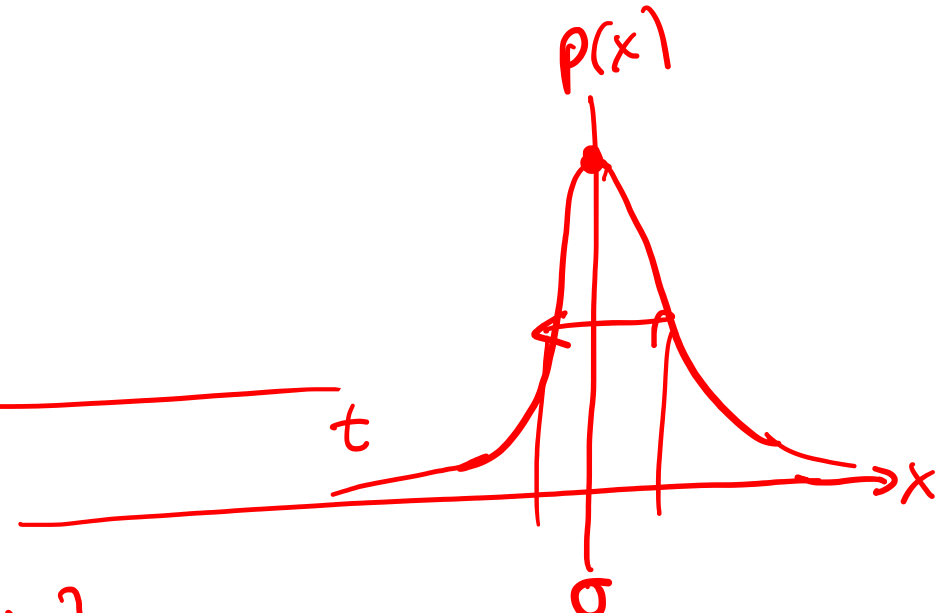


- **Noise is <u>random</u>**
  - Has to be treated statistically – can't predict actual value
- **Deal with mean (average), variance, spectrum**

Signal Histogram

- The measured voltage has a distribution function usually specified through PDF or CDF
- Most electronic noise sources have a Gaussian profile





$$m = E[x]$$

$$\sigma = E[(x-m)^2] = E[x^2]$$

Stationary Process

- If the circuit is stationary, or the noise statistics don't change in time, then one would expect that if we plot the distribution of successive samples of the signal, it should also follow the same distribution

$m \neq f(t) = E[x]$ for any

$\sigma = E[x^2]$ t

Correlation

- For Stationary Systems, the auto-covariance is only a function of the time difference between samples leading to the autocorrelation function

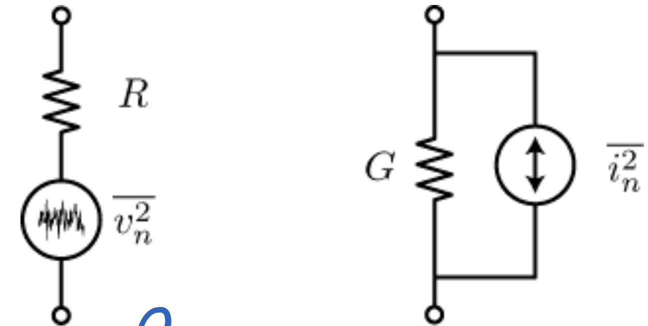
$$E[x(t_1)\,x(t_2)] = E[x(t_1)\,x(t_1 \pm \tau)]$$

$$\tau = t_2 - t_1$$

- If signals measured at time 1 are completely independent and uncorrelated to measurements at time 2, we can say the autocorrelation function as a function of time offset has an expected value that is a delta function …
- What's the spectrum of such a signal ?

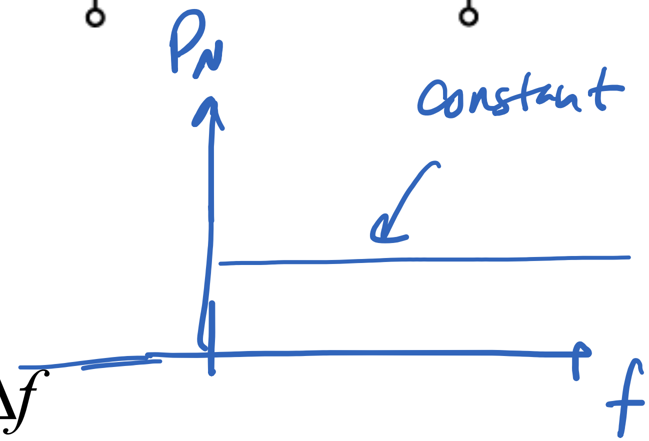
Origin and Properties of Thermal Noise

- **Origin: Brownian Motion**
  - Thermally agitated particles
  - (Everything that can generate heat has this type of noise)



- **Independent of DC current flow**

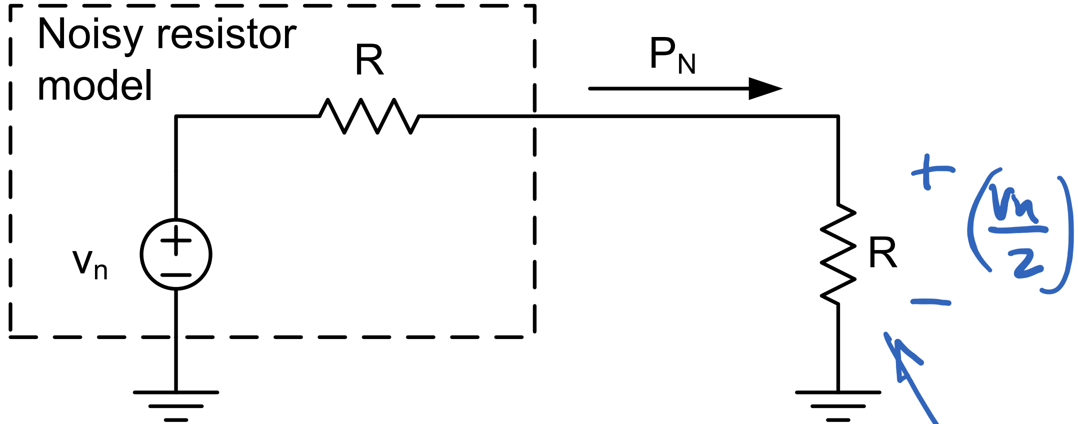
- **Zero mean, Gaussian PDF, i.i.d.**



- **Available noise power:** $P_N = k_B T \Delta f$
  - Noise power in bandwidth Δf delivered to a matched load
  - Power spectral density is "white" up to ~THz frequencies
  - kbT = 4e-21 J (T = 290K)

J.B. Johnson, "Thermal Agitation of Electricity in conductors," Phys. Rev., July 1928

Resistor Noise Model



$$P_R = \frac{\left(\frac{v_n}{2}\right)^2}{R}$$

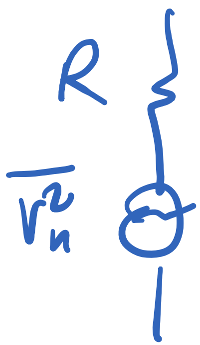
$$P_N = k_B T \Delta f = \frac{\overline{v_n^2}}{4R}$$

$$4k\Delta f = \frac{v_n^2}{4R}$$

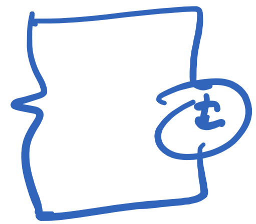
Mean square noise voltage:

$$\boxed{\overline{v_n^2} = 4k_B TR\Delta f}$$

Resistor Noise Model (current)



$$\frac{1}{R} = G$$



$$\overline{i_n^2} = 4kTG\Delta f$$

ONE-SIDED SPECTRUM

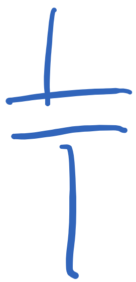
$$P = \int_0^f \frac{\overline{v_n^2}(f)}{1\Omega}\, df$$

TWO-SIDED SPECTRUM

$$P = \int_{-f/2}^{f/2} \frac{\overline{v_n^2}(f)}{1\Omega}\, df$$

$$\overline{v_n^2} = 2kTG\Delta f$$

Thermal Noise in Capacitors?



Ideal capacitor only stores energy ...

so it does not generate noise

same argument for inductors

Noise Power Spectral Density (PSD)

R

$$\overline{v^2(t)} = \int_0^B 4kTR\,df = 4kTR \cdot B$$

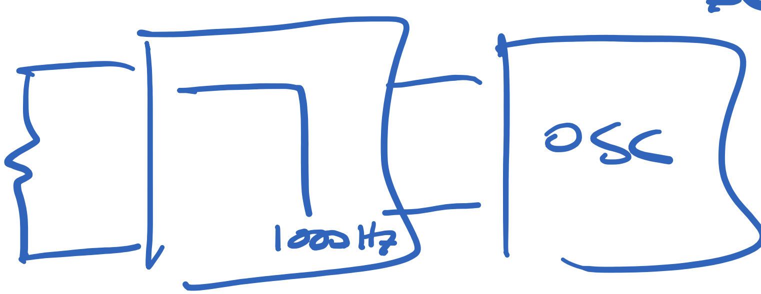
$1k\Omega$

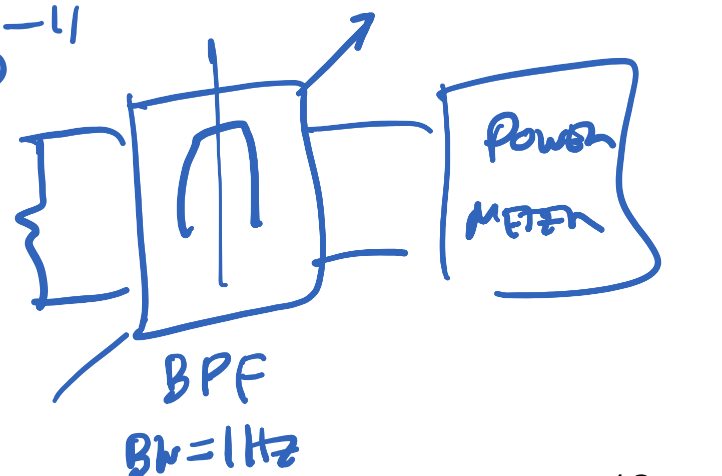
$B = 1000\text{ Hz}$

$$\overline{v^2(t)} = 4\left(26mV \times 1.6\times10^{-19}\right) \cdot \underbrace{1000 \times 1000}_{10^6}$$

$$= 10^8 \; 1.6\times10^{-19}$$

$$\approx 10^{-11}$$





Spectral Density

The power of any signal is computed or measured over a large time interval

$$P = \lim_{T\to\infty} \frac{1}{T}\int_0^T |x(t)|^2 dt$$

- The truncated Fourier transform (which is practically the only thing we can measure) is defined

$$\hat{x}(\omega) = \frac{1}{\sqrt{T}}\int_0^T x(t)e^{-j\omega t}dt$$

- The spectral density is defined as follows (***E*** is the expected value)

$$S_{xx}(\omega) = \lim_{T\to\infty} E[|\hat{x}(\omega)|^2]$$

Spectral Density Computation

- If we expand the expected value

$$E[|\hat{x}(\omega)|^2] = E\left[\frac{1}{T}\int_0^T x^*(t)e^{j\omega t}dt \int_0^T x(t')e^{j\omega t'}dt'\right]$$

$$= \frac{1}{T}\int_0^T\int_0^T E[x^*(t)x(t')]e^{j\omega(t-t')}dt\,dt'$$

- If the process is stationary, then we can re-write this as

$$= \int_0^T \frac{1}{T}\int_0^T E[x^*(t)x(t+\tau)]dt\; e^{-j\omega\tau}\,d\tau$$

$$= \int_0^T R_{xx}(\tau)\; e^{-j\omega\tau}d\tau$$

Wiener-Khinchin Theorem

- The Spectral Density and Autocorrelation Function are Fourier Transform Pairs

$$S_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau)\, e^{-j\omega\tau} d\tau = R_{xx}(\omega)$$

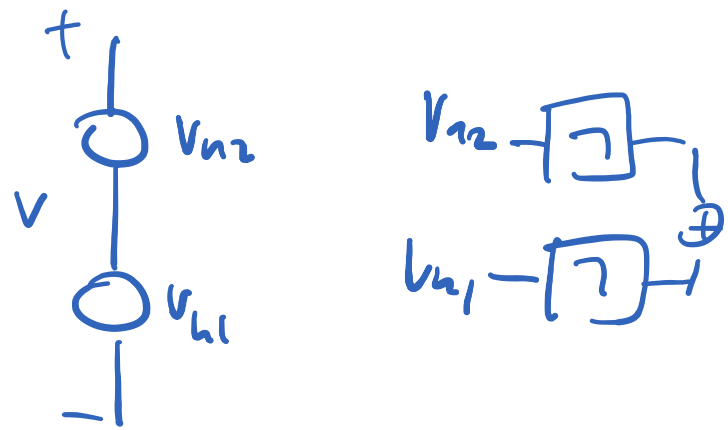
- The autocorrelation function is a measure of how much a signal is related to itself at a different time. If every sample in time is completely unrelated (independent) from every other sample [*R* = delta function], the Power Spectral Density is going to be white

Low Passing a Signal

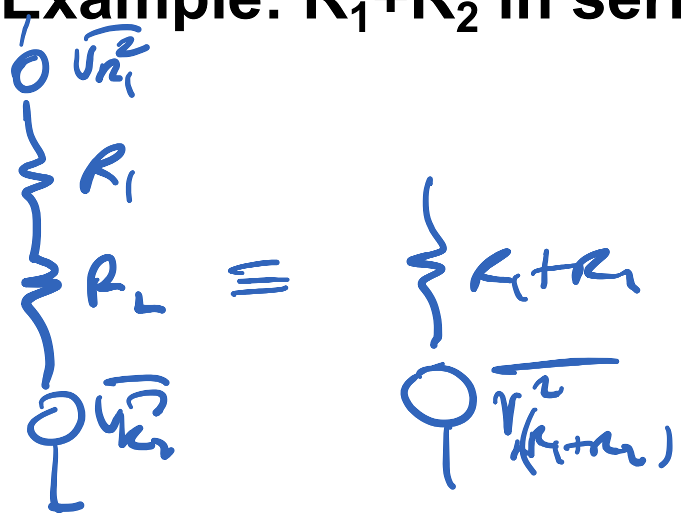
- When we filter a signal, we are spreading out each sample in time.
- LPF: A delta function is turned into a decaying exponential. The "correlation time" is related to the bandwidth of the filter
- It's therefore clear that filtering a signal is going to color the Power Spectral Density

Noise Calculations

- **Noise calculations**
  - **Instantaneous voltages add**
  - **Power spectral densities add**
  - **RMS voltages do <u>NOT</u> add**



- **Example: R_1+R_2 in series**



$$v = v_{n1} + v_{n2}$$

$$E[v^2] = E[v_{n1}^2 + v_{n2}^2 + 2v_{n1}v_{n2}]$$

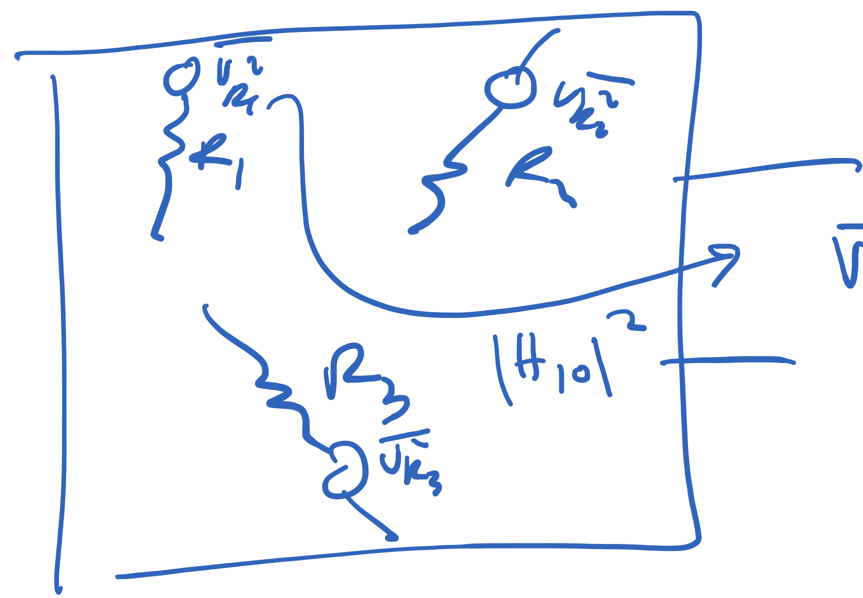
$$= E[v_{n1}^2] + E[v_{n2}^2] + 2E[v_{n1}v_{n2}]$$

$$= \overline{v_{n1}^2} + \overline{v_{n2}^2} + 0$$

Calculating Noise in Passive Networks

- **Capacitors and inductors only shape spectrum:**

$$\overline{v_{on,T}^2(f)} = \sum_x \left|H_x(s)\right|^2_{s=2\pi j f} \overline{v_x^2(f)}$$



$$\overline{V_o^2} = \sum_j |H_j(s)|^2 \cdot \overline{V_j^2(f)}$$

Side Note on Shot Noise (Diodes)

- **Shot noise**
  - **Zero mean, Gaussian pdf, white**
  - **Proportional to current**
  - **Independent of temperature**

$$\overline{i_n^2} = 2qI_D\Delta f$$

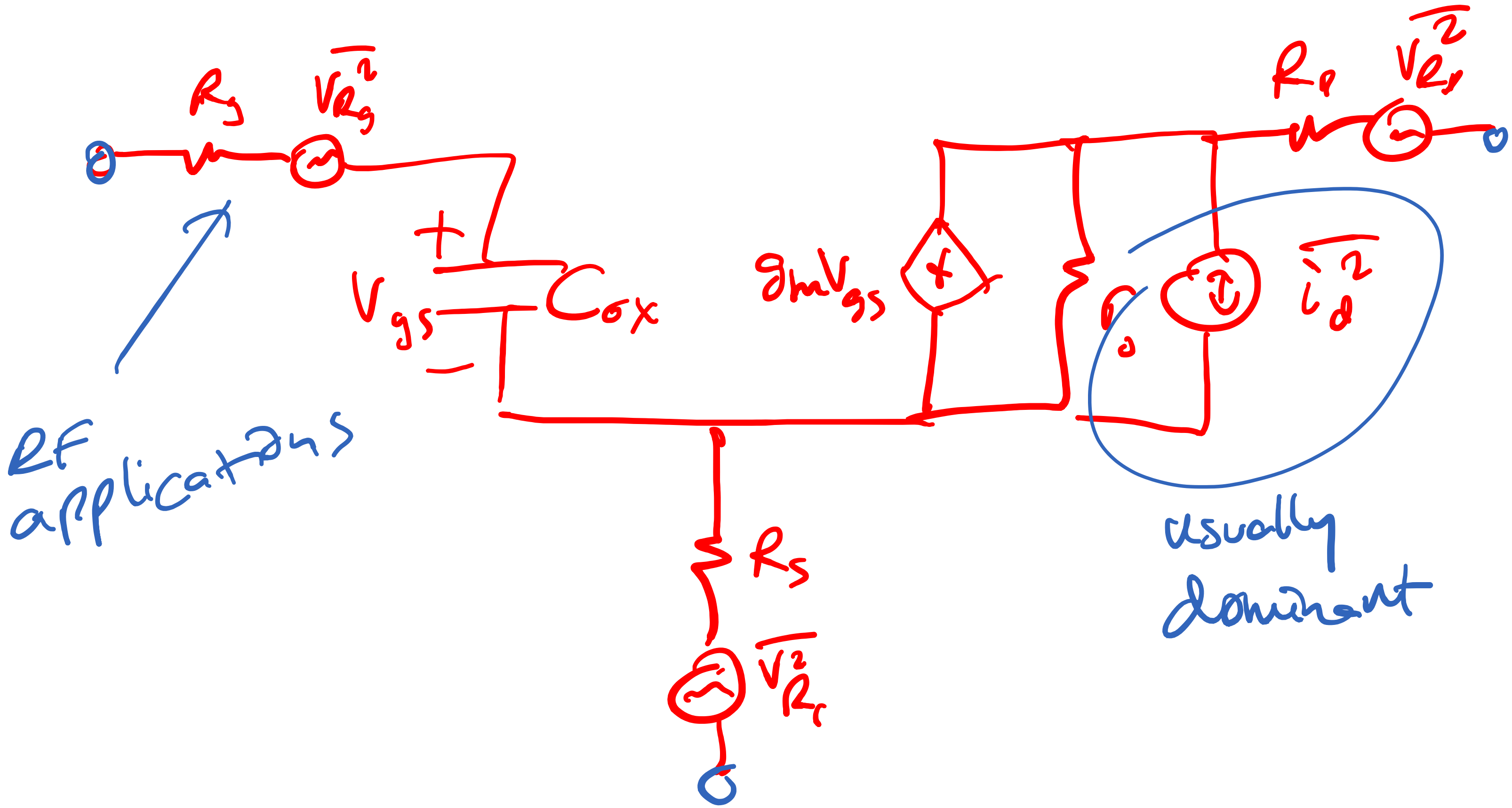
- **Example:**
  I_D = 1mA → <u>17.9pA/rt-Hz</u>
  1MHz bandwidth → σ = <u>17.9nA</u>

$g_{diag} = \frac{qI}{kT}$

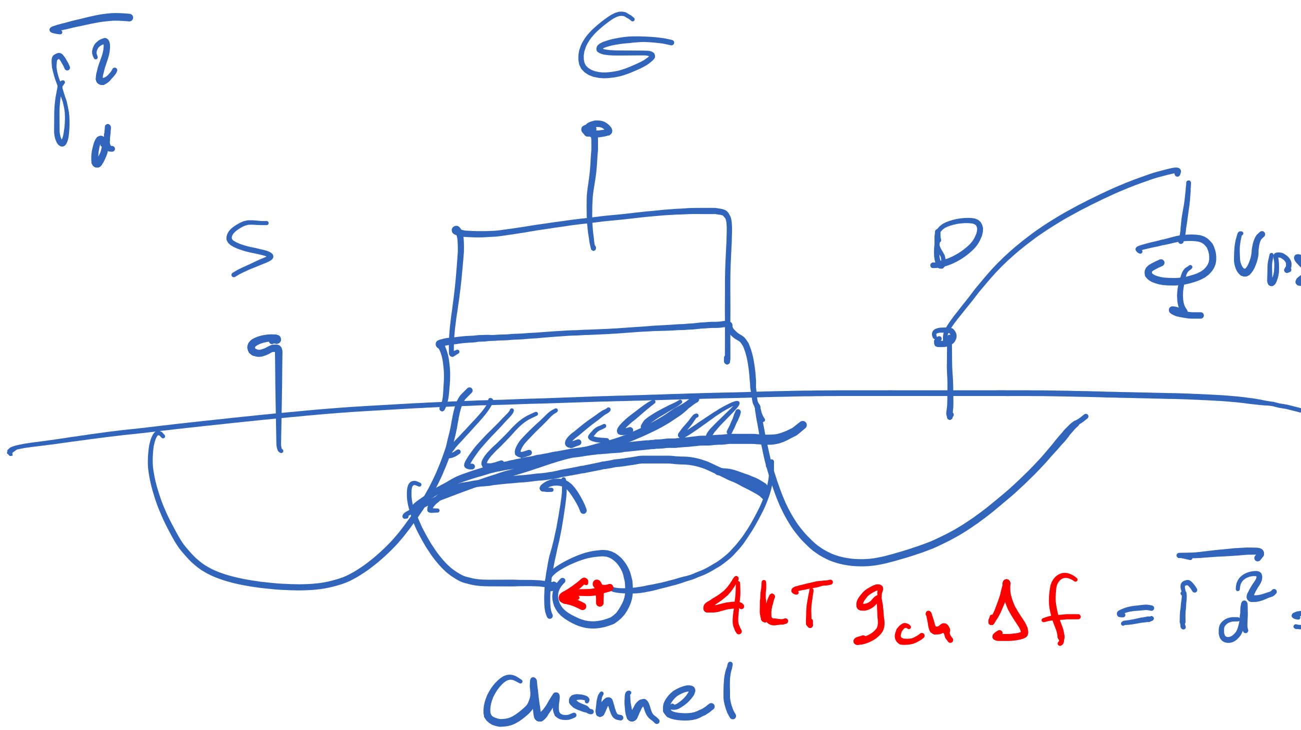
- **Shot noise versus thermal noise**
  - **$g_{diode} = I_d/(k_bT/q)$**
  - **Thermal noise density: $4k_bTg_{diode} = 4qI_d$**
  - **Shot noise half of this (current flow in 1 direction)**

$\frac{4\cancel{kT}}{2} \times \frac{qI}{\cancel{kT}} = \frac{4qI}{2}$

Simplified FET Noise Model



Simplified FET Noise Model



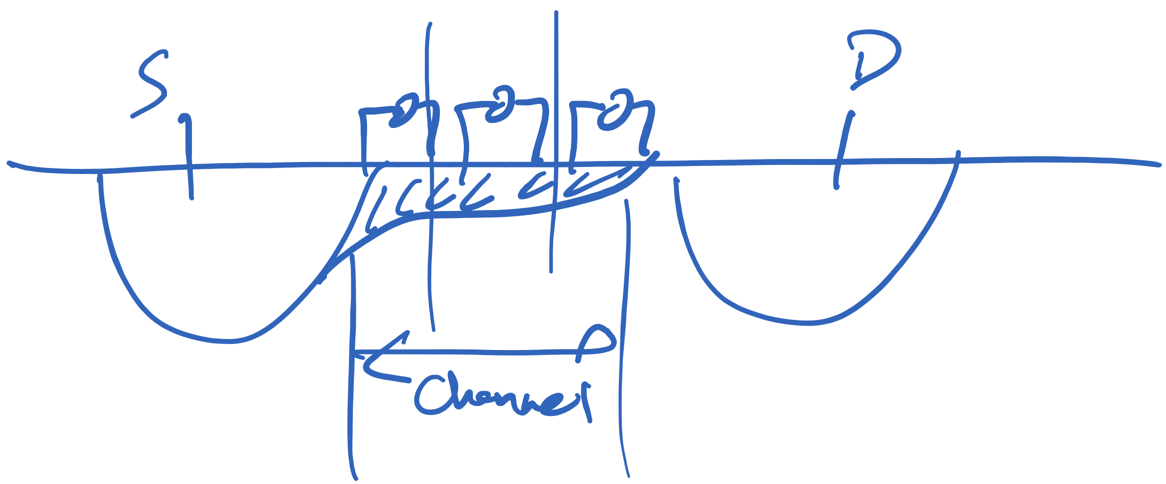
$$4kT\, g_{ch}\, \Delta f = \overline{i_d^2} = 4kT\, g_m\, \Delta f + \overline{i_f^2}$$

sq. law

$$g_{ch} = \frac{\partial I_D}{\partial V_{DS}} \simeq \mu C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_t)$$

$= g_{msat}$ ✓

~~Word of Caution~~ Saturation



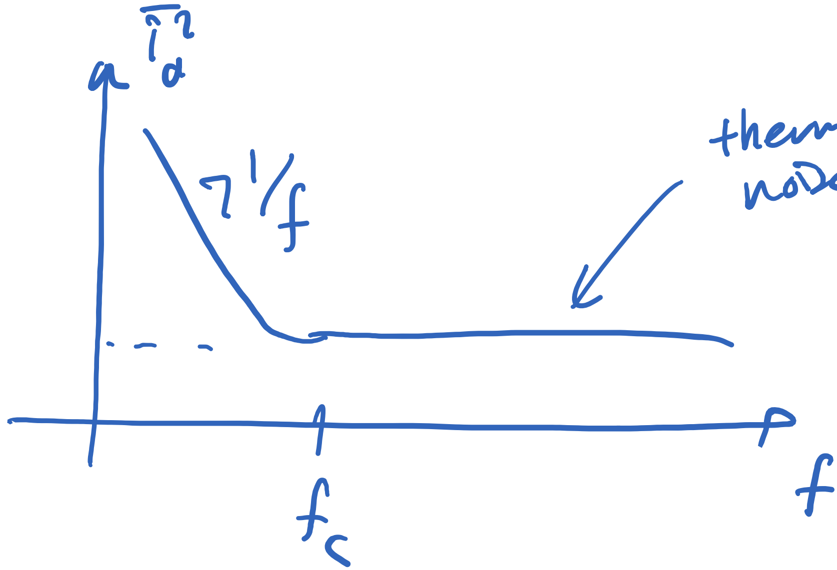
$\alpha \triangleq \frac{g_m}{g_{ch}}$

Channel is no longer uniform

$$\overline{i_d^2} = 4kT g_{ch} \cdot \frac{2}{3} \Delta f$$

$$= 4kT g_{ch} \cdot \gamma \cdot \Delta f = 4kT g_m \frac{\gamma}{\alpha} \Delta f$$

Flicker Noise



thermal noise $\sim \left(4kT\frac{\gamma}{\alpha}g_m\right)$

- PMOS is better... why?

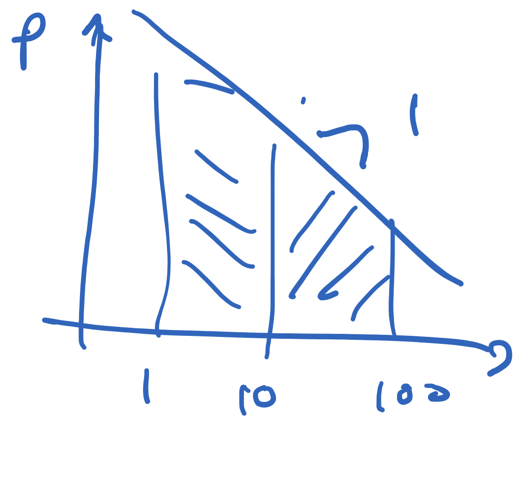
- BJT's have Flicker Noise but orders of mag less

- Only happens when there's current

1/f Noise Modeling

$$\overline{i^2_{1/f}} = \frac{K_f I_D}{L^2 C_{ox}} \frac{\Delta f}{f}$$

- **Flicker noise**
  - **$K_{f,NMOS}$ = 6 x 10^{-29} A*F**
    $K_{f,PMOS}$ = 3 x 10^{-29} A*F
  - **Strongly process dependent**



- **<u>Example</u>: I_D = 10μA, L = 1μm,**
  - **C_{ox} = 15fF/μm², f_{hi} = 1MHz**

    f_{lo} = 1Hz → σ = 0.74nA
    f_{lo} = 1/year → σ = 1.11nA

$$\overline{i^2_{1/f,total}} = \int_{f_{lo}}^{f_{hi}} \frac{K_f I_D}{L^2 C_{ox}} \frac{df}{f} = \frac{K_f I_D}{L^2 C_{ox}} \ln \frac{f_{hi}}{f_{lo}}$$

Random Telegrapher's Signal (RTS)

- For a <u>very small device</u>, if we observe the drain current, we observe that the current jumps in a random manner
- Essentially, we are observing a single capture and release event of an electron from the channel into a "trap" in the oxide.

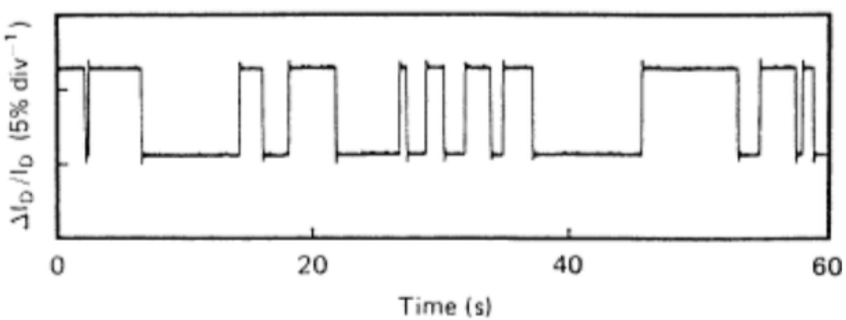


Figure 9.1: Random telegraph signal. Change in current against time. Active area of MOSFET is 0.4 μm^2. V_D = 10 mV, V_G = 0.94 V, I_D = 6.4 nA, T = 293 K.

https://www.nii.ac.jp/qis/first-quantum/forStudents/lecture/

By studying the bias-voltage dependence of the current, one can infer that the traps are located within a few nanometers of the channel – within the tunneling distance

RTS → 1/f Noise

- In a large device, there are many traps. The oxide traps have a wide range of time constants which gives rise to different RTS signals
- In a large device, the summation of many RTS signals leads to 1/f noise.
- One can show analytically that 1/f noise spectrum is a result of Poissonian Telegraphics Events and a uniformly distributed time constant for the traps

What about 1/f^power Noise?

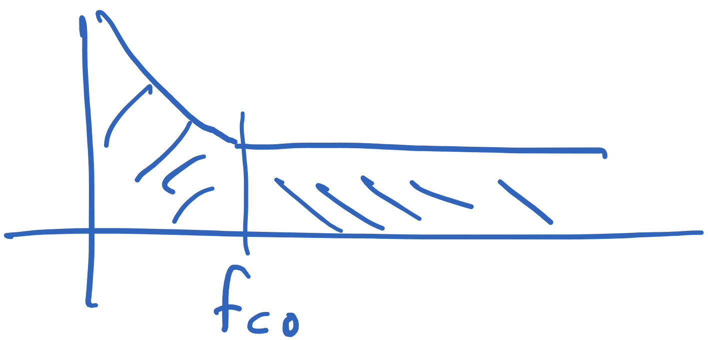
- If very small devices are observed, one does not see 1/f noise but more generally $1/f^{\alpha}$ to some power $^{\alpha}$
- This is consistent with RTS theory if there are few traps and they are not uniformly distributed
- It can be shown that the average noise performance for a large number of small devices is still 1/f

1/f Noise Corner Frequency

- **Definition (MOS)**

$$\frac{K_f I_D}{L^2 C_{ox}} \frac{\Delta f}{f_{co}} = 4 k_B T_r \gamma g_m \Delta f$$

$$f_{co} = \frac{K_f I_D}{L^2 C_{ox}} \frac{1}{4 k_B T_r \gamma g_m} = \frac{K_f}{4 k_B T_r \gamma C_{ox}} \frac{1}{L^2} \frac{1}{g_m/I_D} = \frac{K_f}{8 k_B T_r \gamma C_{ox}} \frac{V^*}{L^2}$$



- **<u>Example</u>:**
  - V* = 100mV, γ = 1.16

		NMOS	PMOS
L = 100nm	→	2MHz	233kHz
L = 300nm	→	222kHz	25.9kHz