EE240B HW3

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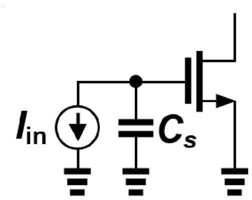
1 Amplifier Noise

For the circuit below calculate the transconductance g_m that minimizes the minimum detectable signal (when I_{in} equals to input-referred current noise) as a function of γ, ω_T, and C_s. In the plot of MDS vs g_m, comment briefly on why there is a minimum and how the slope relates to g_m. Consider C_{gs} and ignore C_{gd}.



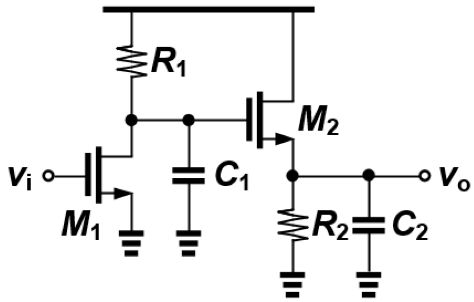
I think the answer is just that, to first order, the MDS is independent of g_m; this is because $\omega_T = \frac{g_m}{C_s}$ and ω_T is a process and bias constant. Therefore any increase in C_s would also necessitate a proportional increase in g_m. Notice that the output noise caused by a current noise at the input is $\overline{i_s^2}\left(\frac{\omega_T}{\omega}\right)^2$ which is a constant!

The solution doesn't make sense. Firstly the 2 expressions given for $\overline{i_{out}^2}$ aren't equal; they are off by a factor of 4.

But that shouldn't change the optimal g_m anyways. Indeed if you differentiate and set to 0 the expression in the solution, the solution $g_{m,opt} = C_s\omega_T$ is what comes out, but that just equals g_m again! How is it legitimate to differentiate the expression in the solution wrt g_m if ω_T itself can be written in terms of g_m?

2 Amplifier Design

(a) What is the total noise at the output of the common-source-common-drain cascade shown below? Ignore flicker noise, r_o, and capacitor except those drawn in the diagram. You should provide your answer in terms of $kT, C_1, C_2, \gamma, A_{v1}, A_{v2}, \omega_{p1}, \omega_{p2}$. $A_{v1,v2}$ and $\omega_{p1,p2}$ are the low-frequency voltage gains and dominant poles of the two stages.



$$\overline{v_{n,o}^2} = \underbrace{4kT\left(g_{m2}\gamma + \frac{1}{R_2}\right)}_{\text{Current noise from } M_2 \text{ and } R_2} \cdot \overbrace{\left(R_2 || \frac{1}{g_{m2}}\right)^2}^{\text{Converted to voltage by } R_{out}} \cdot \underbrace{\int_0^\infty \left|\frac{1}{1+s/\omega_{p2}}\right|^2 df}_{\text{Shaped by output pole}} +$$

$$\underbrace{4kT\left(g_{m1}\gamma + \frac{1}{R_1}\right)}_{\text{Current noise from } M_1 \text{ and } R_1} \cdot R_1^2 \int_0^\infty \left|\frac{1}{1+s/\omega_{p1}}\right|^2 df \cdot A_{v2}^2 \cdot \int_0^\infty \left|\frac{1}{1+s/\omega_{p2}}\right|^2 df$$

$$= 4kT\left(g_{m2}\gamma + \frac{1}{R_2}\right)\left(\frac{R_2}{1+R_2 g_{m2}}\right)^2 \frac{\omega_{p2}}{4} + 4kT\left(g_{m1}\gamma + \frac{1}{R_1}\right) R_1^2 A_{v2}^2 \frac{\omega_{p1}\omega_{p2}}{16}$$

Sub $R_1 = \frac{1}{\omega_{p1} C_1}$ and R_2 from solving $\omega_{p2} = \frac{1}{(R_2||1/g_{m2})C_2}$. Just some painful algebra.

(b) We have a fixed power budget for the two stages of amplifier and we would like to minimize the total noise at the output. To simplify the analysis, we can assume the V^*'s and GBW of the two stages are identical, $R_1C_1 = R_2C_2$, and $g_{m1}R_1, g_{m2}R_2 \gg 1$. What portion of power would you allocate to the M_1 in order to minimize the total noise at the output? (Hint: In order to maintain fixed GBW, the power consumption of each stage is linearly proportional to its load capacitance)

With the simplifying assumptions:

$$A_{v1} = -g_{m1}R_1$$
$$A_{v2} = 1$$
$$\omega_{p1} = \frac{1}{R_1 C_1}$$
$$\omega_{p2} = \frac{g_{m2}}{C_2} = \frac{2I_{d2}}{V^* C_2}$$

Setting GBW equal:

$$A_{v1}\omega_{p1} = A_{v2}\omega_{p2}$$
$$-\frac{g_{m1}}{C_1} = \frac{g_{m2}}{C_2}$$

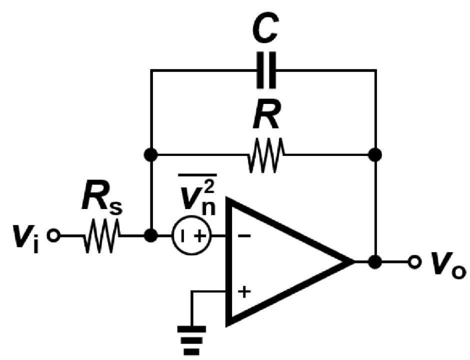
This proves the hint which is that power is proportional to input capacitance for equal GBW.

I substituted the values above to the noise equation, and then performed the substitution $g_{m1} = I_{d1}$ and $g_{m2} = KI_{d1}$. After differentiating wrt I_{d1}, setting to zero, and solving for K, I get something on the order of:

$$K = \frac{\sqrt{1/I_{d1}}}{I_{d1}}$$

3 Filter Noise

You are given an active RC low pass filter with noisy resistors and noisy op-amp shown below.



(a) Assume the op-amp has infinite DC gain and infinite GBW (ω_u). Calculate v_o/v_i and total output noise. What issue did you find?

$$\frac{v_o}{v_i} = \frac{R||C}{R_s} = \frac{\frac{R}{1+sRC}}{R_s} = \frac{R}{R_s}\frac{1}{1+sRC}$$

$$\overline{v^2_{n,out,R_s}} = \int_0^\infty \overline{v^2_{n,R_s}} \left(\frac{v_o}{v_i}\right)^2 df$$

$$= 4kTR_s\left(\frac{R}{R_s}\right)^2 \int_0^\infty \left|\frac{1}{1+sRC}\right|^2 df$$

$$= 4kTR_s\left(\frac{R}{R_s}\right)^2 \frac{1}{4RC}$$

$$\overline{v^2_{n,out,R}} = \int_0^\infty \overline{i^2_{n,R}}(R||C)^2$$

$$= 4kT\frac{1}{R}\int_0^\infty \left|\frac{R}{1+sRC}\right|^2 df = 4kT\frac{1}{R}R\frac{1}{4RC} = \frac{kT}{RC}$$

$$\overline{v^2_{n,out,amp}} = \overline{v^2_{n,amp}}\int_0^\infty \left|1+\frac{R}{R_s}\frac{1}{1+sRC}\right|^2 df = \infty$$

The amplifier noise is infinite at the output assuming GBW $= \infty$.

(b) Assume the op-amp has infinite DC gain and finite GBW (i.e. You can model the op-amp gain $A(s) = -\omega_u/s$ where $\omega_u \gg 1/RC$). Calculate v_o/v_i and total output noise. How does the noise relate to ω_u?

Let V_x be the node at the negative terminal of the op-amp. Then these equations can be written and we can solve for $\frac{v_o}{v_i}$.

$$V_x = V_o - iZ$$

$$V_o = \frac{V_x \omega_u}{s}$$

$$V_x - V_i = R_s i_1$$

$$Z = \frac{1}{1 + sRC}$$

$$\frac{v_o}{v_i} = \frac{R\omega_u}{-CRR_s\omega_u s + CRR_s s^2 + Rs - R_s\omega_u + R_s s}$$

To perform noise analysis with feedback, cut the loop and find the noise at the amplifier input:

$$\left(\frac{v_o}{v_i}\right)_{cut} = \frac{R||C}{R_s + (R||C)} A(s)$$

$$v_{in,n,R_s} = v_{n,Rs}$$

$$v_{out,n,R} = \frac{R_s||C}{R + (R_s||C)} v_{n,R} A(s)$$

$$v_{in,n,R} = \frac{v_{out,n,R}}{(v_o/v_i)_{cut}}$$

$$v_{in,n,amp} = \frac{v_{n,amp} A(s)}{(v_o/v_i)_{cut}}$$