Problem 1

Equation for a pulse:

$$p(t) := u(t) - u(t - \tau)$$

In the frequency domain:

$$H(s) := \frac{1}{s} - \frac{e^{-s\cdot\tau}}{s}$$

$$\left|H(j\cdot\omega)\right| = \frac{2\sin\left(\frac{\omega\cdot\tau}{2}\right)}{\omega}$$

[we ignore the pulse train scaling factor, since we normalize it out anyway]

We normalize H(jω) by the gain at DC:

$$\left|H_{norm}(j\cdot\omega)\right| = \frac{\left|H(j\cdot\omega)\right|}{\left|H(0)\right|} = \frac{2\sin\left(\frac{\omega\cdot\tau}{2}\right)}{\omega\tau}$$

The input frequency is fs/2:

$$\omega = 2\cdot\pi\frac{f_s}{2} = \frac{\pi}{T_s}$$

Define:

$$T_{norm} = \frac{\tau}{T_s} = \tau\cdot f_s \quad \text{(duty cycle)}$$

$$\omega\cdot\tau = \pi\cdot T_{norm}$$

Set up the equation and solve:

$$\frac{2\sin\left(\frac{\pi\cdot T_{norm}}{2}\right)}{\pi\cdot T_{norm}} = 10^{\frac{-1}{20}} \text{ solve } \rightarrow -0.52300039000913476872$$

[this function is symmetric around the y axis, and the numerical solver has trouble finding the positive solution]

Final answer: $\tau = \frac{T_{norm}}{f_s} = \frac{0.523}{f_s}$

[Full credit for having a correct equation and the final answer for τ in terms of the unknown, if you couldn't get the numerical solution.]

Problem 2

Input signal: $x(t) := \frac{1}{2}\cos(\omega\cdot t)$ 1V peak to peak!!

Distortion function: $\mathrm{Dist}\left(V_{in}\right) := V_{in} + \beta\cdot V_{in}^{\ 3}$

$$\mathrm{Dist}(x(t)) \rightarrow \frac{\beta\cdot\cos(\omega\cdot t)^3}{8} + \frac{\cos(\omega\cdot t)}{2}$$

Apply trig identity: $\frac{\beta}{8}\cdot\cos(\omega\cdot t)^3 + \frac{1}{2}\cdot\cos(\omega\cdot t) = \left(\frac{1}{2} + \frac{\beta}{8}\cdot\frac{3}{4}\right)\cdot\cos(\omega\cdot t) + \frac{\beta}{8}\cdot\frac{1}{4}\cdot\cos(3\omega\cdot t)$

$\beta := 0.05$

Calculate SFDR:

$$\mathrm{SFDR} := 20\cdot\log\left(\frac{\frac{1}{2}+\frac{3\cdot\beta}{32}}{\frac{\beta}{32}}\right) = 50.184 \qquad \text{(dB)}$$

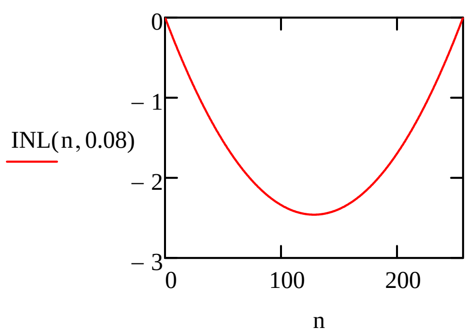
Problem 3

$$R_{sh}(x) := R_o\cdot(1+\alpha\cdot x)$$

$$R(n) := \int_0^{\frac{n}{255}} \frac{R_{sh}(x)}{W}\,dx \rightarrow \frac{R_o\cdot n\cdot(\alpha\cdot n+510)}{130050\cdot W} \qquad \text{(resistance to ground as a function of code)}$$

$$Vo(n,\alpha) := \frac{R(n)}{R(255)} \rightarrow \frac{n\cdot(\alpha\cdot n+510)}{255\cdot(255\cdot\alpha+510)} \qquad \text{(voltage divider)}$$

$$INL(n,\alpha) := \frac{Vo(n,\alpha)-\frac{n}{255}}{\frac{1}{256}} \qquad INL(127,0.08) = -2.462 \qquad \text{(LSB, peak INL at midscale)}$$



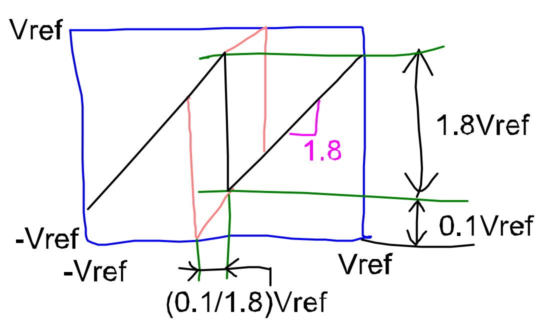
Problem 4

The worst-case DNL will occur when the next unit element turns on in the MSB section, and all of the elements in the LSB section turn off. The expression is therefore:

$$\sigma_{DNLmax} = \sqrt{\sigma_1^{\,2} + \left(2^{B_2}-1\right)\cdot\sigma_2^{\,2}}$$

Note: depending on how you interpreted the problem, you may have multiplied σ1 by 2^B2. This answer will also be given full credit.

Problem 5



The ADC can tolerate up to 0.056 * Vref of comparator offset (if Vref is defined as shown in the picture) without the residue leaving the box.