EE 240C Project: ADC Modeling and Non-Ideality Analysis

1 Choice of Architecture

The desired specs for the ADC are listed in Table 1.

Table 1: ADC Specifications

Parameter	Value
Bandwidth	≥ 12.5 MHz
Dynamic Range	≥ 70 dB
Signal to Noise and Distortion Ratio	≥ 65 dB
Power Consumption	minimize

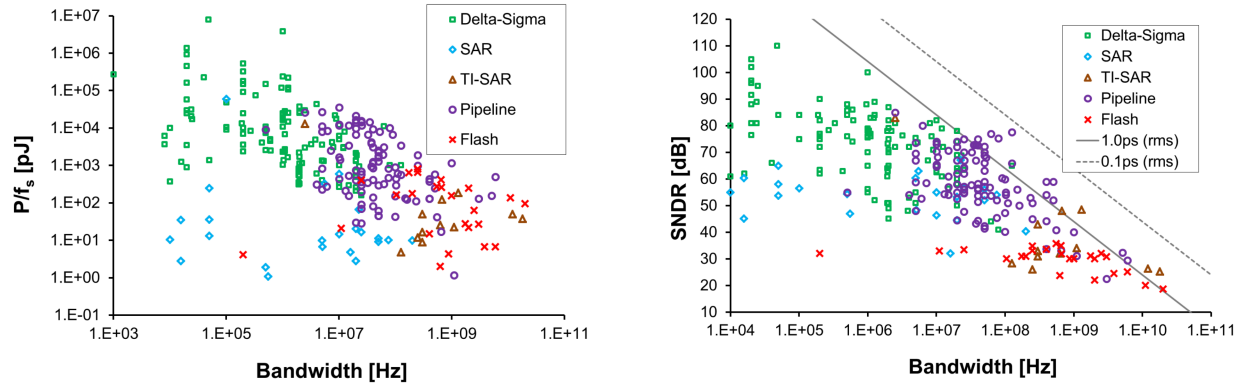I looked at Boris Murmann's 2012 ADC survey. His plots are in Figure 1.

Figure 1: Plots from Murmann's (2012) ADC survey

Clearly a flash architecture cannot reasonably meet the SNDR requirement and would be more suitable for a higher bandwidth requirement. The SAR architecture seems to deliver the best power efficiency, but it looks like the SOTA SAR ADCs cannot achieve a SNDR higher than 70 dB which makes achieving 65 dB a difficult proposition for this architecture.

The dynamic range spec translates to a 12-bit ADC, and scanning the table of ADC papers from Murmann's spreadsheet, either a pipeline or sigma-delta architecture seems appropriate. According to figures from Pavan ("Understanding Delta-Sigma ADCs"), the sigma-delta ADC requires an oversampling ratio of 32-64 for a low-order modulator (2-3) and an OSR of 16-32 for a high-order modulator (4-6) to achieve SQNR above 65 dB. This figure doesn't account for other noise sources from the SC stages used for error feedback, the sampling capacitor, or distortion. This means the ADC has to run at $f_{adc} = 12.5 \cdot 2 \cdot$ OSR which is 400 - 800 MHz, imposing additional power requirements on the SAH stage to achieve settling time requirements.

In contrast the SAH and gain stages of the pipeline ADC can run at the Nyquist rate, and it should be able to achieve the SNDR spec. I will design a model of a pipeline ADC.

2 Pipeline ADC Analysis

The dynamic range spec sets the number of bits for the ADC:

$$\text{DR} = (6.02 \cdot B) \text{ dB} \geq 70$$
$$B \geq 11.6$$

At least a 12-bit ADC is required to meet the dynamic range spec. To determine each stage's optimal resolution and gain requires power and per-stage noise analysis first.

The bandwidth spec sets the sampling frequency of the pipeline ADC to at least $f_{samp} = 25$ MHz, and since the OTAs must settle the residue within half of the sample clock cycle this sets their required g_m. This also places limits on the decision time of the flash ADC (within each stage) to 20ns.

The SNDR is determined by the quantization noise (from the selection of B), thermal noise from the SAH stage and every OTA gain stage, and distortion induced from non-linearities in the OTAs.

2.1 Noise Analysis

Consider the MDAC presented in lecture (Figure 2) with 2 clock phases for sampling (ϕ_1) and residual amplification (ϕ_2).
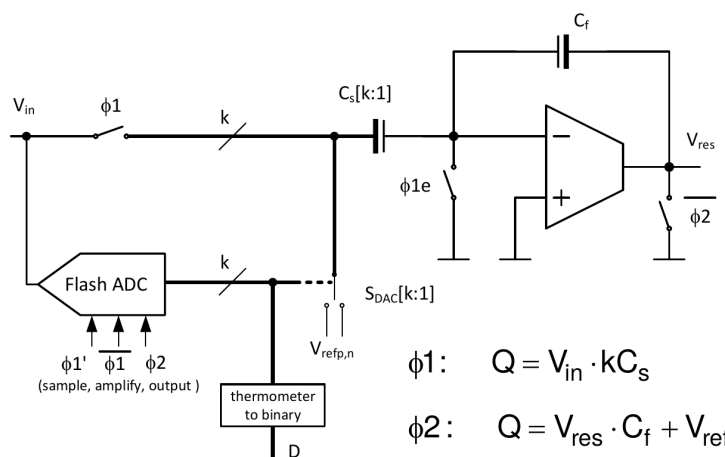

Figure 2: MDAC presented in lecture

2.1.1 Sampling Phase

During the first clock phase ϕ_1, V_{in} is sampled on C_s and the switches S_{ϕ_1} and $S_{\phi_{1e}}$ inject voltage noise on C_s. I'm not considering any noise injected due to charge injection caused by fast gating since bottom-plate sampling from $S_{\phi_{1e}}$ should alleviate this.

Both switches contribute thermal noise when they are on and the noise spectrum is shaped by C_s. The total integrated noise across C_s after sampling is:

$$\overline{v_{samp}^2} = \frac{kT}{C_s}$$

2.1.2 MDAC Amplification Phase

During the second clock phase ϕ_2, the OTA develops V_{res} by opening the ϕ_1 switches and closing the switches to the DAC supply voltages.

OTA Noise Assume that the OTA is a simple 5 transistor circuit with one current source NMOS (M5), an NMOS input pair (M1/M2), and a PMOS active load (M3/M4). The current noise of each transistor is $\overline{I_n^2} = 4kT\gamma g_m$.

$$\overline{i_{out,n,inp}^2} = 2 \cdot 4kT\gamma g_{m1}$$
$$\overline{i_{out,n,mir}^2} = 2 \cdot 4kT\gamma g_{m3}$$
$$\overline{v_{out,n,tot}^2} = (2 \cdot 4kT\gamma g_{m1} + 2 \cdot 4kT\gamma g_{m3}) \cdot (r_{o2}||r_{o4})^2$$
$$\overline{v_{in,n,tot}^2} = \overline{v_{out,n,tot}^2} \cdot \frac{1}{g_{m1}^2(r_{o2}||r_{o4})^2} = 2\left(\frac{4kT\gamma}{g_{m1}}\right) + 2\left(\frac{4kT\gamma g_{m3}}{g_{m1}^2}\right)$$

OTA With Capacitive Feedback Now consider the OTA with C_s connected to the (-) terminal, C_f connected between the output and (-) terminal, and a load cap C_L at the OTA's output representing the C_s of the next stage.

$$\beta(\text{FB factor}) = \frac{C_f}{C_{s,k} + C_F}$$
$$G_{CL} = \frac{1}{\beta}$$
$$C_{out,eff} = C_L + (1-\beta)C_f = C_{s,k+1} + (1-\beta)C_f$$
$$\omega_p = \frac{\beta g_m}{C_{out,eff}}$$
$$\overline{v_{n,ota,out}^2} = \int_0^\infty \overline{v_{in,n,tot}^2} \frac{G}{1 + s/\omega_p} df$$
$$= \overline{v_{in,n,tot}^2} \frac{\omega_p}{4} G^2$$
$$= \overline{v_{in,n,tot}^2} \frac{g_{m1}}{4C_{out,eff}\beta^2}$$

Total Thermal Noise The ADC consists of a sample and hold stage and N pipeline stages consisting of a flash ADC and MDAC.

$$\overline{V_{n,in}^2} = \overline{v_{samp,1}^2} + \overline{v_{n,ota,sah}^2} + \frac{\overline{v_{n,ota,1}^2} + \overline{v_{n,samp,2}^2}}{G_1^2} + \frac{\overline{v_{n,ota,2}^2} + \overline{v_{n,samp,3}^2}}{G_1^2 G_2^2} + \cdots + \frac{\overline{v_{n,ota,N}^2}}{G_1^2 G_2^2 \ldots G_N^2}$$

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2.1.3 Quantization Noise

The quantization noise power is:

$$\overline{v_{n,quant}^2} = \frac{\Delta^2}{12} = \frac{V_{DD}/2^{12^2}}{12} = 7.15 \text{ nV/Hz}$$

2.1.4 SNDR

The SNDR captures quantization, thermal, and distortion induced noise.

$$\text{SNDR} = \frac{P_{sig}}{P_{noise}} = \frac{(1.2/\sqrt{2})^2}{\overline{v_{n,quant}^2} + \overline{v_{n,in}^2}}$$

2.2 OTA g_m

The OTA needs to drive V_{res} to the next stage's C_s. The static error on C_s should be under an LSB.

For an initial guess for g_m, I assume the OTA has a 1-pole transfer function and assume the settling to within 1 LSB must happen within 1 time constant.

$$\begin{aligned}
\tau &= \frac{C_{out,eff}}{\beta g_m} \\
&= \frac{t_{settle}}{\ln(1 \text{ LSB})} \\
&= \frac{T_s/2}{\ln(1 \text{ LSB})}
\end{aligned}$$

2.3 Power Analysis

The power consumption is dominated by the OTA. The flash ADC, its comparators, and the switches consume relatively less power. I'm neglecting the power required for the clock drivers and distribution.

$$P = V_{DD}I_d = V_{DD} \left(\sum_{i=1}^{N} \frac{g_{m,i}}{\frac{g_{m,i}}{I_d}} \right)$$

2.4 Numbers

For a first cut design, we will consider a 12-bit pipeline ADC built out of 12 1.5 bps stages to accomodate comparator offset varitions.

$$P_{noise,adc} \approx \frac{4kT}{C_s}$$

$$SNR \approx \frac{(V_{DD}/sqrt2)^2}{P_{noise,adc}}$$

$$C_s \approx 73 \text{ fF}$$

3 Pipeline ADC Modeling

The pipeline ADC is modeled in Python using separate classes for the flash ADC and the MDAC, then combining those into a stage and stages into a pipeline. A few caveats were that each stage could only use a 1.2V supply and the input was single-ended. This meant the OTA's (+) terminal needed to be biased at midrail. The SAH stage was treated specially to produce a midrail-biased residual while the rest of the pipeline stages assumed that a midrail input voltage meant there was no residual left and values above and below the midrail were treated as positive and negative residuals.

The first check is to make sure the quantization noise gave sufficient room for the additional thermal noise. The ADC was coherently sampled with $f_{in} = \frac{J}{N} f_s$ with $J = 1223$ and $N = 2^{12}$. This produced a spectrum with only quantization noise:
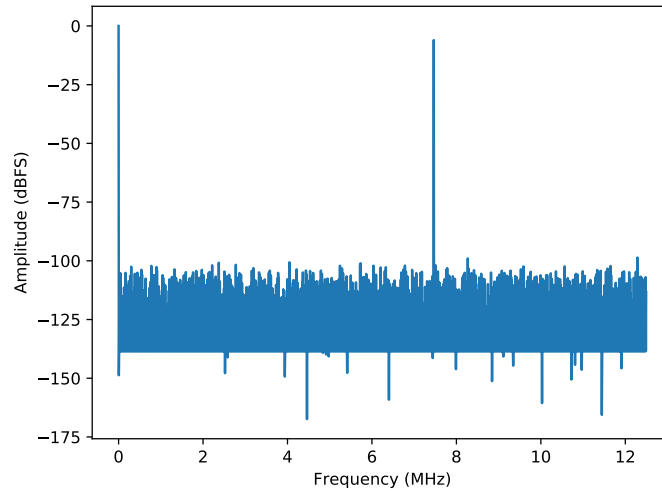

Figure 3: FFT of ADC output only considering quantization noise

This spectrum had an SNDR of 74 dB, which is in line with the expected number.

The next step was to add input referred thermal noise as calculated above per OTA stage. This noise was added in the time domain as normally distributed noise using the sampling cap value

and the equations from the analysis section.

$$g_{m1} = g_{m3} = 118\mu S$$
$$\overline{v_{n,samp}^2} = 5.67e\text{-}8 \quad V^2/Hz$$
$$\overline{v_{n,ota}^2} = 3.72e\text{-}16 \quad V^2/Hz$$
$$\overline{v_{n,ota,fb}^2} = 1.02e\text{-}6 \quad V^2/Hz$$
$$\overline{v_{n,tot}^2} = 4.94e\text{-}6 \quad V^2/Hz$$

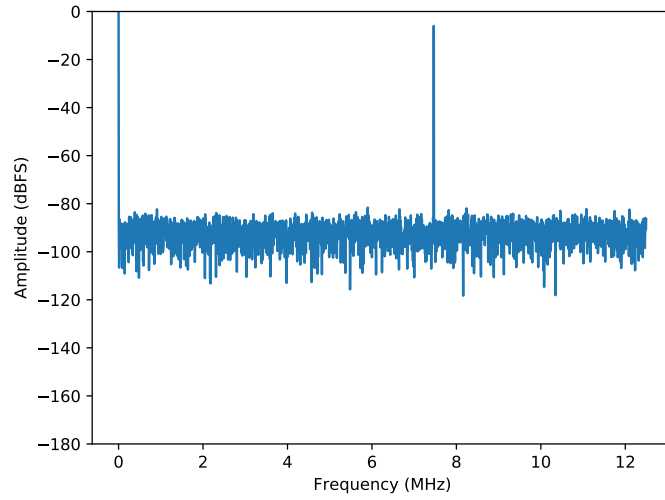This results in a noise spectrum that is clearly larger than desired: with a SNDR of 51dB.

Figure 4: FFT of ADC output considering thermal noise of OTAs and sampling switches

Since this does not meet the spec, the knobs available for tuning are the per-stage gain, the sampling capacitance value, and the setting error. To recover the SNDR loss, the sampling capacitance of the first stage needs to be bumped up to 512 fF for an SNDR of 65 dB. To compensate for the excessive power draw of the first stage, the later stages have their capacitances scaled down by a 1.5x taper factor.

The other modeled non-idealities were comparator offsets and capacitor mismatches. The comparator offset errors were handled by making the inter-stage gain smaller than full-scale, similar to the 1.5bps technique. This increases the number of required stages, but the additional stage consumes a modest amount of power and doesn't contribute much to noise.

The capacitor mismatches were handled by measuring the analog gain of each OTA stage in isolation and using this gain when adding up the ADC codes to produce the final output. There is no significant power or noise issue introduced since the only source of additional noise are more switches around the OTA for out-of-band OTA testing.

3.1 Summary

Table 2: ADC Parameters

Parameter	Value
Bandwidth	12.5 MHz
Dynamic Range	12 bits
Signal to Noise and Distortion Ratio	65.2 dB
Power Consumption	4 mW
Configuration	3-3-3-3-3 with equal gains per stage and 1.5x C_s taper