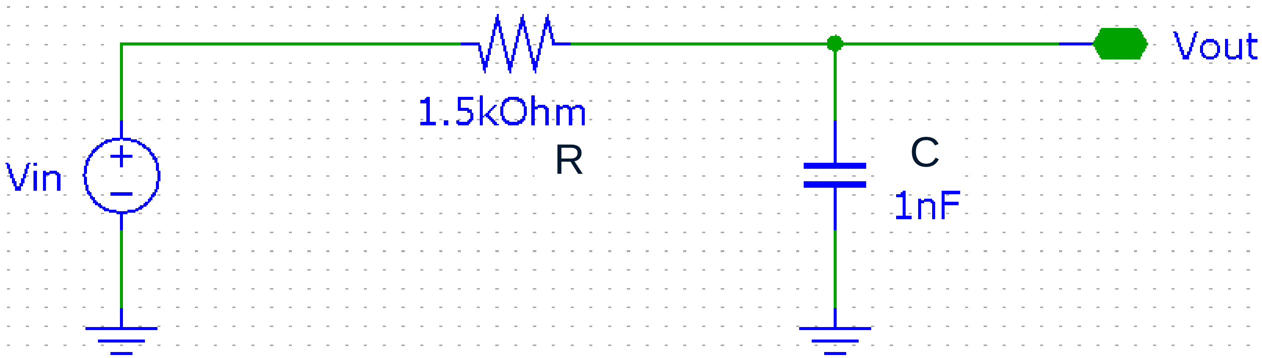
EE 240C
Analog-Digital Interface Integrated Circuits

Filters

EE 240C
Analog-Digital Interface Integrated Circuits

First Order Filter

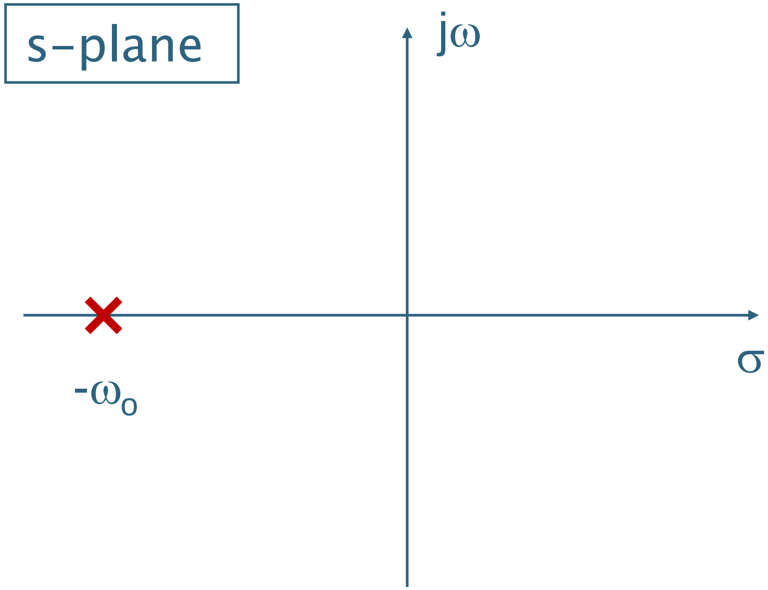
First-Order RC Filter



$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1 + \frac{s}{\omega_0}} \qquad \omega_0 = \frac{1}{RC}$$

Poles and Zeros

$$H(s) = \frac{N(s)}{D(s)} = \frac{1}{1 + \frac{s}{\omega_o}}$$



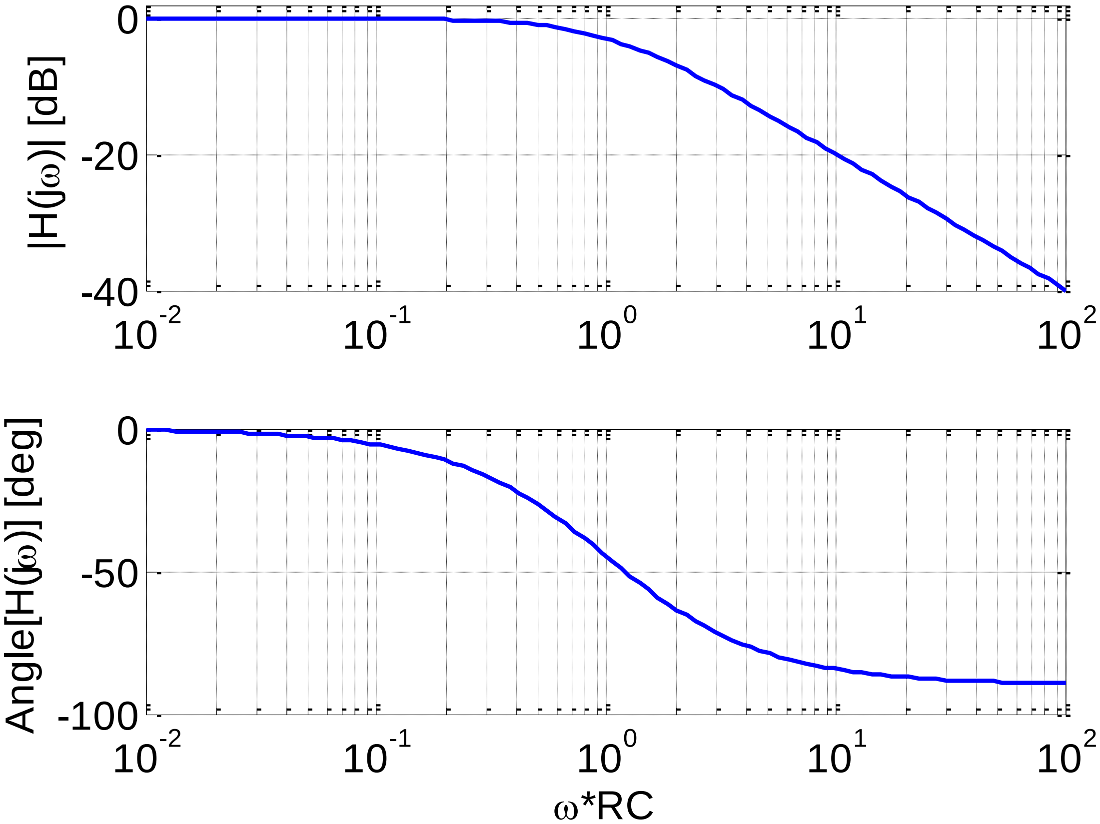
$$s = \sigma + j\omega$$

Pole $\quad p = -\omega_o$

Zero $\quad z \to \infty$

- In general, the "finite" poles and zeros follow from the roots of D(s) and N(s)
  - When the order (n) of N(s) is smaller than the order (d) of D(s), there will be zero(s) at infinity with multiplicity d-n

Bode Plot



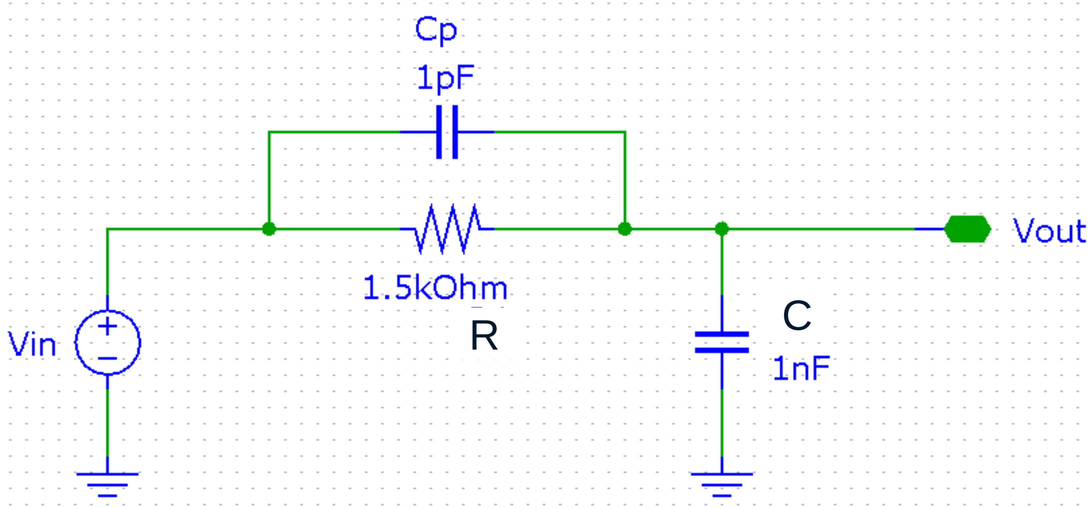
At ω = 1/RC

$$|H(j\omega)| = \frac{1}{\sqrt{1+(1)^2}} = \frac{1}{\sqrt{2}}$$

$$20 \cdot \log\left(\frac{1}{\sqrt{2}}\right) \cong -3 \;\; [\text{dB}]$$

$$\angle H(j\omega) = \tan^{-1}(-1) = -45^\circ$$

Practical Considerations: Feedthrough Capacitance

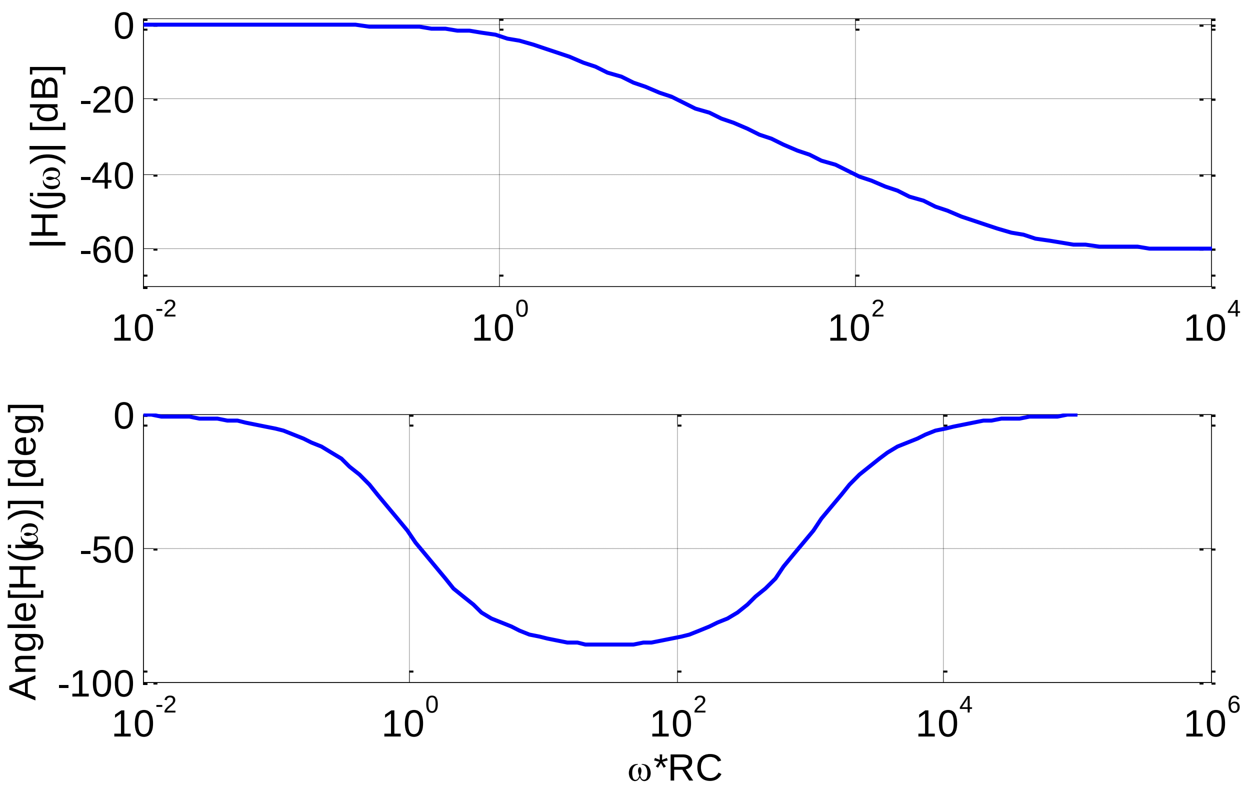


$$H(s) = \frac{1 + sRC_P}{1 + sR\left(C + C_P\right)}$$

Pole $\quad p = -\frac{1}{R\left(C + C_P\right)} \approx -\frac{1}{RC}$

Zero $\quad z = -\frac{1}{RC_P}$

Bode Plot



Maximum Attenuation: $\left|H(j\omega)\right|_{\omega\to\infty} = \dfrac{C_P}{C + C_P} \cong \dfrac{C_P}{C} = 10^{-3} = -60\text{dB}$

EE 240C
Analog-Digital Interface Integrated Circuits

Second Order Filters

Second-Order Lowpass Filter

- Better attenuation (compared to first order)
- General two-pole transfer function

$$H(s) = \frac{1}{1 + \frac{s}{\omega_P Q_P} + \frac{s^2}{\omega_P^2}}$$

$$\left|H(j\omega)\right|_{\omega=0} = 1$$

$$\left|H(j\omega)\right|_{\omega\to\infty} = 0$$

$$\left|H(j\omega)\right|_{\omega=\omega_P} = Q_P$$

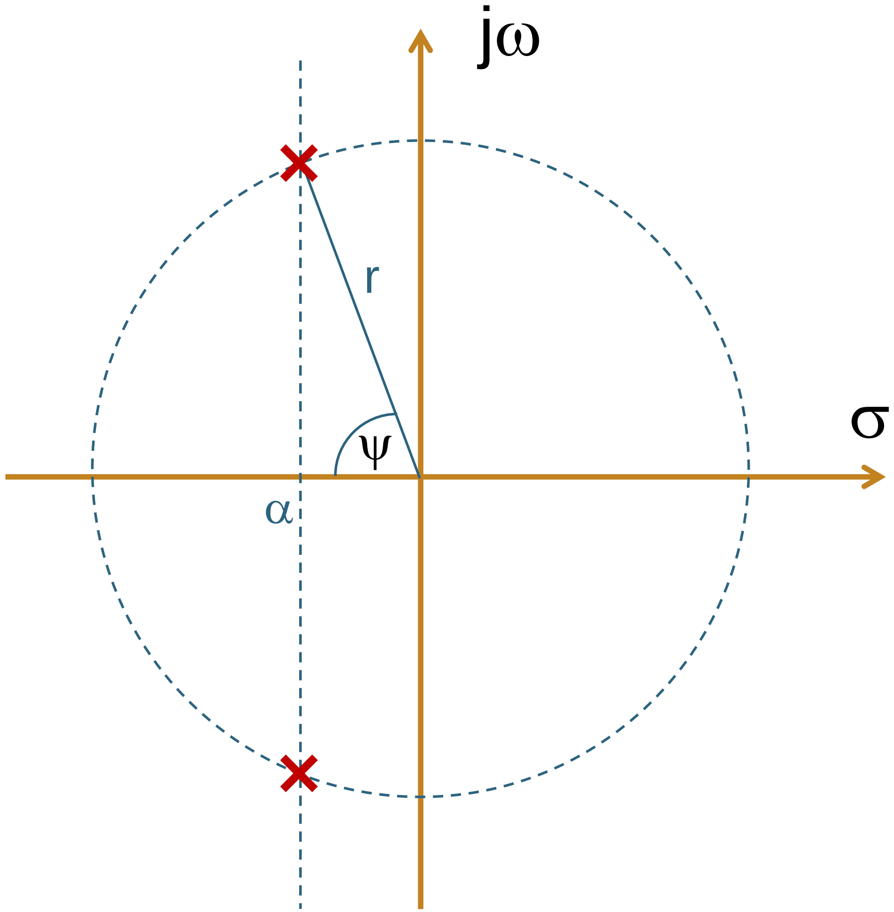
Pole Locations

$$H(s) = \frac{1}{1 + \frac{s}{\omega_P Q_P} + \frac{s^2}{\omega_P^2}}$$

for $Q_P \le \frac{1}{2}$ $\quad s_{1,2} = -\frac{\omega_P}{2Q_P}\left(1 \pm \sqrt{1 - 4Q_P^2}\right)$ (poles are real)

for $Q_P > \frac{1}{2}$ $\quad s_{1,2} = -\frac{\omega_P}{2Q_P}\left(1 \pm j\sqrt{4Q_P^2 - 1}\right)$ (poles are complex conjugate)

Complex Pole Locations in the s-Plane



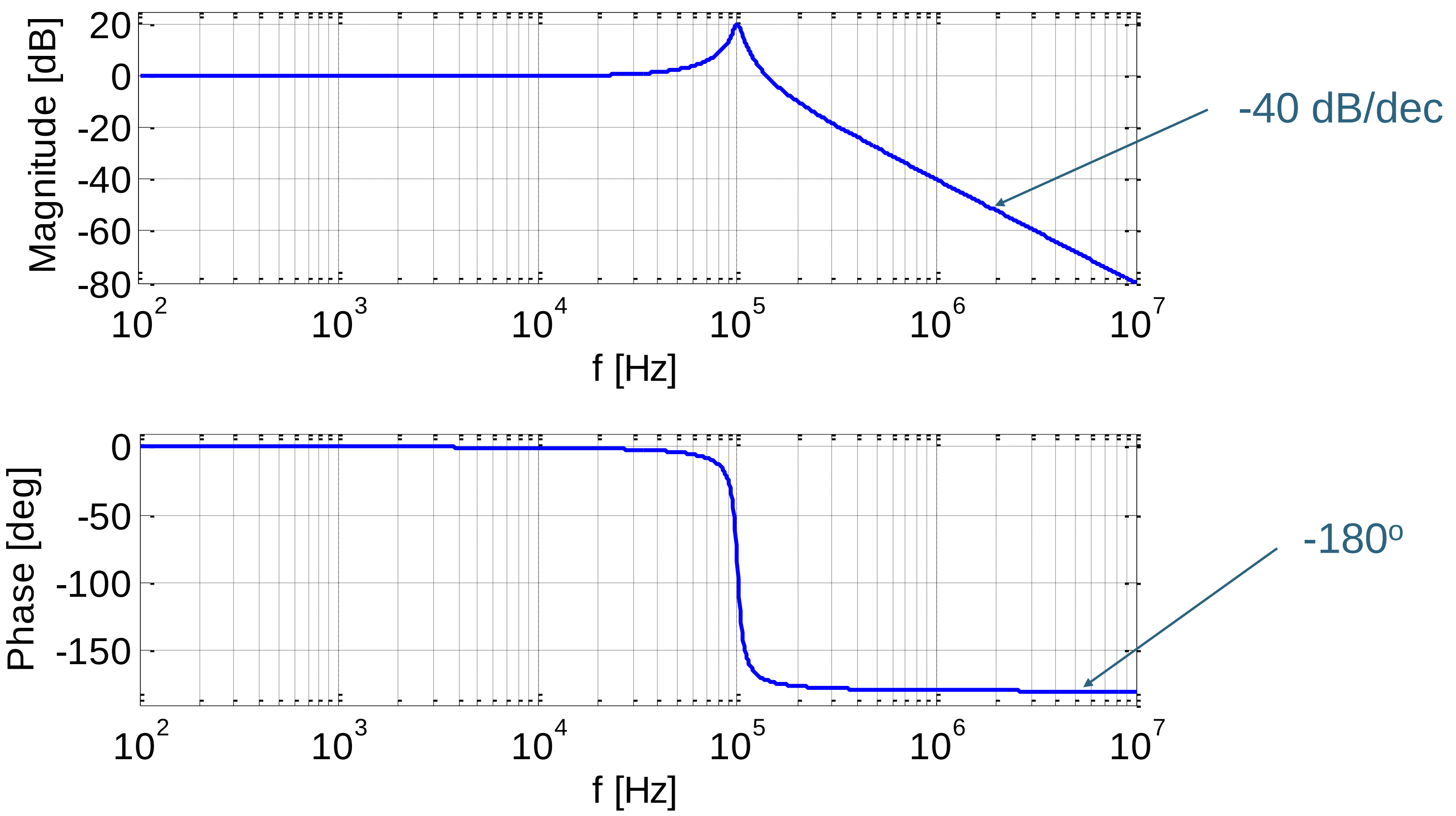
$$s_{1,2} = -\frac{\omega_P}{2Q_P}\left(1 \pm j\sqrt{4Q_P^2 - 1}\right)$$

$$r^2 = \left(\frac{\omega_P}{2Q_P}\right)^2 \left(1 + 4Q_P^2 - 1\right) = \omega_P^2$$

$$\alpha = -\frac{\omega_P}{2Q_P}$$

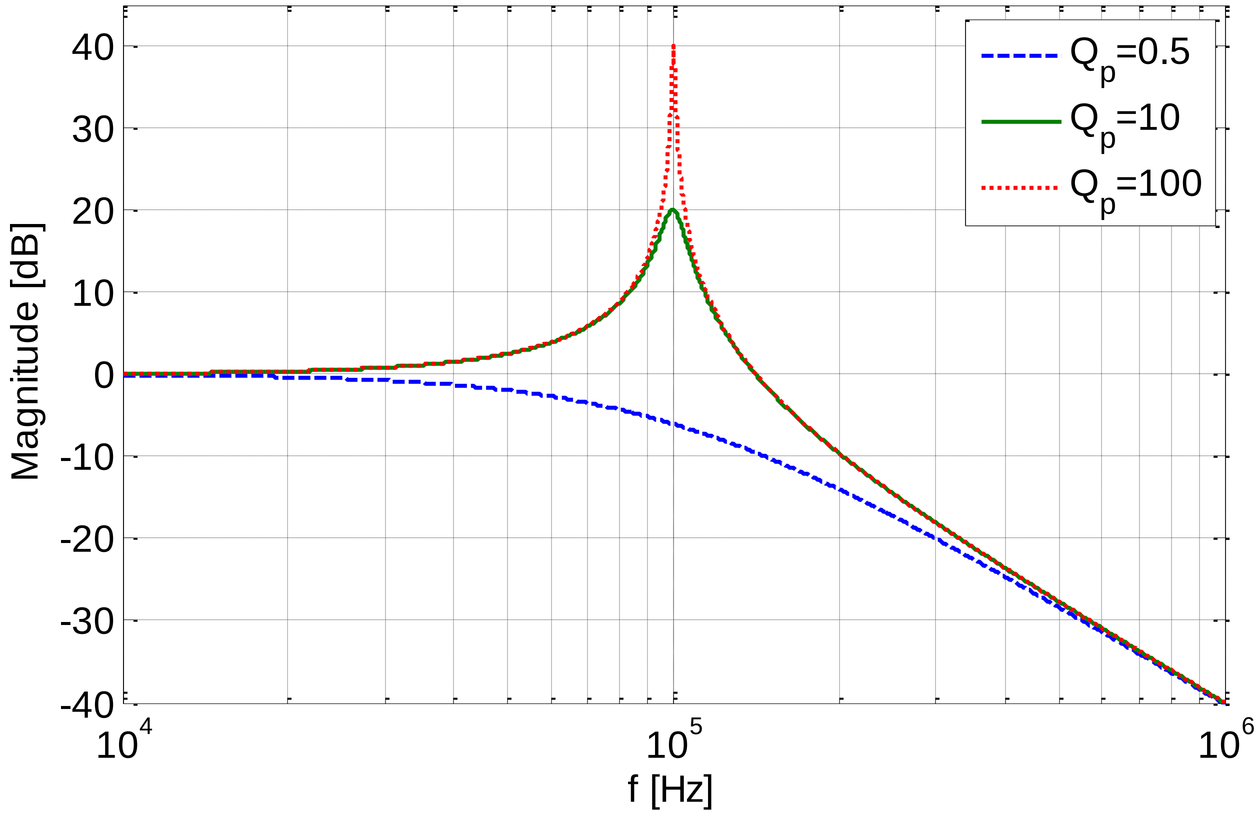
$$\psi = \cos^{-1}\left(-\frac{\alpha}{r}\right) = \cos^{-1}\left(\frac{1}{2Q_P}\right)$$

Bode Plot



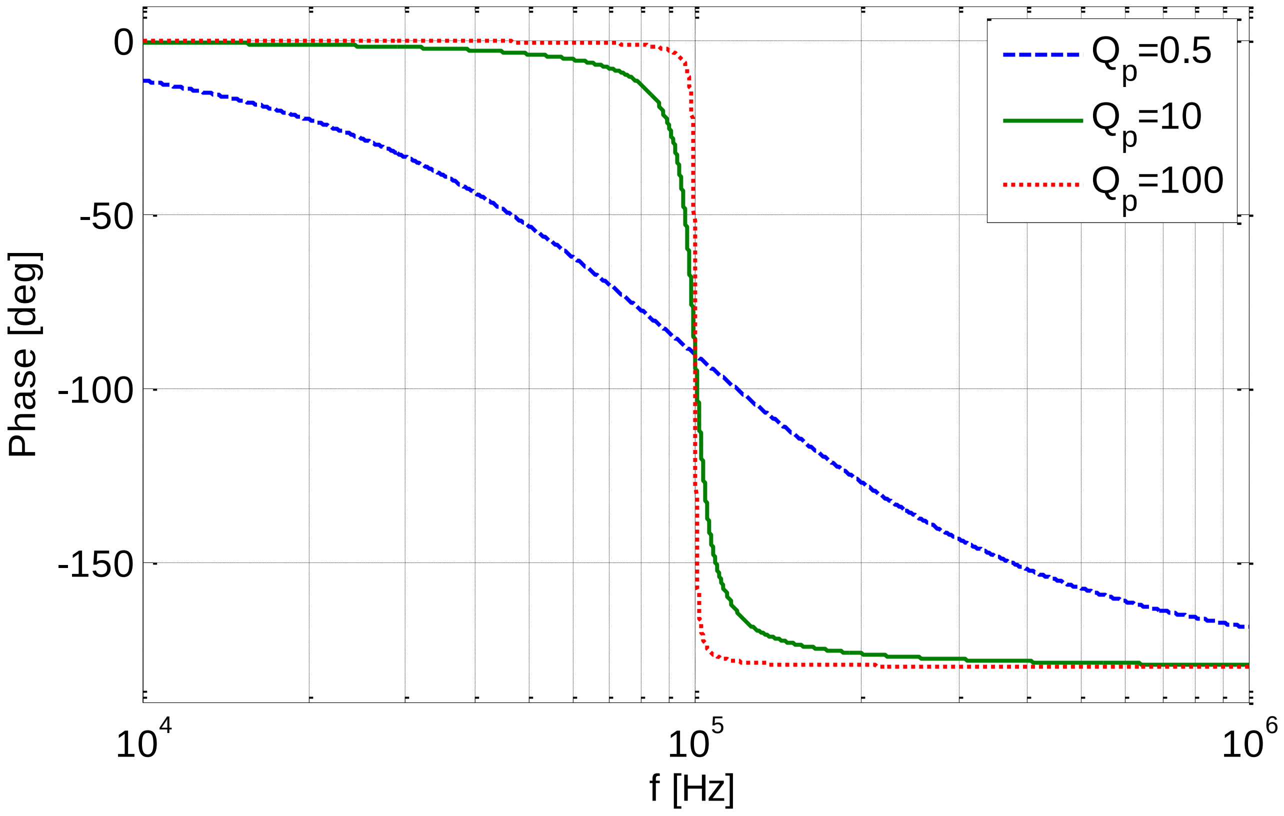
Magnitude [dB]
20
0
-20
-40
-60
-80
10^2
10^3
10^4
10^5
10^6
10^7
f [Hz]
-40 dB/dec
Phase [deg]
0
-50
-100
-150
10^2
10^3
10^4
10^5
10^6
10^7
f [Hz]
-180º

Varying Q → Magnitude



$$\left|H(j\omega)\right|_{\omega=\omega_P} = Q_P$$

Varying Q → Phase



Slope of phase at ω_p is given by -45°/decade $\cdot$ Q_p

Matlab Code

```
% Bode plot of second order lowpass
clear all;

fp = 100e3;
Qp = 10;
f = logspace(2, 7, 1000);
s = tf('s');
wp = 2*pi*fp;
h = 1 / (1 + s/wp/Qp + s^2/wp^2);

[mag, phase] = bode(h, 2*pi*f);
n = length(f);
magdb =20*log10(reshape(mag, 1, n));
angle =reshape(phase, 1, n);
```

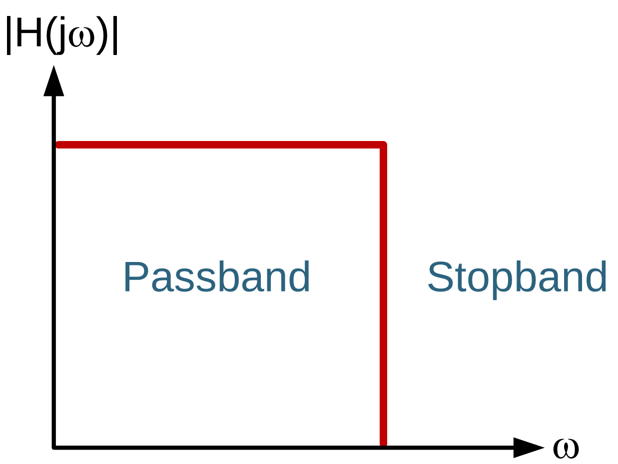
```
figure(1);
subplot(2, 1, 1)
semilogx(f, magdb, 'linewidth', 2);
set(gca, 'fontsize', 14);
xlabel('f [Hz]')
ylabel('Magnitude [dB]');
axis([min(f) max(f) -80 max(magdb)+5])
grid;

subplot(2, 1, 2)
semilogx(f, angle, 'linewidth', 2);
set(gca, 'fontsize', 14);
xlabel('f [Hz]')
ylabel('Phase [deg]');
axis([min(f) max(f) -190 10])
grid;
```

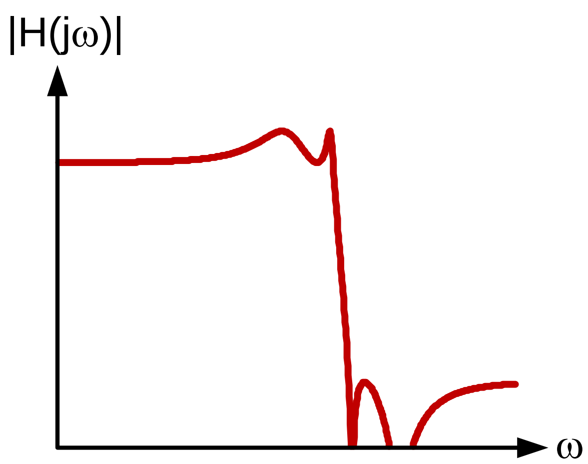
EE 240C
Analog-Digital Interface Integrated Circuits

Approximation Problem

The Filter Approximation Problem

- Ideal Filter
  - Brick-wall characteristic
  - Flat magnitude response in the passband
  - Infinite attenuation in the stopband
- Practical filter
  - Ripple in either or both the passband and stopband
  - Limited attenuation in the stopband

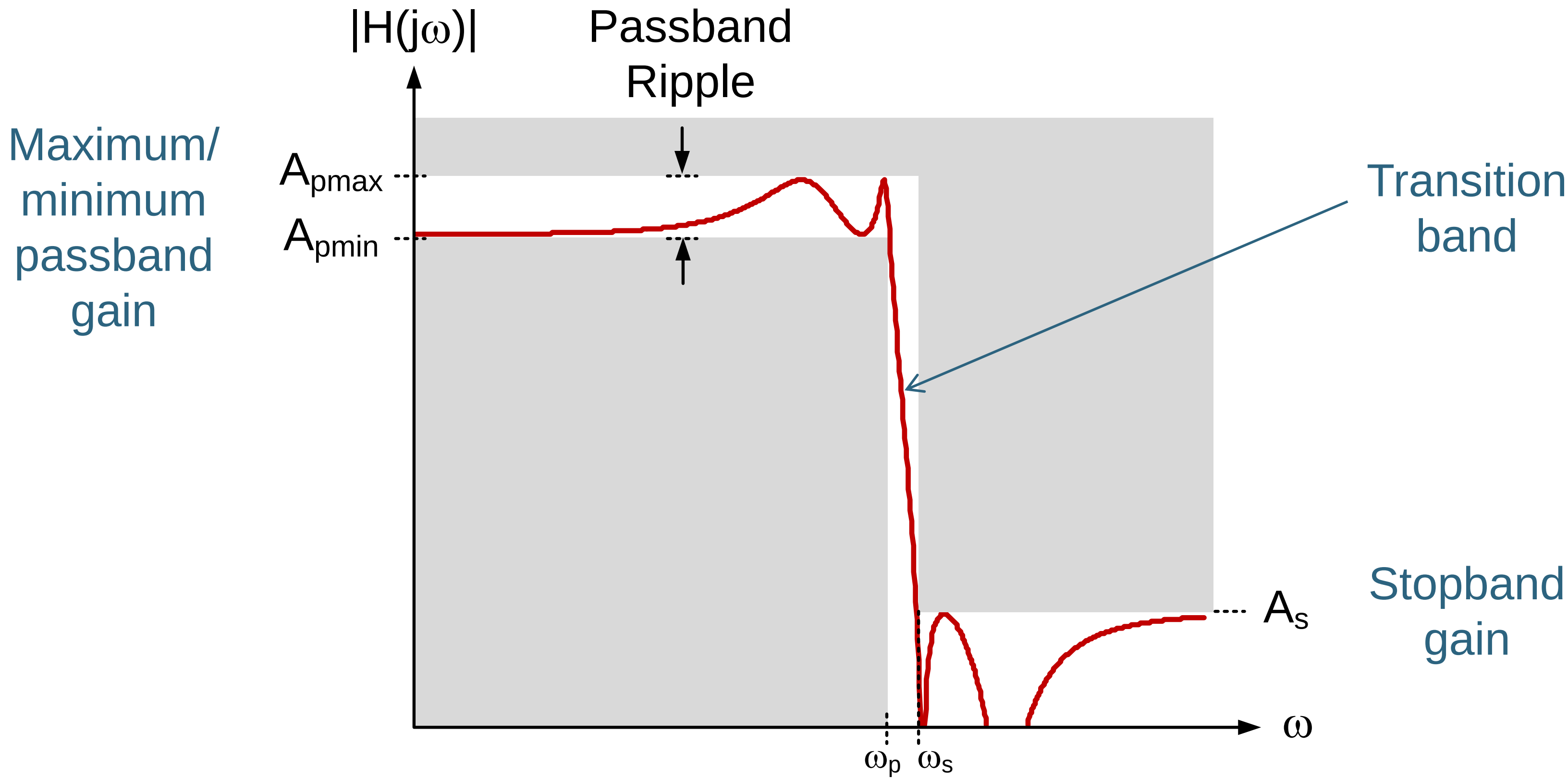




Filter Design

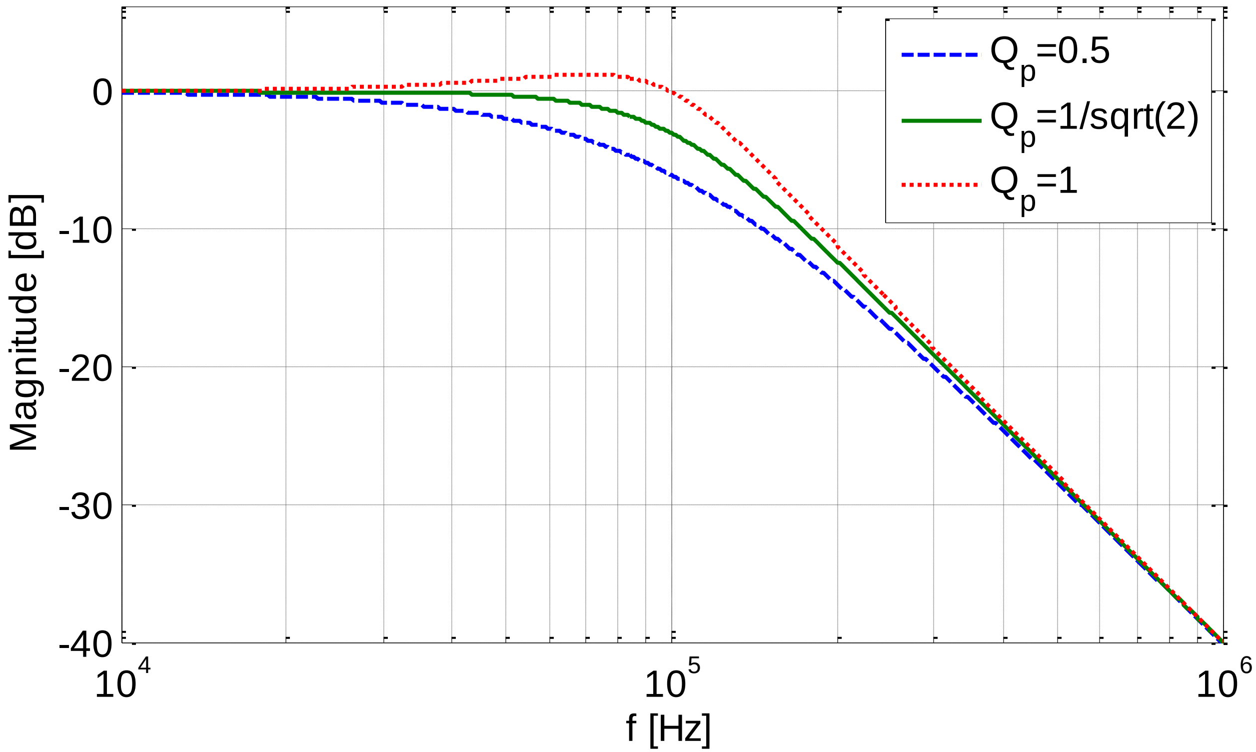
- Ideal filters are non-causal or otherwise impractical
- No global optimization techniques known
- In practice, chose from several known solutions
  - Butterworth, Elliptic, Bessel, …
- The overall goal of filter design is to approximate the ideal response by one that implements a reasonable compromise between filter complexity (number of poles and zeros) and approximation error
- Filter design, in general, requires a compromise between magnitude response, phase response, step response, complexity, etc.

Lowpass Filter Template



- Magnitude response is fully specified by A_{pmin}, A_{pmax}, A_s, ω_p, ω_s

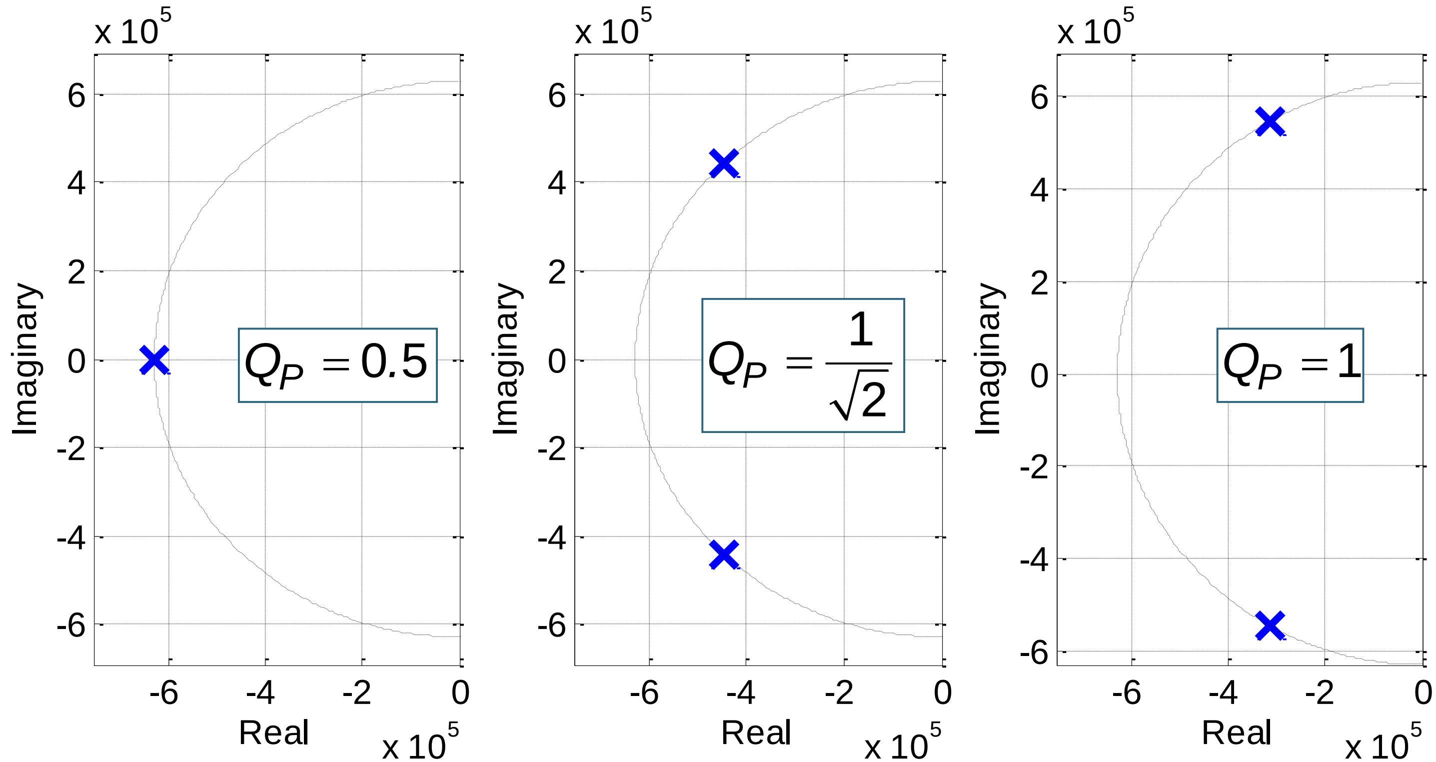
Second Order Lowpass Filter



$$H(s) = \frac{1}{1 + \frac{s}{\omega_P Q_P} + \frac{s^2}{\omega_P^2}}$$

- Magnitude response is “maximally flat” (no peaking) for $Q_P = 1/\mathrm{sqrt}(2)$

Pole Positions



$$\psi = \cos^{-1}\left(\frac{1}{2Q_P}\right) = 0^\circ \qquad \psi = \cos^{-1}\left(\frac{1}{2Q_P}\right) = 45^\circ \qquad \psi = \cos^{-1}\left(\frac{1}{2Q_P}\right) = 60^\circ$$

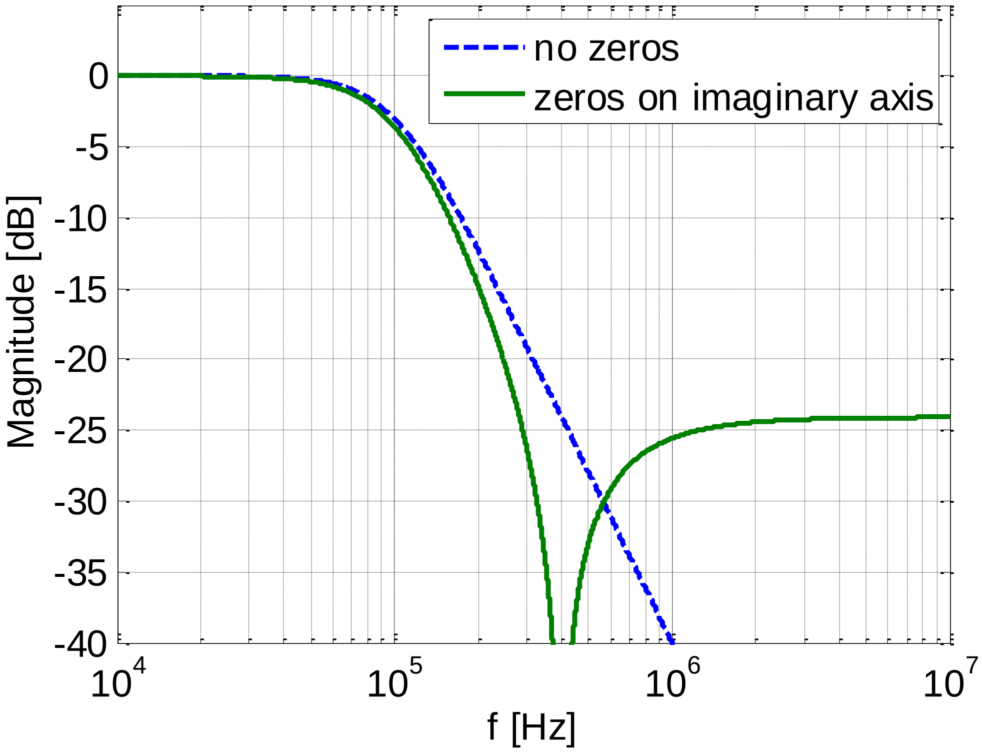
EE 240C
Analog-Digital Interface Integrated Circuits

Zeros

Improvements

- A maximally flat response is great, but how can we make the roll-off steeper?
- Let's look at
  - Imaginary zeros
  - Increasing the filter order
  - High-Q poles
  - High-Q poles and imaginary zeros

Bode Plot

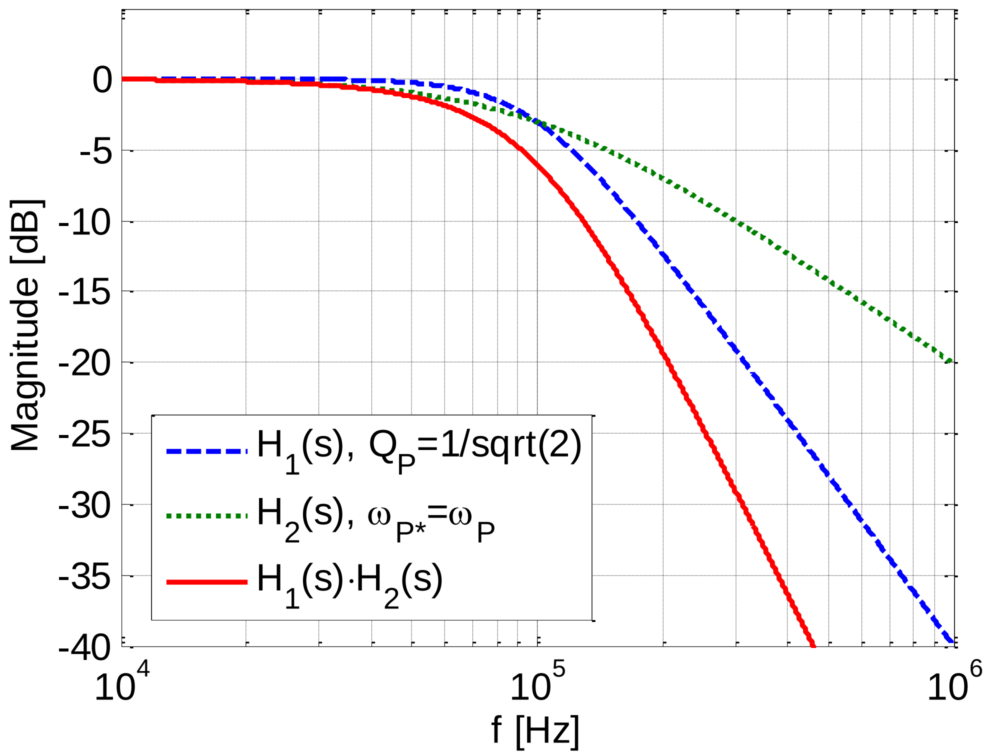


$$H(s) = \frac{1 + \left(\frac{s}{\omega_Z}\right)^2}{1 + \frac{s}{\omega_P Q_P} + \left(\frac{s}{\omega_P}\right)^2}$$

$$\left|H(j\omega)\right|_{\omega\to\infty} = \left(\frac{\omega_P}{\omega_Z}\right)^2$$

- Steeper roll-off at the expense of reduced stopband rejection

Adding Another Pole



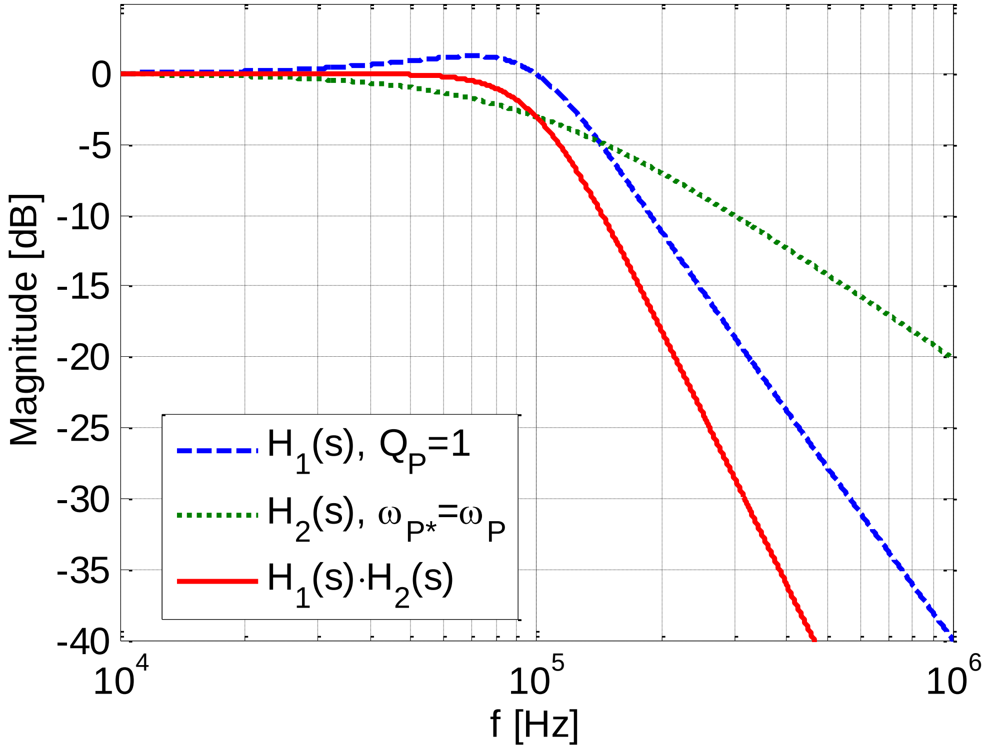
$$H(s) = H_1(s) \cdot H_2(s)$$

$$H_1(s) = \frac{1}{1 + \frac{s}{\omega_P Q_P} + \left(\frac{s}{\omega_P}\right)^2}$$

$$H_2(s) = \frac{1}{1 + \left(\frac{s}{\omega_{P*}}\right)}$$

- As expected, steeper roll-off, but transition is not all that sharp
- Can fix this issue by increasing Q of $H_1(s)$!

Utilizing Peaking in $H_1(s)$



$$H(s) = H_1(s) \cdot H_2(s)$$

$$H_1(s) = \frac{1}{1 + \frac{s}{\omega_P Q_P} + \left(\frac{s}{\omega_P}\right)^2}$$

$$H_2(s) = \frac{1}{1 + \left(\frac{s}{\omega_{P*}}\right)}$$

- Win-win improvement
  - Passband more flat, roll-off steeper

nth Order Generalization

- Stephen Butterworth showed in 1930 that the magnitude response of an nth order maximally flat lowpass filter is given by

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_P}\right)^{2n}}}$$

- This magnitude response is monotonically decreasing and satisfies

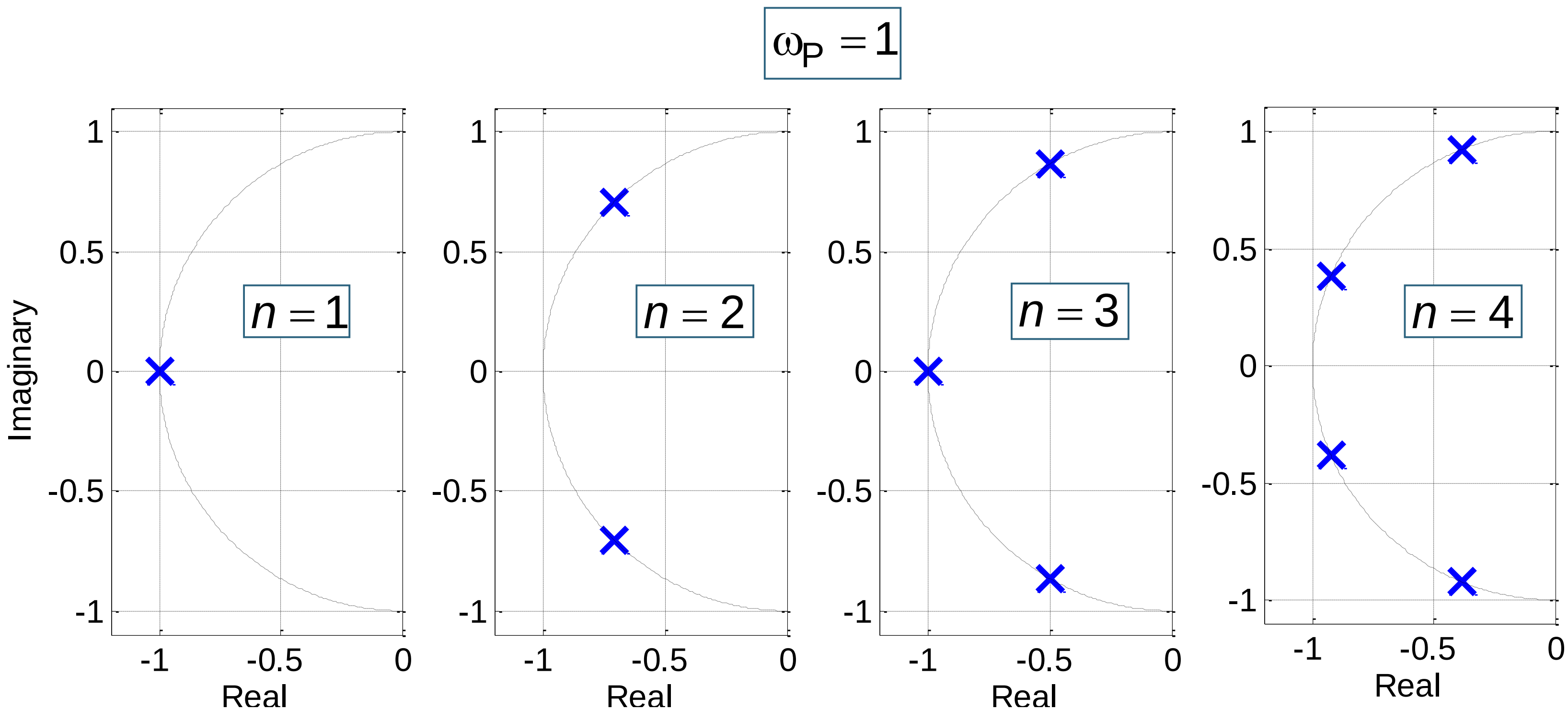
$$\left.\frac{d^k |H(j\omega)|}{d\omega^k}\right|_{\omega=0} = 0 \quad \text{for} \quad 1 \le k \le 2n-1$$

- The corresponding pole locations can be determined using

$$|H(s)|^2 = H(s)\cdot H(-s) = \frac{1}{1+\left(\frac{-s^2}{\omega_P^2}\right)^n} \qquad \frac{-s^2}{\omega_P^2} = (-1)^{1/n} = e^{\frac{j(2k-1)\pi}{n}} \qquad k = 1,2,3\ldots,n$$

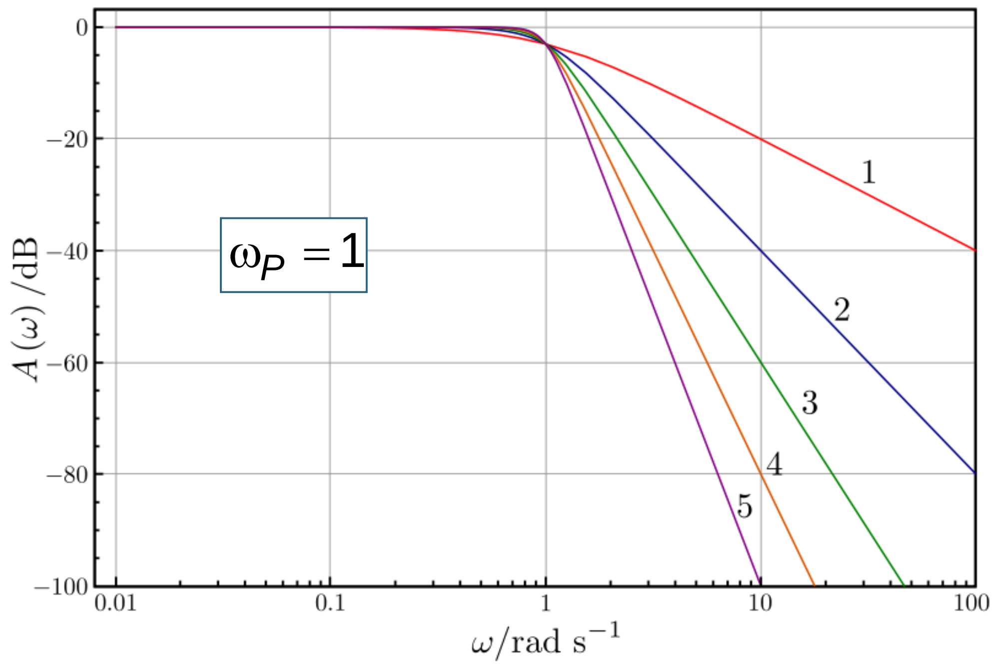
Pole Locations

- The poles lie equally spaced (in angle) on a circle in the s-plane centered at the origin with radius ω_P
- The LHP roots are taken to be the poles of H(s), while those in the RHP are regarded as the poles of H(–s)



Magnitude Response and Coefficients

http://en.wikipedia.org/wiki/Butterworth_filter

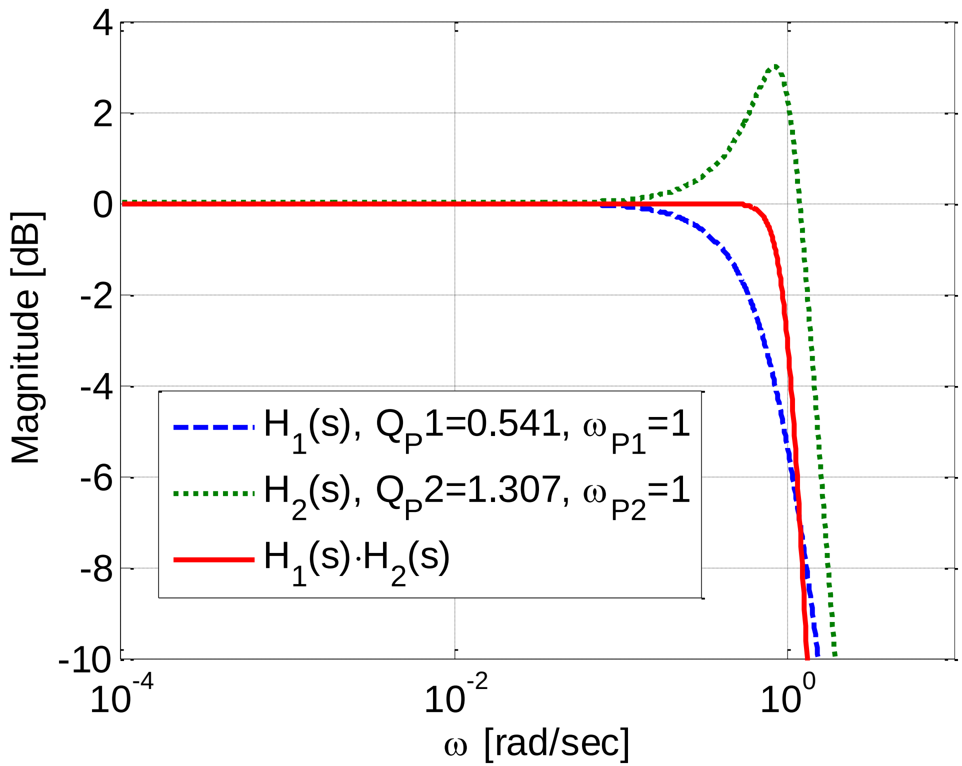


n	Denominator Polynomial
1	$(s + 1)$
2	$s^2 + 1.4142s + 1$
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.7654s + 1)(s^2 + 1.8478s + 1)$
5	$(s + 1)(s^2 + 0.6180s + 1)(s^2 + 1.6180s + 1)$
6	$(s^2 + 0.5176s + 1)(s^2 + 1.4142s + 1)(s^2 + 1.9319s + 1)$
7	$(s + 1)(s^2 + 0.4450s + 1)(s^2 + 1.2470s + 1)(s^2 + 1.8019s + 1)$
8	$(s^2 + 0.3902s + 1)(s^2 + 1.1111s + 1)(s^2 + 1.6629s + 1)(s^2 + 1.9616s + 1)$

EE 240C
Analog-Digital Interface Integrated Circuits

Passband Ripple

A Closer Look at n=4



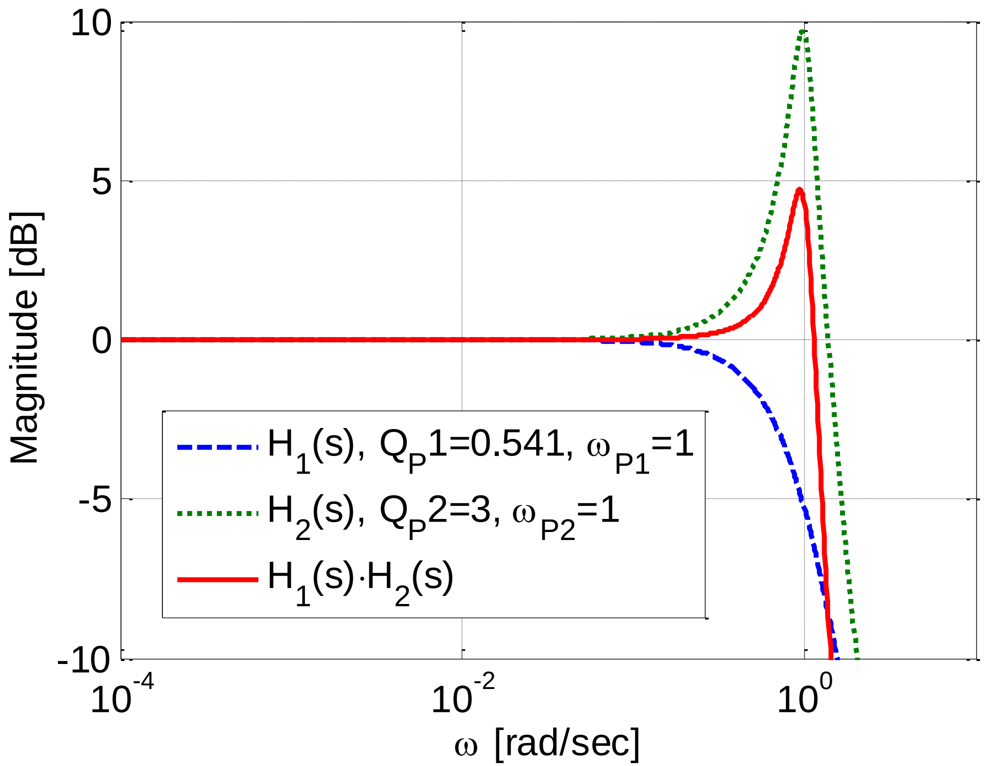
$$H(s) = H_1(s) \cdot H_2(s)$$

$$H_1(s) = \frac{1}{1 + \frac{s}{\omega_{P1} Q_{P1}} + \left(\frac{s}{\omega_{P1}}\right)^2}$$

$$H_2(s) = \frac{1}{1 + \frac{s}{\omega_{P2} Q_{P2}} + \left(\frac{s}{\omega_{P2}}\right)^2}$$

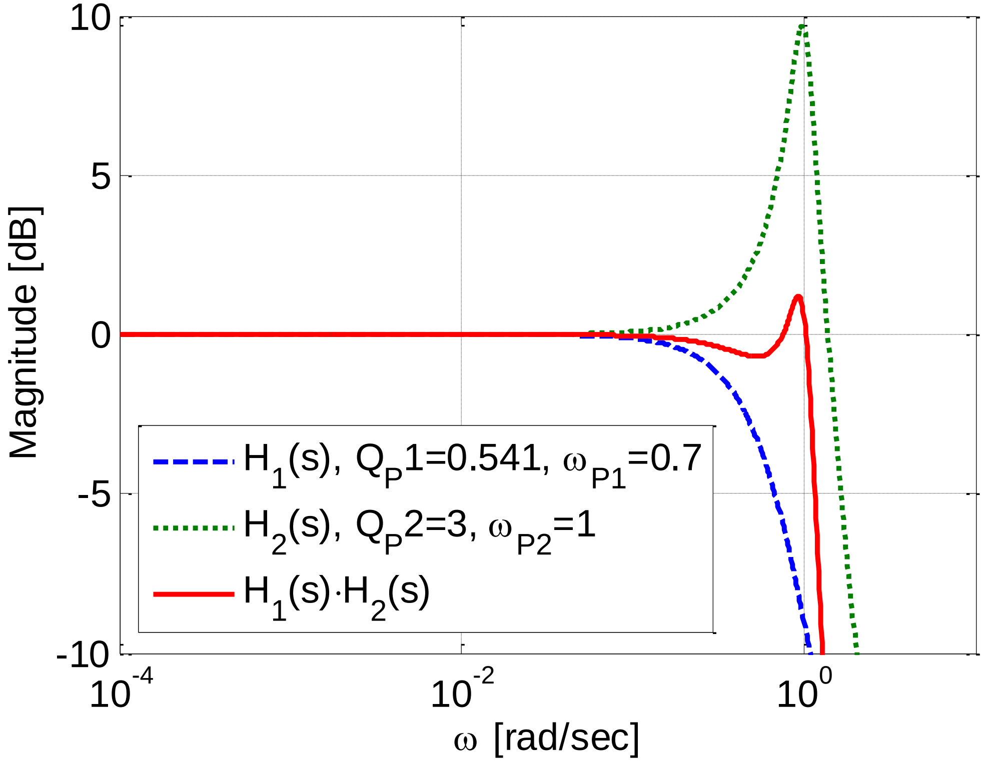
$$\psi = \cos^{-1}\left(\frac{1}{2Q_P}\right) \qquad Q_{P1} = \frac{1}{2\cos\left(22.5^\circ\right)} = 0.541 \qquad Q_{P2} = \frac{1}{2\cos\left(67.5^\circ\right)} = 1.307$$

Increasing Q_{P2}



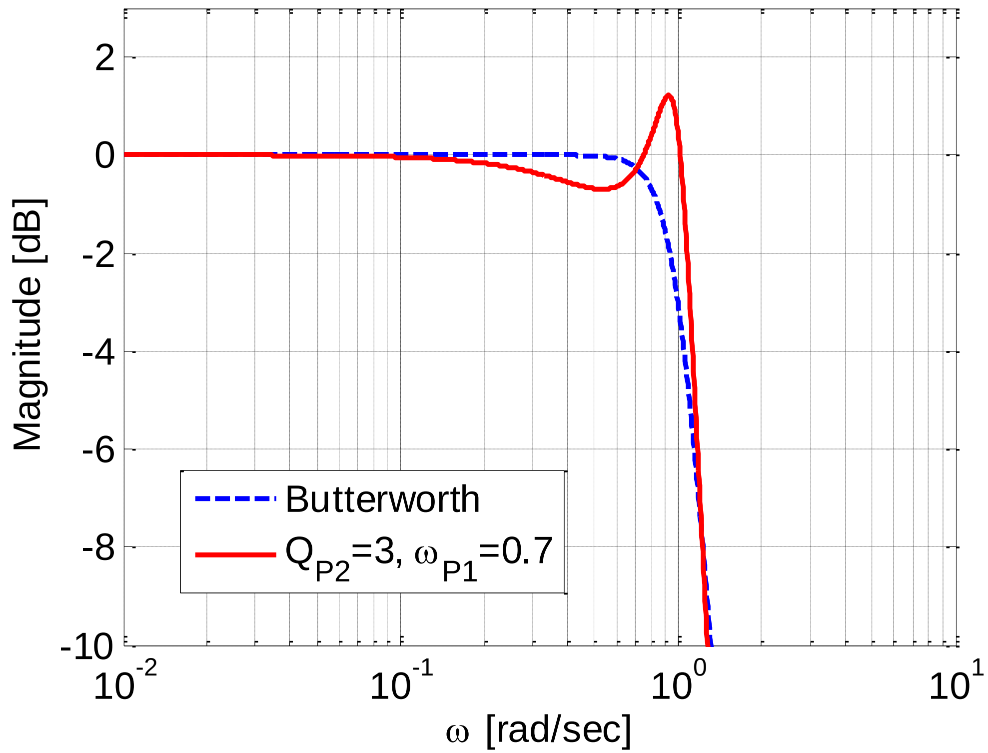
- Helps make the roll-off steeper, but introduces peaking
- We can try to alleviate this problem this by reducing ω_{P1}

Increased Q_{P2}, Reduced ω_{P1}



- This may not a bad choice of we can tolerate some peaking or ripple

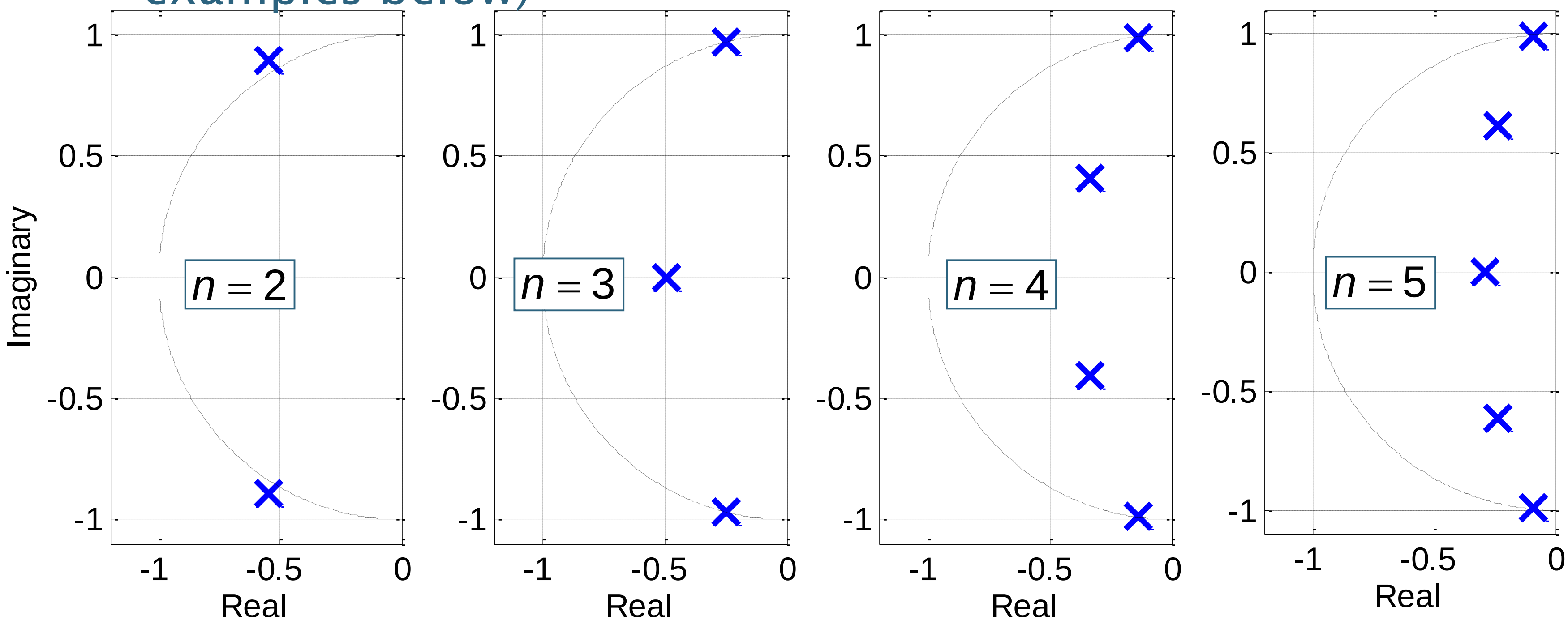
Comparison with Original Butterworth



- How can we optimize this situation, i.e. minimize the transition band for a given tolerable peaking (or "ripple") in the passband?

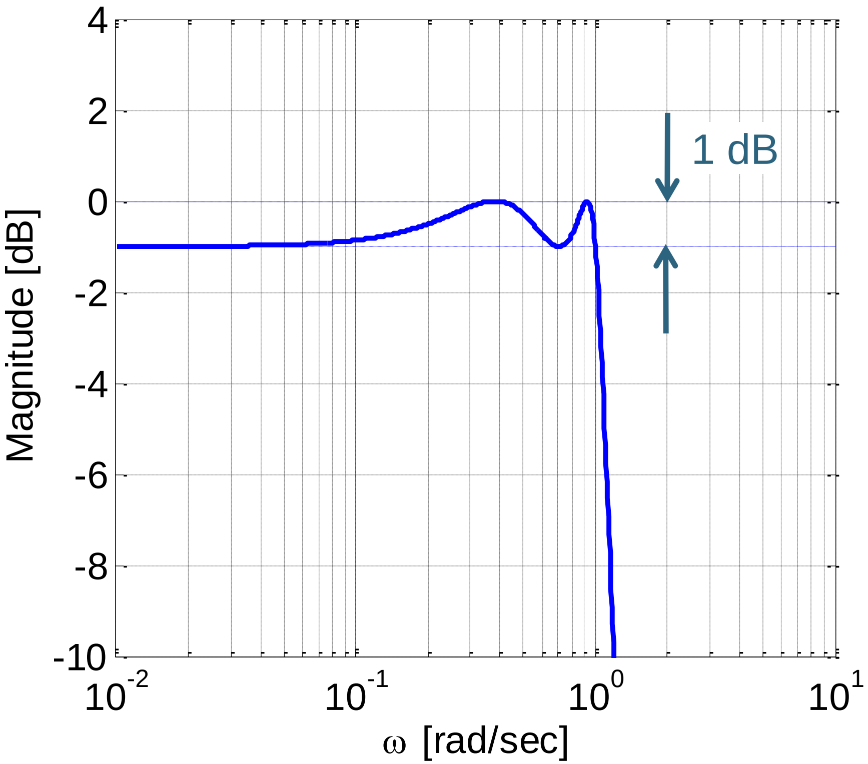
Chebyshev1 Filter Approximation

- Fortunately someone has already figure this out
- The "Chebyshev1" filter approximation minimizes the error between the idealized response and the actual filter, with the passband ripple as a parameter (1dB for examples below)



Matlab Code

```
wp = 1; % Edge of passband
R = 1;  % Passband ripple in dB
[z, p, k] = cheby1(4, R, wp, 's');
sys = zpk(z, p, k);
w = logspace(-2, 1, 1000);
[mag, phase] = bode(sys, w);
db = 20*log10(reshape(mag, 1, length(w)));

figure(1)
semilogx(w, db, 'linewidth', 2); hold on;
plot([w(1) w(end)], [0 0], '--');
plot([w(1) w(end)], [-1 -1], '--');
set(gca, 'fontsize', 14);
xlabel('\omega [rad/sec]')
ylabel('Magnitude [dB]');
axis([min(w) max(w) -10 4])
grid;
```



EE 240C
Analog-Digital Interface Integrated Circuits

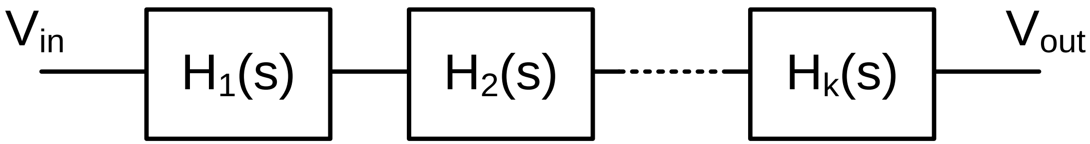
Biquad Filters

EE 240C
Analog-Digital Interface Integrated Circuits

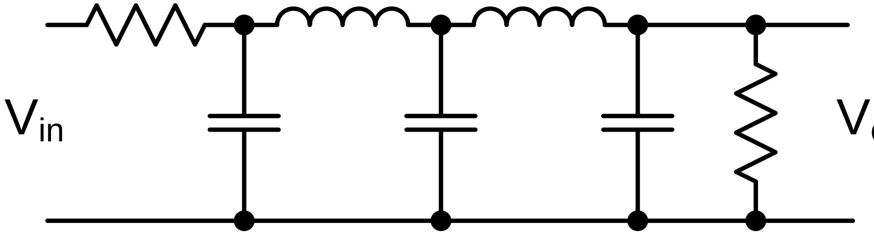
Filter Architectures

Architectural Options for High-Order Filters

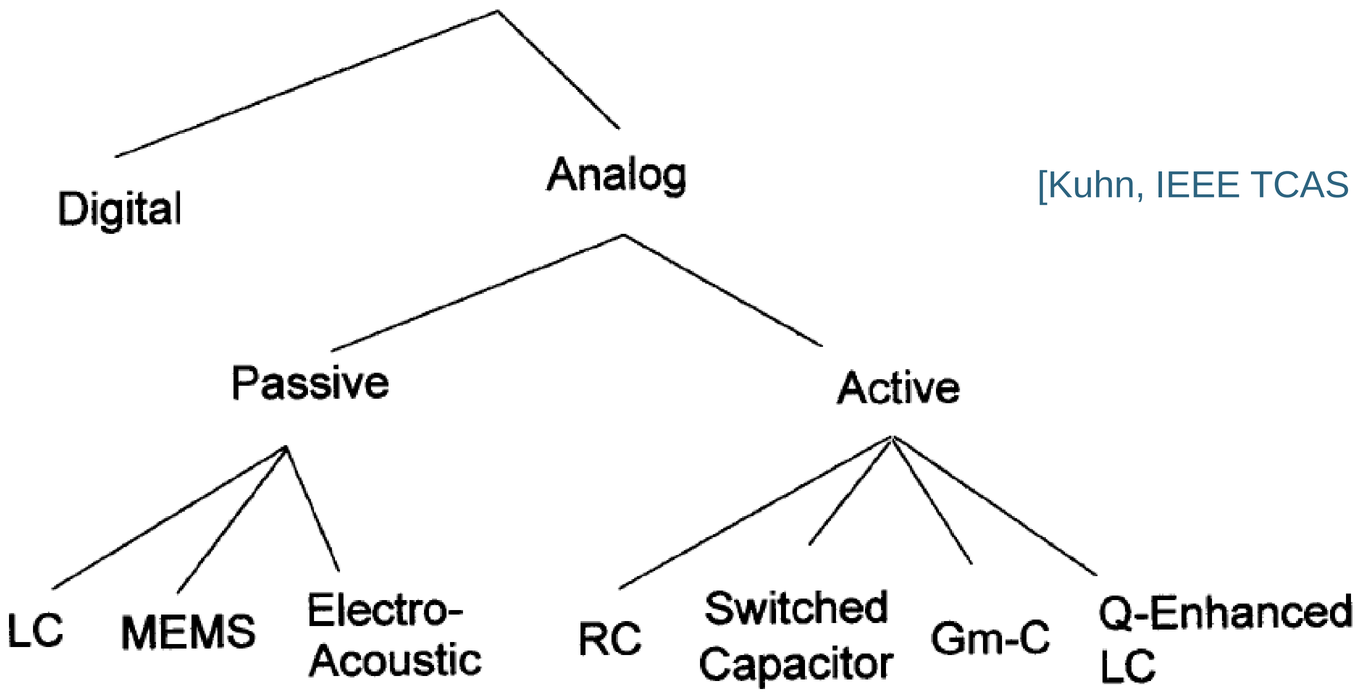
- Cascades of (active) first and second-order sections (biquads)



- Ladder filters (passive or emulated using active components)

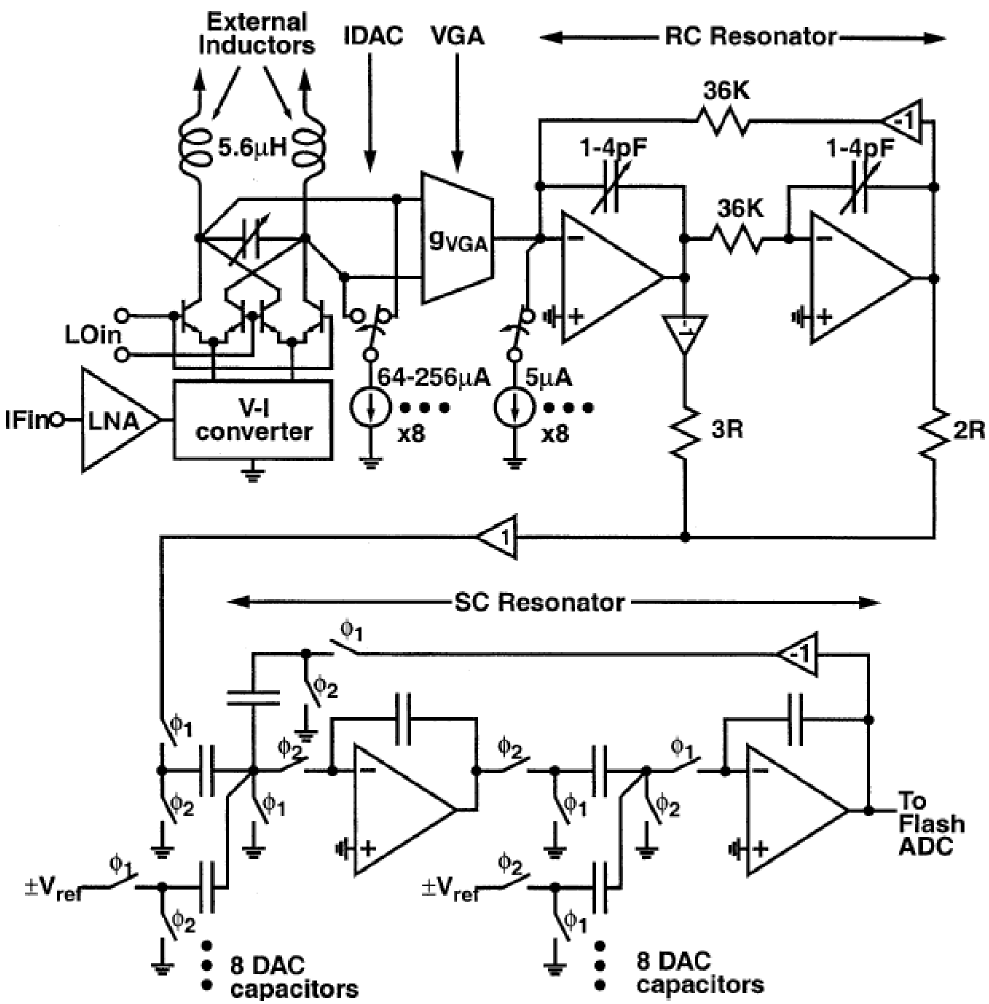


Building Block Options



[Kuhn, IEEE TCAS II, 10/2003]

Example



- An interesting filter that combines three different approaches
  - Passive LC
  - Active RC
  - Switched capacitor

[Schreier, JSSC 12/2002]

The Challenge

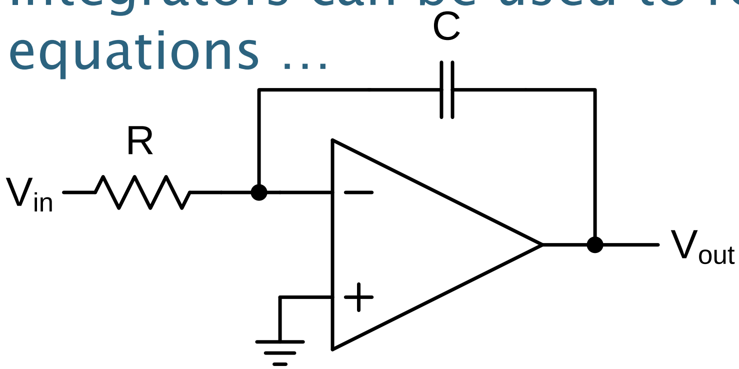
- Way <u>too many</u> options available
- Deciding on which implementation is “best” may only be possible once several options have been thoroughly compared
  - In terms of both first-order properties and second-order nonidealities, which aren’t always easy to understand
- The following discussion starts from the most basic ideas, and derives some of the most popular solutions used in practice
- For now, we will focus on the realization of biquads, and cover ladder-based implementations later
  - The treatment of biquads will help us understand why we may ultimately want to go for a ladder implementation

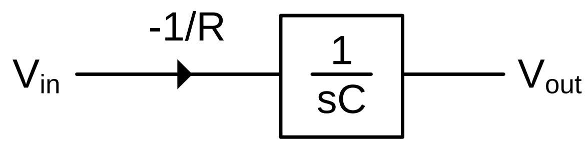
EE 240C
Analog-Digital Interface Integrated Circuits

Active Biquads

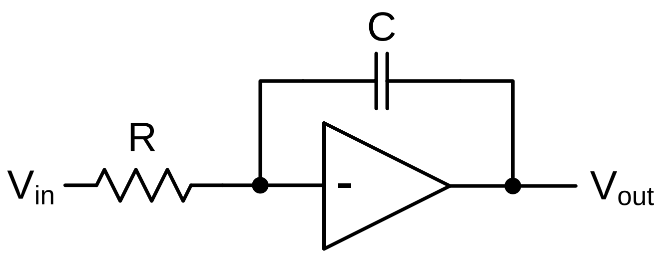
Integrators

- If we can't use inductors, can we use another component to realize filters?
- Mathematical view:
  - Filter = physical realization of (linear) differential equations
  - Integrators can be used to realize and solve differential equations …



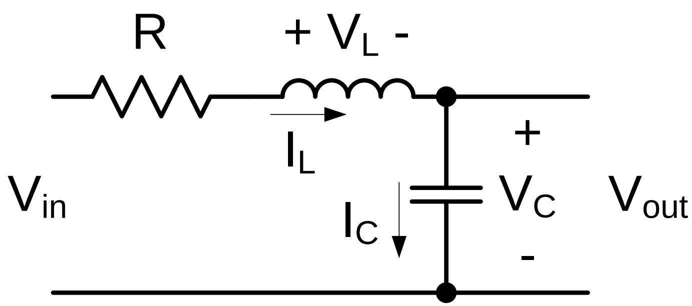


$$v_{out}(t) = -\frac{1}{C}\int \frac{v_{in}(t)}{R}\,dt$$



$$V_{out}(s) = -\frac{1}{sRC}V_{in}(s)$$

State-Space Realization



$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1 + sRC + s^2LC}$$

State variables
(integrator outputs)

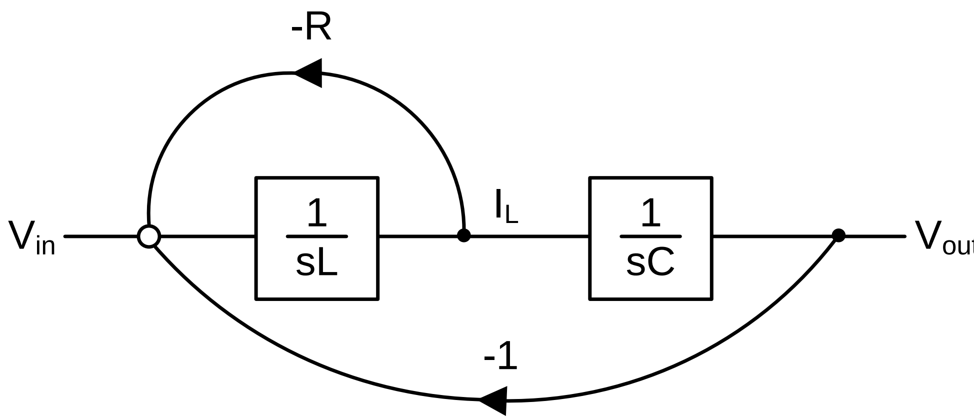
$$v_c(t) = \frac{1}{C}\int i_c(t)dt \qquad i_L(t) = \frac{1}{L}\int v_L(t)dt$$

$$V_c(s) = \frac{1}{sC} I_c(s) \qquad I_L(s) = \frac{1}{sL} V_L(s)$$

$$V_c = \frac{1}{sC} I_c = \frac{1}{sC} I_L = V_{out}$$

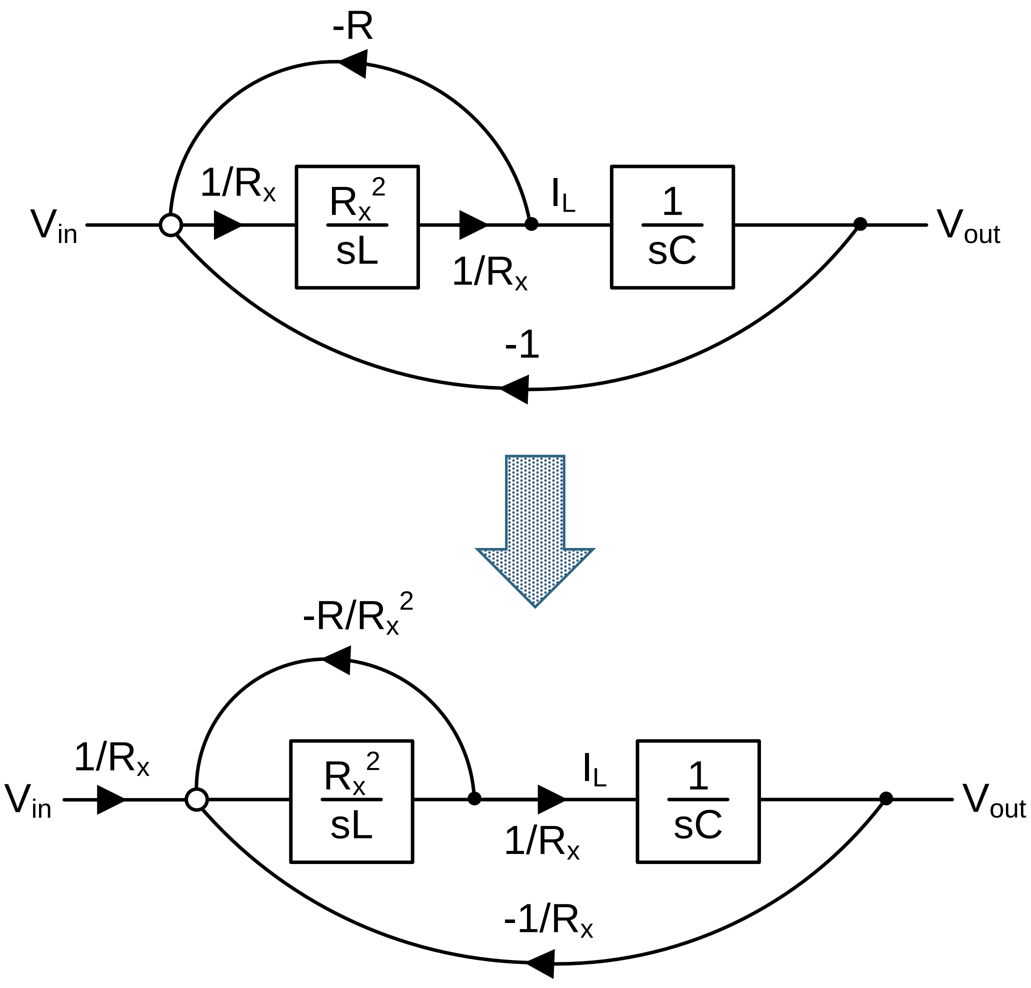
$$I_L = \frac{1}{sL} V_L = \frac{1}{sL}\left(V_{in} - I_L R - V_{out}\right)$$

Block Diagram

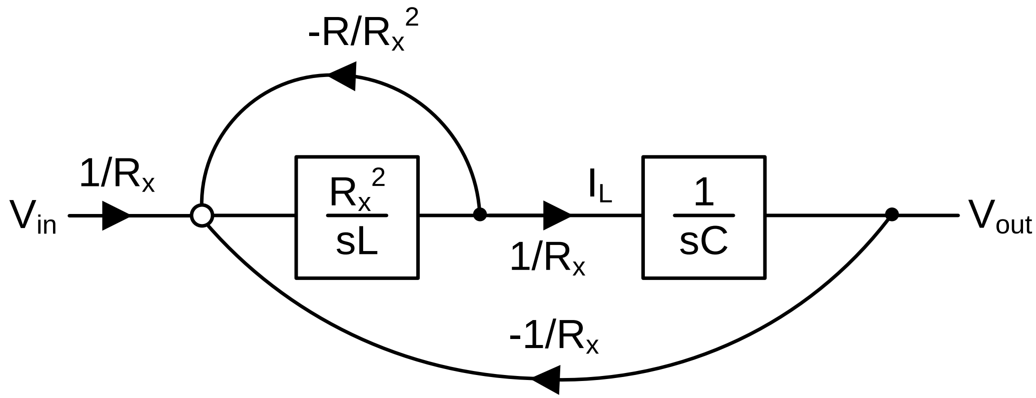


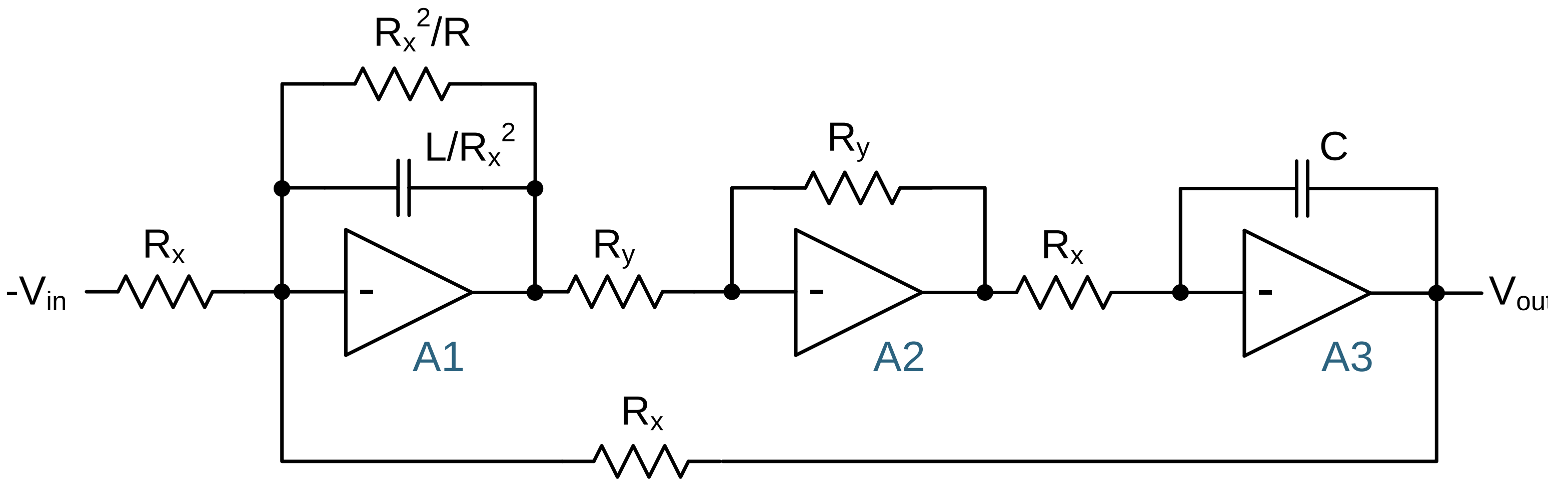
- Looks promising, but the problem with this realization is that the first integrator takes a voltage at the input and produces a current at the output
  - We need the opposite if we want to realize the circuit with an RC integrator

Modified (Equivalent) Block Diagrams



Implementation





$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{-1}{1 + sRC + s^2LC}$$

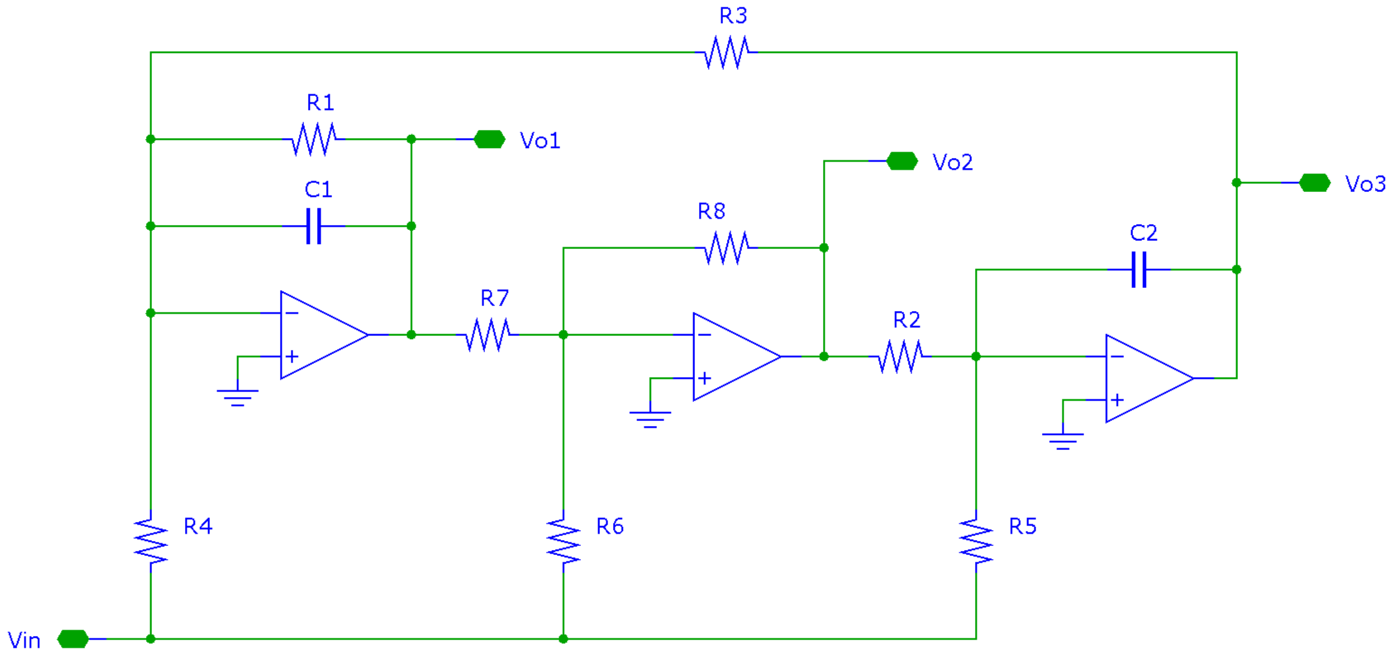
EE 240C
Analog-Digital Interface Integrated Circuits

Practical Biquads

Practical Biquads

- Textbooks list dozens of biquad realizations
  - Typically under their inventor's name
- Varying complexity & performance
  - Component spread
  - Sensitivity to component mismatch (see later)
- Two "good" representative options:
  - Tow-Thomas
    - Arbitrary zeroes
    - Low sensitivity
  - Sallen-Key
    - Single opamp
    - High component spread and/or sensitivity for high Q
    - Zeroes only at f=0 or infinity

Tow-Thomas Biquad



P. E. Fleischer and J. Tow, “Design Formulas for biquad active filters using three operational amplifiers,” Proc. IEEE, vol. 61, pp. 662-3, May 1973.

- General biquad using only three op-amps
  (2 in fully differential implementations)

Tow-Thomas Transfer Functions

$$\frac{V_{o1}}{V_{in}} = -k_2 \frac{(b_2a_1 - b_1)s + (b_2a_0 - b_0)}{s^2 + a_1s + a_0}$$

$$\frac{V_{o2}}{V_{in}} = \frac{b_2s^2 + b_1s + b_0}{s^2 + a_1s + a_0}$$

$$\frac{V_{o3}}{V_{in}} = -\frac{1}{k_1\sqrt{a_0}} \frac{(b_0 - b_2a_0)s + (a_1b_0 - a_0b_1)}{s^2 + a_1s + a_0}$$

- V_{o2}/V_{in} implements a general biquad section with arbitrary poles and zeros
- V_{o1}/V_{in} and V_{o3}/V_{in} realize the same poles but are limited to at most one finite zero

Tow-Thomas Design Equations

$$b_0 = \frac{R_8}{R_3 R_5 R_7 C_1 C_2}$$

$$b_1 = \frac{1}{R_1 C_1}\left(\frac{R_8}{R_6} - \frac{R_1 R_8}{R_4 R_7}\right)$$

$$b_2 = \frac{R_8}{R_6}$$

$$a_0 = \frac{R_8}{R_2 R_3 R_7 C_1 C_2}$$

$$a_1 = \frac{1}{R_1 C_1}$$

$$k_1 = \sqrt{\frac{R_2 R_8 C_2}{R_3 R_7 C_1}}$$

$$k_2 = \frac{R_7}{R_8}$$

For given a_i, b_i, k_i, C_1, C_2 and R_8 :

$$R_1 = \frac{1}{a_1 C_1}$$

$$R_2 = \frac{k_1}{\sqrt{a_0} C_2}$$

$$R_3 = \frac{1}{k_1 k_2} \frac{1}{\sqrt{a_0} C_1}$$

$$R_4 = \frac{1}{k_2} \frac{1}{a_1 b_2 - b_1} \frac{1}{C_1}$$

$$R_5 = \frac{k_1 \sqrt{a_0}}{b_0 C_2}$$

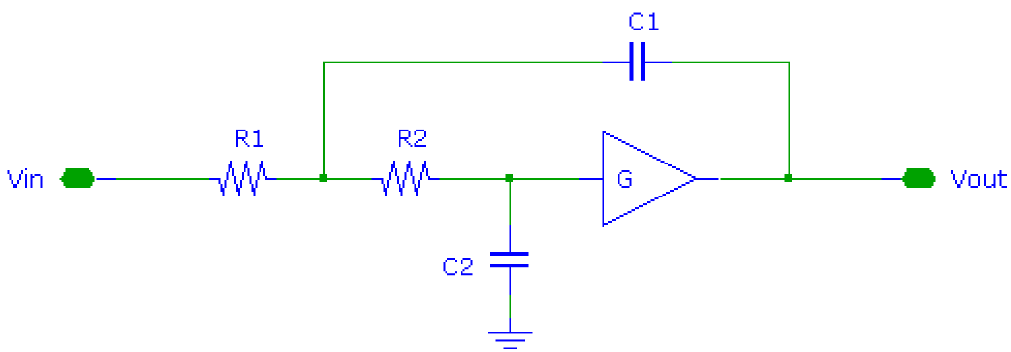
$$R_6 = \frac{R_8}{b_2}$$

$$R_7 = k_2 R_8$$

$$\omega_P = \sqrt{\frac{R_8}{R_2 R_3 R_7 C_1 C_2}}$$

$$Q_P = \omega_P R_1 C_1$$

Sallen-Key LPF



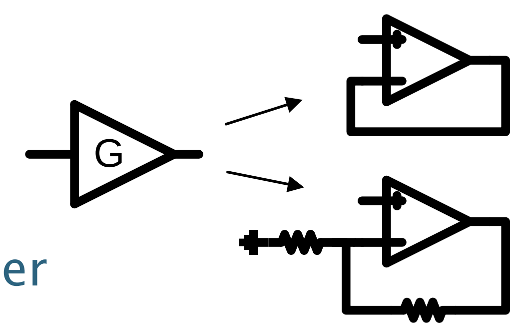
$$H(s) = \frac{G}{1 + \frac{s}{\omega_p Q_p} + \frac{s^2}{\omega_p^2}}$$

$$\omega_p = \frac{1}{\sqrt{R_1 C_1 R_2 C_2}}$$

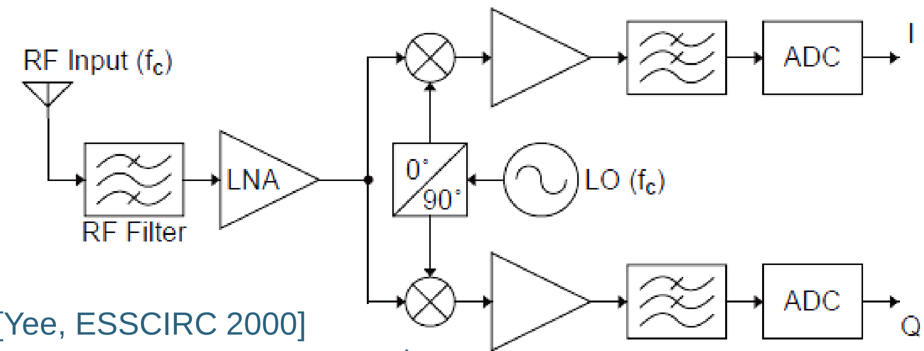
$$Q_p = \frac{\omega_p}{\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1-G}{R_2 C_2}}$$

R.P. Sallen and E. L. Key "A Practical Method of Designing RC Active Filters." *IRE Trans. Circuit Theory,* Vol. CT-2, pp. 74-85, 1955

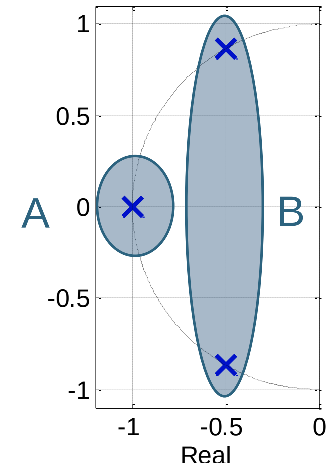
- Single gain element
  - typically $1 \leq G \leq 10$
- Poles only, no zeros
- Sensitive to parasitic capacitances
- Versions exist for HP, BP, …
  - http://en.wikipedia.org/wiki/Sallen_Key_filter

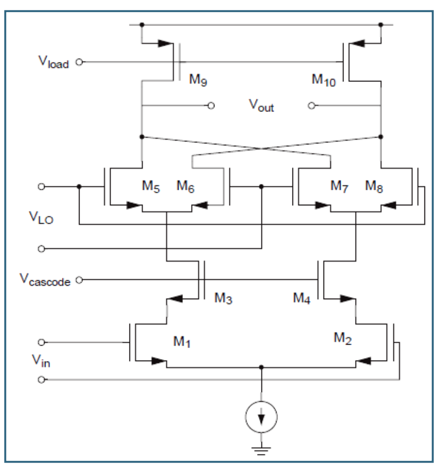


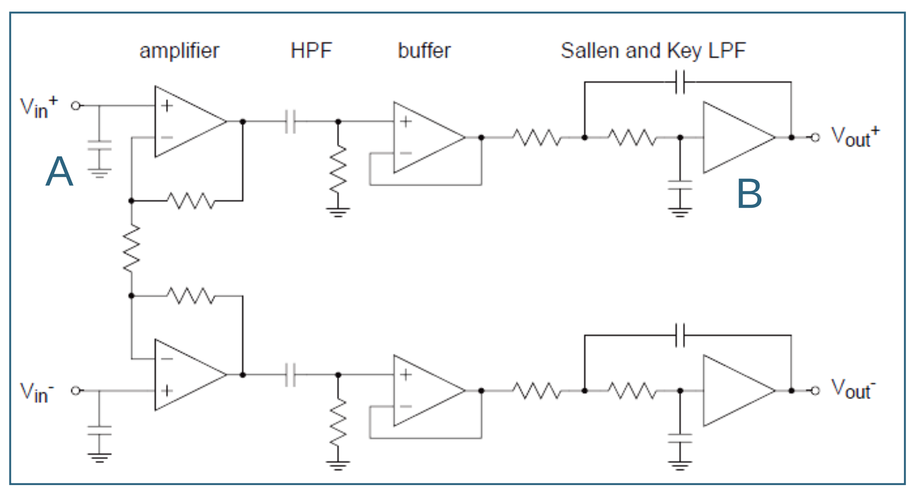
Example1: WCDMA Receiver



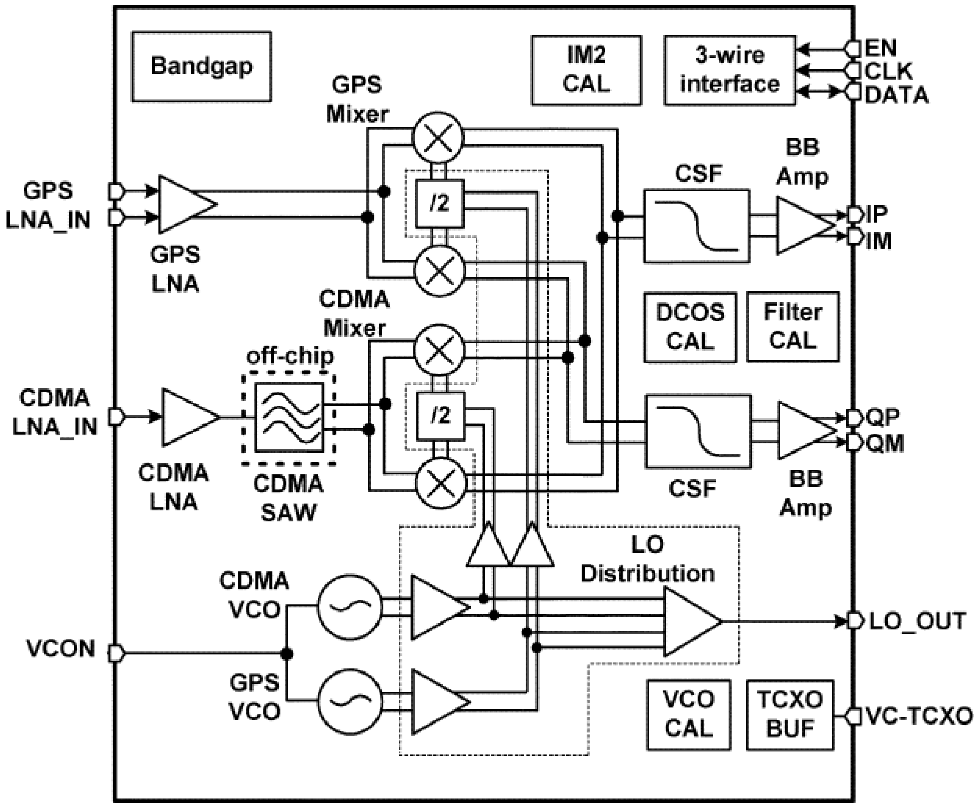
[Yee, ESSCIRC 2000]







Example 2: CDMA/GPS Receiver



Lim et al., "A Fully Integrated Direct-Conversion Receiver for CDMA and GPS Applications," IEEE JSSC, Nov. 2006

- Channel select filters (CSF)
  - 640 kHz passband, lowpass
  - 0.5 dB passband ripple
  - > 40 dB stopband attenuation at 900 kHz
- 5th order elliptical filter
- Phase distortion can be tolerated in this application

EE 240C
Analog-Digital Interface Integrated Circuits

Sensitivity

Tow-Thomas or Sallen-Key?

- Suppose we now wanted to realize an all-pole filter with biquads
- Should we use a Tow-Thomas or Sallen-Key realization?
- Clearly, from the perspective of complexity, we would probably want to go for Sallen-Key
- Unfortunately, the Sallen-Key realization comes with disadvantages in terms of sensitivity to component variations
- Let's take a closer look …

Sensitivity

- The sensitivity of any variable y to any parameter x is defined as (See e.g. Gray & Meyer, Section 4.2)

$$S_x^y = \lim_{\Delta x \to 0}\left(\frac{\Delta y / y}{\Delta x / x}\right) = \frac{x}{y}\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \frac{x}{y}\frac{\partial y}{\partial x}$$

- In order to relate fractional changes in y to fractional changes in x we can then write

$$\frac{\Delta y}{y} \cong S_x^y \frac{\Delta x}{x}$$

- Example

$$S_x^y = 10 \qquad \frac{\Delta x}{x} = 2\% \qquad \Rightarrow \frac{\Delta y}{y} \cong 20\%$$

- Common sense: sensitivity is a first order approximation, accurate only for “small” errors

Sensitivity to Mismatch (Sallen-Key)

$$\omega_P = \frac{1}{\sqrt{R_1 C_1 R_2 C_2}}$$

$$Q_P = \frac{\omega_P}{\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1-G}{R_2 C_2}}$$

$$S_{R_1}^{\omega_P} = S_{R_2}^{\omega_P} = S_{C_1}^{\omega_P} = S_{C_2}^{\omega_P} = -\frac{1}{2}$$

$$S_{R_1}^{Q_P} = -S_{R_2}^{Q_P} = -\frac{1}{2} + Q_P \sqrt{\frac{R_2 C_2}{R_1 C_1}}$$

$$S_{C_1}^{Q_P} = -S_{C_2}^{Q_P} = -\frac{1}{2} + Q_P \left(\sqrt{\frac{R_1 C_2}{R_2 C_1}} + \sqrt{\frac{R_2 C_2}{R_1 C_1}} \right)$$

$$S_G^{Q_P} = Q_P G \sqrt{\frac{R_1 C_1}{R_2 C_2}}$$

- Sensitivity depends on Q_P and "component spread" i.e. the ratios of the resistors and capacitors, respectively

Example (1)

- Want to design a Sallen-Key filter with Q_P=10
- Choice 1: All R and C are the same $\Rightarrow$ G = 3 –(1/Q_P) = 2.9
  - Very nice from the perspective of component spread, but bad for sensitivity, e.g.

$$S_{R_1}^{Q_P} = -S_{R_2}^{Q_P} = -\frac{1}{2} + Q_P = 9.5$$

- Choice 2: Reduce sensitivity by accepting large component spread
  - E.g. G=1
    - See e.g. http://www.maxim-ic.com/appnotes.cfm/an_pk/738
      - Note: The expression for S^Q_K is incorrect in the application note (R_3 and R_1 should be interchanged in this expression to match the result from the previous slide)

Example (2)

- For G=1, we have

$$Q_P = \frac{\omega_P}{\frac{1}{R_1C_1} + \frac{1}{R_2C_1}}$$

and it follows that

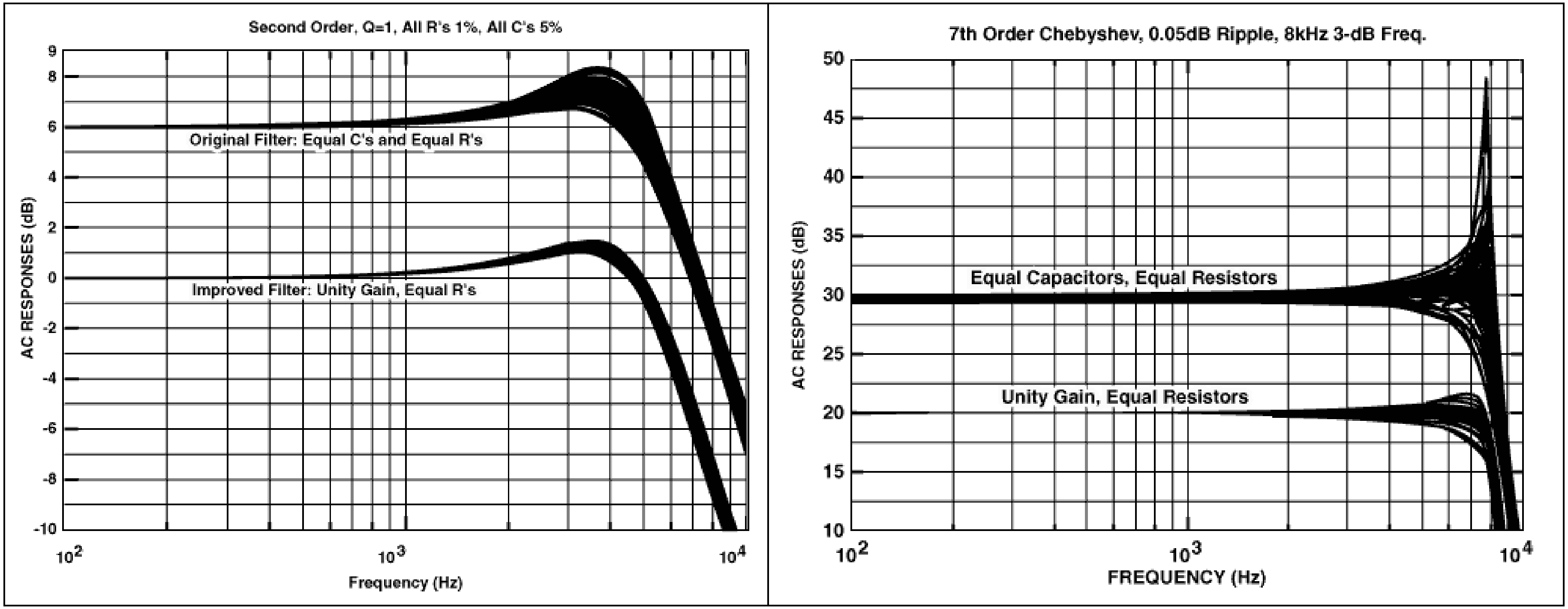
$$S_{R_1}^{Q_P} = -S_{R_2}^{Q_P} = -\frac{1}{2} + \frac{R_2}{R_1 + R_2} = 0 \qquad for \quad R_1 = R_2$$

- Unfortunately, in this case

$$\frac{C_1}{C_2} = 4Q_P^2 = 400 \qquad for \quad Q_P = 10$$

- Bottom line: The Sallen-Key realization suffers from a strong tradeoff between sensitivity and component spread, especially for high Q_P

Case Studies



MAXIM APPLICATION NOTE 738
Minimizing Component-Variation Sensitivity in Single Op Amp Filters
http://www.maxim-ic.com/appnotes.cfm/an_pk/738/

Sensitivity to Mismatch (Tow-Thomas)

$$\omega_P = \sqrt{\frac{R_8}{R_2 R_3 R_7 C_1 C_2}}$$

$$S_{R2}^{\omega_P} = S_{R3}^{\omega_P} = S_{R3}^{\omega_P} = -S_{R8}^{\omega_P} = S_{C1}^{\omega_P} = S_{C2}^{\omega_P} = -\frac{1}{2}$$

$$S_{R_1}^{Q_P} = 1$$

$$Q_P = \omega_P R_1 C_1 = R_1 \sqrt{\frac{R_8 C_1}{R_2 R_3 R_7 C_2}}$$

$$S_{R_2}^{Q_P} = S_{R_3}^{Q_P} = S_{R_7}^{Q_P} = -S_{R_8}^{Q_P} = -S_{C_1}^{Q_P} = S_{C_2}^{Q_P} = -\frac{1}{2}$$

- Constant sensitivities, independent of Q_P and component spread
  - Big improvement over Sallen-Key

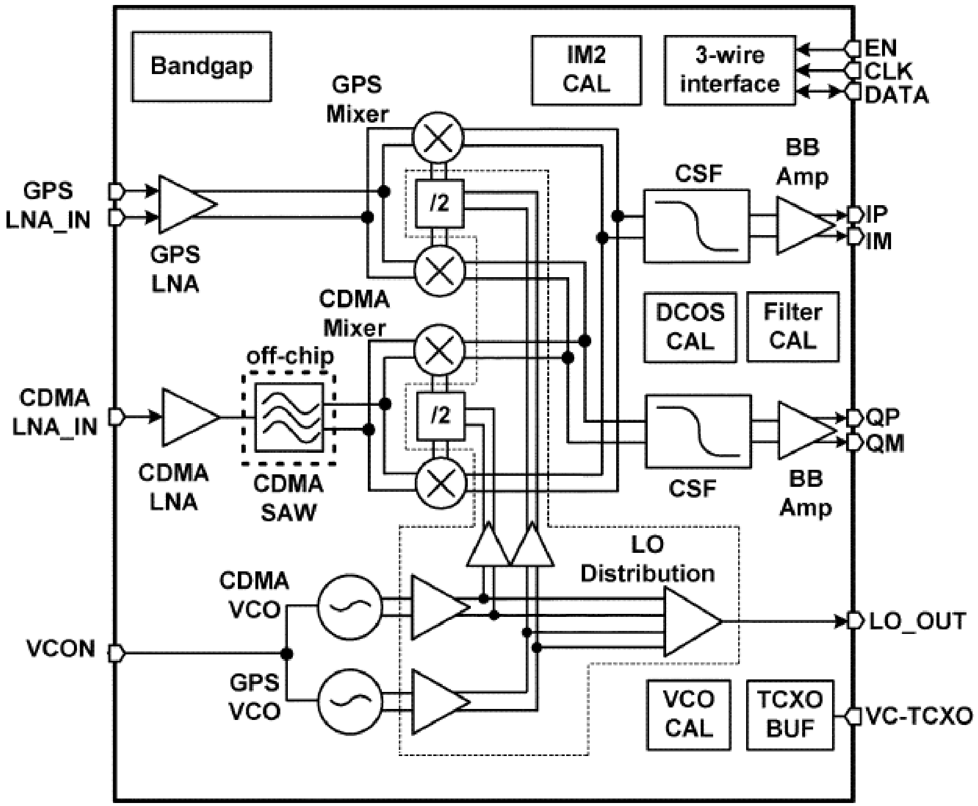
Conclusions

- Biquads can be realized in numerous different ways
- Implementation and component sizing have a big impact on sensitivity to variations
  - Of course, we must avoid high-sensitivity circuits in practice
- No theory for finding a low-sensitivity architecture
  - Use proven circuits & check!
- Tow-Thomas biquad
  - Arbitrary poles and zeros, three amplifiers
  - Well-behaved sensitivities
- Sallen-Key biquad
  - No arbitrary zeroes, one amplifier
  - Sensitivities trade off with component spread
    - Typically use G=1 and use this circuit only for “low Q” poles

EE 240C
Analog-Digital Interface Integrated Circuits

Biquad Filter Design Flow
Synthesis

Example: CDMA/GPS Receiver

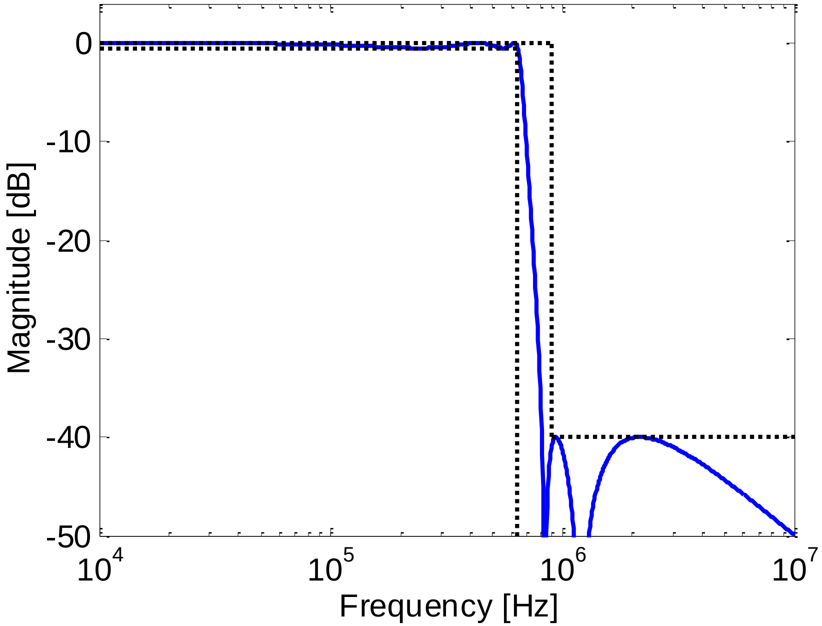


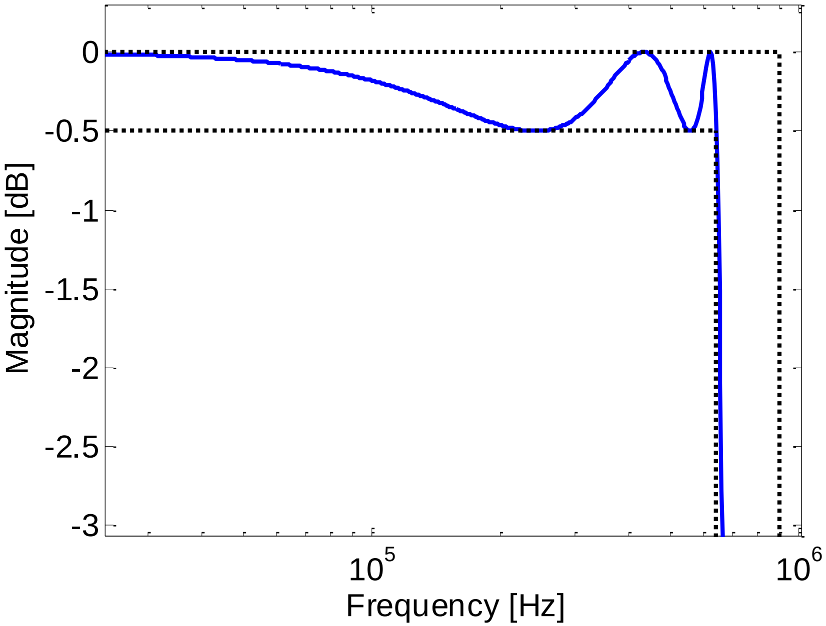
Lim et al., "A Fully Integrated Direct-Conversion Receiver for CDMA and GPS Applications," IEEE JSSC, Nov. 2006

- Channel select filter (CSF)
  - 640 kHz passband, lowpass
  - 0.5 dB passband ripple
  - > 40 dB stopband attenuation at 900 kHz
- 5th order elliptical filter

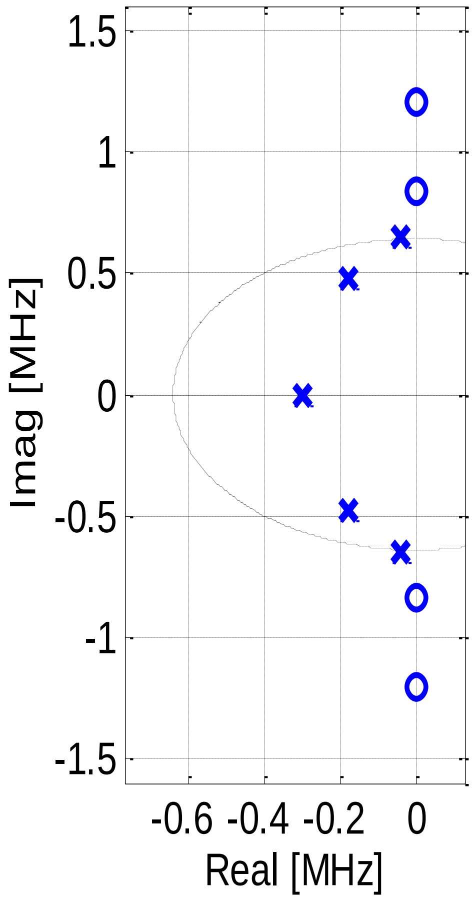
Matlab Synthesis

```
>> [b,a] = ellip(5, 0.5, 40, 2*pi*640e3, 's');
>> H = zpk(tf(b, a))

Zero/pole/gain:
                204155.1855 (s^2 + 2.786e013) (s^2 + 5.715e013)
--------------------------------------------------------------------------
(s+1.89e006) (s^2 + 2.217e006s + 1.034e013) (s^2 + 5.315e005s + 1.664e013)
```





Pole and Zero Locations

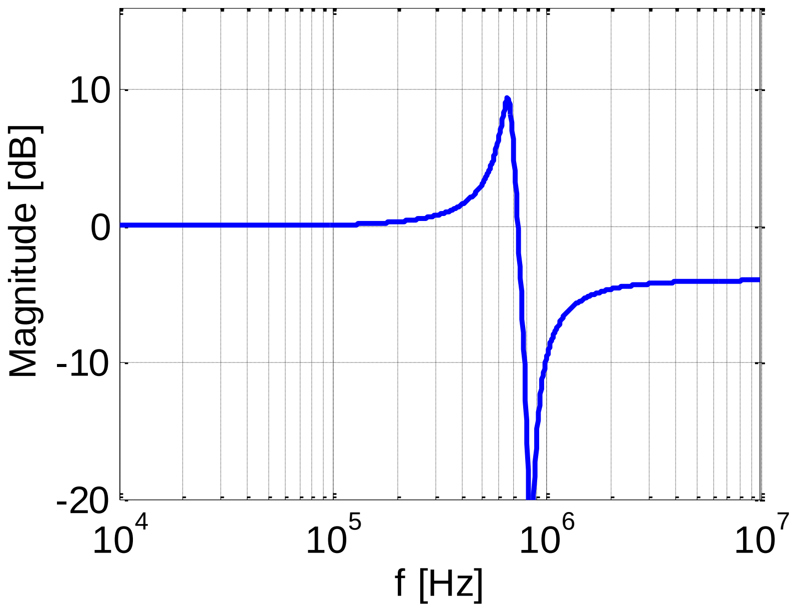


		ω_P	Q_P
$p_{1,2}$	−42.30 ± j6.4783 kHz	649.21 kHz	7.6748
$p_{3,4}$	−176.45 ± j4.8030 kHz	511.68 kHz	1.4499
p_5	−300.80 kHz		
$z_{1,2}$	± j1203.2 kHz		
$z_{3,4}$	± j840.1 kHz		

Pairing Options for $p_{1,2}$ (High-Q)

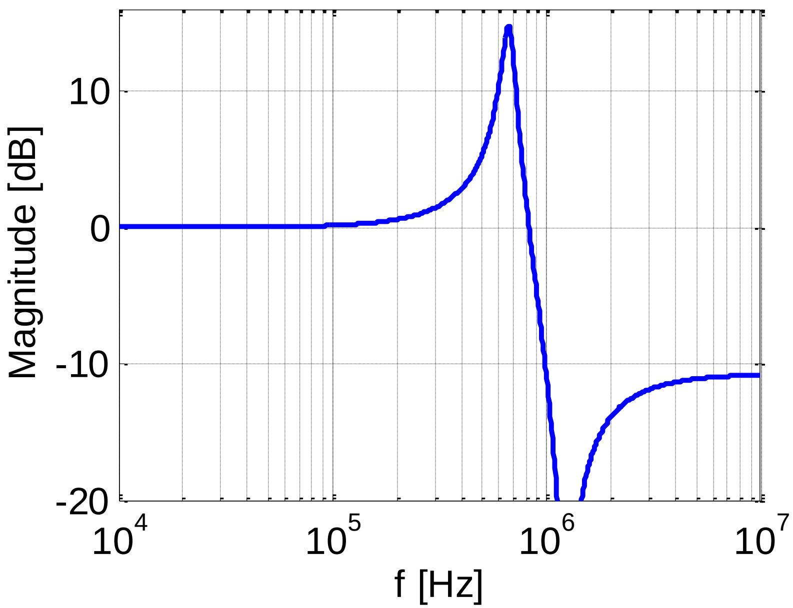
- Pairing with nearby zero

```
          (s^2/2.786e013 + 1)
------------------------------------
(s^2/1.664e013 + s/3.1308e+007 + 1)
```



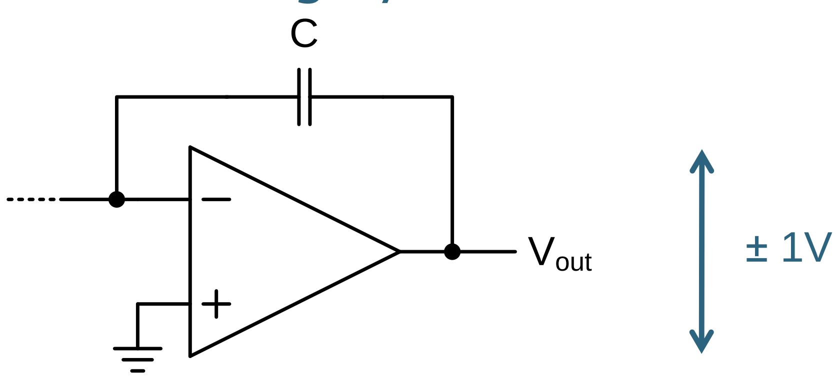
- Pairing with remote zero

```
          (s^2/5.715e013 + 1)
------------------------------------
(s^2/1.664e013 + s/3.1308e+007 + 1)
```

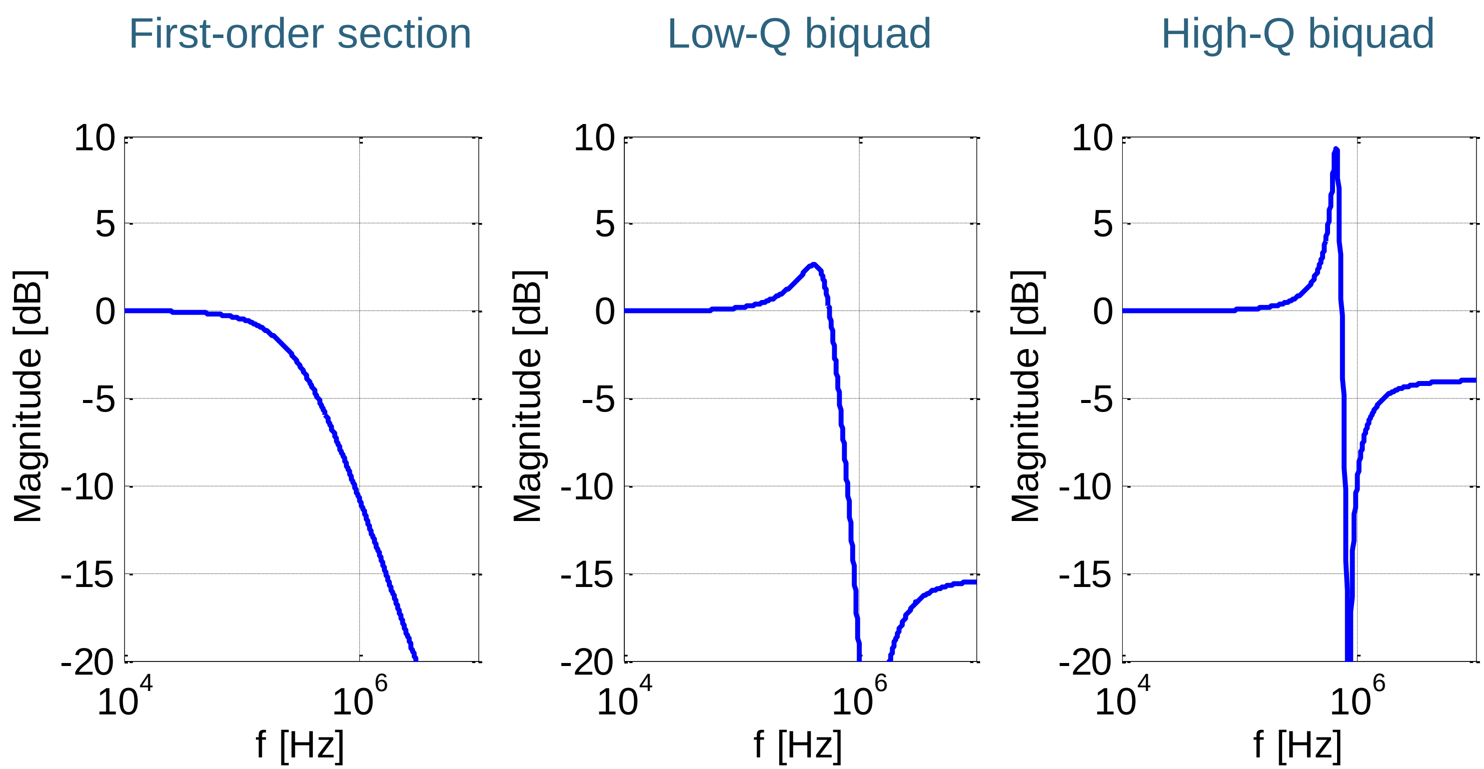


Pole-Zero Pairing

- Pairing high-Q poles with nearby zeros is desirable from a dynamic range perspective
  - Say that the amplifier at the output of the biquad can handle a maximum signal of 1 V_{peak}
  - If the biquad response peaks 20 dB above unity, this means that we can only process inputs with 100 mV amplitude near the frequency of the peak (which lies in the passband)
  - The signal is therefore reduced relative to the thermal noise of the circuit, which is highly undesirable

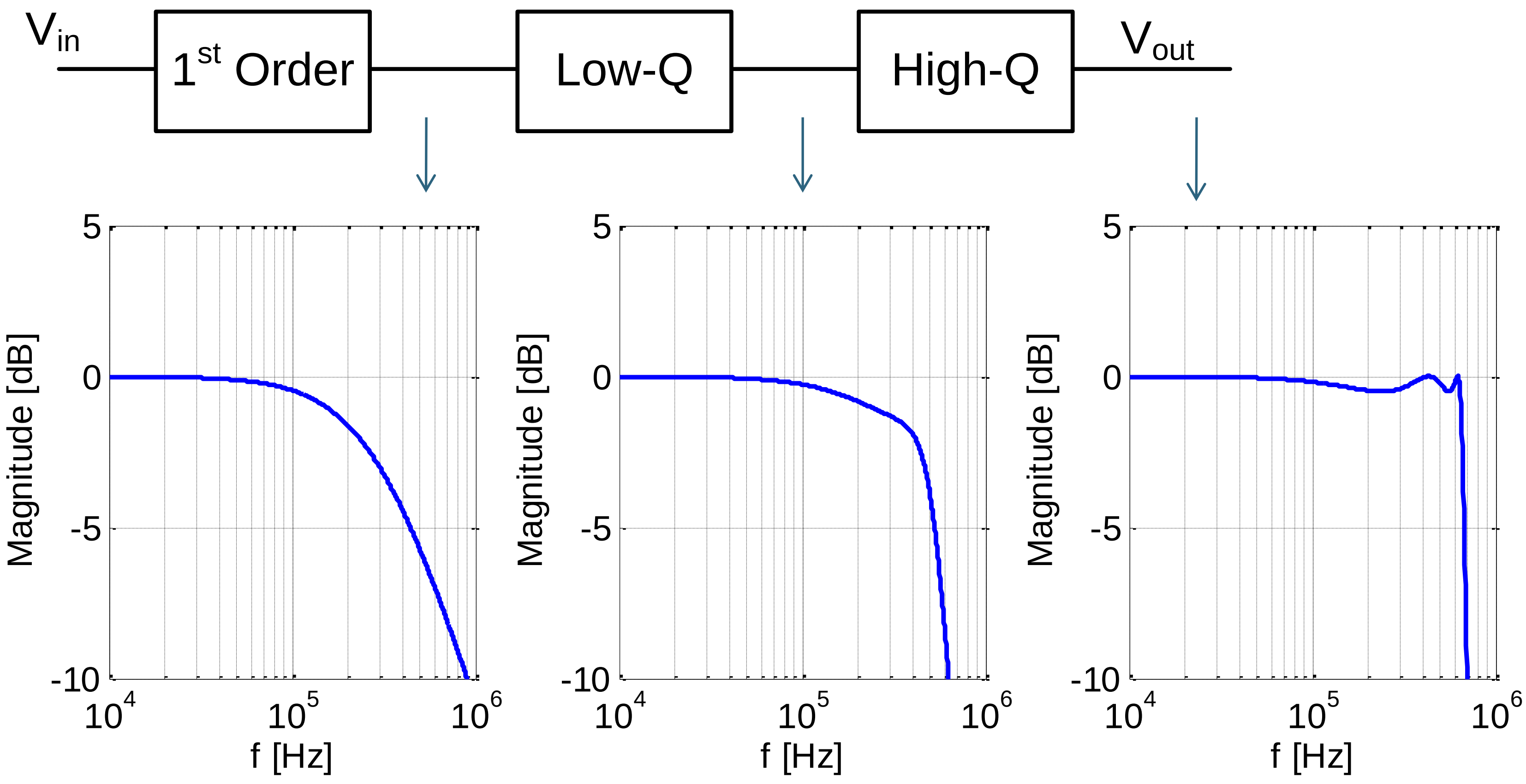


Response of the Individual Sections



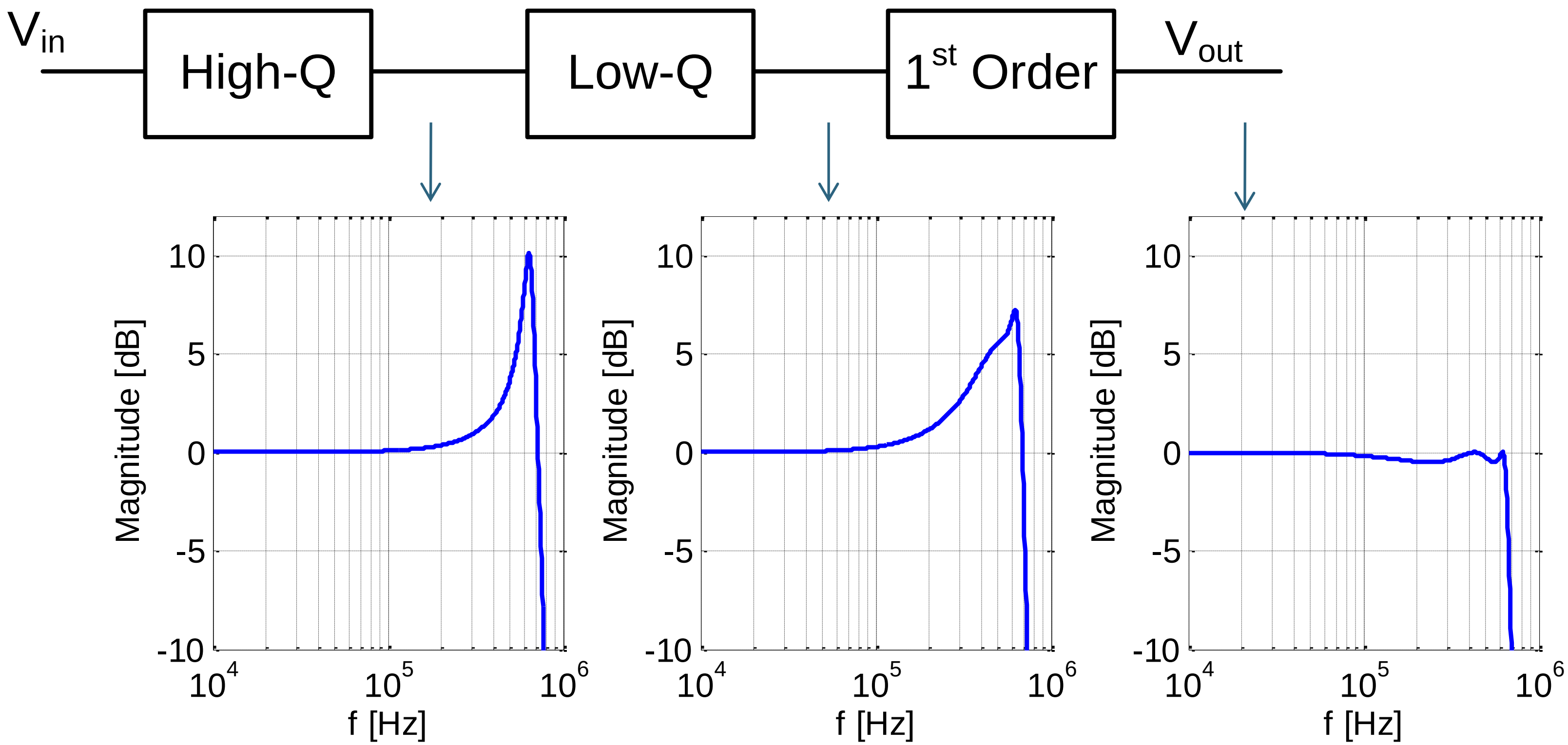
- In which order should we cascade these sections?
  - No “perfect” answer

Intermediate Outputs for Low-Q → High-Q



- This ordering is most frequently used in practice
- Noise from high-Q section unfiltered at the output

Intermediate Outputs for High-Q → Low-Q



- At first glance this looks bad, but the noise from the high-Q biquad is filtered before it reaches the output
  - We will revisit this situation in the context of noise analysis

EE 240C
Analog-Digital Interface Integrated Circuits

Biquad Design Flow
Dynamic Range Scaling

Dynamic Range Scaling

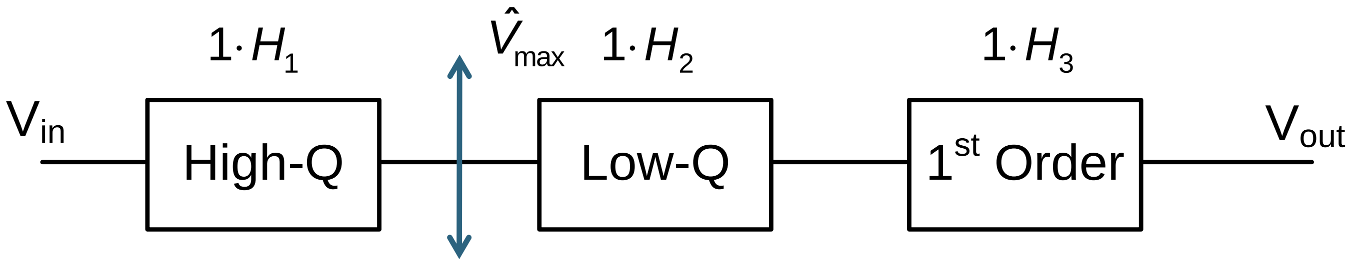
- Suppose we decided "high-Q biquad first" order
- In this case, we need to think about a proper gain distribution that avoids "clipping" in the individual amplifiers
- For this purpose, we introduce gain scale factors for each section, while keeping the overall gain constant ($K_1K_2K_3 = 1$ in this example)

```
         (s^2/2.786e013 + 1)                    (s^2/5.715e013 + 1)                   1
----------------------------------   ----------------------------------   ----------------
(s^2/1.664e013 + s/3.1308e+007 + 1)  (s^2/1.034e013 + s/4.6640e+006 + 1)  (s/1.89e006 + 1)
```

$\Downarrow$

```
       K1*(s^2/2.786e013 + 1)                K2*(s^2/5.715e013 + 1)                  K3
----------------------------------   ----------------------------------   ----------------
(s^2/1.664e013 + s/3.1308e+007 + 1)  (s^2/1.034e013 + s/4.6640e+006 + 1)  (s/1.89e006 + 1)
```

Analysis (1)



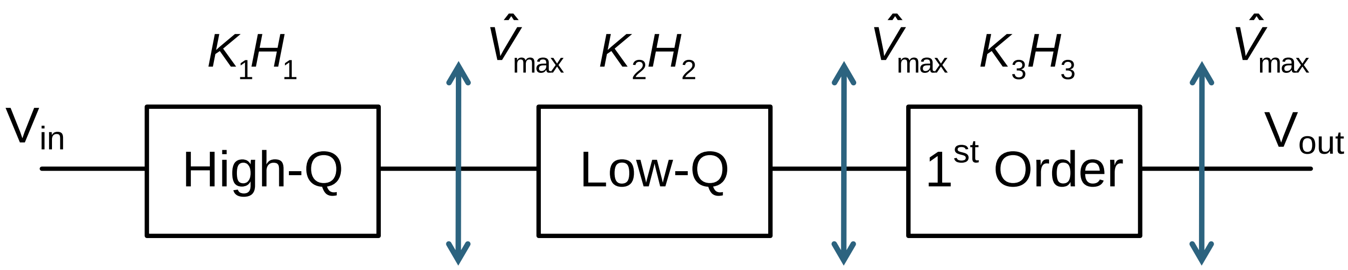
- Suppose we chose $K_1=K_2=K_3=1$ and assume that we will apply single sine waves with arbitrary frequencies to the input
- Since H_1 has significant peaking ($|H1|_{max} \cong 3.19 \cong 10$ dB), we can guarantee proper operation only for input amplitudes up to

$$\frac{\hat{V}_{max}}{|H_1|_{max}} \quad \text{e.g.} \quad \frac{1V}{3.19} = 314mV$$

- Since the overall gain is unity (with no peaking above 1), this means V_{out} swings only 314mV, meaning that we are “wasting” available signal range

Analysis (2)

- A more desirable outcome may be to scale K_1, K_2, K_3 such that all stages utilize the maximum available swing as the input tone is swept across all frequencies
  - Note that in general, the maximum output swings for each stage may not occur at the same frequency



Analysis (3)

- This is achieved for

$$K_1 \left|H_1\right|_{max} = K_1 K_2 K_3 \left|H_1 H_2 H_3\right|_{max}$$ Amplitude at output of H_1 same as filter out

$$K_1 K_2 \left|H_1 H_2\right|_{max} = K_1 K_2 K_3 \left|H_1 H_2 H_3\right|_{max}$$ Amplitude at output of H_2 same as filter out

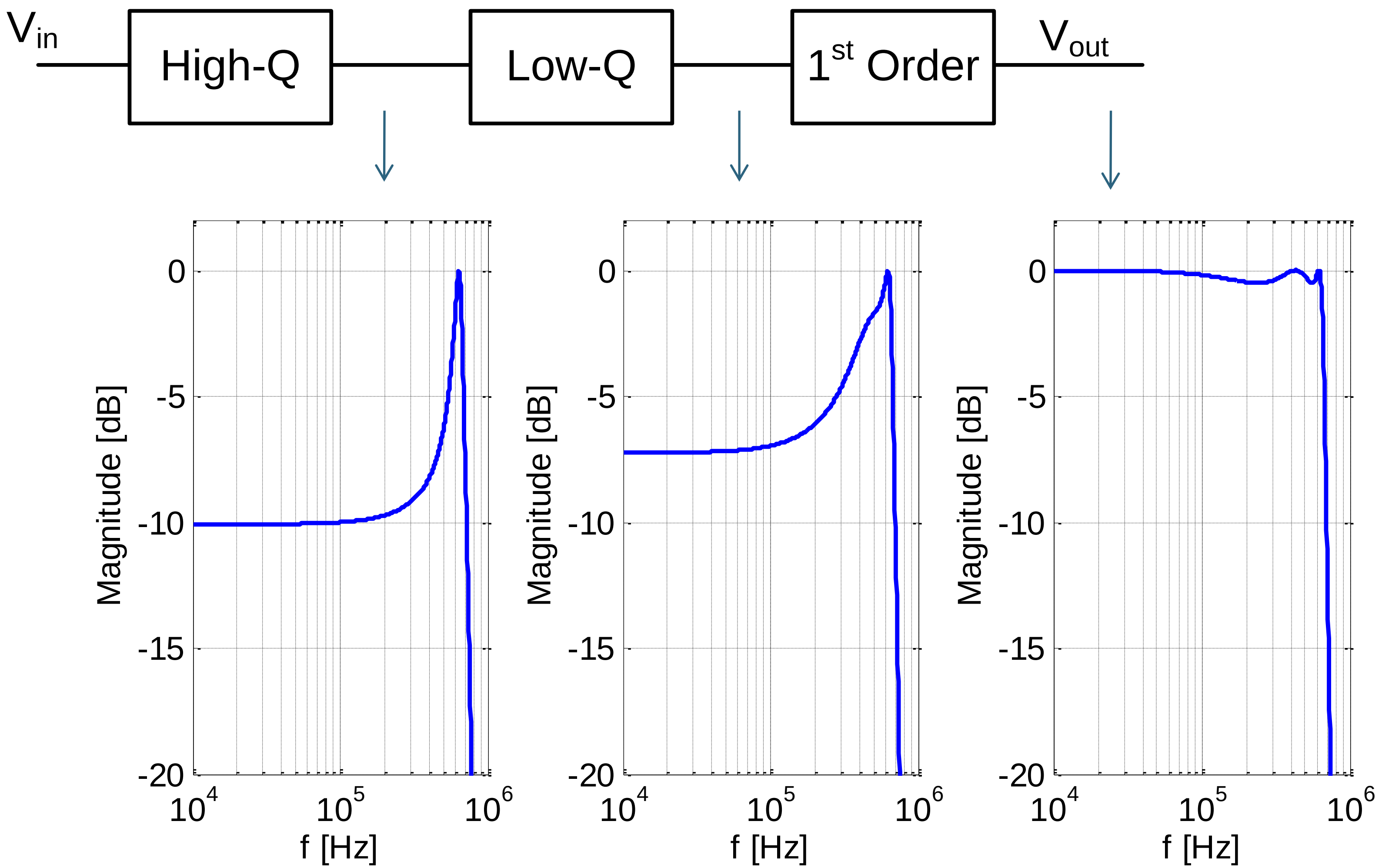
- In our example

$$K_1 K_2 K_3 = 1 \qquad \left|H_1\right|_{max} = 3.19 \qquad \left|H_1 H_2\right|_{max} = 2.3 \qquad \left|H_1 H_2 H_3\right|_{max} = 1$$

and therefore

$$K_1 = \frac{1}{\left|H_1\right|_{max}} = \frac{1}{3.19} \qquad K_2 = \frac{1}{K_1 \left|H_1 H_2\right|_{max}} = \frac{3.19}{2.3} \qquad K_3 = \frac{1}{K_1 K_2} = \frac{3.19 \cdot 2.3}{3.19}$$

Intermediate Outputs After DR Scaling



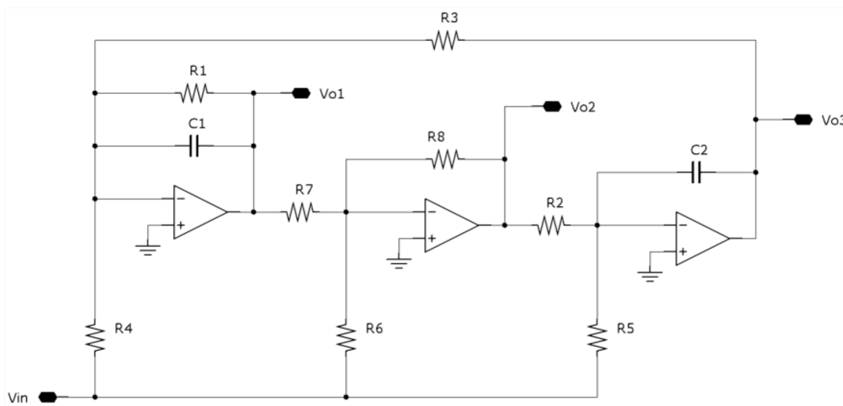
Arguments Against “Sinusoidal” DR Scaling

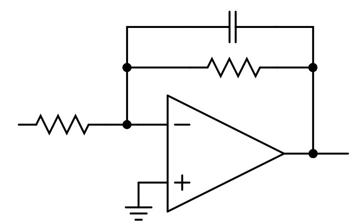
- If the input signal is wide-band (as in many telecommunication systems), the node with peaking may not saturate due to limited signal power in that frequency region
  - May want to optimize the gain distribution based on a power spectral density “template” of the incoming signal
- Aligning the peaks for each output perfectly will require non-integer component ratios
  - But we may want to use integer ratios to be able to use unit elements for better matching
- For a discussion on why sinusoidal dynamic range scaling may not always the best choice, see Behbahani, JSSC 4/2000

Expressions for Implementation

```
 0.3133*(s^2/2.786e013 + 1)              1.3865*(s^2/5.715e013 + 1)            2.3021
-----------------------------------    -----------------------------------    ----------------
(s^2/1.664e013 + s/3.1308e+007 + 1)    (s^2/1.034e013 + s/4.6640e+006 + 1)    (s/1.89e006 + 1)
```

2x Tow-Thomas





$$\frac{V_{o2}}{V_{in}} = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0} \qquad b_1 = 0$$

Tow-Thomas Component Values (b_1=0)

$$R_1 = \frac{1}{a_1 C_1} \qquad R_2 = \frac{k_1}{\sqrt{a_0} C_2} \qquad R_3 = \frac{1}{k_1 k_2} \frac{1}{\sqrt{a_0} C_1}$$

$$R_4 = \frac{1}{k_2} \frac{1}{a_1 b_2} \frac{1}{C_1} \qquad R_5 = \frac{k_1 \sqrt{a_0}}{b_0 C_2} \qquad R_6 = \frac{R_8}{b_2} \qquad R_7 = k_2 R_8$$

$$\omega_Z = \sqrt{\frac{R_6}{R_3 R_5 R_7 C_1 C_2}} \qquad \omega_P = \sqrt{\frac{R_8}{R_2 R_3 R_7 C_1 C_2}} \qquad Q_P = \omega_P R_1 C_1$$

- a_0, a_1, b_0, b_1, b_2 are known; can pick k_1, k_2, C_1, C_2 and R8
- Reasonable starting values
  - $k_1 = k_2 = 1$
  - Set $C_1 = C_2$ to a reasonable value that is easily implemented, e.g. 1pF
  - Set R_8 to a reasonable value that is easily implemented and represents an integer multiple or fraction of R_2, R_3 or R_7

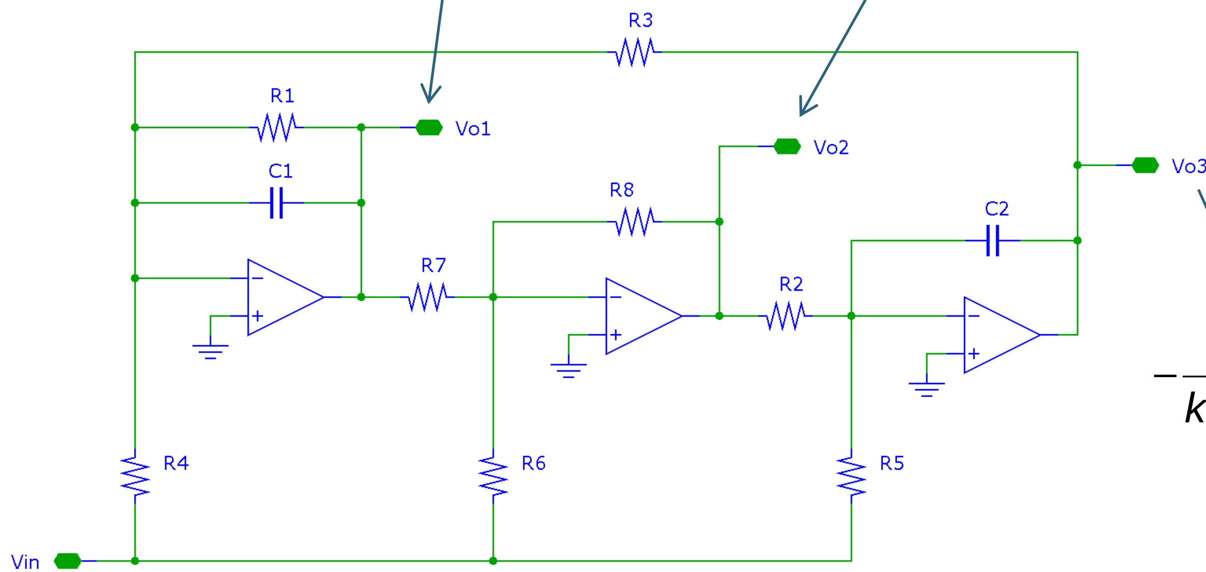
Example Design Procedure

- First cut component calculation using reasonable starting values for k_1, k_2, C_1, C_2 and R_8
- Dynamic range scaling of internal amplifier outputs by adjusting k_1 and k_2
- Thermal noise scaling using ideal amplifiers
  - Increase all capacitors and reduce all resistors until noise specification is met
- Design amplifiers
- Repeat thermal noise scaling to accommodate amplifier noise
- Analyze sensitivity to component variations and devise tuning mechanism (if needed)

Dynamic Range Scaling of Internal Nodes

$$-k_2 \frac{(b_2 a_1 - b_1)s + (b_2 a_0 - b_0)}{s^2 + a_1 s + a_0}$$

$$\frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$$



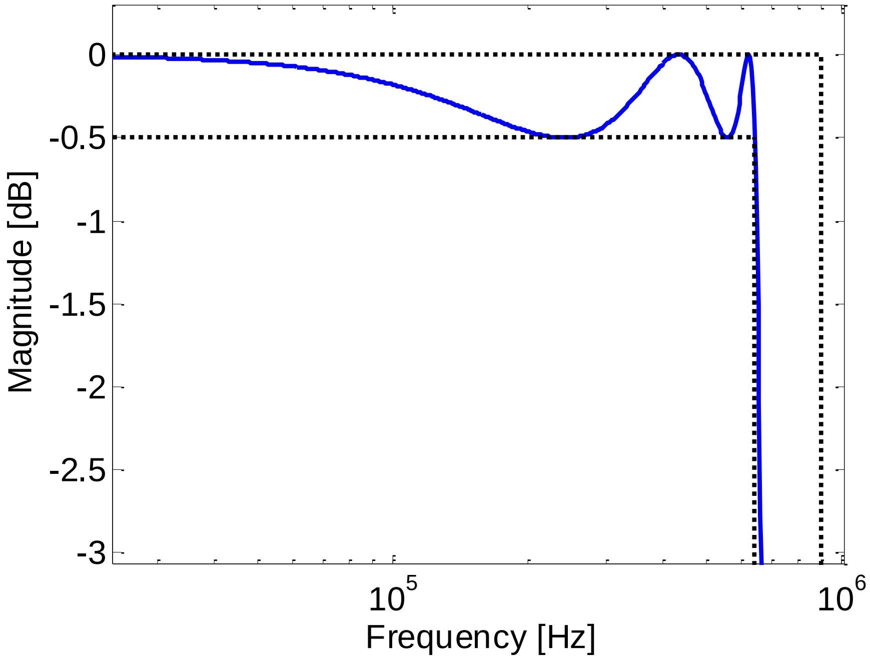
$$-\frac{1}{k_1\sqrt{a_0}} \frac{(b_0 - b_2 a_0)s + (a_1 b_0 - a_0 b_1)}{s^2 + a_1 s + a_0}$$

- Scale k_1 and k_2 such that peak magnitude at V_{o1} and V_{o2} corresponds to maximum available amplifier swing

EE 240C
Analog-Digital Interface Integrated Circuits

Biquad Design Flow
Sensitivity

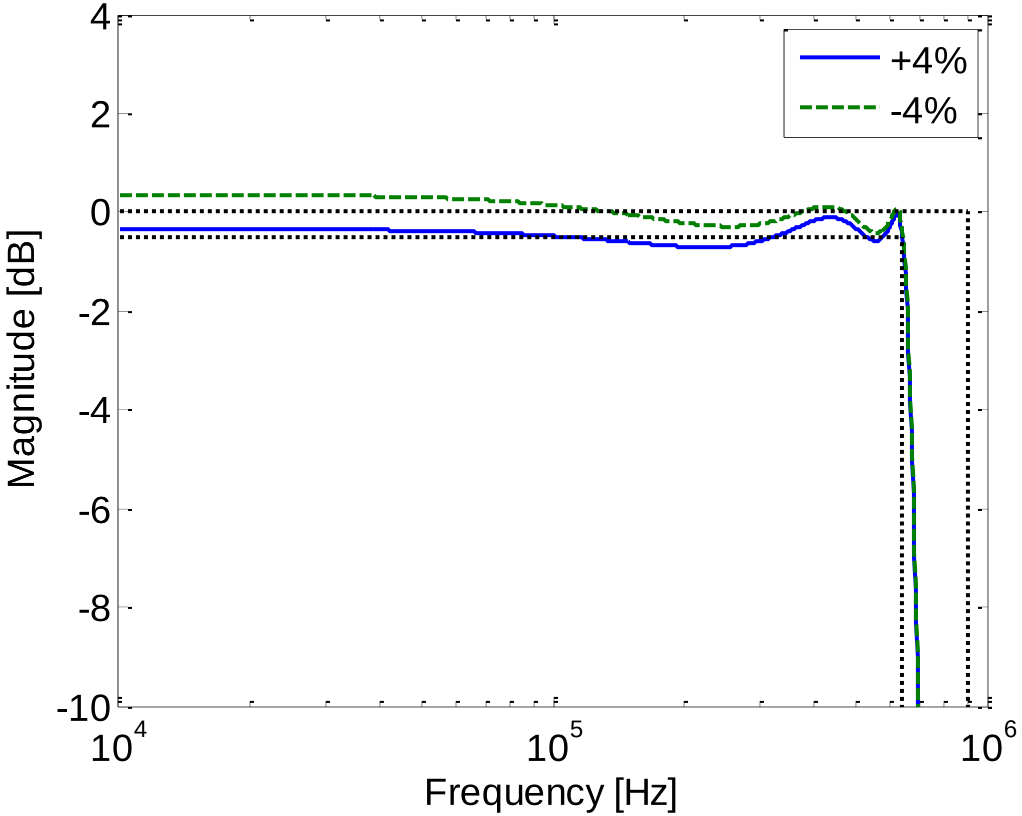
Sensitivity Analysis



- Ideally, we would like to have an analytical expression that relates "interesting points" of the response to variations in all components
  - E.g. calculate variations in the passband ripple as a function of the percent error in R_2
- This is almost impossible or at least impractical to do in practice

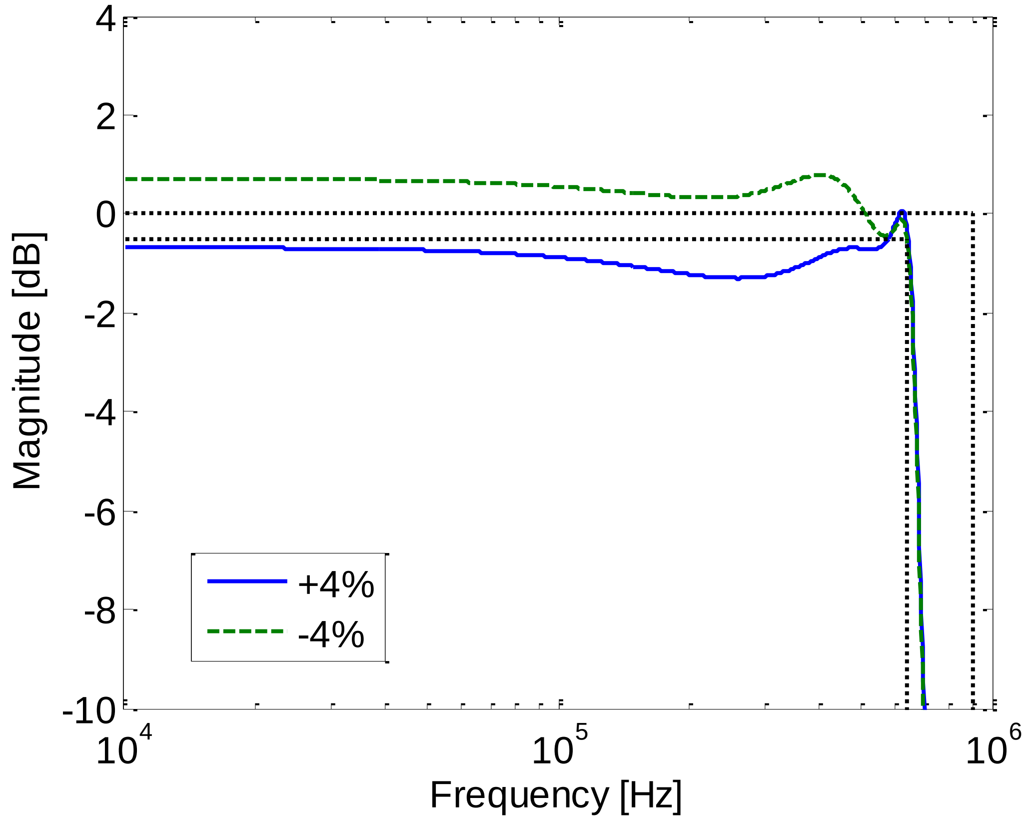
Passband with Pole Errors (1)

- $\pm$ 4% change in ω_p of first order section



Passband with Pole Errors (2)

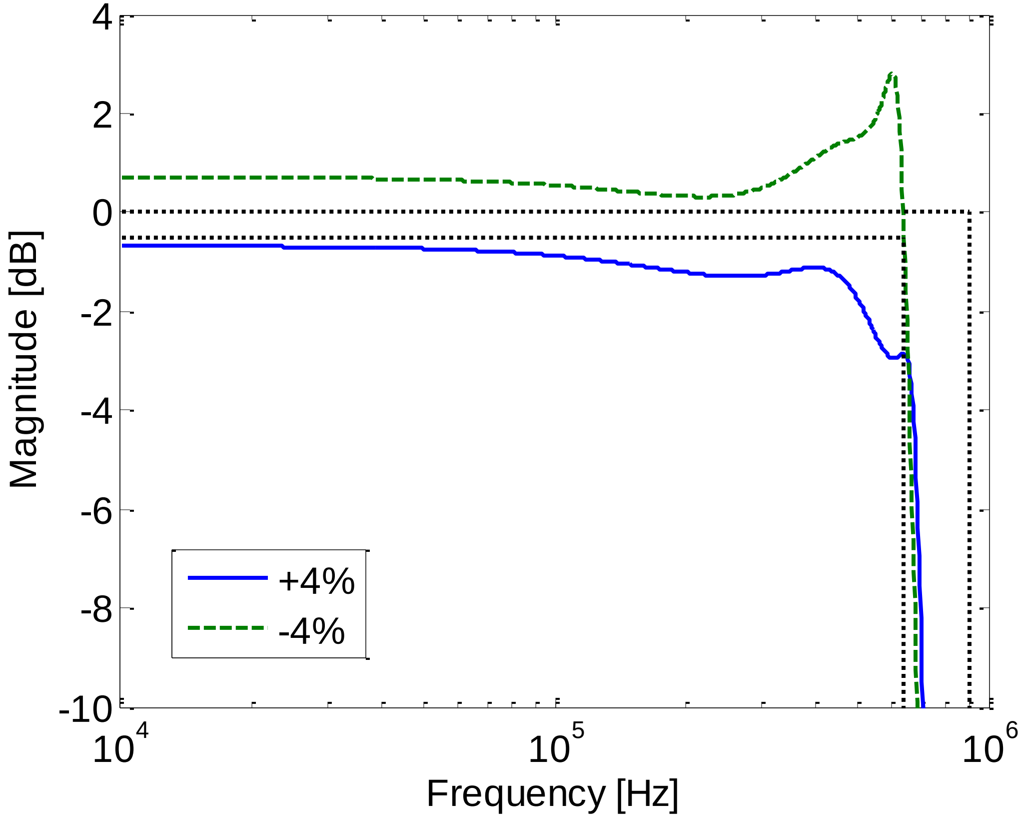
- $\pm$ 4% change in ω_p of low-Q section



Worse.

Passband with Pole Errors (3)

- ± 4% change in ω_p of high-Q section



Bad !

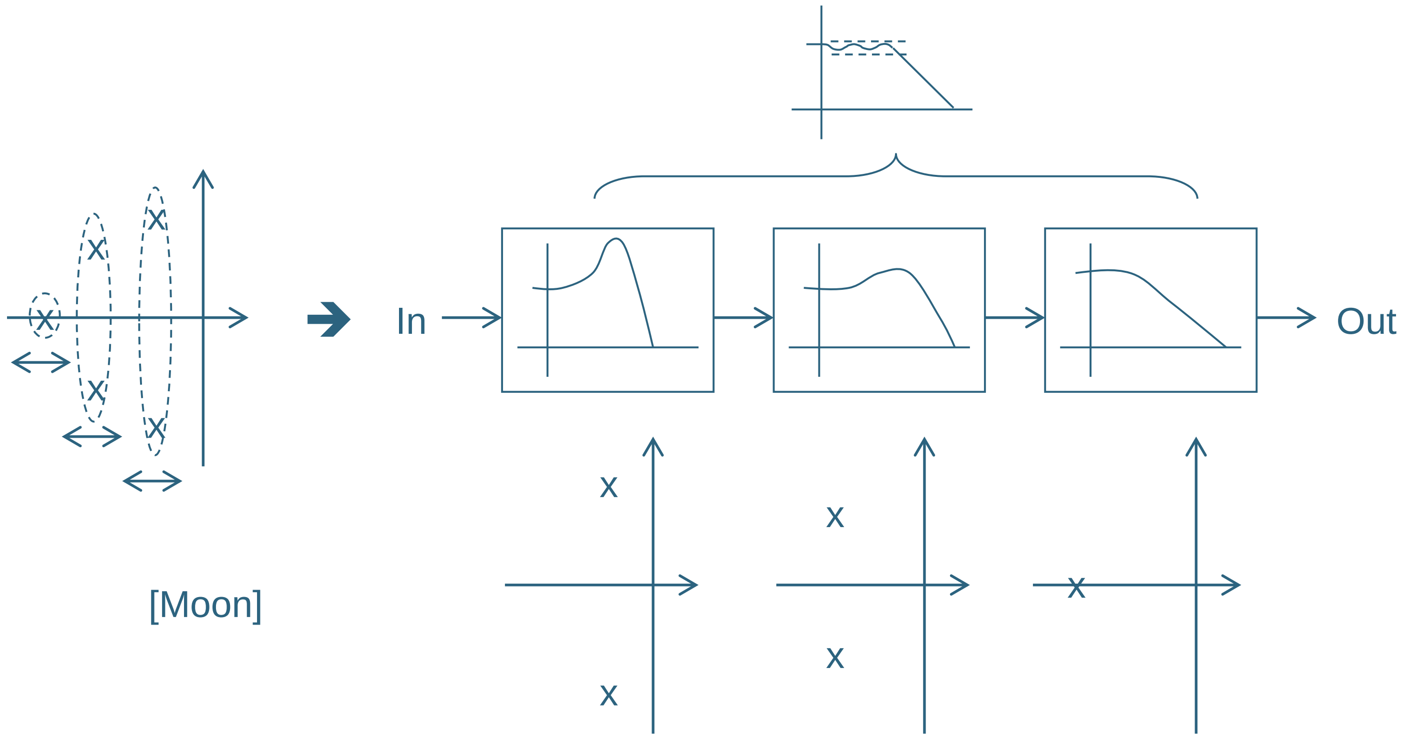
Solutions?

- "Tune" all (?) component values?
- Different topology?

EE 240C
Analog-Digital Interface Integrated Circuits

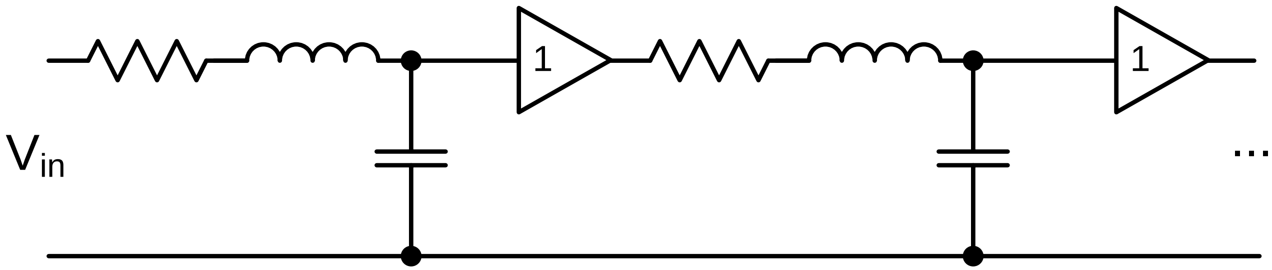
Ladder Filters

Sensitivity Problem with Cascaded Biquads



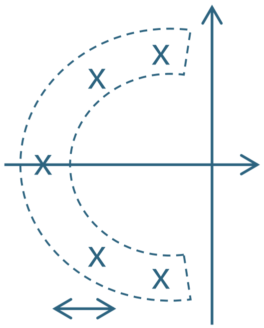
- Passband response is sensitive to shifts in the pole positions
  - Especially for high Q
- Sets practical limit on analog biquad filter order (typically ≤5)

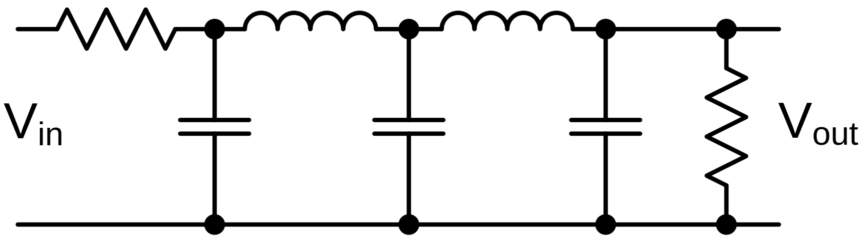
Conceptual View of a Biquad Cascade



- Individual sections are actively decoupled
  - Variations in individual components affect only one pair of poles (and/or zeros)
- Ideally, we would like all the poles (and zeros) to "move together"
  - This would at least preserve the "shape" of the filter response

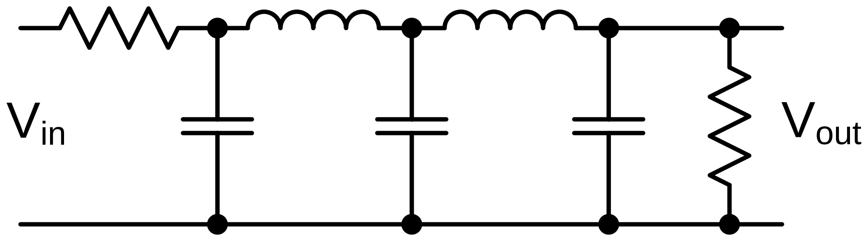
Doubly Terminated LC Ladder Filters





- The passband response of ladder filters is much less sensitive to component variations when compared to a biquad cascade
  - Poles "tend" to move together
- For a sensitivity analysis, see e.g.
  - G. C. Temes and H. J. Orchard, "First order sensitivity and worst-case analysis of doubly terminated reactance two-ports," IEEE Trans. Circuit Theory, 20 (5), pp. 567-571, 1973.

Basic Intuition



- In the passband, the gain from V_{in} to V_{out} is maximum (0.5 for equal termination resistors)
- Any detuning of L and C can only reduce the passband gain
- Therefore, the passband gain is convex in L and C, and the sensitivity is zero around the nominal design point!