EE 240C Homework 1

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Problem 1: Aliasing

A sinusoidal signal with a frequency of 4 MHz and with second and third harmonics is sampled by a 6 MS/s system.

a) Draw the resulting spectrum. What happens to each of the distortion components? Assume that the signal has a fundamental component with amplitude α and 2nd and 3rd harmonics with amplitudes β and γ respectively. The resulting spectrum contains:

- the 4 MHz fundamental aliased to 1 MHz with amplitude α
- the 8 MHz 2nd harmonic aliased to 2 MHz with amplitude β
- the 12 MHz 3rd harnomic aliased to DC with amplitude γ

b) What is the minimum sampling frequency that avoids aliasing?

$f_s \geq BW \cdot 2 \rightarrow f_s \geq 24$ MS/s

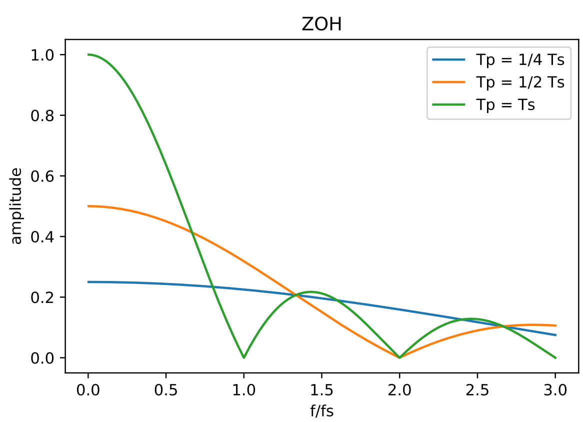
Problem 2: Reconstruction

A discrete time signal with a sample rate of $f_s = 1$ MHz is converted to continuous time using zero-order hold pulses.

a) Plot the frequency response corresponding to reconstruction using zero-order hold pulses for pulse widths of 250ns, 500ns, and 1μs.

We derived the amplitude envelope corresponding to using ZOH pulses with pulse width T_p as:

$$|H_{ZOH}(f)| = \left| \frac{T_p}{T_s} \frac{\sin(\pi f T_p)}{\pi f T_p} \right|$$



b) Plot the output spectrum up to 2 MHz when a discrete-time sine wave of 200 kHz is converted to continuous-time using a 1μs zero-order hold pulse.

This corresponds to convolving the 200 kHz tone with the frequency response of the ZOH pulse. The output spectrum has these components:

- $|H_{ZOH}(200\text{ kHz})|$ at 200 kHz
- $|H_{ZOH}(800\text{ kHz})|$ at 800 kHz
- $|H_{ZOH}(1200\text{ kHz})|$ at 1200 kHz
- $|H_{ZOH}(1800\text{ kHz})|$ at 1800 kHz

Problem 3: ADC DNL and INL

DNL:

```
hist = [28, 21, 19, 23, 22, 20, 15, 21, 23, 18, 19, 19, 23, 18, 24, 29]
inner_range = sum(hist[1:-1])
wavg = (sum(hist[0:-1]) - hist[0]) / (2**4 - 2)
dnl = (np.array(hist[1:-1]) - wavg) / wavg
print(dnl)
print(np.max(dnl))
print(np.min(dnl))
```

```
[ 0.03157895 -0.06666667  0.12982456  0.08070175 -0.01754386 -0.26315789
  0.03157895  0.12982456 -0.11578947 -0.06666667 -0.06666667  0.12982456
 -0.11578947  0.17894737]
0.1789473684210526
-0.26315789473684215
```

INL:

```
inl = np.cumsum(dnl)
print(np.max(inl))
print(np.min(inl))
```

```
0.17543859649122795
-0.1929824561403512
```

Problem 4: ADC DNL and INL

a) Monotonic ADC with output codes 0, 1, 2, $\ldots, M$. Show:

$$
\begin{aligned}
\text{INL}(k) &= \sum_{i=1}^{k-1} \text{DNL}(i) \\
&= \sum_{i=1}^{k-1} \frac{\text{Step}(k) - \text{Step}_{\text{avg}}}{\text{Step}_{\text{avg}}} \\
&= \sum_{i=1}^{k-1} \frac{\text{T}(k+1) - \text{T}(k) - \text{Step}_{\text{avg}}}{\text{Step}_{\text{avg}}} \\
&= \frac{1}{\text{Step}_{\text{avg}}} T(n) - T(0) - (n-2)\text{Step}_{\text{avg}} \\
&= \frac{T(k) - T_{uniform}(k)}{W_{avg}}
\end{aligned}
$$

b) Sum of DNLs is 0.

We know INL(M) = 0.

$$\text{INL}(M) = 0 = \sum_{i=1}^{M-1} \text{DNL}(i)$$