L2 : Number System 1

Q.1   **What will be the last digit of** $2^{3^{4^{5}}} \times 3^{16^{3^{5}}}$ **?**
a **6**
b **4**
C **2**
d **8**
e **None of these**

Q.2
Every integer of the form $(n^3 - n)(n - 2)$, (for n = 3, 4, 5, ...) is

- **a** **divisible by 6 but not always divisible by 12**
- **b** **divisible by 12 but not always divisible by 24**
- **C** **divisible by 24 but not always divisible by 48**
- **d** **divisible by 9**
- **e** **divisible by 16.**

Q.3

Let T be the set of integers {2, 12, 22, 32 ..., 542, 552} and S be a subset of T such that the sum of no two elements of S is divisible by 3. The maximum possible number of elements in S is

a	18
b	19
C	20
d	28
e	21

Q.4

If $f(y) = y^{y^{y^{y^{y^{y^{\cdots\cdots}}}}}}$, $g(y) = y \times y \times y \times \ldots$ and $h(y) = y + y + y + \ldots$ where, y is a whole number. What is the remainder, when f(2) is divided by 5?

- **a** 2
- **b** 3
- **c** 4
- **d** 1
- **e** None of these

Q.5

f(y) = $y^{y^{y^{y^{y^{y^{\cdots}}}}}}$, g(y) = y × y × y × … and h(y) = y + y + y + … where, y is a whole number. What is the remainder, when g(2) is divided by 5?

a **2**

b **3**

c **4**

d **1**

e **Cannot be determined**

Q.6

$N = 1^1 \times 2^2 \times 3^6 \times 4^{12} \times 5^{20}$ ..... 25 terms. Find the highest power of 75 that can divide N.

a 1900

b 1850

c 2526

d 1200

e None of these

Q.7

If n is an odd natural number and n! ends with 32 zeros, then how many values of n are possible? Note : n! means product of first n natural numbers.

- **a** ○ **2**
- **b** ○ **3**
- **c** ○ **1**
- **d** ○ **4**
- **e** ○ **None of these**

Q.8

If n! ends with 29 zeros and n is an even natural number, then how many values of n are possible?
Note : n! means product of first n natural numbers.

a	**2**
b	**3**
c	**1**
d	**4**
e	**None of these**

Q.9

If X is the smallest number that is divisible by both 6 and 5, than find the maximum possible power of 10 that would completely divide the product of the first 20 multiples of X.

- **a** 23
- **b** 24
- **c** 25
- **d** 26
- **e** 27

Q.10

The auto fare in Ahmedabad has the following formula based upon the metre-reading. The metre-reading is rounded up to the next higher multiple of 4. For instance, if the metre-reading is 37 paise, it is rounded up to 40 paise. The resultant is multiplied by 12. The final result is rounded off to the nearest multiple of 25 paise. If 53 paise is the metre-reading, then what will be the actual fare?

a **Rs. 6.75**

b **Rs. 6.50**

c **Rs. 6.25**

d **Rs. 7.50**

e **None of these**

SOLUTIONS

Question1

What will be the last digit of $2^{3^{4^5}} \times 3^{15^{3^5}}$?

Answer:

Correct Option is: "**b**"

Solution:

3^{4^5} when divided by 4 gives remainder

1. Units digit is 2 for $2^{3^{4^5}}$.

Cyclicity of 3 is 4:

$$\frac{15^{3^5}}{4} = \frac{(16-1)^{3^5}}{4}.$$

15^{3^5} when divided by 4 given a remainder –1 or 3.

Unit digit is 7 (as $3^3 = 27$).

$\therefore$ Unit's digit of product is 4.

Further explanation of the concepts involved :
If you analyze the powers of 2, which means 2, 2^2, 2^3 etc. you'll get their numerical values as 2, 4, 8, 16, 32, 64 and so on.
Here, you can see that the last digit of these numbers repeat after every four numbers (this is called cyclicity).

What this means is that the digits 2, 4, 8 and 6 (in that order) appear in a cyclic order as the last digit of numbers which are higher powers of 2.

So, if we know exactly what type of power 2 has, we can find out the last digit of the entire number itself. Let me explain -
If the number is of type 2^{4k} (where k is a natural number) like 2^4, 2^8 etc. then the last digit of this number will be 6.
If its $2^{(4k+1)}$ then last digit will be 2.
If its $2^{(4k+2)}$ then last digit will be 4.
If its $2^{(4k+3)}$ then last digit will be 6.

So, in this question we are trying to figure out the remainder left when the power of $2^{3^{4^5}}$, which is 3^{45}, is divided by 4.
Now 3 leaves a remainder of -1 when divided by 4.
You can also see that in 3^{45}, 3 is multiplied 4^5 times which means some even number of times.
So, the remainder -1 will be multiplied even number of times and hence will give 1.
Hence, 3^{45} when divided by 4, should give a remainder of 1.
So, 3^{45} is of type 4k+1 and 2^{345} is of type 2^{4k+1}.
Hence last digit of 2^{345} will be 2 (as explained above)
Similarly we have calculated the last digit for 3^{1535} See that the cyclicity of last digit of powers of 3 is 4 again (in the order 3, 9, 7 and 1).

Question2	Every integer of the form $(n^3 - n)(n - 2)$, (for n = 3, 4, 5, …) is
Answer:	Correct Option is: "**c**"
Solution:	**The expression can be expanded and written as** **$(n - 2)(n - 1)n(n + 1)$** **Since product of 4 consecutive terms is divisible by 24 but not always divisible by 48.**
Question3	Let T be the set of integers {2, 12, 22, 32 ..., 542, 552} and S be a subset of T such that the sum of no two elements of S is divisible by 3. The maximum possible number of elements in S is
Answer:	Correct Option is: "**c**"
Solution:	**From 2 to 552 there are 56 numbers. Numbers of the form 3n are 12, 42, 72, ... 552, i.e. 19 numbers 3n + 1 are 22, 52, 82, ... 532, i.e. 18 numbers. 3n + 2 are 2, 32, ... 542, i.e. 19 numbers. In order that sum of any 2 numbers is not divisible by 3. Choose 19 numbers of the form 3n + 2 and 1 number of the form 3n. Hence, there are 20 numbers possible**
Question4	If f(y) = $y^{y^{y^{y^{y^{y^{y^{\cdots}}}}}}}$, g(y) = y × y × y × ... and h(y) = y + y + y + ... where, y is a whole number. What is the remainder, when f(2) is divided by 5?
Answer:	Correct Option is: "**d**"
Solution:	$2^{2^{2^{\cdots}}} = 2^{4k} = 4^{2k} = (5 - 1)^{2k}$ $= 5N + 1$ (N is a natural number) When 5N + 1 is divided by 5, the remainder will be 1.
Question5	f(y) = $y^{y^{y^{y^{y^{y^{y^{\cdots}}}}}}}$, g(y) = y × y × y × ... and h(y) = y + y + y + ... where, y is a whole number. What is the remainder, when g(2) is divided by 5?
Answer:	Correct Option is: "**e**"
Solution:	**As number of terms can be even or odd, the remainder cannot be determined uniquely.**

Question6	$N = 1^1 \times 2^2 \times 3^6 \times 4^{12} \times 5^{20}$ ..... 25 terms. Find the highest power of 75 that can divide N.
Answer:	Correct Option is: "**e**"
Solution:	**Powers are distributed as $n^2 - n$. So indices of 1 is 12 – 1 = 0 or $1^0 = 1^1$ 2's power is $2^2 - 2 = 4 - 2 = 2$ 3's power is $3^2 - 3 = 9 - 3 = 6$ 4's power is $4^2 - 4 = 16 - 4 = 12$ 5's power is $5^2 - 5 = 25 - 5 = 20$ and so on. $75 = 25 \times 3 = 5^2 \times 3$ We have to determine highest power of 5 greater than 5^2 which can divide this number. To determine the power of 3 is immaterial because obviously power of 5^2 will be less than power of 3. So we are concerned about finding highest power of 5^2 only. 5 will contribute = $5^2 - 5 = 20$. 10 will contribute= $10^2 - 10 = 90$ 15 will contribute = 225 – 15 = 210 power of 5. 20 contributes = 400 – 20 = 380 power of 5. 25 contributes = (625 – 25) × 2 = 600 × 2 = 1200 (as $25 = 5^2$) So highest power of 5 is 1900. So highest power of 52 is 950. So highest power of 75 that can divide the number is 950.**
Question7	If n is an odd natural number and n! ends with 32 zeros, then how many values of n are possible? **Note :** n! means product of first n natural numbers.
Answer:	Correct Option is: "**a**"
Solution:	**n can be only 131 and 133. Please check how many times 5, 25 & 125 is there within 131, you will realise that there will be 32 'zeros' in the end of 131!**
Question8	If n! ends with 29 zeros and n is an even natural number, then how many values of n are possible? **Note :** n! means product of first n natural numbers.
Answer:	Correct Option is: "**e**"
Solution:	**This is not possible, n! can never end with 29 zeros. 120! to 124! will have 24 + 4 = 28 zeroes and 125! to 129! will have 31 zeros.**
Question9	If X is the smallest number that is divisible by both 6 and 5, than find the maximum possible power of 10 that would completely divide the product of the first 20 multiples of X.
Answer:	Correct Option is: "**b**"
Solution:	**Product of first 20 multiples of 30 can be written as $(30)^{20}$[20!]. $(30)^{20}$has 20 zeros and 20! has 4 zeros. Hence total number of zeros is 24.**

Question10	The auto fare in Ahmedabad has the following formula based upon the metre-reading. The metre- reading is rounded up to the next higher multiple of 4. For instance, if the metre-reading is 37 paise, it is rounded up to 40 paise. The resultant is multiplied by 12. The final result is rounded off to the nearest multiple of 25 paise. If 53 paise is the metre-reading, then what will be the actual fare?
Answer:	Correct Option is: "**a**"
Solution:	**First, 53 paise has to be rounded off to next higher multiple of 4, i.e. 56 paise. This is then multiplied by 12, viz. 56 × 12 = 672. This is then rounded off to the closest multiple of 25 paise, which in this case is Rs. 6.75.**