L3 : Number System 1

Q.1 **Bill and Clinton take the square of a certain decimal number and express it in base 5 and 6 respectively. Then Bush comes and he takes the two representations and assuming that the expressions are in base 10, adds the numbers. Which of the following cannot be the value of the unit's digit of the sum obtained?**
a **0**
b **2**
c **8**
d **6**
e **3**

Q.2
Find the sum of all the natural numbers from 1 to 100 that are neither a multiple of 2 nor a multiple of 5.

a	**2000**
b	**2275**
c	**2515**
d	**2845**
e	**2125**

Q.3

The sequence 2, 3, 5, 6, 7, 10, ... consists of all natural numbers that are neither perfect squares nor perfect cubes. Find the 76th term of this sequence.

a	89
b	87
c	86
d	90
e	88

Q.4
The numbers from 1 to m are written one after another as follows 1234567.......m. The resulting number is divisible by 3 if m is of the form (n is a natural number):
I. 3n
II. 3n – 1
III. 3n – 2

a ○ **I only**
b ○ **I or II only**
c ○ **I or III only**
d ○ **II or III only**
e ○ **Cannot be determined**

Q.5

The number of values of ‘x’ less than 30 for which 50! – x! ends in 6 zeroes is:

a	6
b	5
c	4
d	3
e	0

Q.6
LCM of two numbers is ab^3, where a and b are prime numbers. If one of the numbers is b^3, then which of the following cannot be the other number?

a	ab^3
b	ab
c	a
d	ab^2
e	None of these

Q.7

P denotes the sum of two three-digit numbers such that P is 606. How many of the 10 digits from 0 to 9 cannot appear as the last digit of the product of these two three-digit numbers?

a	4
b	3
c	5
d	2
e	None of these

Q.8

When a natural number "p" is multiplied by 4, it gives a perfect cube and when it is multiplied by 9, it gives a perfect square. Find the minimum possible value of the expression $p^2 - 16p + 1$.

a ○ 4

b ○ 8

c ○ 12

d ○ 16

e ○ None of these.

Q.9
What is the 12th digit from the left of $(6007)^3$?

a	0
b	1
c	2
d	3
e	9

Q.10

The product of two numbers '231' and 'ABA' is 'BA4AA' in a certain base system (where base is less than 10), where A and B are distinct digits. What is the base of that system?

a	5
b	6
c	7
d	8
e	4

SOLUTIONS

Question1	Bill and Clinton take the square of a certain decimal number and express it in base 5 and 6 respectively. Then Bush comes and he takes the two representations and assuming that the expressions are in base 10, adds the numbers. Which of the following cannot be the value of the unit's digit of the sum obtained?
Answer:	Correct Option is: "**d**"
Solution:	**Unit's digits in n^2 is one of {0, 1, 4 , 5, 6 , 9}** **That means, in base 5, units digit possible is one of {0, 1 , 4} only.** **Now, note that any natural number is of the form either 3n - 1 or 3n or 3n + 1** **⇒Square of any natural number is of the form** **$9n^2 \pm 6n + 1$ or $9n^2$** **⇒In base 6, units digit could be one of {0, 1, 3, 4}** only. So, unit's digit of the sum can never be 6 or 9.** ****: '$9n^2 \pm 6n + 1$' or '$9n^2$' when converted to base 6 will have only digit 0, 1, 3 or 4 as at unit's place. '$9n^2$' will leave remainder 3 or 0 on division by 6. '$9n^2 \pm 6n + 1$' will leave remainder 0 + 1 = 1 and 3 + 1 = 4 on division by 6.**
Question2	Find the sum of all the natural numbers from 1 to 100 that are neither a multiple of 2 nor a multiple of 5.
Answer:	Correct Option is: "**a**"
Solution:	**Sum of all numbers from 1 to 100 = 5050** **Sum of all multiples of 2 = 2550** **Sum of all multiples of 5 = 1050** **Sum of all multiples of 10 = 550** **⇒required value = 5050 – 2550 – 1050 + 550 = 2000** **Alternative Method:** **If multiples of 2 were removed, 50 numbers would remain, all odd. Out of those 50, removal of multiples of 5 would leave 40 odd numbers. Sum of 40 odd numbers = even. Only option (a) satisfies.**
Question3	The sequence 2, 3, 5, 6, 7, 10, … consists of all natural numbers that are neither perfect squares nor perfect cubes. Find the 76th term of this sequence.
Answer:	Correct Option is: "**b**"
Solution:	**76th natural number = 76** **We have 7 perfect squares and 3 perfect cubes from 2 to 75 in which 64 occur twice (because of being both a perfect square and a perfect cube)** **Hence, 9 numbers must have been removed.** **The number 76, if we start a series of natural numbers from 2, will be the 75th number.** **If we do not include the above 9 numbers in this series then 76 becomes the 75 - 9 = 66th number.** **Subsequently 80 will be the 70th number.** **But 81 being a perfect square cannot be included.** **Hence, 82 will be the 71st number and subsequently the 76th number will be 87.**

Question4	The numbers from 1 to m are written one after another as follows 1234567…….m. The resulting number is divisible by 3 if m is of the form (n is a natural number): I. 3n II. 3n – 1 III. 3n – 2
Answer:	Correct Option is: "**b**"
Solution:	**Sum of the first two natural numbers in any three consecutive natural numbers 3k + 1, 3k + 2 and 3k + 3 or sum of all the three such numbers would be divisible by 3. Hence m must be either of the form 3n or 3n – 1**
Question5	The number of values of 'x' less than 30 for which 50! – x! ends in 6 zeroes is:
Answer:	Correct Option is: "**b**"
Solution:	**For 50! – x! to end in 6 zeroes, x! must also end in 6 zeroes. There are 5 numbers x = 25 to 29 which would all have 6 zeroes in their factorials. Hence the answer is 5.**
Question6	LCM of two numbers is ab^3, where a and b are prime numbers. If one of the numbers is b^3, then which of the following cannot be the other number?
Answer:	Correct Option is: "**e**"
Solution:	**Start checking with option (a). LCM of ab^3 and b^3 will be ab^3. Similarly for option (b) also. LCM of ab and ab^3 is again ab^3. If we examine the options one by one, we can see that all of them individually when paired up with b^3, will give an LCM as ab^3. Therefore the correct choice should be none of these.**
Question7	P denotes the sum of two three-digit numbers such that P is 606. How many of the 10 digits from 0 to 9 cannot appear as the last digit of the product of these two three-digit numbers?
Answer:	Correct Option is: "**a**"
Solution:	**The last digits of the 2 three digit numbers can be (0, 6), (1, 5), (2, 4), (3, 3), (8, 8) an (7, 9) $\therefore$ The last digit of product of these numbers can be 0, 5, 8, 9, 4 and 3. $\therefore$ Four digits i.e. 7, 6, 2 and 1 cannot be the last digit of the product of these numbers.**
Question8	When a natural number "p" is multiplied by 4, it gives a perfect cube and when it is multiplied by 9, it gives a perfect square. Find the minimum possible value of the expression $p^2 - 16p + 1$.
Answer:	Correct Option is: "**e**"
Solution:	**$9p = 3^2p$ is a perfect square. As 3 is a prime number p must also be a perfect square. Let $p = k^2$ Now, 4p is a perfect cube and $4p = (2k)^2$ So, $k = 2^2\alpha^3$, where α is a natural number Minimum possible value of $p = 4(2^2) = 16$ The expression $p^2 – 16p + 1$ will have minimum possible value at p = 16 only as any other value of p is much greater than 16 and correspondingly the expression will carry a higher value. $p^2 – 16p + 1$ for p = 16 is equal to $(16)^2 – 16(16) + 1 = 1$**

Question9 What is the 12th digit from the left of $(6007)^3$?

Answer: Correct Option is: "**d**"

Solution:

To simplify the problem, lets say
$6007 \approx 6000$
$6000^2 = 36000000$ (8 digits)
$6000^3 = 216000000000$ (12 digits)
Similarly, $(6007)^3$ will also contain 12 digits
∴ the 12th digit will be the last digit of $(6007)^3$ and it will depend upon '7'
$7^3 = 343$
∴ The 12th digit in $(6007)^3$ will be 3.

Question10 The product of two numbers '231' and 'ABA' is 'BA4AA' in a certain base system (where base is less than 10), where A and B are distinct digits. What is the base of that system?

Answer: Correct Option is: "**b**"

Solution:

Here,

$$\begin{array}{ccccc} & & A & B & A \\ & \times & 2 & 3 & 1 \\ \hline & & A & B & A \\ & A & B & A & \times \\ & A & B & A & \times \\ & A & B & A & \times \\ A & B & A & \times & \times \\ A & B & A & \times & \times \\ \hline \end{array}$$

For, the digit at the tens place of the product,
$B + 3A = \text{Base} + A$ or $2A + B = \text{Base}$ ...(i)
[In case of carry over 2, hundred digit '4' is not possible]
For the digit at the hundreds place of the product,
$3A + 3B$,
$= \text{Base} + \text{Base} + 4 + 1$(carry over from the tens place)
$\Rightarrow 2B + A + 1 = \text{Base} + 4$ ...(ii)
from (i) and (ii) we get
$2B + A + 1 = 2A + B + 4$
$\Rightarrow B = A + 3$
For thousands place of the product: 2(carry over from the hundreds place) + $3A + 2B = \text{Base} + \text{Base} + A$

$2 + 3A + 2B = 2(2A + B) + A$

$\Rightarrow A = 1 \Rightarrow B = A + 3 = 1 + 3 = 4$
Hence, $\text{Base} = 2A + B = 2 \times 1 + 4 = 6$
[In case of other bases 5, 7, 8 or 9, this product is not possible]