L3 : Number System 2

Q.1 How many sets of three distinct factors of the number $N = 2^6 \times 3^4 \times 5^2$ can be made such that the factors in each set has a highest common factor of 1 with respect to every other factor in that set?
a 236
b 360
c 104
d 380
e None of these

Q.2

How many digits cannot be the unit's digit of the product of 3 three-digit numbers whose sum is 989?

a	2
b	4
c	1
d	3
e	None of these

DIRECTIONS for Question 3 : A two-digit number having distinct digits when divided by the sum of the digits gives the same remainder as when a two-digit number that is formed by reversing the digits of original number is divided by the sum of the digits.

Q.3
Out of all such possible two-digit numbers, a number is randomly picked. What is the probability that this number is divisible by 4?

a $\frac{3}{8}$

b $\frac{5}{12}$

c $\frac{2}{7}$

d $\frac{7}{12}$

e **None of these**

Q.4
What is the remainder when $723^{243} + 318^{243}$ is divided by 17?

a	2
b	3
c	8
d	9
e	None of these

Q.5
A natural number N, less than 3000, has the following properties:
I. The number N when successively divided by 5, 4 and 3 gives remainder 3, 1 and 2 respectively.
II. The number when divided by 72 gives remainder 12.

Find how many such natural numbers exist.

a	8
b	7
c	6
d	9
e	None of these

Q.6

S is a set containing all the integers less than 21000, which are the product of three consecutive prime numbers. N is a non-empty subset of S, in which all the elements are relatively prime to each other.If the number of elements in N is maximum possible, then how many such distinct subsets are possible?

a 1

b 10

c 3

d 6

e None of these

Q.7

P is the product of first 30 multiples of 30. N is the total number of factors of P. In how many ways N can be written as the product of two natural numbers such that the HCF of these two natural numbers is 19?

- a 3
- b 4
- c 5
- d 6
- e None of these

Q.8

Let M be a three-digit number denoted by 'ABC' where A, B and C are numerals from 0 to 9. Let N be a number formed by reversing the digits of M. It is known that M − N + (396 × C) is equal to 990. How many possible values of M are there which are greater than 300?

a 10

b 18

c 30

d 20

e None of these

Q.9
g(P) represents the product of all the digits of P, e.g. g(45) = 4 × 5. What is the value of g(67) + g(68) + g(69) + ... + g(122) + g(123)?

a	1381
b	1281
c	1481
d	1581
e	None of these

Q.10

The product of three positive integers is 6 times their sum. One of these integers is the sum of the other two integers. If the product of these three numbers is denoted by P, then find the sum of all distinct possible values of P.

a) 432

b) 252

c) 144

d) 336

e) None of these

SOLUTIONS

Question1

How many sets of three distinct factors of the number $N = 2^6 \times 3^4 \times 5^2$ can be made such that the factors in each set has a highest common factor of 1 with respect to every other factor in that set?

Answer: Correct Option is: "**a**"

Solution:

$N = 2^6 \times 3^4 \times 5^2$

Case [A]: when none of the elements is 1.

$\Rightarrow$ the three factors must be some powers of 2, 3 & 5 respectively.

$\Rightarrow$ total number of such sets $= 6 \times 4 \times 2 = 48$

Case [B]: when one of the elements is 1.

$\Rightarrow$ the other two factors could be of the form

$(2^a), (3^b)$ – number of sets $= 6 \times 4 = 24$

$(2^a), (5^b)$ – number of sets $= 6 \times 2 = 12$

$(3^a), (5^b)$ – number of sets $= 4 \times 2 = 8$

$(2^a \times 3^b), (5^c)$ – number of sets $= 6 \times 4 \times 2 = 48$

$(2^a \times 5^b), (3^c)$ – number of sets $= 6 \times 2 \times 4 = 48$

$(3^a \times 5^b), (2^c)$ – number of sets $= 4 \times 2 \times 6 = 48$

$\therefore$ Total number of such sets = 188.

Combining both the cases, total sets possible
$= 188 + 48 = 236$.

Question2

How many digits cannot be the unit's digit of the product of 3 three-digit numbers whose sum is 989?

Answer: Correct Option is: "**a**"

Solution:

We can have the following possible combinations of the unit's digit of the three 3-digit numbers whose sum is 9 or 19.

1. (0, 0 and 9): Last digit of the product is 0.
2. (1, 9 and 9): Last digit of the product is 1.
3. (1, 2 and 6): Last digit of the product is 2.
4. (2, 3 and 4): Last digit of the product is 4.
5. (1, 3 and 5): Last digit of the product is 5.
6. (1, 4 and 4): Last digit of the product is 6.
7. (1, 1 and 7): Last digit of the product is 7.
8. (3, 7 and 9): Last digit of the product is 9.

In total there are 8 possible digits which can be the last digit of the product of three 3-digit numbers.

So there are only two digits (3 and 8), which are not possible.

DIRECTIONS for Question 3 : Answer the question on the basis of the information given below.

A two-digit number having distinct digits when divided by the sum of the digits gives the same remainder as when a two-digit number that is formed by reversing the digits of original number is divided by the sum of the digits.

Question3	Out of all such possible two-digit numbers, a number is randomly picked. What is the probability that this number is divisible by 4?
Answer:	Correct Option is: "**a**"
Solution:	Out of the 16 such possible two-digit numbers only 12, 24, 36, 48, 72 and 84 are divisible by 4. $\text{Probability} = \frac{6}{16} = \frac{3}{8}$.
Question4	What is the remainder when $723^{243} + 318^{243}$ is divided by 17?
Answer:	Correct Option is: "**d**"

Solution:

$$\frac{(723^{243} + 318^{243})}{17}$$

$$= \frac{\left((731-8)^{243} + (323-5)^{243}\right)}{17}.$$

731 and 323 are divisible by 17; the net remainder is the same as when $(-8)^{243} + (-5)^{243}$ is divided by 17.

$$\frac{\left\{(-8)^{243} + (-5)^3 \times (-5)^{240}\right\}}{17}$$

$$= \frac{\left\{-2^{729} + (-5)^3 \times (625)^{60}\right\}}{17}$$

$$= \frac{\left\{(-2) \times (2^4)^{182} - (125) \times (612+13)^{60}\right\}}{17}$$

$$= \frac{\left\{(-2) \times (17-1)^{182} - (125) \times \left(17 \times 36k + (170-1)^{30}\right)\right\}}{17}$$

(where k is a natural number)

$$= \frac{\left\{(-2) \times (17-1)^{182} - (17 \times 7 + 6) \times \left(17 \times 36k + (17 \times 10 - 1)^{30}\right)\right\}}{17}$$

$\therefore$ The required remainder would be 17 – (2 + 6) = 9.

Question5	A natural number N, less than 3000, has the following properties: **I.** The number N when successively divided by 5, 4 and 3 gives remainder 3, 1 and 2 respectively. **II.** The number when divided by 72 gives remainder 12. Find how many such natural numbers exist.
Answer:	Correct Option is: "**a**"
Solution:	I. Any natural number when divided by 3 giving a remainder 2 can be represented by $3k + 2$. Any natural number when divided by 4 giving a remainder 1 and quotient '$3k + 2$' can be represented by $4(3k + 2) + 1$ and any natural number when divided by 5 giving a remainder 3 and quotient $[4(3k + 2) + 1]$ can be represented by $5[4(3k + 2) + 1] + 3 = 60k + 48$. ('k' is a natural number). II. Any natural number when divided by 72 giving a remainder 12 can be represented by $72n + 12$. Therefore, $72n + 12 = 60k + 48$. Or, $6n = 5k + 3$. First set of values of n and k that satisfy the above equation is 3 and 3 respectively. Now values of n will increase with a difference of 5 and values of k will increase with a difference of 6. Smallest such natural number $= 72 \times 3 + 12 = 228$ and second smallest such natural number $= 72 \times 8 + 12 = 588$. Values of n will increase with a difference of $72 \times 5 = 360$. So, $n = 228 + (m - 1) \times 360 = 360m - 132 < 3000$ Or, $360m < 3132$ or $m < 8.7$ Therefore 8 such natural numbers exist.

Question6

S is a set containing all the integers less than 21000, which are the product of three consecutive prime numbers. N is a non-empty subset of S, in which all the elements are relatively prime to each other.If the number of elements in N is maximum possible, then how many such distinct subsets are possible?

Answer: Correct Option is: "**b**"

Solution:

S = {(2 × 3 × 5), (3 × 5 × 7), (5 × 7 × 11), (7 × 11 × 13), (11 × 13 × 17), (13 × 17 × 19), (17 × 19 × 23), (19 × 23 × 29) and (23 × 29 × 31)}

So, S contains 9 elements.

Given that the number of elements in the set N is maximum possible.

Since, all the elements of the set N are relatively prime to each other, therefore the maximum possible number of elements in the set N could be 3.

Also, in the set N, there will be 1 element out of the first 3 elements of the set S, 1 element out of the next 3 elements of the set S and 1 element out of the last 3 elements of the set S.

This is possible in ten ways:

1.(2 × 3 × 5, 7 × 11 × 13, 17 × 19 × 23)
2.(2 × 3 × 5, 7 × 11 × 13, 19 × 23 × 29)
3.(2 × 3 × 5, 7 × 11 × 13, 23 × 29 × 31)
4.(2 × 3 × 5, 11 × 13 × 17, 19 × 23 × 29)
5.(2 × 3 × 5, 11 × 13 × 17, 23 × 29 × 31)
6.(2 × 3 × 5, 13 × 17 × 19, 23 × 29 × 31)
7.(3 × 5 × 7, 11 × 13 × 17, 19 × 23 × 29)
8.(3 × 5 × 7, 11 × 13 × 17, 23 × 29 × 31)
9.(3 × 5 × 7, 13 × 17 × 19, 23 × 29 × 31)
10.(5 × 7 × 11, 13 × 17 × 19, 23 × 29 × 31)

Question7

P is the product of first 30 multiples of 30. N is the total number of factors of P. In how many ways N can be written as the product of two natural numbers such that the HCF of these two natural numbers is 19?

Answer: Correct Option is: "**b**"

Solution:

$P = (1 \times 30) \times (2 \times 30) \times (3 \times 30) \times (4 \times 30) \times \ldots \times (29 \times 30) \times (30 \times 30).$

$P = 30^{30} \times 30!$

$P = 2^{56} \times 3^{44} \times 5^{37} \times 7^4 \times 11^2 \times 13^2 \times 17 \times 19 \times 23 \times 29.$

$N = 57 \times 45 \times 38 \times 5 \times 3 \times 3 \times 2^4 = 2^5 \times 3^5 \times 5^2 \times 19^2$

Let the two numbers be '19a' and '19b' respectively such that 'a' and 'b' are relatively prime to each other.

$\Rightarrow (19a) \times (19b) = N = 2^5 \times 3^5 \times 5^2 \times 19^2$

$\Rightarrow ab = 2^5 \times 3^5 \times 5^2$

Possible pairs (a, b) are

$(2^5 \times 3^5 \times 5^2, 1)$; $(2^5, 3^5 \times 5^2)$; $(2^5 \times 3^5, 5^2)$; $(2^5 \times 5^2, 3^5)$.

Therefore, N can be written as the product of two numbers, such that their HCF is 19, in 4 ways.

Short cut method: Here, the answer has to be a power of 2. (Why?)

Question8	Let M be a three-digit number denoted by 'ABC' where A, B and C are numerals from 0 to 9. Let N be a number formed by reversing the digits of M. It is known that M − N + (396 × C) is equal to 990. How many possible values of M are there which are greater than 300?
Answer:	Correct Option is: "**d**"
Solution:	see below

M = ABC and N = CBA

$\Rightarrow$ M – N + 396C

= (100A + 10B + C) – (100C + 10B + A) + 396C

$\Rightarrow$ M – N + 396 C = 99(A – C + 4C)

$\therefore$ 99(A + 3C) = 990 $\Rightarrow$ A + 3C = 10

Possible values of A and C that satisfy A + 3C = 10 or

$C = \frac{1}{3}(10 - A)$ are given by :

A	1	4	7
C	3	2	1

Now, since the three digit number is greater than 300. A cannot be equal to 1. B can take any value from 0 to 9.

Therefore the number of such three digit numbers

= 10 × 2 = 20.

Question9	g(P) represents the product of all the digits of P, e.g. g(45) = 4 × 5. What is the value of g(67) + g(68) + g(69) + ... + g(122) + g(123)?
Answer:	Correct Option is: "**b**"
Solution:	see below

$$S = g(67) + g(68) + ... + g(123)$$

$$\Rightarrow S = [g(67) + g(68) + g(69)] + [g(70) + g(71) + ... + g(99)]$$
$$+ [g(100) + g(101) + ... + g(123)]$$

$$\Rightarrow S = [6 \times (7+8+9)] + [(7+8+9) \times (1+2+3+.....+9)]$$
$$+ [1 \times 1 \times (1+2+3+.....+9) + 1 \times 2(1+2+3)]$$

$$\Rightarrow S = 6 \times 24 + 24 \times 45 + 45 + 2 \times 6$$

$$\therefore S = 1281.$$

Question10	The product of three positive integers is 6 times their sum. One of these integers is the sum of the other two integers. If the product of these three numbers is denoted by P, then find the sum of all distinct possible values of P.
Answer:	Correct Option is: "**d**"
Solution:	Let the numbers be K, L, K + L, $\Rightarrow$ KL (K + L) = 12 (K + L) $\Rightarrow$ KL = 12. $\Rightarrow$ {K, L} = {1, 12}, {2, 6} or {3, 4} Hence, product P = 156, 96 or 84 $\therefore$ Required sum = 156 + 96 + 84 = 336.