NOTES FOR GAME THEORY - CONTRACT PART

AARON NOTES SERIES

Aaron Xia
Department of Electronic Engineering
Tsinghua University
HaiDian District, Peking
muranqz@gmail.com

May 23, 2019

1 Contract theory in continuous time

1.1 Introduciton

The main topic of this part is mathematical modeling and analysis of contracting between two parties, **Principal** and **Agent**, in an uncertain environment. As a typical example of a **Principal–Agent problem**, henceforth the **PA problem**, we can think of the principal as an investor (or a group of investors), and of the agent as a portfolio manager who manages the investors' money. Another interesting example from Finance is that of a company (as the principal) and its chief executive (as the agent). As may be guessed, the principal offers a contract to the agent who has to perform a certain task on the principal's behalf (in our model, it's only one type of task).

The economic problem is for the principal to construct a contract in such a way that: (i) the agent will accept the contract; this is called an *individual rationality (IR) constraint*, or a *participation constraint*; (ii) the principal will get the most out of the agent's performance,in terms of expected utility. How this should be done in an optimal way, depends crucially on the amount of information that is available to P and to A. There are three classical cases studied in the literature, and which we also focus on in this part: *Risk Sharing (RS)* with symmetric information, *Hidden Action (HA)* and *Hidden Type (HT)*.

1.2 The general risk sharing problem

1.2.1 The model and the PA problem

Let $\{B_t\}_{t\geqslant 0}$ be a d-dimensional Brownian motion on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and denote by $\boldsymbol{F} := \{\mathcal{F}_t\}_{t\leqslant T}$ its augmented filtration on the interval $[0, T]$. The output process is denoted $X = X^{u,v}$ and its dynamics are given by

$$dX_t = b(t, X_t, u_t, v_t)dt + v_t dB_t \tag{1.2.1}$$

where (u, v) take values in $A_1 \times A_2 \subset \mathbb{R}^m \times \mathbb{R}^d$, and b is a function taking values in $\mathbb{R}$, possibly random and such that, as a process, it is $\boldsymbol{F}$-adapted. The notation xy for two vectors $x, y \in \mathbb{R}^d$ indicates the inner product.

The principal offers the agent compensation $C_T = C(\omega, X)$ at time T, where $C : \Omega \times C[0, T] \to A_3 \subset \mathbb{R}$ is a mapping such that C_T is $\mathcal{F}_T$ measurable. Introduce the accumulated cost of the agent,

$$G_T = G_T^{u,v} := \int_0^T g(t, X_t, u_t, v_t)dt \tag{1.2.2}$$

The risk sharing problem is

$$\max_{C,u,v} J(C_T, u, v) := \max_{C,u,v} \mathbb{E}[U_P(X_T - C_T) + \lambda U_A(C_T, G_T)]. \tag{1.2.3}$$

The functions U_A and U_P are utility functions of the agent and the principal. The function g is a cost function. Typical cases studied in the literature are the *separable utility case* with $U_A(x, y) = U_A(x) - y$, and the *non-separable utility case* with $U_A(x, y) = U_A(x - y)$, where, with a slight abuse of notation, we use the same notation U_A also for the function of one argument only.

References