

# Assignment 2

## ICSE 2018 Class 12 Q.5(a)

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# Question

Show that the function,

$$f(x) = \begin{cases} x^2, & x \leq 1 \\ \frac{1}{x}, & x > 1 \end{cases}$$

is continuous at  $x = 1$  but not differentiable.

# Solution

Given

$$f(x) = \begin{cases} x^2, & x \leq 1 \\ \frac{1}{x}, & x > 1 \end{cases}$$

We can say  $f$  is continuous at  $x = 1$ , iff

$$\lim_{x \rightarrow 1} f(x) = f(1) \quad (1)$$

In other words  $f$  should satisfy,

$$f(1^-) = f(1^+) = f(1) \quad (2)$$

where,

$$f(1^-) = \lim_{h \rightarrow 0} f(1 - h) \quad (3)$$

$$f(1^+) = \lim_{h \rightarrow 0} f(1 + h) \quad (4)$$

$$f(1) = 1 \quad (5)$$

# Continuity at $x = 1$

Now,

$$f(1^-) = \lim_{h \rightarrow 0} f(1 - h) \quad (6)$$

$$= \lim_{h \rightarrow 0} (1 - h)^2 \quad (7)$$

$$\implies f(1^-) = 1 \quad (8)$$

And,

$$f(1^+) = \lim_{h \rightarrow 0} f(1 + h) \quad (9)$$

$$= \lim_{h \rightarrow 0} \frac{1}{(1 + h)} \quad (10)$$

$$\implies f(1^+) = 1 \quad (11)$$

Using eq 5 ,eq 8,eq 11,we can say that  $f$  is continuous at  $x = 1$ .

# Differentiability at $x = 1$

We can say that  $f$  is differentiable at  $x = 1$  iff the limit,

$$\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$$

exists.

In that case  $f$  should satisfy,

$$\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{f(1) - f(1-h)}{h} \quad (12)$$

## LHD

$$\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{\left(\frac{1}{(1+h)}\right) - 1}{h} \quad (13)$$

$$= \lim_{h \rightarrow 0} \frac{(1 - (1+h))}{h(1+h)} \quad (14)$$

$$= \lim_{h \rightarrow 0} \frac{-h}{h(1+h)} \quad (15)$$

$$= \lim_{h \rightarrow 0} \frac{-1}{(1+h)} \quad (16)$$

$$= -1 \quad (17)$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = -1. \quad (18)$$

## RHD

RHS:

$$\lim_{h \rightarrow 0} \frac{f(1) - f(1-h)}{h} = \lim_{h \rightarrow 0} \frac{1 - ((1-h)^2)}{h} \quad (19)$$

$$= \lim_{h \rightarrow 0} \frac{-(2h - h^2)}{h} \quad (20)$$

$$= \lim_{h \rightarrow 0} \frac{h(2-h)}{h} \quad (21)$$

$$= \lim_{h \rightarrow 0} -(2-h) \quad (22)$$

$$= 2 \quad (23)$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{f(1) - f(1-h)}{h} = 2. \quad (24)$$

$\therefore LHS \neq RHS$

Hence, function  $f(x)$  is not differentiable at  $x = 1$ .



# Conclusion

Therefore, we proved that  $f(x) = \begin{cases} x^2, & x \leq 1 \\ \frac{1}{x}, & x > 1 \end{cases}$  is continuous at  $x = 1$  but not differentiable.