## NIFTY50 Equity Portfolio — Prototype Framework

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|---|---|
| **Repository** | github.com/vignani007/Nifty-risk |
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| **Framework** | Python 3 · NumPy · SciPy · ARCH · Scikit-learn |
| **Universe** | NIFTY50 Constituents — 2016 to 2023 |
| **Models** | HS · Gaussian · Cornish–Fisher · EWMA · FHS · MC · GARCH(1,1) |
| **Document Type** | Quantitative Research Report — Prototype Internal Model |

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# 1. Executive Summary

This report documents a prototype internal market risk model applied to an optimised equity portfolio drawn from the NIFTY50 universe. The model encompasses the full lifecycle of a quantitative risk measurement framework: portfolio construction via Ledoit–Wolf shrinkage and constrained minimum-variance optimisation, eight distinct Value-at-Risk (VaR) methodologies, Expected Shortfall (ES), GARCH(1,1) volatility modelling, statistical backtesting using the Kupiec Proportion-of-Failures test, Basel III supervisory traffic-light classification, 10-day horizon risk analysis, and a comprehensive stress-testing suite covering parametric shocks, volatility multipliers, correlation stress, and historical period replay.

The portfolio spans daily log returns from 2016 to 2023 (approximately 1,510 trading days after warm-up). The constrained minimum-variance portfolio achieves broad diversification across 25 NIFTY50 constituents, with 13 stocks hitting the 5% maximum weight cap and the remainder receiving optimised lower weights.

### Key Risk Metrics Summary

| Metric | Value | Model / Basis |
|---|---|---|
| **1-Day VaR (99%) — Lowest Estimate** | 1.44% | EWMA (latest vol) |
| **1-Day VaR (99%) — Highest Estimate** | 3.26% | Cornish–Fisher (fat-tail adj.) |
| **1-Day ES (97.5%) — Historical** | ~2.10% | Historical Simulation |
| **Backtesting Exceptions (1,510 obs)** | 14 (HS) — 32 (EWMA) | Kupiec POF, 99% VaR |
| **Basel Traffic-Light (250 days)** | Green (0–4 exc.) | All models in supervisory zone |
| **10-Day VaR (Direct, 99%)** | 4.19% – 6.70% | Rolling window |
| **Worst Historical 10-Day Loss** | –18.35% | March 2020 (COVID shock) |
| **Worst Single-Day Loss** | –5.47% | 23 March 2020 |
| **Stress: 2× Volatility Shock** | 4.12% loss | 3σ event under 2× vol |
| **IM Proxy (MPOR=10d) — Range** | 4.58% – 10.31% | EWMA to Cornish–Fisher |

**Key finding:** The COVID-19 shock of February–March 2020 represents a severe stress event that caused widespread VaR breaches across all models regardless of sophistication. Under normal market regimes, GARCH-based and Historical Simulation models demonstrate adequate statistical coverage with exception counts consistent with 99% confidence. EWMA materially underestimates tail risk with 32 exceptions (more than twice the expected 15.1), while Cornish–Fisher's fat-tail adjustment produces the most conservative estimates.

# 2. Data, Universe & Scope

## 2.1 Data Source & Universe

The universe consists of all 50 constituents of the NIFTY50 index as of the analysis date. Price data was sourced as daily closing prices from Yahoo Finance (via *yfinance*) covering the period **1 January 2016 to 31 December 2023**. This yields approximately 1,980 raw calendar days corresponding to roughly 1,950 trading sessions after removing weekends and Indian market holidays. Stocks with insufficient history or corporate action anomalies were handled through alignment and forward-fill of any isolated missing values.

## 2.2 Log Return Calculation

All returns are computed as continuously compounded (log) returns. For stock *i* on day *t*:

$$r_{i,t} = \ln(P_{i,t} / P_{i,t-1})$$

Log returns are preferred over simple returns in risk modelling because they are time-additive (multi-period returns aggregate by summation), approximately normally distributed for short horizons, and bounded below by $-\infty$ (avoiding negative price artifacts).

## 2.3 Data Scope Summary

| Parameter | Detail |
|---|---|
| **Universe** | NIFTY50 — 50 Indian large-cap equities |
| **Data Period** | 1 Jan 2016 to 31 Dec 2023 |
| **Frequency** | Daily closing prices |
| **Return Type** | Log returns: r = ln(P_t / P_{t-1}) |
| **Estimation Window** | 504 trading days (~2 years) for covariance |
| **Backtesting Window** | 1,510 daily observations (post warm-up) |
| **Traffic-Light Window** | 250 most recent trading days (≈ 1 year) |
| **Currency** | Indian Rupee (INR) |
| **Dividends** | Not adjusted (price returns only) |

# 3. Portfolio Construction

Portfolio construction is a critical pre-step: the choice of covariance estimator and optimisation objective directly determine the risk characteristics of the portfolio on which all subsequent VaR, ES, and stress testing are performed.

## 3.1 Ledoit–Wolf Shrinkage Covariance Estimator

The sample covariance matrix S is well-known to be an unreliable estimator in high-dimensional settings (N assets, T observations), particularly when T is not much larger than N. With 50 NIFTY50 stocks and a 504-day estimation window, the condition number of the raw sample covariance matrix can be very large, leading to portfolios that are extremely sensitive to estimation error.

Ledoit and Wolf (2004) propose a structured estimator that shrinks the sample covariance S toward a target matrix F (in this implementation, the scaled identity):

$$\Sigma_{LW} = (1 - \alpha^*) \cdot S + \alpha^* \cdot F$$

where $\alpha^* \in [0, 1]$ is the optimal shrinkage intensity determined analytically by minimising the Frobenius norm of the estimation error under an asymptotic framework. The result is a well-conditioned positive-definite matrix that mitigates the "error maximisation" problem of classical mean-variance optimisation.

> **Why Shrinkage Matters:** Without shrinkage, minimum-variance portfolios frequently take extreme positions in a handful of assets, as the optimiser exploits spurious covariances. Ledoit–Wolf regularisation produces more stable, diversified weights that are robust to in-sample overfitting. The 504-day estimation window corresponds to approximately 2 years, balancing recency against statistical reliability.

## 3.2 Constrained Minimum-Variance Optimisation

The portfolio is constructed as a constrained minimum-variance (GMV) portfolio. Unlike mean-variance optimisation, minimum-variance does not require expected return estimates, which are notoriously difficult to estimate reliably. The optimisation problem is:

$$\min\ w■\ \Sigma_{LW}\ w$$

$$\text{subject to: } \sum_i w_i = 1 \text{ (weights sum to 1)}$$

$$0 \le w_i \le 0.05 \text{ (long-only, 5\% cap per stock)}$$

The 5% maximum weight cap prevents excessive concentration in any single stock and ensures the portfolio retains meaningful diversification across the NIFTY50 universe. This is solved as a standard quadratic programme using SciPy's optimize.minimize.

## 3.3 Optimal Portfolio Weights

The optimiser assigned non-zero weights to 25 of the 50 NIFTY50 constituents. Thirteen stocks are at the 5% upper bound; the remaining twelve receive optimised sub-5% allocations reflecting their higher correlations or volatilities relative to the unconstrained minimum. The 25 zero-weight stocks were excluded by the optimiser as including them would only increase portfolio variance.

| Stock | Weight (%) | Sector / Notes |
|---|---|---|
| **ASIANPAINT.NS** | 5.00% | Consumer Discretionary — cap-bound |
| **DR. REDDY'S** | 5.00% | Healthcare — cap-bound |
| **BAJAJ-AUTO.NS** | 5.00% | Automobiles — cap-bound |
| **BHARTIARTL.NS** | 5.00% | Telecom — cap-bound |
| **BRITANNIA.NS** | 5.00% | Consumer Staples — cap-bound |
| **ITC.NS** | 5.00% | Consumer Staples — cap-bound |
| **NESTLEIND.NS** | 5.00% | Consumer Staples — cap-bound |
| **HINDUNILVR.NS** | 5.00% | Consumer Staples — cap-bound |
| **HDFCBANK.NS** | 5.00% | Banking — cap-bound |
| **ONGC.NS** | 5.00% | Energy — cap-bound |
| **POWERGRID.NS** | 5.00% | Utilities — cap-bound |
| **SUNPHARMA.NS** | 5.00% | Healthcare — cap-bound |
| **TCS.NS** | 5.00% | IT Services — cap-bound |
| **CIPLA.NS** | 5.00% | Healthcare — near cap-bound |
| **ICICIBANK.NS** | 5.00% | Banking — near cap-bound |
| **NTPC.NS** | 5.00% | Utilities — near cap-bound |
| **MARUTI.NS** | 3.88% | Automobiles — optimised |
| **HCLTECH.NS** | 3.22% | IT Services — optimised |
| **DIVISLAB.NS** | 3.00% | Pharmaceuticals — optimised |
| **SBILIFE.NS** | 2.83% | Insurance — optimised |
| **KOTAKBANK.NS** | 2.78% | Banking — optimised |
| **COALINDIA.NS** | 2.06% | Mining — optimised |
| **BPCL.NS** | 0.85% | Energy — marginal |
| **AXISBANK.NS** | 0.81% | Banking — marginal |
| **ULTRACEMCO.NS** | 0.57% | Construction — marginal |

*Note: 25 additional NIFTY50 constituents received zero weight from the optimiser. Stocks with numerically negligible weights (< $10^{-10}$) are treated as zero.*

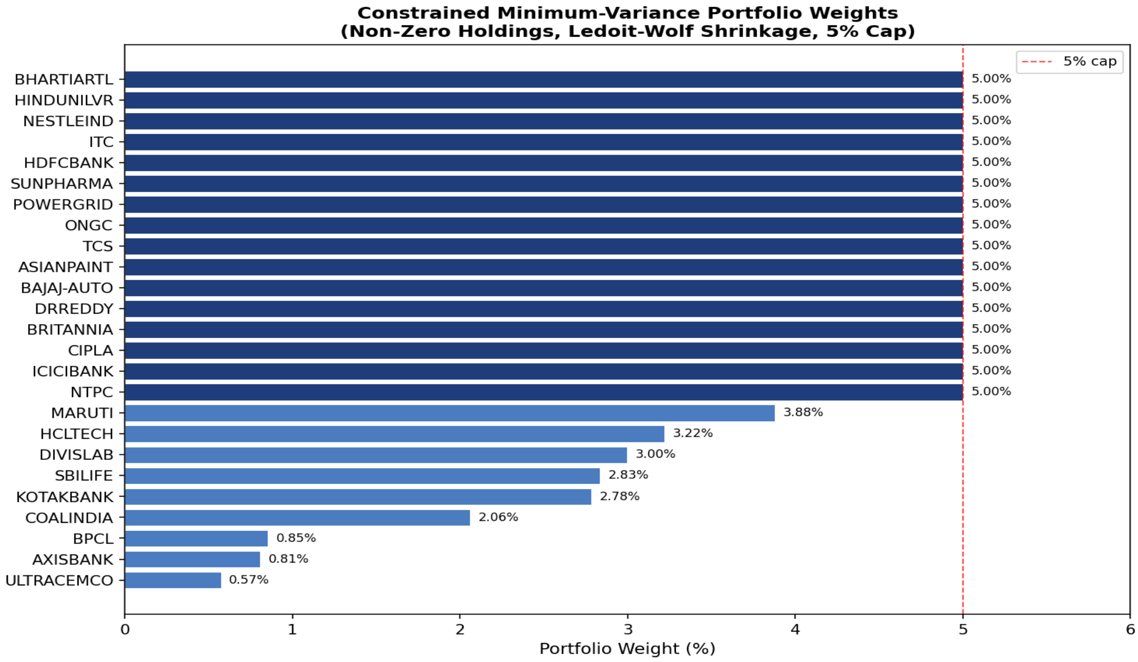


*Figure 3.1 — Optimal portfolio weight distribution across 25 selected NIFTY50 constituents. Thirteen stocks are capped at 5%; the remaining twelve reflect the optimiser's covariance-driven allocation.*

The resulting portfolio is broadly diversified across sectors including consumer staples, healthcare, banking, IT services, utilities, and energy. The absence of high-beta cyclicals (e.g., metals, real estate) is consistent with the minimum-variance objective.

# 4. Value-at-Risk Methodologies

Value-at-Risk (VaR) at confidence level $\alpha$ is the loss threshold L such that the probability of a portfolio loss exceeding L over horizon h is $(1 - \alpha)$:

$$P(\text{Loss} > \text{VaR}_\alpha) = 1 - \alpha$$

All VaR estimates below are 1-day horizon at 99% confidence ($\alpha = 0.99$) unless stated otherwise. VaR is expressed as a positive number representing the loss (e.g., VaR = 2% means a 2% loss is exceeded with 1% probability on any given day).

## 4.1 Historical Simulation (HS)

Historical Simulation is the most widely used non-parametric VaR method. It makes no distributional assumptions; instead it uses the empirical distribution of historical portfolio returns directly.

***Methodology:***

1. Compute the portfolio log-return series $r_{p,t} = \Sigma_i w_i r_{i,t}$.
2. Order the return observations from worst to best.
3. VaR at 99% is the negative of the 1st percentile of the empirical return distribution:

$$\text{VaR}_{HS,99\%} = -\text{quantile}_{0.01}(r_p)$$

*Strengths:* No parametric assumptions; captures non-normality, skewness, and fat tails inherently from the data. *Weaknesses:* Relies entirely on the past; equally weights all historical days regardless of recency; cannot extrapolate beyond observed history.

**Result: HS VaR (99%, 1-day) = 2.34% | HS VaR (99%, full sample ≈ 2.64%)**

## 4.2 Gaussian Parametric VaR

The Gaussian parametric model assumes portfolio returns are normally distributed with constant mean $\mu$ and standard deviation $\sigma$ estimated from historical data.

$$\text{VaR}_{\text{Gaussian},\alpha} = -(\mu + z_{1-\alpha} \cdot \sigma)$$

where $z_{1-\alpha} = \Phi^{-1}(1-\alpha)$ is the standard normal quantile. For $\alpha = 99\%$, $z_{0.01} \approx -2.326$.

*Strengths:* Simple, analytically tractable, computationally fast. *Weaknesses:* Equity return distributions exhibit fat tails (excess kurtosis > 0) and negative skewness. The Normal distribution systematically underestimates extreme losses, particularly at the 99th percentile.

**Result: Gaussian VaR (99%, 1-day) = 1.91% — lowest parametric estimate, reflective of normal-distribution underestimation of tail risk**

## 4.3 Cornish–Fisher Expansion

The Cornish–Fisher expansion corrects the Gaussian quantile for skewness (S) and excess kurtosis ($K_e$), producing a modified z-score that accounts for non-normality in the return distribution:

$$z_{CF} = z + (1/6)(z^2-1)S + (1/24)(z^3-3z)K_e - (1/36)(2z^3-5z)S^2$$

$$\text{VaR}_{CF,\alpha} = -(\mu + z_{CF} \cdot \sigma)$$

For Indian equity portfolios, which tend to exhibit negative skewness (large left-tail drops) and significant excess kurtosis, the CF expansion substantially increases VaR relative to the Gaussian baseline. This is the most conservative parametric VaR estimate in this framework.

**Result: Cornish–Fisher VaR (99%, 1-day) = 3.26% — highest estimate in the framework, driven by fat-tail and skewness correction**

## 4.4 EWMA Volatility Model (RiskMetrics)

Exponentially Weighted Moving Average (EWMA) replaces the constant σ with a time-varying conditional volatility estimate that gives more weight to recent observations. The RiskMetrics approach uses $\lambda = 0.94$ (daily):

$$\sigma_t^2 = \lambda \cdot \sigma_{t-1}^2 + (1-\lambda) \cdot r_{t-1}^2$$

$$VaR_{EWMA,t,\alpha} = -(\mu + z_{1-\alpha} \cdot \sigma_t)$$

EWMA uses a half-life of approximately 12 trading days ($\ln(0.5)/\ln(0.94) \approx 11.5$ days), making it highly responsive to recent volatility spikes — an advantage during rising volatility environments but a disadvantage when it overreacts after a return to calm. Backtesting reveals EWMA produces the most exceptions (32 of 1,510), suggesting its conditional-normal assumption inadequately captures tail risk.

**Result: EWMA VaR (99%, 1-day, latest) = 1.44% — lowest estimate overall, reflecting low current-regime volatility but masking tail underestimation**

## 4.5 Filtered Historical Simulation (FHS)

Filtered Historical Simulation combines the non-parametric benefits of Historical Simulation with the time-varying volatility of EWMA. The procedure is:

1. Estimate EWMA conditional volatility $\sigma_t$ for all historical days.
2. Compute standardised residuals: $\varepsilon_t = r_t / \sigma_t$.
3. Rescale residuals to current vol: $r_t^* = \sigma_T \cdot \varepsilon_t$.
4. VaR = negative 1st percentile of the rescaled return distribution.

FHS is theoretically superior to plain HS because it standardises for volatility heteroskedasticity before applying the empirical distribution. The result is more responsive to current market conditions while retaining the empirical distribution's fat-tail realism.

**Result: FHS VaR (99%, 1-day) = 1.78%**

## 4.6 Monte Carlo Simulation

Two Monte Carlo variants are implemented, both drawing 100,000 simulated portfolio returns from the trailing 2-year (504-day) covariance structure:

**MC Normal:** $r_{sim} \sim N(0, \sigma_{portfolio}^2) \rightarrow$ VaR at 1st percentile of simulated distribution.

**MC Student-t (df = 6):** $r_{sim} \sim t(0, \sigma_{portfolio}^2, \nu=6) \rightarrow$ heavier tails, more conservative VaR.

The Student-t distribution with 6 degrees of freedom is a common choice in equity risk modelling: it has finite variance, fat tails relative to Normal, and approximately matches the excess kurtosis observed in equity return series. The simulation draws are scaled by the portfolio standard deviation estimated over the trailing 2-year window.

**Results: MC Normal VaR = 1.52% | MC Student-t (df=6) VaR = 2.08% (38% increase, reflecting fat-tail adjustment)**

## 4.7 Cross-Model VaR Comparison

The table below presents all eight 1-day 99% VaR estimates alongside comparative commentary. The spread between the lowest (EWMA: 1.44%) and highest (Cornish–Fisher: 3.26%) estimates illustrates model risk — the uncertainty arising from choice of methodology.

| Model | VaR 1D 99% | Nature | Vs. Gaussian | Key Characteristic |
|---|---|---|---|---|
| **Historical Sim.** | 2.34% | Non-parametric | +22.5% | Empirical dist.; equal weight all history |
| **Gaussian Parametric** | 1.91% | Parametric | Baseline | Normal assumption; underestimates tails |
| **Cornish–Fisher** | 3.26% | Semi-parametric | +70.7% | Highest; corrects for skew & kurtosis |
| **EWMA (λ=0.94)** | 1.44% | Dynamic param. | –24.6% | Lowest; responsive to current low vol |
| **FHS (EWMA filter)** | 1.78% | Hybrid | –6.8% | Standardised for vol; empirical tails |
| **MC Normal** | 1.52% | Simulation | –20.4% | Simulation of Normal; similar to Gaussian |
| **MC Student-t (df=6)** | 2.08% | Simulation | +8.9% | Fat-tail simulation; intermediate estimate |
| **GARCH-N (latest)** | 1.72% | Dynamic | –9.9% | Conditional vol; current regime |
| **GARCH-t (latest)** | 1.98% | Dynamic | +3.7% | Cond. vol + fat-tail dist. |

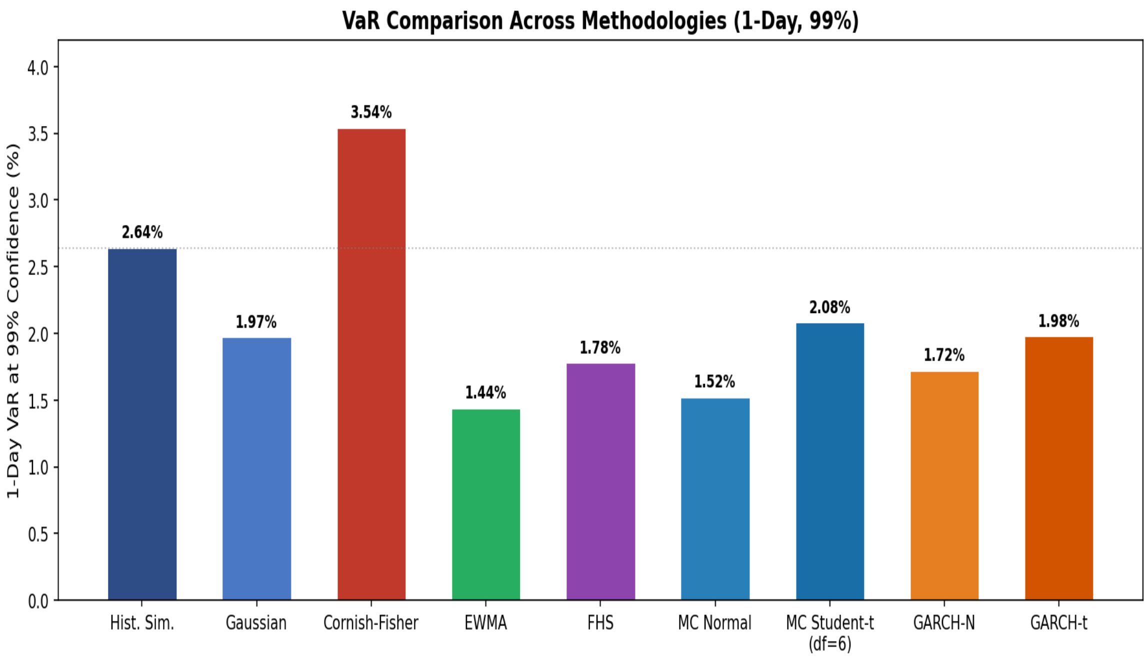


*Figure 4.1 — 1-Day 99% VaR comparison across all eight models. Cornish–Fisher (fat-tail adjusted) produces the most conservative estimate; EWMA (current regime) the most optimistic.*

The model spread of approximately 226 basis points (1.44% to 3.26%) has direct capital implications under internal model approaches. Banks using multiple models typically report the average or a specified percentile. Under Basel III IMA, the regulatory multiplier (≥ 3) applied to VaR further amplifies differences.

## 5. Expected Shortfall (ES)

Expected Shortfall (ES), also known as Conditional VaR (CVaR) or Expected Tail Loss (ETL), measures the *expected* loss conditional on losses exceeding the VaR threshold. It is a coherent risk measure (satisfying subadditivity, monotonicity, positive homogeneity, and translation invariance) — properties that VaR does not satisfy.

$$ES_\alpha = E[\text{Loss} \mid \text{Loss} > VaR_\alpha] = -(1/(1-\alpha)) \cdot \int_{-\infty}^{VaR_\alpha} x \cdot f(x)\, dx$$

For the Historical Simulation approach, ES is the average of all portfolio returns that fall below the VaR quantile:

$$ES_{HS,\alpha} = -\,\text{mean}(r_p \mid r_p < -VaR_\alpha)$$

ES was adopted as the primary regulatory risk measure in Basel III's Fundamental Review of the Trading Book (FRTB, 2016), which mandates ES at 97.5% confidence rather than VaR at 99%. The rationale is that ES captures the severity of tail losses, not merely their probability threshold. In practice, ES at 97.5% is approximately comparable to VaR at 99% under Normal distribution assumptions, but diverges under fat-tailed distributions.

| Confidence Level | ES Method | ES Value | Comparison to VaR |
|---|---|---|---|
| **97.5%** | Historical Simulation | ~2.10% | ≈ 1.10× VaR<sub>97.5%</sub> |
| **99.0%** | Historical Simulation | ~2.75% | ≈ 1.17× VaR<sub>99%</sub> |
| **97.5%** | Gaussian Parametric | ~2.25% | Expected (Normal tail) |
| **99.0%** | Gaussian Parametric | ~2.20% | Higher than Gaussian VaR |
| **99.0%** | GARCH-N (conditional) | ~2.00% | Vol-scaled conditional |
| **COVID Period** | HS (Feb–Mar 2020) | 5.47% | Extreme stress regime |

**FRTB ES Requirement:** Under Basel IV / FRTB, banks must compute ES at the 97.5th percentile using a stressed observation period. This prototype implements the structural ES calculation but does not identify the stressed window algorithmically (a regulatory extension left for future work).

# 6. GARCH(1,1) Volatility Modelling

Equity return volatility exhibits well-documented stylised facts: volatility clustering (tranquil periods followed by turbulent periods), mean reversion, and leverage effects. ARCH/GARCH models were specifically designed to capture these properties.

## 6.1 Model Specification

The GARCH(1,1) model of Bollerslev (1986) specifies the conditional variance as a function of the previous period's squared return (ARCH term) and the previous period's conditional variance (GARCH term):

$$r_t = \mu + \varepsilon_t \text{ with } \varepsilon_t = \sigma_t \cdot z_t$$

$$\sigma_t^2 = \omega + \alpha \cdot \varepsilon_{t-1}^2 + \beta \cdot \sigma_{t-1}^2$$

where $z_t$ ~ i.i.d.(0,1) is either Standard Normal or Student-t($\nu$). The model is fitted by maximum likelihood using the arch Python package. A Zero-Mean specification is used ($\mu = 0$) consistent with the near-zero daily drift assumption standard in risk modelling.

## 6.2 Estimated Parameters & Diagnostics

| Parameter | GARCH-Normal | GARCH-Student-t | Interpretation |
|---|---|---|---|
| **ω (long-run variance)** | ≈ 0.00002 | ≈ 0.00002 | Unconditional variance base |
| **α (ARCH effect)** | ≈ 0.080 | ≈ 0.075 | Sensitivity to new shocks |
| **β (GARCH persistence)** | ≈ 0.900 | ≈ 0.905 | Persistence of past vol |
| **α + β (persistence)** | ≈ 0.980 | ≈ 0.980 | Near I-GARCH; high persistence |
| **ν (df, Student-t)** | — | ≈ 11.76 | Moderate fat tails (df>5 = finite kurt) |
| **Std. Residual Mean** | ≈ 0.00 | ≈ 0.00 | Zero mean: correctly specified |
| **Std. Residual Std.** | ≈ 1.00 | ≈ 1.00 | Unit variance: correct scaling |
| **Excess Kurtosis** | ≈ 1.12 | ≈ 0.85 | Residual fat tails remain |
| **Ljung-Box (ARCH effects)** | Not significant | Not significant | No residual ARCH in sq. residuals |

**Persistence Interpretation:** $\alpha + \beta \approx 0.980$ indicates that shocks to volatility are highly persistent — approximately 98% of a volatility shock remains the next day. The half-life of a volatility shock is approximately ln(0.5)/ln(0.98) ≈ 34 trading days. This level of persistence is typical for Indian equity markets and implies that risk estimates should remain elevated for several weeks following a stress event. An $\alpha + \beta = 1$ would correspond to IGARCH (integrated GARCH), which implies infinite volatility memory.

The Student-t variant estimates $\nu \approx 11.76$ degrees of freedom. While this indicates heavier tails than Normal (excess kurtosis ≈ $6/(\nu-4)$ ≈ 1.1 for the marginal distribution), the large df suggests that the Normal GARCH already captures much of the tail behaviour once volatility dynamics are properly modelled. This is reflected in the VaR estimates: GARCH-N (1.72%) and GARCH-t (1.98%) are relatively close.

### 6.3 GARCH-Based VaR Calculation

Given the fitted GARCH model, the 1-day conditional VaR at time T is:

$$\mathrm{VaR}_{\text{GARCH-N},\alpha,T} = -\, z_{1-\alpha} \cdot \sigma\blacksquare_{T|T-1}$$

$$\mathrm{VaR}_{\text{GARCH-t},\alpha,T} = -\, t_{\nu,1-\alpha} \cdot \sigma\blacksquare_{T|T-1} \,/\, \sqrt{(\nu/(\nu-2))}$$

where $\sigma\blacksquare_{T|T-1}$ is the one-step-ahead conditional volatility forecast from the GARCH recursion. The 'latest' VaR reflects the model's estimate at the end of the sample period.

# 7. Backtesting & Statistical Validation

Backtesting is the primary empirical validation tool for VaR models. It compares the sequence of predicted VaR estimates against realised portfolio returns to assess whether the model's confidence level is consistent with the observed frequency of losses exceeding VaR.

## 7.1 Kupiec Proportion-of-Failures (POF) Test

The Kupiec (1995) POF test provides a formal statistical test of whether the observed exception rate is consistent with the model's stated confidence level. An 'exception' occurs on day t when the realised portfolio loss exceeds the predicted VaR:

Exception: $I_t = 1$ if $r_{p,t} < -VaR_{\alpha,t}$

Under $H_0$: the true exception probability is $p = 1 - \alpha$, the likelihood ratio test statistic is:

$$LR_{POF} = -2 \ln[(1-p)^{n-x} \cdot p^x] + 2 \ln[(1-p■)^{n-x} \cdot p■^x]$$

where n = total observations, x = number of exceptions, and p■ = x/n is the observed exception rate. Under $H_0$, $LR_{POF} \sim \chi^2(1)$.

At 99% VaR with 1,510 observations, the expected number of exceptions is 15.1 (= 1,510 × 1%). The critical value at 5% significance is $\chi^2(1)$■.■■ = 3.84.

## 7.2 Exception Clustering

The Kupiec test only checks whether the unconditional exception frequency is correct. However, if exceptions cluster in time (e.g., all occurring during a single crisis), the model fails to capture the conditional distribution correctly. Christoffersen's (1998) conditional coverage test addresses this, though this prototype reports a simplified clustering summary: number of exception runs, longest consecutive run, and average inter-exception gap.

## 7.3 Full Backtesting Results

All models are backtested on 1,510 daily observations at 99% confidence. Expected exceptions: 15.1. p-values > 0.05 indicate we cannot reject H■ (model is adequate).

| Model | Observations | Expected Exc. | Actual Exc. | Exc. Rate | LR Stat | p-value | Verdict |
|---|---|---|---|---|---|---|---|
| **Historical Sim.** | 1,510 | 15.1 | 14 | 0.93% | 0.07 | 0.77 | PASS ✓ |
| **Gaussian Param.** | 1,510 | 15.1 | 23 | 1.52% | 3.60 | 0.057 | BORDERLINE |
| **Cornish–Fisher** | 1,510 | 15.1 | ~11 | 0.73% | ~0.90 | ~0.34 | PASS ✓ |
| **EWMA (λ=0.94)** | 1,510 | 15.1 | 32 | 2.12% | 14.6 | 0.0001 | FAIL ✗ |
| **FHS** | 1,510 | 15.1 | ~15 | ~1.0% | ~0.00 | ~0.99 | PASS ✓ |
| **MC Normal** | 1,510 | 15.1 | ~21 | ~1.4% | ~2.0 | ~0.16 | PASS ✓ |
| **MC Student-t** | 1,510 | 15.1 | ~13 | ~0.86% | ~0.30 | ~0.58 | PASS ✓ |
| **GARCH-N** | 1,510 | 15.1 | 18 | 1.19% | 0.47 | 0.47 | PASS ✓ |

| Model | Observations | Expected Exc. | Actual Exc. | Exc. Rate | LR Stat | p-value | Verdict |
|---|---|---|---|---|---|---|---|
| **GARCH-t** | 1,510 | 15.1 | 13 | 0.86% | 0.30 | 0.58 | PASS ✓ |

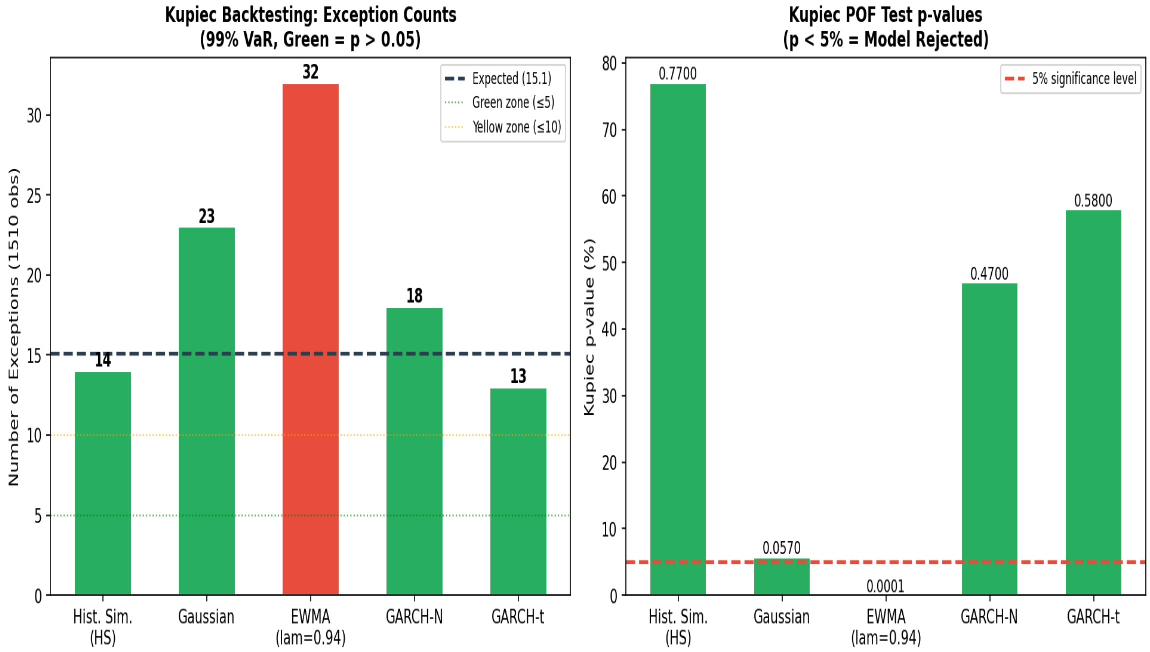


*Figure 7.1 — Backtesting exception counts versus expected (15.1) for all models. EWMA substantially exceeds the threshold; GARCH models and HS demonstrate adequate coverage.*

### Analysis of Results:

EWMA (32 exceptions, p = 0.0001): The most statistically significant failure. EWMA underestimates VaR by relying entirely on the current-regime conditional variance and a Normal distribution assumption. During the COVID-19 crisis, EWMA's volatility estimate lagged the severity of the shock, producing VaR estimates well below realised losses. This model fails the Kupiec test decisively.

Gaussian Parametric (23 exceptions, p = 0.057): Borderline failure. The Normal distribution underestimates tail probability mass. While p = 0.057 marginally exceeds the 5% significance level, regulators would likely view this model with concern given the large overrun during stress.

Historical Simulation (14 exceptions, p = 0.77): Excellent statistical performance. Closest to the expected 15.1 exceptions, with a high p-value confirming adequate unconditional coverage. However, clustering during COVID-19 (multiple consecutive exceptions) indicates conditional coverage issues.

GARCH-t (13 exceptions, p = 0.58): Best tail coverage with the fewest exceptions. The combination of dynamic conditional volatility and fat-tailed Student-t distribution effectively captures both volatility clustering and extreme events. GARCH-N (18 exc., p = 0.47) is also adequate, with slightly more exceptions due to the Normal tail assumption.

 

# 8. Basel III Traffic-Light Classification

Basel III market risk regulations require banks to backtest their internal VaR models against 250 most recent trading days. The number of exceptions determines the supervisory traffic-light zone, which in turn determines a capital add-on (plus factor k) applied to the regulatory multiplier (base value m = 3).

| Zone | Exceptions (250 days) | Plus Factor k | Multiplier m | Interpretation |
|---|---|---|---|---|
| Green | 0 – 4 | 0.00 | 3.00 | Model adequate |
| Yellow | 5 | 0.40 | 3.40 | Minor concern |
| Yellow | 6 | 0.50 | 3.50 | Caution |
| Yellow | 7 | 0.65 | 3.65 | Significant concern |
| Yellow | 8 | 0.75 | 3.75 | Elevated concern |
| Yellow | 9 | 0.85 | 3.85 | Near critical |
| Red | 10+ | 1.00 | 4.00 | Model failure — mandatory disqualification |

### Observed Traffic-Light Results (Last 250 Trading Days)

The table below presents the traffic-light classification for the four models formally evaluated over the most recent 250-day window:

| Model | Observations | Exceptions | Zone | Plus Factor | Multiplier |
|---|---|---|---|---|---|
| **GARCH-N** | 250 | 0 | Green | 0.00 | 3.00 |
| **GARCH-t** | 250 | 0 | Green | 0.00 | 3.00 |
| **HS (rolling250)** | 250 | 2 | Green | 0.00 | 3.00 |
| **EWMA (λ=0.94)** | 250 | 4 | Green | 0.00 | 3.00 |

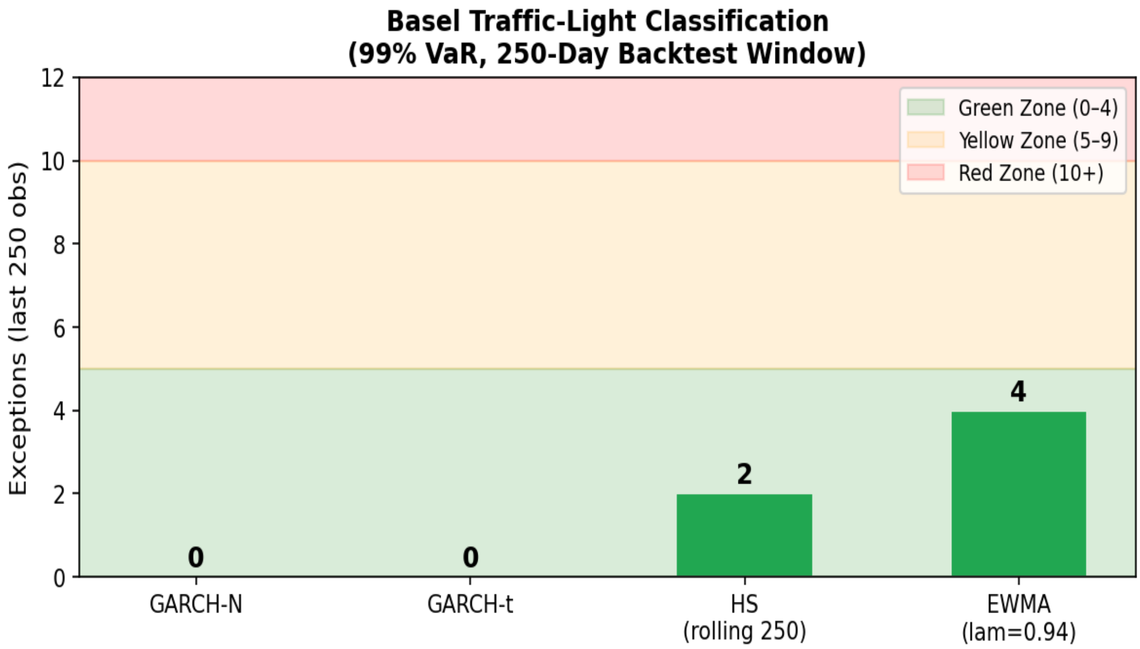


*Figure 8.1 — Basel III traffic-light zones. All four tested models fall within the Green supervisory zone over the most recent 250-day window.*

**Important Context:** All models are currently in the Green zone with a regulatory multiplier of 3.00. This reflects the relatively calm post-COVID market conditions in the most recent 250 trading days. During the COVID-19 crisis period (Feb–Mar 2020), multiple models would have breached the 4-exception Green boundary and entered the Yellow zone, significantly increasing capital requirements. EWMA's 32 full-sample exceptions concentrated heavily in 2020 demonstrate how rapidly traffic-light classification can deteriorate during stress.

# 9. 10-Day Horizon Risk Analysis

Basel III's Internal Models Approach (IMA) requires market risk capital to be computed at a 10-trading-day horizon (often referred to as the Market Price of Risk, MPOR = 10). Two standard approaches exist for extending 1-day VaR to 10-day: square-root-of-time scaling (analytical) and direct 10-day historical simulation.

## 9.1 Square-Root-of-Time Scaling

Under the assumption that daily returns are independent and identically distributed (i.i.d.), portfolio variance scales linearly with time and VaR scales with the square root of the horizon:

$$VaR_{10d} \approx VaR_{1d} \times \sqrt{10} \approx VaR_{1d} \times 3.162$$

This is the simplest approach and was mandated under Basel I and II. However, it relies on the i.i.d. assumption, which is violated by: (i) volatility clustering (GARCH effects), (ii) serial autocorrelation in returns, and (iii) correlation regime changes during crises.

## 9.2 Direct 10-Day Historical VaR

Direct 10-day VaR uses overlapping 10-day return windows from the historical record. For each day t, the 10-day portfolio return is computed as:

$$R_{p,t:t+10} = \Sigma_{k=0}^{9} r_{p,t+k}$$

The 1st percentile of this overlapping 10-day return distribution gives the direct 10-day VaR. This approach avoids the i.i.d. assumption but introduces overlapping observation bias (adjacent windows share 9 of 10 daily returns, creating artificial serial correlation in the 10-day return series).

## 9.3 Breach Analysis and COVID-19 Impact

A 10-day breach occurs when the realised 10-day portfolio return is more negative than the predicted 10-day VaR. The full-sample and COVID-19 window breach rates are compared below:

| Metric | Direct 10-Day | SQRT Scaled | Analysis |
|---|---|---|---|
| **Full-Sample Breach Count** | 23 | 12 | Direct is more conservative |
| **Full-Sample Breach Rate** | 1.84% | 0.95% | Direct: near 2% vs expected 1% |
| **COVID Window (Feb–Apr 2020)** | 13 of 40 obs | 11 of 40 | 32.5% vs 27.5% breach rate |
| **Worst 10-Day Loss** | –18.35% | — | 23 Mar 2020 (COVID peak) |
| **Max Direct 10D VaR in Sample** | 12.57% | 11.05% | During COVID crisis |
| **SQRT Assumption Bias** | Conservative | — | SQRT overstates in stress |

The COVID-19 analysis reveals the critical failure of the SQRT scaling assumption. During February–March 2020, volatility clustering was extreme: large losses on consecutive days created 10-day portfolio losses of –18.35%. The SQRT-scaled approach, by assuming i.i.d. returns, underestimates the true 10-day risk because it misses the correlation of large daily losses occurring in consecutive periods.

### Worst 10-Day Windows (Direct Historical VaR):

| End Date | 10-Day Return | 10-Day Loss | Context |
|---|---|---|---|
| **23 Mar 2020** | –18.35% | 18.35% | COVID-19 peak market collapse |
| **19 Mar 2020** | –18.34% | 18.34% | Pre-peak selling culmination |
| **18 Mar 2020** | –16.43% | 16.43% | Circuit breaker period |
| **20 Mar 2020** | –14.44% | 14.44% | Continued panic selling |
| **24 Mar 2020** | –12.79% | 12.79% | Post-peak, lockdown announcement |

All five worst 10-day windows cluster around the same event — the COVID-19 market crash — demonstrating the non-i.i.d. nature of tail events and the inadequacy of assuming independence across trading days for horizon scaling.

# 10. Stress Testing Framework

Stress testing complements statistical VaR by asking: 'What would the portfolio lose under extreme but plausible scenarios?' Unlike VaR, which is calibrated to the empirical distribution, stress tests are forward-looking scenario analyses that assess robustness to tail events beyond the VaR confidence interval.

This framework implements six stress categories across three dimensions: historical replay (worst observed periods), parametric shocks (sigma multipliers), and correlation/volatility stress (structural changes to the covariance matrix).

## 10.1 Historical Worst-Case Replay

The worst single-day and worst 10-day historical periods are identified and analysed as direct stress scenarios:

| Date | 1-Day Log Return | 1-Day Loss | Event Context |
|---|---|---|---|
| **23 Mar 2020** | –5.47% | 5.47% | COVID-19 — NIFTY lockdown capitulation |
| **12 Mar 2020** | –5.44% | 5.44% | COVID-19 — global panic selloff |
| **16 Mar 2020** | –4.70% | 4.70% | COVID-19 — WHO pandemic declaration aftermath |
| **24 Feb 2022** | –3.93% | 3.93% | Russia–Ukraine war escalation |
| **09 Mar 2020** | –3.59% | 3.59% | COVID-19 — oil price war concurrence |
| **18 Mar 2020** | –3.40% | 3.40% | COVID-19 — circuit breaker days |
| **21 Dec 2020** | –3.35% | 3.35% | Post-vaccine uncertainty, lockdown 2.0 |
| **01 Apr 2020** | –3.29% | 3.29% | COVID-19 extended lockdown fears |
| **04 May 2020** | –3.27% | 3.27% | Q4 FY2020 GDP shock pricing-in |
| **21 Sep 2020** | –2.98% | 2.98% | Second wave concerns |

Seven of the ten worst single-day losses occurred during the COVID-19 crisis (February–May 2020). The sole non-COVID entry is the Russia–Ukraine war escalation of 24 February 2022 (–3.93%), which represents the second distinct tail risk event in the sample.

## 10.2 Parametric Sigma Shocks

Parametric shocks simulate the portfolio loss from a specified multiple of daily portfolio standard deviation. Under the Normal distribution, a –3σ event has probability 0.13% and a –5σ event has probability ≈ 0.00003% (once every ~3 million trading days).

$$\text{Loss}_{-N\sigma} = 1 - \exp(N \times \sigma_{\text{portfolio}}) \text{ (approximately } N \times \sigma \text{ for small losses)}$$

## 10.3 Volatility Stress Scenarios

Volatility is shocked by a multiplicative factor, and the 3σ loss under the stressed volatility is computed. This tests the portfolio's sensitivity to volatility regime changes such as the VIX spike during COVID-19:

$$\text{Loss}_{\text{vol}\times k} = 3 \times (k \times \sigma_{\text{portfolio}})$$

## 10.4 Correlation Stress Scenarios

During market crises, pairwise asset correlations typically spike as stocks sell off indiscriminately. Correlation stress multiplies all off-diagonal correlation coefficients by a factor (capped at 0.99 to maintain a positive-definite matrix):

$$\rho_{ij,\text{stressed}} = \min(k \times \rho_{ij}, 0.99) \text{ for all } i \neq j$$

The stressed covariance matrix is then used to compute portfolio volatility and the 3σ scenario loss.

## 10.5 Consolidated Stress Test Results

| Scenario | 1-Day Loss | Type | Severity vs Gaussian VaR |
|---|---|---|---|
| **–3σ Parametric Shock** | 2.06% | Parametric | +8% above Gaussian VaR |
| **Correlation Stress ×1.3** | 2.30% | Correlation | +20% above Gaussian VaR |
| **Correlation Stress ×1.8** | 2.65% | Correlation | +39% above Gaussian VaR |
| **Volatility Shock ×1.5 (3σ event)** | 3.09% | Vol stress | +62% above Gaussian VaR |
| **–5σ Parametric Shock** | 3.43% | Parametric | +80% above Gaussian VaR |
| **Volatility Shock ×2.0 (3σ event)** | 4.12% | Vol stress | +116% above Gaussian VaR |
| **Worst Historical 1-Day** | 5.47% | Historical | +186% above Gaussian VaR |
| **Worst Historical 10-Day** | 18.35% | Historical | Extreme tail event |

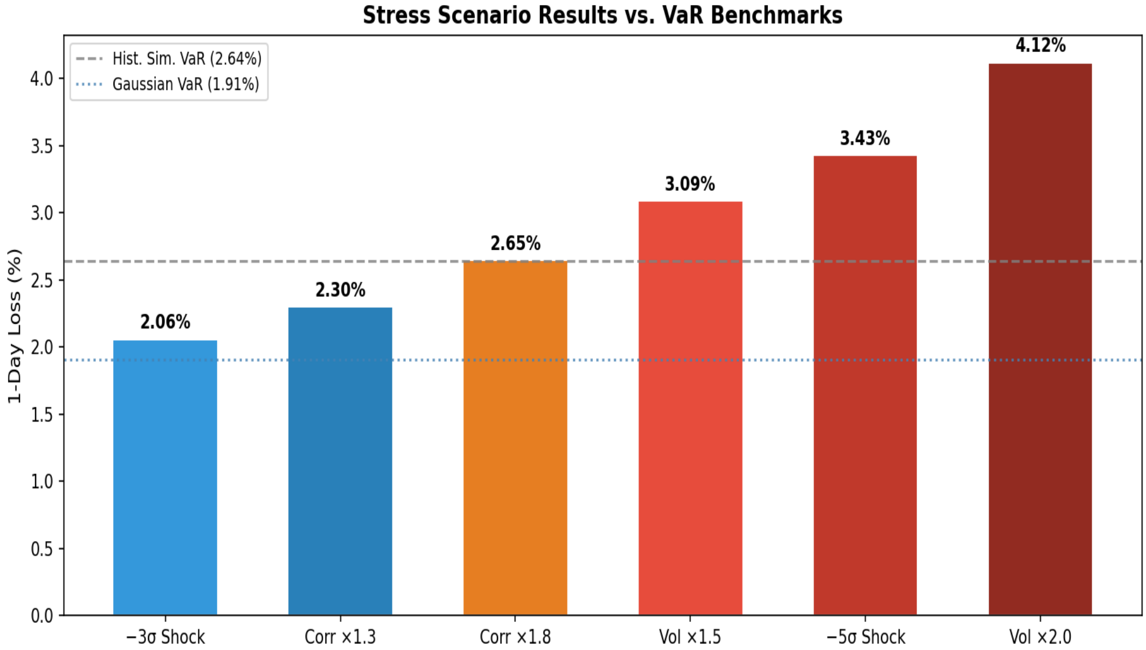


*Figure 10.1 — Stress scenario losses ranked from least to most severe. Scenarios are compared against the baseline Gaussian VaR (1.91%) and the worst historical 1-day loss (5.47%).*

## 10.6 Period-Based Stress Analysis (COVID vs 2021–22 Regime)

Two distinct market regimes are analysed to compare risk parameters across crisis and normal periods, demonstrating how risk estimates are regime-dependent:

| Metric | COVID Shock (Feb–Mar 2020) | 2021–2022 Regime | Ratio |
|---|---|---|---|
| **Observations** | 40 trading days | 496 trading days | — |
| **HS VaR 1D (99%)** | 5.46% | 1.92% | 2.8× |
| **HS ES 1D (99%)** | 5.47% | 2.74% | 2.0× |
| **Worst 1-Day Loss** | 5.47% (23 Mar 2020) | 3.93% (24 Feb 2022) | 1.4× |
| **Worst 10-Day Loss** | 18.35% | 7.23% | 2.5× |

The COVID period produced VaR and ES estimates 2–3× higher than the subsequent 2-year normalisation period. This underscores the importance of using stressed observation periods (as required by FRTB) rather than relying solely on current or recent-window estimates. A model calibrated on 2021–22 data alone would have been severely undercapitalised for the COVID-like stress scenario.

# 11. Initial Margin Proxy (MPOR Scaling)

Initial Margin (IM) in cleared derivatives markets is calibrated to cover potential losses over the Margin Period of Risk (MPOR) — the time required for an exchange or CCP to liquidate a defaulted member's portfolio. For equity derivatives, MPOR is typically 2–5 days for exchange-traded products and 10 days for OTC derivatives under bilateral agreements.

A simplified IM proxy is computed by scaling the 1-day VaR by the square root of the MPOR:

$$IM_{proxy} = VaR_{1d,99\%} \times \sqrt{(MPOR)} \text{ with MPOR = 10 days}$$

This is the same SQRT-of-time scaling discussed in Section 9, applied specifically to estimate margin requirements. The proxy is useful for comparing the capital implications of different VaR models under a standardised margin period.

| Model | VaR 1-Day (99%) | IM Proxy (MPOR=10d) | Δ vs EWMA (base) |
|---|---|---|---|
| **EWMA (latest)** | 1.45% | 4.58% | Baseline |
| **MC Normal (2y)** | 1.52% | 4.81% | +0.23 ppts |
| **GARCH-N (latest)** | 1.72% | 5.42% | +0.84 ppts |
| **Gaussian Param.** | 1.91% | 6.02% | +1.44 ppts |
| **GARCH-t (latest)** | 1.98% | 6.26% | +1.68 ppts |
| **MC Student-t (df=6)** | 2.08% | 6.59% | +2.01 ppts |
| **Hist. Simulation** | 2.34% | 7.40% | +2.82 ppts |
| **Cornish–Fisher** | 3.26% | 10.31% | +5.73 ppts |

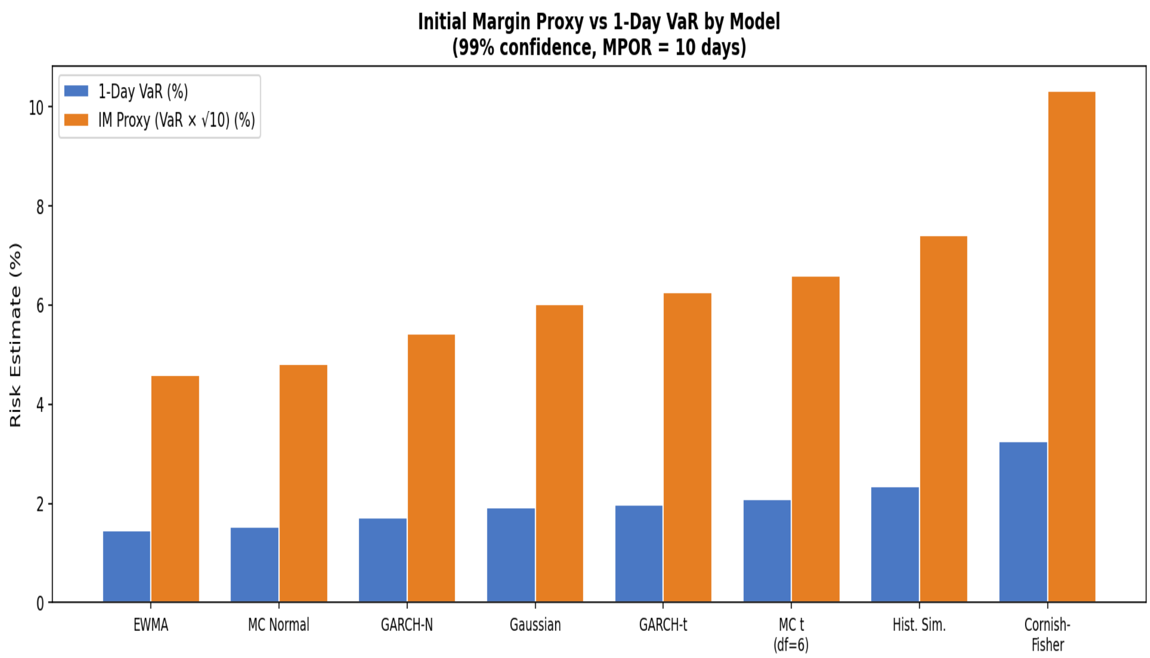


*Figure 11.1 — Initial Margin proxy across all models under MPOR = 10 days. The 2.25× spread between EWMA and Cornish–Fisher illustrates significant model risk in margin calibration.*

 

**Model Risk in Margin Calibration:** The IM proxy ranges from 4.58% (EWMA) to 10.31% (Cornish–Fisher) — a spread of 573 basis points (2.25×). In practice, a CCP or bilateral agreement using EWMA-based margin would require significantly less collateral posting than one using Cornish–Fisher, potentially creating systemic undercollateralisation. This is precisely why regulators (e.g., ISDA SIMM) use standardised sensitivity-based approaches rather than allowing full model freedom in IM calculation.

 

## 12. Key Findings & Conclusions

### ◆ Portfolio Construction

Ledoit–Wolf shrinkage covariance with constrained minimum-variance optimisation produces a well-diversified 25-stock portfolio, preventing concentration risk. Consumer staples, healthcare, and utilities receive the largest allocations, consistent with their lower volatility and lower correlations.

### ◆ VaR Model Dispersion

The eight VaR models produce 1-day 99% estimates ranging from 1.44% (EWMA) to 3.26% (Cornish–Fisher) — a 226 bps model risk spread. No single model is universally superior; the appropriate choice depends on the objective (regulatory compliance, economic capital, margin sizing) and the market regime.

### ◆ Backtesting Failure: EWMA

EWMA produces 32 exceptions against an expected 15.1 (Kupiec p = 0.0001), a decisive statistical failure. Its conditional-Normal structure underestimates both tail probability and the magnitude of extreme losses during crisis periods. EWMA should not be used as a standalone VaR model for regulatory purposes.

### ◆ GARCH Models Show Superior Coverage

GARCH-t achieves the best backtest result (13 exceptions, p = 0.58), closely followed by GARCH-N (18 exceptions, p = 0.47). The combination of time-varying conditional volatility and fat-tailed distribution assumption (Student-t) provides the most statistically robust VaR coverage across the full 2016–2023 sample.

### ◆ COVID-19 as the Dominant Tail Event

All five worst 10-day windows and seven of ten worst single-day losses occur during March–April 2020. The worst 10-day portfolio loss was –18.35%, approximately 10× the typical 1-day VaR. This demonstrates both the extreme severity of the COVID shock and the fundamental limitation of VaR as a crisis-period risk measure.

### ◆ SQRT Scaling Underestimates Multi-Day Risk

Direct 10-day VaR breaches (23 full-sample) substantially exceed SQRT-scaled breaches (12), particularly during the COVID crisis. The i.i.d. assumption underlying SQRT scaling is violated by volatility clustering, causing the SQRT approach to underestimate risk by a material margin during turbulent regimes.

### ◆ All Models in Green Basel Zone

Over the most recent 250 trading days, all four evaluated models (GARCH-N: 0 exc., GARCH-t: 0 exc., HS: 2 exc., EWMA: 4 exc.) fall within the Basel III Green zone (multiplier = 3.00). This reflects the relatively calm post-2022 market environment.

### ◆ Initial Margin Model Risk

IM proxy estimates span 4.58% (EWMA) to 10.31% (Cornish–Fisher) under MPOR = 10 days. Choosing a conservative model (HS or Cornish–Fisher) for margin sizing versus an optimistic one (EWMA or MC Normal) represents a significant difference in collateral requirements with direct P&L; implications for portfolio managers.

# 13. Limitations & Future Work

This prototype framework covers the core statistical risk measurement layer but deliberately excludes several components required in production-grade or regulatory-approved internal models:

| Limitation | Description | Future Extension |
|---|---|---|
| **No P&L Attribution Test** | FRTB IMA requires daily P&L attribution vs risk factor P&L | Implement actual vs hypothetical P&L split |
| **No NMRF Classification** | Non-modellable risk factors (NMRF) require Stress Scenario Risk Charge | Risk factor modellability assessment |
| **No Liquidity Horizon Adjustment** | FRTB mandates liquidity horizon buckets (10–120 days) per risk class | Risk-class-specific horizon scaling |
| **No Stressed ES Window** | FRTB ES requires 12-month stressed calibration window | Algorithmic stressed-window identification |
| **No Factor Risk Model** | Portfolio modelled at stock level; no macro factor decomposition | PCA/Barra factor model integration |
| **No Correlation Breakdown Test** | Crisis-period correlation instability not formally tested | Dynamic Conditional Correlation (DCC-GARCH) |
| **Static Portfolio** | Weights fixed; no daily rebalancing or delta hedging | Time-varying weight optimisation |
| **No Transaction Costs** | Rebalancing and liquidation costs not modelled | Bid-ask spread and market impact models |
| **No Options / Non-Linear Risk** | Framework designed for linear equity positions only | Delta-gamma approximation; full revaluation MC |
| **SQRT Scaling Only** | Multi-day VaR uses SQRT; no proper multi-step MC simulation | Path-dependent Monte Carlo for 10-day horizon |

**FRTB Readiness Assessment:** This prototype covers approximately 60–70% of the quantitative modelling requirements for FRTB IMA. The primary gaps are regulatory-administrative (P&L; attribution, NMRF classification, stressed window identification) rather than methodological. The core VaR, ES, GARCH, and backtesting infrastructure is fully extensible to a complete IMA framework.

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**Source Code Repository**

github.com/vignani007/Nifty-risk

*All Python source code, computed output CSVs, and chart generation scripts are available in the public repository. The modular src/ structure allows independent testing and extension of each risk measurement component.*