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FILTERING OF SURFACE PROFILES USING FAST FOURIER TRANSFORM

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Abstract – An essential preliminary to numerical assessment of roughness is the elimination of waviness. The waviness is usually filtered using analog filters of the capacitance–resistance type. This paper deals with a digital technique to remove the waviness. FFT, one of the important tools of digital signal processing was used to filter surface profiles. This technique was compared with the existing methods available for filtering surface profiles.

INTRODUCTION

SURFACE roughness measurements are usually made by stylus type instruments. The estimation of any roughness parameter should be done only after processing the signal to eliminate waviness and form error. The roughness is normally considered to be superimposed on the waviness. Hence a correct estimate of roughness can be obtained only after the waviness is removed. This is generally accomplished in two ways. The first method involves establishing a mean line which excludes the effect of waviness. In the second method, signal representing the profile is processed in a high pass filter to remove the low frequencies, namely, the waviness. The transmission characteristics of such a filter is standardized. The meter in the roughness measuring instrument which shows the reading is usually calibrated using sine wave type profiles. Though the meter readings in subsequent measurements of irregular profiles diverge from the graphically determined values, the instrument reading has been accepted as correct in practice since the variation in the value is small. The necessity to determine the reading that ought to be given by the instrument, especially for the correct calibration and for the standardization of the filter, led to the development of analytical methods for calculating the roughness value [1–3]. Most of these analyses were basically for computation of the filtering characteristics of RC filters, commonly used in surface roughness instruments. One of the major limitations is the limited choice of cut-off lengths for elimination of waviness. Though filtering has been done on surface profiles for a long period it is a well known fact that even today no comprehensive report regarding the role played by filtering exists, i.e. elimination of long wavelengths on the performance of the components. For such studies it is essential to have freedom to select the separation of waviness conveniently. This paper deals with one such method where FFT (Fast Fourier Transforms) are used to do the filtering [4, 7]. It is possible to approximate surface profiles using Fourier series and therefore removal of low or high frequencies can be easily achieved using FFT [4, 5]. The technique of spectral analysis employing the Fourier transform and series have long represented an area of application in surface profile processing [6]. The development of Cooley–Tuckey and other algorithms for rapid computation of the spectrum, paved the way for varied and new applications of spectral analysis. In fact, it has become quite possible to filter signals by FFT transformation, numerical alteration of the spectrum and inverse computation. This paper deals with the application of the above technique to process the surface profiles.

FFT FILTERING

Fast Fourier transform

The discrete Fourier transform of a sequence $\{X_n\}$ is given by

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$$A_p = \sum_{M=0}^{N-1} X_n W_N^{np} \qquad n, p = , 1, 2, \ldots (N-1),$$

where $W_N = e^{-j2\pi/N}$.

$\{X_n\}$ are, in general, complex numbers. The resulting sequence $\{A_p\}$ is the DFT of the input sequence. A class of algorithms to compute the DFT have been developed and they are generally known as FFT algorithms. The straightforward computation of the DFT of a sequence of length N, requires a computation rate that grows as N, but for the FFT algorithms the computation grows only as $N \log_2 N$. This represents a drastic reduction in the number of computations performed. For FFT, N is chosen such that it is an integral power of 2, i.e. $N = 2^L$, with L an integer. The key to the development of the FFT algorithm is the fact that the sequence $\{A_P\}$ can be obtained by a suitable combination of two transforms of length $N/2$ and the combination requires only N multiplications. Since N is a power of 2, this decomposition process can be repeated L times, $L = \log_2 N$. Basically, one can form $N/2$ transforms of length 2, combine them to get $N/4$ transforms of length 4, combine these to get $N/8$ transforms of length 8 and so on until finally after L such stages the transform of length N is obtained. Since in taking the transform of length 2, $W_2 = e^{-j2\pi/2} = -1$, only addition and subtraction are needed. The computation of the DFT using this approach requires multiplication only in the steps of combining half-length transforms to form full-length transforms. As a result, the total number of multiplications is $N \cdot L = N \log_2 N$. The above algorithm is known as radix 2 decimation in time [7]. Many other algorithms are also available for computing FFT based on the above method and a computer program can be written to perform FFT.

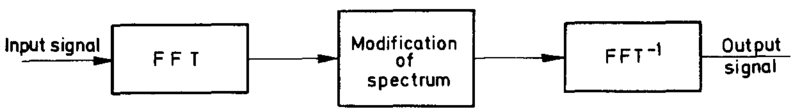

FIG. 1. Filtering using FFT.

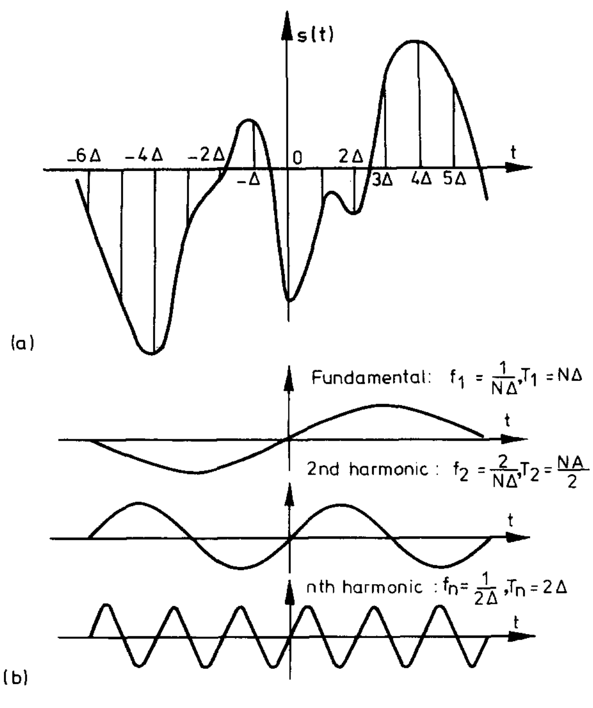

FIG. 2. An irregular profile and its frequency decomposition.

Filtering procedure

The roughness, waviness and form errors are separated generally on a basis of crest spacing by mechanical or electrical devices which are able to distinguish between crest spacing longer or shorter than some given value which can be referred to as cut-off. The FFT is one such device which can be used effectively for separation of roughness or waviness. Figure 1 shows the procedure for filtering of surface profiles using FFT [4]. From among the library of surface profiles available in the laboratory, turned and ground profiles were selected for analysis. Appropriate cut-off lengths were selected based on the manufacturing process, but this value can also be selected according to requirement. The first step in the filtering process is the computation of the spectrum using FFT. The computation was carried out using an IBM 370/155 and a built-in program was used for the purpose. The second step involves the numerical alteration of the calculated spectrum in the frequency domain to do the desired filtering. The alteration of the spectrum is the most important operation in this filtering technique. Figure 2 shows an irregular profile and its frequency decomposition using a Fourier series. The profile could be considered as a time series with length replacing time in the x-axis. The frequencies can be calculated by

$$f_1 = \frac{1}{N\Delta}, \qquad T_1 = N\Delta,$$

$$f_2 = \frac{2}{N\Delta}, \qquad T_2 = \frac{N\Delta}{2},$$

$$f_n = \frac{1}{2\Delta}, \qquad T_n = 2\Delta,$$

where N is the number of ordinates and Δ is the ordinate spacing.

Depending on the cut-off selected, the number of harmonics that are to be retained in the spectrum to get the waviness or roughness can be found out. Once the number of terms is known, the spectrum is numerically altered in the frequency domain. The modification of the spectrum involves weighting the frequency or wavelength terms in the spectrum that are to be retained by one and the remaining with zero. The weighting is one till the cut-off and it is then gradually reduced to zero in a narrow transition band. The weighting function used for the above modification is shown in Fig. 3 and it resembles the transmission characteristics of a filter [2]. The weighting function is a complex one and the imaginary part of the weights were made zero. This procedure eliminates any possible phase lag due to the filtering process. If the aim is to get a mean line, then the long wavelength, i.e. the low frequency terms are multiplied

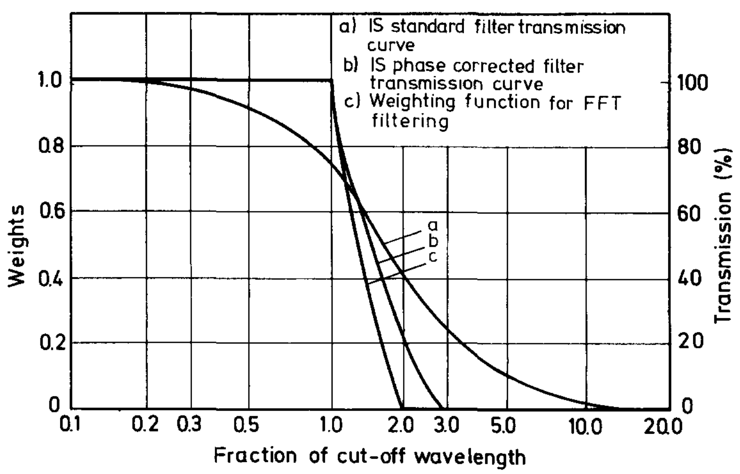

FIG. 3. Transmission characteristics and weighting function.

by a weight of unity. The terms in the transition band, which is one cut-off length, are multiplied by weights gradually varying from one to zero and the remaining high frequency terms representing the roughness are multiplied by zero. On the other hand if roughness is to be extracted the weights are interchanged. After multiplying the spectrum with the weighting function the inverse FFT of the modified spectrum is computed and the result is the required filtered signal in the time domain. In order to show that the attenuation characteristics can be changed easily, weighting functions or transmission characteristics which have unit transmission up to the cut-off and then falling to zero at two times and three times the cut-off length were selected. A very sharp transmission characteristic is not desirable as it could lead to oscillation when the inverse transform is computed. Excessively high rates of attenuation however, need not be realistic even from the mechanical point of view because considerable variation is not expected in the functional behaviour of two surfaces having equal amplitude but slightly different wavelength. The number of harmonics selected sometimes may not correspond to the exact cut-off since the number of points for FFT should be a power of 2, but for filtering purposes it is adequate. Moreover the profile obtained from the instrument would have undergone mechanical filtering due to the stylus guidance and the shape of the stylus tip. Therefore the selected transmission characteristics could be considered as practical ones.

ANALYSIS OF THE FILTERED PROFILES

The estimation of roughness parameters can be easily done from the filtered signal. Both low pass and high pass filtering were tried to remove waviness and roughness respectively. Two types of transmission characteristics mentioned earlier were used for analysis. The filtering procedure is based on spectrum modification in the frequency domain. Since the spectrum is a discrete one a smaller transition band would give better results. The results

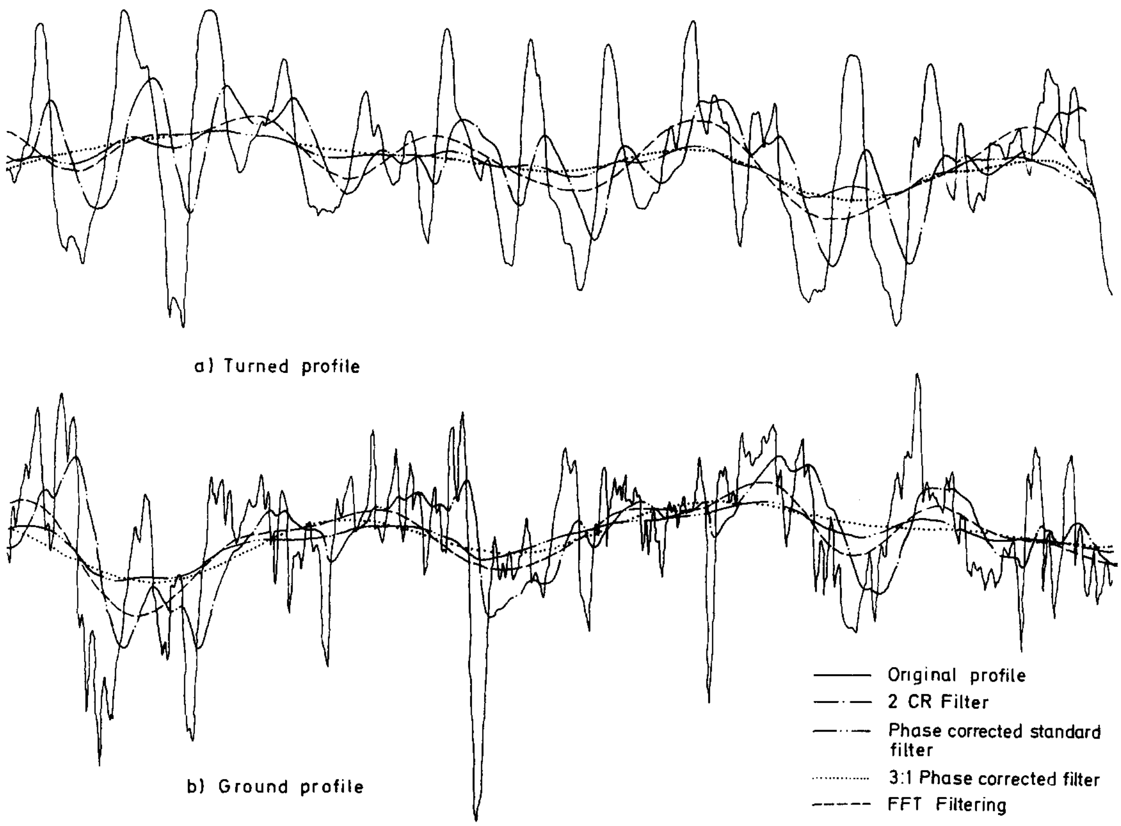

FIG. 4. Original profile together with mean line obtained from different filters.

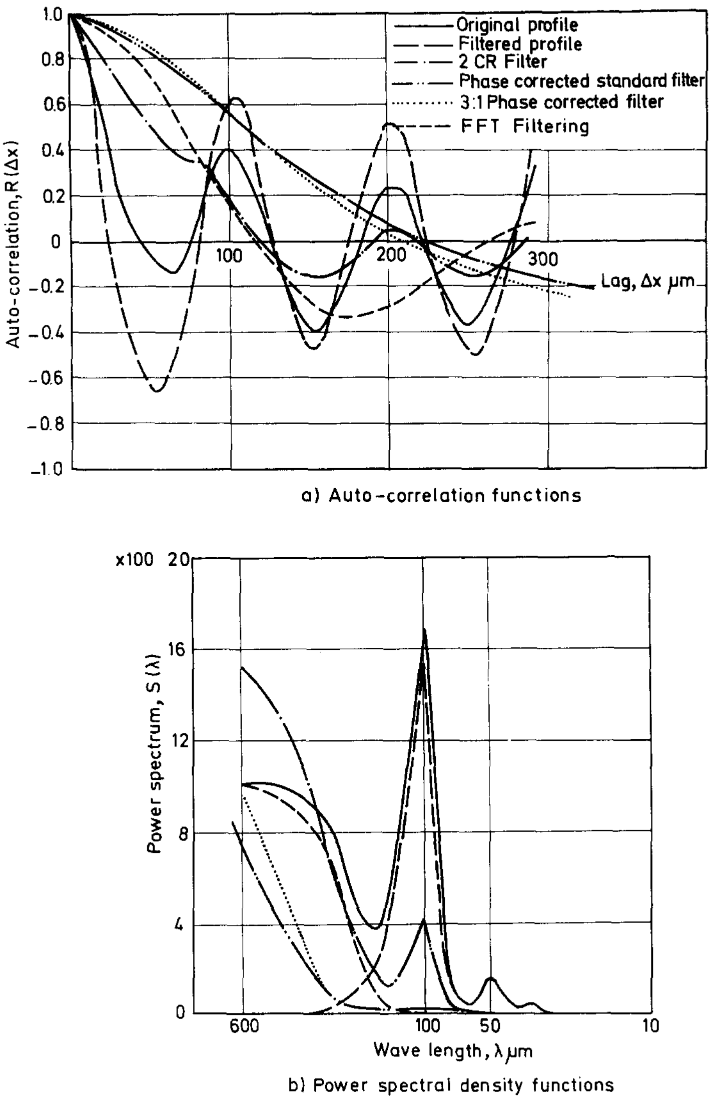

FIG. 5. Auto-correlation and power spectral densities of original profile, mean lines from different filters and filtered profile for a turned surface.

presented were obtained from a filter having one cut-off length as its transition band. The effectiveness of the filtering operation was evaluated by the computing the auto-correlation and power spectral density of the profile before and after filtering. The existing filters, namely the 2 CR filter, phase corrected standard filter and 3 : 1 phase corrected filter were compared with the proposed filter. The comparison was based on the shape of the mean line, transmission characteristics, phase lag and the power spectral density of the filtered profile and the mean line. The cross-correlation between the mean line and the profile was also evaluated to find out the phase difference, if any, between the mean line and the profile. Cla values were computed by processing the profile by the available filters.

The performance of various filters were compared by processing sine waves of different wavelengths. Figure 9 shows the response of the filters to the sine waves. The first four waves have been selected within the pass band. The FFT filtering and 3 : 1 phase corrected filter give the same response in this region. At two times the cut-off, the difference is only 5% between FFT filtering and the 3 : 1 phase corrected filter. Turned and ground profiles were processed by the filters and the results are discussed below.

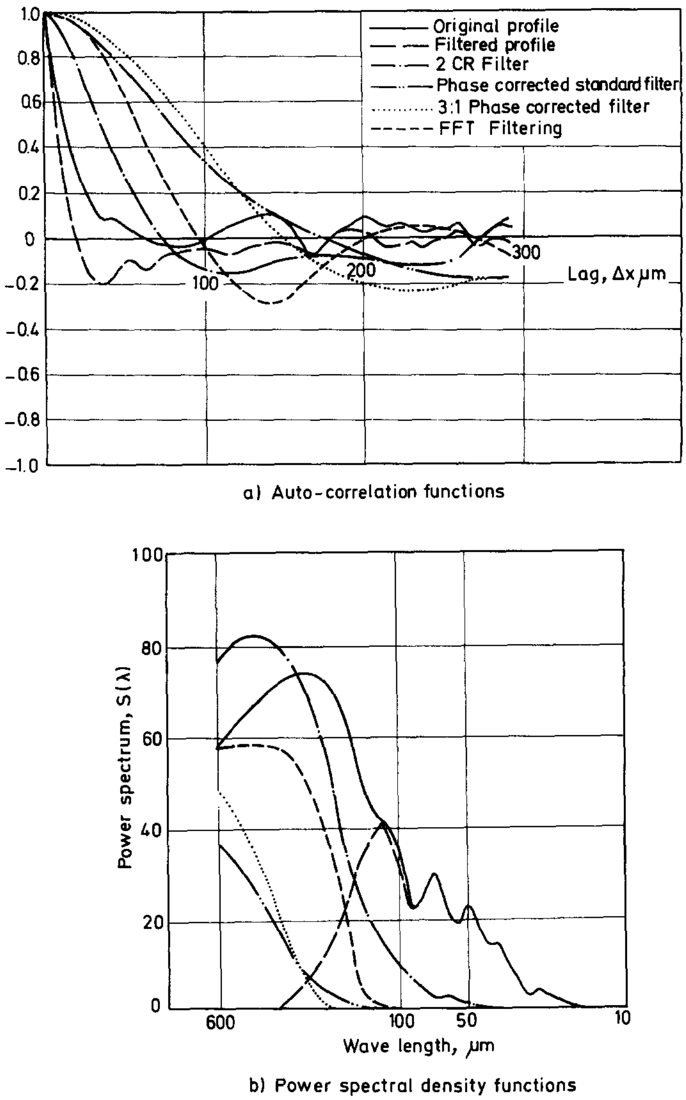

FIG. 6. Auto-correlation and power spectral densities of original profile, mean lines from different filters and filtered profile for a ground surface.

DISCUSSIONS

The mean line obtained from the existing filters and FFT filtering together with the original profiles in turned and ground surfaces are shown in Fig. 4. Among the already available filters, the mean line obtained from 3 : 1 phase filter gives the best representation of waviness [2, 8]. The amplitude undulation of this filter is low when compared with the other filters and also the mean line is in phase with the profile. The above conclusions are also supported by the power spectral density and auto-correlation curves, shown in Figs. 5 and 6 for turned and ground profiles. The 2 CR mean line contains some of the roughness and this is clearly revealed both by the auto-correlation and power spectral density curves in the case of turned profile. On the other hand the power spectrum of the mean line obtained from the 3 : 1 phase corrected filter shows that it mainly constitutes wavelengths longer than the cut-off namely the waviness. The mean line obtained from the FFT filtering also gives an equally good representation of waviness and the calculation of Cla value based on this mean line is more realistic since it resembles a graphical mean line. The spectrum of the mean line obtained from the FFT filtering when compared with others shows that the longer wavelengths are retained without much attenuation. In order to compare the performance of

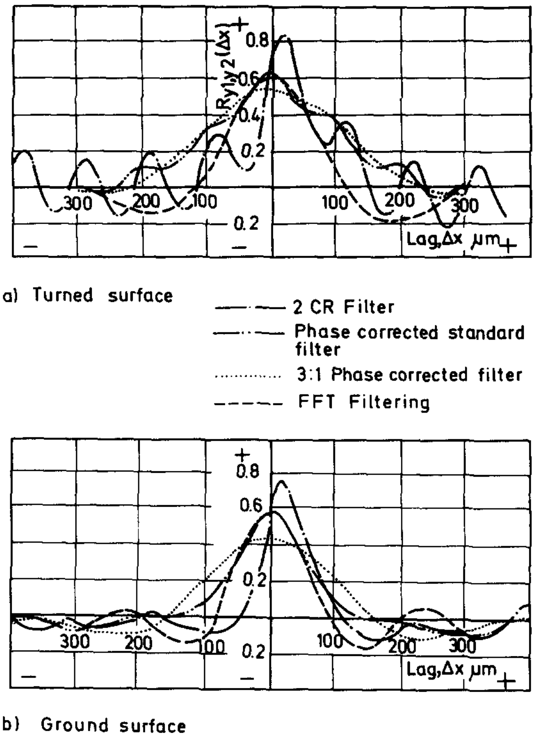

FIG. 7. Cross-correlation function between mean line and original profile for different filters.

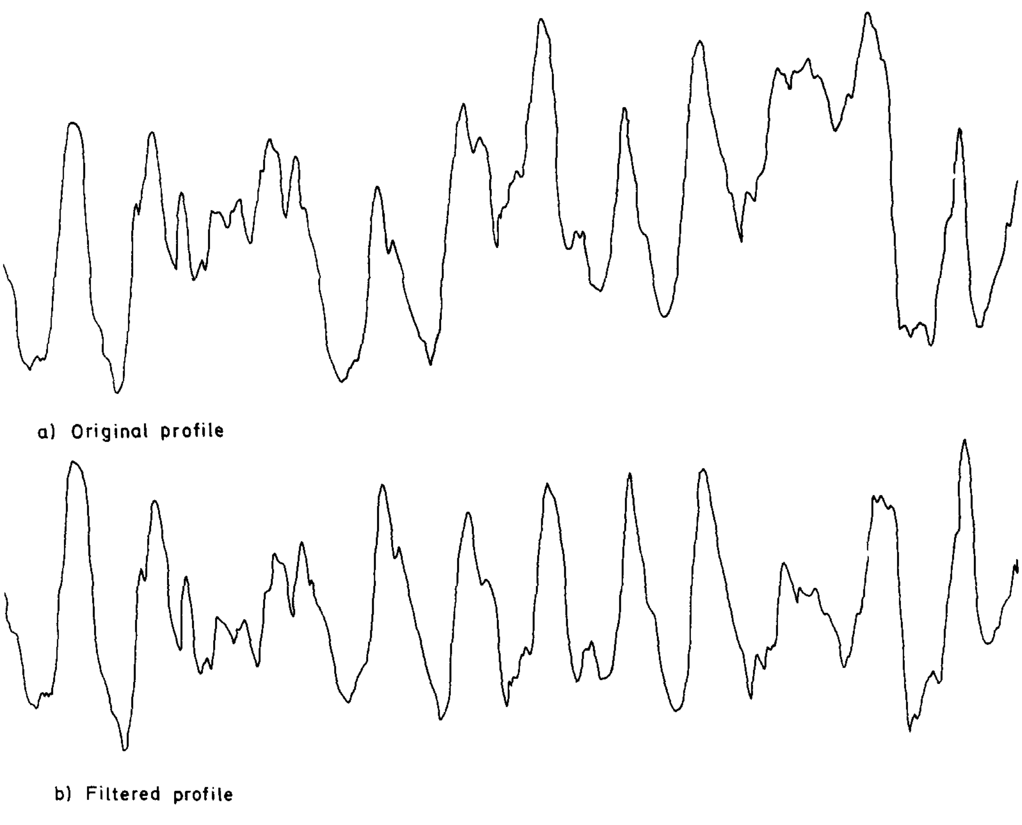

FIG. 8. Original profile and roughness filtered using FFT filtering.

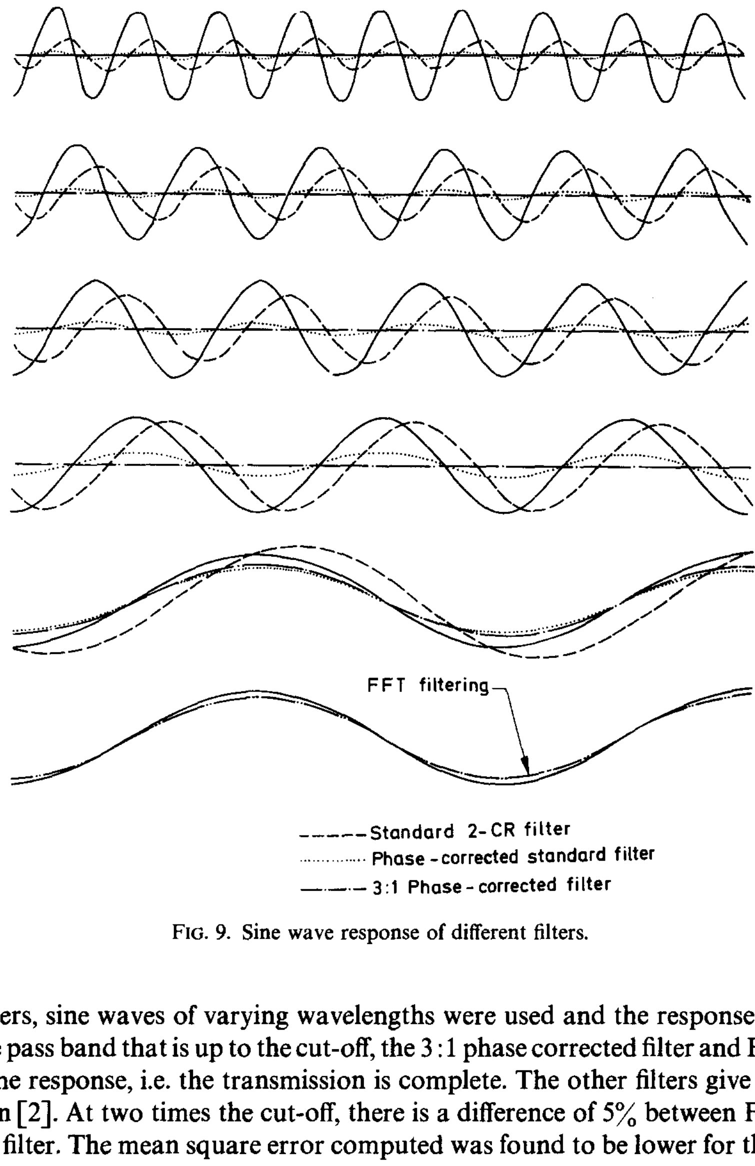

FIG. 9. Sine wave response of different filters.

different filters, sine waves of varying wavelengths were used and the response is shown in Fig. 9. in the pass band that is up to the cut-off, the 3:1 phase corrected filter and FFT filtering give the same response, i.e. the transmission is complete. The other filters give only partial transmission [2]. At two times the cut-off, there is a difference of 5% between FFT filtering and the 3:1 filter. The mean square error computed was found to be lower for the 3:1 phase corrected filter, than for FFT filtering. The cross-correlation function between the mean line and the profile shown in Fig. 7 reveals that the FFT mean line is in phase with the profile. The analytical techniques so far developed were useful only for computation of mean line but this technique can be used to obtain the roughness directly. Figure 8 shows the original profile together with the filtered profile obtained from a high pass FFT filtering technique. The spectrum of the filtered profile coincides with the spectrum of the original profile in the low wavelength region and hence the above method can be used for filtering roughness directly. The main advantage of this technique is that it can be performed in a general purpose computer and the time taken is very short. The physical realization of this technique

TABLE 1.

Type of profile	Cut-off	High pass* FFT filtering	Low pass† FFT filtering	2 CR filter
Turned	250 μm	2.309 μm	2.299 μm	2.282 μm
Ground	250 μm	0.659 μm	0.659 μm	0.711 μm

* Cla through direct filtering.
† Cla through meanline.

requires FFT processes for a specified length. From the point of view of realization and real-time processing, the convolution method is faster and requires less memory requirements. The FFT technique can be used as an alternative in situations where the attenuation characteristics and cut-off are varied for investigating machined surface profiles for their functional performance.

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