$(a+b)^2 = a^2 + b^2 + 2ab$ **Logarithms.**

$(a-b)^2 = a^2 + b^2 - 2ab$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$(a-b)^3 = a^3 + b^3 - 3ab(a-b)$

$(a+b)(a-b) = a^2 - b^2$

---

1 $3\frac{1}{2} - 4\frac{1}{2}$

$= \frac{6+1}{2} - \frac{8+1}{2}$

$= \frac{7}{2} - \frac{9}{2} \Rightarrow \frac{7-9}{2} \Rightarrow \frac{-2}{2} = -1$

---

2 $\frac{5}{2} + \frac{11}{2}$

$= \frac{5+11}{2} \Rightarrow \frac{16}{2} \Rightarrow 8$

---

3 $4\frac{1}{2} + 7\frac{1}{2}$

$= \frac{8+1}{2} + \frac{14+1}{2}$

$= \frac{9}{2} + \frac{15}{2} \Rightarrow \frac{24}{2} \Rightarrow 12$

---

4 $\frac{7}{2} + \frac{6}{2}$

$= \frac{13}{2}$

$= \underline{6.5}$

5 $3\frac{1}{2} + 4\frac{1}{2}$

$= \frac{6+1}{2} + \frac{8+1}{2}$

$= \frac{7}{2} + \frac{9}{2}$

$= \frac{16}{2}$

$= \underline{\underline{8}}$

Natural Numbers N

1, 2, 3, 4, 5, 6, 7, 8, 9, 10 ....

Integer Numbers Z

....., −3, −2, −1, 0, 1, 2, 3 ....

Rational Numbers Q

$\frac{3}{11}$, $\frac{7}{8}$, ... Integer also

Irrational Numbers

$\sqrt{2}$, $\sqrt{3}$, $\sqrt{9}$ ....

Real Numbers [R]

1, 2, −3, $\frac{3}{11}$, $\sqrt{3}$ ... All the above

Complex Number [C]

$a + bi$

Real Imaginary

$7 + 2i$

Multiplication

$$\frac{7}{3} \times \frac{4}{3} = \frac{28}{9}$$

Rationalising a dinominator

1 $\frac{7}{\sqrt{2}}$

$$= \frac{7 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} \Rightarrow \frac{7\sqrt{2}}{2}$$

2 $\frac{6}{\sqrt{3}}$

$$= \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow \frac{6\sqrt{3}}{3}$$

3 $\frac{7}{\sqrt{7}}$

$$= \frac{7}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} \Rightarrow \frac{7\sqrt{7}}{7}$$

4 $\frac{3+\sqrt{5}}{\sqrt{3}}$

$$= \frac{3+\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{3\sqrt{3}+\sqrt{15}}{3}$$

5 $\dfrac{4-\sqrt{3}}{2\sqrt{7}}$

$$= \frac{4-\sqrt{3}}{2\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} \Rightarrow \frac{4\sqrt{7}-\sqrt{21}}{2\times 7} \Rightarrow \frac{4\sqrt{7}-\sqrt{21}}{14} //$$

6 $\dfrac{3+\sqrt{5}}{2-\sqrt{3}}$

$$= \frac{3+\sqrt{5}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{(3+\sqrt{5})(2+\sqrt{3})}{2^2-(\sqrt{3})^2}$$

$$= \frac{6+3\sqrt{3}+2\sqrt{5}+\sqrt{15}}{4-3}$$

$$= 6+3\sqrt{3}+2\sqrt{5}+\sqrt{15} //$$

7 $\dfrac{4\sqrt{2}-7}{\sqrt{5}+3}$

$$= \frac{4\sqrt{2}-7}{\sqrt{5}+3} \times \frac{\sqrt{5}-3}{\sqrt{5}-3}$$

$$= \frac{(4\sqrt{2}-7)(\sqrt{5}-3)}{(\sqrt{5})^2-3^2}$$

$$= \frac{4\sqrt{10}-7\sqrt{5}-12\sqrt{2}+21}{5-9}$$

$$= \frac{4\sqrt{10}-7\sqrt{5}-12\sqrt{2}+21}{-4} //$$

Laws of indicies

1. $a^m \cdot a^n = a^{m+n}$

2. $\frac{a^m}{a^n} = a^{m-n}$

3. $(a^m)^n = a^{mn}$

$\sqrt{2} = (2)^{1/2}$

$\sqrt[3]{2} = (2)^{1/3}$

$\frac{1}{6} = 6^{-1}$

Logarithm

Logarithm of a number to a given base is the index or the power to which the base must be raised to produce the number that is to make it equal to the given number. i.e $a^x = n$ (exponential form) or equivalently $x = \log_a n$ (logarithm form)

Laws of Logarithams

Logarithm of the ~~two~~ product of two no. is equal to the sum of the logarithms of the no's to the same base.

i.e $\log_a mn = \log_a m + \log_a n$

2. The logarithm of the quotients of two no's is equal to the difference of their logarithm to the equal base

i.e $\log_a \frac{m}{n} = \log_a m - \log_a n$

3. Logarithm of the no. raised to a power is equal to the index of the power multiplied by the logarithm of the no to the same base

i.e $\log_a m^n = n \log_a m$

<u>Examples</u>

$$\frac{1}{2} \log_{10} 25 - 2 \log_{10} 3 + \log_{10} 18$$

$$\log_a m^n = n \log_a m$$

$$\log_{10} (25)^{1/2} - \log_{10} (3)^2 + \log_{10} 18$$

$$\log_{10} 5 - \log_{10} 9 + \log_{10} 18$$

$\log_{10} 10 = 1$

$\log_{10} 100 = 2$

$$\log_{10} 5 + \log_{10} 18 - \log_{10} 9$$

$a^x = n$

$x = \log_a n$

$$\log_{10} (5 \times 18) - \log_{10} 9$$

$$\log_{10} \left(\frac{5 \times \cancel{18}^{2}}{\cancel{9}} \right)$$

$\log 10^1 = 10$

$$\log_{10} 10$$

$$= 1$$

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2 without using log tables find x. If

$$\frac{1}{2}\log_{10}(11+4\sqrt{7}) = \log_{10}(2+x)$$

$$\log_{10}(11+4\sqrt{7})^{1/2} = \log_{10}(2+x)$$

$$\cancel{\log(11)^{1/2} + \log(4\sqrt{7})^{1/2} = \log_{10}\{2+\log_{10}x}$$

$$\frac{1}{2}\log_{10}(11+4\sqrt{7}) = \log\underline{(11+4\sqrt{7})^{\frac{1}{2}}}$$

$$(11+4\sqrt{7})^{1/2} = \sqrt{x}+\sqrt{y}$$

$$\left[(11+4\sqrt{7})^{1/2}\right]^2 = (\sqrt{x}+\sqrt{y})^2$$

$$11+4\sqrt{7} = x+y+2\sqrt{x}\sqrt{y}$$

$$\underline{11 = x+y} \text{ - } ① \quad \overset{2}{\cancel{4}}\sqrt{7} = \cancel{2}\sqrt{xy}$$

$$2\sqrt{7} = \sqrt{xy}$$

$$4\times 7 = xy$$

$$xy = \underline{\underline{28}}$$

$$(x-y)^2 = (x+y)^2 - 4xy \qquad (a-b)^2 = a^2+b^2-2ab$$

$$(x-y)^2 = (11)^2 - 4\times 28 \qquad (a-b)^2 = a^2+b^2-2ab+2ab-2ab$$

$$(x-y)^2 = 121 - 112 \qquad (a-b)^2 = a^2+b^2+2ab-4ab$$

$$(x-y)^2 = 9 \qquad (a-b)^2 = (a+b)^2-4ab$$

$$x-y = 3 \text{ —— } ②$$

$$x+y = 11 \quad [\text{Add eq}^n\ ① + ②]$$

$$x-y = 3$$

$$2x = 14$$

$$x = \underline{\underline{7}}$$

Sub the value of x in (1)

we get $11 = x + y$

$y = 4$

$\therefore (11 + 4\sqrt{7})^{1/2} = \sqrt{7} + \sqrt{4}$

$\log_{10} (11 + 4\sqrt{7})^{1/2} = \log_{10} (\sqrt{7} + \sqrt{4})$

$\log_{10} (\sqrt{7} + \sqrt{4}) = \log_{10} (2 + x)$

$\sqrt{7} + \sqrt{4} = 2 + x$

$\sqrt{7} + 2 = 2 + x$

$\sqrt{7} + \cancel{2} - \cancel{2} = x$

$x = \sqrt{7}$

3 Change into logarithm form

i) $6^{-1} = \frac{1}{6}$

$a^x = n$ [exponential form]

$x = \log_a n$ [logarithm form]

$6^{-1} = \frac{1}{6}$

$x = \log_a n$

$-1 = \log_6 \frac{1}{6}$

ii) $2^4 = 16$

$x = \log_a n$

$4 = \log_2 16$

iii] $\sqrt[3]{8} = 2$

$(8)^{1/3} = 2$

$x = \log_a n$

$\frac{1}{3} = \log_8 2$

iv] $a^0 = 1$

$x = \log_a n$

$0 = \log_a 1$

---

4 Change into Exponential form

i] $\log_4 64 = 3$

$x = 3 \quad a = 4 \quad n = 64$

$a^x = n$

$4^3 = 64$

ii] $\log_5 \frac{1}{625} = -4$

$a = 5 \quad n = \frac{1}{625} \quad x = -4$

$a^x = n$

$5^{-4} = \frac{1}{625}$

iii] $\frac{1}{3} = \log_8 2$

$8^{\frac{1}{3}} = 2$

iii] $\log_{\sqrt{2}} 16 = 8$

$a = \sqrt{2} \quad n = 16 \quad x = 8$

$a^x = n$

$(\sqrt{2})^8 = 16$

5 Find the Value of x

i] $\log_5 x = 3$

$5^3 = x$

ii] $\log_a x = 0$

$x = a^0$

$x = 1$

6 Show that

1 $\log_3\left(1+\frac{1}{3}\right) + \log_3\left(1+\frac{1}{4}\right) + \log_3\left(1+\frac{1}{5}\right) + \cdots + \log_3\left(1+\frac{1}{242}\right) = 4$

$\rightarrow \log_3\left(\frac{3+1}{3}\right) + \log_3\left(\frac{4+1}{4}\right) + \log_3\left(\frac{5+1}{5}\right) + \cdots + \log_3\left(\frac{242+1}{242}\right)$

$\log_3\left(\frac{4}{3}\right) + \log_3\left(\frac{5}{4}\right) + \log_3\left(\frac{6}{5}\right) + \cdots \log_3\left(\frac{243}{242}\right)$

$\log_3\left(\frac{\cancel{4}}{3} \times \frac{\cancel{5}}{\cancel{4}} \times \frac{6}{\cancel{5}} \cdots \frac{\cancel{242}}{241} \times \frac{243}{\cancel{242}}\right)$

$$\log_3\left(\frac{\cancel{243}^{81}}{\cancel{3}}\right)$$

$$= \log_3(81)$$

$$= \log_3(3^4)$$

$$= 4\log_3 3 \qquad \log_3 3 = \underline{1}$$

$$= \underline{\underline{4}}$$

---

2) $$\frac{2\log 6 + 6\log 2}{4\log 2 + \log 27 - \log 9}$$

→ Soln

$$= \frac{2\log(2\times 3) + 6\log 2}{4\log 2 + \log 3^3 - \log 3^2}$$

$$= \frac{2\log 2 + 2\log 3 + 6\log 2}{4\log 2 + \underline{3\log 3} - \underline{2\log 3}}$$

$$= \frac{8\log 2 + 2\log 3}{4\log 2 + \log 3}$$

$$= \frac{2\cancel{(4\log 2 + \log 3)}}{\cancel{4\log 2 + \log 3}}$$

$$= \underline{2}$$

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5 Without using tables show that

* $$\frac{\log\sqrt{27}+\log\sqrt{8}-\log\sqrt{125}}{\log 6-\log 5}=\frac{3}{2}$$

$$=\frac{\log(27)^{1/2}+\log(8)^{1/2}-\log(125)^{1/2}}{\log 6-\log 5}$$

$$=\frac{\frac{1}{2}\log 27+\frac{1}{2}\log 8-\frac{1}{2}\log 125}{\log(2\times 3)-\log 5}$$

$$=\frac{\frac{1}{2}\log 3^3+\frac{1}{2}\log 2^3-\frac{1}{2}\log 5^3}{\log 2+\log 3-\log 5}$$

$$=\frac{\frac{3}{2}\log 3+\frac{3}{2}\log 2-\frac{3}{2}\log 5}{\log 2+\log 3-\log 5}$$

$$=\frac{\frac{3}{2}\left[\log 3+\log 2-\log 5\right]}{\log 2+\log 3-\log 5}$$

$$=\frac{3}{2}$$

$\frac{1}{2}\log 3^3$

$3\times\frac{1}{2}\log 3$

4 without using table evaluate

$\log \frac{41}{35} + \log 70 - \log \frac{41}{2} + 2\log 5$

$= \log 41 - \log 35 + \log (35 \times 2) - [\log 41 - \log 2] + \log 5^2$

$= \cancel{\log 41} - \cancel{\log 35} + \cancel{\log 35} + \log 2 - \cancel{\log 41} + \log 2 + \log 5^2$

$= \log 2 + \log 2 + \log 5^2$

$= \log (2 \times 2 \times 5^2)$

$= \log (4 \times 25)$

$= \log 100$

$= \underline{2}$

---

5 $\log \frac{81}{8} - 2\log \frac{3}{2} + 3\log \frac{2}{3} + \log \frac{3}{4} = 0$

*

$\log \frac{3^4}{2^3} - 2\log \frac{3}{2} + 3\log \frac{2}{3} + \log \frac{3}{4}$

$\log 3^4 - \log 2^3 - \log \left[\frac{3}{2}\right]^2 + \log \left[\frac{2}{3}\right]^3 + \log \frac{3}{4}$

$\log 3^4 - \log 2^3 - \log \frac{3^2}{2^2} + \log \frac{2^3}{3^3} + \log \frac{3}{4}$

$\log 3^4 - \log 2^3 - [\log 3^2 - \log 2^2] + \log 2^3 - \log 3^3 + \log 3/4$

$$\log 3^4 - \cancel{\log 2^3} - \log 3^2 + \cancel{\log 2^2} + \cancel{\log 2^3} + \log 3^3 + \log 3 - \cancel{\log 2^2}$$

$$= \log 3^4 - \log 3^2 - \log 3^3 + \log 3$$

$$= \log 3^4 - \log 3^2 - \log 3^3 + \log 3$$

$$= \log\left(\frac{3^4 \times 3}{3^2 \times 3^3}\right)$$

$$= \log\left(\frac{3^5}{3^5}\right)$$

$$= \log 1$$

$$= \underline{\underline{0}}$$

Change of base

If the logarithm of a number to any base is given then the logarithm of the same number to any other base can be determined from the following relations.

$$\log_a m = \log_b m + \log_a b$$

$$\log_b m = \frac{\log_a m}{\log_a b}$$

1 Find $\log_8 25$ given that $\log_{10} 2 = 0.3010$

$$\log_b m = \frac{\log_a m}{\log_a b}$$

$\log_{10} 10 = 1$

$$\log_8 25 = \frac{\log_{10} 25}{\log_{10} 8}$$

$$= \frac{\log_{10}\left(\frac{100}{4}\right)}{\log_{10} 2^3}$$

$$= \frac{\log_{10}(100) - \log_{10} 4}{\log_{10} 2^3}$$

$$= \frac{\log_{10} 10^2 - \log_{10} 2^2}{3 \log_{10} 2}$$

$$= \frac{2 \log_{10} 10 - 2 \log_{10} 2}{3 \log_{10} 2}$$

$$= \frac{2 \times 1 - 2 \times 0.3010}{3 \times 0.3010}$$

$$= \frac{2 - 0.602}{0.903}$$

$$= \frac{1.390}{0.903}$$

$$= 1.5481 \Rightarrow 1.55$$

2 If $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$ find x

$$\log_b m = \frac{\log_a m}{\log_a b}$$

$$\log_2 x = \frac{\log_{10} x}{\log_{10} 2} \qquad \log_4 x = \frac{\log_{10} x}{\log_{10} 4} \qquad \log_{16} x = \frac{\log_{10} x}{\log_{10} 16}$$

$$\frac{\log_{10} x}{\log_{10} 2} + \frac{\log_{10} x}{\log_{10} 4} + \frac{\log_{10} x}{\log_{10} 16} = \frac{21}{4}$$

$$\frac{\log_{10} x}{\log_{10} 2} + \frac{\log_{10} x}{\log_{10} 2^2} + \frac{\log_{10} x}{\log_{10} 2^4} = \frac{21}{4}$$

$$\frac{\log_{10} x}{\log_{10} 2} + \frac{\log_{10} x}{2\log_{10} 2} + \frac{\log_{10} x}{4\log_{10} 2} = \frac{21}{4}$$

LCM

$$\frac{4\log_{10} x + 2\log_{10} x + \log_{10} x}{4\log_{10} 2} = \frac{21}{4}$$

$$\frac{\log_{10} x^4 + \log_{10} x^2 + \log_{10} x}{\log_{10} 2^4} = \frac{21}{4}$$

$$\frac{\log_{10}(x^4 \cdot x^2 \cdot x)}{\log_{10} 2^4} = \frac{21}{4}$$

$$\frac{\log_{10} x^7}{\log_{10} 2^4} = \frac{21}{4}$$

$$4 \cdot \log_{10} x^7 = 21 \log_{10} 2^4$$

$$\cancel{\log_{10}} x^{28} = \cancel{\log_{10}} 2^{84}$$

$$\frac{28 \times 3}{84}$$

$$x^{28} = 2^{84}$$

$$x^{28} = 2^{28 \times 3}$$

$$x^{28} = (2^3)^{28}$$

$$x = 2^3$$

$$x = \underline{\underline{8}}$$

---

3 <u>Solve the following equations</u>

1 $\log_x 3 + \log_x 9 + \log_x 729 = 9$

<u>Soln</u>

$$\log_x 3 + \log_x 3^2 + \log_x 3^6) = 9$$

$$\log_x (3 \cdot 3^2 \cdot 3^6) = 9$$

$$\log_x (3^9) = 9 \qquad \log_a n = x$$

$$x^9 = 3^9 \qquad a^x = n$$

$$x = 3$$

2 $\log_x 4 + \log_x 16 + \log_x 64 = 12$

$\log_x 2^2 + \log_x 2^4 + \log_x 2^6 = 12$ $2\times2\times2\times2$ $2\times2\times2\times$

$\log_x (2^2 \cdot 2^4 \cdot 2^6) = 12$ $\log_a n = x$

$\log_x (2^{12}) = 12$ $a^x = n$

$x^{12} = 2^{12}$

$x = \underline{\underline{2}}$

---

3 $\log_{10} x + \log_{10} (x-3) = 1$ find the value of x

$\log_{10} (x(x-3)) = 1$

$\log_{10} (x^2 - 3x) = 1$

$\cancel{\log_{10}} (x^2 - 3x) = \cancel{\log_{10}} 10$

$x^2 - 3x = 10$

$x^2 - 3x - 10 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{9 - 4\times1\times(-10)}}{2\times1}$$

$$= \frac{3 \pm \sqrt{9+40}}{2}$$

$$= \frac{3 \pm \sqrt{49}}{2} \Rightarrow \frac{3 \pm 7}{2}$$

$$\frac{3+7}{2} \quad \text{or} \quad \frac{3-7}{2}$$

$$\frac{10}{2} \quad \text{or} \quad \frac{-4}{2}$$

$$x = \underline{\underline{5}} \quad \text{or} \quad \underline{\underline{-2}}$$

Value of $x = \underline{\underline{5}}$

4 $\log_{10}(x+3)^2 - 2 = \log_{10}\frac{1}{x^2}$ find x

$$\log_{10}(x+3)^2 - \log_{10}100 = \log_{10}\frac{1}{x^2}$$

$$\log_{10}(x+3)^2 - \log_{10}\frac{1}{x^2} = \log_{10}100$$

$$\cancel{\log_{10}}\left[\frac{(x+3)^2}{1/x^2}\right] = \cancel{\log_{10}}100$$

$$\frac{(x+3)^2}{1/x^2} = 100$$

$$(x+3)^2 \times \frac{x^2}{1} = 100$$

$$(x+3)^2 = \frac{10^2}{x^2}$$

$$(x+3)^{\cancel{2}} = \left(\frac{10}{x}\right)^{\cancel{2}}$$

$$x+3 = \frac{10}{x}$$

$$x(x+3) = 10$$

$$x^2 + 3x = 10$$

$$x^2 + 3x - 10 = 0$$

$$x^2 + 3x - 10 = 0$$

$a = 1 \quad b = 3 \quad c = -10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{9 - 4 \times 1 \times (-10)}}{2 \times 1}$$

$$= \frac{-3 \pm \sqrt{9 + 40}}{2}$$

$$= \frac{-3 \pm \sqrt{49}}{2}$$

$$= \frac{-3 \pm 7}{2}$$

$$= \frac{-3 + 7}{2} \quad \text{or} \quad \frac{-3 - 7}{2}$$

$$= \frac{4}{2} \quad \text{or} \quad \frac{-10}{2}$$

$x = 2$ or $x = -5$

Value of $x = \underline{2}$

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5 $\log_{10}(x-9) + \log_{10} x = 1$

$\log a + \log b = \log a \cdot b$

$\log_{10}(x-9)x = 1$

$\cancel{\log_{10}}(x-9)x = \cancel{\log_{10}} 10$

$(x-9)x = 10$

$x^2 - 9x = 10.$

$x^2 - 9x - 10 = 0$

$a = 1 \quad b = -9 \quad c = -10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-9) \pm \sqrt{81 - 4 \times 1 \times (-10)}}{2 \times 1}$$

$$= \frac{9 \pm \sqrt{81 + 40}}{2}$$

$$= \frac{9 \pm \sqrt{121}}{2}$$

$$= \frac{9 \pm 11}{2}$$

$$= \frac{9+11}{2} \text{ or } \frac{9-11}{2}$$

$$= \frac{20}{2} \text{ or } \frac{-2}{2}$$

$\therefore$ Value of $x = \underline{10}$

$x = 10 \text{ or } x = -1$

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6 $\log_8 x + \log_4 x + \log_2 x = 11$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

$$\frac{\log_{10} x}{\log_{10} 8} + \frac{\log_{10} x}{\log_{10} 4} + \frac{\log_{10} x}{\log_{10} 2} = 11$$

$$\frac{\log_{10} x}{\log_{10} 2^3} + \frac{\log_{10} x}{\log_{10} 2^2} + \frac{\log_{10} x}{\log_{10} 2^1} = 11$$

$$\frac{\log_{10} x}{3 \cdot \log_{10} 2} + \frac{\log_{10} x}{2 \log_{10} 2} + \frac{\log_{10} x}{\log_{10} 2} = 11$$

$$\frac{2 \log_{10} x + 3 \log_{10} x + 6 \log_{10} x}{6 \log_{10} 2} = 11$$

$$\frac{\log_{10} x^2 + \log_{10} x^3 + \log_{10} x^6}{\log_{10} 2^6} = 11$$

$$\frac{\log_{10} (x^2 \cdot x^3 \cdot x^6)}{\log_{10} 2^6} = 11$$

$$\log_{10} x^{11} = 11 \cdot \log_{10} 2^6$$

$$\cancel{\log_{10}} (x^{11}) = \cancel{\log_{10}} (2^6)^{11}$$

$$x = 2^6 \Rightarrow x = 64$$

7 If $\log \frac{x+y}{7} = \frac{1}{2}(\log x + \log y)$ Show that

$$\frac{x}{y} + \frac{y}{x} = 47$$

$\rightarrow$ $$\log \frac{x+y}{7} = \frac{1}{2}(\log xy)$$

$$\cancel{\log} \frac{x+y}{7} = \cancel{\log}(xy)^{1/2}$$

$$\frac{x+y}{7} = (xy)^{1/2}$$

Squaring both side

$$\frac{(x+y)^2}{7^2} = \left[(xy)^{1/2}\right]^2$$

$$\frac{(x+y)^2}{7^2} = xy$$

$$(x+y)^2 = 49xy.$$

$$x^2+y^2+2xy = 49xy$$

$$x^2+y^2 = 49xy - 2xy$$

$$x^2+y^2 = 47xy.$$

$\div xy$, $$\frac{x^2}{xy} + \frac{y^2}{xy} = \frac{47\cancel{xy}}{\cancel{xy}}$$

$$\frac{x}{y} + \frac{y}{x} = 47$$

Scanned with CamScanner

8 Prove that $\log\left(\frac{1}{3}(a+b)\right) = \frac{1}{2}(\log a + \log b)$ if $a^2 + b^2 = 7ab$

$\rightarrow$ $a^2 + b^2 = 7ab$

$a^2 + b^2 = 9ab - 2ab$

$a^2 + b^2 + 2ab = 9ab$

$(a+b)^2 = 9ab$

$$\frac{(a+b)^2}{9} = ab$$

$$\frac{(a+b)^2}{3^2} = ab$$

$$\left[\frac{a+b}{3}\right]^2 = ab$$

$$\log\left(\frac{a+b}{3}\right)^2 = \log ab$$

$$2\log\left(\frac{a+b}{3}\right) = \log ab$$

$$2\log\left(\frac{1}{3}(a+b)\right) = \log a + \log b$$

$\div 2$,

$$\log\left(\frac{1}{3}(a+b)\right) = \frac{1}{2}\log a + \log b$$

9 If $\log 2 = 0.3010$ $\log 3 = 0.4771$

find $\log \dfrac{(16)^{1/5} (5)^2}{(108)^3}$

$$\begin{array}{r|l} 2 & 108 \\ 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$\rightarrow$ $\log \dfrac{(24)^{1/5} \left(10/2\right)^2}{(2^2 \times 3^3)^3}$

$\dfrac{10^2}{2^2} \Rightarrow 10^2 \times \dfrac{1}{2^2}$

$= 10^2 \times 2^{-2}$

$\log \dfrac{(2)^{4/5} (10)^2 (2)^{-2}}{2^6 \times 3^9}$

$\log \dfrac{a}{b} = \log a - \log$

$\log \left[(2)^{4/5} \cdot (10)^2 \cdot (2)^{-2}\right] - \left[\log 2^6 \times 3^9\right]$

$\log (2)^{4/5} + \log (10)^2 + \log (2)^{-2} - \left[\log 2^6 + \log 3^9\right]$

$\frac{4}{5} \log 2 + 2 \log 10 + (-2) \log 2 - 6 \log 2 - 9 \log$

$\frac{4}{5} \times 0.3010 + 2 \times 1 - 2 \times 0.3010 - 6 \times 0.3010 - 9 \times 0$

$= 0.2408 + 2 - 0.602 - 1.806 - 4.2939$

$= 2.2408 - 6.7019$

$= -4.4611$

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2. Permutations And Combinations

$\{a, b, c\}$

	Permutations	Combinat
1 at a time :	$\{a\}, \{b\}, \{c\}$	$\{a\}, \{b\},$
2 at a time :	$\{a, b\}, \{b, c\}$	$\{a, b$
	$\{c, a\}, \{b, a\}$	$\{b, c$
	$\{c, b\}, \{a, c\}$	$\{c, a$
3 at a time :	$\{a, b, c\}$	$\{a, b, c$
	$\{b, c, a\}$	
	$\{c, a, b\}$	

Fundamental Rules of Counting

If one thing can be done in m ways and when it has been done in any of the m ways, a Second thing can be done in n ways, then the two things together can be done in $m \times n$ ways.

Permutations

Permutations refered to different arrengment of things from given log taken one or more at a time

Kramp's factorial Notation

The product of the first n natural numbers 1, 2, 3 . . . n is called fractional n or n factorial and it is return as $\lfloor n$ or $n!$

Thus $n! = 1 \times 2 \times \ldots \ldots (n-2) \times (n-1) \times n$

1 Show that $\frac{10!}{8!} = 90$

→ $\frac{10!}{8!} = \frac{10 \times 9 \times \cancel{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}}{\cancel{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}}$

$= 10 \times 9$

$= \underline{\underline{90}}$

---

2 Find How many 4 letter words can be formed out of the word LOGARITHMS

$^nP_r = \frac{n(n-1)(n-2)\ldots(n-r+1)(n-r)!}{(n-r)!}$

$= \frac{n!}{(n-r)!}$

$n = 10 \quad r = 4$

$^nP_r = {}^{10}P_4 = \frac{10!}{(10-4)!}$

$= \frac{10 \times 9 \times 8 \times 7 \times \cancel{6!}}{\cancel{6!}}$

$= 10 \times 9 \times 8 \times 7$

$= \underline{\underline{5{,}040}}$

Note : Permutation of n different things taken r at a time where $r \leq n$ are

$n(n-1) \dots (n-r+1)$

* The number of permutations of n different things taken all at a time is ${}^nP_n$

$${}^nP_n = n(n-1) \dots \times 3 \times 2 \times 1 = n!$$

* $${}^nP_r = n(n-1) \dots (n-r+1)$$

$$= \frac{n(n-1)(n-2) \dots (n-r+1)(n-r)!}{(n-r)!}$$

$$= \frac{n!}{(n-r)!}$$

* $${}^nP_n = n!$$

$${}^nP_n = \frac{n!}{(n-n)!}$$

$$n! = \frac{n!}{0!}$$

$$0! = \frac{n!}{n!}$$

$$0! = 1$$

1 Indicate How many 4 digit Numbers greater than 7,000 can be formed from the digits 3, 5, 7, 8, 9

→ 3, 5, 7, 8, 9 > 7,000 So we have to choose 7, 8, 9 for first place.

__ __ __ __
1

$$^3P_1 = \frac{3!}{(3-1)!}$$

$$= \frac{3 \times 2 \times 1}{2 \times 1}$$

$$= \underline{\underline{3}}$$

If the digits are to be greater than 7,000 then the first digit can be any of the 7, 8 and 9. Now the first digit can be choosen in 3 ways.

$$3P_1 = \frac{3!}{(3-1)!}$$

$$= \frac{3 \times 2 \times 1}{2 \times 1}$$

$$= \underline{\underline{3}}$$

And the remaining 3 digit can be any of the 4 digits left which can choosen in $^4P_3$ ways.

Therefore total number of ways $= 3 \times {}^4P_3$

$$3 \times \frac{4!}{(4-3)!}$$

$$3 \times \frac{4 \times 3 \times 2 \times 1}{1!}$$

$$3 \times (4 \times 3 \times 2)$$

$$3 \times 24$$

$$= \underline{72}$$

Permutations of things not all different the number of Permutations of n things of which p things are of One kind, q things are of a Second kind, r things are of a third kind and all the rest are different is given by $x = \frac{n!}{p!\, q!\, r!}$ If letters are diff

1 Find the number of Permutations of the word ACCOUNTANT

→ ACCOUNTANT

$n = 10$ $C = 2$ $A = 2$ $T = 2$ $N = 2$

Solution

The word ACCOUNTANT has 10 letters of which 2 are A's, 2 are C's, 2 are N's and 2 are T's the rest are different therefore the number permutation is

$$= \frac{10!}{2!\,2!\,2!\,2!}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times \overset{3}{\cancel{6}} \times 5 \times \overset{2}{\cancel{4}} \times 3 \times \cancel{2!}}{\cancel{2} \times 1 \times \cancel{2} \times 1 \times \cancel{2} \times 1 \times \cancel{2!}}$$

$$= 2,26,800$$

2 Find the number of Permutations of the word ENGINEERING

→ The word ENGINEERING has 11 letters of which 3 are E's, 3 are N's, 2 are G's 2 are I's the rest are different

∴ The Number of Permutation is

$$= \frac{11!}{3!\,3!\,2!\,2!}$$

$$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times \overset{2}{\cancel{6}} \times 5 \times \overset{2}{\cancel{4}} \times \cancel{3!}}{\cancel{3!}\ \cancel{3} \times \cancel{2} \times 1 \times \cancel{2} \times 1 \times \cancel{2} \times 1}$$

$$= 11 \times 10 \times 9 \times 8 \times 7 \times 5$$

$$= 2,77,200$$

3 Find the Number of arrangments that can be made out of the letters of the word ASSASSINATION

→ The word ASSASSINATION has 13 letters of which 3 are A's, 4 are S's, 2 are I's 2 are N's the rest are different

$\therefore$ The Number of Permutations is

$$= \frac{13!}{3!\,4!\,2!\,2!}$$

$$= \frac{13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4!\; 3 \times 2 \times 2 \times 1 \times 2 \times 1}$$

$$= 13 \times 12 \times 11 \times 10 \times 9 \times 2 \times 7 \times 5$$

$$= \underline{1,08,10,800}$$

4 Combinations

Combinations reffer to different sets or groups made out of a given log, without repeating An element, taking One or more of them at a time

Theorem

1. The number of combinations of n different things taken r at a time. r given by

$$^nC_r = \frac{n!}{(n-r)!\,r!} \quad [r \le n]$$

1 In an Examination in Paper On advanced accountance, 10 questions are set. In How many different ways can Examinee choose 7 questions.

→ $n = 10 \quad r = 7$

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

$$^{10}C_7 = \frac{10!}{(10-7)!\,7!}$$

$$= \frac{10 \times \overset{3}{\cancel{9}} \times \cancel{8} \times \cancel{7!}}{\cancel{3 \times 2} \times 1\ \cancel{7!}}$$

$$= \underline{\underline{120}}$$

The number of different choices is evid equal to the number of ways in which 7 places can be filled up by 10 different things. Therefore required number of ways is 120 ways.

---

2 In a mercantile firm 4 post for vacant and 35 candidats apply for the post. In How many ways can a selection be make.
① If one perticular candidate is always included.
② If one perticular candidate is always excluded.

→ ① Since a perticular person is always to be selected we must select be remaining 3 candidats out of the remaining 35.
Therefore the required number of selections is $^{34}C_3 = \frac{34!}{(34-3)!\,3!}$

$$= \frac{34 \times \overset{11}{\cancel{33}} \times \overset{16}{\cancel{32}} \times \cancel{31!}}{\cancel{31!}\ \cancel{3 \times 2} \times 1}$$

$= 34 \times 11 \times 16$

$= \underline{5984}$

(2) Since a perticular person is always to be excluded. The choice is restricted to 4 candidates out of the remaining 34

Therefore required number of selection is

$$^{34}C_4 = \frac{34!}{(34-4)!\,4!}$$

$$= \frac{34 \times 33 \times 32 \times 31 \times 30!}{30!\;4 \times 3 \times 2 \times 1}$$

$= 17 \times 11 \times 8 \times 31$ 46,376

$= \underline{46,376}$

Binomial theorem

If $(x+a)$ is a binomial expression the expansion of $(x+a)^n$ is given by

2M

$$(x+a)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 + {}^nC_3 x^{n-3} a^3 + \dots\dots + {}^nC_r x^{n-r} a^r + {}^nC_n a^n.$$

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* $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$= a^3 + b^3 + 3a^2b + 3ab^2$

$(a+b)^3 = {}^3C_0\, a^3 + {}^3C_1\, a^2b + {}^3C_2\, a b^2 + {}^3C_3\, b^3$

$= \frac{3!}{(3-0)!\,0!} a^3 + \frac{3!}{(3-1)!\,1!} a^2b + \frac{3!}{(3-2)!\,2!} a b^2$

$+ \frac{3!}{(3-3)!\,3!} b^3$

$= a^3 + 3a^2b + 3ab^2 + b^3$

$= a^3 + b^3 + 3a^2b + 3ab^2$

$= a^3 + b^3 + 3ab(a+b)$

---

1 Expand $(x - 1/x)^5$

$n = 5 \quad a = (-1/x)$

$(x - 1/x)^5 = {}^5C_0\, x^5 + {}^5C_1\, x^4(-1/x) + {}^5C_2\, x^3(-1/x)^2 +$

${}^5C_3\, x^2(-1/x)^3 + {}^5C_4\, x^1(-1/x)^4 + {}^5C_5\, (-1/x)^5$

$= x^5 + 5x^4(-1/x) + 10x^3(-1/x)^2 + 10x^2(-1/x)^3$

$+ 5x^1(-1/x)^4 + (-1/x)^5$

$= x^5 - 5\frac{x^4}{x} + 10\frac{x^3}{x^2} - 10\frac{x^2}{x^3} + 5\frac{x}{x^4} - \frac{1}{x^5}$

$= x^5 - 5x^3 + 10x^1 - \frac{10}{x} + \frac{5}{x^3} - \frac{1}{x^5}$

2 $(y+z)^4$

$(y+z)^4 = {}^4C_0 y^4 + {}^4C_1 y^3 z + {}^4C_2 y^2 z^2 + {}^4C_3 y z^3 + {}^4C_4 z^4$

$= \frac{4!}{(4-0)!\,0!} y^4 + \frac{4!}{(4-1)!\,1!} y^3 z + \frac{4!}{(4-2)!\,2!} y^2 z^2 + \frac{4!}{(4-3)!\,3!} y z^3 + \frac{4!}{(4-4)!\,4!} z^4$

$= 1.y^4 + 4y^3 z + 6y^2 z^2 + 4yz^3 + z^4$

3 Write clone the Expansion of $(3x - \frac{1}{2}y)^4$ by the binomial theorem by giving value to X and Y obtain the Value of $(29.5)^4$ correct to 4 Significant figures.

$\rightarrow$ $(3x - \frac{1}{2}y)^4 = {}^4C_0 (3x)^4 + {}^4C_1 (3x)^3 (-\frac{1}{2}y) + {}^4C_2 (3x)^2 (-\frac{1}{2}y)^2 + {}^4C_3 (3x)^1 (-\frac{1}{2}y)^3 + {}^4C_4 (-\frac{1}{2}y)^4$

$= 1.81x^4 + 4.27x^3(-\frac{1}{2}y) + 6.9x^2 (\frac{1}{2^2}y^2) + 4.3x(-\frac{1}{2^3}y^3) + 1.(\frac{1}{2}y)^4$

$= 81x^4 - \overset{54}{\cancel{108}} x^2 (\frac{y}{\cancel{2}}) + 54x^2(\frac{y^2}{2^2}) + 12x(\frac{y^3}{2^3}) + \frac{y^4}{2^4}$

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$$= 81x^4 - 54x^3y + 54x^2\left(\frac{y^2}{2^2}\right) - 12x\left(\frac{y^3}{8}\right) + \frac{y^4}{16}$$

$$= 81x^4 - 54x^3y + \frac{27x^2y^2}{2} - \frac{3xy^3}{2} + \frac{y^4}{16}$$

Consider this expression as (*)

(*)

4. $(29.5)^4 = (30 - 0.5)^4 = \left(3 \times 10 - \frac{1}{2} \times 1\right)^4$

$x = 10$ and $y = 1$

$$\left(3\times 10 - \frac{1}{2}\times 1\right)^4 = 81 \times (10)^4 - 54 \times (10)^3 \times (1) + \frac{27 \times (10)^2 (1)^2}{2} - \frac{3 \times 10 \times 1^3}{2} + \frac{1^4}{16}$$

$$= 81 \times 10000 - 54 \times 1000 \times 1 + \frac{27 \times 100 \times 1}{2} - \frac{3 \times 10 \times 1}{2} + \frac{1}{16}$$

$$= 81 \times 10000 - 54 \times 1000 + \frac{2700}{2} - \frac{3}{2} + \frac{1}{16}$$

48.5625

0.0625

$$= 810000 - 54000 + 1350 - 1.5 + 0.0625$$

$$= 757335.0625 \Rightarrow 7573$$

4 HW

$$\left(\frac{x}{3}+\frac{2}{y}\right)^4$$

$$= {}^4C_0\left(x/3\right)^4 + {}^4C_1\left(x/3\right)^3\left(2/y\right) + {}^4C_2\left(x/3\right)^2\left(2/y\right)^2$$

$$+ {}^4C_3\left(\frac{x}{3}\right)\left(2/y\right)^3 + {}^4C_4\left(2/y\right)^4$$

$$= \frac{4!}{(4-0)!\,0!}\cdot\frac{x^4}{3^4} + \frac{4!}{(4-1)!\,1!}\cdot\frac{x^3}{3^3}\cdot\frac{2}{y}$$

$$+ \frac{4!}{(4-2)!\,2!}\cdot\frac{x^2}{3^2}\,\frac{4}{y^2} + \frac{4!}{(4-3)!\,3!}\cdot\frac{x}{3}\cdot\frac{8}{y^3}$$

$$+ \frac{4!}{(4-4)!\,4!}\cdot\frac{16}{y^4}$$

$$= 1\cdot\frac{x^4}{3^4} + \frac{4\times\cancel{3!}}{\cancel{3!}\,1!}\cdot\frac{x^3}{3^3}\cdot\frac{2}{y} + \frac{\overset{2}{\cancel{4}}\times 3\times\cancel{2!}}{\cancel{2!}\,\cancel{2}\times 1}$$

$$\cdot\frac{x^2}{3^2}\cdot\frac{4}{y^2} + \frac{4\times\cancel{3!}}{1!\,\cancel{3!}}\cdot\frac{x}{3}\cdot 8/y^3 + 1\cdot\frac{16}{y^4}$$

$$= \frac{x^4}{81} + 4\cdot\frac{x^3}{27}\cdot\frac{2}{y} + 6\cdot\frac{x^2}{9}\cdot\frac{4}{y^2} + 4\cdot\frac{x}{3}\cdot\frac{8}{y^3}$$

$$+ 16/y^4$$

$$= \frac{x^4}{81} + \frac{8x^3}{27y} + \frac{24x^2}{9y^2} + \frac{32x}{3y^3} + \frac{16}{y^4}$$

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5 write down and Simplify

1 The 11th term in a expansion of $(y+4x)^{30}$

$\rightarrow (y+4x)^{30} = {}^{30}C_0\, y^{30} + {}^{30}C_1\, y^{29}(4x) + {}^{30}C_2\, y^{28}(4x)^2$
$+ {}^{30}C_3\, y^{27}(4x)^3 + {}^{30}C_4\, y^{26}(4x)^4 + {}^{30}C_5\, y^{25}(4x)^5$
$+ {}^{30}C_6\, y^{24}(4x)^6 + {}^{30}C_7\, y^{23}(4x)^7 + {}^{30}C_8\, y^{22}(4x)^8$
$+ {}^{30}C_9\, y^{21}(4x)^9 + {}^{30}C_{10}\, y^{20}(4x)^{10} + {}^{30}C_{11}$

$${}^{30}C_{10}\, y^{(30-10)}(4x)^{10}$$

$$= {}^{30}C_{10}\, y^{20}(4x)^{10}$$

6 Find the 5th term in the expansion of

$$\left(\frac{3x}{4} + \frac{4}{3x}\right)^{12}$$

$$\left(\frac{3x}{4} + \frac{4}{3x}\right)^{12} = {}^{12}C_4 \left[\frac{3x}{4}\right]^{(12-4)} \left[\frac{4}{3x}\right]^4$$

$$= {}^{12}C_4 \left(\frac{3x}{4}\right)^8 \left(\frac{4}{3x}\right)^4$$

$$= \frac{12!}{8!\,4!} \left(\frac{3x}{4}\right)^8 \left(\frac{4}{3x}\right)^4$$

$$= \frac{\overset{3}{\cancel{12}} \times 11 \times \overset{5}{\cancel{10}} \times 9 \times \cancel{8!}}{\cancel{8!}\ \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \left(\frac{3x}{4}\right)^8 \left(\frac{4}{3x}\right)^4$$

7

$$= 495 \frac{(3x)^4 (3x)^4}{4^4 \times \cancel{4^4}} \times \frac{\cancel{4^4}}{\cancel{(3x)^4}}$$

$$= 495 \frac{(3x)^4}{4^4}$$

$$= 495 \left(\frac{3x}{4}\right)^4$$

$$= 495 \times \frac{81 \times x^4}{256}$$

$$= \frac{40095\, x^4}{256}$$

---

7 Find the middle term in the expansion of $\left(\frac{a}{x} - bx\right)^{12}$

$12 + 1 \rightarrow ({}^nC_0)$

13

middle of $13 = 7$

$n = 12$

$${}^{12}C_6 \left(\frac{a}{x}\right)^{(12-6)} (-bx)^6$$

$$t_7 = {}^{12}C_6 \frac{a^6}{\cancel{x^6}} \times b^6 \cancel{x^6}$$

$$= {}^{12}C_6\, a^6 b^6$$

If n is even then 1 middle t

If n is odd then 2 middle ter

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b$

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8 Find the two middle terms in the expansion of $\left(3x - \frac{2x^2}{3}\right)^7$

$\rightarrow$ $\left(3x - \frac{2x^2}{3}\right)^7 = {}^7C_3 \, (3x)^{7-3} \left(\frac{2x^2}{3}\right)^3$

$t_4 = \frac{7!}{(7-3)!\,3!} (3x)^4 \left(\frac{2^3x^6}{3^3}\right)$

$= \frac{7 \times \cancel{6}^{2} \times 5 \times \cancel{4!}}{\cancel{4!}\, \cancel{3 \times 2} \times 1} \, 3^4x^4 \times \frac{2^3x^6}{3^3}$

$= \frac{35 \times 3^4 \times x^4 \times 2^3 \times x^6}{3^3}$

$= 35 \times 3 \times x^4 \times 8 \times x^6$

$= 840x^{10}$

$t_5 = {}^7C_4 \, (3x)^{7-4} \left(\frac{2x^2}{3}\right)^4$

$= \frac{7!}{(7-4)!\,4!} (3x)^3 \left(\frac{2^4x^8}{3^4}\right)$

$= \frac{7 \times \cancel{6}^{3} \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}\, 4 \times 3 \times 2 \times 1} \, 3^3x^3 \left(\frac{2^4x^8}{3^4}\right)$

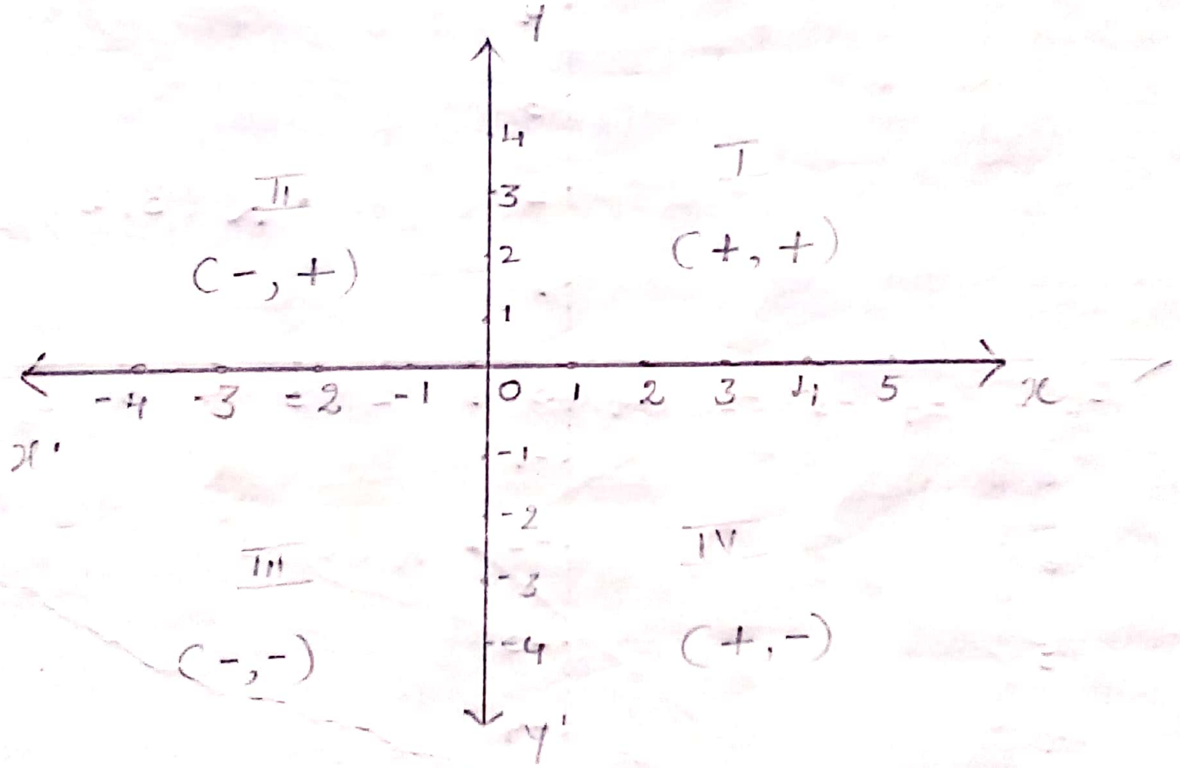
$= \frac{35 \times 3^3 \times x^3 \times 2^4 x^8}{3^4}$

$$= \frac{35 \times 16 \times x''}{3}$$

$$= \frac{560}{3} x''$$

Analytical Geometry

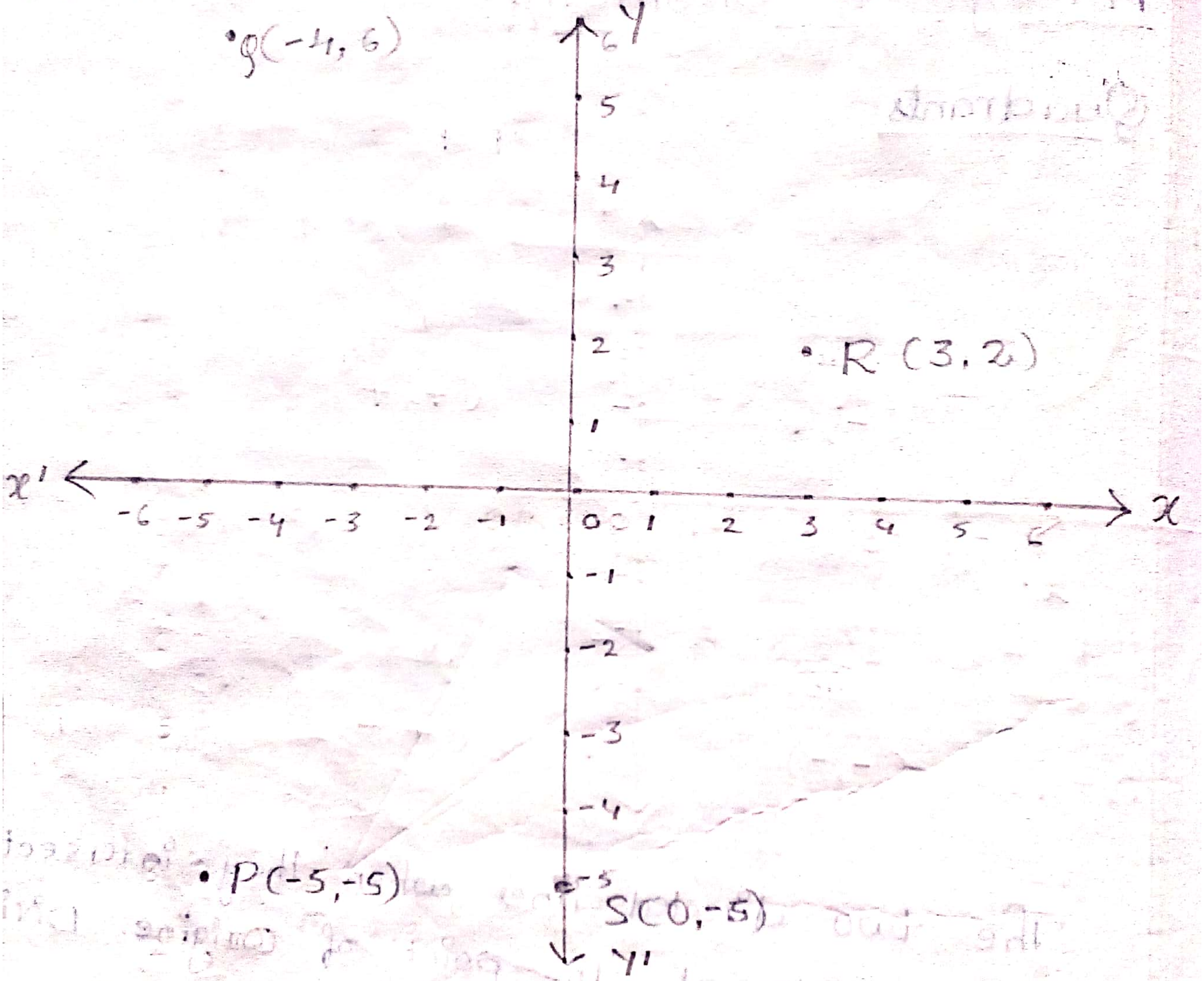
Quadrants



The two directed lines when they intersect at right angle at the point of Origine. Divide their plane into 4 parts on regions namely XOY, YOX', X'OY', Y'OX these parts are respectivily indicates as First (I), Second (II), third (III) and fourth (IV) Quadrants.

The position of the Quadrenates in a parti-
war Quadrant would defined on the positive
and negative values of the co-ordinates.

Plote the points with the following coordinate
P(-5,-5) R(3,2) Q(-4,6) S(0,-5)



Co-ordinates of Midpoint

We can find out the coordinates of the midpoint from the coordinates of any two points using the formula

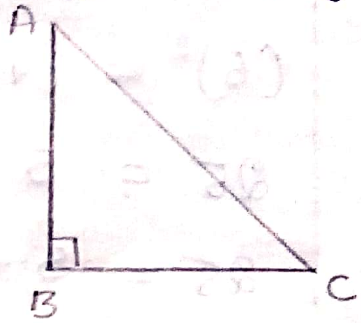
$$x_m = \frac{x_1 + x_2}{2} \qquad Y_m = \frac{Y_1 + Y_2}{2}$$

Distance between two Points

The Distance say d between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by the formula.
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

---

1 Show that the points $(6, 6)$, $(2, 3)$ and $(4, 7)$ are the vertices of Rightangle triangle



$$AC^2 = AB^2 + BC^2$$

→ $A(6, 6)$ $B(2, 3)$ $C(4, 7)$
$\quad x_1, y_1 \qquad x_2, y_2 \qquad x_3, y_3$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$(AB) = \sqrt{(6-2)^2 + (6-3)^2}$$
$$= \sqrt{4^2 + 3^2}$$
$$= \sqrt{16 + 9}$$
$$(AB) = \sqrt{25}$$
$$AB = \underline{5}$$

$$BC = \sqrt{(2-4)^2 + (3-7)^2}$$
$$= \sqrt{(-2)^2 + (-4)^2}$$
$$= \sqrt{2^2 + 4^2}$$
$$= \sqrt{4 + 16}$$
$$= \sqrt{20}$$
$$= \sqrt{4 \times 5}$$
$$= \underline{2\sqrt{5}}$$

$$AC = \sqrt{(4-6)^2 + (7-6)^2}$$

$$AC = \sqrt{(-2)^2 + (1)^2}$$

$$= \sqrt{4+1}$$

$$= \underline{\underline{\sqrt{5}}}$$

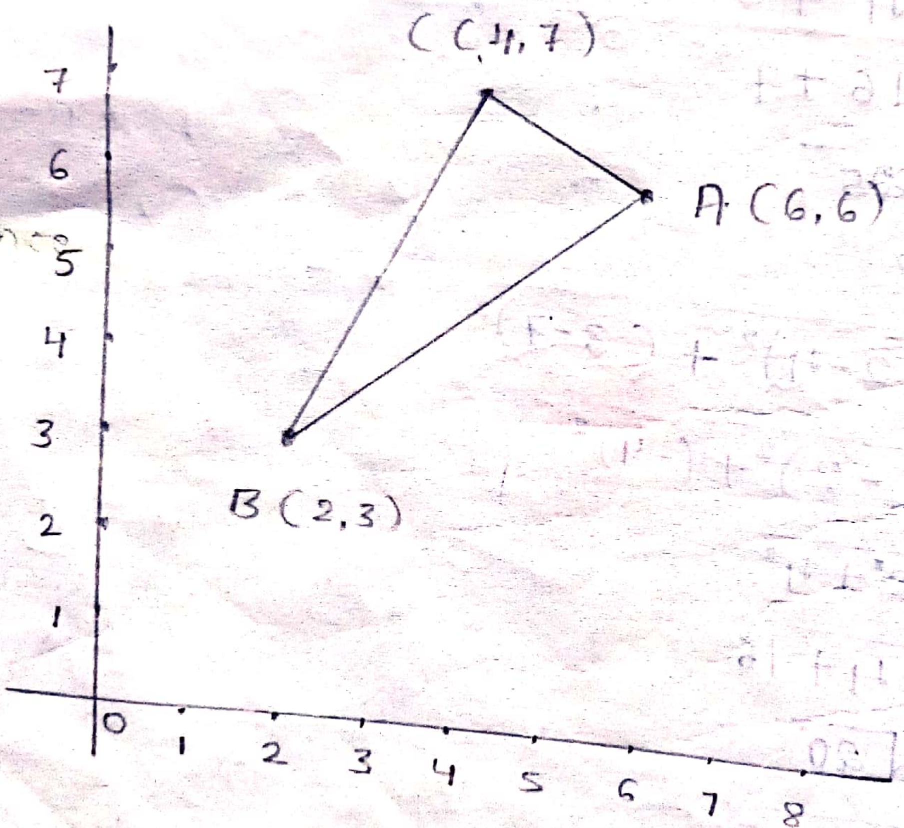
$$AB^2 = AC^2 + BC^2$$

$$(5)^2 = (\sqrt{5})^2 + (2\sqrt{5})^2$$

$$25 = 5 + 4 \times 5$$

$$25 = 5 + 20$$

$$\underline{\underline{25}} = \underline{\underline{25}}$$



Hence the points (6,6), (2,3) and (4,7) are vertices of a right angle triangle.

Q. Prove that Points $(4, 3)$ $(7, -1)$ $(9, 3)$ are the vertices of an isoceles triangle

-> we know that properties of an isoceles triangle is that two of its sides are equal using the distance formula.

$A(4, 3)$ $B(7, -1)$ $C(9, 3)$

$$AB = \sqrt{(4-7)^2 + (3+1)^2}$$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{9 + 16} = \sqrt{25} = \underline{\underline{5}}$$

$$BC = \sqrt{(7-9)^2 + (-1-3)^2}$$

$$= \sqrt{2^2 + (-4)^2}$$

$$= \sqrt{4 + 16} = \sqrt{20} = \sqrt{4 \times 5} = \underline{\underline{2\sqrt{5}}}$$

$$CA = \sqrt{(9-4)^2 + (3-3)^2}$$

$$= \sqrt{5^2 + 0^2}$$

$$= \sqrt{25}$$

$$= \underline{5}$$

Since two of the sides that is AB and CA are equal that triangle is an isoceles triangle.

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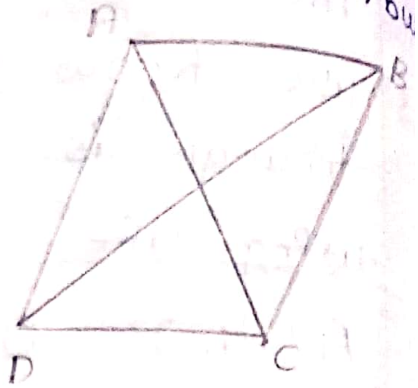
3 Prove that Quadrilateral with Vertices $A(2,-1)$, $B(3,4)$, $C(-2,3)$, $D(-3,-2)$ are rhombus



4 sides are equal but $AC \neq DB$

$$AB = \sqrt{(2-3)^2 + (-1-4)^2}$$
$$= \sqrt{1^2 + (-5)^2}$$
$$= \sqrt{1+25} = \sqrt{26}$$

$$BC = \sqrt{(3+2)^2 + (4-3)^2}$$
$$= \sqrt{5^2 + 1^2}$$
$$= \sqrt{25+1} = \sqrt{26}$$

$$CD = \sqrt{(-2+3)^2 + (3+2)^2}$$
$$= \sqrt{1^2 + 5^2}$$
$$= \sqrt{1+25} = \sqrt{26}$$

$$DA = \sqrt{(-3-2)^2 + (-2+1)^2}$$
$$= \sqrt{(-5)^2 + 1^2}$$
$$= \sqrt{25+1} = \sqrt{26}$$

$$AC = \sqrt{(2+2)^2 + (-1-3)^2}$$
$$= \sqrt{4^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32}$$

$$BD = \sqrt{(3+3)^2 + (4+2)^2}$$
$$= \sqrt{6^2 + 6^2} = \sqrt{36+36} = \sqrt{72}$$

$$AB = BC = CD = DA = \sqrt{26}$$
$$AC \neq BD$$

4 Show that the points $(4,-5)$ $(8,1)$ $(14,-3)$ and $(10,-9)$ are the vertices of a Square

→ $d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$AB = \sqrt{(4-8)^2+(-5-1)^2}$

$= \sqrt{(-4)^2+(-6)^2} = \sqrt{16+36} = \sqrt{52}$

$BC = \sqrt{(8-14)^2+(1+3)^2}$

$= \sqrt{(-6)^2+(4)^2} = \sqrt{36+16} = \sqrt{52}$

$CD = \sqrt{(14-10)^2+(-3+9)^2}$

$= \sqrt{4^2+(+6)^2} = \sqrt{16+36} = \sqrt{52}$

$DA = \sqrt{(10-4)^2+(-9-(-5))^2}$

$= \sqrt{(10-4)^2+(-9+5)^2}$

$= \sqrt{6^2+(-4)^2}$

$= \sqrt{36+16}$

$= \sqrt{52}$

$AB = BC = CD = DA$

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5 Show that the points (2, -2) (8, 4) (5, 7) and (-1, 1) are the vertices of a rectangle

$\rightarrow$ $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$AB = \sqrt{(2-8)^2 + (-2-4)^2}$

$= \sqrt{(-6)^2 + (-6)^2}$

$= \sqrt{36+36} \Rightarrow \sqrt{72}$

$BC = \sqrt{(8-5)^2 + (4-7)^2}$

$= \sqrt{3^2 + (-3)^2}$

$= \sqrt{9+9} \Rightarrow \sqrt{18}$

$CD = \sqrt{(5+1)^2 + (7-1)^2}$

$= \sqrt{6^2 + 6^2}$

$= \sqrt{36+36} \Rightarrow \sqrt{72}$

$DA = \sqrt{(-1-2)^2 + (1+2)^2}$

$= \sqrt{(-3)^2 + 3^2}$

$= \sqrt{9+9} \Rightarrow \sqrt{18}$

$AB = CD$ and $BC = DA$

6 Prove that points are vertices of the Parallelogram

ⓐ (−2, −1) (1, 0) (4, 3) and (1, 2)

ⓑ (2, 1) (3, 2) (6, 4) and (3, 3)

→ ⓐ A(−2, −1) B(1, 0) C(4, 3) D(1, 2)

Mid point of AC = $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$

$$AC = \left(\frac{-2+4}{2}, \frac{-1+3}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{2}{2}\right)$$

$$= (1, 1) \quad — ①$$

Mid point of BD = $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$

$$= \left(\frac{1+1}{2}, \frac{0+2}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{2}{2}\right)$$

$$= (1, 1) \quad — ②$$

From ① and ② we conclude that the AC and BD bisect each other by the at Same point (1, 1) hence the quadrailateral ABCD is a parallelogram.

ⓑ (2, 1) (3, 2) (6, 4) (3, 3)

Mid point of AC $= \left(\frac{2+6}{2}, \frac{1+4}{2}\right)$

$= \left(\frac{8}{2}, \frac{5}{2}\right)$

$= (4, 5/2)$ — ①

Mid point of BD $= \left(\frac{5+3}{2}, \frac{2+3}{2}\right)$

$= \left(\frac{8}{2}, \frac{5}{2}\right)$

$= (4, 5/2)$ — ②

From ① and ② we conclude that AC and BD biset each other at same poin $(4, 5/2)$ hence the quadrilateral ABCD is parallelogram.

Section Formula

The coordinates of the point R (x, y) dividing a line in the ratio of m:n connecting the point P (x_1, y_1) and Q (x_2, y_2) is given by $x = \frac{mx_2 + nx_1}{m+n}$

$$y = \frac{my_2 + ny_1}{m+n}$$

External division

In the above it was assumed that R divides PQ line internaly in the ratio of m:n this means the point Re lies between P and Q. If line PQ divided externally by R then R lies outside PQ then the formula is given by $x = \frac{mx_2 - nx_1}{m-n}$

$$y = \frac{my_2 - ny_1}{m-n}$$

1 Find the coordinates of the point which divides the point P (8,9) Q(-7,4) internally in the ratio of 2:3 and externally in the ratio of 4:3.

→ Internally

P(8,9) Q(-7,4)

$m = 2 \quad n = 3 \quad x_1 = 8 \quad x_2 = -7 \quad y_1 = 9 \quad y_2 = 4$

$x = \frac{mx_2 + nx_1}{m+n}$ $\qquad$ $y = \frac{my_2 + ny_1}{m+n}$

$x = \frac{2(-7) + 3(8)}{2+3}$ $\qquad$ $y = \frac{2(4) + 3(9)}{2+3}$

$= \frac{-14 + 24}{5}$ $\qquad$ $= \frac{8 + 27}{5}$

$= \frac{10}{5}$ $\qquad$ $= \frac{35}{5}$

$= 2$ $\qquad$ $= 7$

Externally

$m = 4 \quad n = 3 \quad x_1 = 8 \quad x_2 = -7 \quad y_1 = 9 \quad y_2 = 4$

$$x = \frac{mx_2 - nx_1}{m - n}$$

$$= \frac{4(-7) - 3(8)}{4 - 3}$$

$$= \frac{-28 - 24}{1}$$

$$= -52$$

$$y = \frac{my_2 - ny_1}{m - n}$$

$$= \frac{4(4) - 3 \times 9}{4 - 3}$$

$$= \frac{16 - 27}{1}$$

$$= -11$$

R $(-52, -11)$

2 Find the coordinates of the point which divides internally join the pair of points.

(a) $(6, -5)$ and $(-7, -15)$ ratio $4:7$

(b) $(5, 2)$ and $(7, 9)$ ratio $2:7$

(a) $(6, -5) \quad (-7, -15)$

$x_1 = 6 \quad x_2 = -7 \quad y_1 = -5 \quad y_2 = -15 \quad m = 4 \quad n = 7$

Internally

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$= \frac{4(-7) + 7(6)}{4 + 7}$$

$$= \frac{-28 + 42}{11}$$

$$y = \frac{my_2 + ny_1}{m + n}$$

$$= \frac{4(-15) + 7(-5)}{4 + 7}$$

$$= \frac{-60 - 35}{11}$$

$x = \frac{14}{11}$ $\qquad y = \frac{-95}{11}$

External

(b) $(5, 2) \quad (7, 9)$

$x_1 = 5 \quad x_2 = 7 \quad y_1 = 2 \quad y_2 = 9 \quad m = 2 \quad n = 7$

$x = \frac{mx_2 + nx_1}{m+n}$ $\qquad y = \frac{my_2 + ny_1}{m+n}$

$= \frac{2(7)+7(5)}{2+7}$ $\qquad = \frac{2(9)+7(2)}{2+7}$

$= \frac{14+35}{+9}$ $\qquad = \frac{18+14}{9}$

$x = \frac{49}{9}$ $\qquad y = \frac{32}{9}$

3 Find the coordinates of the point which divides Externally in the joint pair of point

(a) $(4, 7)$ and $(1, -2)$ $\quad 3:2$

(b) $(-3, 2)$ and $(4, -3)$ $\quad 5:3$

(a) $x = \frac{mx_2 - nx_1}{m-n}$ $\qquad y = \frac{my_2 - ny_1}{m-n}$

$= \frac{3(1)-2(4)}{3-2}$ $\qquad = \frac{3(-2)-2(7)}{3-2}$

$= \frac{3-8}{1}$ $\qquad = \frac{-6-14}{1}$

$= -5$ $\qquad = -20$

(b) $x = \dfrac{mx_2 - nx_1}{m-n}$ $\quad$ $y = \dfrac{my_2 - ny_1}{m-n}$

$= \dfrac{5(4) - 3(-3)}{5-3}$ $\quad$ $= \dfrac{5(-3) - 3(2)}{5-3}$

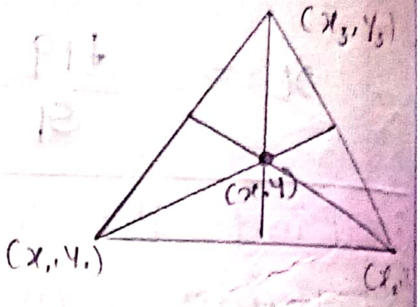
$= \dfrac{20+9}{2}$ $\quad$ $= \dfrac{-15-6}{+2}$

$= \dfrac{29}{2}$ $\quad$ $= -\dfrac{21}{+2}$

Coordinates of a Centroid

The ~~coo~~ centroid of a triangle is the point of intersection of 3 medians of a triangle.

The centroid of a triangle is calculated by the formula.



$x = \dfrac{x_1 + x_2 + x_3}{3}$ $\quad$ $y = \dfrac{y_1 + y_2 + y_3}{3}$

1 Find the coordinates of the centroid of the triangle whose vertices are $(3, 2)$ $(-1, 4)$ $(-5, 6)$

→ $x = \dfrac{x_1 + x_2 + x_3}{3}$ $\quad$ $y = \dfrac{y_1 + y_2 + y_3}{3}$

$= \dfrac{3 - 1 - 5}{3}$ $\quad$ $= \dfrac{2+4+6}{3}$

$= \dfrac{3-6}{3}$ $\quad$ $= 12/3$

$= -3/3 = -1$ $\quad$ $= 4$

∴ Coordinats of the centroid $(-1, 4)$

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Area of a triangle

We can find out the area of a triangle with the vertices given as $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ by the formula

$$\text{Area of } \triangle ABC = \frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$$

Remark: The sign of the area of the triangle is positive or negative as the arrangment of the vertices are counter clockvise (anti clockvise) or clockvise

$$\text{area of } \triangle ABC = \pm\frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$$

1 Find the areas of triangle whose vertices are (a) (0, 0) (1, 2) and (−1, 2)
(b) (2, −1) (−3, −4) and (0, 2)

(a) (0, 0) (1, 2) (−1, 2)

$$\text{Area of } \triangle ABC = \frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$$

$$= \frac{1}{2}\{0(2-2)+1(2-0)-1(0-2)\}$$

$$= \frac{1}{2}\{2+2\}$$

$$= \frac{1}{\cancel{2}} \times \cancel{4}^{\,2}$$

$$= 2 \text{ Sq units}$$

(b) $(2,-1)$ $(-3,-4)$ $(0,2)$

area of $\Delta ABC = \frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$

$= \frac{1}{2}\{2(-4-2)-3(2+1)+0(-1+4)\}$

$= \frac{1}{2}\{2(-6)-3(3)+0\}$

$= \frac{1}{2}\{-12-9\}$

$= \frac{1}{2}\{-21\}$

$= -\frac{21}{2}$ sq units.

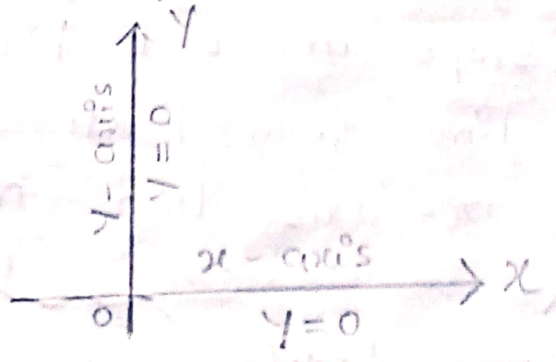
The Straight Line

Different forms of equation of the Straight line :

(1) Equation of the coordinate axis

(a) If $P(x, y)$ be any point on the x axis then its ordinate y is always zero. for any position of the point P on the X axis. and for no other position. Therefore $y = 0$ is the equation of x-axis.

(b) If $P(x, y)$ is any point on the Y axis then x is always zero for any position of the point P on the Y axis and for no other point therefore $x = 0$ is the equation of Y-axis.



(2) Equations of the lines parallel to the coordinate axis

(a) Let $P(x, y)$ be any point on a line parallel to Y axis at a distance 'a' from it for any position of the point P be lying on this line and for no other point the value of x is always constant and is equal to a therefore $x = a$ is the equation of the line

General Equation of the Straight line

An equation of the form $ax+by=-c$ or $ax+by+c=0$ where a, b, c are constant and x, y are Variable is called the general eqn of the Straight line

Slope of the line $ax+by+c=0$ is given by $-\dfrac{\text{coeff } x}{\text{coeff } y}$

1. The coordinates of two points a and b are $(-1, 2)$ and $(2,-1)$ respectivily. find the equation and slope of the line AB.

$\rightarrow$ $(-1, 2)$ $(2,-1)$

$$(y-y_1) = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$(y-2) = \frac{-1-2}{2+1}(x+1)$$

$$y-2 = \frac{-3}{3}(x+1)$$

$$y-2 = -1(x+1)$$

$$y-2 = -x-1$$

$$x+y = -1+2$$

$$x+y-1=0$$

$$\text{Slope} = -\frac{\text{Coeff } x}{\text{Coeff } y} = \frac{-1}{1} = -1$$

2 Firm inversted Rs 10, million in a new factory that has a net return of 5 lakh per year. And invesment of Rs 20 million would yield a net income of 2 ~~million~~ lakh per year what is the linear Relationship b/w investment and annual income. what would be annual return on a inveistment of 15 million

→ Investment [x] Return [Y]

10 million (100,00,000) → 5,00,000

20 million (200,00,000) → 20,00,000

$$A\ (\overset{x_1}{100,00,000},\ \overset{y_1}{5,00,000})$$

$$B\ (\overset{x_2}{200,00,000},\ \overset{y_2}{20,00,000})$$

$$(y-y_1) = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$(y-5,00,000) = \frac{20,00,000 - 5,00,000}{200,00,000 - 100,00000}(x-100,00,000)$$

$$(y-500,000) = \frac{15,\cancel{00,000}}{100,\cancel{00,000}}(x-100,00,000)$$

$$y-5,00,000 = \frac{\overset{3}{\cancel{15}}}{\underset{20}{\cancel{100}}}(x-100,00,000)$$

$$y-5,00,000 = \frac{3}{20}(x-100,00,000)$$

$$20y - 100,00,000 = 3x - 300,00,000$$

$$3x - 2y = +300,00,000 - 100,00,0$$

$$3x - 2y = +200,00,000$$

or

$$3x - 20y - 200,00,000 = 0$$

$$3 \times 150,00,000 - 20y = 200,00,000$$

$$20y = 450,00,000 - 200,00,000$$

$$20y = 250,00,000$$

$$y = \frac{250,00,000}{20}$$

$$y = 125.0000$$

---

3 Find the eqⁿ of the straight line passing through the point $(-3, 1)$ and perpendicular to the line $5x - 2y + 7 = 0$.

→ $5x - 2y + 7 = 0$

$$m = -\frac{\text{coeff } x}{\text{coeff } y} = -\frac{5}{-2} \Rightarrow \frac{5}{2}$$

$$m = -\frac{2}{5}$$ [If there are 2 perpendicular line)

$$(y - y_1) = m(x - x_1)$$

$$(y - 1) = \frac{-2}{5}(x + 3)$$

$$5(y - 1) = -2(x + 3)$$

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$$5y - 5 = -2x - 6$$

$$2x + 5y = -1$$

$$2x + 5y + 1 = 0$$

4 Find the eqn of the straight line parallel to the X-axis and passing through

i] $(-5,-7)$ ii] $(-8,5)$

→ i] Given that line is parallel to the X-axis that is y remains constant for every x.

Therefore eqn of the straight line passing through $(-5,-7)$ is $y = -7$ or $y + 7 = 0$

ii] Given that is parallel to the x-axis that is y remains constant for every x.

Therefore eqn of the straight line passing through $(-8,5)$ is $y = 5$ or $y - 5 = 0$.

5 Find the eqn of the straight line passing through the origin and making with X axis and angle of i] 45° ii] 60° iii] 90°

→ i] 45°

$$m = \tan\theta$$

$$m = \tan 45$$

$$m = \underline{1}$$

$$(y - y_1) = m(x - x_1)$$

$$(y - 0) = 1 \quad (x - 0)$$

$$\underline{y = x}$$

ii] 60°

$m = \tan\theta$

$m = \tan 60$

$m = \sqrt{3}$

$(y - y_1) = m(x - x_1)$

$(y - 0) = \sqrt{3}\,(x - 0)$

$y = \sqrt{3}x$

iii] 90°

$m = \tan\theta$

$m = \tan 90^\circ$

$m = \infty$ then $x = 0$

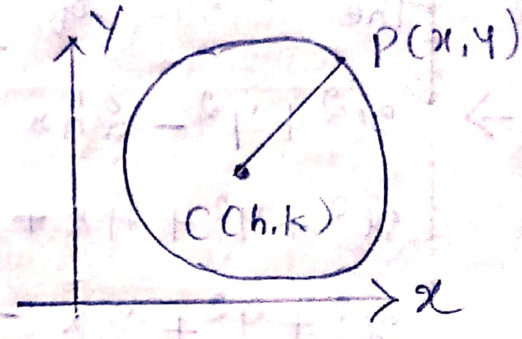
Remark

1. If two lines are parallel then their Slope will be equal
2. If two lines are perpendicular slope of one line is the negative reciprocal of other line.

Circle

The circle is the locus of a point which moves in such a way that its distance from a fixed point always remains constant. The fixed point is called the centre of the circle and the constant distance is termed as the radius of the circle.

Equation of the Circle



$$r = \sqrt{(x-h)^2+(y-k)^2}$$

$$r^2 = (x-h)^2 + (y-k)^2$$

Let the moving point be $P(x,y)$ centre $C(h,k)$ and a fixed distance that is radius is r then $r = \sqrt{(x-h)^2+(y-k)^2}$

$$r^2 = (x-h)^2 + (y-k)^2$$

Find the equation of the circle whose centre is $(4,5)$ and radius is 7

$$r^2 = (x-h)^2 + (y-k)^2$$

$$7^2 = (x-4)^2 + (y-5)^2$$

$$49 = (x-4)^2 + (y-5)^2$$

$$49 = x^2 + 4^2 - 2\times4\times x + y^2 + 5^2 + 2\times5\times y$$

$$49 = x^2 + 16 - 8x + y^2 + 25 + 10y$$

$$x^2 + y^2 - 8x - 10y + 16 + 25 - 49 = 0$$

$$x^2 + y^2 - 8x - 10y - 8 = 0$$

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General Equation of The Circle

$(x-h)^2+(y-k)^2=r^2$

$x^2+h^2-2xh+y^2+k^2-2yk=r^2$

or

$x^2+y^2-2hx-2yk+(h^2+k^2-r^2)=0$

1 Find the equation of the circle whose centre is $(4,5)$ and which passes through the centre of the circle $x^2+y^2+4x+6y-12=0$

→ $x^2+y^2-2hx-2yk+(h^2+k^2-r^2)=0$

$x^2+y^2+4x+6y-12=0$

$x^2+y^2+2\times(-2)\times x+2\times(-3)\times y+((-2)^2+(-3)^2-(-2)^2-(-3)^2)-12=0$

$x^2+y^2+4x+6y+\underbrace{4+9-4-9-12}_{-25}=0$

$(x+2)^2+(y+3)^2=25$

$(x+2)^2+(y+3)^2=5^2$

$(x-(-2))^2+(y-(-3))^2=5^2$

$(-2,-3)$

Distance formula

$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$r=\sqrt{(4+2)^2+(5+3)^2}$

$r=\sqrt{6^2+8^2} \Rightarrow \sqrt{36+64} \Rightarrow \sqrt{100}$

$r=10$

$(x-h)^2+(y-k)^2=r^2$

$(x-4)^2+(y-5)^2=10^2$

$x^2+16-8x+y^2+25-10y=100$

$x^2+\underline{16}-8x+y^2+\underline{25}-10y-\underline{100}=0$

$x^2+y^2-8x-10y-59=0$

---

2 Write down the coordinates of the centre and length of the radius of the circle.

$x^2+y^2+7x-9y-20=0$

$\rightarrow$ $x^2+y^2-2hx-2yk+(h^2+k^2-r^2)=0$

$x^2+y^2+7x-9y-20=0$

$$\underbrace{x^2}_{a^2}+y^2-\underbrace{2\times\left(-\tfrac{7}{2}\right)\times x}_{2ab}-2\times\left(\tfrac{9}{2}\right)\times y+\left(\left(-\tfrac{7}{2}\right)^2+\left(\tfrac{9}{2}\right)^2-\left(-\tfrac{7}{2}\right)^2-\left(\tfrac{9}{2}\right)^2\right)-20=0$$

b^2

$$x^2+y^2+\frac{14}{2}x-\frac{18}{2}y+\frac{49}{4}+\frac{81}{4}-\frac{49}{4}-\frac{81}{4}-20=0$$

$$\left(x-\left(-\tfrac{7}{2}\right)\right)^2+\left(y-\left(\tfrac{9}{2}\right)\right)^2-20-\frac{49}{4}-\frac{81}{4}=0$$

$$\left(x-\left(-\tfrac{7}{2}\right)\right)^2+\left(y-\left(\tfrac{9}{2}\right)\right)^2-\frac{80-49-81}{4}=0$$

$$\left(x-\left(-\tfrac{7}{2}\right)\right)^2+\left(y-\left(\tfrac{9}{2}\right)\right)^2=\frac{210}{4}$$

$$\left(x-\left(-\tfrac{7}{2}\right)\right)^2+\left(y-\left(\tfrac{9}{2}\right)\right)^2=\left(\sqrt{\frac{210}{4}}\right)^2$$

$$\text{radius} = \sqrt{\frac{210}{4}}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\left(x - \left(-\tfrac{7}{2}\right)\right)^2 + \left(y - \tfrac{9}{2}\right)^2 = \left(\sqrt{\frac{210}{4}}\right)^2$$

$$\left(-\tfrac{7}{2}, \tfrac{9}{2}\right)$$

3 Find the equation of the circle whose centre is $(2,-3)$ and passing through the point $(5,1)$

→ distance formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$r = \sqrt{(5-2)^2 + (1+3)^2}$$

$$r = \sqrt{3^2 + 4^2}$$

$$= \sqrt{9+26}$$

$$= \sqrt{25}$$

$$r = \underline{\underline{5}}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-2)^2 + (y-(-3))^2 = r^2$$

$$(x-2)^2 + (y+3)^2 = r^2$$

$$x^2 + 4 - 4x + y^2 + 9 + 6y = 25$$

$$x^2 + y^2 - 4x + 6y - 25 + 13 = 0$$

$$x^2 + y^2 - 4x + 6y - 12 = 0$$

Equation of a tangent

The eqⁿ of a tangent at any point (x_1, y_1) on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

tangent $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

$$xx_1 + yy_1 + gx + gx_1 + fy + fy_1 + c = 0$$

$$(x_1 + g)x + (y_1 + f)y + gx_1 + fy_1 + c = 0$$

Slope of tangent $= -\dfrac{\text{coeff } x}{\text{coeff } y}$

$$= -\frac{x_1 + g}{y_1 + f}$$

Slope of normal $= y_1 + f$

Equation of the Normal

A normal line to a curve at a point is the line perpendicular to the tangent line at the point of contact.

The equation of the tangent at x, y is given as $xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$

Therefore the slope of the tangent is

$$-\frac{x_1+g}{y_1+f}$$

Therefore the slope of the normal is equal

$\frac{y_1+f}{+x_1+g}$, hence the slope of the normal

at x_1, y_1 is $(y-y_1) = \frac{y_1+f}{x_1+g}(x-x_1)$

---

1 Find the equation of tangent and normal at the point $(-2, 5)$ on $x^2+y^2+3x-8y+17=0$

→ $(x_1, y_1) = (-2, 5)$

$x^2+y^2+3x-8y+17=0$

$x^2+y^2+2\left(\frac{3}{2}\right)x + 2(-4)y + 17 = 0$

tangent = $xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$

$x(-2)+y(5)+\frac{3}{2}(x-2)+(-4)(y+5)+17=0$

$$-2x + 5y + \frac{3}{2}x - 3 - 4y - 20 + 17 = 0$$

$$\frac{-4x + 3x}{2} + y - 6 = 0$$

$$-4x + 3x = 2(-y + 6)$$

$$-x = -2y + 12$$

$$-x + 2y - 12 = 0$$

Slope of tangent $= -\left(\frac{x_1 + g}{y_1 + f}\right)$

$$= -\left(\frac{-2 + 3/2}{5 - 4}\right) \Rightarrow -\left(\frac{\frac{-4+3}{2}}{1}\right)$$

$$= -\left(\frac{-4+3}{2}\right) = \underline{+1/2}$$

Slope of normal $= \frac{y_1 + f}{x_1 + g}$

$$= \left(\frac{5-4}{-2 + 3/2}\right) \Rightarrow \left(\frac{1}{\frac{-4+3}{2}}\right)$$

$$= \left(\frac{1}{-1/2}\right) = \frac{2}{-1} \Rightarrow \underline{-2}$$

Hence the slope of the normal.

$$(y - y_1) = \frac{y_1 + f}{x_1 + g}(x - x_1)$$

$$(y - 5) = -2(x + 2)$$

$$y - 5 = -2x - 4 \Rightarrow 2x + y = 1 \text{ or } \underline{2x + y - 1 = 0}$$

Ellipse

An Ellipse is the sort of alongated circle. It is formed by a locus of a point which moves in such a way that the sum of its distances from a two fixed point is always constant. These points are called foci of the elipse. If the major and minor axis of lies on the x-axis and y-axis respectivily then the centre at the origin & the eqn in terms of cartisic coordinates

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where a and b are length of the semi major and minor axis.

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1 Construct the graph of an equation.

$4x^2 + 9y^2 = 36$.

$$\frac{4x^2}{36} + \frac{9y^2}{36} = 1$$

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

→ $$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$4x^2 + 9y^2 = 36$$

$$9y^2 = 36 - 4x^2$$

$$y^2 = \frac{36 - 4x^2}{9}$$

$$y = \pm \sqrt{\frac{36-4x^2}{9}}$$

$$y = \pm \sqrt{\frac{4(9-x^2)}{3^2}}$$

$-3, -2, -1, 0$

$1 \quad 2 \quad 3$

$$y = \pm \frac{2}{3}\sqrt{9-x^2}$$

If $x = -3$

$$y = \pm \frac{2}{3}\sqrt{9-(-3)^2}$$

$$= \pm \frac{2}{3}\sqrt{0}$$

$$y = 0$$

If $x = -2$

$$y = \pm \frac{2}{3}\sqrt{9-(-2)^2}$$

$$= \pm \frac{2}{3}\sqrt{9-4}$$

$$= \pm \frac{2}{3}\sqrt{5}$$

$$= \pm 1.4$$

If $x = -1$

$$y = \pm \frac{2}{3}\sqrt{9-(-1)^2}$$

$$= \pm \frac{2}{3}\sqrt{9-1}$$

$$= \pm \frac{2}{3}\sqrt{8}$$

$$= \pm 1.88$$

If $x = 0$ $y = \pm \frac{2}{3} \sqrt{9 - 0^2} \Rightarrow \pm \frac{2}{3}\sqrt{9} \Rightarrow \pm \frac{2}{3}$

$y = 2$

If $x = 1$ $y = \pm 1.88$

If $x = 2$ $y = \pm 1.4$

If $x = 0$ $y = \pm 0$

x	-3	-2	-1	0	1	2	3
y	0	± 1.4	± 1.88	± 2	± 1.88	± 1.4	0