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Matrix Algebra

A matrix is a rectangular array of numbers arranged in rows and columns enclosed by a pair of brackets and subject to certain rules of presentation.

eg:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \rightarrow \text{rows}$$

$\downarrow$

columns

A general form of a matrix

A matrix of order $m \times n$ (i.e, one having m rows and n columns) can be written as

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

where $a_{11}, a_{12} \ldots$ stand for real numbers. The above matrix can also be written as $A = [a_{ij}]_{m \times n}$ where $i = 1, 2 \ldots m$, $j = 1, 2 \ldots n$, where a_{ij} is the element in the i^{th} row & j^{th} column and is refered as $(i, j)^{th}$ element.

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Problem:

$$\begin{bmatrix} 0 & 1 & -1 \\ 2 & 6 & 5 \\ 9 & 7 & -8 \end{bmatrix}_{3\times3}$$

Here a_{12} element is 1

$a_{23} = 5$.

$a_{31} = 9$

$a_{33} = -8$

a_{24} = not possible.

H.W

$$\begin{bmatrix} 5 & 8 & 9 & 12 \\ -1 & 3 & 0 & 4 \\ 6 & 8 & 7 & 3 \end{bmatrix}$$

Find the elements of the position a_{23}, a_{31}, a_{14}, a_{21}, a_{35}, a_{41}

Types of matrices.

a) **Square matrix.**

A matrix in which the number of rows is equal to the number of columns is called a square matrix. Thus a $m \times n$ matrix will be a square matrix if $m = n$ and it will be refered as a square matrix of order n or n-ordered matrix

eg:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times2} \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & -1 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3} \quad \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{n\times n}$$

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Remark:

In a square matrix all those elements a_{ij} for which $i=j$. i.e those occur in the same row and same column namely $a_{11}, a_{12} \ldots a_{nn}$ are called the diagonal elements. A square matrix has of couse two diagonals. Diagonal extending from the upper left to the lower right is more important than the other diagonal. This is known as the principal or main diagonal and its elements are called the diagonal elements.

$$\begin{bmatrix} 1 & 2 & -3 \\ 6 & 8 & 5 \\ 2 & -1 & 6 \end{bmatrix}.$$

principal diagonal is (1, 8, 6).

b) Row and column matrices

A row matrix is defined as a matrix having a single row and a column matrix is one having a single column.

eg:

$$[a_{11} \; a_{12} \ldots a_{1n}]_{1\times n} \text{ — row matrix}$$

$$\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}_{m\times 1} \text{ — column matrix}$$

c) Diagonal Matrix

A square matrix all of whose elements except those in the leading diagonal are zero is called a diagonal matrix.

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$$A = \begin{bmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ 0 & a_{22} & 0 & \cdots & 0 \\ 0 & 0 & a_{33} & \cdots & 0 \\ 0 & 0 & 0 & \cdots & a_{nn} \end{bmatrix}_{n \times n}$$

is a diagonal matrix and may be written as $A = \text{diag}(a_{11}, a_{12}, \ldots a_{nn})$.

Remark:

1. The square matrix A will be a diagonal matrix if all elements a_{ij} for which $i \neq j$ are zero.

2. A diagonal matrix whose all the diagonal elements are equal is called a scalar matrix.

eg:

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \text{diag}(2, 2, 2)$$

d) Unit matrix

A scalar matrix each of whose diagonal element is unity (or one) is called a unit matrix or an identity matrix.

A unit matrix of order n is written as I_n.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Remark: In general for a unit matrix

$$\begin{cases} a_{ij} = 0, & i \neq j \\ a_{ij} = 1, & i = j \end{cases}$$

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e) Zero matrix or a null matrix

A matrix rectangular or square, each of whose elements are zero is called a zero matrix or a null matrix and is denoted by O.

eg:

$$O = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{3\times4} \qquad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times2},$$

f) Triangular matrices

A square matrix $A = (a_{ij})_{m\times n}$ is called upper triangular matrix if $a_{ij} = 0$ for $i>j$ and is called lower triangular matrix if $a_{ij} = 0$ for $i<j$

eg:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix} \qquad \begin{bmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ a_{21} & a_{22} & 0 & \cdots & 0 \\ \vdots & & & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix}$$

upper triangular matrix   lower triangular.

g) Sub matrix

A matrix obtained by deleting some rows or columns or both of a given matrix is called a submatrix of a given matrix.

eg:

Let
$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times4}$$

If we delete the 1^{st} row and column, the submatrix of A is.

$$\begin{bmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \end{bmatrix}_{2\times3}$$

h) Scalar matrix:

A square matrix when given in the form of a scalar multiplication to an identity matrix is called a scalar matrix.

eg:

$$3I = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$aI = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

i) Symmetric matrices

A symmetric matrix is a special kind of a square matrix $A = [a_{ij}]$ for which $a_{ij} = a_{ji}$ for all i and j

i.e the $(i, j)^{th}$ element = $(j, i)^{th}$ element.

eg:

$$\begin{bmatrix} 5 & 2 & 1 \\ 2 & 6 & -1 \\ 1 & -1 & 5 \end{bmatrix} \qquad \begin{bmatrix} a & k & l \\ k & g & t \\ l & t & u \end{bmatrix}$$

j) Complex conjugate of a matrix

It is a matrix obtained by replacing all its elements by their respective complex conjugates.

eg:

$$A = \begin{bmatrix} 2+3i & 4 \\ 5-3i & 7 \end{bmatrix} \text{ then } \bar{A} = \begin{bmatrix} 2-3i & 4 \\ 5+3i & 7 \end{bmatrix}$$

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k) Skew- symmetric matrix.

It is a square matrix A if $A^t = -A$ i.e the transpose of a square matrix is equal to the negative of that matrix.

or,

defn:

A square matrix A is called a skew symmetric matrix if $a_{ij} = -a_{ji}$ for all i and j. In a skew-symmetric matrix all the diagonal elements are zeros

eg:

$$\begin{bmatrix} 0 & 9 & 2 \\ -9 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$$

Scalar multiplication of a matrix.

A real number is referred to as a scalar when it occurs in operations involving matrices. The scalar multiple kA of a matrix A by scalar k, is a matrix obtained by multiplying every element of A by the scalar k, i.e. the scalar multiple of the matrix $A = [a_{ij}]_{m \times n}$ by scalar k is the matrix $C = [c_{ij}]_{m \times n}$ where $c_{ij} = k a_{ij}$ $i = 1, 2 .. m$ $j = 1, 2 .. n$.

eg:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad \text{then } kA = \begin{bmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & & & \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{bmatrix}$$

Illustrations:

1. $$3\begin{bmatrix} 4 & -3 \\ 8 & -2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 12 & -9 \\ 24 & -6 \\ -3 & 0 \end{bmatrix}$$

2. $$5\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ 0 \end{bmatrix}$$

3. If $$A = \begin{bmatrix} 3 & 7 & 6 & -5 \\ 2 & -6 & 0 & 4 \\ 5 & 2 & 8 & 8 \\ -1 & 6 & 5 & -3 \end{bmatrix}$$

then $$-A = \begin{bmatrix} -3 & -7 & -6 & 5 \\ -2 & 6 & 0 & -4 \\ -5 & -2 & -8 & -8 \\ 1 & -6 & -5 & 3 \end{bmatrix}$$

$$4A = \begin{bmatrix} 12 & 28 & 24 & -20 \\ 8 & -24 & 0 & 16 \\ 20 & 8 & 32 & 32 \\ -4 & 24 & 20 & -12 \end{bmatrix}$$

Equality of Matrices.

Two matrices are said to be equal iff

i) They are comparable. i.e they are of the same order.

ii) Each element of one is equal to the ~~corrend~~ corresponding element of the other

i.e if $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$

then $A = B$ iff $a_{ij} = b_{ij}$ $\forall$ i $1 \le i \le m$

j $1 \le j \le n$.

Illustrations:

1. If $A = \begin{bmatrix} 4 & 7 & 0 \\ 7 & -2 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 7 & 0 \\ 7 & -2 & 5 \end{bmatrix}$

then $A = B$.

2. If $A = \begin{bmatrix} 3 & 4 \\ 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 4 & 1 \end{bmatrix}$.

Here $a_{22} \neq b_{22}$ $\therefore A \neq B$.

3. If $A = \begin{bmatrix} 3 & 4 & 7 \\ 2 & 8 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 4 & 7 \\ 2 & 8 & 6 \\ 1 & 2 & 5 \end{bmatrix}$.

Here A and B are not of the same order $\therefore A \neq B$

4. Find x, y, z, w

$$\begin{bmatrix} x+y & 2z+w \\ x-y & z-w \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix}$$

$x+y = 3$ — (1) $\quad 2z+w = 5$ — (3)

$x-y = 1$ — (2) $\quad z-w = 4$ — (4)

add eqn (1) & (2)

$x+\cancel{y} = 3$

$x-\cancel{y} = 1$

$2x = 4$

$x = 2$

sub x in (1)

$x+y = 3$

$2+y = 3$

$y = 3-2 = 1$

add eqn (3) & (4).

$2z+\cancel{w} = 5$

$z-\cancel{w} = 4$

$3z = 9$

$z = 9/3 = 3.$

sub z in (4).

$z - w = 4$

$3 - w = 4$

$-w = 4-3. = 1$

$w = -1$

$\therefore x = 2, \; y = 1, \; z = 3, \; w = -1$

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Exercise

1 Find the elements a_{31}, a_{24}, a_{34}, a_{11} in each of the ~~matri~~ following matrices. Also give their diagonal element.

a) $\begin{bmatrix} 8 & 7 & -4 & 2 \\ 3 & 2 & 0 & 5 \\ 7 & 6 & 3 & 1 \\ -5 & 12 & 5 & 9 \end{bmatrix}$.

Here $a_{31} = 7$ $a_{24} = 5$ $a_{34} = 1$ $a_{11} = 8$.

H.W b) $\begin{bmatrix} -1 & 0 & 3 \\ 3 & 2 & 5 \\ 7 & 0 & -6 \end{bmatrix}$

$a_{31} =$ ___ $a_{24} =$ ___ $a_{34} =$ ___ $a_{11} =$ ___

a_{24} & a_{34} not possible.

2. Find x and y if

$$\begin{bmatrix} x+y & 2 \\ 1 & x-y \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}$$

soln:

$x+y = 3$ — ① $x-y = 7$ — ②.

add ① & ②.

$x+y = 3$
$x-y = 7$.

$2x = 10$
$x = 5$.

sub x in ①.

$x+y = 3$
$5+y = 3$
$y = 3-5 = -2$.

Here $x = 5$. $y = -2$.

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3. classify the following matrices.

i) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ – identity matrix

ii) $\begin{bmatrix} 3 & 0 & 0 \\ 1 & -4 & 0 \\ 9 & 5 & 10 \end{bmatrix}$ – lower triangular matrix

iv) $\begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} = \underline{?}$ v) $\begin{bmatrix} -1, & 2, & 3 \end{bmatrix} = \underline{?}$

v) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \underline{?}$

vi) $\begin{bmatrix} 3 & -1 & 2 \\ 0 & 5 & 6 \\ 0 & 0 & 8 \end{bmatrix} = \underline{?}$

vii) $\begin{bmatrix} 8 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix} = \underline{?}$

Matrix operations

1. **Addition and substraction**

i) matrices can be added or substracted if and only if they are of the same order.

ii) The sum or difference of two $(m \times n)$ matrices is another matrix $(m \times n)$ whose elements are the sum or differences of the corresponding elements in the component matrix.

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3}$ a_{44} $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ a_{21} & b_{22} & b_{23} \end{bmatrix}_{2\times3}$

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Soln:

Here A and B of same order i.e A and B matrixes of order (2×3). hence we can add or substract matrices A and B.

$$A+B = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \end{bmatrix}_{2\times3}.$$

$$A-B = \begin{bmatrix} a_{11}-b_{11} & a_{12}-b_{12} & a_{13}-b_{13} \\ a_{21}-b_{21} & a_{22}-b_{22} & a_{23}-b_{23} \end{bmatrix}_{2\times3}.$$

Eg:

If $A = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$

find the value of $2A+3B$.

Soln:

$$2A = 2\begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix}$$

$$3B = 3\begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 21 & 18 & 9 \\ 3 & 12 & 15 \end{bmatrix}$$

$$2A+3B = \begin{bmatrix} 0 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix} + \begin{bmatrix} 21 & 18 & 9 \\ 3 & 12 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 0+21 & 4+18 & 6+9 \\ 4+3 & 2+12 & 8+15 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 22 & 15 \\ 7 & 14 & 23 \end{bmatrix}$$

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Exercise:

1. Find $[x \quad y]$ if.

i) $[4 \quad 5] + [x \quad y] = [7 \quad 3]$.

Soln:

$$[4 \quad 5] + [x \quad y] = [7 \quad 3]$$

$$[4 + x \quad 5 + y] = [7 \quad 3].$$

$\rightarrow$

$4 + x = 7 \quad -① \qquad 5 + y = 3 \quad -②$

from $x = 7 - 4 \qquad y = 3 - 5$

$\quad = 3 \qquad\qquad = -2.$

Here $x = 3 \quad y = -2$

$$[x \quad y] = [3 \quad -2].$$

H.work ii) $[x \quad y] - [0 \quad -1] = [5 \quad 4]$.

Ans: $[x \quad y] = [5 \quad 3]$

iii) $[1 \quad -9] - [2 \quad -3] = [x \quad y]$

Soln:

$$[1 - 2 \quad -9 + 3] = [x \quad y]$$

$$[-1 \quad -6] = [x \quad y].$$

$$[x \quad y] = [-1 \quad -6].$$

2. Given

$$A = \begin{bmatrix} 2 & 0 & 4 \\ 6 & 2 & 8 \\ 2 & 4 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 8 & 4 & -2 \\ 0 & -2 & 0 \\ 2 & 2 & 6 \end{bmatrix} \quad C = \begin{bmatrix} 8 & 2 & 0 \\ 0 & 2 & -6 \\ -8 & 4 & -10 \end{bmatrix}$$

Compute the following:

a) $A+B$ b) $A-B$ c) $A+(B+C)$ d) $(A+B)+C$

e) $(A-B)+C$ f) $A-B-C$ g) $2(A+B)$

h) $2A+2B$ i) $3A+2B-3C$ j) $3B+2A$

k) $2B+3A$

Soln:

f) $A-B-C$

$$\begin{bmatrix} 2 & 0 & 4 \\ 6 & 2 & 8 \\ 2 & 4 & 6 \end{bmatrix} - \begin{bmatrix} 8 & 4 & -2 \\ 0 & -2 & 0 \\ 2 & 2 & 6 \end{bmatrix} - \begin{bmatrix} 8 & 2 & 0 \\ 0 & 2 & -6 \\ -8 & 4 & -10 \end{bmatrix}$$

$$\begin{bmatrix} 2-8-8 & 0-4-2 & 4+2-0 \\ 6-0-0 & 2-(-2)-2 & 8-0-(-6) \\ 2-2-(-8) & 4-2-4 & 6-6-(-10) \end{bmatrix}$$

$$\begin{bmatrix} -14 & -6 & 6 \\ 6 & 2 & 14 \\ 8 & -2 & 10 \end{bmatrix}$$

i) $3A+2B-3C$

$$3A = 3\begin{bmatrix} 2 & 0 & 4 \\ 6 & 2 & 8 \\ 2 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 12 \\ 18 & 6 & 24 \\ 6 & 12 & 18 \end{bmatrix}$$

$$2B = 2\begin{bmatrix} 8 & 4 & -2 \\ 0 & -2 & 0 \\ 2 & 2 & 6 \end{bmatrix} = \begin{bmatrix} 16 & 8 & -4 \\ 0 & -4 & 0 \\ 4 & 4 & 12 \end{bmatrix}$$

$$3C = 3\begin{bmatrix} 8 & 2 & 0 \\ 0 & 2 & -6 \\ -8 & 4 & -10 \end{bmatrix} = \begin{bmatrix} 24 & 6 & 0 \\ 0 & 6 & -18 \\ -24 & 12 & -30 \end{bmatrix}$$

$3A + 2B - 3C$

$$= \begin{bmatrix} 6 & 0 & 12 \\ 18 & 6 & 24 \\ 6 & 12 & 18 \end{bmatrix} + \begin{bmatrix} 16 & 8 & -4 \\ 0 & -4 & 0 \\ 4 & 4 & 12 \end{bmatrix} - \begin{bmatrix} 24 & 6 & 0 \\ 0 & 6 & -18 \\ -24 & 12 & -30 \end{bmatrix}$$

$$= \begin{bmatrix} 6+16-24 & 0+8-6 & 12-4-0 \\ 18+0-0 & 6-4-6 & 24+0+18 \\ 6+4+24 & 12+4-12 & 18+12+30 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 2 & 8 \\ 18 & -4 & 42 \\ 34 & 4 & 60 \end{bmatrix}$$

<u>Ans:</u>

a) $\begin{bmatrix} 10 & 4 & 2 \\ 6 & 0 & 8 \\ 4 & 6 & 12 \end{bmatrix}$ b) $\begin{bmatrix} -6 & -4 & 6 \\ 6 & 4 & 8 \\ 0 & 2 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 18 & 6 & 2 \\ 6 & 2 & 2 \\ -4 & 10 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 18 & 6 & 2 \\ 6 & 2 & 2 \\ -4 & 10 & 2 \end{bmatrix}$

e) $\begin{bmatrix} 2 & -2 & 6 \\ 6 & 6 & 2 \\ -8 & 6 & -10 \end{bmatrix}$ g) $\begin{bmatrix} 20 & 8 & 4 \\ 12 & 0 & 16 \\ 8 & 12 & 24 \end{bmatrix}$.

h) $\begin{bmatrix} 20 & 8 & 4 \\ 12 & 0 & 16 \\ 8 & 12 & 24 \end{bmatrix}$ j) $\begin{bmatrix} 28 & 18 & 2 \\ 12 & -2 & 16 \\ 10 & 14 & 30 \end{bmatrix}$

k) $\begin{bmatrix} 22 & 8 & 8 \\ 18 & 2 & 24 \\ 10 & 16 & 30 \end{bmatrix}$

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multiplication.

Two matrices are conformable for multiplication if the number of columns of first matrix is equal to the number of rows of the second matrix.

if the matrix $A = [a_{ij}]_{m\times n}$ and $B = [b_{jk}]_{n\times p}$ then the product AB is the matrix

$C = [c_{ik}]_{m\times p}$.

If
$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{3\times 2}$$

$$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \end{bmatrix}_{2\times 2}$$

Eg:

1. Write the product AB of the matrices A and B where $A = [1 \ 2 \ 3 \ 4]$ and $B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$.

Soln:

Since A is 1×4 matrix, B is 4×1 matrix, AB will be 1×1 matrix.

$$AB = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}_{1\times 4} \times \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}_{4\times 1}$$

$$= \begin{bmatrix} 1\times 1 + 2\times 2 + 3\times 3 + 4\times 4 \end{bmatrix} = \begin{bmatrix} 1 + 4 + 9 + 16 \end{bmatrix}$$

$$= [30]_{1\times 1}$$

8. If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$ find AB and BA. Is AB = BA?

Soln:

$$AB = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 2\times1+5\times(-3) & 2\times(-1)+5\times2 \\ 1\times1+3\times(-3) & 1\times(-1)+3\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-15 & -2+10 \\ 1-9 & -1+6 \end{bmatrix}$$

$$= \begin{bmatrix} -13 & 8 \\ -8 & 5 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1\times2+(-1)\times1 & 1\times5+(-1)\times3 \\ -3\times2+2\times1 & -3\times5+2\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 5-3 \\ -6+2 & -15+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ -4 & -9 \end{bmatrix}$$

Thus $AB \neq BA$.

9. Obtain the product

$$\begin{bmatrix} 2 & 1 & 0 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 0 & 1 & 2 \\ 3 & 1 & 0 & 5 \end{bmatrix}$$

Soln:

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}_{3\times3} \qquad B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 0 & 1 & 2 \\ 3 & 1 & 0 & 5 \end{bmatrix}$$

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$$AB = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 0 & 1 & 2 \\ 3 & 1 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times1+1\times2+0\times3 & 2\times2+1\times0+0\times1 & 2\times3+1\times1+0\times0 & 2\times4+1\times2+0\times5 \\ 3\times1+2\times2+1\times3 & 3\times2+2\times0+1\times1 & 3\times3+2\times1+1\times0 & 3\times4+2\times2+1\times5 \\ 1\times1+0\times2+1\times3 & 1\times2+0\times0+1\times1 & 1\times3+0\times1+1\times0 & 1\times4+0\times2+1\times5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2+0 & 4+0+0 & 6+1+0 & 8+2+0 \\ 3+4+3 & 6+0+1 & 9+2+0 & 12+4+5 \\ 1+0+3 & 2+0+1 & 3+0+0 & 4+0+5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 & 7 & 10 \\ 10 & 7 & 11 & 21 \\ 4 & 3 & 3 & 9 \end{bmatrix}$$

4. If $A = \begin{bmatrix} 9 & 1 \\ 4 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ find matrix X such that $3A + 5B + 2X = 0$.

Soln:

We have $3A + 5B + 2X = 0$.

$$\Rightarrow 2X = -[3A + 5B]$$

$$\Rightarrow X = -\frac{1}{2}[3A + 5B].$$

$$X = -\frac{1}{2}\left[3\begin{bmatrix} 9 & 1 \\ 4 & 3 \end{bmatrix} + 5\begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}\right]$$

$$= -\frac{1}{2}\left[\begin{bmatrix} 27 & 3 \\ 12 & 9 \end{bmatrix} + \begin{bmatrix} 5 & 25 \\ 35 & 60 \end{bmatrix}\right]$$

$$= -\frac{1}{2}\begin{bmatrix} 32 & 28 \\ 47 & 69 \end{bmatrix} = \begin{bmatrix} -\frac{32}{2} & -\frac{28}{2} \\ -\frac{47}{2} & -\frac{69}{2} \end{bmatrix}$$

$$= \begin{bmatrix} -16 & -14 \\ -\frac{47}{2} & -\frac{69}{2} \end{bmatrix}$$

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5. Given the matrices A, B, C.

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \quad C = \begin{bmatrix} 1, & -2 \end{bmatrix}$$

verify that $(AB)C = A(BC)$

soln:

$$AB = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix}_{2\times 3} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}_{3\times 1}$$

$$= \begin{bmatrix} 2+3-2 \\ 3+0+4 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}_{2\times 1}$$

$$(AB)\,C = \begin{bmatrix} 3 \\ 7 \end{bmatrix}_{2\times 1} \begin{bmatrix} 1 & -2 \end{bmatrix}_{1\times 2}$$

$$= \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}_{2\times 2} \quad \text{— (a)}$$

$$BC = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}_{3\times 1} \begin{bmatrix} 1 & -2 \end{bmatrix}_{1\times 2}$$

$$= \begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix}_{3\times 2}$$

$$A\,(BC) = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix}_{2\times 3} \begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix}_{3\times 2}$$

$$= \begin{bmatrix} 2+3-2 & -4-6+4 \\ 3+0+4 & -6+0-8 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}_{2\times 2} \quad \text{— (b)}$$

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from (a) & (b) $(AB)C = A(BC)$

6. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ show that $\cancel{A^2 = 3A^2}$

$A^3 - 3A^2 - A + 9I = 0$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+3 & 2+2-1 & 1-2+1 \\ 0+0-3 & 0+1+1 & 0-1-1 \\ 3+0+3 & 6-1-1 & 3+1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & 0 \\ -3 & 2 & -2 \\ 6 & 4 & 5 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 4 & 3 & 0 \\ -3 & 2 & -2 \\ 6 & 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 11 & 1 \\ -9 & -2 & -7 \\ 21 & 11 & 7 \end{bmatrix}$$

Now. $A^3 - 3A^2 - A + 9I$

$$\begin{bmatrix} 4 & 11 & 1 \\ -9 & -2 & -7 \\ 21 & 11 & 7 \end{bmatrix} - 3\begin{bmatrix} 4 & 3 & 0 \\ -3 & 2 & -2 \\ 6 & 4 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$

$$+ 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$\begin{bmatrix} 4 & 11 & 1 \\ -9 & -2 & -7 \\ 21 & 11 & 7 \end{bmatrix} - \begin{bmatrix} 12 & 9 & 0 \\ -9 & 6 & -6 \\ 18 & 12 & 15 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Exercise

H.W 1. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 1 & 1 & 0 \\ -1 & 4 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 4 & 9 \end{bmatrix}$

Show that $AB = 0$

H.W 2) If $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$

$C = \begin{bmatrix} -1 & -1 & 1 \\ 2 & 2 & -2 \\ -3 & -3 & 3 \end{bmatrix}$

Show that AB and CA are null matrices (i.e $AB = 0$ & $CA = 0$) But $BA \neq 0$, $AC \neq 0$.

(Find AB, CA, BA & AC)

3) If $M = \begin{bmatrix} 1 & a & b & c \\ a^{-1} & 1 & a^{-1}b & a^{-1}c \\ b^{-1} & ab^{-1} & 1 & b^{-1}c \\ c^{-1} & ac^{-1} & bc^{-1} & 1 \end{bmatrix}$ prove that $M^2 = 4M$.

Soln:

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$$4M = 4\begin{bmatrix} 1 & a & b & c \\ a^{-1} & 1 & a^{-1}b & a^{-1}c \\ b^{-1} & ab^{-1} & 1 & b^{-1}c \\ c^{-1} & ac^{-1} & bc^{-1} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4a & 4b & 4c \\ 4a^{-1} & 4 & 4a^{-1}b & 4a^{-1}c \\ 4b^{-1} & 4ab^{-1} & 4 & 4b^{-1}c \\ 4c^{-1} & 4ac^{-1} & 4bc^{-1} & 4 \end{bmatrix} \quad -(a)$$

$$M^2 = M \cdot M = \begin{bmatrix} 1 & a & b & c \\ a^{-1} & 1 & a^{-1}b & a^{-1}c \\ b^{-1} & ab^{-1} & 1 & b^{-1}c \\ c^{-1} & ac^{-1} & bc^{-1} & 1 \end{bmatrix} \begin{bmatrix} 1 & a & b & c \\ a^{-1} & 1 & a^{-1}b & a^{-1}c \\ b^{-1} & ab^{-1} & 1 & b^{-1}c \\ c^{-1} & ac^{-1} & bc^{-1} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times 1 + a\times a^{-1} + b\times b^{-1} + c\times c^{-1} & a + a + b\times ab^{-1} + c\times ac^{-1} & - & - \\ & & & \\ & & & \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+1+1 & a+a+a+a & b+b+b+b & c+c+c+c \\ a^{-1}+a^{-1}+a^{-1}+a^{-1} & 1+1+1+1 & a^{-1}b+a^{-1}b+a^{-1}b+a^{-1}b & a^{-1}c+a^{-1}c+a^{-1}c+a^{-1}c \\ b^{-1}+b^{-1}+b^{-1}+b^{-1} & ab^{-1}+ab^{-1}+ab^{-1}+ab^{-1} & 1+1+1+1 & b^{-1}c+b^{-1}c+b^{-1}c+b^{-1}c \\ c^{-1}+c^{-1}+c^{-1}+c^{-1} & ac^{-1}+ac^{-1}+ac^{-1}+ac^{-1} & bc^{-1}+bc^{-1}+bc^{-1}+bc^{-1} & 1+1+1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4a & 4b & 4c \\ 4a^{-1} & 4 & 4a^{-1}b & 4a^{-1}c \\ 4b^{-1} & 4ab^{-1} & 4 & 4b^{-1}c \\ 4c^{-1} & 4ac^{-1} & 4bc^{-1} & 4 \end{bmatrix} \quad -(b)$$

$\left[\because aa^{-1} = 1 = a^{-1}a, \; bb^{-1} = 1 = b^{-1}b\right]$

From (a) & (b) $M^2 = 4M$

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Transpose of a matrix

The matrix obtained by interchanging rows and columns of the matrix A is called the transpose of A and is denoted by A' or A^t

eg:

$$A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \\ 7 & -5 \end{bmatrix}_{3\times 2} \quad \text{then} \quad A^t = \begin{bmatrix} 3 & 4 & 7 \\ 2 & 1 & -5 \end{bmatrix}_{2\times 3}$$

Remark:

1. If A is $m\times n$ matrix, then A^t will be a $n\times m$ matrix.
2. The transpose of a row (column) matrix is a column (row) matrix.
3. $(A^t)^t = A$.
4. The transpose of the sum of two matrices is the sum of their transposes.
   i.e $(A+B)^t = A^t + B^t$.
5. The transpose of the product AB is equal to the product of the transposes taken in the reverse order.
   i.e $(AB)^t = B^t A^t$.
6. For a symmetric matrix, the transpose of matrix is equal the matrix itself.
   i.e $A^t = A$.
7. For a skew symmetric matrix, the transpose of a matrix is equal to the negative of the matrix.
   i.e $A^t = -A$.

Problem:

1. Let $A = \begin{bmatrix} 2 & -3 & 1 \\ 4 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 3 & -5 \end{bmatrix}$

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Show that $(A+B)^t = A^t + B^t$.

soln:

$$A+B = \begin{bmatrix} 2 & -3 & 1 \\ 4 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 4 \\ 1 & 3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+3 & -3-2 & 1+4 \\ 4+1 & 2+3 & 3-5 \end{bmatrix} = \begin{bmatrix} 5 & -5 & 5 \\ 5 & 5 & -2 \end{bmatrix}$$

$$(A+B)^t = \begin{bmatrix} 5 & 5 \\ -5 & 5 \\ 5 & -2 \end{bmatrix} \quad \text{— (a)}$$

$$A^t = \begin{bmatrix} 2 & 4 \\ -3 & 2 \\ 1 & 3 \end{bmatrix} \qquad B^t = \begin{bmatrix} 3 & 1 \\ -2 & 3 \\ 4 & -5 \end{bmatrix}$$

$$A^t + B^t = \begin{bmatrix} 2 & 4 \\ -3 & 2 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -2 & 3 \\ 4 & -5 \end{bmatrix} = \begin{bmatrix} 2+3 & 4+1 \\ -3-2 & 2+3 \\ 1+4 & 3-5 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ -5 & 5 \\ 5 & -2 \end{bmatrix} \text{ — (b)}$$

from (a) & (b) $(A+B)^t = A^t + B^t$

2 Verify that $(A^t)^t = A$.

$$A = \begin{bmatrix} 2 & 8 & 4 \\ 8 & 6 & -1 \\ 4 & -1 & 0 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 2 & 8 & 4 \\ 8 & 6 & -1 \\ 4 & -1 & 0 \end{bmatrix}$$

$$(A^t)^t = \begin{bmatrix} 2 & 8 & 4 \\ 8 & 6 & -1 \\ 4 & -1 & 0 \end{bmatrix}$$

Here $(A^t)^t = A$.

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3. compute $(AB)^t = B^t A^t$.

$$A = \begin{bmatrix} 8 & 16 & -4 \\ -4 & 0 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} 12 & 16 & 20 \\ -4 & 8 & 28 \\ 8 & 4 & 0 \end{bmatrix}$$

Solⁿ:

$$AB = \begin{bmatrix} 8 & 16 & -4 \\ -4 & 0 & 8 \end{bmatrix} \begin{bmatrix} 12 & 16 & 20 \\ -4 & 8 & 28 \\ 8 & 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 96-64-32 & 128+128-16 & 160+448+0 \\ -48+0+64 & -64+0+32 & -80+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 240 & 608 \\ 16 & -32 & -80 \end{bmatrix}$$

$$(AB)^t = \begin{bmatrix} 0 & 16 \\ 240 & -32 \\ 608 & -80 \end{bmatrix} \quad \text{— (a)}$$

$$A^t = \begin{bmatrix} 8 & -4 \\ 16 & 0 \\ -4 & 8 \end{bmatrix} \qquad B^t = \begin{bmatrix} 12 & -4 & 8 \\ 16 & 8 & 4 \\ 20 & 28 & 0 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} 12 & -4 & 8 \\ 16 & 8 & 4 \\ 20 & 28 & 0 \end{bmatrix} \begin{bmatrix} 8 & -4 \\ 16 & 0 \\ -4 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 96-64-32 & -48+0+32 \\ 128+128-16 & -64+0+32 \\ 160+448+0 & -80+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -16 \\ 240 & -32 \\ 608 & -80 \end{bmatrix} \quad \text{— (b)}$$

from (a) & (b) $(AB)^t = B^t A^t$.

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Determinants of a square matrix

Let $A = [a_{ij}]$ be a square matrix. A determinant formed by the same array of elements of the same square matrix A is called the determinant of the square matrix A and is denoted by the symbol det A or $|A|$.

determinants of order two

The determinant of a 2×2 matrix is denoted by the following way -

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb \quad \text{or} \quad ad - bc.$$

eg:

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix} \quad \text{then} \quad |A| = \begin{vmatrix} 5 & 3 \\ 2 & 3 \end{vmatrix}$$

$$= 15 \times 1 - 2 \times 3$$

$$= 15 - 6 = 9.$$

Cramer's Rule:

It is a simple rule using determinants to express the solution of a system of linear equations for which the number of equations is equal to the number of variables.

Let the given equations be written in the form

$$a_1 x + b_1 y = c_1 \quad \text{—(1)}$$

$$a_2 x + b_2 y = c_2 \quad \text{—(2)}$$

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Then the solution can be written in the determinant form as

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} (N_x)}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} (D)} \qquad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} (N_y)}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$x = \frac{N_x}{D} \qquad y = \frac{N_y}{D}.$$

Eg:

Solve the following simultaneous linear equations using determinants.

$2x - y = 5$

$3x + 2y = -3.$

Soln:

$$D = \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = 2\times 2 - 3\times(-1)$$
$$= 4 + 3 = 7.$$

$D \neq 0$, the system has a unique solution.

$$N_x = \begin{vmatrix} 5 & -1 \\ -3 & 2 \end{vmatrix} = 10 - (-3)\times(-1)$$
$$= 10 - 3 = 7.$$

$$N_y = \begin{vmatrix} 2 & 5 \\ 3 & -3 \end{vmatrix} = 2\times(-3) - 5\times 3$$
$$-6 - 15 = -21$$

$$x = \frac{N_x}{D} = \frac{7}{7} = 1$$

$$y = \frac{N_y}{D} = \frac{-21}{7} = -3.$$

$x = 1 \quad y = -3$

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Determinant of order three

consider 3×3 matrix, then determinants are defined as follows.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)$$

Problem:

compute the determinant of the following matrices.

1. $A = \begin{bmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{bmatrix}$

$$|A| = 2\begin{vmatrix} -4 & 2 \\ -1 & 5 \end{vmatrix} - 3\begin{vmatrix} 0 & 2 \\ 1 & 5 \end{vmatrix} - 4\begin{vmatrix} 0 & -4 \\ 1 & -1 \end{vmatrix}$$

$$= 2(-4\times5 - 2\times(-1)) - 3(0\times5 - 1\times2) - 4(0\times(-1)-(-4)$$

$$= 2(-20+2) - 3(0-2) - 4(0+4)$$

$$= 2(-18) - 3(-2) - 4(4)$$

$$= -36 + 6 - 16$$

$$= -46$$

2) $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$

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$$|B| = 1\begin{vmatrix} 1 & 4 \\ 2 & 1 \end{vmatrix} - 2\begin{vmatrix} 0 & 4 \\ 3 & 1 \end{vmatrix} + 3\begin{vmatrix} 0 & 1 \\ 3 & 2 \end{vmatrix}$$

$$= 1(1-8) - 2(0-12) + 3(0-3)$$

$$= 1(-7) - 2(-12) + 3(-3)$$

$$= -7 + 24 - 9$$

$$= 8$$

Expansion of the determinants.

Determinants can be represented as linear combinations of order two with coefficients from second row or third row in terms of the elements of any column. Further the signs accompanying the coefficient in the original determinant will follow the following checker boad pattern.

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

Problem:

1\. Find the determinants with co-efficients from i) first column & ii) the third row in the following co-efficients of the determinant

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Soln:

i) $$\Delta = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & c_2 \\ b_3 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}.$$

$$= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_2) + a_3(b_1c_2 - b_2c_1)$$

$$= a_1b_2c_3 - a_1b_3c_2 - a_2b_1c_3 + a_2b_3c_2 + a_3b_1c_2 - a_3b_2c_1$$

(ii) $\Delta = a_3\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} - b_3\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} + c_3\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

$= a_3(b_1c_2 - b_2c_1) - b_3(a_1c_2 - c_1a_2) + c_3(a_1b_2 - b_1a_2)$

$= a_3b_1c_2 - a_3b_2c_1 - b_3a_1c_2 + b_3c_1a_2 + c_3a_1b_2 - c_3b_1a_2$

minors of a matrix

consider a matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

when we delete any one row and any one column of A, then we get a 2x2 matrix which is called a submatrix of A. The determinant of any such submatrix is called a minor of determinant A.

The det minor of a_{11} is $\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

denoted by m_{11}

m_{23} i.e minor of a_{23} is $\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$.

Cofactor of a matrix

If we multiply the minor of the element in the i^{th} row and j^{th} column of the determinant of the matrix by $(-1)^{i+j}$ the product is called the cofactor of the element.

$A_{ij} = (-1)^{i+j} \times$ minor of a_{ij} in $|A|$

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Eg:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

cofactor of a_{11} : $A_{11} = (-1)^{1+1} \times$ minor of a_{11}

$$= (-1)^2 \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$= (-1)^2 (a_{22}a_{33} - a_{23}a_{32})$$

$$= 1 (a_{22}a_{33} - a_{23}a_{32})$$

$$= a_{22}a_{33} - a_{23}a_{32}$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= (-1)^5 (a_{11}a_{32} - a_{31}a_{12})$$

$$= -1 (a_{11}a_{32} - a_{31}a_{12})$$

$$= a_{31}a_{12} - a_{11}a_{32}.$$

Problem:

If $A = \begin{bmatrix} 3 & 4 & 7 \\ -2 & 5 & 6 \\ 7 & 3 & -9 \end{bmatrix}$

find the co-factors of element 6, -9.

Soln:

The element 6 in the position: a_{23}

then cofactor of 6 is

$$A_{23} = (-1)^{2+3} \times M_{23}$$

$$= (-1) \begin{vmatrix} 3 & 4 \\ 7 & 3 \end{vmatrix} = (-1)(3 \times 3 - 7 \times 4)$$

$$= (-1)(9 - 28)$$

$$= (-1)(-19) = 19$$

The co-factor of the element a_{33} i.e -9

$$A_{33} = (-1)^{3+3} \cdot M_{33}$$

$$= (-1)^{6} \begin{vmatrix} 3 & 4 \\ -2 & 5 \end{vmatrix}$$

$$= 1(3 \times 5 + 2 \times 4) = 1(15+8) = \underline{23}$$

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Adjoint of a square matrix

Let $A = [a_{ij}]_{n\times n}$ be a square matrix of order n. then adjoint of A is defined to be transpose of matrix $[A_{ij}]_{n\times n}$ where A_{ij} is cofactor of a_{ij} in $|A|$

Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\text{adj } A = \text{transpose of } \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}$$

Here A_{ij} are the cofactors of a_{ij}.

Remarks:

1 If A be a $n\times n$ square matrix. then $A(\text{adj } A) = (\text{adj } A) A = |A| I_n$ where I_n is a unit matrix of order n.

2. For a 2×2 matrix. $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$.

$$\text{adj } A = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

1 Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$

Verify $A(\text{adj } A) = (\text{adj } A) A = |A| I_2$.

Soln:

$$\text{adj } A = \begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A(\text{adj } A) = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -5-6 & -2+2 \\ -15+15 & -6-5 \end{bmatrix} = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix} \text{ --- (a)}$$

$$(\text{adj } A) A = \begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix} = \begin{bmatrix} -5-6 & -10+10 \\ -3+3 & -6-5 \end{bmatrix} = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix} \text{ --- (b)}$$

$$|A| = 1\times(-5) - 3\times 2 = -5-6 = -11$$

$$|A| I_2 = -11 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix} \text{ --- (c)}$$

from (a), (b) & (c) condition is verified.

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2. If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ show that $\text{Adj } A = 3A'$

Soln:

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = 1(1-4) = -3.$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} = -1(2+4) = -6.$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} = 1(-4-2) = -6.$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & -2 \\ -2 & 1 \end{vmatrix} = -1(-2-4) = +6.$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = 1(-1+4) = 3$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -1(+2+4) = -6.$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & -2 \\ 1 & -2 \end{vmatrix} = 1(4+2) = 6$$

$$A_{32} = (-1)^{2+2} \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -1(2+4) = -6$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = +1(-1+4) = 3.$$

$$\begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix} \qquad \text{adj } A = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix} \text{ — (a)}$$

$$3A^t = 3 \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}^t = 1 \begin{bmatrix} \cancel{-3} & \cancel{-6} & \cancel{-6} \\ \cancel{6} & \cancel{3} & \cancel{-6} \\ 6 & -6 & \cancel{3} \end{bmatrix}$$

$$= 3 \begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix} \text{ — (b)}$$

from (a) & (b) $\text{Adj } A = 3A'$

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3 Find the adjoint of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ and verify $A(\text{adj} A) = (\text{adj} A)A = |A| I_3$.

Soln:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$A_{11} = (-1)^2 \begin{vmatrix} 2 & -3 \\ -1 & 3 \end{vmatrix} = 1(6-3) = 3$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & -3 \\ 2 & 3 \end{vmatrix} = -1(3+6) = -9$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = 1(-1-4) = -5$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} = -1(3+1) = -4$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1(3-2) = 1$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -1(-1-2) = +3$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = 1(-3-2) = -5$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} = -1(-3-1) = +4$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2-1 = 1$$

$$\begin{bmatrix} 3 & -9 & -5 \\ -4 & 1 & 3 \\ -5 & 4 & 1 \end{bmatrix} \qquad \text{adj} A = \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$$

$$A \,\text{adj} A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 3-9-5 & -4+1+3 & -5+4+1 \\ 3-18+15 & -4+2-9 & -5+8-3 \\ 6+9-15 & -8-1+9 & -10-4+3 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{bmatrix}$$

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$$\text{adj}A \cdot A = \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 3-4-10 & 3-8+5 & 3+12-15 \\ -9+1+8 & -9+2-4 & -9-3+12 \\ -5+3+2 & -5+6-1 & -5-9+3 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{bmatrix} \quad -(b)$$

$$|A| = 1\begin{vmatrix} 2 & -3 \\ -1 & 3 \end{vmatrix} - 1\begin{vmatrix} 1 & -3 \\ 2 & 3 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$$

$$= 1(6-3) - 1(3+6) + 1(-1-4)$$

$$= 3 - 9 - 5 = -11$$

$$|A|.I_3 = -11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -11 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{bmatrix} \quad -(c)$$

from (a), (b) & (c) $\text{adj}A.A = A.\text{adj}A = |A| I_3$

Inverse of a matrix

defn:

Let A be any $n \times n$ matrix. The n-square matrix B is called inverse of A if $AB = BA = I_n$.

The inverse of A is denoted by A^{-1}

i.e $B = A^{-1}$ so that $AA^{-1} = A^{-1}A = I_n$.

Remark:

A square matrix A has an inverse if and only if $|A| \neq 0$. i.e only non-singular matrix possesses an inverse

The inverse of A is given by

$$A^{-1} = \frac{1}{|A|} \text{adj}A.$$

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Problem:

1. compute the inverse of the matrix

$$\begin{bmatrix} 1 & 0 & -4 \\ -2 & 2 & 5 \\ 3 & -1 & 2 \end{bmatrix}$$

Soln:

We know that $A^{-1} = \frac{1}{|A|} \text{adj} A$.

$$|A| = 1\begin{vmatrix} 2 & 5 \\ -1 & 2 \end{vmatrix} - 0\begin{vmatrix} -2 & 5 \\ 3 & 2 \end{vmatrix} - 4\begin{vmatrix} -2 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= 1(4+5) - 0 - 4(2-6) = 9 + 16 = 25$$

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 2 & 5 \\ -1 & 2 \end{vmatrix} = 1(4+5) = 9$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} -2 & 5 \\ 3 & 2 \end{vmatrix} = -1(-4-15) = 19$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} -2 & 2 \\ 3 & -1 \end{vmatrix} = 1(2-6) = -4$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 0 & -4 \\ -1 & 2 \end{vmatrix} = -1(0-4) = +4.$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & -4 \\ 3 & 2 \end{vmatrix} = 1(2+12) = 14$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 0 \\ 3 & -1 \end{vmatrix} = -1(-1+0) = 1$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 0 & -4 \\ 2 & 5 \end{vmatrix} = 1(0+8) = 8$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & -4 \\ -2 & 5 \end{vmatrix} = -1(5-8) = 3.$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 0 \\ -2 & 2 \end{vmatrix} = +1(2-0) = 2.$$

$$\begin{bmatrix} 9 & 19 & -4 \\ 4 & 14 & 1 \\ 8 & 3 & 2 \end{bmatrix} \quad \text{adj} A = \begin{bmatrix} 9 & 4 & 8 \\ 19 & 14 & 3 \\ -4 & 1 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{25}\begin{bmatrix} 9 & 4 & 8 \\ 19 & 14 & 3 \\ -4 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 9/25 & 4/25 & 8/25 \\ 19/25 & 14/25 & 3/25 \\ -4/25 & 1/25 & 2/25 \end{bmatrix}$$

2) compute the adjoint and inverse of the matrix

i) $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

Soln:

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

$$|A| = 2(12-2) - 3(16-1) + 4(8-3) = -5$$
$$= 2(10) - 3(15) + 4(5)$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} = 1(12-2) = 10.$$

$$A_{12} = (-1)^{1+2}(16-1) = -15$$

$$A_{13} = (-1)^{1+3}(8-3) = +5$$

$$A_{21} = (-1)^{2+1}(12-8) = -1 \times 4 = -4$$

$$A_{22} = (-1)^{2+2}(8-4) = +1 \times 4 = +4$$

$$A_{23} = (-1)^{2+3}(4-3) = -1 \times 1 = -1$$

$$A_{31} = (-1)^{3+1}(3-12) = 1 \times (-9) = -9$$

$$A_{32} = (-1)^{3+2}(2-16) = -1 \times (-14) = 14$$

$$A_{33} = (-1)^{3+3}(6-12) = 1 \times 6 = 6.$$

$$\text{Adj } A = \begin{bmatrix} 10 & -15 & 5 \\ -4 & 4 & -1 \\ -9 & 14 & 6 \end{bmatrix}^t = \begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & 6 \end{bmatrix}$$

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$$A^{-1} = \frac{1}{|A|}\ \text{adj}\ A$$

$$= \frac{1}{-5}\begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & 6 \end{bmatrix} = \begin{bmatrix} -10/5 & 4/5 & 9/5 \\ -15/5 & -4/5 & -14/5 \\ -5/5 & 1/5 & 6/5 \end{bmatrix}$$

ii) H.W

$$\begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$$

Ans:

$$A^{-1} = \begin{bmatrix} 3/14 & -1/14 & 5/14 \\ 5/14 & 3/14 & -1/14 \\ -1/14 & 5/14 & 3/14 \end{bmatrix}$$

Rank of a matrix

A non-zero matrix is said to have a rank say r if at least one of its minor (r-square) is different from zero while (r+1) square minor, if any, is zero.

Illustrations:

1. The rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 0 & 5 \end{bmatrix}$ is 2.

since $\begin{vmatrix} 1 & 2 \\ -4 & 0 \end{vmatrix} = 0 + 4 = 4 \neq 0$;

and there is no minor of order three.

2. $B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \\ 2 & 4 & 8 \end{bmatrix}$ Find the rank of the matrix

soln:

$|B| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \\ 2 & 4 & 8 \end{vmatrix} = 1(16-20) - 2(8-10) + 3(4-4)$

$= -4 + 4 = 0.$

consider any 2x2 submatrix of matrix B.

$\begin{vmatrix} 2 & 3 \\ 2 & 5 \end{vmatrix} = 10 - 6 = 4 \neq 0.$

hence determinant of the above 2x2 matrix is nonzero.

rank of B is 2.

3. $C = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & 6 & 9 \end{bmatrix}$ find the rank of the matrix

soln:

$|C| = \begin{vmatrix} 0 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & 6 & 9 \end{vmatrix}$

$= 0(36-36) - 2(0-0) + 3(0-0) = 0$

$|C| = 0.$

Also determinant of every 2×2 submatrix of C is zero.

$\therefore$ rank of C is 1

Problem:

Find the rank of the following matrices.

1. $\begin{bmatrix} -1 & 1 & 2 \\ 1 & -1 & 2 \\ -1 & 1 & 10 \end{bmatrix}$

Soln:

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & -1 & 2 \\ -1 & 1 & 10 \end{bmatrix}$$

$$|A| = -1(-10-2) - 1(10+2) + 2(1-1)$$

$$= -1(-12) - 1(12) + 0$$

$$= 12 - 12 = 0.$$

Consider any 2×2 submatrix

$$\begin{vmatrix} -1 & 2 \\ 1 & 10 \end{vmatrix} = -10 - 2 = -12 \neq 0.$$

Hence rank of A is 2.

3) $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Soln:

$|B| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$

$= 1(1-0) - 0(0-0) + 0(0-0)$

$= 1 \neq 0.$

Hence rank of B is 3

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