Trignometry

Angles

An Angle in trignometry is defined as the amount of rotation made by a straight line from one position to another above the point

Measurment of Angles

In trignometry there are 3 Systems for the measurement of angles

1. English System (Sexagesimal)

1 right. angle $= 90^\circ$

$1^\circ = 60$ minutes $(60')$

$1' = 60$ Seconds $(60'')$

2. French System (centesimal)

1 rt. angle $=$ 100 grades (100^g)

$1^g = 100'$

$1' = 100''$

The Relationship b/w the English and French System is $1^\circ = \frac{10^g}{9}$

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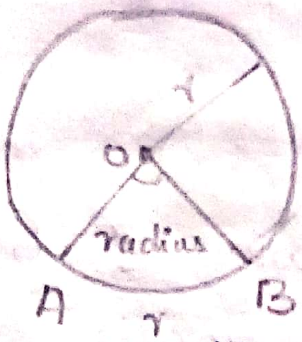
3. Circular System

In this System the unit of measure is radian.

The radian is defined as the an angle Substanded at the Centre of Circle by an arc equal in length of the radius of the circle.



The angles at the Centre of the circle are prapotion to the arc Substanded through them. We have $\frac{\angle AOB}{\text{Total angle at}}$

$$\frac{\angle AOB}{\text{Total angle at } O} = \frac{\text{arc } AB}{\text{Circumferance}} = \frac{r}{2\pi r} = \frac{1}{2\pi}$$

$$\frac{1 \text{ radian}}{4 \text{ rt. angle}} = \frac{1}{2\pi}$$

$$\pi \text{ radian} = 2 \text{rt angle} = 2 \times 90^\circ = 180^\circ = 200$$

$$\boxed{\pi \text{ radian} = 180^\circ = 200^g}$$

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1 Find the circular measure of i] 60° ii] $112^\circ 30'$ iii] 135° iv] $40^\circ 27' 30''$ v] $65^\circ 6' 7''$

→ i] 60°

π radian $= 180^\circ$

$$60^\circ = \frac{\pi}{\cancel{180}_3} \times \cancel{60}^{1}$$

$$= \frac{\pi}{3}$$

ii] $112^\circ 30'$

$1^\circ - 60'$

$? - 30'$

$1^\circ = 60'$

$? = 30'$

$\frac{30}{60} - 1/2$

$$\frac{30}{60} \Rightarrow 1/2$$

$$112 + \frac{1}{2} \Rightarrow \frac{224+1}{2} \Rightarrow \frac{225^\circ}{2}$$

π radian $= 180^\circ$

$$\frac{225^\circ}{2} = \frac{\pi}{\cancel{180}} \times \frac{\cancel{225}}{2}$$

15

~~45~~

36

12

$$= \frac{15\pi}{24}$$

iii] $135°$

π radian $= 180°$

$$135° = \frac{\pi}{\cancel{180}_{\cancel{36}_{4}}} \times \cancel{135}^{\cancel{27}\ \cancel{9}\ 3}$$

$$= \frac{3\pi}{4}$$

iv] $40° 27' 30''$

$$30'' = \frac{30}{60} = \frac{1}{2} \text{ minutes}$$

$$27' + \frac{1}{2}' \Rightarrow \frac{54+1}{2} = \frac{55}{2}' \text{ minute}$$

$1° = 60'$

$? \; - \; \frac{55}{2}'$

$$\frac{\frac{55}{2}}{60} \Rightarrow \frac{55}{2 \times 60} \Rightarrow \frac{55°}{120} \text{ Degree}$$

$$40° + \frac{55°}{120} \Rightarrow \frac{4800 + 55}{120} \Rightarrow \frac{\cancel{4855}^{971}}{\cancel{120}_{24}}$$

$$= \frac{971°}{24} = \frac{971}{24} \times \frac{\pi}{180} = \frac{971\pi}{4,320}$$

$$= 0.224760\pi$$

v] $65^g\ 6'\ 7''$

$1^g = 100'$

$1' = 100''$

$7'' = \frac{7'}{100}$ minutes

$6' + \frac{7'}{100} \Rightarrow \frac{600' + 7'}{100} \Rightarrow \frac{607'}{100}$

$1^g = 100'$

$\frac{607'}{100} = \frac{607}{100 \times 100} = \frac{607^g}{10,000}$ grade

$65^g + \frac{607^g}{10,000} \Rightarrow \frac{6,50,000 + 607}{10,000}$

$= \frac{6,50,607}{10,000} \Rightarrow \underline{\underline{65.0607^g}}$

π radian $= 200^g$

$65.0607^g = \frac{\pi}{200} \times 65.0607$

$= \frac{65.0607\pi}{200}$

$= \underline{\underline{0.32530\pi}}$

1 Express in radian i] 225° ii] 375°
iii] 225^g iv] $47^\circ 48' 45''$

→ i] 225°

π radian $= 180^\circ$

$225^\circ = \frac{\pi}{\cancel{180}} \times \cancel{225}^{\cancel{45}\,15}$
$\cancel{36}$
12

$= \frac{15\pi}{12}$

ii] 375°

π radian $= 180^\circ$

$375^\circ = \frac{\pi}{\cancel{180}} \times \cancel{375}^{\cancel{75}\,25}$
$\cancel{36}\ 12$

$= \frac{25\pi}{12}$

iii] 225^g

π radian $= 200^g$

$225^g = \frac{\pi}{\cancel{200}} \times \cancel{225}^{\cancel{45}\,9}$
$\cancel{40}$
8

$= \frac{9\pi}{8}$

iv] $47^\circ 48' 45''$

$$45'' = \frac{45}{60} = \frac{9}{12} = \frac{3}{4} \text{ minutes}$$

$$48' + \frac{3}{4}' \Rightarrow \frac{192+3}{4}$$

$$= \frac{195}{4} \text{ minutes.}$$

$1^\circ - 60'$

$? - \frac{195}{4}$

$$\frac{\frac{195}{4}}{60} \Rightarrow \frac{195}{60 \times 4} \Rightarrow \frac{\cancel{195}^{\,\cancel{39}\,13}}{\cancel{240}_{\,\cancel{48}\,16}} = \frac{13}{16}^\circ \text{ (deg}$$

$$47^\circ + \frac{13}{16}^\circ \Rightarrow \frac{752+13}{16} \Rightarrow \frac{765}{16}^\circ$$

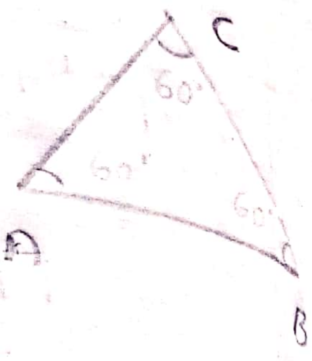
$$\frac{765}{16}^\circ = \frac{\pi}{180} \times \frac{765}{16}^\circ$$

$$= \frac{765\pi}{2{,}880}$$

$$= 0.2656\pi$$

Express both in degrees and Radian the angles of the triangle whose angles are to each other as 1 : 2 : 3

1 : 2 : 3



$\angle CAB = x \quad \angle CBA = 2x. \quad \angle BCA = 3x$

$$x + 2x + 3x = 180^\circ$$

$$6x = 180^\circ$$

$$x = \frac{180}{6} = 30^\circ$$

$2x \Rightarrow 2 \times 30^\circ = 60^\circ$

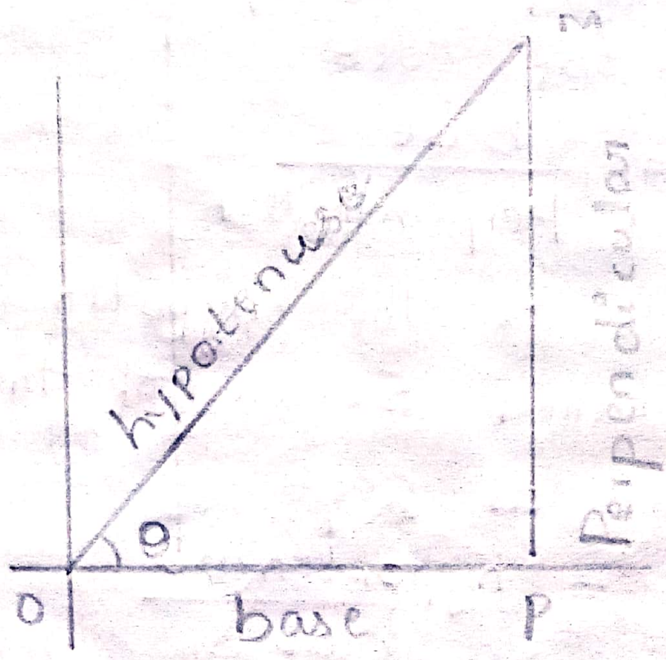
$3x \Rightarrow 3 \times 30^\circ = 90^\circ$

$\angle CAB = 30^\circ \Rightarrow 30^\circ \times \frac{\pi}{180} \Rightarrow \frac{\pi}{6}$ radian

$\angle CBA = 60^\circ \Rightarrow 60^\circ \times \frac{\pi}{180} \Rightarrow \frac{\pi}{3}$ radian

$\angle BCA = 90^\circ \Rightarrow 90^\circ \times \frac{\pi}{180} \Rightarrow \frac{\pi}{2}$ radian

Trignometric Ratio's :



The Ratio of the perpendicular to the hypotencuese is called the Sin of the angle θ and it written as Sinθ

$$\sin\theta = \frac{\text{Perpendicular}}{\text{hypotenuse}}$$

The Ratio of the base to the hypotenuse is called the cos of the angle θ and it written as cosθ.

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$$

The Ratio of the Perpendicular to the base is called the tan of the angle θ and it written as tanθ

$$\tan\theta = \frac{\text{Perpendicular}}{\text{base}}$$

The Ratio of the base to the perpendicular and is called cot of the angle θ and it written as $\cot\theta$

$$\cot\theta = \frac{\text{base}}{\text{Perpendicular}}$$

The Ratio of the hypotenuse to the base is called Secant of the angle θ and it written as $\sec\theta$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{base}}$$

The Ratio of the hypotenuse to the perpendicular is called cosecant of the angle θ and it written as $\operatorname{cosec}\theta$

$$\operatorname{cosec}\theta = \frac{\text{hypotenuse}}{\text{Perpendicular}}$$

Relations between trignometric functions

1. Reciprocal relation

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}$$

2. Quotient Relations

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

3. Square Relation

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta + \tan^2\theta = 1$

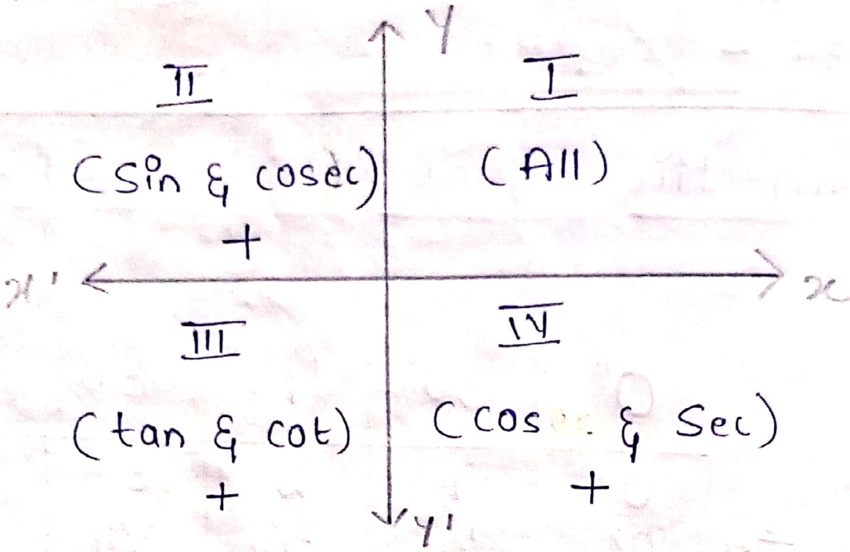
$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

Trignometric functions of Standard angles

	$0°$	$30°$	$45°$	$60°$	$90°$
	0	1	2	3	4
$\div 4$	$0/4$	$1/4$	$2/4$	$3/4$	$4/4$
Root	0	$1/\sqrt{4}$	$\sqrt{2/4}$	$\sqrt{3/4}$	$\sqrt{1}$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
$\operatorname{cosec}\theta$	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1

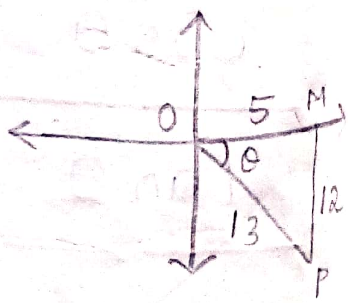
Signs of a trignometric function

In the I quadrant all functions are positive. In the II quadrant sin and cosecant are positive. In III quadrant tan and cot are Positive. In VI quadrant cos and sec are positive.



If θ is in the ~~II~~ IV quadrant and $\cos\theta = \frac{5}{13}$ find the value of $\frac{13\sin\theta + 5\sec\theta}{5\tan\theta + 6\operatorname{cosec}\theta}$

$\cos\theta = \frac{\text{base}}{\text{hyp}}$



$\frac{5}{13} = \frac{OM}{OP}$

$(OP)^2 = (OM)^2 + (MP)^2$

$(MP)^2 = (OP)^2 - (OM)^2$

$(MP)^2 = 13^2 - 5^2$

$(MP)^2 = 169 - 25$

$(MP)^2 = 144$

$MP = 12$

$\sin\theta = \dfrac{\text{Per}}{\text{hyp}} \Rightarrow \dfrac{12}{13} \Rightarrow -\dfrac{12}{13}$ (IV Q) −

$\tan\theta = \dfrac{\text{Per}}{\text{base}} \Rightarrow \dfrac{12}{5} \Rightarrow -\dfrac{12}{5}$ (IV Q) −

$\sec\theta = \dfrac{\text{hyp}}{\text{base}} \Rightarrow \dfrac{13}{5}$ (IV Q +)

$\text{cosec}\,\theta = \dfrac{\text{hyp}}{\text{per}} \Rightarrow -\dfrac{13}{12}$ (IV Q −)

$$\frac{13\sin\theta + 5\sec\theta}{5\tan\theta + 6\,\text{cosec}\,\theta}$$

$$= \frac{\cancel{13}\left(\frac{-12}{\cancel{13}}\right) + \cancel{5}\left(\frac{13}{\cancel{5}}\right)}{\cancel{5}\left(\frac{-12}{\cancel{5}}\right) + \cancel{6}\left(\frac{-13}{\cancel{12}_{2}}\right)}$$

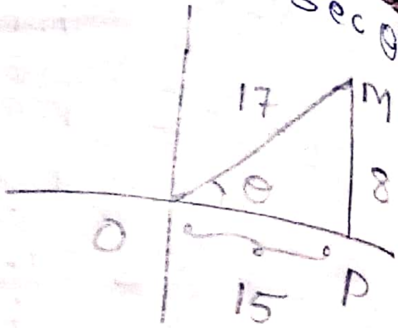
$$= \frac{-12+13}{-12 - \frac{13}{2}}$$

$$= \frac{-12+13}{\frac{-24-13}{2}}$$

$$= \frac{1}{-\frac{37}{2}}$$

$$= -\frac{2}{37}$$

2 If $\sin\theta = \frac{8}{17}$ find $\tan\theta$ and $\sec\theta$



→ $\sin\theta = \frac{\text{Per}}{\text{hyp}}$

$\frac{8}{17} = \frac{PM}{OM}$

If Quadrant is not there then consider it as I quad.

$(OM)^2 = (MP)^2 + (OP)^2$

$(OP)^2 = (OM)^2 - (MP)^2$

$(OP)^2 = 17^2 - 8^2$

$(OP)^2 = 225$

$OP = \underline{\underline{15}}$

$\tan\theta = \frac{\text{Per}}{\text{base}} \Rightarrow \frac{8}{15}$

$\sec\theta = \frac{\text{hyp}}{\text{base}} = \frac{17}{15}$

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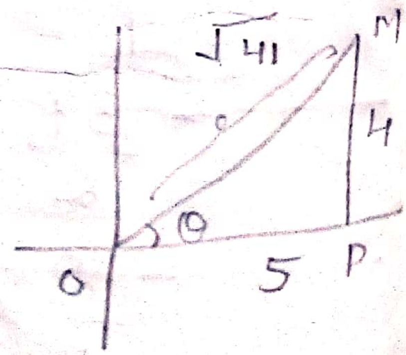
3 If $\tan\theta = \frac{4}{5}$ find the value of

$\frac{2\sin\theta + 3\cos\theta}{4\cos\theta + 3\sin\theta}$

→ $\tan\theta = \frac{4}{5}$



$\tan\theta = \frac{\text{Per}}{\text{base}}$

$\frac{4}{5} = \frac{MP}{OP}$

$$(OM)^2 = (MP)^2 + (OP)^2$$

$$(OM)^2 = 4^2 + 5^2$$

$$(OM)^2 = 16 + 25$$

$$(OM)^2 = 41$$

$$OM = \sqrt{41}$$

$$\sin\theta = \frac{\text{Per}}{\text{hyp}} \Rightarrow \frac{4}{\sqrt{41}}$$

$$\cos\theta = \frac{\text{base}}{\text{hyp}} \Rightarrow \frac{5}{\sqrt{41}}$$

$$= \frac{2\sin\theta + 3\cos\theta}{4\cos\theta + 3\sin\theta}$$

$$= \frac{2\left(\frac{4}{\sqrt{41}}\right) + 3\left(\frac{5}{\sqrt{41}}\right)}{4\left(\frac{5}{\sqrt{41}}\right) + 3\left(\frac{4}{\sqrt{41}}\right)}$$

$$= \frac{\frac{8}{\sqrt{41}} + \frac{15}{\sqrt{41}}}{\frac{20}{\sqrt{41}} + \frac{12}{\sqrt{41}}}$$

$$= \frac{\frac{23}{\cancel{\sqrt{41}}}}{\frac{32}{\cancel{\sqrt{41}}}}$$

$$= \frac{23}{32}$$

(4) Find the values of $\frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} -$

1] $\frac{5}{2} \times \frac{\sin 90^\circ}{\cos 0^\circ}$

$\rightarrow$ $\frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} - \frac{5}{2} \times \frac{\sin 90^\circ}{\cos 0^\circ}$

$\frac{1}{2} + \frac{2}{1} - \frac{5}{2} \times \frac{1}{1}$

$= \frac{1}{2} - \frac{5}{2} + 2$

$= \frac{1-5+4}{2}$

$= 0/2$

$= \underline{\underline{0}}$

2] $\cos^2 0^\circ + \cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90$

$\rightarrow$ $\cos^2 x = (\cos x)^2$

$(\cos 0^\circ)^2 + (\cos 30)^2 + (\cos 45^\circ)^2 + (\cos 60)^2 + (\cos 90)$

$1 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + (0)^2$

$= 1 + \frac{3}{4} + \frac{1}{2} + \frac{1}{4} + 0$

$= \frac{4+3+2+1}{4}$

$= \frac{10}{4}$

$= 5/2$

Prove that

$$4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) - 2 = 0$$

$$4[(\sin 30^\circ)^4 + (\cos 60^\circ)^4] - 3[(\cos 45^\circ)^2 - (\sin 90^\circ)^2] - 2$$

$$4[(1/2)^4 + (1/2)^4] - 3[(1/\sqrt{2})^2 - (1)^2] - 2$$

$$4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[1/2 - 1\right] - 2$$

$$4\left[\frac{\cancel{2}^{1}}{\cancel{16}_{8}}\right] - 3\left[\frac{1-2}{2}\right] - 2$$

$$\cancel{4}^{1}\left[\frac{1}{\cancel{8}_{2}}\right] - 3\left[-1/2\right] - 2$$

$$= \frac{1}{2} + \frac{3}{2} - 2$$

$$= \frac{1+3-4}{2}$$

$$= 0/2$$

$$= \underline{\underline{0}}$$

4 If $\tan^2 45° - \cos^2 60° = x \sin 45° \tan 60°$ find x.

$\rightarrow (\tan 45°)^2 - (\cos 60°)^2 = x \sin 45° \tan 60°$

$$1^2 - \left(\frac{1}{2}\right)^2 = x \times \frac{1}{\sqrt{2}} \times \sqrt{3}$$

$$1 - \frac{1}{4} = x \frac{\sqrt{3}}{\sqrt{2}}$$

$$\frac{4-1}{4} = x \frac{\sqrt{3}}{\sqrt{2}}$$

$$x = \frac{3}{4} \times \frac{\sqrt{2}}{\sqrt{3}}$$

$$x = \frac{\sqrt{3}\,\cancel{\sqrt{3}}}{2\sqrt{2}\,\cancel{\sqrt{2}}} \times \frac{\cancel{\sqrt{2}}}{\cancel{\sqrt{3}}}$$

$$x = \frac{\sqrt{3}}{2\sqrt{2}}$$

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5 Find x from the equation

$$x \sin 30° \cos^2 45° = \frac{\cot^2 30° \sec 60° \tan 45°}{\operatorname{cosec}^2 45° \operatorname{cosec} 30°}$$

$\rightarrow$ $x \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{(\sqrt{3})^2 \times 2 \times 1}{(\sqrt{2})^2 \times 2}$ cosec = (s

$$x \frac{1}{2} \times \frac{1}{2} = \frac{3 \times 2 \times 1}{2 \times 2}$$

$$x \times \frac{1}{4} = \frac{6}{4}$$

$$x = \frac{6}{\cancel{4}} \times \frac{\cancel{4}}{1}$$

$$x = \underline{\underline{6}}$$

CALCULUS

$\in$ [Absolon]

Limit of a function

If corresponding to a positive no. ϵ however small, we are able to find the number δ (delta) such that $|f(x) - l| < \epsilon$ for all values of x satisfying $|x - a| < \delta$ then we say that $f(x)$ approaches to l $(f(x) \to l)$ as x approaches to a $(x \to a)$ and writ

$$\lim_{x \to a} f(x) = l$$

Some Important Limits

1. $\lim\limits_{x \to a} \dfrac{x^n - a^n}{x - a} \quad n \cdot a^{n-1}$

2. $\lim\limits_{n \to 0} (1+n)^{1/n} = \lim\limits_{n \to \infty} \left(1 + \dfrac{1}{n}\right)^n = e \quad e.a=$

3. $\lim\limits_{x \to 1} \dfrac{(1+x)^n - 1}{x} = n$

4. $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$

Find the limit of $\frac{4x^4+3x^2-1}{x^3+7}$ when $x=1$ and $x=-1$

$$\lim_{x\to 1} \frac{4x^4+3x^2-1}{x^3+7} = \frac{4\times 1+3\times 1-1}{1+7}$$

$$= \frac{6}{8} \Rightarrow \frac{3}{4}$$

$$\lim_{x\to -1} \frac{4x^4+3x^2-1}{x^3+7} = \frac{4(-1)^4+3(-1)^2-1}{(-1)^3+7}$$

$$= \frac{4+3-1}{-1+7} \Rightarrow \frac{6}{6} \Rightarrow 1$$

2. $\lim_{x\to 2} \frac{2x^2-1}{2x+2}$

$$\lim_{x\to 2} \frac{2x^2-1}{2x+2}$$

$$= \frac{2(2)^2-1}{2(2)+2}$$

$$= \frac{2(4)-1}{4+2}$$

$$= \frac{8-1}{6}$$

$$= 7/6$$

3 Evaluate $\lim_{x \to 3} \dfrac{x^2+2x-15}{x^2-9}$

$\rightarrow$ $\lim_{x \to 3} \dfrac{x^2+2x-15}{x^2-9}$

$$= \frac{3^2+2(3)-15}{9^2-9}$$

$$= \frac{9+6-15}{9-9} = \frac{0}{0} = \underline{\underline{0}}$$ Indeterminate form

Factorizing

$$\frac{x^2+2x-15}{x^2-9}$$

-15 → -3, $+5$

$$\lim_{x \to 3} = \frac{x^2+5x-3x-15}{(x+3)(x-3)}$$

$$\lim_{x \to 3} = \frac{x(x+5)-3(x+5)}{(x+3)(x-3)}$$

$$\lim_{x \to 3} = \frac{\cancel{(x-3)}(x+5)}{(x+3)\cancel{(x-3)}}$$

$$\lim_{x \to 3} = \frac{x+5}{x+3}$$

$$= \frac{3+5}{3+3}$$

$$= \cancel{\frac{8}{6}}\ {}^{4}_{3}$$

$$= 4/3$$

$$\lim_{x \to 2} \frac{2x^2 - 7x + 6}{5x^2 - 11x + 2}$$

$$\lim_{x \to 2} \frac{2(2)^2 - 7(2) + 6}{5(2)^2 - 11(2) + 2}$$

$$\frac{2(4) - 7(2) + 6}{5(4) - 22 + 2} \Rightarrow \frac{8 - 14 + 6}{22 - 22} = \frac{0}{0}$$

Factorizing

10
-4 -3

$$\lim_{x \to 2} \frac{2x^2 - 7x + 6}{5x^2 - 11x + 2}$$

$$\lim_{x \to 2} \frac{2x^2 - 4x - 3x + 6}{5x^2 - 10x - 1x + 2}$$

$$\lim_{x \to 2} \frac{2x(x-2) - 3(x-2)}{5x(x-2) - 1(x-2)}$$

$$\lim_{x \to 2} \frac{(2x-3)\cancel{(x-2)}}{(5x-1)\cancel{(x-2)}}$$

$$\lim_{x \to 2} \frac{2x-3}{5x-1}$$

$$= \frac{2(2) - 3}{5(2) - 1}$$

$$= \frac{4 - 3}{10 - 1}$$

$$= \frac{1}{9}$$

5 Evaluate $\lim_{x \to 2} \dfrac{2x^2-5x+2}{x^2-3x+2}$

$\rightarrow$ $\lim_{x \to 2} \dfrac{2x^2-5x+2}{x^2-3x+2}$

$= \dfrac{2(4)-5(2)+2}{4-3(2)+2} \Rightarrow \dfrac{8-10+2}{4-6+2} = \dfrac{0}{0}$

$\lim_{x \to 2} \dfrac{2x^2-5x+2}{x^2-3x+2}$

$$\begin{array}{cc} 4 & 2 \\ -4 \quad -1 & -1 \quad -2 \end{array}$$

$\lim_{x \to 2} \dfrac{2x^2-4x-x+2}{x^2-1x-2x+2}$

$\lim_{x \to 2} \dfrac{2x(x-2)-1(x-2)}{x(x-1)-2(x-1)}$

$\lim_{x \to 2} \dfrac{\cancel{(x-2)}(2x-1)}{\cancel{(x-2)}(x-1)}$

$\lim_{x \to 2} \dfrac{2x-1}{x-1}$

$= \dfrac{2(2)-1}{2-1}$

$= \dfrac{4-1}{1}$

$= \underline{\underline{3}}$

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6 $\lim_{x \to 0} \dfrac{4x^4 + 5x^3 + 7x^2 + 6x}{5x^5 + 7x^2 + x}$

$\lim_{x \to 0} \dfrac{x(4x^3 + 5x^2 + 7x + 6)}{x(5x^4 + 7x + 1)}$

$\lim_{x \to 0} \dfrac{4x^3 + 5x^2 + 7x + 6}{5x^4 + 7x + 1}$

$= \dfrac{4(0)^3 + 5(0)^2 + 7(0) + 6}{5(0)^4 + 7(0)^1 + 1}$

$= \dfrac{6}{1}$

$= \underline{\underline{6}}$

Continuity

A function $f(x)$ is said to be continuous ~~and~~ at $x = a$ corresponding to any arbitary assigned positive number absolon $\in$ However Small (but not zero) there exist a +ve no. δ Such that $f(x) - f(a)$ is less than. $\in$ $[|f(x) - f(a)| < \in$ for all $|x - a| < \delta$

1 $f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{for } x < 2 \\ 4 & \text{for } x = 2 \\ 2 & \text{for } x > 2 \end{cases}$

→ Check the continuity at $x = 2$

Here

i] $f(2) = 4$

ii] limit existancy

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{x^2 - 4}{x - 2} = \frac{(x+2)(x-2)}{(x-2)}$$

$$\lim_{x \to 2^-} (x + 2) = 2 + 2 = \underline{\underline{4}}$$

$$\lim_{x \to 2^+} f(x) = \underline{\underline{2}}$$

LHL $\neq$ RHL

$\therefore$ limit does not exist

Here i] $f(2) = 4$

ii] lim existancy.

Here left hand side limit is not equal to the Right hand side limit.

$\therefore$ Limit does not exist in this case

$\therefore$ Function is not continous at a point $x = 2$.

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{for } x < 2 \\ 2 & \text{for } x \geq 2 \end{cases}$$

Check the continuity at $x = 2$

Here i] $f(2) = 2$

ii] Limit existancy.

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{x^2-4}{x-2} = \lim_{x \to 2^-} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= \lim_{x \to 2^-} x+2 = 2+2 = \underline{\underline{4}}$$

$$\lim_{x \to 2^+} f(x) = 2$$

Here left hand side limit is not equal to the Right hand side limit

$\therefore$ Limit does not exist in this case

$\therefore$ Function is not continous at a point $x = 2$.

3 $f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{for} \quad 0 \le x \le 2. \\ 2 & \text{for} \quad x = 2 \\ x+1 & \text{for} \quad x > 2 \end{cases}$

Check the continuity at $x = 2$

→ Here i] $f(2) = 2$

ii] limit existancy.

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{x^2-4}{x-2} \Rightarrow \lim_{x\to 2^-} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= \lim_{x\to 2^-} (x+2) = 2+2 = \underline{\underline{4}}.$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x+1 = 2+1 = \underline{\underline{3}}$$

LHL ≠ RHL

∴ Limit does not exist in this case

∴ Function is not continous at the point $x = 2$.

Prove that the function $f(x) = x^2 + 4x - 2$ is continous at $x = 1$

i] $f(1) = 1$

$\lim_{x \to 1} f(x) = \lim_{x \to 1} x^2 + 4x - 2 = 1 + 4(1) - 2 = \underline{\underline{3}}$

ii] $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} (x-h)^2 + 4(x-h) - 2$

$= \lim_{h \to 0} (1-h)^2 + 4(1-h) - 2$

$= \lim_{h \to 0} 1^2 + h^2 - 2h + 4 - 4h - 2$

$= \lim_{h \to 0} h^2 - 6h + 3$

$= 0 - 6(0) + 3$

$= \underline{\underline{3}}$

iii] $\lim_{x \to 1^+} f(x) = \lim_{x + 1^+} x^2 + 4x - 2$

$= \lim_{h \to 0} (1+h)^2 + 4(1+h) - 2$

$= \lim_{h \to 0} 1^2 + h^2 + 2h + 4 + 4h - 2$

$= \lim_{h \to 0} h^2 + 6h + 3$

$= 0^2 + 6(0) + 3$

$= \underline{\underline{3}}$

Here LHL = RHL

$\therefore$ Limit exist in this case also

$f(1) = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$ $\therefore$ fn is continous at $x = 1$

5 Prove that the function $f(x) = \frac{x^2-9}{x-3}$ is discontinous at $x = 3$.

→ [i] $f(3) = \frac{3^2-9}{3-3} = \frac{9-9}{0} = \underline{\underline{0}}$

[ii] $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \frac{x^2-9}{x-3} = \lim_{x \to 3^+} \frac{(x+3)(\cancel{x-3})}{(\cancel{x-3})}$

$= \lim_{x \to 3^+} (x+3) = \lim_{\substack{x \to 3^+ \\ h \to 0}} (x+h)+3$

$= \lim_{h \to 0} (3+h)+3 = \lim_{h \to 0} 6+h = 6+0 = \underline{\underline{6}}$

[iii] $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{x^2-9}{x-3} = \lim_{x \to 3^-} \frac{(x+3)(\cancel{x-3})}{(\cancel{x-3})}$

$= \lim_{x \to 3^-} x+3 = \lim_{\substack{x \to 3^- \\ h \to 0}} (x-h)+3$

$= \lim_{h \to 0} (3-h)+3 = \lim_{h \to 0} 6-h = 6-0 = \underline{\underline{6}}$

Here LHL = RHL

∴ Limit exist.

But $f(3) \neq \lim_{x \to 3^-} = \lim_{x \to 3^+}$

∴ Function is not continous at the point $x = 3$.

Differentiation

A derivate is the limit of the Ratio the increament in the fn correspondin in a small increament in argument as

we can say $\frac{\delta y}{\delta x}$ is the change in y with respective to a small change in. If the

we may write $\frac{dy}{dx}$ or $\lim_{\delta x \to 0}\left(\frac{\delta y}{\delta x}\right)$

Derivative of a Power function.

i] $y = x^n$

then $\frac{dy}{dx} = \frac{d(x^n)}{dx} = n \cdot x^{n-1}$

$\frac{dx^2}{dx} = 2 \cdot x^{2-1} = \underline{2 \cdot x^1}$

Derivative of a Constant with any fn

$y = \underline{c}\, f(x)$

$\frac{dy}{dx} = \frac{d\,c f(x)}{dx} = c \cdot \frac{d f(x)}{dx}$

$y = 2x^3 = \frac{d\,2x^3}{dx} = 2 \cdot \frac{dx^3}{dx} = 2 \cdot 3x^2 = \underline{6x^2}$

3. Derivate of Sum of functions

$Y = f(x) + \phi(x) + g(x) + \ldots$

$$\frac{dy}{dx} = \frac{d\,f(x)}{dx} + \frac{d\,\phi(x)}{dx} + \frac{d\,g(x)}{dx} + \ldots$$

$Y = x^4 + x^3 + x^2$ Diff w.r.to x

$$\frac{dy}{dx} = \frac{dx^4}{dx} + \frac{dx^3}{dx} + \frac{dx^2}{dx}$$

$$= 4x^3 + 3x^2 + 2x^1$$

4. Derivative of the product of two functions

$Y = f(x)\,g(x)$

$d(u.v) = v.d(u) + u\,d(v)$

$$\frac{dy}{dx} = g(x)\frac{d}{dx}f(x) + f(x).\frac{d}{dx}(g(x))$$

$$= g(x)\,f'(x) + f(x).g(x)'$$

$Y = x^5 . x^2$ w.r. to x

$$\frac{dy}{dx} = x^2 . \frac{d}{dx}(x^5) + x^5 . \frac{d}{dx}(x^2)$$

$$= x^2 . 5x^4 + x^5 . 2x^1$$

$$= 5x^6 + 2x^6$$

$$= 7x^6$$

5. Derivative of the Quotient of two functions

$$y = \frac{f(x)}{g(x)}$$

$$d\left(\frac{u}{v}\right) = \frac{v \cdot \frac{d(u)}{dx} - u \cdot \frac{d(v)}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{g(x)\frac{d}{dx}(f(x)) - f(x) \cdot \frac{d}{dx}(g(x))}{[g(x)]^2}$$

Example

$$f(x) = \frac{x^2-1}{x^2+1} \quad \text{w.r.t } x$$

$$\frac{df(x)}{dx} = \frac{(x^2+1)\frac{d}{dx}(x^2-1) - (x^2-1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{x^2+1 \cdot \frac{dx^2}{dx} + \frac{d(-1)}{dx} - x^2-1 \cdot \frac{dx^2}{dx} + \frac{d1}{dx}}{(x^2+1)^2}$$

0

$$= \frac{(x^2+1) \cdot 2(x) - (x^2-1)(2x+0)}{(x^2+1)^2}$$

$$= \frac{2x^3+2x - 2x^3+2x}{(x^2+1)^2}$$

$$= \frac{4x}{(x^2+1)^2}$$

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1 $y = 2x + x^2$ diff w.r. to x

→ $y = 2x + x^2$

$$\frac{dy}{dx} = \frac{d}{dx}(2x + x^2)$$

$$= \frac{d}{dx}(2x) + \frac{d}{dx}(x^2)$$

$$= 2 \cdot \frac{dx}{dx} + \frac{d}{dx}(x^2)$$

$$= 2 \cdot 1 + 2x'$$

$$= \underline{2 + 2x}$$

3

→

---

2 Diff $(3x^2 + 5)(2x^3 + x + 7)$ w.r. to x

→ take $y = (3x^2+5)(2x^3+x+7)$

$$\frac{dy}{dx} = (2x^3+x+7)\frac{d}{dx}(3x^2+5) + (3x^2+5) \cdot \frac{d}{dx}(2x^3+x+7)$$

$$= (2x^3+x+7) \cdot \frac{d}{dx}(3x^2) + \frac{d}{dx}(5) + (3x^2+5) \cdot \frac{d}{dx}(2x^3) + \frac{dx}{dx} + \frac{d}{dx}(7)$$

$$= (2x^3+x+7) \cdot (3 \times 2 \times x + 0) + (3x^2+5) \cdot (2 \times 3 \times x^2 + 1 + 0)$$

$$= (2x^3+x+7)(6x) + (3x^2+5)(6x^2+1)$$

$$= 12x^4 + 6x^2 + 42x + 18x^4 + 30x^2 + 3x^2 + 5$$

$= 30x^4 + 39x^2 + 42x + 5$

Diff. w.r. to x

3 ① $\dfrac{x^{1/2} + 2}{x^{1/2}}$

$\Rightarrow \dfrac{d}{dx}\left[\dfrac{u}{v}\right] = \dfrac{v \cdot \dfrac{d(u)}{dx} - u \cdot \dfrac{dv}{dx}}{v^2}$

$= \dfrac{x^{1/2} \cdot \dfrac{d}{dx}(x^{1/2}+2) - (x^{1/2}+2)\dfrac{d}{dx}(x^{1/2})}{(x^{1/2})^2}$

$= \dfrac{x^{1/2} \cdot (\frac{1}{2} x^{\frac{1}{2}-1} + 0) - (x^{1/2}+2) \cdot (\frac{1}{2} x^{\frac{1}{2}-1})}{x}$

$= \dfrac{\frac{1}{2} x^0 + 0 - \frac{1}{2} \cdot x^0 - 2 \cdot \frac{1}{2} x^{-1/2}}{x}$

$= \dfrac{\frac{1}{2} \cdot 1 - \frac{1}{2} \cdot 1 - x^{-1/2}}{x}$

$= \dfrac{-x^{-1/2}}{x}$

$= -x^{-1/2} \cdot x^{-1}$ $\qquad \dfrac{-1-2}{2}$

$= -x^{-1/2-1}$ $\qquad -\dfrac{3}{2}$

$= -x^{-3/2} = -\dfrac{1}{x^{3/2}}$

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4 $y = x + \frac{4}{x} - \frac{2}{x^7}$

$$\frac{dy}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}\left[\frac{4}{x}\right] - \frac{d}{dx}\left[\frac{2}{x^7}\right]$$

$$= 1 + \frac{1}{4}\frac{d}{dx}\left[\frac{1}{x}\right] - 2\frac{d}{dx}\left[\frac{1}{x^7}\right]$$

$$= 1 + \frac{1}{4}\left[\frac{-1}{x^2}\right] - 2\left[\frac{-7}{x^8}\right]$$

$$= 1 - \frac{4}{x^2} + \frac{14}{x^8}$$

5 $y = 9x^4 - 7x^3 + 8x^2 - \frac{8}{x} + \frac{10}{x^3}$

*

$$\frac{dy}{dx} = \frac{d}{dx}(9x^4) - \frac{d}{dx}(7x^3) + \frac{d}{dx}(8x^2) - \frac{d}{dx}\left[8/x\right] + \frac{d}{dx}\left[\frac{10}{x^3}\right]$$

$$= 9\frac{d}{dx}(x^4) - 7\frac{d}{dx}(x^3) + 8\frac{d}{dx}(x^2) - 8\frac{d}{dx}\left[\frac{1}{x}\right] + 10\frac{d}{dx}\left[\frac{1}{x^3}\right]$$

$$= 9 \cdot 4x^3 - 7 \cdot 3x^2 + 8 \cdot 2x^1 - 8\left[\frac{-1}{x^2}\right] + 10\left[\frac{-3}{x^4}\right]$$

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$$= 36x^3 - 21x^2 + 16x + \frac{8}{x^2} - \frac{30}{x^4}$$ //

6 $y = x^3 + 2x - \frac{1}{x^8}$

7 $y = x^6 - 6$

8 $y = \frac{1}{x^2} - 5x^4 + \frac{7}{x^3} + 4$

9 $y = \frac{15}{x^2} - 3x + 4 - \frac{1}{x} + x^3$

10 $y = 2x + \frac{3}{x^2 - 1}$

11 $y = \frac{4x}{2x^3 - x^2}$

12 $y = x^3 \cdot x^5$

Integral Calculus:

If $\phi(x)$ be Any differentiable function of x. Such that $\frac{d}{dx}[\phi(x)] = f(x)$ then $\phi(x)$ called anti derivative or An indefinate integral or Simply an intergal of $f(x)$. Simbolicaly we can write $\phi(x) = \int f(x).dx$. And is read as $\phi(x)$ is the integral of $f(x)$ with respect to x. The proses of finding the integration of a given function is called integration. And the given function is called the integrand.

Rules of Integration

① The integral of a product of a constant and function is equal to the product of the constant and the integral of the function. That is,

$$\int k\, f(x).dx = k \int f(x).dx.$$

(2) The integral of the sum or difference of the function is equal to the sum or and difference of their integrals.

$\int (f_1(x).dx + f_2(x).dx + \dots f_n(x).dx) =$
$\int f_1(x).dx + \int f_2(x).dx + \dots \int f_n(x).dx.$

where $f_1, f_2 \dots f_n$ functions of f.

Some standard

$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$ $\qquad \int x^n.dx = \frac{x^{n+1}}{n+1}$

$\frac{d}{dx}(\log x) = \frac{1}{x}$ $\qquad \int \frac{1}{x}.dx = \log x$

$\frac{d}{dx}(e^x) = e^x$ $\qquad \int e^x.dx = e^x$

1 $f(x) = x^2$

$\int f(x).dx = \int x^2.dx = \frac{x^{2+1}}{2+1} = \frac{x^3}{3}$

2 $f(x) = x^5 + x^3$

$\int f(x).dx = \int (x^5 + x^3)dx = \int x^5.dx + \int x^3.dx$

$= \frac{x^{5+1}}{5+1} + \frac{x^{3+1}}{3+1} = \frac{x^6}{6} + \frac{x^4}{4}$

3 $f(x) = 3$

$\int f(x) \cdot dx = \int 3 \cdot dx$

$= 3 \int 1 \cdot dx = 3 \int x^0 \cdot dx = 3 \cdot \frac{x^{0+1}}{0+1}$

$= 3 \cdot \frac{x^1}{1} = 3x$

4 $f(x) = 2x^7$

$\int f(x) \cdot dx = \int 2x^7 \cdot dx$

$= 2 \cdot \int x^7 \cdot dx = 2 \cdot \frac{x^{7+1}}{7+1} = \cancel{2} \frac{x^8}{\cancel{8}_4} = \frac{x^8}{4}$

1 $\int 5x^2 \cdot dx$

$= 5 \cdot \int x^2 \cdot dx$

$= 5 \cdot \frac{x^{2+1}}{2+1} \Rightarrow 5 \cdot \frac{x^3}{3}$

2 $\int (3 - 2x - x^4) \cdot dx$

$= \int 3 \cdot dx - \int 2x \cdot dx - \int x^4 \cdot dx$

$= 3 \int 1 \cdot dx - 2 \int x \cdot dx - \int x^4 \cdot dx$

$= 3 \int x^0 \cdot dx - 2 \int x \cdot dx - \int x^4 \cdot dx$

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$$= 3\cdot\frac{x^{0+1}}{0+1} - 2\,\frac{x^{1+1}}{1+1} - \frac{x^{4+1}}{4+1}$$

$$= \frac{3x^1}{1} - \frac{2x^2}{2} - \frac{x^5}{5}$$

$$= 3x - x^2 - \frac{x^5}{5}$$

3 $\int (4x^3 + 3x^2 - 2x + 5)\cdot dx$

$$= \int 4x^3\cdot dx + \int 3x^2\cdot dx - \int 2x\cdot dx + \int 5\cdot dx$$

$$= 4\int x^3\cdot dx + 3\int x^2\cdot dx - 2\int x\cdot dx + 5\int \overset{x^0}{1}\cdot dx$$

$$= 4\cdot\frac{x^{3+1}}{3+1} + 3\cdot\frac{x^{2+1}}{2+1} - 2\,\frac{x^{1+1}}{1+1} + 5\cdot\frac{x^{0+1}}{0+1}$$

$$= \cancel{4}\cdot\frac{x^4}{\cancel{4}} + \cancel{3}\cdot\frac{x^3}{\cancel{3}} - \cancel{2}\,\frac{x^2}{\cancel{2}} + 5\,\frac{x^1}{1}$$

$$= x^4 + x^3 - x^2 + 5x$$

4 $\int x^{6/5}\cdot dx$

$$\frac{6}{5}+1 = \frac{6+5}{5} = \frac{11}{5}$$

$$= \frac{x^{6/5+1}}{\frac{6}{5}+1}$$

$$= \frac{x^{11/5}}{\frac{11}{5}}$$

$$= \frac{5}{11}\cdot x^{11/5}$$

5 $\int \sqrt[3]{x^4}\,.dx$

$= \int (x^4)^{1/3}.\,dx$

$= \int x^{4/3}.\,dx$

$= \dfrac{x^{4/3+1}}{\frac{4}{3}+1}$

$= \dfrac{x^{7/3}}{7/3} \Rightarrow \dfrac{3}{7}.\,x^{7/3}.$

$\dfrac{4}{3}+1$

$\dfrac{4+3}{3} = \dfrac{7}{3} \rightarrow$

7.

6 $\int \dfrac{1}{\sqrt{x}}\,.dx$

$= \int \dfrac{1}{(x)^{1/2}}\,.dx.$

$= \int x^{-1/2}.\,dx$

$= \dfrac{x^{-1/2+1}}{-1/2+1}$

$= \dfrac{x^{1/2}}{1/2}$

$= 2.\,x^{1/2}$

$\int (x^2-1)^2$

$\int (x^4+1-2x^2)$

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$$\int \left(7x^2 - 3x + 8 - \frac{1}{\sqrt{x}} + \frac{1}{x} + \frac{1}{x^2}\right) dx.$$

$$\int 7x^2 \cdot dx - \int 3x \cdot dx + \int 8 \cdot dx - \int \frac{1}{x^{1/2}} \cdot dx + \int \frac{1}{x} \cdot dx$$
$$+ \int x^{-2} \cdot dx.$$

$$= 7\int x^2 \cdot dx - 3\int x \cdot dx + 8\int x^0 \cdot dx - \int x^{-1/2} \cdot dx +$$
$$\int \frac{1}{x} \cdot dx + \int x^{-2} \cdot dx.$$

$$= 7 \cdot \frac{x^{2+1}}{2+1} - 3\,\frac{x^{1+1}}{1+1} + 8 \cdot \frac{x^{0+1}}{0+1} - \frac{x^{-1/2+1}}{-1/2+1} + \log x$$
$$+ \frac{x^{-2+1}}{-2+1}$$

$$= 7\,\frac{x^3}{3} - 3\,\frac{x^2}{2} + 8x - \frac{x^{+1/2}}{+1/2} + \log x + \frac{x^{-1}}{-1}$$

$$= 7\,\frac{x^3}{3} - 3\,\frac{x^2}{2} + 8x - 2 \cdot x^{+1/2} + \log x - \frac{1}{x}$$

Definate Integrals.

In Geometrical & other applications of Calculs it becomes neccessary to find the difference in the Values of integral of a function $f(x)$ between two assign values a, b. The difference is called the definate integral of $f(x)$ over the integral $[a, b]$ and it is denoted by $\int_a^b f(x)$ if $\phi(x)$ is integral of $f(x)$.

we write $\int_a^b f(x) \cdot dx = \left[\phi(x)\right]_a^b$

$$= \phi(b) - \phi(a)$$

---

* $\int_2^3 x^2 \cdot dx$

$$= \left[\frac{x^{2+1}}{2+1}\right]_2^3$$

$$= \left[\frac{x^3}{3}\right]_2^3$$

$$= \frac{3^3}{3} - \frac{2^3}{3}$$

$$= \frac{27}{3} - \frac{8}{3} \Rightarrow \frac{27-8}{3} \Rightarrow \frac{19}{3}$$

1 $\int_{-1}^{1} (2x^2 - x^3) \cdot dx$

$\rightarrow \int_{-1}^{1} 2x^2 \cdot dx - \int_{-1}^{1} x^3 \cdot dx$

$2 \cdot \left[\frac{x^3}{3}\right]_{-1}^{1} - \left[\frac{x^4}{4}\right]_{-1}^{1}$

$2 \cdot \left[\frac{+1^3}{3} - \frac{(-1)^3}{3}\right] - \left[\frac{+1^4}{4} - \frac{(-1)^4}{4}\right]$

$2\left[\frac{1}{3} + \frac{1}{3}\right] - \left[\frac{1}{4} - \frac{1}{4}\right]$

$2 \left[\frac{2}{3}\right] - 0$

$= \frac{4}{3}$

---

2 $\int_{6}^{10} \left[\frac{1\,dx}{x+2}\right.$

$\frac{1}{x} \to \log x$

$\frac{1}{x+2} = \log(x+2)$

$\rightarrow \log(x+2) \Big|_{6}^{10}$

$= \log(10+2) - \log(6+2)$

$= \log 12 - \log 8$

$= \log\left[\frac{12}{8}\right]$

$= \log 3/2$

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Differentiation

HW 1

$$Y = x^3 + 2x - \frac{1}{x^8}$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) + \frac{d}{dx}(2x) - \frac{d}{dx}\left[\frac{1}{x^8}\right]$$

$$= 3x^2 + 2 \cdot \frac{dx}{dx} - \left[\frac{-8}{x^9}\right]$$

$$= 3x^2 + 2 + \frac{8}{x^9}$$

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$$y = x^6 - 6$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^6) - \frac{d}{dx}(6)$$

$$= 6 \cdot x^5 - 0$$

$$= \underline{6x^5}$$

$$y = \frac{1}{x^2} - 5x^4 + \frac{7}{x^3} + 4$$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{1}{x^2}\right] - \frac{d}{dx}[5x^4] + \frac{d}{dx}\left[\frac{7}{x^3}\right] + \frac{d}{dx}(4)$$

$$= \frac{-2}{x^3} - 5 \cdot 4x^3 + 7 \cdot \frac{d}{dx}\left[\frac{1}{x^3}\right] + 0$$

$$= \frac{-2}{x^3} - 20x^3 + 7 \cdot \left[\frac{-3}{x^4}\right]$$

$$= \frac{-2}{x^3} - 20x^3 - \frac{21}{x^4}$$

$$y = \frac{15}{x^2} - 3x + 4 - \frac{1}{x} + x^3$$

$$\frac{dy}{dx} = 15 \cdot \frac{d}{dx}\left[\frac{1}{x^2}\right] - 3\frac{d}{dx}(x) + \frac{d}{dx}(4) - \frac{d}{dx}\left[\frac{1}{x}\right] + \frac{d}{dx}x^3$$

$$= 15 \cdot \left[\frac{-2}{x^3}\right] - 3 \cdot 1 + 0 - \left[\frac{-1}{x^2}\right] + 3 \cdot x^2$$

$$= \frac{-30}{x^3} - 3 + \frac{1}{x^2} + 3x^2$$

5 $y = 2x + \frac{3}{x^2-1}$

$$\frac{dy}{dx} = \frac{d}{dx}(2x) + \frac{d}{dx}\left[\frac{3}{x^2-1}\right]$$

$$= 2 \cdot \frac{dx}{dx} + 3 \cdot \frac{d}{dx}\left[\frac{3}{x^2-1}\right]$$

$$= 2 + 3 \cdot \left[\frac{(x^2-1)\frac{d}{dx}(3) - 3\frac{d}{dx}(x^2-1)}{(x^2-1)^2}\right]$$

$$= 2 + \frac{(x^2-1) \cdot 0 - 3 \cdot 2x - 0}{(x^2-1)^2} = \frac{2+0-6x}{(x^2-1)^2}$$

$$= 2 \cancel{+} - \frac{6x}{(x^2-1)^2}$$

6 $y = \frac{4x}{2x^3 - x^2}$

$$\frac{dy}{dx} = \frac{(2x^3-x^2)\frac{d}{dx}(4x) - 4x \cdot \frac{d}{dx}(2x^3-x^2)}{(2x^3-x^2)^2}$$

$$= \frac{(2x^3-x^2) \cdot 4 - 4x \cdot (2 \cdot 3x^2 - 2x)}{(2x^3-x^2)^2}$$

$$= \frac{8x^3 - 4x^2 - 24x^3 + 8x^2}{(2x^3-x^2)^2}$$

$$= \frac{-16x^3 + 4x^2}{(2x^3-x^2)^2}$$

$y = x$

$$\frac{dy}{dx} = x^5 \cdot \frac{d}{dx}(x^3) + x^3 \cdot \frac{d}{dx}(x^5)$$

$$= x^5 (3x^2) + x^3 (5x^4)$$

$$= 3x^7 + 5x^7$$

$$= +8x^7$$