BCAC 181

BASIC MATHEMATICS

UNIT - 1

Logarithms

Introduction, Law of Logarithms (statements only), Illustrations 2, 3, 4 Examples 2, 4, 5, 7, 11 (a) and 11 (b), 14 (P 195, 197 - 199, 201, 202, 204), 19(a) P(206), Exercise (I) 1, 2 (I, III), 3 (I) (II), 11 (a), (b), 17 (a)

Permutations and Combinations

Introduction, Fundamental rules of Counting (statement), Example 1, 5 Permutations, Illustrations 1 (P 303), Remark 1 (P304), Example 6, 7 (304, 305), Permutations of things not all different

Example 12 (a), b, 13 (P 307, 308) Combination Formulae, statement of Theorem (P139) Example 31 and 34 (P 139, 320)

Binomial Theorem

Statements only (P 334), Example 1, 2 (P 336), Exercise 1 (ii) 2 (i) & (ii) P (338)

Positions of terms.

Examples 5 (P 337), 7 (a) & 7 (b) (P 339)

Analytical Geometry

Introduction, Directed Line, Quadrants, Example 1 (P 555), Coordinates of the midpoints. (statement and example) (P 556), Distance between two points (only formula no proof), Section Formula, External Division, Coordinates of Centroid, Area of a tringle (only statements).

Examples 2 (a) & (b) (P 557), 3, 4, 7, 11 (P 558, 559, 562, 565)

Exercise 1 (i, ii), 8, 9(i), 15 (a) and (b), 16 (a) and (b) 21 (a), 24 (i) & (ii)

Straight Line

Diffrent forms of equations of stright line (statements), General equation of a straight Line (statement Only), Example 18 (P.. Example 23 (581), Example 29 (587) Exercise 2 (a), 3(b) (i), (ii) and iii (P 592)

Circle

The equation of a Circle (only Formula), Illustration (P 597), General Equation of the Circle (statement only), Finding centre and radius Example (39) (P 601) Exercise 5 (i) (P 612), 6(a) Equation of tangent and normal (statement only, P 605 and 606) Example 50.

Ellipse Example 53.

UNIT II

Trigonometry

Quadrants, Measurement of Angles, Circular measure, Example 2, Exercise 3 (a) i and ii, 4 (P 483), Trigonometric functions (defintion only), trignomatric Ratios, relation between trignometric function I, II & III only formulae (P 487), Sign of Trigonometric functions, T-ratios of standarad angles (only table P 503) Example 25 (P 493), Exercise (II) 12 (a), (b), 13 (d), (e) (P 499) Exercise (III) 1 (i) (ii), 2 (a) 4 (a) (b)

Calculus

Limit of a function definition, Some Important Limits, Example 3, 4 (P 635) Exercise 1 (a), (e) (P 645).

Continuity of a Function statement only, Example 16 (a) (b) (c) (P 641, 642), Exercise 5, 6 (P 645)

Differentiation

Definition, Derivative of a power function, derivative of a constant with any function, derivative of sum of functions, derivative of product of two function, derivative of the quotient of the two functions (only statements), Illustraction 1 and 3 (P 656, 657)

Integration

Diffinition (P 724), Indefinite Integrals, Rules of integration, Some standard Results (Formula only) (I II, & IX)

Illustration 1, 2, 3 (P727), Exercise 1, 2(i) 2(ii) (P 730)

Definite Integrals (Definition), Illustration 1, 2, 3, 5, (P 758, 759), Exercise (VI) (4 i)

UNIT - III

Set Theory

Basic Concepts of set theory, Inclusion and Equality of sets.

The Power Set, Definitions

Exercises 2 - 1.3 1, 2 a to g, 4

Some Operations on sets.

Definitions

Example 1, 3, 5, 7 (P(113 to 115)

Exercise 2 - 1.4 2, 3 and 7 (P 115 & 116)

Venn Diagrams

Ordered Pairs and n - Tuples

Cartesian Products definition

Examples 1 & 2 (P 124) Exercises (using Examples Not using Postulates) 2 - 1 1, 2, 3, 4, 5, 8, 9, 13 (P 126)

Relations

Definitions

Example 1 (P 151), Exercise 2-3, 1.1

Properties o Binary Relations in sets

Definitions

Exercise 2-3, 25

Relation Matrix and Graph of the Relation.

Example 1, 2 and 3 (P 158 - 160), Equivalence

Relation, Definition

Example 1 and 2 (P 165), Compatibility Relations

Definition

Composition of Binary relations

Definition (2-3, 13, 2-3, 14),

Example 1, 2, 3 and 4 (P 177 - 180)

Partial ordering

Definition (2-3, 16)

Functions

Definitions (2-4.1, 2-4.3, 2-4.4, 2-4.5), Example (P 196)

Composition of functions, definition (2-4, 6), Example 1 and 2 (P 198, 199), Inverse Functions, Example 1, 2 (202),

Exercises 2-4, 3 1, 3, 4

Binary and n-ary operations, Definition (2-4, 8)

Characteristic Function of a set, Definition (2-4, 17)

UNIT - IV

Logical statements and Truth tables

Introduction, definition, truth tables, negation, Compounding, Negation of compound statements, Tautologies and Fallacies, Prepositions, Conditional statements, Biconditional statements, Arguments, Joint Denial

Example : 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17

Exercise : 1, 2, 3, 5, 6, 7

Matrix Algebra

Introduction, definition, types of matrices.

Illustrations

Scalar multiplication of matrices, Illustrations, equality of matrices,

Exercise (I) 1, 2, 3

matrix operations, Addtion, and subtraction,

Example 1.

Multiplication, Example 2, 3, 4, 6, 12, 13

Exercise : 1 (i, ii, iii), 2, 13.

Transpose of a matrix, determinants of order two, Cramer's rule, Example : 16, 17 Determinants of order three, expansion of the determinants, minors of a matrix, co-factors of a matrix,

Example : 23, 24, 25 exercise : 1, 3

Adjoint of a square matrix,

inverse of a matrix (using adjoint matrices - cofactor method), Example : 27 Exercise : 6

Rank of a matrix. Illustration : 1, 2, 3 Exercise : 4 (i, ii)

Set

Definition :

A set is a well defined class or collection of objects. It can be represented in 2 forms.

1. Roaster form:

In this form a set is described by listing the elements seperated by commas with in the braces { }

Ex: { a, e, i, o, u }

2. Set Builder form :

All the elements of the set are posess a single common property which is not possessed by any element outside the ~~class~~ set.

Ex : The set of ovals in english alphabet.

{ x | x is a vowel in the english alphabet }

Give another description of the following set

1) { x | x is a integer and $5 \leq x \leq 12$

Ans = { 5, 6, 7, 8, 9, 10, 11, 12 }

2) { 2, 4, 8 . . . }

Ans : { x | x is an even positive integer }

3) { All the countries of the world }

Ans = { India, America, China . . . .

. . . .

}

Inclusion and Equality of sets :

Definition :

Let A & B be any two sets if every element of a is an element of B then A is called A subset of B or A is said to be included in B or B include A. Symbolically this relation is denoted by $a \leq b$ or $b \geq A$.

$\in$ → belongs to

$\notin$ → does not belongs to

Ex : 1) $A = \{1, 2, 3\}$ $b = \{1, 2, 3, 4, 5, 6, 7\}$

$A \leq B$

2) $C = \mathbb{N}$ $D = \mathbb{Z}$

$C \subseteq D$

Given $S = \{2, a, \{3\}, 4\}$

$R = \{\{a\}, 3, 4, 1\}$

Indicate whether the following true or false.

a) set contains $\{a\} \in$ - False

b) $\{a\} \in R$ - True

c) $\{a, 4, \{3\}\} \subseteq S$ - True

d) $\{\{a\}, 1, 3, 4\} \subset R$ - False

e) $R = S$ - False

f) $\{a\} \subseteq S$ - True

g) $\{a\} \subseteq R$ - False

Definition:

Two SETS A & B are said to be equal if and only if (iif) a (is contained) $\subseteq$ b $B \subseteq A$ or

symbolically $a = b \Leftrightarrow (A \subseteq B \wedge B \subseteq A)$

$A = \{1, 3, 2\}$ $B = \{1, 2, 3\}$

$\boxed{\begin{array}{l} \wedge - \text{And} \\ \vee - \text{Or} \end{array}}$

$A \subseteq B$
$\because A = B$
$B \subseteq A$

Definition:

a SEA A is called proper subset of a SET B if A (is a $\subseteq$ subset of) B and A (is not equal to) B.

Symbolically we can write $A \subset B \Leftrightarrow (A \subseteq B \wedge A \neq B)$

Ex: $A = \{a, e, i\}$ $B = \{a, e\}$

$A \neq B$
$B \in A$ $\because B \subset A$

Definition:

A SET is called a universal set if it includes every SET under the discussion. A universal SET will be denoted by 'E'.

Definition:

A SET which does not contain any element is called NULL / empty SET And it is denoted by ϕ.

The Power SET :

for a SET A, a collection or family of all subsets of A is called power SET of A. The power set of A is denoted by $P(A)$ or 2^A, so that

$P(A) = 2^A = \{ x \mid x \subseteq A \}$

Ex : $B = \{1, 2, 3\}$

$P(B) = \{ \emptyset, B, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\} \}$

A number of possible subset is 2^n where n denotes the number of elements in the SET.

$2^3 = 8$

Give the Power set of the following:

(a) $\{a, \{b\}\}$ → $\{ \phi, \{a\} \{\{b\}\}, \{a, \{b\}\} \}$

(b) $\{1, \phi\}$ → $\{ \phi, \{1\} \}$

(c) $\{x, y, z\}$ → $\{ \phi, \{x\}, \{y\}, \{z\}, \{x,y\}, \{x,z\} \{y,z\}, \{x,y,z\} \}$

Some Operations on SETS :

1) **Definition :**

The intersection of any 2 sets A & B written as $A \cap B$ is the SET consisting of all the elements which belong to both A & B. symbolically

$A \cap B = \{ x \mid (x \in A \wedge x \in B \}$

Ex : $\{a, b, c\}$ $B = \{f, b, c, g\}$

$A \cap B = \{b, c\}$

Properties :

- $A \cap B = B \cap A$
- $A \cap A = A$
- $A \cap \phi = \phi$

Definition :

Two SETS A & B are called disjoint iff $A \cap B = \phi$

i.e A & B have no elements in common.

Ex : $A = \{1, 2\}$ $B = \{c, d\}$

$A \cap B = \phi$

Definition :

A collection of SETS is called a disjoint collection if for every pair of SETS in the collection that 2 sets are disjoint. The elements of a disjoint collection are said to be mutual disjoint.

$A_1 = \{\{1,2\}, \{3\}\}$

$A_2 = \{\{1\}, \{2,3\}\}$

$A_3 = \{\{1,2,3\}\}$

Then show that A_1, A_2, A_3 are mutually disjoint.

Here $A_1 \cap A_2 = \phi$ & $A_1 \cap A_3 = \phi$ & $A_2 \cap A_3 = \phi$

therefore A_1, A_2, A_3 are mutually disjoint.

Definition :

For any two SETS A & B the union of A & B written as $A \cup B$ is the set of all elements which are members of the SET A or SET B or Both. Symbolically it written as $A \cup B = \{x \mid x \in A \vee x \in B\}$

Ex: $A = \{1, 2, 3\}$ $B = \{a, e\}$

$A \cup B = \{1, 2, 3, a, e\}$

Properties :

- $A \cup B = B \cup A$
- $A \cup \phi = A$
- $A \cup A = A$

Problems :

1) $S = \{a, b, p, q\}$ $Q = \{a, p, t\}$

$S \cup Q = \{a, b, p, q, t\}$

$S \cap Q = \{a, p\}$

2) $A_1 = \{1, 2\}$ $A_2 = \{2, 3\}$ $A_3 = \{1, 2, 3, 6\}$

$\cup^3 A_i = \{1, 2, 3, 6\}$ $(A_1 \cup A_2 \cup A_3)$

$\cap^3 A_i = \{2\}$ $(A_1 \cap A_2 \cap A_3)$

3) Let A & B be any 2 SETS the relative complement B in A or (of B with respect to A) written as $A - B$ is the SET consisting of all elements of A which are not element of B. that is

$A - B = \{x \mid x \in A \wedge x \notin B\}$

Ex: $A = \{1, 2, 3, 5\}$ $B = \{3, 5, 7\}$

$A . B = \{1, 2\}$

$B . A = \{7\}$

Definition :

IMP

Let E be the universal set. for any SET A the relative complement of A with respect to E. that is, $E - A$ is called the absolute complement of A. The Absolute Complement of a SET A is often called the complement of A & it is denoted by $\sim A$. That

$\sim A = \{x \mid x \in E \wedge x \notin A\}$

$= \{x \mid x \notin A\}$

Note : $A \cup \sim A = E$
$A \cap \sim A = \phi$
$\sim \phi = E$
$\sim E = \phi$

Given $A = \{2, 5, 6\}$

$B = \{3, 4, 2\}$

$C = \{1, 3, 4\}$.

find $A - B$ & $B - A$ show that $A - B \neq B - A$

and $A - C = A$

$A - B = \{5, 6\}$

$B - A = \{3, 4\}$

Here $A - B \neq B - A$

$A - C = \{2, 5, 6\}$

Here $A - C = A$

Show that for any 2 SET A & B $A - (A \cap B) = A - B$.

<u>Ans:</u>

$A - A \cap B \iff \{x \mid x \in A \wedge x \notin (A \cap B)\}$ ($\because x \notin A \iff \sim(x \in A)$)

$\iff \{x \mid x \in A \wedge \sim(x \in A \cap B)\}$

$\iff \{x \mid x \in A \ (x \in \sim A \cup x \in \sim B)\}$ ($\because$ Demorgan's: $\sim(A \cup B) = \sim A \cap \sim B$, $\sim(A \cap B) = \sim A \cup \sim B$)

$\iff \{x \mid x \in A \cap (x \notin A \vee x \notin B)\}$

$\iff \{x \mid (x \in A \wedge x \notin A) \vee (x \in A \wedge x \notin B)\}$

$\iff \{x \mid \phi \vee (x \in A \wedge x \notin B)\}$

$\iff \{x \mid x \in A \wedge x \notin B\}$

$\iff A - B$

<u>Definition:</u>

Let A & B be any 2 SETS. The symmetric difference (or boolean sum of A & B is the SET $A + B$ defined by

$A + B = (A - B) \cup (B - A)$)

<u>Ex:</u> $A = \{1, 2, 3\}$ $B = \{3, 4\}$

$A - B = \{1, 2\}$

$B - A = \{4\}$

$A + B = (A - B) \cup (B - A)$

$= \{1, 2, 4\}$

Given $A = \{ x \mid x \text{ is an integer and } x \le x \le 5 \}$

$B = \{ 3, 4, 5, 1, 7 \}$ $C = \{ 1, 2, 3 \ldots \}$

find $A \cap B$ $A \cap C$ $A \cup B$ & $A \cup C$

$A = \{ 1, 2, 3, 4, 5 \}$

$A \cap B = \{ 3, 4, 5 \}$

$A \cap C = \{ 1, 2, 3, 4, 5 \}$

$A \cup B = \{ 1, 2, 3, 4, 5, 7 \}$

$A \cup C = \{ 1, 2, 3, \ldots \ldots \}$

$A = \{ 2, 3, 4 \}$ $B = \{ 1, 2 \}$ $C = \{ 4, 5, 6 \}$

find $A + B$, $B + C$, $A + B + C$, $(A+B) + (B+C)$

$A - B = \{ 3, 4 \}$

$B - A = \{ 1 \}$

$B - C = \{ 1, 2 \}$

$C - B = \{ 4, 5, 6 \}$

$A + B = (A - B) \cup (B - A)$

$= \{ 3, 4 \} \cup \{ 1 \}$

$= \underline{\{ 1, 3, 4 \}}$

$B + C = (B - C) \cup (C - B)$

$= \{ 1, 2 \} \cup \{ 4, 5, 6 \}$

$= \underline{\{ 1, 2, 4, 5, 6 \}}$

$A + B + C = (A + B) + C$

$= \{ 1, 3, 4 \} + \{ 4, 5, 6 \}$

$= (A + B - C) \cup (C - A + B)$

$= \{ 1, 3 \} \cup \{ 5, 6 \}$

$= \underline{\{ 1, 3, 5, 6 \}}$

$(A+B)+(B+C) = (A+B)-(B+C) \cup (B+C - A+B)$

$= \{3\} \cup \{2,5,6\}$

$= \{2,3,5,6\}$

$P = \{1, a, c, d\}$ $Q = \{c, d, e\}$

$R = \{1, 2, 3\}$ $S = \{c, 2, e\}$ find

$P+R$, $Q+S$, $R+S$, $P+R+S$, $(Q+S)+(R+S)$, $P+Q$

$P-R = \{a, c, d\}$ $R-S = \{1, 3\}$

$R-P = \{2, 3\}$ $S-R = \{c, e\}$

$Q-S = \{d, e\}$ $P-Q = \{1, a\}$

$S-Q = \{2\}$ $Q-P = \{e\}$

$P+R = (P-R) \cup (R-P)$

$= \{a, c, d\} \cup \{2, 3\}$

$= \{a, c, d, 2\}$

$Q+S = (Q-S) \cup (S-Q)$

$= \{d\} \cup \{2\}$

$= \{d, 2\}$

$R+S = (R-S) \cup (S-R)$

$= \{1, 3\} \cup \{C, E\}$

$= \{1, 3, C, E\}$

$P+Q = (P-Q) \cup (Q-P)$

$= \{1, a\} \cup \{e\}$

$= \{1, a, e\}$

$P+R+S = (P+R)-S \cup S-(P+R)$

$= \{a, d, 3\} \cup \{e\}$

$= \{a, d, e, 3\}$

$(Q+S)+(R+S) = (Q+S - R+S) \cup (R+S - Q+S)$

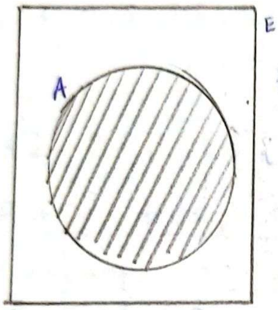
$= \{d, e, 2\} \cup \{1, 3, c, e\}$

$= \{c, d, e, 1, 2, 3\}$

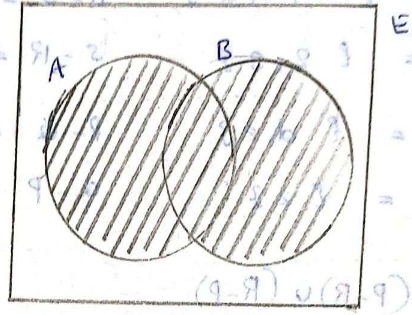
Venn Diagram :

A venn diagram is a schematic represent of a SET by a set of points. The universal set E is represented by a SET of points in a rectangle and a subset say A of E is represented by the interior of a circle or some other closed curve inside the rectangle.

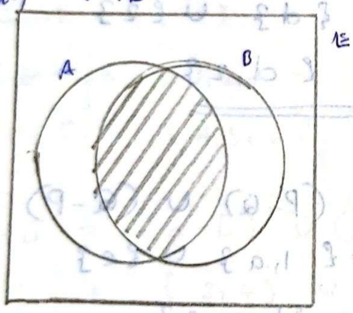
Ex : i) SET A



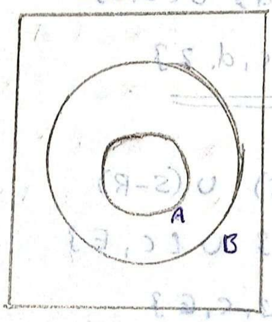
ii) A∪B



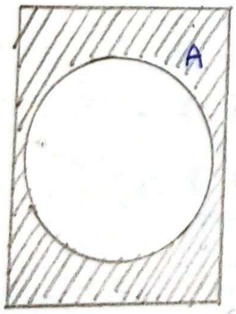
iii) A∩B



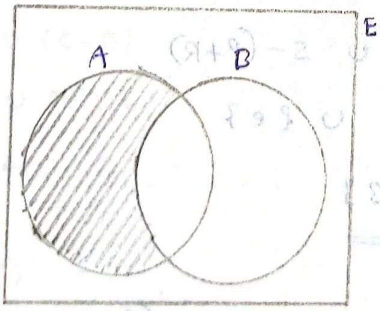
iv) A ⊆ B



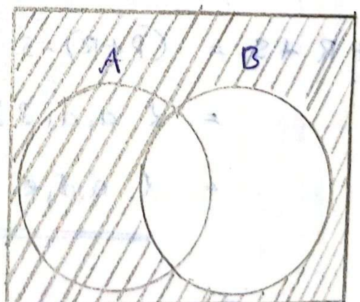
(v) ∼A



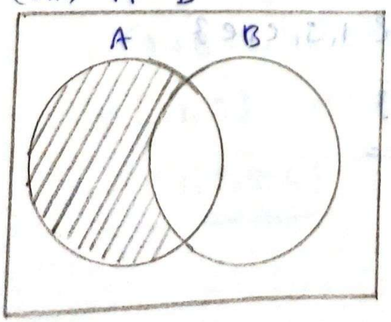
(vi) A ∩ ∼B



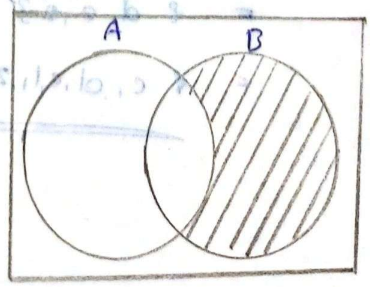
(vii) A ∪ ∼B



(viii) A − B

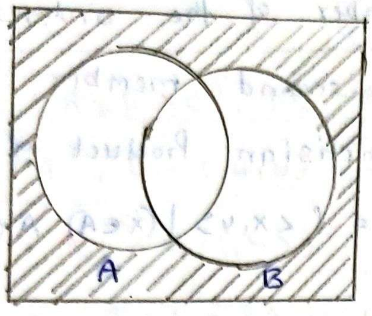


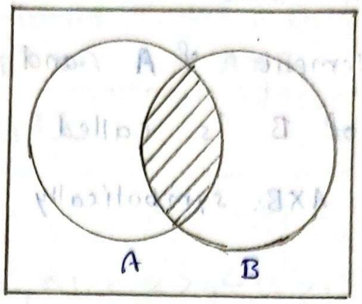
(ix) B − A



Draw the Venn Diagram for the following SETS

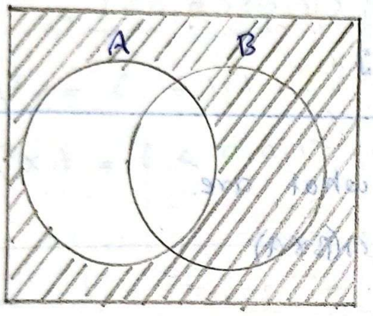
i) $\sim(A \cup B)$ (ii) $B - (\sim A)$

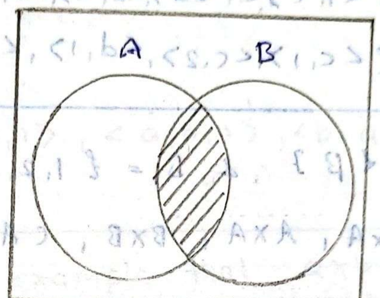




(iii) $\sim A \cap B$

(iv) $\sim A \cap B$





Order Pairs and n- Tuples :

An ordered pairs consist of 2 objects in a given fixed order note that n ordered pair is not a set consisting of 2 elements. The ordering of the 2 objects is important. we shall denote an ordered pair $\langle x, y \rangle$.

The equality of 2 ordered pair $\langle x, y \rangle$ and $\langle u, v \rangle$

$$\langle x, y \rangle = \langle u, v \rangle \iff x = u \wedge y = v$$

An ordered tuple is an ordered pair whose first member is itself an ordered pair. Thus an ordered tuple can be written as

$$\langle \langle x, y \rangle, z \rangle = \langle x, y, z \rangle$$

Cartision Products :

Let A & B any 2 sets, the set of all ordered pairs such that the first member of the ordered pair is an element of A and the second member is an element of B is called a Cartision Product of A & B. written as $A \times B$. symbolically $A \times B = \{ \langle x, y \rangle \mid (x \in A) \wedge (y \in B) \}$

Ex:

$A = \{1, 2\}$ $B = \{c, d\}$

$A \times B = \{ \langle 1, c \rangle, \langle 1, d \rangle, \langle 2, c \rangle, \langle 2, d \rangle \}$

$B \times A = \{ \langle c, 1 \rangle, \langle c, 2 \rangle, \langle d, 1 \rangle, \langle d, 2 \rangle \}$

Q if $A = \{ \alpha \beta \}$ & $B = \{1, 2, 3\}$ what are

$A \times B$, $B \times A$, $A \times A$, $B \times B$, $(A \times B) \cap (B \times A)$

Ans:

$A \times B = \{ \langle \alpha, 1 \rangle \langle \alpha, 2 \rangle, \langle \alpha, 3 \rangle, \langle \beta, 1 \rangle, \langle \beta, 2 \rangle \langle \beta, 3 \rangle \}$

$B \times A = \{ \langle 1, \alpha \rangle \langle 1, \beta \rangle, \langle 2, \alpha \rangle, \langle 2, \beta \rangle, \langle 3, \alpha \rangle, \langle 3, \beta \rangle \}$

$A \times A = \{ \langle \alpha, \beta \rangle, \langle \alpha, \alpha \rangle \langle \beta, \beta \rangle \langle \beta, \alpha \rangle \}$

$B \times B = \{ \langle 1,1 \rangle, \langle 1,2 \rangle, \langle 1,3 \rangle, \langle 2,1 \rangle, \langle 2,2 \rangle \langle 2,3 \rangle, \langle 3,1 \rangle, \langle 3,2 \rangle, \langle 3,3 \rangle \}$

$(A \times B) \cap (B \times A) = \phi$

Q If $A = \phi$ $B = \{1, 2, 3\}$ what are $(A \times B) \cap (B \times A)$

Ans:

$(A \times B) \cap (B \times A) = \phi$

Q Give example of a set A, B, C, such that $A \cup B = A \cup C$, but $B \neq C$

Ans:

$A = \{1, 2, 3\}$

$B = \{4, 5, 6\}$

$C = \{3, 4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$A \cup C = \{1, 2, 3, 4, 5, 6\}$

But $B \neq C$

Q Write the members of $\{\alpha, \beta\} \times \{1, 2, 3, 4\}$

Ans:

$= \{\langle\alpha, 1\rangle \langle\alpha, 2\rangle \langle\alpha, 3\rangle \langle\beta, 1\rangle, \langle\beta, 2\rangle, \langle\beta, 3\rangle, \langle\alpha, 4\rangle, \langle\beta, 4\rangle\}$

Q. Write $A \times B \times C$, B^2, A^3, $B^2 \times A$ & $A \times B$ where

$A = \{1\}$, $B = \{a, b\}$ $C = \{2, 3\}$

Ans:

$A \times B = \{\langle 1, a\rangle \langle 1, b\rangle\}$

$A \times B \times C = \{\langle 1, a, 2\rangle, \langle 1, a, 3\rangle, \langle 1, b, 2\rangle, \langle 1, b, 3\rangle\}$

$B^2 = \{\langle a, a\rangle \langle a, b\rangle \langle b, a\rangle \langle b, b\rangle\}$

$A^3 = \{\langle 1, 1, 1\rangle\}$

$B^2 \times A = \{\langle a, a, 1\rangle, \langle b, b, 1\rangle, \langle a, b, 1\rangle, \langle b, a, 1\rangle\}$

Q. Show by means of an example that $a \times b \neq b \times a$,

$(A \times B) \times C \neq A \times (B \times C)$

$A = \langle P, Q, R\rangle$

$B = \langle 1, 2\rangle$

$A \times B = \{\langle P, 1\rangle, \langle P, 2\rangle, \langle Q, 1\rangle, \langle Q, 2\rangle, \langle R, 1\rangle, \langle R, 2\rangle\}$

$B \times A = \{\langle 1, P\rangle, \langle 1, Q\rangle, \langle 1, R\rangle, \langle 2, P\rangle, \langle 2, Q\rangle, \langle 2, R\rangle\}$

$A \times B \neq B \times A$

$A = \{p, q, r\}$

$B = \{1, 2\}$

$C = \{\alpha, \beta\}$

$(A \times B) \times C = \{\langle P, 1, \alpha\rangle, \langle P, 1, \beta\rangle, \langle P, 2, \alpha\rangle, \langle P, 2, \beta\rangle, \langle Q, 1, \alpha\rangle, \langle Q, 1, \beta\rangle$
$\langle Q, 2, \alpha\rangle \langle Q, 2, \beta\rangle \langle R, 1, \alpha\rangle, \langle R, 1, \beta\rangle, \langle R, 2, \alpha\rangle, \langle R, 2, \beta\rangle\}$

$A \times (B \times C)$

$B \times C = \{\langle 1, \alpha\rangle \langle 1, \beta\rangle, \langle 2, \alpha\rangle, \langle 2, \beta\rangle\}$

$A \times (B \times C) = \{\langle P, 1, \alpha\rangle, \langle P, 1, \beta\rangle, \langle P, 2, \alpha\rangle, \langle p, 2, \beta\rangle$
$\langle q, 1, \alpha\rangle, \langle q, 1, \beta\rangle, \langle q, 2, \alpha\rangle, \langle q, 2, \beta\rangle$
$\langle r, 1, \alpha\rangle, \langle r, 1, \beta\rangle \langle r, 2, \alpha\rangle, \langle r, 2, \beta\rangle\}$

Show that $A\times(B\cap C) = (A\times B)\cap(A\times C)$

$A\times(B\cap C) = \{\langle x,y\rangle \mid (x\in A) \wedge (y\in B\cap C)\}$

$= \{\langle x,y\rangle \mid (x\in A)\wedge(y\in B \cap y\in C)\}$

$= \{\langle x,y\rangle \mid (x\in A \cap y\in B)\wedge(x\in A\cap y\in C)\}$

$= (A\times B)\cap(A\times C)$

True that $A\cap(\sim A\cup B) = A\cap B$

$A\cap(\sim A\cup B) = \{x \mid (x\in A)\wedge(x\in(\sim A\cup B))\}$

$= \{x\mid (x\in A)\wedge(x\in\sim A)\vee(x\in B)\}$

$= \{x \mid (x\in A)\wedge(x\in\sim A\vee(x\in A\wedge x\in B)\}$

$= \{x\mid x\in A \cap x\in B\}$

$= A\cap B$

True that $(A\cap B)\cup(A\cap\sim B) = A$

$(A\cap B)\cup(A\cap\sim B) = \{x\mid (x\in A\cup B) \vee (x\in A\cap\sim B)\}$

$= \{x \mid (x\in A)\vee(x\in B)\vee(x\in A)\wedge(x\in\sim B)\}$

$= \{x\mid (x\in A \vee (x\in A)\}$

$= \{x\mid (x\in A)\}$

$= A$

Relations :

Any SET of ordered pair defines bynary relations.

Ex :

$X = \{1, 2, 3, 4\}$

$R = \{ \langle x, y \rangle \mid (x \in X) \wedge (y \in Y) \wedge (x - y = 0) \}$

$R = \{ \langle 1,1 \rangle, \langle 2,2 \rangle, \langle 3,3 \rangle, \langle 4,4 \rangle$.

$X \times X = \{ \langle 1,1 \rangle \langle 2,1 \rangle, \langle 3,1 \rangle, \langle 4,1 \rangle, \langle 1,2 \rangle \langle 4,2 \rangle \langle 3,2 \rangle, \langle 4,2 \rangle$
$\langle 3,1 \rangle \langle 3,2 \rangle, \langle 3,3 \rangle \langle 3,4 \rangle, \langle 4,1 \rangle, \langle 4,2 \rangle, \langle 4,3 \rangle \langle 4,4 \rangle$

$R = \{ \langle 1,3 \rangle, \langle 2,4 \rangle, \langle 3,1 \rangle, \langle 4,2 \rangle \}$

Definition :

Let S be a Binary relation. The set $D(S)$ of all objects x such that for some y order pair $\langle x, y \rangle \in S$ is called the domain of S. That is

$D(S) = \{ x \mid (\exists y) \ \langle x, y \rangle \in S \}$

Similarly the set $R(S)$ of all objects y such that for some x order pair $\langle x, y \rangle \in S$ is called Range of S that is $R(S) = \{ y \mid (\exists x) \ \langle x, y \rangle \in S \}$

Problems :

$P = \{ \langle 1,2 \rangle, \langle 2,4 \rangle, \langle 3,3 \rangle \}$ and $Q = \{ \langle 1,3 \rangle, \langle 2,4 \rangle, \langle 4,2 \rangle \}$ find $P \cup Q$, $P \cap Q$, $D(P)$, $D(Q)$, $D(P \cup Q)$, $R(P)$, $R(Q)$ & $R(P \cap Q)$ show that

$D(P \cup Q) = D(P) \cup D(Q)$

$R(P \cap Q) \subseteq R(P) \cap R(Q)$

$P \cup Q = \{ \langle 1,2 \rangle, \langle 2,4 \rangle, \langle 3,3 \rangle \langle 1,3 \rangle, \langle 4,2 \rangle \}$

$P \cap Q = \{ \langle 2,4 \rangle \}$

$D(P) = \{ 1,2,3 \}$

$D(Q) = \{ 1,2,4 \}$

$D(P \cup Q) = \{ 1,2,3,4 \}$

$R(P) = \{ 2,4,3 \}$

$R(Q) = \{ 3,4,2 \}$

$R(P \cap Q) = \{ 4 \}$

$D(P \cup Q) = D(P) \cup D(Q)$

$= \{1,2,3\} \cup \{1,2,4\}$

$= \{1,2,3,4\}$

$R(P \cap Q) \subseteq R(P) \cap R(Q)$

$\langle 4 \rangle \leq \langle 2,4,3 \rangle \cap \langle 3,4,2 \rangle$

$\langle 4 \rangle \leq \langle 2,4,3 \rangle$

Properties of Binary Relations :

Reflexsive :

A binary relation R in a SET X is reflecsive for every $x \in X$, $\langle x,x \rangle \in R$, or R is reflexsive in $X \Leftrightarrow (x)\ (x \in X \rightarrow (x R x))$.

Symmetric Relation :

A relation are in X is symmetric for every x and y in X. whenever xRy then yRx that is R is symmetric in $X \Leftrightarrow (x)(y)\ (x \in X \wedge y \in X \wedge xRy \rightarrow yRx)$

Transitive :

A relation are in a SET x is transitive for every x, y & z x whenever xRy & yRz the xRz that is

R is transitive in X <-> (x)(y) (x ∈ X ∧ y ∈ X ∧ z ∈ X ∧ (xRy) ∧ yRz → xRz

---

X = { 1, 2, 3, 4 }

R = { <1,1>, <1,2>, <2,1>, <4,1>, <2,2>, <3,3>, <4,4> }

Refexsive : <1,1> <2,2> <3,3>, <4,4> ∈ R

Symmetry : <1,2> ∈ R → <2,1> ∈ R
<4,1> ∈ R → <1,4> ∈ R

Trasitive : <1,2> ∧ <2,1> ∈ R → <1,1> ∈ R
<4,1> ∧ <1,4> ∈ R → <4,4> ∈ R

" ≤ " This relation ≤ is reflexsive, transitive relation but not a symmetric relation.

Irreflexsive :

A relation are in a SET x is a irreflexsive. If for every x ∈ X ordered pair x, y ∈ R

X = { 1, 2, 3, 4 } 1 ≤ 2 ∧ 2 ≤ 3

R = { <1,1>, <1,2>, <2,1>, <4,1>, <2,2> }

This relation R is not A reflexsive or irreflexsive relationship.

Anti symmetric :

A relation are in a SET X is anty symmetric if for every x & y in X whenever $x \in y$ & $y \in x$ then $x = y$. symbolically R is anti symmetric in $X \leftrightarrow$

$(x)(y)\ (x \in y \land y \in x \land xRy \land yRx) \rightarrow x = y)$

Problem :

Show whether the following relations are transitive.

$R_1 = \{ <1,1> \}$

$R_2 = \{ <1,2>, <2,2> \}$

$R_3 = \{ <1,2>, <2,3>, <1,3>, <2,1> \}$

1st relation is transitive

2nd relation is transitive

3rd relation is not an transitive.

Relation Matrix and Graph of a relation :

A relation R from a finite SET X to a finite SET Y can also represented by a matrix called the relation matrix of R

Let $X = \{ x_1, x_2, \ldots\ldots, x_n \}$

$Y = \{ y_1, y_2, \ldots\ldots, y_n \}$

and R be a relation from X to Y the relation matrix of R can be obtained by first constructing a table whoose column are produced by column consisting of succesesive elements of X and whoose role are headed by a row consisting of the successive elements of y. If $x_i R y_j$ the we enter a 1 in the i^{th} row & j^{th} column. If $x_k \not R\, y_l$ then we enter zero in the k^{th} row and l^{th} column that is

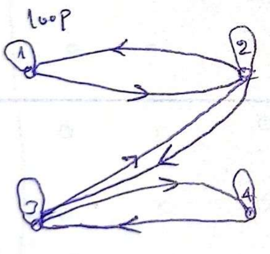
$$A_{ij} = \begin{cases} 1 & \text{i< } x_i R y_j \\ 0 & \text{i< } x_i \not R\, y_j \end{cases}$$

$R = \{ \langle 1,1\rangle, \langle 1,2\rangle, \langle 2,3\rangle, \langle 3,3\rangle, \langle 4,4\rangle, \langle 4,3\rangle, \langle 3,4\rangle, \langle 2,1\rangle, \langle 4,2\rangle, \langle 2,4\rangle \}$

Relation Matrix :

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Graph of a relation :



vertex or nods,

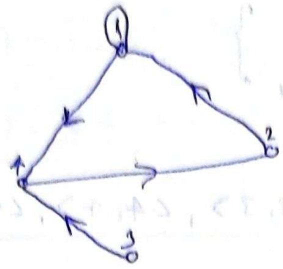
Definition :

A relation can also be represented pictorially by drawing its graph. Let R be a relation in a set X $X = \{ x_1, x_2, \ldots x_n \}$ the elements of X are represented by points or circle called nodes (or vertex). If $x_i R x_j$ that is $\langle x_i, x_j \rangle \in R$ then we connect nodes x_i and x_j by means of an ark and put an arrow on the ark in the direction from x_i to x_j

Ex: Draw the graph of the relation r

$X = \{1, 2, 3, 4\}$

$e = \{ \langle 1,1 \rangle, \langle 2,1 \rangle, \langle 1,4 \rangle, \langle 3,4 \rangle, \langle 4,2 \rangle \}$



$X = \{1, 2\}$ $\quad Y = \{3, 4\}$

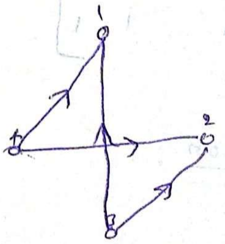
$R = \{ \langle x, y \rangle \mid (x-y)$ is an non negative integer $\}$

$R = \phi$

$X = \{3, 4\}$ $\quad Y = \{1, 2\}$

$R = \{ \langle 3,1 \rangle, \langle 4,2 \rangle, \langle 3,2 \rangle, \langle 4,1 \rangle$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$



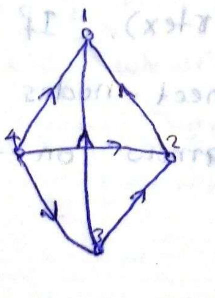
$X = \{1, 2, 3, 4\}$

$R = \{ \langle x, y \rangle \mid x > y \}$

Draw the graph of r and also give its matrix.

$R = \{ \langle 2,1 \rangle, \langle 3,1 \rangle, \langle 4,1 \rangle$

$\langle 3,2 \rangle, \langle 4,2 \rangle, \langle 4,3 \rangle \}$



$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$A = \{a, b, c\}$ and denote the subset a by $B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7$ according to the convension given below.

$B_0 = \phi$ $\qquad$ $B_4 = \{a\}$

$B_1 = \{c\}$ $\qquad$ $B_5 = \{a, c\}$

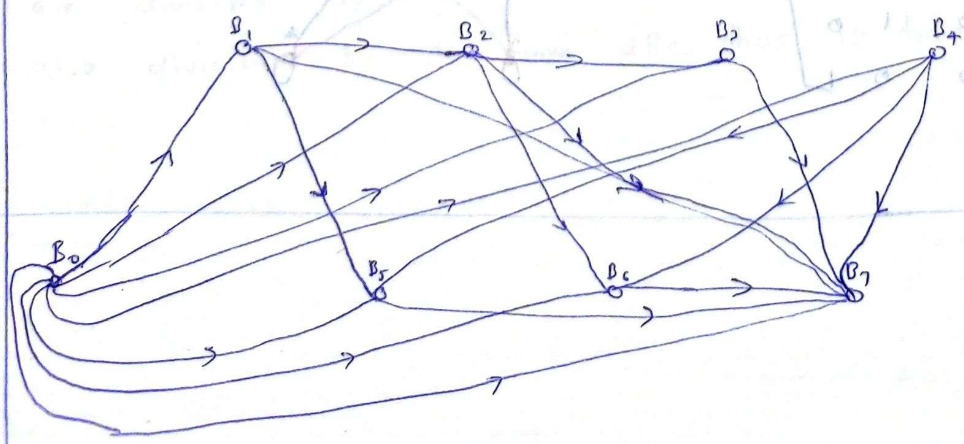
$B_2 = \{b\}$ $\qquad$ $B_6 = \{a, b\}$

$B_3 = \{b, c\}$ $\qquad$ $B_7 = \{a, b, c\}$

If R is the relation of proper inclusion on the subset $B_0 \ldots B_7$ then give the matrix of the relation also draw the graph.

$R = \{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$

$$\begin{array}{c|c}
 & \begin{matrix} B_0 & B_1 & B_2 & B_3 & B_4 & B_5 & B_6 & B_7 \end{matrix} \\
\begin{matrix} B_0 \\ B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \\ B_7 \end{matrix} &
\begin{bmatrix}
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}$$



Determine the properties of the relation given by the corresponding relation matrixes

a) $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

a) i) irresplexsive

b) i) reflexsive

c) i) reflexsive

d) i) Transitive.

Equilance Relation :

A relation R is a SET X is called an equialance relation. If it is reflexsive, symmetric, & transitive.

Ex: $X = \{1, 2, 3, 4\}$

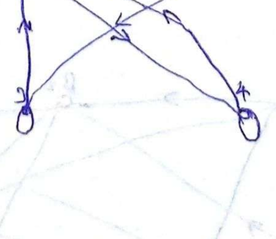
$R = \{\langle 1,1\rangle, \langle 1,4\rangle, \langle 4,1\rangle, \langle 4,4\rangle, \langle 2,2\rangle, \langle 3,1\rangle, \langle 2,3\rangle, \langle 3,3\rangle\}$

Write the matrix of R and sketch its graph.

Solution :

It is an equilance relation.

$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$



$X = \{ 1, 2, \ldots 7 \}$

$R = \{ \langle x, y \rangle \mid x - y$ is divisible by $3 \}$

Show that R is equialance relation draw the graph of R and matrix.

$R = \{ \langle 4, 4 \rangle, \langle 4, 1 \rangle, \langle 1, 7 \rangle, \langle 7, 1 \rangle, \langle 2, 5 \rangle, \langle 5, 2 \rangle, \langle 3, 6 \rangle$
$\langle 6, 3 \rangle, \langle 7, 4 \rangle, \langle 4, 7 \rangle, \langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle, \langle 4, 4 \rangle,$
$\langle 5, 5 \rangle, \langle 6, 6 \rangle, \langle 7, 7 \rangle,$

reflexsive : $3 / a - a \;\forall\; a \in X$ i.e aRa.

symmetric : $aRb \Rightarrow 3 | a - b \Rightarrow 3 | -(a-b) = 3 / a - \bar{a} \Rightarrow bRa$

Transitive : aRb & $bRc \Rightarrow 3 (a - b$ & $3 | b - c$

$a - c = a - b + b - c$

$3 / a - c$

$\Rightarrow aRc.$

Solution :

for any $a \in X$ $a - a$ is divisible by 3 therefore aRa or R is reflexsive.

for any a, b belongs to x if $a - b$ is divisible by 3 then $B - A$ also divisible by 3 that is aRb implies bRa or R is symmetric.

for $a, b, c \in X$ if aRb & bRc then both $a - b$ & $b - c$ are divisible by 3 so that $a - c = a - b + b + c$ is also divisible by 3 hense aRc thus is transitive.

Imp **Compatibility Relation :**

A relation R in X is said to be compatibility relation if it is reflexsive and symmetric.

Imp **Partially ordering :**

A binary relation R is a SET P is called a Partially order relation or partially order in P iff R is reflexsive, antisymmetric & transitive.

Composition of Binary relation

Let R be a relation from X to Y and S be a relation from Y to Z then a relation R o [composition] X is called composite relation of R & S

$$RoS = \{\langle x, z\rangle \mid x \in X \wedge z \in Z \wedge (\exists y) (y \in Y \wedge \langle x, y\rangle \in R \wedge \langle y, z\rangle \in S\}$$

The operation of obtaining RoS from R&S is called composition of relation.

Ex:

R = {<1,2>, <3,4>, <2,2>} & S = {<4,2>, <2,5>, <3,1>, <1,3>}

find RoS, SoR, Ro(SoR), (RoS)oR, RoR, SoS, RoRoR.

RoS (1) = R(1) = 2, S(2) = 5 RoS (3) = R(3) = 4, S(4) = 2 RoS (2) = R(2) = (2), S(2) = (5)

RoS = {<1,5>, <3,2>, <2,5>}

SoR(4) = S(4) = 2, R(2) = 2 SoR(2) = S(2) = 5, R(5) = ∅ SoR(3) = S(3) = 1, R(1) = 2 SoR(1) = S(1) = 3, R(3) = 4

SoR = {<4,2>, <3,2> <1,4>}

$Ro(SoR)$ = $R(1) = 2$ $R(3) = 4$ $R(2) = 2$

$SoR(2)$ = not defined $SoR(4) = 2$ $SoR(2)$ = not defined.

$Ro(SoR) = \{ \langle 3, 2 \rangle \}$

$(RoS)oR$ = $RoS(1) = 5$ $RoS(3) = 2$ $RoS(2) = 5$

$R(5) = 8$ $R(2) = 2$ $R(5)$ = not defined

$(RoS)oR = \{ \langle 3, 2 \rangle \}$

RoR $R(1) = 2$ $R(3) = 4$ $R(2) = 2$

$R(2) = 2$ $R(4) = 3$ $R(2) = 2$

$RoR = \{ \langle 1, 2 \rangle \ \langle 2, 2 \rangle \}$

SoS = $S(4) = 2$ $S(2) = 5$ $S(3) = 1$ $S(1) = 3$

$S(2) = 5$ $S(5) = 3$ $S(1) = 3$ $S(3) = 1$

$SoS = \{ \langle 4, 5 \rangle, \langle 3, 3 \rangle, \langle 1, 1 \rangle \}$

$(RoR)oR$ = $RoR(1) = 2$ $RoR(2) = 2$

$R(2) = 2$ $R(2) = 2$

$RoRoR = \{ \langle 1, 2 \rangle, \langle 2, 2 \rangle \}$

Let R & S be 2 relations on a positive integer

I = $R = \{ \langle x, 2x \rangle \mid x \in I \}$

$S = \{ \langle x, 7x \rangle \mid x \in I \}$

Find . RoR, $RoRoR$, RoS, $RoSoR$.

$RoS = \{ \langle x, 14x \rangle \mid x \in I \}$

$RoR = \{ \langle x, 4x \rangle \mid x \in I \}$

$RoRoR = \{ \langle x, 8x \rangle \mid x \in I \}$

$RoSoR = \{ \langle x, 28x \rangle \mid x \in I \}$

Imp

Give the relation matrixes M_R, M_S find M_{ROS}, $M_{\tilde{R}}$, $M_{\tilde{S}}$, M_R, M_S, $M_{\widetilde{ROS}}$, $M_{\widetilde{ROS}} = M_{\widetilde{SOR}}$

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Ans:

$$M_{ROS} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \qquad M_{\tilde{R}} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \qquad M_{\tilde{S}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$M_{\widetilde{ROS}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad M_{\tilde{R}O\tilde{S}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$\therefore M_{\widetilde{ROS}} = M_{\tilde{R}O\tilde{S}}$

Write the relation matrix M_R and M_S find M_{ROS} and M_{SOR}

Functions :

Definition :

Let X and Y be any 2 sets. A relation f from $X \to Y$ is called a function if for every $x \in X$ there is a unique $y \in Y$ such that $\langle x, y \rangle \in f$.

Here x is called argument and the corresponding y is called image of x under f.

Definition :

A maping f from $X \to Y$ is called onto (surjective) if the range $R_f = Y$ otherwise it is called into.

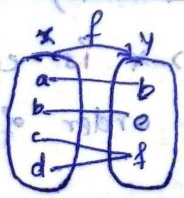
Definition :

A maping f from $X \to Y$ is called one-to-one (injective) (1-1) if distinct elements of X are maped into distinct element of Y. In other words f is one-to-one if $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

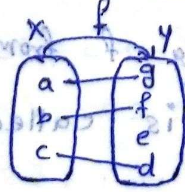
$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

Definition :

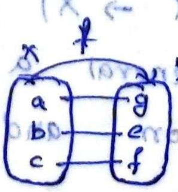
A maping f from $X \to Y$ is called one-to-one onto (bijective) if it is both one-to-one and onto. Such a maping is also called a one-to-one correspondance between X and Y.



onto
(nothing to left)



one to one



bijective
(satisfy both)

Inverse function :

A maping f from $X \to Y$ is a bijective function then $f^{-1} : Y \to X$ is called Inverse of a function f.

Identity function :

A maping I from $X \to X$ is called identity map or function. if $I = \{<x, x> | x \in X\}$

$I : X \to X$

Note :

If f from $X \to Y$ is invertible then f^{-1} of $I = fof^{-1}$

Problem :

Show that the function $f(x) = x^3$ $g(x) = x^{1/3}$ for $x \in \mathbb{R}$ are inverse of each other.

$(fog)(x) = f(g(x))$ | $(gof)(x) = g(f(x))$
$= f(x^{1/3})$ | $= g(x^3)$
$= (x^{1/3})^3$ | $= (x^3)^{1/3}$
$= x$ | $= x$

i.e $gof = I$

i.e $fog = I$

Hence f & g are inverse of each other.

Binary and n-ary Operations :

Let X be a set and f is a maping from $X \times X$ to X ($f : X \times X \to X$) then f is called a binary operation on X. In general a maping f from $X^n \to X$ is called n-ary operation and n is called the order of the operation.

for $n = 1$ f from $X \to X$ is called Uniary operation

Definition :

Characterstic function of A SET :

Let E be a universal SET and A be a subset of E the function $\psi_A : E \to \{0, 1\}$ defined by $\psi_A(x) = \begin{cases} 1 \text{ if } x \in A \\ 0 \text{ if } x \notin A \end{cases}$

is called the characterestic function of the x.

Definition :

Let us $f : X \to Y$ and $g : Y \to Z$ be 2 function the composite relation gof such that $gof = \{ \langle x, z \rangle \mid (x \in X \wedge z \in Z) \wedge (\exists y)$ (there exist) $(y \in Y \wedge y = f(x) \wedge z = g(y)) \}$

Problem

$X = \{1, 2, 3\}$, $Y = \{p, q\}$, $Z = \{a, b\}$ also let $f : X \to Y$ be $f = \{\langle 1, p \rangle, \langle 2, 1 \rangle, \langle 3, 2 \rangle\}$ $g : X \to Z$ be $g = \{\langle p, b \rangle, \langle q, b \rangle\}$

$gof, X \to Z$

$gof(1) = g(f(1)) = g(p) = b$

$gof(2) = g(f(2)) = g(q) = b$

$gof(3) = g(f(3)) = g(q) = b$

Let $X = \{1, 2, 3\}$ and f, g, h & s be a function from $X \to X$ given by

$f = \{\langle 1, 2 \rangle, \langle 2, 3 \rangle, \langle 3, 1 \rangle\}$

$g = \{\langle 1, 2 \rangle, \langle 2, 1 \rangle, \langle 3, 3 \rangle\}$

$h = \{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 1 \rangle\}$

$s = \{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle\}$ find fog, gof, foh, fohog, sog, gos, sos and fos.

$fog(1) = f(g) = 1 \quad g(2) = 3 \quad \langle 1,3 \rangle$

$fog(2) = f(g(2)) = 3 : f(3) = 1 \quad \langle 2,1 \rangle$

$fog(3) = f(g(3)) = 3 \quad f(3) = 1 \quad \langle 3,1 \rangle$

$fog = \{ \langle 1,3 \rangle, \langle 2,1 \rangle, \langle 3,1 \rangle \}$

$gof(1) = g(f(1)) = 2 \quad g(2) = 1 \quad = \langle 1,1 \rangle$

$gof(2) = g(f(2)) = 3 \quad g(3) = 3 \quad = \langle 2,3 \rangle$

$gof(3) = g(f(3)) = 1 \quad g(1) = 2 \quad = \langle 3,2 \rangle$

$gof = \{ \langle 1,1 \rangle \langle 2,3 \rangle, \langle 3,2 \rangle \}$

$foh(1) = f(h(1)) = 1 \quad f(1) = 2 \quad \langle 1,2 \rangle$

$foh(2) = f(h(2)) = 2 \quad f(2) = 3 \quad \langle 2,3 \rangle$

$foh(3) = f(h(3)) = 1 \quad f(1) = 2 \quad \langle 3,2 \rangle$

$foh = \{ \langle 1,2 \rangle, \langle 2,3 \rangle, \langle 3,2 \rangle \}$

$(foh)og(1) = (foh)(g(1)) = 2 \quad foh(2) = 3 \quad \langle 1,3 \rangle$

$fohog(2) = (foh)(g(2)) = 1 \quad foh(1) = 2 \quad \langle 2,2 \rangle$

$fohog(3) = (foh)(g(3)) = 1 \quad foh(1) = 2 \quad \langle 3,2 \rangle$

$fohog = \{ \langle 1,3 \rangle, \langle 2,3 \rangle, \langle 3,1 \rangle \}$

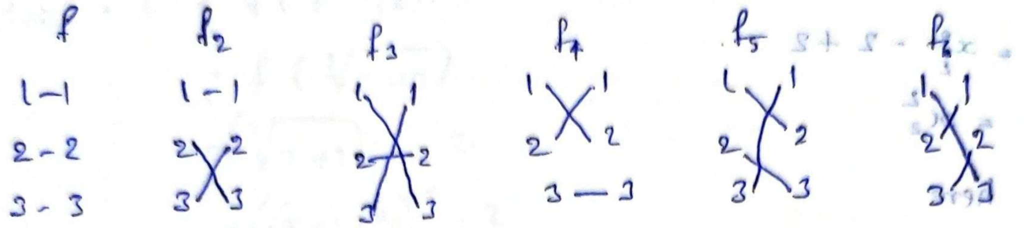
$sog = \{ \langle 1,2 \rangle, \langle 2,1 \rangle, \langle 3,5 \rangle \}$

$gos = \{ \langle 1,2 \rangle, \langle 2,1 \rangle, \langle 3,3 \rangle \}$

$sos = \{ \langle 1,1 \rangle, \langle 2,2 \rangle, \langle 3,3 \rangle \}$

$fos = \{ \langle 1,2 \rangle, \langle 2,3 \rangle, \langle 3,1 \rangle \}$

Let F_2 be the set of all $1 \to 1$ maping from X onto X where $X = \{1,2,3\}$ find all the elements of F_2 and find inverse of each element.



o	f_1	f_2	f_3	f_4	f_5	f_6
f_1	f_1	f_2	f_3	f_4	f_5	f_6
f_2	f_2	f_1	f_5			
f_3	f_3					
f_4	f_4					
f_5	f_5					
f_6	f_6					

Let $f : R \to R$ and $G : R \to R$ where R is a set of real numbers find fog and gof where $f(x) = x^2 - 2$ and $g(x) = x + 4$ state whether these function are injective, sujective and bijective.

$fog(x) = f(g(x)) = x+4 = (x+4)^2 - 2$

$= x^2 + 16 + 8x - 2$

$= x^2 + 14 + 8x$

$gof(x) = g(f(x)) = g(x^2 - 2)$

$= x^2 - 2 + 4$

$= x^2 + 2$

$f(x_1) = f(x_2)$

$f(x) = x^2 - 2$ $\quad$ $f(x_1) = x_1^2 - 2$

$f(x_1) = f(x_2)$ $\quad$ $f(x_2) = x_2^2 - 2$

$x_1^2 = x_2^2 - 2 + 2$

$x_1^2 = \cancel{x_2^2}$

but here

$x_1 \neq x_2$ in general

not injective

$f(x) = x^2 - 2$

$(-2\ \infty)$

$(-\infty, \infty)$

is not a sujective function

Therefore $f(x)$ is not a byjective function

$g(x) = x + 4$

$g(x_1) = g(x_2)$

$x_1 + 4 = x_4 + 4$

$x_1 = x_2$

g is injective

$g : R - R$

Here g is a sujective function

Therefore g is byjective.

Problem

F: R → R given by $f(x) = x^3 - 2$ find f inverse

$f(x) = x^3 - 2$ $\qquad f^{-1}{}_o f = \sqrt[3]{x+2}$

$f \circ f^{-1} = I = f(f^{-1}(x))$

$= f(\sqrt[3]{x+2})$

$= (\sqrt[3]{x+2})^3 - 2$

$= ((x+2)^{1/3})^3 - 2$

$= x + 2 - 2$

$= x$

$f^{-1}{}_o f(x) = f^{-1}(f(x))$

$= f^{-1}(x^3 - 2)$

$= \sqrt[3]{x^3 - 2}$

$= \sqrt[3]{x^3 \cancel{-2} \cancel{+2}}$

$= \sqrt[3]{x^3} = (x^3)^3$

$= \underline{x}$

Therefore inverse of 'x is $\sqrt[3]{x+2}$