Influence Examples

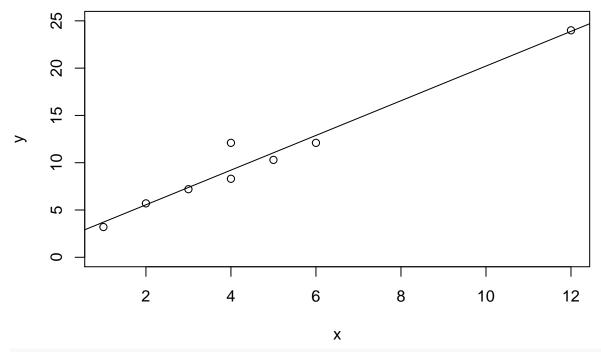
In this example we use two small data sets to illustrate influence diagnostics.

The primary influence diagnostic is Cook's distance. We can get Cook's D and several other diagnostics using influence.measures(). A common rule of thumb says Cook'D > 1 (or > 0.5) indicated an influential observation.

Using the plot() function we can get diagnostic plots including Cook's distance and leverage.

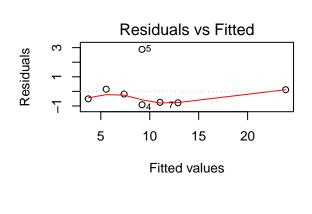
Example 1

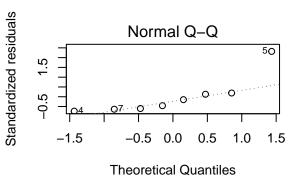
```
library(MASS)
library(car)
## Loading required package: carData
Ex1 <- read.csv("~/Dropbox/STAT512/Lectures/MultReg4/MR4_Influence1.csv")</pre>
##
           У
## 1 1 3.2
## 2 2 5.7
## 3 3 7.2
     4 8.3
## 4
## 5 4 12.1
## 6 5 10.3
## 7 6 12.1
## 8 12 24.0
Model1 <- lm(y \sim x, data = Ex1)
plot(y \sim x, ylim = c(0,25), data = Ex1)
abline(Model1)
```

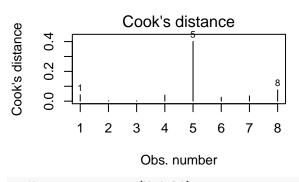


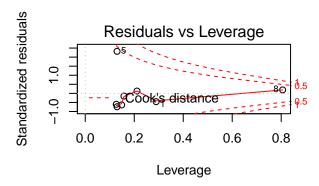
summary(Model1)

```
##
## Call:
## lm(formula = y ~ x, data = Ex1)
##
## Residuals:
##
      Min
               1Q Median
                               ЗQ
                                      Max
## -0.9163 -0.7587 -0.3484 0.1220 2.8837
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                1.8804
                           0.8357
                                     2.25
                                            0.0654 .
## x
                1.8340
                           0.1492
                                    12.29 1.77e-05 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.333 on 6 degrees of freedom
## Multiple R-squared: 0.9618, Adjusted R-squared: 0.9554
## F-statistic: 151.1 on 1 and 6 DF, p-value: 1.766e-05
par(mfrow=c(2,2))
plot(Model1, which = c(1:2,4:5))
```









influence.measures(Model1)

```
## Influence measures of
##
     lm(formula = y \sim x, data = Ex1) :
##
##
      dfb.1
               dfb.x
                       dffit cov.r cook.d
## 1 -0.2696  0.2047 -0.2715  1.8877  0.04268  0.290
     0.0583 -0.0387  0.0606  1.8157  0.00220  0.211
## 3 -0.0519 0.0270 -0.0591 1.6977 0.00208 0.158
## 4 -0.1944 0.0529 -0.2725 1.3691 0.04051 0.130
## 5 1.8096 -0.4920 2.5358 0.0179 0.40126 0.130
## 6 -0.1000 -0.0255 -0.2161 1.4558 0.02632 0.127
## 7 -0.0473 -0.1005 -0.2518 1.4701 0.03548 0.149
## 8 -0.1911 0.3272 0.3560 7.3309 0.07558 0.806
```

Resids1 <- data.frame(RawRes = resid(Model1), StdRes = stdres(Model1), RStdRes = rstudent(Model1))
Resids1</pre>

```
## RawRes StdRes RStdRes
## 1 -0.5143975 -0.4577025 -0.4253142
## 2 0.1516432 0.1280619 0.1170641
## 3 -0.1823161 -0.1490206 -0.1362890
## 4 -0.9162754 -0.7367175 -0.7051787
## 5 2.8837246 2.3186154 6.5631678
## 6 -0.7502347 -0.6021330 -0.5670699
## 7 -0.7841941 -0.6374358 -0.6026606
## 8 0.1120501 0.1907716 0.1746804
```

Example 2

(Intercept)

x

5.5049

0.7260

```
Ex2 <- read.csv("~/Dropbox/STAT512/Lectures/MultReg4/MR4_Influence2.csv", header=TRUE)</pre>
Ex2
##
      Х
        3.2
## 1
     1
## 2
      2 5.7
## 3 3 7.2
## 4 4 8.3
## 5 4 12.1
## 6 5 10.3
## 7 6 12.1
## 8 12 12.0
Model2 \leftarrow lm(y \sim x, data = Ex2)
plot(y \sim x, ylim = c(0,25), data = Ex2)
abline(Model2)
     20
     15
                                0
                                            0
                                                                                   0
     10
                                      0
                   0
     2
            0
     0
                   2
                                4
                                            6
                                                         8
                                                                     10
                                                                                  12
                                               Χ
summary(Model2)
##
## Call:
## lm(formula = y ~ x, data = Ex2)
##
## Residuals:
       Min
                1Q Median
                                 ЗQ
                                        Max
## -3.0308 -1.4968 -0.2958 1.4338 3.6912
##
## Coefficients:
```

0.0115 *

0.0379 *

3.590

2.652

Estimate Std. Error t value Pr(>|t|)

1.5335

0.2738

```
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 2.447 on 6 degrees of freedom
## Multiple R-squared: 0.5396, Adjusted R-squared: 0.4628
## F-statistic: 7.032 on 1 and 6 DF, p-value: 0.03794
par(mfrow=c(2,2))
plot(Model2, which = c(1:2,4:5))
                                                 Standardized residuals
                Residuals vs Fitted
                                                                    Normal Q-Q
                                                      \alpha
Residuals
                                                                                   0....
     2
                                                      0
     7
                                                                  01
                                                      7
                 8
                        10
                                12
         6
                                        14
                                                         -1.5
                                                                    -0.5
                                                                         0.0
                                                                               0.5
                                                                                     1.0
                                                                                          1.5
                    Fitted values
                                                                 Theoretical Quantiles
                                                 Standardized residuals
                 Cook's distance
                                                               Residuals vs Leverage
Cook's distance
                                                      ^{\circ}
                                                                05
8
     ω
                                                                                             0.5
                                                      0
     4
                                                                                             9.5
                                                                  Cooles distance
                                                      ņ
                                                                                          gΘ
     0
          1
               2
                   3
                            5
                                6
                                     7
                                         8
                                                          0.0
                                                                  0.2
                                                                          0.4
                                                                                  0.6
                                                                                         8.0
                    Obs. number
                                                                       Leverage
influence.measures (Model2)
## Influence measures of
     lm(formula = y \sim x, data = Ex2) :
##
##
                          dffit cov.r
##
      dfb.1_{-}
                 dfb.x
                                          cook.d
## 1 -1.0630 0.80687 -1.0704 0.830 0.440003 0.290
## 2 -0.2704  0.17969 -0.2812  1.628  0.044800  0.211
## 3 -0.0751 0.03905 -0.0854 1.684 0.004340 0.158
## 4 -0.0120  0.00326 -0.0168  1.654  0.000169  0.130
     0.5420 -0.14737
                        0.7595 0.527 0.195229 0.130
     0.0839 0.02136
                        0.1812 1.509 0.018851 0.127
## 7 0.0777 0.16512 0.4138 1.182 0.085904 0.149
## 8 3.7803 -6.47311 -7.0422 0.646 8.782265 0.806
Resids2 <- data.frame(RawRes = resid(Model2), StdRes = stdres(Model2), RStdRes = rstudent(Model2))
Resids2
##
         RawRes
                       StdRes
                                   RStdRes
## 1 -3.0308294 -1.46955159 -1.67679736
```

2 -1.2568075 -0.57836868 -0.54333921

```
## 3 -0.4827856 -0.21503762 -0.19706243
## 4 -0.1087637 -0.04765383 -0.04351003
## 5 3.6912363 1.61728188 1.96575742
## 6 1.1652582 0.50963159 0.47563628
## 7 2.2392801 0.99188254 0.99028264
## 8 -2.2165884 -2.05648471 -3.45554911
#Fit the model with Obs 8 omitted
Model3 <- update(Model2, subset = -8)</pre>
summary(Model3)
##
## Call:
## lm(formula = y \sim x, data = Ex2, subset = -8)
##
## Residuals:
##
                            3
         1
                                              5
## -0.63548 0.08387 -0.19677 -0.87742 2.92258 -0.65806 -0.63871
##
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                  2.055
                           1.353 1.519 0.18920
                             0.346 5.147 0.00362 **
## x
                  1.781
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.456 on 5 degrees of freedom
## Multiple R-squared: 0.8412, Adjusted R-squared: 0.8095
## F-statistic: 26.49 on 1 and 5 DF, p-value: 0.003625
beta0hat <- Model3$coef[[1]]</pre>
beta1hat <- Model3$coef[[2]]</pre>
beta0hat; beta1hat
## [1] 2.054839
## [1] 1.780645
lht(Model2, rbind(c(1,0), c(0,1)), rhs=c(betaOhat, betaIhat))
## Linear hypothesis test
##
## Hypothesis:
## (Intercept) = 2.05483870967742
## x = 1.78064516129032
##
## Model 1: restricted model
## Model 2: y ~ x
##
##
   Res.Df
                RSS Df Sum of Sq
                                    F Pr(>F)
## 1
         8 141.077
## 2
         6 35.921 2
                       105.16 8.7823 0.01651 *
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
par(mfrow=c(1,1))
plot(y \sim x, ylim = c(0,25), data = Ex2)
```

```
abline(Model2, lty = 1)
abline(Model3, lty = 2)
legend("topleft", lty = c(1,2), c("All Data", "Remove Obs #8"))
```

