

Design Matrix Example (For Illustration)

We look at the design matrix in the context of simple linear regression. We start with the standard analysis in R and get the estimated coefficients.

```
Corn <- read.csv("~/Dropbox/STAT512/Lectures/R_Stuff & Intro/Corn.csv")
Corn
```

```
##      Yield X
## 1      12 2
## 2      13 2
## 3      13 3
## 4      14 3
## 5      15 4
## 6      15 4
## 7      14 5
## 8      16 5
## 9      17 6
## 10     18 6
```

```
Fit <- lm(Yield ~ X, data = Corn)
summary(Fit)
```

```
##
## Call:
## lm(formula = Yield ~ X, data = Corn)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.8500 -0.3000  0.2250  0.4125  1.0000
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  10.1000    0.7973   12.67 1.42e-06 ***
## X            1.1500    0.1879    6.12 0.000283 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8404 on 8 degrees of freedom
## Multiple R-squared:  0.824, Adjusted R-squared:  0.802
## F-statistic: 37.45 on 1 and 8 DF, p-value: 0.0002832
```

fertilizer

$H_0: \beta_1 = 0$

Now we look at the design matrix and estimate the coefficients “by hand”. `solve()` takes the inverse of a matrix. `t()` transposes a matrix. `%*%` is used for matrix multiplication.

```
model.matrix(Fit)
```

```
##      (Intercept) X
## 1              1 2
## 2              1 2
## 3              1 3
## 4              1 3
## 5              1 4
## 6              1 4
## 7              1 5
```

```
## 8          1 5
## 9          1 6
## 10         1 6
## attr("assign")
## [1] 0 1

X <- model.matrix(Fit)
is.matrix(X)

## [1] TRUE

Y <- Corn$Yield
BetaHat <- solve(t(X)%*(X))%*t(X)%*Y
BetaHat
```

[,1]

(Intercept) 10.10 - β_0

X 1.15 - β_1