Date: Statistics - Assignment 1

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Q1) Given: I agent → # of customers per hour
II agent → Time of arrival
III agent → Whether the product was sold

(a) (i) From the givendata, I can figure out that the data reported by the first agent is discrete and it must be a poisson random Variable.

It consists of counting of no. of times a Certain event occurs during a given unit of time. The mean or expected no. of events is denoted by λ.

$$P(x=j|\lambda) = \frac{e^{-\lambda}\cdot\lambda^{j}}{j!}, \quad j = 0, 1, 2, \dots, \lambda > 0$$

$$E(x) = Var(x) = \lambda$$

(ii) The type of data reported by the second agent is Continuous and it must be a exponential random Variable.

It describes the time between events in a poisson point process. (i.e.,) a process in which events occur continuously and independently at a constant average rate.

$$f(x|\lambda) = \lambda e^{-\lambda x}, \quad x \geq 0, \lambda > 0; \quad E(x) = 1/\lambda, \quad Var(x) = 1/\lambda^2$$

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For an exponential distribution with $\lambda = 0.01$ and $x \leq 2$

$$p(x \leq 2) = \int_0^2 \frac{1}{100} e^{(-1/\cancel{100}_{50})(\cancel{2})} dx$$

$$= \int_0^2 \frac{1}{100} e^{-1/50} dx$$

$$= \left[e^t\right]_{-1/50}^{0}$$

$$= e^0 - e^{-1/50}$$

$$= 1 - e^{-1/50}$$

$$= 1 - 0.9801$$

$$\therefore p(x \leq 2) = 0.0198 //$$

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(iii) The third agent reports a type of data that is discrete and is a binomial random variable.

This is because it satisfies the condition that there can be only two possible outcomes on each trial for 'n' no. of events.

$$p(x=j\,|\,p) = {}^{n}C_{j}\, p^{j}(1-p)^{n-j}, \quad j=0,\dots,n,$$
$$p \in (0,1)$$

$$E(x) = np \quad ; \quad Var(x) = np(1-p)$$

Q1) b) Given data: $n = 10$

$p = 0.8$

To find: $p(x>1) = ?$

Sol: $p(x>1) = 1 - p(x \le 1)$

$= 1 - [p(x=0) + p(x=1)]$

$$= 1 - \left[\binom{10}{0}(0.8)^0(0.2)^{10} + \binom{10}{1}(0.8)^1(0.2)^9\right]$$

$$= 1 - \left[\frac{10!}{0!\,10!}(1)(0.2)^{10} + \frac{10!}{1!\,9!}(0.8)(0.2)^9\right]$$

$$= 1 - \left[(0.2)^{10} + 8(0.2)^9\right]$$

$$p(x>1) = 0.9999958$$

q2) Given data: Beta $(2, 4)$

$\therefore \alpha = 2$; $\beta = 4$.

To find: $E(x) = ?$; $Var(x) = ?$

$p(x \geq 2) = ?$

$E(x) = \alpha / (\alpha + \beta) = 2/(2+4) = 2/6 = 0.33$

The expectation of the security of this sensor is 33% (0.33)

$$Var(x) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{2 \times 4}{(2+4)^2(2+4+1)}$$

$$Var(x) = 8/(6)^2(7) = 0.0317$$

It's given that the network is stable if at least 2 sensors are secured out of the 5 sensors. As per the hint, we have to find the $p(x \geq 2)$ using binomial distribution. where, $n = 5$ and $p = E(x) = 0.33$

$$p(x \geq 2) = 1 - p(x < 2)$$

$$= 1 - [p(x=0) + p(x=1)]$$

$$= 1 - \left[\binom{5}{0}(0.33)^0(0.67)^5 + \binom{5}{1}(0.33)^1(0.67)^4\right]$$

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$$p(x \geq 2) = 1 - \left[\frac{5!}{0!\,5!} (1)(0.67)^5 + \frac{5!}{1!\,4!} (0.33) \cdot (0.67)^4 \right]$$

$$= 1 - \left[(0.67)^5 + 5(0.33)(0.67)^4 \right]$$

$$= 1 - \left[0.13501 + 0.33249 \right]$$

$$= 1 - 0.46750$$

$$\therefore p(x \geq 2) = 0.53249.$$

The probability of the security of network is 53%

Q3) Given data: Normal distribution

$\mu = 500$; $\sigma = 100$

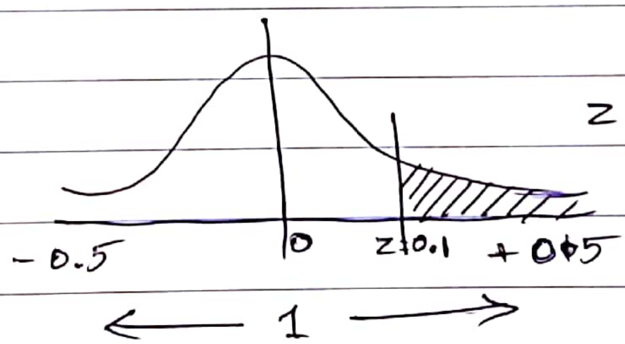
$\sigma^2 = 10,000$

To find: $p(x > 510) = ?$

$$p(x > 510) = p(z > ?)$$

$$z = \frac{x - \mu}{\sigma} = \frac{510 - 500}{100} = 0.1$$

$$\therefore \; p(x > 510) = p(z > 0.1)$$



$z(0.1) = 0.5398$

↳ from z-table for left tail.

we can find it by substracting the area ~~from~~ from -0.5 to 0.1 from 1.

i.e., $p(x > 510) = 1 -$ area from -0.5 to 0.1

$$= 1 - 0.5398$$

$$\therefore \; p(x > 510) = 0.4602$$

q 4) It's given that, of 4 features extracted, we assume that the probability of extraction of each feature is same,

(i.e,) $p = 1/4 = 0.25$

a) Given data : $n = 10$; ~~8/10~~
To find : $p(x=8)$; $\forall, j = 8$

using binomial distribution, we find $p(x=8)$

$$p(x=8) = {}^{10}C_{8}\,(0.25)^8\,(0.75)^2$$

$$= \frac{10!}{8!\,2!}\,(0.25)^8\,(0.75)^2$$

$$= \frac{\overset{5}{\cancel{10}} \times 9 \times \cancel{8!}}{\cancel{2} \times \cancel{8!}}\,(0.25)^8\,(0.75)^2$$

$$= 45 \times 0.25^8 \times 0.75^2$$

$$p(x=8) = 0.000386$$

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Q4) b) To find : $Cor(x_1, x_3)$

Given data : $n = 10$, $P_1 = P_2 = P_3 = P_4 = 0.25$

Sol: $$Cor(x_1, x_3) = \frac{Cov(x_1, x_3)}{Sd(x_1) \cdot Sd(x_3)}$$

where, $$Cov(x_1, x_3) = E(x_1 x_3) - E(x_1) E(x_3)$$

$$= \frac{-P_1 \cdot P_3}{\sqrt{P_1(1-P_1) \cdot P_3(1-P_3)}}$$

$$= \frac{-(0.25)(0.25)}{\sqrt{(0.25)(0.75)(0.25)(0.75)}}$$

$$= \frac{-0.625}{\sqrt{0.03515625}}$$

$$= -0.625 / 0.1875$$

$$\therefore Cov(x_1, x_3) = -0.3333$$

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$$Sd(x_1) = \sqrt{np(1-p)}$$

$$= \sqrt{10 \times 0.25 \times 0.75}$$

$$Sd(x_1) = 1.369306$$

$$|||ly \quad Sd(x_3) = 1.369306$$

$$\therefore Cor(x_1, x_3) = \frac{Cov(x_1, x_3)}{Sd(x_1) \cdot Sd(x_3)}$$

$$= \frac{-0.3333}{(1.369)(1.369)}$$

$$\therefore Cor(x_1, x_3) = -1.7778$$

The Correlation between the first and third feauture is -1.7778.