Date: 26 - 04 - 2018
Day

Vignesh Muthumani

Statistics - CA2. (10385771)

Q1) Using the dataset - CA2, testing if the proportion of males is equal to proportion of female.

Step 1 : $H_o : P_m = P_f = 0.5$
$H_a : P_m \neq P_f \neq 0.5$

where, P_m - prop. of male
P_f - prop. of female.

from our dataset, we can see that, $n = 27$

Step 2 : Assuming $\alpha = 0.05$

Step 3 : finding t- Value.

$$t\text{- Value} = \frac{\hat{P} - P_o}{\sqrt{\frac{P_o(1-P_o)}{n}}}$$

where, $\hat{P} = \frac{\text{no. of males}}{n} = \frac{14}{27}$

$\therefore \hat{P} = 0.5185$
and $P_o = 0.5$

Date:
Day

t-Value = $(0.5185 - 0.50) / \sqrt{0.5(1-0.5)/27}$

$= 0.0185 / \sqrt{0.5(0.5)/27}$

$\therefore$ t-Value = 0.1922

Step 4 : Finding C-Value.

C-Value = $Z_{\alpha/2}$ (Since the population is two-tail)

$\therefore$ ~~C-Vaclebe~~ = $Z_{0.025}$

$\therefore$ C-Value = 1.96 (from z-table)

Step 5 : Decision making :

Since, t-Value < C-Value,
(0.1922) < 1.96

We accept H_0 (i.e., $P_0 = 0.5$; $P_m = P_f$)

So, the proportion of males is equal to the proportion of females.

Date:

Day

Q3) Given: $x_1, x_2, \ldots, x_{10} \sim Bin(5, p)$

where, $n = 5$

a) Finding likelihood function:

$$L(p \mid x_1, x_2, \ldots, x_{10}) = \prod_{i=1}^{10} \binom{5}{x_i} p^{x_i} \cdot (1-p)^{5-x_i}$$

$$= \prod_{i=1}^{10} \binom{5}{x_i} \cdot p^{\sum_{i=1}^{10} x_i} \cdot (1-p)^{50 - \sum_{i=1}^{10} x_i}$$

b) Conjugate prior:

$$p(p) = \frac{p^{(\alpha-1)} \cdot (1-p)^{(\beta-1)}}{\frac{\Gamma(\alpha)\,\Gamma(\beta)}{\Gamma(\alpha+\beta)}}$$

Since the denominator of the above equation is independent of 'p', we are neglecting it.

$$\therefore\ p(p) = p^{(\alpha-1)} \cdot (1-p)^{(\beta-1)}$$

Date:
Day

c) Posterior distribution of p:

$$p(p|x_1, x_2, \dots, x_{10}) = \text{Beta}(\alpha', \beta')$$

$$p(p|x_1, \dots, x_{10}) \propto p(p) \times L(p|x_1, x_2, \dots, x_{10})$$

$$\propto p^{(\alpha-1)} \cdot (1-p)^{(\beta-1)} \times \left[\prod_{i=1}^{10} \binom{5}{x_i} \cdot p^{\sum_{i=1}^{10} x_i} \cdot (1-p)^{50-\sum_{i=1}^{10} x_i}\right]$$

$$p(p|x_1, \dots x_{10}) \propto p^{\sum_{i=1}^{10} x_i + \alpha + 1} \cdot (1-p)^{\beta - \sum_{i=1}^{10} x_i + 49}$$

We are neglecting the $\left(\prod_{i=1}^{10} \binom{5}{x_i}\right)$ as it's independent of 'p'.

from the above eqn, we can see the α' & β'

$$\alpha' = \sum_{i=1}^{10} x_i + \alpha \quad ; \quad \beta' = \beta - \sum_{i=1}^{10} x_i + 50$$

d) Computing minimum Bayesian risk estimator:

$$E(p|x_1, x_2, \dots, x_{10}) = \frac{\alpha'}{\alpha' + \beta'}$$

$$\therefore E(p|x_1, \dots x_{10}) = \frac{\sum_{i=1}^{10} x_i + \alpha}{\sum_{i=1}^{10} x_i + \alpha + \beta - \sum_{i=1}^{10} x_i + 50} = \frac{\sum_{i=1}^{10} x_i + \alpha}{\alpha + \beta + 50}$$