$$L(\theta, x, z) = \begin{cases} \pi\left(p^h (1-p)^{m-h}\right) & z=0 \\ (1-\pi)\left(q^h (1-q)^{m-h}\right) & z=1 \end{cases}$$

Let $A = p^h (1-p)^{m-h}$, $B = q^h (1-q)^{m-h}$

$$P_{z|x} = \frac{P_{xz}}{P_x} = \frac{\begin{cases} \pi A & z=0 \\ (1-\pi)B & z=1 \end{cases}}{\pi A + (1-\pi)B}$$

$$Q(\theta|\theta^t) = E\left[L(\theta, x, z) \mid X\right]$$

$$= \sum_{i=1}^{n} L(\theta, x_i, 0) P_{z_i|x_i}(0|x_i) + L(\theta, x_i, 1) P_{z_i|x_i}(1|x_i)$$

$$= \sum_{i=1}^{n} \log\left(\pi p^{h_i} (1-p)^{m-h_i} \cdot \frac{\pi^t p_t^{h_i} (1-p_t)^{m-h_i}}{\pi^t p_t^{h_i} (1-p_t)^{m-h_i} + (1-\pi) q_t^{h_i} (1-q_t)^{m-h_i}}\right)$$

$$+ \log\left(\cdots \cdot \cdots\right)$$

k_i^t

$$\therefore Q(\theta|\theta^t) = \sum_{i=1}^{n} k_i^t \log \pi p^{h_i}(1-p)^{m-h_i} + \sum_{i=1}^{n} (1-k_i^t) \log (1-\pi) q^{h_i}(1-q)^{m-h_i}$$

$$= \sum_i k_i^t \log \pi + \sum_i (1-k_i^t) \log(1-\pi)$$

$$+ \sum_i k_i^t h_i \log p + \sum_i k_i^t (m-h_i) \log(1-p)$$

$$+ \sum_i (1-k_i^t) h_i \log q + \sum_i (1-k_i^t)(m-h_i) \log(1-q)$$

$$\frac{\partial Q}{\partial \pi} = 0 \Rightarrow \sum k_i^t (1-\pi) - \sum_i (1-k_i^t)\pi = 0$$

$$\Rightarrow \pi = \frac{\sum k_i^t}{n} = \pi^{t+1}$$

$$\frac{\partial Q}{\partial p} = 0 \Rightarrow \frac{\sum k_i^t h_i}{p} - \frac{\sum k_i^t (m-h_i)}{1-p} = 0$$

$$\Rightarrow \sum k_i^t h_i \cancel{- p \sum k_i^t h_i} \cancel{+ p \sum k_i^t h_i}$$

$$- p m \sum k_i^t = 0$$

$$\Rightarrow p = \frac{\sum k_i^t h_i}{m \sum k_i^t} = p^{t+1}$$

$$\Rightarrow q^{t+1} = \frac{\sum (1-k_i^t) h_i}{m \sum (1-k_i^t)}$$

With Prior :

$$Q(\theta|\theta^t) = \sum_i k_i^t \log\pi + \sum_i (1-k_i^t)\log(1-\pi)$$

$$+ \sum_i k_i^t h_i \log p + \sum_i k_i^t (m-h_i)\log(1-p)$$

$$+ \sum_i (1-k_i^t) h_i \log q + \sum_i (1-k_i^t)(m-h_i)\log(1-q)$$

$$+ \log \frac{\pi^{\alpha-1}(1-\pi)^{\beta-1}}{B(\alpha,\beta)}$$

$\frac{\partial Q}{\partial p}$, $\frac{\partial Q}{\partial q}$ are same.

$$\frac{\partial Q}{\partial \pi} = 0 \Rightarrow \frac{\sum k_i^t}{\pi} - \frac{\sum(1-k_i^t)}{1-\pi}$$

$$+ \frac{\alpha-1}{\pi} - \frac{\beta-1}{1-\pi} = 0$$

$$\Rightarrow (1-\pi)\left(\sum k_i^t + \alpha - 1\right)$$

$$- \pi\left(\sum(1-k_i^t) + \beta - 1\right) = 0$$

$$\Rightarrow \sum k_i^t + \alpha - 1 = \pi\left(m + \alpha + \beta - 2\right)$$

$$\Rightarrow \pi = \frac{\alpha - 1 + \sum k_i^t}{m + \alpha + \beta - 2} = \pi^{t+1}$$