EE5111 : Estimation Theory

Vignesh S

EE16B127

Take home exam

Bayesian approach to stock trading

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Contents

1 Abstract

In this report, we take a look at the underlying mathematics in the daily activities of investors, i.e. trading of stocks, and the strategies involved during the process. In particular, we try to model the task and the factors involved using a Bayesian inference approach. Along with this, we also look at a unique scenario of trading stocks in a once in a 100-year event under specific constraints.

2 Influencing factors and model

A stock represents the shares of the company by which an investor can gain fractional ownership in the company. The price of any stock varies based on a number of factors, past performance, future promise to name a few. These variations are kept track by the stock market, a place where investors trade their stocks. Investors when they buy stocks, typically hope to make revenue by 2 means, firstly by the dividends which the company pays them periodically from the profit it makes, and secondly from the profit margin they obtain by selling their stocks at a higher price from what they originally bought. Since the stock market varies continuously, investors monitor the stock movements carefully and strategize their moves to capitalize on the situation. Among the many strategies that are used, we look at the Bayesian inference approach which allows us to update our beliefs regularly based on the stock market trends. Below we list several factors that influence our decision and categorize them into prior knowledge (estimated from past knowledge) and evidence (determines the likelihood).

2.1 Buying

- **Future promise** : *as prior*
  Being in the market for many years, we would have seen situations where a well-established company has increased its share values over the years, credit to the wise decisions made by the company (For eg. Reliance increasing its share prices by collaborating with Facebook and Google). We would have also seen new companies which go onto make significant progress due to their innovative ideas. Thus the evaluation of this (promise) factor's influence on the stock depends on the company's vision, the decisions that the company will take in the future and the estimate of how similar companies have fared in the recent past. This is quantified in the stock market using Price-to-Earnings ratio (higher P/E ratio indicates better future yield), but we will stick to simple terms here.

- **Stability** : *as prior*
  Usually small or new companies have volatile stocks (whose prices are susceptible to market fluctuations) due to their deals, which can make or break. They tend to have a high Beta value (stock market term which quantifies volatility of a stock). High Beta stocks are usually risky (dividends may also reduce if stock value goes down) but can yield very high returns if bought and sold at the right time. Settled companies usually have less volatility and hence low Beta value. Low Beta stocks are less risky and ensure constant income flow. Hence we need to assess the risk involved and take a calculated decision.

- **Past performance and Current situation** : *as prior*
  We should also consider the recent performance of the stock and see the current scenario as to why it is being sold. There maybe rise in the stock's price of late, but that might be the cap and it won't increase significantly later. Then we should avoid buying such stocks. There maybe some stocks which have seen downturn of fortunes in recent past, but due to some sudden changes (like change in CEO) we expect the company to do well over next period. Then we should be inclined to buying such stocks.

- **Current demand** : *as evidence*
  Demand for a stock is something we obtain as information from the market scenario. Higher demand for a stock generally implies that stock is more valuable (good future performance), but it also indicates that the price for the stock will soar up. Similarly, lower demand implies the stock prices will fall. Hence it is better if we spot the promise in a stock earlier so that we can buy it before the demand increases for it. This will ensure a higher revenue conversion.

- **Market gossip** : *as evidence*
  Market gossip is the rumors or news that we hear around the market that is not officially from the company. The stocks of a company may go up if there is a new collaboration with a another company, or if there is a recent agreement/deal between two countries from which the import prices can reduce, etc. This news will not be broken out by the company immediately, but if it's neglected, then the prices will soar and we will miss the opportunity to buy. But one should also take these news and rumors with a hint of caution. If the source is not reliable or if they have not done enough background research, then the stock prices may not change or worse may even fall down. Hence we should keep a careful eye out for these.

2.2 Selling

- **Future performance** : *as prior*
  Usually when we are selling a stock, either we are expecting to fetch returns fully after earning sufficient dividends from it, or we feel that the stock price is going downhill and hence fetch the returns before it reaches the bottom. Unless there is an urgency to liquidate the stock, we would not want to sell a stock which has potential to rise in the future. Apart from these, the performance depends on factors similar to ones mentioned in buying. Thus assessing future performance of the stock price is critical.

- **Stability** : *as prior*
  The idea of stability in case of selling stocks is similar to that of buying. In case of a volatile stock one should take care to sell it when the prices are high (high enough to make sufficient profit) since they might stay high for long.

- **Past performance and Current situation** : *as prior*
  As mentioned before if the stock has been falling in the recent past and sees no signs of recovering then it better to sell it and make most of the situation. Also if we feel the a rising stock has almost reached its peak, then better to sell it. Apart from this, past information can help us indicate the trajectory of similar companies, giving us a better estimate of the achievable revenue. We also need to ensure currently it has been more than a year since we bought the stock, so that we can avoid tax.

- **Current demand** : *as evidence*
  The idea is again similar to that of buying except the outcome is reversed, i.e. if there is a high demand for the stock then the prices will rise, which will increase our revenue, and if the demand is low, the prices will reduce, which will decrease our income. Thus it better to sell the stocks when the demand is high.

- **Market gossip** : *as evidence*
  Market gossip should be looked at carefully like in the previous case of buying. For example if a reliable source predicts the fall of some of our stock prices, then it is better to sell them and gain as much revenue as possible.

2.3 Model

Here we try to model the above problem of stock trading into Bayesian inference problem. We use the revenue as the variable we want to estimate given the evidences mentioned above. The revenue is calculated as follows,

$$R = V + D - C$$

where

R : Total revenue from a stock
V : Current valuation (price) of stock
C : Original cost of the stock
D : Dividends earned over the period

Let's look at the case of buying (selling is similar with few minor changes). If R indicates the possible revenue that can be accumulated it N time (years or months) by buying the stock under consideration, and E is the evidence given to us, then the posterior (probability density) is given by,

$$P_{R|E}(r|\underline{e}) = cL(r)\pi(r)$$

$$L(r) = P(\underline{e} = (d, g)|r)$$

$$\pi(r) = \pi(f(st, pr, pa))$$

Where

$L(r)$: Likelihood
$\pi(r)$: Prior
c : Normalizing constant
$\underline{e}$: evidence
d : Demand
g : gossip estimated price
st : Stability
pr : Promise
pa : past performance
f : well-defined function

Now based on our needs we need to choose a parameter r_{th} : Threshold revenue, and based on our prior experience another parameter ϵ_{th} : Threshold confidence. We will buy a particular stock if the posterior probability ($P_{R|E}$) of the revenue being greater than r_{th} is more than ϵ_{th}, i.e mathematically,

$$\int_{r_{th}}^{\infty} P(r|\underline{e})dr = \epsilon \geq \epsilon_{th}$$

This is the strategy that will be used for the stock trading problem. In case of selling, if the ϵ_{future} (after N years) is less than ϵ_{th} or if it is significantly lesser than $\epsilon_{current}$, then we should sell the stock. If there are multiple stocks which satisfy the above constraint, then the simplest solution will be to choose the stock which has the highest ϵ. Other possible solutions will be discussed in the Optimization section.

3 Illustration

Consider the following examples given below for buying and selling stocks. We make a few assumptions which help us understand and illustrate the above problem in a much easier way.

3.1 Assumptions

- All random variables are assumed to continuous
- Though the minimum value of R is -C we extend it to $-\infty$ so that we can represent it using known random variables (this is reasonable assumption since very negative values of R usually have low probabilities)
- Demand is taken to be the difference between the number of buyers and sellers of the stock. Again for convenience of representation, we assume it to extend from $-\infty$ to ∞
- We assume that the demand and gossip likelihoods are independent given R, i.e. $P(d, g|r) = P(d|r)P(g|r)$
- While replicating stock volatility we assume only the extreme cases of the revenue swing for simplicity of the model
- We will consider most of the distributions to the positive side of R since we are interested to gain positive revenue (profit)

3.2 Buying

For this example we consider a volatile market and not a very reliable source of information for market gossip. Since it is a volatile market, lot of buyers and sellers will be joining and dropping, thereby creating a significant variance in the amount of demand. As we already have seen above, higher demand indicates lesser revenue (due to increase in stock price) and vice versa. Thus if we try to represent the random variable $D|R = r$ as a normal distribution then the mean will be proportional to $(r_d - r)$ where r_d is a suitably chosen parameter. If the mean is chosen as mentioned above, then it will replicate the property of higher demand for lower revenue and vice versa. The variance of this normal distribution is dependant on the stock volatility and can be estimated using previous data. Thus,

$$(D|R = r) \sim N(\alpha(r_d - r), \sigma_d^2)$$

If a market gossip is from a reliable source then the revenue value estimated by it will have a higher probability to be close to the actual value. If it is not a reliable source then predicted value will have a lower probability of being closer to the actual value. If we were to model the variable $G|R = r$ as a Gaussian distribution, then the mean will be proportional to $\sigma_g r$. This choice of mean justifies the above requirements since less reliable source has higher variance and vice versa. Thus,

$$(G|R = r) \sim N(\beta(\sigma_g r), \sigma_g^2)$$

Now let us consider modelling the prior as a Gaussian. The future promise indicates the mean of the distribution and we denote this by r_p. Since the market is volatile there are chances of the revenue swaying to the other side yielding a loss. Thus we can model this a Gaussian mixture model (only 2 mixtures here as mentioned in the assumption). The standard deviation of the first mixture will include the variation due to company's decisions, other similar company trajectories and market fluctuations and is denoted by σ_p. The other mixture has a mean and standard deviation r_f, σ_f both driven mainly by market fluctuation of the stock. Thus,

$$r_\pi \sim w_p N(r_p, \sigma_p^2) + w_f N(r_f, \sigma_f^2)$$

Since all the above distributions are normal (w.r.t. R) their product which yields the posterior will also be normal. Thus,

$$(R|E = (d, g)) \sim w_0 N(r_0, \sigma_0) + \tilde{w_0} N(\tilde{r_0}, \tilde{\sigma_0})$$

where $r_0, \tilde{r_0}, \sigma_0, \tilde{\sigma_0}, w_0, \tilde{w_0}$ are functions of $(d, g, r_p, r_f, r_d, \sigma_d, \sigma g, \sigma p, \sigma f, \alpha, \beta, w_p, w_f)$ We calculate the ϵ for this, which is nothing but

$$\epsilon = w_0 Q\left(\frac{r_{th} - r_0}{\sigma_0}\right) + \tilde{w_0} Q\left(\frac{r_{th} - \tilde{r_0}}{\tilde{\sigma_0}}\right)$$

(where Q is the CCDF of normal distribution) and check if it is greater than ϵ_{th}. The plots on the left column replicate the above example. The value orange area in 4 denotes the ϵ value

3.3 Selling

For this example, let us consider a stable (non-volatile) stock and a reliable source of market gossip. The demand variance will be less since it is a non-volatile stock. By using the knowledge of demand variation with revenue and from the buying model (for demand) we can say that the mean of the normal distribution for the variable $D|R = r$ is proportional to r. We will consider similar demand situations for current and future prices for simplicity. Thus,

$$(D|R = r) \sim N(\alpha r, \sigma_d^2)$$

Based on the market gossip about the current prices and future prices we evaluate our stock. The model for the variable $G|R = r$ is similar to the buying case except that the variance is lesser since we have a reliable source. Thus,

$$(G|R = r) \sim N(\beta(\sigma_g r), \sigma_g^2)$$

As mentioned above we are considering a non-volatile stock and hence we can use a unimodal Gaussian distribution for the prior. The current revenue variation depends mainly on the market fluctuation (since we are already aware of its value). We also estimate the mean and variance of the future revenue based on the expected future performance. Thus the expression for the prior will be,

$$r_\pi \sim N(r_f, \sigma_f^2)$$

We are considering variances in the current prices since we are yet to sell the stock. Similar to the buying case here also we have all Gaussian distributions being multiplied which will in turn give us a Gaussian posterior distribution which can be expressed as,

$$(R|E = (d, g)) \sim N(r_0, \sigma_0)$$

where r_0, σ_0 depend on the above (similar to buying case) mentioned parameters. We calculate ϵ as,

$$\epsilon = Q\left(\frac{r_{th} - r_0}{\sigma_0}\right)$$

We compare the $\epsilon_{current}$ with the ϵ_{future} as estimated by us. If the ϵ_{future} is less than ϵ_{th} or of it is significantly lesser than $\epsilon_{current}$, then we should sell the stock.

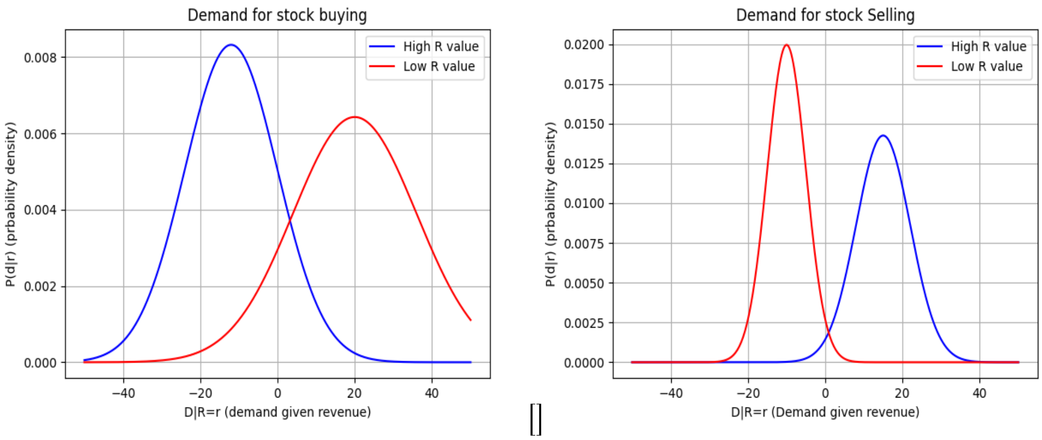

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Figure 1: Demand curves for buying(left) and selling

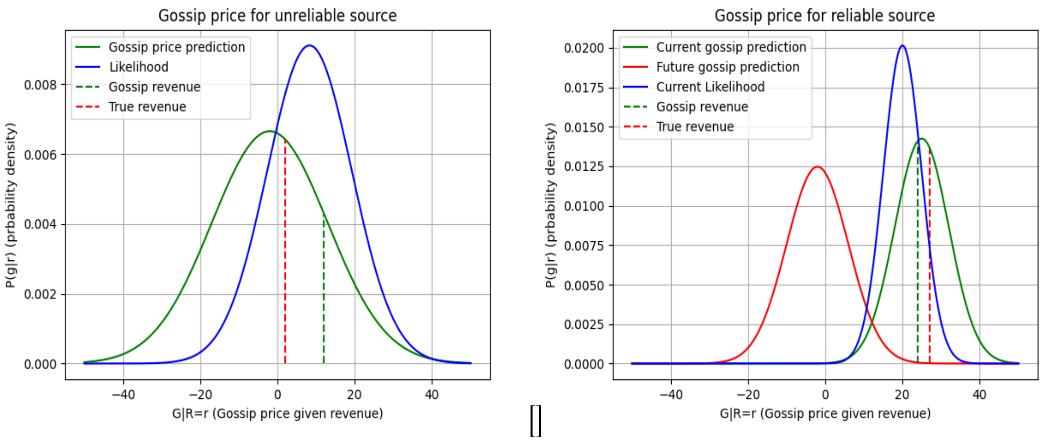

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Figure 2: Gossip price estimates for buying(left) and selling

3.4 Optimization

There are two kinds of optimization that we will look at. First we try to minimize the variance so that we can increase our ϵ_{th} to have a higher confidence. The variance of the posterior depends mainly on the variances of the likelihood and the prior and hence reducing those will help effectively reducing the posterior variance. The demand likelihood variance (σ_d) mainly depends on the stock volatility as explained in the above examples. Thus if we are keen on having a high confidence, then it is better to estimate in low volatility stocks (this makes intuitive sense as well). The gossip price variance (σ_g) depends on the reliability of the gossip source and hence finding more reliable sources of information can help reduce the uncertainty. We can also help reduce the variance by thorough research of past companies (which are similar) and other previous data. Since sample number (of data) is generally inversely proportional to variance, this will help reduce the uncertainty to a some degree.
Secondly we try to optimize over the stocks, so that we can buy/sell the one

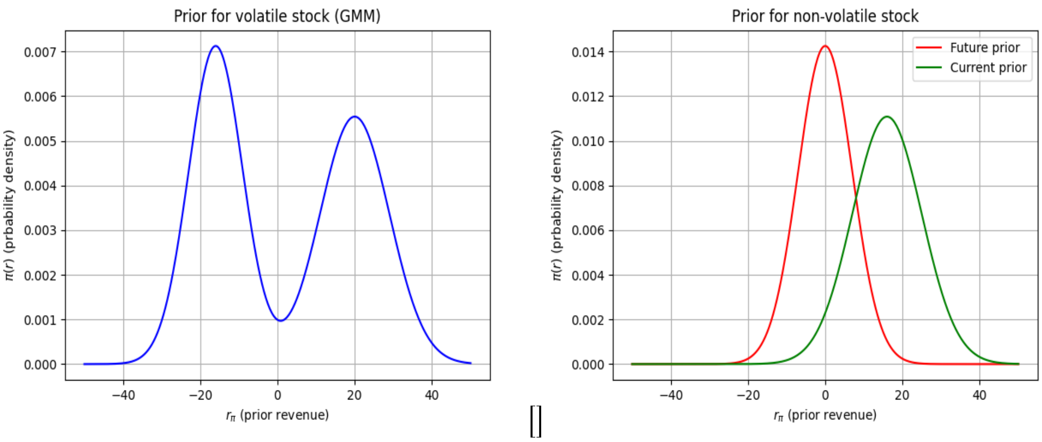

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Figure 3: Prior curves for buying(left) and selling

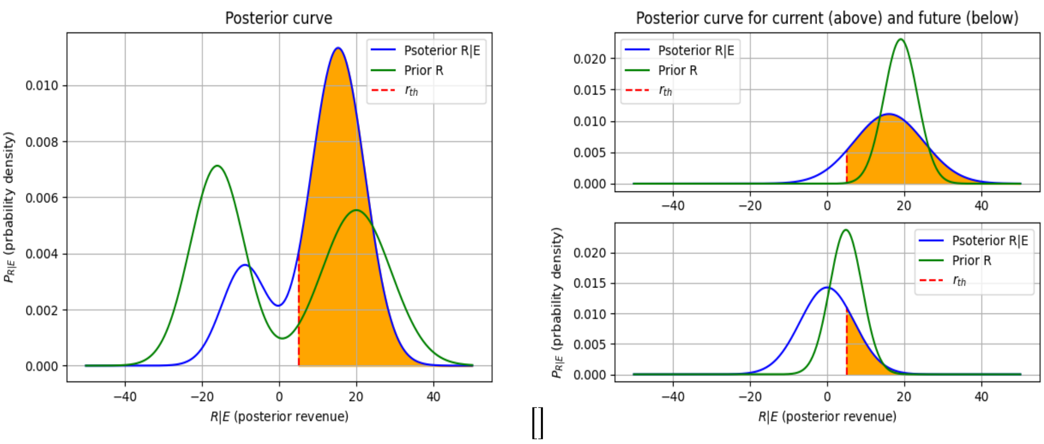

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Figure 4: Posterior curves for buying(left) and selling; Orange area : ϵ

which is best suited for our needs. We have 2 parameters in our control viz. r_{th}, ϵ_{th}. We consider the mean and the confidence estimated from the posterior $(R|E)$ to be $r_0(s), \epsilon(s)$ which are functions of the stock under consideration. The optimal stock S^* over all the stocks S can be found as,

$$S^* = \arg\max_{S} \quad \theta(r_0(s) - r_{th}) + (\epsilon(s) - \epsilon_{th})$$

where θ accounts for the normalizing and also the preference. Some may prefer to have a higher average posterior revenue and willing to compromise on ϵ (like in the case of volatile stocks). While others may prefer to play it safe and have higher confidence and moderate posterior average. Thus we have the leverage to choose θ based on our needs.

4 Once-in-100-year event

The given situation is similar to the COVID-19 pandemic outbreak at present and I will use this as illustration so that it will be more convenient to explain. The situation indicates that the demand for luxury items, consumption have reduced, and economies are depressed. Though the demand for luxury items has gone down, there is a surge in demand for many other commodities (like masks, sanitizers) which can be capitalized upon. We consider the following two cases below.

4.1 Case 1 : No urgency of liquidity

If we are not in an urgent need for liquidity, then there is no need to panic and take rash decisions. We don't need to sell any of the stocks immediately. We have the liberty to wait and watch if the company is able to adapt to the new norms in the next one year and decide based on that. But we definitely need to buy some stocks since the prices of many stocks have gone down due to lack of demand. For example, the price of oil has gone down significantly due to the pandemic, but after 5 years the oil market will again flourish as before, hence making it a good investment. Apart from this we should invest in something that can solve this once-in-100-year event. In the case of the pandemic, we should invest in some labs which are trying find a vaccine or some vaccine manufacturing companies, since upon success they will have a meteoric rise in stock prices due to the large-scale requirements. We should also look out for low-volatility stocks which can provide us good dividends that is necessary, since we are not selling stocks to liquidate. Considering these factors into our model, we need to estimate the revenue after 5 years. If we invest in low demand high prospect stocks, then we will have a good ϵ even for a higher than normal r_{th} value.

4.2 Case 2 : Urgency of liquidity

If we need liquidity within the next 2 years, then the only way is to sell the stocks. But the question is which stocks to sell and which to keep? And also should we buy any stocks? Demand for most commodities have gone down, does that imply we should sell all those?
It would be sensible to only sell those stocks which do not have a good chance to recover from this crisis. To know that we need to estimate the future promise (pr) of our stocks. If we have some stocks in cab services like Uber/Ola or in Air services, then the stocks would have taken a severe hit. But these are definitely going to recover once things start to settle down since people will need to travel from one place to another and one country to another, and are bound to start using these transport services later on (also the new norms are applicable only for a year). Hence it would be better to hold onto these stocks and wait for them to raise, rather than selling them and experiencing loss. But we do need to sell certain commodities and especially those which cannot adjust to these new norms (there will be long-term investors who do not need liquidity and will be looking to buy these stocks). For example, if we have invested in cultural activities or sports-oriented companies which require large gatherings for their functioning, then it will be a long time (around 2-3 years until a authentic vaccine arrives) before they make up money for their losses. Hence it is better to sell them now, while they can still provide a decent return. If we look from our model's perspective, then if the stock's ϵ value estimated for 2 years later (current ϵ is most likely less than ϵ_{th}) is still less than ϵ_{th} then we should sell the stock.
Since we are in need of liquidity we should take a gamble and invest in high volatile stocks. If we are careful and sell the stocks at the right time, then we can get the required returns. We should also invest in stocks which are seeing a rise during these times (like hygiene-oriented companies), so that not only do we get good dividends due to their profit, but also we can sell it within two years which is when they will reach their peak price. Based on our model, we should buy stocks with a higher than usual r_{th} (since we need more revenue to liquidate) and lower than usual ϵ_{th} (since we are willing to take more risk)

5 Takeaways

The key thing to takeaway from this would be, why does Bayesian inference work for this problem? The stock market is a constantly changing environment, and one needs to change his beliefs accordingly to thrive in the market. The Bayesian setup is meant to do exactly this, i.e. update beliefs. We see that even in a once-in-100-year event we could make use of the Bayesian model to update the information about current demands, future promise and decide which stocks to trade. In our model we have kept things quite simple, however the model can be made more robust and yield better performance by adding other complex factors or by using Bayesian Neural Networks.

-End of report-