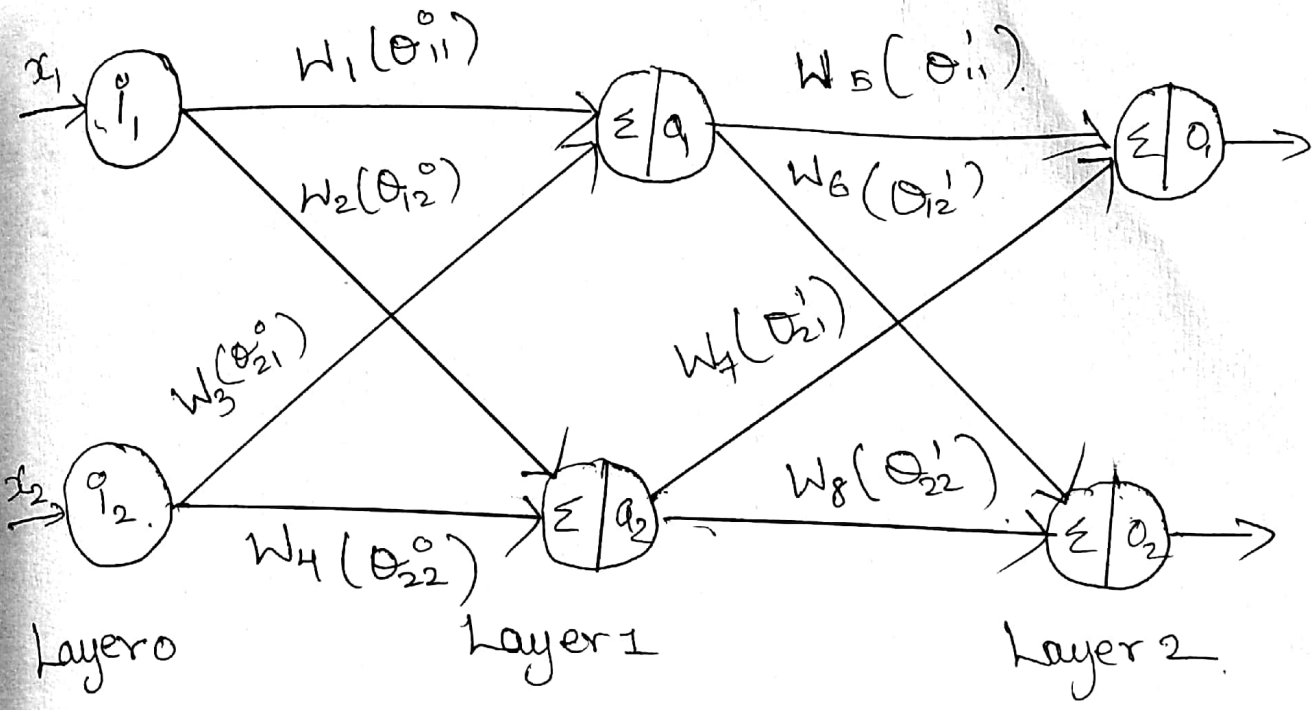
Deep Learning :- Neutral Network



Random Initialisation

(θ_{11}^0) $W_1 = 0.2$

(θ_{12}^0) $W_2 = 0.1$

(θ_{21}^0) $W_3 = 0.4$.

(θ_{22}^0) $W_4 = 0.1$

(θ_{11}') $W_5 = 0.5$.

(θ_{12}') $W_6 = 0.1$

(θ_{21}') $W_7 = 0.8$

(θ_{22}') $W_8 = 0.9$.

Input

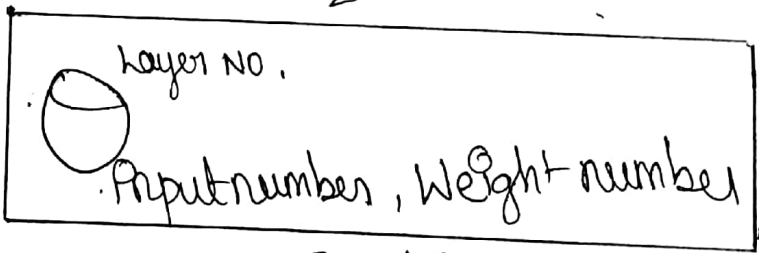
$x_1 \rightarrow 0.3$

$x_2 \rightarrow 0.9$.

Desired Output

Output $O_1 \rightarrow 0.8$

$O_2 \rightarrow 0.2$



Activation Function

Sigmoid

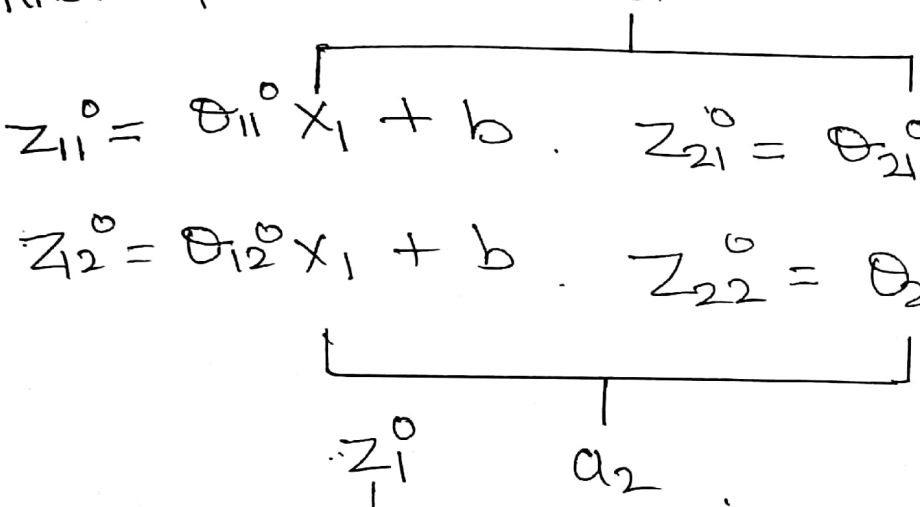
$$g(z) = \frac{1}{1+e^{-z}}.$$

$$z = Wx + b$$

Where $b = 0$.

FORWARD PROPAGATION:-

To find a_1 & a_2.



$a_1 = g(z_{11}^0 + z_{21}^0)$ $\quad a_2 = g(z_{12}^0 + z_{22}^0)$

$z_1^0 = \theta_{11}^0 x_1 + \theta_{21}^0 x_2$ $\quad z_2^0 = \theta_{12}^0 x_1 + \theta_{22}^0 x_2$

$= (0.2)(0.3) + (0.4)(0.9)$ $\quad = (0.1)(0.3) + (0.9)(0.1)$

$= 0.06 + 0.36$ $\quad = 0.03 + 0.09$

$z_1^0 = 0.42$ $\quad z_2^0 = 0.12$

$a_1 = g(z_1^0)$ $\quad a_2 = g(z_2^0)$

$= \frac{1}{1 + e^{-0.42}}$ $\quad = \frac{1}{1 + e^{-0.12}}$

$a_1 = 0.603483249886$ $\quad a_2 = 0.529964051766$

To find O_1 & O_2.

$$z_{11}' = \theta_{11}' a_1 + b \quad \Big] \rightarrow O_1.$$
$$z_{21}' = \theta_{21}' a_2 + b$$

$$z_{21}' = \theta_{21}\theta_{12}' a_1 + b \quad \Big] \rightarrow O_2.$$
$$z_{22}' = \theta_{22}' a_2 + b$$

$a_1 = 0.6084$
$a_2 = 0.5299$
$\theta_{11}' = 0.5$
$\theta_{12}' = 0.1$
$\theta_{21}' = 0.8$
$\theta_{22}' = 0.9$

$O_1 = g(z_{11}' + z_{21}')$.

$O_2 = g(z_{12}' + z_{22}')$.

$z_1' = z_{11}' + z_{21}'$

$z_2' = z_{12}' + z_{22}'$.

$z_1' = \theta_{11}' a_1 + \theta_{21}' a_2$

$= (0.5)(0.6034) + (0.8)(0.5299)$

$= 0.3017 + 0.42392$

$z_1' = 0.72562$.

$z_2' = \theta_{12}' a_1 + \theta_{22}' a_2$.

$= (0.1)(0.6034) + (0.9)(0.5299)$

$= 0.06034 + 0.47691$

$z_2' = 0.53725$.

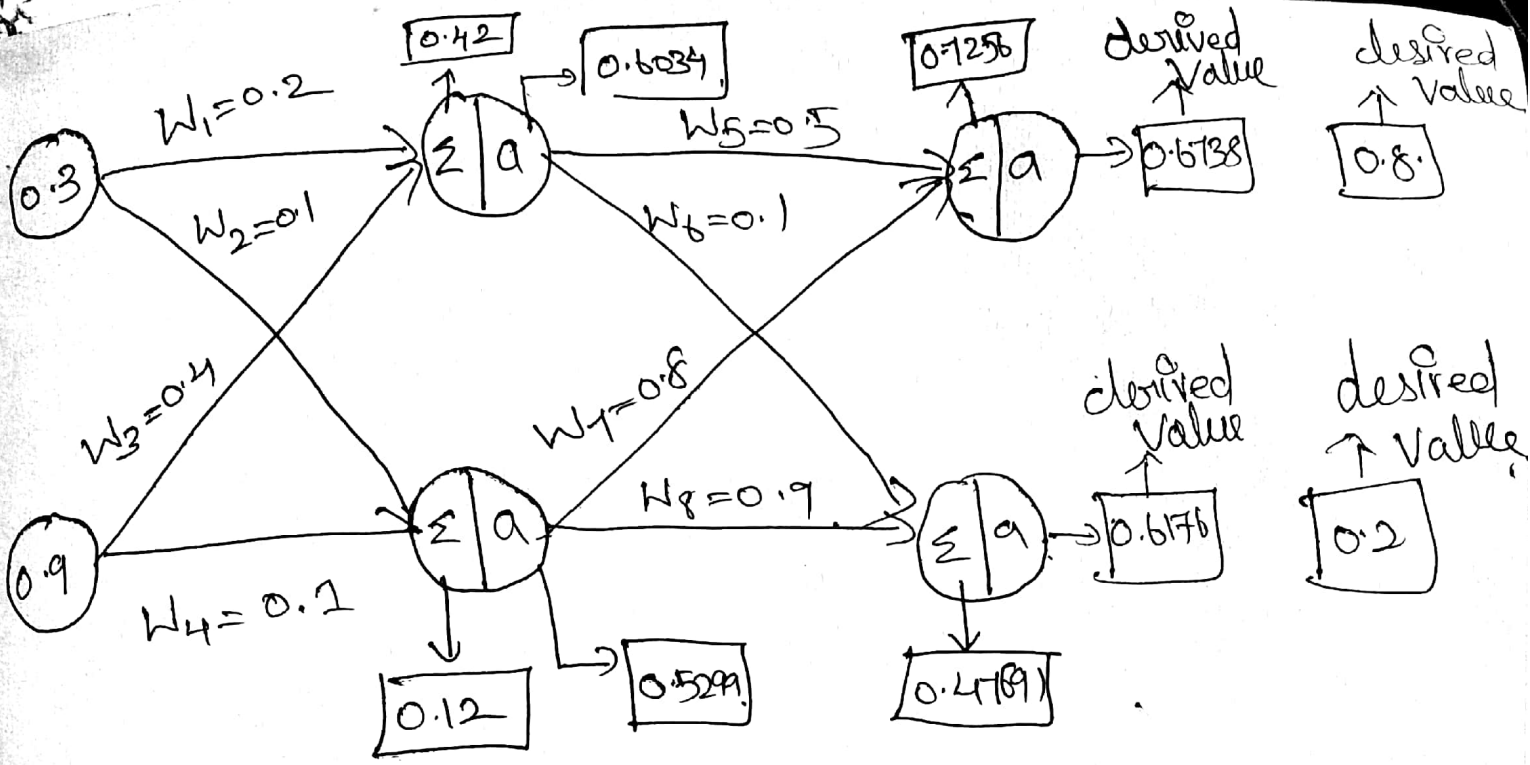
$O_1 = g(z_1') \Rightarrow \frac{1}{1 + e^{-0.72562}}$.

$= 0.673843376$

$O_2 = g(z_2')$

$= \frac{1}{1 + e^{-0.53725}}$

$= 0.629749906$.



$$E_{total} = \sum \frac{1}{2} (Target - Output)^2$$

$$E_{01} = \frac{1}{2} (0.8 - 0.6738)^2$$

$$= 0.00796322.$$

$$E_{02} = \frac{1}{2} (0.2 - 0.6176)^2$$

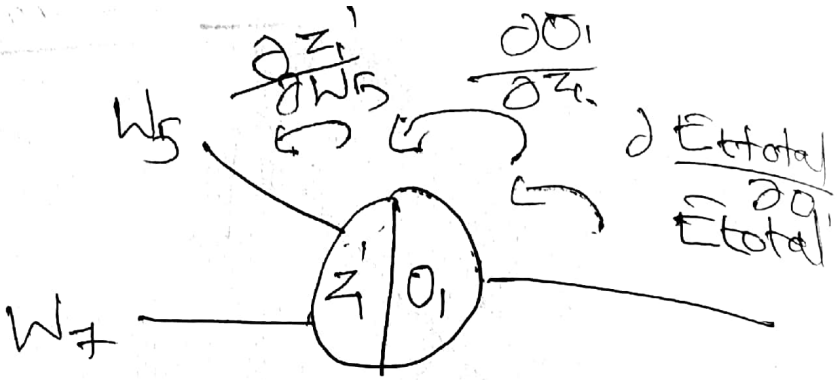
$$= 0.08719488.$$

$$E_{total} = E_{01} + E_{02}$$

$$= 0.00796322 + 0.08719488$$

$$E_{total} = 0.0951581.$$

Backpropagation.



To find w_5.

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial o_1} * \frac{\partial o_1}{\partial z_1'} * \frac{\partial z_1'}{\partial w_5}$$

$$E_{total} = E_{o1} + E_{o2}.$$

$$\frac{\partial E_{total}}{\partial o_1} = \frac{1}{2}(target_{o1} - output_{o1})^2 + \frac{1}{2}(target_{o2} - output_{o2})^2.$$

$$= \frac{2}{2}(target_{o1} - output_{o1}) * -(1) + 0.$$

$$\frac{\partial E_{total}}{\partial o_1} = -(target_{o1} - output_{o1}).$$

$$= -(0.8 - 0.6738)$$

$$= -0.1262.$$

$$\frac{\partial o_1}{\partial z_1'} = g(z_1')$$

$$= \frac{1}{1+e^{-z_1'}}$$

$$= \frac{1}{1+e^{-z_1'}}\left(1 - \frac{1}{1+e^{-z_1'}}\right)$$

$$\frac{\partial o_1}{\partial z_1'} = \frac{1}{1+e^{-0.72562}}\left(1 - \frac{1}{1+e^{-0.6738}}\right)$$

$$= 0.6738 \times (1 - 0.6738)$$

$$= 0.21979356.$$

$$\frac{\partial z_1'}{\partial w_5} = \theta_{11}' a_1 + \theta_{21}' a_2 \quad \text{where } w_5 = \theta_{11}'$$

$$= a_1 + 0$$

$$\frac{\partial E_{total}}{\partial w_5} = -0.1262 \times 0.21979356 \times 0.603483$$

$$= -0.016759.$$

$$w_5 = w_5 - \alpha\left(\frac{\partial E_{total}}{\partial w_5}\right) \rightarrow \text{Gradient descent}$$

$$= 0.5 - (0.5)(-0.016739)$$

$$= 0.5083695.$$

$\alpha = 0.5 \Rightarrow$ learning rate

To find W_6.

$$\frac{\partial E_{total}}{\partial W_6} = \frac{\partial E_{total}}{\partial O_2} \frac{\partial O_2}{\partial Z_2'} \frac{\partial Z_2'}{\partial W_6}$$

$$\frac{\partial E_{total}}{\partial O_2} = \frac{1}{2}(target_{O_1} - Output_{O_1})^2 + \frac{1}{2}(target_{O_2} - Output_{O_2})^2.$$

$$= 0 + (-1)(target_{O_2} - Output_{O_2})$$

$$= -(target_{O_2} - Output_{O_2})$$

$$= -(0.2 - 0.61760)$$

$$= 0.41760.$$

$$\frac{\partial O_2}{\partial Z_2'} = g(Z_2').$$

$$= \left(\frac{1}{1+e^{-Z_2'}}\right)\left(1 - \frac{1}{1+e^{-Z_2'}}\right)$$

$$= \left(\frac{1}{1+e^{-0.47891}}\right)\left(1 - \frac{1}{1+e^{-0.47891}}\right)$$

$$= 0.23616083$$

$$\frac{\partial z_2'}{\partial w_6} = \theta_{12}' a_1 + \theta_{22}' a_2$$ where $\theta_{12}' = w_6$.

$$= a_1 + 0.$$

$$= 0.603483.$$

$$\frac{\partial E_{total}}{\partial w_6} = (0.4176) \times (0.23616083) \times (0.603483)$$

$$= 0.059510036.$$

$$w_6 = w_6 - \alpha \left(\frac{\partial E_{total}}{\partial w_6} \right)$$

$$= (0.1) - (0.5)(0.059510036)$$

$$= 0.1 - 0.02975$$

$$= 0.07025$$

To find w_7

$$\frac{\partial E_{total}}{\partial w_7} = \frac{\partial E_{total}}{\partial o_1} \frac{\partial o_1}{\partial z_1'} \frac{\partial z_1'}{\partial w_7}$$

$$\frac{\partial E_{total}}{\partial o_1} = -(\text{target}_{o_1} - \text{Output}_{o_1})$$

$$= -((0.8) - (0.6738))$$

$$\frac{\partial E_{o_1}}{\partial o_1} = -0.1262.$$

$$\frac{\partial o_1}{\partial z_1'} = g(z_1')$$

$$= g(z_1')\,(1 - g(z_1'))$$

$$= 0.21979356.$$

$$\frac{\partial z_1'}{\partial w_7} = \theta_{11}' a_1 + \theta_{21}' a_2$$

where $w_7 = \theta_{21}'$

$$= 0 + a_2(1)$$

$$= 0.5299$$

$$\frac{\partial E_{total}}{\partial w_7} = (-0.1262) * (0.21979356) * (0.5299)$$

$$= -0.014698338$$

$\alpha = 0.5 \rightarrow$ Learning rate.

$$W_7 = W_7 - \alpha \left(\frac{\partial E_{total}}{\partial W_7} \right)$$

$= 0.8 - (0.5)(-0.046983386)$

$= 0.8 - (-0.007349169)$.

$= 0.807349169$.

$= 0.808$

To find W_8

$$\frac{\partial E_{total}}{\partial W_8} = \frac{\partial E_{total}}{\partial O_2} * \frac{\partial O_2}{\partial Z_2'} * \frac{\partial Z_2'}{\partial W_8}.$$

$\frac{\partial E_{total}}{\partial O_2} = 0.4176.$

$\frac{\partial O_2}{\partial Z_2'} = 0.23616083.$

$\frac{\partial Z_2'}{\partial W_8} = \theta_2' a_1 + \theta_{22}' a_2$

$= 0 + a_2$

$W_8 = \theta_{22}' = 0.9.$

$a_2 = 0.5299.$

$\frac{\partial E_{total}}{\partial W_8} = 0.4176 \times 0.23616083 \times 0.5299.$

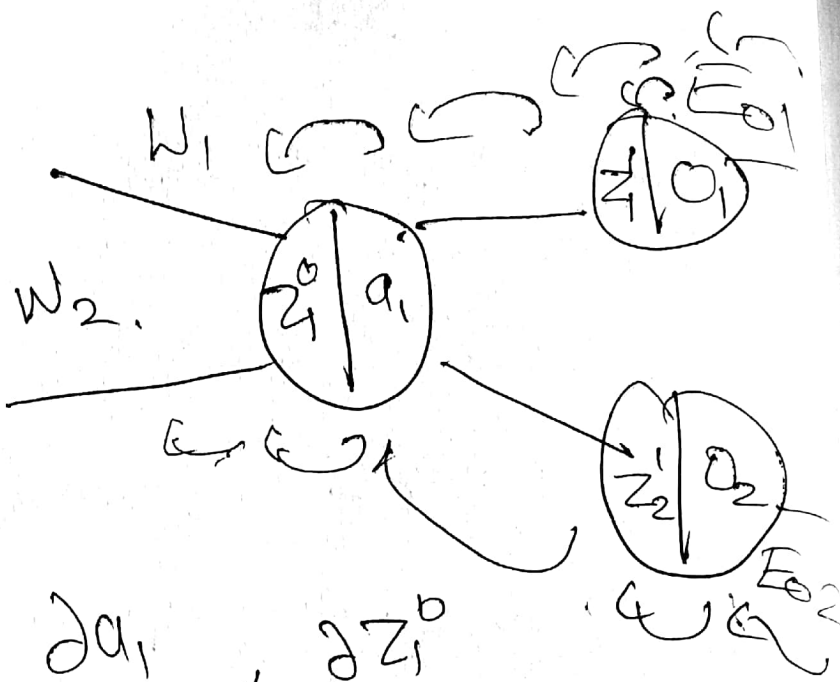
$= 0.052259142.$

$W_8 = W_8 - \alpha \left(\frac{\partial E_{total}}{\partial W_8} \right)$ $\alpha = 0.5 \rightarrow$ Learning rate.

$= 0.9 - (0.5)(0.052259142)$

$= 0.9 - 0.026129571 \Rightarrow 0.873870429.$

To find w_1



$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{a_1} * \frac{\partial a_1}{\partial z_1^0} * \frac{\partial z_1^0}{\partial w_1}$$

$$\frac{\partial E_{total}}{a_1} = \frac{\partial E_{o1}}{\partial a_1} + \frac{\partial E_{o2}}{\partial a_1}$$

$$\frac{\partial E_{o1}}{\partial a_1} = \frac{\partial E_{o1}}{\partial z_1} * \frac{\partial z_1^1}{\partial a_1}$$

$$\frac{\partial E_{o1}}{a_1} = \frac{\partial E_{o1}}{\partial z_1'} * \frac{\partial z_1'}{\partial a_1}$$

$$\frac{\partial E_{o1}}{\partial z_1'} = \frac{\partial E_{o1}}{\partial o_1} * \frac{\partial o_1}{\partial z_1'}$$

$$= (-0.1262) * (0.21979356)$$

$$= -0.027737947$$

$$\frac{\partial E_{total}}{\partial w_7} = \frac{\partial E_{total}}{\partial o_1} * \frac{\partial o_1}{\partial z_1'} = \frac{\partial E_{total}}{\partial z_1'}$$

$$\frac{\partial z_1'}{\partial a_1} = \theta_{11}' a_1 + \theta_{21}' a_2$$

$$= \theta_{11}' + 0 \Rightarrow w_5$$

$$= 0.5$$

$$\frac{\partial E_{total}}{\partial w_7} \left(\frac{\partial E_{total}}{\partial o_1} \frac{\partial o_1}{\partial z_1'} \right) * \frac{\partial z_1'}{\partial w}$$

$$\frac{\partial E_{total}}{\partial z_1'} = \frac{\partial E_{o1}}{\partial z_1'}$$

$$\frac{\partial E_{a1}}{\partial z_1'} = -0.027737947 \times 0.5$$

$$= -0.013868973$$

because O_2 become $= 0$

$$\left(\frac{\partial E_{o2}}{\partial o_2} * \frac{\partial o}{\partial z_1'} * \frac{\partial z}{\partial a_1} \right)$$

$$\frac{\partial E_{o2}}{\partial a_1} = \frac{\partial E_{o2}}{\partial z_2'} * \frac{\partial z_2'}{\partial a_1}$$

$$\frac{\partial E_{o2}}{\partial z_2'} = \frac{\partial E_{o2}}{\partial z_2'} * \frac{\partial O_2}{\partial z_2'}$$

$$= (0.4176) \times (0.23616083)$$

$$= 0.09862076^2.$$

$$\frac{\partial z_2'}{\partial a_1} = w_{12}' a_1 + w_{22}' a_2$$

$$\frac{\partial z_2'}{\partial a_1} = w_{12}' = w_6$$

$$= 0.1$$

$$\frac{\partial E_{o2}}{\partial a_1} = 0.09862076 \times 0.1$$

$$= 0.009862076.$$

$$\frac{\partial E_{total}}{a_1} = -0.01386897 3 + 0.009862076$$

$$= -0.00400689 7$$

$\frac{\partial E_{o2}}{\partial O_2} \frac{\partial O_2}{\partial z_2'}$

4.

E_{o1}

c_1

become

zero.

$\left(\frac{\partial E_{o1}}{\partial O_2} \times \frac{\partial O_2}{\partial z_2'} \frac{\partial z_2'}{\partial a_2} \right)$

$$\frac{\partial a_1}{\partial z_1^0} = g(z_1^0)$$

$$= \left(\frac{1}{1+e^{-z_1^0}}\right)\left(1 - \frac{1}{1+e^{-z_1^0}}\right)$$

$$= (0.60348329)(1 - 0.60348329)$$

$$= 0.239291219.$$

$$\frac{\partial z_1^0}{\partial w_1} = \theta_{11}^0 x_1 + \theta_{21}^0 x_2$$

$$+ 0$$

$$\partial w_1 = \theta_{11}^0 = x_1$$

$$= 0.2 \quad = (0.3).$$

$$\frac{\partial E_{total}}{\partial w_1} = -0.00400689 7 * 0.239291219$$

$$* 0.3$$

$$= -0.000287644.$$

$$\alpha = 1$$

$$w_1 = w_1 - \alpha \frac{\partial E_{total}}{\partial w_1}$$

$$= w_1 - 1(-0.000287644)$$

$$= 0.2 + 0.000287644$$

$$= 0.2002876 44$$

To find W_3.

$$\frac{\partial E_{total}}{\partial W_3} = \frac{\partial E_{total}}{\partial a_1} * \frac{\partial a_1}{\partial z_1^0} * \frac{\partial z_1^0}{\partial W_3}$$

$$\frac{\partial E_{total}}{\partial a_1} = \frac{\partial E_{o1}}{\partial a_1} + \frac{\partial E_{o2}}{\partial a_1}$$

$$= -0.004006897$$

$$\frac{\partial E_{o1}}{\partial a_1} = \frac{E_{o1}}{\partial o_1} \frac{\partial o_1}{\partial z_1^1}$$

$$\frac{\partial E_{o2}}{\partial a_1} = \frac{E_{o2}}{\partial o_1} * \frac{\partial o_1}{\partial z_2^1}$$

$$\frac{\partial a_1}{\partial z_1^0} = 0.239291219$$

$$\frac{\partial z_1^0}{\partial W_3} = o_{11}^0 x_1 + o_{21}^0 x_2$$

$$0 + x_2$$

$$= 0.9$$

$W_3 = 0.4$

$\theta_{21}^0 = 0.4$

$$\frac{\partial E_{total}}{\partial W_3} = -0.004006897 * 0.239291219 * 0.9$$

$$= -0.000862933$$

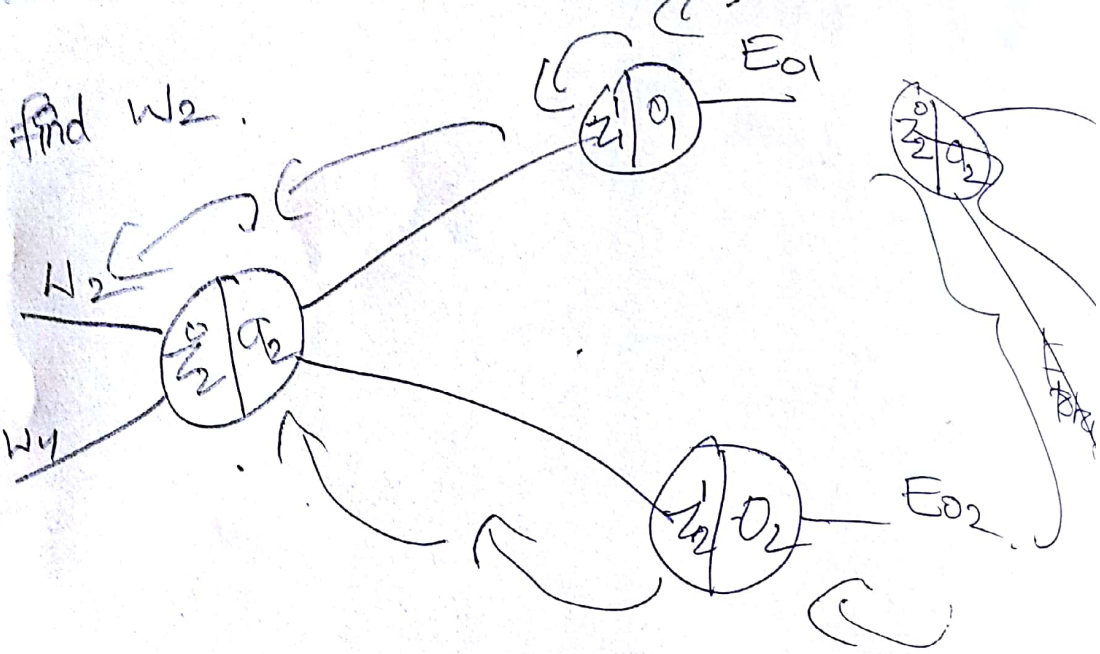
$$= -0.009$$

$\alpha = 1$

$$W_3 = W_3 - \alpha \frac{\partial E_{total}}{\partial W_3} \Rightarrow 0.4 - (1)(-0.009)$$

$$= 0.409$$

To find W_2.



$$\frac{\partial E_{total}}{\partial W_2} = \frac{\partial E_{total}}{\partial a_2} * \frac{\partial a_2}{z_2^0} * \frac{\partial z_2^0}{\partial W_2}$$

$$\frac{\partial E_{total}}{\partial W a_2} = \frac{\partial E_{O1}}{\partial a_2} + \frac{\partial E_{O2}}{\partial a_2}$$

$$\frac{\partial E_{O1}}{\partial a_2} = \frac{\partial E_{O1}}{\partial O_1} * \frac{\partial O_1}{\partial z_1'} * \frac{\partial z_1'}{\partial a_2}$$

$$= -0.1262 * \frac{\partial z_1'}{\partial a_2}$$

$$\frac{\partial z_1'}{\partial a_2} = \theta_{11}' a_1 + \theta_{21}' a_2$$

$$= 0 + \theta_{21}' \qquad \theta_{21}' = W_4 = 0.8$$

$$= -0.1262 * 0.8$$

$$\frac{\partial E_{O1}}{\partial a_2} = -0.10096$$

$$\frac{\partial E_{o2}}{\partial a_2} = \frac{\partial E_{o2}}{\partial O_2} * \frac{\partial O_2}{\partial z_2'} * \frac{\partial z_2'}{\partial a_2}$$

$$= 0.09862076 * \frac{\partial z_2'}{\partial a_2}$$

$$= 0.09862076 * \partial$$

$$\frac{\partial z_2'}{\partial a_2} = \theta_{12}' a_1 + \theta_{22}' a_2$$

$$= 0 + \theta_{22}' \qquad \theta_{22}' = w_8 = 0.9$$

$$= 0.9$$

$$\frac{\partial E_{o2}}{\partial a_2} = 0.09862076 * 0.9$$

$$= 0.088758684$$

$$\frac{\partial E_{total}}{\partial a_2} = \frac{\partial E_{o1}}{\partial a_2} + \frac{\partial E_{o2}}{\partial a_2}$$

$$= -0.10096 + 0.088758684$$

$$= -0.012201316$$

$$\frac{\partial a_2}{\partial z_2^0} = g(z_2^0)$$

$$= (0.529964)(1 - 0.529964)$$

$$= 0.249102158$$

$$\frac{\partial Z_2^o}{\partial W_2} = \theta_{12}^o x_1 + \theta_{22}^o x_2$$

$$\frac{\partial Z_2^o}{\partial W_2} = x_1 + 0$$

$\theta_{12}^o = 0.1 = 0.3$

$$\frac{\partial E_{total}}{\partial W_2} = \frac{\partial E_{total}}{\partial a_2} * \frac{\partial a_2}{\partial Z_2^o} * \frac{\partial Z_2^o}{\partial W_2}$$

$= -0.012201316 * 0.24910258 * 0.3$

$= -0.000091812$

$\alpha = 1$

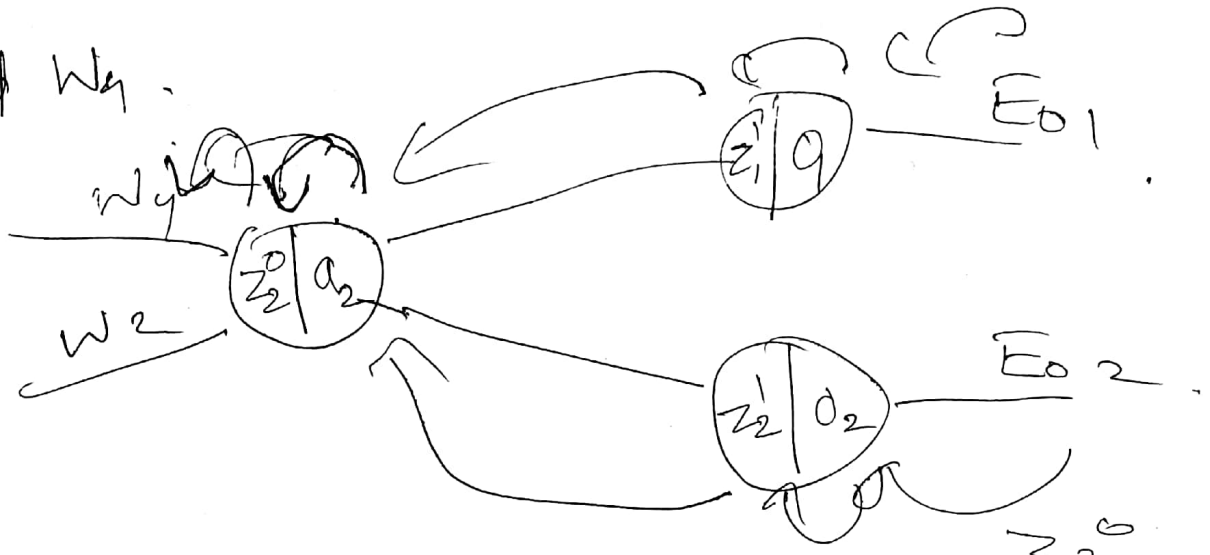
$$W_2 = W_2 - \alpha \frac{\partial E_{total}}{\partial W_2}$$

$= 0.1 - (1)(-0.000091812)$

$W_2 = 0.100091812$

$W_2 = 0.101$

To find W_4.



$$\frac{\partial E_{total}}{\partial W_4} = \frac{\partial E_{total}}{\partial a_2} * \frac{\partial a_2}{\partial z_2^0} * \frac{\partial z_2^0}{\partial W_4}$$

$$= -0.01220131 6 * 0.249102158 * \frac{\partial z_2^0}{\partial W_4}$$

$$\frac{\partial z_2^0}{\partial W_4} = \theta_{12}^0 x_1 + \theta_{22}^0 x_2$$

$W_4 = \theta_{22}^0 = 0 + x_2$

$= 0.9$

$= -0.01220131 6 * 0.249102158 * 0.9$

$= -0.002735436$

$$W_4 = W_2 - \alpha \frac{\partial E_{total}}{\partial W_4}, \quad \alpha = 1$$

$= W_4 0.9 - (-0.002735436)$

$= 0.902735436 = 0.102735$

$z_1^0 = w_1 x_1 + w_3 x_2$

$= 0.209 \times 0.3 + 0.409 \times 0.9$

$= 0.4308$

$a_1 = g(z_1^0)$

$= \left(\frac{1}{1 + e^{-z_1^0}}\right)$

$= 0.60606$

$z_2^0 = w_2 x_1 + w_4 x_2$

$= 0.0303 + 0.0918$

$= 0.1221$

$a_2 = g(z_2^0)$

$= \left(\frac{1}{1 + e^{z_2^0}}\right)$

$= 0.530487$

$z_1^1 = w_5 a_1 + w_7 a_2$

$= 0.30909 + 0.737704$

$= 1.047112$

$a_1 = g(z_1^1)$

$= \left(\frac{1}{1 + e^{-z_1^1}}\right)$

$= 0.74$

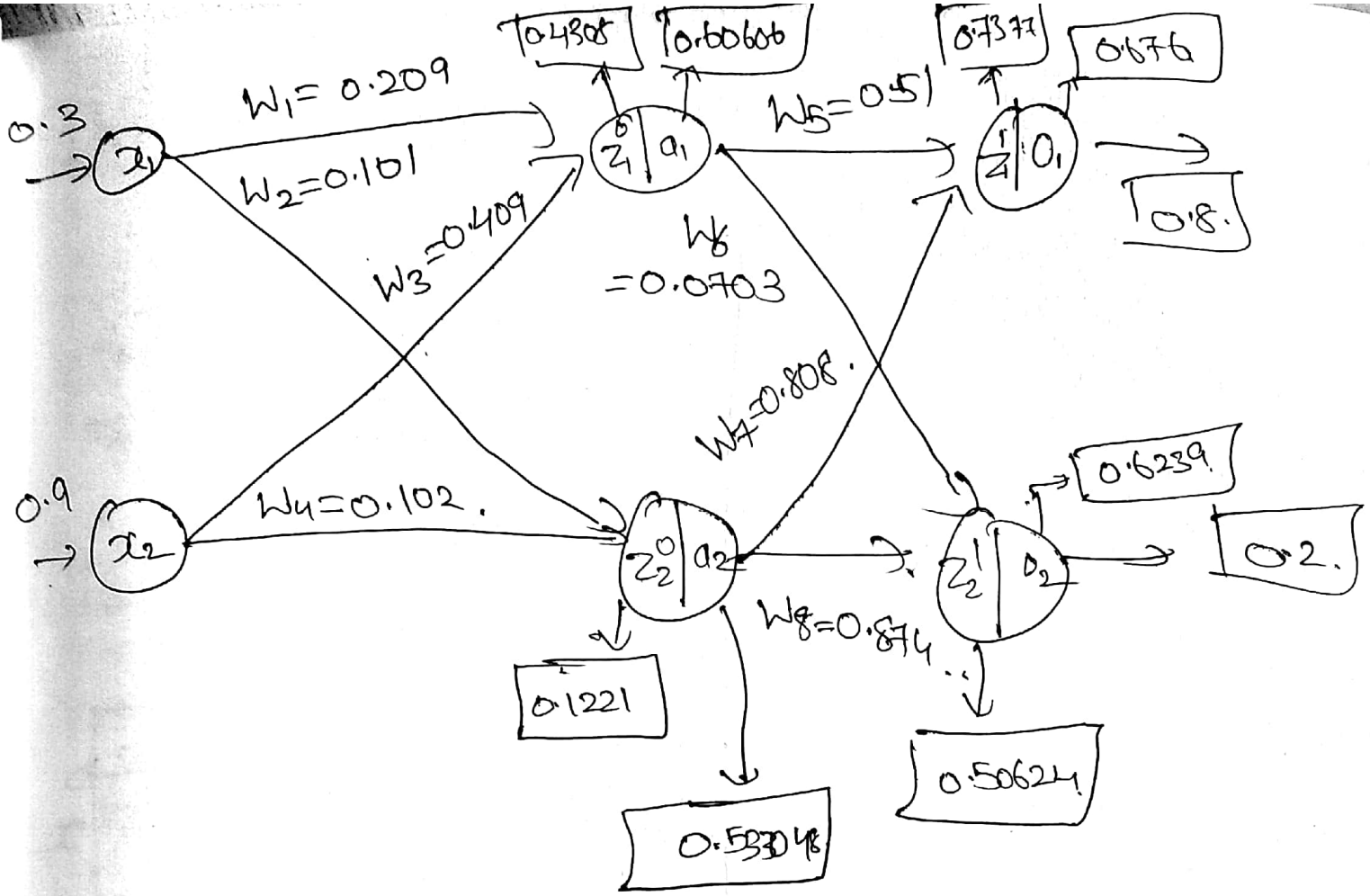
$z_2^1 = w_6 a_1 + w_8 a_2$

$= 0.042606 + 0.4636$

$= 0.50624$

$O_2 = \frac{1}{1 + e^{-g(z_2^1)}}$

$= 0.6239$



New weights.

$\alpha = 1$

$W_1 = 0.209.$

$W_2 = 0.101$

$W_3 = 0.409.$

$W_4 = 0.102.$

$\alpha = 0.5$ $W_5 = 0.51$

$W_6 = 0.0703$

$W_7 = 0.808$

$W_8 = 0.874.$