Collaborative Filtering–Based Recommender System: Approaches and Research Challenges

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Abstract— **Due to information explosion, huge number of items are present over web which makes it difficult for user to find appropriate item from available set of options. Recommender System (RS) overcomes the problem of information overload and suggests items that interest to a user. It has gained a lot of popularity in past decades and huge amount of work has been done in this field. Collaborative Filtering (CF) is the most popular and widely used approach for RS which tries to analyze the user's interest over the target item on the basis of views expressed by other like-minded users. This paper gives a brief idea of various approaches used for Recommender System and provides an insight of Collaborative Filtering technique. Here, we also discuss well-known methods for CF i.e. Memory-based, Model-based, and hybrid approaches and at last we focus on research challenges that need to be addressed.**

Keywords—Recommender systems, collaborative filtering, memory-based methods, model-based methods, user-based CF, item-based CF

I. Introduction

Earlier, people were the information consumers but now they used to publish information in the form of articles, posts, blogs, forums etc. which leads to information explosion. When huge number of choices is available, it becomes difficult for consumer to arrive at the most appropriate choices. To deal with the task of choosing the correct product that satisfies the customer requirements, recommender systems were developed to provide suggestions for a set of items that are of high interest to the active user. Recommendations can be applied to several domains as entertainment, shopping sites, social networking, job portals, finding relevant web pages and many more. Thus, it helps in various decision-making processes, like which movies to watch, what items to buy, what music to listen to, which articles to read and so on.

Users have preferences, likes and dislikes for certain items and the degree of preference is evaluated either by analyzing the user sentiment expressed through reviews or by numeric assigned by a user to an item which is represented as rating matrix. User can provide values on 5-point or 10-point scale where 1 is treated as highly dissatisfied, 2 as dissatisfied and so on. Users explore limited number of items and have rated few of them. The matrix becomes sparse, which means most of the entries are not known. These missing values indicate that no information is available about user's preference for the item.

Recommendation process is split into two tasks. One is considered as the problem of predicting ratings or evaluating usefulness of items that have not been discovered by a user. Usually, this prediction is based on the past behavior of current user or other users. Another task is to provide recommendations for the items that are most preferable. Recommendation is made after estimating the rating for unrated items i.e. items with highest rating value are shown as recommendation.

To accomplish the task of recommendation, various approaches have been proposed in literature that is depicted in Fig. 1. Collaborative Filtering is among the most popular algorithms for recommender system which identifies user opinion for an item based on the interest of other like-minded people. D. Goldberg et al. were the first who coined the term 'Collaborative Filtering' where they have proposed manual collaborative filtering technique for Tapestry mailing system [1].

Besides collaborative filtering, other widely used algorithm for RSs is content-based information filtering techniques where recommendations are made by matching the user profile to that of item descriptions. These algorithms employ Information Filtering/Information Retrieval techniques as LSI, TF-IDF and so on. Content-based techniques have been used for Netnews filtering [2], improving web page searching via hyperlinks [3], recommending interesting websites based on topics [4] etc.

Demographic-based recommender system [5] considers user's demographic profile (age, gender, location etc.) to find similarity between users. It believes that people who grouped according to factors like age, gender, location etc. are more likely to share similar interests and recommendations are made for these homogenous groups.

Knowledge-based RS uses domain knowledge for producing recommendations by gathering information that finds usefulness of an item for a particular user [6]. Knowledge-based systems can be divided into two: Case-based and Constraint-based recommender systems [5].

Community-based RS is based on the fact that user rely more on their friend's suggestion instead of random similar users. This system analyzes the social network of the target user and determines his friend's preference over the given item, then produces recommendations for high rated items. Basically, this technique is applied over social networks and known as social recommender systems [7].

Hybrid approaches are used to combine two or more techniques to take advantage of individual approaches [8]. Several ways have been proposed to combine various techniques to create a new hybrid system [9].

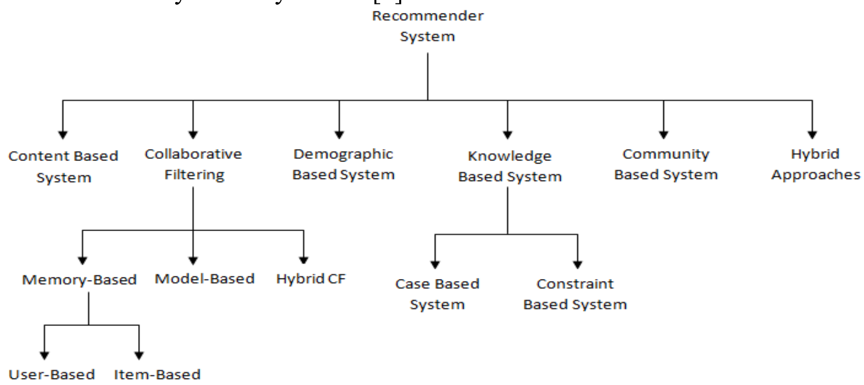

Fig.1. Classification of Recommender System Approaches

II. Collaborative Filtering Overview

Collaborative filtering is used to filter choices based on the interest of other people in the system. A collaborative filtering system consists of m users denoted as U = [u_1, u_2, u_3.....u_m] and n items as I = [i_1, i_2, i_3.....i_n] which together forms $m \times n$ rating matrix R as shown in Table 1. Each user experiences item of their choice and expresses opinion over these items as rating score. This rating database is used to find similarity between users.

TABLE I. SAMPLE RATING MATRIX

	i_1	i_2	i_3	i_4	i_5
u_1	4	?	5	3	1
u_2	5	2	5	?	?
u_3	2	4	?	1	3

Each row corresponds to user rating distribution over item set and column represent rating pattern for that item. To recommend most preferable items to the current user, the process follows the two given steps:

- Prediction- In order to find user u_1 interest over item i_2, either similarity between u_1 and other users is measured or similar items to that of i_2 are identified. A rating value r_{12} is predicted using similarity metrics or machine learning algorithms.
- Recommendation- After predicting the u_1 likeliness for all unseen items, items that seem most interesting i.e. high-rated items are recommended to u_1.

Basically, CF approaches try to model the user-item interaction based on the different ratings provided. Sometimes, the rating provided by user is not genuine because a lenient user may give higher rating than other users in contrast to a strict user who gives less than others. This tendency may affect the overall prediction value and decreases the recommendation quality. A baseline prediction [5] denoted by b_{ui} for unknown rating r_{ui} is calculated as shown in (1)

$$b_{ui} = \bar{r} + b_u + b_i \quad (1)$$

Where $\bar{r}$ refers to average rating, b_u and b_i denotes user and item biases respectively. This baseline prediction is subtracted while predicting the value for r_{ui} in order to nullify the effect of item and user biases.

III. Collaborative Filtering Approaches

Collaborative filtering recommends item based on the interest of other like-minded users or identify items similar to those previously rated by the target user. It uses statistical techniques [10] to find the similarity between user or item vector. CF approaches can be classified into three categories- Memory-Based, Model-Based and Hybrid approaches.

A. *Memory-Based Algorithms*

These methods memorize user-item rating matrix and use full rating database to determine item/user similarity [11]. These methods are easy to implement but faces memory issues as it requires a large space to store full-fledge rating matrix. Also, memory-based algorithms are lazy-learner and susceptible to scalability issues.

1) *Category-* Memory-based approaches can be further categorized into following two techniques:

a) User-Based Collaborative filtering

This approach is based on the fact that people who shared similar opinion in the past are likely to share the same in future. This was first introduced in GroupLens for recommending news article based on other users' interest or rating [12]. Further this concept was used for recommending music albums [13] and for video recommendation [14].

In user-based CF, user-vector for common items is used to evaluate the rating score. To predict how u_2 will rate item i_4, similarity between u_2 and u_1 is calculated over i_1 and i_3. Likewise, similarity between u_2 and other users is measured to identify similar users. From Table 1, we see that u_2 is more similar to u_1 rather than u_3, so u_1 will have more impact on predicted rating score.

b) Item-Based Collaborative Filtering

To extend the scope of CF so it can be applied to large datasets, a need arises for designing a more scalable algorithm. The basic concept behind this approach considers that user possesses the same view for similar items which was first used by [15]. It is explained that user might want to add items which are similar to the items present in shopping cart [16].

Contrary to user-based CF, it finds similarity between rating pattern of target item to that of other items. To identify the user preference for a specific item, item similarity is measured by analyzing how other people have rated that particular item. For example, if most of the users have provided similar ratings to different items shows that the items are similar.

In Item-based CF, item-vector is used for making predictions. To predict user u_3 rating value for item i_3, we measure the similarity of i_3 with other items and their weighted impact is used to calculate rating value r_{33}. From table 1, we know that item i_3 is more similar to i_1 than other items.

2) *Similarity Measures & Prediction Computation:* System tries to identify the nearest neighbor by using similarity measures then predictions are made using rating score and Top-N recommendations are produced. Similarity between target user and other users is determined and.

a) Similarity Calculation- Pearson-correlation similarity measure tries to find a linear relationship between two vectors [17]. Similarity between user u and user v is determined by using Pearson's coefficient as given in (2).

$$C_{uv} = \frac{\sum_{i=1}^{n}(r_{ui}-\bar{r}_u)(r_{vi}-\bar{r}_v)}{\sqrt{\sum_{i=1}^{n}(r_{ui}-\bar{r}_u)^2(r_{vi}-\bar{r}_v)^2}} \qquad (2)$$

Here, $\bar{r}_u$ and $\bar{r}_v$ are the average rating of user u and v respectively. Likewise, item similarity can be calculated over all users. Another most commonly used similarity measure is cosine-vector similarity. Other methods for measuring similarity are Adjusted Cosine Similarity, Jaccard Similarity, Conditional Probability-Based Similarity, Default Voting, Inverse User Frequency, Case Amplification etc.

b) Weighted Sum of Ratings- To get a prediction for user u over an item i, weighted sum of ratings is used to predict current user's (u) interest over an item (i) as shown in (3).

$$P_{ui} = \bar{r}_u + \frac{\sum_{v=1}^{m}(r_{vi}-\bar{r}_v).C_{uv}}{\sum_{v=1}^{m}|C_{uv}|} \qquad (3)$$

Other methods such as simple weighted average, weighted majority prediction techniques can also be used for computing prediction. P_{ui} is calculated for all unseen items, Top-N recommendations are used to produce a ranked list of N most relevant items that seems useful to the user u. This ranked list contains item with higher predicted value.

B. Model-Based Agorithms:

To overcome the space and scalability issues of memory-based approaches, model-based algorithms were developed. These methods build a model which learns or observes user-item interactions by the factor of low dimensional representations (user and item feature vectors). These methods also known as Latent factor models which can be realized as Matrix Factorization (MF) and its variants like Singular Value Decomposition (SVD) [18] and so on.

Matrix Factorization- It tries to characterize both items and users by their feature vectors of low dimension inferred from the rating patterns of user. For example, consider a set of users denoted by U and a set of items as I. R would be the rating matrix of size |U|×|I|. Then, our goal is to discover L latent features and find two matrices P and Q as shown in (4).

$$\hat{R} = P \times Q^T \approx R \qquad (4)$$

Basically, matrix R is factorized into matrix P (|U|×L) and matrix Q (|I|×L). Here, for matrix P each factor determines users' interest over items that get high score for the corresponding feature vector. This close association between user and item latent features results in recommendation. Rating for an item i by user u is predicted using (5).

$$\hat{r}_{ui} = p_u q_i^T \qquad (5)$$

Where p_u and q_i corresponds to user and item feature vector. Feature vectors are learned by minimizing the difference between the actual and the estimated rating. Equation (5) is extended by introducing regularization terms to avoid over-fitting as given in (6).

$$min_{p,q} \sum_{u,i}(r_{ui} - \hat{r}_{ui})^2 + \lambda(\|p_u\|^2 + \|q_i\|^2) \qquad (6)$$

The constant λ is the regularization term to avoid over-fitting which generally varies between 0 and 1. Items containing high value for features that are of high interest to the target user are recommended.

Generally, if rating data are in categorical form, classification algorithms like Bayesian methods [19], probabilistic model [20, 21], and clustering approaches [22, 23] can be used to model collaborative filtering tasks. Otherwise, for numerical ratings, Matrix Factorization (MF), Singular Value Decomposition (SVD) [24], regression techniques etc. can be used.

C. Hybrid Collaborative Filtering:

To avoid the problem of sparsity and scalability issues, memory-based and model-based techniques are combined to produce better recommendations.

We have studied several methods used to apply collaborative filtering approaches to recommender system. In literature, many researchers have applied CF to various domains like movies, songs, books etc and evaluated RS using different measures. RSs can be evaluated using online measures, offline measures and user-studies [5]. Generally, CF based approaches rely on offline evaluation where ratings for random items have been removed and then recommendation algorithm is evaluated by predicting rating for the removed items. Lower the difference between actual and estimated rating, higher is the accuracy of the system. A comparative study of the collaborative filtering techniques has been presented in Table II which lists a number of techniques applied in different domain and also shows the evaluation metrics used by authors to measure system performance, accuracy, efficiency etc.

IV. RESEARCH CHALLENGES

A. User Cold-Start problem

When a new user enters in the system, he has rated no or only few items so it becomes very difficult to find similar users and therefore, degrades the quality of recommendations.

B. Item Cold-Start problem

When new item arrives in the system, less number of users have experienced or rated that item. When less rating data are available, CF approaches fails to predict the similarity between items. This is a cold-start for new items as rating for new items cannot be predicted and therefore, system will not be able to recommend new items till some people have rated it.

TABLE II. A COMPARATIVE ANALYSIS OF COLLABORATIVE FILTERING RESEARCH

Author, Year	Domain/ Dataset	Technique	Evaluation Metric
(D. Goldberg et al., 1992) [1]	Mail System	Tapestry Query Language (TQL)	-
(P. Resnick et al., 1994) [12]	UseNet news	Correlation Coefficient	-
(L. H. Ungar et al., 1998) [10]	Movie Domain	Probabilistic Approach	Accuracy (RMSE)
(Delgado et al., 1999) [11]	-	Combination of Memory-based and online learning algorithms	-
(Y. H. Chen & E. I. George, 1999)	EachMovie	Bayesian approach with Markov Chain Monte Carlo methods.	Accuracy (MSE)
(K. Goldberg et al., 2000) [25]	Jester	Pearson correlation coefficient, PCA, Recursive Rectangular clustering	Accuracy (Normalized Mean Absolute Error (NMAE))
(B. Sarwar et. al, 2001) [15]	MovieLens	Memory-based techniques	Accuracy (MAE)
(G. Linden et al., 2003) [16]	Amazon Dataset	Memory-based techniques	-
(Thomas Hofmann, 2003) [20]	EachMovie	Probabilistic model	Accuracy (RMSE)
(Benjamin Marlin, 2003) [21]	EachMovie and MovieLens	Probabilistic model	Accuarcy (NMAE)
(Z. Huang et al., 2005) [26]	Book Sales Dataset	Graph-based approach	Performance (Recall, Precision Rank Score, F-Measure)
(X. Su et al., 2006) [19]	MovieLens and other real world multi-class dataset	Bayesian Belief Model	Accuracy (MAE)
(X, Wan et al., 2008) [27]	-	Markov Chain Model and multi-dimensional approach	Efficiency (conformity rate)
(P. S. Chakraborty, 2009) [22]	MovieLens	Incremental clustering approach	Accuracy (MAE)
(K. Alodhaibi et al., 2011) [28]	MovieLens	Randomized algorithm with Greedy approach	MAE, Mean Actual Rating (MAR) & Diversity Measurement
(C. Chen et al., 2013) [29]	Epinions	Social trust with probabilistic approach	Accuracy (MAE)
(D. Zhang et al., 2013) [23]	MovieLens and Netflix	Bi-clustering with smoothing and fusion technique	Accurcy (RMSE & MAE)
(X. Luo et al., 2015) [18]	MovieLens and Jester	Latent Factor model based on Hessian-free optimization technique	Accuracy (RMSE & MAE)
(H. Wang et al., 2015) [30]	CiteULike Dataset	Hierarchical Bayesian Model	Performance (Recall)
(G. Guo et al., 2016) [24]	FilmTrust, Ciao, Epinions and Flixster	Trust based SVD	Accuracy (RMSE & MAE)
(R. Lopes et al., 2016) [31]	BookCrossing, Amazon, MovieLens, FilmTrust	Graph-based approach with Bayesian paradigm	Precision, Mean Average Precision (MAP), Mean Reciprocal Rank (MRR), Normalized Discounted Cumulative Gain (NDCG)
(A. Davoudi & M. Chatterjee, 2016) [32]	Epinions	Trust model with probabilistic Matrix Factorization approach & Vector space similarity method with centrality measures	Accuracy (MAE)
(S. Deng et al., 2016) [33]	Flixster & Epinions	Deep Learning based Matrix Factorization	RMSE, Coverage, Precision, F-measure

C. Sparsity

Collaborative filtering approaches depend solely on rating database whether to predict similartity between user/item or to train a model. Most of the items are experienced by only a few users therefore, user-item matrix formed is extremely sparse due to insufficient rating data which makes algorithms inefficient to measure similarity between users. One of the reasons for sparsity is huge item-to-user ratio.

D. Scalability

As system gets mature with time, number of users and items increases, leads to millions of ratings which results in slow computations and degrades the quality of recommendation system.

E. Gray Sheep Problem

'Gray sheep' term is used for the users those who possess unusual taste that rarely matches to any other group of people. For these users, it becomes very difficult to determine like-minded people and thus, CF based approaches fails to generate promising recommendations which deteriorates the quality of recommender system and results in dissatisfaction of user.

F. Ramp-Up Problem

A large amount of rating data should be initialized for CF approaches to perform well, because it is not useful with a small base of ratings. And also, the accuracy of CF-based system depends on the number of items rated [13]. This refers to ramp-up problem i.e. the system will not be able to produce useful recommendations for most of the users until there is a sufficient number of people whose interests are known as well as the large set of rated items.

G. Shared Account Problem

Sometimes, it happens that one account is shared by two or more people with varied set of interests like an account is shared by a lady and her children. That account is used by lady to order crockery and her children order games CDs using the same account. This makes RSs inefficient to analyze the needs of person and it may end up recommending new games CDs, play station etc to the lady and vase, decorative items to children which may results in customer dissatisfaction.

H. Evaluation Measures

Most of the people emphasize on accuracy of RSs which means how closely it predicts the user rating. To realize the full strength of RSs, significant amount of effort is needed in the direction of novelty, serendipity and diversity. New evaluation techniques should be developed that can measure these attributes along with accuracy.

I. Limited Scope of RSs

RSs have been used in limited areas as shopping sites, movies, songs, websites, and so on. It can be applied to other domains like financial investment, medical and health related areas, diet recommendation etc.

V. CONCLUSION & FUTURE WORK

Recommender Systems are very helpful in providing solution to information explosion. Several techniques are available to accomplish the task of recommendations. Among them, Collaborative Filtering is one of the most widely used and successful approaches. Memory-based CF techniques focus on finding the similar user pairs and item pairs for user-based CF and item-based CF respectively. Pearson Correlation, Cosine similarity etc. can be used to determine similarity between user and item vectors. Prediction is made using weighted sum of ratings, simple weighted average and so on. Top-N Recommendations can be produced based on the highly-rated items. Model-based CF techniques develop a model using user rating pattern; further this model is used to make predictions and recommendations. Bayesian networks, clustering, MF, SVD etc. can be used to predict missing values and for recommending items of keen interest.

We have also discussed some challenges related to CF, many researchers have provided solution to these problems but there is a long way to go to achieve satisfactory results. One can also explore new methods to combine collaborative filtering with other approaches to resolve scalability and cold-start issues. CF based approaches can be enhanced by incorporating trust, cross-domain information, context and time-variant features.

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