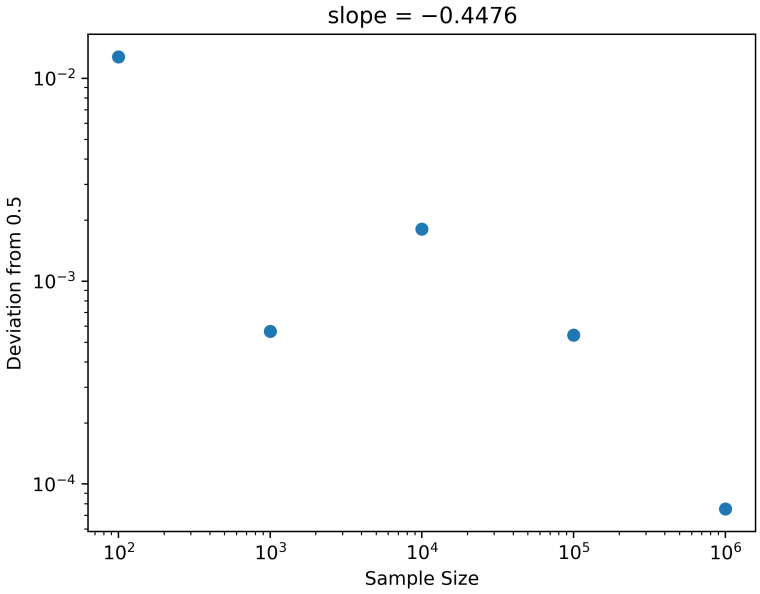
Assignment 1

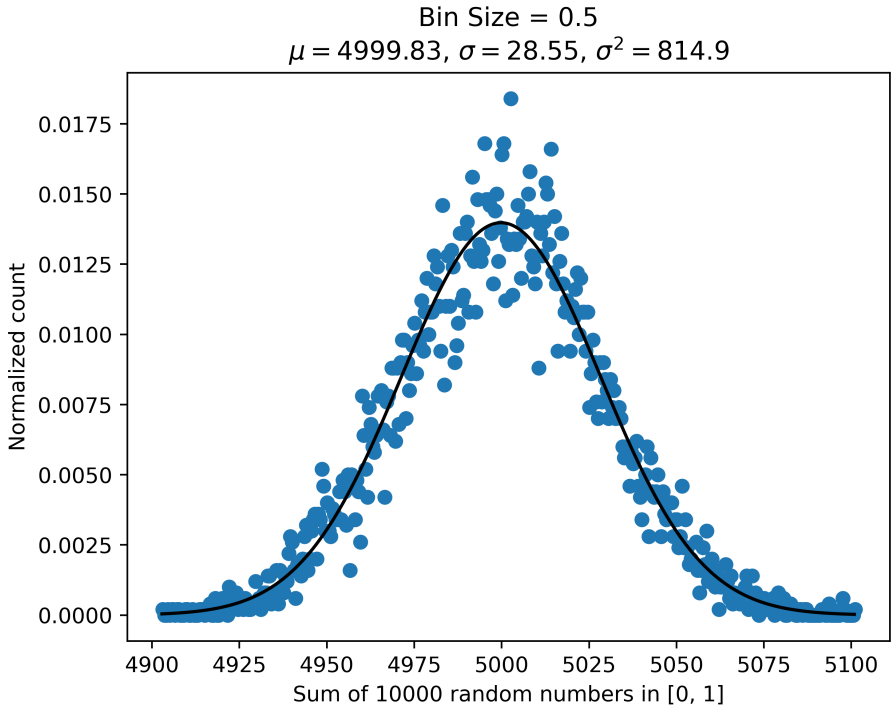
Vignesh M Pai (20211132)

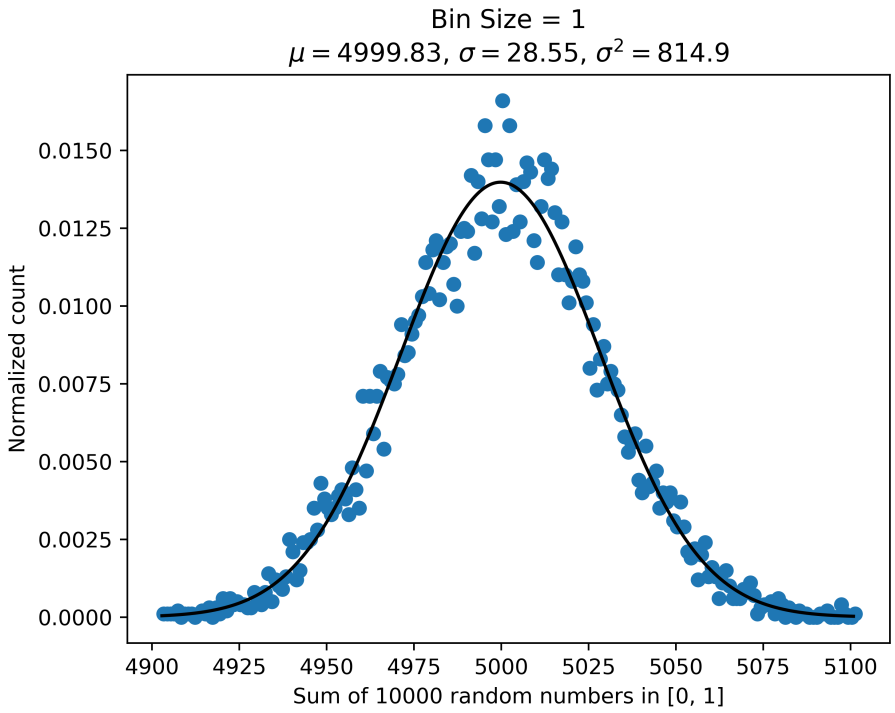
1. g)

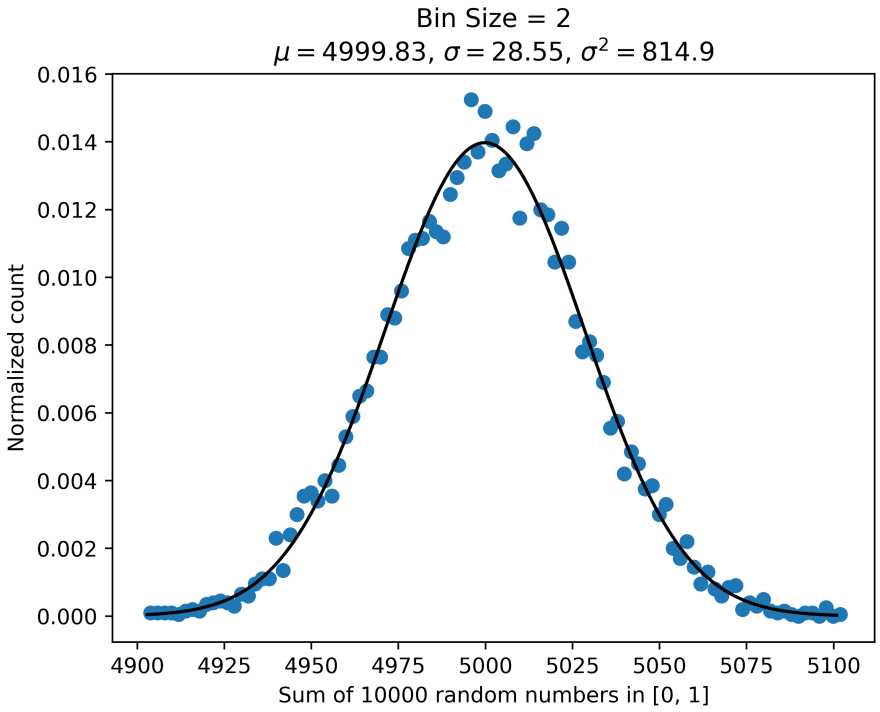


The deviation of the average from the expectation falls off approximately as $N^{-1/2}$ where N is the sample size. This follows from CLT which states that the standard deviation of the sample mean falls at this same rate.

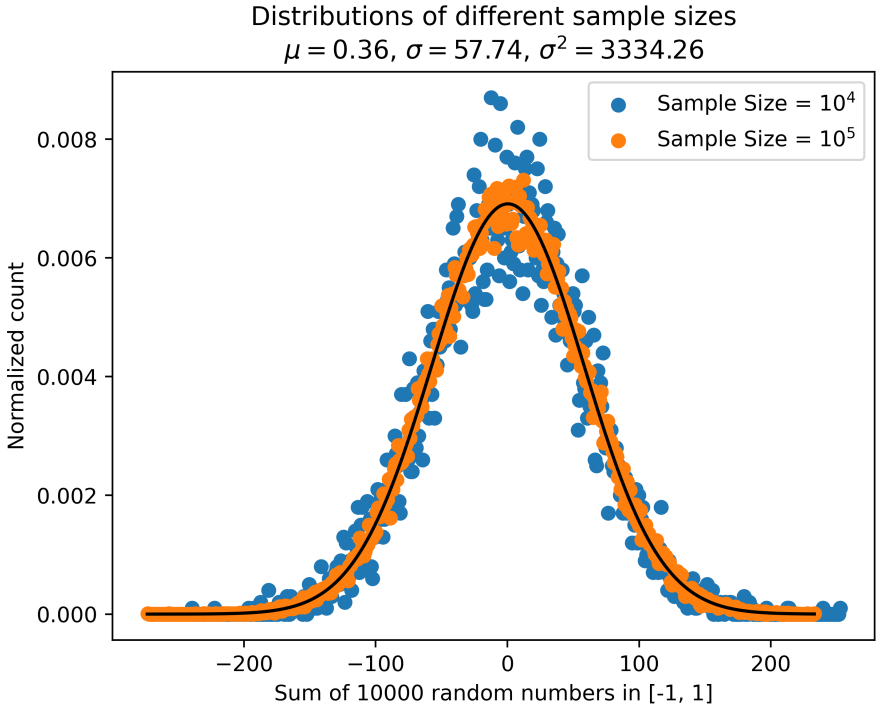
1. h)


Bin Size = 0.5
$\mu = 4999.83$, $\sigma = 28.55$, $\sigma^2 = 814.9$
Normalized count
0.0175
0.0150
0.0125
0.0100
0.0075
0.0050
0.0025
0.0000
4900
4925
4950
4975
5000
5025
5050
5075
5100
Sum of 10000 random numbers in [0, 1]


Bin Size = 1
$\mu = 4999.83$, $\sigma = 28.55$, $\sigma^2 = 814.9$
Normalized count
0.0150
0.0125
0.0100
0.0075
0.0050
0.0025
0.0000
4900
4925
4950
4975
5000
5025
5050
5075
5100
Sum of 10000 random numbers in [0, 1]

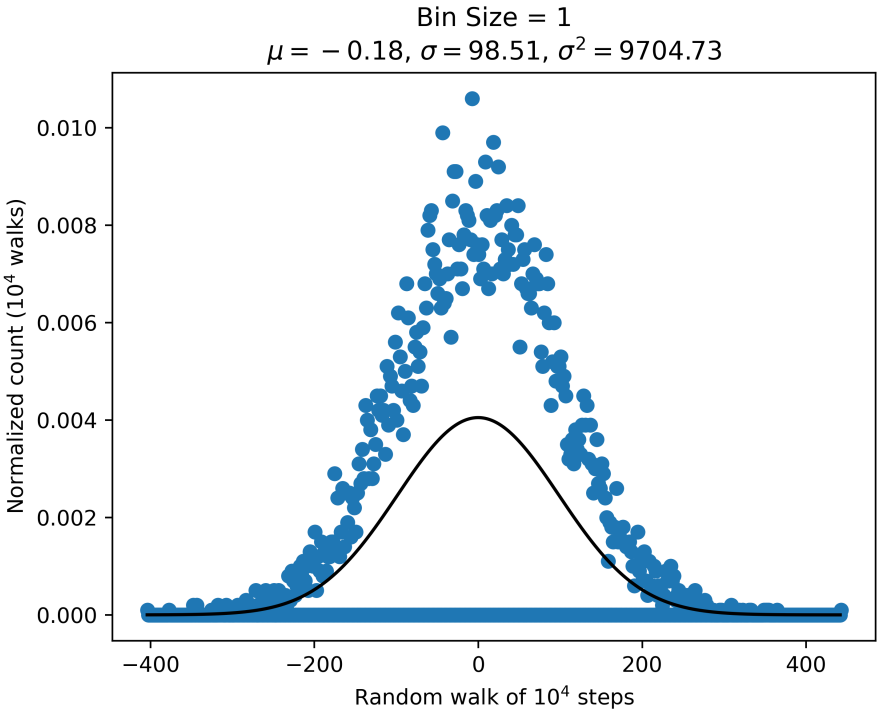


Increasing the bin size reduces the spikes in the distribution and makes it appear smoother.



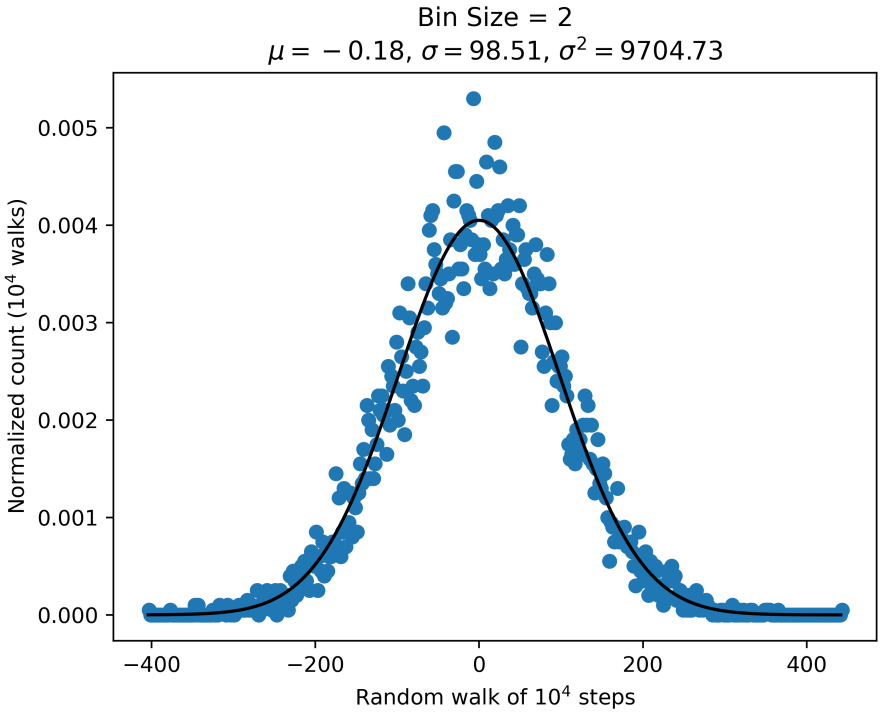
The mean and standard deviations in larger samples appears to be similar, however the distribution with more samples is smoother.

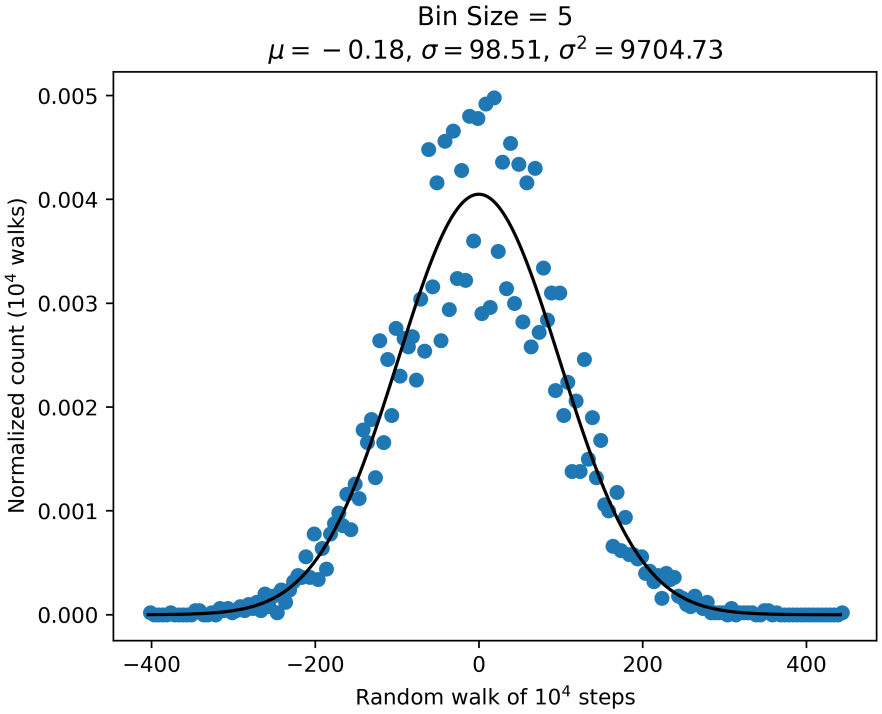
1. i)

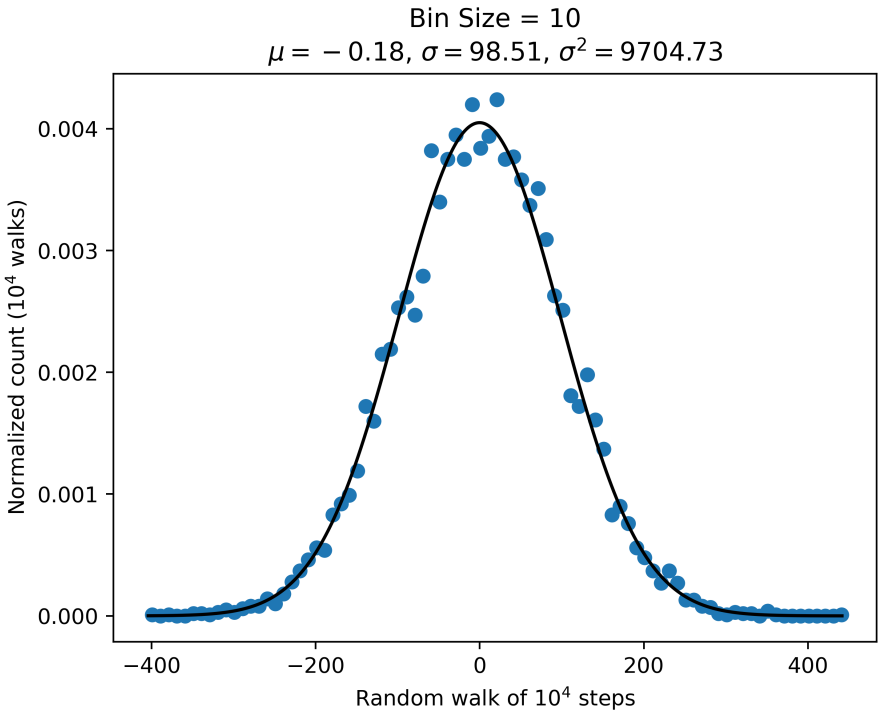


Alternate bins in the distribution are empty, this is because we have taken an even number of steps, thus the endpoint must be even. Hence, bins such as $[1, 2)$ will be empty. To make up for these bins, bins such as $[0, 1)$ will have twice the count estimated by CLT.

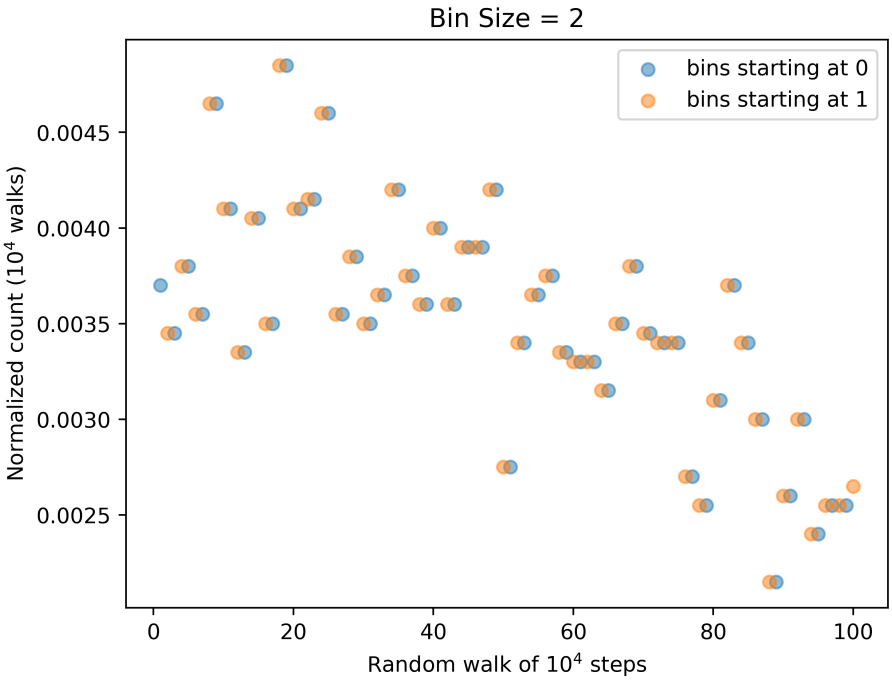
1. j)


Bin Size = 2
$\mu = -0.18, \sigma = 98.51, \sigma^2 = 9704.73$
Normalized count (10^4 walks)
0.000
0.001
0.002
0.003
0.004
0.005
−400
−200
0
200
400
Random walk of 10^4 steps


Bin Size = 5
$\mu = -0.18, \sigma = 98.51, \sigma^2 = 9704.73$
Normalized count (10^4 walks)
0.000
0.001
0.002
0.003
0.004
0.005
−400
−200
0
200
400
Random walk of 10^4 steps

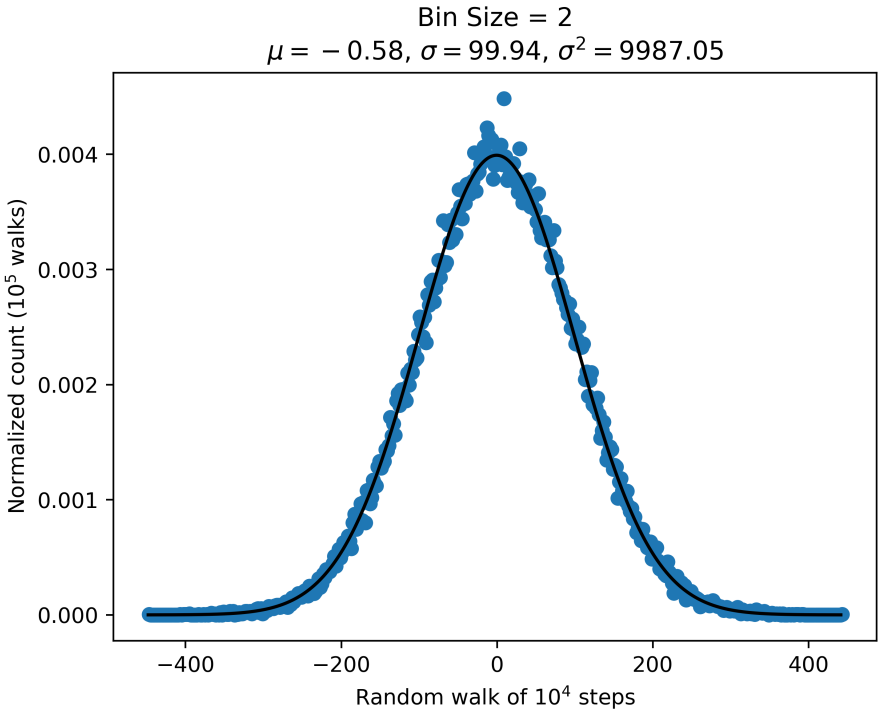


These larger bin size distribution do not have the properties of bin size 1.

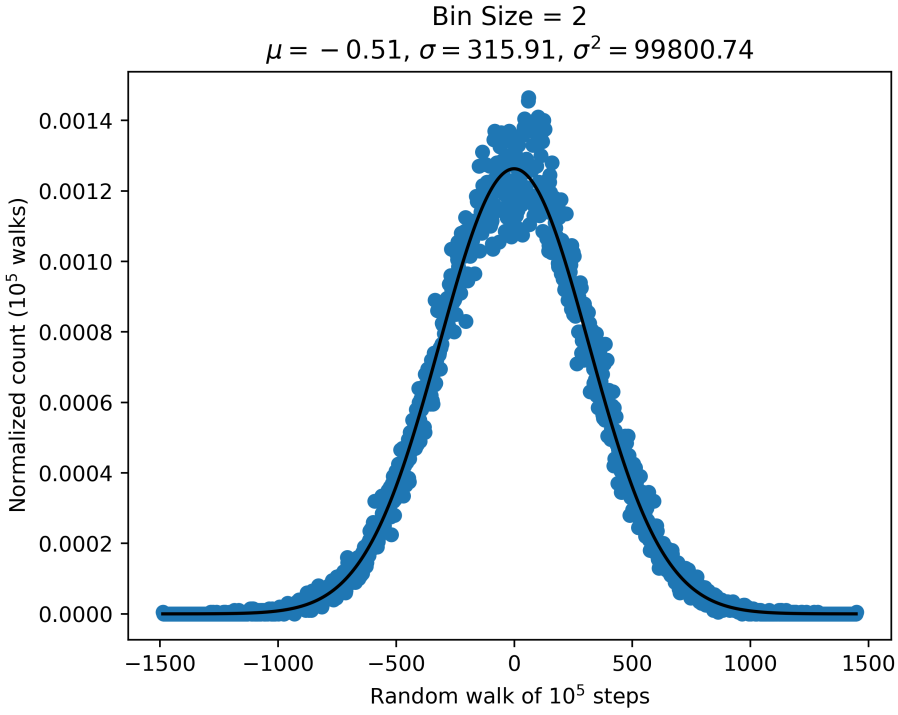


Shifting the bins shifts the histogram.

1. k)


Bin Size = 2
$\mu = -0.58, \sigma = 99.94, \sigma^2 = 9987.05$
Normalized count (10^5 walks)
0.004
0.003
0.002
0.001
0.000
−400
−200
0
200
400
Random walk of 10^4 steps

1. l)


Bin Size = 2
$\mu = -0.51, \sigma = 315.91, \sigma^2 = 99800.74$
Normalized count (10^5 walks)
0.0014
0.0012
0.0010
0.0008
0.0006
0.0004
0.0002
0.0000
−1500
−1000
−500
0
500
1000
1500
Random walk of 10^5 steps