Monte Carlo Integration: Assignment 1 **Due date: 30/01/2023**

Module 2: https://apratimchatterji.wixsite.com/apratim/computational-physics-nptel-videos

Q1a. By trapezoidal method calculate $\int_0^1 \frac{4}{1+x^2} dx$. You know the answer will be π.

b. Choose dx =0.01d0, 0.001d0, 0.0001d0, 0.00001d0 (you need to use double precision real*8). Does the error go as (1/n^2), n is the number of intervals as mentioned in lectures? Do a log-log plot of error versus n, fit a straight line in log-log plot, and confirm for yourself that the slope is 2.

c. Change the integrand function to sin(x) between 0 and π (double precision). The value of π is generated in fortran as 2*asin(1) (=pi). Choose suitable values of dx and convince yourself that integration scheme works.

d. Now take a normalized Gaussian function $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ with standard deviation (SD=1). Integrate between -3 and +3. What value do you expect to get? Does your answer match?

Q2. Use a random number generator which generates uniform random numbers between 0 and 1.

a. Plot probability distribution data to prove that you have random numbers with uniform deviate.

b. Do a scatter plot to show that the random numbers are uncorrelated.

c. Calculate the correlation function to convince yourself that random numbers have no correlation.

d. Calculate the standard deviation (SD) of the random numbers about the mean.

Q4. Generate random numbers with (a) exponential (e^{-2x}) and (b) Gaussian (with SD=2) distributions.

Q5. Write a program that computes the following multi-dimensional integral using the (a) brute force and (b) importance sampling Monte Carlo integration methods:

$$I = \int_{-\infty}^{\infty} d^3\vec{x} d^3\vec{y} g(\vec{x}, \vec{y})$$

where $g(\vec{x}, \vec{y}) = \exp(-\vec{x}^2 - \vec{y}^2 - \frac{(\vec{x}-\vec{y})^2}{2})$

Suppose you do not know the exact value of the integral. How do you compute the error? Compare the efficiency of the two methods.