Assignment 4 + 5: Solution of Differential equations (ODE + PDE) to be completed by 16th Feb 2023.

Also plot the solution of D.E. for the solutions to compare as shown in lectures.

Solve the differential equation **$dy/dx = y^2 + 1$** from the interval 0 to 1.55 using Euler method, Modified and Improved Euler methods with dx =0.001. Carry out the same calculation using RK4 method with dx=0.01. At x=0, y =0 (the initial condition). Save the data in 4 different files. The solution to the differential is y = tan(x). The actual value of y (x=1.55) = y_A = 48.078.

The solutions obtained using Euler, Modified Euler, Improved Euler and RK4 methods will be referred to as y_E, y_{ME} y_{IE} and y_{RK4} respectively. Now answer the following questions.

Q1. The value of the difference $y_A - y_E$ at x=1.550 is:

Q2. The value of the difference $y_A - y_{ME}$ at x=1.55 is:

Q3. The value of the difference $y_A - y_{IE}$ at x=1.55 is :

Q4. The value of the difference $y_A - y_{RK4}$ at x=1.55 is :

Q5. Solve the differential equation **$d^2x / dt^2 = -\sin(x)$** with initial values x_0 =0.1 and v_0 =1.9 at time t=0. The integration interval dt =0.01. Run the calculation for 5000 iterations, i.e. for elapsed time =50. The value of x at the end of 5000 iterations is :

Q6. For the previous problem if the initial conditions were changed to x_0 =0 and v_0 = 1.999, then the value of x at the end of 5000 iterations is :

Q7. Why does the solution look so different when v_0 > 2.0d0

Q8. Suppose you have 50 particles in a circular ring of radius 5.0 placed on the x-z plane. There is periodic boundary conditions such that the *50*-th particle has particle number *1* and *49* as its neighbours. The particles can only be displaced from the x-z plane along the y-direction. The spring constant kappa =1, and the mass of each of the particles is 1, such that kappa/m = km =1.

The force acting on particle *i* in the y direction is **$f_i^y = km *(y_{i+1} + y_{i-1} - 2\, y_i)$.**

The initial conditions are (i) velocity v_i (t=0) = 0 for all the particles AND (ii) y_i (t=0) = 0 for all *i,* except for two particles, i.e. 1 and 26: y_1 *(t=0)* **=** y_{26} *(t=0)* = 0.8 .

Using RK4, with dt =0.02, find y_i as a function of time for all *i*. The position of the 1-st particle after 2000 iterations (i.e. at time t=40) is:

9. Solve the differential equation **y'' - 5 y' + 10y = 10x** using Gauss Seidel method and with the Boundary conditions y(x=0) =0.d0 and y (x=1.0) = 2.0d0 with dx =0.01 and convergence condition as 0.0001. Use double precision (real*8) for all real variables.

The boundary condition y (x=1.0) = 2.0d0 implies the value of y =2.0.0d0 at x=1.0. The value of y at x=0.80 is :