Part 1

1

$$\|b + c\| = \left\| \begin{bmatrix} -1 \\ -5 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ -3 \\ -6 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 2 \\ -8 \\ -5 \end{bmatrix} \right\| = \sqrt{4+64+25} = \sqrt{93}$$

2

$$2a+b = 2\begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix} + \begin{bmatrix} -1 \\ -5 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \\ 4 \end{bmatrix} + \begin{bmatrix} -1 \\ -5 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ -7 \\ 5 \end{bmatrix}$$

3

$$a \cdot c = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ -3 \\ -6 \end{bmatrix} = 12 + 3 - 12 = 3$$

4

$$\left\| \frac{a}{2} - c \right\| = \left\| \frac{1}{2}\begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ -3 \\ -6 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 2 \\ -1/2 \\ 1 \end{bmatrix} - \begin{bmatrix} 3 \\ -3 \\ -6 \end{bmatrix} \right\| = \left\| \begin{bmatrix} -1 \\ 2.5 \\ 7 \end{bmatrix} \right\|$$

$$= \sqrt{1 + 2.5^2 + 49}$$

$$= \sqrt{56.25} = 7.5$$

5

$$b \cdot (a+c) = \begin{bmatrix} -1 \\ -5 \\ 1 \end{bmatrix} \cdot \left(\begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ -3 \\ -6 \end{bmatrix} \right) = \begin{bmatrix} -1 \\ -5 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ -4 \\ -4 \end{bmatrix}$$

Part 2

① $$AB^T = \begin{bmatrix} -3 & 8 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 2 & 6 & -9 \\ -7 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -6 + 8 \cdot 7 & -3 \cdot 6 + 8 \cdot -1 & 27 + 4 \cdot 8 \\ 10 & -5 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} -62 & -26 & 59 \\ -35 & -5 & 20 \end{bmatrix}$$

② $$CD = \begin{bmatrix} -6 & 0 & 5 \\ 1 & 3 & -2 \\ 7 & -5 & -8 \\ 4 & 9 & -10 \end{bmatrix} \begin{bmatrix} -4 & 0 & 3 \\ 8 & -2 & 5 \\ 6 & -3 & 1 \end{bmatrix} = \begin{bmatrix} -6 \cdot -4 + 30 & -15 & -18 + 5 \\ -4 + 24 - 12 & -6 + 6 & 3 + 15 - 2 \\ -28 - 40 - 48 & +10 + 24 & 21 + -25 - 8 \\ -16 + 72 - 60 & -16 + 72 - 96 & 12 + 45 - 10 \end{bmatrix} = \begin{bmatrix} 54 & -15 & -13 \\ 8 & 0 & 16 \\ -116 & 34 & -12 \\ -4 & 12 & 47 \end{bmatrix}$$

③ $$DB = \begin{bmatrix} -4 & 0 & 3 \\ 8 & -2 & 5 \\ 6 & -3 & 1 \end{bmatrix} \begin{bmatrix} 2 & -7 \\ 6 & -1 \\ -9 & 4 \end{bmatrix} = \begin{bmatrix} -8 - 27 & +28 \ \ 12 \\ 16 - 12 \ \ 15 & -56 \ \ 1 \ +20 \\ 12 \ \ -9 & -42 \ \ 3+4 \end{bmatrix} = \begin{bmatrix} -35 & 40 \\ -41 & -34 \\ -15 & -35 \end{bmatrix}$$

④ Ce $4 \times 3 \cdot 4 \times 1$ so dime n mismatch

⑤ $$e^T C = \begin{bmatrix} 7 & -8 & 10 & 1 \end{bmatrix} \begin{bmatrix} -6 & 0 & 5 \\ 1 & 3 & -2 \\ 7 & -5 & -8 \\ 4 & 9 & -10 \end{bmatrix} = \begin{bmatrix} -42 - 8 + 70 + 4 & -24 - 50 + 9 & 35 + 16 - 80 - 10 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & -83 & -19 \end{bmatrix}$$

part 3

1 $a\begin{bmatrix}8\\-2\end{bmatrix}+b\begin{bmatrix}1\\4\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$8a+b=0 \qquad a=b/8$

$-2a+4b=0 \qquad a=2b$

$\rightarrow \frac{b}{8}=2b \qquad b=0 \Rightarrow a=0$

Since a and b are independent they span $\mathbb{R}^2$

2 $a\begin{bmatrix}8\\-2\end{bmatrix}+b\begin{bmatrix}-3\\0.75\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$8a-3b=0 \qquad 8a=3b \qquad a=\frac{3}{8}b$

$-2a+0.75b=0 \qquad a=\frac{0.75}{-2}b \qquad \frac{0.75}{+2}b=\frac{3}{8}b$

$m=\frac{8}{-2}=-4$

$b=b$

Since a and b are dependent they span the line $y=\frac{-1}{4}x$

3 from part 1 we have shown they are linearly independent

4 Since a, b are linearly dependent that is enough to say the set of all are linearly dependent

5 $a\cdot b=\begin{bmatrix}8\\-2\end{bmatrix}\cdot\begin{bmatrix}1\\4\end{bmatrix}=8+-8=0$ So a,b are orthogonal

$b\cdot c=\begin{bmatrix}1\\4\end{bmatrix}\cdot\begin{bmatrix}-3\\0.75\end{bmatrix}=-3+3=0$ So b,c are orthogonal

$a\cdot c=\begin{bmatrix}8\\-2\end{bmatrix}\cdot\begin{bmatrix}-3\\0.75\end{bmatrix}=-24+-1.5\neq 0$ So a,c are not

Part 4

1)
$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 4 \\ 3 & 5 & 5 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 4 \\ 0 & -1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 0 & -1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

part 5

$$\text{proj}_a(b) = \frac{b \cdot a}{a \cdot a} \cdot a = \frac{\begin{bmatrix} 0 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix}}{\begin{bmatrix} -1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix}} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \frac{9}{10} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -0.9 \\ 2.7 \end{bmatrix}$$

so $(-0.9, 2.7)$ is the closest point to b from span a

part 6

$$3x + y - 5z = 27$$
$$-x + 4y + z = -15$$
$$x + 0y + 2z = -5$$

$$\left[\begin{array}{ccc|c} 3 & 1 & 5 & 27 \\ -1 & 4 & 1 & -15 \\ 1 & 0 & 2 & 5 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ -1 & 4 & 1 & -15 \\ 3 & 1 & -5 & 27 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 4 & 3 & -20 \\ 3 & 1 & -5 & 27 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 1 & -11 & 42 \\ 0 & 4 & 3 & -20 \end{array}\right] \Rightarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 1 & -11 & 42 \\ 0 & 0 & 47 & -171 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -4 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -4 \end{array}\right]$$

$x = 3, y = -2, z = -4$

part 7

a)

i) $\det(A - \lambda I) = 0$

$$\det\left(\begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 0$$

$$\det\begin{pmatrix} -6-\lambda & 3 \\ 4 & 5-\lambda \end{pmatrix} = 0$$

$$(-6-\lambda)(5-\lambda) - (12) = 0$$

$$-30 - 5\lambda + 6\lambda + \lambda^2 - 12 = 0 \Rightarrow \lambda^2 + \lambda - 42 = 0$$

$$\lambda = -7, 6$$

ii)

$$(A - \lambda I)\,v = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) v = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -6-\lambda & 3 \\ 4 & 5-\lambda \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$v_1(-6-\lambda) + 3v_2 = 0$$

$$4v_1 + v_2(5-\lambda) = 0$$

Now solve for $\lambda = \{6, -7\}$:

6:

$$-12v_1 + 3v_2 = 0$$

$$4v_1 - v_2 = 0 \qquad v_2 = 4v_1 \qquad \frac{v_1}{v_2} = \frac{1}{4} \quad \text{So } v = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

-7:

$$v_1 + 3v_2 = 0 \qquad v_1 = -3v_2$$

$$4v_1 + 12v_2 = 0 \qquad \frac{v_1}{v_2} = \frac{-1}{3} \quad \text{So } v = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

So we have eigen vectors $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ or a scalar multiple of them

)

i) $\det(B - \lambda I) = 0$

$$\det\left(\begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0$$

$$\det\begin{pmatrix} 5-\lambda & 6 \\ 2 & 1-\lambda \end{pmatrix} = 0$$

$$(5-\lambda)(1-\lambda) - 12 = 0$$

$$5 - 5\lambda - \lambda + \lambda^2 - 12 = 0$$
$$\lambda^2 - 6\lambda - 7 = 0$$
$$\lambda = \{-1, 7\}$$

ii)

$$(B - \lambda I)\,v = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5-\lambda & 6 \\ 2 & 1-\lambda \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

$$v_1(5-\lambda) + 6v_2 = 0$$
$$2v_2 + v_2(1-\lambda) = 0$$

-1:

$6v_1 + 6v_2 = 0 \qquad v_1 = -v_2 \qquad \frac{v_1}{v_2} = \frac{-1}{1}$ So $v = \begin{bmatrix} +1 \\ -1 \end{bmatrix}$

$+7$:

$-2v_1 + 6v_2 = 0 \qquad \frac{v_1}{v_2} = \frac{6}{2} = \frac{3}{1}$ So $v = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$2v_2 - 6v_2 = 0$

So the eigenvectors are $\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ or a scalar multiple of them