

a)

$$A = \begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$\det\left(\begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0$$

$$\det\begin{pmatrix} 2-\lambda & -2 \\ -1 & 3-\lambda \end{pmatrix} = 0$$

$$(2-\lambda)(3-\lambda) - 2 = 0$$

$$6 - 2\lambda - 3\lambda + \lambda^2 - 2 = 0$$

$$\lambda^2 - 5\lambda + 4 = 0 \quad \boxed{\lambda = \{1, 4\}}$$

$$(A - \lambda I)V = 0$$

$$\left(\begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)V = 0$$

$$\begin{bmatrix} 2-\lambda & -2 \\ -1 & 3-\lambda \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$v_1(2-\lambda) - 2v_2 = 0 \Rightarrow 2v_1 - 4v_2 - 2v_2 = 0$$

$$-v_1 + v_2(3-\lambda) = 0$$

$$\lambda = 1:$$

$$\begin{aligned} 2v_1 - v_1 - 2v_2 &= 0 \\ v_1 - 2v_2 &= 0 \quad \frac{v_1}{v_2} = 2 \end{aligned}$$

$$\text{So } v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\lambda = 4:$$

$$\begin{aligned} 2v_1 - 4v_1 - 2v_2 &= 0 \\ -2v_1 - 2v_2 &= 0 \quad \frac{v_1}{v_2} = \frac{+1}{-1} \end{aligned}$$

$$\text{So } v = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \text{ and } \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

b)

$$B = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$$

$$\det(B - \lambda I) = 0$$

$$\det\left(\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0$$

$$\det\begin{pmatrix} 7-\lambda & 3 \\ 3 & -1-\lambda \end{pmatrix} = 0$$

$$(7-\lambda)(-1-\lambda) - 9 = 0$$

$$-7 - 7\lambda + \lambda + \lambda^2 - 9 = 0$$

$$\lambda^2 - 6\lambda - 16 = 0 \quad \boxed{\lambda = \{-2, 8\}}$$

$$(B - \lambda I)V = 0$$

$$\left(\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)V = 0$$

$$\begin{bmatrix} 7-\lambda & 3 \\ 3 & -1-\lambda \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$v_1(7-\lambda) + 3v_2 = 0$$

$$3v_1 + v_2(-1-\lambda) = 0$$

$$\lambda = -2:$$

$$9v_1 + 3v_2 = 0$$

$$\frac{v_1}{v_2} = \frac{-1}{3}$$

$$\text{So } v = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$\lambda = 8:$$

$$-v_1 + 3v_2 = 0$$

$$\frac{v_1}{v_2} = \frac{3}{1} \quad \text{So } v = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\text{So } v = \begin{bmatrix} -1 \\ 3 \end{bmatrix} \text{ and } \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

or scalar multiples of the two

②

$$\text{Proj}_b(a) = \frac{a \cdot b}{b \cdot b} \times b$$

$$= \frac{\begin{bmatrix} -4 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}}{\begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{-4 + 2}{1 + 4} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= \frac{-2}{5} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2/5 \\ -4/5 \end{bmatrix}$$