DSA 5303. HW1

①. The money invested is X_0. The money recieved at the end of year is $X_0 - X_1 + X_0$.

Hence, $R = \dfrac{2X_0 - X_1}{X_0}$.

④. Let α, β equal to the percent of investment in stock 1 and stock 2 respectively.

The problem is

$$\min_{\alpha,\beta} \alpha^2\sigma_1^2 + \beta^2\sigma_2^2 + 2\alpha\beta\sigma_{12}.$$

subject to $\alpha + \beta = 1$.

Setting up Langrangian L, we have

$$L = \alpha^2\sigma_1^2 + \beta^2\sigma_2^2 + 2\alpha\beta\sigma_{12} - \lambda(\alpha+\beta-1)$$

The first order necessary conditions are

$$0 = \frac{\partial L}{\partial \alpha} = 2\alpha\sigma_1^2 + 2\beta\sigma_{12} - \lambda.$$

$$0 = \frac{\partial L}{\partial \beta} = 2\beta\sigma_2^2 + 2\alpha\sigma_{12} - \lambda.$$

$$1 = \alpha + \beta$$

which imply $\alpha = \left[\dfrac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}\right]$

The mean rate of return is just

$\alpha m_1 + \beta m_2$

---

(8).

(a) $\mathrm{var}(r - r_m) = \mathrm{var}(r) - 2\,\mathrm{cov}(r, r_m)$
$+ \mathrm{var}(r_m)$.

$$= \sum_{i,j=1}^{n} \alpha_i \alpha_j \sigma_{ij} - 2 \sum_{i=1}^{n} \alpha_i \sigma_{im} + \sigma_m^2.$$

So, to minimize $\mathrm{var}(r - r_m)$ subject to $\sum_{i=1}^{n} \alpha_i = 1$ set up the Langrangian.

$$L = \sum_{i,j=1}^{n} \alpha_i \alpha_j \sigma_{ij} - 2 \sum_{i=1}^{n} \alpha_i \sigma_{im} + \sigma_m^2$$
$$+ \lambda \left(\sum_{i=1}^{n} \alpha_i - 1 \right)$$

The first order necessary conditions imply.

$$2 \sum_{j=1}^{n} \alpha_i \sigma_{ij} - 2\sigma_{im} + \lambda = 0, \text{ for all } i.$$

$$\sum_{i=1}^{n} \alpha_i = 1.$$

(b) Similar to (a) with added constraint

$$\sum_{i=1}^{n} \alpha_i r_i = m$$

so the first order necessary conditions imply.

$$2\sum_{j=1}^{n} \alpha_i \sigma_{ij} - 2\sigma_{im} + \lambda + \mu r_i = 0 \text{ for all } i.$$

$$\sum_{i=1}^{n} \alpha_i = 1 \quad , \quad \sum_{i=1}^{n} \alpha_i r_i = m.$$

(6). The market consists of \$150 in shares of A and \$300 in shares of B. Hence the market return is.

$$r_m = \left(\frac{150}{450}\right) r_A + \left(\frac{300}{450}\right) r_B$$

$$r_m = 1/3\, r_A + 2/3\, r_B.$$

(a) $\bar{r}_m = 1/3 * 0.15 + 2/3 * 0.12 = 0.13$

(b). $\sigma_m = \left[1/9 (0.15)^2 + \frac{4}{9*3} (0.15)(0.09) + 4/9 (0.09)^2 \right]^{1/2} = 0.09$

standard deviation of market portfolio = 0.09.

(c) $\sigma_{AM} = \frac{1}{3}\sigma_A^2 + \frac{2}{3}\rho_{AB}\sigma_A\sigma_B$

$= \frac{1}{3}(0.15)^2 + \frac{2}{9}(0.15)(0.09)$

$= \underline{0.0105}$.

$\beta_A = \frac{\sigma_{AM}}{\sigma_M^2} = \underline{1.2963}$ = Beta of stock A.

(d) Since Simpleland satisfies the CAPM exactly, stock A and B plot on the security market line, specifically,

$\bar{r}_A - r_f = \beta_A(\bar{r}_M - r_f)$.

Hence, $r_f = \frac{\bar{r}_A - \beta_A \bar{r}_M}{1 - \beta_A} = \underline{0.0625}$.

= risk free rate of Simpleland.