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The Pearson Guide to

Objective Arithmetic

for competitive examinations

Second Edition

- Short-cut methods explained and illustrated
- MCQs with explanatory answers
- Timed exercises for practice
- Model test papers based on original examinations
- Bonus feature: data interpretation & data sufficiency

Dinesh Khattar

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The Pearson Guide to

OBJECTIVE ARITHMETIC

For Competitive Examinations

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OBJECTIVE ARITHMETIC

For Competitive Examinations

SECOND EDITION

SBI Officers'/Bank PO, RBI Officers' Grade Examination; RBI/Bank Clerk Examination; LIC/GIC-AAO Examination; SSC (Graduate and Matric Level); Assistants' Grade, Income Tax Inspectors, SI Delhi Police/CBI Examination; AC/DSP in BSF/CRPF/ITBP; BBA/MBA Entrance/MAT/GMAT/GRE/XAT/JMET; NIFT Entrance Examination; Clerks' Grade Examination of Railway Recruitment Board; NDA/CDS/SCRA Examination; Teachers' Recruitment Test for Kendriya Vidyalaya and Government Schools; NSTSE/National Science Olympiad/National Cyber Olympiad/Math Olympiad

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To my wife, Anita
for her love, support and encouragement
for her patience in going through the manuscript and proofs
for her efforts in bringing out this book in the shortest possible time…

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PREFACE TO THE SECOND EDITION

The overwhelming response to the first edition of this book from students has not only boosted my confidence but their feedback has inspired me to bring forth this second edition. Every effort has been made to weed out various typographical errors in this book. In order to provide a more focussed approach, the obsolete previous years question papers have been replaced with new ones.

I expect further suggestions for improvement of this book.

Best of luck.

DINESH KHATTAR

PREFACE

With so many books available in the market on objective arithmetic for students appearing in different competitive examinations, the publication of yet another volume on the subject requires some explanation. It has been my experience that the average student needs the treatment of theory in a manner easily understandable to him. An effort, therefore, has been made, in this book to put across concepts in a lucid and unambiguous manner. The book aims at helping students enhance their knowledge of arithmetic and equipping them with the skills that will enable them to succeed in any competitive examination.

The book is noteworthy in the following aspects:

1. Each chapter contains concise definitions and explanations of basic/fundamental principles, which are further augmented with illustrative examples to enable students to learn and recall fast.
2. Each chapter has a separate section on short-cut methods covering all kinds of questions asked in competitive examinations. In addition, each short-cut method is explained with the help of illustrative examples.
3. Completely worked-out solutions to a large range of problems have been included in the text. The number of questions in each chapter has been kept sufficiently large to provide rigorous practice.
4. A large number of problems that have been asked in the competitive examinations in recent times are included in every chapter with explanatory answers.
5. Practice exercises covering all the topics in each chapter are provided at the end for self-assessment
6. Model test papers based on different competitive examinations are provided at the end to facilitate students in understanding the pattern and the type of questions asked in the examinations.

The answers to almost all unsolved problems have been checked and every care has been taken to minimise typographical as well as factual errors. However, it is possible that a few errors might have managed to dodge the vigilant eye. The author will be grateful to the readers for bringing these errors to his notice as also for their valuable suggestions. It is earnestly hoped that the book will help the students grasp the subject and help them in obtaining a commendable score in the examination.

It would have been difficult to prepare this book without the aid and support from different people. Mr Amit Hasija from S. P. Jain Management and Research Institute, Mumbai and Dr Rudrashish Chakraborty from Kirori Mal College, University of Delhi, with whom the author had fruitful discussions, deserve special mention. My deepest appreciation goes to Mr Sanjay Singh for his incisive suggestions and observations about the style and content of the book.

I am thankful to the entire staff of Add Computers, Delhi, for typesetting the book.

I am deeply indebted to my parents without whose encouragement this dream could not have been translated into reality. And last, but not the least, it was the cherubic smiles of my daughters Nikita and Nishita that inspired me to treat my work as worship.

DINESH KHATTAR

COMPETITIVE EXAMINATIONS
AN OVERVIEW

DEFENCE SERVICES EXAMINATIONS

Combined Defence Services Examination

Combined Defence Services examination is conducted by the Union Public Service Commission (UPSC) twice every year for recruitment to the officer cadres of the Army, Navy and Air Force. The CDS examination is conducted generally in the months of May and October.

Educational Qualifications

For IMA/OTA: A degree from a recognised university or equivalent.
For Naval Academy: B.Sc. with physics and/or mathematics or a Bachelor of Engineering degree.
For Air Force Academy: B.Sc. from a recognised university or equivalent with physics and/or mathematics as subjects or a BE degree.

Age

For IMA: Unmarried male candidates between 19 and 24 years on 1 January or 1 July of the year succeeding the year of examination.
For Naval Academy: Unmarried male candidates between 19 and 22 years on 1 January or 1 July of the year succeeding the year of examination.
For Air Force Academy: Unmarried male candidates between 19 and 23 years on 1 January or 1 July of the year succeeding the year of examination.
For Officers Training Academy: Male candidates (married or unmarried) between 19 and 25 years on 1 January or 1 July of the year succeeding the year of examination.

Examination

Plan of the Examination: The examination comprises (*a*) written examination and (*b*) interview for intelligence and personality test of such candidates as may be called for interview at one of the Services Selection Centres.

Examination Subjects: The following will be the subjects for the written examination (each of 2 hours duration):

(*a*) For admission to Indian Military Academy, Naval Academy and Air Force Academy:
English – This paper tests the understanding of the English language.
General Knowledge – This tests the general knowledge, current events, history, Indian geography, and other matters of everyday observation.
Elementary Mathematics – This paper covers arithmetic, algebra, geometry, trigonometry, and statistics.

(*b*) For Admission to Officers Training Academy (OTA):
English – This paper tests the understanding of the English language.
General Knowledge – This tests the general knowledge, current events, history of India, geography of nature and other matters of everyday observation.

Notes: (*i*) The papers in all subjects will consist of objective-type (multiple-choice) questions only. The question papers will be set in English only.
(*ii*) Each paper will be of 2 hours duration and will carry 100 marks.

Standard of Examination: The standard of the paper in Elementary Mathematics will be of the matriculation examination. The standard of papers in other subjects will approximately be such as may be expected of a graduate of an Indian university.

Syllabus for Elementary Mathematics: *Arithmetic* – number system, natural numbers, integers, rational and real numbers, fundamental operations—addition, subtraction, multiplication, division, square roots, decimal fractions. *Unitary method* – Time and distance, time and work, percentage, applications to simple and compound interest, profit and loss, ratio and proportion, variation. *Elementary Number Theory* – Division algorithm, prime and campsite number, test of divisibility by 2, 3, 4, 5, 9 and 11, multiples and factors, factorisation theorem, H.C.F. and L.C.M. *Algebra* – Basic operations, simple factors, remainder theorem, H.C.F., L.C.M., theory of polynomials, solutions of quadratic equations, considered). Simultaneous linear equations in two unknowns – analytical and graphical solutions. Simultaneous linear inequations in two variables and their solutions. Practical problems leading to two simultaneous linear equations or inequations in two variables or quadratic equations in one variable and their solutions. Set language and set national, rational expressions and conditional identities, laws of indices. *Trigonometry* – Sine x, Cosine x, Tangent x when $0° \le x \le 90°$. Values of sin x,

cos x and tan x, for $x = 0°, 30°, 45°, 60°$ and $90°$. Simple trigonometric identities. Use of trigonometric tables. Simple cases of heights and distances. ***Geometry*** – Lines and angles, Plane and plane figures. Theorems on (*a*) properties of angles at a point, (*b*) parallel lines, (*c*) sides and angles of a triangle, (*d*) congruency of triangles, (*e*) similar triangles, (*f*) concurrence of medians and altitudes, (*g*) properties of angles, sides and diagonals of a parallelogram, rectangle and square, (*h*) circle and its properties, including tangents and normals, and (*i*) loci. ***Mensuration*** – Areas of squares, rectangles, parallelograms, triangle and circle. Areas of figures, which can be split up into the figures (field book). Surface area and volume of cuboids, lateral surface and volume of right circle cones and cylinders. Surface area and volume of spheres. ***Statistics*** – Collection and tabulation of statistical data. Graphical representation—frequency polygons, histograms, bar charts, pie charts, etc. Measures of central tendency.

NDA Entrance Examination

Recruitment of candidates to Army, Navy and Air Force wings of the National Defence Academy (NDA), is through National Defence Academy Entrance Examination which is held twice a year, generally in the month of April and September. NDA examination is conducted by the UPSC.

Age and marital status: A candidate must be an unmarried male and have attained the age of 16½ years and must not have attained the age 19 years on the 1st January or 1st of July of the year succeeding the year of examination.

Educational Qualifications: (1) For Army Wing of National Defence Academy: 12th Class pass of 10 + 2 pattern of School Education or equivalent examination conducted by a State education Board or a University. (2) For Air Force and Naval Wings of National Defence Academy and for the 10 + 2 (Executive Branch) course at the Naval Academy: 12th Class pass of the 10 + 2 pattern of a School Education or equivalent with Physics and Mathematics conducted by a State Education Board or a University. Candidates who are appearing in the 12th Class under the 10 + 2 pattern of School Education or equivalent examination can also apply.

Examination

Plan of the Examination: The examination comprises (*i*) written examination and (*ii*) intelligence, obstacles and group tests of the candidates qualifying the written examination.

Examination Subjects: The subjects of the written examination, the time allowed and the maximum marks allotted to each subject are as follows:

S.No	*Subject*	*Duration*	*Max. Marks*
1.	Mathematics	2½ hours	300
2.	General Ability Test (English, GK and Science)	2½ hours	600
	Total		900

Notes

- The papers in all subjects will consist of objective-type questions only.
- The question papers (test booklets) will be set in English only.

Syllabus of Arithmetic: ***Number Systems*** – natural numbers, integers, rational and real numbers. ***Fundamental operations*** – addition, subtraction, multiplication, division, square roots, decimal fractions. ***Unitary method*** – time and distance, time and work. ***Percentage*** – applications to simple and compound interest, profit and loss, ratio of proportion, variation. Elementary number theory, division algorithm prime and composite numbers. Tests of divisibility by 2, 3, 4, 6, 9, and 11. Multiples and factors. Factorisation theorem. H.C.F. and L.C.M. Euclidean algorithm. Logarithms to base 10, laws of logarithms. ***Mensuration*** – areas of squares, rectangles, parallelograms, triangle and circles. Area of figures, which can be split up into these figures (field book). Surface area and volume of cuboids, lateral surface and volume of right circular cones and cylinders. Surface area and volume of spheres. ***Algebra*** – basic operations, simple factors, remainder theorem, H.C.F., L.C.M. of polynomials. Solutions of quadratic equations, relation between its roots and coefficients (only real roots to be considered). Simultaneous linear equations to two unknown analytical and graphical solutions. Practical problems leading to two simultaneous linear equations in two variable or quadratic equations in one variable and their solutions. Set language and set nation. Rational expression and conditional identities. Law of indices. ***Geometry*** – lines and angles. Plane and plane figure. Theorems on properties of angles at a point, parallel lines, sides and angles of a triangle, congruency of triangles, similar triangles, concurrence of medians and altitudes, properties of angles, sides and diagonals of a parallelogram, rectangle and square, circles and their properties including tangents and normals, loci. ***Trigonometry*** – sine x, cosine , tangent x when $0° \leq x \leq 90°$. Value of sin x, cos x and tan x for $x = 0°, 30°, 45°, 60$ and $90°$. Simple trigonometric identities. Use of trigonometrical tables. Simple cases of heights and distances. ***Statistics*** – Collection and tabulation of

statistical data. Graphical representation, frequency polygons, histograms, bar charts, pie charts, etc. Calculation of mean of raw and grouped data.

SSC COMPETITIVE EXAMINATIONS

Assistants' Grade Examination

The Assistants' Grade Examination is conducted by the Staff Selection Commission for recruitment to (*i*) Grade IV (Assistant) of the General Cadre of the Indian Foreign Services (B); (*ii*) Grade II (Cypher Assistant) of Cypher Cadre of Indian Foreign Service Branch (B); (*iii*) Assistants' Grade of the Central Secretariat Service; (*iv*) Assistants' Grade of the Armed Forces Headquarters Civil Service; and (*v*) Posts of Assistants in other departments/organisations and the attached offices of the Government of India, not participating in IFS (B)/Railway Board Secretariat Service.

Age: A candidate must have attained the age of 20 years and must not have attained the age of 25 years on the first day of the August of the previous year. The upper age limit may be relaxed in respect of scheduled caste, scheduled tribes, other backward classes and such other categories of persons as may, from time to time, be notified in this behalf by the government of India to the extent and subject to the conditions notified in respect of each category. The supper age limit is also relaxed to 30 years in respect of LDCs/UDCs, Stenographers Grade 'D' and Grade II with not less than 3 years continuous and regular service in various departments/offices of Government of India.

Educational Qualifications: Degree from a recognised University or equivalent

Examination

Plan of Examination: The examination consists of two stages (*a*) preliminary examination and (*b*) main examination.

(a) Preliminary Examination (objective type): The preliminary examination will be of a test for selection of candidates for the main examination and would comprise the following subjects:

S.No.	*Test*	*No. of questions*	*Marks*	*Duration*
1.	Reasoning Ability	75	75	
2.	General Awareness	75	75	
3.	Arithmetic	75	75	3 hours
4.	Language Paper English or Hindi	75	75	
	Total	300	300	

Notes: (i) The papers in all the subjects will be of objective type (multiple choice type).
(ii) Question on reasoning ability, general awareness and arithmetic will be set in both English and Hindi.
(iii) Part (iv) language paper will be of qualifying nature and the marks secured by the candidates in this part will not be counted for the preparation of the list of candidates to be called for the main examination.
(iv) The aggregate marks secured in Part (i), (ii) and (iii) will be taken for preparation of the list of candidates to be called for main examination.

(b) Main Examination (Conventional Type): The main examination consists of the following papers:

Paper	*Test*	*Marks*	*Duration*
I	Language Comprehension		
	(a) General English	50	3 hours
	(b) Communication/Writing Skill	150	
II	Arithmetic	100	2 hours
	Total	300	

Notes: (i) The question papers for the Main Examination will be of conventional (Essay) Type.
(ii) In Paper I, Part (a) of Main Examination will be compulsory for all the candidates. However, this will be of a qualifying nature. The marks secured in this part will not be counted for preparation of the final selection list. The Commission have the discretion to fix a qualifying standard in this part.

(iii) Candidates will have the option to answer Paper I Part (b) (Communication/Writing Skill) and Paper II (Arithmetic) either in Hindi or in English.
(iv) Scripts of Paper I Part (b) (Communication/Writing Skill) and Paper II (Arithmetic) will be evaluated of only those candidates who secure minimum qualifying standard in Part (a) of paper I of the Main Examination.

Syllabus of Preliminary Examination (Arithmetic): This test will include questions on problems relating to Number Systems, Percentages, Ratio and Proportion, Averages, Estimation, Use of Tables and Graphs, Mensuration, Time and Distance, Ratio and time etc.

Clerks' Grade Examination

Clerks Grade Examination is conducted by the Staff Selection Commission for recruitment to the posts of clerks for the following groups of services/offices:

Group X

- Indian foreign Service (B) Grade VI
- Railway Board Secretarial Clerical Service Grade II
- Central Secretariat Clerical Service—Lower division Grade
- Armed Forces Headquarters Clerical Service—Lower Division Grade
- Ministry of Parliamentary Affairs
- President's Secretariat

Group Y

- Equivalent/comparable posts in Subordinate offices of Government of India located throughout India
- Offices of the Controller and Auditor General of India, Accountants General (Audit) and Accountants General (Accounts and Estt.) in various States
- Controller General of Defence Accounts
- Central Vigilance Commission
- Equivalent/comparable posts in other Departments and Attached Offices of Government of India not mentioned in group 'X'
- Equivalent/comparable posts in the offices of public sector undertakings, autonomous bodies, like Employees State Insurance Corporation
- Delhi Administration
- Municipal Corporation of Delhi and New Delhi Municipal Corporation
- University of Delhi

Age limit: 18 to 25 years on the 1st August of the year of examination. Upper age limit relaxable for scheduled castes/ scheduled tribes, etc.

Educational Qualification: Matriculation or equivalent.

Examination

Plan of Examination: The examination shall consist of two parts. Part I-written Examination and Part II-Typewriting Test for those candidates who attain such minimum standards in the written test as may be fixed by the commission in their discretion.

Written examination: The subjects of written examination, the time allowed and the maximum marks for each subject will be as follows:

S. No.	*Test*	*Max. Marks*	*Duration*
1.	General Intelligence and Clerical Aptitude	50	
2.	English Language	50	2 hours
3.	Numerical Aptitude	50	
4.	General Awareness	50	

Notes: (i) The questions in all the four tests will be Objective-Multiple Choice Type
(ii) Candidates will be required to qualify in each of the tests separately.

Syllabus of Numerical Aptitude Test: Questions will be designed to test the ability of arithmetical computation of whole numbers, decimals and fractions and relationship between numbers. The questions would be based on arithmetical concepts and relationship between numbers and not on complicated arithmetical computation

Combined Graduate Preliminary Examination

The Combined Graduate Level Examination is conducted by the Staff Selection Commission for recruitment to various subordinate services like (*i*) Assistants Grade [Group 'B' Non-Gazetted) for the groups of services/offices like AFHQ CSS, Railway Board, IFS (B) and Posts in offices like Research Designs and Standards Organisation, Central vigilance Commission, etc.; (*ii*) Inspector of Posts (Group 'B' Non-Gazetted); (*iii*) Inspectors of Central Excise, Income Tax, Preventive Officer in Custom Houses, Assistant Enforcement Officers, etc.; (*iv*) Sub-Inspectors of Police in Central Police Organisations, [BSF, CRPF, CISF, ITBP); and (*v*) Divisional Accountants/Auditors/Junior Accountants/UDCs, various Central Government departments.
Age: Candidate's age must be between (*i*) 20 and 27 years for Assistant Grade and SIs in CBI (*ii*) 18 and 27 years for inspectors of CE/IT and Div. Acctts/Auditors/UDCs, etc., (*iii*) 20 to 25 years for SIs in CPOs on the first day of August of the year of examination. The upper age limit may be relaxed in respect of Scheduled Castes, Scheduled Tribes, Other Backward Classes and such other categories of person as may, from time to time, notified in the behalf by the Government of India to the extent and subject to the conditions notified in respect of each categories.
Educational Qualification: Degree of a recognised university or equivalent.

Examination

Plan of Examination: The examination is held in two phases. The first phase of the examination is called the Preliminary Examination. Those who qualify in the preliminary Examination are eligible to appear in the Mains.
Preliminary Examination (Objective Type): The Combined Preliminary Examination is a test for selection of candidates for the main examination for the different categories of posts, mentioned in the first paragraphs, and would comprise the following subjects:

	Paper	*No. of Questions*	*Marks*	*Duration*
Part A	General Intelligence and General Awareness	100	100	2 hours
Part B	Arithmetic	100	100	2 hours

Notes:
(i) Both Parts A and B will be of objective type (multiple choice).
(ii) Question papers (test booklets) for the preliminary examination will be set both in English and Hindi.

Main Examination (Conventional Type)

Time Allowed	*Paper*	*Name of the Test*	*Examination*	*Max. Marks*
(A) For Asstt. Grade, Insp. of Central Excise, Income-Tax, Sub-Inspectors of COP/CBI				
2 hours	I	English Language	All candidates	100
2 hours	II	Language Comprehension (English)	All candidates except Assistants Grade	100
2 hours	III	Communication Skill and Writing Ability	Assistants Grade	200
3 hours	IV	Arithmetic	All candidates	300
3 hours	V	General Studies	All candidates	200
(B) For divisional Accountants, Auditors, UDCs Post				
2 hours	I	(a) General English		50
		(b) General Studies		50
2 hours	II	Arithmetic		100

Notes: (i) All the question papers for the main examination will be of conventional essay type.
(ii) All the question papers except that of Paper I and Paper II with be printed both in English and in Hindi. Only those candidates who qualify in all the papers as well as in the aggregate would be eligible to be considered for being called for the personality test which will carry a maximum of 100 marks for all candidates.
(iii) Paper I will be compulsory for all the candidates. However, this will be of qualifying nature only for the candidates who are opting for the post of Assistants. (For candidates who are opting for the post of Assistants, the marks secured in this paper will not be counted for preparation of the final select list. The commission shall have the discretion to fix the qualifying standard in this paper. Scripts of the remaining papers will be evaluated only of those candidates who secure the minimum qualifying standard in Paper I of the Main Examination.)
(iv) Candidates will have the option to answer Paper III, Paper IV and paper V either in Hindi or English. Medium opted in Preliminary Examination (Hindi or English) will remain the same in Examination.

Syllabus of Arithmetic: This part will include questions on problems relating to number systems, computation of whole numbers, decimals and fractions and relationships between numbers, fundamental arithmetical operations, percentages, ratio and proportion, averages, interest, profit and loss, discount, use of table and graphs, mensuration, time and distance, ratio and time, etc.

Divisional Accountants/Auditors/U.D.D. Examination

Divisional Accountants/Auditors/U.D.D. Examination is held every year for recruitment to the following posts:

1. Divisional accountants in the various offices of Accountant General under Comptroller and Auditor General of India;
2. Auditors in the offices of the Accountant General (Audit) in various States and other Audit and Accounts Offices under the Comptroller and Auditor General;
3. Auditors in the Defence Accounts Department;
4. Junior accountants in the various departmentalised accounts offices in the Central Ministries/Departments;
5. Upper division clerks in the offices under the charge of various Commissioners of Income Tax;
6. Upper division clerks in the offices under the formation of various Collectorates of Central Excise and Customs.

Age: A candidate must have attained the age of 18 years and must not have attained the age of 25 years on the 1st January of the year of examination. The upper age is readable in respect of Scheduled Castes/Scheduled Tribes, Departmental Candidates and such other categories of persons as may be decided from time to time.

Educational Qualifications: A candidate must hold degree of a recognised University or its equivalent.

Application: The examination is held usually in the month of March. Blank application forms and other particulars are published in the *Employment News* in the month of October of the previous year. The last date for the submission of application is usually in the second week of November every year.

Examination

Plan of the Examination: The subjects of the examination, the time allowed and the maximum marks for each subject will be as follows:

S No.	*Test*	*Questions*	*Marks*	*Time*
1.	General Intelligence	50	50	
2.	General Awareness	50	50	2 hours
3.	Comprehension and writing ability in English	50	50	
4.	Arithmetic	50	50	

Note: Both the question paper will contain objective multiple choice items, each containing four alternative responses. All parts except Comprehension and Writing Ability of English will be printed in both Hindi and English. In arithmetic, questions will be designed to test the ability of the candidates to perform fundamental arithmetic operations and his skill in working out problems relating to and graphs, mensuration, time and distance, ratio and time, etc.

Income Tax/Excise Inspectors Examination

The Staff Selection Commission (SSC) conducts this examination every year for recruitment to the following posts:

1. Inspectors' of Central Excise in different Collectorates of Central Excise.
2. Examiner (Ordinary Grade) in Custom House at Mumbai, Kolkata, Goa, Cochin, Chennai and Visakhapatnam.

3. Preventive Officer (Ordinary Grade) in Custom Houses at Mumbai, Calcutta, Goa, Cochin, Chennai and Visakhapatnam.
4. Inspector of Income Tax in different charges of the Commissioner of Income Tax.
5. Assistant Enforcement Officers in the Directorate of Enforcement (FERA).

Age: A candidate must have attained the age of 18 years and must not have attained the age of 25 years on the first July of the year of examination. The upper age limit is relaxable in respect of Scheduled Castes, Scheduled Tribes Other Backward Classes, Departmental candidates and such other categories of persons as may be decided from time to time.

Educational Qualifications: A candidate must hold degree of a recognised university,

Examination

Plan of Examination: The examination comprises (*i*) written examination and (*ii*) interview for personality test for candidates who qualify in the written test. The subjects of the examination, the time allowed and the maximum marks for each subject will be as follows:

S. No.	*Test*	*Marks*	*Time*
1.	General Intelligence	120	
2.	General English	80	2 hours
3.	Arithmetical Ability	80	
4.	General Awareness	120	

Syllabus of Arithmetical Ability: Questions will be designed to test the ability of the candidates to performs fundamental arithmetical operations and his skill in working our problems relating to percentages, ratio and proportion, average estimation, rounding numbers, use of tables and graphs, mensuration, time and distance, ratio and time.

L.I.C/G.I.C. COMPETITIVE EXAMINATIONS

L.I.C. Officers' Examination

A competitive examination for the recruitment of the Assistant Administrative Officers etc, in Life Insurance Corporation (LIC) is held once a year, generally in the month of June. The blank application form and particulars are published in the Employment News, usually the second week of May every year.

Educational Qualifications: Bachelor's/Master's Degree from a recognised Indian or Foreign University with a minimum of 50% marks (relaxable in the case of SC/ST candidates to 40%) in aggregate in either of the degrees.

Age Limits: 21 to 28 years on the 1st April of the year of examination. Upper age limit is relaxable for Scheduled Castes, Scheduled Tribes, confirmed L.I.C. employees, etc.

Examination

Scheme of Examination: The examination comprises (*i*) written examination and (*ii*) interview of candidates who qualify in the written test.

Examination Papers: The written examination consists of the following papers:

Paper I (Objective)

1. Reasoning Ability (Bilingual)
2. General Knowledge and current Affairs (Bilingual)
3. Numerical Ability (Bilingual) The purpose of this test is to ascertain how quick you are in working at numerical calculations
4. English Language with special emphasis on grammar and vocabulary.

Paper II (Descriptive)

1. Test on Essay (can be written in Hindi or English)
2. Precise and Comprehension in English

LIC Development Officers' Examination

A competitive examination for the recruitment of Assistant Development Officers' in the Life Insurance Corporation is held once a year, generally in the month of September. The blank application forms and particulars are published in the

Employment News, generally in the month of July and the last date for submission of applications is generally the first week of August.
Educational Qualification: Candidates must hold a bachelor's degree in arts, science, commerce, agriculture or law of an Indian or foreign university or an equivalent qualification.
Age Limits: The applicants should have completed the age of 21 years on the 1st July of the year of examination.

Examination
Scheme of the Examination: The examination comprises (*i*) written examination (*ii*) interview of such candidates, who qualify in the written test.
Examination Papers: The written test will consist of (*i*) test of reasoning and numerical ability and (*ii*) general English/ Hindi and general knowledge. The test papers will be set bilingual and the candidates will have choice to write answers either in English or in Hindi.

G.I.C. Assistants' Examination

This examination is held once a year, generally in the month of August. The blank application and particulars are published in the Employment News, usually in the month of March. The last date for the submission of applications is usually first week of April every year.
Age Limit: 18 to 28 years on 1 June of the year of examination.
Educational Qualification: Pass in higher secondary with 60% marks of graduate of a recognised university.

Examination
Scheme of the Examination: The examination comprises (*i*) written examination and (*ii*) interview of candidates qualifying the written test.
Examination Papers: (*i*) The written test will be objective type, consisting of test of reasoning, numerical ability, clerical aptitude, English language and general knowledge. (*ii*) There will also be a descriptive test on essay, letter and precise writing in English. Objective tests except English will be bilingual, i.e., both in English and Hindi.

BANKING SERVICES

Banking has emerged as one of the most challenging sectors in the country. Openings are available at various levels, from bank clerical to probationary officers (PO).
Recruitment for the public sector banks is done through the Banking Service Recruitment Boards (BSRBs). The advertisements for recruitment appear in newspapers as well as the *Employment News*. Recruitment is done on the basis of a written test, which consists of (*a*) test of reasoning, (*b*) quantitative aptitude, (*c*) general awareness, (d) English language and (*e*) descriptive test.
The test is qualifying in nature and the marks obtained are not added in the final merit list. It is held on Sundays. Except for the descriptive portion, all other sections contain objective-type questions. In the reasoning test, there are verbal and non-verbal sections. In English, the test is aimed at judging the overall comprehension and understanding of the language. The descriptive paper can be answered in English or Hindi. It judges the written expression of the candidates. All sections must be qualified.

Reserve Bank of India

Staff Officers Grade A Examination
Selection for this class I post is made on all-India basis by Reserve Bank of India Services Board, Hong Kong Building, 6th Floor, M.G. Road, P.O. Box 10009, Hutatma Chow, Mumbai 400 001. Set up in July 1968, the Board functions on the lines of the UPSC and conducts various examinations for recruitment of officers grade. There is reservation of posts for SC/ST categories.
Qualifications: Bachelor'/Master's Degree (50%) or Chartered/Cost Accountant with Bachelor's Degree or Degree in Management.
Age: 21–26 years.
Subjects: Written test consists of (*i*) Paper I (objective type) – test of reasoning, test of quantitative aptitude, test of English language, general awareness, (*ii*) Paper II (descriptive type) – English essay, précis writing/comprehension and (*iii*) Paper III (descriptive type) – economic and social problems. Those who qualify the written test are called for interview.

Staff Officer Grade B Examination

Qualifications: MA/M.Sc./M.Com. (55%), Ph.D./Chartered or Cost Accountants with a bachelor's degree (60%) or postgraduate diploma in management with a bachelor's degree. For SC/ST candidates the eligibility is a bachelor's degree with 50% marks or a simple postgraduate degree.
Age: 21–28 years (maximum 26 years for degree holders). Relaxable by 5 years for SC/ST and ex-servicemen, and by 3 years for OBC candidates.
Subjects: ***Paper I (Objective type)*** – test of reasoning, quantitative aptitude, general awareness. ***Paper II*** – English. ***Paper III*** – economic and social problems. Paper II and III are assessed only if Paper I is cleared. Written test is followed by interview.

State Bank Group

Probationary Officers Examination

This examination is held by Central Recruitment Board (State Bank Group) Madhuli, Second Floor, H/2 Shiv Nagar Estate, Dr. Annie Besant Road, Worli, Mumbai 400 018 for recruitment of Probationary Officers in State Bank of India and its associates banks, namely, State Bank of Bikaner and Jaipur, State Bank of Hyderabad, State Bank of Indore, State Bank of Mysore, State Bank of Patiala, State Bank of Saurashtra and State Bank of Travancore. There is reservation of posts for SC/ST, ex-servicemen, OBC, etc.
Qualifications: Degree
Age: 18–26 years, relaxable for SC/ST, OBC, Ex-Servicemen.
Subjects: ***Paper I (objective type)*** – Test of reasoning ability, quantitative aptitude, English comprehension, general awareness. ***Paper II (descriptive type)*** – Essay, letter writing or précis writing in English. Those who qualify the written test are called for interview.

Clerical Cadre Examination (Regional Recruitment Board)

This examination is generally held annually to recruit the clerical cadre in State Bank of India and its associate banks. There is resonation of posts for SC/ST, ex-servicemen and OBC, etc.
Qualifications: Degree
Age: 18–26 years, relaxable for various categories as per rules.
Subjects: ***Paper I*** – General awareness, reasoning ability, English language, numerical ability. ***Paper II*** – essay writing, letter writing, précis writing.

Nationalised Banks

In nationalised banks, the recruitment is made through competitive test held by Banking Service Recruitment Board.

Probationary Officers Examination

There are 15 Banking Service Recruitment Boards in the country. These Boards recruit probationary officers and clerical cadre for the nationalised banks. There is reservation of posits for SC/ST, ex-servicemen etc.
Qualifications: Degree
Age: 21–28 years, relaxable for SC/ST, OBC, Ex-servicemen.
Subjects: ***Paper I (objective type)*** – Test of reasoning ability, quantitative aptitude, English comprehension, general awareness. ***Paper II (descriptive type)*** – Essay (in English or regional language), letter writing or précis writing in English.

Clerical Cadre Examination

Qualification: ***For clerks, typists*** – Degree or 10 + 2 (50%) or diploma in banking (50%) or matriculation (60%). ***For Stenographers*** – matriculation. Relaxation in percentage of marks in qualifying examination for SC/ST, ex-servicemen and physical handicapped.
Age: 18–26 years, relaxable for certain categories as per rules.
Subjects: Written examination (200 marks)
Objective Test: Reasoning ability, English language, numerical ability, clerical aptitude
Descriptive Test: Three out of four questions of short essay or exposition type on a given proposition, situation to be answered in Hindi or English.
For the posts of typists and stenographers, proficiency in typing and shorthand with the following minimum speed is required: (*i*) English typing speed 30 wpm, (*ii*) English shorthand speed 80 wpm, (*iii*) Hindi typing speed 25 wpm, (*iv*) Hindi shorthand speed 60 wpm. Proficiency test for typist/stenographer is held if they qualify the written test. Those ranking high in the written examination are called for an interview (100 marks). Final selection is on the basis of candidate's performance in written tests and interview taken together. Probation period is six months.

National Bank for Agriculture and Rural Development (NABARD)

Development Officer Grade B Examination

Qualifications: Master's degree in economics/agricultural economics (50%), Ph.D, desirable.
Age: 24–32 years, relaxable for certain categories as per rules.
Subjects: Preliminary and main examinations. ***Preliminary examination (objective type)*** – general awareness, English language, quantitative aptitude, reasoning ability. ***Main examination, Paper I*** – General English and general awareness. ***Paper II*** – economic and social problems. ***Paper III*** – economics/agricultural economics.
Examination Fee: Rs. 50/-

Assistant Development Officer Grade 'A' Examination

Qualifications: Degree/Master's Degree (50%)
Age: 21–26 years. Relaxable for certain categories as per rules.
Subjects: Preliminary and main examinations. ***Preliminary Examination (Objective type)*** – General awareness, English language, quantitative aptitude, reasoning ability. ***Main Examination, Paper I*** – General English and general awareness. ***Paper II*** – Economic and social problems.
Address: Recruitment Section, Poonam Chambers, Dr. Annie Besant Road, Post Box Number 6552, Worli, Mumbai, 400018

MBA ENTRANCE EXAMINATIONS

Each business school has its own admission policies and practices, but all programmes evaluate a candidate's application on a range of objective and subjective criteria. Usually, the admission process consists of the following:
Step 1: Written test: multiple choice tests in quantitative ability (QA), data interpretation, verbal reasoning, logical reasoning, reading comprehension and business awareness. These tests are usually conducted at centres located across the country.

- **Quantitative Ability (QA):** Essentially covers high school mathematics to test a candidate's numerical ability.
- **Data Interpretation (DI):** Designed to test the ability of the candidate to analyse and interpret information from Tables, Activity Charts, Bar Charts, Pie Charts and Graphs.
- **Verbal Ability (VA):** Designed to test a candidate's usage of the English language – Grammar, Vocabulary and Verbal Reasoning.
- **Reading Comprehension (RC):** Designed to test a candidate's reading comprehension skills. This section contains a few descriptive passages followed by questions that test the candidate about his comprehension of the content of the passage.
- **General Awareness/Business Awareness (GA/BA):** Appears in tests other than the CAT. This section tests a candidate's awareness of current affairs, business events and general awareness.
- **Logical Reasoning (LR):** Designed to test the candidate's analytical skills through Data Sufficiency exercises and Numerical/Analytical reasoning.

Step 2: Personality Assessment: Subsequent to the written test, a candidate is required to undergo certain stages of personality assessment comprising Group Discussion and Personal Interview.

- **Group Discussion:** A Topic of general interest is assigned to a group of 10–15 participants in the group Discussion. The group is required to discuss the topic freely while their discussion is observed and evaluated by a panel. Participants are evaluated on a variety of parameters ranging from their articulation skills, leadership abilities, listening ability and team skills.
- **Personal Interview:** This is the most subjective part of the evaluation process. The Business Schools evaluate a candidate on the basis of his personal characteristics. Although different schools emphasise different qualities most seek candidates who demonstrate leadership, maturity, integrity, responsibility and teamwork.

Examinations

Most prominent management institutes coming under the purview of UGC/AICTE and deemed universities admit students through any of the following examinations:

- **Common Admission Test:** CAT is conducted for admission to the postgraduate programmes at the six IIM's and more than 50 other B-Schools. The CAT is a two-hour test and the number of questions ranging between 150–180. Wrong answers carry negative marks. Apart form the IIMs, other prominent institutes that consider CAT scores for evaluation include MDI, Gurgaon; SP Jain, Mumbai; MICA, Ahmedabad; TA Pai Management Institute, Manipal.

- **Xavier Admission Test:** the Xavier Labour Relations' Institute (XLRI) Jamshepur conducts XAT. XAT is a two-hour test comprising 80 questions on English, 60 questions on reasoning and quantitative ability and 60 questions on general awareness. In the last 20 minutes the candidate is required to write a 250–300–word essay on topics ranging form social relevance to abstract.
- **Joint Management Entrance Test:** JMET is conducted to admit students to post graduate degree programmes in management at IITs and ISc, Bangalore. The question paper carries 150 questions spread over four sections: verbal communication – 40 questions, logical reasoning – 40 questions, quantitative ability – 40 questions and data interpretation – 30 questions.
- **AIMS Test for Management Admission:** ATMA is conducted by the association of Indian Management Schools (AIMS) which is a country wide network of over 300 business schools for admission to post graduate management programmes including functional; sectorial and computer managements programmes offered by various institutes or universities. The test comprises three sections: quantitative ability – 70 questions, verbal ability – 60 questions and analytical reasoning – 70 questions, with each section of one-hour duration.
- **Management Aptitude Test (MAT):** The all India Management Association (AIMA) conducts MAT to admit students in over 100 institutes and universities. This examination is conducted four times in any year and is a 150–minute test comprising 200 questions.
- **State-level joint entrance tests:** Conducted by the states

The Ministry of Human Resource Development (MHRD) has recently decided that each B-school or any university/deemed university running PG level management courses must necessarily admit students from one of these six entrance examinations. However, the institutions admitting students through state-level tests shall restrict admission to students from within the state only. They are barred from admitting students from outside the state.

The B-schools/institutions wishing to admit students from outside the state will have to subscribe to one of the five all-India admission tests described above (i.e., CAT. JMET, MAT, ATMA or XAT). The states that do not want to hold their own tests could use the merit determined by any one of these all-India tests.

Nowadays, more and more B-schools are turning to CAT. In fact, the Institute of Management Technology, Ghaziabad, FORE, and the Management Development Institute, Gurgaon, have recently decided to subscribe to CAT scores. This will certainly give those appearing for CAT a wider choice.

OTHER COMPETITIVE EXAMINATIONS

National Institute of Fashion Technology

National Institute of Fashion Technology (NIFT) offers 4-year undergraduate and 2-year postgraduate programmes in the areas of design, management and technology.

Eligibility for UG Programme in Design: 10 + 2 or its equivalent in any stream.

Entrance Test: The entrance test at NIFT consists of (*i*) creative ability test (CAT), (*ii*) general ability test (GAT) and (*iii*) managerial ability test (MAT). The General Ability Test (2 hours) is designed to test general ability of the applicant and also the ability to work under time pressure. The UG tests will contain only 100 questions but the PG tests will contain 120 questions. This test consists of questions on addition, subtraction, multiplication, division, fractions, percentages, interest, work and task, ratio and proportion, distance, profit and loss, algebra and geometry, etc. The weightage assigned to each test in the final merit list of the entrance examination will be as under:

Programme	*Test*	*Weightage*
Design (Undergraduate)	GAT-UG	40%
	CAT-UG	40%
	Situation Test	20%
Management (Postgraduate)	GAT-PG	35%
	MAT-PG	35%
	Group Discussion	10%
	Interview	20%

National Science Talent Search Examination (NSTSE)

National Science Talent Search Examination is conducted by the Unified Council.
Eligibility: Students of classes 4, 5, 6, 7, 8, 9, 10, 11 and 12 are eligible to participate in this examination.
Medium and Syllabus: This examination is conducted in only English medium and suitable for all the students following CBSE/ICSE/State Board Syllabi.
Examination Pattern: There will be a separate question paper for each class. All questions are objective type multiple choice with no negative marking for wrong answers.
No. of questions: 100
Marks: 100
Duration: 90 minutes
Date: Conducted every year on the last Sunday of January
Test centres: Over 200 centres across India.

Distribution of Marks

Classes IV and V	*Classes VI to X*
Mathematics: 45 marks General Questions: 10 marks General Science: 45 marks	Mathematics: 25 marks Physics: 25 marks Chemistry: 20 marks Biology: 20 marks General Questions: 10 marks

CONTENTS

Preface ..... *vii*
Competitive Examinations: An Overview ..... *ix*

1 Numbers 1
2 H.C.F. and L.C.M. of Numbers 19
3 Simplification 33
4 Square Root and Cube Root 56
5 Percentage 69
6 Average 93
7 Ratio and Proportion 112
8 Partnership 137
9 Simple Interest 148
10 Compound Interest 169
11 Discount (True and Banker's) 186
12 Profit and Loss 196
13 Time and Work, Work and Wages 219
14 Pipes and Cisterns 239
15 Time and Distance 249
16 Boats and Streams 270
17 Alligation or Mixture 280
18 Races and Games of Skill 294
19 Stocks, Shares and Debentures 301
20 Clocks and Calendar 314
21 Problems on Ages 325
22 Mensuration I: Area and Perimeter 336
23 Mensuration II: Volume and Surface Area 376
24 Permutations and Combinations 408
25 Probability 430
26 Series 452
27 Progression 466
28 Heights and Distances 481
29 Logarithms 492
30 Data Interpretation 502
31 Data Sufficiency 536
32 Binary Number System 555
33 Miscellaneous Topics 562

Model Test Paper I based on CDS Examination 2007 573
Model Test Paper II based on Andra Bank (PO) Examination 2007 579
Model Test Paper III based on IDBI Bank Officers' Examination 2007 581
Model Test Paper IV based on Bank Clerical Examination 2007 585
Model Test Paper V based on MAT Examination 2007 588
Model Test Paper VI based on Common Admission Test 2006 592

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1

NUMBERS

INTRODUCTION

In Hindu Arabic System, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called *digits* to represent any number. This is the *decimal system* where we use the numbers 0 to 9. 0 is called *insignificant digit* whereas 1, 2, 3, 4, 5, 6, 7, 8, 9 are called *significant digits*.

A group of figures, denoting a number is called a *numeral*. For a given numeral, we start from extreme right as Unit's place, Ten's place, Hundred's place and so on.

Illustration 1 We represent the number 309872546 as shown below:

Ten Crore 10^8	Crores 10^7	Ten Lacs (million) 10^6	Lacs 10^5	Ten Thousand 10^4	Thousand 10^3	Hundred 10^2	Ten's 10^1	Units 10^0
3	0	9	8	7	2	5	4	6

We read it as

"Thirty crores, ninety-eight lacs, seventy-two thousands five hundred and forty-six."

In this numeral:

The place value of 6 is $6 \times 1 = 6$

The place value of 4 is $4 \times 10 = 40$

The place value of 5 is $5 \times 100 = 500$

The place value of 2 is $2 \times 1000 = 2000$ and so on.

The face value of a digit in a number is the value itself wherever it may be.

Thus, the face value of 7 in the above numeral is 7. The face value of 6 in the above numeral is 6 and so on.

NUMBER SYSTEM

Natural Numbers

Counting numbers 1, 2, 3, 4, 5, ... are known as *natural numbers*.

The set of all natural numbers can be represented by

$$N = \{1, 2, 3, 4, 5, ...\}.$$

Whole Numbers

If we include 0 among the natural numbers, then the numbers 0, 1, 2, 3, 4, 5, ... are called *whole numbers*.

The set of whole numbers can be represented by

$$W = \{0, 1, 2, 3, 4, 5, ...\}$$

Clearly, every natural number is a whole number but 0 is a whole number which is not a natural number.

INTEGERS

All counting numbers and their negatives including zero are known as *integers*.

The set of integers can be represented by

$$Z \text{ or } I = \{... -4, -3, -2, -1, 0, 1, 2, 3, 4, ...\}$$

Positive Integers

The set $I^+ = \{1, 2, 3, 4, ...\}$ is the set of all *positive integers*. Clearly, positive integers and natural numbers are synonyms.

Negative Integers

The set $I^- = \{-1, -2, -3, ...\}$ is the set of all *negative integers*. 0 is neither positive nor negative.

Non-negative Integers

The set $\{0, 1, 2, 3, ... \}$ is the set of all *non-negative integers*.

Rational Numbers

The numbers of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, are known as *rational numbers*, e.g. $\frac{4}{7}, \frac{3}{2}, -\frac{5}{8}, \frac{0}{1}, -\frac{2}{3}$, etc.

The set of all rational numbers is denoted by Q.

i.e. $Q = \{x : x = \frac{p}{q}; p, q \in I, q \neq 0\}$.

Since every natural number 'a' can be written as $\frac{a}{1}$, every natural number is a rational number. Since 0 can be written as $\frac{0}{1}$ and every non-zero integer 'a' can be written as $\frac{a}{1}$, every integer is a rational number.

Every rational number has a peculiar characteristic that when expressed in decimal form is expressible either in terminating decimals or in non-terminating repeating decimals.

For example, $\frac{1}{5} = 0.2$, $\frac{1}{3} = 0.333\ldots$, $\frac{22}{7} = 3.1428714287$, $\frac{8}{44} = 0.181818\ldots$, etc.

The recurring decimals have been given a short notation as

$$0.333\ldots = 0.\overline{3}$$

$$4.1555\ldots = 4.0\overline{5}$$

$$0.323232\ldots = 0.\overline{32}.$$

Irrational Numbers

Those numbers which when expressed in decimal form are neither terminating nor repeating decimals are known as *irrational numbers*, e.g. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π, etc.

Note that the exact value of π is not $\frac{22}{7}$. $\frac{22}{7}$ is rational while π is irrational number. $\frac{22}{7}$ is approximate value of π. Similarly, 3.14 is not an exact value of it.

Real Numbers

The rational and irrational numbers combined together are called *real numbers,* e.g. $\frac{13}{21}$, $\frac{2}{5}$, $-\frac{3}{7}$, $\sqrt{3}$, $4 + \sqrt{2}$, etc. are real numbers.

The set of all real numbers is denoted by R.

Note that the sum, difference or product of a rational and irrational number is irrational, e.g. $3 + \sqrt{2}$, $4 - \sqrt{3}$, $\frac{2}{3} - \sqrt{5}$, $4\sqrt{3}$, $-7\sqrt{5}$ are all irrational.

Even Numbers

All those numbers which are exactly divisible by 2 are called *even numbers*, e.g. 2, 6, 8, 10, etc., are even numbers.

Odd Numbers

All those numbers which are not exactly divisible by 2 are called *odd numbers*, e.g. 1, 3, 5, 7 etc., are odd numbers.

Prime Numbers

A natural number other than 1, is a *prime number* if it is divisible by 1 and itself only.

For example, each of the numbers 2, 3, 5, 7 etc., are prime numbers.

Composite Numbers

Natural numbers greater than 1 which are not prime, are known as composite numbers.

For example, each of the numbers 4, 6, 8, 9, 12, etc., are composite numbers.

Note:

1. The number 1 is neither a prime number nor a composite number.
2. 2 is the only even number which is prime.
3. Prime numbers up to 100 are:
   2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, i.e. 25 prime numbers between 1 and 100.
4. Two numbers which have only 1 as the common factor are called *co-primes* or *relatively prime* to each other, e.g. 3 and 5 are co-primes.

Note that the numbers which are relatively prime need not necessarily be prime numbers, e.g. 16 and 17 are relatively prime although 16 is not a prime number.

ADDITION AND SUBTRACTION (SHORT-CUT METHODS)

The method is best illustrated with the help of following example:

Illustration 2 54321 – (9876 + 8967 + 7689) = ?

Step 1 Add Ist column:

$$6 + 7 + 9 = 22$$

To obtain 1 at unit's place add 9 to make 31. In the answer, write 9 at unit's place and carry over 3.

$$\begin{array}{r} 54321 \\ \left.\begin{array}{r} 9876 \\ 8967 \\ 7689 \end{array}\right\} \\ \hline 27789 \end{array}$$

Step 2 Add 2nd column:

$$3 + 7 + 6 + 8 = 24$$

To obtain 2 at ten's place, add 8 to make 32. In the answer, write 8 at ten's place and carry over 3.

Step 3 Add 3rd column:

$$3 + 8 + 9 + 6 = 26$$

To obtain 3 at hundred's place, add 7 to make 33. In the answer, write 7 at hundred's place and carry over 3.

Step 4 Add 4th column:

$$3 + 9 + 8 + 7 = 27$$

To obtain 4 at thousand's place add 7 to make 34. In the answer, write 7 at thousand's place and carry over 3.

Step 5 5th column:

To obtain 5 at ten-thousand's place add 2 to it to make 5. In the answer, write 2 at the ten-thousand's place.

$\therefore$ 54321 – (9876 + 8967 + 7689) = 27789.

MULTIPLICATION (SHORT-CUT METHODS)

1. Multiplication of a given number by 9, 99, 999, etc., that is by $10^n - 1$

Method: Put as many zeros to the right of the multiplicant as there are nines in the multiplier and from the result subtract the multiplicant and get the answer.

Illustration 3 Multiply:

(a) 3893 by 99 (b) 4327 by 999

(c) 5863 by 9999.

Solution (a) $3893 \times 99 = 389300 - 3893 = 385407.$

(b) $4327 \times 999 = 4327000 - 4327 = 4322673$

(c) $5863 \times 9999 = 58630000 - 5863 = 58624137.$

2. Multiplication of a given number by 11, 101, 1001, etc., that is, by $10^n + 1$.

Method: Place n zeros to the right of the multiplicant and then add the multiplicant to the number so obtained.

Illustration 4 Multiply:

(a) 4782×11 (b) 9836×101

(c) 6538×1001.

Solution (a) $4782 \times 11 = 47820 + 4782 = 52602$

(b) $9836 \times 101 = 983600 + 9836 = 993436$

(c) $6538 \times 1001 = 6538000 + 6538 = 6544538$

3. Multiplication of a given number by 15, 25, 35, etc.

Method: Double the multiplier and then multiply the multiplicant by this new number and finally divide the product by 2.

Illustrations 5 Multiply:

(a) 7054×15 (b) 3897×25

(c) 4563×35

Solution (a) $7054 \times 15 = \frac{1}{2}(7054 \times 30)$

$= \frac{1}{2}(211620) = 105810.$

(b) $3897 \times 25 = \frac{1}{2}(3897 \times 50) = \frac{1}{2}(194850)$

$= 97425.$

(c) $4536 \times 35 = \frac{1}{2}(4563 \times 70) = \frac{1}{2}(319410)$

$= 159705.$

4. Multiplication of a given number by 5, 25, 125, 625, etc., that is, by a number which is some power of 5.

Method: Place as many zeros to the right of the multiplicant as is the power of 5 in the multiplier, then divide the number so obtained by 2 raised to the same power as is the power of 5.

Illustration 6 Multiply:

(a) 3982×5 (b) 4739×25

(c) 7894×125 (d) 4863×625

Solution (a) $3982 \times 2 = \frac{39820}{2} = 19910.$

(b) $4739 \times 25 = \frac{473900}{2^2} = \frac{473900}{4} = 118475.$

(c) $7894 \times 125 = \frac{7894000}{2^3} = \frac{7894000}{8}$

$= 986750$

(d) $4863 \times 625 = \frac{48630000}{2^4} = \frac{48630000}{16}$

$= 3039375.$

Distributive Laws

For any three numbers a, b, c, we have

(a) $a \times b + a \times c = a \times (b + c)$

(b) $a \times b - a \times c = a \times (b - c)$

Illustration 7 $438 \times 637 + 438 \times 367 = ?$

Solution $438 \times 637 + 438 \times 367 = 438 \times (637 + 367)$

$= 430 \times 1000 = 438000.$

Illustration 8 $674 \times 832 - 674 \times 632 = ?$

Solution $674 \times 832 - 674 \times 632 = 674 \times (832 - 632)$

$= 674 \times 200 = 134800.$

SQUARES (SHORT-CUT METHODS)

1. To square any number ending with 5.

Method: $(A5)^2 = A(A + 1)/25$

Illustration 9

(a) $(25)^2 = 2\ (2 + 1)/25 = 6/25 = 625$

(b) $(45)^2 = 4\ (4 + 1)/25 = 20/25 = 2025$

(c) $(85)^2 = 8\ (8 + 1)/25 = 72/25 = 7225.$

2. To square a number in which every digit is one.

Method: Count the number of digits in the given number and start writing numbers in ascending order from one to this number and then in descending order up to one.

Illustration 10

(a) $11^2 = 121$ (b) $111^2 = 12321$

(c) $1111^2 = 1234321$

(d) $222^2 = 2^2\,(111)^2 = 4\,(12321) = 49284$

(e) $3333^2 = 3^2\,(1111)^2 = 9\,(1234321) = 11108889$

3. To square a number which is nearer to $10x$.

Method: Use the formula:

$$x^2 = (x^2 - y^2) + y^2 = (x + y)(x - y) + y^2$$

Illustration 11

(a) $(97)^2 = (97 + 3)\,(97 - 3) + 3^2$
$= 9400 + 9 = 9409$

(b) $(102)^2 = (102 - 2)\,(102 + 2) + 2^2$
$= 10400 + 4 = 10404$

(c) $(994)^2 = (994 + 6)\,(994 - 6) + 6^2$
$= 988000 + 36 = 988036$

(d) $(1005)^2 = (1005 - 5)\,(1005 + 5) + 5^2$
$= 1010000 + 25 = 1010025.$

DIVISION

Division is repeated subtraction.

For example, when we divide 63289 by 43, it means 43 can be repeatedly subtracted 1471 times from 63289 and the remainder 36 is left.

```
                 1471 ← Quotient
Divisor → 43) 63289  ← Dividend
              43
              202
              172
               308
               301
                79
                43
                36   ← Remainder
```

Dividend = (Divisor × Quotient) + Remainder

or, $\text{Divisor} = \dfrac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$

Illustration 12 On dividing 7865321 by a certain number, the quotient is 33612 and the remainder is 113. Find the divisor.

Solution $\text{Divisor} = \dfrac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$

$$= \frac{7865321 - 113}{33612} = \frac{7865208}{33612} = 234.$$

Illustration 13 A number when divided by 315 leaves remainder 46 and the value of quotient is 7. Find the number.

Solution Number = (Divisor × Quotient) + Remainder
$= (315 \times 7) + 46 = 2205 + 46 = 2251.$

Illustration 14 Find the least number of 5 digits which is exactly divisible by 632.

Solution The least number of 5 digits is 10000. Dividing this number by 632, the remainder is 520. So, the required number = 10000 + (632 + 520) = 10112.

```
        15
632) 10000
      632
      3680
      3160
       520
```

Illustration 15 Find the greatest number of 5 digits which is exactly divisible by 463.

Solution The greatest number of 5 digits is 99999. Dividing this number by 463, the remainder is 454. So, the required number = 99999 – 454 = 99545.

```
        215
463) 99999
      926
       739
       463
       2769
       2315
        454
```

Illustration 16 Find the number nearest to 13700 which is exactly divisible by 235.

Solution On dividing the number 13700 by 235, the remainder is 70. Therefore, the nearest number to 13700, which is exactly divisible by 235 = 13700 – 70 = 13630.

```
        58
235) 13700
      1175
       1950
       1880
         70
```

TESTS OF DIVISIBILITY

1. **Divisibility by 2** A number is divisible by 2 if the unit's digit is zero or divisible by 2.

   For example, 4, 12, 30, 18, 102, etc., are all divisible by 2.

2. **Divisibility by 3** A number is divisible by 3 if the sum of digits in the number is divisible by 3.

   For example, the number 3792 is divisible by 3 since 3 + 7 + 9 + 2 = 21, which is divisible by 3.

3. **Divisibility by 4** A number is divisible by 4 if the number formed by the last two digits (ten's digit and unit's digit) is divisible by 4 or are both zero.

   For example, the number 2616 is divisible by 4 since 16 is divisible by 4.

4. **Divisibility by 5** A number is divisible by 5 if the unit's digit in the number is 0 or 5.

   For example, 13520, 7805, 640, 745, etc., are all divisible by 5.

5. **Divisibility by 6** A number is divisible by 6 if the number is even and sum of its digits is divisible by 3.

For example, the number 4518 is divisible by 6 since it is even and sum of its digits 4 + 5 + 1 + 8 = 18 is divisible by 3.

6. **Divisibility by 7** The unit digit of the given number is doubled and then it is subtracted from the number obtained after omitting the unit digit. If the remainder is divisible by 7, then the given number is also divisible by 7.

   For example, consider the number 448. On doubling the unit digit 8 of 448 we get 16.

   Then, 44 – 16 = 28.

   Since 28 is divisible by 7, 448 is divisible by 7.

7. **Divisibility by 8** A number is divisible by 8, if the number formed by the last 3 digits is divisible by 8.

   For example, the number 41784 is divisible by 8 as the number formed by last three digits, i.e. 784 is divisible by 8.

8. **Divisibility by 9** A number is divisible by 9 if the sum of its digits is divisible by 9.

   For example, the number 19044 is divisible by 9 as the sum of its digits 1 + 9 + 0 + 4 + 4 = 18 is divisible by 9.

9. **Divisibility by 10** A number is divisible by 10, if it ends in zero.

   For example, the last digit of 580 is zero, therefore, 580 is divisible by 10.

10. **Divisibility by 11** A number is divisible by 11 if the difference of the sum of the digits at odd places and sum of the digits at even places is either zero or divisible by 11.

    For example, in the number 38797, the sum of the digits at odd places is 3 + 7 + 7 = 17 and the sum of the digits at even places is 8 + 9 = 17. The difference is 17 – 17 = 0, so the number is divisible by 11.

11. **Divisibility by 12** A number is divisible by 12 if it is divisible by 3 and 4.

12. **Divisibility by 25** A number is divisible by 25 if the number formed by the last two digits is divisible by 25 or the last two digits are zero.

    For example, the number 13675 is divisible by 25 as the number formed by the last two digits is 75 which is divisible by 25.

13. **Divisibility by 125** A number is divisible by 125 if the number formed by the last three digits is divisible by 125 or the last three digits are zero.

    For example, the number 5250 is divisible by 125 as 250 is divisible by 125.

14. **Divisibility by 18** An even number satisfying the divisibility test of 9 is divisible by 18.

15. **Divisibility by 88** A number is divisible by 88 if it is divisible by 11 and 8.

SOME USEFUL SHORT-CUT METHODS

1. **Test to find whether a given number is a prime**

   Step 1 Select a least positive integer n such that $n^2 >$ given number.

   Step 2 Test the divisibility of given number by every prime number less than n.

   Step 3 The given number is prime only if it is not divisible by any of these primes.

Illustration 17 Investigate whether 571 is a prime number

Solution Since $(23)^2 = 529 < 571$ and $(24)^2 = 576 > 571$

$\therefore \quad n = 24.$

Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23. Since 24 is divisible by 2, 571 is not a prime number.

Illustration 18 Investigate whether 923 is a prime number.

Solution Since $(30)^2 = 900 < 923$ and $(31)^2 = 961 > 923$

$\therefore \quad n = 31.$

Prime numbers less than 31 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. Since 923 is not divisible by any of these primes, therefore 923 is a prime number.

2. The least number, which when divided by d_1, d_2 and d_3 leaves the remainders r_1, r_2 and r_3 respectively such that $(d_1 - r_1) = (d_2 - r_2) = (d_3 - r_3)$, is = (L.C.M. of d_1, d_2 and d_3) – $(d_1 - r_1)$ or $(d_2 - r_2)$ or $(d_3 - r_3)$.

Illustration 19 Find the least number which when divided by 9, 10 and 15 leaves the remainders 4, 5 and 10, respectively.

Solution Here, 9 – 4 = 10 – 5 = 15 – 10 = 5.

Also, L.C.M. (9, 10, 15) = 90,

$\therefore$ the required least number = 90 – 5 = 85.

3. A number on being divided by d_1 and d_2 successively leaves the remainders r_1 and r_2, respectively. If the number is divided by $d_1 \times d_2$, then the remainder is = $(d_1 \times r_2 + r_1)$.

Illustration 20 A number on being divided by 10 and 11 successively leaves the remainders 5 and 7, respectively. Find the remainder when the same number is divided by 110.

Solution The required remainder

$$= d_1 \times r_2 + r_1 = 10 \times 7 + 5 = 75.$$

4. To find the number of numbers divisible by a certain integer.

   The method is best illustrated with the help of following example.

Illustration 21 How many numbers up to 532 are divisible by 15?

Solution We divide 532 by 15.

$$532 = \underline{35} \times 15 + 7$$

The quotient obtained is the required number of numbers. Thus, there are 35 such numbers.

Illustration 22 How many numbers up to 300 are divisible by 5 and 7 together?

Solution L.C.M. of 5 and 7 = 35.

We divide 300 by 35.

$$300 = \underline{8} \times 35 + 20$$

Thus, there are 8 such numbers.

5. Two numbers when divided by a certain divisor give remainders r_1 and r_2. When their sum is divided by the same divisor, the remainder is r_3. The divisor is given by $r_1 + r_2 - r_3$.

Illustration 23 Two numbers when divided by a certain divisor give remainders 473 and 298, respectively. When their sum is divided by the same divisor, the remainder is 236. Find the divisor.

Solution The required divisor

$$= 437 + 298 - 236 = 499.$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. $7372 \times 7372 + 7372 \times 628 = ?$
(a) 58976000 (b) 58967000
(c) 5897600 (d) None of these

2. $9999 + 8888 + 777 + ? = 19700$
(a) 36 (b) 16
(c) 64 (d) 26

3. $60 ? 6 \times 111 = 666666$
(a) 0 (b) 2
(c) 1 (d) 6

4. $3149 \times 1 ? 5 = 425115$
(a) 3 (b) 2
(c) 4 (d) 6

5. If the two digits of the age of Mr. Manoj are reversed then the new age so obtained is the age of his wife. $\frac{1}{11}$ of the sum of their ages is equal to the difference between their ages. If Mr. Manoj is older than his wife then find the difference between their ages.
(a) Cannot be determined
(b) 8 years (c) 10 years
(d) 9 years (e) 7 years

6. If in a long division sum, the dividend is 380606 and the successive remainders from the first to the last are 434, 125 and 413, then divisor is
(a) 451 (b) 843
(c) 4215 (d) 3372

7. If $\frac{x}{y} = \frac{3}{4}$, then the value of $\left(\frac{6}{7} + \frac{y-x}{y+x}\right)$ equals
(a) $\frac{5}{7}$ (b) $1\frac{1}{7}$
(c) 1 (d) 2

8. The largest natural number by which the product of three consecutive even natural numbers is always divisible, is
(a) 16 (b) 24
(c) 48 (d) 96

9. Which number should replace both the '*'s in $\left(\frac{*}{21}\right) \times \left(\frac{*}{189}\right) = 1$?
(a) 21 (b) 63
(c) 3969 (c) 147

10 In a division sum, the divisor is 12 times the quotient and 5 times the remainder. If the remainder be 48, then the dividend is
(a) 240 (b) 576
(c) 4800 (d) 4848

11. What least number must be subtracted from 1294 so that the remainder when divided by 9, 11, 13 will leave in each case the same remainder 6?
(a) 0 (b) 1
(c) 2 (d) 3

12. 24 is divided into two parts such that 7 times the first part added to 5 times the second part makes 146. The first part is
(a) 11 (b) 13
(c) 16 (d) 17

13. $\frac{1}{4}$ of a number subtracted from $\frac{1}{3}$ of the number gives 12. The number is
(a) 144 (b) 120
(c) 72 (d) 63

14. $\frac{4}{3}$ of a certain number is 64. Half of that number is
(a) 32 (b) 40
(c) 80 (d) 16

15. A fraction becomes 4 when 1 is added to both the numerator and denominator; and it becomes 7 when 1 is subtracted from both the numerator and denominator. The numerator of the given fraction is
(a) 2 (b) 3
(c) 7 (d) 15

16. Three numbers are in the ratio 3 : 4 : 5. The sum of the largest and the smallest equals the sum of the third and 52. The smallest number is
(a) 20 (b) 27
(c) 39 (d) 52

17. The sum of three numbers is 68. If the ratio between first and second is 2 : 3 and that between second and third is 5 : 3, then the second number is
(a) 30 (b) 20
(c) 58 (d) 48

18. If 1 is added to the denominator of a fraction, the fraction becomes $\frac{1}{2}$. If 1 is added to the numerator, the fraction becomes 1. The fraction is
(a) $\frac{4}{7}$ (b) $\frac{5}{9}$
(c) $\frac{2}{3}$ (c) $\frac{10}{11}$

19. $\frac{4}{5}$ of a number exceeds its $\frac{2}{3}$ by 8. The number is
(a) 30 (b) 60
(c) 90 (d) None of these

20. What is the sum of all prime numbers from 60 to 80?
(a) 361 (b) 341
(c) 351 (d) 349

21. The quotient arising from the division of 24446 by a certain divisor is 79 and the remainder is 35, what is the divisor?
(a) 39 (b) 309
(c) 390 (c) 3009

22. In a division sum, the quotient is 120, the divisor 456 and the remainder 333, find the dividend.
(a) 5533 (b) 50553
(c) 56053 (d) 55053

23. The quotient arising from a division of a number by 62 is 463 and the remainder is 60, what is the number?
(a) 28666 (b) 28766
(c) 28576 (c) 28676

24. A number when divided by 221 gives a remainder 43. What remainder will be obtained by dividing the same number by 17?
(a) 11 (b) 8
(c) 9 (d) 13

25. Which one of the following is the largest prime number of three digits?
(a) 997 (b) 999
(c) 991 (d) 993

26. When a certain number is multiplied by 7, the product consists entirely of fives; find the least value of such a number.
(a) 79365 (b) 78365
(c) 77365 (d) 79265

27. In a division sum, the divisor is 10 times the quotient and five times the remainder. What is the dividend, if the remainder is 46?
(a) 5636 (b) 5536
(c) 5336 (d) 5436

28 Which one of the following is the least number of four digits divisible by 71?
(a) 1006 (b) 1065
(c) 1094 (d) 1056

29. How many numbers up to 100 are dvisible by 7?
(a) 14 (b) 107
(c) 93 (d) 100

30. How many numbers up to 500 are divisible by 23?
(a) 23 (b) 27
(c) 21 (d) 19

31. How many numbers up to 200 are divisible 2 and 3 both?
(a) 35 (b) 33
(c) 29 (d) 27

32. How many numbers between 100 and 300 are divisible by 11?
(a) 11 (b) 10
(c) 12 (d) 18

33. How many numbers between 150 and 500 are divisible by 2, 3 and 7 together?
(a) 9 (b) 8
(c) 10 (d) 11

34. The number of five figures to be added to a number of four fives to obtain the least number of six figures exactly divisible by 357 is
(a) 94762 (a) 94802
(c) 94485 (d) None of these

35. The nearest figure to 58701 which is divisible by 567 is
(a) 58968 (b) 58434
(c) 58401 (d) None of these

36. The digits indicated by * in 3422213 ** so that this number is divisible by 99 are
(a) 1, 9 (b) 3, 7
(c) 4, 6 (d) 5, 5

37. The least value to be given to * so that the number 5 * 3457 is divisible by 11 is

(a) 2 (b) 3
(c) 0 (d) 4

38. The nearest whole number to one million which is divisible by 537 is
(a) 1000106 (b) 999894
(c) 1000437 (d) 999563

39. The smallest number between 400 and 500 which is divisible by 9 is
(a) 414 (b) 405
(c) 423 (d) None of these

40. Which one of the following is the greatest number of five digits divisible by 231?
(a) 99792 (b) 99892
(c) 99692 (d) 99972

41. Find the number nearest to 16386 which is exactly divisible by 425.
(a) 16575 (b) 16375
(c) 16050 (d) 16450

42. Find the least number which must be subtracted from 9269 so that resulting number is exactly divisible by 73?
(a) 17 (b) 57
(c) 71 (d) 63

43. Find the least number which must be added to 15463 so that the resulting number is exactly divisible by 107?
(a) 52 (b) 71
(c) 55 (d) 19

44. What is the number just more than 5000 which is exactly divisible by 73?
(a) 5001 (b) 5009
(c) 5037 (d) 5027

45. The sum of two numbers is 100 and their difference is 37. The difference of their squares is
(a) 37 (b) 100
(c) 63 (d) 3700

46. The number of times 79 be subtracted from 50000, so that the remainder be 43759; is
(a) 69 (b) 79
(c) 59 (d) None of these

47. The ratio between two numbers is 3 : 4 and their sum is 420. The greater of the two numbers is
(a) 175 (b) 200
(c) 240 (d) 315

48. The difference between the squares of two consecutive numbers is 35. The numbers are
(a) 14, 15 (b) 15, 16
(c) 17, 18 (d) 18, 19

49. Three-fourths of one-fifth of a number is 60. The number is
(a) 300 (b) 400
(c) 450 (d) 1200

50. The sum of squares of two numbers is 80 and the square of their difference is 36. The product of the two numbers is
(a) 22 (b) 44
(c) 58 (d) 116

51. A number when divided by 357 gives a remainder 37. By dividing the same number by 17, the remainder would be
(a) 3 (b) 4
(c) 2 (d) None of these

52. The product of two numbers is 120. The sum of their squares is 289. The sum of the two numbers is
(a) 20 (b) 23
(c) 169 (d) None of these

53. Three numbers are in the ratio 4 : 5 : 6 and their average is 25. The largest number is
(a) 42 (b) 36
(c) 30 (d) 32

54. A number exceeds 20% of itself by 40. The number is
(a) 50 (b) 60
(c) 80 (d) 320

55. If 16% of 40% of a number is 8, the number is
(a) 200 (b) 225
(c) 125 (d) 320

56. 4767 exactly divides *** 341, the missing digits are
(a) 468 (b) 586
(c) 363 (d) None of these

57. A number when divided by a certain divisor left remainder 241, when twice the number was divided by the same divisor, the remainder was 112. Find the divisor.
(a) 370 (b) 365
(c) 380 (d) 456

58. Two numbers when divided by a certain divisor give remainders 43 and 37 respectively, when their sum is divided by the same divisor, the remainder is 13. Find the divisor.
(a) 71 (b) 67
(c) 57 (d) 77

59. Two numbers are such that the ratio between them is 3 : 5; but if each is increased by 10, the ratio between them becomes 5 : 7. The numbers are
(a) 3, 5 (b) 7, 9
(c) 13, 22 (d) 15, 25

60. Divide 50 into two parts so that the sum of their reciprocals is 1/12.
(a) 20, 30 (b) 24, 26
(c) 28, 22 (d) 36, 14

61. The sum of seven numbers is 235. The average of the first three is 23 and that of last three is 42. The fourth number is
(a) 40 (b) 126
(c) 69 (d) 195

62. The sum of squares of two numbers is 68 and the squares of their difference is 36. The product of the two numbers is
(a) 16 (b) 32
(c) 58 (d) 104

63. What is the least value of K so that the number 6735K1 is divisible by 9?
(a) 5 (b) 7
(c) 4 (d) 3

64. For what value of K, the number 7236K2 is divisible by 8?
(a) 7 (b) 5
(c) 4 (d) 9

65. Find the least values of x and y so that the number $5x423y$ is divisible by 88.
(a) 8, 2 (b) 7, 3
(c) 9, 4 (d) 6, 5

66. 24 is divided into two parts such that 7 times the first part added to 5 times the second part makes 146. The first part is
(a) 13 (b) 15
(c) 17 (d) 19

67. Sum of three numbers is 132. First number is twice the second and third number is one-third of the first. Find the second number.
(a) 18 (b) 36
(c) 20 (d) 16

68. What least number must be added to 7231 so that the resulting number is exactly divisible by 5 and 9 together?
(a) 20 (b) 18
(c) 14 (d) 16

69. Find a number nearest to 9231 which is exactly divisible by 3 as well as by 11.
(a) 9240 (b) 9340
(c) 9540 (d) 9440

70. Find a nearest number to 12199 which is exactly divisible by the product of the first four prime numbers.
(a) 12181 (b) 12179
(c) 11281 (d) 11279

71. The sum of squares of two numbers is 90 and the squares of their difference is 46. The product of the two numbers is
(a) 22 (b) 24
(c) 26 (d) 28

72. If 40% of a number is 360, what will be 15% of that number?
(a) $20\frac{1}{4}$ (b) $20\frac{1}{2}$
(c) $22\frac{1}{4}$ (d) $22\frac{1}{2}$

73. The sum of the digits of a two-digit number is 8. If the digits are reversed the number is increased by 54. Find the number.
(a) 17 (b) 19
(c) 21 (d) 23

EXPLANATORY ANSWERS

1. Ans. (a)

Solution Given Expression $= 7372 \times (7372 + 628)$
$= 7372 \times 8000$
$= 58976000.$

2. Ans. (a)

Solution Let $9999 + 8888 + 777 + x = 19700$
$\therefore \quad x = 19700 - 19664 = 36.$

3. Ans. (a)

Solution Let $x \times 111 = 666666$
$\Rightarrow \quad x = \frac{666666}{111} = 6006$
$\therefore$ Missing figure = 0

4. Ans. (a)

Solution Let $3149 \times x = 425115$
$\Rightarrow \quad x = \frac{425115}{3149} = 135$
$\therefore$ Missing digit = 3.

5. Ans. (d)

Solution Let the age of Mr. Manoj be $(10x + y)$ years.
$\therefore$ His wife's age = $(10y + x)$ years
Then, $(10x + y + 10y + x)\frac{1}{11} = 10x + y - 10y - x$
or, $x + y = 9x - 9y$, or, $8x = 10y$
or, $\frac{x}{y} = \frac{5}{4}$
$\therefore \quad x = 5$ and $y = 4$
[$\because$ any other multiple of 5 will make x of two digits]
$\therefore$ Difference $= 10x + y - 10y - x$
$= 9x - 9y = 9(x - y)$
$= 9(5 - 4) = 9$ years.

6. Ans. (b)

7. Ans. (c)

8. Ans. (c)

9. Ans. (b)

Solution Let $\frac{x}{21} \times \frac{x}{189} = 1$.

Then, $x^2 = 21 \times 189 = 21 \times 21 \times 3 \times 3$

$\therefore \quad x = 21 \times 3 = 63$.

10. Ans. (d)

Solution Let Quotient = Q and remainder = R

Then, given $12Q = 5R$

Now, $R = 48 \Rightarrow 12Q = 5 \times 48 \Rightarrow Q = 20$

$\therefore$ Dividend = 20 × 240 + 48 = 4848.

11. Ans. (b)

Solution The number when divided by 9, 11 and 13 leaving remainder 6 = (L.C.M. of 9, 11, 13) + 6 = 1293

$\therefore$ Required number = 1294 – 1293 = 1.

12. Ans. (b)

Solution Let these parts be x and $(24 - x)$.

Then, $7x + 5(24 - x) = 146 \Rightarrow x = 13$

So the first part is 13.

13. Ans. (a)

Solution $\frac{1}{3}x - \frac{1}{4}x = 12 \Rightarrow \frac{1}{12}x = 12 \Rightarrow x = 144$.

14. Ans. (b)

Solution $\frac{4}{5} \times x = 64 \Rightarrow x = \frac{64 \times 5}{4} = 80$

$\therefore \quad \frac{1}{2} \times x = \frac{1}{2} \times 80 = 40$.

15. Ans. (d)

Solution Let the required fraction be $\frac{x}{y}$

Then, $\frac{x+1}{y+1} = 4 \Rightarrow x - 4y = 3$

and, $\frac{x-1}{y-1} = 7 \Rightarrow x - 7y = -6$

Solving these equations, we get $x = 15$, $y = 3$.

16. Ans. (c)

Solution Let the numbers be $3x$, $4x$ and $5x$

Then, $5x + 3x = 4x + 52 \Rightarrow 4x = 52 \Rightarrow x = 13$

$\therefore$ The smallest number = $3x = 3 \times 13 = 39$.

17. Ans. (a)

Solution Let the numbers be a, b, c. Then,

$$\frac{a}{b} = \frac{2}{3}, \frac{b}{c} = \frac{5}{3} \Rightarrow \frac{a}{b} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}, \frac{b}{c} = \frac{5 \times 3}{3 \times 3} = \frac{15}{9}$$

$$\Rightarrow a : b : c = 10 : 15 : 9$$

Let the numbers be $10x$, $15x$, $9x$,

Then, $10x + 15x + 9x = 68 \Rightarrow 34x = 68 \Rightarrow x = 2$

$\therefore$ Second number = $15x = 15 \times 2 = 30$.

18. Ans. (c)

Solution Let the required fraction be $\frac{x}{y}$. Then,

$$\frac{x}{y+1} = \frac{1}{2} \Rightarrow 2x - y = 1$$

and, $$\frac{x+1}{y} = 1 \Rightarrow x - y = -1$$

Solving $2x - y = 1$ and $x - y = -1$,

we get $x = 2$, $y = 3$

$\therefore$ The fraction is $\frac{2}{3}$.

19. Ans. (b)

Solution Let the number be x. Then,

$$\frac{4}{5}x - \frac{2}{3}x = 8 \Rightarrow \frac{12x - 10x}{15} = 8 \Rightarrow 2x = 120$$

or, $x = 60$.

20. Ans. (c)

Solution 61 + 67 + 71 + 73 + 79 = 351.

21. Ans. (b)

Solution Divisor = (24446 – 35) ÷ 79 = 309 .

22. Ans. (d)

Solution Dividend = 456 × 120 + 33 = 55053.

23. Ans. (b)

Solution Number = 463 × 62 + 60 = 28766.

24. Ans. (c)

Solution Dividing 43 by 17, the remainder is 9.

25. Ans. (a)

26. Ans. (a)

Solution

7	555555
	79365

$\therefore$ The least number is 79365.

27. Ans. (c)

Solution Remainder = 46

Divisor = 5 × 46 = 230

Also 10 × quotient = 5 × remainder

$\therefore$ Remainder = 2 × quotient

i.e., quotient = 23

Dividend = 23 × 230 + 46 = 5336.

28. Ans. (b)

Solution

```
71) 1000 (14
    -71
     290
    -284
       6
```

$\therefore$ least number = 1000 – 6 + 71 = 1065.

29. Ans. (a)

Solution Quotient when 100 is divided by 7 is 14.

30. Ans. (c)

Solution Quotient when 500 is divided by 23 is 21.

31. Ans. (b)

Solution Quotient when 200 is divided by the L.C.M. of 2 and 3, i.e. 6 is 33.

32. Ans. (d)

Solution Quotient when 300 and 100 are divided by 11 are 27 and 9. ∴ Between 300 and 100, there are 27 – 9, i.e. 18 numbers.

33. Ans. (a)

Solution A number divisible by 2, 3, and 7 is divisible by their L.C.M., i.e. 42. Up to 100, there are 2 numbers divisible by 42. Up to 500, there are 11 numbers divisible by 42.

∴ Between 100 and 500, there are 11 – 2, i.e. 9 numbers divisible by 42.

34. Ans. (a)

Solution The least no. of six figures is 100000

On dividing 100000 by 357, remainder = 40

∴ Least number of six figures which is divisible by 357

= 100000 + (357 – 40) = 100317

∴ Required number = 100317 – 5555 = 94762.

35. Ans. (a)

Solution On dividing 58701 by 567

Remainder = $300 > \frac{1}{2}(567)$

∴ Integer nearest to 58701 and divisible by 567

= 58701 + (567 – 300)

= 58701+ 267 = 58968.

36. Ans. (a)

Solution Let x, y be the required digits.

The number is to be divisible by 99, i.e. 9 and 11 both.

∴ Sum of digits is to be divisible by 9, i.e.

$3 + 4 + 2 + 2 + 2 + 1 + 3 + x + y = 17 + x + y$

is to be divisible by 9

and, $(y + 3 + 2 + 2 + 3) - (x + 1 + 2 + 4) = 0$

or, multiple of 11, i.e. $y - x + 3 = 0$ or multiple of 11

∴ $x = 1, y = 9$.

37. Ans. (a)

Solution Let the least value to be given to * be x

Then, $x + 4 + 7 = 5 + 3 + 5$

$x = 2$.

38. Ans. (b)

Solution On dividing 100000 by 537, remainder

$= 106 < \frac{537}{2}$

∴ Nearest whole number to one million which is divisible by 537 = 100000 – 106 = 999894.

39. Ans. (b)

Solution On dividing 400 by 9, remainder = 4

∴ Number nearest to 400 and divisible by 9

= 400 + (9 – 4) = 405.

40. Ans. (a)

Solution Greatest number of five digits = 99,999

Dividing this by 231, the remainder = 207

∴ Required greatest number

= 99,999 – 207 = 99792.

41. Ans. (d)

Solution Dividing 16386 by 425, the remainder is 361 which is more than half the divisor, therefore, the required number is 16368 + (425 – 361) = 16450.

42. Ans. (c)

Solution Divide 9269 by 73, the remainder is 71.

∴ 71 is the required least number.

43. Ans (a)

Solution Divide 15463 by 107, the remainder is 55, therefore, the number to be added = 107 – 55 = 52.

44. Ans. (c)

Solution Dividing 5000 by 73, the remainder is 36. The number greater than 5000 is obtained by adding to 5000 the difference of divisor and the remainder.

∴ The required number = 5000 + (73 – 36) = 5037.

45. Ans. (d)

Solution Let the numbers be a and b.

Then, $a + b = 100$ and $a - b = 37$

∴ $a^2 - b^2 = (a + b)(a - b) = 100 \times 37 = 3700$.

46. Ans. (b)

Solution 50000 = 79 × quotient + 43759

∴ 50000 – 43759 = 79 × quotient

or, 6241 = 79 × quotient

∴ Required number of times = $\frac{6241}{79} = 79$.

47. Ans. (c)

Solution Let the numbers be $3x$ and $4x$.

Then, $3x + 4x = 420 \Rightarrow 7x = 420 \Rightarrow x = 60$

∴ Greater number = 4 × 60 = 240.

48. Ans. (c)

Solution Let the numbers be x and $(x + 1)$. Then,

$$(x + 1)^2 - x^2 = 35 \Rightarrow x^2 + 2x + 1 - x^2 = 35$$
$$\Rightarrow 2x = 34$$

or $x = 17$

So, the numbers are 17 and 18.

49. Ans. (b)

Solution Let the number be x. Then,

$$\frac{3}{4} \times \frac{1}{5} \times x = 60 \Rightarrow 3x = 60 \times 5 \times 4$$

or, $x = 400$.

50. Ans. (a)

Solution Let the numbers be a and b. Then,

$$a^2 + b^2 = 80 \text{ and } (a - b)^2 = 36$$
$$(a - b)^2 = 36 \Rightarrow a^2 + b^2 - 2ab = 36$$
$$\Rightarrow 2ab = (a^2 + b^2) - 36 = 80 - 36 = 44$$
$$\Rightarrow ab = 22.$$

51. Ans. (a)

Solution Let K be the quotient. Then, number

$$= 357 \times K + 37$$
$$= 17 \times 21K + 17 \times 2 + 3$$
$$= 17 \times (21K + 2) + 3$$

So, required remainder = 3 and new quotient = $21K + 2$.

52. Ans. (b)

Solution Let the numbers be a and b. Then,

$$(a + b)^2 = (a^2 + b^2) + 2ab$$
$$= 289 + 2 \times 120 = 289 + 240 = 529$$
$$\therefore \quad a + b = \sqrt{529} \Rightarrow a + b = 23.$$

53. Ans. (c)

Solution Let the numbers be $4x$, $5x$ and $6x$. Then,

$$\frac{4x + 5x + 6x}{3} = 25 \quad \text{or} \quad 15x = 75 \Rightarrow x = 5$$

$\therefore$ The largest number = $6x = 6 \times 5 = 30$.

54. Ans. (a)

Solution Let the required number be x. Then,

$$x - \frac{20}{100}x = 40 \quad \text{or,} \quad 5x - x = 200 \quad \text{or,} \quad x = 50.$$

55. Ans. (c)

Solution Let $\frac{16}{100} \times \frac{40}{100} \times x = 8$

Then, $x = \frac{8 \times 100 \times 100}{16 \times 40} = 125.$

56. Ans. (b)

Solution Last digit of dividend = 1

Last digit of divisor = 7

$\therefore$ Last digit of quotient should be 3

$$4767 \times 3 = 14301$$
$$4767 \times 20 = 95340$$
$$4767 \times 100 = 476700$$
$$\therefore \quad 4767 \times (3 + 20 + 100) = 586341$$

Missing digits are 586.

57. Ans. (a)

Solution The divisor $= r_1 + r_2 - r_3$

$$= 241 + 241 - 112 = 370.$$

58. Ans. (b)

Solution The divisor $= r_1 + r_2 - r_3 = 43 + 37 - 13 = 67$.

59. Ans. (d)

Solution Let the numbers be $3x$ and $5x$. Then,

$$\frac{3x + 10}{5x + 10} = \frac{5}{7} \Rightarrow 7(3x + 10) = 5(5x + 10)$$
$$\Rightarrow x = 5$$

$\therefore$ The numbers are 15 and 25.

60. Ans. (a)

Solution Let the numbers be x and $(50 - x)$. Then,

$$\frac{1}{x} + \frac{1}{50 - x} = \frac{1}{12} \Rightarrow \frac{50 - x + x}{x(50 - x)} = \frac{1}{12}$$
$$\Rightarrow x^2 - 50x + 600 = 0$$
$$\Rightarrow x = 30 \text{ or } 20.$$

$\therefore$ The numbers are 20, 30.

61. Ans. (a)

Solution $23 \times 3 + x + 42 \times 3 = 235 \Rightarrow x = 40$

$\therefore$ Fourth number = 40.

62. Ans. (a)

Solution Let the numbers be a and b. Then,

$$a^2 + b^2 = 68 \text{ and } (a - b)^2 = 36$$

Now, $(a - b)^2 = 36 \Rightarrow a^2 + b^2 - 2ab = 36$

$$\Rightarrow 68 - 2ab = 36$$
$$\Rightarrow 2ab = 32 \Rightarrow ab = 16.$$

63. Ans. (a)

Solution $6 + 7 + 3 + 5 + K + 1 = 22 + K$

The least number greater than 22 and divisible by 9 is 27

$\therefore \quad 27 = 22 + K \Rightarrow K = 5.$

64. Ans. (a)

Solution The last three digits $6K2$ is divisible by 8 if K is 3 or 7 since 632 and 672 are divisible by 8.

65. Ans. (a)

Solution Test of 8 is independent of the value of x. First, apply the test of 8. Last three digit of the given number are $23y$ which is divisible by 8 if y is 2. Substitute for y.

The number now becomes $5x4232$.

Apply the test of 11.

Sum of the digits at odd and even places is

$5 + 4 + 3$, i.e. 12 and $x + 2 + 2$, i.e. $x + 4$

$\therefore \quad x + 4 = 12 \Rightarrow x = 8$

Hence, $x = 8$ and $y = 2$.

66. Ans. (a)

Solution Let x be the first part so that the other part is $24 - x$.

$\therefore \quad 7x + 5(24 - x) = 146 \Rightarrow x = 13.$

67. Ans. (b)

Solution Let the second number be $3x$, so that the first number is $6x$ and the third one is $2x$.

$\therefore \quad 6x + 3x + 2x = 132 \Rightarrow 11x = 132$ or $x = 12$.

Second number $= 3x = 3 \times 12 = 36$.

68. Ans. (c)

Solution Divide 7231 by 45, the remainder is 31.

$\therefore$ Required number $= 45 - 31 = 14$.

69. Ans. (a)

Solution A number which is divisible by 3 and 11 is also divisible by 33. Dividing 9331 by 33, the remainder is 24 which is more than half the divisor.

```
33)9331(279
   66
   263
   231
    321
    297
     24
```

Since the remainder 24 is more than half the divisor 33,

$\therefore$ The nearest number $= 9231 + (33 - 24) = 9240$.

70. Ans. (a)

Solution Product of first four prime numbers is

$$2 \times 3 \times 5 \times 7 = 210$$

Dividing 12199 by 210, we find the remainder 19, which is less than half the divisor.

```
210)12199(58
    1050
     1699
     1680
       19
```

$\therefore$ The number nearer to 12199 divisible by 210 is $12199 - 18 = 12181$.

71. Ans. (a)

Solution Let the numbers be x and y.

According to the question:

$x^2 + y^2 = 90$ ....(1)

and, $(x - y)^2 = 46$ ....(2)

From equation (2),

$(x - y)^2 = 46$ or, $x^2 + y^2 - 2xy = 46$

or, $90 - 2xy = 46$ or, $xy = \dfrac{90 - 46}{2} = 22$

$\therefore$ Product of two numbers = 22.

72. Ans. (a)

Solution Let the number be x. Then, we have 40% of $x = 360$

$\therefore \quad x = \dfrac{360 \times 100}{40} = 900$

Now, 15% of $x = \dfrac{15}{100} \times 900 = 135$

Again, 15% of $135 = \dfrac{15}{100} \times 135 = 20.25$.

73. Ans. (a)

Solution Let the two digit number be $10x + y$

Then, we have, $x + y = 8$ ....(1)

and, $10y + x = 10x + y + 54$

or, $y - x = 6$ ...(2)

From equations (1) and (2),

the required number $= 1 \times 10 + 7 = 17$.

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. If $\frac{1}{8}$ of $\frac{2}{3}$ of $\frac{4}{5}$ of a number is 12, then 30 per cent of the number will be

(a) 48 (b) 64
(c) 54 (d) 42
(e) None of these

[Based on BSRB Chennai Bank PO 2000]

2. When any number is divided by 12 then, dividend becomes $\frac{1}{4}$ of the other number. By how much per cent is first number greater than the second number?

(a) 200 (b) 150
(c) 300 (d) Data inadequate
(e) None of these

[Based on BSRB Chennai Bank PO 2000]

3. What approximate value should come in place of the question mark (?) in the following equation?

$$6.39 \times 15.266 + 115.8 \text{ of } \frac{2}{5} = ?$$

(a) 145 (b) 165
(c) 180 (d) 130

[Based on BSRB Chennai Bank PO 2000]

4. What should come in place of question mark (?) in the following equation?

$$8597 - ? = 7429 - 4358$$

(a) 5706 (b) 5526
(c) 5426 (d) 5626
(e) None of these

[Based on BSRB Chennai Bank PO 2000]

5. In a set of 5 numbers the sum of two of these numbers is 6 more than the sum of the remaining three numbers, whereas the sum of these three numbers is two times one of those two numbers. What definitely is one of those two numbers?

(a) 18 (b) 20
(c) 16 (d) Data inadequate
(e) None of these

[Based on BSRB Chennai Bank PO 2000]

6. The sum of the digits of a two digit number is $\frac{1}{5}$ of the difference between the number and the number obtained by interchanging the positions of the digits. What definitely is the difference between the digits of that number?

(a) 5 (b) 9
(c) 7 (d) Data inadequate
(e) None of these

[Based on BSRB Chennai Bank PO 2000]

7. A number gets reduced to its one-third when 48 is subtracted from it. What is two-thirds of that number?

(a) 24 (b) 72
(c) 36 (d) 46
(e) None of these

[Based on BSRB Bhopal Bank PO 2000]

8. The sum of three consecutive numbers is given, what is the difference between first and third number?

(a) 1 (b) 3
(c) either 1 or 2 (d) 2
(e) None of these

[Based on BSRB Bhopal Bank PO 2000]

9. What will come in place of the question mark (?) in the following equation?

$0.001 + 9.909 \times 1.01 \div 0.1 = ?$

(a) 99.0819 (b) 100.0819
(c) 100.091 (d) 100.0919
(e) None of these

[Based on BSRB Bhopal Bank PO 2000]

10. $(299.99999)^3 = ?$

(a) 27000000 (b) 9000000000
(c) 180000 (c) 2.7×10^9
(e) 27000000

11. If the two digits of the age of Mr. Manoj are reversed then, the new age so obtained is the age of his wife. $\frac{1}{11}$ of the sum of their ages is equal to the difference between their ages. If Mr. Manoj is elder than his wife then find the difference between their ages?

(a) Cannot be determined
(b) 10 years
(c) 8 years (d) 7 years
(e) 9 years

[Based on BSRB Bangalore Bank PO 2000]

12. If $x = 9$, then what will be the value of following expression?

$$\frac{20x^3 + 12x + 3 + 5x^2}{10x^3 + 3 + 5x^2 + 6x}$$

(a) $1\frac{18}{19}$ (b) $\frac{188}{969}$
(c) $1\frac{88}{89}$ (d) Cannot be determined
(e) None of these

[Based on BSRB Bangalore Bank PO 2000]

13. A number is greater than the square of 44 but smaller than the square of 45. If one part of the number is the square of 6 and the number is a multiple of 5, then find the number.

(a) 1940 (b) 2080
(c) 1980 (d) Cannot be determined
(e) None of these

[Based on BSRB Bangalore Bank PO 2000]

14. Which least number should be subtracted from 427396 so that the remainder would be divisible by 15?

(a) 6 (b) 1
(c) 16 (d) 11
(e) None of these

[Based on BSRB Delhi Bank PO 2000]

15. A number is decreased by 4 and divided by 6, the result is 9. What would be the result if 3 is subtracted from the number and then it is divided by 5?

(a) $9\frac{2}{5}$ (b) $10\frac{1}{5}$
(c) $11\frac{2}{5}$ (d) 11
(e) None of these

[Based on BSRB Delhi Bank PO 2000]

16. A two digits number is seven times the sum of its digits. If each digit is increased by 2, the number thus obtained is 4 more than six times the sum of its digits. Find the number.

(a) 42 (b) 24
(c) 48 (d) data inadequate
(e) None of these

[Based on BSRB Patna Bank PO 2001]

17. If A and B are positive integers such that $9A^2 = 12A + 96$ and $B^2 = 2B + 3$, then which of the following is the value of $5A + 7B$?

(a) 31 (b) 41
(c) 36 (d) 43
(e) 27 **[Based on BSRB Patna Bank PO 2001]**

18. The digit in the units place of a number is equal to the digit in the tens place of half of that number and the digit in the tens place of that number is less than the digit in units place of half of the number by 1. If the sum of the digits of the number is seven, then what is the number?
(a) 52 (b) 16
(c) 34 (d) Data inadequate
(e) None of these **[Based on SBI Bank PO 2001]**

19. What will come in place of question mark (?) in the following question:
$\frac{3}{5}$ of $\frac{4}{7}$ of $\frac{5}{12}$ of 1015 = ?
(a) 220 (c) 340
(c) 240 (d) 145
(e) None of these
[Based on Andhra Bank Specialist Officers Exam 2002]

20. Two fifths of one-third of three-sevenths of a number is 15. What is 40 per cent of that number?
(a) 136 (b) 140
(c) 72 (d) 84
(e) None of these
[Based on IBPS Jr. Executive Exam 2002]

21. If 3167 is added to 4093 and the sum is divided by 145, approximately what will be the outcome?
(a) 50 (b) 75
(c) 60 (d) 90
(e) 80 **[Based on IBPS Jr. Executive Exam 2002]**

22. When a number is divided by 31 the remainder is 29. When the same number is divided by 16, what will be the remainder?
(a) Data inadequate (b) 13
(c) 15 (d) 11
(e) None of these
[Based on IBP Jr. Executive Exam 2002]

23. If the digit in the units place of a two-digit number is halved and the digit in the tens place is doubled, the number thus obtained is equal to the number obtained by interchanging the digits. Which of the following is definitely true?
(a) Digits in the units place and the tens place are equal.
(b) Sum of the digits is a two-digit number.
(c) Digit is the units place is half of the digit in the tens place.
(d) Digit in the unit's place is twice the digit in the tens place.
(e) None of these.
[Based on PNB Management Trainee Exam 2003]

24. Twenty times a positive integer is less than its square by 96. What is the integer?
(a) 24 (b) 20
(c) 30 (d) Cannot be determined
(e) None of these
[Based on Bank of Maharashtra PO Exam 2003]

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Let the number be x

and, $\frac{1}{8}$ of $\frac{2}{3}$ of $\frac{4}{5} \times x = 12$

$\therefore \frac{3x}{10} = \frac{3}{10} \times 12 \times 15 = 54.$

2. Ans (d)

Solution Here, neither the remainder nor the dividend nor the second number is given, so number cannot be determined.

3. Ans. (a)

Solution $? = 6.39 \times 15.266 + 115.8 \text{ of } \frac{2}{5}$
$\approx 6.50 \times 15 + 115 \times 0.4$
$= 97.50 + 46 \approx 145.$

4. Ans. (b)

Solution $8597 - ? = 7429 - 4358$

or, $8597 - ? = 3071$

$\therefore \quad ? = 8597 - 3071 = 5526.$

5. Ans. (d)

6. Ans. (a)

Solution Let the two digit number be $10x \times y$

Then, $x + y = \frac{1}{5}(10x + y - 10y - x)$

or, $x + y = \frac{9}{5}(x - y)$

or, $4x - 14y = 0 \Rightarrow \frac{x}{y} = \frac{7}{2}$

Using componendo and dividendo, we have

$$\frac{x+y}{x-y} = \frac{7+2}{7-2} = \frac{9}{5}$$

i.e. $x - y = 5k$

Here, k has the only possible value, $k = 1$

Because the difference of two single-digit numbers will always be of a single digit.

7. Ans. (e)

Solution Let the number be x.

Then, $x - \frac{x}{3} = 48$

$\therefore \quad \frac{2}{3}x = 48.$

8. Ans. (d)

Solution Let the three consecutive numbers be x, $x + 1$ and $x + 2$ respectively.

$\therefore$ Difference between first and third number

$= x + 2 - x = 2.$

9. Ans. (b)

Solution $? = 0.001 + 9.909 \times 1.01 \div 0.1$

$= 0.001 + 9.909 \times 1.01 \times \frac{1}{0.1}$

$= 0.001 + 9.909 \times 10.1$

$= 0.001 + 100.0809$

$= 100.0819.$

10. Ans. (a)

Solution $? = (299.99999)^3$

$\approx (300)^3$

$= 27000000.$

11. Ans. (e)

Solution Let the age of Mr. Manoj be $(10x + y)$ years.

$\therefore$ His wife's age $= (10y + x)$ years

Then, $(10x + y + 10y + x)\frac{1}{11}$

$= 10x + y - 10y - x$

or, $x + y = 9x - 9y$ or, $8x = 10y$

or, $\frac{x}{y} = \frac{5}{4}$

$\therefore \quad x = 5$ and $y = 4$

[$\because$ any other multiple of 5 will make x of two digits]

$\therefore$ Difference $= 10x + y - 10y - x = 9x - 9y$

$= 9(x - y) = 9(5 - 4) = 9$ years.

12. Ans. (a)

Solution Given expression is

$$\frac{20x^3+12x+3+5x^2}{10x^3+3+5x^2+6x} = \frac{(20x^3+12x)+(3+5x^2)}{(10x^3+5x^2)+(3+6x)}$$

$$= \frac{4x(5x^2+3)+1(3+5x^2)}{5x^2(2x+1)+3(2x+1)}$$

$$= \frac{(5x^2+3)(4x+1)}{(2x+1)(5x^2+3)} = \frac{4x+1}{2x+1}$$

$$= \frac{37}{19} = 1\frac{18}{19}.$$

13. Ans (a)

Solution Let the number be x

$(44)^2 < x < (45)^2 \Rightarrow 1936 < x < 2025$ ...(1)

From equation (1), the required number will be any number between 1936 and 2025. Since one part of the number is the square of 6 means one factor is 36.

$\therefore$ L.C.M. of 36 and 5 = 180

$\therefore$ Number will be a multiple of 180,

i.e. $180 \times 11 = 1980$, the only value which satisfies the equation (1).

14. Ans. (b)

15. Ans. (d)

Solution Let the number be x

$\therefore \quad \frac{x-4}{6} = 9 \Rightarrow x = 58$

Again, $\frac{x-3}{5} = \frac{58-3}{5} = \frac{55}{5} = 11.$

16. Ans. (a)

Solution Let the two digit number be $10x + y$

$10x + y = 7(x + y) \Rightarrow x = 2y$ ...(1)

$10(x + 2) + y + 2 = 6(x + y + 4) + 4$

or, $10x + y + 22 = 6x + 6y + 28$

$\Rightarrow \quad 4x - 5y = 6$ ...(2)

Solving equations, (1) and (2), we get $x = 4$, $y = 2$.

17. Ans. (b)

Solution $9A^2 = 12A + 96 \Rightarrow 3A^2 - 4A - 32 = 0$

$\therefore \quad A = \frac{4 \pm \sqrt{16+384}}{6} = 4, \frac{-8}{3}$

$B^2 = 2B + 3 \Rightarrow B^2 - 2B - 3 = 0$

$\therefore \quad B = \frac{2 \pm \sqrt{4+12}}{2} = 3, -1$

$\therefore \quad 5A + 7B = 5 \times 4 + 7 \times 3 = 20 + 21 = 41.$

18. Ans. (a)

Solution Let $\frac{1}{2}$ of the number $= 10x + y$ and the number $= 10V + W$

From the given conditions,

$W = x$ and $V = y - 1$

Thus, the number $= 10(y - 1) + x$ ...(A)

$\therefore \quad 2(10x + y) = 10(y - 1) + x$

$\Rightarrow \quad 8y - 19x = 10$ ...(1)

Again, from the question,

$V + W = 7 \Rightarrow y - 1 + x = 7$

$\therefore \quad x + y = 8$ ...(2)

Solving equations (1) and (2), we get

$x = 2$ and $y = 6$ $\quad \therefore$ from equation (A)

Number $= 10(y - 1) + x = 52.$

19. Ans. (d)

Solution $\frac{3}{5} \times \frac{4}{7} \times \frac{5}{12} \times 1015 = \frac{1015}{7} = 145.$

20. Ans. (e)

Solution Let the number be 'x'. Then,

$$\frac{2}{5} \times \frac{1}{3} \times \frac{3}{7} \times x = 15$$

or, $\frac{2x}{35} = 15$ or, $x = \frac{15 \times 35}{2}$

$\therefore$ 40% of $x = \frac{2}{5} \times \frac{15 \times 35}{2} = 105.$

21. Ans. (a)

Solution Required number $= \frac{3167 + 4093}{145} = \frac{7260}{145} \approx 50.$

22. Ans. (a)

Solution Quotient is not given. Hence, remainder cannot be determined.

23. Ans. (d)

Solution Suppose the two digit number is $10x + y$

Then, $10y + x = 20x + y/2$

or, $20y + 2x = 40x + y$ or, $y = 2x$.

24. Ans. (a)

Solution Let the positive integer be x.

Now, $x^2 - 20x = 96$

or, $x^2 - 20x - 96 = 0$

or, $x^2 - 24x + 4x - 96 = 0$

or, $x(x - 24) + 4(x - 4) = 0$

or, $(x - 24)(x + 4) = 0$

or, $x = 24, -4$.

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. Out of three numbers, the first is twice the second and is half of the third. If the average of three numbers is 56, the smallest number is
(a) 24 (b) 36
(c) 40 (d) 48

2. How many numbers up to 450 are divisible by 4, 6 and 8 together?
(a) 19 (b) 18
(c) 17 (d) 16

3. The ratio of the sum and the difference of two numbers is 7 : 1. Find the ratio of those two numbers.
(a) 4 : 3 (b) 3 : 4
(c) 4 : 5 (d) 5 : 3

4. Find the greatest number that will divide 55, 127 and 175 so as to leave the same remainder in each case.
(a) 24 (b) 25
(c) 26 (d) 27

5. If the sum of a number and its square is 182, what is the number?
(a) 15 (b) 13
(c) 26 (d) 19

6. The sum of two numbers is 75 and their difference is 20. Find the difference of their squares.
(a) 1500 (b) 1300
(c) 1100 (d) 1600

7. How many numbers between 100 and 300 are divisible by 9?
(a) 11 (b) 13
(c) 19 (d) 22

8. The denominator of a rational number is 3 more than its numerator. If the numerator is increased by 7 and the denominator is decreased by 2, we obtain 2. The rational number is
(a) $\frac{6}{5}$ (b) $\frac{8}{5}$
(c) $\frac{5}{8}$ (d) $\frac{7}{5}$

9. A number on being divided by 8 and 9 successively leaves the remainder 5 and 7, respectively. Find the remainder when the same number is divided by 72.
(a) 61 (b) 8
(c) 71 (d) 9

10. If one-fifth of one-third of one-half of a number is 15, find the number.
(a) 550 (b) 350
(c) 650 (d) 450

Test II **[Time: 10 min]**

11. The difference between the squares of two consecutive numbers is 37. Find the numbers
(a) 17, 14 (b) 15, 16
(c) 18, 19 (d) 14, 15

12. The sum of three consecutive even numbers is 15 less than three-fourths of 60. What is the middle number?
(a) 5 (b) 10
(c) 20 (d) 15

13. The difference between a two digit number and the number obtained by interchanging the digits is 27. What are the sum and the difference of the two digits of the number?

(a) 3, 4
(b) Cannot be determined; 3
(c) 3, 6
(d) 3, 5

14. Find the number which when multiplied by 16 is increased by 225.
(a) 15 (b) 20
(c) 25 (d) 30

15. If the places of last two digits of a three digit number are interchanged, a new number greater than the original number by 54 is obtained. What is the difference between the last two digits of that number?
(a) 9 (b) 6
(c) 12 (c) 3

16. A number when divided by 899 gives a remainder 63. What remainder will be obtained by dividing the same number by 29?
(a) 5 (b) 6
(c) 7 (d) 8

17. The product of two numbers is 192 and the sum of these two numbers is 28. What is the smaller of these two numbers?
(a) 5 (b) 12
(c) 9 (d) 3

18. The difference between squares of two numbers is 256000 and sum of the numbers is 1000. The numbers are
(a) 628, 372 (b) 600, 400
(c) 640, 630 (b) None of these

19. The ratio between a two-digit number and the sum of the digits of that number is 4 : 1. If the digit at the units place is 3 more than the digit at the tens place, what is the number?
(a) 24 (b) 52
(c) 48 (d) 36

20. The largest natural number by which the product of three consecutive even natural numbers is always divisible is
(a) 48 (b) 16
(c) 96 (d) 24

Answers

1. (a)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (b)	**6.** (a)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (d)
11. (c)	**12.** (b)	**13.** (b)	**14.** (a)	**15.** (b)	**16.** (a)	**17.** (b)	**18.** (a)	**19.** (d)	**20.** (a)

2

H.C.F. AND L.C.M. OF NUMBERS

COMMON FACTOR

A *common factor* of two or more numbers is a number which divides each of them exactly.

For example, 4 is a common factor of 8 and 12.

HIGHEST COMMON FACTOR

Highest common factor of two or more numbers is the greatest number that divides each one of them exactly. For example, 6 is the highest common factor of 12, 18 and 24. Highest Common Factor is also called *Greatest Common Divisor* or *Greatest Common Measure*.

Symbolically, these can be written as H.C.F. or G.C.D. or G.C.M., respectively.

METHODS OF FINDING H.C.F.

I. Method of Prime Factors

Step 1 Express each one of the given numbers as the product of prime factors.

[A number is said to be a *prime number* if it is exactly divisible by 1 and itself but not by any other number, e.g. 2, 3, 5, 7, etc. are prime numbers]

Step 2 Choose Common Factors.

Step 3 Find the product of these common factors. This is the required H.C.F. of given numbers.

Illustration 1 Find the H.C.F. of 70 and 90.

Solution $70 = \mathbf{2} \times \mathbf{5} \times 7$

$90 = \mathbf{2} \times \mathbf{5} \times 9$

Common factors are 2 and 5.

$\therefore$ H.C.F. $= 2 \times 5 = 10$.

Illustration 2 Find the H.C.F. of 3332, 3724 and 4508

Solution $3332 = \mathbf{2} \times \mathbf{2} \times \mathbf{7} \times \mathbf{7} \times 17$

$3724 = \mathbf{2} \times \mathbf{2} \times \mathbf{7} \times \mathbf{7} \times 19$

$4508 = \mathbf{2} \times \mathbf{2} \times \mathbf{7} \times \mathbf{7} \times 23$

$\therefore$ H.C.F. $= 2 \times 2 \times 7 \times 7 = 196$.

Illustration 3 Find the H.C.F. of 360 and 132.

Solution $360 = 2^3 \times 3^2 \times 5$

$132 = 2^2 \times 3^1 \times 11$

$\therefore$ H.C.F. $= 2^2 \times 3^1 = 12$.

Illustration 4 If $x = 2^3 \times 3^5 \times 5^9$ and $y = 2^5 \times 3^7 \times 5^{11}$, find H.C.F. of x and y.

Solution The factors common to both x and y are 2^3, 3^5 and 5^9.

$\therefore$ H.C.F. $= 2^3 \times 3^5 \times 5^9$.

II. Method of Division

***A. For two numbers*:**

Step 1 Greater number is divided by the smaller one.

Step 2 Divisor of (1) is divided by its remainder.

Step 3 Divisor of (2) is divided by its remainder. This is continued until no remainder is left.

H.C.F. is the divisor of last step.

Illustration 5 Find the H.C.F. of 3556 and 3444.

```
3444)3556(1
     3444
      112)3444(30
          3360
            84)112(1
               84
               28)84(3
                  84
                  ×
```

$\therefore$ H.C.F. = 28.

***B. For more than two numbers*:**

Step 1 Any two numbers are chosen and their H.C.F. is obtained.

Step 2 H.C.F. of H.C.F. (of (1)) and any other number is obtained.

Step 3 H.C.F. of H.C.F. (of (2)) and any other number (not chosen earlier) is obtained.

This process is continued until all numbers have been chosen. H.C.F. of last step is the required H.C.F.

Illustration 6 Find the H.C.F. of 13915, 9499 and 2553 by division method.

Solution

```
9499) 13915 (1
      9499
      ----
      4416 ) 9499 (2
             8832
             ----
              667 ) 4416 (6
                    4002
                    ----
                     414 ) 667 (1
                           414
                           ---
                           253 ) 414 (1
                                 253
                                 ---
                                 161 ) 253 (1
                                       161
                                       ---
                                        92 ) 161 (1
                                             92
                                             --
                                             69 ) 92 (1
                                                  69
                                                  --
                                                  23 ) 69 (3
                                                       69
                                                       --
                                                       ×
```

Now, in the next step, we will find the H.C.F. of 23 and 2553.

```
23) 2553 (111
    23
    --
    25
    23
    --
     23
     23
     --
     ×
```

Thus, H.C.F. of 13915, 9499 and 2553 = 23.

Illustration 7 Find the greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm.

Solution Required length

= (H.C.F. of 700, 385, 1295) cm = 35 cm.

COMMON MULTIPLE

A *common multiple* of two or more numbers is a number which is exactly divisible by each one of them.

For example, 32 is a common multiple of 8 and 16.

$8 \times 4 = 32$

$16 \times 2 = 32.$

LEAST COMMON MULTIPLE

The *least common multiple* of two or more given numbers is the least or lowest number which is exactly divisible by each of them.

For example, consider the two numbers 12 and 18.

Multiples of 12 are 12, 24, 36, 48, 72, ...

Multiples of 18 are 18, 36, 54, 72, ...

Common multiples are 36, 72, ...

$\therefore$ Least common multiple, i.e. L.C.M. of 12 and 18 is 36.

METHODS OF FINDING L.C.M.

A. Method of Prime Factors

Step 1 Resolve each given number into prime factors.

Step 2 Take out all factors with highest powers that occur in given numbers.

Step 3 Find the product of these factors. This product will be the L.C.M.

Illustration 8 Find the L.C.M. of 32, 48, 60 and 320.

Solution

$32 = 2^5 \times 1$

$48 = 2^4 \times 3$

$60 = 2^2 \times 3 \times 5$

$320 = 2^6 \times 6$

$\therefore$ L.C.M. $= 2^6 \times 3 \times 5 = 960.$

B. Method of Division

Step 1 The given numbers are written in a line separated by common.

Step 2 Divide by any one of the prime numbers 2, 3, 5, 7, 11, ... which will divide at least any two of the given numbers exactly. The quotients and the undivided numbers are written in a line below the first.

Step 3 Step 2 is repeated until a line of numbers (prime to each other) appears.

Step 4 Find the product of all divisors and numbers in the last line which is the required L.C.M.

Illustration 9 Find the L.C.M. of 12, 15, 20 and 54.

Solution

2	12,	15,	20,	54
2	6,	15,	10,	27
3	3,	15,	5,	27
5	1,	5,	5,	9
	1,	1,	1,	9

L.C.M. = 2 × 2 × 3 × 5 × 1 × 1 × 1 × 9 = 540.

Note: Before finding the L.C.M. or H.C.F., we must ensure that all quantities are expressed in the same unit.

SOME USEFUL SHORT-CUT METHODS

1. H.C.F. and L.C.M. of Decimals

Step 1 Make the same number of decimal places in all the given numbers by suffixing zero(s) if necessary.

Step 2 Find the H.C.F./L.C.M. of these numbers without decimal.

Step 3 Put the decimal point (in the H.C.F./L.C.M. of step 2) leaving as many digits on its right as there are in each of the numbers.

Illustration 10 Find the L.C.M. of 1.2, 0.24 and 6.

Solution The given numbers can be written as

1.20, 0.24 and 6.00.

Now, ignoring the decimal we find the L.C.M. of

120, 24 and 600.

2	120,	24,	600
2	60,	12,	300
2	30,	6,	150
3	15,	3,	75
5	5,	1,	25
	1,	1,	5

∴ L.C.M. = 2 × 2 × 2 × 3 × 5 × 1 × 5 = 600

Thus, the required L.C.M. = 6.00, i.e. 6.

Illustration 11 Find the H.C.F. of 6·16 and 13.

Solution The given numbers can be written as

6.16 and 13.00.

Now, ignoring the decimals we find the H.C.F. of 616 and 1300.

```
616 ) 1300 (2
      1232
      ----
        68 ) 616 (9
             612
             ---
               4 ) 68 (17
                   68
                   --
                    ×
```

∴ H.C.F. of 616 and 1300 is 4.

Thus, the required H.C.F. = 0.04.

2. L.C.M. and H.C.F. of Fractions

$$\text{L.C.M} = \frac{\text{L.C.M. of the numbers in numerators}}{\text{H.C.F. of the numbers in denominators}}$$

$$\text{H.C.F.} = \frac{\text{H.C.F. of the numbers in numerators}}{\text{L.C.M. of the numbers in denominators}}$$

Illustration 12 Find the L.C.M. of $\frac{2}{5}$, $\frac{3}{10}$ and $\frac{6}{25}$.

Solution L.C.M. of numerators 2, 3 and 6 is 6.

H.C.F. of denominators 5, 10 and 25 is 5.

$$\therefore \quad \text{Required L.C.M.} = \frac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}} = \frac{6}{5}.$$

Illustration 13 Find the H.C.F. of $\frac{4}{9}$, $\frac{10}{21}$ and $\frac{20}{63}$.

Solution H.C.F. of numerators 4, 10 and 20 is 2.

L.C.M. of denominators 9, 21 and 63 is 63.

$$\therefore \quad \text{Required H.C.F.} = \frac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}} = \frac{2}{63}.$$

Notes:

1. If the given set of numbers includes fractions as well as whole numbers, treat whole number too as fraction with 1 in its denominator.
2. The H.C.F. of a number of fractions is always a fraction, but the L.C.M. may be a fraction or an integer.

3. Product of two numbers

= L.C.M. of the numbers × H.C.F. of the numbers

Illustration 14 The H.C.F. and the L.C.M. of any two numbers are 63 and 1260, respectively. If one of the two numbers is 315, find the other number.

Solution The required number

$$= \frac{\text{L.C.M.} \times \text{H.C.F.}}{\text{First Number}} = \frac{1260 \times 63}{315} = 252.$$

4. To find the greatest number that will exactly divide x, y and z.

   Required number = H.C.F. of x, y and z.

Illustration 15 Find the greatest number that will exactly divide 200 and 320.

Solution The required greatest number
= H.C.F. of 200 and 320 = 40.

5. To find the greatest number that will divide x, y and z leaving remainders a, b and c, respectively.

   Required number = H.C.F. of $(x - a)$, $(y - b)$ and $(z - c)$.

Illustration 16 Find the greatest number that will divide 148, 246 and 623 leaving remainders 4, 6 and 11, respectively.

Solution The required greatest number

= H.C.F. of (148 – 4), (246 – 6) and (623 – 11),

i.e. H.C.F. of 144, 240 and 612 = 12.

6. To find the least number which is exactly divisible by x, y and z.

   Required number = L.C.M. of x, y and z.

Illustration 17 What is the smallest number which is exactly divisible by 36, 45, 63 and 80?

Solution The required smallest number
= L.C.M. of 36, 45, 63 and 80
= 5040.

7. To find the least number which when divided by x, y and z leaves the remainders a, b and c, respectively. It is always observed that $(x - a) = (y - b) = (z - c) = k$ (say)

   $\therefore$ Required number = (L.C.M. of x, y and z) – k.

Illustration 18 Find the least number which when divided by 36, 48 and 64 leaves the remainders 25, 37 and 53 respectively.

Solution Since, (36 – 25) = (48 – 37) = (64 – 53) = 11, therefore, the required smallest number
= (L.C.M. of 36, 48 and 64) – 11
= 576 – 11 = 565.

8. To find the least number which when divided by x, y and z leaves the same remainder r in each case.
   Required number = (L.C.M. of x, y and z) + r.

Illustration 19 Find the least number which when divided by 12, 16 and 18, will leave in each case a remainder 5.

Solution The required smallest number
= (L.C.M. of 12, 16 and 18) + 5
= 144 + 5 = 149.

9. To find the greatest number that will divide x, y and z leaving the same remainder in each case.

   (a) When the value of remainder r is given:
   Required number = H.C.F. of $(x - r)$, $(y - r)$ and $(z - r)$.

   (b) When the value of remainder is not given:
   Required number = H.C.F. of $|(x - y)|$, $|(y - z)|$ and $|(z - x)|$

Illustration 20 Find the greatest number which will divide 772 and 2778 so as to leave the remainder 5 in each case.

Solution The required greatest number
= H.C.F. of (772 – 5) and (2778 – 5)
= H.C.F. of 767 and 2773
= 59.

Illustration 21 Find the greatest number which on dividing 152, 277 and 427 leaves equal remainder.

Solution The required greatest number
= H.C.F. of $|(x - y)|$, $|(y - z)|$ and $|(z - x)|$
= H.C.F. of $|(152 - 277)|$, $|(277 - 427)|$ and $|(427 - 152)|$
= H.C.F. of 125, 275 and 150
= 25.

10. To find the n-digit greatest number which, when divided by x, y and z,

    (a) leaves no remainder (i.e. exactly divisible)

    Step 1 L.C.M. of x, y and $z = L$

    Step 2 $L\overline{)n\text{-digit greatest number}(}$
    remainder = R

    Step 3 Required number
    = n-digit greatest number – R

    (b) leaves remainder K in each case
    Required number
    = (n-digit greatest number – R) + K.

Illustration 22 Find the greatest number of 4 digits which, when divided by 12, 18, 21 and 28 leaves 3 as a remainder in each case.

Solution L.C.M. of 12, 18, 21 and 28 = 252.

$$252\overline{)9999}\,(39$$
$$\underline{9828}$$
$$171$$

$\therefore$ The required number = (9999 – 171) + 3 = 9931.

Illustration 23 Find the greatest number of 4 digits which, when divided by 12, 15, 20 and 35 leaves no remainder.

Solution L.C.M. of 12, 15, 20 and 35 = 420.

$$\begin{array}{r} 420\overline{)9999}(\ 23 \\ \underline{9660} \\ 339 \end{array}$$

$\therefore$ The required number = 9999 – 339 = 9663.

11. To find the n-digit smallest number which when divided by x, y and z

(a) leaves no remainder (i.e. exactly divisible)

Step 1 L.C.M. of x, y and $z = L$

Step 2 $\dfrac{L\overline{)n-\text{digit smallest number}\ (}}{\text{remainder} = R}$

Step 3 Required number

= n-digit smallest number + $(L - R)$.

(b) leaves remainder K in each case.

Required number

= n-digit smallest number + $(L - R) + k$.

Illustration 24 Find the least number of four digits which is divisible by 4, 6, 8 and 10.

Solution L.C.M. of 4, 6, 8 and 10 = 120.

$$\begin{array}{r} 120\overline{)1000}(8 \\ \underline{960} \\ 40 \end{array}$$

$\therefore$ The required number = 1000 + (120 – 40) = 1080.

Illustration 25 Find the smallest 4-digit number, such that when divided by 12, 18, 21 and 28, it leaves remainder 3 in each case.

Solution L.C.M. of 12, 18, 21 and 28 = 252.

$$\begin{array}{r} 252\overline{)1000}(\ 3 \\ \underline{756} \\ 244 \end{array}$$

$\therefore$ The required number

= 1000 + (252 – 244) + 3

= 1011.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. What is the H.C.F. of 27, 18 and 36?
(a) 7 (b) 11
(c) 9 (d) None of these

2. Determine the L.C.M of $\frac{2}{5}$, $\frac{3}{10}$ and $\frac{6}{25}$.
(a) $\frac{6}{5}$ (b) $\frac{11}{5}$
(c) $\frac{9}{5}$ (d) None of these

3. What is the L.C.M. of 25, 30, 35 and 40?
(a) 3800 (b) 4200
(c) 4400 (d) None of these

4. What is the greatest number which divides 852, 1065 and 1491 exactly?
(a) 193 (b) 183
(c) 223 (d) 213

5. What is the H.C.F. of $\frac{4}{9}$, $\frac{10}{21}$ and $\frac{20}{63}$?
(a) $\frac{4}{189}$ (b) $\frac{6}{63}$
(c) $\frac{2}{63}$ (d) None of these

6. Find the least number which when divided by 16, 18, 20 and 25 leaves 4 as remainder in each case but when divided by 7 leaves no remainder.
(a) 8004 (b) 13004
(c) 18004 (d) 18014

7. Area of three fields is 165 sq m, 195 sq m and 85 sq m respectively. In each of the fields a flower bed of equal length has to be made. If flower bed in each of the fields is 3 m wide then what is the maximum length of the flower bed in each of the fields?
(a) 7 m (b) 9 m
(c) 5 m (d) None of these

8. Find the greatest number which will divide 2112 and 2792 leaving the remainder 4 in each case.
(a) 78 (b) 68
(c) 65 (d) 63

9. The H.C.F. of two numbers is 12 and their difference is 12. The numbers are:
(a) 66, 78 (b) 70, 82
(c) 94, 106 (d) 84, 96

10. A merchant has three different kinds of milk: 435 litres, 493 litres and 551 litres. Find the least number of casks of equal size required to store all the milk without mixing.
(a) 51 (b) 61
(c) 47 (d) 45

11. Find the greatest number which will divide 25, 73 and 97 so as to leave the same remainder in each case.
(a) 12 (b) 18
(c) 24 (d) 32

12. The sum of two numbers is 216 and their H.C.F. is 27. The numbers are:
(a) 54, 162 (b) 108, 118
(c) 27, 189 (d) None of these

13. How often will five bells toll together in one hour if they start together and toll at intervals of 5, 6, 8, 12, 20 seconds, respectively?
(a) 29 (b) 30
(c) 31 (d) 120

14. Find the greatest number that will divide 964, 1238 and 1400 leaving remainders 41, 31 and 51, respectively.
(a) 71 (b) 81
(c) 61 (d) 73

15. Find the side of the largest square slabs which can be paved on the floor of a room 5 metres 44 cm long and 3 metres 74 cm broad.
(a) 56 (b) 42
(c) 38 (d) 34

16. The traffic lights at three different road crossings change after every 48 sec., 72 sec. and 108 sec., respectively. If they all change simultaneously at 8 : 20 : 00 hrs; then they will again change simultaneously at:
(a) 8 : 27 : 12 hrs (b) 8 : 27 : 24 hrs
(c) 8 : 27 : 36 hrs (d) 8 : 27 : 48 hrs

17. The product of two numbers is 6760 and their H.C.F. is 13. How many such pairs can be formed?
(a) 2 (b) 3
(c) 4 (d) only one

18. Find the greatest number of four digits which when divided by 10, 15, 21 and 28 leaves 4, 9, 15 and 22 as remainders, respectively.
(a) 9654 (b) 9666
(c) 9664 (d) 9864

19. The number of prime factors in the expression $(6)^{10} \times (7)^{17} \times (11)^{27}$ is:
(a) 54 (b) 64
(c) 71 (d) 81

20. Find the greatest number which will divide 3962, 4085 and 4167 leaving the same remainder in each case.
(a) 37 (b) 39
(c) 41 (d) 43

21. A wholesale tea dealer has 408 kilograms, 468 kilograms and 516 kilograms of three different qualities of tea. He wants it all to be packed into boxes of equal size without mixing. Find the capacity of the largest possible box.
(a) 50 (b) 36
(c) 24 (d) 12

22. A room is 4 metres 37 m long and 3 metres 23 m broad. It is required to pave the floor with minimum square slabs. Find the number of slabs required for this purpose?
(a) 485 (b) 431
(c) 391 (d) 381

23. The least perfect square number which is divisible by 3, 4, 5, 6 and 8:
(a) 900 (b) 1200
(c) 2500 (d) 3600

24. Find the least number of five digits which when divided by 12, 16, 21, 36 and 40 leaves in each case a remainder 8.
(a) 10088 (b) 10072
(c) 10080 (d) None of these

25. Three pieces of timber 42 m, 49 m and 63 m long have to be divided into planks of the same length. What is the greatest possible length of each plank?
(a) 7 m (b) 14 m
(c) 42 m (d) 63 m

26. Three men start together to travel the same way around a circular track of 11 kilometres in circumference. Their speeds are 4, $5\frac{1}{2}$ and 8 kilometres per hour, respectively. When will they meet at the starting point?
(a) 11 hours (b) 12 hours
(c) 23 hours (d) 22 hours

27. Five bells begin to toll together and toll at intervals of 36, 45, 72, 81 and 108 seconds. After what interval of time will they keep on tolling together?
(a) 3240 seconds (b) 3080 seconds
(c) 3140 seconds (d) 3200 seconds

28. Three different containers contain different quantities of mixture of milk and water, whose measurements are 403 kg, 434 kg and 465 kg. What biggest measure must be there to measure all the different quantities exactly?
(a) 1 kg (b) 7 kg
(c) 31 kg (d) 41 kg

29. The L.C.M. and G.C.D. of two numbers are 1530 and 51, respectively. Find how many such pairs are possible?
(a) 2 (b) 3
(c) 4 (d) only one

30. Find the least number of five digits which when divided by 63, 56 and 42 leaves a remainder 1 in each case.
(a) 10082 (b) 10081
(c) 10001 (d) 10071

31. The H.C.F. and L.C.M. of two numbers are 44 and 264, respectively. If the first number is divided by 2, the quotient is 44. The other number is:
(a) 33 (b) 66
(c) 132 (d) 264

32. The largest natural number which exactly divides the product of any four consecutive natural numbers, is
(a) 6 (b) 12
(c) 24 (d) 120

33. Find the least number of six digits which is exactly divisible by 15, 21 and 28:
(a) 100480 (b) 100270
(c) 100380 (d) 100340

34. Find the greatest number of five digits which when divided by 12, 15, 21, 25 and 28 leaves 5, 8, 14, 18 and 21 as remainders, respectively.
(a) 98696 (b) 98700
(c) 97693 (d) 98693

35. What is the smallest number which when increased by 3 is divisible by 16, 24, 30 and 32?
(a) 480 (b) 475
(c) 472 (d) 477

36. The least number of square tiles required to pave the ceiling of a room 15 m 17 cm long and 9 m, 2 cm broad, is:
(a) 656 (b) 738
(c) 814 (d) 902

37. Find the least number which when divided by 2, 3, 4, 5 and 6 leaves 1, 2, 3, 4 and 5 as remainders, respectively, but when divided by 7 leaves no remainder.
(a) 210 (b) 119
(c) 126 (d) 154

38. Find the greatest number of five digits which when divided by 4, 6, 10 and 15 leaves the same remainder 3 in each case.
(a) 99993 (b) 99063
(c) 90093 (d) 99963

39. Find the least number which is a multiple of 31 and when divided by 15, 24 and 32 leaves the remainders 2, 11 and 19, respectively.
(a) 2418 (b) 2387
(c) 2356 (d) 2325

40. Find the two largest numbers of four digits having 531 as their H.C.F.
(a) 9231, 9762 (b) 9027, 9558
(c) 9037, 9568 (d) 9127, 9658

41. Find the greatest number of five digits which becomes exactly divisible by 10, 12, 15 and 18 when 3769 is added to it.
(a) 99811 (b) 99911
(c) 98911 (d) 99011

42. Find the least number which when decreased by 11 is divisible by 14, 15, 21, 32 and 60.
(a) 4371 (b) 3271
(c) 3371 (d) 3360

43. Find the least number of five digits which when divided by 8, 12, 16 and 20 leaves remainders 1, 5, 9 and 13, respectively.
(a) 10003 (b) 10093
(c) 10073 (d) 10013

44. The H.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the numbers is 77, find the other.
(a) 909 (b) 119
(c) 66 (d) 99

45. Find the greatest number of four digits which is exactly divisible by 24, 28, 30 and 35.
(a) 9225 (b) 9240
(c) 9250 (d) 9260

46. Find the greatest number of four digits which must be added to 5231 so that the final number becomes exactly divisible by 12, 15, 27, 32 and 40.
(a) 7929 (b) 7829
(c) 9729 (d) 7729

47. A heap of stones can be made up into groups of 21. When made up into groups of 16, 20, 25 and 45, there are 3 stones left in each case. How many stones at least can there be in the heap?
(a) 7203 (b) 2403
(c) 3603 (d) 4803

48. Find the greatest number of five digits which when divided by 8, 9 and 10 leaves 3 as remainder in each case.
(a) 99996 (b) 99723
(c) 99983 (d) None of these

49. What is the least number of cut pieces of equal length that can be cut out of two lengths 10 metres 857 millimetres and 15 metres 87 millimetres?
(a) 174 (b) 172
(c) 164 (d) 184

EXPLANATORY ANSWERS

1. Ans. (c)

Solution H.C.F. of 27, 18 and 36

$$\begin{array}{r|l} 18 & 27 \;(1 \\ & 18 \\ \hline 9 & 18 \;(2 \\ & 18 \\ \hline & \times \end{array}$$

∴ H.C.F. of 27 and 18 is 9

Now, H.C.F. of 9 and 36

$$\begin{array}{r|l} 9 & 36 \;(4 \\ & 36 \\ \hline & \times \end{array}$$

∴ H.C.F. of 9 and 36 is 9

Therefore, the required H.C.F of 27, 18 and 36 is 9.

2. Ans. (a)

Solution L.C.M. of $\frac{2}{5}$, $\frac{3}{10}$ and $\frac{6}{25}$

$$= \frac{\text{L.C.M. of 2, 3 and 6}}{\text{H.C.F. of 5, 10 and 25}}$$

$\because$ L.C.M. of 2, 3 and 6 = 6

and, H.C.F. of 5, 10 and 25 = 5

$\therefore$ Required L.C.M. $= \frac{6}{5}$.

3. Ans. (b)

Solution

2	25,	30,	35,	40
5	25,	15,	35,	20
	5,	3,	7,	4

$\therefore$ Required L.C.M. $= 2 \times 5 \times 5 \times 3 \times 7 \times 4$
$= 4200$.

4. Ans. (d)

Solution H.C.F. of 852 and 1065 is 213

H.C.F. of 213 and 1491 is 213

5. Ans. (c)

Solution H.C.F. of $\frac{4}{9}$, $\frac{10}{21}$ and $\frac{20}{63}$

$$= \frac{\text{H.C.F. of 4,10 and 20}}{\text{L.C.M. of 9,21 and 63}}$$

$\because$ H.C.F. of 4, 10 and 20 = 2

and L.C.M. of 9, 21 and 63 = 63

$\therefore$ Required H.C.F. $= \frac{2}{63}$.

6. Ans. (c)

Solution L.C.M. of 16, 18, 20 and 25 is 3600

Required number $= 3600 \times K + 4$
$= (7 \times 514 + 2)K + 4$
$= (7 \times 514)K + 2K + 4$

Now $(2K + 4)$ is divisible by 7 for $K = 5$.

$\therefore$ Required number $= 5 \times 3600 + 4$
$= 18004$.

7. Ans. (c)

Solution H.C.F. of 165, 195 and 85 will be maximum area of each of the flower beds.

H.C.F. of 165 and 195:

```
165 ) 195 ( 1
      165
       30 ) 165 ( 2
            150
             15 ) 35 ( 2
                  30
                   ×
```

$\therefore$ H.C.F. of 165 and 195 is 15.

Also, now, H.C.F. of 15 and 85 is 5.

8. Ans. (b)

Solution Subtract 4 from each of the numbers 2112 and 2792 and then take the H.C.F, i.e. H.C.F of 2108 and 2788.

9. Ans. (d)

Solution The difference of requisite numbers must be 12 and each one must be divisible by 12. So, the numbers are 84, 96.

10. Ans. (a)

Solution Since minimum number of casks are required, the size of the cask is greatest. Also the cask in three cases are of equal size. The size of the cask is the H.C.F. of 435, 493 and 551 which is 29.

Now, the number of casks required for storing the milk

$= (493 + 435 + 551) \div 29 = 51$.

11. Ans. (c)

Solution $73 - 25 = 48$

$97 - 73 = 24$

$97 - 25 = 72$

H.C.F. of 48, 24 and 72 is 24.

12. Ans. (c)

Solution Let the numbers be $27a$ and $27b$

Then, $27a + 27b = 216$ or $27(a + b) = 216$

or, $a + b = \frac{216}{27} = 8$

$\therefore$ Values of co-primes (with sum 8) are (1, 7) and (3, 5)

So, the numbers are $(27 \times 1, 27 \times 7)$,

i.e. (27, 189).

13. Ans. (c)

Solution The time after which the bells will ring together is the L.C.M. of 5, 6, 8, 12 and 20 seconds, i.e. 120 seconds. The number of times they will toll together in one hour

$= (3600 \div 120) + 1$
$= 30 + 1 = 31$.

14. Ans. (a)

Solution $964 - 41 = 923$

$1238 - 31 = 1207$

$1400 - 51 = 1349$

H.C.F. of 923 and 1207 is 71

H.C.F. of 71 and 1349 is 71.

15. Ans. (d)

Solution The side of the square slab is the H.C.F. of 544 and 374 cm i.e. 34.

16. Ans. (a)

Solution Interval of change = (L.C.M. of 48, 72, 108) sec.

= 432

So, the lights will change after every 432 seconds, i.e. 7 min. 12 sec.

So, the next simultaneous change will take place at 8 : 27 : 12 hrs.

17. Ans. (a)

Solution Let the numbers be $13x$ and $13y$

$13x \cdot 13y = 6760$

$\therefore \quad x \times y = 6760 \div (13 \times 13) = 40$

Possible values of (x, y) are

(1, 40); (2, 20); (4, 10); (5, 8)

Only two acceptable values are (1, 40) and (5, 8).

18. Ans. (a)

Solution First, find the greatest number of four digits that is divisible by the L.C.M. of 10, 15, 21 and 28 and then subtract 6 from it to get the required number.

19. Ans. (b)

Solution Since 2, 3, 7, 11 are prime numbers and the given expression is $2^{10} \times 3^{10} \times 7^{17} \times 11^{27}$, the number of prime factors in the given expression is (10 + 10 + 17 + 27) = 64.

20. Ans. (c)

Solution 4085 – 3962 = 123

4167 – 4085 = 82

4167 – 3962 = 205

H.C.F. of 123, 82 and 205 is 41.

21. Ans. (d)

Solution The capacity of the box is H.C.F. of 408, 468 and 516, i.e. 12.

22. Ans. (c)

Solution Length = 437 cm

Breadth = 323 cm.

The side of the square slab is the H.C.F. of 437 and 323, i.e. 19 cm.

$\therefore$ Area of square slab = 19 cm × 19 cm = 361 cm^2

$$\text{The number of slabs} = \frac{\text{Area of the room}}{\text{Area of the slab}}$$

$$= \frac{437 \times 323 \text{ cm}^2}{361 \text{ cm}^2}$$

$$= 391.$$

23. Ans. (d)

2	3,	4,	5,	6,	8
2	3,	2,	5,	3,	4
3	3,	1,	5,	3,	2
	1,	1,	5,	1,	2

L.C.M. of 3, 4, 5, 6, 8 = 120

Required number = 2 × 2 × 2 × 2 × 3 × 3 × 5 × 5

= 3600.

24. Ans. (a)

Solution Required number = the least number of 5 digits divisible by the L.C.M. of 12, 16, 21, 36, 40 + the remainder 8.

25. Ans. (a)

Solution Greatest possible length of each plank

= (H.C.F. of 42, 49, 63) m = 7 m.

26. Ans. (d)

Solution Time for one revolution by each of three men

$$= \frac{11}{4}, \frac{11}{5\frac{1}{2}}, \frac{11}{8} \text{ hours}$$

$$= \frac{11}{4}, \frac{2}{1}, \frac{11}{8} \text{ hours}$$

$\therefore$ The time when they will meet at the starting point = L.C.M. of $\frac{11}{4}, \frac{2}{1}, \frac{11}{8}$ which is $\frac{22}{1}$, i.e. 22 hours.

27. Ans. (a)

Solution The interval of time is L.C.M. of the numbers 36, 45, 72, 81 and 108.

2	36,	45,	72,	81,	108
2	18,	45,	36,	81,	54
2	9,	45,	18,	81,	27
3	9,	45,	9,	81,	27
3	3,	15,	3,	27	9
3	1,	5,	1,	9,	3
	1,	5,	1,	3,	1

L.C.M (36, 45, 72, 108) = 3240.

28. Ans. (c)

Solution Biggest measure = H.C.F. of (403, 434, 465)

= 31 kg.

29. Ans. (c)

Solution Let the numbers be $51x$ and $51y$ where x and y are co-prime.

Now, $51x \times 51y = 51 \times 1530$

$\therefore \quad x \times y = 30$

Possible pairs are (1, 30); (2, 15); (3, 10) and (5, 6).

30. Ans. (b)

Solution L.C.M. of 63, 56 and 42 is 504.

Least number of 5 digits divisible by 504:

```
504 ) 10000 ( 19
      -504
       4960
      -4536
        424
```

= 1000 + (504 – 424) = 10080

∴ Required number = 10080 + 1 = 10081.

31. Ans. (c)

Solution First number = 2 × 44 = 88

Second number = $\frac{44 \times 264}{88}$ = 132.

32. Ans. (c)

Solution 1 × 2 × 3 × 4 = 24

∴ Required number = 24.

33. Ans. (c)

Solution L.C.M. of 15, 21 and 28 is 420.

Least number of 6 digits = 100000

```
420 ) 100000 ( 238
      -840
       1600
      -1260
        3400
      -3360
          40
```

Remainder = 40.

∴ Least number = 100000 + (420 – 40)
= 100380.

34. Ans. (d)

Solution Find the greatest number of five digits which is divisible by the L.C.M. of 12, 15, 21, 25 and 28 and then subtract 7 from it to get the required number.

Required number = 98700 – 7 = 98693.

35. Ans. (d)

Solution Required number
= (L.C.M. of 16, 24, 30 and 32) – 3
= 480 – 3 = 477.

36. Ans. (c)

Solution Side of each tile = (H.C.F. of 1517 and 902) cm
= 41 cm

∴ Number of tiles = $\frac{1517 \times 902}{41 \times 41}$ = 814.

37. Ans. (b)

Solution L.C.M. of 2, 3, 4, 5, 6 is 60.

One of the numbers satisfying the first condition is 60 – 1 = 59

60 + 59 = 119, etc.

But 119 is also divisible by 7.

38. Ans. (d)

Solution L.C.M. of 4, 6, 10, 15 = 60

Greatest number of five digits which is divisible by 60 = 99960.

∴ Required number = 99960 + 3 = 99963.

39. Ans. (b)

Solution L.C.M. of 15, 24, 32 is 480

Required number = 480K – 13
= 15 × 31K + (15K – 13)

(15K – 13) is divisible by 31 for K = 5

∴ Least number = 480 × 5 – 13 = 2387.

40. Ans. (b)

Solution The greatest number of four digits divisible by 531 is 9558, so the other number is 9558 – 531 = 9027. Thus, the numbers are 9558 and 9027.

41. Ans. (b)

Solution L.C.M. of 10, 12, 15 and 18 = 540. Dividing (99999 + 3769) by 540, the remainder is 88.

∴ Required number = 99999 – 88 = 99911.

42. Ans. (c)

Solution Required number
= (L.C.M. of 14, 15, 21, 32, 60) + 11
= 3360 + 11 = 3371.

43. Ans. (c)

Solution Least number of five digits divisible by L.C.M. of 8, 12, 16, 20 is 10080.

∴ Required number = 10080 – 7 = 10073.

44. Ans. (d)

Solution Required number = $\frac{\text{L.C.M.} \times \text{H.C.F.}}{\text{Given number}}$

= $\frac{693 \times 11}{77}$ = 99.

45. Ans. (b)

Solution L.C.M. of 24, 28, 30 and 35

2	24,	28,	30,	35
2	12,	14,	15,	35
3	6,	7,	15,	35
5	2,	7,	5,	35
7	2,	7,	1,	7
	2,	1,	1,	1

= 2 × 2 × 2 × 3 × 5 × 7 = 840

Greatest number of four digits = 9999.

Quotient when 9999 is divided by 840 is 11 and remainder is 759.

∴ Greatest number of four digits in this case = 9999 – 759 = 9240.

46. Ans. (d)

Solution L.C.M. of 12, 15, 27, 32, 40 = 4320. Let us add 5231 to the greatest number of four digits and then divide by 4320 to find the remainder.

```
4320)15230(3
     12960
     -----
      2270
```

Required greatest number of four digits

= 9999 – 2270

= 7729.

47. Ans. (a)

Solution L.C.M. of 16, 20, 25, 45 = 3600

1st number = 3600 × 1 + 3 = 3603 which is not divisible by 21.

2nd number = 3600 × 2 + 3 = 7203 which is divisible by 21.

48. Ans. (b)

Solution L.C.M. of 8, 9, 10 = 360

```
360)99999(277
    720
    ----
    2799
    2520
    ----
     2799
     2520
     ----
      279
```

Greatest number of 5 digits which is divisible by 360 = 99999 – 279 = 99720

∴ Required number = 99729 + 3 = 99723.

49. Ans. (d)

Solution H.C.F. of 10857 and 15087 is 141.

The least number of cut pieces

= (10857 + 15087) ÷ 141

= 184.

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. The H.C.F. of two numbers is 8. Which one of the following can never be their L.C.M.?
(a) 24 (b) 48
(c) 56 (d) 60
[Based on SSC (GL) Prel. Exam 2000]

2. The L.C.M. of three different numbers is 120. Which of the following can not be their H.C.F.?
(a) 8 (b) 12
(c) 24 (d) None of these
[Based on SSC (GL) Prel. Exam 2000]

3. The L.C.M. and H.C.F. of the numbers 28 and 42 are in the ratio:
(a) 6 : 1 (b) 2 : 3
(c) 3 : 2 (d) 7 : 2
[Based on SSC (GL) Prel. Exam 2000]

4. The L.C.M. of two numbers is 1820 and their H.C.F. is 26. If one number is 130 then the other number is:
(a) 70 (b) 1690
(c) 364 (d) 1264
[Based on SSC (GL) Prel. Exam 2002]

5. H.C.F. and L.C.M. of two numbers are 7 and 140, respectively. If the numbers are between 20 and 45, the sum of the numbers is:
(a) 70 (b) 77
(c) 63 (d) 56
[Based on SSC (GL) Prel. Exam 2003]

6. The L.C.M. of two numbers is 14560 and their H.C.F. is 13. If one of them is 416, then the other is:
(a) 460 (b) 455
(c) 450 (d) 446
[Based on SSC (GL) Prel. Exam 2002]

7. The ratio of two numbers is 3 : 4 and their H.C.F. is 4. Their L.C.M. is:
(a) 12 (b) 16
(c) 24 (d) 48
[Based on SSC (GL) Prel. Exam 2002]

8. The H.C.F. of two numbers is 16 and their L.C.M. is 160. If one of the numbers is 32, then other number is:
(a) 48 (b) 80
(c) 96 (d) 112
[Based on SI of Police Rec. Exam 1997]

9. The H.C.F. of two numbers is 24. The number, which can be their L.C.M., is:
(a) 84 (b) 120
(c) 128 (d) 148
[Based on Assitant's Grade Exam 1997]

10. H.C.F. of 4 × 27 × 3125, 8 × 9 × 25 × 7 and 16 × 81 × 5 × 11 × 49 is:
(a) 180 (b) 360
(c) 540 (d) 1260
[Based on SI of Police Rec. Exam 1997]

11. The sum of two numbers is 528 and their H.C.F. is 3. The number of such pairs is:
(a) 2 (b) 3
(c) 4 (d) 5
[Based on SI of Police Rec. Exam 1997]

12. The L.C.M. of two numbers is 1920 and their H.C.F., is 16. If one of the numbers is 128, find the other number.
(a) 204 (b) 240
(c) 260 (d) 320
[Based on SI of Police Rec. Exam 1997]

13. The H.C.F. and L.C.M. of two numbers are 11 and 385 respectively. If one number lies between 75 and 125, then that number is:
(a) 77 (b) 88
(c) 99 (d) 110
[Based on SI Rec. COP Exam 1998]

14. The sum of two numbers is 2000 and their L.C.M. is 21879. The two numbers are:
(a) 1993, 7 (b) 1991, 9
(c) 1989, 11 (d) 1987, 13
[Based on Assistant's Grade Exam 1998]

15. The L.C.M. of two numbers is 495 and their H.C.F. is 5. If the sum of two numbers is 100 then their difference is:
(a) 10 (b) 46
(c) 70 (d) 90
[Based on SSC (GL) Prel. Exam 1999]

16. L.C.M. of two numbers is 225 and their H.C.F. is 5. If one number is 25, the other number will be:
(a) 5 (b) 25
(c) 45 (d) 225
[Based on SSC (GL) Prel. Exam 1999]

EXPLANATORY ANSWERS

1. Ans. (d)

Solution Non-multiples of 8 are not the L.C.M.

2. Ans. (d)

3. Ans. (a)

Solution L.C.M. of 28, 42 = 84
H.C.F. of 28, 42 = 14
$\therefore$ Required ratio = 84 : 14 = 6 : 1.

4. Ans. (c)

Solution Required number = $\frac{1820 \times 26}{130}$ = 364.

5. Ans. (c)

Solution Since the H.C.F. is 7 therefore, possible numbers could be 21, 28, 35 and 42. L.C.M. of the numbers does not have the factor of 3. But 21 and 42 are the numbers which have 3 as a factor. So, 21 and 42 will not be the numbers. If 21 and 42 are not the numbers, then the numbers are 28 and 35.
$\therefore$ Sum of the numbers = 28 + 35 = 63.

6. Ans. (b)

Solution $\frac{14560 \times 13}{416}$ = 455.

7. Ans. (d)

8. Ans. (b)

Solution H.C.F. × L.C.M. = Product of the numbers, i.e. $16 \times 160 = 32 \times K \Rightarrow K = 80$

9. Ans. (b)

10. Ans. (c)

Solution $4 \times 27 \times 3125$
$= 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
$8 \times 9 \times 25 \times 7$
$= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7$
$16 \times 81 \times 5 \times 11 \times 49$
$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 7 \times 7 \times 11$
$\therefore$ H.C.F. $= 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 540$.

11. Ans. (c)

Solution (33, 495); (99, 429); (165, 363); (231, 297)

12. Ans. (b)

Solution $128 \times K = 1920 \times 16 \Rightarrow K = 240$.

13. Ans. (a)

Solution Let the numbers be x and y
$\therefore$ $xy = 11 \times 385 = 11 \times 5 \times 7 \times 11 = 77 \times 55$.

14. Ans. (c)

Solution From the given choices we can see that L.C.M. of 1989 and 11 is 21879.
Therefore, required numbers are 1989, 11.

15. Ans. (a)

Solution Let the two numbers be x and $(100 - x)$
L.C.M. × H.C.F. = Product of the numbers
$495 \times 5 = x(100 - x)$
or, $x^2 - 100x + 2475 = 0$
or, $x^2 - 55x - 45x + 2475 = 0$
or, $(x - 55)(x - 45) = 0$ or $x = 45$ or $x = 55$
Thus, the numbers are 45 and 55.
When $x = 55$, we get $100 - x = 45$ and vice versa.
Hence, their difference = 55 – 45 = 10.

16. Ans. (c)

Solution Other number = $\frac{225 \times 5}{25}$ = 45.

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. Find the least multiple of 13 which when divided by 6, 8 and 12 leaves 5, 7 and 11 as remainders, respectively.
(a) 143 (b) 169
(c) 260 (d) 221

2. Find the H.C.F. of $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{7}{15}$.
(a) $\frac{7}{90}$ (b) $\frac{1}{30}$
(c) $\frac{1}{60}$ (d) $\frac{7}{60}$

3. Three measuring rods are 64 cm, 80 cm and 96 cm in length. The least length of cloth that can be measured exact number of times using any one of the above rods is:
(a) 0.96 m (b) 19.20 m
(c) 9.60 m (d) 96.00 m

4. The L.C.M. of two numbers is 45 times their H.C.F. One number is 125; and the sum of their H.C.F. and L.C.M. is 1150. Find the other number.
(a) 275 (b) 215
(c) 230 (d) 225

5. Find the L.C.M. of $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{7}{15}$.
(a) 140 (b) 120
(c) 70 (d) 210

6. About the number of pairs which have 16 as their H.C.F and 136 as their L.C.M., we can definitely say that
(a) Only one such pair exists
(b) Only two such pairs exist
(c) Many such pairs exist
(d) No such pair exists.

7. Find the least number which is exactly divisible by 32, 36, 45, 60 and 80.
(a) 1440 (b) 1400
(c) 1380 (d) 1360

8. Find the least square number which is exactly divisible by 10, 15, 18, 25 and 30.
(a) 8100 (b) 900
(c) 3600 (d) 6400

9. The H.C.F of two numbers of 3 digits is 310 and their L.C.M. is 1860. Find the numbers.
(a) 310, 1860 (b) 620, 310
(c) 620, 930 (d) None of these

10. Two numbers 4242 and 2903 when divided by a certain number of three digits, leave the same remainder. Find the number.
(a) 101 (b) 103
(c) 107 (d) 113

Test II **[Time: 10 min]**

11. Find the least number which when divided by 12, 15, 20 and 54 leaves in each case a remainder 7.
(a) 547 (b) 533
(c) 540 (d) 587

12. A gardener was asked to plant flowers in a row containing equal number of plants. He tried to plant 6, 8, 10 and 12 in each row, but 5 plants were left in each case. When he planted 13 in a row, no plant was left. Find the number of plants with him.
(a) 245 (b) 125
(c) 485 (d) 845

13. The sum and difference of the L.C.M. and the H.C.F. of two numbers are 312 and 264, respectively. Find the numbers if their sum is 168.
(a) 96, 72 (b) 72, 96
(c) 108, 84 (d) None of these

14. Three persons begin to walk round a circular track. The first completes revolution in $15\frac{1}{6}$ seconds, the second in $16\frac{1}{4}$ seconds and the third in $18\frac{2}{3}$ seconds. After what time will they be together at the starting point again?
(a) 1 hr 40 min. (b) 1 hr 40 sec
(c) 1.40 hr (d) 1 hr 3 min. 40 sec.

15. Find the least number which when diminished by 6 is divisible by 14, 35, 75, 96 and 210.
(a) 16800 (b) 16794
(c) 16086 (d) None of these

16. What is the least number which when divided by 8, 12, 18 and 24 leaves the remainders 4, 8, 14 and 20, respectively?
(a) 78 (b) 68
(c) 58 (d) None of these

17. The L.C.M. of 54, 90 and a third number is 1890 and their G.C.D. is 18. What is the third number?
(a) 36 (b) 180
(c) 126 (d) 108

18. H.C.F. of three numbers is 12. If they be in the ratio 1 : 2 : 3, the numbers are:
(a) 12, 24, 36 (b) 10, 20, 30
(c) 5, 10, 15 (d) 4, 8, 12

19. 21 mango trees, 42 apple trees and 56 orange trees have to be planted in rows such that each row contains the same number of trees of one variety only. Minimum number of rows in which the above trees may be planted is:
(a) 15 (b) 17
(c) 3 (d) 20

20. The product of two numbers is 2160, their H.C.F. is 12. Find the number of such possible pairs.
(a) 2 (b) only one
(c) 3 (d) More than 3

Answers

1. (a)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (a)	**6.** (d)	**7.** (a)	**8.** (b)	**9.** (c)	**10.** (b)
11. (a)	**12.** (d)	**13.** (a)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (c)	**18.** (a)	**19.** (b)	**20.** (a)

3

SIMPLIFICATION

SIMPLE ARITHMETIC OPERATIONS

It is a common need to simplify the expressions formulated according to the statements of the problems relating to practical life. To do this, it is essential to follow in sequence the mathematical operations given by the term "BODMAS".

BODMAS

Each letter of the word BODMAS stands as follows:

B for Bracket	:	[{(–)}]
O for Of	:	of
D for Division	:	÷
M for Multiplication	:	×
A for Addition	:	+
S for Subtraction	:	–

There are four brackets, namely, – bar, ( ), { } and [ ]. They are removed, strictly in the order –, ( ), { } and [ ].

The order of various operations in exercises involving brackets and fractions must be performed strictly according to the order of the letters of the word BODMAS.

Note: Here, $-\overline{5-8} = -(-3) = 3$.

Illustration 1 Simplify:

$$8\frac{1}{2}-\left[3\frac{1}{5}\div 4\frac{1}{2}\text{ of }5\frac{1}{3}+\left\{11-\left(3-\overline{1\frac{1}{4}-\frac{5}{8}}\right)\right\}\right]$$

Solution Given expression

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2}\text{ of }\frac{16}{3}+\left\{11-\left(3-\overline{\frac{5}{4}-\frac{5}{8}}\right)\right\}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2}\text{ of }\frac{16}{3}+\left\{11-\left(3-\frac{5}{8}\right)\right\}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2}\text{ of }\frac{16}{3}+\left\{11-\frac{19}{8}\right\}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2}\text{ of }\frac{16}{3}+\frac{69}{8}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2}\times\frac{16}{3}+\frac{69}{8}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{5}\div\frac{24}{1}+\frac{69}{8}\right]$$

$$=\frac{17}{2}-\left[\frac{16}{5}\times\frac{1}{24}+\frac{69}{8}\right]=\frac{17}{2}-\left[\frac{16}{120}+\frac{69}{8}\right]$$

$$=\frac{17}{2}-\left[\frac{16+1035}{120}\right]=\frac{17}{2}-\frac{1051}{120}$$

$$=\frac{1020-1051}{120}=-\frac{31}{120}.$$

Illustration 2 Simplify:

$$5\frac{1}{3}-\left\{4\frac{1}{3}-\left(3\frac{1}{3}-\overline{2\frac{1}{3}-\frac{1}{3}}\right)\right\}$$

Solution Given expression

$$=\frac{16}{3}-\left\{\frac{13}{3}-\left(\frac{10}{3}-\overline{\frac{7}{3}-\frac{1}{3}}\right)\right\}$$

$$=\frac{16}{3}-\left\{\frac{13}{3}-\left(\frac{10}{3}-\frac{6}{3}\right)\right\}=\frac{16}{3}-\left\{\frac{13}{3}-\frac{4}{3}\right\}$$

$$=\frac{16}{3}-\left\{\frac{9}{3}\right\}=\frac{16}{3}-\frac{9}{3}=\frac{7}{3}=2\frac{1}{3}.$$

Use of Algebraic Formulae

The following important formulae are sometimes found useful in dealing with the simplifications:

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$
4. $(a+b)^2 - (a-b)^2 = 4ab$
5. $a^2 - b^2 = (a+b)(a-b)$
6. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
   $= a^3 + b^3 + 3ab(a+b)$
7. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
   $= a^3 - b^3 - 3ab(a-b)$
8. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
9. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

10. $\dfrac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca} = (a + b + c).$

11. $a^4 - a^4 = (a^2 + b^2)(a + b)(a - b).$

Illustration 3 Simplify the following:

(i) $0.32 \times 0.32 + 0.64 \times 0.68 + 0.68 \times 0.68$

Solution Given expression

$= 0.32 \times 0.32 + 2 \times 0.32 \times 0.68 + 0.68 \times 0.68$

$= (0.32)^2 + 2 \times 0.32 \times 0.68 + (0.68)^2$

$= (0.32 + 0.68)^2 \quad [\because a^2 + 2ab + b^2 = (a+b)^2]$

$= 1^2 = 1.$

(ii) $2.45 \times 2.45 - 0.9 \times 2.45 + 0.45 \times 0.45$

Solution Given expression

$= 2.45 \times 2.45 - 2 \times 2.45 \times 0.45 + 0.45 \times 0.45$

$= (2.45)^2 - 2 \times 2.45 \times 0.45 + (0.45)^2$

$= (2.45 - 0.45)^2 \quad [\because a^2 - 2ab + b^2 = (a-b)^2]$

$= (2)^2 = 4.$

(iii) $\dfrac{7\times\{(146+92)^2+(146-92)^2\}}{(146)^2+(92)^2}$

Solution Given expression

$= \dfrac{7\times 2\{(146)^2+(92)^2\}}{(146)^2+(92)^2}$

$[\because (a + b)^2 + (a - b)^2 = 2(a^2 + b^2)]$

$= 14.$

(iv) $\dfrac{(0.345+0.255)^2-(0.345-0.255)^2}{0.345\times 1.02}$

Solution Given expression

$= \dfrac{(0.345+0.255)^2-(0.345-0.255)^2}{4\times 0.345\times 0.255}$

$= \dfrac{4\times 0.345\times 0.255}{4\times 0.345\times 0.255} \quad [\because (a + b)^2 - (a - b)^2 = 4ab]$

$= 1.$

(v) $\dfrac{0.682\times 0.682-0.318\times 0.318}{0.682-0.318}$

Solution Given expression

$= \dfrac{(0.682)^2-(0.318)^2}{0.682-0.318}$

$= (0.682 + 0.318) \quad \left[\because \dfrac{a^2-b^2}{a-b} = a+b\right]$

$= 1.$

(vi) $\dfrac{(3.29)^2-(0.81)^2}{4}$

Solution Given expression

$= \dfrac{(3.29)^2-(0.81)^2}{3.29+0.81}$

$= (3.29 - 0.81) \quad \left[\because \dfrac{a^2-b^2}{a+b} = a-b\right]$

$= 2.48.$

(vii) $(2.35)^3 + 1.95 \times (2.35)^2 + 7.05 \times (0.65)^2 + (0.65)^3$

Solution Given expression

$= (2.35)^3 + 3 \times 0.65 \times (2.35)^2$

$+ 3 \times 2.35 \times (0.65)^2 + (0.65)^3$

$= (2.35 + 0.65)^3$

$[\because a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3]$

$= (3)^3 = 27.$

(viii) $\dfrac{(4.32)^3-0.96\times(4.32)^2+12.96\times(0.32)^2-(0.32)^3}{4\times 4\times 4}$

Solution Given expression

$= \dfrac{(4.32)^3-3\times 0.32\times(4.32)^2+3\times 4.32\times(0.32)^2-(0.32)^3}{4\times 4\times 4}$

$= \dfrac{(4.32-0.32)^3}{4^3} \quad [\because a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3]$

$= \left(\dfrac{4}{4}\right)^3 = 1.$

(ix) $\dfrac{885\times 885\times 885+115\times 115\times 115}{885\times 885+115\times 115-885\times 115}$

Solution Given expression

$= \dfrac{(885)^3+(115)^3}{(885)^2+(115)^2-885\times 115}$

$= (885 + 115) \quad \left[\because \dfrac{a^3+b^3}{a^2-ab+b^2} = a+b\right]$

$= 1000.$

(x) $\dfrac{0.62\times 0.62\times 0.62-0.41\times 0.41\times 0.41}{0.62\times 0.62+0.62\times 0.41+0.41+0.41}$

Solution Given expression

$= \dfrac{(0.62)^3-(0.41)^3}{(0.62)^2+0.62\times 0.41+(0.41)^2}$

$= (0.62 - 0.41) \quad \left[\because \dfrac{a^3-b^3}{a^2+ab+b^2} = a-b\right]$

$= 0.21.$

(xi)

$$\frac{(2.3)^3+(1.5)^3+(1.2)^3-3\times 2.3\times 1.5\times 1.2}{2.3\times 2.3+1.5\times 1.5+1.2\times 1.2-2.3\times 1.5-2.3\times 1.2-1.5\times 1.2}$$

Solution Given expression

$= \dfrac{(2.3)^3+(1.5)^3+(1.2)^3-3\times 2.3\times 1.5\times 1.2}{(2.3)^2+(1.5)^2+(1.2)^2-2.3\times 1.5-2.3\times 1.2-1.5\times 1.2}$

$= (2.3 + 1.5 + 1.2)$

$\left[\because \dfrac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-ac-bc} = a+b+c\right]$

$= 5.$

Surds and Indices

a^n is called a **surd** if n is a fraction and a^n is called an **index** if n is an integer. a is called the **base**.

SOME USEFUL FORMULAE

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = (a^n)^m = a^{mn}$
4. $\left(\frac{a}{b}\right)^{-\frac{m}{n}} = \left(\frac{b}{a}\right)^{\frac{m}{n}}$
5. $a^m \div b^{-n} = a^m \times b^n$
6. $(\sqrt[n]{a})^n = a$, where 'n' is a +ve integer and 'a' a +ve rational number
7. $\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}$, where '$n$' is a +ve integer and '$a$', '$b$' are rational numbers
8. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$, where '$n$' is a +ve integer and '$a$', '$b$' are rational numbers
9. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$, where '$m$', '$n$' are +ve integers and '$a$' is a +ve rational number
10. $\sqrt[n]{\sqrt[m]{(a^k)^m}} = \sqrt[n]{a^k} = \sqrt[mn]{a^{km}}$, where '$m$', '$n$', '$k$' are +ve integers and '$a$' is a +ve rational number
11. $\sqrt{a} \times \sqrt{a} = a$
12. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
13. $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$
14. $(\sqrt{a} - \sqrt{b})^2 = a + b - 2\sqrt{ab}$
15. $a + \sqrt{b} = c + \sqrt{d} \Rightarrow a = c$ and $b = d$.
16. $\frac{1}{\sqrt{a} - \sqrt{b}} = \frac{\sqrt{a} + \sqrt{b}}{(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})} = \frac{\sqrt{a} + \sqrt{b}}{a - b}$
17. $\frac{1}{\sqrt{a} + \sqrt{b}} = \frac{\sqrt{a} - \sqrt{b}}{(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})} = \frac{\sqrt{a} - \sqrt{b}}{a - b}$
18. If $x = n(n + 1)$, then
    (a) $\sqrt{x - \sqrt{x - \sqrt{x - \ldots \infty}}} = n$ and
    (b) $\sqrt{x + \sqrt{x + \sqrt{x + \ldots \infty}}} = (n + 1)$.

Illustration 4 Find the value of $(243)^{0.8} \div (243)^{0.4}$.

Solution $(243)^{0.8} \div (243)^{0.4} = (243)^{0.8-0.4}$ $[\because a^m \div a^n = a^{m-n}]$

$= (243)^{0.4}$

$= (3^5)^{\frac{2}{5}} = 3^2 = 9.$

Illustration 5 Find the value of $(27)^{2/3} \div (64)^{-4/3}$

Solution $(27)^{2/3} \div (64)^{-4/3} = (3^3)^{2/3} \times (64)^{4/3}$ $[\because a^m \div b^{-n} = a^m \times b^n]$

$= 3^2 \times (4^3)^{4/3}$

$= 9 \times (4^4) = 9 \times 256 = 2304.$

Illustration 6 Find the value of $(-3)^{(-2)^{(-2)^{(-4)}}}$

Solution $(-3)^{(-2)^{(-2)^{(-4)}}} = \left(-\frac{1}{3}\right)^{(2)^{(-2)^{(-4)}}} = \left(\frac{1}{9}\right)^{(-2)^{(-4)}}$

$= (9)^{(2)^{(-4)}}$

$= (81)^{-4} = \left(\frac{1}{81}\right)^4$

$= \left(\frac{1}{3^4}\right)^4 = \frac{1}{3}.$

Illustration 7 Find the value of x if $\sqrt[5]{2x-7} - 3 = 0$.

Solution We have,

$\sqrt[5]{2x-7} - 3 = 0 \Rightarrow \sqrt[5]{2x-7} = 3$

$\Rightarrow (\sqrt[5]{2x-7})^5 = 3^5$

$\Rightarrow 2x - 7 = 243$ $[\because (\sqrt[n]{a})^n = a]$

$\Rightarrow 2x = 250$ or $x = 125$.

Illustration 8 Find the value of $\sqrt[5]{64} \times \sqrt[5]{512}$.

Solution $\sqrt[5]{64} \times \sqrt[5]{512}$

$= \sqrt[5]{64 \times 512}$ $[\because \sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}]$

$= \sqrt[5]{8^2 \times 8^3} = \sqrt[5]{8^5} = 8.$ $[\because \sqrt[n]{a^n} = a]$

Illustration 9 Find the value of $\sqrt[3]{\sqrt[2]{729}}$.

Solution $\sqrt[3]{\sqrt[2]{729}} = \sqrt[6]{729}$ $[\because \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}]$

$= \sqrt[6]{3^6} = 3.$ $[\because \sqrt[n]{a^n} = a]$

Illustration 10 Find the value of $\frac{\sqrt[7]{\sqrt[5]{(21^7)^5}}}{\sqrt[5]{\sqrt[3]{(7^5)^3}}}$.

Solution Given expression

$= \frac{\sqrt[7]{(21)^7}}{\sqrt[5]{(7)^5}}$ $\left[\because \sqrt[n]{\sqrt[m]{(a^p)^m}} = \sqrt[n]{a^p}\right]$

$= \frac{21}{7} = 3.$ $[\because \sqrt[n]{a^n} = a]$

Illustration 11 Find the value of $\sqrt{5} \times \sqrt{125}$.

Solution $\sqrt{5} \times \sqrt{125} = \sqrt{625}$ $[\because \sqrt{a} \times \sqrt{b} = \sqrt{ab}]$

$= 25.$

Illustration 12 Simplify each of the following by rationalising the denominator.

(i) $\frac{1}{2 + \sqrt{3}}$ (ii) $\frac{7\sqrt{3} - 5\sqrt{2}}{\sqrt{48} + \sqrt{18}}$

Solution (i) $\dfrac{1}{2+\sqrt{3}} = \dfrac{1}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$$

$$= \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}.$$

(ii) $\dfrac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \dfrac{7\sqrt{3}-5\sqrt{2}}{\sqrt{4^2\times 3}+\sqrt{3^2\times 2}}$

$$= \frac{7\sqrt{3}-5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} = \frac{7\sqrt{3}-5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} \times \frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}$$

$$= \frac{(7\sqrt{3}-5\sqrt{2})(4\sqrt{3}-3\sqrt{2})}{(4\sqrt{3}+3\sqrt{2})(4\sqrt{3}-3\sqrt{2})}$$

$$= \frac{7\sqrt{3}\times 4\sqrt{3}-7\sqrt{3}\times 3\sqrt{2}-5\sqrt{2}\times 4\sqrt{3}+5\sqrt{2}\times 3\sqrt{2}}{(4\sqrt{3})^2-(3\sqrt{2})^2}$$

$$= \frac{28\sqrt{3\times 3}-21\sqrt{3\times 2}-20\sqrt{2\times 3}+15\sqrt{2\times 2}}{16\times 3-9\times 2}$$

$$= \frac{28\times 3-21\times\sqrt{6}-20\sqrt{6}+15\times 2}{48-18}$$

$$= \frac{84-(21\times 20)\sqrt{6}+30}{30} = \frac{114-41\sqrt{6}}{30}.$$

Illustration 13 If a and b are rational numbers, find the values of a and b in the following equation:

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6}.$$

Solution $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$= \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$= \frac{3+2+2\sqrt{3}\times\sqrt{2}}{3-2} = \frac{5+2\sqrt{6}}{1}$$

$$= 5 + 2\sqrt{6}$$

$$\therefore \quad \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6} \Rightarrow 5 + 2\sqrt{6} = a + b\sqrt{6}.$$

On equating rational and irrational parts, we get

$$a = 5 \text{ and } b = 2.$$

Illustration 14 Find the value of

$$\left(\sqrt{72+\sqrt{72+\sqrt{72+\ldots\infty}}}\right) \div \left(\sqrt{12-\sqrt{12-\sqrt{12-\ldots\infty}}}\right)$$

Solution Since $72 = 9 \times 8$,

therefore, $72 + \sqrt{72+\sqrt{72+\ldots\infty}} = 9$

Also, since $12 = 4 \times 3$

therefore, $\sqrt{12-\sqrt{12-\sqrt{12-\ldots\infty}}} = 3.$

Thus, the given expression $= \dfrac{9}{3} = 3.$

Fractions

1. Continued Fraction

Fractions of the form $7 + \cfrac{2}{5+\cfrac{3}{4+\cfrac{2}{3+\cfrac{1}{4}}}}$

are called continued fractions.

To simplify a continued fraction, we start from the bottom and work upwards.

Illustration 15 Simplify $3 - \cfrac{1}{4+\cfrac{7}{9-\cfrac{5}{6+\cfrac{2}{3}}}}$.

Solution $3 - \cfrac{1}{4+\cfrac{7}{9-\cfrac{5}{6+\cfrac{2}{3}}}} = 3 - \cfrac{1}{4+\cfrac{7}{9-\cfrac{15}{20}}}$

[Multiply the numerator and denominator of lowest term $\dfrac{5}{\left(6+\frac{2}{5}\right)}$ by 3 to get $\dfrac{15}{20}$.]

$$= 3 - \cfrac{1}{4+\cfrac{7}{9-\cfrac{3}{4}}} = 3 - \cfrac{1}{4+\cfrac{28}{33}}$$

[Multiply the numerator and denominator of the lowest term $\dfrac{7}{\left(9-\frac{3}{4}\right)}$ by 4 to get $\dfrac{28}{33}$]

$$= 3 - \frac{33}{160}$$

[Multiply the numerator and denominator of the term $\dfrac{1}{\left(4+\frac{28}{33}\right)}$ by 33 to get $\dfrac{33}{160}$]

$$= \frac{480-33}{160} = \frac{447}{160} = 2\frac{127}{160}.$$

2. Comparison of Fractions

The following points are found useful while comparing two or more fractions:

(a) If the denominators of the fractions are same, the largest is one whose numerator is the largest.

Illustration 16 Which is the largest fraction among

$$\frac{3}{8}, \frac{7}{8} \text{ and } \frac{5}{8}?$$

Solution $\dfrac{7}{8}$.

(b) If the numerators of the fractions are same, the largest is one whose denominator is the smallest.

Illustration 17 Which is the largest fraction among

$$\frac{5}{2}, \frac{5}{7} \text{ and } \frac{5}{9}?$$

Solution $\frac{5}{2}$.

(c) If neither the numerators nor denominators of the fractions are same then they are converted into equivalent fractions of the same denominator by taking the L.C.M. of the denominators of the given fractions. Then, the fractions are compared according to (1).

Illustration 18 Which is the largest fraction among

$$\frac{1}{2}, \frac{2}{3}, \frac{4}{5} \text{ and } \frac{5}{8}?$$

Solution L.C.M. of 2, 3, 5 and 8 = 120.

Then, $\frac{1}{2} = \frac{1\times60}{2\times60} = \frac{60}{120}$

$\frac{2}{3} = \frac{2\times40}{3\times40} = \frac{80}{120}$

$\frac{4}{5} = \frac{4\times24}{5\times24} = \frac{96}{120}$

and, $\frac{5}{8} = \frac{5\times15}{8\times15} = \frac{75}{120}$

Now, the denominator of these fractions are same and the largest numerator is 96. Hence, the largest fraction is $\frac{96}{120}$, that is $\frac{4}{5}$.

(d) Two fractions can also be compared by cross multiplication method.

Illustration 19 Which is greater $\frac{6}{13}$ or $\frac{5}{7}$?

Solution **Step 1.** Cross multiply the two given fractions

$$\frac{6}{13} \times \frac{5}{7},$$

we get $6 \times 7 = 42$ and $13 \times 5 = 65$.

Step 2. Since 65 is greater than 42 and in 65, the numerator of $\frac{5}{7}$ is included, $\therefore$ $\frac{5}{7}$ is greater than $\frac{6}{13}$.

(e) If the difference of the numerator and denominator of each of the given fractions be same then the fraction of the largest numerator is the smallest.

Illustration 20 Which one of the following fractions is the largest?

$$\frac{2}{3}, \frac{3}{4}, \frac{5}{6} \text{ and } \frac{9}{10}$$

Solution Since in each of the given fractions the difference between the numerator and denominator is same and the largest numerator is 9, therefore, the largest fraction is $\frac{9}{10}$.

(f) In the given fractions, $\frac{x}{y}, \frac{x+a}{y+b}, \frac{x+2a}{y+2b}, \ldots, \frac{x+na}{y+nb}$, where $a < b$

(a) If $\frac{\text{Increase in Numerator}}{\text{Increase in Denominator}}$ > first fraction, the last value is the greatest

(b) If $\frac{\text{Increase in Numerator}}{\text{Increase in Denominator}}$ < first fraction, the last value is the least.

(c) If $\frac{\text{Increase in Numerator}}{\text{Increase in Denominator}}$ = first fraction, all values are equal.

Illustration 21 Which one the following fractions is the greatest?

$$\frac{3}{8}, \frac{4}{11}, \frac{5}{14}, \frac{6}{17}, \frac{7}{20}.$$

Solution Since, $\frac{\text{increase in numerator}}{\text{increase in denominator}} = \frac{1}{3}$ is less than the first fraction $\frac{3}{8}$, therefore, the first fraction $\frac{3}{8}$ is the greatest.

Illustration 22 Which of the following fractions is the least?

$$\frac{2}{5}, \frac{4}{11}, \frac{6}{17}, \frac{8}{23}$$

Solution Since, $\frac{\text{increase in numerator}}{\text{increase in denominator}} = \frac{2}{6} = \frac{1}{3}$ is less than the first fraction $\frac{2}{5}$, therefore, the last fraction $\frac{8}{23}$ is the least.

3. Inserting a fraction between two given fractions

To insert a fraction between two given fractions $\frac{a_1}{b_1}$ and $\frac{a_2}{b_2}$, the following steps may be useful:

Step 1 The numerators of the two given fractions are added to get the numerator of the resulting fraction, that is, $a_1 + a_2$.

Step 2 The denominators of the two given fractions are added to get the denominator of the resulting fraction, that is, $b_1 + b_2$.

Step 3. Resulting fraction = $\frac{a_1 + a_2}{b_1 + b_2}$.

Illustration 23 Insert one fraction between $\frac{2}{5}$ and $\frac{4}{7}$.

Solution Using the above method,

$$\frac{2}{5}, \frac{2+4}{5+7}, \frac{4}{7} = \frac{2}{5}, \frac{6}{12}, \frac{4}{7} \text{ or, } \frac{2}{5}, \frac{1}{2}, \frac{4}{7}.$$

Illustration 24 Insert three fractions between $\frac{5}{7}$ and $\frac{9}{11}$.

Solution Using the above method,

$$\frac{5}{7}, \frac{5+9}{7+11}, \frac{9}{11} = \frac{5}{7}, \frac{14}{18}, \frac{9}{11} \text{ or } \frac{5}{7}, \frac{7}{9}, \frac{9}{11}.$$

Further,

$$\frac{5}{7}, \frac{5+7}{7+9}, \frac{7}{9}, \frac{7+9}{9+11}, \frac{9}{11} = \frac{5}{7}, \frac{12}{16}, \frac{7}{9}, \frac{16}{20}, \frac{9}{11}$$

or $$\frac{5}{7}, \frac{3}{4}, \frac{7}{9}, \frac{4}{5}, \frac{9}{11}.$$

Thus, the three fractions inserted between $\frac{5}{7}$ and $\frac{9}{11}$ are $\frac{3}{4}, \frac{7}{9}$ and $\frac{4}{5}$.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Simplify

$$\frac{3}{10} \div \frac{3}{7} \text{ of } \left(2\frac{3}{10} + 2\frac{3}{5}\right) + \frac{1}{5} \div 1\frac{2}{5} - \frac{2}{7}$$

(a) 1 (b) 2
(c) 0 (d) 3

2. $1 + 1 \div \left\{1 + 1 \div \left(1 - \frac{1}{3}\right)\right\} = ?$

(a) $\frac{7}{5}$ (b) $\frac{2}{3}$
(c) $\frac{4}{5}$ (d) None of these

3. $48 \div 12 \times \left(\frac{9}{8} \text{ of } \frac{4}{3} + \frac{3}{4} \text{ of } \frac{2}{3}\right) = ?$

(a) 9 (b) 12
(c) 15 (d) None of these

4. Simplify:

$2 \div [2 + 2 \div \{2 + 2 \div (2 + 2 \div 3)\}]$

(a) 13/15 (b) 17/15
(c) 11/15 (d) None of these

5. $7\frac{1}{2} - \left[2\frac{1}{4} \div \left\{1\frac{1}{4} - ?\left(1\frac{1}{2} - \frac{1}{3} - \frac{1}{6}\right)\right\}\right] = 3$

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$
(c) $\frac{4}{3}$ (d) None of these

6. The simplification of $\frac{0.8 \times 0.8 \times 0.8 - 0.5 \times 0.5 \times 0.5}{0.8 \times 0.8 + 0.8 \times 0.5 + 0.5 \times 0.5}$ gives

(a) 0.8 (b) 0.4
(c) 0.3 (d) 0.13

7. The simplification of $\left[\frac{1}{2} + \frac{1}{2}\left\{\frac{3}{4} - \frac{1}{2}\left(\frac{7}{8} - \frac{3}{4}\right)\right\}\right]$ yields

(a) $\frac{27}{16}$ (b) $\frac{27}{32}$
(c) $\frac{27}{64}$ (d) $\frac{107}{112}$

8. Simplify: $1 - [2 - \{5 - (4 - \overline{3-2})\}]$

(a) 1 (b) 2
(c) 3 (d) 4

9. $3 \div \left[(8-5) \div \left\{(4-2) \div \left(2 + \frac{8}{13}\right)\right\}\right] = ?$

(a) $\frac{33}{71}$ (b) $\frac{55}{17}$
(c) $\frac{13}{17}$ (d) None of these

10. $\frac{69842 \times 69842 - 30158 \times 30158}{69842 - 30158} = ?$

(a) 100000 (b) 69842
(c) 39684 (d) 30158

11. Simplify $\frac{2\frac{1}{7} - 2\frac{1}{2}}{2\frac{1}{4} + 1\frac{1}{7}} \div \frac{1}{2 + \frac{1}{2 + \frac{1}{2 - \frac{1}{2}}}}$

(a) $\frac{-1}{2}$ (b) $-\frac{1}{8}$
(c) $-\frac{1}{6}$ (d) $-\frac{1}{4}$

12. The value of $\frac{2.75 \times 2.75 \times 2.75 - 2.25 \times 2.25 \times 2.25}{2.75 \times 2.75 + 2.75 \times 2.25 + 2.25 \times 2.25}$ is

(a) 0.30 (b) 0.50
(c) 3.00 (d) 5.00

13. $\dfrac{\frac{1}{2}\div 4+20}{\frac{1}{2}\times 4+20} = ?$

(a) $\frac{81}{88}$ (b) $2\frac{3}{11}$
(c) $\frac{161}{176}$ (d) 1

14. Evaluate $\dfrac{0.53\times 0.53-2\times 0.53\times 0.41+0.41\times 0.41}{0.53\times 0.41}$

(a) 0.16 (b) 0.8
(c) 0.12 (d) None of these

15. The value of $\dfrac{9^2\times 18^4}{3^{16}}$ is

(a) $\frac{2}{3}$ (b) $\frac{4}{9}$
(c) $\frac{16}{81}$ (d) $\frac{32}{243}$

16. The simplification of $1+\dfrac{1}{2+\dfrac{1}{1-\frac{1}{3}}}$ yields the result

(a) $\frac{2}{7}$ (b) $\frac{7}{9}$
(c) $\frac{9}{7}$ (d) $\frac{13}{7}$

17. $108 + ?$ of $\frac{1}{3} + \frac{2}{5}\times 3\frac{3}{4} = 10\frac{1}{2}$

(a) 15 (b) 63
(c) 24 (d) 36

18. The value of

$1+\dfrac{1}{4\times 3}+\dfrac{1}{4\times 3^2}+\dfrac{1}{4\times 3^3}$ up to four places of decimals is:

(a) 1.1202 (b) 1.1203
(c) 1.1204 (d) None of these

19. $\frac{3}{48}$ is what part of $\frac{1}{12}$?

(a) $\frac{3}{7}$ (b) $\frac{1}{12}$
(c) $\frac{3}{4}$ (d) None of these

20. The simplification of

$1+\dfrac{1}{1+\dfrac{1}{1-\frac{2}{3}}}$ yields the result

(a) $\frac{7}{4}$ (b) $\frac{4}{5}$
(c) $\frac{5}{4}$ (d) None of these

21. Which of the following fractions is less than $\frac{7}{8}$ and greater than $\frac{1}{3}$?

(a) $\frac{1}{4}$ (b) $\frac{23}{24}$
(c) $\frac{11}{12}$ (d) $\frac{17}{24}$

22. How many $\frac{1}{8}$'s are there in $37\frac{1}{2}$?

(a) 300 (b) 400
(c) 500 (d) Cannot be determined

23. In a college, $\frac{1}{5}$th of the girls and $\frac{1}{8}$th of the boys took part in a social camp. What of the total number of students in the college took part in the camp?

(a) $\frac{13}{40}$ (b) $\frac{13}{80}$
(c) $\frac{2}{13}$ (d) Data inadequate

24. $\left\{7\frac{1}{2}+\frac{1}{2}\div\frac{1}{2} \text{ of } \frac{1}{4}-\frac{2}{5}\times 2\frac{1}{3}\div 1\frac{7}{8} \text{ of } \left(1\frac{2}{5}-1\frac{1}{3}\right)\right\} = ?$

(a) $3\frac{1}{5}$ (b) $2\frac{1}{24}$
(c) $4\frac{1}{30}$ (d) None of these

25. When simplified, the product

$\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right)\ldots\left(2-\frac{999}{1001}\right)$ is equal to:

(a) $\frac{991}{1001}$ (b) $\frac{1001}{13}$
(c) $\frac{1003}{3}$ (d) None of these

26. The value of $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\frac{1}{4.5.6}$ is

(a) $\frac{7}{30}$ (b) $\frac{1}{3}$
(c) $\frac{13}{30}$ (d) None of these

27. $15\frac{2}{3}\times 3\frac{1}{6}+6\frac{1}{3} = 11\frac{7}{18} + ?$

(a) $39\frac{5}{9}$ (b) $137\frac{4}{9}$
(c) $29\frac{7}{9}$ (d) None of these

28. $3\div\left[(8-5)\div\left\{(4-2)\div\left(2+\frac{8}{13}\right)\right\}\right] = ?$

(a) $\frac{13}{17}$ (b) $\frac{68}{13}$
(c) $\frac{17}{13}$ (d) $\frac{13}{68}$

29. If the numbers $\frac{3}{5}, \frac{2}{3}, \frac{3}{4}$ are given, then we can say that

(a) $\frac{3}{4} > \frac{3}{5} > \frac{2}{3}$ (b) $\frac{2}{3} > \frac{3}{5} > \frac{3}{4}$
(c) $\frac{3}{4} > \frac{2}{3} > \frac{3}{5}$ (d) $\frac{3}{5} > \frac{2}{3} > \frac{3}{4}$

30. $\frac{(272-32)(124+176)}{17\times15-15} = ?$
(a) 0 (b) 2.25
(c) 300 (d) None of these

31. If $\frac{a}{b} = \frac{1}{3}$, then $\frac{3a+2b}{3a-2b}$ is equal to
(a) 3 (b) –3
(c) –5 (d) –1

32. $(20 \div 5) \div 2 + (16 \div 8) \times 2 + (10 \div 5) \times (3 \div 2) = ?$
(a) 9 (b) 12
(c) 15 (d) 18

33. $\frac{5}{6} \div \frac{6}{7} \times ? - \frac{8}{9} \div 1\frac{3}{5} + \frac{3}{4} \times 3\frac{1}{3} = 2\frac{7}{9}$
(a) $\frac{7}{6}$ (b) $\frac{6}{7}$
(c) 1 (d) None of these

34. $4\frac{1}{2} + 3\frac{1}{6} + ? + 2\frac{1}{3} = 13\frac{2}{5}$
(a) $3\frac{2}{5}$ (b) $1\frac{2}{5}$
(c) $4\frac{1}{5}$ (d) $4\frac{1}{6}$

35. The simplification of
$\frac{0.67\times0.67\times0.67-0.001}{0.67\times0.67+0.067+0.01}$ gives
(a) 0.57 (b) 0.66
(c) 0.68 (d) 0.77

36. In a certain college the number of girls is twice the number of boys. $\frac{1}{5}$th of the girls and $\frac{1}{8}$th of the boys took part in a social camp. What part of the total number of students took part in the camp?
(a) $\frac{7}{40}$ (b) $\frac{7}{80}$
(c) $\frac{2}{12}$ (d) $\frac{1}{24}$

37. If we multiply a fraction by itself and divide the product by its reciprocal, the fraction thus obtained is $18\frac{26}{27}$. The fraction is
(a) $\frac{8}{27}$ (b) $2\frac{2}{3}$
(c) $1\frac{1}{3}$ (d) None of these

38. Which of the following numbers is the greatest?
(a) $(0.3)^2$ (b) $1 \div 0.3$
(c) $\frac{1}{8}$ (d) $\sqrt{0.49}$

39. What fraction must be subtracted from the sum of $\frac{1}{4}$ and $\frac{1}{6}$ to have an average of $\frac{1}{12}$ of all the three fractions?
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) $\frac{1}{6}$

40. A person was to multiply a fraction by $\frac{6}{7}$. Instead, he divided and got an answer which exceeds the correct answer by $\frac{1}{7}$. The correct answer was
(a) $\frac{6}{13}$ (b) $\frac{36}{91}$
(c) $\frac{7}{13}$ (d) None of these

41. $2 + \sqrt{2} + \frac{1}{2+\sqrt{2}} + \frac{1}{\sqrt{2}-2} = ?$
(a) 2 (b) 4
(c) 0 (d) Cannot be determined

42. The value of $\frac{1}{2} + \frac{1}{2.3} + \frac{1}{2.3.4} + \frac{1}{2.3.4.5}$ correct to three places of decimal is
(a) 0.713 (b) 0.715
(c) 0.717 (d) 0.718

43. $\frac{?\div12}{0.2\times3.6} = 2$
(a) 17.82 (b) 17.22
(c) 17.28 (d) 17.12

44. $\sqrt{?\times7} \times 18 = 84$
(a) 3.11 (b) 3.12
(c) 3.13 (d) 3.14

45. The difference between the sum of $1\frac{3}{4}, 2\frac{1}{3}, 3\frac{5}{12}, 5\frac{1}{5}$ and $2\frac{1}{6}$ and the nearest whole number is
(a) $\frac{2}{15}$ (b) $\frac{13}{15}$
(c) $\frac{11}{60}$ (d) None of these

46. $\left(2\frac{3}{x}\right)\times\left(y\frac{1}{2}\right) = 7\frac{3}{4}$, find the values of x and y
(a) (3, 19) (b) (3, 14)
(c) (14, 3) (d) (24, 6)

47. If we multiply a fraction by itself and divide the product by the square of its reciprocal, the fraction so obtained is $3\frac{13}{81}$. The original fraction is

(a) $\frac{16}{9}$ (b) $\frac{8}{9}$
(c) $\frac{4}{3}$ (d) $\frac{1}{3}$

48. If $x * y = (x + 2)^2(y - 2)$ then $7 * 5 = ?$
(a) 234 (b) 243
(c) 343 (d) 423

49. If m and n are whole numbers such that $m^n = 121$, then $(m - 1)^{n+1} = ?$
(a) 10 (b) 10^2
(c) 10^3 (d) 10^4

50. Between two fractions $\frac{1}{2}$ and $\frac{1}{8}$, how many fractions are there in all?
(a) four (b) zero
(c) sixteen (d) Infinite

51. A boy was asked to multiply a given number by 8/17. Instead, he divided the given number by $\frac{8}{17}$ and got the result 225 more than what he should have got if he had multiplied the number by 8/17. The given number was
(a) 8 (b) 17
(c) 64 (d) 136

52. The value of $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{9.10}$ is
(a) $\frac{11}{10}$ (b) $\frac{8}{9}$
(c) $\frac{9}{10}$ (d) $\frac{25}{1089}$

53. $\frac{\sqrt{1296}}{?} = \frac{?}{2.25}$
(a) 6 (b) 3
(c) 9 (d) 12

54. If we multiply a fraction by itself and divide the product by its reciprocal, the fraction thus obtained is $18\frac{26}{27}$. The original fraction is
(a) $\frac{8}{27}$ (b) $2\frac{2}{3}$
(c) $1\frac{1}{3}$ (d) None of these

55. If $\frac{a}{a+b} = \frac{17}{23}$, what is $\frac{a+b}{a-b}$ equal to?
(a) $\frac{11}{23}$ (b) $\frac{17}{32}$
(c) $\frac{23}{11}$ (d) $\frac{23}{17}$

56. A tin of oil was $\frac{4}{5}$ full, when six bottles of oil were taken out and four bottles of oil were poured into it, it was $\frac{3}{4}$ full. How many bottles of oil can it contain?
(a) 10 (b) 20
(c) 30 (d) 40

57. In an examination a student was asked to find (3/14) of a certain number. By mistake he found (3/4) of it. His answer was 150 more than the correct answer. The given number is
(a) 180 (b) 240
(c) 280 (d) 290

58. The value of $\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right)\ldots\left(2-\frac{999}{1001}\right)$ is
(a) $\frac{1003}{3}$ (b) $\frac{1003}{1001}$
(c) $\frac{1}{1001}$ (d) None of these

59. If $\sqrt{2^n} = 64$, then find the value of n
(a) 8 (b) 10
(c) 12 (d) 16

60. If $10^{2y} = 25$ then what is the value of 10^y?
(a) –5 (b) 5
(c) $\frac{1}{25}$ (d) $\sqrt{\frac{1}{25}}$
(e) None of these

61. $11\frac{1}{3} \times 4\frac{8}{10} \div ? = 22\frac{2}{3}$
(a) 2.4 (b) 4.2
(c) 2.6 (d) 2.8

62. Simplify

$$\frac{a^{1/2}+a^{-1/2}}{1-a}+\frac{1-a^{-1/2}}{1+\sqrt{a}}$$

(a) $\frac{a}{a-1}$ (b) $\frac{a-1}{2}$
(c) $\frac{2}{a-1}$ (d) $\frac{2}{1-a}$

63. If $a^2 + b^2 = 45$ and $ab = 18$, find $\frac{1}{a}+\frac{1}{b}$.
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{1}{2}$ (d) Cannot be determined

64. If $\frac{a^2+b^2}{c^2+d^2} = \frac{ab}{cd}$, then find the value of $\frac{a+b}{a-b}$ in terms of c and d only
(a) $\frac{c+d}{cd}$ (b) $\frac{cd}{c+d}$
(c) $\frac{c-d}{c+d}$ (d) $\frac{c+d}{c-d}$

65. $(1.06 + 0.04)^2 - ? = 4 \times 1.06 \times 0.04$
(a) 1.04 (b) 1.4
(c) 1.5 (d) Cannot be determined

66. The highest score in an inning was $\frac{2}{9}$ of the total score and the next highest was $\frac{2}{9}$ of the remainder. These scores differ by 8 runs. What was the total score in the innings?
(a) 162 (b) 152
(c) 142 (d) 132

67. Simplify $\left(\frac{1}{64}\right)^0 + (64)^{-1/2} + (-32)^{4/5}$
(a) $17\frac{1}{8}$ (b) $17\frac{3}{8}$
(c) $11\frac{7}{8}$ (d) $17\frac{7}{8}$

68. $\dfrac{\frac{64}{121} - \frac{9}{64}}{\frac{8}{11} + \frac{3}{8}} = ?$
(a) $\frac{88}{31}$ (b) $\frac{31}{88}$
(c) $\frac{41}{99}$ (d) $\frac{99}{41}$

69. When $\frac{1}{4}$ of a number is subtracted from $\frac{1}{3}$ of the same number, the remainder obtained is 12. The number is
(a) 144 (b) 72
(c) 120 (d) 63
(e) None of these

70. What is the difference between the largest and the smallest fractions

$$\frac{5}{8}, \frac{21}{35}, \frac{9}{16} \text{ and } \frac{6}{7}?$$

(a) $\frac{33}{112}$ (b) $\frac{11}{37}$
(c) $\frac{13}{41}$ (d) $\frac{9}{35}$
(e) None of these

71. If a man spends $\frac{5}{6}$th part of money and then again earns $\frac{1}{2}$ part of the remaining money, what part of his money is with him now?
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) $\frac{2}{3}$ (d) $\frac{3}{4}$
(e) $\frac{1}{3}$

72. Manmohan spends $\frac{1}{5}$ part of his money as pocket money and $\frac{4}{5}$ of the remainder in other affairs. If he is left with Rs 48 per month, what is monthly income?
(a) Rs 360 (b) Rs 400
(c) Rs 320 (d) Rs 300
(e) None of these

73. If the difference between $\frac{4}{5}$ part and $\frac{3}{4}$ part of a number is 4, what is the number?
(a) 60 (b) 100
(c) 80 (d) 40
(e) None of these

74. If $\frac{2}{3}$ part of a number is 96, what is the value of $\frac{3}{4}$ part of the same number?
(a) 48 (b) 192
(c) 108 (d) 72
(e) None of these

75. A man completes $\frac{2}{15}$ of his journey by aeroplane, $\frac{2}{5}$ by train and the rest by taxi. What part of his journey does he complete by taxi?
(a) $\frac{8}{15}$ (b) $\frac{7}{15}$
(c) $\frac{9}{15}$ (d) None of these

76. If $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\left(1-\frac{1}{70}\right) = \frac{x}{70}$, then what is the value of x ?
(a) 69 (b) 35
(c) 20 (d) 15
(e) 1

77. The value of

$$\frac{1.073 \times 1.073 - 0.927}{1.073 - 0.927} + \frac{(3^4)^4 \times 9^6}{(27)^7 \times (3)^9} \text{ is}$$

(a) 2 (b) $\frac{1}{9}$
(c) $2\frac{1}{9}$ (d) $3\frac{1}{9}$
(e) None of these

78. The value of

$$\frac{2^{1/2}.3^{1/3}.4^{1/4}}{10^{-1/5}.5^{3/5}} \div \frac{3^{4/3}.5^{-7/5}}{4^{-3/5}.6} \text{ is}$$

(a) 5 (b) 6
(c) 10 (d) 15
(e) None of these

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Given expression:

$$= \frac{3}{10} \div \frac{3}{7} \text{ of } \left(\frac{23}{10}+\frac{13}{5}\right)+\frac{1}{5}\times\frac{5}{7}-\frac{2}{7}$$

$$= \frac{3}{10} \div \frac{3}{7} \text{ of } \frac{49}{10}+\frac{1}{7}-\frac{2}{7} = \frac{3}{10}\div\frac{21}{10}-\frac{1}{7}$$

$$= \frac{3}{10}\times\frac{10}{21}-\frac{1}{7} = \frac{1}{7}-\frac{1}{7} = 0.$$

2. Ans. (a)

Solution Given expression

$$= 1 + 1 \div \left\{1+1\div\left(\frac{2}{3}\right)\right\}$$

$$= 1 + 1 \div \left\{1+1\times\frac{3}{2}\right\}$$

$$= 1 + 1 \div \left\{1+\frac{3}{2}\right\} = 1 + 1 \div \frac{5}{2}$$

$$= 1 + 1 \times \frac{2}{5} = 1 + \frac{2}{5} = \frac{7}{5}.$$

3. Ans. (b)

Solution Given expression

$$= 48 \div 12 \times \left(\frac{9}{8} \text{ of } \frac{4}{3}\div\frac{3}{4} \text{ of } \frac{2}{3}\right)$$

$$= \frac{48}{12}\times\left\{\left(\frac{9}{8}\times\frac{4}{3}\right)\div\left(\frac{3}{4}\times\frac{2}{3}\right)\right\}$$

$$= \frac{48}{12}\times\left(\frac{3}{2}\times 2\right) = 4 \times 3 = 12.$$

4. Ans. (c)

Solution Given expression

$$= 2 \div \left[2+2\div\left\{2+2\div\left(2+\frac{2}{3}\right)\right\}\right]$$

$$= 2 \div \left[2+2\div\left\{2+2\times\frac{3}{8}\right\}\right]$$

$$= 2 \div \left[2+2\div\frac{11}{4}\right] = 2 \div \left[2+2\times\frac{4}{11}\right]$$

$$= 2 \div \frac{30}{11} = 2 \times \frac{11}{30} = \frac{11}{15}.$$

5. Ans. (b)

Solution Let missing figure = x

$$\frac{15}{2}-\left[\frac{9}{4}\div\left\{\frac{5}{4}-x\left(\frac{3}{2}-\frac{1}{3}-\frac{1}{6}\right)\right\}\right] = 3$$

$$\frac{15}{2}-\left[\frac{9}{4}\div\left\{\frac{5}{4}-x\right\}\right] = 3$$

$$\frac{15}{2}-3 = \frac{9/4}{5/4-x}$$

$$\frac{9}{2} = \frac{9}{5-4x}$$

$$5 - 4x = 2$$

$$x = \frac{3}{4}.$$

6. Ans. (c)

Solution We know that $\dfrac{a^3-b^3}{a^2+ab+b^2} = a - b$

$\therefore$ The given expression = 0.8 – 0.5 = 0.3.

7. Ans. (b)

8. Ans. (a)

Solution Given expression

$$= 1 - [2 - \{5 - (4 - 1)\}]$$

$$= 1 - [2 - \{5 - 3\}]$$

$$= 1 - [2 - 2] = 1 - 0 = 1.$$

9. Ans. (c)

Solution Given expression

$$= 3 \div \left[3\div\left\{2\div\frac{34}{13}\right\}\right]$$

$$= 3 \div \left[3\div\left\{2\times\frac{13}{34}\right\}\right] = 3 \div \left[3\times\frac{17}{13}\right]$$

$$= 3 \div \frac{51}{13} = 3 \times \frac{13}{51} = \frac{13}{17}.$$

10. Ans. (a)

Solution Given expression

$$= \frac{(69842)^2-(30158)^2}{69842-30158}$$

$$= \frac{(69842-30158)(69842+30158)}{69842-30158}$$

$$= 100000.$$

11. Ans. (d)

Solution Given expression

$$= \frac{\frac{15}{7}-\frac{5}{2}}{\frac{9}{4}+\frac{8}{7}} \div \frac{1}{2+\frac{1}{2+\frac{2}{3}}}$$

$$= \frac{-5}{14}\times\frac{28}{95}\div\frac{1}{2+\frac{3}{8}}$$

$$= \frac{-2}{19}\div\frac{8}{19} = \frac{-2}{19}\times\frac{19}{8} = \frac{-1}{4}.$$

12. Ans. (b)

Solution The given expression

$$= 2.75 - 2.25 = 0.50.$$

13. Ans. (c)

Solution Given expression

$$= \frac{\frac{1}{2}\times\frac{1}{4}+20}{2+20} = \frac{161}{8}\times\frac{1}{22} = \frac{161}{176}.$$

14. Ans. (c)

Solution Given expression

$$= \frac{(0.53)^2 - 2\times0.53\times0.41+(0.41)^2}{0.12}$$

$$= \frac{(0.53-0.41)^2}{0.12} = \frac{(0.12)^2}{0.12} = 0.12.$$

15. Ans. (c)

Solution Given expression

$$= \frac{(3^2)^2\times(3\times3\times2)^4}{3^{16}}$$

$$= \frac{3^4\times3^8\times2^4}{3^{16}} = \frac{2^4}{3^4} = \frac{16}{81}.$$

16. Ans. (c)

Solution Given expression

$$= 1 + \frac{1}{2+\frac{1}{\frac{2}{3}}} = 1 + 1 + \frac{1}{2+\frac{3}{2}}$$

$$= 1 + \frac{1}{\frac{7}{2}} = 1 + \frac{2}{7} = \frac{9}{7}.$$

17. Ans. (d)

Solution Let x be the missing number

$$= 108 \div x \text{ of } \frac{1}{3}+\frac{2}{5}\times3\frac{3}{4} = 10\frac{1}{2}$$

$$= 108 \div \frac{x}{3}+\frac{2}{5}\times\frac{15}{4} = \frac{21}{2}$$

$$= \frac{3\times108}{x}+\frac{3}{2} = \frac{21}{2}$$

$$= \frac{3\times108}{x} = \frac{21}{2}-\frac{3}{2}$$

$$= \frac{3\times108}{x} = 9$$

$$x = \frac{3\times108}{9}$$

$$x = 36.$$

18. Ans. (b)

Solution Given expression

$$= \frac{108+9+3+1}{108} = \frac{121}{108} = 1.1203.$$

19. Ans. (d)

Solution Let x of $\frac{1}{12} = \frac{3}{48}$

Then, $x = \frac{3}{48}\times12 = \frac{3}{4}$.

20. Ans. (c)

Solution Given expression

$$= 1 + \frac{1}{1+3} = 1 + \frac{1}{4} = \frac{5}{4}.$$

21. Ans. (d)

Solution $\frac{1}{3} = 0.33$ and $\frac{7}{8} = 0.875$

$\frac{1}{4} = 0.25$ does not lie between 0.33 and 0.875

$\frac{23}{24} = 0.96$ which exceeds 0.875

$\frac{11}{12} = 0.92$ which exceeds 0.875

$\frac{17}{24} = 0.708$ which lies between 0.33 and 0.875.

22. Ans. (a)

Solution Number of $\frac{1}{8}$'s $= \frac{75}{2}\div\frac{1}{8} = \frac{75}{2}\times8 = 300.$

23. Ans. (c)

Solution Out of the 5 girls, 1 took part in the camp.
Out of the 8 boys, 1 took part in the camp.
Out of the 13 students, 2 took part in the camp.

$\therefore \frac{2}{13}$ of total number of students took part in the camp.

24. Ans. (c)

Solution Given expression

$$= \frac{15}{2}+\frac{1}{2}\div\frac{1}{8}-\frac{2}{5}\times\frac{7}{3}\div\frac{15}{8} \text{ of } \left(\frac{7}{5}-\frac{4}{3}\right)$$

$$= \frac{15}{2}+4-\frac{2}{5}\times\frac{7}{3}\div\frac{15}{8} \text{ of } \frac{1}{15}$$

$$= \frac{15}{2}+4-\frac{2}{5}\times\frac{7}{3}\div\frac{1}{8}$$

$$= \frac{15}{2}+4-\frac{2}{5}\times\frac{7}{3}\times\frac{8}{1}$$

$$= \frac{15}{2}+4-\frac{112}{15} = \frac{23}{2}-\frac{112}{15}$$

$$= \frac{121}{30} = 4\frac{1}{30}.$$

25. Ans. (c)

Solution Given expression

$$= \frac{5}{3}\times\frac{7}{5}\times\frac{9}{7}\times\ldots\times\frac{1003}{1001} = \frac{1003}{3}.$$

26. Ans. (a)

Solution Given expression

$$= \frac{1}{6}+\frac{1}{24}+\frac{1}{60}+\frac{1}{120}$$

$$= \frac{20+5+2+1}{120} = \frac{28}{120} = \frac{7}{30}.$$

27. Ans. (d)

Solution Let $\frac{47}{3}\times\frac{19}{6}+\frac{19}{3} = \frac{205}{18} + x$

Then, $x = \frac{893}{18}+\frac{19}{3}-\frac{205}{18}$

$$= \frac{893+114-205}{18} = \frac{802}{18} = 44\frac{5}{9}.$$

28. Ans. (a)

Solution Given expression

$$= 3 \div \left[3\div\left\{2\div\frac{34}{13}\right\}\right]$$

$$= 3 \div \left[3\div\left(2\times\frac{13}{34}\right)\right]$$

$$= 3 \div \left[3\times\frac{17}{13}\right] = 3 \times \frac{13}{51} = \frac{13}{17}.$$

29. Ans. (c)

Solution $\frac{3}{5} = 0.60$

$\frac{3}{4} = 0.75$

$\frac{2}{3} = 0.66$

$\therefore \quad \frac{3}{4} > \frac{2}{3} > \frac{3}{5}.$

30. Ans. (c)

Solution Given expression

$$= \frac{240\times300}{240} = 300.$$

31. Ans. (b)

Solution $\frac{3a+2b}{3a-2b} = \frac{3\left(\frac{a}{b}\right)+2}{3\left(\frac{a}{b}\right)-2} = \frac{3\left(\frac{1}{3}\right)+2}{3\left(\frac{1}{3}\right)-2}$

$$= \frac{3}{-1} = -3.$$

32. Ans. (a)

Solution Given expression

$$= \frac{4}{2}+2\times2+2\times\frac{3}{2} = 2 + 4 + 3 = 9.$$

33. Ans. (b)

Solution Let $\frac{5}{6}\div\frac{6}{7}\times x-\frac{8}{9}\div\frac{8}{5}+\frac{3}{4}\times\frac{10}{3} = \frac{25}{9}$. Then,

$$\frac{5}{6}\times\frac{7}{6}\times x-\frac{8}{9}\times\frac{5}{8}+\frac{3}{4}\times\frac{10}{3} = \frac{25}{9}$$

or, $\frac{35}{36}x-\frac{5}{9}+\frac{10}{4} = \frac{25}{9}$ or, $\frac{35}{36}x = \frac{25}{9}+\frac{5}{9}-\frac{5}{2}$

or, $\frac{35}{36}x = \frac{30}{9}-\frac{5}{2}$ or, $\frac{35}{36}x = \frac{60-45}{18}$

or, $x = \frac{15}{18}\times\frac{36}{35}$

$\therefore \quad x = \frac{6}{7}.$

34. Ans. (a)

Solution Let $\frac{9}{2}+\frac{19}{6}+x+\frac{7}{3} = \frac{67}{5}$. Then,

$$x = \frac{67}{5}-\left(\frac{9}{2}+\frac{19}{6}+\frac{7}{3}\right)$$

$$= \frac{67}{5}-10 = \frac{17}{5} = 3\frac{2}{5}.$$

35. Ans. (a)

Solution $0.001 = (0.1)^3$; $0.067 = 0.1 \times 0.67$

The given expression

$$= \frac{a^3-b^3}{a^2+ab+b^2} = a - b$$

$$= 0.67 - 0.10 = 0.57.$$

36. Ans. (a)

Solution Let total number of students = x

Number of girl students = $\frac{2x}{3}$

Number of boy students = $\frac{x}{3}$

Number of girls who took part in camp

$$= \frac{1}{5}\left(\frac{2x}{3}\right) = \frac{2}{15}x$$

Number of boys who took part in camp

$$= \frac{1}{8}\left(\frac{x}{3}\right) = \frac{x}{24}$$

Total number of students who took part in camp

$$= \frac{2}{15}x+\frac{x}{24} = \left(\frac{16+5}{120}\right)x = \frac{7}{40}x.$$

37. Ans. (b)

Solution Let the fraction be $\frac{a}{b}$. Then,

$$\left(\frac{a}{b}\times\frac{a}{b}\right)\div\frac{b}{a} = 18\frac{26}{27} = \frac{512}{27}$$

or, $\left(\frac{a}{b}\right)^3 = \left(\frac{8}{3}\right)^3$

$\therefore \frac{a}{b} = \frac{8}{3} = 2\frac{2}{3}$.

38. Ans. (b)

39. Ans. (d)

Solution Let $\frac{1}{4}+\frac{1}{6}-x = 3 \times \frac{1}{12}$. Then,

$\frac{1}{4}+\frac{1}{6}-x = \frac{1}{4}$ or, $x = \frac{1}{6}$.

40. Ans. (b)

Solution Let x be the fraction

$$\frac{7}{6}x-\frac{6}{7}x = \frac{1}{7} \Rightarrow x = \frac{6}{13}$$

Correct answer $= \frac{6}{7}x = \frac{6}{7}\times\frac{6}{13} = \frac{36}{91}$.

41. Ans. (a)

Solution $2 + \sqrt{2} + \left[\frac{\sqrt{2}-2+2+\sqrt{2}}{(\sqrt{2})^2-(2)^2}\right]$

$= 2 + \sqrt{2} + \frac{2\sqrt{2}}{2-4}$

$= 2 + \sqrt{2} - \sqrt{2} = 2.$

42. Ans. (c)

Solution $\frac{1}{2} = 0.50000$ ...(i)

$\frac{1}{2.3} = 0.16667$ ...(ii) (divide (i) by 3)

$\frac{1}{2.3.4} = 0.04167$ ...(iii) (divide (ii) by 4)

$\frac{1}{2.3.4.5} = 0.00833$ ...(iv) (divide (iii) by 5)

Adding, we have 0.71667 or 0.717 up to three places.

43. Ans. (c)

Solution Putting x in place of ?

$\frac{x\div 12}{0.2\times 3.6} = 2$ or, $x \div 12 = 2 \times 0.2 \times 3.6$

$\Rightarrow x = 2 \times 0.2 \times 3.6 \times 12$ or, $x = 17.28$.

44. Ans. (a)

Solution Substituting x for ?, we get

$\sqrt{x\times 7} \times 18 = 84$

or, $\sqrt{x\times 7} = \frac{84}{18}$ or, $(\sqrt{x\times 7})^2 = \left(\frac{84}{18}\right)^2$

or, $x \times 7 = \frac{84\times 84}{18\times 18}$ or, $x = \frac{84\times 84}{18\times 18\times 7} = 3.11$.

45. Ans. (a)

Solution Sum $= \frac{7}{4}+\frac{7}{3}+\frac{41}{12}+\frac{26}{5}+\frac{13}{6}$

$= \frac{105+140+205+312+130}{60}$

$= \frac{892}{60} = 14\frac{13}{15}$

which is nearer to 15 than 14

Difference : $15 - 14\frac{13}{15} = \frac{2}{15}$.

46. Ans. (c)

Solution Taking the quotient 2, y and 7, we get $2y = 7$, which gives the quotient as 3

$\therefore$ $y = 3$. Substituting the value of y, we get

$2\frac{3}{x}\times 3\frac{1}{2} = 7\frac{3}{4}$

Now, $\frac{7\frac{3}{4}}{3\frac{1}{2}} = 2\frac{3}{x} \Rightarrow 2\frac{3}{14} = 2\frac{3}{x}$

$\therefore$ $x = 14$, $y = 3$.

47. Ans. (c)

Solution Let x be the fraction

$x \times x \div \left(\frac{1}{x}\right)^2 = 3\frac{13}{81} \Rightarrow x^4 = \frac{256}{81} = \left(\frac{4}{3}\right)^4$

$\therefore$ $x = \frac{4}{3}$.

48. Ans. (b)

Solution Substituting $x = 7$ and $y = 5$, we get

$7 * 5 = (7 + 2)^2(5 - 2) = (9)^2 \times 3 = 243.$

49. Ans. (c)

Solution Given that $m^n = 121 \Rightarrow m^n = 11^2$

Hence, $m = 11$, $n = 2$. Substituting these values

$(m - 1)^{n+1} = (11 - 1)^{2+1} = 10^3 = 1000.$

50. Ans. (d)

51. Ans. (d)

Solution $x \times \frac{17}{8} - x \times \frac{8}{17} = 225$ or, $\frac{225}{136}x = 225$

$\therefore$ $x = 136$.

52. Ans. (c)

Solution $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{9.10}$

$= \left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right) + \ldots + \left(\frac{1}{9}-\frac{1}{10}\right)$

$= 1 - \frac{1}{10} = \frac{9}{10}$.

53. Ans. (c)

Solution Putting x for ?

$\sqrt{1296} \times 2.25 = x^2$ or, $36 \times 2.25 = x^2$

or, $x = \sqrt{36 \times 2.25}$ or, $x = 6 \times 1.5$

$\therefore$ $x = 9$.

54. Ans. (b)

Solution $x \times x \div \frac{1}{x} = 18\frac{26}{27}$ or, $x^3 = \frac{512}{27}$

$\therefore$ $x^3 = \left(\frac{8}{3}\right)^3$ and so $x = \frac{8}{3} = 2\frac{2}{3}$.

55. Ans. (c)

Solution Given that $\frac{a}{a+b} = \frac{17}{23}$

i.e. if $a = 17$, then $a + b = 23$ or, $b = 6$

$a - b = 17 - 6 = 11$

$\therefore$ $\frac{a+b}{a-b} = \frac{23}{11}$.

56. Ans. (d)

57. Ans. (c)

Solution $\frac{3}{4}x - \frac{3}{14}x = 150$ or, $\frac{15}{28}x = 150$

$\therefore$ $x = \frac{150 \times 28}{15} = 280$.

58. Ans. (a)

Solution Given product

$= \frac{5}{3} \times \frac{7}{5} \times \frac{9}{7} \times \ldots \times \frac{1003}{1001} = \frac{1003}{3}$.

59. Ans. (c)

Solution $\sqrt{2^n} = 64$ or, $(2^n)^{1/2} = 2^6$

$\Rightarrow$ $\frac{n}{2} = 6 \Rightarrow n = 12$.

60. Ans. (b)

Solution $10^y = \sqrt{10^{2y}} = \sqrt{25} = 5$.

61. Ans. (a)

Solution Putting x for ? and solving

$11\frac{1}{3} \times 4\frac{8}{10} \div x = 22\frac{2}{3}$

or, $11\frac{1}{3} \times 4\frac{8}{10} = 22\frac{2}{3}x$

or, $x = \frac{11\frac{1}{3} \times 4\frac{8}{10}}{22\frac{2}{3}}$ or, $x = \frac{1}{2} \times 4\frac{8}{10}$

$\therefore$ $x = \frac{1}{2} \times \frac{48}{10} = \frac{24}{10} = 2.4$.

62. Ans. (d)

Solution $\frac{a^{1/2} + a^{-1/2}}{1-a} + \frac{1-a^{-1/2}}{1+\sqrt{a}}$

$= \frac{a^{1/2} + a^{-1/2}}{(1+a^{1/2})(1-a^{1/2})} + \frac{1-a^{-1/2}}{1+a^{1/2}}$

$= \frac{a^{1/2} + a^{-1/2} + (1-a^{-1/2})(1-a^{1/2})}{(1+a^{1/2})(1-a^{1/2})}$

$= \frac{a^{1/2} + a^{-1/2} + 1 - a^{-1/2} - a^{1/2} + 1}{1-a}$

$= \frac{2}{1-a}$.

63. Ans. (c)

Solution $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{\sqrt{a^2 + b^2 + 2ab}}{ab}$

$= \frac{\sqrt{45 + 2 \times 18}}{18} = \pm \frac{9}{18} = \pm \frac{1}{2}$.

64. Ans. (d)

Solution $\frac{a^2 + b^2}{c^2 + d^2} = \frac{ab}{cd}$ or, $\frac{a^2 + b^2}{c^2 + d^2} = \frac{2ab}{2cd}$

or, $\frac{a^2 + b^2 + 2ab}{a^2 + b^2 - 2ab} = \frac{c^2 + d^2 + 2cd}{c^2 + d^2 - 2cd}$

[by componendo and dividendo]

or, $\left(\frac{a+b}{a-b}\right)^2 = \left(\frac{c+d}{c-d}\right)^2$

$\therefore$ $\frac{a+b}{a-b} = \frac{c+d}{c-d}$.

65. Ans. (a)

Solution Putting x for ? and solving

$(1.06 + 0.04)^2 - x = 4 \times 1.06 \times 0.04$

Here, $1.06 = a$ and $0.04 = b$

$\therefore$ $(a + b)^2 - x = 4ab$

$\therefore$ $x = (a + b)^2 - 4ab = (a - b)^2 = (1.06 - 0.04)^2$

$= (1.02)^2 = 1.0404$.

66. Ans. (a)

Solution Let the total score be x runs, such that

$\frac{2}{9}x - \frac{2}{9} \times \left(x - \frac{2}{9}x\right) = 8$ or, $\frac{2}{9}x - \frac{2}{9} \times \frac{7}{9}x = 8$

or, $\frac{2}{9}x \times \frac{2}{9} = 8$ or, $x = 162$.

67. Ans. (a)

Solution $\left(\frac{1}{64}\right)^0 + (64)^{-1/2} + (-32)^{4/5}$

$= 1 + (8^2)^{-1/2} + (-1 \times 32)^{4/5}$

$= 1 + 8^{-1} + [(-1)^{4/5} \times (32)^{4/5}]$

$$= 1 + 8^{-1} + [((-1)^2)^{2/5} \times (2^5)^{4/5}]$$

$$= 1 + \frac{1}{8} + [1 \times 16] = 17\frac{1}{8}.$$

68. Ans. (b)

Solution $x = \dfrac{(64)^2 - 9\times121}{121\times64} \times \dfrac{8\times11}{(8)^2+3\times11}$

$$x = \frac{(64)^2 - 3\times3\times11\times11}{11\times11\times8\times8} \times \frac{8\times11}{64+33}$$

or, $x = \dfrac{(64+33)(64-33)}{88} \times \dfrac{1}{64+33}$

or, $x = \dfrac{31}{88}$.

69. Ans. (a)

Solution Let the number be 1

$\therefore$ $\frac{1}{3}$ of $1 = \frac{1}{3}$ and, $\frac{1}{4}$ of $1 = \frac{1}{4}$

$\therefore$ $\frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12}$

$\therefore$ Number $12 \div \frac{1}{12} = 144$.

70. Ans. (a)

Solution L.C.M. of 7, 8, 16 and 35 = 560

$\therefore$ $\frac{5}{8} = \frac{5\times70}{8\times70} = \frac{350}{560}$

$\frac{21}{35} = \frac{21\times16}{35\times16} = \frac{336}{560}$

$\frac{9}{16} = \frac{9\times35}{16\times35} = \frac{315}{560}$

and, $\frac{6}{7} = \frac{6\times80}{7\times80} = \frac{480}{560}$

$\therefore$ Difference between the largest and the smallest fractions

$$= \frac{6}{7} - \frac{9}{16} = \frac{480}{560} - \frac{315}{560} = \frac{165}{560} = \frac{32}{112}.$$

71. Ans. (b)

Solution Let the money with the man at first be Re 1

$\therefore$ Money spent $= \frac{5}{6}$ of $1 =$ Rs $\frac{5}{6}$

$\therefore$ Remaining money $= 1 - \frac{5}{6} =$ Rs $\frac{1}{6}$

and money earned $= \frac{1}{2}$ of Rs $\frac{1}{6} =$ Rs $\frac{1}{12}$

$\therefore$ Total money with him now

$$= \frac{1}{6} + \frac{1}{12} = \text{Rs } \frac{3}{12} = \text{Rs } \frac{1}{4}$$

$\therefore$ $\frac{1}{4}$th part of his money is with him now.

72. Ans. (d)

Solution Let the monthly income of Manmohan be Re 1

$\therefore$ Pocket money $= \frac{1}{5}$ of Re $1 =$ Rs $\frac{1}{5}$

and remainder $= 1 - \frac{1}{5} =$ Rs $\frac{4}{5}$

$\therefore$ Other expenses $= \frac{4}{5}$ of Rs $\frac{4}{5} =$ Rs $\frac{16}{25}$

$\therefore$ Saving $= \frac{4}{5} - \frac{16}{25} =$ Rs $\frac{4}{25}$

$\therefore$ Monthly income $= 48 \div \frac{4}{25} =$ Rs 300.

73. Ans. (c)

Solution Let the number be 1

$\therefore$ $\frac{4}{5}$ of $1 = \frac{4}{5}$ and, $\frac{3}{4}$ of $1 = \frac{3}{4}$

$\therefore$ Difference $= \frac{4}{5} - \frac{3}{4} = \frac{1}{20}$

$\therefore$ Number $= 4 \div \frac{1}{20} = 80$.

74. Ans. (c)

Solution $\because$ $\frac{2}{3}$ part $= 96$

$\therefore$ $\frac{3}{4}$ part $= 96 \times \frac{3}{2}\times\frac{3}{4} = 108$.

75. Ans. (b)

Solution Journey completed by aeroplane and train

$$= \frac{2}{15} + \frac{2}{5} = \frac{2+6}{15} = \frac{8}{15}$$

$\therefore$ Remaining journey $= 1 - \frac{8}{15} = \frac{7}{15}$

$\therefore$ He completed $\frac{7}{15}$ part of his journey by taxi.

76. Ans. (e)

Solution $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\left(1-\frac{1}{70}\right) = \frac{x}{70}$

$\therefore$ $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\ldots\frac{69}{70} = \frac{x}{70}$

$\therefore$ $\frac{1}{70} = \frac{x}{70}$

$\therefore$ $x = 1$.

77. Ans. (c)

Solution $\dfrac{1.073\times1.073 - 0.927\times0.927}{1.073-0.927} + \dfrac{(3^4)^4\times(9)^6}{(27)^7\times(3)^9}$

$$= \frac{(1.073)^2 - (0.927)^2}{1.073-0.927} + \frac{(3^4)^4\times(3^2)^6}{(3^3)^7\times(3)^9}$$

$$= \frac{(1.073+0.927)(1.073-0.927)}{1.073-0.927} + \frac{3^{28}}{3^{30}}$$

$$= 2 + \frac{1}{3^2} = 2 + \frac{1}{9} = 2\frac{1}{9}.$$

78. Ans. (c)

Solution $\dfrac{2^{1/2}\times 3^{1/3}\times 4^{1/4}}{10^{-1/5}\times 5^{3/5}}\div\dfrac{3^{4/3}\times 5^{-7/5}}{4^{-3/5}\times 6}$

$=\dfrac{2^{1/2}\times 3^{1/3}\times (2^2)^{1/4}\times 10^{1/5}}{5^{3/5}}\div\dfrac{3^{4/3}\times 4^{3/5}}{5^{7/5}\times 6}$

$=\dfrac{2^{1/2}\times 3^{1/3}\times 2^{1/2}\times 2^{1/5}\times 5^{1/5}}{5^{3/5}}\times\dfrac{5^{7/5}\times 2\times 3}{3^{4/3}\times 2^{6/5}}$

$=2^{\frac{1}{2}+\frac{1}{2}+\frac{1}{5}-\frac{6}{5}+1}\times 3^{\frac{1}{3}-\frac{4}{3}+1}\times 5^{\frac{1}{5}-\frac{3}{5}+\frac{7}{5}}$

$=2^1\times 3^0\times 5^1=2\times 5=10.$

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. $(36)^{1/6}$ is equal to
(a) 1 (b) 6
(c) 6 (d) $3\sqrt{6}$
[Based on SSC (GL) Prel. Exam 2000]

2. A common factor of $(13^7 + 11^7)$ and $(13^5 + 11^5)$ is
(a) 24 (b) $13^5 + 11^5$
(c) $13^2 + 11^2$ (d) None of the above
[Based on SSC (GL) Prel. Exam 2000]

3. $\left(\dfrac{8}{125}\right)^{-4/3}$ simplifies to
(a) $\dfrac{625}{16}$ (b) $\dfrac{625}{8}$
(c) $\dfrac{625}{32}$ (d) $\dfrac{16}{625}$
[Based on SSC (GL) Prel. Exam 2000]

4. $(6.5 \times 6.5 - 45.5 + 3.5 \times 3.5)$ is equal to
(a) 10 (b) 9
(c) 7 (d) 6
[Based on SSC (GL) Prel. Exam 2000]

5. $\left(\dfrac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\dfrac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}\right)$ simplifies to
(a) $\sqrt{5}+\sqrt{6}$ (b) $2\sqrt{5}+\sqrt{6}$
(b) $\sqrt{5}-\sqrt{6}$ (d) $2\sqrt{5}-3\sqrt{6}$
[Based on SSC (GL) Prel. Exam 2000]

6. If $x = 7 - 4\sqrt{3}$, then the value of $x+\dfrac{1}{x}$ is
(a) $3\sqrt{3}$ (b) $8\sqrt{3}$
(c) $14 + 8\sqrt{3}$ (d) 14
[Based on SSC (GL) Prel. Exam 2000]

7. $(5.5)^3 - (4.5)^3$ is equal to
(a) 1 (b) 75
(c) 74.25 (d) 75.25
[Based on SSC (GL) Prel. Exam 2000]

8. Which one of the numbers 33^{33}, 3^{333}, 333^3 and $3^{3^{3^3}}$ is the greatest?
(a) 33^{33} (b) 3^{333}
(c) 333^3 (d) $3^{3^{3^3}}$
[Based on SSC (GL) Prel. Exam 2000]

9. A number when divided by 6 leaves a remainder 3. When the square of the same number is divided by 6, then remainder is
(a) 0 (b) 1
(c) 2 (d) 3
[Based on SSC (GL) Prel. Exam 2000]

10. The numbers 1, 2, 3, 4, ..., 100 are multiplied together. The number of zeros at the end (on the right) of the product must be
(a) 30 (b) 130
(c) 200 (d) 249
[Based on SSC (GL) Prel. Exam 2000]

11. $7^{1/4} \times (343)^{0.25}$ is equal to
(a) $\sqrt{7}$ (b) 49
(c) 7 (d) $7\sqrt{7}$

12. One less then $(49)^{15}$ is exactly divisible by
(a) 50 (b) 51
(c) 14 (d) 8
[Based on SSC (GL) Prel. Exam 2000]

13. $(-2)^{(-2)^{(-2)}}$ is equal to
(a) 16 (b) 8
(c) –8 (d) –16
[Based on SSC (GL) Prel. Exam 2000]

14. If the sum and difference of two numbers are 20 and 8, respectively, then the difference of their squares is
(a) 12 (b) 28
(c) 180 (d) 160
[Based on SSC (GL) Prel. Exam 2000]

15. The value of
$\left(1+\dfrac{1}{x}\right)\left(1+\dfrac{1}{x+1}\right)\left(1+\dfrac{1}{x+2}\right)\left(1+\dfrac{1}{x+3}\right)$ is
(a) $1+\dfrac{1}{x+4}$ (b) $x + 4$
(c) $\dfrac{1}{x}$ (d) $\dfrac{x+4}{x}$
[Based on SSC (GL) Prel. Exam 2000]

16. When $\left(\dfrac{1}{2}-\dfrac{1}{4}+\dfrac{1}{5}-\dfrac{1}{6}\right)$ is divisible by $\dfrac{2}{5}-\dfrac{5}{9}+\dfrac{3}{5}-\dfrac{7}{18}$ the result is
(a) $5\dfrac{1}{10}$ (b) $2\dfrac{1}{18}$
(c) $3\dfrac{1}{6}$ (d) $3\dfrac{3}{10}$
[Based on SSC (GL) Prel. Exam 2000]

17. $\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)^2 + \left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)^2$ is equal to
(a) 64 (b) 62
(c) 66 (d) 68

[Based on SSC (GL) Prel. Exam 2000]

18. If 25^{25} is divisible by 26, the remainder is
(a) 1 (b) 2
(c) 24 (d) 25

[Based on SSC (GL) Prcl. Exam 2000]

19. $\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}$
(a) $6^{2/3}$ (b) 6
(c) $3^{1/3}$ (d) 3

[Based on SSC (GL) Prel. Exam 2000]

20. First 100 multiples of 10 are multiplied together. The number of zeros at the end of the product must be
(a) 125 (b) 120
(c) 111 (d) 110

[Based on SSC (GL) Prel. Exam 2000]

21. $(16)^{0.16} \times (16)^{0.04} \times (2)^{0.2}$ is equal to
(a) 1 (b) 2
(c) 4 (d) 16

[Based on SSC (GL) Prel. Exam 2000]

22. A number divided by 68 gives the quotient 269 and remainder zero. If the same number is divided by 67, the remainder is
(a) 0 (b) 1
(c) 2 (d) 3

[Based on SSC (GL) Prel. Exam 2000]

23. The product of two fractions is $\frac{14}{15}$ and their quotient is $\frac{35}{24}$. The greater fraction is
(a) $\frac{7}{4}$ (b) $\frac{7}{6}$
(c) $\frac{7}{3}$ (d) $\frac{4}{5}$

[Based on SSC (GL) Prel. Exam 2002]

24. Which of the following numbers is least?

$(0.5)^2$, $\sqrt{0.49}$, $\sqrt[3]{0.008}$, 0.23

(a) $(0.5)^2$ (b) $\sqrt{0.49}$
(c) $\sqrt[3]{0.008}$ (d) 0.23

[Based on SSC (GL) Prel. Exam 2002]

25. Find the value of * in the following:

$1\frac{2}{3} \div \frac{2}{7} \times \frac{*}{7} = 1\frac{1}{4} \times \frac{2}{3} \div \frac{1}{6}$

(a) $\frac{1}{6}$ (b) 0.6
(c) 0.006 (d) 6

[Based on SSC (GL) Prel. Exam 2002]

26. A certain amount of money is distributed among A, B and C. A gets $\frac{3}{16}$ and B gets $\frac{1}{4}$ of the whole amount. If C gets Rs. 81, then B gets
(a) Rs 30 (b) Rs 36
(c) Rs 32 (d) Rs 40

[Based on SSC (GL) Prel. Exam 2002]

27. The greatest number which when divides 989 and 1327 leave remainders 5 and 7, respectively, is
(a) 8 (b) 16
(c) 24 (d) 32

[Based on SSC (GL) Prel. Exam 2002]

28. If * means adding 6 times the second number to the first number then (1 * 2) * 3 equals
(a) 121 (b) 31
(c) 93 (d) 91

[Based on SSC (GL) Prel. Exam 2003]

29. Find the value of $\frac{2}{1+\frac{1}{1-\frac{1}{2}}} \times \frac{3}{\frac{5}{6}\text{of}\,\frac{3}{2} \div 1\frac{1}{4}}$
(a) 6 (b) 8
(c) 4 (d) 2

[Based on SSC (GL) Prel. Exam 2003]

30. If $x * y = x^2 + y^2 - xy$, the value of 9 * 11 is
(a) 93 (b) 103
(c) 113 (d) 121

[Based on SSC (GL) Prel. Exam 2003]

31. A man spends $\frac{1}{3}$ of his income on food, $\frac{2}{5}$ of his income on house rent and $\frac{1}{5}$ of his income on clothes. If he still has Rs 400 left with him, his income is
(a) Rs 4000 (b) Rs 5000
(c) Rs 6000 (d) Rs 7000

[Based on SSC (GL) Prel. Exam 2003]

32. If $a * b = 2a + 3b$, then the value If 2 * 3 + 3 * 4 is
(a) 24 (b) 31
(c) 32 (d) 34

[Based on SSC (GL) Prel. Exam 2002]

33. The simplified value of $\left[\sqrt[3]{\sqrt[6]{2^9}}\right]^4 \times \left[\sqrt[6]{\sqrt[3]{2^9}}\right]^4$ is
(a) 2^{16} (b) 2^{12}
(c) 2^8 (d) 2^4

[Based on SBI PO Exam 2000]

34. The value of the following is

$$\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+\sqrt{225}}}}}$$

(a) 10 (b) 8
(c) 6 (d) 4

[Based on Bank of Baroda PO 1999]

35. What will come in place of the question mark (?) in the following equation

$25^{7.5} \times 5^{2.5} \div 125^{1.5} = 5\ ?$

(a) 16 (b) 17.5
(c) 8.5 (d) 13
(e) None of these

[Based on PNB Management Trainee Exam 2003]

36. Two-fifths of one-third for three-sevenths of a number is 15. What is 40 per cent of that number?

(a) 136 (b) 140
(c) 72 (d) 84
(e) None of these

[Based on IBPS Jr. Executive Exam 2002]

37. By how much is two-fifths of 200 greater than three-fifths of 125?

(a) 15 (b) 3
(c) 5 (d) 30
(e) None of these **[Based on Canara Bank PO 2003]**

38. Which of the following has fractions in ascending order?

(a) $\frac{2}{3}, \frac{3}{5}, \frac{7}{9}, \frac{9}{11}, \frac{8}{9}$ (b) $\frac{3}{5}, \frac{2}{3}, \frac{7}{9}, \frac{9}{11}, \frac{8}{9}$
(c) $\frac{8}{9}, \frac{9}{11}, \frac{7}{9}, \frac{3}{5}, \frac{2}{3}$ (d) $\frac{3}{5}, \frac{2}{3}, \frac{9}{11}, \frac{7}{9}, \frac{8}{9}$
(e) $\frac{8}{9}, \frac{9}{11}, \frac{7}{9}, \frac{2}{3}, \frac{3}{5}$

[Based on NABARD Asst. Manager Exam 2002]

39. What should come in place of the question mark (?) in the following equation

$47^{7.5} \div 47^{3/2} \times 47^{-3} = \left(\sqrt{47}\right)$

(a) 3 (b) $2\frac{1}{2}$
(c) 6 (d) 3.5
(e) None of these **[Based on BSRB Patana PO 2001]**

40. What should come in the place of the question mark (?) in the following equation

$$\frac{(7\times ?)^2}{49} = \sqrt{81}\ ?$$

(a) 9 (b) 2
(c) 4 (d) None of these

[Based on BSRB Delhi PO 2000]

41. $\frac{(10008.99)^2}{10009.001} \times \sqrt{3589} \times 0.4987 = ?$

(a) 3000 (b) 300000
(c) 3000000 (d) 5000
(e) 9000000 **[Based on BSRB Bhopal PO 2000]**

42. In a set of 5 numbers the sum of two of these numbers is 6 more than the sum of the remaining three numbers, whereas the sum of these two numbers is two times one of those numbers. What definitely is one of those two numbers?

(a) 18 (b) 12
(c) 16 (d) Data inadequate
(e) None of these

[Based on BSRB Chennai PO 2000]

43. Multiply the difference between the two lowest numbers with the difference between the two highest numbers in the following sequences:

89 7 91 72 31 25 18 89 16 58 38 42 86

(a) 18 (b) 77
(c) 81 (d) 16
(e) None of these **[Based on NABARD 1999]**

44. $\frac{1}{5}$ of a number is equal to $\frac{5}{8}$ of the second number. If 35 is added to the first number it becomes 4 times of second number. What is the value of the second number?

(a) 125 (b) 70
(c) 40 (d) 25
(e) None of these **[Based on NABARD 1999]**

EXPLANATORY ANSWERS

1. Ans. (d)

Solution $(36)^{1/6} = (6^2)^{1/6} = 6^{1/3} = \sqrt[3]{6}$.

2. Ans. (a)

3. Ans. (a)

Solution $\left(\frac{8}{125}\right)^{-4/3} = \left(\frac{2}{5}\right)^{-4} = \left(\frac{5}{2}\right)^{4} = \frac{625}{16}$.

4. Ans. (b)

Solution Let $a = 6.5$, $b = 3.5$

$\therefore$ Given expression $= (a - b)^2 = (6.5 - 3.5)^2 = 9$.

5. Ans. (c)

Solution $\frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3})+(1-\sqrt{2})(\sqrt{5}+\sqrt{3})}{5-3}$

$= \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}+\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{2}$

$= \frac{2\left(\sqrt{5}-\sqrt{6}\right)}{2} = \sqrt{5}-\sqrt{6}$.

6. Ans. (d)

Solution $x = 7 - 4\sqrt{3}$

$$\frac{1}{x} = \frac{1}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} = 7 + 4\sqrt{3}$$

$$\therefore \quad x + \frac{1}{x} = 14.$$

7. Ans. (d)

Solution $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$
$= (a - b)[(a + b)^2 - ab]$
$= 1 \times [(10)^2 - 24.75] = 75.25.$

8. Ans. (d)

9. Ans. (d)

10. Ans. (d)

11. Ans. (c)

Solution $7^{1/4} \times (7^3)^{1/4} = 7.$

12. Ans. (d)

Solution $a^n - 1$ is always divisible by $a - 1$

$\therefore$ $49^{15} - 1$ is divisible by 48, i.e. 8.

13. Ans. (a)

14. Ans. (d)

Solution $x + y = 20$ and $x - y = 8$

$\Rightarrow x = 14, y = 6$

$\therefore x^2 - y^2 = 14^2 - 6^2 = 196 - 36 = 160.$

15. Ans. (d)

Solution $\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x+1}\right)\left(1+\frac{1}{x+2}\right)\left(1+\frac{1}{x+3}\right)$

$$= \left(\frac{x+1}{x}\right)\left(\frac{x+2}{x+1}\right)\left(\frac{x+3}{x+2}\right)\left(\frac{x+4}{x+3}\right)$$

$$= \frac{x+4}{x}.$$

16. Ans. (a)

Solution $\frac{1}{2} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} = \frac{30-15+12-10}{60} = \frac{17}{60}$

$\frac{2}{5} - \frac{5}{9} + \frac{3}{5} - \frac{7}{18} = \frac{36-50+54-35}{90} = \frac{5}{90}$

According to question $\frac{17/60}{5/90} = \frac{51}{10} = 5\frac{1}{10}.$

17. Ans. (b)

Solution $\left[\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right]^2 + \left[\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right]^2$

$$= \left[\frac{(\sqrt{5}+\sqrt{3})(\sqrt{3}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}\right]^2 + \left[\frac{(\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}\right]^2$$

$$= \left[\frac{5+3+2\sqrt{15}}{5-3}\right]^2 + \left[\frac{5+3-2\sqrt{15}}{5-3}\right]^2$$

$$= \left[4+\sqrt{15}\right]^2 + \left[4-\sqrt{15}\right]^2$$

$= 16 + 15 + 8\sqrt{15} + 16 + 15 - 8\sqrt{15}$

$= 16 + 15 + 16 + 15$ or, 62.

18. Ans. (d)

Solution $25^{25} = (26 - 1)^{25}$

$= 26^{25} + 25_{C_1} \times 26^{24} \times (-1)^1$

$+ 25_{C_2} \times 26^{23} \times (-1)^2 + \ldots + (-1)^{25}$

[using Binomial theorem]

Now, all the terms are divisible by 26 except the last term $(-1)^{25}$. So, the remainder is $26 - 1 = 25$.

19. Ans. (d)

Solution Let $x = \sqrt{6+\sqrt{6+\sqrt{6+\cdots}}}$

On squaring, we get

$x^2 = 6 + x$ or $x^2 - x - 6 = 0$

or, $(x - 3)(x + 2) = 0$ or, $x = 3, -2$

But –ve value cannot be accepted.

$\therefore$ $x = 3.$

20. Ans. (c)

21. Ans. (b)

Solution Given expression $= (2^4)^{0.16} \times (2^4)^{0.04} \times (2)^{0.2}$

$= 2^{0.64} \times 2^{0.16} \times 2^{0.2}$

$= 2^1 = 2.$

22. Ans. (b)

Solution Number $= 269 \times 68 = 18292$

$= 67 \times 273 + 1.$

23. Ans. (b)

Solution $\frac{14}{15} \times \frac{35}{24} = \frac{7}{6}.$

24. Ans. (c)

25. Ans. (d)

Solution $\frac{5}{3} \div \frac{2}{7} \times \frac{*}{7} = \frac{5}{4} \times \frac{2}{3} \div \frac{1}{6}$

or, $\frac{5}{3} \times \frac{7}{2} \times \frac{*}{7} = \frac{5}{4} \times \frac{2}{3} \times \frac{6}{1}$

or, $* = \frac{5}{4} \times \frac{2}{3} \times \frac{6}{1} \times \frac{3}{5} \times \frac{2}{7}$

Hence, $* = 6.$

26. Ans. (b)

Solution Suppose that certain amount is Rs x

Then, $\left(x - \frac{3}{16}x - \frac{x}{4}\right) = 81$

or, $16x - 3x - 4x = 81 \times 16$

or, $x = \frac{81 \times 16}{9} = 144$

Hence, B gets $= 144 \times \frac{1}{4} =$ Rs 36.

27. Ans. (c)

Solution Required greatest number is *HCF* of (989 – 5) and (1327 – 7)

Hence, *HCF* of 989 and 1320 is 24.

28. Ans. (b)

Solution $1 * 2 = 1 + 2 \times 6 = 1 + 12 = 13$

$(1 * 2) * 3 = 13 * 3 = 13 + 3 \times 6 = 31.$

29. Ans. (d)

Solution The given expression

$$= \frac{2}{1+\frac{2}{2-1}} \times \frac{3}{\frac{5}{4}\div\frac{5}{4}} = \frac{2}{3} \times 3 = 2.$$

30. Ans. (b)

Solution $x * y = x^2 + y^2 - xy$

$\therefore\ 9 * 11 = (9)^2 + (11)^2 - 9 \times 11$

$= 81 + 121 - 99 = 103.$

31. Ans. (c)

Solution Man has $1 - \left(\frac{1}{3}+\frac{2}{3}+\frac{1}{5}\right) = \frac{1}{15}$

$\therefore$ Man's income = 400 × 15 = Rs 6000.

32. Ans. (b)

Solution $2 * 3 + 3 * 4$

$= [2\ (2) + 3\ (3)] + [2\ (3) + 3\ (4)]$

$= [4 + 9] + [6 + 12] = 31.$

33. Ans. (d)

Solution Given expression

$$= \left[\left\{(2^9)^{1/6}\right\}^{1/3}\right]^4 \times \left[\left\{(2^9)^{1/3}\right\}^{1/6}\right]^4$$

$$= (2^{1/2})^4 \times (2^{1/2})^4 = 2^2 \times 2^2 = 2^4.$$

34. Ans. (d)

Solution Given expression

$$=\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+15}}}} = \sqrt{10+\sqrt{25+\sqrt{108+13}}}$$

$$= \sqrt{10+\sqrt{25+11}} = \sqrt{10+6} = \sqrt{16} = 4.$$

35. Ans. (d)

Solution $25^{7.5} \times 5^{2.5} \div 125^{1.5} = 5^?$

or, $5^{2\times7.5} \times 5^{2.5} \div 5^{3\times1.5} = 5^?$

or, $5^{15} \times 5^{2.5} \div 5^{4.5} = 5^?$

or, $5^{17.5} \times \frac{1}{5^{4.5}} = 5^?$ or, $5^{13} = 5^?$

or, $? = 13.$

36. Ans. (e)

Solution Let the number be x

Then, $\frac{2}{5}\times\frac{1}{3}\times\frac{3}{7}\times x = 15$

or, $\frac{2x}{35} = 15$ or, $x = \frac{15\times35}{2}$

$\therefore$ 40 % of $x = \frac{40}{100}\times\frac{15\times35}{2} = 105.$

37. Ans. (c)

Solution Required number

$$= \frac{2}{5} \times 200 - \frac{3}{5} \times 125$$

$$= 80 - 75 = 5.$$

38. Ans. (b)

Solution $\frac{2}{3} = \frac{2\times165}{3\times165} = \frac{330}{495}$

$\frac{3}{5} = \frac{3\times99}{5\times99} = \frac{297}{495}$

$\frac{7}{9} = \frac{7\times55}{9\times55} = \frac{385}{495}$

$\frac{9}{11} = \frac{9\times45}{11\times45} = \frac{405}{495}$

$\frac{8}{9} = \frac{8\times55}{9\times55} = \frac{440}{495}$

Ascending order $\frac{3}{5},\frac{2}{3},\frac{7}{9},\frac{9}{11},\frac{8}{9}.$

39. Ans. (c)

Solution $((47)^{1/2})^{15} \div ((47)^{1/2})^3 \times ((47)^{1/2})^{-6}$

$= \left(\sqrt{47}\right)^? = \left(\sqrt{47}\right)^{15} \div \left(\sqrt{47}\right)^3 \times \left(\sqrt{47}\right)^{-6} = \left(\sqrt{47}\right)^?$

$\Rightarrow \left(\sqrt{47}\right)^{15-3-6} = \left(\sqrt{47}\right)^?$

$\therefore\ ? = 6.$

40. Ans. (c)

41. Ans. (b)

Solution $? = \frac{(10008.99)^2}{10009.001} \times \sqrt{3589} \times 0.4987$

$\approx (10009) \times \sqrt{3600} \times 0.50$

$\approx (10009) \times 60 \times 0.50 \approx 300000.$

42. Ans. (d)

43. Ans. (a)

Solution Difference between two lowest numbers

= 16 – 7 = 9

Difference between two highest numbers

= 91 – 89 = 2

$\therefore$ Product of these two numbers = 9× 2 = 18.

44. Ans. (c)

Solution $\frac{1}{5}\ I = \frac{5}{8}\ II \quad \therefore\ \frac{I}{II} = \frac{25}{8}$ ..(i)

$I + 35 = 4\ II$ or, $\frac{25}{8}\ II + 35 = 4\ II$

$\therefore\ II = 40.$

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. $\frac{31}{10} \times \frac{3}{10} + \frac{7}{5} \div 20 = ?$
(a) 0 (b) 1
(c) 100 (d) $\frac{107}{200}$

2. The value of $1 + \frac{1}{1.2} + \frac{1}{1.2.4} + \frac{1}{1.2.4.8}$ up to 2 places of decimal is
(a) 1.64 (b) 1.65
(c) 1.79 (d) 1.78

3. $\dfrac{\frac{1}{5} \div \frac{1}{5} \text{ of } \frac{1}{5}}{\frac{1}{5} \text{ of } \frac{1}{5} \div \frac{1}{5}} = ?$
(a) 1 (b) 5
(c) $\frac{1}{5}$ (d) 25

4. In a certain office, $\left(\frac{1}{3}\right)$ of the workers are women, $\left(\frac{1}{2}\right)$ of the women are married and $\left(\frac{1}{3}\right)$ of the married women have children. If $\left(\frac{3}{4}\right)$ of the men are married and $\frac{2}{3}$ of the married men have children, what part of the workers are without children?
(a) $\frac{5}{18}$ (c) $\frac{4}{9}$
(c) $\frac{11}{18}$ (d) $\frac{17}{36}$

5. The value of $48 \div 12 \times \left(\frac{9}{8} \text{ of } \frac{4}{3} \div \frac{3}{4} \text{ of } \frac{2}{3}\right)$ is
(a) $1\frac{1}{3}$ (b) $5\frac{1}{3}$
(c) 3 (d) 12

6. In a bank there are equal number of male and female workers. $\left(\frac{1}{2}\right)$ of the female are married and $\left(\frac{1}{3}\right)$ of the married women have children, $\left(\frac{3}{4}\right)$ of the males are married and $\left(\frac{2}{3}\right)$ of the married men have children, what part of workers are without children?
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) $\frac{1}{12}$

7. If we multiply a fraction by itself and divide the product by its reciprocal, the fraction thus obtained is $18\frac{26}{27}$. The original fraction is
(a) $\frac{16}{27}$ (b) $\frac{8}{3}$
(c) $\frac{8}{27}$ (d) $\frac{4}{3}$

8. $\left(\frac{?}{18}\right) \times \left(\frac{?}{162}\right) = 1$
(a) 64 (b) 34
(c) 54 (d) 94

9. Ravi earns twice as much in January as in each of the other months. What part of his annual earnings he earns in that month?
(a) $\frac{2}{13}$ (c) $\frac{1}{10}$
(c) $\frac{5}{7}$ (d) $\frac{5}{7}$

10. $\dfrac{(0.55)^2 + (0.07)^2 + (0.027)^2}{(0.055)^2 + (0.007)^2 + (.0027)^2} = ?$
(a) 10 (b) 100
(c) 50 (d) 1000

Test II **[Time: 10 min]**

11. The value of $\dfrac{1}{3 + \dfrac{2}{2 + \frac{1}{2}}}$ is
(a) $\frac{5}{19}$ (b) $\frac{19}{5}$
(c) $\frac{4}{5}$ (d) $\frac{5}{4}$

12. $7\frac{1}{2} - \left[2\frac{1}{4} \div \left\{1\frac{1}{4} - \frac{1}{2}\left(1\frac{1}{2} - \frac{1}{3} - \frac{1}{6}\right)\right\}\right]$ is equal to
(a) $\frac{2}{9}$ (b) 1
(c) $4\frac{1}{2}$ (d) $1\frac{77}{288}$

13. $\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right)\text{----}\left(1 - \frac{1}{1001}\right) = ?$

(a) $\frac{1}{1001}$ (b) $\frac{1000}{1001}$

(c) $\frac{2}{1001}$ (d) None of these

14. A number divided by 68 gives the quotient 269 and the remainder zero. If the same number is divided by 67, the remainder is

(a) 3 (b) 2

(c) 1 (d) 0

15. $(4^3)^4 \div (4^2)^3 \times (4^5)^0 = ?$

(a) 6^4 (b) 4^6

(c) 2^6 (d) 6^2

16. $\sqrt{\frac{67.6}{?}} = 0.26$

(a) 10 (b) 10^2

(c) 10^3 (d) 50^2

17. In a family, the father took $\frac{1}{4}$ of the cake and he had 3 times as much as others had. The total number of family members is

(a) 3 (b) 7

(c) 10 (d) 12

18. If $\sqrt{18 \times 14 \times x} = 84$, x is equal to?

(a) 82 (b) 28

(c) 32 (d) 42

19. The smallest fraction which should be subtracted from the sum of $1\frac{3}{4}$, $2\frac{1}{2}$, $5\frac{7}{12}$, $3\frac{1}{3}$ and $2\frac{1}{4}$ to make the result a whole number is

(a) $\frac{5}{12}$ (b) $\frac{7}{12}$

(c) $\frac{1}{2}$ (d) 7

20. The value of $1 + \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{9}}}$ is

(a) $\frac{29}{19}$ (b) $\frac{10}{19}$

(c) $\frac{29}{10}$ (d) $\frac{10}{9}$

Answers

1. (b)	**2.** (a)	**3.** (d)	**4.** (c)	**5.** (d)	**6.** (a)	**7.** (b)	**8.** (c)	**9.** (a)	**10.** (b)
11. (a)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (c)	**17.** (c)	**18.** (b)	**19.** (a)	**20.** (a)

4

SQUARE ROOT AND CUBE ROOT

SQUARE

A number multiplied by itself is known as the *square* of the given number. For example, square of 6 is 6 × 6 = 36.

Square Root

Square root of a given number is that number which when multiplied by itself is equal to the given number.

For example, square root of 81 is 9 because $9^2 = 9 \times 9 = 81$.

The square root of a number is denoted by the symbol √ or $\sqrt{\ }$, called the *radical sign*.

Thus, $\sqrt{81} = 9$, $\sqrt{64} = 8$ and so on.

Note that $\sqrt{1} = 1$.

Methods of Finding the Square Root

I. Prime Factorisation Method

1. Find the prime factors of the given number.
2. Group the factors in pairs.
3. Take one number from each pair of factors and then multiply them together.

   This product is the square root of the given number.

Illustration 1 Find the square root of:

(i) 4761 (ii) 207025

Solution (i) $4761 = \underbrace{23 \times 23} \times \underbrace{3 \times 3}$

$\therefore \sqrt{4761} = 23 \times 3 = 69.$

(ii) $207025 = \underbrace{5 \times 5} \times \underbrace{7 \times 7} \times \underbrace{13 \times 13}$

$\therefore \sqrt{207025} = 5 \times 7 \times 13 = 455.$

Note: The above method is used when the given number is a perfect square or when every prime factor of that number is repeated twice.

II. Method of Division

This method is used when the number is large and the factors cannot be easily determined.

The working rule is explained with the help of following example:

Step 1: The digits of a number, whose square root is required, are separated into periods of two beginning from the right. The last period may be either single digit or a pair.

```
          476
      ------------
   8 | 22   65   76
  87 | 16
     |  665
     |  609
 946 |   5676
     |   5676
     |    ×
```

Step 2: Find a number (here, 4) whose square may be equal to or less than the first period (here, 22).

Step 3: Find out the remainder (here, 6) and bring down the next period (here, 65).

Step 4: Double the quotient (here, 4) and write to the left (here, 8).

Step 5: The divisor of this stage will be equal to the above sum (here, 8) with the quotient of this stage (here, 7) suffixed to it (here, 87).

Step 6: Repeat this process (step 4 and step 5) till all the periods get exhausted.

The quotient (here, 476) is equal to the square root of the given number (here, 226576).

Illustration 2 Find the square root of:

(i) 180625 (ii) 1498176

Solution (i)

```
          425
      ------------
   8 | 18   06   25
  82 | 16
     |  206
     |  164
 845 |   4225
     |   4225
     |    ×
```

Thus, $\sqrt{180625} = 425.$

(ii)

```
        1224
1   | 1   49   81   76
    | 1
22  |     49
    |     44
242 |      581
    |      484
2444|       9776
    |       9776
    |          ×
```

Thus, $\sqrt{1478176} = 1224$.

Square Root of a Decimal

If the given number is having decimal, we separate the digits of that number into periods of two to the right and left starting from the decimal point and then proceed as in the following illustration:

Illustration 3 Find the square root of :

(i) 12.1801 (ii) 127.0129

(iii) 0.1790136 (iv) 0.000625.

Solution (i)

```
        3.49
3   | 12.  18   01
    | 16
64  |  665
    |  609
689 |   6201
    |   6201
    |      ×
```

$\therefore \sqrt{12.1801} = 3.49.$

(ii)

```
        11.27
1   | 1   27.   01   29
    | 1   .
21  |     27
    |     21
222 |      601
    |      444
2447|       15729
    |       15729
    |          ×
```

$\therefore \sqrt{127.0129} = 11.27$

(iii) Since the number of decimal places is odd, we make it even by affixing one 0 to the right.

```
        0.423
4   | 0.   17   90   13   60
    |      16
82  |       190
    |       164
843 |        2613
    |        2529
846 |          8460
```

In the above solution, after bringing down the last period, we note that the remainder is not zero. So, a pair of zeros can be annexed and process can be continued to find the square root up to 4 places of decimals. The above process can be continued still further and square root up to required number of decimal places can be obtained.

Note: If a decimal has an odd number of decimal places, its square root cannot be found exactly.

(iv)

```
        .025
    | 0.   00   06   25
    |      00
02  |       06
    |       04
45  |        225
    |        225
    |          ×
```

$\therefore \sqrt{0.000625} = 0.025.$

Square Root of a Fraction

(a) *If the denominator is a perfect square*:

The square root is found by taking the square root of the numerator and denominator separately.

(b) *If the denominator is not a perfect square*:

The fraction is converted into decimal and then square root is obtained or the denominator is made perfect square by multiplying and dividing by a suitable number and then its square root is obtained.

Illustration 4 Find the square root of:

(i) $\frac{2704}{49}$ (ii) $\frac{44}{25}$

(iii) $\frac{354}{43}$ (iv) $\frac{461}{32}$

Solution (i) $\sqrt{\frac{2704}{49}} = \frac{\sqrt{2704}}{\sqrt{49}} = \frac{\sqrt{52 \times 52}}{\sqrt{7 \times 7}} = \frac{52}{7}$

$= 7\frac{3}{7}.$

(ii) $\sqrt{\frac{44}{25}} = \frac{\sqrt{44}}{\sqrt{25}} = \frac{\sqrt{44}}{\sqrt{5 \times 5}} = \frac{\sqrt{44}}{5} = \frac{6.6332}{5}$

$= 1.3266$ (nearly).

(iii) $\sqrt{\frac{354}{43}} = \sqrt{8.2325} = 2.8692$ (nearly)

(iv) $\sqrt{\frac{461}{32}} = \sqrt{\frac{461 \times 2}{32 \times 2}} = \frac{\sqrt{922}}{\sqrt{64}} = \frac{30.3644}{8}$

$= 3.7955$ (nearly).

Cube

Cube of a number is obtained by multiplying the number itself thrice.

For example, 27 is the cube of 3 as $27 = 3 \times 3 \times 3$.

Cube Root

The *cube root* of a given number is that number which when raised to the third power produces the given number, that is the cube root of a number x is the number whose cube is x.

The cube root of x is written as $\sqrt[3]{x}$.

For example, cube root of 64 is 4 as $4 \times 4 \times 4 = 64$.

Methods to find Cube Root

I. Method of Factorisation

1. Write the given number as product of prime factors.
2. Take the product of prime numbers, choosing one out of three of each type.

   This product gives the cube root of the given number.

Illustration 5 Find the cube root of 42875.

Solution Resolving 42875 into prime factors, we get

$$42875 = \underbrace{5\times5} \times \underbrace{5\times7} \times \underbrace{7\times7}$$

$$\therefore \quad \sqrt[3]{42875} = 5 \times 7 = 35.$$

II. Short-cut Method to Find Cube Roots of Exact Cubes Consisting of up to 6 Digits:

Before we discuss the method to find the cube roots of exact cubes, the following two remarks are very useful and must be remembered by heart.

1. $1^3 = 1$; $2^3 = 9$; $3^3 = 27$; $4^3 = 64$; $5^3 = 125$; $6^3 = 216$; $7^3 = 343$; $8^3 = 512$; $9^3 = 729$; $10^3 = 1000$.
2. If the cube ends in 1, then its cube root ends in 1

If the cube ends in 2, then its cube root ends in 8
If the cube ends in 3, then its cube root ends in 7
If the cube ends in 4, then its cube root ends in 4
If the cube ends in 5, then its cube root ends in 5
If the cube ends in 6, then its cube root ends in 6
If the cube ends in 7, then its cube root ends in 3
If the cube ends in 8, then its cube root ends in 2
If the cube ends in 9, then it cube root ends in 9
If the cube ends in 0, then its cube root ends in 0

Clearly, from above

$1 \leftrightarrow 1$, $4 \leftrightarrow 4$, $5 \leftrightarrow 5$, $6 \leftrightarrow 6$, $9 \leftrightarrow 9$, $0 \leftrightarrow 0$
$2 \leftrightarrow 8$, $3 \leftrightarrow 7$.

The method of finding the cube root of a number up to 6 digits which is actually a cube of some number consisting of 2 digits is best illustrated with the help of following examples:

Illustration 6 Find the cube roots of the following:

(i) 2744 (ii) 9261
(iii) 19684 (iv) 54872
(v) 614125

Solution (i) Make groups of 3 digits from the right side. $\underline{2}\ \underline{744}$

2 lies between 1^3 and 2^3, so left digit is 1.
744 ends in 4, so right digit is 4.
Thus, cube root of 2744 is 14.

(ii) $\underline{9}\ \underline{261}$

9 lies between 2^3 and 3^3, so left digit is 2.
261 ends in 1, so right digit is 1.
Thus, cube root of 9261 is 21.

(ii) $\underline{19}\ \underline{683}$

19 lies between 2^3 and 3^3, so left digit is 2.
683 ends in 3, so right digit is 7.
Thus, cube root of 19683 is 27.

(iv) $\underline{54}\ \underline{872}$

54 lies between 3^3 and 4^3, so left digit is 3.
872 ends in 2, so right digit is 8.
Thus, cube root of 19683 is 38.

(iv) $\underline{614}\ \underline{125}$

614 lies between 8^3 and 9^3, so left digit is 8.
125 ends in 5, so right digit is 5.
Thus, cube root of 614125 is 85.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Find the square root of 4356
(a) 68 (b) 64
(c) 66 (d) None of these

2. Find the square root of 104976
(a) 324 (b) 424
(c) 326 (d) None of these

3. Find the square root of 211600
(a) 460 (b) 440
(c) 480 (d) None of these

4. Find the value of $\sqrt{6492304}$
(a) 2384 (b) 2484
(c) 2548 (d) 2684

5. Find the least number which when multiplied with 74088 will make it a perfect square.
(a) 42 (b) 44
(c) 46 (d) 48

6. $\sqrt{10} \times \sqrt{250} = ?$
(a) 46.95 (b) 43.75
(c) 50.25 (d) 50.00

7. $\sqrt{80} + 3\sqrt{245} - \sqrt{125} = ?$
(a) $20\sqrt{5}$ (b) $25\sqrt{2}$
(c) $15\sqrt{2}$ (d) None of these

8. $\frac{250}{\sqrt{?}} = 10$
(a) 25 (b) 250
(c) 625 (d) 2500

9. If $\sqrt{256} \div \sqrt{x} = 2$, then x is equal to
(a) 64 (b) 128
(c) 512 (d) 1024

10. Find the smallest number by which 216 should be divided to make the result a perfect square.
(a) 4 (b) 3
(c) 6 (d) 2

11. $\frac{\sqrt{?}}{200} = 0.02$
(a) 0.4 (b) 4
(c) 16 (d) 1.6

12. $\frac{\sqrt{6727}}{\sqrt{7}} = ?$
(a) 30.79 (b) 32.29
(c) 31 (d) None of these

13. $\sqrt{.09} = ?$
(a) 0.3 (b) 0.03
(c) 0.003 (d) None of these

14. $\frac{14}{3+\sqrt{2}} = ?$
(a) 3.172 (b) 4.586
(c) 8.828 (d) None of these

15. Find the smallest number which when added to 3579 gives a perfect square.
(a) 27 (b) 24
(c) 21 (d) 18

16. If $\sqrt{\left(1+\frac{27}{169}\right)} = 1 + \frac{x}{13}$, then x equals
(a) 1 (b) 3
(c) 5 (d) 7

17. $\frac{\sqrt{4375}}{\sqrt{7}} = ?$
(a) 24.75 (b) 27.75
(c) 25 (d) 35

18. If $\sqrt{0.04 \times 0.4 \times a} = 0.4 \times 0.04 \times \sqrt{b}$, then the value of $\frac{a}{b}$ is.
(a) 0.016 (b) 1.60
(c) 0.16 (d) None of these

19. $\sqrt{\sqrt[3]{\sqrt[3]{\sqrt[3]{\sqrt[3]{\sqrt[3]{3}}}}}} = ?$
(a) $3^{31/64}$ (b) $3^{31/32}$
(c) $3^{1/64}$ (d) None of these

20. $\frac{\sqrt{1296}}{?} = \frac{?}{2.25}$
(a) 6 (b) 7
(c) 8 (d) 9

21. $\sqrt{176 + \sqrt{2401}} = ?$
(a) 14 (b) 15
(c) 18 (d) 24

22. $\sqrt{10} \times \sqrt{15} = ?$
(a) $5\sqrt{6}$ (b) $6\sqrt{5}$
(c) 5 (d) $\sqrt{30}$

23. $\sqrt{\frac{4}{3}} - \sqrt{\frac{3}{4}} = ?$
(a) $\frac{1}{2\sqrt{3}}$ (b) $-\frac{1}{2\sqrt{3}}$
(c) 1 (d) $\frac{5\sqrt{3}}{6}$

24. $\sqrt{248 + \sqrt{52 + \sqrt{144}}} = 1$
(a) 14 (b) 16
(c) 16.6 (d) 18.8

25. $\sqrt{0.0009} \div \sqrt{0.01} = ?$
(a) 3 (b) 0.3
(c) $\frac{1}{3}$ (d) None of these

26. $\frac{1}{\sqrt{9} - \sqrt{8}} = ?$
(a) $\frac{1}{2}(3 - 2\sqrt{2})$ (b) $\frac{1}{3+2\sqrt{2}}$
(c) $3 - 2\sqrt{2}$ (d) $3 + 2\sqrt{2}$

27. If $\sqrt{\frac{x}{169}} = \frac{54}{39}$, then x is equal to:
(a) 108 (b) 324
(c) 2916 (d) 4800

28. $\sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots}}} = ?$
(a) 3 (b) 4
(c) 6 (d) greater than 6

29. $\frac{112}{\sqrt{196}} \times \frac{\sqrt{576}}{12} \times \frac{\sqrt{256}}{8} = ?$

(a) 8 (b) 12
(c) 16 (d) 32

30. If $\sqrt{12}$ = 3.464, value of $\sqrt{\frac{3}{4}} + 2\sqrt{\frac{4}{3}}$ is
(a) 3.17 (b) 3.464
(c) 3.1753 (d) None of these

31. If $\sqrt{15625}$ = 125, then the value of
$\sqrt{15625} + \sqrt{156.25} + \sqrt{1.5625}$ is
(a) 1.3875 (b) 13.875
(c) 138.75 (d) 156.25

32. If $\sqrt{0.03\times0.3\times a} = 0.03 \times 0.3 \times \sqrt{b}$, value of $\frac{a}{b}$ is
(a) 0.009 (b) 0.03
(c) 0.09 (d) None of these

33. Given that $\sqrt{4096}$ = 64, the value of $\sqrt{4096} + \sqrt{40.96} + \sqrt{.004096}$ is
(a) 70.4 (b) 70.464
(c) 71.104 (d) 71.4

34. If $\sqrt{1+\sqrt{1-\frac{2176}{2401}}} = 1+\frac{x}{7}$, value of x is
(a) 3 (b) 1
(c) 5 (d) 7

35. Which of the following numbers, wherein some of the digits have been suppressed by symbols, can possibly be the perfect square of a 3 digit odd number?
(a) 65 ××× 1 (b) 9 ×× 1
(c) 10 ××× 4 (d) 9 ×××××× 5

36. $\sqrt{\frac{0.324\times0.081\times4.624}{1.5625\times0.0289\times72.9\times64}}$ = ?
(a) 24 (b) 2.40
(c) 0.024 (d) None of these

37. Find the cube root of $\frac{512}{3375}$
(a) 12/15 (b) 16/25
(c) 8/15 (d) None of these

38. $\sqrt{0.01+\sqrt{0.0064}}$ = ?
(a) 0.3 (b) 0.03
(c) $\sqrt{0.18}$ (d) None of these

39. Find the cube root of 15.625
(a) 3.5 (b) 2.5
(c) 4.5 (d) 5.5

40. What is the value of $\sqrt[3]{.000064}$?
(a) 0.4 (b) 0.08
(c) 0.04 (d) 0.16

41. What is the value of $\sqrt[3]{\sqrt{441}+\sqrt{16}+\sqrt{4}}$
(a) 3 (b) 5
(c) 7 (d) 9

42. The least number by which 14175 be divided to make it a perfect square is
(a) 3 (b) 5
(c) 7 (d) 15

43. $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ is equal to
(a) $4 + \sqrt{15}$ (b) $4 - \sqrt{15}$
(c) $\frac{1}{2}$ (d) 1

44. $\frac{\sqrt{24}+\sqrt{216}}{\sqrt{96}}$ = ?
(a) $2\sqrt{6}$ (b) $6\sqrt{2}$
(c) 2 (d) $\frac{2}{\sqrt{6}}$

45. Given that $\sqrt{20}$ = 4. 472, find the square root of $2\frac{2}{9}$ up to two places of decimals
(a) 1.56 (b) 1.69
(c) 1.49 (d) None of these

46. If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, then the value of $\frac{a^2+ab+b^2}{a^2-ab+b^2}$ is
(a) $\frac{3}{4}$ (b) $\frac{4}{3}$
(c) $\frac{3}{5}$ (d) $\frac{5}{3}$

47. The least number by which 10584 be multiplied to make it a perfect square is:
(a) 2 (b) 3
(c) 6 (d) 8

48. The smallest number which is a perfect square and contains 7936 as a factor is
(a) 12008 (b) 246016
(c) 61504 (d) 240616

49. $\sqrt{0.00059049}$ = ?
(a) 0.243 (b) 0.0243
(c) 0.00243 (d) 0.000243

50. Given that $\sqrt{10}$ = 3.16, what is the value of $\sqrt{\frac{4}{12.1}}$ to one place of decimal?
(a) 0.16 (b) 0.06
(c) 0.6 (d) 0.016

51. $\sqrt{\frac{0.256\times0.081\times4.356}{1.5625\times0.0121\times129.6\times64}}$ = ?
(a) 0.0124 (b) 0.124
(c) 0.0024 (d) 0.024

52. A general wishing to draw up his 16160 men in the form of a solid square found that he had 31 men over. The number of men in the front row is
(a) 127 (b) 123
(c) 137 (d) 129

53. The areas of two square fields are 420.25 sq. m. and 441 sq. m., respectively. The ratio of their sides is
(a) 20 : 21 (b) 40 : 41
(c) 41 : 42 (d) 40 : 42

54. A general wishing to draw up his 5180 men in the form of a solid square found that he had 4 men less. If he could get four more men and form the solid square, the number of men in the front row is
(a) 68 (b) 72
(c) 78 (d) 82

55. The largest number of three digits which is a perfect square is
(a) 900 (b) 841
(c) 961 (c) 784

56. What least number should be subtracted from the square root of $21\frac{15}{289}$ so that the result is a whole number?
(a) 15/289 (b) 7/17
(c) 10/17 (d) 5/17

57. The smallest number which when subtracted from the number 62512 makes it a perfect square is
(a) 22 (b) 32
(c) 12 (d) 2

58. The largest number of five digits which is a perfect square is
(a) 97344 (b) 98596
(c) 99856 (d) None of these

59. By what least number, 2450 be multiplied, so that the resulting number is perfect square ?
(a) 8 (b) 10
(c) 5 (d) 2

60. The smallest number by which 3600 must be multiplied to make it a perfect cube is
(a) 40 (b) 60
(c) 20 (d) 15

EXPLANATORY ANSWERS

1. Ans. (c)

Solution The prime factors of 4356 are

$2 \times 2 \times 3 \times 3 \times 11 \times 11$

$4356 = 2 \times 2 \times 3 \times 3 \times 11 \times 11$

2	4356
2	2178
3	1089
3	363
11	121
11	11
	1

$\sqrt{4356} = \sqrt{2^2 \times 3^2 \times 11^2}$

$= 2 \times 3 \times 11 = 66.$

2. Ans. (a)

Solution

	324		
3	$\overline{10}$	$\overline{49}$	$\overline{76}$
	9		
62	149		
	124		
644	2576		
	2576		
	×		

∴ Square root of 104976 is 324.

3. Ans. (a)

Solution

	460		
4	$\overline{21}$	$\overline{16}$	$\overline{00}$
	16		
86	516		
	516		
	×		

∴ Square root of 211600 is 460.

4. Ans. (c)

Solution

	2548			
2	6	$\overline{49}$	$\overline{23}$	$\overline{04}$
	4			
45	249			
	225			
644	2423			
	2016			
5088	40704			
	40704			
	×			

∴ $\sqrt{6492304} = 2548.$

5. Ans. (a)

Solution $74088 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$

$= (2 \times 2) \times (3 \times 3) \times (7 \times 7) \times (2 \times 3 \times 7)$

Therefore, required number

$= 2 \times 3 \times 7 = 42.$

2	74088
2	37044
2	18522
3	6261
3	3087
3	1029
7	343
7	49
7	7
	1

6. Ans. (d)

Solution $\sqrt{10} \times \sqrt{250} = \sqrt{2500} = 50.$

7. Ans. (a)

Solution $\sqrt{80} + 3\sqrt{245} - \sqrt{125} = 4\sqrt{5} + 21\sqrt{5} - 5\sqrt{5}$

$= 20\sqrt{5}.$

8. Ans. (c)

Solution Let $\frac{250}{\sqrt{x}} = 10$. Then, $\sqrt{x} = \frac{250}{10} = 25$

$\therefore \quad x = (25)^2 = 625.$

9. Ans. (a)

Solution $\frac{\sqrt{256}}{\sqrt{x}} = 2$ or, $\frac{16}{\sqrt{x}} = 2$

$\therefore \quad 16 = 2\sqrt{x} \Rightarrow \sqrt{x} = 8$ or, $x = 64.$

10. Ans. (c)

Solution We know that

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^2 \times 6$

Thus, $\frac{216}{6} = 2^2 \times 3^2 = 6^2$

Therefore, 216 should be divided by 6, so that the result is a perfect square.

11. Ans. (c)

Solution Let $\frac{\sqrt{x}}{200} = 0.02$. Then,

$\sqrt{x} = 200 \times 0.02$ or $\sqrt{x} = 4$

So, $x = 16.$

12. Ans. (c)

Solution $\frac{\sqrt{6727}}{\sqrt{7}} = \frac{\sqrt{6727}}{\sqrt{7}} = \sqrt{961} = 31.$

13. Ans. (a)

Solution $\sqrt{0.09} = \sqrt{\frac{9}{100}} = \frac{3}{10} = 0.3.$

14. Ans. (a)

Solution $\frac{14}{3+\sqrt{2}} = \frac{14(3-\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{14(3-\sqrt{2})}{9-2}$

$= 2(3 - \sqrt{2}) = 2(3 - 1.414)$

$= 2 \times 1.586 = 3.172.$

15. Ans. (c)

Solution The number nearest to 3579 which is a perfect square is 3600.

$\therefore$ Required number $= 60^2 - 3579 = 21.$

16. Ans. (a)

Solution $\sqrt{\left(1+\frac{27}{169}\right)} = 1 + \frac{x}{13}$

$\therefore \quad \sqrt{\frac{196}{169}} = 1 + \frac{x}{13}$

or, $\frac{14}{13} = 1 + \frac{x}{13}$ or, $\frac{x}{13} = \frac{14}{13} - 1$

or, $\frac{x}{13} = \frac{1}{13}$ or, $x = 1.$

17. Ans. (c)

Solution $\frac{\sqrt{4375}}{\sqrt{7}} = \frac{\sqrt{4375}}{\sqrt{7}} = \sqrt{625} = 25.$

18. Ans. (a)

Solution $\sqrt{0.016a} = 0.016 \times \sqrt{b}$

$\Rightarrow \sqrt{\frac{a}{b}} = \frac{0.016}{\sqrt{0.016}} \Rightarrow \sqrt{\frac{a}{b}} = \sqrt{0.016} \Rightarrow \frac{a}{b} =$ 0.016.

19. Ans. (b)

Solution $\sqrt{\sqrt[3]{\sqrt[3]{\sqrt[3]{3.3^{1/2}}}}} = \sqrt{\sqrt[3]{\sqrt[3]{3.3^{3/4}}}} = \sqrt{\sqrt[3]{3.3^{7/8}}}$

$= \sqrt{3.3^{15/16}} = 3^{31/32}.$

20. Ans. (d)

Solution Let $\frac{\sqrt{1296}}{x} = \frac{x}{2.25}$

Then, $\frac{36}{x} = \frac{x}{2.25}$

or, $x^2 = 36 \times \frac{225}{100}$

$\therefore \quad x = \sqrt{\frac{36 \times 225}{100}} = \frac{6 \times 15}{10} = 9.$

21. Ans. (b)

Solution $\sqrt{176 + \sqrt{2401}} = \sqrt{176 + 49} = \sqrt{225} = 15.$

22. Ans. (a)

Solution $\sqrt{10} \times \sqrt{15} = \sqrt{150} = \sqrt{25 \times 6}$

$= \sqrt{25} \times \sqrt{6} = 5\sqrt{6}.$

23. Ans. (a)

Solution $\frac{\sqrt{4}}{\sqrt{3}}-\frac{\sqrt{3}}{\sqrt{4}} = \frac{2}{\sqrt{3}}-\frac{\sqrt{3}}{2} = \frac{4-3}{2\sqrt{3}} = \frac{1}{2\sqrt{3}}$.

24. Ans. (b)

Solution $\sqrt{248+\sqrt{52+\sqrt{144}}} = \sqrt{248+\sqrt{52+12}}$

$= \sqrt{248+\sqrt{64}}$

$= \sqrt{248+8}$

$= \sqrt{256} = 16.$

25. Ans. (b)

Solution Given expression $= \frac{\sqrt{0.0009}}{\sqrt{0.01}} = \sqrt{\frac{0.0009}{0.0100}}$

$= \sqrt{\frac{9}{100}} = \frac{3}{10} = 0.3.$

26. Ans. (d)

Solution $\frac{1}{\sqrt{9}-\sqrt{8}} = \frac{1}{\sqrt{9}-\sqrt{8}}\times\frac{\sqrt{9}+\sqrt{8}}{\sqrt{9}+\sqrt{8}} = \frac{3+2\sqrt{2}}{9-8}$

$= 3 + 2\sqrt{2}.$

27. Ans. (b)

Solution $\sqrt{\frac{x}{169}} = \frac{54}{39} \Rightarrow \frac{x}{169} = \frac{54}{39}\times\frac{54}{39}$

$\therefore\ x = \frac{54}{39}\times\frac{54}{39}\times 169 = 324.$

28. Ans. (b)

Solution Let, given expression $= x$

Then, $\sqrt{12+x} = x \Rightarrow 12 + x = x^2$

$\therefore\ x^2 - x - 12 = 0$ or, $(x - 4)(x + 3) = 0$

So, $x = 4$ (neglecting $x = -3$).

29. Ans. (d)

Solution Given expression

$= \frac{112}{14}\times\frac{24}{12}\times\frac{16}{8} = 32.$

30. Ans. (c)

Solution $\sqrt{\frac{3}{4}}+2\sqrt{\frac{4}{3}} = \sqrt{12}\left(\frac{1}{4}+\frac{2}{3}\right)$

$= \frac{3.464\times 11}{12} = 3.1753.$

31. Ans. (c)

Solution Given expression

$= \sqrt{15625}+\sqrt{\frac{15625}{100}}+\sqrt{\frac{15625}{10000}}$

$= 125+\frac{125}{10}+\frac{125}{100}$

$= 125 + 12.5 + 1.25 = 138.75.$

32. Ans. (a)

Solution $\sqrt{0.03\times 0.3\times a} = 0.03 \times 0.3 \times \sqrt{b}$

$\Rightarrow \sqrt{\frac{a}{b}} = \sqrt{0.03\times 0.3}$

$\Rightarrow \frac{a}{b} = 0.03 \times 0.3$

or, $\frac{a}{b} = 0.009.$

33. Ans. (b)

Solution $\sqrt{4096}+\sqrt{40.96}+\sqrt{.004096}$

$= \sqrt{4096}+\sqrt{\frac{4096}{100}}+\sqrt{\frac{4096}{1000000}}$

$= 64 + \frac{64}{10}+\frac{64}{1000}$

$= 64 + 6.4 + 0.064 = 70.464.$

34. Ans. (b)

Solution $\sqrt{1+\sqrt{1-\frac{2176}{2401}}} = 1 + \frac{x}{7}$

$\Rightarrow 1 + \frac{x}{7} = \sqrt{1+\sqrt{\frac{225}{2401}}} = \sqrt{1+\frac{15}{49}}$

$= \sqrt{\frac{64}{49}} = \frac{8}{7} = 1 + \frac{1}{7}$

$\therefore\ x = 1.$

35. Ans. (a)

Solution The square of an odd number cannot have 4 as the unit digit. The square of a 3-digit number will have at least 5 digits and at the most 6 digits.

36. Ans. (c)

Solution Given expression

$= \sqrt{\frac{324\times 81\times 4624}{15625\times 289\times 729\times 64}}$

(Sum of decimal places being equal in Num. and Denom.)

$= \frac{18\times 9\times 68}{125\times 17\times 27\times 8} = \frac{3}{125} = 0.024.$

37. Ans. (c)

Solution $\sqrt[3]{\frac{512}{3375}} = \sqrt[3]{\frac{2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2}{3\times 3\times 3\times 5\times 5\times 5}}$

$= \frac{2\times 2\times 2}{3\times 5} = \frac{8}{15}.$

38. Ans. (a)

Solution Given expression

$= \sqrt{0.01+0.08} = \sqrt{0.09}$

$= \sqrt{\frac{9}{100}} = \frac{3}{10} = 0.3.$

39. Ans. (b)

Solution $\sqrt[3]{15.625} = \sqrt[3]{\frac{15625}{1000}} = \sqrt[3]{\frac{5\times5\times5\times5\times5\times5}{2\times2\times2\times5\times5\times5}}$

$= \frac{5\times5}{2\times5} = \frac{5}{2} = 2.5.$

40. Ans. (c)

Solution $\sqrt[3]{0.000064} = \sqrt[3]{\frac{64}{1000000}}$

$= \sqrt[3]{\frac{4\times4\times4}{100\times100\times100}}$

$= \frac{4}{100} = 0.04.$

41. Ans. (a)

Solution $\sqrt[3]{\sqrt{441}+\sqrt{16}+\sqrt{4}}$

$= \sqrt[3]{21+4+2} = \sqrt[3]{27}$

$= \sqrt[3]{3\times3\times3} = 3.$

42. Ans. (c)

Solution $14175 = 5 \times 5 \times 3 \times 3 \times 3 \times 3 \times 7$

$= 5^2 \times 3^4 \times 7$

It must be multiplied by 7.

43. Ans. (b)

Solution $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$= \frac{(\sqrt{5}-\sqrt{3})^2}{5-3} = \frac{5+3-2\sqrt{15}}{2}$

$= \frac{2(4-\sqrt{15})}{2} = 4 - \sqrt{15}.$

44. Ans. (c)

Solution $\frac{\sqrt{24}+\sqrt{216}}{\sqrt{96}} = \frac{\sqrt{4\times6}+\sqrt{36\times6}}{\sqrt{16\times6}}$

$= \frac{2\sqrt{6}+6\sqrt{6}}{4\sqrt{6}} = \frac{8\sqrt{6}}{4\sqrt{6}} = 2.$

45. Ans. (c)

Solution $\sqrt{2\frac{2}{9}} = \frac{\sqrt{20}}{\sqrt{9}} = \frac{\sqrt{20}}{3}$

$= \frac{4.472}{3} = 1.491 \approx 1.49.$

46. Ans. (b)

Solution $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}\times\frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{(\sqrt{5}+1)^2}{(\sqrt{5})^2-(1)^2}$

$= \frac{5+1+2\sqrt{5}}{5-1}$

$= \frac{6+2\sqrt{5}}{4} = \frac{3+\sqrt{5}}{2}$

$b = \frac{\sqrt{5}-1}{\sqrt{5}+1}\times\frac{\sqrt{5}-1}{\sqrt{5}-1} = \frac{(\sqrt{5}-1)^2}{(\sqrt{5})^2-(1)^2}$

$= \frac{5+1-2\sqrt{5}}{5-1} = \frac{6+2\sqrt{5}}{4} = \frac{3-\sqrt{5}}{2}$

$a^2 + b^2 = \frac{(3+\sqrt{5})^2+(3-\sqrt{5})^2}{4}$

$= \frac{9+5+6\sqrt{5}+9+5-6\sqrt{5}}{5} = \frac{28}{4} = 7$

$ab = 1$

$\therefore \quad \frac{a^2+ab+b^2}{a^2-ab+b^2} = \frac{7+1}{7-1} = \frac{8}{6} = \frac{4}{3}.$

47. Ans. (c)

Solution $10584 = 4 \times 9 \times 2 \times 7 \times 7 \times 3$

$= 2^2 \times 3^2 \times 7^2 \times 2 \times 3$

This must be multiplied by 6.

48. Ans. (b)

Solution $7936 = 4 \times 4 \times 4 \times 4 \times 31$

To make it a perfect square, we multiply it by 31

$\therefore$ Required smallest number

$= 7936 \times 31 = 246016.$

49. Ans. (b)

Solution $\sqrt{.00059049} = \sqrt{\frac{59049}{100000000}}$

$= \frac{\sqrt{59049}}{\sqrt{100000000}}$

$= \frac{243}{10000} = 0.0243.$

50. Ans. (c)

Solution $\sqrt{\frac{4}{12.1}} = \sqrt{\frac{4\times10}{12.1\times10}} = \frac{\sqrt{4}\times\sqrt{10}}{\sqrt{121}}$

$= \frac{2}{11}\times\sqrt{10} = \frac{2}{11}\times3.16$

$= \frac{6.32}{11} = 0.57 \approx 0.6.$

51. Ans. (d)

Solution Given expression

$= \sqrt{\frac{256\times81\times4356}{15625\times121\times1296\times64}}$

$= \frac{16\times9\times66}{125\times11\times36\times8} = 0.024.$

52. Ans. (a)

Solution The number of men in the front row is the square root of 16160 – 31, that is 16129 which is 127.

53. Ans. (c)

Solution Ratio of their sides is the ratio of their square roots

$$= \sqrt{420.25} : \sqrt{441}$$
$$= 20.5 : 21$$
$$= 41 : 42.$$

54. Ans. (b)

Solution The number of men in the front row is the square root of 5180 + 4, that is 5184 which is 72.

55. Ans. (c)

Solution The largest number of three digits is 999

	31
3	999
	9
61	99
	61
	38

$\therefore$ Required number is $(31)^2 = 961$.

56. Ans. (c)

Solution $21\frac{15}{289} = \frac{6084}{289} = \left(\frac{78}{17}\right)^2$

$\therefore$ Square root $= \frac{78}{17} = 4\frac{10}{17}$.

$\therefore$ Least fraction to be subtracted $= \frac{10}{17}$.

57. Ans. (c)

Solution

	250
2	$6\overline{2}\ 5\ \overline{12}$
	4
45	225
	225
50	12

So, 12 is the smallest number which when subtracted from 62512 makes it a perfect square

$$\sqrt{62500} = 250.$$

58. Ans. (c)

Solution Largest no. of 5 digits = 99999

	316
3	$9\ \overline{99}\ \overline{99}$
	9
61	99
	61
625	3899
	3756
	143

Reqd. number $= (316)^2 = 99856$.

59. Ans. (d)

Solution $2450 = 5 \times 5 \times 7 \times 7 \times 2$

$\therefore$ 2450 must be multiplied by 2.

60. Ans. (b)

Solution $3600 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$
$= 2^2 \times 2^2 \times 3^2 \times 5^2$

$\therefore$ 3600 should be multiplied by $2 \times 2 \times 3 \times 5$ That is 60 to make it a perfect cube.

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. The digit in the unit's place in the square root of 15876 is
(a) 8 (b) 6
(c) 4 (d) 2
[Based on SSC (GL) Prel. Exam 2000]

2. $\sqrt[3]{\frac{72.9}{0.4096}}$ is equal to
(a) 0.5625 (b) 5.625
(c) 182 (d) 13.6
[Based on SSC (GL) Prel. Exam 2000]

3. If the square root of 5 is 2.236, then the square root of 80 equals = 2.236 times of
(a) 2 (b) 2.5
(c) 4 (d) 5
[Based on SSC (GL) Prel. Exam 2000]

4. $(169 - 144)^{1/2} \div (64 + 36)^{1/2} = ?$
(a) 0.5 (b) 0.25
(c) 2.5 (d) 5
[Based on SSC (GL) Prel. Exam 2000]

5. The digit in the unit's place in the cube root of 21952 is
(a) 8 (b) 6
(c) 4 (d) 2
[Based on SSC (GL) Prel. Exam 2000]

6. Given $\sqrt{5} = 2.2361$, $\sqrt{3} = 1.7321$, then $\frac{1}{\sqrt{5}-\sqrt{3}}$ is equal to
(a) 1.984 (b) 1.9841
(c) 1.98 (d) 2
[Based on SSC (GL) Prel. Exam 2000]

7. $\frac{\sqrt{625}}{11} \times \frac{14}{\sqrt{25}} \times \frac{11}{\sqrt{196}}$ is equal to
(a) 100 (b) 80
(c) 60 (d) 30
[Based on SSC (GL) Prel. Exam 2000]

8. The square root of $(272^2 - 128^2)$ is
(a) 256 (b) 200
(c) 240 (d) 144
[Based on SSC (GL) Prel. Exam 2000]

9. The square root of 0.9 is equal to
(a) 0.3 (b) 0.03
(c) 0.94 (d) 0.81
[Based on SSC (GL) Prel. Exam 2000]

10. The value of $\frac{\sqrt{0.441}}{\sqrt{0.625}}$ is equal to
(a) 0.048 (b) 0.84
(c) 0.48 (d) 0.084
[Based on SSC (GL) Prel. Exam 2002]

11. The square root of $\frac{0.342 \times 0.684}{0.000342 \times 0.000171}$ is
(a) 250 (b) 2500
(c) 2000 (d) 4000
[Based on SSC (GL) Prel. Exam 2002]

12. The sum of $\sqrt{0.01} + \sqrt{0.81} + \sqrt{1.21} + \sqrt{0.0009}$ is
(a) 2.1 (b) 2.13
(c) 2.03 (d) 2.11

13. If cube root of 175616 is 56, then the value of $\sqrt[3]{175.616} + \sqrt[3]{0.175616} + \sqrt[3]{0.000175616}$ is equal to
(a) 0.168 (b) 62.16
(c) 6.216 (d) 6.116
[Based on SSC (GL) Prel. Exam 2002]

14. Given $\sqrt{2} = 1.414$, then the value of $\sqrt{8} + \sqrt[2]{32} - \sqrt[3]{128} + \sqrt[4]{50}$ is
(a) 8.484 (b) 8.526
(c) 8.426 (d) 8.876
[Based on SSC (GL) Prel. Exam 2003]

15. $\sqrt{\sqrt[3]{0.004096}}$ is equal to
(a) 4 (b) 0.4
(c) 0.04 (d) 0.004
[Based on SSC (GL) Prel. Exam 2003]

16. If $\sqrt{15} = 3.88$, then what is the value of $\sqrt{\frac{5}{3}}$?
(a) $1.29\overline{3}$ (b) 1.2934
(c) 1.29 (d) 1.295
[Based on SSC (GL) Prel. Exam 2003]

17. If the square root of 5625 is 75, then $\sqrt{5625} + \sqrt{56.25} + \sqrt{0.5625} =$
(a) 9 (b) 83.25
(c) 82.80 (d) 8.325
[Based on SSC (GL) Prel. Exam 2002]

18. What approximate value should come in place of the question mark (?) ?
$36.0001 \div 5.9998 \times \sqrt{?} = 108.0005$
(a) 18 (b) 16
(c) 256 (d) 316
(e) 325
[Based on Bank of Maharashtra PO Exam 2003]

19. $\sqrt{1223.9975} = ?$
(a) 110 (b) 144
(c) 34 (d) 12.55
(e) 125 **[Based on IBPS Jr. Executive Exam 2002]**

20. $\sqrt{\sqrt{20800}} = ?$
(a) 12 (b) 120
(c) 140 (d) 102
(e) 1020 **[Based on IBPS Jr. Executive Exam 2002]**

21. $\sqrt{10000} + \frac{3.001}{4.987}$ of $1891.992 = ?$
(a) 2500 (b) 1230
(c) 1640 (d) 1525
(e) 2130 **[Based on Canara Bank PO 2003]**

22. $\sqrt{?} = \pm 75$
(a) –562.5 (b) 75 × – 75
(c) 1500 (d) Cannot be determined
(e) None of these
[Based on NABARD Assistant Manager Exam 2002]

23. $(1.5)^2 \times \sqrt{0.0225} = ?$
(a) 0.3375 (b) 3.275
(c) 32.75 (d) 0.0373
(e) None of these
[Based on Andhra Bank Specialist Officer's Exam 2002]

EXPLANATORY ANSWERS

1. Ans. (b)

Solution $\sqrt{15876} = 126.$

2. Ans. (b)

Solution $\sqrt[3]{\frac{.9 \times .9 \times .9 \times 100}{.16 \times .16 \times .16 \times 100}} = \frac{.9}{.16} = 5.625.$

3. Ans. (c)

Solution $\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5} = 4 \times 2.236.$

4. Ans. (a)

Solution $? = 25^{1/2} \div 100^{1/2} = 5 \div 10 = 0.5.$

5. Ans. (a)

Solution $21952 = 4 \times 4 \times 4 \times 7 \times 7 \times 7$

$\therefore \quad \sqrt[3]{21952} = 4 \times 7 = 28.$

6. Ans. (b)

Solution $\frac{1}{\sqrt{5}-\sqrt{3}} = \frac{1}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \frac{\sqrt{5}+\sqrt{3}}{5-3} = \frac{2.2361+1.7321}{2}$

$= \frac{3.9682}{2} = 1.9841.$

7. Ans. (b)

Solution $\frac{\sqrt{625}}{11} \times \frac{14}{\sqrt{25}} \times \frac{11}{\sqrt{196}} = \frac{25}{11} \times \frac{14}{5} \times \frac{11}{14} = 5.$

8. Ans. (c)

Solution $\sqrt{272^2 - 128^2} = \sqrt{(272+128)(272-128)}$

$= \sqrt{400 \times 144}$

$= 20 \times 12 = 240.$

9. Ans. (c)

Solution $\sqrt{0.9} = \sqrt{\frac{9}{10}} = \frac{3}{10} \times \frac{\sqrt{10}}{\sqrt{10}}$

$= \frac{3 \times 3.16}{10} = \frac{9.48}{10} = 0.94.$

10. Ans. (b)

Solution $\frac{\sqrt{0.441}}{\sqrt{0.625}} = \frac{\sqrt{441}}{\sqrt{625}} = \frac{21}{25} = 0.84.$

11. Ans. (c)

Solution $\frac{0.342 \times 0.684}{0.000342 \times 0.000171} = \frac{342 \times 684 \times 10^6}{342 \times 171}$

$= 4 \times 10^6$

Square root of $4 \times 10^6 = 2 \times 10^3 = 2000.$

12. Ans. (b)

Solution $\sqrt{0.01} + \sqrt{0.81} + \sqrt{1.21} + \sqrt{0.0009}$

$= 0.1 + 0.9 + 1.1 + 0.03 = 2.13.$

13. Ans. (c)

Solution $\sqrt[3]{175.616} = \sqrt[3]{\frac{175616}{10000}} = \frac{56}{10} = 5.6$

$\sqrt[3]{0.175616} = \sqrt[3]{\frac{175616}{1000000}} = \frac{56}{100} = 0.56$

$\sqrt[3]{0.000175616} = \sqrt[3]{\frac{175616}{1000000000}} = \frac{56}{1000} = 0.056$

$\therefore$ Required answer $= 5.6 + 0.56 + 0.056 = 6.216.$

14. Ans. (a)

Solution $\sqrt{8} + 2\sqrt{32} - \sqrt[3]{128} + \sqrt[4]{50}$

$= 2\sqrt{2} + 2 \times 4\sqrt{2} - 3 \times 8\sqrt{2} + 4 \times 5\sqrt{2}$

$= 2\sqrt{2} + 8\sqrt{2} - 24\sqrt{2} + 20\sqrt{2}$

$= 6\sqrt{2} = 6 \times 1.414 = 8.484.$

15. Ans. (b)

Solution $\sqrt{\sqrt[3]{0.004096}} = \sqrt{((0.16)^3)^{1/3}}$

$= \sqrt{0.16} = 0.4.$

16. Ans. (a)

Solution $\sqrt{\frac{5}{3}} = \sqrt{\frac{5 \times 3}{3 \times 3}} = \sqrt{\frac{15}{3}} = \frac{3.88}{3} = 1.29\overline{3}.$

17. Ans. (b)

Solution $\sqrt{5625} + \sqrt{56.25} + \sqrt{0.5625}$

$= \sqrt{5625} + \sqrt{\frac{5625}{100}} + \sqrt{\frac{5625}{1000}} = 75 + \frac{75}{10} + \frac{75}{100}$

$= 75 + 7.5 + 0.75 = 83.25.$

18. Ans. (e)

Solution $\frac{36}{6} \times \sqrt{?} = 108$

or, $\sqrt{?} = \frac{108}{6}$ or, $\sqrt{?} = 18$ or, $? = 324 \approx 325.$

19. Ans. (c)

Solution $? = \sqrt{1223.9975} \approx 34.$

20. Ans. (a)

Solution $? = \sqrt{\sqrt{20\,800}} \approx \sqrt{144} = 12.$

21. Ans. (b)

Solution $\sqrt{10000} + \frac{3.001}{4.987}$ of $1891.992 = ?$

or, $? \approx 100 + \frac{3}{5}$ of $1900 = 100 + 1140 \approx 1230.$

22. Ans. (e)

Solution $\sqrt{?} = \pm 75$

Squaring on both sides, we get

$? = 75 \times 75 = 5625.$

23. Ans. (a)

Solution $2.25 \times 0.15 = 0.3375.$

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. $\frac{\sqrt{324}}{1.5} = \frac{?}{\sqrt{256}}$

(a) 192 (b) 432
(c) 288 (d) 122

2. Find the value of $\sqrt[3]{50+\sqrt{121+\sqrt{9}}}$
(a) 6 (b) 8
(c) 4 (d) 12

3. If $\sqrt{2^n}$ = 64, then the value of n is
(a) 2 (b) 4
(c) 6 (d) 12

4. Find the least number by which when 750 is divided or multiplied, then it becomes a perfect square.
(a) 25 (b) 15
(c) 30 (d) 35

5. $\dfrac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}$ = ?
(a) $\sqrt{2}$ (b) 2
(c) 4 (d) 8

6. Each student in a class contributed as many paise as the number of students in the class. The teacher contributed Rs. 13 to make the total collection of Rs. 49. How many students were there in the class?
(a) 36 (b) 69
(c) 70 (d) None of these

7. $2+\dfrac{1}{\sqrt{2}}+\dfrac{1}{2+\sqrt{2}}+\dfrac{1}{\sqrt{2}-2}$ = ?
(a) $2-\dfrac{1}{\sqrt{2}}$ (b) $2+\dfrac{1}{\sqrt{2}}$
(c) $\dfrac{1}{\sqrt{2}}$ (d) $-\dfrac{1}{\sqrt{2}}$

8. The least perfect square number divisible by 3, 4, 5, 6, 8 is
(a) 900 (b) 1200
(c) 2500 (d) 3600

9. If $\sqrt{3}$ = 1.732 and $\sqrt{2}$ = 1.414, then the value of $\dfrac{1}{\sqrt{3}+\sqrt{2}}$ is
(a) 0.064 (b) 0.308
(c) 0.318 (d) 2.146

10. If $\sqrt{3}$ = 1.732, value of $\dfrac{3}{2+\sqrt{3}}$ is
(a) 0.268 (b) 11.196
(c) 0.804 (d) None of these

Test II

[Time: 10 min]

11. If $\sqrt{6}$ = 2.55, then the value of $\sqrt{\dfrac{2}{3}}+3\sqrt{\dfrac{3}{2}}$ is
(a) 4.48 (b) 4.49
(c) 4.50 (d) None of these

12. Find the least number by which when 2205 is divided or multiplied, then it becomes a perfect square
(a) 25 (b) 15
(c) 10 (d) 5

13. If $\sqrt{(75.24+x)}$ = 8.71, then the value of x is
(a) 0.6241 (b) 6.241
(c) 62. 41 (d) None of these

14. A man after a tour, finds that he had spent every day as many Rs. 5 notes as the number of days he has been away from home, and he spent in all Rs. 9245. How long did his tour last?
(a) 95 (b) 45
(c) 43 (d) 47

15. The smallest number that should be added to 13218 to make it a perfect square is
(a) 11 (b) 3
(c) 17 (d) 7

16. By what least number, 6728 is divided so that the resulting number is the perfect square?
(a) 2 (b) 4
(c) 7 (d) None of these

17. If $\sqrt{24}$ = 4.899, then the value of $\sqrt{\dfrac{8}{3}}$ is
(a) 0.544 (b) 2.666
(c) 1.633 (d) 1.333

18. By what smallest number must the number 21600 be multiplied in order that the resulting number may be a perfect cube?
(a) 100 (b) 10
(c) 5 (b) 20

19. The smallest number of six digits which is a perfect square is
(a) 100489 (b) 100000
(c) 101124 (d) 101761

20. Find the square root of $\dfrac{5}{6+\dfrac{7}{8\frac{10}{11}}}$ up to two decimal places.
(a) 0.85 (b) 0.56
(c) 0.71 (d) 0.32

Answers

1. (a)	**2.** (c)	**3.** (d)	**4.** (c)	**5.** (b)	**6.** (d)	**7.** (a)	**8.** (d)	**9.** (c)	**10.** (c)
11. (d)	**12.** (d)	**13.** (a)	**14.** (c)	**15.** (d)	**16.** (a)	**17.** (c)	**18.** (a)	**19.** (a)	**20.** (a)

5

PERCENTAGE

INTRODUCTION

The term *per cent* means per hundred or for every hundred. It is the abbreviation of the Latin phrase *per centum*.

Scoring 60 per cent marks means out of every 100 marks the candidate scored 60 marks.

The term per cent is sometimes abbreviated as p.c. The symbol % is often used for the term per cent.

Thus, 40 per cent will be written as 40%.

A fraction whose denominator is 100 is called a *percentage* and the numerator of the fraction is called *rate per cent*, e.g. $\frac{5}{100}$ and 5 per cent means the same thing, i.e. 5 parts out of every hundred parts.

SOME BASIC FORMULAE

1. To Convert a Fraction into a Per cent:

   To convert any fraction $\frac{l}{m}$ to rate per cent, multiply it by 100 and put % sign, i.e. $\frac{l}{m} \times 100\%$.

Illustration 1 What percentage is equivalent to $\frac{3}{5}$?

Solution $\frac{3}{5} \times 100 = 60\%$.

2. To Convert a Per cent into a Fraction:

   To convert a per cent into a fraction, drop the per cent sign and divide the number by 100.

Illustration 2 What fraction is $16\frac{2}{3}\%$?

Solution $16\frac{2}{3}\% = \frac{\left(\frac{50}{3}\right)}{100} = \left(\frac{50}{3} \times \frac{1}{100}\right) = \frac{1}{6}$.

3. To find a percentage of a given number:

   x % of given number (N) $= \frac{x}{100} \times$ N.

Illustration 3 75% of 400 = ?

Solution 75% of 400 $= \frac{75}{100} \times 400 = 300$.

Illustration 4 Find a number whose 4% is 72.

Solution Let the required number be x.

Then, 4% of $x = 72$

$\Rightarrow \frac{4}{100} \times x = 72 \Rightarrow x = \frac{100}{4} \times 72 = 1800$.

Illustration 5 What per cent of 25 kg is 3·5 kg?

Solution Let x% of 25 kg be 3·5 kg.

Then, x% of 25 kg = 3·5 kg

$\Rightarrow \frac{x}{100} \times 25 = 3{\cdot}5 \Rightarrow x = \frac{3{\cdot}5 \times 100}{25} = 14$.

Hence, 3·5 kg is 14% of 25 kg.

SOME USEFUL SHORT-CUT METHODS

1. (a) If A is x% more than that of B, then B is less than that of A by

$$\left[\frac{x}{100+x} \times 100\right]\%.$$

(b) If A is x% less than that of B, then B is more than that of A by

$$\left[\frac{x}{100-x} \times 100\right]\%.$$

Explanation

Given: $A = B + \frac{x}{100}B = \frac{100+x}{100}B$

$$\therefore \quad A - B = \frac{100+x}{100}B - B$$

$$= \left(\frac{100+x}{100} - 1\right)B = \frac{x}{100}B.$$

$$\text{So,} \quad \frac{A-B}{A} = \frac{\frac{x}{100}B}{\frac{100+x}{100}B} = \frac{x}{100+x}$$

$$\Rightarrow \quad A - B = \left(\frac{x}{100+x} \times 100\right)\% \text{ of } A.$$

Therefore, B is less than that of A by

$$\left(\frac{x}{100+x} \times 100\right)\%.$$

Similarly, (b) can be proved.

Illustration 6 If Mohan's salary is 10% more than that of Sohan, then how much per cent is Sohan's salary less than that of Mohan?

Solution Here, $x = 10$.

$$\therefore \quad \text{Required answer} = \left(\frac{x}{100+x} \times 100\right)\%$$

$$= \left(\frac{10}{100+10} \times 100\right)\%$$

$$= 11\frac{1}{9}\%.$$

Illustration 7 If A's income is 40% less than B's income, then how much per cent is B's income more than A's income?

Solution Here, $x = 40$.

$$\therefore \quad \text{Required answer} = \left(\frac{x}{100-x} \times 100\right)\%$$

$$= \left(\frac{40}{100-40} \times 100\right)\%$$

$$= 66\frac{2}{3}\%.$$

2. If A is x% of C and B is y% of C, then

$$A = \frac{x}{y} \times 100\% \text{ of } B.$$

Explanation

Given:

$$A = \frac{x}{100}C \Rightarrow C = 100\frac{A}{x}$$

$$\text{and,} \quad B = \frac{y}{100}C \Rightarrow C = 100\frac{B}{y}$$

$$\therefore \quad C = 100\frac{A}{x} = 100\frac{B}{y} \Rightarrow A = \frac{x}{y}B$$

$$\text{or,} \quad \frac{x}{y} \times 100\% \text{ of } B.$$

Illustration 8 If A is 20% of C and B is 25% of C, then what percentage is A of B?

Solution Here, $x = 20$ and $y = 25$.

$$A = \frac{x}{y} \times 100\% \text{ of } B$$

$$= \frac{20}{25} \times 100\% \text{ of } B, \text{ i.e. } 80\% \text{ of } B.$$

3. (a) If two numbers are respectively x% and y% more than a third number, then the first number is $\left(\frac{100+x}{100+y} \times 100\right)\%$ of the second and the second is $\left(\frac{100+y}{100+x} \times 100\right)\%$ of the first.

(b) If two numbers are respectively x% and y% less than a third number, then the first number is $\left(\frac{100-x}{100-y} \times 100\right)\%$ of the second and the second is $\left(\frac{100-y}{100-x} \times 100\right)\%$ of the first.

Explanation

Let A, B and C be the three numbers.

Given:

$$A = C + \frac{x}{100}C = \left(\frac{100+x}{100}\right)C \Rightarrow C = A\left(\frac{100}{100+x}\right)$$

$$\text{and,} \quad B = C + \frac{y}{100}C = \left(\frac{100+y}{100}\right)C \Rightarrow C = B\left(\frac{100}{100+y}\right)$$

$$\therefore \quad A\left(\frac{100}{100+x}\right) = B\left(\frac{100}{100+y}\right)$$

$$\Rightarrow \quad A = \left(\frac{100+x}{100+y}\right)B \text{ or } \left(\frac{100+x}{100+y}\right) \times 100\% \text{ of } B$$

$$\text{and,} \quad B = \left(\frac{100+y}{100+x}\right)A \text{ or } \left(\frac{100+y}{100+x}\right) \times 100\% \text{ of } A.$$

Similarly, (b) can be proved.

Illustration 9 Two numbers are respectively 20% and 50% more than a third number. What per cent is the first of the second?

Solution Here, $x = 20$ and $y = 50$.

$\therefore$ First number $= \left(\frac{100+x}{100+y}\right) \times 100\%$ of the second

$= \left(\frac{100+20}{100+50}\right) \times 100\%$ of the second

i.e. 80% of the second.

Illustration 10 Two numbers are respectively 32% and 20% less than a third number. What percent is the first of the second?

Solution Here, $x = 32$ and $y = 20$.

$\therefore$ First number $= \left(\frac{100-x}{100-y} \times 100\right)\%$ of the second

$= \left(\frac{100-32}{100-20} \times 100\right)\%$ of the second

i.e. 85% of the second.

4. (a) If the price of a commodity increases by P%, then the reduction in consumption so as not to increase the expenditure is

$$\left(\frac{P}{100+P} \times 100\right)\%.$$

(b) If the price of a commodity decreases by P%, then the increase in consumption so as not to decrease the expenditure is

$$\left(\frac{P}{100-P} \times 100\right)\%.$$

Explanation

Let the original price of the commodity be Rs. 100.

Then, the increased price $= 100 + \frac{P}{100} \times 100$

$=$ Rs. $(100 + P)$.

Therefore, to keep the price unchanged, there should be a reduction in the consumption of the commodity by Rs. P.

$\because$ Decrease in Rs. $(100 + P)$ = Rs. P

$\therefore$ Decrease in Rs. 100 $= \frac{P}{100+P} \times 100$

$\therefore$ Required reduction in consumption is

$$\left(\frac{P}{100+P} \times 100\right)\%.$$

Similarly, (b) part can be proved.

Illustration 11 If the price of sugar increases by 25%, find how much per cent its consumption be reduced so as not to increase the expenditure.

Solution Reduction in consumption

$$= \left(\frac{P}{100+P} \times 100\right)\%$$

$$= \left(\frac{25}{100+25} \times 100\right)\% \text{ or } 20\%.$$

Illustration 12 If the price of a commodity decreases by 25%, find how much per cent its consumption be increased so as not to decrease the expenditure.

Solution Increase in consumption

$$= \left(\frac{P}{100-P} \times 100\right)\%$$

$$= \left(\frac{25}{100-25} \times 100\right)\% \text{ or } 33\frac{1}{3}\%.$$

5. If a number is changed (increased/decreased) successively by x% and y%, then net % change is given by $\left(x + y + \frac{xy}{100}\right)\%$ which represents increase or decrease in value according as the sign is +ve or –ve.

If x or y indicates decrease in percentage, then put –ve sign before x or y, otherwise +ve sign.

Explanation

Let the given number be N.

If it is increased by x%, then it becomes

$$N + x\% \text{ of } N = N + \frac{Nx}{100} = \frac{N(x+100)}{100}.$$

If it is further increased by y%, then it becomes

$$\frac{N(x+100)}{100} + \frac{y}{100} \times \frac{N(x+100)}{100}$$

$$= \frac{N(x+100)(y+100)}{(100)^2}$$

$$\therefore \text{ Net change} = \frac{N(x+100)(y+100)}{(100)^2} - N$$

$$= \frac{N(100x+100y+xy)}{(100)^2}$$

$$\therefore \text{ \% change} = N\left(x + y + \frac{xy}{100}\right) \times \frac{1}{100} \times \frac{100}{N}$$

$$= \left(x + y + \frac{xy}{100}\right)\%.$$

Illustration 13 If salary of a person is first increased by 15% and thereafter decreased by 12%, what is the net change in his salary?

Solution Here, $x = 15$ and $y = -12$.

$\therefore$ The net % change in the salary

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(15 - 12 - \frac{15 \times 12}{100}\right)\% \text{ or } 1.2\%.$$

Since the sign is +ve, the salary of the person increases by 1.2%.

Illustration 14 The population of a town is decreased by 25% and 40% in two successive years. What per cent population is decreased after two years?

Solution Here, $x = -25$ and $y = -40$.

$\therefore$ The net % change in population

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(-25 - 40 + \frac{25 \times 40}{100}\right)\% \text{ or } -55\%.$$

Since the sign is –ve, there is decrease in population after two years by 55%.

6. If two parameters A and B are multiplied to get a product and if A is changed (increased/decreased) by x% and another parameter B is changed (increased/decreased) by y%, then the net % change in the product ($A \times B$) is given $\left(x + y + \frac{xy}{100}\right)\%$ which represents increase or decrease in value according as the sign in +ve or –ve.

   If x or y indicates decrease in percentage, then put –ve sign before x or y, otherwise +ve sign.

Illustration 15 If the side of a square is increased by 20%, its area is increased by k%. Find the value of k.

Solution Since side × side = area

$\therefore$ Net % change in area

$$= \left(x + y + \frac{xy}{100}\right)\% = \left(20 + 20 + \frac{20 \times 20}{100}\right)\%$$

[Here, $x = 20$ and $y = 20$]

$= 44\%$.

Therefore, the area is increased by 44%.

Here, $k = 44$.

Illustration 16 The radius of a circle is increased by 2%. Find the percentage increase in its area.

Solution Since π × radius × radius = area

$\therefore$ Net % change in area

$$= \left(x + y + \frac{xy}{100}\right)\% = \left(2 + 2 + \frac{2 \times 2}{100}\right)\%$$

[Here, $x = 2$ and $y = 2$]

$$= 4\frac{1}{25}\%.$$

Therefore, the percentage increase in area is $4\frac{1}{25}$%.

Illustration 17 The tax on a commodity is diminished by 15% and its consumption increases by 10%. Find the effect on revenue.

Solution Since tax × consumption = revenue

$\therefore$ Net % change in revenue

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(-15 + 10 - \frac{15 \times 10}{100}\right)\%$$

[Here, $x = -15$ and $y = 10$]

$= -6.5\%$.

$\therefore$ The revenue decreases by 6.5%.

7. If the present population of a town (or value of an item) be P and the population (or value of item) changes at r% per annum, then

   (a) Population (or value of item) after n years

   $$= P\left(1 + \frac{r}{100}\right)^n$$

   (b) Population (or value of item) n years ago

   $$= \frac{P}{\left(1 + \frac{r}{100}\right)^n}.$$

   where r is +ve or –ve according as the population (or value of item) increases or decreases.

Explanation

Population at the end of first year

$$= P + \frac{r}{100}P = P\left(1 + \frac{r}{100}\right).$$

Now, the population at the beginning of second year

$$= P\left(1 + \frac{r}{100}\right).$$

$\therefore$ Population at the end of second year

$$= P\left(1 + \frac{r}{100}\right) + \frac{r}{100}P\left(1 + \frac{r}{100}\right) = P\left(1 + \frac{r}{100}\right)^2$$

$$\vdots \qquad \vdots \qquad \vdots$$

Population at the end of n years $= P\left(1 + \frac{r}{100}\right)^n$.

Illustration 18 The population of a town increases 5% annually. If its present population is 84000, what will it be in 2 years time?

Solution Here, $P = 84000$, $r = 5$ and $n = 2$.

$\therefore$ Population after 2 years

$$= P\left(1+\frac{r}{100}\right)^n$$

$$= 84000\left(1+\frac{5}{100}\right)^2$$

$$= 84000 \times \frac{105}{100}\times\frac{105}{100} = 92610.$$

Illustration 19 The population of a town increases at the rate of 5% annually. If the present population is 4410, what it was 2 years ago?

Solution Here, $P = 4410$, $r = 5$ and $n = 2$.

$\therefore$ Population of the town 2 years ago

$$= \frac{P}{\left(1+\frac{r}{100}\right)^n}$$

$$= \frac{4410}{\left(1+\frac{5}{100}\right)^2} = \frac{4410}{\frac{105}{100}\times\frac{105}{100}} = 4000.$$

8. If a number A is increased successively by x% followed by y% and then by z%, then the final value of A will be

$$A\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)\left(1+\frac{z}{100}\right)$$

In case a given value decreases by any percentage, we will use a negative sign before that.

Illustration 20 The population of a town is 144000. It increases by 5% during the first year. During the second year, it decreases by 10% and increases by 15% during the third year. What is the population after 3 years?

Solution Here, $P = 144000$, $x = 5$, $y = -10$ and $z = 15$.

$\therefore$ Population after 3 years

$$= A\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)\left(1+\frac{z}{100}\right)$$

$$= 144000\left(1+\frac{5}{100}\right)\left(1-\frac{10}{100}\right)\left(1+\frac{15}{100}\right)$$

$$= \frac{144000\times105\times90\times115}{100\times100\times100}$$

$$= 156492.$$

9. In an examination, the minimum pass percentage is x%. If a student secures y marks and fails by z marks, then the maximum marks in the examination is $\frac{100(y+z)}{x}$.

Explanation

Let the maximum marks be m.

Given: x% of $m = y + z$

$$\Rightarrow \frac{x}{100} \times m = y + z \text{ or } m = \frac{100(y+z)}{x}.$$

Illustration 21 In an examination, a student must get 60% marks to pass. If a student who gets 120 marks, fails by 60 marks, find the maximum marks.

Solution Here, $x = 60$, $y = 120$ and $z = 60$.

$$\therefore \text{ Maximum marks} = \frac{100(y+z)}{x} = \frac{100(120+60)}{60}$$

$$= \frac{100\times180}{60} = 300.$$

10. In an examination x% and y% students respectively fail in two different subjects while z% students fail in both the subjects, then the percentage of students who pass in both the subjects will be $(100 - (x + y - z))$%.

Explanation

%age of students who failed in one subject only $= (x - z)$%.

%age of students who failed in other subject only $= (y - z)$%.

%age of students who failed in both the subjects $= z$%.

$\therefore$ % age of students who passed in both the subjects

$= [100 - [(x - z) + (y - z) + z]]$%

$= (100 - (x + y - z))$%.

Illustration 22 In an examination, 42% students failed in Mathematics and 52% failed in Science. If 17% failed in both the subjects, find the percentage of those who passed in both the subjects.

Solution Here, $x = 42$, $y = 52$ and $z = 17$.

$\therefore$ Percentage of students passing both the subjects.

$= (100 - (x + y - z))$%

$= (100 - (42 + 52 - 17))$% or 23%.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. What percentage is equivalent to $5\frac{1}{4}$?
(a) 525% (b) 425%
(c) 625% (d) None of these

2. $0 \cdot 005$ = (... ? ...)%
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$%
(c) $\frac{1}{3}$% (d) None of these

3. $6\frac{2}{3}$% expressed as a fraction in its lowest term is
(a) $\frac{2}{15}$ (b) $\frac{1}{15}$
(c) $\frac{3}{20}$ (d) None of these

4. What fraction is 0.6%?
(a) $\frac{7}{500}$ (b) $\frac{9}{500}$
(c) $\frac{3}{500}$ (d) None of these

5. $\cdot 025$ in terms of rate per cent is
(a) 3.5% (b) 2.5%
(c) 1.5% (d) None of these

6. What per cent of 12 is 84?
(a) 800% (b) 600%
(c) 700% (d) None of these

7. Express $\frac{7}{8}$ as percentage.
(a) $67\frac{1}{2}$ (b) $87\frac{1}{2}$%
(c) $97\frac{1}{4}$% (c) None of these

8. Express $8\frac{1}{3}$% as a fraction.
(a) $\frac{1}{12}$ (b) $\frac{1}{16}$
(c) $\frac{1}{18}$ (d) None of these

9. $37\frac{1}{2}$% of Rs. 48 is
(a) Rs. 20 (b) Rs. 16
(c) Rs. 18 (d) None of these

10. What per cent of $\frac{2}{7}$ is $\frac{1}{35}$?
(a) 15% (b) 18%
(c) 10% (d) None of these

11. 75% of 480 = (?) × 15
(a) 12 (b) 36
(c) 24 (d) None of these

12. If 200% of a number is 90, then what is the 80% of that number?
(a) 48 (b) 36
(c) 24 (d) None of these

13. If $37\frac{1}{2}$% of a number is 45, then $87\frac{1}{2}$% of the number will be
(a) 115 (b) 135
(c) 105 (d) None of these

14. ? × 15 = 37·5% of 220.
(a) 5.5 (b) 7.5
(c) 6.5 (d) None of these

15. What per cent of 4 km is 8 metres?
(a) 0·4 (b) 0·2
(c) 0·8 (d) None of these

16. x% of y + y% of x = ?
(a) 3% of xy (b) 2% of xy
(c) 5% of xy (d) None of these

17. 0·35% of a number is equivalent to multiplying it by the number
(a) 0.0025 (b) 0.0045
(c) 0.0035 (d) None of these

18. If 8% of x = 4% of y, then 20% of x is
(a) 15% of y (b) 10% of y
(c) 20% of y (d) None of these

19. x% of y + ?% of x = x% of $(x + y)$.
(a) $x + y$ (b) x
(c) y (d) None of these

20. A number x is 125% of y. To compute y, the number x has to be multiplied by
(a) 0.08 (b) 0.4
(c) 0.8 (d) None of these

21. 25% of 25% = ?
(a) 6.25 (b) 0.0625
(c) 0.625 (d) None of these

22. Which number is 60% less than 80?
(a) 24 (b) 36
(c) 32 (d) None of these

23. 20% of 30% of 20% of Rs. 850 is
(a) Rs. 9.50 (b) Rs. 10.20
(c) Rs. 10.50 (d) None of these

24. The greatest of $16\frac{2}{3}\%$, $6\frac{2}{3}\%$, 0·3 is
(a) $16\frac{2}{3}\%$ (b) $6\frac{2}{3}\%$
(c) $0 \cdot 3$ (d) Cannot be compared

25. 40% of 20% + 30% of 25% + 50% of 28% is equivalent to
(a) 29.5% (b) 28.5%
(c) 30.5% (d) None of these

26. If 90% of A = 30% of B and $B = x\%$ of A, then the value of x is
(a) 800 (b) 300
(c) 700 (d) None of these

27. 1 quintal 25 kg is what per cent of 1 metric tonne?
(a) $16\frac{1}{2}\%$ (b) $8\frac{1}{2}\%$
(c) $12\frac{1}{2}\%$ (d) None of these

28. If 12% of x is equal to 6% of y, then 18% of x will be equal to how much per cent of y?
(a) 7% (b) 9%
(c) 11% (d) None of these

29. If a number is 20% more than the other, how much per cent is the second number less than the first?
(a) $12\frac{1}{3}\%$ (b) $16\frac{2}{3}\%$
(c) $16\frac{1}{3}\%$ (d) None of these

30. If A's income is 25% less than that of B, then how much per cent is B's income more than that of A?
(a) $33\frac{1}{3}\%$ (b) $66\frac{2}{3}\%$
(c) $11\frac{2}{3}\%$ (d) None of these

31. If the given two numbers are respectively 7% and 28% of a third number, then what percentage is the first of the second?
(a) 20% (b) 25%
(c) 18% (d) None of these

32. Two numbers are respectively 60% and 20% more than a third number. Second number expressed as a percentage of first is
(a) 75% (b) 90%
(c) 80% (d) None of these

33. Two numbers are less than a third number by 30% and 37% respectively. How much per cent is the second number less than the first?
(a) 15% (b) 10%
(c) 20% (d) None of these

34. Two numbers are respectively 20% and 10% more than a third number. How much per cent is the first number more than the second?
(a) $9\frac{1}{11}\%$ (b) $7\frac{1}{11}\%$
(c) $11\frac{1}{11}\%$ (d) None of these

35. The price of cooking oil has increased by 15%. The percentage of reduction that a family should effect in the use of cooking oil so as not to increase the expenditure on this account is
(a) $15\frac{2}{23}\%$ (b) $13\frac{1}{23}\%$
(c) $17\frac{1}{23}\%$ (d) None of these

36. If the price of apples goes down by 10%, find the percentage of increase that a family should effect in its consumption so as not to increase expenditure on this account
(a) $13\frac{1}{9}\%$ (b) $15\frac{1}{9}$
(c) $11\frac{1}{9}\%$ (d) None of these.

37. A number is increased by 20% and then decreased by 20%, the final value of the number
(a) does not change (b) decreases by 2%
(c) increases by 4% (d) decreases by 4%.

38. A man's wages were decreased by 50%. Again, the reduced wages were increased by 50%. He has a loss of
(a) 35% (b) 25%
(c) 20% (d) None of these.

39. The population of a town is decreased by 20% and 25% in two successive years. What per cent population is decreased after two years?
(a) 50% (b) 40%
(c) 60% None of these

40. The difference between a discount of 35% and two successive discounts of 20% and 20% on a certain bill was Rs. 22. Find the amount of the bill.
(a) Rs. 3200 (b) Rs. 2200
(c) Rs. 1800 (d) None of these

41. A shopkeeper marks the prices of his goods at 25% higher than the original price. After that, he allows a discount of 12%. What profit or loss did he get?
(a) 10% profit (b) 15% profit
(c) 10% loss (d) 15% loss

42. Two shopkeepers sell a ratio of similar brand and type at the same list price of Rs. 1000. The first allows two successive discounts of 20% and 10% and the second allows the successive discounts of 15% and 15%. Find the difference in discounts offered by the two shopkeepers.
(a) Rs. 3.50 (b) Rs. 1.50
(c) Rs. 2.50 (d) None of these

43. The tax on a commodity is diminished by 10% and its consumption increases by 10%. Find the effects on revenue.
(a) 1% (b) 2%
(c) 3% (d) None of these

44. The radius of a sphere is increased by 10%. The surface area increases by
(a) 21% (b) 31%
(c) 41% (d) None of these

45. When the price of an article is reduced by 15%, the sales increases by 35%. The percentage change in the total amount of receipts is
(a) $14\frac{3}{4}$% decrease (b) $14\frac{3}{4}$% increase
(c) $13\frac{3}{4}$% decrease (d) None of these

46. If the side of a square is increased by 30%, its area is increased by
(a) 49% (b) 69%
(c) 79% (d) None of these

47. The length and breadth of a square are increased by 30% and 20%, respectively. The area of the rectangle so formed exceeds the area of the square by
(a) 56% (b) 46%
(c) 66% (d) None of these

48. In measuring the sides of a rectangle, one side is taken 10% in excess and the other 20% in deficit. Find the error per cent in area calculated from the measurement.
(a) 12% deficit (b) 10% deficit
(c) 12% excess (d) None of these

49. For a rectangle, the length and breadth are increased by 10% and 20%, respectively. The percentage increase in area is
(a) 24% (b) 48%
(c) 32% (d) None of these

50. Water tax is increased by 20% but its consumption is decreased by 20%. The increase or decrease in the expenditure is
(a) 4% decrease (b) 4% increase
(c) 8% decrease (d) 8% increase

51. On decreasing the price of a colour TV by 30%, its sale is increased by 20%. The effect on the revenue is
(a) 16% decrease (b) 16% increase
(c) 20% increase (d) None of these

52. The population of a city increases at the rate of 10% annually. Its present population is 90.51 lacs. The population 3 years ago was nearly
(a) 72 lacs (b) 68 lacs
(c) 80 lacs (d) None of these

53. The value of a machine depreciates at the rate of 10% every year. It was purchased 3 years ago. If its present value is Rs. 8748, its purchase price was
(a) Rs. 16000 (b) Rs. 18000
(c) Rs. 12000 (d) None of these

54. The income of a company increases 20% per annum. If its income is Rs. 2664000 in the year 1999 what was its income in the year 1997?
(a) Rs. 1750000 (b) Rs. 1650000
(c) Rs. 1850000 (d) None of these

55. The population of a town is 32000. It increases 15% annually. What will it be in 2 years?
(a) 52340 (b) 42320
(c) 62430 (d) None of these

56. The value of a machine is Rs. 6250. It decreases by 10% during the first year, 20% during the second year and 30% during the third year. What will be the value of the machine after 3 years?
(a) Rs. 2650 (b) Rs. 3050
(c) Rs. 3150 (d) None of these

57. The population of a town increases by 12% during first year and decreases by 10% during second year. If the present population is 50400, what it was 2 years ago?
(a) 40000 (b) 35000
(c) 50000 (d) None of these

58. Ramesh loses 20% of his pocket money. After spending 25% of the remainder he has Rs. 480 left. What was his pocket money?
(a) Rs. 600 (b) Rs. 800
(c) Rs. 900 (d) None of these

59. An army lost 10% its men in war, 10% of the remaining due to diseases and 10% of the rest were disabled. Thus, the strength was reduced to 729000 active men. Find the original strength.
(a) 1000000 (b) 1200000
(c) 1500000 (d) None of these

60. The daily wage is increased by 25% and a person now gets Rs. 25 per day. What was his daily wage before the increase?
(a) Rs. 25 (b) Rs. 20
(c) Rs. 30 (d) None of these

61. A student has to secure 15% marks to get through. If he gets 80 marks and fails by 70 marks, find the maximum marks set for the examination.
(a) 900 (b) 1000
(c) 1200 (d) None of these

62. In an examination, 30% and 35% students respectively failed in History and Geography while 27% students failed in both the subjects. If the number of students passing the examination is 248, find the total number of students who appeared in the examination.
(a) 425 (b) 380
(c) 400 (d) None of these

63. Mr. Katial buys a house for Rs. 100000 and rents it. He puts 12.5% of each month's rent aside for upkeep and repairs, pays Rs. 325 per year as taxes and realizes 5.5% annually on his investment. Find the monthly rent.
(a) Rs. 634.76 (b) Rs. 554.76
(c) Rs. 654.76 (d) None of these

64. In an examination, there were 2000 candidates, out of which 900 candidates were boys and rest were girls. If 32% of the boys and 38% of the girls passed, then the total percentage of failed candidates is
(a) 35.3% (b) 64.7%
(c) 68.5% (d) 70%

65. From the salary of an officer, 10% is deducted as house rent, 15% of the rest he spends on children's education and 10% of the balance, he spends on clothes. After this expenditure he is left with Rs. 1377. His salary is
(a) Rs. 2000 (b) Rs. 2040
(c) Rs. 2100 (d) Rs. 2200

66. If the price of gold increases by 30%, find by how much the quantity of ornaments must be reduced so that the expenditure may remain the same as before?
(a) $27\frac{2}{13}\%$ (b) $23\frac{1}{13}\%$
(c) 30% (d) 19%

67. The price of sugar has fallen by 10%. How many quintals can be bought for the same money which was sufficient to buy 18 quintals at the higher price?
(a) 20 (b) 22
(c) 25 (d) 30

68. In an examination, there are 1000 boys and 800 girls, 60% of boys and 40% girls passed. The percentage of candidates that failed is
(a) 48·88 (b) 45·88
(c) 50·00 (d) 49·88

69. The price of an article is cut by 20%. To restore it to its original price, the new price must be increased by
(a) 20% (b) $22\frac{1}{2}\%$
(c) 25% (d) 40%

70. In a fraction, numerator is increased by 25% and the denominator is diminished by 10%. The new fraction obtained is $\frac{5}{9}$. The original fraction is
(a) $\frac{2}{5}$ (b) $\frac{5}{9}$
(c) $\frac{3}{5}$ (d) None of these

71. One side of a square is increased by 30%. To maintain the same area the other side will have to be decreased by
(a) $23\frac{1}{13}\%$ (b) $76\frac{12}{13}\%$
(c) 30% (d) 15%

EXPLANATORY ANSWERS

1. Ans. (a)

Solution $5\frac{1}{4} = \frac{21}{4} = \frac{21}{4} \times 100 = 525\%.$

2. Ans. (b)

Solution $0 \cdot 005 = \frac{5}{1000} = \frac{5}{1000} \times 100 = \frac{1}{2}\%.$

3. Ans. (b)

Solution $6\frac{2}{3}\% = \frac{\left(\frac{20}{3}\right)}{100} = \left(\frac{20}{3} \times \frac{1}{100}\right) = \frac{1}{15}.$

4. Ans. (c)

Solution $0 \cdot 6\% = \frac{0 \cdot 6}{100} = \frac{6}{1000} = \frac{3}{500}.$

5. Ans. (b)

Solution $.025 = \left(\frac{25}{1000} \times 100\right)\% = 2.5\%.$

6. Ans. (c)

Solution Let x% of 12 = 84

$\Rightarrow \frac{x}{100} \times 12 = 84 \Rightarrow x = \frac{84 \times 100}{12} = 700.$

$\therefore$ 700% of 12 is 84.

7. Ans. (b)

Solution $\frac{7}{8} = \left(\frac{7}{8} \times 100\right)\% = \frac{175}{2}\% = 87\frac{1}{2}\%.$

8. Ans. (a)

Solution $8\frac{1}{3}\% = \frac{25}{3}\% = \frac{25}{3} \times \frac{1}{100} = \frac{1}{12}.$

9. Ans. (c)

Solution $37\frac{1}{2}\%$ of Rs. 48 = $48 \times \frac{75}{2 \times 100}$ = Rs. 18.

10. Ans.

Solution Let x% of $\frac{2}{7} = \frac{1}{35} \Rightarrow x = \frac{100\times7}{2\times35} = 10.$

$\therefore$ 10% of $\frac{2}{7}$ is $\frac{1}{35}$.

11. Ans. (c)

Solution Let 75% of 480 = $x \times 15$.

Then, $\frac{75}{100} \times 480 = 15x$ or $x = \frac{75\times480}{100\times15} = 24.$

12. Ans. (b)

Solution Let 200% of $x = 90 \Rightarrow \frac{200}{100} \times x = 90$

$\Rightarrow x = \frac{100\times90}{200} = 45.$

$\therefore$ 80% of 45 = $\frac{80}{100} = 45 = 36.$

13. Ans. (c)

Solution Let the number be x, then

$37\frac{1}{2}$% of $x = 45 \Rightarrow \frac{75}{2}\times\frac{1}{100} \times x = 45$

or, $\frac{3}{8}x = 45 \Rightarrow x = \frac{45\times8}{3} = 120.$

$\therefore$ $87\frac{1}{2}$% of 120 = $\frac{175}{2}\times\frac{1}{100} \times 120 = 105.$

14. Ans. (a)

Solution Let $x \times 15$ = 37.5% of 220

$\Rightarrow 15x = \frac{37.5}{100} \times 220$

$\Rightarrow x = \frac{37.5\times220}{15\times100} = 5.5.$

15. Ans. (b)

Solution Let x% of 4 km = 8 metre

$\Rightarrow \frac{x}{100} \times 4000 = 8$ ($\because$ 1 km = 1000 metre)

$\Rightarrow x = \frac{8\times100}{4000} = \frac{1}{5} = 0.2$

$\therefore$ 0.2% of 4 km = 8 metre.

16. Ans. (b)

Solution x% of y + y% of $x = \left(\frac{x}{100}\times y\right)+\left(\frac{y}{100}\times x\right)$

$= \frac{2}{100}xy$ = 2% of xy.

17. Ans. (c)

Solution $0.35\% = 0{\cdot}35 \times \frac{1}{100} = 0.0035.$

18. Ans. (b)

Solution We have, 8% of x = 4% of y

$\Rightarrow \frac{8}{100} \times x = \frac{4}{100} \times y \Rightarrow x = \left(\frac{4}{100}\times\frac{100}{8}\right)y = \frac{y}{2}.$

$\therefore$ 20% of $x = \frac{20}{100} \times x = \frac{20}{100} \times \frac{y}{2}$

$= \frac{10}{100} \times y$

= 10% of y.

19. Ans. (b)

Solution Let the missing figure be a.

We have, $\frac{x}{100} \times y + \frac{a}{100} \times x = \frac{x}{100} \times (x + y)$

$\Rightarrow xy + ax = x(x + y) \Rightarrow ax = x^2. \quad \therefore a = x.$

20. Ans. (c)

Solution We have, x = 125% of $y \Rightarrow x = \frac{125}{100}y = \frac{5}{4}y$

or, $y = \frac{4}{5}x = 0.8x.$

21. Ans. (b)

Solution 25% of 25% = $\frac{25}{100}\times\frac{25}{100} = \frac{625}{10000} = 0.0625.$

22. Ans. (c)

Solution Required number = 80 – 60% of 80

$= 80 - \frac{60}{100} \times 80 = 32.$

23. Ans. (b)

Solution 20% of 30% of 20% of 850

$= \frac{20}{100}\times\frac{30}{100}\times\frac{20}{100}\times850 = \frac{1020}{100}$ = Rs. 10.20.

24. Ans. (c)

Solution $16\frac{2}{3}\% = \frac{50}{3}\times\frac{1}{100} = \frac{1}{6} = 0.167 < 0.3.$

$6\frac{2}{3}\% = \frac{20}{3}\times\frac{1}{100} = \frac{1}{15} = 0.067 < 0.3.$

$\therefore$ 0.3 is greatest.

25. Ans. (a)

Solution 40% of 20% = $\frac{40}{100}\times\frac{20}{100} = \frac{8}{100} = 8\%$

30% of 25% = $\frac{30}{100}\times\frac{25}{100} = \frac{75}{100} = 7.5\%$

and, 50% of 28% = $\frac{50}{100}\times\frac{28}{100} = \frac{14}{100} = 14\%$

$\therefore$ (40% of 20% + 30% of 25% + 50% of 28%)

= 8% + 7.5% + 14% = 29.5%.

26. Ans. (b)

Solution We have, $\frac{90}{100} \times A = \frac{30}{100} \times B$

$$= \frac{30}{100} \times \frac{x}{100} \times A$$

$$\therefore \quad x = \left(100 \times \frac{100}{30} \times \frac{90}{100}\right) = 300.$$

27. Ans. (c)

Solution Let 1 quintal 25 kg = x% of 1 metric tonne

$$\Rightarrow \quad x\% = \frac{125}{1000} = \frac{1}{8} = \frac{1}{8} \times 100\% = 12\frac{1}{2}\%.$$

28. Ans. (b)

Solution We have, 12% of x = 6% of y

$\Rightarrow$ 2% of x = 1% of y

$\Rightarrow$ (2 × 9)% of x = (1 × 9)% of y

$\Rightarrow$ 18% of x = 9% of y.

29. Ans. (b)

Solution Here, x = 20.

$$\therefore \quad \text{Required answer} = \left(\frac{x}{100+x} \times 100\right)\%$$

$$= \left(\frac{20}{100+20} \times 100\right)\%$$

$$= 16\frac{2}{3}\%.$$

30. Ans. (a)

Solution Here, x = 25.

$$\therefore \quad \text{Required answer} = \left(\frac{x}{100-x} \times 100\right)\%$$

$$= \left(\frac{25}{100-25} \times 100\right)\%$$

$$= 33\frac{1}{3}\%.$$

31. Ans. (b)

Solution Here, l = 7 and m = 28.

$$\therefore \quad \text{First number} = \frac{l}{m} \times 100\% \text{ of second number}$$

$$= \frac{7}{28} \times 100\% \text{ of second number}$$

or 25% of second number.

32. Ans. (a)

Solution Here, x = 60 and y = 20.

$$\therefore \text{ Second number} = \left(\frac{100+y}{100+x} \times 100\right)\% \text{ of the first}$$

$$= \left(\frac{100+20}{100+60} \times 100\right)\% \text{ of the first}$$

i.e. 75% of the first.

33. Ans. (b)

Solution Here, x = 30 and y = 37.

$$\therefore \text{ Second number} = \left(\frac{100-y}{100-x} \times 100\right)\% \text{ of the first}$$

$$= \left(\frac{100-37}{100-30} \times 100\right)\% \text{ of the first}$$

i.e. 90% of the first.

34. Ans. (a)

Solution Here x = 20 and y = 10.

$$\therefore \text{ First number} = \left(\frac{100+x}{100+y} \times 100\right)\% \text{ of the second}$$

$$= \left(\frac{100+20}{100+10} \times 100\right)\% \text{ of the second}$$

i.e. $109\frac{1}{11}$% of the second.

$\therefore$ The first number is $9\frac{1}{11}$% more than the second.

35. Ans. (b)

Solution Reduction in consumption

$$= \left(\frac{P}{100+P} \times 100\right)\%$$

$$= \left(\frac{15}{100+15} \times 100\right)\% \text{ or } 13\frac{1}{23}\%.$$

36. Ans. (c)

Solution Increase in consumption

$$= \left(\frac{P}{100-P} \times 100\right)\%$$

$$= \left(\frac{10}{100-10} \times 100\right)\% \text{ or } 11\frac{1}{9}\%.$$

37. Ans. (d)

Solution Here, x = 20 and y = – 20.

$\therefore$ The net % change in value

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(20 - 20 - \frac{20 \times 20}{100}\right)\% \text{ or } -4\%.$$

Since the sign is –ve, there is decrease in value by 4%.

38. Ans. (b)

Solution Here, x = –50 and y = 50.

$\therefore$ The net % change in wages

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(-50 + 50 - \frac{50 \times 50}{100}\right)\% \text{ or } -25\%.$$

Since the sign is –ve, he has a loss of 25%.

39. Ans. (b)

Solution Here, $x = -20$ and $y = -25$.

$\therefore$ The net % change in population

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(-20 - 25 + \frac{20 \times 25}{100}\right) \text{ or } -40\%.$$

Since the sign is –ve, the population is decreased by 40% after two years.

40. Ans. (b)

Solution The equivalent discount of two successive discounts of 20% and 20%

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(-20 - 20 + \frac{20 \times 20}{100}\right)\% \text{ or } -36\%.$$

Given : 36% –35% = Rs. 22.

$\therefore$ Amount of the bill = 22 × 100 = Rs. 2200.

41. Ans. (a)

Solution Here, $x = 25$ and $y = -12$.

$\therefore$ The net % change in original price

$$= \left(x + y + \frac{xy}{100}\right)$$

$$= \left(25 - 12 - \frac{25 \times 12}{100}\right)\% \text{ or } 10\%.$$

Since the sign is +ve, there is a profit of 10%.

42. Ans. (c)

Solution The equivalent discount of two successive discounts of 20% and 10%

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(-20 - 10 + \frac{20 \times 10}{100}\right)\% \text{ or } 28\%.$$

$\therefore$ Discount on the list price of radio offered by the first shopkeeper

$$= 28\% \text{ of } 1000 = \frac{28}{100} \times 1000 = \text{Rs. } 280.$$

Also, the equivalent discount of two successive discounts of 15% and 15%

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(-15 - 15 + \frac{15 \times 15}{100}\right)\% \text{ or } 27\frac{3}{4}\%.$$

$\therefore$ Discount on the list price of radio offered by the second shopkeeper

$$= 27\frac{3}{4}\% \text{ of } 1000 = \frac{111}{400} \times 1000$$

$$= \text{Rs. } 277.50.$$

$\therefore$ Difference in discounts offered by the two shopkeepers = Rs. 280 – Rs. 277.50 = Rs. 2.50.

43. Ans. (a)

Solution Since tax × consumption = revenue

$\therefore$ Net % change in revenue

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(-10 + 10 - \frac{10 \times 10}{100}\right)\%$$

[Here $x = -10$ and $y = 10$]

$$= -1\%.$$

$\therefore$ The revenue decreases by 1%.

44. Ans. (a)

Solution Since 4π × radius × radius = surface area

$\therefore$ Net % change in area $= \left(x + y + \frac{xy}{100}\right)\%$

$$= \left(10 + 10 + \frac{10 \times 10}{100}\right)\%$$

$$= 21\%.$$

45. Ans. (b)

Solution We have, receipts = price × sales.

$\therefore$ Net % change in receipts

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(-15 + 35 - \frac{15 \times 35}{100}\right)\%$$

$$= 14\frac{3}{4}\%.$$

46. Ans. (b)

Solution Since side × side = area

$\therefore$ Net % change in area

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(30 + 30 + \frac{30 \times 30}{100}\right)\%$$

$$= 69\%.$$

$\therefore$ The area is increased by 69%.

47. Ans. (a)

Solution Since $\text{side}_1 \times \text{side}_2$ = area

∴ Net % change in area

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(30 + 20 + \frac{30 \times 20}{100}\right)\%$$

[Here, x = 30 and y = 20]

= 56%.

∴ The area of the rectangle so formed exceeds the area of the square by 56%.

48. Ans. (a)

Solution Since side$_1$ × side$_2$ = area

∴ Error % in area $= \left(x + y + \frac{xy}{100}\right)\%$

$$= \left(10 - 20 - \frac{10 \times 20}{100}\right)\%$$

[Here, x = 10 and y = –20]

= –12%, i.e. 12% deficit.

49. Ans. (c)

Solution Since side$_1$ × side$_2$ = area

∴ Net % change in area

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(10 + 20 + \frac{10 \times 20}{100}\right)\%$$

= 32%.

∴ The area of the rectangle increases by 32%.

50. Ans. (a)

Solution Since tax × consumption = expenditure

∴ Net % change in expenditure

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(20 - 20 - \frac{20 \times 20}{100}\right)\%$$

[Here, x = 20 and y = –20]

= –4%.

∴ Expenditure decreases by 4%.

51. Ans. (a)

Solution Net % change in revenue

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(-30 + 20\frac{30 \times 20}{100}\right)\%$$

[Here, x = –30 and y = 20]

= –16%.

52. Ans. (b)

Solution We have, P = 90.51, r = 10 and n = 3.

∴ The population 3 years ago

$$= \frac{P}{\left(1 + \frac{r}{100}\right)^n} = \frac{90 \cdot 51}{\left(1 + \frac{10}{100}\right)^3}$$

$$= \frac{9051}{100} \times \frac{100}{110} \times \frac{100}{110} \times \frac{100}{110}$$

= 68 lacs.

53. Ans. (c)

Solution Here, P = 8748, r = –10 and n = 3.

∴ Purchase price of the machine

$$= \frac{P}{\left(1 + \frac{r}{100}\right)^n} = \frac{8748}{\left(1 - \frac{10}{100}\right)^3}$$

$$= \frac{8748 \times 100 \times 100 \times 100}{90 \times 90 \times 90} = \text{Rs. } 12000.$$

54. Ans. (c)

Solution Here, P = 2664000, r = 20 and n = 2.

∴ Company's income in the year 1997

$$= \frac{P}{\left(1 + \frac{r}{100}\right)^n} = \frac{2664000}{\left(1 + \frac{20}{100}\right)^2}$$

$$= \frac{2664000 \times 5 \times 5}{6 \times 6}$$

= Rs. 1850000.

55. Ans. (b)

Solution Here, P = 32000, r = 15 and n = 2.

∴ Population of the town in 2 years

$$= P\left(1 + \frac{r}{100}\right)^n = 32000\left(1 + \frac{15}{100}\right)^2$$

$$= 32000 \times \frac{115}{100} \times \frac{115}{100} = 42320.$$

56. Ans. (c)

Solution Here, A = 6250, x = –10, y = –20 and z = –30.

∴ Value of the machine after 3 years

$$= A\left(1 + \frac{x}{100}\right)\left(1 + \frac{y}{100}\right)\left(1 + \frac{z}{100}\right)$$

$$= 6250\left(1 - \frac{10}{100}\right)\left(1 - \frac{20}{100}\right)\left(1 - \frac{30}{100}\right)$$

$$= \frac{6250 \times 90 \times 80 \times 70}{100 \times 100 \times 100} = \text{Rs. } 3150.$$

57. Ans. (c)

Solution Here, A = 50400, x = 12 and y = –10.

$\therefore$ Population of the town 2 years ago

$$= \frac{A}{\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)}$$

$$= \frac{50400}{\left(1+\frac{12}{100}\right)\left(1-\frac{10}{100}\right)}$$

$$= \frac{50400\times100\times100}{112\times90}$$

$= 50000.$

58. Ans. (b)

Solution Let Rs. A be the pocket money.

Then, $A\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right) = 480$ (Given)

Here, $x = -20$ and $y = -25$.

$\therefore$ $A\left(1-\frac{20}{100}\right)\left(1-\frac{25}{100}\right) = 480$

$\Rightarrow$ $A = \frac{480\times100\times100}{80\times75}$ = Rs. 800.

59. Ans. (a)

Solution Let A be the original strength.

Then, $A\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)\left(1+\frac{z}{100}\right) = 729000$ (Given)

Here, $x = -10$, $y = -10$ and $z = -10$.

$\therefore$ $A\left(1-\frac{10}{100}\right)\left(1-\frac{10}{100}\right)\left(1-\frac{10}{100}\right) = 729000$

$\Rightarrow$ $A = \frac{729000\times100\times100\times100}{90\times90\times90}$ = 1000000 men.

60. Ans. (b)

Solution Let the daily wage before the increase was Rs. A.

Then, $A\left(1+\frac{x}{100}\right) = 25.$

Here, $x = 25$.

$\therefore$ $A\left(1+\frac{25}{100}\right) = 25 \Rightarrow A = \frac{25\times100}{125}$ = Rs. 20.

61. Ans. (b)

Solution Here, $x = 15$, $y = 80$ and $z = 70$.

$\therefore$ Maximum marks $= \frac{100(y+z)}{x} = \frac{100(80+70)}{15}$

$= 1000.$

62. Ans. (c)

Solution Percentage of students passing the examination

$= (100 - (30 + 35 - 27))\%$

[Here, $x = 30$, $y = 35$ and $z = 27$]

$= 62(100 - 38)\% = 62\%.$

Let the total number of students appearing in the examination be x.

Given: 62% of $x = 248$

or, $\frac{62}{100} \times x = 248$ or $x = \frac{248\times100}{62} = 400.$

Therefore, 400 students appeared in the examination.

63. Ans. (b)

Solution Let the monthly rent be Rs. x.

We have, 5.5% of 100000 = x – 12.5% of x – 325

$\Rightarrow$ $\frac{5500}{12} = x - \frac{x}{8} - \frac{325}{12}$

$\Rightarrow$ $\frac{5500}{12} + \frac{325}{12} = \frac{7}{8} \times x$

$\Rightarrow$ $x = \frac{5825}{12}\times\frac{8}{7}$ = Rs. 554.76 per month.

64. Ans. (b)

Solution Boys = 900, Girls = 1100

Passed = (32% of 900) + (38% of 1100)

= 288 + 418 = 706

Failed = 2000 – 706 = 1294

Failed % $= \left(\frac{1294}{2000}\times100\right)\% = 64.7\%$

65. Ans. (a)

Solution Suppose that his salary = Rs. 100

House rent = Rs. 10, balance = Rs. 90

Expenditure on education = Rs. $\left(\frac{15}{100}\times90\right)$

= Rs. 13.50

Balance = Rs. 76.50.

Expenditure on clothes = Rs. $\left(\frac{10}{100}\times76\cdot50\right)$.

= Rs. 7.65

Balance now = Rs. 68.85

If balance is Rs. 68.85, salary = Rs. 100

If balance is Rs. 1377, salary = Rs. $\frac{100}{68\cdot85} \times 1377$

= Rs. 2000.

66. Ans. (b)

Solution Reduction $= \frac{30}{100+30} \times 100\% = 23\frac{1}{13}\%$

67. Ans. (a)

Solution 90% of original price can buy = 18 quintals

$\therefore$ He can buy $\frac{18\times100}{90}$ = 20 quintals at the lower price.

68. Ans. (a)

Solution Number of candidates who failed

$= 40\%$ of $1000 + 60\%$ of 800

$= 400 + 480$

$= 880$

$\therefore$ Fail % $= \left(\frac{880}{1800} \times 100\right)\% = 48.88\%$

69. Ans. (c)

Solution New price must be increased by

$$\left(\frac{20}{100-20} \times 100\right)\% = 25\%.$$

70. Ans. (a)

Solution Let the fraction be $\frac{x}{y}$

Then, $\frac{x+0.25x}{y-0.10y} = \frac{5}{9}$

$$\frac{x(1.25)}{y(0.9)} = \frac{5}{9}$$

$$\frac{x}{y} = \frac{5}{9} \times \frac{90}{125}$$

$$\frac{x}{y} = \frac{2}{5}.$$

71. Ans. (a)

Solution Let the side of the square $= x$

After increase, length of one side $= 1.3x$

Let after decrease, length of other side $= y$

Then, $(1.3x)(y) = x^2$

$$y = \frac{10x}{13}$$

Decrease in other side $= x - \frac{10x}{13} = \frac{3x}{13}$

Percentage decrease in other side

$$= \frac{\frac{3x}{13}}{x} \times 100 = \frac{300}{13} = 23\frac{1}{13}\%.$$

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. What approximate value should come in place of the question mark (?) in the following equation?

$158.25 \times 4.6 + 21\%$ of $847 + ? = 950.93$

(a) 35 (b) 40
(c) 25 (d) 50
(e) 45 **[Based on SBI PO Exam 2000]**

2. In a recent survey 40% houses contained two or more people. Of those houses containing only one person 25% were having only a male. What is the percentage of all houses which contain exactly one female and no males?

(a) 75 (b) 40
(c) 15 (d) Cannot be determined
(e) None of these **[Based on SBI PO Exam 2000]**

3. What approximate value should come in place of the question mark (?) in the following equation?

159% of $6531.8 + 5.5 \times 1015.2 = ? + 5964.9$

(a) 10,000 (b) 10,900
(c) 11,000 (d) 10,600
(e) 12,000 **[Based on BSRB Mumbai PO 1999]**

4. What approximate value should come in place of the question mark (?) in the following equation?

$3\frac{3}{5}$ of $157.85 + 39\%$ of $1847 = ? - 447.30$

(a) 1200 (b) 1500
(c) 1600 (d) 1800
(e) 2100 **[Based on BSRB Mumbai PO 1999]**

5. If the height of a triangle is decreased by 40% and its base is increased by 40%, what will be the effect on its area?

(a) No change (b) 16% increase
(c) 8% decrease (d) 16% decrease
(e) None of these **[Based on SBI PO 1999]**

6. In 1 kg mixture of sand and iron, 20% is iron. How much sand should be added so that the proportion of iron becomes 10%?

(a) 1 kg (b) 200 gms
(c) 800 mgs (d) 1.8 kg
(e) None of these **[Based on SBI PO Exam 1999]**

7. What approximate value should come in place of the question mark (?) in the following equation?

35% of $1478 + 29\%$ of $3214 = ?$

(a) 1600 (b) 1250
(c) 1300 (d) 1450
(e) 1500 **[Based on Bank of Baroda PO 1999]**

8. There is a ratio of 5 : 4 between two numbers. If forty per cent of the first number is 12 then what would be the 50% of second number?

(a) 12 (b) 24
(c) 18 (d) Data inadequate
(e) None of these above
[Based on Bank of Baroda PO 1999]

9. When 30% of a number is added to another number the second number increase to its 140 per cent. What is the ratio between the first and the second number?

(a) 3 : 4 (b) 4 : 3
(c) 3 : 2 (d) Data inadequate
(e) None of these
[Based on Bank of Baroda PO 1999]

10. Suresh's monthly income is 30% more than that of Vinod. Vinod's monthly income is 20% less than that of Vinay. If the difference between the monthly incomes of Suresh and Vinay is Rs. 800. What is the monthly income of Vinod?
(a) Rs. 16000 (b) Rs. 20000
(c) Rs. 12000 (d) Data inadequate
(e) None of these
[Based on Bank of Baroda PO 1999]

11. If 25% of a number is subtracted from a second number. If second number reduces to its five-sixths. What is the ratio between the first number and the second number?
(a) 2 : 3 (b) 3 : 2
(c) 1 : 3 (d) Data inadequate
(e) None of these
[Based on SBI Associates PO 1999]

12. A petrol pump owner mixed leaded and unleaded petrol in such a way that the mixture contains 10% unleaded petrol. What quantity of leaded petrol should be added to 1 litre mixture so that the percentage of unleaded petrol becomes 5%?
(a) 1000 ml (b) 900 ml
(c) 1800 ml (d) None of these
[Based on SBI Associates PO 1999]

13. Out of a total of 85 children playing badminton or table tennis or both, total number of girls in the group is 70% of the total number of boys in the group. The number of boys playing only badminton is 50% of the number of boys and the total number of boys playing badminton is 60% of the total number of boys. The number of children playing only table tennis is 40% of the total number of children and a total of 12 children play badminton and table tennis both. What is the number of girls playing only badminton?
(a) 16 (b) 14
(c) 17 (d) Data inadequate
(e) None of these
[Based on SBI Associates PO 1999]

14. If the numerator of a fraction is increased by 2 and the denominator is increased by 1, the fraction becomes $\frac{5}{8}$ and if the numerator of the same fraction is increased by 3 and the denominator is increased by 1 the fraction becomes $\frac{3}{4}$. What is the original fraction?
(a) Data inadequate (b) $\frac{2}{7}$
(c) $\frac{4}{7}$ (d) $\frac{3}{7}$
(e) None of these **[Based on Guwahati PO 1999]**

15. When 50% of one number is added to a second number, the second number increases to its four-thirds. What is the ratio between the first number and the second number?
(a) 3 : 2 (b) 3 : 4
(c) 2 : 3 (d) Data inadequate
(e) None of these **[Based on Guwahati PO 1999]**

16. If the length of a rectangle is increased by 20% and the breadth is reduced by 20%, what will be the effect on its area?
(a) 4% increase (b) 6% increase
(c) 4% decrease (d) No change
(e) None of these **[Based on Guwahati PO 1999]**

17. The difference between a discount of 35% and two successive discounts of 20% and 20% on a certain bill was Rs. 22. Find the amount of the bill.
(a) Rs. 1100 (b) Rs. 200
(c) Rs. 2200 (d) Data inadequate
(e) None of these
[Based on BSRB Mumbai PO 1999]

18. What approximate value should come in place of the question mark (?) in the following equation?
9% of 64 + 32% of 90 = ?
(a) 40 (b) 30
(c) 35 (d) 45
(e) 50 **[Based on BSRB Mumbai PO 1999]**

19. Four of the five parts numbered (1), (2), (3), (4) and (5) in the following equation are exactly equal. Which of the parts is not equal to the other four? The number of that part is the answer.
(a) 45% of 1600 + $\frac{2}{3}$ of 270
(b) 80% of 1000 + 100% of 100
(c) 140% of 500 + 150% of 160
(d) 60% of 1200 + $\frac{1}{4}$ of 720
(e) $6\frac{1}{2}$ of 200 – $\frac{1}{3}$ of 1200
[Based on BSRB Calcutta PO 1999]

20. What approximate value should come in place of the question mark (?) in the following equation?
143% of 3015 + 1974 = 9500 – ?
(a) 3500 (b) 3200
(c) 4100 (d) 3800
(e) 2800 **[Based on BSRB Calcutta PO 1999]**

21. Two-thirds of three-fifths of one-eighth of a certain number is 268.50. What is 30% of that number?
(a) 1611.0 (b) 716.0
(c) 1342.5 (d) 596.60
(e) None of these **[Based on NABARD 1999]**

22. What approximate value should come in place of the question mark (?) in the following equation?
31% of 3581 + 27% of 9319 = ?
(a) 2630 (b) 3625
(c) 2625 (d) 3635
(e) 3824 **[Based on NABARD 1999]**

23. A solution of sugar syrup has 15% sugar. Another solution has 5% sugar. How many litres of second solution must be added to 20 litres of the first solution to make a solution of 10% sugar?
(a) 10 (b) 2
(c) 15 (d) 20
(e) None of these **[Based on NABARD 1999]**

24. Ram gave 40% of the amount to Deepak. Deepak in turn gave one-fourth of what he received from Ram to Subhash. After paying Rs. 200 to taxi driver out of the amount he got from Deepak, Subhash now has Rs. 600 left with him. How much amount did Ram have?
(a) Rs. 1200 (b) Rs. 4000
(c) Rs. 8000 (d) Data inadequate
(e) None of these
[Based on BSRB Chennai PO 2000]

25. An article when sold for Rs. 960 fetches 20% profit. What would be the per cent profit/loss if 5 such articles are sold for Rs. 825 each?
(a) 3.125% profit
(b) 3.125% loss
(c) Neither profit nor loss
(d) 16·5% profit
(e) None of these **[Based on BSRB Bhopal PO 2000]**

26. Rakesh solved 80% of the questions in an examination correctly. If out of 41 questions solved by Rakesh 37 questions are correct and of the remaining questions out of 8 questions 5 questions have been solved by Rakesh correctly then find the total number of questions asked in the examination.
(a) 75 (b) 65
(c) 60 (d) Cannot be determined
(e) None of these
[Based on BSRB Bangalore PO 2000]

27. In a class of 60 children, 30% children can speak only English, 20% Hindi and English both and rest of the children can speak only Hindi. How many children can speak Hindi?
(a) 42 (b) 36
(c) 30 (d) 48
(e) None of these **[Based on BSRB Patna PO 2001]**

28. The ratio of males and females in a city is 7 : 8 and the percentage of children among males and females is 25% and 20%, respectively. If the number of adult females in the city is 156800, what is the total population?
(a) 245000 (b) 367500
(c) 196000 (d) 171500
(e) None of these **[Based on BSRB Patna PO 2001]**

29. The income of a company increases 20% per annum. If its income is Rs. 2664000 in the year 1999, what was its income in the year 1997?
(a) Rs. 2220000 (b) Rs. 2850000
(c) Rs. 2121000 (d) Rs. 1855000
(e) None of these **[Based on BSRB Patna PO 2001]**

30. Mr. *X*, a businessman, had income in the year 1995 such that he earned a profit of 20% on his investment in the business. In the year 1996 his investment was less by Rs. 5000 but still had the same income (Income = Investment + Profit) as that in 1995. Thus the per cent profit earned in 1996 increased by 26%. What was his investment in 1995?
(a) Rs. 100000 (b) Rs. 100500
(c) Rs. 105000 (d) Data inadequate
(e) None of these **[Based on SBI PO 2001]**

31. The production of a company has ups and downs every year. The production increase for two consecutive years consistently by 15% and in the third year it decrease by 10%. Again, in the next two years it increases by 15% each years and decreases by 10% in the third year. If we start counting from the year 1990 approximately what will be the effect on the production of the company in 1994?
(a) 37% increase (b) 42% increase
(c) 52% increase (d) 32% increase
(e) 27% increase
[Based on Corporation Bank PO 2002]

32. 125% of 260 + ? % of 700 = 500
(a) 32 (b) 56
(c) 25 (d) 46
(e) None of these
[Based on Andhra Bank SO 2002]

33. 45% of 750 – 25% of 480 =
(a) 216 (b) 217.50
(c) 245 (d) 236·50
(e) None of these **[Based on Andhra Bank SO 2001]**

34. If 40% of a number is equal to two-thirds of another number, what is the ratio of the first number to the second?
(a) 7 : 3 (b) 3 : 7
(c) 2 : 5 (d) 5 : 3
(e) None of these **[Based on Andhra Bank SO 2001]**

35. A shopkeeper earns a profit of 15% after selling a book at 20% discount on the printed price. The ratio of the cost price and the printed price of the book is
(a) 20 : 23 (b) 23 : 20
(c) 16 : 23 (d) 23 : 16
(e) None of these
[Based on SSC (GL) Prel. Exam 2002]

36. Successive discounts of 20% and 15% are equivalent to a single discount of
(a) 68% (b) 65%
(c) 35% (d) 32%
[Based on SSC (GL) Prel. Exam 2002]

37. The market price of a watch is Rs. 800. A shopkeeper gives two successive discounts and sells the watch for Rs. 612. If the first discount is 10%, then the second discount is
(a) 12% (b) 20%
(c) 15% (d) 10%
[Based on SSC (GL) Prel. Exam 2002]

38. 5% of 5% of Rs 100 is
(a) Re $0 \cdot 25$ (b) Re $0 \cdot 50$
(c) Rs 10 (d) Rs 25
[Based on SSC (GL) Prel. Exam 2002]

39. If 20% of an electricity bill is deducted, then Rs. 100 are still to be paid. How much was the original bill?
(a) Rs. 110 (b) Rs. 115
(c) Rs. 120 (d) Rs. 125
[Based on SSC (GL) Prel. Exam 2002]

40. If x% of y is 100 and y% of z is 200, then find a relation between x and z.
(a) $z = \frac{x}{2}$ (b) $z = 2x$
(c) $x = \frac{z}{4}$ (d) $z = 4x$
[Based on SSC (GL) Prel. Exam 2002]

41. 40% of the greater number is equal to 60% of the smaller. If their sum is 150, then the greater number is
(a) 70 (b) 80
(c) 90 (d) 60
[Based on SSC (GL) Prel. Exam 2002]

42. Two numbers are less than a third number by 30% and 37% respectively. How much percent is the second number less than the first?
(a) 7% (b) 10%
(c) 4% (d) 3%
[Based on SSC (GL) Prel. Exam 2002]

43. 8% of the voters in an election did not cast their votes. In this election, there were only two candidates. The winner by obtaining 48% of the total votes defeated his rival by 1100 votes. The total number of voters in the election was
(a) 2100 (b) 23500
(c) 22000 (d) 27500
[Based on SSC (GL) Prel. Exam 2003]

44. The present population of a city is 180000. If it increases at the rate of 10% per annum, its population after 2 years will be
(a) 207800 (b) 227800
(c) 217800 (d) 237800
[Based on SSC (GL) Prel. Exam 2003]

45. A reduction of 20% in the price of salt enabled a purchaser to obtain 4 kg more for Rs. 100. The reduced price of salt per kg is
(a) Rs. 4 (b) Rs. 5
(c) Rs. 6.25 (d) Rs. 6.50
[Based on SSC (GL) Prel. Exam 2003]

46. If 60% of *A's* income is equal to 75% of *B's* income, then *B's* income is equal to x% of *A's* income. The value of x is
(a) 70 (b) 60
(c) 80 (d) 90
[Based on SSC (GL) Prel. Exam 2003]

47. A discount of 14% on the marked price of an article is allowed and then the article is sold for Rs. 387. The marked price of the article is
(a) Rs. 450 (b) Rs. 427
(c) Rs. 500 (d) Rs. 440
[Based on SSC (GL) Prel. Exam 2003]

48. A trader marked the selling price of an article at 10% above the cost price. At the time of selling he allows certain discount and suffers a loss of 1%. He allowed a discount of
(a) 11% (b) 10%
(c) 9% (d) 10.5%
[Based on SSC (GL) Prel. Exam 2003]

49. By giving a discount of 10% on the marked price of Rs. 1100 of a cycle, a dealer gains 10%. The cost price of the cycle is
(a) Rs. 1100 (b) Rs. 900
(c) Rs. 1089 (d) Rs. 891
[Based on SSC (GL) Prel. Exam 2003]

50. A trader marks his goods at 20% above the cost price. If he allows a discount of 5% on the marked price, what profit per cent does he make?
(a) 14% (b) 16%
(c) 18% (d) 20%
[Based on SSC (GL) Prel. Exam 2003]

51. A person gave 20% of his income to his elder son, 30% of the remaining to the younger son and 10% of the balance he denoted to a trust. He is left with Rs. 10080. His income was
(a) Rs. 50000 (b) Rs. 40000
(c) Rs. 30000 (d) Rs. 20000
[Based on SSC (GL) Prel. Exam 2003]

52. In an examination there were 640 boys and 360 girls. 60% of boys and 80% of girls were successful. The percentage of failure was
(a) 20 (b) 60
(c) 30.5 (d) 32.8
[Based on SSC (GL) Prel. Exam 2003]

53. A number is increased by 10% and then decreased by 10%. Finally the number
(a) does not change (b) decreases by 1%
(c) increases by 1% (d) increases by 0.1%
[Based on SSC (GL) Prel. Exam 2003]

54. A reduction of 20% in the price of rice enables a person to buy 3·5 kg more rice for Rs 385. The original price of rice per kg is
(a) Rs. 20 (b) Rs. 22.50
(c) Rs. 25 (d) Rs. 27.50
[Based on SSC (GL) Prel. Exam 2003]

55. Difference of two numbers is 1660. If $6\frac{1}{2}$% of one number is $8\frac{1}{2}$% of the other number, the smaller number is
(a) 7055 (b) 5395
(c) 3735 (d) 2075
[Based on SSC (GL) Prel. Exam 2003]

56. The price of an article is raised by 30% and then two successive discounts of 10% each are allowed. Ultimately the price of the article is
(a) increased by 10% (b) increased by 5.3%
(c) decreased by 3% (d) decreased by 5.3%
[Based on SSC (GL) Prel. Exam 2003]

57. A tradesman gives 4% discount on the marked price and gives 1 article free for buying every 15 articles and thus gains 35%. The marked price is increased above the cost price by
(a) 40% (b) 39%
(c) 50% (d) 20%
[Based on SSC (GL) Prel. Exam 2003]

58. Radha spends 40% of her salary on food, 20% on houre rent, 10% on entertainment and 10% on conveyance. If her savings at the end of a month are Rs. 1500, then her salary per month (in Rs.) is
(a) 8000 (b) 7500
(c) 6000 (d) 10000
[Based on SSC (GL) Prel. Exam 2003]

EXPLANATORY ANSWERS

1. Ans. (e)

2. Ans. (e)

Solution Houses containing only one person

$= 100 - 40 = 60\%$

Houses containing only a male

$= 60 \times \frac{25}{100} = 15\%$

$\therefore$ Houses containing only one female

$= 60 - 15 = 45\%$.

3. Ans. (a)

Solution $? \approx 160\%$ of $6530 + 5{\cdot}5 \times 1010 - 5965$

$\approx 10448 + 5555 - 5965 = 10000$.

4. Ans. (d)

Solution $? \approx \frac{18}{5} \times 160 + \frac{40}{100} \times 1850 + 450$

$\approx 576 + 704 + 450$

≈ 1730

≈ 1800.

5. Ans. (d)

Solution Required effect $= \left(+40 - 40 - \frac{40 \times 40}{100}\right)\%$

$= -16\%$.

i.e. the area will decrease by 16%.

6. Ans. (a)

Solution In 1 kg mixture quantity of iron = 20 gm

Let x gm sand should be added, then 10% of $(1000 + x) = 200$

$\therefore \quad x = 1000$ gm $= 1$ kg.

7. Ans. (d)

Solution $? = 35\%$ of $1478 + 29\%$ of 3214

$\approx 35\%$ of $(1500 - 22) + (30\% - 1\%)$ of 3200

$\approx 35 \times 15 - 8 + 30 \times 32 - 32$

$= 525 + 960 - 40$

$= 525 + 920 = 1445$

≈ 1450.

8. Ans. (a)

Solution $\frac{a}{b} = \frac{5}{4}$, $b = \frac{4}{5}a$ (Given)

Given, (40% of a) $= \frac{2}{5}a = 12$

$\therefore \quad a = 5 \times 6$ and $b = \frac{4}{5} \times 5 \times 6 = 24$

$\therefore \quad 50\%$ of $b = \frac{24}{2} = 12$.

9. Ans. (b)

Solution Let the first and the second number be x and y respectively, then

$y + 30\%$ of $x = 140\%$ of y

or, $y + 0.3x = 1.4y$

or, $0.3x = 0.4y$

$\therefore \quad x : y = 4 : 3$

10. Ans. (a)

Solution Suresh = Vinod + 30% of Vinod

= 1·3 Vinod

Vinod = Vinay – 20% of Vinay

= 80% of Vinay

= 0.8 Vinay

$\therefore$ Suresh = 1.3 × 0.8 Vinay

= 1.04 Vinay

Now, Suresh – Vinay = 1.04 Vinay – Vinay

= 0.04 Vinay

= Rs. 800 (given)

$\therefore$ Vinay = Rs. 20000

$\therefore$ Vinod = 0.8 × 20000 = Rs. 16000.

11. Ans. (a)

Solution Let the first and second numbers be x and y, respectively.

$$y - x \times \frac{25}{100} = y \times \frac{5}{6}$$

or, $$y - \frac{x}{4} = \frac{5}{6}y \quad \text{or,} \quad \frac{1}{6}y = \frac{x}{4}$$

$\therefore \quad x : y = 2 : 3.$

12. Ans. (a)

Solution In 1 litre mixture quantity of unleaded petrol = 100 ml

Let x ml leaded petrol be added, then 5% of $(1000 + x)$ = 100 ml

or, $5(1000 + x) = 100 \times 100 \Rightarrow x = \frac{5000}{5} = 1000$ ml.

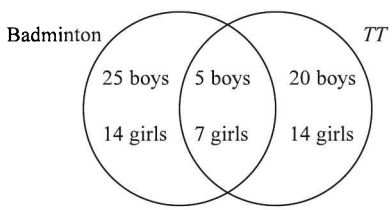
13. Ans. (b)

Solution Let the number of boys = x

Then, $x + \frac{7x}{10} = 85$

$\Rightarrow \quad x = 50$

No. of girls = 85 – 50 = 35.



14. Ans. (d)

Solution Let the original fraction be $\frac{x}{y}$

Then, $\frac{x+2}{y+1} = \frac{5}{8}$ or, $8x - 5y = -11$ ...(i)

Again, $\frac{x+3}{y+1} = \frac{3}{4}$ or, $4x - 3y = -9$ ...(ii)

Solving, we get $x = 3$ and $y = 7$

$\therefore$ Fraction = $\frac{3}{7}$

15. Ans. (c)

Solution Let the numbers be y and x, respectively.

$$x + 50\% \text{ of } y = \frac{4x}{3} \text{ or, } \frac{y}{2} = \frac{4x}{3} - x$$

or, $$\frac{y}{2} = \frac{x}{3} \text{ or, } \frac{y}{x} = \frac{2}{3}.$$

16. Ans. (c)

Solution Percentage change $= x - y - \frac{xy}{100}$

$= 20 - 20 - \frac{20 \times 20}{100}$

$= -4\%$

= 4% decrease.

17. Ans. (c)

Solution Successive discounts $= 20\% + \frac{20 \times 80}{100}$

$= 20 + 16 = 36\%$

Difference in discounts = 36 – 35 = 1%

$\therefore$ Bill amount = 22 × 100

= Rs. 2200.

18. Ans. (c)

Solution $? = \frac{9}{100} \times 64 + \frac{32}{100} \times 90$

$= 34.56$

$\approx 35.$

19. Ans. (c)

Solution Others are equal to 900

20. Ans. (b)

Solution ? = 9500 – 4300 – 2000 = 3200

21. Ans. (a)

Solution Let the number be x

According to the question,

$$\frac{2}{3} \text{ of } \frac{3}{5} \text{ of } \frac{1}{8} \times x = 268.50$$

$\therefore \quad x = \frac{268 \cdot 50 \times 3 \times 5 \times 8}{2 \times 3} = 5370$

$\therefore \quad 30\% \text{ of } x = \frac{30}{100} \times 5370 = 1611.$

22. Ans. (b)

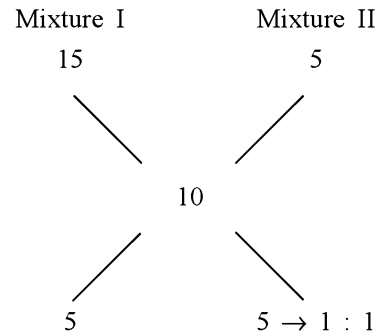
Solution ? = 31% of 3581 + 27% of 9319

$= 1110 \cdot 11 + 2516 \cdot 13$

$\approx 3625.$

23. Ans. (d)

Solution By Alligation rule,



$\therefore$ 20 litres of the 2nd solution be added.

24. Ans. (c)

Solution $D = \frac{2}{5}R \qquad S = \frac{1}{4} \times \frac{2}{5}R = \frac{1}{10}R$

and, $\frac{1}{10}R - 200 = 600$

$\therefore \quad \frac{1}{10}R = 800$

$\therefore \quad R = \text{Rs. } 8000$

25. Ans. (a)

Solution C.P. of the article $= 960 \times \frac{100}{120} = \text{Rs. } 800$

$\therefore$ C.P. of 5 articles = Rs. 800 × 5 = Rs. 4000

$\therefore$ S.P. of 5 articles = Rs. 825 × 5 = Rs. 4125

$\therefore$ Gain % $= \frac{4125-4000}{4000} \times 100 = 3.125\%$.

26. Ans. (b)

Solution Suppose there are $8x$ questions apart from 41 questions. Then,

$$\frac{37+5x}{41+8x} = 80\% = \frac{4}{5} \Rightarrow 185 + 25x = 164 + 32x$$

$$\Rightarrow 7x = 21 \Rightarrow x = 3$$

$\therefore$ Total number of questions $= 41 + 8x$

$= 41 + 8 \times 3$

$= 41 + 24 = 65$.

27. Ans. (a)

Solution Number of students who speak only English

= 30% of 60 = 18

Number of students who speak Hindi and English

= 20% of 60 = 12

$\therefore$ Number of students who speak only Hindi

= (60 – 30) = 30

$\therefore$ Number of students who speak Hindi

= 30 + 12 = 42.

28. Ans. (b)

Solution Number of females $= 156800 \times \frac{100}{80} = 196000$

$\therefore$ Number of males $= \frac{7}{8} \times 196000 = 171500$

$\therefore$ Total population = 196000 + 171500 = 367500.

29. Ans. (e)

Solution Income of company in 1997

$$= \frac{2664000}{\left(1+\frac{20}{100}\right)^2}$$

$$= 2664000 \times \frac{25}{36} = \text{Rs. } 1850000.$$

30. Ans. (c)

Solution Let the investment of X in 1995 be Rs. x

$\therefore$ Profit = Rs. $\frac{x}{5}$ $\therefore$ Income = Rs. $\left(x+\frac{x}{5}\right)$ = Rs. $\frac{6x}{5}$

Investment of company X in 1996 would be $(x - 5000)$

From the question,

$$(x - 5000) \times \frac{126}{100} = \frac{6}{5}x \Rightarrow x = \text{Rs. } 105000.$$

31. Ans. (a)

Solution Suppose the production of the company in the year 1990 be x

Then, production of the company in year 1994

$$= x \times \frac{115}{100} \times \frac{115}{100} \times \frac{90}{100} \times \frac{115}{100}$$

$$= 1{\cdot}368x$$

$\therefore$ Increase % in the production in year 1994

$$= \frac{(1{\cdot}368x - x) \times 100}{x}$$

$$= 36.8\%$$

$$\approx 37\%.$$

32. Ans. (c)

Solution ? % of 700 = 500 – 125% of 260

$$? \% \text{ of } 700 = 500 - \frac{125}{100} \times 260$$

$$\therefore \quad ? = \frac{175 \times 100}{700} = 25.$$

33. Ans. (b)

Solution 45% of 750 – 25% of 480

$$= \frac{45}{100} \times 750 - \frac{25}{100} \times 480$$

$$= 337{\cdot}50 - 120$$

$$= 217.5.$$

34. Ans. (d)

Solution Suppose the first number is x and the second number is y

Therefore, 40% of $x = \frac{2}{3}$ of y

$$\therefore \quad \frac{x}{y} = \frac{2}{3} \times \frac{100}{40} = \frac{5}{3}.$$

35. Ans. (c)

Solution Let the printed price be Rs. x

Discount = 20%

$\therefore$ S.P. $= \frac{4x}{5}$

Profit = 15%

$$\therefore \quad \text{C.P.} = \frac{100 \times \frac{4x}{5}}{100+15} = \frac{80x}{115} = \frac{16x}{23}$$

$$\therefore \quad \frac{\text{C.P.}}{\text{Printed Price}} = \frac{16}{23}.$$

36. Ans. (d)

Solution $-20 - 15 + \frac{(-20)\times(-15)}{100} = -35 + 3 = -32$

$\therefore$ Two successive discounts of 20% and 15% are equivalent to a single discount of 32%.

37. Ans. (c)

Solution Let the second discount be x%

$\therefore$ 800 – 10% = 720

720 – x% of 720 = 612

$\Rightarrow \frac{x}{100} \times 720 = 108$

$\Rightarrow x = \frac{108\times100}{720} = 15.$

38. Ans. (a)

Solution 5% of (5% of 100) = 5% of 5

$= \frac{1}{4} = 0.25.$

39. Ans. (d)

Solution Let the original bill was for Rs. x

$\therefore$ x – 20% of x = 100

$\Rightarrow \frac{4x}{5} = 100 \Rightarrow x = 125.$

40. Ans. (b)

Solution x% of y = 100, y% of z = 200

$\Rightarrow xy = 10000,\ yz = 20000$

$\Rightarrow \frac{xy}{yz} = \frac{10000}{20000} = \frac{1}{2}$

$\Rightarrow z = 2x.$

41. Ans. (c)

Solution Let the numbers be x and y

$\therefore$ $x + y = 150$

40% of x = 60% of $y \Rightarrow 2x = 3y$

$\therefore \frac{3y}{2} + y = 150 \Rightarrow 5y = 300 \Rightarrow y = 60$

$\Rightarrow x = 90$

Hence, the greater number = 90.

42. Ans. (b)

Solution Let the numbers be x, y and z

$\therefore$ $x = z$ – 30% of $z = \frac{7z}{10}$

$y = z$ – 37% of $z = \frac{63z}{100}$

$\Rightarrow \frac{10x}{7} = \frac{100y}{63} \Rightarrow x = \frac{10y}{9}$

$\Rightarrow y = \frac{9x}{10} = x - \frac{x}{10}$

$\Rightarrow y = x$ – 10% of x.

43. Ans. (d)

Solution 8% votes are invalid.

Winner got 48% of the total votes.

Loser will get

100 – (8 + 48) = 44% of the total votes

Now, the total number of voters in the election

$= \frac{1100}{4} \times 100 = 27500.$

44. Ans. (c)

Solution Population of a city after 2 years

$= 180000\left(1+\frac{10}{100}\right)^2 = 217800.$

45. Ans. (b)

Solution Reduced Price = $\frac{20\% \text{ of } 100}{4}$ = Rs. 5 per kg.

46. Ans. (c)

Solution 60% of A = 75% of B or, $\frac{3A}{5} = \frac{3B}{4}$

$\Rightarrow \frac{A}{B} = \frac{5}{4}$

Now, B = x% of A

or, $x = \frac{B}{A} \times 100 = \frac{4}{5} \times 100 = 80\%.$

47. Ans. (a)

Solution Marked price of the article

$= \frac{387}{100-14} \times 100 = \text{Rs } 450.$

48. Ans. (b)

Solution Let the C.P. of the article be Rs. 100, then M.P. of the article = Rs. 110

If the trader has a loss of 1%, it means that the trader sold the article at Rs. 99

$\therefore$ % discount = $\frac{110-99}{110} \times 100 = 10\%.$

49. Ans. (b)

Solution C.P. of cycle = $1100 \times \frac{90}{100} \times \frac{100}{110}$ = Rs. 900.

50. Ans. (a)

Solution Profit % = $+20 - 5 - \frac{20\times5}{100} = 15 - 1 = 14\%.$

51. Ans. (d)

Solution Let the income = Rs. x

Given: $\frac{90}{100}\times\frac{70}{100}\times\frac{80}{100}x = 10080$

$\Rightarrow$ $504x = 10080000$

or, x = Rs. 20000.

52. Ans. (d)

Solution Number of failure students

= 40% of 640 + 20% of 360

= 256 + 72 = 328

% of failure = $\frac{328}{1000} \times 100 = 32{\cdot}80\%$.

53. Ans. (b)

54. Ans. (d)

Solution Let the original price of rice per kg be Rs. x

Now, $\frac{385}{4x/5} - \frac{385}{x} = 3.5$ or, x = Rs. 27·50.

55. Ans. (b)

Solution Ratio of two numbers = $\frac{13}{2}\% : \frac{17}{2}\% = 13 : 17$

Smaller number = $\frac{13}{4} \times 1660 = 5395$.

56. Ans. (b)

Solution Two successive discounts of 10%

$= -10 - 10 + \frac{(-10)\times(-10)}{100} = -19\%$

So, resultant = $+ 30 - 19 + \frac{30\times(-19)}{100} = +5{\cdot}3\%$

57. Ans. (c)

Solution Let C.P. of one article be Rs. x

$\therefore$ C.P. of 16 articles = Rs. $16x$

S.P. of 16 articles = Rs. $16x\left(\frac{135}{100}\right)$

Let the marked price of the article be increased by y% above the cost price.

Then, $15x\left(\frac{100+y}{100}\right)\times\left(\frac{96}{100}\right) = 16x\left(\frac{135}{100}\right)$

or, $(100 + y)6 = 900$ or, $y = 150$

i.e. M.P. of the article is 50% above the cost price.

58. Ans. (b)

Solution Salary = $\frac{1500}{20} \times 100$ = Rs 7500.

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. In a certain examination there were 2500 candidates of whom 20% were girls and the rest boys. Suppose 5% of boys and 40% of girls failed. The percentage of candidates who passed was
(a) 78% (b) 88%
(c) 68% (d) 70%

2. In an election between two candidates, one got 55% of the total valid votes, 20% of the votes were invalid. If the total number of votes was 7500, the number of valid votes that the other candidate got was
(a) 2700 (b) 2900
(c) 3000 (d) 3100

3. The population of a town increases 4% annually but is decreased by emigration annually to the extent of $\frac{1}{2}$%. What will be the increase per cent in three years?
(a) 9.8 (b) 10
(c) 10.5 (d) 10.8

4. The current birth rate per thousand is 32 whereas corresponding death rate is 11 per thousand. The net growth rate in terms of population increase in percent is given by
(a) 0.021% (b) 0.0021%
(c) 21% (d) 2.1%

5. A man loses $12\frac{1}{2}$% of his money and after spending 70% of the remainder, is left with Rs. 210. Find the amount of money he had.
(a) Rs. 800 (b) Rs. 600
(c) Rs. 840 (d) Rs. 900

6. The value of a machine depreciates from Rs 32768 to Rs. 21952 in three years. What is the rate percentage of depreciation?
(a) 11% (b) 12.5%
(c) 33% (d) 12.25%

7. A monthly return railway ticket costs 25 per cent more than a single ticket. A week's extension can be had for the former by paying 5 per cent of the monthly ticket's cost. If the money paid for the monthly ticket (with extension) is Rs 84, the price of the single ticket is

(a) Rs. 48 (b) Rs. 80
(c) Rs. 64 (d) Rs. 72

8. Vineet spends 40% of his salary on food articles and $\frac{1}{3}$rd of the remaining on transport. If he saves Rs. 450 per month, which is half of the balance after spending on food items and transport, what is his monthly salary?
(a) Rs. 1125 (b) Rs. 2250
(c) Rs. 2500 (d) Rs. 4500

9. A mixture of 50 litres of milk and water contains 10% water. The quantity of water to be added, so that new mixture contains 20% of water, is
(a) 6·25 litres (b) 6 litres
(c) 5·4 litres (d) None of these

10. The price of a commodity is raised by 20%. For the expenditure not to increase, the consumption must be reduced in the ratio :
(a) 1 : 2 (b) 1 : 3
(c) 1 : 6 (d) 1 : 4

Test II

[Time: 10 min]

11. The price of tea increases from Rs. 8 per kg to Rs. 9.50 per kg. How much consumption of tea should be reduced by a housewife so that the expenditure on it may not increase?
(a) 16% (b) $15\frac{15}{19}\%$
(c) $14\frac{4}{5}\%$ (d) 15%.

12. Due to increase of 15% in the price of milk, a family reduces its consumption of milk by 15%. What was the effect on expenditure of that family on account of milk?
(a) 2.50% decrease (b) 2.25% decrease
(c) 3% decrease (d) 3.5% decrease

13. In an examination, 20% students passed in 1st division, 25% in 2nd division and 45% passed in 3rd division. If 124 students failed in the examination, what is the total number of students who appeared in the examination?
(a) 1216 (b) 1240
(c) 800 (d) 1364

14. An author gets a royalty of 13% on the printed price of a book written by him. If the printed price of a copy of his book is Rs. 4.50 and 600 copies of the book are sold, how much royalty did he earn?
(a) Rs 2349 (b) Rs 2700
(c) Rs 312 (d) Rs 351

15. A papaya tree was planted 2 years ago. It increases at the rate of 20% every year. If at present, the height of the tree is 540 cm, what was it when the tree was planted?
(a) 324 cm (b) 375 cm
(c) 400 cm (d) 432 cm

16. One litre of water is evaporated from 6 litres of a solution containing 5% salt. The percentage of salt in the remaining solution is:
(a) $4\frac{4}{9}\%$ (b) $5\frac{5}{7}\%$
(c) 5% (d) 6%

17. A mixture of 40 litres of milk and water contains 10% water. How much water should be added to this so that water may be 20% in the new mixture?
(a) 4 litres (b) 5 litres
(c) 6.5 litres (d) 7.5 litres

18. In an examination of n questions, a student answered 15 of the first 20 correctly. Of the remaining questions he answered one-third correctly. All the questions have the same credit. If the student scored 50% marks, how many different values of n can be there?
(a) 4 (b) 3
(c) 2 (d) 1

19. The amount of water (in ml) that should be added to reduce 9 ml lotion, containing 50% alcohol, to a lotion containing 30% alcohol, is
(a) 3 ml (b) 4 ml
(c) 5 ml (d) 6 ml

20. A person saves every year 20% of his income, if his income increases every year by 10%, then his savings increase every year by
(a) 10% (b) 5%
(c) 2% (d) 1%

Answers

1. (b)	**2.** (a)	**3.** (d)	**4.** (d)	**5.** (a)	**6.** (b)	**7.** (c)	**8.** (b)	**9.** (a)	**10.** (c)
11. (b)	**12.** (b)	**13.** (b)	**14.** (d)	**15.** (b)	**16.** (d)	**17.** (b)	**18.** (d)	**19.** (d)	**20.** (a)

6

AVERAGE

INTRODUCTION

Whenever we are asked the marks scored by us in any examination, we usually tell the marks in percentage, taking the percentage of total marks of all subjects. This percentage is called *average percentage*. Also, in a class, if there are 100 students, instead of knowing the age of individual student, we usually talk about average age.

The *average* or *mean* or *arithmetic mean* of a number of quantities of the same kind is equal to their sum divided by the number of those quantities. For example, the average of 3, 9, 11, 15, 18, 19 and 23 is

$$\frac{3+9+11+15+18+19+23}{7} = \frac{98}{7} = 14.$$

SOME BASIC FORMULAE

1. Average $= \dfrac{\text{Sum of quantities}}{\text{Number of quantities}}$
2. Sum of quantities = Average × Number of quantities
3. Number of quantities $= \dfrac{\text{Sum of quantities}}{\text{Average}}$

Illustration 1 A man purchased 5 toys at the rate of Rs 200 each, 6 toys at the rate of Rs 250 each and 9 toys at the rate of Rs 300 each. Calculate the average cost of one toy.

Solution Price of 5 toys = 200 × 5 = Rs 1000

Price of 6 toys = 250 × 6 = Rs 1500

Price of 9 toys = 300 × 9 = Rs 2700

Total number of toys = 5 + 6 + 9 = 20.

$$\text{Average price of 1 toy} = \frac{1000+1500+2700}{20}$$

$$= \frac{5200}{20} = \text{Rs } 260.$$

Illustration 2 The average marks obtained by 200 students in a certain examination is 45. Find the total marks.

Solution Total marks

= Average marks × Number of students

= 200 × 45 = 900.

Illustration 3 Total temperature for the month of September is 840°C. If the average temperature of that month is 28°C, find of how many days is the month of September.

Solution Number of days in the month of September

$$= \frac{\text{Total temperature}}{\text{Average temperature}} = \frac{840}{28}$$

= 30 days.

SOME USEFUL SHORT-CUT METHODS

1. Average of two or more groups taken together
   (a) If the number of quantities in two groups be n_1 and n_2 and their average is x and y, respectively, the combined average (average of all of them put together) is

$$\frac{n_1x+n_2y}{n_1+n_2}$$

Explanation

No. of quantities in first group = n_1

Their average = x

∴ Sum = $n_1 \times x$

No. of quantities in second group = n_2

Their average = y

∴ Sum = $n_2 \times y$

No. of quantities in the combined group = $n_1 + n_2$

Total sum (sum of quantities of first group and second group)

$$= n_1x + n_2y.$$

$\therefore$ Average of the two groups

$$= \frac{n_1x + n_2y}{n_1 + n_2}.$$

(b) If the average of n_1 quantities is x and the average of n_2 quantities out of them is y, the average of remaining group (rest of the quantities) is

$$\frac{n_1x - n_2y}{n_1 - n_2}.$$

Explanation

No. of quantities = n_1

Their average = x

$\therefore$ Sum = $n_1 x$

No. of quantities taken out = n_2

Their average = y

$\therefore$ Sum = n_2y

Sum of remaining quantities = $n_1x - n_2y$

No. of remaining quantities = $n_1 - n_2$

$\therefore$ Average of remaining group = $\dfrac{n_1x - n_2y}{n_1 - n_2}$.

Illustration 4 The average weight of 24 students of section A of a class is 58 kg whereas the average weight of 26 students of section B of the same class is 60.5 kg. Find the average weight of all the 50 students of the class.

Solution Here, $n_1 = 24$, $n_2 = 26$, $x = 58$ and $y = 60.5$.

$\therefore$ Average weight of all the 50 students

$$= \frac{n_1x + n_2y}{n_1 + n_2}$$

$$= \frac{24 \times 58 + 26 \times 60.5}{24 + 26}$$

$$= \frac{1392 + 1573}{50} = \frac{2965}{50} = 59.3 \text{ kg.}$$

Illustration 5 Average salary of all the 50 employees including 5 officers of a company is Rs 850. If the average salary of the officers is Rs 2500, find the average salary of the remaining staff of the company.

Solution Here, $n_1 = 50$, $n_2 = 5$, $x = 850$ and $y = 2500$.

$\therefore$ Average salary of the remaining staff

$$= \frac{n_1x - n_2y}{n_1 - n_2} = \frac{50 \times 850 - 5 \times 2500}{50 - 5}$$

$$= \frac{42500 - 12500}{45} = \frac{30000}{45}$$

= Rs 667 (approx.)

2. If $\bar{x}$ is the average of $x_1, x_2, \ldots, x_n$, then
   (a) The average of $x_1 + a, x_2 + a, \ldots, x_n + a$ is $\bar{x} + a$.
   (b) The average of $x_1 - a, x_2 - a, \ldots x_n - a$ is $\bar{x} - a$.
   (c) The average of $ax_1, ax_2, \ldots, ax_n$ is $a\bar{x}$, provided $a \neq 0$.
   (d) The average of $\dfrac{x_1}{a}, \dfrac{x_2}{a}, \ldots, \dfrac{x_n}{a}$ is $\dfrac{\bar{x}}{a}$, provided $a \neq 0$.

Illustration 6 The average value of six numbers 7, 12, 17, 24, 26 and 28 is 19. If 8 is added to each number, what will be the new average?

Solution The new average = $\bar{x} + a$

$= 19 + 8 = 27.$

Illustration 7 The average of x numbers is $5x$. If $x - 2$ is subtracted from each given number, what will be the new average?

Solution The new average= $\bar{x} - a$

$= 5x - (x - 2) = 4x + 2.$

Illustration 8 The average of 8 numbers is 21. If each of the numbers is multiplied by 8, find the average of a new set of numbers.

Solution The average of a new set of numbers

$= a\bar{x} = 8 \times 21 = 168.$

3. The average of n quantities is equal to x. If one of the given quantities whose value is p, is replaced by a new quantity having value q, the average becomes y, then $q = p + n(y - x)$

Illustration 9 The average weight of 25 persons is increased by 2 kg when one of them whose weight is 60 kg, is replaced by a new person. What is the weight of the new person?

Solution The weight of the new person

$= p + n(y - x)$

$= 60 + 25(2) = 110$ kg.

4. (a) The average of n quantities is equal to x. When a quantity is removed, the average becomes y. The value of the removed quantity is $n(x - y) + y$.
   (b) The average of n quantities is equal to y. When a quantity is added, the average becomes y. The value of the new quantity is $n(y - x) + y$.

Illustration 10 The average age of 24 students and the class teacher is 16 years. If the class teacher's age is excluded, the average age reduces by 1 year. What is the age of the class teacher?

Solution The age of class teacher

$= n(x - y) + y$

$= 25(16 - 15) + 15 = 40$ years.

Illustration 11 The average age of 30 children in a class is 9 years. If the teacher's age be included, the average age becomes 10 years. Find the teacher's age.

Solution The teacher's age

$= n(y - x) + y$

$= 30(10 - 9) + 100 = 40$ years.

5. (a) The average of first n natural numbers is $\frac{n+1}{2}$.
   (b) The average of square of natural numbers till n is $\frac{(n+1)(2n+1)}{6}$.
   (c) The average of cubes of natural numbers till n is $\frac{n(n+1)^2}{4}$.
   (d) The average of odd numbers from 1 to n is $\frac{\text{last odd number} + 1}{2}$.
   (e) The average of even numbers from 1 to n is $\frac{\text{last even number} + 2}{2}$.

Illustration 12 Find the average of first 81 natural numbers.

Solution The required average

$$= \frac{n+1}{2} = \frac{81+1}{2} = 41.$$

Illustration 13 What is the average of squares of the natural numbers from 1 to 41?

Solution The required average

$$= \frac{(n+1)(2n+1)}{6} = \frac{(41+1)(2\times 41+1)}{6}$$

$$= \frac{42\times 83}{6} = \frac{3486}{6} = 581.$$

Illustration 14 Find the average of cubes of natural numbers from 1 to 27.

Solution The required average

$$= \frac{n(n+1)^2}{4} = \frac{27\times(27+1)^2}{4}$$

$$= \frac{27\times 28\times 28}{4} = \frac{21168}{4} = 5292.$$

Illustration 15 What is the average of odd numbers from 1 to 40?

Solution The required average

$$= \frac{\text{last odd number} + 1}{2} = \frac{39+1}{2} = 20.$$

Illustration 16 What is the average of even numbers from 1 to 81?

Solution The required average

$$= \frac{\text{last even number} + 2}{2} = \frac{80+2}{2} = 41.$$

6. (a) If n is odd: The average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the middle number.
   (b) If n is even: The average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the average of the middle two numbers.
   (c) The average of first n consecutive even numbers is $(n + 1)$.
   (d) The average of first n consecutive odd numbers is n.
   (e) The average of squares of first n consecutive even numbers is $\frac{2(n+1)(2n+1)}{3}$.
   (f) The average of squares of consecutive even numbers till n is $\frac{(n+1)(n+2)}{3}$.
   (g) The average of squares of consecutive odd numbers till n is $\frac{n(n+2)}{3}$.
   (h) If the average of n consecutive numbers is m, then the difference between the smallest and the largest number is $2(n - 1)$.

Illustration 17 Find the average of 7 consecutive numbers 3, 4, 5, 6, 7, 8, 9.

Solution The required average = middle number = 6.

Illustration 18 Find the average of consecutive odd numbers 21, 23, 25, 27, 29, 31, 33, 35.

Solution The required average

= average of middle two numbers

= average of 27 and 29

$$= \frac{27+29}{2} = 28.$$

Illustration 19 Find the average of first 31 consecutive even numbers.

Solution The required average = $(n + 1) = 31 + 1 = 32$.

Illustration 20 Find the average of first 50 consecutive odd numbers.

Solution The required average = $n = 50$.

Illustration 21 Find the average of squares of first 19 consecutive even numbers.

Solution The required average $= \dfrac{2(n+1)(2n+1)}{3}$

$$= \frac{2(19+1)(2\times 19+1)}{3}$$

$$= \frac{2\times 20\times 39}{3}$$

$$= \frac{1560}{3} = 520.$$

Illustration 22 Find the average of squares of consecutive even numbers from 1 to 25.

Solution The required average

$$= \frac{(n+1)(n+2)}{3} = \frac{(25+1)(25+2)}{3}$$

$$= \frac{26\times 27}{3} = \frac{702}{3} = 234.$$

Illustration 23 Find the average of squares of consecutive odd numbers from 1 to 31.

Solution The required average

$$= \frac{n(n+2)}{3} = \frac{31\times(31+2)}{3}$$

$$= \frac{31\times 33}{3} = 341.$$

Illustration 24 If the average of 6 consecutive numbers is 48, what is the difference between the smallest and the largest number?

Solution The required difference

$$= 2(n-1) = 2(6-1) = 10.$$

7. Geometric Mean or Geometric Average

   Geometric mean of $x_1, x_2, \ldots, x_n$ is denoted by

   $$\text{G.M.} = \sqrt[n]{x_1 \times x_2 \times \ldots \times x_n}.$$

   Geometric mean is useful in calculating averages of ratios such as average population growth rate, average percentage increase and so on.

Illustration 25 The production of a company for three successive years has increased by 10%, 20% and 40%, respectively. What is the average annual increase of production?

Solution Geometric mean of x, y and $z = (x \times y \times z)^{1/3}$.

$\therefore$ Average increase $= (10 \times 20 \times 40)^{1/3}\% = 20\%$.

Illustration 26 The population of a city in two successive years increases at the rates of 16% and 4%, respectively. Find the average increase of two years.

Solution In case of population increase, the geometric mean is required.

$\therefore$ Geometric mean of 16% and 4% is

$= (16 \times 4)^{1/2}\%$, i.e. 8%.

8. Harmonic Mean or Harmonic Average

   Harmonic means of $x_1, x_2, \ldots, x_n$ is denoted by

   $$\text{H.M.} = \frac{1}{\frac{1}{n}\left(\frac{1}{x_1}+\frac{1}{x_2}+\ldots+\frac{1}{x_n}\right)}$$

   Harmonic mean is useful for finding out average speed of a vehicle, average production per day and so on.

Illustration 27 A man runs 1 km at 15 km per hour and another 1 km he walks at 5 km per hour. Find his average speed for the whole distance in covering 2 km.

Solution When the distance is constant and the speed varies, harmonic mean is used. Harmonic mean of x and y is $\dfrac{2}{\frac{1}{x}+\frac{1}{y}}$ or $\dfrac{2xy}{x+y}$.

$\therefore$ Average speed for the whole distance

$$= \frac{2\times 15\times 5}{15+5} = 7.5 \text{ km/hr.}$$

9. If a certain distance is covered at a speed of x km/hr and the same distance is covered at a speed of y km/hr, the average speed during the entire journey is

   $$\left(\frac{2xy}{x+y}\right)\text{km/hr.}$$

Illustration 28 If half of the journey is travelled at a speed of 15 km/hr and the next half at a speed of 12 km/hr, find the average speed during the entire journey

Solution The average speed

$$= \left(\frac{2xy}{x+y}\right) = \left(\frac{2\times 15\times 12}{15+12}\right)$$

$$= \frac{360}{27} = 13\frac{1}{3} \text{ km/hr.}$$

Illustration 29 A man goes to a certain place at a speed of 30 km/hr and returns to the original place at a speed of 20 km/hr, find the average speed during up and down journey.

Solution The average speed

$$= \left(\frac{2xy}{x+y}\right) = \left(\frac{2\times 30\times 20}{30+20}\right) = \frac{1200}{50}$$

$$= 24 \text{ km/hr.}$$

10. If a person or a motor car covers three equal distances at the speed of x km/hr, y km/hr and z km/hr, respectively, then for the entire journey average speed of the person or motor car is

    $$\left(\frac{3xzy}{xy+yz+zx}\right)\text{km/hr.}$$

Illustration 30 A train covers the first 160 kms at a speed of 120 km/hr, another 160 kms at 140 km/hr and the last 160 kms at 80 km/hr. Find the average speed of the train for the entire journey.

Solution Average speed $= \dfrac{3xyz}{xy+yz+zx}$

$$= \frac{3\times120\times140\times80}{120\times140+140\times80+80\times120}$$

$$= \frac{360\times140\times80}{16800+11200+9600} = \frac{4032000}{37600}$$

$$= 107\frac{11}{47} \text{ km/hr.}$$

11. If a person covers A km at a speed of x km/hr, B km at a speed of y km/hr and C km at a speed of z km/hr, the average speed during the entire journey is

$$\left(\frac{A+B+C}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\right)\text{km/hr.}$$

Illustration 31 A person covers 9 km at a speed of 3 km/hr, 25 km at a speed of 5 km/hr and 30 km at a speed of 10 km/hr. Find the average speed for the entire journey.

Solution The average speed $= \left(\dfrac{A+B+C}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\right)$

$$= \left(\frac{9+25+30}{\frac{9}{3}+\frac{25}{5}+\frac{30}{10}}\right)$$

$$= \frac{64}{11} = 5\frac{9}{11} \text{ km/hr.}$$

12. If a person covers Ath part of the distance at x km/hr, Bth part of the distance at y km/hr and the remaining Cth part at z km/hr, then the average speed during the entire journey is

$$\left(\frac{1}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\right)\text{km/hr.}$$

Illustration 32 A person covers the first $\frac{1}{4}$th of the distance at 8 km/hr, the next $\frac{3}{5}$th at 6 km/hr and the remaining distance at 15 km/hr. Find the average speed during the entire journey.

Solution The average speed

$$= \frac{1}{\left(\frac{A}{x}+\frac{B}{y}+\frac{C}{z}\right)}$$

$$= \left(\frac{1}{\frac{1/4}{8}+\frac{3/5}{6}+\frac{3/20}{15}}\right)$$

$$\left[\text{Here, } A=\frac{1}{4}, B=\frac{3}{5} \text{ and } C=1-\left(\frac{1}{4}+\frac{3}{5}\right)=\frac{3}{20}\right]$$

$$= \frac{1}{\frac{1}{32}+\frac{1}{10}+\frac{1}{100}} = \frac{3200}{452} = 7\frac{9}{113}\text{km/hr.}$$

Illustration 33 A train covers 50% of the journey at 30 km/hr, 25% of the journey at 25 km/hr and the remaining at 20 km/hr. Find the average speed of the train during the entire journey.

Solution The average speed

$$= \left(\frac{100}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\right) = \left(\frac{100}{\frac{50}{30}+\frac{25}{25}+\frac{25}{20}}\right)$$

[Here, A = 50, B = 25 and C = 25]

$$= \frac{100}{47/12} = \frac{1200}{47}$$

$$= 25\frac{25}{47}\text{km/hr.}$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. The daily earnings of a taxi driver during a week are : Rs 60, Rs 65, Rs 70, Rs 52. 50, Rs 63, Rs 73 and Rs 68. What is his average daily earning for the week?
(a) Rs 74.50 (b) Rs 54.50
(c) Rs 64.50 (d) Rs 84.50

2. The average of 10 numbers is 7. What will be the new average if each of the numbers is multiplied by 8?
(a) 45 (b) 52
(c) 56 (d) 55

3. The average weight of 5 persons sitting in a boat is 38 kg. If the average weight of the boat and the persons sitting in the boat is 52 kg, what is the weight of the boat?
(a) 228 kg (b) 122 kg
(c) 232 kg (d) 242 kg

4. There are 35 students in a hostel. If the number of students increased by 7, the expenses of the mess were increased by Rs 42 per day while the average expenditure per head diminished by Re 1. Find the original expenditure of the mess.
(a) Rs 480 (b) Rs 440
(c) Rs 520 (d) Rs 420

5. The daily maximum temperature in Delhi for 7 consecutive days in May 1988 were 42.7°C, 44.6°C, 42.0°C, 39.1°C, 43.0°C, 42.5°C and 38.5°C. Find the average daily maximum temperature.
(a) 42.63° C (b) 45.65° C
(c) 41.77° C (d) 39.60° C

6. The average salary per head of all the workers in a workshop is Rs 850. If the average salary per head of 7 technicians is Rs 1000 and the average salary per head of the rest is Rs 780, find the total number of workers in the workshop.
(a) 26 (b) 24
(c) 28 (d) 22

7. An aeroplane travels distances 2500 km, 1200 km and 500 km at the rate of 500 km/hr, 400 km, and 250 km/hr, respectively. The average is
(a) 420 km/hr (b) 410 km/hr
(c) 405 km/hr (d) 575 km/hr

8. In a class of 20 students in an examination in Mathematics 2 students scored 100 marks each, 3 got zero each and the average of the rest was 40. What is the average of the whole class?
(a) 40 marks (b) 35 marks
(c) 32 marks (d) 45 marks

9. The average weight of 24 students of section A of a class is 58 kg whereas the average weight of 26 students of section B of the same class is 60.5 kg. Find the average weight of all the 50 students of the class.
(a) 57.4 kg (b) 59.3 kg
(c) 58.9 kg (d) 59.7 kg

10. The average age of 5 members is 21 years. If the age of the youngest member be 5 years, find the average age of the family at the birth of the youngest member.
(a) 24 years (b) 25 years
(c) 20 years (d) 28 years

11. The average of 7 numbers is 5. If the average of first six of these numbers is 4, the seventh number is
(a) 14 (b) 12
(c) 11 (d) 15

12. Three years ago the average age of a family of 5 members was 27 years. On the addition of a child to the family, the present average age of the family is still 27 years. Find the present age of the child.
(a) 16 years (b) 12 years
(c) 24 years (d) 20 years

13. The average weight of 10 students is increased by half a kg when one of the students weighing 50 kg is replaced by a new student. Find the weight of the new student.
(a) 55 kg (b) 60 kg
(c) 45 kg (d) 40 kg

14. The average monthly salary of a staff of 9 persons is Rs 2450. One member of the staff whose monthly salary is Rs 2650 is transferred. Find the average salary of the remaining 8 persons of the staff.
(a) Rs 2425 (b) Rs 2625
(c) Rs 3025 (d) Rs 2825

15. The mean marks of 10 boys in a class is 70% whereas the mean marks of 15 girls is 60%. The mean marks of all the 25 students is
(a) 64% (b) 60%
(c) 55% (d) 52%

16. The average income of A for 15 days is Rs 70. The average for first five days is Rs 60 and that for the last nine days is Rs 80. The income for the sixth day is
(a) Rs 80 (b) Rs 60
(c) Rs 40 (d) Rs 30

17. The average of five consecutive even numbers starting with 4, is
(a) 6 (b) 7
(c) 8 (d) 7.5

18. 3 years ago the average age of a family of 5 members was 17 years. With the birth of a new baby, the average remains the same even today. Find the age of the baby.
(a) 1 years (b) 3 years
(c) $2\frac{1}{2}$ years (d) 2 years

19. The average of 17 numbers is 10.9. If the average of first nine numbers is 10.5 and that of the last nine numbers is 11.4, the middle no. is
(a) 11.8 (b) 11.4
(c) 10.9 (d) 11.7

20. A batsman has a certain average of runs for 12 innings. In the 13th innings he scores 96 runs there-by increasing his average by 5 runs. What is his average after the 13th innings?
(a) 48 (b) 64
(c) 36 (d) 72

21. A batsman in his 17th innings makes a score of 85, and thereby increases his average by 3. What is his average after the 17th innings? He had never been 'not out'.
(a) 47 (b) 37
(c) 39 (d) 43

22. The sum of three numbers is 98. If the ratio between first and second be 2 : 3 and that between second and third be 5 : 8, then the second number is
(a) 30 (b) 20
(c) 58 (d) 48

23. The average weight of 8 sailors in a boat is increased by 1 kg if one of them weighing 56 kg is replaced by a new sailor. The weight of the new sailor is
(a) 57 kg (b) 60 kg
(c) 64 kg (d) 62 kg

24. A number x equals 80% of the average of 5, 7, 14 and a number y. If the average of x and y is 26, the value of y is
(a) 13 (b) 26
(c) 39 (d) None of these

25. The average age of A, B, C, D five years ago was 45 years. By including x, the present average age of all the five is 49 years. The present age of x is
(a) 64 years (b) 48 years
(c) 45 years (d) 40 years

26. It rained as much on Wednesday as on all the others days of the week combined. If the average rainfall for the whole week was 3 cm, how much did it rain on Wednesday?
(a) 2.625 cm (b) 3 cm
(c) 10.5 cm (d) 15 cm

27. The average monthly expenditure of a family for the first four months is Rs 2750, for the next three months is Rs 2940 and for the last five months Rs 3130. If the family saves Rs 5330 during the whole year, find the average monthly income of the family during the year.
(a) Rs 3800 (b) Rs 3500
(c) Rs 3400 (d) Rs 4200

28. The average age of 8 men is increased by 2 years. When two of them, whose ages are 20 years and 24 years respectively, are replaced by two women. What is the average age of these women?
(a) 36 years (b) 30 years
(c) 40 years (d) 42 years

29. The average of 50 numbers is 38. If two numbers 45 and 55 are discarded, the average of the remaining set of numbers is
(a) 38.5 (b) 37.5
(c) 37.0 (d) 36.5

30. The average speed of a train running at a speed of 30 km/hr during the first 100 kilometres, at 40 km/hr during the second 100 kilometres and at 50 km/hr during the last 100 kilometres is nearly
(a) 38.5 km/hr (b) 38.3 km/hr
(c) 40.0 km/hr (d) 39.2 km/hr

31. The average of 6 observations is 12. A new seventh observation is included and the new average is decreased by 1. The seventh observation is
(a) 1 (b) 3
(c) 5 (d) 6

32. The average age of 20 boys in the class is 15.6 years. Five new boys join and the new average becomes 15.56 years. What was the average age of five new boys?
(a) 15.5 (b) 15.4
(c) 15.25 (d) 15.3
(e) None of these

33. The average weight of 3 men A, B and C is 84 kg. Another man D joins the group and the average now becomes 80 kg. If another man E, whose weight is 3 kg more than that of D, replaces A, then average weight of B, C, D and E becomes 79 kg. The weight of A is
(a) 70 kg (b) 72 kg
(c) 75 kg (d) 80 kg

34. There was one mess for 30 boarders in a certain hostel. On the number of boarders being increased by 10, the expenses of the mess were increased by Rs 40 per month while the average expenditure per head diminished by Rs 2. Find the original monthly expenses.
(a) Rs 390 (b) Rs 410
(c) Rs 360 (d) Rs 480

35. Of the three numbers the first is twice the second and the second is thrice the third. If the average of the three numbers is 10, the numbers are
(a) 18, 3, 9 (b) 9, 3, 18
(c) 3, 9, 18 (d) 18, 9, 3

36. The average weight of 36 students is 50 kg. It was found later that the figure of 37 kg was misread as 73 kg. What is the correct average?
(a) 49 kg (b) 51 kg
(c) 50.5 kg (d) None of these

37. The average earning of a mechanic for the first four days of a week is Rs 18 and for the last four days is Rs 22. If he earns Rs 20 on the fourth day, his average earning for the whole week is
(a) Rs 18.95 (b) Rs 16
(c) Rs 20 (d) Rs 25.71

38. The average of marks obtained by 120 candidates was 35. If the average of marks of passed candidates was 39 and that of failed candidates was 15, the number of candidates who passed the examination is
(a) 100 (b) 110
(c) 120 (d) 150

39. In a class, there are 20 boys whore average age is decreased by 2 months, when one boy aged 18 years in replaced by a new boy. The age of the new boy is
(a) 14 yrs 8 months (b) 15 yrs
(c) 16 yrs 4 months (d) 17 yrs 10 months

40. The average temperature from Monday to Thursday is 48° and from Tuesday to Friday is 52°. If the temperature on Monday is 42°, what was it on Friday?
(a) 52° (b) 55°
(c) 58° (d) 51°

41. A man spends on an average Rs 269.47 for the first 7 months and Rs 281.05 for the next 5 months. Find his monthly salary if he saved Rs 308.46 during the whole year.
(a) Rs 400 (b) Rs 500
(c) Rs 300 (d) Rs 600

42. The average of two numbers is 62. If 2 is added to the smaller number, the ratio between the numbers becomes 1 : 2. The smaller number is
(a) 60 (b) 30
(c) 84 (d) 40

43. In a coconut grove, $(x + 2)$ trees yield 60 nuts per year, x trees yield 120 nuts per year and $(x - 2)$ trees yield 180 nuts per year. If the average yield per year per tree be 100, find x.
(a) 4 (b) 2
(c) 8 (d) 6

44. Average temperature of first 4 days of a week is 38.6°C and that of the last 4 days is 40.3°C. If the average temperature of the week be 39.1°C, the temperature on 4th day is.
(a) 36.7°C (b) 38.6°C
(c) 39.8°C (d) 41.9°C

45. The average daily wages of A, B and C is Rs 120. If B earns Rs 40 more than C per day and A earns double of what C earns per day, the wages of A per day is
(a) Rs 80 (b) Rs 120
(c) Rs 160 (d) Rs 100

46. With an average speed of 40 km/hr a train reaches its destination on time. If it goes with an average speed of 35 km/hr, it is late by 15 minutes. The total journey is
(a) 30 km (b) 40 km
(c) 70 km (d) 80 km

47. In a competitive examination, the average marks obtained was 45. It was later discovered that there was some error in computerisation and the marks of 90 candidates had to be changed from 80 to 50 and the average came down to 40 marks. The total no. of candidates who appeared in examination is
(a) 520 (b) 550
(c) 540 (d) 525

48. Visitors to a show were charged Rs 15.00 each on the first day, Rs 7.50 on the second, Rs 2.50 on the third day and total attendance on the three days were in the ratio 2 : 5 : 13, respectively. Find the average charge per person for the whole show.
(a) Rs 7 (b) Rs 5
(c) Rs 9 (d) Rs 11

49. The mean daily profit made by a shopkeeper in a month of 30 days was Rs 350. If the mean profit for the first fifteen days was Rs 275, then the mean profit for the last fifteen days would be
(a) Rs 200 (b) Rs 275
(c) Rs 350 (d) Rs 425

50. A man whose bowling average is 12.4 takes 5 wickets for 26 runs and thereby decreases his average by 0.4. The number of wickets, taken by him, before his last match is
(a) 85 (b) 78
(c) 72 (d) 64

51. Out of three numbers, the first is twice the second and is half of the third. If the average of the three numbers is 56, the three numbers in order are
(a) 48, 96, 24 (b) 48, 24, 96
(c) 96, 24, 48 (d) 96, 48, 24

52. There were 35 students in a hostel. If the number of students increases by 7, the expenses of the mess increase by Rs 42 per day while the average expenditure per head diminishes by Re 1. Find the original expenditure of the mess.
(a) Rs 480 (b) Rs 420
(c) Rs 520 (d) Rs 460

53. The average of 50 numbers is 38. If two numbers, namely 45 and 55 are discarded, what is the average of the remaining numbers?
(a) 37.5 (b) 38.5
(c) 39.5 (d) 36.5

54. In a cricket team of 11 boys, one player weighing 42 kg is injured and his place is taken by another player. If the average weight of the team is increased by 100 gm as a result of this, then what is the weight of the new player?
(a) 42.1 kg (b) 45.1 kg
(c) 44.1 kg (d) 43.1 kg

55. The average of these consecutive numbers is n. If the next two consecutive numbers are also included, the average of the five numbers will
(a) remain the same (b) increase by 0.5
(c) increase by 1 (d) increase by 1.5

56. The average salary of 20 workers in an office is Rs 1900 per month. If the manager's salary is added, the average becomes Rs 2000 per month. The manager's salary (in Rs) is
(a) 24000 (b) 25200
(d) 45600 (d) None of these

57. The average age of students of a class is 15.8 years. The average age of boys in the class is 16.4 years and that of the girls is 15.4 years. The ratio of number of boys to the number of girls in the class is
(a) 1 : 2 (b) 3 : 4
(c) 3 : 5 (d) 2 : 3

58. The average expenditure of a man for the first five months is Rs 3600 and for next seven months it is Rs 3900. If he saves Rs 8700 during the year, his average income per month is
(a) Rs 4500 (b) Rs 4200
(c) Rs 4050 (d) Rs 3750

59. Of the three numbers, second is twice the first and is also thrice the third. If the average of the three numbers is 44, the largest number is
(a) 24 (b) 36
(c) 72 (d) 108

60. The average age of a committee of 8 members is 40 years. A member, aged 55 years, retired and he was replaced by a member aged 39 years. The average age of the present committee is
(a) 39 years (b) 38 years
(c) 36 years (d) 35 years

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Total earning for 7 days
= Rs (60 + 65 + 70 + 52.50 + 63 + 73 + 68)
= Rs 451.50

Average daily earning = Rs $\frac{451.50}{7}$ = Rs 64.50.

2. Ans. (c)

Solution The average of 10 numbers = 7
Total of 10 numbers = 10 × 7 = 70
New total of 10 numbers after each of given numbers is multiplied by 8 = 70 × 8 = 560

∴ New average = $\frac{560}{10}$ = 56.

3. Ans. (b)

Solution Average weight of 5 persons = 38 kg
∴ Total weight of these five persons
= 38 × 5 = 190 kg
Now, average weight of (the boat + 5 persons)
= 52 kg
∴ Total weight of (the boat + 5 persons)
= 52 × 6 = 312 kg
∴ Weight of the boat = 312 – 190 = 122 kg.

4. Ans. (d)

Solution Let the original expenditure = Rs x

Original average expenditure = $\frac{x}{35}$

New average expenditure = $\frac{x+42}{42}$

$\Rightarrow \frac{x}{35} - \frac{x+42}{42} = 1 \Rightarrow x = 420$

∴ Original expenditure = Rs 420.

5. Ans. (c)

Solution Average daily maximum temperature

$$= \frac{42.7+44.6+42.0+39.1+43.0+42.5+38.5}{7}$$

$$= \frac{292.4}{7} = 41.77°\text{C}.$$

6. Ans. (d)

Solution Let the total number of workers be x
$\Rightarrow$ 850 × x = 7 × 1000 + (x – 7) × 780
$\Rightarrow$ x = 22.

7. Ans. (a)

Solution The total time taken can be calculated as shown below:

Distance	*Speed*	*Time*
2500 km	500 km/hr	5 hrs.
1200 km	400 km/hr	3 hrs.
500 km	250 km/hr	2 hrs.
Total 4200 km		10 hrs.

Average speed = $\frac{4200}{10}$ = 420 km/hr.

8. Ans. (a)

Solution Marks scored by 2 students = 100 × 2 = 200
Marks scored by 3 students = 3 × 0 = 0
Marks scored by 15 students = 15 × 40 = 600
∴ Marks scored by 20 students
= 200 + 0 + 600 = 800

∴ Average marks = $\frac{800}{20}$ = 40.

9. Ans. (b)

Solution Average weight of 24 students of section A = 58 kg

Total weight of 24 students of section A = 58 × 24 = 1392 kg

Average weight of 26 students of section B = 60.5 kg

Total weight of 26 students of section B

= 60.5 × 26 = 1573 kg

Total weight of 50 students = (1392 + 1573) kg

= 2965 kg

Average weight of students in the class

$$= \frac{2965}{50} = 59.3 \text{ kg.}$$

10. Ans. (c)

Solution Total age of 5 members = 21 × 5 = 105 years

Total age of 4 members at the birth of the younger member, that is, 5 years ago

= 105 – (5 × 5) = 80 years

Before the birth of the youngest member, the family consisted of only 4 members.

Average age of 4 members 5 years ago

$$= \frac{80}{4} = 20 \text{ years.}$$

11. Ans. (c)

Solution Sum of seven numbers = 7 × 5 = 35

Sum of first six numbers = 6 × 4 = 24

Therefore, the seventh number = 35 – 24 = 11.

12. Ans. (b)

Solution Present average age of 5 members

= 27 + 3 = 30 years

Sum of present age of 5 members

= 30 × 5 = 150 years

Let the present age of the child be x years

Present average age of 6 members

$$= \frac{150+x}{6} \text{ and this is equal to 27 yrs.}$$

So, $\frac{150+x}{6} = 27$

or, $x = 27 \times 6 - 150$ or, $x = 12$ yrs.

13. Ans. (a)

Solution Weight of the new student

= 50 + 10 × 112 = 55 kg.

14. Ans. (a)

Solution Average salary of 9 persons = Rs 2450

Total salary of 9 persons

= Rs 2450 × 9 = Rs 2650

Total salary of the person who is transferred

= Rs 2650

Thus, the total salary of remaining 8 persons

= Rs 22050 – Rs 2650 = Rs 19400

The average salary of the remaining 8 persons

$$= \text{Rs } \frac{19400}{8} = \text{Rs } 2425.$$

15. Ans. (a)

Solution The mean marks of 10 boys = 70%

Total marks of 10 boys = 70% × 10 = 700%

The mean marks of 15 girls = 60%

Total marks of 15 girls = 60% × 15 = 900%

∴ Sum of the total marks of 25 students

= 700 + 900 = 1600%

∴ The mean marks of all the 25 students

$$= \frac{1600}{25} = 64\%.$$

16. Ans. (d)

Solution Income for 6th day in rupees

= 15 × 70 – 5 × 60 – 9 × 80 = 30.

17. Ans. (c)

Solution The five even consecutive numbers are

4, 6, 8, 10 and 12

$$\text{Their average} = \frac{4+6+8+10+12}{5} = \frac{40}{5} = 8.$$

18. Ans. (d)

Solution Present age of 5 members

= (5 × 17 + 3 × 5) years.

= 100 years.

Present age of 5 members and a baby

= 17 × 6 = 102 yrs.

∴ Age of the baby = (102 – 100) years.

= 2 years.

19. Ans. (a)

Solution Sum of first nine numbers + sum of last nine numbers = 10.5 × 9 + 11.4 × 9 = 21.9 × 9 = 197.1

Hence, the middle number

= 197.1 – 17 × 10.9

= 197.1 – 185.3 = 11.8.

20. Ans. (c)

Solution To improve his average by 5 runs per innings he has to contribute 12 × 5 = 60 runs for the previous 12 innings. Thus, the average after the 13th innings

= 96 – 60 = 36.

21. Ans. (b)

Solution Average score before 17th innings

= 85 – 3 × 17 = 34

∴ Average score after 17th innings

= 34 + 3 = 37.

22. Ans (a)

Solution Let the numbers be x, y and z. Then,

$$x + y + z = 98, \frac{x}{y} = \frac{2}{3} \text{ and } \frac{y}{z} = \frac{5}{8}$$

$$\therefore \quad x = \frac{2y}{3} \text{ and } z = \frac{8y}{5}$$

So, $$\frac{2y}{3} + y + \frac{8y}{5} = 98$$

or, $$\frac{49y}{15} = 98 \quad \text{or,} \quad y = 30.$$

23. Ans. (c)

Solution The sailor weighing 56 kg is replaced and the average is increased by 1 kg. Hence, the weight of the new sailor is (56 + increase in total weight) = 56 + 1 × 8

= 56 + 8 = 64 kg.

24. Ans. (c)

Solution Average of 5, 7, 14 and $y = \frac{5+7+14+y}{4}$

Therefore, $x = 80\%$ of $\frac{5+7+14+y}{4} = \frac{80}{100} \times \frac{26+y}{4}$

$\Rightarrow x = \frac{26+y}{5}$ ...(i). Also $\frac{x+y}{2} = 26$ ...(ii)

From (i) and (ii), we get $52 - y = \frac{26+y}{5} \Rightarrow y = 39.$

25. Ans. (c)

Solution Present age of x

= [(49 × 5) – (4 × 45 + 4 × 5)] years

= 45 years.

26. Ans. (c)

Solution Let the rainfall on Wednesday be x cm so that on the other 6 days, the total is also x.

Since average rainfall for the week = 3 cm

$\therefore \quad x + x = 3 \times 7 \quad$ or, $\quad x = 10.5$ cm.

27. Ans. (c)

Solution Average monthly expenditure of 4 months

= Rs 2700

Total expenditure for 4 months = Rs 2700 × 4

= Rs 11000

Average monthly expenditure of 3 months

= Rs 2940

Total expenditure of 3 months = Rs 2940 × 5

= Rs 8820 ...(i)

Average monthly expenditure of 5 months

= Rs 3130

Total expenditure of 5 months = Rs 31020

= Rs 1560 ...(ii)

Total expenditure in the whole year

= Rs 11000 + 8820 + Rs 15650

= Rs 35470.

Saving during the whole year = Rs. 5330

Total income of the family during the year

= Rs 35470 + Rs 5330 = Rs 40800

$\therefore$ Average monthly income during the year

$$= \frac{40800}{12} = \text{Rs } 3400.$$

28. Ans. (b)

Solution Let the average age of 8 men be x years

$\therefore$ Sum of the ages of 8 men = 84 years

Now, according to the condition of the question, average age of (6 men + 2 women) = $(x + 2)$ years

$\therefore$ Sum of the ages of (6 men + 2 women)

$= 8(x + 2) = 8x + 16$ yrs

Hence, it is clear that on replacing two men by two women, sum of their ages increased by 16 years

Therefore, sum of the ages of two women

= (20 + 24) + 16 = 60 yrs

$\therefore$ Average age of the women $= \frac{60}{2} = 30$ yrs.

29. Ans. (b)

Solution Average of the remaining set of numbers

$$= \frac{50 \times 38 - (45+55)}{50-2}$$

$$= \frac{1900-100}{48} = 37.5.$$

30. Ans. (b)

Solution Time taken to cover first 100 kilometres

$$= \frac{100}{30} = 3\frac{1}{3}\text{hrs}$$

Time taken to cover second 100 kilometres

$$= \frac{100}{40} = 2\frac{1}{2}\text{hrs}$$

Time take to cover last 100 kilometres

$$= \frac{100}{5} = 2 \text{ hrs}$$

Total time taken $= 3\frac{1}{3} + 2\frac{1}{2} + 2 = \frac{10}{3} + \frac{5}{2} + 2$

$$= \frac{47}{6}\text{hrs}$$

Total distance covered = 300 km

$\therefore$ Average speed $= \frac{300}{47/6} = \frac{300 \times 6}{47} = 38.3$ km/hr.

31. Ans. (c)

Solution Seventh observation

= 7 × 11 – 6 × 12 = 5.

32. Ans. (b)

Solution Average age of five new boys

$= (25 \times 15.56 - 20 \times 15.6) \div 5$

$= 15.4.$

33. Ans. (c)

Solution Weight of $D = (80 \times 4 - 84 \times 3)$ kg = 68 kg

Weight of $E = (68 + 3)$ kg = 71 kg

$(B + C + D + E)$'s weight = (79×4) kg = 316 kg

$\therefore$ $(B + C)$'s weight = $[316 - (68 + 71)]$ kg

= 177 kg

Hence, A's weight = $[(84 \times 3) - 177]$ kg = 75 kg.

34. Ans. (c)

Solution Let x rupees be the average of 30 boarders

$\therefore$ $30x + 40 = (x - 2) \times 40$ or, $x = 12$

Hence, original expenditure

= Rs 12×30 = Rs 360.

35. Ans. (d)

Solution Let the numbers be $2x$, x and $\frac{x}{3}$

Then, average $= \dfrac{2x + x + \frac{x}{3}}{3} = 10 \Rightarrow \dfrac{9x + x}{3 \times 3} = 10$

or, $\dfrac{10x}{9} = 10$ or, $x = 9$

Hence, the numbers are 18, 9 and 3.

36. Ans. (a)

Solution Correct average

$= \dfrac{50 \times 36 - 73 + 37}{36}$

$= \dfrac{1764}{36} = 49$ kg.

37. Ans. (c)

Solution Total earning for the week

= Rs $(4 \times 18 + 4 \times 22 - 20)$ = Rs 140

$\therefore$ Average earning = Rs $\dfrac{140}{7}$ = Rs 20.

38. Ans. (a)

Solution Let the number of candidates who passed = x.

Then, $39 \times x + 15 \times (120 - x) = 120 \times 35$

$\therefore$ $24x = 4200 - 1800$

or, $x = \dfrac{2400}{24}$

$x = 100.$

39. Ans. (a)

Solution Total decrease = (20×2) months

= 3 years 4 months

$\therefore$ Age of the new boy = 18 years – 3 years 4 months

= 14 years 8 months.

40. Ans. (c)

Solution Temperature on Monday + Tuesday + Wednesday + Thursday = $4 \times 48° = 192°$

Temperature on Monday = 42°

$\therefore$ Tuesday + Wednesday + Thursday

$= (192° - 42°) = 150°$

Temperature on Tuesday + Wednesday + Thursday + Friday = $4 \times 52° = 208°$

$\therefore$ Friday's temperature = 208° – 150° = 58°.

41. Ans. (c)

Solution Total spending in 12 months

= Rs $[269.47 \times 7 + 281.05 \times 5]$

= Rs 3291.54

Total income = spending + saving

= Rs 3291.54 + Rs 308.46

= Rs 3600.00

$\therefore$ Monthly salary = Rs $\dfrac{3600}{12}$ = Rs 300.

42. Ans. (d)

Solution Let the numbers be x and y, $x < y$

Then, $x + y = 124$; $\dfrac{x+2}{y} = \dfrac{1}{2} \Rightarrow y = 2x + 4.$

Solving above equations, we get

$y = 84$, $x = 40.$

43. Ans. (a)

Solution $\dfrac{(x+2) \times 60 + x \times 120 + (x-2) \times 180}{x + 2 + x + x - 2} = 100$

$\therefore$ $x = 4.$

44. Ans. (d)

Solution Let temperature on 4th day be x°C

$\therefore$ $4 \times 38.6 + 4 \times 40.3 - x = 7 \times 39.1$

or, $x = 41.9$

$\therefore$ Temperature on 4th day = 41.9°C.

45. Ans. (c)

Solution Let daily wages of $C = x$

Then, daily wages of $A = 2x$

and, daily wages of $B = x + 40$

Hence, average daily wages of A, B and C

$= \dfrac{x + 2x + x + 40}{3} = \dfrac{4x + 40}{3}$

$\therefore$ $\dfrac{4x + 40}{3} = 120$ or, $4x + 40 = 360$

$\Rightarrow$ $4x = 320$ or, $x = 80$

$\therefore$ Wages of A per day = 2×80 = Rs 160.

46. Ans. (c)

Solution $\frac{x}{35}-\frac{x}{40}=\frac{15}{60}$ or, $\frac{5x}{35\times 40}=\frac{1}{4}$

or, $x=\frac{35\times 40}{5\times 4}=70$

$\therefore$ Total journey = 70 km.

47. Ans. (c)

Solution Let the number of candidates be x. Then, total marks obtained by all the candidates = $45x$

Marks reduced for 90 candidates

$= 30 \times 90 = 2700$

Total reduced marks = $45x - 2700$

Reduced average = $\frac{45x-2700}{x}$

$\therefore$ $40 = \frac{45x-2700}{x}$ or, $40x = 45x - 2700$

$\Rightarrow$ $5x = 2700$ or, $x = 540$.

48. Ans. (b)

Solution Let attendance on first, second and third day be 2, 5 and 13, respectively.

Total attendance of three days = 2 + 5 + 13 = 20

Total money collected

$= 2 \times 15 + 5 \times 7.50 + 13 \times 2.50$

$= 30 + 37.5 + 32.5 = 100$

Average charge per person = $\frac{100}{20} = 5$.

49. Ans. (d)

Solution Total profit for 30 days = 30 × 350

= Rs 10500

Profit for first 15 days = 15 × 275 = Rs 4125

$\therefore$ Profit for last 15 days = Rs 10500 – 4125

= Rs 6375

Average profit for the last 15 days

= Rs $\frac{6375}{15}$ = Rs 425.

50. Ans. (a)

Solution Let the number of wickets taken before the last match = x

Then, $\frac{12.4x+26}{x+5}=x \Rightarrow x = 85$.

51. Ans. (b)

Solution Let the numbers be $2x$, x and $4x$

Average = $\frac{2x+x+4x}{3} \Rightarrow \frac{7x}{3} = 56$

$\therefore$ $x = \frac{3\times 56}{7} = 24$

Hence, the numbers in order are 48, 24 and 96.

52. Ans. (b)

Solution Suppose the average expenditure was Rs x

Then, total expenditure = $35x$

When 7 more students join the mess, total expenditure

$= 35x + 42$

Now, the average expenditure

$= \frac{35x+42}{35+7} = \frac{35x+42}{42}$

Now, we have $\frac{35x+42}{42} = x - 1$

$\therefore$ $x = 12$

Thus, the original expenditure of the mass

= 35 × 12 = Rs 420.

53. Ans. (a)

Solution Sum total of 50 numbers = 50 × 38 = 1900

Sum total of the remaining 48 numbers

= 1900 – (45 + 55) = 1800

$\therefore$ Average = $\frac{1800}{48} = 37.5$.

54. Ans. (d)

Solution Average weight of 11 boys is increased by 100 gm (= 0.1 kg.)

$\therefore$ Their total increase in weight

= 0.1 × 11 = 1.1 kg

Hence, the weight of new boy = 42 + 1.1 = 43.1kg

55. Ans. (c)

Solution Let the numbers be $n - 1$, n and $n + 1$. Their average = n

Next two consecutive numbers are $n + 2$ and $n + 3$

Therefore, the average of five numbers

$= \frac{(n-1)+n+(n+1)+(n+2)+(n+3)}{5}$

$= \frac{5n+5}{5} = n + 1$.

56. Ans. (d)

Solution Total monthly salary of 21 persons

= Rs (21 × 2000) = Rs 42000

Total monthly salary of 20 persons

= Rs (20 × 1900) = Rs 38000

Monthly salary of the manager = Rs 4000

Annual salary of the manager = Rs 48000.

57. Ans. (d)

Solution Let the number of boys be x and the number of girls be y.

Sum of ages of boys = $16.4x$

Sum of ages of girls = $15.4y$

Average age of all the students

$$= \frac{16.4x + 15.4y}{x + y} = 15.8$$

$\Rightarrow$ $16.4x + 15.4y = 15.8x + 15.8y$

or, $16.4x - 15.8x = 15.8y - 15.4y$

or, $0.6x = 0.4y$

or, $\frac{x}{y} = \frac{0.4}{0.6} = \frac{2}{3}$ or $x : y = 2 : 3$.

58. Ans. (a)

Solution Total expenditure for the first five months

$= 5 \times 3600 =$ Rs 18000

Total expenditure for the next seven months

$= 7 \times 3900 =$ Rs 27300

Savings = Rs 8700

Total income during the year

$= 18000 + 27300 + 8700 =$ Rs 54000

Average income per month $= \frac{54000}{12} =$ Rs 4500.

59. Ans. (c)

Solution Let the numbers be x, $2x$, $\frac{2}{3}x$

$$\text{Average} = \frac{x + 2x + \frac{2}{3}x}{3} \Rightarrow \frac{11x}{9} = 44$$

$$\therefore \quad x = \frac{44 \times 9}{11} = 36$$

So, the numbers are 36, 72 and 24

Hence, the largest one is 72.

60. Ans. (b)

Solution New average of the committee (in years)

$$= \frac{8 \times 40 - 55 + 39}{8} = \frac{320 - 16}{8} = \frac{304}{8}$$

= 38 years.

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. The average of 7 consecutive numbers is 20. The largest of these numbers is
(a) 24 (b) 23
(c) 22 (d) 20
[Based on SSC (GL) Prel. Exam 2000]

2. The average age of 14 girls and their teacher is 15 years. If the teacher's age is excluded, the average reduces by 1. What is the teacher's age?
(a) 35 years (b) 32 years
(c) 30 years (d) 29 years
[Based on SSC (GL) Prel. Exam 2000]

3. The average age of four brothers is 12 years. If the age of their mother is also included, the average is increased by 5 years. The age of the mother (in years) is
(a) 37 years (b) 43 years
(c) 48 years (d) 53 years
[Based on SSC (GL) Prel. Exam 2000]

4. The average age of A and B is 20 years, that of B and C is 19 years and that of A and C is 21 years. What is the age (in years) of B?
(a) 39 (b) 21
(c) 20 (d) 18
[Based on SSC (GL) Prel. Exam 2000]

5. There are, in all 10 balls; some of them are red and others white. The average cost of all balls is Rs 28. If the average cost of red balls is Rs 25 and that of white balls is Rs 30, then the number of white balls is
(a) 3 (b) 5
(c) 6 (d) 7
[Based on SSC (GL) Prel. Exam 2000]

6. In an experiment, the average of 11 observations was 90. If the average of the first five observations is 87, and that of the last five is 84, then the sixth observation is
(a) 135 (b) 145
(c) 150 (d) 165
[Based on SSC (GL) Prel. Exam 2000]

7. The average weight of 20 students in a class is increased by 0.75 kg when one of the students weighing 30 kg is replaced by a new student. Weight of the new student (in kg) is
(a) 35 (b) 40
(c) 45 (d) 50
[Based on SSC (GL) Prel. Exam 2000]

8. The average of 9 numbers is 30. The average of first 5 numbers is 25 and that of the last 3 numbers is 35. What is the 6th number?
(a) 20 (b) 30
(c) 40 (d) 50
[Based on SSC (GL) Prel. Exam 2000]

9. The average of marks of 14 students was calculated as 71. But it was later found that the marks of one student had been wrongly entered as 42 instead of 56 and of another as 74 instead of 32. The correct average is
(a) 67 (b) 68
(c) 69 (d) 71
[Based on SSC (GL) Prel. Exam 2000]

10. Of the four numbers whose average is 60, the first is $\frac{1}{4}$th of the sum of the last three. The first number is
(a) 15 (b) 45

(c) 48 (d) 60.25

[Based on SSC (GL) Prel. Exam 2000]

11. Average age of father and his two sons is 27 years. Five years ago, the average age of the two sons was 12 years. If the difference between the ages of the two sons is 4 years, then the present age of the father is
(a) 34 yrs (b) 47 yrs
(c) 64 yrs (d) 27 yrs

[Based on SSC (GL) Prel. Exam 2000]

12. The average age of 30 boys in a class is 15 years. One boy, aged 20 years, left the class, buy two new boys came is his place whose ages differ by 5 years. If the average age of all the boys now in the class becomes 15 years, the age of the younger newcomer is
(a) 20 yrs (b) 15 yrs
(c) 10 yrs (d) 8 yrs

[Based on SSC (GL) Prel. Exam 2002]

13. Out of three numbers, the first is twice the second and is half of the third. If the average of the three numbers is 56, then difference of first and third numbers is
(a) 12 (b) 20
(c) 24 (d) 48

[Based on SSC (GL) Prel. Exam 2002]

14. Of the three numbers second is twice the first and is also twice the third. If the average of the three numbers is 44, the largest number is
(a) 24 (b) 72
(c) 36 (d) 108

[Based on SSC (GL) Prel. Exam 2002]

15. The average of 8 men is increased by 2 years when 2 of them whose ages are 21 and 23 years are replaced by two new men. The average age of two new men is
(a) 22 yrs (b) 24 yrs
(c) 28 yrs (d) 30 yrs

[Based on SSC (GL) Prel. Exam 2002]

16. The average age of A and B is 30 years, that of B and C is 32 years, and the average age of C and A is 34 years. The age of C is
(a) 33 yrs (b) 34 yrs
(c) 35 yrs (d) 36 yrs

[Based on SSC (GL) Prel. Exam 2002]

17. In a certain year, the average monthly income of a person is Rs 3400 and that for the first eight months is Rs 3160 and for the last five months is Rs 4120. The income in the eight month of the year is
(a) Rs 5080 (b) Rs 6080
(c) Rs 5180 (d) Rs 3880

[Based on SSC (GL) Prel. Exam 2003]

18. Average age of 6 sons of a family is 8 years. Average age of sons together with their parents is 22 years. If the father is older than the mother by 8 years, the age of the mother (in years) is
(a) 44 (b) 52
(c) 60 (d) 68

[Based on SSC (GL) Prel. Exam 2003]

19. A grocer has a sale of Rs 6435, Rs 6927, Rs 6855, Rs 7230 and Rs 6562 for 5 consecutive months. How much sale must he have in the sixth month so that he gets an average sale of Rs 6500?
(a) Rs 4991 (b) Rs 5991
(c) Rs 6991 (d) Rs 6001

[Based on SSC (GL) Prel. Exam 2003]

20. The average weight of three men A, B and C is 84 kg. D joins them and the average weight of the four becomes 80 kg. If E, whose weight is 3 kg more than that of D, replaces A, the average weight of B, C, D and E becomes 79 kg. The weight of A is
(a) 65 kg (b) 70 kg
(c) 75 kg (d) 80 kg

[Based on SSC (GL) Prel. Exam 2003]

21. The average salary of all the workers in a workshop is Rs 8000. The average salary of 7 technicians is Rs 12000 and the average salary of the rest is Rs 6000. The total number of workers in the workshop is
(a) 20 (b) 21
(c) 23 (b) 22

[Based on SSC (GL) Prel. Exam 2003]

22. The average marks scored by Ganesh in English, Science, Mathematics and History is less than 15 from that scored by him in English, History, Geography and Mathematics. What is the difference of marks in Science and Geography secured by him?
(a) 40 (b) 50
(c) 60 (d) Data inadequate
(e) None of these

[Based on BSRB Chennai PO 2000]

23. A Mathematics teacher tabulated the marks secured by 35 students of 8th class. The average of their marks was 72. If the marks secured by Reema was written as 36 instead of 86 then find the correct average marks up to two decimal places.
(a) 73.41 (b) 74.3
(c) 72.43 (d) 73.43
(e) Cannot be determined

[Based on BSRB Bangalore PO 2000]

24. The average weight of 8 persons increases by 1.5 kg. If a person weighting 65 kg is replaced by a new person, what could be the weight of the new person?
(a) 76 kg (b) 77 kg
(c) 76.5 kg (d) Data inadequate
(e) None of these

25. The average of four consecutive even numbers is one-fourth of the sum of these numbers. What is

the difference between the first and the last number?
(a) 4 (b) 6
(c) 2 (d) Cannot be determined
(e) None of these

[Based on BSRB Delhi PO 2000]

26. Of the three numbers, the average of the first and the second is greater than the average of the second and the third by 15. What is the difference between the first and the third of the three numbers?
(a) 15 (b) 45
(c) 60 (d) Data inadequate
(e) None of these

[BABARD Asst. Manager Exam 2002]

27. The average of 25 results is 18, that of first 12 is 14 and of the last 12 is 17. Thirteenth result is
(a) 72 (b) 78
(c) 85 (d) 28

[Based on CANARA BANK PO 2003]

28. Average age of seven persons in a group is 30 years. The average age of five persons of this group is 31 years. What is the average age of the other two persons in the group?
(a) 55 years (b) 26 years
(c) 15 years (d) Cannot be determined
(e) None of these

[Based on PNB Management Trainee Exam 2003]

29. Average weight of three boys P, T and R is $54\frac{1}{3}$kg while the average weight of three boys T, F and G is 53 kg. What is the average weight of P, T, R, F and H?
(a) 53.8 kg (b) 52.4 kg
(c) 53.2 kg (d) Data inadequate
(e) None of these

[Based on Bank of Maharashtra 2003]

30. In a class of 52 students the number of boys is two less than the number of girls. Average weight of the boys is 42 kg, while the average weight of all the 52 students is 40 kg. Approximately what is the average weight of the girls?
(a) 41 kg (b) 29 kg
(c) 40 kg (d) 38 kg
(e) 42 kg **[Based on IBPS 2003]**

EXPLANATORY ANSWERS

1. Ans. (b)

Solution Let the 7 consecutive numbers be $x - 3$, $x - 2$, $x - 1$, x, $x + 1$, $x + 2$ and $x + 3$

$$\therefore \frac{(x-3)+(x-2)+(x-1)+x+(x+1)+(x+2)+(x+3)}{7} = 20$$

$$\Rightarrow \frac{7x}{7} = 20 \Rightarrow x = 20$$

$\therefore$ largest number $= x + 3 = 20 + 3 = 23$.

2. Ans. (d)

Solution Let the teacher's age be x years

$$\therefore \frac{15\times15-x}{14} = 14 \Rightarrow 225 - x = 196 \Rightarrow x = 29.$$

3. Ans. (a)

Solution Let the age of the mother be x years

$$\therefore \frac{4\times12+x}{5} = 17 \Rightarrow x = 37.$$

4. Ans. (d)

Solution Total age of A and B = 40
Total age of B and C = 38
Total age of A and C = 42
Total age of $A + B + C$ = 60
$\therefore$ Age of $B = 60 - 40 = 18$.

5. Ans. (c)

Solution Total cost of all the balls = Rs 280
Let no. of red balls = x
$\therefore$ No. of white balls $= 10 - x$
$\therefore$ $25x + 30(10 - x) = 280 \Rightarrow x = 4$
$\therefore$ No. of white balls $= 10 - 4 = 6$.

6. Ans. (a)

Solution Sixth observation
$= 11 \times 90 - 5 \times 87 - 5 \times 84$
$= 990 - 435 - 420 = 135$.

7. Ans. (c)

8. Ans. (c)

Solution 6th number $= 9 \times 30 - (5 \times 25 + 3 \times 35)$
$= 270 - (230) = 40$.

9. Ans (c)

Solution Marks obtained by 14 students
$= 14 \times 71 = 994$
Exact marks of 14 students
$= 994 + \{(56 - 42) + (32 - 74)\}$
$= 994 + \{14 + (-42)\} = 994 + \{-28\}$
$= 994 - 28 = 966$

$\therefore$ Correct average $= \frac{966}{14} = 69$.

10. Ans. (c)

Solution Let the numbers be x, y, z and t

$\therefore$ $x + y + z + t = 240$ and $x = \frac{1}{4}(y + z + t)$

$\therefore$ $x = 48$

$\therefore$ First no. = 48.

11. Ans. (b)

Solution $F + S_1 + S_2 = 81$

$(S_1 - 5) + (S_2 - 5) = 24 \Rightarrow S_1 + S_2 = 34$

Also $S_1 - S_2 = 4$

$\therefore$ $S_1 = 19$, $S_2 = 15$

$\therefore$ Age of father = 47.

12. Ans. (b)

Solution $31 \times 15 - (30 \times 15 - 20) = x + x + 5$

$\therefore$ $x = 15$

13. Ans. (d)

Solution $2x + x + 4x = 56 \times 3$

$\therefore$ $2x = 48$.

14. Ans. (b)

Solution Let the first number be x

$\therefore$ second number = $2x$

and third number = $\frac{2}{3}x$

$\therefore$ $\frac{x + 2x + \frac{2}{3}x}{3} = 44$ or, $\frac{11x}{9} = 44$

$\therefore$ $x = 36$

$\therefore$ Second no. is the largest = $2x = 2 \times 36 = 72$.

15. Ans. (b)

Solution When 2 new men are replaced, average age is increased by 2 years.

$\therefore$ Increase in the total age of 8 men

$= 8 \times 2 = 16$ years

$\therefore$ Total age of 2 new men = 16 + 44 = 60 years

$\therefore$ Average age of 2 new men = $\frac{60}{2}$ = 30 years.

16. Ans. (d)

Solution $A + B = 60$, $B + C = 64$, $A + C = 68$

$\therefore$ $C - A = 4$, $C + A = 68$

$\Rightarrow$ $C = 36$, $A = 32$, $B = 28$.

17. Ans. (a)

Solution Income in the eighth month

$= (8 \times 3160 + 5 \times 4120) - (12 \times 3400)$

$= (25280 + 20600 - 40800)$

= Rs 5080.

18. Ans. (c)

Solution Age of mother + father $= 22 \times 8 - 8 \times 6$

$= 176 - 48$

$= 128$

father − mother = 8

$\therefore$ Age of mother = $\frac{128 - 8}{2}$ = 60 yrs.

19. Ans. (a)

Solution Sale amount of grocer in the sixth month

$= 6 \times 6500 - (6435 + 6927 + 6855 + 7230 + 6562)$

$= 39000 - 34009$

= Rs 4991.

20. Ans. (c)

Solution Sum of weights of A, B and $C = 84 \times 3$ kg

= 252 kg

Weight of $D = 4 \times 80 - 252 = 320 - 252 = 68$ kg

Weight of $E = 68 + 3 = 71$ kg

$A + B + C + D = 320$

$B + C + D + E = 79 \times 4 = 316$

$\Rightarrow$ $A - E = 4$ kg

$\therefore$ $A = 4 + E = 4 + 71 = 75$ kg.

21. Ans. (b)

Solution By the method of Alligation:

Technicians	:	Rest
12000	:	6000

8000

1	:	2

Total no. of workers = $7 \times 3 = 21$.

22. Ans. (c)

Solution $\frac{E + S + M + H}{4} - \frac{E + H + G + M}{4} = 15$

or, $E + S + M + H - E - H - G - M = 60$

$\therefore$ $S - G = 60$.

23. Ans. (d)

Solution Correct average $= \frac{35 \times 72 + (86 - 36)}{35}$

$\approx 72 + 1.43$

$= 73.43$.

24. Ans. (b)

25. Ans. (b)

Solution Let the four consecutive even numbers be $2x$, $2x + 2$, $2x + 4$ and $2x + 6$, respectively.

Required difference $= 2x + 6 - 2x = 6$.

26. Ans. (e)

Solution Let the three numbers be x, y and z.

or, $\frac{x+y}{2} - \frac{y+z}{2} = 15$

or, $\frac{x+y-y-z}{2} = 15$ or, $x - z = 30$.

27. Ans. (b)

Solution Total of 25 results $= 25 \times 18 = 450$

Total of first 12 results $= 12 \times 14 = 168$

Total of last 12 results $= 12 \times 17 = 204$

$\therefore$ Thirteenth result $= 450 - 168 - 204 = 78$.

28. Ans. (e)

Solution Total age of seven persons $= 30 \times 7 = 210$ yrs.

Total age of five persons $= 31 \times 5 = 155$ yrs.

$\therefore$ Total age of two persons $= (210 - 155)$ yrs $= 55$ yrs.

$\therefore$ Average age of two persons $= \frac{55}{2} = 27.5$ yrs.

29. Ans. (d)

Solution We are to determine the average weight of P, T, R, F and H.

Obviously, this cannot be determined as we do not know the weight of H.

30. Ans. (d)

Solution Let number of boys $= x$

and number of girls $= y$

Now, $y - 2 = x$ and $x + y = 52$

$y - 2 + y = 52$

$2y = 54$

$\therefore$ $y = 27$ $\therefore$ $x = 25$

Let the average weight of the girls be 'a'.

Then, $\frac{(25\times42)+(27\times a)}{52} = 40$

or, $27 \times a = (52 \times 40) - (25 \times 42)$

or, $27 \times a = 2080 - 1050$

$\therefore$ $a = \frac{1030}{27} = 38.148 \approx 38$ kg.

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. The average of marks obtained by 120 candidates in a certain examination is 35. If the average marks of passed candidates is 39 and that of the failed candidates is 15, what is the number of candidates who passed the examination?
   (a) 200 (b) 100
   (c) 300 (d) 400

2. The average monthly salary of 8 workers and one supervisor in a factory was Rs 430. When the supervisor, whose salary was Rs 870 per month, retired, a new person was appointed and then the average salary of 9 persons was Rs 400 per month. The salary of the new supervisor is
   (a) Rs 700 (b) Rs 600
   (c) Rs 430 (d) Rs 400

3. The average of 9 results is 50. The average of first four results is 52 and average of last 4 results is 49. What is the fifth result?
   (a) 48 (b) 46
   (c) 52 (d) 44

4. The average height of 30 girls out of a class of 40 is 160 cm and that of the remaining girls is 156 cm. What is the average height of the whole class?
   (a) 161 cm (b) 163 cm
   (c) 169 cm (d) 159 cm

5. The average salary of a staff of 18 officers and 32 clerks is Rs 800. If the average salary of the officers is Rs 1200, find the average salary of the clerks
   (a) Rs 565 (b) Rs 550
   (c) Rs 575 (d) Rs 600

6. The average temperature of Monday, Tuesday, Wednesday and Thursday was 38°C and that of Tuesday, Wednesday, Thursday and Friday was 40°C. If the temperature on Monday was 30°C, what was the temperature on Friday?
   (a) 39°C (b) 36°C
   (c) 38°C (d) 42°C

7. The average age of an adult class is 40 years. 12 new students with an average age of 32 years join the class, thereby decreasing the average by 4 years. The original strength of the class was
   (a) 10 (b) 11
   (c) 12 (d) 15

8. The average income of a group of 50 persons is Rs 1690. It was later discovered that one person's

income was wrongly taken as Rs 1340 instead of Rs 1430. The correct average income is
(a) Rs 1680 (b) Rs 1698
(c) Rs 1691.80 (d) None of these

9. The average of the first and the second of three numbers is 10 more than the average of the second and the third of these numbers. What is the difference between the first and the third of these three numbers?
(a) 30 (b) 20
(c) 40 (d) 50

10. The average age of four children in a family is 12 yrs. If the spacing between their ages is 4 yrs, the age of the youngest child is
(a) 6 yrs (b) 7 yrs
(c) 8 yrs (d) 9 yrs

Test II

[Time: 10 min]

11. An investor earns 3% return on $\frac{1}{4}$th of his capital, 5% on $\frac{2}{3}$rd and 11% on the remainder. What is the average rate of return he earns on his total capital?
(a) 5% (b) 10%
(c) 5.5% (d) 10.5%

12. The average of three consecutive numbers is n. If the next two consecutive numbers are also included, the average of the five numbers will
(a) remain the same (b) increase by 0.5
(c) increase by 1 (d) increase by 1.5

13. The average age of 8 persons in a committee is increased by 2 yrs when two men aged 35 yrs and 45 yrs are substituted by two women. Find the average of these two women.
(a) 48 yrs (b) 56 yrs
(c) 52 yrs (d) 44 yrs

14. The average salary of the entire staff in an office is Rs 120 per month. The average salary of officers is Rs 460 and that of non-officers is Rs 110. If the number of officers is 15, then find the number of non-officers in the office.
(a) 410 (b) 610
(c) 510 (d) 710

15. The average temperature of the first three days of a week is 27°C and that of next three days is 29°C. If the average temperature of the whole week is 28.5°C, find the temperature of the last day.
(a) 32.5°C (b) 31.5°C
(c) 33.5°C (d) 34.5°C

16. A candidate scores an aggregate of 60% marks, scoring an average of 56% in 4 of the papers and 68% in the others. How many papers were there totally?
(a) 6 (b) 7
(c) 10 (d) None of these

17. The average of five consecutive even numbers is 38. The largest of these is
(a) 40 (b) 38
(c) 44 (d) 42

18. The average score of 35 students in a class is 37. If every student is given 3 grace marks, the new average of the class is
(a) 40 (b) 34
(c) 43 (d) None of these

19. The average weight of 29 students is 40 kg. If the weight of their teacher be included, the average weight is increased by 300 gm. The weight of the teacher is
(a) 49 kg (b) 48.7 kg
(c) 49.3 kg (d) None of these

20. The average of 5 consecutive numbers is 43. The average of the three smallest of these numbers is
(a) 43 (b) 38
(c) 44 (d) 42

Answers

1. (b)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (c)	**7.** (c)	**8.** (c)	**9.** (b)	**10.** (a)
11. (a)	**12.** (c)	**13.** (a)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (d)	**18.** (a)	**19.** (a)	**20.** (b)

7

RATIO AND PROPORTION

RATIO

A *ratio* is a comparison of two quantities by division. It is a relation that one quantity bears to another with respect to magnitude. In other words, ratio means what part one quantity is of another. The quantities may be of same kind or different kinds. For example, when we consider the ratio of the weight 45 kg of a bag of rice to the weight 29 kg of a bag of sugar we are considering the quantities of same kind but when we talk of allotting 2 cricket bats to 5 sportsmen, we are considering quantities of different kinds. Normally, we consider the ratio between quantities of the same kind.

If a and b are two numbers, the ratio of a to b is $\frac{a}{b}$ or $a \div b$ and is denoted by $a : b$. The two quantities that are being compared are called *terms*. The first is called *antecedent* and the second term is called *consequent*.

For example, the ratio 3 : 5 represents $\frac{3}{5}$ with antecedent 3 and consequent 5.

Note:

1. A ratio is a number, so to find the ratio of two quantities, they must be expressed in the same units.
2. A ratio does not change if both of its terms are multiplied or divided by the same number. Thus,

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} \text{ etc.}$$

TYPES OF RATIOS

1. **Duplicate Ratio** The ratio of the squares of two numbers is called the *duplicate ratio* of the two numbers.

   For example, $\frac{3^2}{4^2}$ or $\frac{9}{16}$ is called the duplicate ratio of $\frac{3}{4}$.

2. **Triplicate Ratio** The ratio of the cubes of two numbers is called the *triplicate ratio* of the two numbers.

   For example, $\frac{3^3}{4^3}$ or $\frac{27}{64}$ is triplicate ratio of $\frac{3}{4}$.

3. **Sub-duplicate Ratio** The ratio of the square roots of two numbers is called the *sub-duplicate ratio* of two numbers.

   For example, $\frac{3}{4}$ is the sub-duplicate ratio of $\frac{9}{16}$.

4. **Sub-triplicate Ratio** The ratio of the cube roots of two numbers is called the *sub-triplicate ratio* of two numbers.

   For example, $\frac{2}{3}$ is the sub-triplicate ratio of $\frac{8}{27}$.

5. **Inverse Ratio or Reciprocal Ratio** If the antecedent and consequent of a ratio interchange their places, the new ratio is called the *inverse ratio* of the first. Thus, if $a : b$ be the given ratio, then $\frac{1}{a} : \frac{1}{b}$ or $b : a$ is its inverse ratio.

   For example, $\frac{3}{5}$ is the inverse ratio of $\frac{5}{3}$.

6. **Compound Ratio** The ratio of the product of the antecedents to that of the consequents of two or more given ratios is called the *compound ratio*. Thus, if $a : b$ and $c : d$ are two given ratios, then $ac : bd$ is the compound ratio of the given ratios.

   For example, if $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{5}{7}$ be the given ratios, then their compound ratio is $\frac{3 \times 4 \times 5}{4 \times 5 \times 7}$, that is, $\frac{3}{7}$.

PROPORTION

The equality of two ratios is called *proportion*.

If $\frac{a}{b} = \frac{c}{d}$, then a, b, c and d are said to be in proportion and we write $a : b :: c : d$. This is read as "a is to b as c is to d".

For example, since $\frac{3}{4} = \frac{6}{8}$, we write 3 : 4 : : 6 : 8 and say 3, 4, 6 and 8 are in proportion.

Each term of the ratio $\frac{a}{b}$ and $\frac{c}{d}$ is called a *proportional*. a, b, c and d are respectively the first, second, third and fourth proportionals.

Here, a, d are known as *extremes* and b, c are known as *means*.

SOME BASIC FORMULAE

1. If four quantities are in proportion, then
   Product of Means = Product of Extremes
   For example, in the proportion $a : b :: c : d$, we have $bc = ad$.

   Product of means
   $a : b :: c : d$
   Product of extremes

   From this relation we see that if any three of the four quantities are given, the fourth can be determined.

2. **Fourth proportional** If $a : b :: c : x$, x is called the *fourth proportional* of a, b, c.
   We have, $\frac{a}{b} = \frac{c}{x}$ or, $x = \frac{b \times c}{a}$.
   Thus, fourth proportional of a, b, c is $\frac{b \times c}{a}$.

Illustration 1 Find a fourth proportional to the numbers 2, 5, 4.

Solution Let x be the fourth proportional, then

$$2 : 5 :: 4 : x \text{ or } \frac{2}{5} = \frac{4}{x}.$$

$$\therefore \quad x = \frac{5 \times 4}{2} = 10.$$

3. **Third Proportional** If $a : b :: b : x$, x is called the third proportional of a, b.
   We have, $\frac{a}{b} = \frac{b}{x}$ or $x = \frac{b^2}{a}$.
   Thus, third proportional of a, b is $\frac{b^2}{a}$.

Illustration 2 Find a third proportional to the numbers 2.5, 1.5.

Solution Let x be the third proportional, then

$$2{\cdot}5 : 1{\cdot}5 :: 1{\cdot}5 : x \text{ or } \frac{2{\cdot}5}{1{\cdot}5} = \frac{1{\cdot}5}{x}.$$

$$\therefore \quad x = \frac{1{\cdot}5 \times 1{\cdot}5}{2{\cdot}5} = 0.9.$$

4. **Mean Proportional** If $a : x :: x : b$, x is called the *mean* or *second proportional* of a, b.
   We have, $\frac{a}{x} = \frac{x}{b}$ or $x^2 = ab$ or $x = \sqrt{ab}$.
   $\therefore$ Mean proportional of a and b is $\sqrt{ab}$.
   We also say that a, x, b are in *continued proportion*.

Illustration 3 Find the mean proportional between 48 and 12.

Solution Let x be the mean proportional. Then,

$$48 : x :: x : 12 \text{ or, } \frac{48}{x} = \frac{x}{12} \text{ or, } x^2 = 576 \text{ or, } x = 24.$$

5. If $\frac{a}{b} = \frac{c}{d}$, then
   (i) $\frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)
   (ii) $\frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)
   (iii) $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (Componendo and Dividendo)
   (iv) $\frac{a}{b} = \frac{a+c}{b+d} = \frac{a-c}{b-d}$.

Illustration 4 The sum of two numbers is c and their quotient is $\frac{p}{q}$. Find the numbers.

Solution Let the numbers be x, y.

Given : $x + y = c$ ...(1)

and, $\frac{x}{y} = \frac{p}{q}$ ...(2)

$\therefore \quad \frac{x}{x+y} = \frac{p}{p+q} \Rightarrow \frac{x}{c} = \frac{p}{p+q}$ [Using (1)]

$\Rightarrow \quad x = \frac{pc}{p+q}$.

SOME USEFUL SHORT-CUT METHODS

1. (a) If two numbers are in the ratio of $a : b$ and the sum of these numbers is x, then these numbers will be $\frac{ax}{a+b}$ and $\frac{bx}{a+b}$, respectively.
   or
   If in a mixture of x litres, two liquids A and B are in the ratio of $a : b$, then the quantities of liquids A and B in the mixture will be $\frac{ax}{a+b}$

litres and $\frac{bx}{a+b}$ litres, respectively.

(b) If three numbers are in the ratio of $a : b : c$ and the sum of these numbers is x, then these numbers will be $\frac{ax}{a+b+c}$, $\frac{bx}{a+b+c}$ and $\frac{cx}{a+b+c}$, respectively.

Explanation

Let the three numbers in the ratio $a : b : c$ be A, B and C. Then,

$$A = ka,\ B = kb,\ C = kc$$

and, $A + B + C = ka + kb + kc = x$

$\Rightarrow$ $k(a + b + c) = x \Rightarrow k = \frac{x}{a+b+c}$.

$\therefore$ $A = ka = \frac{ax}{a+b+c}$

$B = kb = \frac{bx}{a+b+c}$

$C = kc = \frac{cx}{a+b+c}$.

Illustration 5 Two numbers are in the ratio of 4 : 5 and the sum of these numbers is 27. Find the two numbers.

Solution Here, $a = 4$, $b = 5$ and $x = 27$.

$\therefore$ The first number $= \frac{ax}{a+b} = \frac{4\times27}{4+5} = 12$

and, the second number $= \frac{bx}{a+b} = \frac{5\times27}{4+5} = 15$.

Illustration 6 Three numbers are in the ratio of 3 : 4 : 8 and the sum of these numbers is 975. Find the three numbers.

Solution Here, $a = 3$, $b = 4$, $c = 8$ and $x = 975$.

$\therefore$ The first number $= \frac{ax}{a+b+c} = \frac{3\times975}{3+4+8} = 195$.

The second number $= \frac{bx}{a+b+c} = \frac{4\times975}{3+4+8} = 260$.

and, the third number $= \frac{cx}{a+b+c} = \frac{8\times975}{3+4+8} = 520$.

2. If two numbers are in the ratio of $a : b$ and difference between these numbers is x, then these numbers will be

(a) $\frac{ax}{a-b}$ and $\frac{bx}{a-b}$, respectively (where $a > b$)

(b) $\frac{ax}{b-a}$ and $\frac{bx}{b-a}$, respectively (where $a < b$).

Explanation

Let the two numbers be ak and bk.

Let $a > b$.

Given : $ak - bk = x$

$\Rightarrow$ $(a - b)k = x$ or $k = \frac{x}{a-b}$.

Therefore, the two numbers are $\frac{ax}{a-b}$ and $\frac{bx}{a-b}$.

Illustration 7 Two numbers are in the ratio of 4 : 5. If the difference between these numbers is 24, then find the numbers.

Solution Here, $a = 4$, $b = 5$ and $x = 24$.

$\therefore$ The first number $= \frac{ax}{b-a} = \frac{4\times24}{5-4} = 96$

and, the second number $= \frac{bx}{b-a} = \frac{5\times24}{5-4} = 120$.

3. (a) If $a : b = n_1 : d_1$ and $b : c = n_2 : d_2$, then $a : b : c = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$.

(b) If $a : b = n_1 : d_1$, $b : c = n_2 : d_2$ and $c : d = n_3 : d_3$, then $a : b : c : d = (n_1 \times n_2 \times n_3) : (d_1 \times n_2 \times n_3) : (d_1 \times d_2 \times n_3) : (d_1 \times d_2 \times d_3)$.

Illustration 8 If $A : B = 3 : 4$ and $B : C = 8 : 9$, find $A : B : C$.

Solution Here, $n_1 = 3$, $n_2 = 8$, $d_1 = 4$ and $d_2 = 9$.

$\therefore$ $a : b : c = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$

$= (3 \times 8) : (4 \times 8) : (4 \times 9)$

$= 24 : 32 : 36$ or, $6 : 8 : 9$.

Illustration 9 If $A : B = 2 : 3$, $B : C = 4 : 5$ and $C : D = 6 : 7$, find $A : D$.

Solution Here, $n_1 = 2$, $n_2 = 4$, $n_3 = 6$, $d_1 = 3$, $d_2 = 5$ and $d_3 = 7$.

$\therefore$ $A : B : C : D = (n_1 \times n_2 \times n_3) : (d_1 \times n_2 \times n_3) : (d_1 \times d_2 : n_3) : (d_1 \times d_2 \times d_3)$

$= (2 \times 4 \times 6) : (3 \times 4 \times 6) : (3 \times 5 \times 6) : (3 \times 5 \times 7)$

$= 48 : 72 : 90 : 105$ or, $16 : 24 : 30 : 35$.

Thus, $A : D = 16 : 35$.

4. (a) The ratio between two numbers is $a : b$. If x is added to each of these numbers, the ratio becomes $c : d$. The two numbers are given as:

$$\frac{ax(c-d)}{ad-bc} \text{ and } \frac{bx(c-d)}{ad-bc}.$$

Explanation

Let two numbers be ak and bk.

Given: $\frac{ak+x}{bk+x} = \frac{c}{d} \Rightarrow akd + dx = cbk + cx$

$\Rightarrow k(ad - bc) = x(c - d)$

$\Rightarrow k = \frac{x(c-d)}{ad-bc}$.

Therefore, the two numbers are $\frac{ax(c-d)}{ad-bc}$ and $\frac{bx(c-d)}{ad-bc}$.

(b) The ratio between two numbers is $a : b$. If x is subtracted from each of these numbers, the ratio becomes $c : d$.

The two numbers are given as:

$$\frac{ax(d-c)}{ad-bc} \text{ and } \frac{bx(d-c)}{ad-bc}.$$

Explanation

Let the two numbers be ak and bk.

Given : $\frac{ak-x}{bk-x} = \frac{c}{d} \Rightarrow akd - xd = bck - xc$

$\Rightarrow k(ad - bc) = x(d - c)$

$\Rightarrow k = \frac{x(d-c)}{ad-bc}$.

Therefore, the two numbers are $\frac{ax(d-c)}{ad-bc}$ and $\frac{bx(d-c)}{ad-bc}$.

Illustration 10 Given two numbers which are in the ratio of 3 : 4. If 8 is added to each of them, their ratio is changed to 5 : 6. Find the two numbers.

Solution We have,

$a : b = 3 : 4$, $c : d = 5 : 6$ and $x = 8$.

$\therefore$ The first number $= \frac{ax(c-d)}{ad-bc}$

$= \frac{3\times8\times(5-6)}{(3\times6-4\times5)} = 12$

and, the second number $= \frac{bx(c-d)}{ad-bc}$

$= \frac{4\times8\times(5-6)}{(3\times6-4\times5)} = 16.$

Illustration 11 The ratio of two numbers is 5 : 9. If each number is decreased by 5, the ratio becomes 5 : 11. Find the numbers.

Solution We have,

$a : b = 5 : 9$, $c : d = 5 : 11$ and $x = 5$.

$\therefore$ The first number $= \frac{ax(d-c)}{ad-bc}$

$= \frac{5\times5\times(11-5)}{(5\times11-9\times5)} = 15$

and, the second number $= \frac{bx(d-c)}{ad-bc}$

$= \frac{9\times5\times(11-5)}{(5\times11-9\times5)} = 27.$

5. (a) If the ratio of two numbers is $a : b$, then the numbers that should be added to each of the numbers in order to make this ratio $c : d$ is given by

$$\frac{ad-bc}{c-d}.$$

Explanation

Let the required number be x.

Given : $\frac{a+x}{b+x} = \frac{c}{d} \Rightarrow ad + xd = bc + xc$

$\Rightarrow x(d - c) = bc - ad$

or, $x = \frac{ad-bc}{c-d}$.

(b) If the ratio of two numbers is $a : b$, then the number that should be subtracted from each of the numbers in order to make this ratio $c : d$ is given by

$$\frac{bc-ad}{c-d}.$$

Explanation

Let the required number be x.

Given: $\frac{a-x}{b-x} = \frac{c}{d} \Rightarrow ad - xd = bc - xc$

$\Rightarrow x(c - d) = bc - ad$

or, $x = \frac{bc-ad}{c-d}$.

Illustration 12 Find the number that must be subtracted from the terms of the ratio 5 : 6 to make it equal to 2 : 3.

Solution We have $a : b = 5 : 6$ and $c : d = 2 : 3$.

$\therefore$ The required number $= \frac{bc-ad}{c-d}$

$= \frac{6\times2-5\times3}{2-3} = 3.$

Illustration 13 Find the number that must be added to the terms of the ratio 11 : 29 to make it equal to 11 : 20.

Solution We have, $a : b = 11 : 29$ and $c : d = 11 : 20$.

$\therefore$ The required number $= \frac{ad-bc}{c-d}$

$= \frac{11\times20-29\times11}{11-20} = 11.$

6. There are four numbers a, b, c and d.

(i) The number that should be subtracted from each of these numbers so that the remaining numbers may be proportional is given by

$$\frac{ad-bc}{(a+d)-(b+c)}.$$

Explanation

Let x be subtracted from each of the numbers.

The remainders are $a - x$, $b - x$, $c - x$ and $d - x$.

Given: $\frac{a-x}{b-x} = \frac{c-x}{d-x}$

$\Rightarrow (a - x)(d - x) = (b - x)(c - x)$

$\Rightarrow ad - x(a + d) + x^2 = bc - x(b + c) + x^2$

$\Rightarrow (b + c)x - (a + d)x = bc - ad$

$\therefore \quad x = \frac{bc-ad}{(b+c)-(a+d)}$ or, $\frac{ad-bc}{(a+d)-(b+c)}$

(ii) The number that should be added to each of these numbers so that the new numbers may be proportional is given by

$$\frac{bc-ad}{(a+d)-(b+c)}.$$

Explanation

Let x be added to each of the numbers.

The new numbers are $a + x$, $b + x$, $c + x$ and $d + x$.

Given: $\frac{a+x}{b+x} = \frac{c+x}{d+x}$.

$\Rightarrow (a + x)(d + x) = (b + x)(c + x)$

$\Rightarrow ad + x(a + d) + x^2 = bc + x(b + c) + x^2$

$\Rightarrow (a + d)x - (b + c)x = bc - ad.$

$\therefore \quad x = \frac{bc-ad}{(a+d)-(b+c)}.$

Illustration 14 Find the number subtracted from each of the numbers 54, 71, 75 and 99 leaves the remainders which are proportional.

Solution We have, $a = 54$, $b = 71$, $c = 75$ and $d = 99$.

$$\text{The required number} = \frac{ad-bc}{(a+d)-(b+c)}$$

$$= \frac{54\times 99-71\times 75}{(54+99)-(71+75)} = 3.$$

7. The incomes of two persons are in the ratio of $a : b$ and their expenditures are in the ratio of $c : d$. If the saving of each person be Rs. S, then their incomes are given by

$$\text{Rs. } \frac{aS(d-c)}{ad-bc} \text{ and Rs. } \frac{bS(d-c)}{ad-bc}.$$

and, their expenditures are given by

$$\text{Rs. } \frac{cS(b-a)}{ad-bc} \text{ and Rs. } \frac{dS(b-a)}{ad-bc}.$$

Explanation

Let their incomes be Rs. ak and Rs. bk, respectively.

Since each person saves Rs. S,

$\therefore$ expenditure of first person = Rs. $(ak - S)$

and expenditure of second person = Rs. $(bk - S)$.

Given : $\frac{ak-S}{bk-S} = \frac{c}{d}$

$\Rightarrow akd - Sd = bkc - Sc$

$\Rightarrow k(ad - bc) = (d - c)S$ or, $k = \frac{(d-c)S}{ad-bc}$.

Therefore, the incomes of two persons are

$$\frac{a(d-c)S}{ad-bc} \text{ and } \frac{b(d-c)S}{ad-bc}$$

and their expenditures are

$$ak - S \quad \text{and} \quad bk - S$$

that is, $\frac{a(d-c)S}{ad-bc} - S$ and $\frac{b(d-c)S}{ad-bc} - S$

or, $\frac{cS(b-a)}{ad-bc}$ and $\frac{dS(b-a)}{ad-bc}$.

Illustration 15 Annual income of A and B is in the ratio of 5 : 4 and their annual expenses bear a ratio of 4 : 3. If each of them saves Rs. 500 at the end of the year, then find their annual income.

Solution We have, $a : b = 5 : 4$, $c : d = 4 : 3$ and $S = 500$.

$$\therefore \text{ Annual income of } A = \frac{aS(d-c)}{ad-bc}$$

$$= \frac{5\times 500\times(3-4)}{(5\times 3-4\times 4)}$$

$$= \text{Rs. } 2500.$$

$$\text{and annual income of } B = \frac{bS(d-c)}{ad-bc}$$

$$= \frac{4\times 500\times(3-4)}{(5\times 3-4\times 4)}$$

$$= \text{Rs. } 2000.$$

Illustration 16 The incomes of Mohan and Sohan are in the ratio 7 : 2 and their expenditures are in the ratio 4 : 1. If each saves Rs. 1000, find their expenditures.

Solution We have, $a : b = 7 : 2$, $c : d = 4 : 1$ and $S = 1000$.

$$\therefore \text{ A's expenditure} = \frac{cS(b-a)}{ad-bc} = \frac{4\times 1000\times(2-7)}{(7\times 1-2\times 4)}$$

$$= \text{Rs. } 20000$$

and, B's expenditure $= \dfrac{dS(b-a)}{ad-bc} = \dfrac{1\times1000\times(2-7)}{(7\times1-2\times4)}$

$= \text{Rs. } 5000.$

8. (a) If in a mixture of x litres of two liquids A and B, the ratio of liquids A and B is $a : b$, then the quantity of liquid B to be added in order to make this ratio.

$$c : d \text{ is } \frac{x(ad-bc)}{c(a+b)}.$$

Explanation

Quantity of liquid A in the mixture $= \dfrac{ax}{a+b}$.

Quantity of liquid B in the mixture $= \dfrac{bx}{a+b}$.

Let litres of liquid B to be added in order to make this ratio as $c : d$.

Then, $\dfrac{ax}{a+b} : \dfrac{bx}{a+b} + l = c : d$

or, $\dfrac{ax}{a+b} : \dfrac{bx+l(a+b)}{a+b} = c : d$

or, $\dfrac{ax}{bx+l(a+b)} = \dfrac{c}{d}$ or, $axd = bcx + cl(a+b)$

or, $l = \dfrac{x(ad-bc)}{(a+b)c}$.

(b) In a mixture of two liquids A and B, the ratio of liquids A and B is $a : b$. If on adding x litres of liquid B to the mixture, the ratio of A to B becomes $a : c$, then in the beginning the quantity of liquid A in the mixture was $\dfrac{ax}{c-b}$ litres and that of liquid B was $\dfrac{bx}{c-b}$ litres.

Explanation

Let the quantity of mixture be M litres.

Then, the quantity of liquid $A = \dfrac{aM}{a+b}$ litres

and the quantity of liquid $B = \dfrac{bM}{a+b}$ litres.

If x litres of liquid B is added, then

$$\frac{aM}{a+b} : \frac{bM}{a+b} + x = a : c$$

or, $\dfrac{aM}{a+b} : \dfrac{bM+x(a+b)}{a+b} = a : c$

or, $\dfrac{aM}{bM+x(a+b)} = \dfrac{a}{c}$ or, $cM = bM + x(a+b)$

or, $M = \dfrac{x(a+b)}{c-b}$.

$\therefore$ Quantity of liquid $A = \dfrac{ax(a+b)}{(c-b)(a+b)}$

$= \dfrac{ax}{c-b}$ litres

and, quantity of liquid $B = \dfrac{bx(a+b)}{(c-b)(a+b)}$

$= \dfrac{bx}{c-b}$ litres.

Illustration 17 729 ml of a mixture contains milk and water in the ratio 7 : 2. How much more water is to be added to get a new mixture containing milk and water in the ratio of 7 : 3.

Solution Here, $x = 729$, $a : b = 7 : 2$ and $c : d = 7 : 3$.

$\therefore$ The quantity of water to be added

$$= \frac{x(ad-bc)}{c(a+b)} = \frac{729\times(7\times3-2\times7)}{7(7+2)} = 81 \text{ ml.}$$

Illustration 18 A mixture contains alcohol and water in the ratio of 6 : 1. On adding 8 litres of water, the ratio of alcohol to water becomes 6 : 5. Find the quantity of water in the mixture.

Solution We have, $a : b = 6 : 1$, $a : c = 6 : 5$ and $x = 8$.

$\therefore$ The quantity of water in the mixture

$$= \frac{bx}{c-b} = \frac{1\times8}{5-1} = 2 \text{ litres.}$$

9. When two ingredients A and B of quantities q_1 and q_2 and cost price/unit c_1 and c_2 are mixed to get a mixture c having cost price/unit c_m, then

(a) $\dfrac{q_1}{q_2} = \dfrac{c_2-c_m}{c_m-c_1}$ and

(b) $c_m = \dfrac{c_1\times q_1 + c_2\times q_2}{q_1+q_2}$.

Illustration 19 In what ratio the two kinds of tea must be mixed together into one at Rs. 9 per kg and another at Rs. 15 per kg, so that mixture may cost Rs. 10.2 per kg?

Solution We have, $c_1 = 9$, $c_2 = 15$, $c_m = 10.2$

$$\therefore \frac{q_1}{q_2} = \frac{c_2-c_m}{c_m-c_1} = \frac{15-10.2}{10.2-9} = \frac{4.8}{1.2} = \frac{4}{1}.$$

Thus, the two kinds of tea are mixed in the ratio 4 : 1.

Illustration 20 In a mixture of two types of oils O_1 and O_1, the ratio $O_1 : O_2$ is 3 : 2. If the cost of oil O_1 is Rs. 4 per litre and that of O_2 is Rs. 9 per litre, then find the cost/litre of the resulting mixture.

Solution We have, $q_1 = q_2 = 2$, $c_1 = 4$ and $c_2 = 9$.

$\therefore$ The cost of resulting mixture

$$= \frac{c_1 \times q_1 + c_2 \times q_2}{q_1 + q_2} = \frac{4 \times 3 + 9 \times 2}{3+2} = \frac{30}{5} = \text{Rs. } 6.$$

10. (a) If a mixture contains two ingredients A and B in the ratio $a : b$, then

percentage of A in the mixture $= \frac{a}{a+b} \times 100\%$ and percentage of B in the mixture $= \frac{b}{a+b} \times 100\%$.

(b) If two mixtures M_1 and M_2 contain ingredients A and B in the ratios $a : b$ and $c : d$ respectively, then a third mixture M_3 obtained by mixing M_1 and M_2 in the ratio $x : y$ will contain

$$\left[\frac{\frac{a \times x}{a+b} + \frac{c \times y}{c+d}}{x+y}\right] \times 100\% \text{ ingredient } A, \text{ and}$$

$$\left[100\% - \left\{\frac{\frac{ax}{a+b} + \frac{cy}{c+d}}{x+y}\right\}\right]$$

or, $$\left[\frac{\frac{bx}{a+b} + \frac{dy}{c+d}}{x+y}\right] \times 100\% \text{ ingredient } B.$$

Illustration 21 If a mixture contains water and alcohol in the ratio 2 : 3, what is the percentage quantity of water in the mixture?

Solution Here, $a = 2$, $b = 3$.

$\therefore$ percentage quantity of water in the mixture

$$= \frac{a}{a+b} \times 100\% = \frac{2}{2+3} \times 100\%$$

$$= \frac{2}{5} \times 100\%$$

$$= \frac{200}{5} \text{ or, } 40\%.$$

Illustration 22 Two alloys contain silver and copper in the ratio 3 : 1 and 5 : 3. In what ratio the two alloys should be added together to get a new alloy having silver and copper in the ratio of 2 : 1?

Solution We have, $a : b = 3 : 1$, $c : d = 5 : 3$

Let the two alloys be mixed in the ratio $x : y$.

Then, percentage quantity of sliver in the new alloy

$$= \left[\frac{\frac{ax}{a+b} + \frac{cy}{c+d}}{x+y}\right] \times 100\%$$

$$= \left[\frac{\frac{3x}{4} + \frac{5y}{8}}{x+y}\right] \times 100\%$$

$$= \frac{6x+5y}{8(x+y)} \times 100\% \quad \text{...(1)}$$

Since the ratio of silver and copper in the new alloys is 2 : 1.

$\therefore$ percentage quantity of sliver in the new alloy

$$= \frac{2}{2+1} \times 100\% = \frac{200}{3}\% \quad \text{...(2)}$$

From (1) and (2), we get

$$\frac{6x+5y}{8(x+y)} = \frac{2}{3} \text{ or } 18x + 15y = 16x + 16y$$

or, $2x = y$

or, $x : y = 1 : 2$.

Hence, the two alloys should be mixed in the ratio 1:2.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Find a fourth proportional to the numbers 60, 48, 30.
(a) 36 (b) 24
(c) 48 (d) None of these

2. Find the value of x in the following proportion: 27 : 72 : : x : 8
(a) 5 (b) 7
(c) 3 (d) None of these

3. Find a third proportional to the numbers 4, 42.
(a) 441 (b) 541
(c) 641 (d) None of these

4. If 18 : x = x : 8, then x is equal to
(a) 12 (b) 16
(c) 18 (d) None of these

5. The third proportional to 0.8 and 0.2 is
(a) 0.6 (b) 0.05
(c) 0.7 (d) None of these

6. The fourth proportional to 0.2, 0.12 and 0.3 is
(a) 0.24 (b) 0.16
(c) 0.18 (d) None of these

7. In a ratio 11 : 14, if the antecedent is 55, the consequent is
(a) 70 (b) 90
(c) 60 (d) None of these

8. The mean proportional between 64 and 81 is
(a) 48 (b) 68
(c) 72 (d) None of these

9. The mean proportional of 0.25 and 0.04 is
(a) 0.01 (b) 0.1
(c) $10\sqrt{10}$ (d) None of these

10. The ratio of two numbers is 3 : 4 and their sum is 420. The greater of the two numbers is
(a) 360 (b) 240
(c) 180 (d) None of these

11. The ratio of boys and girls in a school is 9 : 5. If the total number of students in the school is 1050, then the number of boys is
(a) 785 (b) 890
(c) 675 (d) None of these

12. An amount of Rs. 1200 is distributed among A, B and C in the ratio of 5 : 7 : 13. What is the difference between the shares of C and B?
(a) Rs. 288 (b) Rs. 328
(c) Rs. 296 (d) None of these

13. Amit, Sumit and Puneet share an amount of Rs. 660 in the ratio of 3 : 4 : 5. What is the share of Puneet?
(a) Rs. 375 (b) Rs. 275
(c) Rs. 575 (d) None of these

14. Three numbers A, B and C are in the ratio of 12 : 15 : 25. If sum of these numbers is 312, find the ratio between the difference of B and A and the difference of C and B.
(a) 3 : 7 (b) 10 : 3
(c) 3 : 10 (d) None of these

15. The prices of a scooter and a television set are in the ratio of 3 : 2. If a scooter costs Rs. 600 more than the television set, then the price of television set is
(a) Rs. 1800 (b) Rs. 1200
(c) Rs. 2400 (d) None of these

16. Two numbers are in the ratio of 4 : 9. If the larger number is 35 more than the smaller number, then the product of the numbers is
(a) 1764 (b) 1564
(c) 1864 (d) None of these

17. If the income of A, B and C is in the ratio of 2 : 5 : 11 and the income of B is Rs. 291 more than that of A, then the income of C is
(a) Rs. 907 (b) Rs. 1127
(c) Rs. 1067 (d) None of these

18. If $A : B = 7 : 5$ and $B : C = 9 : 11$, then $A : B : C$ is equal to
(a) 55 : 45 : 63 (b) 63 : 45 : 55
(c) 45 : 63 : 55 (d) None of these

19. If $A : B = 3/4$, $B : C = 4/5$, $C : D = 5/6$, then $A : D$ will be
(a) 2 : 3 (b) 4 : 3
(c) 1 : 2 (d) None of these

20. If $3A = 4B = 5C$, then $A : B : C$ is
(a) 16 : 20 : 18 (b) 15 : 20 : 16
(c) 20 : 15 : 12 (d) None of these

21. If $3A = 5B$ and $2B = 3C$, then $A : C$ is
(a) 5 : 2 (b) 7 : 2
(c) 3 : 2 (d) None of these

22. Ajay, Aman, Suman and Geeta rented a house and agreed to share the rent as follows:
Ajay : Aman = 8 : 15, Aman : Suman = 5 : 8 and Suman and Geeta = 4 : 5. The part of rent paid by Suman will be
(a) $\frac{24}{77}$ (b) $\frac{12}{55}$
(c) $\frac{13}{66}$ (d) None of these

23. The ratio of money with Anju and Sanju is 4 : 5 and that with Sanju and Manju is 5 : 6. If Anju has Rs. 280, then the amount of money Manju has
(a) Rs. 320 (b) Rs. 420
(c) Rs. 640 (d) None of these

24. There are three sections A, B and C in a school of class I. The ratio of students in sections A and B is 3 : 5 and that in B and C is 4 : 7. If the total number of students in the class be 201, then the number of students in section A are
(a) 24 (b) 36
(c) 48 (d) None of these

25. The sum of three numbers is 124. If the ratio between the first and second be 2 : 3 and that between the second and third be 7 : 9, then the third number is
(a) 54 (b) 64
(c) 48 (d) None of these

26. A, B, C and D share a property worth Rs. 77500. If $A : B = 3 : 2$, $B : C = 5 : 4$ and $C : D = 3 : 7$, find the share of B.
(a) Rs. 20000 (b) Rs. 15000
(c) Rs. 25000 (d) None of these

27. Two numbers are in the ratio 3 : 5. If each number is increased by 10, the ratio becomes 5 : 7. The numbers are
(a) 15, 25 (b) 30, 45
(c) 48, 60 (d) None of these

28. The ratio between two numbers is 2 : 3. If each number is increased by 4, the ratio becomes 5 : 7. The numbers are
(a) 8, 16 (b) 16, 24
(c) 24, 32 (d) None of these

29. Two numbers are in the ratio of 5 : 6. If 5 is subtracted from each number, the ratio becomes 4 : 5. The numbers are

(a) 25, 30 (b) 30, 45
(c) 15, 20 (d) None of these

30. The ratio of present ages of Suresh and Mahesh is 7 : 5. If after 6 years their ages will be in the ratio of 4 : 3, the present age of Mahesh is
(a) 32 years (b) 36 years
(c) 30 years (d) None of these

31. The ratio of present ages of Sita and Gita is 4 : 3. If, 4 years before, the ratio of their ages was 2 : 1, the present age of Sita is
(a) 8 years (b) 10 years
(c) 12 years (d) None of these

32. Two numbers are in the ratio of 5 : 8. If 12 be added to each, they are in the ratio of 3 : 4. Find the sum of two numbers.
(a) 43 (b) 39
(c) 47 (d) None of these

33. Two numbers are in the ratio of 5 : 7. If 25 be subtracted from each, they are in the ratio of 35 : 59. Find the difference of the two numbers.
(a) 48 (b) 52
(c) 24 (d) None of these

34. When x is added to each term of 7 : 13 the ratio becomes 2 : 3. The value of x is
(a) 7 (b) 11
(c) 5 (d) None of these

35. Find the number which, when subtracted from the terms of the ratio 12 : 17 makes it equal to the ratio 2 : 3.
(a) 2 (b) 6
(c) 8 (d) None of these

36. The value of k that must be added to 7, 16, 43, 79 so that they are in proportion, is
(a) 7 (b) 5
(c) 9 (d) None of these

37. What should be subtracted from 15, 28, 20 and 38 so that the remaining numbers may be proportional?
(a) 2 (b) 4
(c) 6 (d) None of these

38. The number that must be added to each of the numbers 8, 21, 13 and 31 to make the ratio of first two numbers equal to the ratio of last two numbers is
(a) 5 (b) 7
(c) 9 (d) None of these

39. The incomes of A and B are in the ratio 3 : 2 and their expenditures in the ratio 5 : 3. If each saves Rs. 1000, A's income is
(a) Rs. 5000 (b) Rs. 6000
(c) Rs. 8000 (d) None of these

40. The annual incomes and expenditures of man and his wife are in the ratios of 5 : 3 and 3 : 1, respectively. If they decide to save equally and find a balance of Rs. 4000 at the end of year, their incomes were
(a) Rs. 5000, Rs. 3000 (b) Rs. 6000, Rs. 4000
(c) Rs. 3000, Rs. 2000 (d) None of these

41. The incomes of Gupta and Verma are in the ratio 9 : 4 and their expenditures are in the ratio 7 : 3. If each saves Rs. 2000, then Gupta's expenditure is
(a) Rs. 6000 (b) Rs. 8000
(c) Rs. 7000 (d) None of these

42. In a mixture of 60 litres, the ratio of milk and water is 2 : 1. What amount of water must be added to make the ratio of milk and water as 1 : 2?
(a) 75 litres (b) 55 litres
(c) 60 litres (d) None of these

43. A mixture contains alcohol and water in the ratio of 12 : 5. On adding 14 litres of water, the ratio of alcohol to water becomes 4 : 3. The quantity of alcohol in the mixture is
(a) 18 litres (b) 24 litres
(c) 26 litres (d) None of these

44. If an alloy contains copper and silver in the ratio 3 : 7, then the percentage quantity of silver in the alloy is
(a) 90% (b) 70%
(c) 60% (d) None of these

45. Two alloys contain zinc and copper in the ratio of 2 : 1 and 4 : 1. In what ratio the two alloys should be added together to get as new alloy having zinc and copper in the ratio of 3 : 1?
(a) 7 : 5 (b) 5 : 7
(c) 3 : 5 (d) None of these

46. Mixture of milk and water has been kept in two separate containers. Ratio of milk to water in one of the containers is 5 : 1 and that in the other container is 7 : 2. In what ratio the mixtures of these two containers should be added together so that the quantity of milk in the new mixture may become 80%?
(a) 2 : 3 (b) 3 : 2
(c) 4 : 5 (d) None of these

EXPLANATORY ANSWERS

1. Ans. (b)

Solution Let x be the fourth proportional, then

$$60 : 48 : 30 : x \text{ or, } \frac{60}{48} = \frac{30}{x}$$

$$\therefore \quad x = \frac{30 \times 38}{60} = 24.$$

2. Ans. (c)

Solution We have, $27 : 72 :: x : 8$ or, $\frac{27}{72} = \frac{x}{8}$

$$\therefore \quad x = \frac{8 \times 27}{72} = 3.$$

3. Ans. (a)

Solution Let x be the third proportional, then

$$4 : 42 :: 42 : x \text{ or, } \frac{4}{42} = \frac{42}{x}.$$

$$\therefore \quad x = \frac{42 \times 42}{4} = 441.$$

4. Ans. (a)

Solution We have, $\frac{18}{x} = \frac{x}{8}$ or, $x^2 = 18 \times 8$

or, $\quad x = \sqrt{144} = 12.$

5. Ans. (b)

Solution Let x be the third proportional. Then,

$$0.8 : 0.2 :: 0.2 : x \text{ or, } \frac{0{\cdot}8}{0{\cdot}2} = \frac{0{\cdot}2}{x}$$

$$\therefore \quad x = \frac{0{\cdot}2 \times 0{\cdot}2}{0{\cdot}8} = 0{\cdot}05.$$

6. Ans. (c)

Solution Let x be the fourth proportional. Then,

$$0.2 : 0{\cdot}2 :: 0.2\ x \text{ or. } \frac{0.2}{0.12} = \frac{0.3}{x}.$$

$$\therefore \quad x = \frac{0.3 \times 0.12}{0.2} = 0.18$$

7. Ans. (a)

Solution $\frac{11}{14} = \frac{11 \times 5}{14 \times 5} = \frac{55}{70}.$

$\therefore$ Consequent = 70.

8. Ans. (c)

Solution Let x be the mean proportional. Then,

$$64 : x :: x : 81 \text{ or, } \frac{64}{x} = \frac{x}{81} \text{ or, } x^2 = 5184 \text{ or } x = 72.$$

9. Ans. (b)

Solution Let x be the mean proportional. Then,

$$0.25 : x : x : 0{\cdot}04 \text{ or, } \frac{0.25}{x} = \frac{x}{0{\cdot}04}$$

or, $\quad x^2 = 0.01$ or, $x = 0{\cdot}1$

10. Ans. (b)

Solution Here, $a = 3$, $b = 4$ and $x = 420$.

$$\therefore \quad \text{The first number} = \frac{ax}{a+b} = \frac{3 \times 420}{3+4} = 180.$$

$$\text{and, the second number} = \frac{bx}{a+b} = \frac{4 \times 420}{3+4} = 240.$$

11. Ans. (c)

Solution Here, $a = 9$, $b = 5$ and $x = 1050$.

$$\therefore \quad \text{Number of boys} = \frac{ax}{a+b} = \frac{9 \times 1050}{9+5} = 675.$$

12. Ans. (a)

Solution We have, $a = 5$, $b = 7$, $c = 13$ and $x = 1200$.

$$\therefore \quad \text{Share of } B = \frac{bx}{a+b+c} = \frac{7 \times 1200}{5+7+13} = 336$$

$$\text{and, share of } C = \frac{cx}{a+b+c} = \frac{13 \times 1200}{5+7+13} = 624.$$

13. Ans. (b)

Solution Here, $a = 3$, $b = 4$, $c = 5$ and $x = 660$.

$$\therefore \text{Share of Puneet} = \frac{cx}{a+b+c} = \frac{5 \times 660}{3+4+5} = \text{Rs. } 275.$$

14. Ans. (c)

Solution We have, $a = 12$, $b = 15$, $c = 25$ and $x = 312$.

$$\therefore \quad A = \frac{ax}{a+b+c} = \frac{12 \times 312}{12+15+25} = 72,$$

$$B = \frac{bx}{a+b+c} = \frac{15 \times 312}{12+15+25} = 90$$

$$\text{and, } C = \frac{cx}{a+b+c} = \frac{25 \times 312}{12+15+25} = 150$$

$\therefore \quad B - A = 18$ and $C - B = 60$

Thus, their ratio = 18 : 60 or, 3 : 10.

15. Ans. (b)

Solution Here, $a = 3$, $b = 2$ and $x = 600$.

$$\therefore \text{The price of a television set} = \frac{bx}{a-b} = \frac{2 \times 600}{3-2}$$

$$= \text{Rs. } 1200.$$

16. Ans. (a)

Solution Here, $a = 4$, $b = 9$ and $x = 35$.

$$\therefore \quad \text{The first number} = \frac{ax}{b-a} = \frac{4 \times 35}{9-4} = 28$$

$$\text{and, the second number} = \frac{bx}{b-a} = \frac{9 \times 35}{9-4} = 63.$$

Thus, the product of the numbers = 28 × 63 = 1764.

17. Ans. (c)

Solution Ratio of the income of A, B and C = 2 : 5 : 11.

$\therefore$ Ratio of the income of A and B = 2 : 5.

Difference between income of A and B = Rs. 291.

$\therefore$ Income of $C = \frac{cx}{b-a} = \frac{11\times291}{5-2}$ = Rs. 1067.

[Here, a = 2, b = 5, c = 11 and x = 291]

18. Ans. (b)

Solution Here, $n_1 = 7$, $n_2 = 9$, $d_1 = 5$ and $d_2 = 11$.

$\therefore$ $A : B : C = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$

$= (7 \times 9) : (5 \times 9) : (5 \times 11)$

$= 63 : 45 : 55.$

19. Ans. (c)

Solution We have, $n_1 = 3$, $n_2 = 4$, $n_3 = 5$, $d_1 = 4$, $d_2 = 5$ and $d_3 = 6$.

$\therefore$ $A : B : C : D = (n_1 \times n_2 \times n_3) : (d_1 \times n_2 \times n_3)$ $(d_1 \times d_2 \times n_3) : (d_1 \times d_2 \times d_3)$

$= (3 \times 4 \times 5) : (4 \times 4 \times 5) : (4 \times 5 \times 5) : (4 \times 5 \times 6)$

$= 60 : 80 : 100 : 120$ or, $3 : 4 : 5 : 6$.

Thus, $A : D = 3 : 6$ or, $1 : 2$.

20. Ans. (c)

Solution We have $A : B = 4 : 3$ and $B : C = 5 : 4$.

Here, $n_1 = 4$, $n_2 = 5$, $d_1 = 3$ and $d_2 = 4$.

$\therefore$ $A : B : C = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$

$= (4 \times 5) : (3 \times 5) : (3 \times 4)$

$= 20 : 15 : 12.$

21. Ans. (a)

Solution We have, $A : B = 5 : 3$ and $B : C = 3 : 2$.

Here, $n_1 = 5$, $n_2 = 3$, $d_1 = 3$ and $d_2 = 2$.

$\therefore$ $A : B : C = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$

$= (5 \times 3) : (3 \times 3) : (3 \times 2)$

$= 15 : 9 : 6$ or $5 : 3 : 2$.

Thus, $A : C = 5 : 2$.

22. Ans. (a)

Solution We have, $A : B = 8 : 15$, $B : C = 5 : 8$ and $C : D = 4 : 5$.

Here, $n_1 = 8$, $n_2 = 5$, $n_3 = 4$, $d_1 = 15$, $d_2 = 8$ and $d_3 = 5$.

$\therefore$ $A : B : C : D = (n_1 \times n_2 \times n_3) : (d_1 \times n_2 \times n_3) : (d_1 \times d_2 \times n_3) : (d_1 \times d_2 \times d_3)$

$= (8 \times 5 \times 4) : (15 \times 5 \times 4) : (15 \times 8 \times 4) : (15 \times 8 \times 5)$

$= 160 : 300 : 480 : 600$ or, $8 : 15 : 24 : 30$.

Thus, Suman pays $= \frac{24}{8+15+24\times30}$ of the rent

$= \frac{24}{77}$ of the rent.

23. Ans. (b)

Solution We have, $A : B = 4 : 5$ and $B : C = 5 : 6$.

Here, $n_1 = 4$, $n_2 = 5$, $d_1 = 5$ and $d_2 = 6$.

$\therefore$ $A : B : C = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$

$= (4 \times 5) : (5 \times 5) : (5 \times 6)$

$= 20 : 25 : 30$ or, $4 : 5 : 6$.

Thus, ratio of money with Anju, Sanju and Manju is 4 : 5 : 6. Since Anju has Rs. 280, the amount of money Manju has $= \frac{280}{4} \times 6$ = Rs. 420.

24. Ans. (b)

Solution We have, $A : B = 3 : 5$ and $B : C = 4 : 7$.

Here, $n_1 = 3$, $n_2 = 4$, $d_1 = 5$ and $d_2 = 7$.

$\therefore$ $A : B : C = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$

$= (3 \times 4) : (5 \times 4) : (5 \times 7)$

$= 12 : 20 : 35.$

The total number of students = 201.

$\therefore$ The number of students in section A

$= \frac{12}{12+20+35} \times 201 = 36.$

25. Ans. (a)

Solution We have, $A : B = 2 : 3$ and $B : C = 7 : 9$.

Here, $n_1 = 2$, $n_2 = 7$, $d_1 = 3$ and $d_2 = 9$.

$\therefore$ $A : B : C = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$

$= (2 \times 7) : (3 \times 7) : (3 \times 9)$

$= 14 : 21 : 27.$

Since the sum of the numbers is 124, the third number is $\frac{27}{14+21+27} \times 124 = 54.$

26. Ans. (b)

Solution We have, $A : B = 3 : 2$, $B : C = 5 : 4$ and $C : D = 3 : 7$.

Here, $n_1 = 3$, $n_2 = 5$, $n_3 = 3$, $d_1 = 2$, $d_2 = 4$ and $d_3 = 7$.

$\therefore$ $A : B : C : D = (n_1 \times n_2 \times n_3) : (d_1 \times n_2 \times n_3) : (d_1 \times d_2 \times n_3) : (d_1 \times d_2 \times d_3)$

$= (3 \times 5 \times 3) : (2 \times 5 \times 3) : (2 \times 4 \times 3) : (2 \times 4 \times 7)$

$= 45 : 30 : 24 : 56$

$\therefore$ B's share of property worth Rs. 77500 is

$= \left(\frac{30}{45+30+24+56}\right) \times 77500$ = Rs. 15000.

27. Ans. (a)

Solution We have, $a : b = 3 : 5$, $c : d = 5 : 7$ and $x = 10$.

$\therefore$ The first number $= \frac{ax(c-d)}{ad-bc}$

$$= \frac{3\times10\times(5-7)}{(3\times7-5\times5)} = 15$$

and, the second number $= \dfrac{bx(c-d)}{ad-bc}$

$$= \frac{5\times10\times(5-7)}{(3\times7-5\times5)} = 235.$$

28. Ans. (b)

Solution We have, $a : b = 2 : 3$, $c : d = 5 : 7$ and $x = 4$.

$\therefore$ The first number $= \dfrac{ax(c-d)}{ad-bc}$

$$= \frac{2\times4\times(5-7)}{(2\times7-3\times5)} = 16.$$

and, the second number $= \dfrac{bx(c-d)}{ad-bc}$

$$= \frac{3\times4\times(5-7)}{(2\times7-3\times5)} = 24.$$

29. Ans. (a)

Solution We have, $a : b = 5 : 6$, $c : d = 4 : 5$ and $x = 5$.

$\therefore$ The first number $= \dfrac{ax(d-c)}{ad-bc}$

$$= \frac{5\times5\times(5-4)}{(5\times5-6\times4)} = 25$$

and, the second number $= \dfrac{bx(d-c)}{ad-bc}$

$$= \frac{6\times5\times(5-4)}{(5\times5-6\times4)} = 30.$$

30. Ans. (c)

Solution We have, $a : b = 7 : 5$, $c : d = 4 : 3$ and $x = 6$.

$\therefore$ The present age of Mahesh $= \dfrac{bx(c-d)}{ad-bc}$

$$= \frac{5\times6\times(4-3)}{(7\times3-5\times4)}$$

$= 30$ years.

31. Ans. (a)

Solution We have, $a : b = 4 : 3$, $c : d = 2 : 1$ and $x = 4$.

$\therefore$ The present age of Sita $= \dfrac{ax(d-c)}{ad-bc}$

$$= \frac{4\times4\times(1-2)}{(4\times1-3\times2)}$$

$= 8$ years.

32. Ans. (b)

Solution We have, $a : b = 5 : 8$, $c : d = 3 : 4$ and $x = 12$.

$\therefore$ The first number $= \dfrac{ax(c-d)}{ad-bc}$

$$= \frac{5\times12\times(3-4)}{(5\times4-8\times3)} = 15$$

and, the second number $= \dfrac{bx(c-d)}{ad-bc}$

$$= \frac{8\times12\times(3-4)}{(5\times4-8\times3)} = 24.$$

33. Ans. (c)

Solution We have, $a : b = 5 : 7$, $c : d = 35 : 59$ and $x = 25$.

$\therefore$ The first number $= \dfrac{ax(d-c)}{ad-bc}$

$$= \frac{5\times25\times(59-35)}{(5\times59-7\times35)} = 60$$

and, the second number $= \dfrac{bx(d-c)}{ad-bc}$

$$= \frac{7\times25\times(59-35)}{(5\times59-7\times35)} = 84$$

$\therefore$ The difference of two numbers $= 84 - 60 = 24$.

34. Ans. (c)

Solution We have, $a : b = 7 : 13$ and $c : d = 2 : 3$.

$$\therefore \quad x = \frac{ad-bc}{c-d} = \frac{7\times3-13\times2}{2-3} = 5.$$

35. Ans. (a)

Solution We have, $a : b = 12 : 17$ and $c : d = 2 : 3$.

$\therefore$ The required number $= \dfrac{bc-ad}{c-d}$

$$= \frac{17\times2-12\times3}{2-3} = 2.$$

36. Ans. (b)

Solution Here, $a = 7$, $b = 16$, $c = 43$ and $d = 79$.

$$\therefore k = \frac{bc-ad}{(a+d)-(b+c)} = \frac{16\times43-7\times79}{(7+79)-(16+43)} = 5.$$

37. Ans. (a)

Solution Here, $a = 15$, $b = 28$, $c = 20$ and $d = 38$.

$\therefore$ The required number $= \dfrac{ad-bc}{(a+d)-(b+c)}$

$$= \frac{15\times38-28\times20}{(15+38)-(28+20)}$$

$= 2.$

38. Ans. (a)

Solution Here, $a = 8$, $b = 21$, $c = 13$ and $d = 31$.

$\therefore$ The required number $= \dfrac{bc-ad}{(a+d)-(b+c)}$

$$= \frac{21\times13-8\times31}{(8+31)-(21+13)}$$

$= 5.$

39. Ans. (b)

Solution We have, $a : b = 3 : 2$, $c : d = 5 : 3$ and $S = 1000$.

$$\therefore \quad A\text{'s income} = \frac{aS(d-c)}{ad-bc} = \frac{3\times1000\times(3-5)}{(3\times3-2\times5)} = \text{Rs. } 6000.$$

40. Ans. (a)

Solution We have, $a : b = 5 : 3$, $c : d = 3 : 1$ and $S = 2000$.

$$\therefore \quad \text{Income of man} = \frac{aS(d-c)}{ad-bc} = \frac{5\times2000\times(1-3)}{(5\times1-3\times3)} = \text{Rs. } 5000$$

and, the income of his wife

$$= \frac{bS(d-c)}{ad-bc} = \frac{3\times2000\times(1-3)}{(5\times1-3\times3)} = \text{Rs. } 3000.$$

41. Ans. (c)

Solution We have, $a : b = 9 : 4$, $c : d = 7 : 3$ and $S = 2000$.

$$\therefore \quad \text{Gupta's expenditure} = \frac{cS(b-a)}{ad-bc} = \frac{7\times2000\times(4-9)}{(9\times3-4\times7)} = \text{Rs. } 70000.$$

42. Ans. (c)

Solution Here, $x = 60$, $a : b = 2 : 1$ and $c : d = 1 : 2$.

$\therefore$ Required amount of water to be added

$$= \frac{x(ad-bc)}{c(a+b)} = \frac{60\times(2\times2-1\times1)}{1(2+1)} = 60 \text{ litres.}$$

43. Ans. (b)

Solution The two given ratios are 12 : 5 and 4 : 3.

In order to equate the antecedents of the two ratios, we write the second ratio as 12 : 9.

Now, we have, $a : b = 12 : 5$, $c : d = 12 : 9$ and $x = 14$

$\therefore$ The quantity of alcohol in the mixture

$$= \frac{ax}{c-b} = \frac{12\times14}{12-5} = 24 \text{ litres.}$$

44. Ans. (b)

Solution Here, $a = 3$ and $b = 7$.

$\therefore$ percentage quantity of silver in the alloy

$$= \left(\frac{b}{a+b}\right) \times 100\% = \left(\frac{7}{3+7}\right) \times 100\% = 70\%.$$

45. Ans. (c)

Solution We have, $a : b = 2 : 1$, $c : d = 4 : 1$.

Let the two alloys be mixed in the ratio $x : y$.

Then, percentage quantity of zinc in the new alloy

$$= \left[\frac{\frac{ax}{a+b}+\frac{cy}{c+d}}{x+y}\right] \times 100\%$$

$$= \left[\frac{\frac{2x}{3}+\frac{4y}{5}}{x+y}\right] \times 100\%$$

$$= \frac{10x+12y}{15(x+y)} \times 100\% \quad \ldots(1)$$

Since the ratio of zinc and copper in the new alloy is 3 : 1

$\therefore$ percentage quantity of zinc in the new alloy

$$= \frac{3}{3+1}\times100\% = \frac{300}{4}\% \text{ or, } 57\% \quad \ldots(2)$$

From (1) and (2), we get

$$\frac{10x+12y}{15(x+y)} = \frac{3}{4} \quad \text{or,} \quad 40x + 48y = 45(x+y)$$

$$\text{or,} \quad 5x = 3y \quad \text{or, } x : y = 3 : 5.$$

Hence, the alloys should be mixed in the ratio 3 : 5.

46. Ans. (a)

Solution Here, $a : b = 5 : 1$, $c : d = 7 : 2$.

Let the mixtures of the two containers be added together in the ratio of $x : y$.

$\therefore$ percentage quantity of milk in the new mixture

$$= \left[\frac{\frac{ax}{a+b}+\frac{cy}{c+d}}{x+y}\right] \times 100\%$$

$$= \left[\frac{\frac{5x}{6}+\frac{7y}{9}}{x+y}\right] \times 100\% = \frac{45x+42y}{(x+y)} \times 100\%$$

Since the percentage quantity of milk in the new mixture is 80%

$$\therefore \quad \frac{45x+42y}{54(x+y)} \times 100\% = 80\%$$

$\Rightarrow$ $(45x + 42y) \times 5 = 4 \times 54(x + y)$

$\Rightarrow$ $225x + 210y = 216x + 216y$

$\Rightarrow$ $9x = 6y$ or $x : y = 2 : 3$.

QUESTIONS FROM COMPETITIVE EXAMS

1. When a number is added to another number the total becomes $3\frac{1}{3}$ per cent of the second number. What is the ratio between the first and the second number?
(a) 3 : 7 (b) 7 : 4
(c) 7 : 3 (d) Data inadequate
(e) None of these
[Based on BSRB Mumbai PO 1998]

2. An amount of money is to be distributed among *P*, *Q* and *R* in the ratio 6 : 19 : 7, respectively. If *R* gives Rs. 200 of his share to *Q* the ratio among *P*, *Q* and *R* becomes 3 : 10 : 3, respectively. What was the total amount?
(a) Rs. 6400 (b) Rs. 12800
(c) Rs. 3200 (d) Data inadequate
(e) None of these
[Based on Bank of Baroda PO 1999]

3. A man in his will distributed his money in such a way that half of it is for his wife, two-thirds of the remaining equally to three of his sons and the remaining amount equally to four of his daughters. If each of the daughters receives Rs 20000, how much money will each of his sons receive?
(a) Rs. 50333.33 (b) Rs. 48233.33
(c) Rs. 53333.33 (d) Data inadequate
[Based on SBI Associates PO 1999]

4. The ratio between the present ages of *P* and *Q* is 5 : 8. After 4 years, the ratio between their ages will be 2 : 3. What is *Q*'s age at present?
(a) 36 years (b) 20 years
(c) 24 years (d) Data inadequate
(e) None of these **[Based on Guwahati PO 1999]**

5. The ratio of *P's* and *Q's* ages is 5 : 7. If the difference between the present age of *Q* and the age of *P* six years hence is 2, then what is the total of present ages of *P* and *Q*?
(a) 52 years (b) 48 years
(c) 56 hears (d) Data inadequate
(e) None of these **[Based on Guwahati PO 1999]**

6. An amount of money is to be distributed among *P*, *Q* and *R* in the ratio 5 : 8 : 12, respectively. If the total share of *Q* and *R* is four times that of *P*, what is definitely *P*'s share?
(a) Rs. 3000 (b) Rs. 5000
(c) Rs. 8000 (d) Data inadequate
(e) None of these
[Based on BSRB Mumbai PO 1999]

7. In a business *A* and *C* invested amounts in the ratio 2 : 1, whereas the ratio between amounts invested by *A* and *B* was 3 : 2. If Rs. 1,57,300 was their profit, how much amount did *B* receive?
(a) Rs. 72600 (b) Rs. 48400
(c) Rs. 36300 (d) Rs. 24200
(e) None of these
[Based on BSRB Calcutta PO 1999]

8. An amount of money is to be divided among *P*, *Q* and *R* in the ratio 4 : 9 : 16. If *R* gets 4 times more that *P*, What is *Q*'s share in it?
(a) Rs. 1800 (b) Rs. 2700
(c) Rs. 3600 (d) Data inadequate
(e) None of these
[Based on BSRB Hyderabad PO 1999]

9. When 30 per cent of a number is added to another number the second number increases by its 20 per cent. What is the ratio between the first and the second number?
(a) 3 : 2 (b) 2 : 3
(c) 2 : 5 (d) Data inadequate
(e) None of these **[Based on NABARD 1999]**

10. The ratio of *A's* and *B's* salary is 9 : 4. If *A's* salary is increased by 15%, then his total salary becomes Rs. 5175. What is the salary of *B*?
(a) Rs. 2000 (b) Rs. 4000
(c) Rs. 4500 (d) Rs. 2500
(e) None of these
[Based on BSRB Chennai PO 2000]

11. Mohan is younger than Sohan by 10 years. If 5 years back their ages were in the ratio 1 : 2, how old is Sohan?
(a) 20 (b) 15
(c) 25 (d) Data inadequate
(e) None of these
[Based on BSRB Chennai PO 2000]

12. Ratio of present ages of *P* and *Q* is 7 : 3. After four years their ages are in the ratio 2 : 1. What is the present age of *P*?
(a) 24 years (b) 28 years
(c) 32 years (d) Data inadequate
(e) None of these
[Based on BSRB Chennai PO 2000]

13. If 40 per cent of a number is added to another number then it becomes 125 per cent of itself. What will be the ratio of first and second numbers?
(a) 8 : 5 (b) 5 : 7
(c) 5 : 8 (d) Data inadequate
(e) None of these
[Based on BSRB Chennai PO 2000]

14. An amount of Rs 125000 is to be distributed among Raju, Monu and Sonu in the respective ratio of 2 : 3 : 5. What will be the difference between Monu and Raju's share?
(a) Rs. 25000 (b) Rs. 12500

(c) Rs. 18750 (d) Rs. 2500
(e) None of these
[Based on BSRB Bangalore PO 2000]

15. The ratio of present ages of Ram and Shyam is 7 : 8, respectively. Four years hence this ratio becomes 9 : 10, respectively. What is Ram's present age in years?
(a) 18 (b) 14
(c) 17 (d) Data inadequate
(e) None of these
[Based on BSRB Bangalore PO 2000]

16. Salaries of A, B and C were in the ratio 3 : 5 : 7, respectively. If their salaries were increased by 50%, 60% and 50% respectively, what will be the new ratio of their respective salaries?
(a) 3 : 6 : 7 (b) 4 : 5 : 7
(c) 4 : 5 : 8 (d) Data inadequate
(e) None of these **[Based on BSRB Delhi PO 2000]**

17. One year ago the ratio of Karan's and Ankit's ages was 6 : 7, respectively. Four years hence this ratio would become 7 : 8. How old is Ankit?
(a) 35 years (b) 30 years
(c) 31 years (d) Cannot be determined
(e) None of these **[Based on BSRB Delhi PO 2000]**

18. The third proportional of 38 and 15 is
(a) $\frac{38\times38}{15}$ (b) $\frac{15}{38\times38}$
(c) $\frac{15\times15}{38}$ (d) $\frac{38\times15}{2}$
[Based on BSRB Patna PO 2001]

19. Three numbers are in the ratio 1 : 2 : 3; their product is 20250. The numbers are
(a) 15; 30; 45 (b) 25; 50; 75
(c) 35; 70; 105 (d) 45; 90; 135
[Based on Andhra Bank SO 2002]

20. If $\frac{a}{b} = \frac{7}{9}$, $\frac{b}{c} = \frac{3}{5}$, $a : b : c = ?$
(a) 7 : 9 : 15 (b) 7 : 9 : 5
(c) 21 : 35 : 45 (d) 7 : 3 : 15
[Based on RBI Grade 'B' Officers 2002]

21. Two numbers are in the ratio of 5 : 9; with the addition of nine to each, their ratio becomes 16 : 27. Then one of the two numbers would be
(a) 66 (b) 77
(c) 88 (d) 99
[Based on SI of Police Rec. Exam 1997]

22. A and B are two different alloys of gold and copper prepared by mixing metals in the proportion 7 : 2 and 7 : 11, respectively. If equal quantities of the alloys are melted to form a third alloy C, find the ratio of gold and copper in C.
(a) 5 : 7 (b) 6 : 6
(c) 7 : 5 (d) 14 : 13

23. If $\frac{x}{2y} = \frac{6}{7}$, the value of $\frac{x-y}{x+y}+\frac{14}{19}$ equals
(a) $\frac{13}{19}$ (b) $\frac{15}{19}$
(c) 1 (d) $1\frac{1}{19}$
[Based on SI of Police Rec. Exam 1997]

24. The ratio between the annual incomes of A and B is 4 : 3 and between their annual expenditure is 3 : 2. If at the end of a year both save Rs. 600 each, find the difference in their incomes.
(a) Rs. 450 (b) Rs. 500
(c) Rs. 600 (d) Rs. 750
[Based on SI of Police Rec. Exam 1997]

25. If a, b, c and d are real numbers such that $a : b = b : c = c : d$ and $a : d = 8 : 125$, then the value of $a : c$ is
(a) 25 : 4 (b) 125 : 8
(c) 4 : 25 (d) 8 : 25
[Based on SI of Police Rec. Exam 1997]

26. Three pots have the same volume. The ratio of milk and water in first, second and third pots are 3 : 2, 7 : 3 and 11 : 4, respectively. If the liquid of three pots are mixed, then the ratio of milk and water in the mixture is
(a) 61 : 29 (b) 61 : 30
(c) 5 : 4 (d) 29 : 61
[Based on SI of Police Rec. Exam 1997]

27. If $\frac{x}{y} = \frac{6}{5}$, the value of $\frac{x^2+y^2}{x^2-y^2}$ is
(a) $\frac{36}{25}$ (b) $\frac{25}{36}$
(c) $\frac{61}{11}$ (d) $\frac{11}{61}$
[Based on SI of Police Rec. Exam 1997]

28. If $a : 5 = b : 7 = c : 8$, then $\frac{a+b+c}{a}$
(a) 4 (b) 2
(c) 7 (d) 1/4
[Based on SI of Police Rec. Exam 1997]

29. If $a : b : c = 2 : 3 : 4$, then $\frac{1}{a}:\frac{1}{b}:\frac{1}{c}$ is equal to
(a) 4 : 3 : 2 (b) $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$
(c) $\frac{1}{4}:\frac{1}{3}:\frac{1}{2}$ (d) 3 : 4 : 6
[Based on SI of Police Rec. Exam 1997]

30. If $x : y = 8 : 9$, then $5x - 4y : 3x + 2y$ is equal to
(a) 3 : 2 (b) 2 : 3
(c) 3 : 4 (d) 2 : 21
[Based on SI of Police Rec. Exam 1997]

31. Two numbers are in the ratio of 3 : 4. If 5 is subtracted from each, then the ratio will be 2 : 3. What is the smaller number?
(a) 15 (b) 18
(c) 20 (d) 24
[Based on SI of Police Rec. Exam 1997]

32. Monthly income of Anil and Sunil are in the ratio 5 : 4 and their expenses are in the ratio 4 : 3. If each of them saves Rs. 1200 at the end of the month, their monthly incomes, respectively are
(a) Rs. 6000, Rs. 4800
(b) Rs. 8000, Rs. 6400
(c) Rs. 8000, Rs. 7200
(d) Rs. 2000, Rs. 1600
[Based on SI of Police Rec. Exam 1999]

33. A bag contains one rupee, 50-paise and 25-paise coins in the ratio 2 : 3 : 5. Their total value is Rs. 144. The value of 50-paise coins is
(a) Rs. 24 (b) Rs. 36
(c) Rs. 48 (d) Rs. 72
[Based on SI of Police Rec. Exam 1999]

34. A sum of money is divided among 160 males and some females in the ratio 16 : 21. Individually each male gets Rs. 4 and a female Rs. 3. The number of females is
(a) 280 (b) 198
(c) 284 (d) 270
[Based on SI of Police Rec. Exam 1999]

35. Tea at Rs. 126 per kg. and at Rs. 135 per kg are mixed with a third variety in the ratio 1 : 1 : 2. If the mixture is worth Rs. 153 per kg, the price of the third variety (per kg) is
(a) Rs. 169.50 (b) Rs. 175
(c) Rs. 175.50 (d) Rs. 185
[Based on SI of Police Rec. Exam 1999]

36. Two numbers are in the ratio 3 : 4. If 50 is added to each number, the ratio becomes 7 : 9. The sum of the numbers is
(a) 50 (b) 350
(c) 700 (d) 800
[Based on SI of Police Rec. Exam 1999]

37. If $3A = 5B$ and $4B = 6C$, then $A : C$ is equal to
(a) 5 : 2 (b) 3 : 5
(c) 2 : 5 (d) 4 : 5
[Based on SI of Police Rec. Exam 1999]

38. A sum of Rs. 370 is to be divided among A, B and C such that $\frac{A's \text{ share}}{B's \text{ share}} = \frac{B's \text{ share}}{C's \text{ share}} = \frac{3}{4}$
Then A's share (in rupees) is
(a) 240 (b) 120
(c) 100 (d) 90
[Based on Assistant's Grade Exam 1997]

39. If the ratio of boys to girls in a class is B and the ratio of girls to boys is G, then $3(B + G)$ is
(a) equal to 3 (b) less than 3
(c) more than 3 (d) less than $\frac{1}{3}$
[Based on Assistant's Grade Exam 1997]

40. The monthly incomes of two persons are in the ratio 4 : 7 and their expenses are in the ratio 11 : 20. If each of them saves Rs 400 per month then their monthly income must be respectively:
(a) Rs. 3600, Rs. 4200
(b) Rs. 4000, Rs. 7000
(c) Rs. 4200, Rs. 7350
(d) Rs. 4800, Rs. 8400
[Based on Assistant's Grade Exam 1997]

41. What number should be subtracted from both the terms of the ratio 15 : 19 in order to make it 3 : 4?
(a) 9 (b) 6
(c) 5 (d) 3
[Based on SSC (GL) Prel. Exam 2000]

42. If $p : q = r : s : t : u = 2 : 3$, then $(mp + nr + ot) : (mq + ns + ou)$ is equal to
(a) 1 : 3 (b) 1 : 2
(c) 2 : 3 (d) 3 : 2
[Based on SSC (GL) Prel. Exam 2000]

43. If $a : b = c : d = e : f = 1 : 2$, then $(pa + qc + re) : (pb + qd + rf)$ is equal to
(a) $p : (q + r)$ (b) $(p + q) : r$
(c) 2 : 3 (d) 1 : 2
[Based on SSC (GL) Prel. Exam 2000]

44. If $x : y = 3 : 1$, then $x^3 - y^3 : x^3 + y^3 = ?$
(a) 13 : 14 (b) 14 : 13
(c) 10 : 11 (d) 11 : 10
[Based on SSC (GL) Prel. Exam 2000]

45. If 10% of m is the same as 20% of n. Then $m : n$ is equal to
(a) 2 : 1 (b) 1 : 2
(c) 1 : 10 (d) 1 : 20
[Based on SSC (GL) Prel. Exam 2000]

46. The ratio $2^{1 \cdot 5} : 2^{0 \cdot 5}$ is the same as
(a) 2 : 1 (b) 3 : 1
(c) 6 : 1 (d) 3 : 2
[Based on SSC (GL) Prel. Exam 2000]

47. If $m : n = 3 : 2$, then $(4m + 5n) : (4m - 5n)$ is equal to
(a) 4 : 9 (b) 9 : 4
(c) 11 : 1 (d) 9 : 1
[Based on SSC (GL) Prel. Exam 2000]

48. The smallest integer, which when subtracted from both the terms of 6 : 7 gives a ratio less than 16 : 21, is
(a) 5 (b) 4
(c) 3 (d) 2
[Based on SSC (GL) Prel. Exam 2000]

49. If $\frac{a}{b} = \frac{2}{3}$ and $\frac{b}{c} = \frac{4}{5}$, then the ratio $\frac{a+b}{b+c}$ is equal to
(a) $\frac{20}{27}$ (b) $\frac{27}{20}$
(c) $\frac{6}{8}$ (d) $\frac{8}{6}$

50. The sum of two numbers is 40 and their difference is 4. The ratio of the numbers is
(a) 21 : 19 (b) 22 : 9
(c) 11 : 9 (d) 11 : 18
[Based on SSC (GL) Prel. Exam 2000]

51. Neeta bought a book at 30% discount on the listed price. Had she not got the discount, she would have paid Rs. 82.50 extra. At what price did she buy the book?
(a) Rs. 192.50 (b) Rs. 275
(c) Rs. 177.85 (d) Cannot be determined
(e) None of these
[Based on Canara Bank PO 2003]

52. A man spends Rs. 1810 for buying bags at Rs. 200 each and bottles at Rs. 70 each. What will be the ratio of bags to bottles when maximum number of bags are bought?
(a) 3 : 8 (b) 8 : 3
(c) 9 :1 (d) 1 : 9
(e) None of these
[Based on IBPS Jr. Executive Exam 2000]

53. If $a : b = 2 : 3$ and $b : c = 4 : 5$, find $a^2 : b^2 : bc$
(a) 4 : 9 : 45 (b) 16 : 36 : 45
(c) 16 : 36 : 20 (d) 4 : 36 : 20
[Based on SSC (GL) Prel. Exam 2002]

54. If $A : B = \frac{1}{2} : \frac{3}{8}$, $B : C = \frac{1}{3} : \frac{5}{9}$ and $C : D = \frac{5}{6} : \frac{3}{4}$, then the ratio $A : B : C : D$ is
(a) 6 : 4 : 8 : 10 (b) 6 : 8 : 9 : 10
(c) 8 : 6 : 10 : 9 (d) 4 : 6 : 8 : 10
[Based on SSC (GL) Prel. Exam 2002]

55. Two numbers are in the ratio of 5 : 7. On diminishing each of them by 40, they become in the ratio 17 : 27. The difference of the numbers is
(a) 18 (b) 52
(c) 137 (d) 50
[Based on SSC (GL) Prel. Exam 2002]

56. The ratio of the number of boys and girls of a school with 504 students is 13 : 11. What will be the new ratio if 12 more girls are admitted?
(a) 91 : 81 (b) 81 : 91
(c) 9 : 10 (d) 10 : 9
[Based on SSC (GL) Prel. Exam 2002]

57. A and B have monthly incomes in the ratio 5 : 6 and monthly expenditures in the ratio 3 : 4. If they save Rs. 1800 and Rs. 1600 respectively, find the monthly income of B.
(a) Rs. 3400 (b) Rs. 2700
(c) Rs. 1720 (d) Rs. 7200
[Based on SSC (GL) Prel. Exam 2002]

58. A sum of Rs 9000 is to be distributed among A, B and C in the ratio 4 : 5 : 6. What will be the difference between A's and C's shares?
(a) Rs. 600 (b) Rs. 1000
(c) Rs. 900 (d) Rs. 1200
[Based on SSC (GL) Prel. Exam 2002]

59. Zinc and copper are in the ratio of 5 : 3 in 200 gm of an alloy. How many grams of copper should be added to make the ratio 3 : 5?
(a) $133\frac{1}{3}$ (b) $\frac{1}{200}$
(c) 72 (d) 66
[Based on SSC (GL) Prel. Exam 2002]

60. Divide Rs. 7500 among A, B and C such that A's share to B's share is in the ratio 5 : 2 and B's share to C's share is in the ratio 7 : 13. How much will B receive?
(a) Rs. 1400 (b) Rs. 3500
(c) Rs. 2600 (d) Rs. 7000
[Based on SSC (GL) Prel. Exam 2002]

61. The ratio of copper and zinc in brass is 13 : 7. How much zinc will be there in 100 kg of brass?
(a) 20 kg (b) 55 kg
(c) 35 kg (d) 40 kg
[Based on SSC (GL) Prel. Exam 2002]

62. If $A : B : C = 2 : 3 : 4$, then $\frac{A}{B} : \frac{B}{C} : \frac{C}{A}$ is equal to
(a) 8 : 9 : 16 (b) 8 : 9 : 12
(c) 8 : 9 : 24 (d) 4 : 9 : 16
[Based on SSC (GL) Prel. Exam 2002]

63. If $A : B = 1 : 2$, $B : C = 3 : 4$ and $C : D = 5 : 6$, find $D : C : B : A$
(a) 6 : 5 : 4 : 2 (b) 6 : 3 : 2 : 1
(c) 6 : 4 : 2 : 1 (d) 48 : 40 : 30 : 15
[Based on SSC (GL) Prel. Exam 2002]

64. Two numbers are in the ratio $1\frac{1}{2} : 2\frac{2}{3}$. When each of these is increased by 15, they become in the ratio $1\frac{2}{3} : 2\frac{1}{2}$. The greater of the numbers is
(a) 27 (b) 36
(c) 48 (d) 64
[Based on SSC (GL) Prel. Exam 2002]

65. The students in three classes are in the ratio 2 : 3 : 5. If 40 students are increased in each class, the ratio changes to 4 : 5 : 7. Originally, the total number of students was
(a) 100 (b) 180
(c) 200 (d) 400
[Based on SSC (GL) Prel. Exam 2002]

66. The ratio of incomes of two persons is 5 : 3 and that of their expenditures is 9 : 5. Find the income of each person, if they save Rs. 1,300 and Rs. 900, respectively.
(a) Rs. 4000, Rs. 2400 (b) Rs. 3000, Rs. 1800
(c) Rs. 5000 Rs. 300 (d) Rs. 450 Rs. 2700
[Based on SSC (GL) Prel. Exam 2002]

67. If the ratio of the areas of two squares is 1 : 4, then the ratio of their perimeters is
(a) 1 : 2 (b) 1 : 4
(c) 1 : 6 (d) 1 : 8
[Based on SSC (GL) Prel. Exam 2002]

68. An equilational triangle is described on the diagonal of a square. What is the ratio of the area of the triangle to that of the square?
(a) $\sqrt{3}:2$ (b) $2 : \sqrt{3}$
(c) $\sqrt{3} : 4$ (d) $4 : \sqrt{3}$
[Based on SSC (GL) Prel. Exam 2002]

69. If $x : y = 3 : 4$, then $(7x + 3y) : (7x - 3y)$ is equal to
(a) 5 : 2 (b) 4 : 3
(c) 11 : 3 (d) 37 : 19
[Based on SSC (GL) Prel. Exam 2002]

70. If $a : b = 5 : 7$ and $c : d = 2a : 3b$, then $ac : bd$ is
(a) 20 : 38 (b) 50 : 147
(c) 10 : 21 (d) 50 : 151
[Based on SSC (GL) Prel. Exam 2002]

71. If $A : B = 2 : 3$, $B : C = 4 : 5$, and $C : D = 6 : 7$, then $A : B : C : D$ is
(a) 18 : 24 : 30 : 35 (b) 16 : 24 : 30 : 35
(c) 16 : 22 : 30 : 35 (d) 16 : 24 : 15 : 35
[Based on SSC (GL) Prel. Exam 2002]

72. The three numbers are in the ratio $\frac{1}{2}:\frac{2}{3}:\frac{3}{4}$. The difference between the greatest and the smallest numbers is 36. Find the numbers.
(a) 72, 84, 108 (b) 60, 72, 96
(c) 72, 84, 96 (d) 72, 96, 108
[Based on SSC (GL) Prel. Exam 2002]

73. The ratio of the number of girls and boys participating in sports of a school is 4 : 5. If the number of girls is 212, then determine the number of boys participating in the sports:
(a) 256 (b) 265
(c) 251 (d) 263
[Based on SSC (GL) Prel. Exam 2002]

74. The ratio of market price of wheat and paddy is 2 : 3 and the ratio of the quantities consumed in a family is 5 : 4. Find the ratio of the expenditures of wheat and paddy
(a) 6 : 5 (b) 5 : 6
(c) 1 : 1 (d) 8 : 15
[Based on SSC (GL) Pre. Exam 2002]

75. If $a : b = \frac{2}{9}:\frac{1}{3}$, $b : c = \frac{2}{7}:\frac{5}{14}$ and $d : c = \frac{7}{10}:\frac{3}{5}$, then $a : b : c : d$ is
(a) 4 : 6 : 7 : 9 (b) 16 : 24 : 30 : 35
(c) 8 : 12 : 15 : 7 (d) 30 : 35 : 24 : 16
[Based on SSC (GL) Prel. Exam 2003]

76. The incomes of A, B and C are in the ratio 7 : 9 : 12 and their spendings are in the ratio 8 : 9 : 15. If A saves $\frac{1}{4}$ of his income, then the savings of A, B and C are in the ratio of
(a) 56 : 99 : 69 (b) 69 : 56 : 99
(c) 99 : 56 : 69 (d) 99 : 69 : 56
[Based on SSC (GL) Prel. Exam 2003]

77. In an alloy, the ratio of copper and zinc is 5 : 2. If 1.250 kg of zinc is mixed in 17 kg 500 gm of alloy, then the ratio of copper and zinc will be
(a) 2 : 1 (b) 2 : 3
(c) 3 : 2 (d) 1 : 2
[Based on SSC (GL) Prel. Exam 2003]

78. If 378 coins consist of rupee, 50-paise and 25-paise coins whose values are in the ratio of 13 : 11 : 7, the number of 50-paise coins will be
(a) 132 (b) 128
(c) 136 (d) 133
[Based on SSC (GL) Prel. Exam 2003]

79. If $A : B = 2 : 3$ and $B : C = 4 : 5$, then $A : B : C$ is
(a) 2 : 12 : 5 (b) 8 : 12 : 15
(c) 12 : 8 : 15 (d) 15 : 12 : 8
[Based on SSC (GL) Prel. Exam 2000]

80. If $A : B = \frac{1}{2}:\frac{1}{3}$, $B : C = \frac{1}{2}:\frac{1}{3}$, then $A : B : C$ is equal to
(a) 2 : 3 : 3 (b) 1 : 2 : 6
(c) 3 : 2 : 6 (d) 9 : 6 : 4
[Based on SSC (GL) Prel. Exam 2000]

81. Instead of dividing Rs. 117 among P, Q, R in the ratio $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$, it was divided in the ratio 2 : 3 : 4 by mistake. Who gained in this transaction?
(a) Only P (b) Only Q
(c) Only R (d) Both Q and R
[Based on SSC (GL) Prel. Exam 2000]

82. A boy contains 10-paise coins and 25-paise coins in the ratio 17 : 6. If the total money in the bag is Rs. 112, then the number of 10-paise coins is
(a) 35 (b) 210
(c) 490 (d) 595
[Based on SSC (GL) Prel. Exam 2000]

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Let the first and second numbers be x and y, respectively

Then, $x + y = \frac{10}{3}y$ or, $x = \frac{7}{3}y$

$\therefore$ $x : y = 7 : 3$.

2. Ans. (a)

Solution Let the sums of P, Q and R be $6x$, $19x$ and $7x$

$\therefore$ Total sum $= 6x + 19 + 7x = 32x$

From the question

$6x : 19x + 200 : 7x - 200 = 3 : 10 : 3$

that is, $6x = 7x - 200$

$\therefore$ $x = 200$

$\therefore$ Total sum $= 32 \times 200 =$ Rs. 6400.

3. Ans. (c)

Solution Let the man have Rs. x

Amount given to his wife $= \frac{x}{2}$

Remaining $= x - \frac{x}{2} = \frac{x}{2}$

$\therefore$ Amount given to each son $= \frac{x}{2} \times \frac{2}{3} \times \frac{1}{3} = \frac{x}{9}$

$\therefore$ Amount given to each daughter

$= \left(\frac{x}{2} - \frac{x}{3}\right) \times \frac{1}{4} = \frac{x}{24} \Rightarrow \frac{x}{24} = 20000$

$\therefore$ $x =$ Rs. 480000

$\therefore$ Amount gives to each son $= 480000 \times \frac{1}{9}$

$=$ Rs. 53333.33.

4. Ans. (e)

Solution $\frac{P}{Q} = \frac{5}{8}$ or $P = \frac{5Q}{8}$ ...(i)

$\frac{P+4}{Q+4} = \frac{2}{3}$

or, $3P + 12 = 2Q + 8$

or, $2Q - 3P = 4$ ...(ii)

Putting value of P from equation (i), we get

$2Q - 3\left(\frac{5Q}{8}\right) = 4 \Rightarrow Q = 32$ years.

5. Ans (b)

Solution $\frac{P}{Q} = \frac{5}{7}$ or, $Q = \frac{7P}{5}$

Case I: $Q - (P + 6) = 2$ or, $Q = P + 8$

$\therefore$ $\frac{7P}{5} = P + 8$ or, $7P = 5P + 40$

$\therefore$ $P = 20$ and $Q = \frac{7}{5} \times 20$

$Q = 28$

$\therefore$ $P + Q = 20 + 28 = 48$ years.

Case II: $(P + 6) - Q = 2$

or, $P + 6 - \frac{7}{5}P = 2$ or, $P = 10$ and $Q = 14$

$\therefore$ $P + Q = 10 + 14 = 24$ years.

6. Ans. (d)

Solution $P : Q : R = 5 : 8 : 12$

$\frac{\text{Total share of } Q \text{ and } R}{\text{share of } P} = \frac{8+12}{5} = \frac{20}{5} = 4.$

Thus, we see that no new information has been given in the question and therefore P's share cannot be determined.

7. Ans. (b)

Solution Ratio $A : B = 3 : 2$

and, $A : C = 2 : 1$

$\therefore$ $A : B : C = 6 : 4 : 3$

Profit share of $B = \frac{4}{13} \times 1,57,300 =$ Rs. 48400.

8. Ans. (d)

Solution Here, neither the total amount nor the individual amount is given. So the share of Q cannot be determined.

9. Ans. (b)

Solution 30% of I + II = II $\times \frac{120}{100}$

or, $\frac{3}{10}\text{I} + \text{II} = \frac{12}{10}\text{II}$

or, $\frac{3}{10}\text{I} = \frac{2}{10}\text{II} \Rightarrow \text{I} : \text{II} = 2 : 3$.

10. Ans. (a)

Solution Let the salaries of A and B be $9x$ and $4x$

$9x \times \frac{115}{100} = 5175$

$\therefore$ $x = 500$

$\therefore$ Salary of $B = 500 \times 4 =$ Rs. 2000

11. Ans. (c)

Solution Sohan – Mohan = 10 ...(i)

$$\frac{\text{Mohan}-5}{\text{Sohan}-5} = \frac{1}{2}$$

$\Rightarrow$ 2 Mohan – Sohan = 5 ...(ii)

On solving (i) and (ii), we get

Mohan = 15 years and Sohan = 25 years.

12. Ans. (b)

Solution $\frac{P}{Q} = \frac{7}{3} \Rightarrow Q = \frac{3P}{7}$ ...(i)

and, $\frac{P+4}{Q+4} = \frac{2}{1}$ or, $P + 4 = 2Q + 8$

or, $2Q = P - 4$ ...(ii)

Solving equations (i) and (ii), we get

$P = 28$ years.

13. Ans. (e)

14. Ans. (b)

Solution Required difference between Monu's and Raju's shares

$$= \frac{3-2}{2+3+5} \times 125000 = \text{Rs. } 12500.$$

15. Ans. (b)

Solution

	Ram	Shyam
	7	8
4	9	10

Present age of Ram $= \frac{4\times(10-9)}{9\times8-10\times7} \times 7$

$= 14$ years.

16. Ans. (e)

Solution Suppose the salaries of A, B and C were $300\,k$, $500\,k$ and $700\,k$, respectively.

After increment salary of

$A = 300\,k + 50\%$ of $300\,k = 450\,k$

$B = 500\,k + 50\%$ of $500\,k = 800\,k$

$C = 700\,k + 50\%$ of $700\,k = 1050\,k$

Hence, new ratio of the respective salaries of A, B and $C = 450\,k : 800\,k : 1050\,k = 9 : 16 : 21$.

17. Ans. (e)

18. Ans. (c)

Solution Let k be the third proportional of 38 and 15

$$38 : 15 :: 15 : k \Rightarrow k = \frac{15\times15}{38}.$$

19. Ans. (a)

Solution Let the three numbers be k, $2k$ and $3k$

Also, $6k^3 = 20250 \Rightarrow k^3 = 3375 \Rightarrow k = 15$.

20. Ans. (a)

Solution We have, $\frac{a}{7} = \frac{b}{9} = \frac{c}{15}$.

21. Ans. (d)

Solution Let the number be $5k$ and $9k$

$\therefore$ $k = 11$

$\therefore$ The two number are 55 and 99.

22. Ans. (c)

Solution Suppose 18 kg each is melted. Ratio of gold and copper in one alloy will be 14 : 4 and in another 7 : 11.

$\therefore$ Ratio of gold and copper in the new alloys

$C = 14 + 7 : 4 + 11 = 21 : 15 = 7 : 5$.

23. Ans. (c)

Solution $\frac{x}{2y} = \frac{6}{7} \Rightarrow \frac{x}{y} = \frac{12}{7}$

$$\therefore \quad \frac{x-y}{x+y} + \frac{14}{19} = \frac{\frac{x}{y}-1}{\frac{x}{y}+1} + \frac{14}{19} = \frac{\frac{12}{7}-1}{\frac{12}{7}} + \frac{14}{19}$$

$$= \frac{5}{19} + \frac{14}{19} = 1.$$

24. Ans. (c)

Solution Annual incomes of A and B are $4k$ and $3k$, say.

Annual expenditure of A and B are $3L$ and $2L$, say.

$\therefore$ $4k - 3L = 600$

$3k - 2L = 600$

$\Rightarrow$ $k = 600$, $L = 600$

$\therefore$ Difference in incomes of A and B

$= 4k - 3k = k = 600$.

25. Ans. (c)

Solution $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k$, say.

Given: $\frac{a}{d} = \frac{8}{125} = \frac{a}{b}\times\frac{b}{c}\times\frac{c}{d} = k^3 \Rightarrow k = \frac{2}{5}$

Now, $\frac{a}{c} = \frac{a}{b}\times\frac{b}{c} = k^2 = \frac{4}{25}$.

26. Ans. (a)

Solution Ratio of milk and water

$$= \frac{3}{5}+\frac{7}{10}+\frac{11}{15} : \frac{2}{5}+\frac{3}{10}+\frac{4}{15}$$

$$= \frac{18+21+22}{30} : \frac{12+9+8}{30} = 61 : 29.$$

27. Ans. (c)

Solution $$\frac{x^2+y^2}{x^2-y^2} = \frac{\frac{x^2}{y^2}+1}{\frac{x^2}{y^2}-1} = \frac{\frac{36}{25}+1}{\frac{36}{25}-1} = \frac{\frac{61}{25}}{\frac{11}{25}} = \frac{61}{11}.$$

28. Ans. (a)

Solution $\frac{a}{5} = \frac{b}{7} = \frac{c}{8} = k \Rightarrow a = 5k,\ b = 7k,\ c = 8k$

$\therefore \quad \frac{a+b+c}{a} = \frac{20k}{5k} = 4.$

29. Ans. (b)

Solution $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = k$

$\Rightarrow \quad a = 2k,\ b = 3k,\ c = 4k$

$\Rightarrow \quad \frac{1}{a}:\frac{1}{b}:\frac{1}{c} = \frac{1}{2k}:\frac{1}{3k}:\frac{1}{k4} = \frac{1}{2}:\frac{1}{3}:\frac{1}{4}.$

30. Ans. (d)

Solution $\frac{x}{8} = \frac{y}{9} = k \Rightarrow x = 8k,\ y = 9k$

$\therefore \quad \frac{5x-4y}{3x+2y} = \frac{40k-36k}{24k+18k} = \frac{4}{42} = \frac{2}{21}.$

31. Ans. (a)

Solution $x = 3k,\ y = 4k$, say

$\therefore \quad \frac{x-5}{y-5} = \frac{2}{3} \Rightarrow \frac{3k-5}{4k-5} = \frac{2}{3} \Rightarrow k = 5.$

32. Ans. (a)

Solution Suppose income of Anil and Sunil are $5k$ and $4k$, respectively.

Expenses of Anil and Sunil are $4L$ and $3L$.

$\therefore \quad 5k - 4L = 1200$

$4k - 3L = 1200 \Rightarrow k = 1200,\ L = \text{Rs. } 1200$

33. Ans. (b)

Solution Let the number of one-rupee coins, 50-paise coins and 25-paise coins be $2k$, $3k$ and $5k$, respectively.

$\therefore \quad 2k \times 1 + 3k \times 0.50 + 5k \times 0.25 = 114$

$\Rightarrow 2k + 1.50k + 1.25k = 114$

$\Rightarrow 4.75k = 114 \Rightarrow k = 24.$

34. Ans. (a)

Solution Let number of females = F

Suppose 160 males get Rs. $16\,k$ and F females get Rs. $21\,k$

$\therefore \quad 160 \times 4 = 16k$

$\Rightarrow \quad k = 40 \ \therefore F$ females get Rs. 840

$\therefore$ Number of females $= \frac{840}{3} = 280.$

35. Ans. (c)

Solution Let k kg of 1st variety, k kg of 2nd variety and $2k$ kg of 3rd variety of tea are mixed.

Let price of the third variety = Rs. x per kg

$\therefore \quad 126k + 135k + x(2k) = 153(k + k + 2k)$

$\Rightarrow \quad x = 175.50.$

36. Ans. (c)

Solution Let the numbers be $3k$ and $4k$

$\therefore \quad \frac{3k+50}{4k+50} = \frac{7}{9} \Rightarrow k = 100$

$\therefore$ Sum of the numbers $= 7k = 700.$

37. Ans. (a)

Solution $\frac{A}{B} = \frac{5}{3},\ \frac{B}{C} = \frac{6}{4} = \frac{3}{2}$

$\therefore \quad \frac{A}{C} = \frac{A}{B} \times \frac{B}{C} = \frac{5}{3} \times \frac{3}{2} = \frac{5}{2}.$

38. Ans. (d)

Solution Suppose A's share = Rs. $3k$

B's share = Rs. $4k$

C's share = Rs. $\frac{16}{3}$

$\therefore$ Rs. 370 are divided among A, B and C in the ratio 9 : 12 : 16

$\therefore \quad A$'s share $= \frac{9}{37} \times 370 =$ Rs. 90.

39. Ans. (c)

Solution $B = \frac{1}{G} \quad \Rightarrow B + G > 1$

$\therefore \quad 3(B + G) > 3.$

40. Ans. (d)

Solution Let the incomes be $4k$ and $7k$ and expenses be $11L$ and $20L$, respectively.

$\Rightarrow \quad 4k - 11L = 400$

$7k - 20L = 400$

$\Rightarrow \quad k = 1200,\ L = 400$

$\therefore$ Monthly incomes are Rs 4800 and Rs. 8400, respectively.

41. Ans. (d)

Solution $\frac{15-x}{19-x} = \frac{3}{4} \Rightarrow 60 - 4x = 57 - 3x \Rightarrow x = 3.$

42. Ans. (c)

Solution $\frac{p}{q} = \frac{r}{s} = \frac{t}{u} = \frac{2}{3}$

$\therefore \quad \frac{mp+nr+ot}{mq+ns+ou} = \frac{m\frac{2q}{3}+n\cdot\frac{2s}{3}+0\cdot\frac{2u}{3}}{mq+ns+ou} = \frac{2}{3}.$

43. Ans. (d)

Solution $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{1}{2}$

$\Rightarrow \quad \frac{pq+qc+re}{pb+qd+rf} = \frac{\frac{pb}{2}+\frac{qd}{2}+\frac{rf}{2}}{pb+qd+rf} = \frac{1}{2}.$

44. Ans. (a)

Solution $\frac{x}{y} = \frac{3}{1} \Rightarrow \frac{x}{3} = \frac{y}{1} = k \Rightarrow x = 3k,\ y = k$

$\therefore \quad \frac{x^3 - y^3}{x^3 + y^3} = \frac{27k^3 - k^3}{27k^3 + k^3} = \frac{13}{14}.$

45. Ans. (a)

Solution 10% of m = 20% of $n \Rightarrow 10\,m = 20\,n$

$\Rightarrow \frac{m}{n} = \frac{2}{1}.$

46. Ans. (a)

Solution $2^{1.5} : 2^{0.5} = 2\sqrt{2} : \sqrt{2} = 2 : 1.$

47. Ans. (c)

Solution If $m : n = 3 : 2 \Rightarrow \frac{m}{n} = \frac{3}{2} \Rightarrow m = \frac{3}{2}n$

then, $\frac{4m+5n}{4m-5n} = \frac{4\times\frac{3}{2}n+5n}{4\times\frac{3}{2}n-5n} = \frac{22n}{2n} = 11 : 1.$

48. Ans. (c)

Solution Let the two numbers be $6x$ and $7x$, respectively.

$\frac{6x-n}{7x-n} < \frac{16}{21}$ (n = smallest integer)

or, $126x - 21n < 112x - 16n$

or, $14x < 5n$

$\therefore \quad n > \frac{14}{5}x.$

49. Ans. (a)

Solution $\frac{a}{b} = \frac{2}{3}, \frac{b}{c} = \frac{4}{5}$. Then,

$\frac{a+b}{b+c} = \frac{\frac{2}{3}b+b}{b+\frac{5}{4}b} = \frac{\frac{5}{3}b}{\frac{9}{4}b} = \frac{5\times4}{9\times3} = \frac{20}{27}.$

50. Ans. (c)

Solution Let the two numbers be a and b, respectively.

Then, $a + b = 40$ and $a - b = 4$

On solving, we get $a = 22$, $b = 18$

$\therefore$ Ratio = 22 : 18 = 11 : 9.

51. Ans. (a)

Solution List price of book $= \frac{82{\cdot}50}{30}\times100$ = Rs. 275

Neeta bought the book in

275 × 0·70 = Rs. 192·50.

52. Ans. (b)

Solution The man cannot purchase more than 8 bags

$\therefore$ Cost of 8 bags = 8 × 200 = Rs. 1600

Remaining amount = Rs. (1810 – 1600) = Rs. 210

In Rs. 210, the man can purchase

$\frac{210}{70}$ = 3 bottles

$\therefore$ Required ratio = 8 : 3.

53. Ans. (b)

Solution We have, $\frac{a}{b} = \frac{2}{3}$ and $\frac{b}{c} = \frac{4}{5}$

$\Rightarrow \frac{a}{2} = \frac{b}{3}$ and $\frac{b}{4} = \frac{c}{5}$

$\Rightarrow \frac{a}{8} = \frac{b}{12} = \frac{c}{15}$

$\therefore \quad a : b : c = 8 : 12 : 15$

$\therefore \quad a^2 : b^2 : bc = 64 : 144 : 180 = 16 : 36 : 45.$

54. Ans. (c)

55. Ans. (d)

Solution $\frac{(7-5)(27-17)\times40}{27\times5-17\times7} = \frac{2\times10\times40}{16} = 50.$

56. Ans. (a)

Solution $\frac{504\times13}{(13+11)} \div \left(\frac{504\times11}{13+11}+12\right) = 91 : 81.$

57. Ans. (d)

Solution $5x - 3y = 1800$

$6x - 4y = 1600$

Solving the two equations, we get

$x = 1200 \quad \therefore$ Monthly income of B = 6 × 1200

= Rs. 7200.

58. Ans. (d)

Solution Required difference $= \frac{9000}{(4+5+6)} \times (6 - 4)$

= 1200.

59. Ans. (a)

Solution $\frac{200\times(5\times5-3\times3)}{3(5+3)} = \frac{200\times16}{3\times8}$

$= \frac{400}{3}$ gram.

60. Ans. (a)

Solution

	A		B		C
	5		2		
			7		13
	5 × 7	:	2 × 7	:	2 × 13
or,	35	:	14	:	26

Hence, share of $B = \frac{14\times7500}{35+14+26}$

= Rs. 1400.

61. Ans. (c)

Solution Quantity of zinc in brass

$$= \frac{7}{13+7} \times 100 = 7 \times 5 = 35 \text{ kg.}$$

62. Ans. (c)

Solution $A : B : C = 2 : 3 : 4$

or, $\frac{A}{B} = \frac{2}{3}, \frac{B}{C} = \frac{3}{4}$ and $\frac{C}{A} = \frac{4}{2} = 2$

$\therefore \quad \frac{A}{B} : \frac{B}{C} : \frac{C}{A} = \frac{2}{3} : \frac{3}{4} : 2 = 8 : 9 : 24.$

63. Ans. (d)

64. Ans. (c)

Solution Let the numbers be $\frac{3}{2}x$ and $\frac{8}{3}x$

$$\frac{\frac{3}{2}x+15}{\frac{8}{3}x+15} = \frac{\frac{5}{3}}{\frac{5}{2}}, \text{ or, } \frac{\frac{3x+30}{2}}{\frac{8x+45}{3}} = \frac{2}{3}$$

or, $\frac{3x+30}{8x+45} \times \frac{3}{2} = \frac{2}{3}$ or, $\frac{3x+30}{8x+45} = \frac{4}{9}$ or, $x = 18$.

$\therefore$ greater of the numbers $= \frac{8}{3} \times 18 = 48.$

65. Ans. (c)

Solution Let the number of students in three classes be $2x$, $3x$ and $5x$

Now, according to the question

$2x + 40 : 3x + 40 : 5x + 40 = 4 : 5 : 7$

or, $\frac{2x+40}{3x+40} = \frac{4}{5}$ or, $10x + 200 = 12x + 160$

$\therefore \quad x = 20$

$\therefore$ total number of students

$= 2x + 3x + 5x = 10x = 10 \times 20 = 200.$

66. Ans. (a)

Solution Let the incomes of two persons be $5x$ and $3x$ and their expenditures be $9y$ and $5y$.

According to question

$5x - 9y = 1300$ ...(i)

$3x - 5y = 900$ ...(ii)

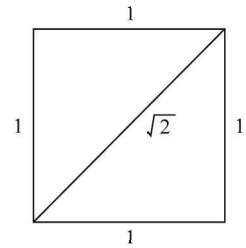
By solving equations (i) and (ii), we have $x = 800$

$\therefore$ persons income $= 5 \times 800 =$ Rs. 4000 and $3 \times 800 =$ Rs. 2400.

67. Ans. (a)

68. Ans. (a)

Solution Area of equilateral triangle with side $\sqrt{2}$



$$= \frac{\sqrt{3}}{4} \times (\sqrt{2})^2 = \frac{\sqrt{3}}{2}$$

Area of the square with side 1 = 1

$\therefore$ Required ratio $= \sqrt{3} : 2$

69. Ans. (c)

Solution $\frac{7x+3y}{7x-3y} = \frac{7\frac{x}{y}+3}{7\frac{x}{y}-3}$

$$= \frac{7 \times \frac{3}{4}+3}{7 \times \frac{3}{4}-3} = \frac{\frac{33}{4}}{\frac{9}{4}} = \frac{33}{9} = \frac{11}{3}.$$

70. Ans. (b)

Solution $\frac{a}{b} = \frac{5}{7}, \frac{c}{d} = \frac{2a}{3b} = \frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$

$\therefore \quad \frac{ac}{bd} = \frac{50}{147}$

$\therefore \quad ac : bd = 50 : 147$

71. Ans. (b)

Solution $\frac{A}{B} = \frac{2}{3}, \frac{B}{C} = \frac{4}{5}, \frac{C}{D} = \frac{6}{7}$

$\therefore \quad \frac{A}{16} = \frac{B}{24} = \frac{C}{30} = \frac{D}{35}.$

72. Ans. (d)

Solution Let the number be $\frac{1}{2}k$, $\frac{2}{3}k$ and $\frac{3}{4}k$

$\therefore \quad \frac{3}{4}k - \frac{1}{2}k = 36 \Rightarrow k = 144$

$\therefore$ The numbers are 72, 96 and 108.

73. Ans. (b)

Solution Let the number of girls and boys be $4k$ and $5k$, respectively.

$\therefore \quad 4k = 212 \Rightarrow k = 53$

$\therefore$ Number of boys $= 5k = 265.$

74. Ans. (b)

75. Ans. (b)

Solution $a : b = \frac{2}{9} : \frac{1}{3} = 2 : 3$

$b : c = \frac{2}{7} : \frac{5}{14} = 4 : 5$

$c : d = \frac{3}{5} : \frac{7}{10} = 6 : 7$

Now, $a : b = 2 : 3$

$b : c = 4 : 5$

$c : d = 6 : 7$

$\therefore$ $a : b : c : d = 48 : 72 : 90 : 105$

$= 16 : 24 : 30 : 35.$

76. Ans. (a)

Solution Let the incomes of A, B and C be $7x$, $9x$ and $12x$ respectively, and the expenditures $8y$, $9y$ and $15y$, respectively. We have to find the value of

$(7x - 8y) : (9x - 9y) : (12x - 15y)$

Also, $7x - 8y = \frac{7x}{4}$ or, $7x - \frac{7x}{4} = 8y$

or, $\frac{x}{y} = \frac{32}{21}$

Now, $(7 \times 32 - 8 \times 21) : (9 \times 32 - 9 \times 21) : (12 \times 32 - 15 \times 21) = 56 : 99 : 69.$

77. Ans. (a)

Solution Amount of copper $= \frac{5}{7} \times 17.5 = 12.5$ kg

Amount of zinc $= \frac{2}{7} \times 17.5 = 5$

Now, the amount of zinc $= 5 + 1.25 = 6.25$

$= 6.25$ kg

$\therefore$ Required ratio $= 12.5 : 6.25 = 2 : 1.$

78. Ans. (a)

Solution The ratio of number of coins

$= 13 \times \frac{100}{100} : 11 \times \frac{100}{50} : 7 \times \frac{100}{25}$

$= 13 : 22 : 28$

No. of 50-paise coins $= \frac{22}{63} \times 378 = 132.$

79. Ans. (b)

Solution $\frac{A}{2} = \frac{B}{3}$ and $\frac{B}{4} = \frac{C}{5} \Rightarrow \frac{A}{8} = \frac{B}{12} = \frac{C}{15}.$

80. Ans. (d)

Solution $A : B = \frac{1}{2} : \frac{1}{3} = 3 : 2$

$B : C = \frac{1}{2} : \frac{1}{3} = 3 : 2$

$\therefore$ $\frac{A}{3} = \frac{B}{2}$ and $\frac{B}{3} = \frac{C}{2}$

$\Rightarrow$ $\frac{A}{9} = \frac{B}{6} = \frac{C}{4}.$

81. Ans. (c)

Solution If Rs. 117 are divided in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$ that is, $6 : 4 : 3$ among P, Q and R, then

Share of P = Rs. 54

Share of Q = Rs. 36

Share of R = Rs. 27

If Rs. 117 are divided in the ratio of $2 : 3 : 4$ among, P, Q and R, then

Share of P = Rs. 26

Share of Q = Rs. 39

Share of R = Rs. 52

82. Ans. (d)

Solution $17k \times 0{\cdot}10 + 6k \times 0{\cdot}25 = 112$

$\Rightarrow 1{\cdot}7k + 1{\cdot}5k = 112 \Rightarrow 3{\cdot}2k = 112 \Rightarrow k = 35$

$\therefore$ No. of 10-paise coins $= 17 \times 35 = 595$

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. Rs. 675 was divided between A, B and C. If each of them had received Rs. 5 less, their shares would have been in the ratio of $1 : 2 : 3$. How much did B receive?
(a) Rs. 112.50 (b) Rs. 215
(c) Rs. 220 (d) Rs. 225

2. A merchant blends two varieties of tea from two different gardens, one costing Rs. 45 per kg and other Rs. 60 per kg, in the ratio of $7 : 3$, respectively. He sells the blended variety at Rs. 54.45 per kg. The profit per cent in the transaction is
(a) 5% (b) 10%
(c) $9\frac{1}{11}\%$ (d) $11\frac{1}{9}\%$

3. The ages of Amit and Puneet are in the ratio $2 : 3$. After 12 years, their ages will be in the ratio $11 : 15$. The age of Puneet is
(a) 32 years (b) 42 years
(c) 48 years (d) 56 years.

4. The proportion of zinc and copper in a brass piece is $13 : 7$. How much zinc will be there in 100 kg of such a piece?
(a) 20 kg (b) 35 kg
(c) 55 kg (d) 65 kg

5. 15 litres of mixture contains 20% alcohol and the rest water. If 3 litres of water be mixed in it, the percentage of alcohol in the new mixture will be
(a) 17 (b) $16\frac{2}{3}$
(c) $18\frac{1}{2}$ (d) 15

6. Rs. 427 is to be divided among A, B and C in such a manner that three times the A's share may be equal to four times the B's share and seven times the C's share. Share of C will be
(a) Rs. 81 (b) Rs. 104
(c) Rs. 84 (d) Rs. 90

7. If incomes of A, B and C are in the ratio of 9 : 3 : 7 and income of C exceeds the income of B by Rs. 1200 then the income of A is
(a) Rs. 2800 (b) Rs. 3100
(c) Rs. 2700 (d) Rs. 3200

8. If Rs. 600 is divided among A, B, C so that Rs. 40 more than $\frac{2}{5}$ of A's share, Rs. 20 more than $\frac{2}{7}$ of B's share and Rs. 10 more than $\frac{9}{17}$ of C's share may all be equal. C's share is
(a) Rs. 170 (b) Rs. 280
(c) Rs. 150 (d) Rs. 210

9. Two numbers are in the ratio 2 : 3 and their H.C.F. is 6. The numbers are
(a) 12 : 18 (b) 18 : 27
(c) 14 : 21 (d) None of these

10. Two numbers are in the ratio 3 : 4. If their L.C.M. is 144, the numbers are
(a) 27, 36 (b) 24, 32
(c) 36, 48 (c) None of these

Test II

[Time: 10 min]

11. Arrange 5 : 6, 7 : 8 and 13 : 16 in ascending order of magnitude.
(a) 13 : 16 < 5 : 6 , 7 : 8
(b) 5 : 6 < 7 : 8 < 13 : 16
(c) 7 : 8 > 5 : 6 > 13 : 16
(d) None of these

12. The ratio of ages of Usha and Madhuri is 4 : 3. The sum of their ages is 28 years. The ratio of their ages after 8 years will be
(a) 2 : 3 (b) 5 : 3
(c) 6 : 5 (d) 3 : 5

13. The average age of 3 girls is 20 years and their ages are in the proportion 3 : 5 : 7. The age of youngest girl is
(a) 4 years (b) 6 years 8 months
(c) 8 years 3 months (d) 12 years

14. A father's age was 5 times his son's age 5 years ago and will be 3 times son's age after 2 years, the ratio of their present ages is
(a) 5 : 2 (b) 5 : 3
(c) 10 : 3 (d) 11 : 5

15. An alloy contains copper and zinc in the ratio 7 : 3. If the alloy contains 10.5 kg zinc, then the quantity of copper in the alloy is:
(a) 17.35 kg (b) 24.5 kg
(c) 28.2 kg (d) 31.5 kg

16. Rs. 1900 is divided between A, B and C so that A's share is $1\frac{1}{2}$ times B's and B's is $1\frac{1}{2}$ times C's. What is C's share?
(a) Rs. 800 (b) Rs. 420
(c) Rs. 400 (d) Rs. 900

17. In a bag there are coins of 50-paise, 25-paise and one rupee in the proportion 5 : 6 : 2. If there are in all Rs. 42, the number of 25-paise coins is
(a) 32 (b) 42
(c) 52 (d) 60

18. Rs. 1300 be divided into three parts A, B and C. How much A is more than C if their ratio is $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$?
(a) Rs. 100 (b) Rs. 200
(c) Rs. 300 (d) Rs. 600

19. In an aptitude test boys and girls both appeared. If $\frac{1}{4}$th of the boys and $\frac{3}{8}$th of the girls qualified, the fractional part of the total number of candidates who qualified is
(a) $\frac{1}{3}$ (b) $\frac{3}{8}$
(c) $\frac{5}{7}$ (d) Data is inadequate

20. A and B are alloys of gold and copper prepared by mining metals in proportions 7 : 2 and 7 : 11, respectively if equal quantities of alloys are melted to form a third alloy C, the proportion of gold and copper in C will be:
(a) 7 : 5 (b) 3 : 5
(c) 5 : 7 (d) 11 : 7

Answers

1. (d)	**2.** (b)	**3.** (c)	**4.** (d)	**5.** (b)	**6.** (c)	**7.** (c)	**8.** (a)	**9.** (a)	**10.** (c)
11. (a)	**12.** (c)	**13.** (d)	**14.** (c)	**15.** (b)	**16.** (c)	**17.** (b)	**18.** (c)	**19.** (d)	**20.** (a)

8

PARTNERSHIP

INTRODUCTION

In *partnership,* two or more persons carry on a business and share the profits of the business at an agreed proportion. Persons who have entered into partnership with one another are individually called *partners* and collectively called a *firm* and the name under which their business is carried on is called the *firm name*. The partnership may be simple or compound.

Simple Partnership is one in which the capital of each partner is in the business for same time.

Compound Partnership is one in which the capitals of partners are invested for different periods.

Again partner can be working partner or sleeping partner.

Sleeping Partner is one who invests the capital in the business but does not actively participate in the conduct of business.

Working Partner besides investing capital, takes part in running the business. For his work, he is either paid some salary or given a certain per cent of profit, in addition.

SOME BASIC FORMULAE

1. (a) If capitals of two partners be Rs. C_1 and Rs. C_2 for the same period and the total profit be Rs. P, then shares of the partners in the profits are

$$\text{Rs.}\left(\frac{C_1 \times P}{C_1 + C_2}\right) \text{ and Rs.}\left(\frac{C_2 \times P}{C_1 + C_2}\right).$$

(b) If the capitals of three partners be Rs. C_1, Rs. C_2 and Rs. C_3 for the same period and the total profit be Rs. P, then shares of the partners in the profits are

$$\text{Rs.}\left(\frac{C_1 \times P}{C_1 + C_2 + C_3}\right), \text{Rs.}\left(\frac{C_2 \times P}{C_1 + C_2 + C_3}\right)$$

$$\text{and Rs.}\left(\frac{C_3 \times P}{C_1 + C_2 + C_3}\right).$$

Illustration 1 A, B and C invested Rs. 20000, Rs. 50000 and Rs. 40000, respectively in a business. The net profit for the year was Rs. 12100 which was divided in proportion to investments. Find the amount of profit each partner earned.

Solution We have, $C_1 = 20000$, $C_2 = 50000$, $C_3 = 40000$ and $P = 12100$.

Therefore, profit share of A

$$= \frac{C_1 \times P}{C_1 + C_2 + C_3}$$

$$= \frac{20000 \times 12100}{20000 + 50000 + 40000}$$

$$= \frac{2}{11} \times 12100 = \text{Rs. } 2200$$

Profit share of B

$$= \frac{C_2 \times P}{C_1 + C_2 + C_3}$$

$$= \frac{50000 \times 12100}{20000 + 50000 + 40000}$$

$$= \frac{5}{11} \times 12100 = \text{Rs. } 5500.$$

and, profit share of C

$$= \frac{C_3 \times P}{C_1 + C_2 + C_3}$$

$$= \frac{40000 \times 12100}{20000 + 50000 + 40000}$$

$$= \frac{4}{11} \times 12100 = \text{Rs. } 4400.$$

2. (a) If the capitals of two partners be Rs. C_1 and Rs. C_2 for the periods t_1 and t_2, respectively and the total profit be Rs. P, then shares of the partners in the profits are:

$$\text{Rs.}\left(\frac{C_1 \times t_1 \times P}{C_1t_1 + C_2t_2}\right) \text{ and Rs.}\left(\frac{C_2 \times t_2 \times P}{C_1t_1 + C_2t_2}\right)$$

(b) If the capitals of three partners be Rs. C_1, Rs. C_2 and Rs. C_3 for the periods t_1, t_2 and t_3, respectively and the total profit be Rs. P, then shares of the partners in the profits are

$$\text{Rs.}\left(\frac{C_1 \times t_1 \times P}{C_1t_1 + C_2t_2 + C_3t_3}\right), \text{Rs.}\left(\frac{C_2 \times t_2 \times P}{C_1t_1 + C_2t_2 + C_3t_3}\right)$$

$$\text{and, Rs.}\left(\frac{C_3 \times t_3 \times P}{C_1t_1 + C_2t_2 + C_3t_3}\right)$$

Illustration 2 A, B are two partners in a business. A contributes Rs. 1200 for 5 months and B Rs. 750 for 4 months. If total profit is Rs. 450, find their respective shares.

Solution We have, $C_1 = 1200$, $C_2 = 750$, $t_1 = 5$, $t_2 = 4$ and $P = 450$.

$\therefore$ Profit share of A

$$= \frac{C_1 \times t_1 \times P}{C_1t_1 + C_2t_2} = \frac{1200 \times 5 \times 450}{1200 \times 5 + 750 \times 4}$$

$$= \frac{2700000}{9000} = \text{Rs. } 300$$

and, profit share of B

$$= \frac{C_2 \times t_2 \times P}{C_1t_1 + C_2t_2}$$

$$= \frac{750 \times 4 \times 450}{1200 \times 5 + 750 \times 4}$$

$$= \frac{1350000}{9000} = \text{Rs. } 150.$$

SOME USEFUL SHORT-CUT METHODS

1. (a) If the capitals of two partners be Rs. C_1 and Rs. C_2 for the periods t_1 and t_2 respectively, then

$$\frac{\text{Profit of } A}{\text{Profit of } B} = \frac{C_1 \times t_1}{C_2 \times t_2}.$$

(b) If the capitals of three partners be Rs. C_1, Rs. C_2 and Rs. C_3 for the periods t_1, t_2 and t_3 respectively, then profit of A : profit of B : profit of $C = C_1 \times t_1 : C_2 \times t_2 : C_3 \times t_3$.

Note: If there is a loss in the business, then

Loss of A : Loss of B : Loss of C

$= C_1 \times t_1 : C_2 \times t_2 : C_3 \times t_3$.

Illustration 3 There are three partners A, B and C in a certain business. A puts in Rs. 2000 for 5 months, B Rs. 1200 for 6 months and C Rs. 2500 for 3 months. Find the ratio of their shares in the profit.

Solution Here, $C_1 \times t_1 = 2000 \times 5 = 10000$, $C_2 \times t_2 = 1200 \times 6 = 7200$ and $C_3 \times t_3 = 2500 \times 3 = 7500$.

$\therefore$ Profit of A : Profit of B : Profit of C

$= C_1 \times t_1 : C_2 \times t_2 : C_3 \times t_3$

$= 10000 : 7200 : 7500$ or $100 : 72 : 75$.

2. If the capitals of three partners are invested in the ratio $C_1 : C_2 : C_3$ and their profits are in the ratio $P_1 : P_2 : P_3$, then the ratio of timing of their investments $= \frac{P_1}{C_1} : \frac{P_2}{C_2} : \frac{P_3}{C_3}$.

Illustration 4 Anu, Manu and Tanu invested capitals in the ratio 4 : 6 : 9. At the end of the business term, they received the profits in the ratio 2 : 3 : 5. Find the ratio of time for which they invested their capitals.

Solution We have, $C_1 : C_2 : C_3 = 4 : 6 : 9$

and, $P_1 : P_2 : P_3 = 2 : 3 : 5$

Therefore, the ratio of time for which Anu, Manu and Tanu invested their capitals

$$= \frac{P_1}{C_1} : \frac{P_2}{C_2} : \frac{P_3}{C_3} = \frac{2}{4} : \frac{3}{6} : \frac{5}{9} \quad \text{or,} \quad \frac{1}{2} : \frac{1}{2} : \frac{5}{9}$$

or, $9 : 9 : 10$.

3. Three partners invested their capitals in a business. If the timing of their investments is in the ratio $t_1 : t_2 : t_3$ and their profits are in the ratio $P_1 : P_2 : P_3$, then the ratio of their capitals invested is $\frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3}$.

Illustration 5 Gupta, Singhal and Kansal start a business. If the ratio of their periods of investments are 1 : 2 : 5 and their profits are in the ratio of 3 : 4 : 5, find the ratio of capitals of Gupta, Singhal and Kansal.

Solution We have, $P_1 : P_2 : P_3 = 3 : 4 : 5$ and $t_1 : t_2 : t_3 = 1 : 2 : 5$.

$$\therefore \text{Required ratio} = \frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3}$$

$$= \frac{3}{1} : \frac{4}{2} : \frac{5}{5} \quad \text{or} \quad 3 : 2 : 1.$$

Thus, Gupta, Singhal and Kansal invested their capitals in the ratio 3 : 2 : 1.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Nikita and Nishita enter into a partnership investing Rs. 50000 and Rs. 40000, respectively. They agree to share profits in the ratio of their capitals. Find the share of Nikita in a profit of Rs. 22500 after one year.
(a) Rs. 1500 (b) Rs. 9500
(c) Rs. 10500 (d) None of these

2. Niki, Nisha and Anu formed a partnership with investments of Rs. 75000, Rs. 60000 and Rs. 40000, respectively. After 3 years of operation, the partnership had a net profit of Rs. 26250. What was the share of Anu in the profit?
(a) Rs. 6000 (b) Rs. 5000
(c) Rs. 8000 (d) None of these

3. Mahesh, Suresh and Ganesh entered into a partnership. Mahesh invested Rs. 16000 for 9 months. Suresh invested Rs. 12000 for 6 months and Ganesh invested Rs. 8000 for 12 months. At the end of a year there was a profit of Rs. 26000. Find the share of Suresh in the profit.
(a) Rs. 8000 (b) Rs. 7500
(c) Rs. 6000 (d) None of these

4. Sita and Gita enter into a partnership. Sita contributes Rs. 5000 while Gita contributes Rs. 4000. After 1 month, Gita withdraws $\frac{1}{4}$ part of her contribution and after 3 months from the starting Sita puts Rs. 2000 more. When Gita withdraws her money at the same time Rita also joins them with Rs. 7000. If at the end of 1 year there is a profit of Rs. 1218, what will be share of Rita in the profit?
(a) Rs. 488.47 (b) Rs. 8447.37
(c) Rs. 588.47 (d) None of these

5. A starts business with Rs. 3500 and 5 months after B joins A as his partner. After a year the profits are divided in the ratio of 2 : 3. How much did B contribute?
(a) Rs. 7000 (b) Rs. 11000
(c) Rs. 9000 (d) None of these

6. Gupta and Bansal enter into a partnership with their capitals in the ratio 5 : 6. At the end of 8 months, Gupta withdraws his capital. If they receive the profits in the ratio of 5 : 9, find how long Bansal's capital was used?
(a) 10 months (b) 12 months
(c) 14 months (d) None of these

7. Arvind began a business with Rs. 550 and was joined afterwards by Brij with Rs. 330. When did Brij join if the profits at the end of the year were divided in the ratio 10 : 3?
(a) After 4 months (b) After 6 months
(c) After 4.5 months (d) None of these

8. A began business with Rs. 3750 and was joined afterwards by B with Rs. 5000. When did B join if the profits at the end of the year were divided equally?
(a) After 5 months (b) After 9 months
(c) After 7 months (d) None of these

9. Anju and Brijesh enter into a partnership with their capitals in the ratio 5 : 9. At the end of 8 months, Anju withdraws her capital. If they receive the profits in the ratio 4 : 9, find how long Brijesh's capital was used?
(a) 8 months (b) 10 months
(c) 12 months (d) None of these

10. A, B and C invested capitals in the ratio 3 : 5 : 9; the timing of their investments being in the ratio 2 : 3 : 1. In what ratio would their profits be distrubuted?
(a) 2 : 5 : 3 (b) 3 : 2 : 5
(c) 7 : 5 : 3 (d) None of these

11. Sumit, Punit and Ramit started a business by investing their capitals in the ratio 1 : 2 : 3. At the end of the business term, they received the profits in the ratio 1 : 2 : 3. Find the ratio of time for which they invested their capitals.
(a) 1 : 1 : 1 (b) 2 : 3 : 4
(c) 2 : 4 : 3 (d) None of these

12. A, B and C start a business. If the ratio of their periods of investments are 2 : 3 : 6 and their profits are in the ratio of 4 : 5 : 6, then the ratio of capitals of A, B and C is
(a) 6 : 8 : 10 (b) 12 : 10 : 6
(c) 10 : 12 : 6 (d) None of these

13. A, B and C rented a pasture. A puts in 12 oxen for 6 months, B 8 oxen for 7 months and C 6 oxen for 8 months. If the rent of the field is Rs. 396, what rent is paid by A?
(a) Rs. 126 (b) Rs. 108
(c) Rs. 162 (d) Rs. 168.

14. A, B, C and D enter into partnership. A subscribes $\frac{1}{3}$ of the capital, B $\frac{1}{4}$, C $\frac{1}{5}$ and D the rest. What is the share of D out of a profit of Rs. 6000?

(a) Rs. 2000 (b) Rs. 1600
(c) Rs. 1200 (d) Rs. 1300

15. A and B enter into partnership for a year. A contributes Rs. 1500 and B Rs. 2000. After 4 months, they admit C who contributes Rs. 2250. If B withdraws his contribution after 9 months, at the end of the year they share profit in the ratio
(a) 2 : 1 : 3 (b) 1 : 3 : 2
(c) 1 : 1 : 2 (d) 1 : 1 : 1

16. A and B started a business with initial investments in the ratio 5 : 7. If after one year their profits were in the ratio 1 : 2 and the period for A's investment was 7 months, B invested the money for
(a) 6 months (b) $2\frac{1}{2}$ months
(c) 10 months (d) 4 months

17. A and B jointly invest Rs. 2100 and Rs. 3100, respectively in a firm. A is an active partner and hence he gets 25% of the profit separately. If their business yields them total Rs. 1040 as profit, what will be the gain of each of them?
(a) Rs. 415, Rs. 625 (b) Rs. 575, Rs. 465
(c) Rs. 515, Rs. 525 (d) Rs. 560, Rs. 480

18. Two partners invested Rs. 12500 and Rs. 8500, respectively in a business and decided that 60% of the profit incurred from the business will be equally divided between them while remaining profit will be assumed as interest on their capitals. If one of the partners gets Rs. 300 more profit than the other, what is the total profit in the business?
(a) Rs. 3937.50 (b) Rs. 4940.50
(c) Rs. 3936.50 (d) Rs. 4156

19. A, B, C enter into a partnership with shares in the ratio $\frac{7}{2}:\frac{4}{3}:\frac{6}{5}$. After 4 months, A increases his share by 50%. If the total profit at the end of one year be Rs. 21600, then B's share in the profit is
(a) Rs. 2100 (b) Rs. 2400
(c) Rs. 3600 (d) Rs. 4000

20. A and B invest in a business in the ratio 3 : 2. If 5% of the total profit goes to charity and A's share is Rs. 855, total profit is
(a) Rs. 1576 (b) Rs. 1537.50
(c) Rs. 1500 (d) Rs. 1425

21. B is a sleeping partner and A working. A puts in Rs. 5000 and B puts in 6000. A received $12\frac{1}{2}\%$ of profit for managing the business and rest is divided in propoftion to their capitals. A's share of profit in a profit of Rs. 880 is
(a) Rs. 350 (b) Rs. 400
(c) Rs. 420 (d) Rs. 460

22. A starts business with a capital of Rs. 1200. B and C join with some investments after 3 and 6 months, respectively. If at the end of a year, the profit is divided in the ratio 2 : 3 : 5 respectively, what is B's investment in the business?
(a) Rs. 2400 (b) Rs. 1800
(c) Rs. 3600 (d) Rs. 6000

23. A, B and C entered into a partnership with Rs. 3512420, Rs. 4222180 and Rs. 4065400. After 2 years A withdrew Rs. 11 lacs and at the same time C invested Rs. 8 lacs more. If at the end of 3 years the profit is Rs. 1053000, what is the share of C in the profit?
(a) Rs. 283117.80 (b) Rs. 379996.20
(c) Rs. 399866 (d) Rs. 299866
(e) None of these

24. A and B enter into partnership. A supplies whole of the capital amounting to Rs. 45000 with the condition that the profits are to be equally divided and that B pays A interest on half of the capital at 10% per annum, but receives, Rs. 120 per month for carrying on the concern. When B's income is one-half of A's income their total yearly profit is
(a) Rs. 9180 (b) Rs. 7150
(c) Rs. 3060 (d) Rs. 1440
(e) None of these

25. A, B and C entered into a partnership by investing Rs. 12000, Rs. 15000 and Rs. 18000, respectively. A is also a working partner and getting 15% of the annual profit for his work. If B and C got Rs. 8500 and Rs. 10200, respectively from the annual profit as their shares, what amount did A get from the annual profit?
(a) Rs. 10,500 (b) Rs. 11,500
(c) Rs. 11,300 (d) Rs. 14,000

EXPLANATORY ANSWERS

1. Ans. (a)

Solution Here, $C_1 = 50000$, $C_2 = 40000$ and $P = 22500$.
$\therefore C_1 + C_2 = 50000 + 40000 = 90000$.

$\therefore$ Nikita's share $= \frac{C_1 \times P}{C_1 + C_2} = \frac{50000 \times 22500}{90000}$

$= \frac{5}{9} \times 22500 =$ Rs. 12500.

2. Ans. (a)

Solution Ratio of the capitals
= 75000 : 60000 : 40000 = 15 : 12 : 8

$\therefore$ Profit-sharing ratio = 15 : 12 : 8

Sum of the profit-sharing ratio = 15 + 12 + 8 = 35

Total profit = Rs. 26250

$\therefore$ Anu's share = $\frac{8}{35} \times 26250$ = Rs. 6000.

3. Ans. (c)

Solution Here, $C_1 = 16000$, $C_2 = 12000$, $C_3 = 8000$, $t_1 = 9$, $t_2 = 6$, $t_3 = 12$ and $P = 26000$.

Suresh's share in the profit

$$= \frac{C_2 \times t_2 \times P}{C_1t_1 + C_2t_2 + C_3t_3}$$

$$= \frac{12000 \times 6 \times 26000}{16000 \times 9 + 12000 \times 6 + 8000 \times 12}$$

$$= \frac{1872000000}{312000} = \text{Rs. } 6000.$$

4. Ans. (a)

Solution Here,

$C_1 \times t_1 = 5000 \times 12 + 2000 \times 9 = 78000$,

$C_2 \times t_2 = 4000 \times 1 + 3000 \times 11 = 37000$,

$C_3 \times t_3 = 7000 \times 11 = 77000$ and $P = 1218$.

$\therefore$ Share of Rita in the profit

$$= \frac{C_3 \times t_3 \times P}{C_1t_1 + C_2t_2 + C_3t_3}$$

$$= \frac{77000 \times 1218}{78000 + 37000 + 77000} = \text{Rs. } 488.47.$$

5. Ans. (c)

Solution We have, $C_1 \times t_1 = 3500 \times 12 = 42000$ and $C_2 \times t_2 = x \times 7 = 7x$.

Then, $\frac{\text{Profit for } A}{\text{Profit for } B} = \frac{C_1 \times t_1}{C_2 \times t_2}$

$\Rightarrow \frac{2}{3} = \frac{42000}{7x}$ or, $x = \frac{42000 \times 3}{2 \times 7}$ = Rs. 9000.

6. Ans. (b)

Solution Let Bansal's capital be used for x months.

Then, we have $\frac{5 \times 8}{6 \times x} = \frac{5}{9}$

$\Rightarrow x = \frac{5 \times 8 \times 9}{6 \times 5}$ = 1 month.

$\therefore$ Bansal's capital was used for 12 months.

7. Ans. (b)

Solution Let Brij remain in the business for x months.

We have, $C_1 \times t_1 = 550 \times 12 = 6600$

$C_2 \times t_2 = 330 \times x = 330x$

$\therefore \frac{\text{Arvind's share of profit}}{\text{Brij's share of profit}} = \frac{C_1 \times t_1}{C_2 \times t_2}$

$\Rightarrow \frac{10}{3} = \frac{6600}{330x} \Rightarrow x = \frac{6600 \times 3}{330 \times 10}$ = 6 months.

8. Ans. (b)

Solution Let B remain in the business for x months.

We have, $C_1 \times t_1 = 3750 \times 12 = 45000$

and, $C_2 \times t_2 = 5000 \times x = 5000x$

$\therefore \frac{A\text{'s share in profit}}{B\text{'s share in profit}} = \frac{C_1 \times t_1}{C_2 \times t_2}$

$\Rightarrow \frac{1}{1} = \frac{45000}{5000x}$ or, $x = \frac{45000}{5000}$ = 9 months

9. Ans. (b)

Solution Let Brijesh's capital be used for x months.

Ratio of capitals of Anju and Brijesh are 5 : 9.

Let the capitals of Anju and Brijesh be Rs. $5y$ and Rs. $9y$.

We have, $C_1 \times t_1 = 5y \times 8 = 40y$

and, $C_2 \times t_2 = 9y \times x = 9yx$

$\therefore \frac{\text{Anju's share of profit}}{\text{Brijesh's share of profit}} = \frac{C_1 \times t_1}{C_2 \times t_2}$

$\Rightarrow \frac{4}{9} = \frac{40y}{9yx} \Rightarrow x = \frac{40 \times 9}{4 \times 9}$ = 10 months.

10. Ans. (a)

Solution Ratio of capitals of A, B and C are 3 : 5 : 9. Let the capitals of A, B and C be $3x$, $5x$ and $9x$, respectively.

Ratio of timing of their investments are 2 : 3 : 1. Let A, B and C invest their capitals for $2y$, $3y$ and y months, respectively.

Then, profit of A : profit of B : profit of C

$= C_1 \times t_1 : C_2 \times t_2 : C_3 \times t_3$

$= 3x \times 2y : 5x \times 3y : 9x \times y$

$= 6 : 15 : 9$ or, 2 : 5 : 3.

11. Ans. (a)

Solution We have, $C_1 : C_2 : C_3 = 1 : 2 : 3$

and, $P_1 : P_2 : P_3 = 1 : 2 : 3$.

$\therefore$ Required ratio = $\frac{P_1}{C_1} : \frac{P_2}{C_2} : \frac{P_3}{C_3} = \frac{1}{1} : \frac{2}{2} : \frac{3}{3}$

or, 1 : 1 : 1.

Thus, Sumit, Punit and Ramit invested their capitals for equal period of time.

12. Ans. (b)

Solution We have, $P_1 : P_2 : P_3 = 4 : 5 : 6$ and $t_1 : t_2 : t_3 = 2 : 3 : 6$.

$\therefore$ Required ratio = $\frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3} = \frac{4}{2} : \frac{5}{3} : \frac{6}{6}$

or, 12 : 10 : 6.

Thus, A, B and C invested their capitals in the ratio 12 : 10 : 6.

13. Ans. (c)

Solution Ratio in which A, B, C pay the rent

$= (6 \times 12) : (8 \times 7) : (6 \times 8)$

$= 72 : 56 : 48$

$= 9 : 7 : 6$

Rent paid by $A = \frac{9}{22} \times$ Rs. 396 = Rs. 162.

14. Ans. (d)

Solution D's capital $= 1 - \frac{1}{3} - \frac{1}{4} - \frac{1}{5} = \frac{13}{60}$

Profit ratio of A, B, C, D is $\frac{1}{3} : \frac{1}{4} : \frac{1}{5} : \frac{13}{60}$

$\Rightarrow$ 20 : 15 : 12 : 13

$\therefore$ Share of $D = \frac{13}{60} \times$ Rs. 6000 = Rs. 1300.

15. Ans. (d)

Solution Obviously, C invests for 12 – 4 = 8 months.

$\therefore$ Equivalent capitals are:

Rs. 1500 × 1; Rs. 2000 × $\frac{9}{12}$ = Rs. 1500,

Rs. 2250 × $\frac{8}{12}$ = Rs. 1500

$\therefore$ Profit is to be shared in the ratio

1500 : 1500 : 1500 = 1 : 1 : 1.

16. Ans. (c)

Solution Let investments of A and B respectively be $5x$ and $7x$ and period of B's investment be y months

Then, $\frac{(5x) \times 7}{(7x) \times y} = \frac{1}{2} \Rightarrow y = 10.$

17. Ans. (b)

Solution Total profit in a business = Rs. 1040

Separate profit for A = 25% of Rs. 1040

$= \frac{1040 \times 25}{100}$ = Rs. 260

Remaining profit = Rs. (1040 – 260) = Rs. 780

The remaining profit will be divided in proportion to their capitals.

$\therefore$ Ratio between capitals of A and B = 2100 : 3100 = 21 : 31

Sum of proportional = 21 + 31 = 52

$\therefore$ A's profit $= \frac{21}{52} \times 780$ = Rs. 315

B's profit $= \frac{31}{52} \times 780$ = Rs. 465

Total profit of A = Rs. (315 + 260) = Rs. 575

Therefore, A and B will make profit of Rs. 575 and Rs. 465, respectively.

18. Ans. (a)

Solution Suppose total profit in the business = Rs. x

$\therefore$ 60% of total profit = 60% of x = Rs. $\frac{3x}{5}$

$\therefore$ The two partners will get profit of Rs. $\frac{3x}{10}$ and Rs. $\frac{3x}{10}$, respectively.

$\therefore$ Remaining profit $= x - \frac{3x}{5}$ = Rs. $\frac{2x}{5}$.

The remaining profit assumed as interest on the capital will be divided in the proportion of their capitals.

Ratio between their capitals = 12500 : 8500 = 25 : 17

Sum of proportionals = 25 + 17 = 42

$\therefore$ Ist partner's profit $= \frac{25}{42} \times \frac{2x}{5}$ = Rs. $\frac{5x}{21}$

2nd partner's profit $= \frac{17}{42} \times \frac{2x}{5}$ = Rs. $\frac{17x}{105}$

Given : $\frac{3x}{10} + \frac{5x}{21} = \frac{3x}{10} + \frac{17x}{105} + 300$

$\therefore$ $\frac{8x}{105} = 300$

or, $x = \frac{105 \times 300}{8}$ = Rs. 3937.50

$\therefore$ Total profit in the business = Rs. 3937.50.

19. Ans. (d)

Solution Given ratio $= \frac{7}{2} : \frac{4}{3} : \frac{6}{5} = 105 : 40 : 36$

Let they initially invest Rs. 105, Rs 40 and Rs. 36, respectively.

Ratio of investments

$= [105 \times 4 + (150\% \text{ of } 105) \times 8] : (40 \times 12) : (36 \times 12)$

$= 1680 : 480 : 432 = 35 : 10 : 9$

B's share = Rs. $\left(21600 \times \frac{10}{54}\right)$ = Rs. 4000.

20. Ans. (c)

Solution Let the total profit be Rs. 100.

After paying the charity, A's share = Rs. $\left(95 \times \frac{3}{5}\right)$

= Rs. 57

If A's share is Rs. 57, total profit = Rs. 100

If A's share is Rs. 855, total profit = Rs. $\left(855 \times \frac{100}{57}\right)$

= Rs. 1500.

21. Ans. (d)

Solution A's share for managing the business

$= 12\frac{1}{2}\%$ of Rs. 880 = Rs. 110

Remaining profit = Rs. 770

Profit ratio of A and B = 5 : 6

A's share = $\frac{5}{11}$ of Rs. 770 = Rs. 350

A's total profit = Rs. 350 + Rs. 110 = Rs. 460.

22. Ans. (a)

Solution Profit ratio of A, B and C is

$(1200 \times 12) : (x \times 9) : (y \times 6) = 2 : 3 : 5$

$\Rightarrow \frac{1200 \times 12}{2} = \frac{9x}{3} \quad \therefore \quad x$ = Rs. 2400.

23. Ans. (e)

Solution Share of C in profit = $\frac{C_3 t_3 \times P}{C_1 t_1 + C_2 t_2 + C_3 t_3}$

[Here, $C_1 \times t_1 = 3512420 \times 2 + 2412420 \times 1$

$C_2 \times t_2 = 4222180 \times 3'C_3 \times t_3$

$= 4065400 \times 2 + 4865400 \times 1$ and $P = 105300$]

$$= \frac{(4065400 \times 2 + 4865400 \times 1) \times 1053000}{(3512420 \times 2 + 2412420 \times 1) + 4222180 \times 3 + (4065400 \times 2 + 4865400 \times 1)}$$

= Rs. 389886.

24. Ans. (a)

Solution Let the total profit be Rs. x

Salary given to B = Rs. 1440

$\therefore$ Net profit = $x - 1440$

$\therefore$ Share of A and B each = $\frac{x - 1440}{2}$

Interest given by B to A = $\frac{10}{100} \times 22500$ = Rs. 2250.

$$\therefore \quad \frac{\frac{x-1440}{2} + 1440 - 2250}{\frac{x-1440}{2} + 2250} = \frac{1}{2}$$

or, $\frac{x - 3060}{x + 3060} = \frac{1}{2}$

$\therefore \quad x$ = Rs. 9180.

25. Ans. (c)

Solution Capital ratio of A, B, C is 12000 : 15000 : 18000 = 4 : 5 : 6

Profit ratio of B and C = 8500 : 10200 = 5 : 6

$\therefore$ A's share of profit = $\frac{4}{5} \times$ Rs. 8500 = Rs. 6800

85% of x = 6800 + 8500 + 10200 = 25500

$\therefore$ 15% of x = $\frac{25500 \times 15}{85}$ = 4500

Total profit of A = Rs. 6800 + Rs. 4500 = Rs. 11300.

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. Three friends X, Y and Z started a partnership business investing money in the ratio of 5 : 4 : 2, respectively for a period of 3 years. What is the amount received by X as the share in the total profit?

I. Total amount invested in the business is Rs. 22000.

II. Profit was distributed after a period of 2 years.

III. The average amount of profit earned per year is Rs. 2750.

(a) I only (b) II only
(c) III only (d) I or III only
(e) I or II or III **[Based on SBI PO 1999]**

2. Two friends A and B started a business investing amounts in the ratio of 5 : 6. R joined them after six months investing an amount equal to that of Q's amount. At the end of the year 20% profit was earned which was equal to Rs. 980000. What was the amount invested by R?

(a) Rs. 210000 (b) Rs. 105000
(c) Rs. 175000 (d) data inadequate
(e) None of these **[Based on SBI PO 1999]**

3. Three friends A, B and C started a business by investing a sum of money in the ratio 5 : 7 : 6. After 6 months C withdraws half of his capital. If the sum invested by 'A' is Rs. 40000, out of total annual profit of Rs. 33000. C's share will be

(a) Rs. 9000 (b) Rs. 12000
(c) Rs. 11000 (d) Rs. 10000
(e) None of these
[Based on BSRB Hyderabad PO 1999]

4. Mr. Saxena started a business investing Rs. 5000. Four months later Mr. Jain joined the business by investing Rs. 9000. If the profit at the end of the year was Rs. 22000 how much amount would Mr. Jain have received as the profit?

(a) Rs. 16000 (b) Rs. 14000
(c) Rs. 12000 (d) Rs. 11000
(e) None of these **[Based on BSRB Delhi PO 2000]**

5. Vinod and Ankit started a business, investing amounts in the ratio 2 : 3. If Vinod had invested an additional amount of Rs. 10000 the ratio of Vinod's investment to Ankit's investment would have been 3 : 2. What was the amount invested by Vinod?

(a) Rs. 8000 (b) Rs. 12000
(c) Rs. 9000 (d) Data inadequate
(e) None of these **[Based on BSRB Patna PO 2001]**

6. Vinay started a business investing Rs. 50000. After one year he invested another Rs. 30000 and Aditya

also joined him with a capital of Rs. 70000. If the profit earned in three years from the starting of business was Rs. 87500, find the share of Aditya in the profit.
(a) Rs. 37500 (b) Rs. 32500
(c) Rs. 38281 (d) Rs. 52500
(e) None of these
[Based on Corporation Bank PO 2002]

7. A started a business with Rs. 10000 and B joined him later with a capital of Rs. 40000. If at the end of the year, they both get an equal share of the profit, how many months after the business started did B join it?
(a) 4 months (b) 6 months
(c) 8 months (d) 9 months
[Based on SI of Police Rec. Exam. 1997]

8. A, B, C enter into a partnership investing Rs. 35000, Rs. 45000 and Rs. 55000 respectively. The respective shares of A, B, C in an annual profit of Rs. 40500 are
(a) Rs. 13500, Rs. 16500, Rs. 10500
(b) Rs. 10500, Rs. 13500, Rs. 16500
(c) Rs. 13500, Rs. 10500, Rs. 16600
(d) None of these
[Based on SI of Police Rec. Exam. 1997]

9. Three men rent a pasture for Rs. 660. The first uses it to graze 50 sheep for 4 months, the second uses it to graze 40 sheep for 3 months and the third 46 sheep for 5 months. How much should the first man pay as rent?
(a) Rs. 276 (b) Rs. 220
(c) Rs. 144 (d) Rs. 240
[Based on SI of Police Rec. Exam. 1997]

10. A began business with Rs. 45000 and B joined afterwards with Rs. 30000. At the end of a year, the profit is divided in the ratio 2 : 1. When did B join?
(a) 3 months after (b) 6 months after
(c) 8 months after (d) 9 months after
[Based on Assistant's Grade Exam. 1997]

11. A, B and C invested Rs. 10000, Rs. 14000 and Rs. 12000, respectively in a business. If at the end of a year, they got a profit of Rs. 5400, the share of B in profit (if profit is divided in proportion to the investment of each) is
(a) Rs. 1500 (b) Rs. 2100
(c) Rs. 1800 (d) Rs. 150
[Based on Assistant's Grade Exam. 1998]

12. A starts business with Rs. 3500 and after 5 months B joins with A as his partner. After a year, the profit is divided in the ratio 2 : 3. What is B's contribution in the capital?
(a) Rs. 8000 (b) Rs. 8500
(c) Rs. 9000 (d) Rs. 7500
[Based on SSC (GL) Prel. Exam. 2000]

13. A, B and C rent a pasture. A puts 10 oxen for 7 months, B puts 12 oxen for 5 months and C puts 15 oxen for 3 months for grazing. If the rent of the pasture is Rs. 175, how much must C pay as his share of rent?
(a) Rs. 45 (b) Rs. 50
(c) Rs. 55 (d) Rs. 60
[Based on SSC (GL) Prel. Exam. 2000]

14. A, B and C enter into a partnership and their shares are in the ratio $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$. After 2 months, A withdraws half of its capital and after 10 months a profit of Rs. 378 is divided among them. What is B's share?
(a) Rs. 129 (b) Rs. 144
(c) Rs. 156 (d) Rs. 168
[Based on SSC (GL) Prel. Exam. 2000]

15. A and B are partners in a business. A contrbutes $\frac{1}{4}$ of the capital for 15 months and B received $\frac{1}{4}$ of the profit. Find for how long B's money was used?
(a) 1 year (b) 9 months
(c) 6 months (d) 10 months
[Based on SSC (GL) Prel. Exam. 2000]

16. A, B and C are partners of a company. During a particular year A received one-third of the profit, B received one-fourth of the profit and C received the remaining Rs. 5000. How much did A receive?
(a) Rs. 5000 (b) Rs. 4000
(c) Rs. 3000 (d) Rs. 1000
[Based on SSC (GL) Prel. Exam. 2000]

17. A began a business with Rs. 4500 and was joined afterwards by B with Rs. 5400. When did B join if the profits at the end of the year were divided in the ratio 2 : 1?
(a) 4 months (b) 5 months
(c) 6 months (d) 7 months
[Based on SSC (GL) Prel. Exam. 2000]

18. Vinay started a business investing Rs. 70000. Ashok joined him after six months with an amount of Rs. 105000 and Sunil joined them with Rs. 1.4 lakhs after another six months. The amount of profit earned should be distributed in what ratio among Vinay, Ashok and Sunil respectively, three years after Vinay started the business?
(a) 42 : 45 : 56 (b) 7 : 6 : 10
(c) 12 : 15 : 16 (d) Cannot be determined
(e) None of these
[Based on PNB Management Trainee 2003]

19. Veena started a business, investing Rs. 75000. After 3 months, Poonam joined her with an amount of Rs. 125000 and after another six months Sarita joined them with an amount of Rs, 150000. Profit earned at the end of three years from when Veena started the business shoud be distributed in what ratio among Veena, Poonam and Sarita, respectively?
(a) 36 : 55 : 54 (b) 18 : 28 : 27
(c) 35 : 54 : 55 (d) Cannot be determined
(e) None of these
[Based on Bank of Maharashtra PO 2003]

EXPLANATORY ANSWERS

1. Ans. (c)

Solution When investment ratio is given, the amount of profit can be found out with the help of III only.

2. Ans. (b)

3. Ans. (a)

Solution Sum invested by A, B and C is

$5 \times 12 : 7 \times 12 : 6 \times 6 + 3 \times 6$

or, $60 : 84 : 54$

or, $10 : 14 : 9$

$\therefore$ Share of $C = \frac{9}{33} \times 33000$ = Rs. 9000.

4. Ans. (c)

Solution Ratio of their investment

$= 50000 \times 12 : 90000 \times 8 = 5 : 6$

$\therefore$ Amount received by Mr. Jain $= \frac{6}{11} \times 22000$

= Rs. 12000.

5. Ans. (a)

Solution Let the initial investments of Vinod and Ankit be $2x$ and $3x$, respectively.

According to question,

$$\frac{2x + 10000}{3x} = \frac{3}{2}$$

or, $4x + 20000 = 9x$

$\therefore$ $x = 4000$

$\therefore$ Amount invested by Vinod $= 2x$ = Rs. 8000.

6. Ans. (e)

Solution Ratio of Vinay and Aditya for one month

$= (50000 \times 12) + (80000 \times 24) : (70000 \times 24)$

$= 60000 + 1920000 : 1680000 = 3 : 2$

Hence, share of Aditya in the profit earned from the business $= \frac{87,500}{3+2} \times 2$ = Rs. 35000.

7. Ans. (d)

Solution Suppose, B joins A after K months.

$\therefore$ Investments of A and B are in the ratio of

$10000 \times 12 : 40000 \times (12 - K)$

$\therefore 10000 \times 12 = 40000 \times (12 - K)$

i.e. $4(12 - K) = 12 \Rightarrow K = 9$.

8. Ans. (b)

Solution A, B, C invest in the ratio

35000 : 45000 : 55000

i.e., 7 : 9 : 11

$\therefore$ A's share in the profit $= \frac{7}{27} \times 40500$

$= 7 \times 1500$ = Rs. 10500

B's share in the profit $= 9 \times 1500$ = Rs. 13500

C's share in the profit $= 11 \times 1500$ = Rs. 16500.

9. Ans. (d)

Solution They pay in the ratio $50 \times 4 : 40 \times 3 : 46 \times 5$

i.e. 200 : 120 : 230

i.e. 20 : 12 : 23

$\therefore$ 1st man should pay Rs. $\frac{20}{55} \times 660$ = Rs. 240.

10. Ans. (d)

Solution Suppose B joined after K months

$\therefore$ Profit is divided in the ratio of

$45000 \times 12 : 30000 \times K$

$\therefore \frac{45000 \times 12}{30000 \times K} = \frac{2}{1} \Rightarrow 60000K = 540000$

$\therefore$ $K = 9$.

11. Ans. (b)

Solution Profit is divided between A, B and C in the ratio 10 : 14 : 12

i.e. 5 : 7 : 6

$\therefore$ B's share $= \frac{7}{18} \times 5400$ = Rs. 2100.

12. Ans. (c)

Solution Let B's contribution be Rs. x

$\therefore \frac{3500 \times 12}{7x} = \frac{2}{3}$

or, $x = \frac{3500 \times 12 \times 3}{7 \times 2}$ $\therefore$ $x = 9000$.

13. Ans. (a)

Solution A, B and C pay in the ratio of

$10 \times 7 : 12 \times 5 : 15 \times 3$

i.e. 70 : 60 : 45

i.e. 14 : 12 : 9

$\therefore$ C pay Rs. $\frac{9}{35} \times 175$ = Rs. 45.

14. Ans. (b)

Solution Suppose, shares of A, B and C are $6K$, $4K$ and $3K$, respectively.

$\therefore$ Profit of 378 is divided among them in the ratio of

$6K \times 2 + 3K \times 10 : 4K \times 12 : 3K \times 12$

i.e. 42 : 48 : 36

i.e. 7 : 8 : 6

$\therefore$ B's share $= \frac{8}{21} \times 378$ = Rs. 144.

15. Ans. (d)

Solution A's capital $= \frac{1}{4}K$

B's capital $= \frac{3}{4}K$

A and B share the total profit in the ratio 1 : 2. Let B contribute for x months

$$\therefore \quad \frac{\frac{1}{4}K \times 15}{\frac{3}{4}K \times x} = \frac{1}{2} \Rightarrow x = 10.$$

16. Ans. (b)

Solution Profit earned by

$$C = 1 - \left(\frac{1}{3} + \frac{1}{4}\right) = 1 - \frac{7}{12} = \frac{5}{12}$$

So, $\frac{5}{12} = 5000 \quad \therefore \quad 1 \rightarrow 12000$

$\therefore$ Profit received by $A = \frac{1}{3} \times 12000$ = Rs. 4000.

17. Ans. (d)

Solution Suppose, B joined after K months

$\therefore$ $4500 \times 12 : 5400 \times (12 - K) = 2 : 1$

$$\Rightarrow \quad \frac{4500 \times 12}{5400 \times (12 - K)} = \frac{2}{1}$$

$$\Rightarrow \quad \frac{60}{6(12 - K)} = \frac{2}{1}$$

$\Rightarrow$ $12(12 - K) = 60 \Rightarrow K = 7.$

18. Ans. (c)

Solution Ratio of their investments

$= 70 \times 36 : 105 \times 30 : 140 \times 24$

$= 12 : 15 : 16.$

19. Ans. (a)

Solution Ratio of their profits

$= 75 \times 36 : 125 \times 33 : 150 \times 27$

$= 3 \times 36 : 5 \times 33 : 6 \times 27$

$= 3 \times 12 : 5 \times 11 : 6 \times 9$

$= 36 : 55 : 54.$

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. A began business with Rs. 6000 and was joined afterwards by B with Rs. 8000. At the end of the year B got one-third of the total profit. When did B join?
(a) After 3 months (b) After 4 months
(c) After 8 months (d) After 9 months

2. Dolly, Veena and Aruna started a shop by making investments in the ratio $\frac{5}{2} : \frac{4}{3} : \frac{7}{5}$. After 4 months Dolly increased her capital by 70%. If the total profit at the end of one year be Rs. 9600, Aruna's share in profit was
(a) Rs. 2100 (b) Rs. 4500
(c) Rs. 2000 (d) Rs. 3200

3. A and B are two business partners. They decided to distribute equally 60% of the profit and remaining 40% as the interest on their capital. If they invested Rs. 1250 and Rs. 850, respectively and A received Rs. 30 more than B, the total profit was
(a) Rs. 371.25 (b) Rs. 423.75
(c) Rs. 325.50 (d) Rs. 393.75

4. A, B and C are partners in a business and their shares are in the proportion $\frac{1}{3} : \frac{1}{4} : \frac{1}{5}$. A withdraws half his capital at the end of 4 months and after 8 months more a profit of Rs. 1694 is divided. A's share is
(a) Rs. 560 (b) Rs. 630
(d) Rs. 440 (d) Rs. 504

5. A, B and C enter partnership by making investments in the ratio 3 : 5 : 7. After a year, C invests another Rs. 337600 while A withdrew Rs. 45600. The ratio of investments then changes to 24 : 59 : 167. How much did A invest initially?
(a) Rs. 45600 (b) Rs. 96000
(c) Rs. 141600 (d) None of these

6. A and B enter into partnership, investing Rs. 12000 and Rs. 16000, respectively. After 8 months, C joins them with a capital of Rs. 15000. The share of C in a profit of Rs. 45600 after 2 years will be
(a) Rs. 21200 (b) Rs. 19200
(c) Rs. 14400 (d) Rs. 12000

7. *A* began business with Rs. 4200 and was joined afterwards by *B* with Rs. 7200. When did *B* join if the profits at the end of the year were divided equally?
(a) After 7 months (b) After 4 months
(c) After 5 months (d) After 8 months

8. *A* and *B* began business with Rs. 3000 and Rs. 4000. After 8 months, *A* withdraws Rs. 1000 and *B* advances Rs. 1000 more. At the end of the year, their profits, amounted to Rs. 630. Find the share of *A*
(a) Rs. 240 (b) Rs. 350
(c) Rs. 340 (d) Rs. 390

9. Rahul started a business with a capital of Rs. 8000. After six months, Sanjay joined him with investment of some capital. If at the end of the year each of them gets equal amount as profit, how much did Sanjay invest in the business?
(a) Rs. 18000 (b) Rs. 17500
(c) Rs. 16000 (d) Rs. 16500

10. Rakesh and Mukesh are partners in a business. They divide between them the profit incurred in the business in such a proportion that $\frac{2}{5}$th portion of the profit gained by Rakesh is equal to $\frac{1}{3}$rd portion of the profit gained by Mukesh. If total profit in the business is Rs. 1210, find the profit earned by Mukesh
(a) Rs. 780 (b) Rs. 550
(c) Rs. 680 (d) Rs. 590

Test I

[Time: 10 min]

11. *A* started a business with Rs. 3200. After sometime *B* joined him with Rs. 4800. If at the end of one year they got equal profit, after how many months did *B* join?
(a) 6 (b) 5
(c) 3 (d) 4
(e) None of these

12. Ramesh, Mahesh and Naresh jointly hired a vedio cassette. If they use it for 6 hours, 10 hours and 12 hours, respectively, what is the share of Naresh for rent?
(a) Rs. 75 (b) Rs. 125
(c) Rs. 35 (d) Rs. 150
(e) None of these

13. Two partners invest Rs. 12500 and Rs. 8500, respectively in a business. If one partner gets Rs. 300 more than the other in the profit, what is the total profit?
(a) Rs. 1475 (b) Rs. 1575
(c) Rs. 1675 (d) Rs. 1570
(e) Rs. 157

14. *A* and *B* enter into partnership investing Rs. 1200 and Rs. 1600 respectively. After 8 months *C* joins them with a capital of Rs. 1500. If in 2 years profit earned is Rs. 4560, *A*'s share will be
(a) Rs. 1440 (b) Rs. 1200
(c) Rs. 1920 (d) Rs. 1320

15. *A* puts in Rs. 600 more in a business than *B*, but *B* has invested his capital for 4 months. If *A* gets Rs. 48 more than *B* out of the total profit of Rs. 528, money invested by *B* was
(a) Rs. 1800 (b) Rs. 600
(c) Rs. 1200 (d) None of these

16. *A*, *B* and *C* have respectively invested Rs. 20, Rs. 18 and Rs. 12 (all in thousands) jointly in a business. *A* and *B* receive respectively, 12% and 8% of the annual profits for services, the remainder being divided among *A*, *B* and *C* in proportion to their capitals. If at the end of the year *A* receives altogether Rs. 648 more than *B*, *C*'s share is
(a) Rs. 3960 (b) Rs. 3312
(c) Rs. 1728 (d) Rs. 2400

17. *A*, *B* and *C* are three partners. They altogether invested Rs. 1400 in business. At the end of the year *A* got Rs. 337.50, *B* Rs. 1125.00 and *C* Rs. 637.50 as profit. The difference between investments of *B* and *A* was
(a) Rs. 4200 (b) Rs. 3200
(c) Rs. 2200 (d) Rs. 5250

18. If 2 (*A*'s capital) = 3 (*B*'s capital) = 5 (*C*'s captial), out of a total profit of Rs. 930, *C* gets
(a) Rs. 180 (b) Rs. 450
(c) Rs. 300 (d) Rs. 350

19. *A* and *B* rent a pasture for 10 months. A puts in 80 oxen for 7 months. How many can *B* put in for the remaining 3 months if he pays half as much again as *A*?
(a) 280 (b) 250
(c) 220 (d) 180

20. *A*, *B* and *C* purchase a form for Rs. 10000 of which *A* pays Rs. 4000. They sell it so as to gain a certain sum of which *B* gets Rs. 275 and *C* Rs. 175. *A*'s share is
(a) Rs. 275 (b) Rs. 300
(c) Rs. 325 (d) Rs. 250

Answers

1. (d)	**2.** (a)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (d)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (b)
11. (d)	**12.** (d)	**13.** (b)	**14.** (a)	**15.** (c)	**16.** (c)	**17.** (d)	**18.** (a)	**19.** (a)	**20.** (a)

9

SIMPLE INTEREST

INTRODUCTION

When a person A borrows some money from another person B, then A has to pay certain amount to B for the use of this money. This amount paid by A is called *Interest*. The total amount of money borrowed by A from B is called the *Principal*. The money paid back to B, which comprises of the principal and the interest is called the *Amount*.

In other words,

Amount = Principal + Interest

The interest is usually charged according to a specified term, which is expressed as some per cent of the principal and is called the *rate of interest* for the fixed period of time. This fixed period may be a year, six months, three months or a month and correspondingly the rate of interest is charged annually, semi-annually, quarterly or monthly. For example, the rate of interest is 5% per annum means the interest payable on Rs. 100 for one year is Rs. 5.

Interest can be of two types :

1. Simple Interest 2. Compound Interest

SIMPLE INTEREST

When the interest is payable on the principal only, it is called *simple interest*. For example, simple interest on Rs. 100 at 5% per annum will be Rs. 5 each year, that is, at the end of one year, total amount will be Rs. 105. At the end of second year, it will be Rs. 110 and so on.

Thus, simple interest is the interest computed on the principal for the entire period it is borrowed.

In this chapter, we shall limit ourselves to simple interest. Compound interest will be discussed in the next chapter.

SOME BASIC FORMULAE

If P stands for Principal, R the rate per cent per annum, T the number of years, I the simple interest and A the amount, then

1. Simple Interest $= \dfrac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$

or, $I = \dfrac{P \times R \times T}{100}$

Illustration 1 Find the simple interest on Rs. 5200 for 2 years at 6% per annum.

Solution Here, P = Rs. 5200, T = 2 years and R = 6%

$\therefore$ Simple interest $= \dfrac{P \times R \times T}{100} = \dfrac{5200 \times 6 \times 2}{100}$

= Rs. 624.

2. Principal $= \dfrac{100 \times \text{Simple Interest}}{\text{Rate} \times \text{Time}}$

or, $P = \dfrac{100 \times I}{R \times T}$

Illustration 2 A man earns Rs. 450 as an interest in 2 years on a certain sum invested with a company at the rate of 12 per cent per annum. Find the sum invested by the man in the company.

Solution We have, I = Rs. 450, T = 2 years, R = 12% per annum

$\therefore \quad P = \dfrac{I \times 100}{R \times T} = \dfrac{450 \times 100}{12 \times 2}$ = Rs. 1875.

Thus, the money invested by the man was Rs. 1875.

3. Rate $= \dfrac{100 \times \text{Simple Interest}}{\text{Principal} \times \text{Time}}$

or, $R = \dfrac{100 \times I}{P \times T}$

Illustration 3 At what rate per annum will a sum of Rs. 5000 amount to Rs. 6000 in 4 years?

Solution Here, P = Rs. 5000, A = Rs. 6000, T = 4 years

So, $I = A - P$ = Rs. (6000 – 5000) = Rs. 1000

$$\therefore \quad R = \frac{100 \times I}{P \times T} = \frac{100 \times 1000}{5000 \times 4} = 5\%.$$

4. $$\text{Time} = \frac{100 \times \text{Simple Interest}}{\text{Rate} \times \text{Principal}}$$

or, $$T = \frac{100 \times I}{R \times P}$$

Illustration 4 In what time will Rs. 1200 earn an interest of Rs. 240 at 5% per annum?

Solution Here, P = Rs. 1200, I = Rs. 240, R = 5%

$$\therefore \quad T = \frac{100 \times I}{P \times R} = \frac{100 \times 240}{1200 \times 5} = 4 \text{ years.}$$

5. Amount = Principal + Simple Interest

$$= \text{Principal} + \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$

$$= \text{Principal}\left(1 + \frac{\text{Rate} \times \text{Time}}{100}\right)$$

or, $$A = P\left(1 + \frac{R \times T}{100}\right)$$

Illustration 5 Mahesh borrowed Rs. 3000 from his friend Suresh at 15 per cent per annum for 3 years. Find the interest and money returned by Mahesh to Suresh.

Solution Here, P = Rs. 3000, R = 15% per annum, T = 3 years

$$\therefore \quad I = \frac{P \times R \times T}{100} = \frac{3000 \times 15 \times 3}{100} = \text{Rs. } 1350.$$

$\therefore$ $A = P + I$ = Rs. 3000 + Rs. 1350 = Rs. 4350.

Thus, Mahesh paid Rs. 1350 as interest to Suresh and the amount returned by Mahesh to Suresh = Rs. 4350.

SOME USEFUL SHORT-CUT METHODS

1. If a certain sum in T years at R% per annum amounts to Rs. A, then the sum will be

$$P = \frac{100 \times A}{100 + R \times T}$$

Explanation

Let the principal be Rs. x

$\therefore$ Simple interest = Rs. $(A - x)$

$$\therefore \quad A - x = \frac{x \times R \times T}{100}$$

$$\Rightarrow \quad 100A - 100x = xRT$$

$$\Rightarrow \quad (100 + RT)x = 100A$$

$$\therefore \quad x = \frac{100 \times A}{100 + R \times T}.$$

Illustration 6 What principal will amount to Rs 570 at 4% per annum in $3\frac{1}{2}$ years?

Solution We have, A = Rs. 570, R = 4% per annum, $T = \frac{7}{2}$ years.

$$\therefore \quad P = \frac{100 \times A}{100 + R \times T} = \frac{100 \times 570}{100 + 4 \times 7/2}$$

$$= \frac{100 \times 570}{114} = \text{Rs. } 500$$

Thus, Rs. 500 will amount to Rs. 570 at 4% per annum in $3\frac{1}{2}$ years.

2. The annual payment that will discharge a debt of Rs. A due in T years at R% per annum is

$$\text{Annual payment} = \text{Rs.} \left(\frac{100A}{100T + \frac{RT(T-1)}{2}}\right)$$

Explanation

Let the annual payment be Rs. x.

Since the first instalment is paid at the end of first year,

$\therefore$ Amount of first instalment at the end of t years

$$= x + \frac{(T-1) \times R \times x}{100}$$

Similarly, amount of second instalment at the end of t years

$$= x + \frac{(T-2) \times R \times x}{100}, \text{ and so on.}$$

Thus, total amount of T instalments

$$A = \left[x + \frac{(T-1) \times R \times x}{100}\right] + \left[x + \frac{(T-2) \times R \times x}{100}\right] + \ldots + x$$

$$= Tx + \frac{Rx}{100}[(T-1) + (T-2) + \ldots + 1]$$

$$= Tx + \frac{Rx}{100}\left[\frac{(T-1) \times T}{2}\right]$$

or, $$100\,Tx + Rx\left[\frac{T(T-1)}{2}\right] = 100A$$

or, $x\left[100T + \frac{RT(T-1)}{2}\right] = 100A$

$\therefore \quad x = \frac{100A}{100T + \frac{RT(T-1)}{2}}.$

Illustration 7 Find the annual instalment that will discharge a debt of Rs. 12900 due in 4 years at 5% per annum simple interest.

Solution Here, A = Rs. 12900, T = 4 years, R = 5% per annum

$$\therefore \text{Annual instalment} = \frac{100 \times A}{100 \times T + \frac{RT(T-1)}{2}}$$

$$= \frac{100 \times 12900}{(100 \times 4) + \frac{5(4-1) \times 4}{2}}$$

$$= \frac{100 \times 12900}{400 + 30} = \frac{100 \times 12900}{430}$$

$$= \text{Rs. } 3000.$$

3. If a certain sum is invested in n types of investments in such a manner that equal amount is obtained on each investment where interest rates are $R_1, R_2, R_3, \ldots, R_n$ respectively and time periods are $T_1, T_2, T_3, \ldots, T_n$ respectively, then the ratio in which the amounts are invested is

$$\frac{1}{100 + R_1T_1} : \frac{1}{100 + R_2T_2} : \frac{1}{100 + R_3T_3} : \ldots \frac{1}{100 + R_nT_n}.$$

Explanation

Let $P_1, P_2, \ldots, P_n$ be invested in n types of investments whose interest rates are $R_1, R_2, \ldots, R_n$ and time periods are $T_1, T_2, \ldots, T_n$.

Then, $P_1 = \frac{100 \times A}{100 + R_1T_1}$

$P_2 = \frac{100 \times A}{100 + R_2T_2}$

$\vdots \qquad \vdots$

$P_n = \frac{100 \times A}{100 + R_nT_n}.$

$$\therefore \quad P_1 : P_2 : \ldots : P_n$$

$$= \frac{100 \times A}{100 + R_1T_1} : \frac{100 \times A}{100 + R_2T_2} : \ldots \frac{100 \times A}{100 + R_nT_n}$$

$$= \frac{1}{100 + R_1T_1} : \frac{1}{100 + R_2T_2} : \ldots \frac{1}{100 + R_nT_n}$$

[$\because$ the amount A is same for all]

Illustration 8 A sum of Rs. 1586 is divided among three such parts that amount obtained on these three parts of money after 2, 3 and 4 years, respectively at the rate of 5% per annum remains equal. Find such three parts of the sum.

Solution Since the amount accured from each of the three parts of Rs. 1586 at the rate of 5% p.a. in 2, 3 and 4 years, respectively remains equal, such three parts of Rs. 1586 will be in the ratio of

$$\frac{1}{100 + R_1T_1} : \frac{1}{100 + R_2T_2} : \frac{1}{100 + R_3T_3}$$

Hence, the ratio

$$= \frac{1}{100 + 5 \times 2} : \frac{1}{100 + 5 \times 3} : \frac{1}{100 + 5 \times 4}$$

$$= \frac{1}{110} : \frac{1}{115} : \frac{1}{120}$$

$$= \frac{1 \times 30360}{110} : \frac{1 \times 30360}{115} : \frac{1 \times 30360}{120}$$

($\because$ L.C.M. of 110, 115 and 120 is 30360)

$\therefore$ ratio = 276 : 264 : 253

Sum of proportionals = 276 + 264 + 253 = 793

$\therefore$ 1st part $= \frac{276}{793} \times 1586$ = Rs. 552,

2nd part $= \frac{264}{793} \times 1586$ = Rs. 528

and, 3rd part $= \frac{253}{793} \times 1586$ = Rs. 506.

4. If a certain sum of money becomes n times itself in T years at simple interest, then the rate of interest per annum is

$$R = \frac{100(n-1)}{T}\%$$

Explanation

Let Rs. P become Rs. nP in t years

$\therefore$ Simple interest I is given by

$$I = nP - P = (n-1)P$$

$\therefore$ Rate of interest R is given by

$$R = \frac{100 \times I}{P \times T} = \frac{100 \times (n-1)P}{P \times T} = \frac{100(n-1)}{T}.$$

Illustration 9 A certain sum of money trebles itself in 5 years simple interest. Find the rate per cent per annum.

Solution Here, n = 3, T = 5 years

$$\therefore \quad R = \frac{100(n-1)}{T}\% = \frac{100(3-1)}{5}\% = 40\%.$$

5. If a certain sum of money becomes n times itself at R% per annum simple interest in T years, then

$$T = \left(\frac{n-1}{R}\right) \times 100 \text{ years.}$$

Illustration 10 In what time a sum of money will double itself at a rate of simple interest of 8% p.a.?

Solution Required time $(T) = \dfrac{(n-1)\times 100}{R}$ years

$= \dfrac{(2-1)\times 100}{8}$ years

$= 12\dfrac{1}{2}$ years.

6. If a certain sum of money becomes n times itself in T years at a simple interest, then the time T in which it will become m times itself is given by

$$T' = \left(\frac{m-1}{n-1}\right)\times T \text{ years.}$$

Explanation

Let the principal be Rs. P.

Let it become m times in T' years.

Then, the amount in T years = Rs. nP

and the amount in T' years = Rs. mP.

$\therefore \quad nP - P = \dfrac{P\times R\times T}{100}$

or, $\quad (n-1)P = \dfrac{P\times R\times T}{100}$ ...(1)

and, $\quad (m-1)P = \dfrac{P\times R\times T'}{100}$ ...(2)

$\therefore \quad \dfrac{(m-1)P}{(n-1)P} = \dfrac{P\times R\times T'}{100}\times\dfrac{100}{P\times R\times T}$

or, $\quad \dfrac{m-1}{n-1} = \dfrac{T'}{T}$

$\therefore \quad T' = \left(\dfrac{m-1}{n-1}\right)T$ years.

Illustration 11 A sum of money put out on simple interest doubles itself in $12\dfrac{1}{2}$ years. In how many years would it treble itself?

Solution Here, $n = 2$, $m = 3$, $T = \dfrac{25}{2}$ years.

$\therefore$ Required time $(T') = \left(\dfrac{m-1}{n-1}\right)\times T$ years

$= \left(\dfrac{3-1}{2-1}\right)\times\dfrac{25}{2}$ years

$= 25$ years.

7. Effect of change of P, R and T on simple interest is given by the following formula :

Change in Simple Interest

$= \dfrac{\text{Product of fixed parameter}}{100}$

$\times$ [difference of product of variable parameters]

For example, if rate (R) changes from R_1 to R_2 and P, T are fixed, then

Change in SI $= \dfrac{PT}{100}\times(R_1 - R_2)$

Similarly, if principal (P) changes from P_1 to P_2 and R, T are fixed, then change in SI $= \dfrac{RT}{100}\times(P_1 - P_2)$

Also, if rate (R) changes from R_1 to R_2 and time (T) changes from T_1 to T_2 but principal (P) is fixed, then change in SI $= \dfrac{P}{100}\times(R_1T_1 - R_2T_2)$.

Illustration 12 If simple interest on Rs. 600 increases by Rs. 30, when the rate % increases by 4% per annum, find the time.

Solution Here, $P = 600$, change in SI = 30, $R_1 - R_2 = 4$, $T = ?$

Using, change in SI $= \dfrac{PT}{100}\times(R_1 - R_2)$

we have, $30 = \dfrac{600T}{100}\times 4 \Rightarrow T = \dfrac{5}{4}$, i.e. $1\dfrac{1}{4}$ years.

Illustration 13 If the simple interest on Rs. 1400 be more than the interest on Rs. 1000 by Rs. 60 in 5 years, find the rate per cent per annum.

Solution Here, change in SI = 60, $P_1 - P_2 = 400$, $T = 5$, $R = ?$

Using change in SI $= \dfrac{RT}{100}\times(P_1 - P_2)$

We have, $60 = \dfrac{5R}{100}\times 400 \Rightarrow R = 3\%$.

Illustration 14 If the simple interest on a certain sum at 4% per annum for 4 years is Rs. 80 more than the interest on the same sum for 3 years at 5% per annum, find the sum.

Solution Here, change in SI = 80, $R_1 = 4$, $R_2 = 5$, $T_1 = 4$, $T_2 = 3$, $P = ?$

Using change in SI $= \dfrac{P}{100}\times(R_1T_1 - R_2T_2)$

We have, $80 = \dfrac{P}{100}\times(4\times 4 - 5\times 3)$

$\Rightarrow \quad P =$ Rs. 8000.

8. If a debt of Rs. Z is paid in 'n' number of instalments and if the value of each instalment is Rs. a, then the borrowed (debt) amount is given by

$$Z = na + \frac{Ra}{100\times b}\times\frac{n(n-1)}{2}$$

where, R = rate of interest per annum

b = no. of instalments/year

$b = 1$, when each instalment is paid yearly

$b = 2$, when each instalment is paid half-yearly

$b = 4$, when each instalment is paid quarterly

$b = 12$, when each instalment is paid monthly.

Illustration 15 A sum of Rs. 2 is lent to be paid back in 3 equal monthly instalments of Re. 1 each. Find the rate per cent.

Solution Here, Z = Rs. 2, a = Re. 1, $n = 3$, $b = 12$, $R = ?$

Using the formula

$$Z = na + \frac{Ra}{100 \times b} \times \frac{n(n-1)}{2},$$

we have, $2 = 3 \times 1 + \frac{R \times 1}{100 \times 12} \times \frac{3 \times 2}{2} \Rightarrow R = 400\%$

$\therefore$ The rate % p.a. is 400%.

9. If a certain sum of money P lent out at SI amounts to A_1 in T_1 years and to A_2 in T_2 years, then

$$P = \frac{A_1T_2 - A_2T_1}{T_2 - T_1}$$

and, $$R = \frac{A_1 - A_2}{A_1T_2 - A_2T_1} \times 100\%$$

Illustration 16 If a certain sum of money at simple interest amounts to Rs. 5184 in 2 years and to Rs. 5832 in 3 years, what is the sum and the rate of interest?

Solution Principal $= \frac{A_1T_2 - A_2T_1}{T_2 - T_1}$

$$\left[\begin{array}{l}\text{Here, } A_1 = 5184, A_2 = 5832 \\ T_1 = 2, T_2 = 3\end{array}\right]$$

$$= \frac{5184 \times 3 - 5832 \times 2}{3 - 2} = \text{Rs. } 3888$$

and, Rate $= \frac{(A_2 - A_1) \times 100}{T_1A_2 - T_2A_1} = \frac{(5832 - 5184) \times 100}{2 \times 5832 - 3 \times 5184}$

$$= \frac{64800}{3888} = 16\frac{2}{3}\%.$$

10. If a certain sum of money P lent out for a certain time T amounts to A_1 at R_1% per annum and to A_2 at R_2% per annum, then

$$P = \frac{A_2R_1 - A_1R_2}{R_1 - R_2}$$

and, $$T = \frac{A_1 - A_2}{A_2R_1 - A_1R_2} \times 100 \text{ years.}$$

Illustration 17 A certain sum is invested for certain time. It amounts to Rs. 450 at 7% per annum. But when invested at 5% per annum, it amounts to Rs. 350. Find the sum and time.

Solution Here, $A_1 = 450$, $R_1 = 7$, $A_2 = 350$, $R_2 = 5$.

Using the formula,

$$P = \frac{A_2R_1 - A_1R_2}{R_1 - R_2}$$

We get, $$P = \frac{350 \times 7 - 450 \times 5}{7 - 5} = \text{Rs. } 100$$

Also, using the formula,

$$T = \left(\frac{A_1 - A_2}{A_2R_1 - A_1R_2}\right) \times 100$$

we get, $$T = \left(\frac{450 - 350}{350 \times 7 - 450 \times 5}\right) \times 100 = 5 \text{ years.}$$

11. If an amount P_1 lent at simple interest rate of R_1% per annum and another amount P_2 at simple interest rate of R_2% per annum, then the rate of interest for the whole sum is

$$R = \left(\frac{P_1R_1 + P_2R_2}{P_1 + P_2}\right).$$

Illustration 18 Mohan deposits Rs. 5000 in NSC at 2% per annum and Rs. 2000 in mutual funds at 4% per annum. Find the rate of interest for the whole sum.

Solution Here, $P_1 = 5000$, $R_1 = 2$, $P_2 = 2000$, $R_2 = 4$.

Using the formula

$$R = \left(\frac{P_1R_1 + P_2R_2}{P_1 + P_2}\right)$$

We get, $$R = \frac{5000 \times 2 + 2000 \times 4}{5000 + 2000} = 2\frac{4}{7}\%.$$

12. If a certain sum of money is lent out in n parts in such a manner that equal sum of money is obtained as simple interest on each part where interest rates are $R_1, R_2, \ldots, R_n$ respectively and time periods are $T_1, T_2, \ldots, T_n$ respectively, then the ratio in which the sum will be divided in n parts is given by

$$\frac{1}{R_1T_1} : \frac{1}{R_2T_2} : \ldots \frac{1}{R_nT_n}.$$

Explanation

Let the n equal parts be $P_1, P_2, \ldots, P_n$ and let I be the equal interest earned on each part.

Then, $$P_1 = \frac{I \times 100}{R_1T_1}$$

$$P_2 = \frac{I \times 100}{R_2T_2}$$

$$\vdots \qquad \vdots$$

$$P_n = \frac{I \times 100}{R_nT_n}.$$

$$\therefore P_1 : P_2 : \ldots : P_n = \frac{I \times 100}{R_1T_1} : \frac{I \times 100}{R_2T_2} : \ldots \frac{I \times 100}{R_nT_n}$$

$$= \frac{1}{R_1T_1} : \frac{1}{R_2T_2} : \ldots \frac{1}{R_nT_n}.$$

Illustration 19 If a sum of Rs. 1600 is divided into two such parts that the simple interest on the first part for $2\frac{1}{2}$ years at the rate of 4% p.a. equals the simple interest on the second part for 5 years at the rate of 3% p.a., then find two such divisions of the sum.

Solution Ratio of one part to other part of Rs. 1600

$$= \frac{1}{R_1T_1} : \frac{1}{R_2T_2}$$

$\therefore$ 1st part : 2nd part $= \dfrac{1}{4 \times 5/2} : \dfrac{1}{3 \times 5}$

[Here, R_1 = 4% p.a., $T_1 = \frac{5}{2}$ years, R_2 = 3% p.a., T_2 = 5 years]

or, 1st part : 2nd part $= \dfrac{1}{10} : \dfrac{1}{15} = 3 : 2$

Sum of proportionals = 3 + 2 = 5

$\therefore$ 1st part $= \dfrac{3}{5} \times 1600$ = Rs. 96

and, 2nd part $= \dfrac{2}{5} \times 1600$ = Rs. 640.

13. When there is a change in principal (P), Rate (R) and Time (T), then the value of simple interest I also changes and is given by

$$\frac{I_1}{I_2} = \frac{P_1 \times R_1 \times T_1}{P_2 \times R_2 \times T_2}$$

$$\Rightarrow \quad \frac{A_1 - P_1}{A_2 - P_2} = \frac{P_1 \times R_1 \times T_1}{P_2 \times R_2 \times T_2}$$

as $I_1 = A_1 - P_1$ and $I_2 = A_2 - P_2$.

Illustration 20 If Rs. 85 amounts to Rs. 95 in 3 years, what Rs. 102 will amount to in 5 years at the same rate per cent?

Solution Here, P_1 = Rs. 85, A_1 = Rs. 95, T_1 = 3 years, P_2 = Rs. 102, T_2 = 5 years, $R_1 = R_2 = R$ (say).

Then, using the formula

$$\frac{A_1 - P_1}{A_2 - P_2} = \frac{P_1 \times R_1 \times T_1}{P_2 \times R_2 \times T_2}$$

We have, $\dfrac{95 - 85}{A_2 - 102} = \dfrac{85 \times R \times 3}{102 \times R \times 5}$

$\Rightarrow \quad A_2 - 102 = 20$

$\Rightarrow \quad A_2 = 122$

$\therefore$ The amount is Rs. 122.

14. Out of a certain sum P, $\frac{1}{a}$ part is invested at R_1%, $\frac{1}{b}$ part at R_2% and the remainder $\left(1 - \frac{1}{a} - \frac{1}{b}\right)$ say $\frac{1}{c}$ part at R_3%. If the annual income from all these investments is Rs. A, then the original sum is given by

$$P = \left(\frac{A \times 100}{\frac{R_1}{a} + \frac{R_2}{b} + \frac{R_3}{c}} \right).$$

Illustration 21 Out of a certain sum, $\frac{1}{3}$rd is invested at 3%, $\frac{1}{6}$th at 6% and the rest at 8%. If the annual income is Rs. 300, then the original sum is

Solution Here, $\frac{1}{a} = \frac{1}{3}$, $\frac{1}{b} = \frac{1}{6}$,

$$\frac{1}{c} = 1 - \left(\frac{1}{3} + \frac{1}{6}\right) = \frac{1}{2},$$

R_1 = 3%, R_2 = 6%, R_3 = 8%, A = Rs. 300.

$\therefore$ The original sum $= \dfrac{A \times 100}{\frac{R_1}{a} + \frac{R_2}{b} + \frac{R_3}{c}}$

$$= \frac{300 \times 100}{\frac{3}{3} + \frac{6}{6} + \frac{8}{2}} = \frac{30000}{6}$$

= Rs. 5000.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. The simple interest on Rs. 500 at 6% per annum from May 3rd to July 15th in the same year is
(a) Rs. 9 (b) Rs. 6
(c) Rs. 4 (d) None of these

2. Mr. Irani borrowed a sum of Rs. 10000 from a finance company for 6 years at 8% per annum. The amount returned by Mr. Irani to the finance company is

(a) Rs. 14800 (b) Rs. 12600
(c) Rs. 13300 (d) None of these

3. The principal that will yield Rs. 60 as simple interest at 6% per annum in 5 years is
(a) Rs. 175 (b) Rs. 350
(c) Rs. 200 (d) None of these

4. The sum of money that will produce Rs. 1770 interest in $7\frac{1}{2}$ years at 8% simple interest per annum is
(a) Rs. 2950 (b) Rs. 3120
(c) Rs. 2800 (d) None of these

5. If the simple interest on a certain sum of money after $6\frac{1}{4}$ years is 3/8 of the principal, then the rate of interest per annum is
(a) 5% (b) 6%
(c) 4% (d) None of these

6. Rakesh borrowed Rs. 5000 from Ganesh at simple interest. If Ganesh got Rs. 500 more than his capital after 5 years, then the rate of interest per annum is
(a) 2% (b) 3%
(c) 4% (d) None of these

7. The rate per cent per annum at which Rs. 1200 amount to Rs. 1440 in 4 years, is
(a) 5% (b) 4%
(c) 6% (d) None of these

8. If simple interest on a certain sum of money is Rs. 256 and the rate of interest per annum equals the number of years, then the rate of interest is
(a) 13% (b) 14%
(c) 16% (d) None of these

9. If the simple interest on a certain sum of money for 2 years is one-fifth of the sum, then the rate of interest per annum is
(a) 9% (b) 10%
(c) 8% (d) None of these

10. If the simple interest on a certain sum of money is 4/25th of the sum and the rate per cent equals the number of years, then the rate of interest per annum is
(a) 2% (b) 3%
(c) 4% (d) None of these

11. If a certain sum of money borrowed at 5% per annum simple interest amounts to Rs. 1020 in 4 years, then the sum of money borrowed is
(a) Rs. 850 (b) Rs. 925
(c) Rs. 750 (d) None of these

12. In what time will Rs. 1200 amount to Rs. 1344 at 6% per annum?
(a) $2\frac{1}{2}$ years (b) 3 years
(c) 2 years (d) None of these

13. In what time will Rs. 8100 produce the same income at 3% as Rs. 225 in 4 years at 3%.
(a) $\frac{1}{7}$ years (b) $\frac{1}{9}$ years
(c) $\frac{1}{6}$ years (d) None of these

14. If Rs. 1000 be invested at interest rate of 5% and the interest be added to the principal every 10 years, then the number of years in which it will amount to Rs. 2000 is
(a) $16\frac{2}{3}$years (b) $16\frac{1}{4}$ years
(c) 16 years (d) None of these

15. If Rs. 500 amounts to Rs. 725 at 9% simple interest in some time, what will Rs. 600 amount to at 11% in the same time?
(a) Rs. 870 (b) Rs. 930
(c) Rs. 910 (d) None of these

16. Sumit lends Rs. 10000 for 2 years at 20% per annum simple interest. After 1 year he gets Rs. 6000. How much will he get next year?
(a) Rs. 5900 (b) Rs. 6400
(c) Rs. 7200 (d) None of these

17. What principal will amount to Rs. 15000 at 10% per annum in 5 years?
(a) Rs. 10000 (b) Rs. 8700
(c) Rs. 10500 (d) None of these

18. The annual payment that will discharge a debt of Rs. 47250 due 3 years hence at the rate of 5% simple interest is
(a) Rs. 8000 (b) Rs. 10000
(c) Rs. 15000 (d) None of these

19. The annual instalment that will discharge a debt of Rs. 4200 due in 5 years at 10% simple interest is
(a) Rs. 700 (b) Rs. 750
(c) Rs. 800 (d) None of these

20. If the amount obtained by Mahesh by investing Rs. 1500 for $2\frac{1}{2}$ years at the rate of 8% p.a. is equal to the amount obtained by Suresh by investing a certain sum for 2 years at 5% p.a. simple interest, then the sum invested by Suresh is
(a) Rs. $1636\frac{4}{11}$ (b) Rs. 1636
(c) Rs. $1636\frac{1}{2}$ (d) None of these

21. A man invests Rs. 3965 in the names of his three daughters Neeta, Sita and Gita in such a way that they get the same amount after 2, 3 and 4 years, respectively. If the rate of interest is 5% p.a., then the amount invested for Neeta, Sita and Gita is
(a) Rs. 1380, Rs. 1320, Rs. 1265 (b) Rs. 1330, Rs. 1360, Rs. 1380

(c) Rs. 1265, Rs. 1320, Rs. 1340 (d) None of these

22. A sum of money at simple interest becomes four times in 24 years. The rate per cent of interest per annum is

(a) $13\frac{3}{4}\%$ (b) $12\frac{1}{2}\%$
(c) $11\frac{3}{4}\%$ (d) None of these

23. In how many years will a sum of money treble itself at 10% per annum simple interest?
(a) 15 years (b) 19 years
(c) 20 years (d) None of these

24. A sum of money doubles itself in 8 years. In how many years will it treble?
(a) 16 years (b) 15 years
(c) 14 years (d) None of these

25. A sum was put at simple interest at a certain rate for 4 years. Had it been put at 2% higher rate, it would have fetched Rs. 56 more. Find the sum.
(a) Rs. 680 (b) Rs. 700
(c) Rs. 720 (d) None of these

26. If the interest on Rs. 800 be more than the interest on Rs. 400 by Rs. 40 in 2 years, then the rate of interest per annum is

(a) 5% (b) $5\frac{1}{2}\%$
(c) 6% (d) None of these

27. If the difference between the simple interest on a certain sum for 4 years at $2\frac{1}{2}\%$ per annum and the simple interest on the same sum for the same period at 3% per annum is Rs. 60, then the sum is
(a) Rs. 3000 (b) Rs. 2900
(c) Rs. 3100 (d) None of these

28. If a certain sum of money at simple interest amounts to Rs. 2800 in 2 years and to Rs. 3250 in 5 years, then the rate of interest per annum is
(a) 4% (b) 6%
(c) 5% (d) None of these

29. If a certain sum of money amounts to Rs. 1760 in two years and Rs. 2000 in 5 years at simple interest, then the sum is
(a) Rs. 1960 (b) Rs. 1590
(c) Rs. 1600 (d) None of these

30. A certain sum is invested for certain time. It amounts to Rs. 450 at 7% per annum. But when invested at 5% per annum, it amounts to Rs. 350. Find the sum.
(a) Rs. 60 (b) Rs. 100
(c) Rs. 120 (d) None of these

31. A certain sum is invested for T years. It amounts to Rs. 400 at 10% per annum. But when invested at 4% per annum, it amounts to Rs. 200. Find the time (T).
(a) 41 years (b) 39 years
(c) 50 years (d) None of these

32. If a sum of Rs. 9 is lent to be paid back in 10 equal monthly instalments of Re. 1 each, then the rate of interest is

(a) $266\frac{2}{3}\%$ (b) $265\frac{3}{4}\%$
(c) 266% (d) None of these

33. A sum of Rs. 7700 is to be divided among three brothers Vikas, Vijay and Viraj in such a way that simple interest on each part at 5% per annum after 1, 2 and 3 years, respectively remains equal. The share of Vikas is more than that of Viraj by
(a) Rs. 2800 (b) Rs. 2500
(c) Rs. 3000 (d) None of these

34. If simple interest on a certain sum of money for 4 years at 5% p.a. is same as the simple interest on Rs. 560 for 10 years at the rate of 4% p.a., then the sum of money is
(a) Rs. 1190 (b) Rs. 1120
(c) Rs. 1210 (d) None of these

35. Mr. Mani invested an amount of Rs. 12000 at the simple interest rate of 10% per annum and another amount at the simple interest rate of 20% per annum. The total interest earned at the end of one year on the total amount invested became 14% per annum. Find the total amount invested.
(a) Rs. 20000 (b) Rs. 20800
(c) Rs. 21000 (d) None of these

36. Mr. Gupta deposits Rs. 3000 in a bank at 10% per annum and Rs. 5000 in another bank at 8% per annum. The rate of interest for the whole sum is

(a) $8\frac{1}{2}\%$ (b) $8\frac{3}{4}\%$
(c) 8% (d) None of these

37. A person invested $\frac{2}{3}$rd of his capital at 3%, $\frac{1}{6}$th at 6% and the remainder at 12%. If his annual income is Rs. 25, then the capital is
(a) Rs. 490 (b) Rs. 510
(c) Rs. 500 (d) None of these

38. The simple interest on a sum of money will be Rs. 600 after 10 years. If the principal is trebled after 5 years, what will be the total interest at the end of the tenth year?
(a) Rs. 1200 (b) Rs. 1190
(c) Rs. 1210 (d) None of these

39. Rs. 1500 is invested at a rate of 10% simple interest and interest is added to the principal after every 5 years. In how many years will it amount to Rs. 2500.

(a) $6\frac{1}{9}$ years (b) $6\frac{1}{4}$ years
(c) 7 years (d) None of these

40. Sumit lent some money to Mohit at 5% per annum simple interest. Mohit lent the entire amount to Birju on the same day at $8\frac{1}{2}$% per annum. In this transaction after a year Mohit earned a profit of Rs. 350. Find the sum of money lent by Sumit to Mohit.
(a) Rs. 9000 (b) Rs. 10000
(c) Rs. 10200 (d) None of these

41. Brinda borrowed Rs. 1000 to build a hut. She pays 5% simple interest. She lets the hut to Ramu and receives the rent of Rs. $12\frac{1}{2}$ per month from Ramu. In how many years Brinda ought to clear off the debt?
(a) 10 years (b) $10\frac{1}{4}$ years
(c) $10\frac{1}{2}$ years (d) None of these

42. The rate of interest on a sum of money is 4% per annum for the first 2 years, 6% per annum for the next 4 years and 8% per annum for the period beyond 6 years. If the simple interest accrued by the sum for a total period of 9 years is Rs. 1120, then the sum is
(a) Rs. 2400 (b) Rs. 2200
(c) Rs. 2000 (d) None of these.

EXPLANATORY ANSWERS

1. Ans. (b)

Solution Time from May 3rd to July 15th
= 28 days of May + 30 days of June and 15 days of July
= 73 days = $\frac{73}{365}$ years, i.e. $\frac{1}{5}$ years.

$$\therefore \quad I = \frac{P \times R \times T}{100} = \frac{500 \times 6 \times \frac{1}{5}}{100} = \text{Rs. } 6.$$

2. Ans. (a)

Solution We have, P = Rs. 10000, R = 8% per annum, T = 6 years.

$$\therefore \quad I = \frac{P \times R \times T}{100} = \frac{10000 \times 8 \times 6}{100} = \text{Rs. } 4800$$

$\therefore$ $A = P + I$ = 10000 + 4800 = Rs. 14800

Thus, Mr. Irani returned Rs. 14800 to the finance company.

3. Ans. (c)

Solution Here, I = Rs. 60, R = 6% per annum, T = 5 years.

$$\therefore \quad \text{Principal } (P) = \frac{100 \times I}{R \times T} = \frac{100 \times 60}{6 \times 5} = \text{Rs. } 200.$$

4. Ans. (a)

Solution Here, I = Rs. 1770, R = 8% per annum, $T = \frac{15}{2}$ years.

$$\therefore \quad \text{Principal } (P) = \frac{100 \times I}{R \times T} = \frac{100 \times 1770}{8 \times \frac{15}{2}}$$

= Rs. 2950.

5. Ans. (b)

Solution Let the sum of money be Rs. x.

Then , simple interest = $\frac{3}{8}x$

Also, time = $6\frac{1}{4}$ years, i.e. $\frac{25}{4}$ years

$$\therefore \quad \text{Rate (R)} = \frac{100 \times I}{P \times T} = \frac{100 \times \frac{3x}{8}}{x \times \frac{25}{4}}$$

$$= \frac{100 \times 3}{2 \times 25} = 6\%.$$

6. Ans. (a)

Solution Here, P = Rs. 5000, I = Rs. 500, T = 5 years.
Therefore, using the formula

$$R = \frac{100 \times I}{P \times T}$$

We have, rate of interest $(R) = \frac{100 \times 500}{5000 \times 5}$ = 2% p.a.

7. Ans. (a)

Solution We have, P = Rs. 1200, T = 4 years,
I = 1440 –1200 = Rs. 240.

$$\therefore \text{Rate } (R) = \frac{100 \times I}{P \times T} = \frac{100 \times 240}{1200 \times 4} = 5\% \text{ per annum.}$$

8. Ans. (c)

Solution Here, I = Rs. 256.
Let the principal be Rs. 100.
Let the rate of interest per annum be x%.
Then, time $(T) = x$ years.
Therefore, using the formula

$$R = \frac{100 \times I}{P \times T}$$

We have, $x = \frac{100 \times 256}{100 \times x} \Rightarrow x^2 = 256$ or $x = 16\%$.

$\therefore$ Rate of interest per annum is 16%.

9. Ans. (b)

Solution We have, $T = 2$ years.

Let the principal be Rs. x

Then, simple interest (I) = Rs. $x/5$.

$$\text{Rate of interest } (R) = \frac{100 \times I}{P \times T} = \frac{100 \times \frac{x}{5}}{x \times 2}$$

$$= \frac{100}{5 \times 2} = 10\% \text{ p.a.}$$

10. Ans. (c)

Solution Let the principal be Rs. x, then the simple interest $(I) = \frac{4}{25}x$.

Let the rate of interest p.a. be r% then time $(T) = r$ years.

$$\therefore \quad R = \frac{100 \times I}{P \times T} \Rightarrow r = \frac{100 \times \frac{4}{25}x}{x \times r}$$

$$\Rightarrow r^2 = \frac{400}{25} \text{ or } r = \frac{20}{5} = 4\%.$$

11. Ans. (a)

Solution We have, A = Rs. 1020, $T = 4$ years, $R = 5\%$ p.a.

Let the principal be Rs. x.

Then, Interest $(I) = A - P = 1020 - x$.

Therefore, by using formula,

$$P = \frac{100 \times I}{R \times T}$$

We have, $x = \frac{100 \times (1020 - x)}{5 \times 4}$

$\Rightarrow \quad x = 5100 - 5x$ or, $6x = 5100$

or, $x = \frac{5100}{6}$ = Rs. 850.

$\therefore$ The sum of money borrowed = Rs. 850.

12. Ans. (c)

Solution Here, P = Rs. 1200, A = Rs. 1344, $R = 6\%$ p.a.

$\therefore$ Interest (I) = 1344 –1200 = 144

$$\therefore \quad \text{Time } (T) = \frac{100 \times I}{P \times R} = \frac{100 \times 144}{1200 \times 6} = 2 \text{ years.}$$

13. Ans. (b)

Solution Income on Rs. 225 in 4 years at 3%

$$= \frac{P_1 \times R \times T_1}{100} = \frac{225 \times 3 \times 4}{100} = \text{Rs. } 27.$$

Now, interest of Rs. 27 is earned on Rs. 8100 at 3% simple interest.

$$\therefore \quad \text{Time } (T_2) = \frac{100 \times \text{I}}{P_2 \times R} = \frac{100 \times 27}{8100 \times 3} = \frac{1}{9} \text{ year.}$$

14. Ans. (a)

Solution The interest earned in 10 years on Rs. 1000 at 5% per annum

$$= \frac{1000 \times 5 \times 10}{100} = \text{Rs. } 500.$$

The principal now becomes = Rs. 1000 + Rs. 500 = Rs. 1500. We now find the time in which Rs. 1500 becomes Rs. 2000 at 5% p.a.

P = Rs. 1500, A = Rs. 2000,

$I = A - P = 2000 - 1500$ = Rs. 500, $R = 5\%$ p.a.

$$\therefore \quad \text{Time } (T) = \frac{100 \times I}{R \times P} = \frac{100 \times 500}{5 \times 1500} = 6\frac{2}{3} \text{ years.}$$

$$\therefore \quad \text{Total time} = \left(10 + 6\frac{2}{3}\right) \text{ years} = 16\frac{2}{3} \text{ years.}$$

15. Ans. (b)

Solution Interest on Rs. 500 is = 725 – 500 = Rs. 225.

$$\text{Time} = \frac{225 \times 100}{500 \times 9} = 5 \text{ years.}$$

$$\therefore \quad \text{Required amount } (A) = P\left(1 + \frac{R \times T}{100}\right)$$

$$= 600\left(1 + \frac{11 \times 5}{100}\right)$$

$$= \text{Rs. } 930.$$

16. Ans. (c)

Solution Amount after 1 year $= P\left(1 + \frac{R \times T}{100}\right)$

$$= 10000\left(1 + \frac{20 \times 1}{100}\right)$$

$$= \text{Rs. } 12000.$$

After paying Rs. 6000, the remaining sum = Rs. 6000.

$\therefore$ Amount obtained in the next year

$$= P\left(1 + \frac{R \times T}{100}\right)$$

$$= 6000\left(1 + \frac{20 \times 1}{100}\right) = \text{Rs. } 7200.$$

17. Ans. (a)

Solution We have, A = Rs. 15000, $R = 10\%$ p.a., $T = 5$ years.

$$\therefore \quad P = \frac{100 \times A}{100 + R \times T} = \frac{100 \times 15000}{100 + 10 \times 5} = \text{Rs. } 10000.$$

18. Ans. (c)

Solution We have, A = Rs. 47250, T = 3 years, R = 5% p.a.

$$\therefore \quad \text{Annual payment} = \frac{100 \times A}{100 \times T + \frac{RT(T-1)}{2}}$$

$$= \frac{100 \times 47250}{100 \times 3 + \frac{5 \times 3 \times 2}{2}}$$

= Rs. 15000.

19. Ans. (a)

Solution Here, A = Rs. 4200, T = 5 years. R = 10% p.a.

$$\therefore \text{ Annual instalment} = \frac{100 \times A}{100 \times T + \frac{RT(T-1)}{2}}$$

$$= \frac{100 \times 4200}{100 \times 5 + \frac{10 \times 5 \times 4}{2}}$$

= Rs. 700.

20. Ans. (a)

Solution Let the sum of money invested by Suresh be Rs. x.

Since the amount obtained in both the cases is equal, the ratio in which the sums are invested is

$$\frac{1}{100 + R_1T_1} : \frac{1}{100 + R_2T_2}$$

where R_1 = 8%, $T_1 = \frac{5}{2}$ years, R_2 = 5%, T_2 = 2 years.

That is, $\frac{1}{100 + 8 \times \frac{5}{2}} : \frac{1}{100 + 5 \times 2}$ or, $\frac{1}{120} : \frac{1}{110}$

Given : 1500 : x : : $\frac{1}{120} : \frac{1}{110}$

$$\Rightarrow \quad \frac{1500 \times 1}{110} = \frac{1}{120} \times x$$

or, $$x = \frac{1500 \times 1 \times 120}{110} = 1636\frac{4}{11}.$$

$\therefore$ The sum invested by Suresh is Rs. $1636\frac{4}{11}$.

21. Ans. (a)

Solution We have, T_1 = 2 years, T_2 = 3 years, T_3 = 4 years.

$R_1 = R_2 = R_3$ = 5% p.a.

$\therefore$ The ratio in which the amount is invested

$$= \frac{1}{100 + R_1T_1} : \frac{1}{100 + R_2T_2} : \frac{1}{100 + R_3T_3}$$

i.e. $$\frac{1}{100 + 2 \times 5} : \frac{1}{100 + 3 \times 5} : \frac{1}{100 + 4 \times 5}$$

i.e. $\frac{1}{110} : \frac{1}{115} : \frac{1}{120}$ or, 276 : 264 : 253.

Their sum = 276 + 264 + 253 = 793

$\therefore$ The amount invested for

$$\text{Neeta} = \frac{3965}{793} \times 276 = \text{Rs. } 1380$$

$$\text{Sita} = \frac{3965}{793} \times 264 = \text{Rs. } 1320$$

$$\text{Gita} = \frac{3965}{793} \times 253 = \text{Rs. } 1265.$$

22. Ans. (b)

Solution We have, n = 4 and T = 24 years.

$$\therefore \quad \text{Rate of interest} = \frac{100(n-1)}{T} = \frac{100(4-1)}{24}$$

$$= 12\frac{1}{2}\%.$$

23. Ans. (c)

Solution Here, n = 3, R = 10% p.a.

$$\therefore \quad \text{Required time} = \left(\frac{n-1}{R}\right) \times 100 = \left(\frac{3-1}{10}\right) \times 100$$

= 20 years.

24. Ans. (a)

Solution We have, n = 2, T = 8 years, m = 3.

$$\therefore \quad \text{Required Time } (T') = \left(\frac{m-1}{n-1}\right) \times T$$

$$= \left(\frac{3-1}{2-1}\right) \times 8 = 16 \text{ years.}$$

25. Ans. (b)

Solution Here, change in SI = Rs. 56, $R_1 - R_2$ = 2, T = 4 years, P = ?

Therefore, using the formula

$$\text{Change in } SI = \frac{PT}{100} \times (R_1 - R_2)$$

We get $56 = \frac{P \times 4}{100} \times 2 \Rightarrow P$ = Rs. 700.

$\therefore$ The sum is Rs. 700.

26. Ans. (a)

Solution Here, change in SI = Rs. 40,

$P_1 - P_2$ = 800 – 400 = Rs. 400, T = 2 years.

Using, change in $SI = \frac{RT}{100} \times (P_1 - P_2)$

We have, $40 = \frac{R \times 2}{100} \times 400 \Rightarrow R$ = 5%.

27. Ans. (a)

Solution Here, change in SI = Rs. 60,

$R_1 - R_2 = 3 - \frac{5}{2} = \frac{1}{2}$, T = 4 years, P = ?

Using, change in $SI = \frac{PT}{100} \times (R_1 - R_2)$

We have, $60 = \frac{P \times 4}{100} \times \frac{1}{2} \Rightarrow P$ = Rs. 3000.

28. Ans. (b)

Solution We have, A_1 = Rs. 2800, A_2 = Rs. 3250, T_1 = 2 years, T_2 = 5 years

$\therefore$ Rate of interest per annum (R)

$$= \frac{A_1 - A_2}{A_1T_2 - A_2T_1} \times 100\%$$

$$= \frac{2800 - 3250}{2800 \times 5 - 3250 \times 2} \times 100\%$$

$$= \frac{450 \times 100}{14000 - 6500}\% = \frac{45000}{7500}\% = 6\%.$$

29. Ans. (c)

Solution We have, A_1 = Rs. 1760, A_2 = 2000, T_1 = 2 years, T_2 = 5 years.

$\therefore$ Principal (P) $= \frac{A_1T_2 - A_2T_1}{T_2 - T_1}$

$$= \frac{1760 \times 5 - 2000 \times 2}{5 - 2} = \frac{4800}{3}$$

= Rs. 1600.

30. Ans. (b)

Solution We have, A_1 = Rs. 450, A_2 = Rs. 350, R_1 = 7%, R_2 = 5%.

$\therefore$ Principal (P) $= \frac{A_2R_1 - A_1R_2}{R_1 - R_2}$

$$= \frac{350 \times 7 - 450 \times 5}{7 - 5} = \frac{200}{2}$$

= Rs. 100.

31. Ans. (c)

Solution We have, A_1 = Rs. 400, A_2 = Rs. 200, R_1 = 10%, R_2 = 4%.

$\therefore$ Time (T) $= \frac{A_1 - A_2}{A_2R_1 - A_1R_2} \times 100$

$$= \frac{400 - 200}{200 \times 10 - 400 \times 4} \times 100 = \frac{20000}{400}$$

= 50 years.

32. Ans. (a)

Solution Here, z = Rs. 9, a = Rs. 1, b = 12, n = 10, R = ?

Using the formula

$$z = na + \frac{Ra}{100 \times b} \times \frac{n(n-1)}{2},$$

We get $9 = 10 \times 1 + \frac{R \times 1}{100 \times 12} \times \frac{10 \times 9}{2}$

$$\Rightarrow \frac{90R}{2400} = 1 \Rightarrow R = \frac{2400}{9} = 266\frac{2}{3}\%.$$

33. Ans. (a)

Solution Here, $T_1 = 1$, $T_2 = 2$, $T_3 = 3$,
$R_1 = R_2 = R_3 = 5\%$.

The shares of Vikas, Vijay and Viraj will be in the ratio

$$\frac{1}{R_1T_1} : \frac{1}{R_2T_2} : \frac{1}{R_3T_3} = \frac{1}{1 \times 5} : \frac{1}{2 \times 5} : \frac{1}{3 \times 5}$$

$$= \frac{1}{1} : \frac{1}{2} : \frac{1}{3} = 6 : 3 : 2.$$

Sum of proportionals = 6 + 3 + 2 = 11.

$\therefore$ Share of Vikas $= \frac{6}{11} \times 7700$ = Rs. 4200

Share of Vijay $= \frac{3}{11} \times 7700$ = Rs. 2100

Share of Viraj $= \frac{2}{11} \times 7700$ = Rs. 1400

Therefore, Vikas's share is 4200 – 1400 = Rs. 2800 more than that of Viraj.

34. Ans. (b)

Solution Let the required sum of money be Rs. x. Here R_1 = 5%, T_1 = 4 years, R_2 = 4%, T_2 = 10 years.

Given: $x : 560 = \frac{1}{R_1T_1} : \frac{1}{R_2T_2} = \frac{1}{5 \times 4} : \frac{1}{4 \times 10}$

$\Rightarrow \frac{x}{560} = \frac{2}{1}$ or, x = Rs. 1120.

35. Ans. (a)

Solution Here, P_1 = Rs. 12000, R_1 = 10%, P_2 = ?, R_2 = 20%, R = 14%.

Therefore, using the formula

$$R = \frac{P_1R_1 + P_2R_2}{P_1 + P_2}$$

We get, $14 = \frac{12000 \times 10 + P_2 \times 20}{12000 + P_2}$

or, P_2 = Rs. 8000.

$\therefore$ Total amount invested = Rs. (12000 + 8000)
= Rs. 20000.

36. Ans. (b)

Solution We have, P_1 = Rs. 3000, R_1 = 10%, P_2 = Rs. 5000, R_2 = 8%.

$\therefore$ Required rate of interest

$$= \frac{P_1R_1 + P_2R_2}{P_1 + P_2}$$

$$= \frac{3000 \times 10 + 5000 \times 8}{3000 + 5000} = \frac{70}{8} = 8\frac{3}{4}\%.$$

37. Ans. (c)

Solution We have, $\frac{1}{a} = \frac{2}{3}$, $\frac{1}{b} = \frac{1}{6}$,

$$\frac{1}{c} = 1 - \left(\frac{2}{3} + \frac{1}{6}\right) = \frac{1}{6},$$

$R_1 = 3\%$, $R_2 = 6\%$, $R_3 = 12\%$, A = Rs. 25.

$$\therefore \text{ The capital } = \frac{A \times 100}{\frac{R_1}{a} + \frac{R_2}{b} + \frac{R_3}{c}}$$

$$= \frac{25 \times 100}{3 \times \frac{2}{3} + \frac{6}{6} + \frac{12}{6}} = \frac{2500}{5}$$

= Rs. 500.

38. Ans. (a)

Solution Interest for 5 years on the sum = Rs. 300.

When the principal is trebled, the interest is also trebled.

$\therefore$ Interest for another 5 years on this increased sum = Rs. (300 × 3) = Rs. 900.

$\therefore$ Total interest = Rs. 300 + Rs. 900 = Rs. 1200.

39. Ans. (a)

Solution The simple interest on Rs. 1500 invested at a rate of 10% p.a. for 5 years is

$$= \frac{1500 \times 10 \times 5}{100} = \text{Rs. } 750.$$

Now, principal after 5 years = Rs. 1500 + 750

= Rs. 2250.

Also, final amount = Rs. 2500.

$\therefore$ Simple interest = Rs. 2500 – 2250 = Rs. 250.

$$\therefore \text{ Time } (T) = \frac{250 \times 100}{2250 \times 10} = \frac{10}{9} \text{ years}$$

Hence, total time = $5 + \frac{10}{9} = \frac{55}{9}$ or $6\frac{1}{9}$ years.

40. Ans. (c)

Solution Let the sum of money lent by Sumit to Mohit be Rs. x.

Then, simple interest paid by Mohit after 1 year

$$= \frac{x \times 5 \times 1}{100} = \text{Rs. } \frac{5x}{100}.$$

Also, the simple interest received by Mohit from Birju after 1 year

$$= \frac{x \times \frac{17}{2} \times 1}{100} = \text{Rs. } \frac{17x}{200}.$$

Given: $\frac{5x}{100} + 350 = \frac{17x}{200}$

$\Rightarrow \frac{5x + 35000}{100} = \frac{17x}{200}$

$\Rightarrow 1700x - 1000x = 7000000$

or, $700x = 7000000$

or, $x = \frac{7000000}{700} = \text{Rs. } 10000.$

Thus, the sum of money lent by Sumit to Mohit is Rs. 10000.

41. Ans. (a)

Solution Simple interest paid by Brinda on Rs. 1000 for 1 year = $\frac{1000 \times 5 \times 1}{100}$ = Rs. 50.

Rent received by Brinda from Ramu in 1 year

$$= 12\frac{1}{2} \times 12 = \text{Rs. } 150.$$

$\therefore$ Net savings = Rs. 100.

Thus, Brinda will clear the debt of Rs. 1000 in 10 years.

42. Ans. (c)

Solution Let the sum be Rs. x.

Given: $\frac{x \times 4 \times 2}{100} + \frac{x \times 6 \times 4}{100} + \frac{x \times 8 \times 3}{100} = 1120$

$\Rightarrow 56x = 112000$ or, $x = \frac{112000}{56} = \text{Rs. } 2000.$

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. Arun borrowed a sum of money from Jayant at the rate of 8% per annum simple interest for the first four years, 10% per annum for the next six years and 12% per annum for the period beyond ten years. If he pays a total of Rs. 12160 as interest only at the end of 15 years, how much money did he borrow?

(a) Rs. 8000 (b) Rs. 10000
(c) Rs. 12000 (d) Rs. 9000
(e) None of these **[Based on BSRB Mumbai PO 1998]**

2. Ashok borrowed some money at the rate of 6% per annum for the first two years, at the rate of 9% per annum for the next three years and at the rate of 14% per annum for the period beyond five years. If he pays a total interest of Rs. 11,400 at the end of nine years, how much money did he borrow?

(a) Rs. 16000 (b) Rs. 14000
(c) Rs. 18000 (d) Rs. 12000
(e) None of these

[Based on Bank of Baroda PO 1999]

3. Ramesh invested an amount of Rs. 12000 at the simple interest rate of 10% per annum and another amount at the simple interest rate of 20% per annum. The total interest earned at the end of one year on the total amount invested become 14% per annum. Find the total amount invested

(a) Rs. 22000 (b) Rs. 25000
(c) Rs. 20000 (d) Rs. 24000
(e) None of these

[Based on SBI Associates PO 1999]

4. Amit borrowed a sum of money with simple interest as per the following rate structure.
(a) 6% per annum for the first three years.
(b) 8% per annum for the next five years
(c) 12% per annum for the next four years

If he paid a total of Rs. 5040 as interest at the end of twelve years, how much money did he borrow?
(a) Rs. 8000 (b) Rs. 10000
(c) Rs. 12000 (d) Rs. 6000
(e) None of these.
[Based on BSRB Calcutta PO 1999]

5. Ansh borrowed Rs. 15000 at the rate of 12% and another amount at the rate of 15% for two years. The total interest paid by him was Rs. 900. How much did he borrow?
(a) Rs. 32000 (b) Rs. 33000
(c) Rs. 30000 (d) Rs. 35000
(e) None of these
[Based on BSRB Hyderabad PO 1999]

6. Mr. Roy borrowed some money from Mr. Das. The rate of interest for first two years is 8% per annum, for the next three years is 11% per annum, for the period beyond 5 years is 14% per annum. Mr. Das got an amount of Rs. 10920 as an interest at the end of eight years. Then what amount was borrowed by Mr. Roy?
(a) Rs. 12000 (b) Rs. 15000
(c) Rs. 1400 (d) Data inadequate
(e) None of these
[Based on BSRB Hyderabad PO 1999]

7. Neeta borrowed some money at the rate of 6% per annum for the first three years, 9% per annum for the next five years and 13% per annum for the period beyond eight years. If the total interest paid by her at the end of eleven years is Rs. 8,160, how much money did she borrow?
(a) Rs. 12000 (b) Rs. 10000
(c) Rs. 8000 (d) Data inadequate
(e) None of these
[Based on BSRB Chennai PO 2000]

8. The simple interest in 14 months on a certain sum at the rate of 6% per annum is Rs. 250 more than the interest on the same sum at the rate of 8% in 8 months. How much amount was borrowed?
(a) Rs. 15000 (b) Rs. 25000
(c) Rs. 7500 (d) Rs. 14500
(e) None of these
[Based on BSRB Bangalore PO 1999]

9. At a simple interest Rs. 800 becomes Rs. 956 in three years. If the interest rate is increased by 3%, how much would Rs. 800 become in three years.
(a) Rs. 1020.80 (b) Rs. 1004
(c) Rs. 1028 (d) Data inadequate
(e) None of these **[Based on BSRB Delhi PO 1999]**

10. Ankit deposited two parts of a sum of Rs. 25000 in different banks at the rates of 15% per annum and 18% per annum respectively. In one year he got Rs. 4050 as the total interest. What was the amount deposited at the rate of 18% per annum?
(a) Rs. 9000 (b) Rs. 18000
(c) Rs. 15000 (d) Data inadequate
(e) None of these **[Based on BSRB Patna PO 2001]**

11. Mr. Kalia invested a total amount of Rs. 16500 for two years in two schemes *A* and *B* with rate of simple interest 10% per annum and 12% per annum, respectively. If the total amount of interest earned was Rs. 3620, what was amount invested in scheme *A*?
(a) Rs. 8000 (b) Rs. 8600
(c) Rs. 8150 (d) Data inadequate
(e) None of these **[Based on Andhra Bank PO 2002]**

12. A sum fetched total simple interest of Rs. 4016.25 at the rate of 9% per annum in 5 years. What is the sum?
(a) Rs. 8925 (b) Rs. 8032.50
(c) Rs. 4462.50 (d) Rs. 8900
(e) None of these
[Based on NABARD Asst. Manager 2002]

13. A shopkeeper allows two successive discounts of 10% and 20%. If he sells an article for Rs. 540, then the marked price of the article is
(a) Rs. 750 (b) Rs. 740
(c) Rs. 725 (d) Rs. 700
[Based on SI of Police Rec. Exam 1997]

14. In how many years will a sum of money double itself at 10% simple interest?
(a) 5 years (b) 6 years
(c) 10 years (d) 20 years
[Based on SI of Police Rec. Exam 1997]

15. What is the present worth of Rs. 132 due in 2 years at 5% simple interest per annum?
(a) Rs. 120 (b) Rs. 122
(c) Rs. 112 (d) Rs. 118.80
[Based on SI of Police Rec. Exam 1997]

16. A sum of Rs. 5000 was lent partly at 6% and partly at 9% simple interest. If the total interest received after one year was Rs. 390, the ratio in which the money was lent at 6% and 9% is
(a) 1 : 1 (b) 3 : 2
(c) 2 : 3 (d) 1 : 2
[Based on SI of Police Rec. Exam 1997]

17. Divide Rs. 2379 into 3 parts so that their amount after 2, 3 and 4 years, respectively may be equal, the rate of interest being 5% per annum at simple interest. The first part is given by
(a) Rs. 828 (b) Rs. 792

(c) Rs. 759 (d) Rs. 818

[Based on SI Rec. Exam Delhi Police 1997]

18. The simple interest on a certain sum of money for $2\frac{1}{2}$ years at 12% per annum is Rs. 40 less than the simple interest on the same sum for $3\frac{1}{2}$ years at 10% per annum. The sum is

(a) Rs. 600 (b) Rs. 800
(c) Rs. 700 (d) Rs. 900

[Based on SI Rec. Exam Delhi Police 1997]

19. In how much time would the simple interest on a certain sum be 0.125 times the principal at 10% per annum?

(a) $1\frac{1}{4}$ years (b) $1\frac{3}{4}$ years
(c) $2\frac{1}{4}$ years (d) $2\frac{3}{4}$ years

[Based on Asstt. Grade Exam 1997]

20. The simple interest on a sum of money is $\frac{1}{4}$ of the principal. If number of years is equal to rate per cent per annum, the interest rate is

(a) 5.0% (b) 5.1%
(c) 5.2% (d) 4.8%

[Based on SI of Police Rec. Exam PO 1998]

21. In how many years a certain sum doubles itself at 4% per annum simple interest?

(a) 5 years (b) 10 years
(c) 20 years (d) 25 years

[Based on SSC (GL) Prel. Exam 1999]

22. The difference between the simple interest on a certain sum of money at 6% per annum for 10 years and at 5% per annum for 2 years is Rs. 100. Find the sum

(a) Rs. 100 (b) Rs. 200
(c) Rs. 400 (d) Rs. 500

[Based on SSC (GL) Prel. Exam 1999]

23. Two equal sums of money were invested, one at 4% and the other at $4\frac{1}{2}$%. At the end of 7 years, the simple interest received from the latter exceeds to that received from the former by Rs. 31.50. Each sum was

(a) Rs. 1000 (b) Rs. 500
(c) Rs. 750 (d) Rs. 900

[Based on SSC (GL) Prel. Exam 1999]

24. The simple interest on a certain sum at 5% per annum for 3 years and 4 years differ by Rs. 42. The sum is

(a) Rs. 210 (b) Rs. 280
(c) Rs. 750 (d) Rs. 840

[Based on SSC (GL) Prel. Exam 1999]

25. A sum of Rs. 1600 gives a simple interest of Rs. 252 in 2 years and 3 months. The rate of interest per annum is

(a) $5\frac{1}{2}$% (b) 8%
(c) 7% (d) 6%.

[Based on SSC (GL) Prel. Exam 2000]

26. A sum of Rs. 400 amounts to Rs. 480 in 4 years. What will it amount to if the rate of interest is increased by 2%

(a) Rs. 484 (b) Rs. 560
(c) Rs. 512 (d) None of these

[Based on SSC (GL) Prel. Exam 2000]

27. The simple interest on a sum of money is $\frac{4}{9}$ of the principal and the number of years is equal to the rate per cent per annum. The rate per annum is

(a) 5% (b) $6\frac{2}{3}$%
(c) 6% (d) $7\frac{1}{5}$%

[Based on SSC (GL) Prel. Exam 2000]

28. Rs. 1500 were divided into two parts. One part was put at 6% and the other at 5% interest. If the whole annual interest from both investments was Rs. 85, then the investment at 6% was

(a) Rs. 1200 (b) Rs. 1000
(c) Rs. 1300 (d) Rs. 1150

[Based on Railway Rec. Board Exam 2000]

29. A person borrows Rs. 5000 for two years at 4% per annum simple interest. He immediately lends to another person at $6\frac{1}{4}$% per annum for 2 years. Find his gain in the transaction per year

(a) Rs. 112.50 (b) Rs. 450.00
(c) Rs. 244.53 (d) Rs. 150.00

[Based on SSC (GL) Prel. Exam 2000]

30. A sum of Rs. 1550 was lent partly at 5% and partly at 8% simple interest. The total interest received after 3 year is Rs. 300. The ratio of money lent at 5% to that at 8%

(a) 5 : 8 (b) 8 : 5
(c) 31 : 6 (d) 16 : 5

[Based on SSC (GL) Prel. Exam 2002]

31. In what time will the simple interest be $\frac{2}{5}$ of the principal at 8% per annum?

(a) 8 years (b) 7 years
(c) 5 years (d) 6 years

[Based on SSC (GL) Prel. Exam 2002]

32. A person lent Rs. 5,000 partly at the rate of 4% and partly at the rate of 5% per annum simple

interest. The total interest after 2 years is Rs. 440. To find the sum of money lent at each of the above rates, Rs. 5,000 is to be divided in the ratio

(a) 4 : 5 (b) 3 : 2
(c) 5 : 4 (d) 2 : 3

[Based on SSC (GL) Prel. Exam 2002]

33. At what rate per cent per annum will the simple interest on a sum of money be $\frac{2}{5}$ of the amount in 10 years?

(a) 4% (b) 6%
(c) $5\frac{2}{3}$% (d) $6\frac{2}{3}$%

[Based on SSC (GL) Prel. Exam 2002]

34. If the simple interest on a certain sum for 15 months at $7\frac{1}{2}$% per annum exceeds the simple interest on the same sum for 8 months at $12\frac{1}{2}$% per annum by Rs. 32.50, then the sum (in Rs.) is

(a) 312 (b) 312.50
(c) 3120 (d) 3120.50

[Based on SSC Prel. (L) Exam 2002]

35. In 4 years, the simple interest on a certain sum of money is $\frac{9}{25}$ of the principal. The annual rate of interest is

(a) 4% (b) $4\frac{1}{2}$%
(c) 9% (d) 10%

[Based on SSC (GL) Prel. Exam 2002]

36. What sum of money will amount to Rs. 520 in 5 years and to Rs. 568 in 7 years at simple interest?

(a) Rs. 400 (b) Rs. 120
(c) Rs. 510 (d) Rs. 220

[Based on SSC (GL) Prel. Exam 2003]

37. A lends Rs. 2500 to B and a certain sum to C at the same time at 7% annual simple interest. If after 4 years, A altogether receives Rs. 1120 as interest from B and C, the sum lent to C is

(a) Rs. 700 (b) Rs. 6500
(c) Rs. 4000 (d) Rs. 1500

[Based on SSC (GL) Prel. Exam 2003]

38. A sum of money becomes $\frac{41}{40}$ of itself in $\frac{1}{4}$ years at a certain rate of simple interest. The rate of interest per annum is

(a) 10% (b) 1%
(c) 2.5% (d) 5%

[Based on SSC (GL) Prel. Exam 2003]

39. A man loses Rs. 55.50 yearly when the annual rate of interest falls from 11.5 to 10%. His capital (in rupees) is

(a) 3700 (b) 7400
(c) 8325 (d) 11100

[Based on SSC (GL) Prel. Exam 2003]

40. A certain amount earns simple interest of Rs. 1750 after 7 years. Had the interest been 2% more, how much more interest would it have earned?

(a) Rs. 35 (b) Rs. 350
(c) Rs. 245 (d) Cannot be determined
(e) None of these

[Based on Canara Bank PO 2003]

41. Nikhal invested certain amount in three different schemes A, B and C with the rate of interest 10% per annum, 12% per annum and 15% per annum, respectively. If the total interest accrued in one year was Rs. 3200 and the amount invested in scheme C was 150% of the amount invested in scheme A and 240% of the amount invested in scheme B, what was the amount invested in scheme B?

(a) Rs. 8000 (b) Rs. 5000
(c) Rs. 6500 (d) Cannot be determined
(e) None of these

[Based on PNB Management Trainee Exam 2003]

EXPLANATORY ANSWERS

1. Ans. (a)

Solution Let the principal be Rs. P

Then $\frac{P\times8\times4}{100}+\frac{P\times10\times6}{100}+\frac{P\times12\times5}{100} = 12160$

$\Rightarrow$ $152P = 12160 \times 100$

or, $P = \frac{12160\times100}{152}$

or, P = Rs. 8000.

2. Ans. (d)

Solution Let the Principal be Rs. P

Then, $\frac{P\times6\times2}{100}+\frac{P\times9\times3}{100}+\frac{P\times14\times4}{100} = 11400$

or, $95P = 1140000$

or, $P = \frac{1140000}{95}$ or, P = Rs. 12000.

3. Ans. (c)

Solution Let the amount invested at 20% rate be Rs. x

According to question,

$$12000 \times \frac{10}{100}+x\times\frac{20}{100} = (12000 + x) \times \frac{14}{100}$$

or, $1200 + \frac{x}{5} = 1680 + \frac{7}{50}x$

or, $\frac{x}{5} - \frac{7}{50}x = 480$ or, $\frac{3}{50}x = 480$

$\therefore$ x = Rs. 8000

$\therefore$ Total amount invested = Rs. (12000 + 8000)
= Rs. 20000.

4. Ans. (e)

Solution Let Rs. x be the amount Amit borrowed

$\therefore$ $\frac{x\times6\times3}{100}+\frac{x\times8\times5}{100}+\frac{x\times12\times4}{100} = 5040$

or, $106x = 504000$

or, $x = \frac{504000}{106}$

$\therefore$ x = Rs. 4754.71.

5. Ans. (b)

Solution Let the amount be Rs. x

According to question,

$\frac{15000\times12\times12}{100}+\frac{x\times15\times2}{100} = 9000$

$\Rightarrow$ $360000 + 30x = 900000$

$\Rightarrow$ $30x = 540000$

$\Rightarrow$ x = Rs. 18000.

6. Ans. (a)

Solution Let Rs. x be the amount borrowed by Mr. Roy.

$\therefore$ $\frac{x\times2\times8}{100}+\frac{x\times3\times11}{100}+\frac{x\times3\times14}{100} = 10920$

or, $\frac{91}{100}x = 10920$ or, $x = \frac{10920\times100}{91}$

$\therefore$ $x = 12000$.

7. Ans. (c)

Solution Let 'x' be the amount borrowed by Neeta

Then, $\frac{x\times6\times3}{100}+\frac{x\times9\times5}{100}+\frac{x\times13\times3}{100} = 8160$

or, $\frac{102x}{100} = 8160$ or $x = \frac{8160\times100}{102}$

$\therefore$ x = Rs. 8000.

8. Ans. (a)

Solution Let the amount be Rs. x

According to question,

$\frac{x\times14\times6}{1200}-\frac{x\times8\times8}{1200} = 250$

$\therefore$ x = Rs. 15000.

9. Ans. (c)

Solution Rate of interest $= \frac{956-800}{3\times800}\times100$
$= 6.50\%$

$\therefore$ Amount $= 800 + \frac{800\times9.5\times3}{100}$
$= 800 + 228$ = Rs. 1028.

10. Ans. (e)

Solution Let the amount deposited at the rate of 15% per annum be Rs. x

15% of x + 18% of $(25000 - x) = 4050$

or, 15% of x + 18% of 25000 – 18% of $x = 4050$

or, 3% of $x = 4500 - 4050$

or, $\frac{3}{100}\times x = 450$

or, x = Rs. 15000

$\therefore$ Amount deposited at 18%
= Rs. (25000 – 15000) = Rs. 10000.

11. Ans. (e)

Solution % interest on total amount per annum

$= \frac{3620\times100}{16500\times2} = \frac{362}{33}\%$

Now, by Alligation method

Scheme A 10%		Scheme B 12%
	$\frac{362}{33}\%$	
$12 - \frac{362}{33}$		$\frac{362}{33} - 10$

Hence, ratio of amount invested in schemes

A and $B = \left(12-\frac{362}{33}\right):\left(\frac{362}{33}-10\right) = 17 : 16$

Hence, amount invested in $A = \frac{17\times16500}{17+16}$
= Rs. 8500.

12. Ans. (a)

Solution Let the sum be Rs. P

Now, 45% of $P = 4016.25$ or, P = Rs. 8925.

13. Ans. (a)

Solution Suppose, marked price = Rs. 100

Price after 1st discount = Rs. 90

Price after 2nd discount = Rs. 72

$\therefore$ If S.P. is Rs. 72, then marked price = Rs. 100

$\therefore$ If S.P. is Rs. 540, then marked price

$$= \text{Rs. } \frac{100}{72} \times 540 = \text{Rs. } 750.$$

14. Ans. (c)

Solution Let sum = Rs. P

Simple interest = Rs. P

Rate R = 10%

$$\therefore \quad \text{Time} = \frac{P \times 100}{P \times 10} = 10 \text{ years.}$$

15. Ans. (a)

Solution Suppose present worth = Rs. x

$$\therefore \quad \frac{x \times 5 \times 2}{100} = 132 - x$$

$$\therefore \quad x = 10(132 - x)$$

$$\Rightarrow \quad 11x = 1320 \quad \text{or,} \quad x = 120.$$

16. Ans. (c)

Solution Suppose Rs. x are invested at 6% and Rs. $(5000 - x)$ at 9%.

$$\therefore \quad \frac{x \times 6 \times 1}{100} + \frac{(5000 - x) \times 9 \times 1}{100} = 390 \Rightarrow x = 2000$$

$\therefore$ Rs. 2000 are lent at 6% and Rs. 3000 at 9%.

17. Ans. (a)

Solution Let first part be x

Second part be y

$\therefore$ Third part = $2379 - (x + y)$

$$\therefore \quad x + \frac{x \times 5 \times 2}{100} = y + \frac{y \times 5 \times 3}{100}$$

$$= [2379 - (x + y)] + \frac{[2379 - (x + y)] \times 5 \times 4}{100} \quad \text{...(A)}$$

$$\therefore \quad x + \frac{x}{10} = y + \frac{3y}{20} \text{ (from } A\text{)}$$

$$\Rightarrow \quad x + \frac{x}{10} = y + \frac{3y}{20}$$

$$\Rightarrow \quad 22x = 23y \quad \text{...(B)}$$

Also $y + \frac{3y}{20} = \frac{6}{5}[2379 - (x + y)]$ (from A)

$$\Rightarrow \quad \frac{23y}{20} = \frac{6}{5}\left[2379 - x - \frac{22x}{23}\right] \text{ (from } B\text{)}$$

$$\Rightarrow \quad \frac{22x}{4} = 6\left[2379 - \frac{45x}{23}\right]$$

$$\Rightarrow \quad \frac{11x}{12} = 2379 - \frac{45x}{23}$$

$$\Rightarrow \quad \frac{11x}{12} + \frac{45x}{23} = 2379 \Rightarrow x = 828.$$

18. Ans. (b)

Solution Let P be the sum

$$\therefore \quad \frac{P \times \frac{5}{2} \times 12}{100} = \frac{P \times \frac{7}{2} \times 10}{100} - 40$$

$$\Rightarrow \quad \frac{30P}{100} - \frac{35P}{100} = -40 \Rightarrow -\frac{5P}{100} = -40$$

$$\Rightarrow \quad P = \frac{100 \times 40}{5} = 800.$$

19. Ans. (a)

Solution Suppose, sum = K

$I = 0.125K$

Rate = 10%

$$\therefore \quad \text{Time} = \frac{100 \times 0.125K}{K \times 10} = 1.25 = 1\frac{1}{4} \text{ years.}$$

20. Ans. (a)

Solution Let sum = Rs. K

Time = T

$$\therefore \quad I = \frac{K}{4}$$

$$\therefore \quad \text{Rate \% } (R) = \frac{\frac{K}{4} \times 100}{K \times T} = \frac{25}{T}$$

$$\Rightarrow \quad RT = 25 \Rightarrow R^2 = 25 \quad (\because R = T)$$

$$\Rightarrow \quad R = 5.$$

21. Ans. (d)

Solution Let Principal = Rs. P

I = Rs. P

$$\therefore \quad T = \frac{SI \times 100}{\text{Principal} \times \text{Rate\%}} = \frac{P \times 100}{P \times 4} = 25 \text{ years.}$$

22. Ans. (b)

Solution Let sum = Rs. P

$$\therefore \quad \frac{P \times 6 \times 10}{100} - \frac{P \times 5 \times 2}{100} = 100$$

$$\Rightarrow \quad 60P - 10P = 10000 \Rightarrow P = 200.$$

23. Ans. (d)

Solution Let the sum be Rs. P

$$\therefore \quad \frac{P \times 7 \times \frac{9}{2}}{100} - \frac{P \times 7 \times 4}{100} = 31.50$$

$$\Rightarrow \quad \frac{63P}{2} - 28P = 3150$$

$$\Rightarrow \quad 7P = 6300 \Rightarrow P = 900.$$

24. Ans. (d)

Solution $$\text{Sum} = \frac{\text{Difference} \times 100}{|R_1T_1 - R_2T_2|}$$

$$= \frac{42 \times 100}{5} = \text{Rs. } 840.$$

25. Ans. (c)

Solution $252 = 1600 \times R \times \frac{9}{4} \times \frac{1}{100}$ $\left[I = \frac{P \times R \times T}{100}\right]$

$\therefore \quad R = \frac{252}{36} = 7\%.$

26. Ans. (c)

Solution $I = \frac{P \times R \times T}{100}$

$80 = \frac{400 \times 4 \times R}{100} \Rightarrow R = 5$

If $R = 7$, then

$I = \frac{400 \times 4 \times 7}{100} = 112$

$\therefore$ Amount = Rs. 512.

27. Ans. (b)

Solution Let the sum of money be P

$I = \frac{P \times R \times T}{100} \Rightarrow \frac{4}{9}P = \frac{P \times R \times T}{100}$

$\therefore \quad R = \sqrt{\frac{400}{9}} = \frac{20}{3} = 6\frac{2}{3}\%.$

28. Ans. (b)

Solution Suppose, one part of sum = Rs. x

$\therefore$ Other part of the sum = Rs. $(1500 - x)$

According to question,

$\frac{x \times 6 \times 1}{100} + \frac{(1500 - x) \times 5 \times 1}{100} = 85$

$6x + 7500 - 5x = 8500$ or, $x = 1000.$

29. Ans. (a)

Solution $\frac{5000 \times 2\frac{1}{4} \times 2}{100} = 50 \times \frac{9}{4} \times 2 = 225$

$\therefore$ Gain per year = Rs. 113.50.

30. Ans. (d)

31. Ans. (c)

Solution Required time $= \frac{2 \times 100}{5 \times 8} = 5$ years.

32. Ans. (b)

Solution By the method of Alligation

Rs. 440 is 8.8% of 5000

I	II
8% (= 4 × 2)	10% (= 5 × 2)

8.8%

1.2 (= 10 − 8.8)	0.8 (= 8.8 − 8)

Hence, required ratio $= \frac{1.2}{0.8} = \frac{3}{2} = 3 : 2$

33. Ans. (d)

Solution Here, $P = 3K$, $I = 2K$, $T = 10$ years

Hence, rate $= \frac{2K \times 100}{3K \times 10} = \frac{20}{3} = 6\frac{2}{3}\%.$

34. Ans. (c)

Solution Let the sum be Rs. x

$\therefore \quad \frac{x \times 7\frac{1}{2} \times 1\frac{1}{4}}{100} - \frac{x \times 12\frac{1}{2} \times \frac{2}{3}}{100} = 32.50$

$\Rightarrow \quad \frac{75}{8}x - \frac{25}{3}x = 3250$

$\Rightarrow \quad 25x = 3250 \times 24 \Rightarrow x = 3120.$

35. Ans. (c)

Solution Let the sum be Rs. x

Let the rate be $R\%$

$\therefore \quad \frac{x \times 4 \times R}{100} = \frac{9}{25}x \Rightarrow R = 9.$

36. Ans. (a)

Solution Rate % per annum $= \frac{100[568 - 520]}{520 \times 7 - 568 \times 5}$

$= \frac{100 \times 48}{3640 - 2840}$

$= \frac{100 \times 48}{800} = 6\%$

Sum $= \frac{(568 - 520) \times 100}{(7 - 5) \times 6} = \frac{48 \times 100}{2 \times 6} =$ Rs. 400.

37. Ans. (d)

38. Ans. (a)

Solution Rate % $= \frac{\left(\frac{41}{40} - 1\right) \times 100}{\frac{1}{4}} = 10\%.$

39. Ans. (a)

Solution His capital $= \frac{55.50}{1.5} \times 100 =$ Rs. 3700.

40. Ans. (d)

Solution Let Principal = P and Rate of interest = $R\%$ per annum.

Then $\frac{P \times R \times 7}{100} = 1750$ or $PR = 25000$

Now $I = \frac{P \times (R + 2) \times 7}{100} = \frac{25000 \times 7 + P \times 14}{100}$

When we solve this equation, we find that we have two variables and one equation.

Therefore, cannot be determined is the correct answer.

41. Ans. (b)

Solution Ratio of Nikhil's investment in difference schemes

$$= 100 : \frac{150 \times 100}{240} : 150 = 8 : 5 : 12$$

Now, according to the question,

$$\frac{8K \times 10}{100} + \frac{5K \times 12}{100} + \frac{12K \times 15}{100} = 3200$$

or, $80K + 60K + 180K = 3200 \times 100$

or, $320K = 3200 \times 100$ or $K = 1000$

$\therefore$ Amount invested in scheme $B = 1000 \times 5$
$= \text{Rs. } 5000.$

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. Rs. 1000 is invested at 5% simple interest. If the interest is added to the principal every 10 years, the amount will become Rs. 2000 after
(a) 15 years (b) 18 years
(c) 20 years (d) $16\frac{2}{3}$ years

2. Divide the sum of Rs. 1175 in three such parts that simple interest accured from them after 3, 4 and 5 years, respectively at the rate of 4% per annum remains equal.
(a) Rs. 490, Rs. 375, Rs. 300
(b) Rs. 500, Rs. 375, Rs. 300
(c) Rs. 500, Rs. 350, Rs. 300
(d) None of these

3. In what time will Rs. 4000 lent at 3% per annum on S.I. earn as much interest as Rs. 5000 will earn in 5 years at 4% per annum on S.I.?
(a) $8\frac{1}{3}$ years (b) 9 years
(c) $7\frac{1}{2}$ years (d) $7\frac{1}{3}$ years

4. A man lends Rs. 10000 in four parts. If he gets 8% on Rs. 2000; $7\frac{1}{2}$% on Rs. 4000 and $8\frac{1}{2}$% on Rs. 1400, what per cent must he get for the remainder, if the average interest is 8.13%?
(a) 8% (b) 10%
(c) 9% (d) None of these

5. A man invests an amount of Rs. 15860 in the names of his three sons A, B and C in such a way that they get the same interest after 2, 3 and 4 years, respectively. If the rate of simple interest is 5%, the ratio of amount invested among A, B and C will be
(a) 10 : 15 : 20 (b) 110 : 115 : 120
(c) $\frac{1}{10} : \frac{1}{15} : \frac{1}{20}$ (d) $\frac{1}{110} : \frac{1}{115} : \frac{1}{120}$

6. What is the least number of years for which simple interest must be reckoned at 3% on Rs. 212.50, so that the interest may be an exact number of rupees.
(a) 6 years (b) 8 years
(c) 10 years (d) 7 years

7. A person invested Rs. 500 at a certain rate of interest and Rs. 700 at 1% higher and the total interest from these investments in 3 years was Rs. 165. Find the rate per cent.
(a) 6% (b) 5%
(c) $4\frac{1}{2}$% (d) 4%

8. Amitabh invested an amount of Rs. 29000 in two parts under two different schemes A and B and earned a total interest of Rs. 3840. Schemes A and B offered 15% and 12% interest, respectively. What was the amount invested in scheme 'A'?
(a) Rs. 17000 (b) Rs. 12000
(c) Rs. 14000 (d) data inadequate
(e) None of these

9. Out of a sum of Rs. 625, a part was lent at 5% and the other at 10% simple interest. If the interest on the first part after 2 years is equal to the interest on the second part after 4 years, then the second sum (in Rs.) is
(a) 125 (b) 200
(c) 250 (d) 300

10. Find in what time a given sum of money will quadruple itself at simple interest at the rate of one paise per rupee per month?
(a) 25 years (b) 27 years
(c) 23 years (d) 28 years

Test II **[Time: 10 min]**

11. The simple interest of Rs. 800 is Rs. 216. If the rate per cent of the interest per annum is $\frac{1}{3}$ of the number of years, the rate per cent per annum is
(a) 2% (b) 4%
(c) 6% (d) 3%

12. The simple interest on Rs. 1820 from Oct. 9, 2003 to Dec. 21, 2003 at $7\frac{1}{2}\%$ will be
(a) Rs. 22.50 (b) Rs. 27.30
(c) Rs. 28.80 (d) Rs. 29

13. A sum of Rs. 1550 was lent partly at 5% and partly at 8% simple interest. The total interest received after 3 years is Rs. 300. The ratio of money lent at 5% to that at 8% is
(a) 5 : 8 (b) 8 : 5
(c) 31 : 6 (d) 16 : 15

14. Rs. 800 becomes Rs. 956 in 3 years at certain simple rate of interest. If the rate of interest is increased by 4%, what amount will Rs. 800 become in 3 years ?
(a) Rs. 1020.80 (b) Rs. 1025
(c) Rs. 1052 (d) data inadequate

15. Mr. Rao invested a total amount of Rs. 16500 for two years in two Schemes *A* and *B* with rate of simple interest 10% p.a., and 12% p.a., respectively. If the total amount of interest earned was Rs. 3,620, what was the amount invested in scheme *B*?
(a) Rs. 8000 (b) Rs. 8600
(c) Rs. 8150 (d) data inadequate
(e) None of these

16. Rs. 4000 was divided into two parts in such a way that when first part was invested at 3% and the second at 5% the whole annual interest from both the investments was Rs. 144, how much was put at 3%?
(a) Rs. 2500 (b) Rs. 2700
(c) Rs. 2800 (d) Rs. 3000

17. The simple interest on a sum of money is 1/6 of the principal and the number of months is equal to twice the rate per cent. Find the time and the rate
(a) $3\frac{1}{3}$ years; 10% (b) $1\frac{2}{3}$ years; 10%
(c) 4 years; 8% (d) $5\frac{1}{3}$ years; $10\frac{2}{3}\%$

18. Ram borrowed a sum of Rs. 12000 at 15% per annum from a money lender on 13th January, 2003 and returned the amount on 8th June, 2003 to clear his debt. What was the amount paid by Ram to the money-lender to clear his debt?
(a) Rs. 12520 (b) Rs. 12820
(c) Rs. 12720 (d) None of these

19. Pankaj deposited a certain sum of money in a post office at 12% per annum for 4 years and deposited an equal amount in fixed deposit in a bank for 5 years at 15% per annum. If the difference in the interest from two sources is Rs. 1350, find the sum deposited in each case.
(a) Rs. 4500 (b) Rs. 5000
(c) Rs. 5500 (d) None of these

20. What equal half yearly payment will discharge the debt of Rs. 4360 due in 2 years at 12% per annum.
(a) Rs. 1000 (b) Rs. 1020
(c) Rs. 990 (d) None of these

Answers

1. (d)	**2.** (b)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (d)	**8.** (b)	**9.** (a)	**10.** (a)
11. (d)	**12.** (b)	**13.** (d)	**14.** (c)	**15.** (a)	**16.** (c)	**17.** (b)	**18.** (c)	**19.** (b)	**20.** (a)

10

COMPOUND INTEREST

INTRODUCTION

In the previous chapter we discussed simple interest. A second method of paying interest is the *compound interest* method, where the interest for each period is added to the principal before interest is calculated for the next period. With this method the principal grows as the interest is added to it. This method is used in investments such as savings account and bonds. An understanding of compound interest is important not only for people planning careers with financial institutions, but also for anyone planning to invest money.

SOME BASIC FORMULAE

1. (a) The amount A due after t years, when a principal P is given on compound interest at the rate R% per annum is given by

$$A = P\left(1+\frac{R}{100}\right)^t.$$

(b) Compound interest (CI) = $A - P$

$$= P\left[\left(1+\frac{R}{100}\right)^t - 1\right]$$

(c) Rate of interest (R) = $\left[\left(\frac{A}{P}\right)^{1/t} - 1\right]$% p.a.

Note: Simple interest and compound interest for 1 year at a given rate of interest p.a. are always equal.

Illustration 1 Mohan invested an amount of Rs. 15000 at compound interest rate 5% p.a. for a period of 2 years. What amount will he receive at the end of 2 years?

Solution Here, P = 15000, R = 5 and t = 2.

$$\therefore \text{ Amount} = P\left(1+\frac{R}{100}\right)^t$$

$$= 15000\left(1+\frac{5}{100}\right)^2 = 15000\left(1+\frac{1}{20}\right)^2$$

$$= \frac{15000\times 21\times 21}{20\times 20} = \text{Rs. } 16537.50.$$

Illustration 2 Find compound interest on Rs. 5000 for 2 years at 4% p.a.

Solution Here, P = 5000, R = 4 and t = 2.

$$\therefore \text{ CI} = P\left[\left(1+\frac{R}{100}\right)^t - 1\right]$$

$$= 5000\left[\left(1+\frac{4}{100}\right)^2 - 1\right]$$

$$= 5000\left[\left(\frac{26}{25}\right)^2 - 1\right] = 5000((1.04)^2 - 1)$$

$$= 5000(1.0816 - 1) = \text{Rs. } 408.$$

$\therefore$ The compound interest is Rs. 408.

Illustration 3 Rashi invested Rs. 16000 for two years at compound interest and received an amount of Rs. 17640 on maturity. What is the rate of interest?

Solution Here, P = 16000, t = 2 and A = 17640.

$$\therefore \quad R = 100\left[\left(\frac{A}{P}\right)^{1/t} - 1\right]\% \text{ p.a.}$$

$$= 100\left[\left(\frac{17640}{16000}\right)^{1/2} - 1\right]\% \text{ p.a.}$$

$$= 100\left[\left(\frac{441}{400}\right)^{1/2} - 1\right]\% \text{ p.a.}$$

$$= 100\left[\left(\frac{21}{20}\right)^{2\times\frac{1}{2}} - 1\right]\% \text{ p.a.}$$

$$= 100 \times \frac{1}{20} = 5\% \text{ p.a.}$$

2. If the interest is compounded half-yearly, then

(a) Amount $(A) = P\left(1+\frac{R}{100\times 2}\right)^{2t}$.

(b) Compound interest (CI)

$$= P\left[\left(1+\frac{R}{100\times 2}\right)^{2t}-1\right]$$

(c) Rate $(R) = 2 \times 100\left[\left(\frac{A}{P}\right)^{\frac{1}{t}\times 2}-1\right]$% p.a.

Illustration 4 Find the amount of Rs. 8000 in $1\frac{1}{2}$ years at 5% per annum compound interest payable half-yearly.

Solution Here, $P = 8000$, $R = 5$ and $t = \frac{3}{2}$.

$$\therefore \text{ Amount} = P\left(1+\frac{R}{100\times 2}\right)^{2t}$$

$$= 8000\left(1+\frac{5}{100\times 2}\right)^{2\times\frac{3}{2}} = 8000\left(\frac{41}{40}\right)^{3}$$

$$= \frac{8000\times 41\times 41\times 41}{40\times 40\times 40} = \text{Rs. } 8615.13.$$

3. If the interest is compounded quarterly, then

(a) Amount $(A) = P\left(1+\frac{P}{100\times 4}\right)^{4t}$

(b) Compound interest (CI)

$$= P\left[\left(1+\frac{R}{100\times 4}\right)^{4t}-1\right]$$

(c) Rate $(R) = 4 \times 100\left[\left(\frac{A}{P}\right)^{\frac{1}{t}\times 4}-1\right]$% p.a.

In general, if the interest is compounded n times a year, then

(a) Amount $(A) = P\left(1+\frac{R}{100\times n}\right)^{n\times t}$

(b) Compound interest (CI)

$$= P\left[\left(1+\frac{R}{100\times n}\right)^{n\times t}-1\right]$$

(c) Rate of interest (R)

$$= n \times 100\left[\left(\frac{A}{P}\right)^{\frac{1}{t}\times n}-1\right]\% \text{ p.a.}$$

Illustration 5 Find the compound interest on Rs. 1000 at 40% per annum compounded quarterly for 1 year.

Solution Here, $P = 1000$, $R = 40$ and $t = 1$.

$\therefore$ Compound interest (CI)

$$= P\left[\left(1+\frac{R}{100\times 4}\right)^{4\times t}-1\right]$$

$$= 1000\left[\left(1+\frac{40}{100\times 4}\right)^{4\times 1}-1\right]$$

$$= 1000\left[\left(\frac{11}{10}\right)^{4}-1\right]$$

$$= 1000\left[\frac{14641-10000}{10000}\right]$$

$$= \text{Rs. } 464.10.$$

Illustration 6 Find the compound interest on Rs. 4000 at 24% per annum for 3 months, compounded monthly.

Solution Here, $P = 4000$, $R = 24$ and $t = \frac{3}{12}$.

$$\therefore \quad CI = P\left[\left(1+\frac{R}{100\times 12}\right)^{12\times t}-1\right]$$

$$= 4000\left[\left(1+\frac{24}{100\times 12}\right)^{12\times\frac{3}{12}}-1\right]$$

$$= 4000\left[\left(\frac{51}{50}\right)^{3}-1\right] = \frac{4000\times 7651}{50\times 50\times 50}$$

$$= \text{Rs. } 244.83.$$

SOME USEFUL SHORT-CUT METHODS

1. When the rates of interest are different for different years, say R_1, R_2, R_3 per cent for first, second and third year respectively, then

$$\text{Amount} = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right).$$

Explanation

Let the given sum of money be Rs. P. Amount after first year $= P\left(1+\frac{R_1}{100}\right)$.

This amount will be the principal for the second year.

∴ Amount after second year

$$= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right).$$

This amount will be the principal for the third year.

∴ Amount after third year

$$= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right).$$

Illustration 7 Anu invests Rs. 5000 in a bond which gives interest at 4% per annum during the first year, 5% during the second year and 10% during the third year. How much does she get at the end of the third year.

Solution Here, $P = 5000$, $R_1 = 4$, $R_2 = 5$ and $R_3 = 10$.

∴ Amount at the end of third year

$$= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$$

$$= 5000\left(1+\frac{4}{100}\right)\left(1+\frac{5}{100}\right)\left(1+\frac{10}{100}\right)$$

$$= 5000 \times \frac{26}{25}\times\frac{21}{20}\times\frac{11}{10} = \text{Rs. } 6006.$$

2. When the time is given in the form of fraction, say $3\frac{3}{4}$ years, then

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^3 \times \left(1+\frac{\frac{3}{4}R}{100}\right).$$

Illustration 8 What will be the compound interest on Rs. 15625 for $2\frac{1}{2}$ years at 4% per annum?

Solution $$\text{CI} = 15625\left[\left(1+\frac{4}{100}\right)^2\left(1+\frac{4\times\frac{1}{2}}{100}\right)-1\right]$$

$$= 15625\left[\frac{26}{25}\times\frac{26}{25}\times\frac{51}{50}-1\right]$$

$$= \frac{15625\times 3226}{31250} = \text{Rs. } 1613.$$

3. (a) The difference between the compound interest and the simple interest on a certain sum of money for 2 years at R% per annum is given by

$$\text{CI} - SI = P\left(\frac{R}{100}\right)^2 \quad \text{[in terms of } P \text{ and } R\text{]}$$

$$\text{and, CI} - SI = \frac{R\times SI}{2\times 100} \quad \text{[in terms of } SI \text{ and } R\text{]}$$

Explanation

Let Rs. P be given sum of money. Simple interest on Rs. P for 2 years at R% per annum

$$= \frac{P\times R\times 2}{100}$$

and compound interest on Rs. P for 2 years at R% per annum

$$= P\left[\left(1+\frac{R}{100}\right)^2-1\right].$$

$$\therefore\ CI - SI = P\left[\left(1+\frac{R}{100}\right)^2-1\right]-\frac{P\times R\times 2}{100}$$

$$= P\left[1+\frac{R^2}{10000}+\frac{2R}{100}-1-\frac{2R}{100}\right]$$

$$= P\left(\frac{R}{100}\right)^2.$$

$$\text{Also, } CI - SI = P\left(\frac{R}{100}\right)^2 = \frac{R}{100\times 2}\times\left(\frac{P\times R\times 2}{100}\right)$$

$$= \frac{R\times SI}{2\times 100}.$$

(b) The difference between the compound interest and the simple interest on a certain sum of money for 2 years at R% per annum is given by

$$CI - SI = P\left[\left(\frac{R}{100}\right)^3+3\left(\frac{R}{100}\right)^2\right]$$

[in terms of P and R]

$$\text{and, } CI - SI = \frac{SI}{3}\left[\left(\frac{R}{100}\right)^2+3\left(\frac{R}{100}\right)\right]$$

[in terms of SI and R]

Explanation

Let Rs. P be the given sum of money. Simple interest on Rs. P for 3 years at R% per annum

$$= \frac{P\times R\times 3}{100}$$

and compound interest on Rs. P for 3 years at R% per annum

$$= P\left[\left(1+\frac{R}{100}\right)^3-1\right]$$

$$\therefore\ CI - SI = P\left[\left(1+\frac{R}{100}\right)^3-1\right]-\frac{P\times R\times 3}{100}$$

$$= P\left[1+\frac{R^3}{100000}+\frac{3R^2}{10000}+\frac{3R}{100}-1-\frac{3R}{100}\right]$$

$$= P\left[\frac{R^3}{1000000}+\frac{3R^2}{1000}\right]$$

$$= P\left[\left(\frac{R}{100}\right)^3 + 3\left(\frac{R}{100}\right)^2\right].$$

$$= \frac{P \times R \times 3}{100} \times \frac{1}{3}\left[\left(\frac{R}{100}\right)^2 + 3\left(\frac{R}{100}\right)\right]$$

$$= \frac{SI}{3}\left[\left(\frac{R}{100}\right)^2 + 3\left(\frac{R}{100}\right)\right].$$

Illustration 9 What will be the difference between simple and compound interest on a sum of Rs. 4500 put for 2 years at 5% per annum?

Solution Here, $P = 4500$ and $R = 5$.

$$\therefore \quad CI - SI = P\left(\frac{R}{100}\right)^2 = 4500\left(\frac{5}{100}\right)^2$$

$$= \frac{4500}{20 \times 20} = \text{Rs. } 11{\cdot}25.$$

Illustration 10 If the difference between the compound interest and simple interest on a certain sum of money for 3 years at 5% per annum is Rs. 61, find the sum.

Solution Here, CI $- SI = 61$ and $R = 5$.

$$\therefore \quad \text{CI} - SI = P\left[\left(\frac{R}{100}\right)^3 + 3\left(\frac{R}{100}\right)^2\right]$$

$$\Rightarrow \quad 61 = P\left[\left(\frac{5}{100}\right)^3 + 3\left(\frac{5}{100}\right)^2\right]$$

$$= P\left[\left(\frac{1}{20}\right)^3 + 3\left(\frac{1}{20}\right)^2\right]$$

$$= P\left[\frac{1 + 3 \times 20}{20 \times 20 \times 20}\right] = P\left(\frac{61}{20 \times 20 \times 20}\right)$$

$$\Rightarrow \quad P = \text{Rs. } 8000.$$

4. If a certain sum becomes n times in t years at compound interest, then the same sum becomes n^m times in mt years.

Explanation

Let Rs. P be the given sum of money. We have,

$$nP = P\left(1 + \frac{R}{100}\right)^t \Rightarrow n = \left(1 + \frac{R}{100}\right)^t \quad \ldots(1)$$

Let the sum become n^m times in T years.

Then, $$n^m = \left(1 + \frac{R}{100}\right)^T$$

or, $$n = \left(1 + \frac{R}{100}\right)^{T/m} \quad \ldots(2)$$

On comparing equations (1) and (2), we get

$T/m = t$ or $T = mt$ years.

Therefore, the sum becomes n^m times in mt years.

Illustration 11 A sum of money placed at compound interest doubles itself in 3 years. In how many years will it amount to four times itself?

Solution Here, $n = 2$, $t = 3$ and $m = 2$

$\therefore$ The given sum of money will become four times itself in mt, i.e. $2 \times 3 = 6$ years.

5. If a certain sum becomes n times in t years, then the rate of compound interest is given by

$$R = 100[(n)^{1/t} - 1].$$

Illustration 12 At what rate per cent compound interest does a sum of money become four-fold in 2 years?

Solution The required rate per cent is

$$R = 100[(n)^{1/t} - 1] = 100[(4)^{1/2} - 1]$$

$$= 100(2 - 1) = 100\%.$$

[Here $n = 4$ and $t = 2$]

6. If a certain sum of money at compound interest amounts to Rs. x in A years and to Rs. y in B years, then the rate of interest per annum is

$$R = \left[\left(\frac{y}{x}\right)^{1/B-A} - 1\right] \times 100\%.$$

Explanation

Let the principal be Rs. P and the rate of interest be R% p.a.

$$\text{Given}: x = P\left(1 + \frac{R}{100}\right)^A \text{ and } y = P\left(1 + \frac{R}{100}\right)^B.$$

$$\therefore \quad \frac{y}{x} = \frac{\left(1 + \frac{R}{100}\right)^B}{\left(1 + \frac{R}{100}\right)^A} = \left(1 + \frac{R}{100}\right)^{B-A}.$$

$$\therefore \quad \left(\frac{y}{x}\right)^{1/B-A} = 1 + \frac{R}{100}$$

or, $$\frac{R}{100} = \left(\frac{y}{x}\right)^{1/B-A} - 1$$

or, $$R = \left[\left(\frac{y}{x}\right)^{1/B-A} - 1\right] \times 100.$$

Illustration 13 A sum of money at compound interest amounts to Rs. 4050 in one year and to Rs. 4723.92 in 3 years. Find the rate of interest per annum.

Solution Here, $x = 4050$, $y = 4723.92$, $A = 1$ and $B = 3$.

$$\therefore \quad R = \left[\left(\frac{y}{x}\right)^{1/B-A} - 1\right] \times 100\%$$

$$= \left[\left(\frac{4723\cdot 92}{4050}\right)^{1/2} - 1\right] \times 100\%$$

$$= \left(\frac{27}{25} - 1\right) \times 100\% = 8\%.$$

7. If a loan of Rs. P at R% compound interest per annum is to be repaid in n equal yearly instalments, then the value of each instalment is given by

$$\text{Rs. } \frac{P}{\left(\frac{100}{100+R}\right) + \left(\frac{100}{100+R}\right)^2 + \ldots \left(\frac{100}{100+R}\right)^n}.$$

Explanation

Let each instalment be of Rs. X.

$\therefore$ Principal for the amount of Rs. X due at end of first year at R%

$$= \frac{100X}{100+R}.$$

Principal for the amount of Rs. X due at the end of second year at R%

$$= \left(\frac{100}{100+R}\right)^2 X$$

$\vdots \qquad \vdots \qquad \vdots$

Principal for the amount of Rs. X due at the end of nth year at R%

$$= \left(\frac{100}{100+R}\right)^n X.$$

$$\therefore \frac{100X}{100+R} + \left(\frac{100}{100+R}\right)^2 X + \ldots \left(\frac{100}{100+R}\right)^n X = P.$$

$$\text{or, } X = \text{Rs. } \frac{P}{\left(\frac{100}{100+R}\right) + \left(\frac{100}{100+R}\right)^2 + \ldots \left(\frac{100}{100+R}\right)^n}.$$

Illustration 14 If a sum of Rs. 13040 is to be paid back in two equal annual instalments at $3\frac{3}{4}$% per annum, what is the amount of each instalment?

Solution Each instalment $= \dfrac{P}{\left(\frac{100}{100+R}\right) + \left(\frac{100}{100+R}\right)^2}$

$$= \frac{13040}{\left(\frac{100}{100+\frac{15}{4}}\right) + \left(\frac{100}{100+\frac{15}{4}}\right)^2}$$

$$\left[\text{Here } P = 13040 \text{ and } R = \frac{15}{4}\right]$$

$$= \frac{13040}{\frac{400}{415} + \left(\frac{400}{415}\right)^2} = \frac{13040}{\frac{400}{415}\left(1 + \frac{400}{415}\right)}$$

$$= 13040 \times \frac{415}{400} \times \frac{815}{400} = \text{Rs. } 6889.$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Nikita invested Rs. 8000 for 3 years at 5% CI in a post office. If the interest is compounded once in a year, what sum will she get after 3 years?
(a) Rs. 9261 (b) Rs. 8265
(c) Rs. 9365 (d) None of these

2. The compound interest on Rs. 2000 at 5% per annum, compounded yearly, for 2 years is
(a) Rs. 315 (b) Rs. 425
(c) Rs. 205 (d) None of these

3. At what rate per cent per annum will Rs. 1000 amount to Rs. 1331 in 3 years? The interest is compounded yearly.
(a) 10% p.a. (b) 12% p.a.
(c) 13% p.a. (d) None of these

4. Find the present worth of Rs. 9261 due 3 years hence at 5% per annum compounded yearly.
(a) Rs. 7000 (b) Rs. 8000
(c) Rs. 9000 (d) None of these

5. The compound interest on Rs. 10000 at 20% per annum at the end of 1 year 6 months if the interest is calculated half-yearly will be
(a) Rs. 5320 (b) Rs. 3310
(c) Rs. 4340 (d) None of these

6. A sum put out at 4% compound interest payable half-yearly amounts to Rs. 6632.55 in $1\frac{1}{2}$ years. The sum is
(a) Rs. 6530 (b) Rs. 6250
(c) Rs. 6470 (d) None of these

7. The compound interest on Rs. 12000 for 9 months at 20% per annum, interest being compounded quarterly, is
(a) Rs. 1891.50 (b) Rs. 1901.50
(c) Rs. 1791.50 (d) None of these

8. The difference of compound interest on Rs. 800 for 1 year at 20% per annum when compounded half yearly and quarterly is

(a) Rs. 4.40 (b) Nil
(c) Rs. 6.40 (d) None of these

9. The difference between the simple interest and the compound interest on Rs. 60 for 1 year at 10% per annum, reckoned half-yearly is
(a) Re. 1 (b) Rs. $1\frac{1}{2}$
(c) Rs. 2 (d) None of these

10. Rs. 800 at 5% per annum compound interest amount to Rs. 882 in
(a) 6 years (b) 2 years
(c) 4 years (d) None of these

11. What will be the compound interest on a sum of Rs. 1875 after 2 years if the rate of interest for the first year is 4% and that for the second year is 8%?
(a) Rs. 231 (b) Rs. 341
(c) Rs. 241 (d) None of these

12. What will be the amount if a sum of Rs. 5000 is placed at compound interest for 3 years while rate of interest for the first, second and third years is 2, 3 and 4 per cent, respectively?
(a) Rs. 5643·12 (b) Rs. 5463·12
(c) Rs. 6413·12 (d) None of these

13. What sum will amount to Rs. 15916.59 in 3 years at compound interest, the interest for first, second and third year being 3, 2 and 1 per cent, respectively?
(a) Rs. 18000 (b) Rs. 12000
(c) Rs. 15000 (d) None of these

14. The compound interest on Rs. 800 in $2\frac{1}{2}$ years at 5% is
(a) Rs. 105.05 (b) Rs. 104.05
(c) Rs. 106.05 (d) None of these

15. On what sum will the compound interest for $2\frac{1}{2}$ years at 10% amount to Rs. 6352.50?
(a) Rs. 7000 (b) Rs. 8000
(c) Rs. 5000 (d) None of these

16. The compound interest on a sum of money for 3 years at 5% is Rs. 1324·05. What is the simple interest?
(a) Rs. 1460 (b) Rs. 1365
(c) Rs. 1260 (d) None of these

17. The simple interest on a certain sum at 4% per annum for 2 years is Rs. 80. The compound interest on the same sum for the same period is
(a) Rs. 91.60 (b) Rs. 81.60
(c) Rs. 71.60 (d) None of these

18. If the compound interest on a certain sum for 2 years is Rs. 60·60 and the simple interest is Rs. 60, then the rate of interest per annum is
(a) 2% (b) 3%
(c) 4% (d) None of these

19. If the compound interest on a certain sum for 2 years is Rs. 105 and simple interest is Rs. 100, then the sum is
(a) Rs. 300 (b) Rs. 500
(c) Rs. 400 (d) None of these

20. The difference between simple interest and compound interest on Rs. 1250 for 2 years at 4% p.a. is
(a) Rs. 3 (b) Rs. 4
(c) Rs. 2 (d) None of these

21. On a certain sum of money, the simple interest for 2 years is Rs. 200 at the rate of 7% per annum. Find the difference in *CI* and *SI*.
(a) Rs. 7 (b) Rs. 9
(c) Rs. 11 (d) None of these

22. The difference between the compound interest and simple interest on a certain sum at 5% for 2 years is Rs. 1.50. The sum is
(a) Rs. 700 (b) Rs. 600
(c) Rs. 500 (d) None of these

23. The difference between the compound interest and simple interest on a certain sum at 3% per annum for 3 years is Rs. 27.27. The sum is
(a) Rs. 12000 (b) Rs. 15000
(c) Rs. 10000 (d) None of these

24. The difference between the compound interest and the simple interest on Rs. 8000 for 3 years at 5% per annum is
(a) Rs. 61 (b) Rs. 63
(c) Rs. 65 (d) None of these

25. If a sum of money at compound interest amounts to thrice itself in 3 years, then in how many years will it be 9 times itself?
(a) 9 years (b) 6 years
(c) 7 years (d) None of these

26. At what rate per cent compound interest does a sum of money become 16 times in 4 years?
(a) 75% (b) 100%
(c) 50% (d) None of these

27. A certain sum of money at compound interest grows up to Rs. 12960 in 2 years and up to Rs. 13176 in 3 years. Find the rate per cent per annum.
(a) $1\frac{1}{3}\%$ (b) $2\frac{1}{3}$
(c) $1\frac{2}{3}\%$ (d) None of these

28. What sum of money at compound interest will amount to Rs. 650 at the end of the first year and Rs. 676 at the end of the second year?
(a) Rs. 825 (b) Rs. 925
(c) Rs. 625 (d) None of these

29. A sum of Rs. 1260 is borrowed from a money lender at 10% p.a. compounded annually. If the

amount is to be paid back in two equal annual instalments, find the annual instalment.
(a) Rs. 726 (b) Rs. 626
(c) Rs. 526 (d) None of these

30. A tree increases annually by $\frac{1}{8}$th of its height. By how much will it increase after 2 years, if it stands today 64 cm high?
(a) 72 cm (b) 74 cm
(c) 75 cm (d) 81 cm

31. The least number of completed years in which a sum of money put out at 20% C.I. will be more than doubled is
(a) 3 (b) 4
(c) 5 (d) 6

32. A man borrows Rs. 4000 from a bank at $7\frac{1}{2}$% compound interest. At the end of every year he pays Rs. 1500 as part repayment of loan and interest. How much does he still owe to the bank after three such instalments?
(a) Rs. 123.25 (b) Rs. 125
(c) Rs. 400 (d) Rs. 469.18

33. If in a certain number of years Rs. 3000 amount to Rs. 4320 at compound interest, in half that time Rs. 3000 will amount to:
(a) Rs. 3400 (b) Rs. 3600
(c) Rs. 3800 (d) Rs. 3520

34. Rs. 3757 is to be divided between A and B such that A's share at the end of 7 years may be equal to B's share at the end of 9 years. If rate per cent be 10% p.a. compound interest, B's share is
(a) Rs. 1700 (b) Rs. 1500
(c) Rs. 2057 (d) Rs. 1400

EXPLANATORY ANSWERS

1. Ans. (a)

Solution Here, $P = 8000$, $t = 3$ and $R = 5$.

$$\therefore \text{ Amount } = P\left(1+\frac{R}{100}\right)^t = 8000\left(1+\frac{5}{100}\right)^3$$
$$= 8000\left(\frac{21}{20}\right)^3 = \frac{8000\times21\times21\times21}{20\times20}$$
$$= \text{Rs. } 9261.$$

$\therefore$ Nikita will get Rs. 9261 after 3 years.

2. Ans. (c)

Solution Here, $P = 2000$, $R = 5$ and $t = 2$.

$$\therefore CI = P\left[\left(1+\frac{R}{100}\right)^t - 1\right]$$
$$= 2000\left[\left(1+\frac{5}{100}\right)^2 - 1\right] = 2000\left[\left(\frac{21}{20}\right)^2 - 1\right]$$
$$= 2000\left(\frac{441}{400} - 1\right) = 2000 \times \frac{41}{400} = \text{Rs. } 205.$$

3. Ans. (a)

Solution Here, $P = 1000$, $A = 1331$ and $t = 3$.

$$\therefore P = 100\left[\left(\frac{A}{P}\right)^{1/t} - 1\right]\% \text{ p.a.}$$
$$= 100\left[\left(\frac{1331}{1000}\right)^{1/3} - 1\right]\% \text{ p.a.}$$
$$= 100\left[\left(\frac{11}{10}\right)^{3\times\frac{1}{3}} - 1\right] = 100 \times \frac{1}{10} = 10\% \text{ p.a.}$$

4. Ans. (b)

Solution Here, $A = 9261$, $t = 3$ and $R = 5$.

$$\therefore P = \frac{A}{\left(1+\frac{R}{100}\right)^t} = \frac{9261}{\left(1+\frac{5}{100}\right)^3}$$
$$= \frac{9261\times20\times20\times20}{21\times21\times21} = \text{Rs. } 8000.$$

5. Ans. (b)

Solution Here, $P = 10000$, $R = 20$ and $t = \frac{3}{2}$.

$$\therefore CI = P\left[\left(1+\frac{R}{100\times2}\right)^{2t} - 1\right]$$
$$= 10000\left[\left(1+\frac{20}{100\times2}\right)^{2\times\frac{3}{2}} - 1\right]$$
$$= 10000\left[\left(\frac{11}{10}\right)^3 - 1\right]$$
$$= 10000\left(\frac{11\times11\times11}{10\times10\times10} - 1\right)$$
$$= \frac{10000\times331}{1000} = \text{Rs. } 3310.$$

6. Ans (b)

Solution Let Rs. x be the sum. Then,

$$6632.55 = x\left(1+\frac{4}{2\times100}\right)^{2\times\frac{3}{2}} = x\left(\frac{51}{50}\right)^3$$
$$\therefore x = \frac{6632\cdot55\times50\times50\times50}{51\times51\times51} = \text{Rs. } 6250.$$

7. Ans. (a)

Solution Here, $P = 12000$, $R = 20$ and $t = \frac{9}{12}$

$$\therefore \quad CI = P\left[\left(1+\frac{R}{100\times 4}\right)^{4\times t}-1\right]$$

$$= 12000\left[\left(1+\frac{20}{100\times 4}\right)^{4\times\frac{9}{12}}-1\right]$$

$$= 12000\left[\left(1+\frac{1}{20}\right)^{3}-1\right] = \frac{12000\times 1261}{20\times 20\times 20}$$

$$= \text{Rs. } 1891{\cdot}50.$$

8. Ans. (a)

Solution When compounded half-yearly:

Here, $P = 800$, $R = 20$ and $t = 1$.

$$\therefore \quad CI = P\left[\left(1+\frac{R}{100\times 2}\right)^{2\times t}-1\right]$$

$$= 800\left[\left(1+\frac{20}{100\times 2}\right)^{2\times 1}-1\right]$$

$$= 800\left[\left(\frac{11}{10}\right)^{2}-1\right] = \frac{800\times 21}{10\times 10} = \text{Rs. } 168.$$

When compounded quarterly:

Here, $P = 8000$, $R = 20$ and $t = 1$.

$$\therefore \quad CI = P\left[\left(1+\frac{R}{100\times 4}\right)^{4\times t}-1\right]$$

$$= 800\left[\left(1+\frac{20}{100\times 4}\right)^{4\times 1}-1\right]$$

$$= 800\left[\left(\frac{21}{20}\right)^{4}-1\right] = \frac{800\times 34481}{20\times 20\times 20\times 20}$$

$$= \text{Rs. } 172.40.$$

$\therefore$ Difference = Rs. (172.40 – 168) = Rs. 4.40.

9. Ans. (b)

Solution $SI = \frac{600\times 10\times 1}{100} = \text{Rs. } 60.$

$$CI = 600\left[\left(1+\frac{10}{100\times 2}\right)^{2\times 1}-1\right]$$

$$= 600\left[\left(\frac{21}{20}\right)^{2}-1\right] = \frac{600\times 41}{20\times 20} = \text{Rs. } 61{\cdot}50.$$

$\therefore$ Difference = Rs. (61.50 – 60) = Rs. 1.50.

10. Ans. (b)

Solution Let the time be t years. Then,

$$882 = 800\left(1+\frac{5}{100}\right)^{t} \Rightarrow \frac{882}{800} = \left(\frac{21}{20}\right)^{t}$$

$$\Rightarrow \left(\frac{21}{20}\right)^{2} = \left(\frac{21}{20}\right)^{t}$$

$$\Rightarrow t = 2 \text{ years.}$$

11. Ans. (a)

Solution Here, $P = 1875$, $R_1 = 4$ and $R_2 = 8$.

$$\therefore \quad CI = P\left[\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)-1\right]$$

$$= 1875\left[\left(1+\frac{4}{100}\right)\left(1+\frac{8}{100}\right)-1\right]$$

$$= 1875\left[\frac{26}{25}\times\frac{27}{25}-1\right]$$

$$= \frac{1875\times 77}{625} = \text{Rs. } 231.$$

12. Ans.

Solution Here, $P = 5000$, $R_1 = 2$, $R_2 = 3$ and $R_3 = 4$.

$\therefore$ Amount after 3 years

$$= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$$

$$= 5000\left(1+\frac{2}{100}\right)\left(1+\frac{3}{100}\right)\left(1+\frac{4}{100}\right)$$

$$= 5000 \times \frac{51}{50}\times\frac{103}{100}\times\frac{26}{25}$$

$$= \text{Rs. } 5463{\cdot}12.$$

13. Ans. (c)

Solution Let Rs. P be the required sum.

$$\text{Then,} \quad 15916.59 = P\left(1+\frac{3}{100}\right)\left(1+\frac{2}{100}\right)\left(1+\frac{1}{100}\right)$$

$$= P\left(\frac{103}{100}\times\frac{102}{100}\times\frac{101}{100}\right).$$

$$\therefore \quad P = \frac{1591659\times 100\times 100}{103\times 102\times 101}$$

$$= \text{Rs. } 15000.$$

14. Ans. (b)

Solution $$CI = 800\left[\left(1+\frac{5}{100}\right)^{2}\left(1+\frac{\frac{1}{2}\times 5}{100}\right)-1\right]$$

$$= 800\left[\frac{21\times 21\times 41}{20\times 20\times 40}-1\right]$$

$$= \frac{800\times 2081}{16000}$$

$$= \text{Rs. } 104.05.$$

15. Ans. (c)

Solution We have, $6352\cdot50 = P\left(1+\frac{10}{100}\right)^2\left(1+\frac{\frac{1}{2}\times10}{100}\right)$

$$= P\left(\frac{11}{10}\right)^2\left(\frac{21}{20}\right)$$

$$\Rightarrow \quad P = \frac{6352\cdot50\times10\times10\times20}{11\times11\times21} = \text{Rs. } 5000.$$

16. Ans. (c)

Solution We have, $CI - SI = \frac{SI}{3}\left[\left(\frac{R}{100}\right)^2+3\left(\frac{R}{100}\right)\right]$

$$\Rightarrow \quad 1324\cdot05 - SI = \frac{SI}{3}\left[\left(\frac{5}{100}\right)^2+3\left(\frac{5}{100}\right)\right]$$

$$= \frac{SI}{3}\left[\frac{1+60}{20\times20}\right] = \frac{61SI}{1200}$$

$$\Rightarrow \quad \left(1+\frac{61}{1200}\right)SI = 1324\cdot05$$

$$\Rightarrow \quad SI = \frac{1324\cdot05\times1200}{1261} = \text{Rs. } 1260.$$

17. Ans. (b)

Solution We have, $CI - SI = \frac{R\times SI}{200}$

$$\Rightarrow CI = SI + \frac{R\times SI}{200} = SI\left(1+\frac{R}{200}\right) = 80\left(1+\frac{4}{200}\right)$$

[Here, $R = 4$ and $SI = 80$]

$$= \frac{80\times51}{50} = \text{Rs. } 81.60.$$

18. Ans. (a)

Solution We have,

$$CI - SI = \frac{R\times SI}{200} \Rightarrow 60.60 - 60 = \frac{R\times60}{200}$$

$$\Rightarrow R = \frac{0\cdot60\times200}{60} = 2\%.$$

19. Ans. (b)

Solution We have,

$$CI - SI = \frac{R\times SI}{200} \Rightarrow 105 - 100 = \frac{R\times100}{200}$$

$$\Rightarrow R = 10.$$

Also, $CI - SI = P\left(\frac{R}{100}\right)^2 \Rightarrow 105 - 100 = P\left(\frac{10}{100}\right)^2$

$$\Rightarrow P = \text{Rs. } 500.$$

20. Ans. (c)

Solution We have,

$$CI - SI = P\left(\frac{R}{100}\right)^2 = 1250\left(\frac{4}{100}\right)^2$$

[Here, $P = 1250$ and $R = 4$]

$$= \frac{1250}{15\times25} = \text{Rs. } 2.$$

21. Ans. (a)

Solution We have,

$$CI - SI = \frac{R\times SI}{200} = \frac{7\times200}{200}$$

[Here, $R = 7$ and $SI = 200$]

$$= \text{Rs. } 7.$$

22. Ans. (b)

Solution We have, $CI - SI = P\left(\frac{R}{100}\right)^2$

$$\Rightarrow \quad \frac{3}{2} = P\left(\frac{5}{100}\right)^2 \quad \left[\text{Here, CI} - \text{SI} = \frac{3}{2} \text{ and } R = 5\right]$$

$$\Rightarrow \quad P = \frac{3\times20\times20}{2} = \text{Rs. } 600.$$

23. Ans. (c)

Solution We have, $CI - SI = P\left[\left(\frac{R}{100}\right)^3+3\left(\frac{R}{100}\right)^2\right]$

$$\Rightarrow 27.27 = P\left[\left(\frac{3}{100}\right)^3+3\left(\frac{3}{100}\right)^2\right]$$

[Here, $CI - SI = 27\cdot27$ and $R = 3$]

$$= P\left[\frac{27+2700}{100\times100\times100}\right]$$

$$\Rightarrow P = \frac{27\cdot27\times100\times100\times100}{2727} = \text{Rs. } 10000.$$

24. Ans. (a)

Solution We have,

$$CI - SI = P\left[\left(\frac{R}{100}\right)^3+3\left(\frac{R}{100}\right)^2\right]$$

$$= 8000\left[\left(\frac{5}{100}\right)^3+3\left(\frac{5}{100}\right)^2\right]$$

[Here, $P = 8000$ and $R = 5$]

$$= 8000\left[\frac{125+7500}{100\times100\times100}\right]$$

$$= \text{Rs. } 61.$$

25. Ans. (b)

Solution Here, $n = 3$, $t = 3$ and $m = 2$.

$\therefore$ The given sum will become 9 times itself in mt, i.e. $2 \times 3 = 6$ years.

26. Ans. (b)

Solution The required rate per cent is

$$R = 100[(n)^{1/t} - 1] = 100[(16)^{1/4} - 1]$$

[Here, $n = 16$ and $t = 4$]

$$= 100(2 - 1) = 100\%.$$

27. Ans. (a)

Solution Here, $x = 12960$, $y = 13176$, $A = 2$ and $B = 3$.

$$\therefore \quad R = \left[\left(\frac{y}{x}\right)^{1/B-A} - 1\right] \times 100\%$$

$$= \left(\frac{13176}{12960} - 1\right) \times 100\%$$

$$= \left(\frac{216}{12960} \times 100\right)\%$$

$$= \frac{4}{3}\% \text{ or } 1\frac{1}{3}\%.$$

28. Ans. (c)

Solution Here, $x = 650$, $y = 676$, $A = 1$ and $B = 2$.

$$\therefore \text{ Rate of interest } (R) = \left[\left(\frac{y}{x}\right)^{1/B-A} - 1\right] \times 100\%$$

$$= \left[\frac{676}{650} - 1\right] \times 100\%$$

$$= \frac{26}{650} \times 100\% = 4\%.$$

$$\therefore \quad 650 = P\left(1 + \frac{4}{100}\right) \Rightarrow P = \frac{650 \times 25}{26} = \text{Rs. } 625.$$

29. Ans. (a)

Solution Here, $P = 1260$ and $R = 10$.

$\therefore$ Annual instalment

$$= \frac{P}{\left(\frac{100}{100+R}\right) + \left(\frac{100}{100+R}\right)^2} = \frac{1260}{\frac{100}{110} + \left(\frac{100}{110}\right)^2}$$

$$= \frac{1260}{\frac{100}{110}\left(1 + \frac{100}{110}\right)} = 1260 \times \frac{110}{100} \times \frac{110}{210}$$

$$= \text{Rs. } 726.$$

30. Ans. (d)

Solution Increase % $= \left(\frac{1}{8} \times 100\right)\% = 12.5\%$

$$\text{Height after 2 years} = 64 \times \left(1 + \frac{25}{2 \times 100}\right)^2$$

$$= 64 \times \frac{9}{8} \times \frac{9}{8}$$

$$= 81 \text{ cm.}$$

31. Ans. (b)

Solution $x\left(1 + \frac{20}{100}\right)^n > 2x$ or $\left(\frac{6}{5}\right)^n > 2$

Now, $\left(\frac{6}{5} \times \frac{6}{5} \times \frac{6}{5} \times \frac{6}{5}\right) > 2$

$\therefore \quad n = 4$ years.

32. Ans. (a)

Solution Balance = Rs. $\left[\left\{4000 \times \left(1 + \frac{15}{2 \times 100}\right)^3\right\}\right.$

$$\left. - \left\{1500 \times \left(1 + \frac{15}{2 \times 100}\right)^2 + 1500 \times \left(1 + \frac{15}{2 \times 100}\right) + 1500\right\}\right]$$

$$= \text{Rs. } 123{\cdot}25$$

33. Ans. (b)

Solution Let $r\%$ be the rate and n years be the time.

$$\text{Then, } \quad 4320 = 3000\left(1 + \frac{r}{100}\right)^n$$

$$\therefore \quad \left(1 + \frac{r}{100}\right)^n = \frac{4320}{3000} = 1.44$$

$$\therefore \quad \left(1 + \frac{r}{100}\right)^{n/2} = \sqrt{1.44} = 1.2$$

$\therefore$ In $\frac{n}{2}$ years, Rs. 3000 will amount to

$$3000\left(1 + \frac{r}{100}\right)^{n/2} = 3000 \times 1.2$$

$$= \text{Rs. } 3600.$$

34. Ans. (a)

Solution Let A's share = Rs. x

B's share = Rs. $(3757 - x)$

$$x\left(1 + \frac{10}{100}\right)^7 = (3757 - x)\left(1 + \frac{10}{100}\right)^9$$

$$x = (3757 - x)\left(\frac{11}{10}\right)^2$$

$$\therefore \quad x\left(1 + \frac{121}{100}\right) = \frac{3757 \times 121}{100}$$

$$\therefore \quad x = \frac{375 \times 121}{221} = \text{Rs. } 2057$$

$\therefore$ B's share = Rs. $(3757 - 2057)$ = Rs. 1700.

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. The difference between the simple and the compound interest compounded every six months at the rate of 10 p.c. p.a. at the end of two years is Rs. 124.05. What is the sum?

(a) Rs. 10000 (b) Rs. 6000
(c) Rs. 124.05. (d) Rs. 8000
(e) None of these **[Based on SBI PO 2000]**

2. Ravi invested an amount of Rs 10,000 at compound interest rate of 10 p.c. p.a. for a period of three years. How much amount will Ravi get after three years?
(a) Rs 12310 (b) Rs 13210
(c) Rs 13320 (d) Rs 13120
(e) None of these
[Based on SBI Associates PO 1999]

3. Veena invested an amount of Rs 16000 for two years at compound interest and received an amount of Rs 17640 on maturity. What is the rate of interest?
(a) 8 p.c. p.a. (b) 5 p.c. p.a.
(c) 4 p.c. p.a. (d) Data inadequate
(e) None of these **[Based on Guwahati PO 1999]**

4. The compound interest on any sum at the rate of 5% for two years is Rs 512·50. Find the sum.
(a) Rs 5200 (b) Rs 4800
(c) Rs 5000 (d) Rs 5500
(e) None of these
[Based on BSRB Hyderabad PO 1999]

5. Find the amount on Rs 60,000, if the interest is compounded half-yearly at 4 p.c. p.a. for $1\frac{1}{2}$ years.
(a) Rs 63672.48 (b) Rs 62424.00
(c) Rs 67491.84 (d) Rs 64896.00
(e) None of these **[Based on NABARD 1999]**

6. A person invested a certain amount at simple interest at the rate of 6% p.a. earning Rs 900 as an interest at the end of three years. Had the interest been compounded every year, how much additional interest would he have earned on the same amount with the same interest rate after 3 years?
(a) Rs 38.13 (b) Rs 25.33
(c) Rs 55.08 (d) Rs 35.30
(e) None of these **[Based on NABARD 1999]**

7. What will be the compound interest acquired on a sum of Rs 12000 for 3 years at the rate of 10% p.a.?
(a) Rs 2652 (b) Rs 3972
(c) Rs 3960 (d) Rs 3852
(e) None of these **[Based on BSRB Bhopal PO 2000]**

8. What will be the difference in simple and compound interest on Rs 2000 after three years at the rate of 10% p.a.?
(a) Rs 160 (b) Rs 42
(c) Rs 62 (d) Rs 20
(e) None of these **[Based on IBPS Bank PO 2002]**

9. Mr. Kumar invested money in two schemes *A* and *B*, offering compound interest at 8 p.c.p.a. and 9 p.c.p.a. respectively. If the total amount of interest accrued through the two schemes together in two years was Rs. 4818.30 and the total amount invested was Rs 27,000 what was the amount invested in scheme A?
(a) Rs 15,000 (b) Rs 13,500
(c) Rs 12,000 (d) Cannot be determined
(e) None of these
[Based on Bank of Maharashtra PO 2003]

10. If the simple interest on a sum of money for 2 years at 5% p.a. is Rs. 50. What is the compound interest on the same sum at the same rate and for the same time?
(a) Rs 54.25 (b) Rs. 52
(c) Rs 51.25 (d) Rs. 60
[Based on SI of Police Rec. Exam 1997]

11. The compound interest on a certain sum for 2 years at 10% p.a. is Rs. 525. The simple interest on the same sum for double the time at half the rate per cent per annum is
(a) Rs 400 (b) Rs 600
(c) Rs 500 (d) Rs 800
[Based on SI Rec. Exam (Delhi Police) 1997]

12. The compound interest on a certain sum for 2 years is Rs. 410 and simple interest is Rs. 400. The rate of interest per annum is
(a) 10% (b) 8%
(c) 5% (d) 4%
[Based on SI Rec. Exam (Delhi Police) 1997]

13. The difference between the compound interest and simple interest at the rate of 12% per annum on Rs. 5000 for two years will be
(a) Rs 17.50 (b) Rs 36
(c) Rs 45 (d) Rs 72
[Based on SSC (GL) Prel. Exam 1999]

14. A sum of money doubles itself at compound interest in 15 years. In how many years will it become eight times?
(a) 30 (b) 45
(c) 50 (d) 60
[Based on SSC (GL) Prel. Exam 1999]

15. Compound interest (compounded annually) on a certain sum of money for 2 years at 4% per annum is Rs. 102. The simple interest on the same sum for the same rate and for the same period will be
(a) Rs 99 (b) Rs 101
(c) Rs 100 (d) Rs 98
[Based on SSC (GL) Prel. Exam 1999]

16. On what sum does the difference between the compound interest and the simple interest for 3 years at 10% is Rs. 31?
(a) Rs 1500 (b) Rs 1200
(c) Rs 1100 (d) Rs 1000
[Based on SSC (GL) Prel. Exam 2000]

17. A builder borrows Rs 2550 to be paid back with compound interest at the rate of 4% per annum by the end of 2 years in two equal yearly instalments. How much will each instalment be?
(a) Rs 1352 (b) Rs 1377
(c) Rs 1275 (d) Rs 1283
[Based on SSC (GL) Prel. Exam 2000]

18. At what per cent per annum will Rs 3000 amount to Rs 3993 in 3 years if the interest is compounded annually?
(a) 9% (b) 10%
(c) 11% (d) 13%
[Based on SSC (GL) Prel. Exam 2000]

19. What annual payment will discharge a debt of Rs 1025 due in 2 years at the rate of 5% compound interest?
(a) Rs 551.25 (b) Rs 550.00
(c) Rs 560.00 (d) Rs 560.75
[Based on SSC (GL) Prel. Exam 2000]

20. The compound interest on Rs 10000 in 2 years at 4% per annum, the interest being compounded half-yearly, is
(a) Rs 636.80 (b) Rs 824.32
(c) Rs 912.86 (d) Rs 828.82
[Based on SSC (GL) Prel. Exam 2000]

21. If the difference between the compound interest compounded every six months and the simple interest on a certain sum of money at the rate of 12% per annum for one year is Rs. 36, the sum is
(a) Rs 10000 (b) Rs 12000
(c) Rs 15000 (d) Rs 9000
[Based on SSC (GL) Prel. Exam 2000]

22. In how many years will Rs 2000 amount to Rs 2420 at 10% per annum compound interest?
(a) 3 (b) $2\frac{1}{2}$
(c) 2 (d) $1\frac{1}{2}$
[Based on SSC (GL) Prel. Exam 2000]

23. What is the difference between compound interest on Rs 5000 for $1\frac{1}{2}$ years at 40% per annum according as the interest is compounded yearly or half yearly?
(a) Rs 2.04 (b) Rs 3.06
(c) Rs 8.30 (d) Rs 4.80
(e) None of these
[Based on SSC (GL) Prel Exam 2000]

24. The principal that amounts to Rs 4913 in 3 years at $6\frac{1}{4}$% per annum compound interest compounded annually is
(a) Rs 4096 (b) Rs 4085
(c) Rs 4076 (d) Rs 3096
[Based on SSC (GL) Prel. Exam 2000]

25. What sum lent at 5% per annum compound interest will amount to Rs 441 in 2 years?
(a) Rs 390 (b) Rs 395
(c) Rs 400 (d) Rs 405
[Based on SSC (GL) Prel. Exam 2000]

26. The compound interest on a certain sum for two years is Rs 618 whereas the simple interest on the same sum at the same rate for two years is Rs 600. The rate of interest per annum is
(a) 18% (b) 9%
(c) 6% (d) 3%
[Based on SSC (GL) Prel. Exam 2000]

27. The effective annual rate of interest corresponding to a nominal rate of 6% per annum payable half yearly is
(a) 6.06% (b) 6.07%
(c) 6.08% (d) 6.09%
[Based on SSC (GL) Prel. Exam 2000]

28. The difference between the simple and compound interest on a certain sum of money for 2 years at 4% per annum is Re. 1. The sum is
(a) Rs. 676 (b) Rs. 675
(c) Rs. 625 (d) Rs. 700
[Based on SSC (GL) Prel. Exam 2002]

29. A sum of money doubles itself in 4 years at compound interest. It will amount to 8 times itself at the same rate of interest in
(a) 18 years (b) 12 years
(c) 16 years (d) 24 years
[Based on SSC (GL) Prel. Exam 2002]

30. The difference between the simple and compound interest on a certain sum of money at 5% rate of interest per annum for 2 years is Rs 15. The sum is
(a) Rs 6500 (b) Rs 5500
(c) Rs 6000 (d) Rs 7000
[Based on SSC (GL) Prel. Exam 2002]

31. A sum borrowed under compound interest doubles itself in 10 years. When will it become fourfold of itself at the same rate of interest?
(a) 15 years (b) 20 years
(c) 24 years (d) 40 years
[Based on SSC (GL) Prel. Exam 2002]

32. A sum of money invested at compound interest amounts in 3 years to Rs 2400 and in 4 years to Rs 2520. The interest rate per annum is
(a) 5% (b) 6%
(c) 10% (d) 12%
[Based on SSC (GL) Prel. Exam 2002]

33. A sum of money placed at compound interest doubles itself in 5 years. It will amount to eight times itself at the same rate of interest in
(a) 10 years (b) 15 years
(c) 7 years (d) 20 years
[Based on SSC (GL) Prel. Exam 2002]

34. A sum of money invested at compound interest amounts to Rs 800 in 3 years and to Rs 840 in 4 years. The rate of interest per annum is

(a) $2\frac{1}{2}\%$ (b) 4%

(c) 5% (d) $6\frac{2}{3}\%$

[Based on SSC (GL) Prel. Exam 2002]

35. In what time will Rs 1000 amount to Rs 1331 at 20% per annum, compounded half-yearly?

(a) $1\frac{1}{2}$ years (b) 2 years

(c) 1 year (d) $2\frac{1}{2}$ years

[Based on SSC (GL) Prel. Exam 2003]

36. The difference between simple and compound interest (compounded annually) on a sum of money for 2 years at 10% per annum is Rs 65. The sum is

(a) Rs 6505 (b) Rs 6566

(c) Rs 6565 (d) Rs 6500

[Based on SSC (GL) Prel. Exam 2003]

EXPLANATORY ANSWERS

1. Ans. (d)

Solution Let the sum be Rs. x

Then, $\left[x\left(1+\frac{5}{100}\right)^4 - x\right] - \left[\frac{x\times10\times2}{100}\right] = 124.05$

Solving the above equation, we get x = Rs 8000

2. Ans. (e)

Solution Amount $= 1000\left(1+\frac{10}{100}\right)^3$

$= 10000 \times \frac{11}{10}\times\frac{11}{10}\times\frac{11}{10}$

= Rs 13310.

3. Ans. (b)

Solution $\frac{A}{P} = \left(1+\frac{R}{100}\right)^t \Rightarrow \frac{17640}{16000} = \left(1+\frac{R}{100}\right)^2$

$\Rightarrow \frac{441}{400} = \left(1+\frac{R}{100}\right)^2$

$\Rightarrow \left(\frac{21}{20}\right)^2 = \left(1+\frac{R}{100}\right)^2$

$\Rightarrow 1 + \frac{R}{100} = \frac{21}{20}$

$\Rightarrow \frac{R}{100} = \frac{1}{20}$

or, R = 5% p.a.

4. Ans. (c)

Solution Let the sum be Rs x

$512.50 = x\left[\left(1+\frac{5}{100}\right)^2 - 1\right]$

$512.50 = x\left(\frac{441-400}{400}\right)$

$\therefore \quad x = \frac{512\cdot50\times400}{41}$ = Rs 5000.

5. Ans. (a)

Solution Amount $= 60000\left(1+\frac{2}{100}\right)^3$

$= 60000 \times \frac{51}{50}\times\frac{51}{50}\times\frac{51}{50}$

= Rs 63672.48.

6. Ans. (c)

Solution Certain sum for the person $= \frac{900\times100}{18}$

= Rs. 5000

$\therefore$ Interest on Rs 5000 by C.I.

$= 5000\left(1+\frac{6}{100}\right)^3 - 5000$

= Rs 955.08

$\therefore$ Additional interest = Rs (955.08 – 900)

= Rs 55.08.

7. Ans. (b)

Solution Compound interest $= 12000\left(1+\frac{10}{100}\right)^3 - 12000$

= 15972 – 12000

= Rs 3972.

8. Ans. (c)

Solution Difference $= \frac{\text{Sum}\times(\text{Rate})^2\,(300+\text{Rate})}{(100)^3}$

$= \frac{2000\times10\times10\times310}{100\times100\times100}$ = Rs 62.

9. Ans. (c)

Solution Let, in scheme A, Mr. Kumar invest Rs. x

Then, his investment in scheme B

= Rs 124.05. – x)

Now, $x\left(1+\frac{8}{100}\right)^2 + (27000-x)\left(1+\frac{9}{100}\right)^2 - 27000$

= 4818.30

or, $x(1.08)^2 + (27000 - x)(1.09)2 = 31818.30$

or, $1.1664x + 32078.7 - 1.1881x = 3181.30$

or, $0.0217x = 260.4$

or, $x = \frac{260 \cdot 4}{0 \cdot 0217} = \text{Rs. } 12000.$

10. Ans. (d)

11. Ans. (c)

Solution Let P be the sum.

$$\therefore \quad P\left(1+\frac{10}{100}\right)^2 - P = 525 \Rightarrow P = 2500$$

$$\text{S.I.} = \frac{2500 \times 4 \times 5}{100} = 500.$$

12. Ans. (c)

Solution Let P be the sum and R be the rate %

$$\text{S.I: } 400 = \frac{P \times R \times 2}{100}$$

$$PR = 20000$$

$$\text{C.I: } 410 = P\left(1+\frac{R}{100}\right)^2 - P$$

$$\Rightarrow \quad P\left[1+\frac{R^2}{(100)^2}+\frac{2R}{100}-1\right] = 410$$

$$\Rightarrow \quad PR\left[\frac{R}{(100)^2}+\frac{2}{100}\right] = 410$$

$$\Rightarrow \quad 2000\left[\frac{R+200}{10000}\right] = 410$$

$$\Rightarrow \quad R + 200 = 205 \Rightarrow R = 5.$$

13. Ans. (d)

Solution $\text{S.I.} = \frac{5000 \times 12 \times 2}{100} = \text{Rs } 1200$

$$\text{C.I.} = 5000\left(1+\frac{12}{100}\right)^2 - 5000$$

$$= 5000 \times \frac{56}{50} \times \frac{56}{50} - 5000$$

$$= 5000\left[\frac{(56)^2-(50)^2}{(50)^2}\right]$$

$$= 5000\left[\frac{(56+50)(56-50)}{2500}\right]$$

$$= 2(106)(6) = \text{Rs } 1272$$

$\therefore$ C.I. – S.I. = Rs. 72.

14. Ans. (b)

Solution $P\left(1+\frac{R}{100}\right)^{15} = 2P$

$$\therefore \quad \left(1+\frac{R}{100}\right)^{15} = 2$$

Cubing both sides, we get

$$\left(1+\frac{R}{100}\right)^{45} = 8$$

or, $P\left(1+\frac{R}{100}\right)^{45} = 8P$

Hence, the required time is 45 years.

15. Ans. (c)

Solution C.I. on 1 rupee

$$= \left(1+\frac{4}{100}\right)^2 - 1 = \left(\frac{104}{100}\right)^2 - 1 = \text{Rs } \frac{816}{10000}$$

$$\text{S.I. on 1 rupee} = \frac{4 \times 2}{100} = \text{Rs } \frac{8}{100}$$

$$\therefore \quad \frac{\text{S.I}}{\text{C.I}} = \frac{8}{100} \times \frac{10000}{816} = \frac{800}{816}$$

$$\therefore \quad \text{S.I.} = \frac{800}{816} \times \text{C.I.} = \text{Rs. } \frac{800}{816} \times 102 = \text{Rs } 100.$$

16. Ans. (d)

Solution Suppose sum = Rs 100

$$\therefore \quad \text{S.I.} = \frac{100 \times 3 \times 10}{100} = 30$$

$$\text{C.I.} = 100\left[1+\frac{10}{100}\right]^3 - 100$$

$$= 100 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} - 100$$

$$= \frac{331}{10} = 33.10$$

If the difference is Rs. 3.10, sum = Rs. 100

If the difference is Rs. 31, sum $= \frac{100}{3.10} \times 31$

$= 1000.$

17. Ans. (a)

Solution Let each instalment be Rs. x

$$\therefore \quad x = P_1\left(1+\frac{4}{100}\right)^t \Rightarrow P_1 = \left(\frac{25}{26}\right)^2 x$$

Similarly, $P_2 = \left(\frac{25}{26}\right)^2 \cdot x$

$$\therefore \quad \left(\frac{25}{26}\right)x + \left(\frac{25}{26}\right)^2 x = 2550 \Rightarrow x = \frac{2550 \times 26 \times 26}{25 \times 51}$$

$x = \text{Rs } 1352.$

18. Ans. (b)

Solution $3993 = 3000\left(1+\frac{R}{100}\right)^3$

$$\Rightarrow \quad \frac{1331}{1000} = \left(1+\frac{R}{100}\right)^3 \Rightarrow \left(1+\frac{R}{100}\right)^3 = \left(\frac{11}{10}\right)^3$$

$$\Rightarrow \quad 1 + \frac{R}{100} = \frac{11}{10} \Rightarrow 1 + \frac{R}{100} = 1 + \frac{1}{10}$$

$$\Rightarrow \quad \frac{R}{100} = \frac{1}{10} \Rightarrow R = 10.$$

19. Ans. (a)

Solution Let instalment = Rs x

$$\therefore \quad \left(\frac{20}{21}\right)x+\left(\frac{20}{21}\right)^2 x = 1025$$

$$\Rightarrow \quad \left(\frac{20}{21}\right)x\left[1+\frac{20}{21}\right] = 1025$$

$$\Rightarrow \quad x = \frac{1025\times 21\times 21}{20\times 41} = \text{Rs } 551.25.$$

20. Ans. (b)

Solution $A = P\left(1+\frac{R}{100}\right)^n = 10000\left(1+\frac{2}{100}\right)^4$

$$= 10000 \times \frac{51}{50}\times\frac{51}{50}\times\frac{51}{50}\times\frac{51}{50}$$

$$= \text{Rs } 10824.32$$

Compound Interest = Rs 10824.32 – Rs 10000
= Rs 824.32.

21. Ans. (a)

Solution Compound interest in one year

$$= P\left(1+\frac{6}{100}\right)^2 - P$$

$$= \frac{2809}{2500}P - P = \frac{309}{2500}P$$

Simple interest in one year $= \frac{P\times 12\times 1}{100} = \frac{2}{25}P$

Then, $\frac{309}{2500}P - \frac{3}{25}P = 36 = \frac{309P-300P}{2500} = 26$

$$= \frac{9}{2500}P = 36$$

$$\therefore \quad P = \frac{36\times 2500}{9} = \text{Rs } 10000.$$

22. Ans. (c)

23. Ans. (e)

Solution Required difference

$$= 5000\left(1+\frac{2}{100}\right)^3 - 5000\left(1+\frac{4}{100}\right)^{3/2}$$

$$= 5306.04 - 5252 = \text{Rs } 54.04.$$

24. Ans. (a)

Solution $P\left(1+\frac{25}{400}\right)^3 = 4913$

$$\Rightarrow \quad P = 4913 \times \frac{16}{17}\times\frac{16}{17}\times\frac{16}{17} = 4096.$$

25. Ans. (c)

Solution $P\left(1+\frac{5}{100}\right)^2 = 441$

$$\Rightarrow \quad P = 441 \times \frac{20}{21}\times\frac{20}{21} = 400.$$

26. Ans. (c)

Solution $P\left(1+\frac{R}{100}\right)^2 - P = 618$

$$\frac{P\times R\times 2}{100} = 600$$

Solving these equations, we get $R = 6$.

27. Ans. (d)

Solution $100\left(1+\frac{3}{100}\right)^2 = 100 \times \frac{103}{100}\times\frac{103}{100} = 106.09.$

28. Ans. (c)

Solution Sum = Difference$\left(\frac{100}{R}\right)^2$

$$\therefore \quad \text{Sum} = 1\left(\frac{100}{4}\right)^2 = \text{Rs. } 625.$$

29. Ans. (b)

Solution $P\left(1+\frac{R}{100}\right)^4 = 2P$

$$\therefore \quad \left(1+\frac{R}{100}\right)^4 = 2$$

Cubing both sides, we get

$$\left(1+\frac{R}{100}\right)^{12} = 8 \quad \text{or,} \quad P\left(1+\frac{R}{100}\right)^{12} = 8P.$$

30. Ans. (c)

Solution Let the sum be P

$$\left[P\left(1+\frac{5}{100}\right)^2 - P\right] - \frac{P\times 2\times 5}{100} = 15$$

$$\therefore \quad P = \text{Rs } 6000.$$

31. Ans. (b)

Solution Let the sum be P, rate be r% and required time be t years.

Then, from the equation,

$$2P = P\left(1+\frac{R}{100}\right)^{10} \text{ or, } \left(1+\frac{R}{100}\right)^{10} = 2$$

Again, $4P = P\left(1+\frac{R}{100}\right)^t$ or, $4 = \left[\left(1+\frac{R}{100}\right)^{10}\right]^{\frac{t}{10}}$

or $\quad 2^{\frac{t}{10}} = 4 = 2^2$ or, $\frac{t}{10} = 2$

$\therefore \quad t = 20$ years.

32. Ans. (a)

Solution $2400 = P\left(1+\frac{R}{100}\right)^3$ ...(i)

$$2520 = P\left(1+\frac{R}{100}\right)^4 \quad \text{...(ii)}$$

Equation (i) ÷ Equation (ii), gives

$$\frac{2400}{2520} = \frac{\left(1+\frac{R}{100}\right)^3}{\left(1+\frac{R}{100}\right)^4}$$

or, $\frac{20}{21} = \frac{1}{1+\frac{R}{100}}$ or, $20 + \frac{R}{5} = 21$

$\therefore$ $R = 5\%$.

33. Ans. (b)

34. Ans. (c)

Solution Interest on Rs 800 is Rs 40 (after 1 year).

35. Ans. (a)

Solution Let time be n years

i.e. $1000\left(1+\frac{10}{100}\right)^{2n} = 1331$

or, $\left(\frac{11}{10}\right)^{2n} = \frac{1331}{1000} = \left(\frac{11}{10}\right)^3$

or, $2n = 3$ or, $n = 1\frac{1}{2}$ years.

36. Ans. (d)

Solution Sum = Difference $\left(\frac{100}{R}\right)^2 = 65\left(\frac{100}{10}\right)^2$

$= 65(100) =$ Rs. 6500.

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. If certain sum lent at $3\frac{3}{4}\%$ p.a. for 16 months produces as much simple interest as Rs. 750 lent at 7% p.a. in 5 years, the sum is
(a) Rs. 525 (b) Rs. 5250
(c) Rs. 4250 (d) Rs. 5000

2. A lent Rs. 600 to B for 2 years and Rs. 150 to C for 4 years at the same rate of interest and received altogether from both Rs. 90 as simple interest. The rate of interest was
(a) 5% (b) $4\frac{1}{2}\%$
(c) 3% (d) $2\frac{1}{2}\%$

3. If Rs 5600 amounts to Rs. 6678 in $3\frac{1}{2}$ years, at the same rate p.c. Rs. 9400 in $5\frac{1}{4}$ years will amount to
(a) Rs. 12114.25 (b) Rs. 11214.25
(c) Rs. 10914.50 (d) None of these

4. A sum of money placed at C.I. doubles itself in 5 years. It will amount to eight times itself in
(a) 15 years (b) 20 years
(c) 12 years (d) 10 years

5. What is the principal amount which earns Rs. 132 at 10% per annum?
(a) Rs. 1000 (b) Rs. 1200
(c) Rs. 1320 (d) None of these

6. What will be the amount if a sum of Rs 5000 is placed at compound interest for 3 years while rate of interest for the first, second and third year is 2, 3 and 4%, respectively?
(a) Rs. 5231·20 (b) Rs. 5000.00
(c) Rs. 5463·12 (d) Rs. 5713.20

7. Divide the sum of Rs. 3903 between A and B in such a manner that if both of them put their parts of money at 4% per annum compound interest, amount received by A after 7 years remains equal to the amount received by B after 9 years.
(a) Rs. 1875 (b) Rs. 1587
(c) Rs. 1785 (d) Rs. 1758

8. The amount obtained from an investment of Rs. 3025 at 10% per annum compound interest for 3 years is equal to the amount obtained from the investment of a certain sum of money at 10% per annum compound interest for 5 years. Find the sum of investment in the second case.
(a) Rs 2500 (b) Rs 2415
(c) Rs 2480 (d) Rs 2550

9. Two friends A and B jointly lent out Rs 81600 at 4% per annum compound interest. After 2 years A gets the same amount as B gets after 3 years. The investment made by B was
(a) Rs. 30000 (b) Rs. 40000
(c) Rs. 45000 (d) Rs. 38000

10. A borrowed from a bank Rs 8000 for 2 years at 5% per annum compound interest and B borrowed from a money lender a certain sum of money for 3 years at 4% per annum compound interest. If after expiry of their respective loan period, each of them pay an equal amount to clear off their debts, find the sum of money borrowed by B from the money lender.
(a) Rs. 7840.25 (b) Rs. 7820.75
(c) Rs. 8840.25 (d) Rs. 8200

Test I **[Time: 10 min]**

11. Sanjay put equal amounts of money one at 10% p.a. compound interest payable half-yearly and other at r% p.a. compound interest payable yearly. If he gets equal amounts after 3 years, the value of r is
(a) $10\frac{1}{4}$ (b) $9\frac{1}{2}$%
(c) 10% (d) None of these

12. A certain sum put at compound interest amounts to Rs 699.66 in 2 years. If the rate p.c. for the first year be 4% and $3\frac{1}{2}$% for the second year, the sum is
(a) Rs. 650 (b) Rs. 625
(c) Rs. 599.25 (d) Rs. 602.27

13. If the simple interest on a certain sum of money at 5% per annum for 2 years is Rs 160, what will be the compound interest on the same sum for the same time and the same rate of interest?
(a) Rs. 146 (b) Rs. 168
(c) Rs. 165 (d) Rs. 164
(e) Rs. 170

14. If a certain sum of money amounts to Rs. 800 for 2 years and Rs 880 for 3 years then how much will it amount to in 4 years?
(a) Rs 920 (b) Rs 968
(c) Rs 898 (d) Rs 1000
(e) None of these

15. On what sum of money will be the difference between simple interest and compound interest for 2 years at 4% per annum be equal to Rs. 50?
(a) Rs 31250 (b) Rs 20400
(c) Rs 100000 (b) Rs 25000
(e) None of these

16. If a certain sum of money invested at compound interest amounts to Rs. 1452 in 2 years and Rs 1597.20 in 3 years, what is the rate of interest?
(a) 11% (b) 12%
(c) 10% (d) 9%
(e) None of these

17. A money lender lends Rs. 2000 for 6 months at 20% per annum whereas the interest is compounded quarterly. After the given period he will get the amount of
(a) Rs. 2205 (b) Rs. 2200
(c) Rs. 2160 (d) Rs. 2040
(e) None of these

18. The population of a village decreases at the rate of 20% per annum. If its population 2 years ago was 10,000, what is its present population?
(a) 6000 (b) 9000
(c) 6400 (d) 7600
(e) 19000

19. If the first year's interest on a certain sum of money placed at 5% per annum compound interest is Rs. 1200, what will be the interest for the third year?
(a) Rs. 1220 (b) Rs. 1323
(c) Rs. 1423 (d) Rs. 1330

20. For obtaining the difference between compound and simple interest on a certain sum of money invested for 3 years at 5% p.a., we must multiply the sum of money by
(a) 0.007625 (b) 0.007645
(c) 0.008625 (d) 0.007525

Answers

1. (b)	**2.** (a)	**3.** (a)	**4.** (a)	**5.** (b)	**6.** (c)	**7.** (a)	**8.** (a)	**9.** (b)	**10.** (a)
11. (a)	**12.** (a)	**13.** (d)	**14.** (b)	**15.** (a)	**16.** (c)	**17.** (a)	**18.** (c)	**19.** (b)	**20.** (a)

11

DISCOUNT (TRUE AND BANKER'S)

INTRODUCTION

Suppose, a man buys a pen at a credit of one year for Rs. 105 at 5% simple interest. If the money is to be paid immediately, he shall give Rs. 100. Rs. 100 is the *present value* or *present worth* of Rs. 105 due 1 year hence. The sum due (Rs. 105) is called the *amount* and the reduction made in consideration of making the immediate payment is called *true discount*.

Present Value

The *present value* or *present worth* of a sum of money due at the end of a given time is that sum which with its interest for the given time at the given rate will amount to the sum due.

True Discount (T.D)

The *true discoun*t is the difference between the sum due at the end of a given time and its present worth.

Thus, T.D. = Amount (A) – present worth (P.W.)

In the above case, T.D. = Rs. (105 – 100) = Rs. 5.

Note:

1. Clearly, T.D. is the Interest on P.W. and

$$A = \text{P.W.} + \text{T.D.}$$

2. Interest is reckoned on P.W. and T.D. is reckoned on amount.

Banker's Discount

Suppose, a businessman A purchases goods worth Rs. 10000 from businessman B at a credit of say 3 months. Thus, B prepares a bill, called the *bill of exchange*. On receipt of the goods, A gives an agreement and signs the bill accepting that the money can be withdrawn from his account after 3 months of the date of the bill. Accordingly, A orders his bank to pay Rs. 10000 to B after 3 months. Besides, 3 days grace period is also added to this date (named as *nominally due date*) of expiry of 3 months to arrive at a date called *legally due date*. Thus, if 4 April 2004 is the nominally due date then 7 April 2004 will be legally due date. The amount of Rs. 10000 is called the *face value*.

Now, suppose B needs the money of this bill earlier than 7 April say 7 March. In such a case, B can approach the banker or broker to pay him money against the bill. Obviously, in such a situation, the money paid by the banker will be less than the face value of the bill. Now suppose, the bill is presented to the banker on 7 March 2004, then the banker will deduct the interest on the face value for the period 7 March 2004 to 7 April 2004 and this interest is called the *Banker's Discount* (B.D.) or *Commercial Discount*.

Thus, Banker's Discount is the simple interest on the face value for the period from the date on which the bill was discounted and the legally due date. The amount mentioned in the bill is called the *face value* of the bill. It may be noted that *banker's discount* is greater than *true discount*, because while the true discount is the interest on the present worth, banker's discount is the interest on sum due.

The difference between the present worth and cash value of a bill is called the *banker's gain* (B.G.) Thus, the interest on the bill value (or the face value) is called the banker's discount (B.D) and the difference between the banker's discount and true discount (T.D.) is called banker's gain (B.G.)

We have the following results:

Banker's gain = Banker's discount – True discount

= Interest on sum due – Interest on present worth

= Interest on (sum due – present worth)

= Interest on True Discount.

SOME BASIC FORMULAE

1. If rate = R% p.a. and time = T years, then

(i) $\text{P.W.} = \dfrac{100 \times A}{100 + R \times T} = \dfrac{100 \times \text{T.D.}}{R \times T}$

Illustration 1 Find the present worth of Rs. 8700 due in 3 years at 15% per annum at simple interest.

Also, find the true discount.

Solution $\text{P.W.} = \dfrac{100 \times A}{100 + R \times T} = \dfrac{100 \times 8700}{100 + 15 \times 3}$

$= \dfrac{100 \times 8700}{145} = \text{Rs. } 6000$

T.D. = Amount – P.W.

= 8700 – 6000 = Rs. 2700.

(ii) $\text{T.D.} = \dfrac{\text{P.W.} \times R \times T}{100} = \dfrac{A \times R \times T}{100 + R \times T}$.

Illustration 2 Find the true discount and the sum due 15 months hence whose present value at 8% is Rs. 1000.

Solution $\text{T.D.} = \dfrac{\text{P.W.} \times R \times T}{100} = \dfrac{1000 \times 8 \times 15}{100 \times 12}$

= Rs. 100.

Sum due = P.W. + T.D.

= 1000 + 100 = Rs. 1100.

Illustration 3 Find the true discount reckoning 3% p.a. simple interest on Rs. 1802 due in 2 year's time.

Solution $\text{T.D.} = \dfrac{A \times R \times T}{100 + R \times T} = \dfrac{1802 \times 3 \times 2}{100 + 3 \times 2} = \text{Rs. } 102.$

(iii) $\text{Sum (A)} = \dfrac{\text{S.I.} \times \text{T.D.}}{\text{S.I.} - \text{T.D.}}$

Explanation

$$\frac{\text{S.I.} \times \text{T.D.}}{\text{S.I.} - \text{T.D.}} = \frac{\left(A \times R \times \dfrac{T}{100}\right) \times \text{T.D.}}{\left(\text{T.D.} \times R \times \dfrac{T}{100}\right)} = A$$

$$\therefore \quad A = \frac{\text{S.I.} \times \text{T.D.}}{\text{S.I.} - \text{T.D.}}.$$

Illustration 4 The true discount on a certain sum of money due 2 years hence is Rs. 1800 and the simple interest on the same sum is Rs. 2232. Find the sum.

Solution $\text{Sum (A)} = \dfrac{\text{S.I.} \times \text{T.D.}}{\text{S.I.} - \text{T.D.}} = \dfrac{2232 \times 1800}{2232 - 1800}$

$= \dfrac{2232 \times 1800}{432} = \text{Rs. } 9300.$

(iv) S.I. – T.D. = S.I. on T.D.

Explanation

$$\text{S.I.} - \text{T.D.} = \frac{A \times R \times T}{100} - \frac{\text{P.W.} \times R \times T}{100}$$

$$= (A - \text{P.W.}) \times \frac{R \times T}{100}$$

$$= \frac{\text{T.D.} \times R \times T}{100} = \text{S.I. on T.D.}$$

Illustration 5 The discount on a certain sum due 4 years hence is Rs. 100 but the interest on the same sum for the same period is Rs. 125. Find the sum and the rate per cent.

Solution We have,

S.I. on T.D. = S.I. – T.D.

= 125 – 100 = Rs. 25.

$\therefore \quad \text{Rate } (R) = \dfrac{25 \times 100}{100 \times 4} = \dfrac{25}{4} = 6\dfrac{1}{4}\%$

and, $\text{Sum } (A) = \dfrac{\text{S.I.} \times \text{T.D.}}{\text{S.I.} - \text{T.D.}} = \dfrac{125 \times 100}{125 - 100}$

= Rs. 500.

(v) When the money is invested on compound interest,

$$\text{P.W.} = \frac{A}{\left(1 + \dfrac{R}{100}\right)^T}$$

Illustration 6 Find the present worth of a bill of Rs. 3380 due 2 years hence at 4% compound interest. Also, calculate T.D.

Solution $\text{P.W.} = \dfrac{A}{\left(1 + \dfrac{R}{100}\right)^T} = \dfrac{3380}{\left(1 + \dfrac{4}{100}\right)^2}$

$= \dfrac{3380 \times 25 \times 25}{26 \times 26} = \text{Rs. } 3125.$

T.D. = A – P.W.

= 3380 – 3125 = Rs. 255.

2. B.D. = S.I. on the bill for unexpired time
3. B.G. = B.D. – T.D.
4. B.D. = T.D. + Interest on T.D.
5. $\text{Sum Due} = \dfrac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}}$
6. $\text{T.D.} = \sqrt{\text{P.W.} \times \text{B.G.}}$
7. $\text{T.D.} = \dfrac{\text{B.G.} \times 100}{\text{Rate} \times \text{Time}}$
8. $\text{T.D.} = \dfrac{\text{Amount} \times \text{Rate} \times \text{Time}}{100 + (\text{Rate} \times \text{Time})}$
9. $\text{B.D.} = \dfrac{\text{Amount} \times \text{Rate} \times \text{Time}}{100}$
10. $\text{Sum Due} = \dfrac{\text{B.D.} \times 100}{R \times T}$
11. Money paid by the banker = Amount – B.D.

Illustration 7 A bill is drawn for Rs. 5050 on the 12 June 2004 at 5 months credit. It is discounted on 3 September at 5% per annum. Find:

(i) Banker's Discount,

(ii) Money received by the holder of the bill, and

(iii) Banker's Gain.

Solution Amount = Rs. 5050.

Date of drawing = 12 June, 2004 (for 5 months)

Date of maturing = 15 Nov, 2004 (including 3 days grace)
Date of discounting = 3 September, 2004.
No. of days from 3 Sept. to 15 Nov. =

Sept.	Oct.	Nov.	
27	+ 31	+ 15	= 73 days = $\frac{1}{5}$ year.

$\therefore$ (i) Banker's discount $= \dfrac{\text{Amount} \times \text{Rate} \times \text{Time}}{100}$

$= \dfrac{5050 \times 1 \times 5}{100 \times 5} =$ Rs. 50.50.

(ii) Amount received by the holder of the bill
= Amount – B.D.
= 5050 – 50.50 = Rs. 4999.50.

(iii) True discount on Rs. 5050

$= \dfrac{\text{Amount} \times \text{Rate} \times \text{Time}}{100 + (\text{Rate} \times \text{Time})}$

$= \dfrac{5050 + \frac{1}{5} \times 5}{100 + \frac{1}{5} \times 5} =$ Rs. 50

Banker's gain = B.D. – T.D. = Rs. 0.50.

Illustration 8 The banker's discount and the true discount on a certain sum of money due 4 months hence are Rs. 48 and Rs. 45, respectively. Find the sum and the rate of interest.

Solution Sum $= \dfrac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = \dfrac{48 \times 45}{48 - 45} = \dfrac{48 \times 45}{3}$

= Rs. 720.

Now, the banker's discount is simple interest on the sum due for 4 months.

Rate of interest $= \dfrac{100 \times 48 \times 3}{720 \times 1} = 20\%$ p.a.

Illustration 9 Find the face value of 3 months bill when the banker's discount at 3% per annum is Rs. 18.

Solution B.D. = Rs. 18, Rate (R) = 3%,

Time (T) $= \frac{1}{4}$ years.

$\therefore$ Face value $= \dfrac{\text{B.D.} \times 100}{R \times T} = \dfrac{18 \times 100 \times 4}{3 \times 1}$

= Rs. 2400.

Illustration 10 The present worth of a bill due sometimes hence is Rs. 1500. Find the banker's discount on the bill, if the true discount is Rs. 75.

Solution T.D. $= \sqrt{\text{P.W.} \times \text{B.G.}}$

$\Rightarrow 75 = \sqrt{1500 \times \text{B.G.}}$

$\Rightarrow 75 \times 75 = 1500 \times$ B.G.

$\Rightarrow$ B.G. $= \dfrac{75 \times 75}{1500} =$ Rs. $\dfrac{15}{4}$ or, Rs. 3.75.

B.D. = T.D. + B.G. = 75 + 3.75 = Rs. 78.75.

Illustration 11 The banker's gain on a bill due 1 year hence at 12% per annum is Rs. 6. Find the true discount.

Solution T.D. $= \dfrac{\text{B.G.} \times 100}{R \times T} = \dfrac{6 \times 100}{12 \times 1} =$ Rs. 50.

Illustration 12 If the true discount on a certain sum due 6 months hence at 6% is Rs. 36, what is the banker's discount on the same sum for the same time and at the same rate?

Solution B.D. = T.D. + Interest on T.D.

$= \text{T.D.} + \dfrac{\text{T.D.} \times R \times T}{100} = 36 + \dfrac{36 \times 6 \times 6}{100 \times 12}$

= 36 + 1.08 = Rs. 37.08.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. The true discount on a bill for Rs. 1260 due 6 months hence at 10% per annum is
(a) Rs. 60 (b) Rs. 160
(c) Rs. 80 (d) Rs. 260

2. If the discount on a certain sum in 2 years at a certain rate is Rs. 150 and the interest in 3 years is Rs. 240. Find the sum and the rate %
(a) Rs. 2400, $3\frac{1}{3}\%$ (b) Rs. 2400, $4\frac{1}{3}\%$
(c) Rs. 2200, $5\frac{1}{3}\%$ (d) None of these

3. If the true discount on Rs. 161 due 2 years 6 months hence is Rs. 21, find the rate of interest.
(a) $2\frac{1}{2}\%$ (b) $4\frac{1}{2}\%$
(c) 5% (d) 6%

4. The present worth of Rs. 920 due at the end of 3 years at 5% simple interest per annum is
(a) Rs. 780 (b) Rs. 850
(c) Rs. 800 (d) Rs. 810

5. If the simple interest on a certain sum due some years hence at 6% is Rs. 180 and the discount at 5% on the same sum is Rs. 140. Find the sum and the time.
(a) Rs 2100 and $1\frac{3}{7}$ years

(b) Rs 2200 and $2\frac{3}{7}$ years

(c) Rs 2000 and $2\frac{3}{7}$ years

(d) None of these

6. The banker's gain on a certain sum due 9 months hence at 4% is Rs. 2.25. The sum is
(a) Rs. 2575 (b) Rs. 2500
(c) Rs. 2250 (d) Rs. 3250

7. At a given rate per cent, the simple interest and the true discount on a certain sum for a given time are Rs. 24 and Rs. 22, respectively. The sum is
(a) Rs. 264 (b) Rs. 220
(c) Rs. 288 (d) Rs. 295

8. The present worth of a bill of Rs. 1764 due 2 years hence at 5% compound interest is
(a) Rs. 1650 (b) Rs. 1700
(c) Rs. 1600 (d) Rs. 1714

9. If Rs. 21 in the true discount on Rs. 371 for a certain time, what is the true discount on the same sum for double that time, the rate being the same in both cases?
(a) Rs. 39.00 (b) Rs. 35.75
(c) Rs. 40.00 (d) Rs. 39.75

10. The T.D. on a certain sum of money due 2 years hence is Rs. 1800 and the simple interest on the same sum is Rs. 2232. Find the sum.
(a) Rs. 9300 (b) Rs. 9350
(c) Rs. 9450 (d) Rs. 9400

11. The present worth of Rs. 220.50 due in 2 years reckoning compound interest at 5% is
(a) Rs. 200 (b) Rs. 197.5
(c) Rs. 202 (d) Rs. 192.25

12. The T.D. on Rs. 936 due after a certain time at 8% is Rs. 36. Money is due after
(a) 6 months (b) 3 months
(c) 1 years (d) 9 months

13. A man bought a motor-cycle for Rs. 32500 and sold it for Rs. 35000, allowing the buyer 6 months credit. If the money be worth 4% per annum, the gain per cent is
(a) $8\frac{1}{7}\%$ (b) $7\frac{9}{13}\%$
(c) $7\frac{5}{13}\%$ (d) $8\frac{2}{3}\%$

14. Find the present worth of a bill of Rs. 3720 which is due 2 years hence at 12% compound interest, being compounded annually.
(a) Rs. 3100 (b) Rs. 3150
(c) Rs. 3125 (d) Rs. 3225

15. The holder of a bill for Rs. 17850 nominally due on 21 May, 1991 received Rs. 357 less than the amount of the bill by having it discounted at 5%. When was it doscounted?
(a) 29 Dec. 1990 (b) 30 Dec. 1989
(c) 19 Dec. 1990 (d) None of these

16. The true discount on a certain bill due nine months hence at 4% simple interest is Rs. 150. Find the amount of the bill.
(a) Rs. 5150 (b) Rs. 5250
(c) Rs. 4750 (d) Rs. 5650

17. A banker discounts a 4 months bill at 3% discount. If the proceeds be invested so that nothing is lost, the rate p.c. should be
(a) 3% (b) 4%
(c) $3\frac{1}{33}\%$ (d) None of these

18. The difference between the simple interest and the true discount on a certain sum of money for 2 years at 15% per annum at simple interest is Rs. 45. Find the sum.
(a) Rs. 700 (b) Rs. 650
(c) Rs. 675 (d) Rs. 625

19. The present worth of a sum of money due 146 days hence at 5% is Rs. 400. The sum due is
(a) Rs. 410 (b) Rs. 408
(c) Rs. 415 (d) Rs. 450

20. The present worth of a bill due sometime hence is Rs. 1500. Find the banker's discount on the bill, if the true discount is Rs. 75.
(a) Rs. 78.75 (b) Rs. 77.75
(c) Rs. 82.75 (d) Rs. 76.75

21. If the simple interest on Rs. 2000 at 5% p.a. is equal to the true discount on Rs. 2500 for the same time and at the same rate, the time is
(a) $4\frac{1}{2}$ years (b) 5 years
(c) $7\frac{1}{2}$ years (d) $2\frac{1}{2}$ years

22. Rs. 21 is the true discount on Rs. 371 for a certain time at certain rate p.c. If the rate p.c. is kept same, true discount on the same sum for double that time will be
(a) Rs. 44.38 (b) Rs. 39.75
(c) Rs. 33.25 (d) None of these

23. The true discount on a bill of Rs. 5450 due in 9 months is Rs. 450. Find the rate of interest.
(a) 12% (b) 12.5%
(c) 11.5% (d) 13.1%

24. If Rs. 10 be allowed as true discount on a bill of Rs. 110 due at the end of a certain time, then the discount allowed on the same sum due at the end of double the time is
(a) Rs. 20 (b) Rs. 21.81
(c) Rs. 22 (d) Rs. 18.33

25. A bill which being due at the end of 4 years is now

worth Rs. 575 but if it is due $2\frac{1}{2}$ years hence it would now be worth Rs. 620. The sum of the bill is
(a) Rs. 695 (b) Rs. 725
(c) Rs. 713 (d) None of these

26. Find the present worth (PW) and the true discount reckoning 6% per annum simple interest of Rs. 176 due in 20 months time.
(a) Rs. 160, Rs. 16 (b) Rs. 130, Rs. 46
(c) Rs. 150, Rs. 26 (d) None of these

27. What rate per cent does a man get for his money when in discounting a bill due 10 months hence, he deducts 4% of the amount of the bill?
(a) 5% (b) 6%
(c) 8% (d) 4%

28. The discount on Rs. 5229 due in 1 year 9 months reckoning compound interest at 5% is
(a) Rs. 429.00 (b) Rs. 415.00
(c) Rs. 393.25 (d) None of these

29. A bill is discounted at 5% per annum. If banker's discount be allowed, at what rate perc ent must the proceeds be invested, so that nothing may be lost?
(a) 5% (b) $4\frac{19}{20}\%$
(c) $5\frac{5}{19}\%$ (d) 10%

30. If the compound interest on a certain sum of money for 2 years at 4% is Rs. 45.90, the true discount on the same sum of money due 2 years hence at 4% simple interest is
(a) Rs. 39.69 (b) Rs. 41.67
(c) Rs. 45.00 (d) Rs. 38.45

31. The true discount on a bill of Rs. 2550 due after 3 months is Rs. 50. Find the banker's discount.
(a) Rs. 53 (b) Rs. 51
(c) Rs. 55 (d) Rs. 57

32. A owes *B* Rs. 1350 due 3 months hence and *B* owes *A* Rs. 1078 due 5 months hence. If they agree to settle their account right now at rate 5% p.a., *A* should pay *B*
(a) Rs. $277\frac{1}{3}$ (b) Rs. 288.25
(c) Rs. 302 (d) None of these

33. Rs. 20 is the true discount on Rs. 260 due after a certain time. What will be the true discount on the same sum due after half of the former time, the rate of interest being the same
(a) Rs. 10 (b) Rs. 10.40
(c) Rs. 15.20 (d) Rs. 13

34. What is the rate of interest when the P.W. of Rs. 1245 due 15 months hence is Rs. 1200?
(a) 3% (b) 4%
(c) $4\frac{1}{2}\%$ (d) 5%

35. *A* has to pay Rs. 22 to *B* after 1 year. *B* asks *A* to pay Rs. 110 in cash and defer the payment of Rs. 110 for 2 years. *A* agrees to it. Counting the rate of interest at 10% per annum in this new mode of payment,
(a) there is no gain or loss to anyone
(b) *A* gains Rs. 7.34
(c) *A* loses Rs. 7.34
(d) *A* gains Rs. 11

36. The B.G. on a sum due 3 years hence at 10% is Rs. 180. The B.D. is
(a) Rs. 680 (b) Rs. 780
(c) Rs. 580 (d) Rs. 480

37. If the discount on Rs. 249 at 5% S.I. be Rs. 9, when is the sum due?
(a) 6 months (b) 4 months
(c) 9 months (d) 7 months

38. The banker's gain on a certain sum due 2 years hence at 5% per annum is Rs.8. The present worth is
(a) Rs. 800 (b) Rs. 1600
(c) Rs. 1200 (d) Rs. 880

39. The B.G. on a certain sum due 5 years hence is $\frac{3}{23}$ of B.D. the rate % is
(a) 6% (b) 5%
(c) 4% (d) 3%

EXPLANATORY ANSWERS

1. Ans. (a)

Solution T.D. = Rs $\dfrac{1260\times\frac{1}{2}\times10}{100+\frac{1}{2}\times10}$ = Rs. 60.

2. Ans. (a)

Solution Interest for 2 years = $\dfrac{240\times2}{3}$ = Rs. 160

Discount for 2 years = Rs. 150

Sum due = $\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = \frac{160 \times 150}{160 - 150}$ = Rs. 2400

Rate of interest = $\frac{240 \times 100}{2400 \times 3} = 3\frac{1}{3}\%$.

3. Ans. (d)

Solution Rs. 21 is the interest on Rs. (161 – 21)

or, Rs. 140 for 2 years 6 months

$\therefore$ Rate % = $\frac{21 \times 100}{140 \times 5/2} = 6\%$.

4. Ans. (c)

Solution Present worth = $\frac{A \times 100}{100 + R \times T}$ = Rs. $\frac{920 \times 100}{100 + 3 \times 5}$

= Rs. $\frac{920 \times 100}{115}$ = Rs. 800.

5. Ans. (a)

Solution B.D. or Simple Interest at 5%

= $\frac{180 \times 5}{6}$ = Rs. 150

Discount at 5% = Rs. 140

Sum = $\frac{\text{B.D} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = \frac{150 \times 140}{150 - 140}$ = Rs. 2100.

Rate per cent = $\frac{180 \times 100}{2100 \times 6} = 1\frac{3}{7}$ years.

6. Ans. (a)

Solution B.G. is the interest on T.D.

$\therefore$ T.D. = $\frac{2.25 \times 100}{\frac{3}{4} \times 4}$ = Rs. 75

B.D. = Rs. 75 + Rs. 2.25 = Rs. 77.25

$\therefore$ Sum due = $\frac{\text{B.D.} \times \text{T.D.}}{\text{B.G.}}$ = Rs. $\frac{77.25 \times 75}{2.25}$

= Rs. 2575.

7. Ans. (a)

Solution Sum = $\frac{\text{T.D.} \times \text{S.I.}}{\text{S.I.} - \text{T.D.}} = \frac{24 \times 22}{24 - 22}$ = Rs. 264.

8. Ans. (c)

Solution P.W. = Rs. 1764 ÷ $\left(1 + \frac{5}{100}\right)^2$

= Rs. 1764 × $\frac{400}{441}$ = Rs. 1600.

9. Ans. (d)

Solution Rs. 21 is the interest on Rs. (371 – 21) = Rs. 350

$\therefore$ $\frac{350 \times \text{ no. of years} \times \text{rate}}{100} = 21$

$\Rightarrow$ No. of years × rate = $\frac{2100}{350} = 6$

$\therefore$ Twice no. of years × rate = 12

Now, on Rs. 112, the T.D. is Rs. 12

$\therefore$ on Rs. 371, the T.D. = Rs. $\frac{12}{112} \times 371$

= Rs. 39.75.

10. Ans. (a)

Solution Sum = $\frac{\text{Simple interest on the sum} \times (\text{T.D.})}{\text{Simple interest on the sum} - (\text{T.D.})}$

= Rs. $\left(\frac{2232 \times 1800}{2232 - 1800}\right)$

= Rs. $\left(\frac{2232 \times 1800}{432}\right)$

= Rs. 9300.

11. Ans. (a)

Solution P.W. = $\frac{220.5}{\left(1 + \frac{5}{100}\right)^2} = \frac{220.5 \times 20 \times 20}{21 \times 21}$

= Rs. 200.

12. Ans. (a)

Solution P.W. = Rs. (936 – 36) = Rs. 900

Rs. 36 is S.I. on Rs. 900

$\therefore$ Time = $\frac{36 \times 100}{900 \times 8} = \frac{1}{2}$ year

= 6 months.

13. Ans. (b)

Solution S.P. of motor = cycle = Rs. 35000

Gain = Rs. 35000 – Rs. 32500

= Rs. 2500

$\therefore$ Gain % = $\frac{2500}{32500} \times 100$ %

= $\frac{100}{13}\% = 7\frac{9}{13}\%$.

14. Ans. (c)

Solution P.W. = $\frac{A}{\left(1 + \frac{R}{100}\right)^T}$ = Rs. $\frac{3720}{\left[1 + \frac{12}{100}\right]^2}$

= Rs. 3720 × $\frac{25}{28} \times \frac{25}{28}$ = Rs. 3125.

15. Ans. (a)

Solution Clearly, S.I. on Rs. 17850 at 5% is Rs. 357

$\therefore$ Time = $\left(\frac{100 \times 357}{17850 \times 5}\right) = \frac{2}{5}$ years = 146 days.

So, the bill is 146 days prior to 24 May, the legally due date.

May	April	March	Feb.	Jan.	Dec.	
24	+ 30	+ 31	+ 28	+ 31	+ 2	= 146 days

So, the bill was discounted on 29 Dec. 1990.

16. Ans. (a)

Solution P.W. = T.D. × $\frac{100}{R \times T}$ = Rs. $\frac{150 \times 100}{\frac{9}{12} \times 4}$

= Rs. 5,000

∴ Amount of the bill = Rs. 5000 + Rs. 150

= Rs. 5150.

17. Ans. (c)

Solution 4 months = $\frac{1}{3}$ year

∴ Banker deducts Rs. 3 × $\frac{1}{3}$ = Re. 1 from a bill of Rs. 100

So, the banker pays Rs. (100 – 1) = Rs. 99

So, the bill-holder loses Re. 1

So, for investment Re. 1 should be interest on Rs. 99 for 4 months.

∴ Rate = $\frac{1 \times 100}{99 \times \frac{4}{12}} = \frac{100}{33} = 1\frac{1}{33}\%$.

18. Ans. (b)

Solution Let the sum be Rs. 100

Time = 2 years

Rate = 15% per annum

S.I. = Rs. $\left(\frac{100 \times 15 \times 2}{100}\right)$ = Rs. 30

Time Discount = Rs. $\left[\frac{100 \times R \times T}{100 + (R \times T)}\right]$

= Rs. $\left[\frac{100 \times 15 \times 2}{100 + (15 \times 2)}\right]$

= Rs. $\left[\frac{100 \times 15 \times 2}{130}\right]$ = Rs. $\frac{300}{13}$.

The difference between S.I. and T.D. is

Rs. $\left(30 - \frac{300}{13}\right)$ = Rs. $\frac{90}{13}$

If the difference in S.I. and T.D. is Rs. $\frac{90}{13}$, the sum = Rs. 100.

If the difference in S.I. and T.D. is Re. 1, the sum = Rs. 100 × $\frac{13}{10}$.

If the difference in S.I. and T.D. is Rs. 45, the sum = Rs. 100 × $\frac{13}{90}$ × 45 = Rs. 650.

19. Ans. (b)

Solution T.D. = Rs. 400 × $\frac{146}{365} \times \frac{5}{100}$ = Rs. 8

S.D. = Rs. 400 + Rs. 8 = Rs. 408.

20. Ans. (a)

Solution T.D. = $\sqrt{\text{P.W.} \times \text{B.G.}}$

75 = $\sqrt{1500 \times \text{B.G.}}$

Squaring both the sides

75 × 75 = 1500 × B.G.

B.G. = Rs. $\frac{75 \times 75}{1500}$ = Rs. $\frac{15}{4}$ = Rs. 3.75

B.D. = T.D. + B.G.

= 75 + Rs. 3.75 = Rs. 78.75.

21. Ans. (b)

Solution T.D. = Rs. 500

Rate = 5%

P.W. = Rs. 2000

∴ Time = $\frac{500 \times 100}{2000 \times 5}$ = 5 years.

22. Ans. (b)

Solution P.W. of Rs. 371 = Rs. (371 – 21) = Rs. 350

Also, T.D. = Simple Interest on P.W.

∴ Simple Interest on Rs. 350 for a certain period at certain rate p.c. = Rs. 21

∴ Simple Interest on Rs. 350 for double the period at same rate p.c. = Rs. 42

∴ Rs. 42 is T.D. on Rs. (350 + 42) = Rs. 392 for double the period.

∴ T.D. on Rs. 371 for double the period and same rate p.c. = $\frac{42}{392}$ × 371 = Rs. 39.75.

23. Ans. (a)

Solution Amount = Rs. 5450

P.W. = Amount – T.D.

= Rs. 5450 – Rs. 450

= Rs. 5000

In other words, simple interest on Rs. 5000 for 9 months is Rs. 450

∴ Rate = $\frac{\text{S.I.} \times 100}{\text{P} \times \text{T}} = \frac{450 \times 100 \times 4}{5000 \times 3}$

= 12% per annum.

24. Ans. (d)

Solution S.I. on Rs. (110 – 10) for a given time = Rs. 10

S.I. on Rs. 100 for double the time = Rs. 20

Sum = Rs. (100 + 20) = Rs. 120

T.D. on Rs. 110 = Rs. $\left(\frac{20}{120} \times 110\right)$ = Rs. 18.33.

25. Ans. (c)

Solution Let rate p.c. be r %.

Let Rs. x be the amount of the bill.

Then, $575 = \dfrac{x \times 100}{100 + 4r}$

i.e. $57500 + 2300r = 100x$

$\therefore \quad x = 575 + 23r$ ...(i)

and, $620 = \dfrac{100x}{100 + \frac{5}{2}r}$

$\therefore \quad 62000 + 1550r = 100x$

$\therefore \quad 6200 + 155r = 10x$

$= 5750 + 230r$ [Using (i)]

$\therefore \quad 75r = 450$

$\therefore \quad r = 6$

$\therefore$ (i) $\Rightarrow x = 575 + 138 =$ Rs. 713.

26. Ans. (a)

Solution Present Worth $= \dfrac{100 \times 76}{100 + 6 \times \frac{20}{12}} =$ Rs. 160

True discount = Amount – Present Worth

= Rs. 176 – Rs. 160

= Rs. 16.

27. Ans. (a)

Solution Let the amount of the bill be Rs. 100

Money deducted = Rs. 4

Money received by holder of the bill

= Rs. (100 – 4) = Rs. 96

S.I. on Rs. 96 for 10 months = Rs. 4

Rate $= \dfrac{100 \times 4 \times 6}{96 \times 5} = 5\%$.

28. Ans. (a)

Solution P.W. $= \dfrac{5229}{\left(1 + \frac{5}{100}\right)\left(1 + \frac{3}{4} \cdot \frac{5}{100}\right)}$

$= 5229 \times \dfrac{20}{21} \times \dfrac{80}{83} =$ Rs. 4800

$\therefore$ T.D. = Rs. (5229 – 4800) = Rs. 429.

29. Ans. (c)

Solution Let the sum be Rs. 100. Then, B.D. = Rs. 5

Proceeds = Rs. (100 – 5) = Rs. 95

$\therefore$ Rs. 5 must be the interest on Rs. 95 for 1 year.

So, rate $= \left(\dfrac{100 \times 5}{95 \times 1}\right) = 5\dfrac{5}{19}\%$.

30. Ans. (b)

Solution Let the sum be Rs. x. Then,

$$45.90 = x\left\{\left(1 + \frac{4}{100}\right)^2 - 1\right\}$$

$$= \left\{\left(\frac{26}{25}\right)^2 - 1\right\} = \left\{\frac{676 - 625}{625}\right\}$$

$\therefore \quad x = \dfrac{625 \times 45.9}{51} =$ Rs. 562.5

$\therefore$ T.D. on Rs. 562.5

$= \dfrac{562.5 \times 4 \times 2}{100 + 4 \times 2} = \dfrac{4500}{108} =$ Rs. 41.67.

31. Ans. (b)

Solution T.D. = Rs. 50

P.W. = Rs. 2550 – Rs. 50 = Rs. 2500

Rate of Interest $= \dfrac{50 \times 100 \times 4}{2500 \times 1} = 8\%$ per annum

B.D. = Rs. $\left[\dfrac{2550}{100} \times 8 \times \dfrac{1}{4}\right] =$ Rs. 51.

32. Ans. (a)

Solution P.W. of Rs. 1350

$= \dfrac{1350 \times 100}{100 + \frac{3}{12} \times 5} = \dfrac{1350 \times 400}{405}$

$=$ Rs. $\dfrac{4000}{3}$

P.W. of Rs. 1078 $= \dfrac{1078 \times 100}{100 + \frac{5}{12} \times 5} = \dfrac{1078 \times 1200}{1225}$

= Rs. 1056

$\therefore$ A should pay B = Rs. $\left(\dfrac{4000}{3} - 1056\right)$

= Rs. $\dfrac{832}{3} =$ Rs. $277\dfrac{1}{3}$.

33. Ans. (b)

Solution S.I. on Rs. 240 for a given time = Rs. 20

S.I. on Rs. 240 for half the time = Rs. 10

$\therefore$ Rs. 10 is T.D. on Rs. 250

So, T.D. on Rs. 260 = Rs. $\left(\dfrac{10}{250} \times 260\right) =$ Rs. 10.40.

34. Ans. (a)

Solution T.D. = Rs. 1245 – Rs. 1200 = Rs. 45

P.W. = Rs. 1200

$\therefore$ Rate % $= \dfrac{45 \times 100}{1200 \times 15/12} = 3\%$.

35. Ans. (b)

Solution A has to pay the P.W. of Rs. 220 due 1 year hence, which is

= Rs. $\left[\dfrac{100 \times 220}{100 + (10 \times 1)}\right] =$ Rs. 200

A actually pays = Rs. [110 + P.W. of Rs. 110 due 2 years hence].

$$= \text{Rs. } \left[110 + \frac{100 \times 110}{100 + (8 \times 2)}\right]$$

$$= \text{Rs. } 192.66$$

$\therefore$ *A* gains = Rs. [200 – 192.66] = Rs. 7.34.

36. Ans. (b)

Solution $\text{T.D.} = \frac{\text{B.G.} \times 100}{\text{R} \times \text{T}} = \text{Rs. } \frac{180 \times 100}{10 \times 3} = \text{Rs. } 600$

$\therefore$ B.D. = Rs. (600 + 180) = Rs. 780.

37. Ans. (c)

Solution P.W. = Rs. 249 – Rs. 9 = Rs. 240

T.D. = Rs. 9

Rate = 5%

$$\therefore \quad \text{Time} = \frac{\text{T.D.} \times 100}{\text{P.W.} \times \text{rate}} = \frac{9 \times 100}{240 \times 5} = \frac{3}{4}\text{year}$$

= 9 months.

38. Ans. (a)

Solution $\text{T.D.} = \frac{\text{B.G.} \times 100}{\text{Rate} \times \text{Time}} = \text{Rs. } \left(\frac{8 \times 100}{5 \times 2}\right)$

= Rs. 800.

39. Ans. (d)

Solution Let B.D. = Re. 1

$\because$ B.G. = Rs. $\frac{3}{23}$

$\therefore$ T.D. = Rs. $\left[1 - \frac{3}{23}\right]$ = Rs. $\frac{20}{23}$

$$\therefore \quad \text{Sum} = \frac{\text{B.D} \times \text{T.D.}}{\text{B.D} - \text{T.D.}} = \frac{1 \times \frac{20}{23}}{1 - \frac{20}{23}} = \text{Rs. } \frac{20}{3}$$

$\because$ S.I. on Rs. $\frac{20}{3}$ for 5 years is Re. 1

$$\therefore \quad \text{Rate} = \frac{100 \times 3}{20 \times 5} = 3\%.$$

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. If the discount on Rs. 1050 for one year was Rs. 50, the discount on the same sum for half the year would be
(a) Rs. 25.61 (b) Rs. 27.50
(c) Rs. 22.50 (d) None of these

2. I want to sell my scooter. There are two offers, one at cash payment of Rs. 8100 and another at credit of Rs. 8250 to be paid after 6 months. If money being worth $6\frac{1}{4}\%$ per annum simple interest, which is the better offer?
(a) Rs. 8100 in cash
(b) Rs. 8250 due 6 months hence
(c) Both are equally good.
(d) Cannot be said

3. The simple interest and the true discount on a certain sum for a given time and at a given rate are Rs. 25 and Rs. 20, respectively. The sum is
(a) Rs. 500 (b) Rs. 200
(c) Rs. 250 (d) Rs. 100

4. A sum of Rs. 2550 is borrowed to be paid back in two years by equal annual payments, 4 per cent compound interest being allowed. What is the annual payment?
(a) Rs. 1352 (b) Rs. 1362
(c) Rs. 1382 (d) Rs. 1772

5. The banker's discount on a sum of money for $1\frac{1}{2}$ years is Rs. 60 and the true discount on the same sum for 2 years is Rs. 75. The rate per cent is
(a) 5% (b) 6%
(c) $6\frac{2}{3}\%$ (d) $3\frac{1}{3}$

6. The banker's gain on a sum due 2 years hence at 6% is Rs. 84. The banker's discount is
(a) Rs. 616 (b) Rs. 716
(c) Rs. 784 (d) Rs. 684

7. Find the present worth of a bill of Rs. 2420 due 2 years hence at 10% compound interest. Also, find the true discount.
(a) Rs. 2000, Rs. 420 (b) Rs. 2200, Rs. 520
(c) Rs. 2100, Rs. 460 (d) None of these

8. The difference between the simple interest and true discount on a certain sum of money for $2\frac{1}{2}$ years at 6% per annum is Rs. 90. The sum is
(a) Rs. 4500 (b) Rs. 4600
(c) Rs. 4300 (d) Rs. 4450

9. A bill was drawn on May 14 at two months after due date and was discounted on July 2 at $8\frac{1}{9}\%$. If the banker's gain be Re. 1, the bill was drawn for sum
(a) Rs. 9030 (b) Rs. 90300
(c) Rs. 11200 (d) Rs. 10020

10. The discount on a sum due 4 years hence is $\frac{5}{6}$ of the interest on the same for the same time. Find the rate of interest.
(a) 4% (b) 5%
(c) 6% (d) 8%

Test II **[Time: 10 min]**

11. The true discount on Rs. 1247 due 1 year 6 months hence at 5% per annum is
(a) Rs. 47 (b) Rs. 87
(c) Rs. 127 (d) Rs. 157

12. A shopkeeper gives two prices of his articles, one in terms of cash and the other on credit for 4 months at 3% per annum. If the credit price is Rs. 202, find the cash price.
(a) Rs. 190 (b) Rs. 192
(c) Rs. 195 (d) Rs. 200

13. The banker's discount on a bill due 1 year and 6 months hence is Rs. 50 and the true discount on the same sum and the same rate per cent is Rs. 45. The rate per cent is
(a) 6% (b) $6\frac{2}{3}\%$
(c) $6\frac{1}{2}\%$ (d) $8\frac{44}{59}\%$

14. A bill is discounted at 5% per annum. If banker's discount be allowed, at what rate per cent must the proceeds be invested, so that nothing may be lost?
(a) 5% (b) $4\frac{19}{21}\%$
(c) $5\frac{5}{19}\%$ (d) 10%

15. The banker's discount on Rs. 1600 at 6% is the same as the true discount on Rs. 1624 for the same time and at the same rate. Then, the time is
(a) 3 months (b) 4 months
(c) 6 months (d) 8 months

16. What is the sum due 219 days hence whose present worth at 5% per annum is Rs. 500?
(a) Rs. 515 (b) Rs. 520
(c) Rs. 525 (d) Rs. 530

17. A bill for Rs. 17850 drawn on 21 May 1999, at 7 months was discounted by a banker on the 29 Dec. 1991. If the money being worth 5%, the difference between the banker's discount and the true discount was
(a) Rs. 12 (b) Rs. 20
(c) Rs. 7 (d) Rs. 15

18. The present worth of Rs. 1404 due in two equal half-yearly instalments at 8% per annum simple interest is
(a) Rs. 1325 (b) Rs. 1300
(c) Rs. 1350 (d) Rs. 1500

19. If the banker's discount calculated for 1 year is 21 times his gain, the rate p.c. is
(a) 5% (b) 6%
(c) 4% (d) $3\frac{1}{2}\%$

20. The banker's gain on a bill due 1 year hence at 5% is Re 1. The true discount is
(a) Rs. 15 (b) Rs. 20
(c) Rs. 25 (d) Rs. 5

Answers

1. (a)	**2.** (a)	**3.** (d)	**4.** (a)	**5.** (d)	**6.** (c)	**7.** (a)	**8.** (b)	**9.** (b)	**10.** (b)
11. (b)	**12.** (d)	**13.** (b)	**14.** (c)	**15.** (a)	**16.** (a)	**17.** (c)	**18.** (a)	**19.** (a)	**20.** (b)

12

PROFIT AND LOSS

INTRODUCTION

Business transactions have now-a-days become common feature of life.When a person deals in the purchase and sale of any item, he either gains or loses some amount generally. The aim of any business is to earn profit. The commonly used terms in dealing with questions involving sale and purchase are:

Cost Price The cost price of an article is the price at which an article has been purchased. It is abbreviated as C.P.

Selling Price The selling price of an article is the price at which an article has been sold. It is abbreviated as S.P.

Profit or Gain If the selling price of an article is more than the cost price, there is a gain or profit.

Thus, Profit or Gain = S.P. – C.P.

Loss If the cost price of an article is greater than the selling price, the seller suffers a loss.

Thus, Loss = C.P. – S.P.

Note that profit and loss are always calculated with respect to the cost price of the item.

Illustration 1. (i) If C.P. = Rs. 235, S.P. = Rs. 240, then profit = ?

(ii) If C.P. = Rs. 116, S.P. = Rs. 107, then loss = ?

Solution (i) Profit = S.P. – C.P. = 240 –235 = Rs. 5.

(ii) Loss = C.P. – S.P. = 116 – 107 = Rs. 9.

SOME BASIC FORMULAE

1. Gain on Rs. 100 is *Gain per cent*

$$\text{Gain\%} = \frac{\text{Gain} \times 100}{\text{C.P.}}$$

Loss on Rs. 100 is *Loss per cent*

$$\text{Loss\%} = \frac{\text{Loss} \times 100}{\text{C.P.}}$$

Illustration 2 The cost price of a shirt is Rs. 200 and selling price is Rs. 250. Calculate the % profit.

Solution We have, C.P. = Rs. 200, S.P. = Rs. 250.

Profit = S.P. – C.P. = 250 – 200 = Rs. 50.

$$\therefore \quad \text{Profit\%} = \frac{\text{Profit} \times 100}{\text{C.P.}} = \frac{50 \times 100}{200} = 25\%.$$

Illustraton 3 Anu bought a necklace for Rs. 750 and sold it for Rs. 675. Find her percentage loss.

Solution Here, C.P. = Rs. 750, S.P. = Rs. 675.

Loss = C.P. – S.P. = 750 – 675 = Rs. 75.

$$\therefore \quad \text{Loss\%} = \frac{\text{Loss} \times 100}{\text{C.P.}} = \frac{75 \times 100}{750} = 10\%.$$

2. When the selling price and gain% are given:

$$\text{C.P.} = \left(\frac{100}{100 + \text{Gain\%}}\right) \times \text{S.P.}$$

3. When the cost and gain per cent are given:

$$\text{S.P.} = \left(\frac{100 + \text{Gain\%}}{100}\right) \times \text{C.P.}$$

Explanation

$$\text{Since Profit\%} = \frac{\text{Profit} \times 100}{\text{C.P.}}$$

$$= \left[\frac{(\text{S.P.} - \text{C.P.}) \times 100}{\text{C.P.}}\right]$$

$$\therefore \quad \frac{\text{Profit\%}}{100} = \frac{\text{S.P.}}{\text{C.P.}} - 1$$

$$\text{or,} \quad \frac{\text{S.P.}}{\text{C.P.}} = 1 + \frac{\text{Profit\%}}{100}$$

$$\therefore \quad \text{S.P.} = \left(\frac{100 + \text{Profit\%}}{100}\right) \times \text{C.P.}$$

$$\text{and,} \quad \text{C.P.} = \left(\frac{100}{100 + \text{Profit\%}}\right) \times \text{S.P.}$$

4. When the cost and loss per cent are given:

$$\text{S.P.} = \left(\frac{100 - \text{Loss\%}}{100}\right) \times \text{C.P.}$$

5. When the selling price and loss per cent are given:

$$\text{C.P.} = \left(\frac{100}{100 - \text{Loss\%}}\right) \times \text{S.P.}$$

Explanation

Since $\text{Loss\%} = \dfrac{\text{Loss} \times 100}{\text{C.P.}}$

$$= \left[\frac{(\text{C.P.} - \text{S.P.}) \times 100}{\text{C.P.}}\right]$$

$$\therefore \quad \frac{\text{Loss\%}}{100} = 1 - \frac{\text{S.P.}}{\text{C.P.}} \text{ or } \frac{\text{S.P.}}{\text{C.P.}} = 1 - \frac{\text{Loss\%}}{100}$$

$$\therefore \quad \text{S.P.} = \left(\frac{100 - \text{Loss\%}}{100}\right) \times \text{C.P.}$$

$$\text{and, C.P.} = \left(\frac{100}{100 - \text{Loss\%}}\right) \times \text{S.P.}$$

Illustration 4 Mr. Sharma buys a cooler for Rs. 4500. For how much should he sell so that there is a gain of 8%?

Solution We have, C.P. = Rs. 4500, gain% = 8%.

$$\therefore \quad \text{S.P.} = \left(\frac{100 + \text{Gain\%}}{100}\right) \times \text{C.P.}$$

$$= \left(\frac{100 + 8}{100}\right) \times 4500$$

$$= \frac{108}{100} \times 4500 = \text{Rs. } 4860.$$

Illustration 5 By selling a fridge for Rs. 7200, Pankaj loses 10%. Find the cost price of the fridge.

Solution We have, S.P. = Rs. 7200, loss% = 10%.

$$\therefore \quad \text{C.P.} = \left(\frac{100}{100 - \text{Loss\%}}\right) \times \text{S.P.}$$

$$= \left(\frac{100}{100 - 10}\right) \times 7200$$

$$= \frac{100}{90} \times 7200 = \text{Rs. } 8000.$$

Illustration 6 By selling a pen for Rs. 99, Mohan gains $12\frac{1}{2}$%. Find the cost price of the pen.

Solution Here, S.P. = Rs. 99, gain% = $12\frac{1}{2}$% or $\frac{25}{2}$%.

$$\therefore \quad \text{C.P.} = \left(\frac{100}{100 + \text{Gain\%}}\right) \times \text{S.P.}$$

$$= \left(\frac{100}{100 + \frac{25}{2}}\right) \times 99$$

$$= \left(\frac{100 \times 2}{225}\right) \times 99 = \text{Rs. } 88.$$

SOME USEFUL SHORT-CUT METHODS

1. If a man buys x items for Rs. y and sells z items for Rs. w, then the gain or loss per cent made by him is

$$\left(\frac{xw}{zy} - 1\right) \times 100\%.$$

Explanation

S.P. of z iterms = Rs. w

S.P. of x items = Rs. $\dfrac{w}{z}x$

Net profit = $\dfrac{w}{z}x - y$.

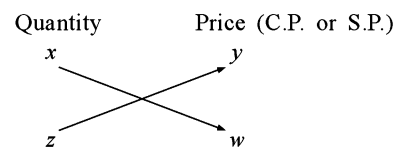
$$\therefore \quad \%\ \text{Profit} = \frac{\frac{w}{z}x - y}{y} \times 100\%$$

i.e. $\left(\dfrac{xw}{zy} - 1\right) \times 100\%$,

which represents loss, if the result is negative.

Note: In the case of gain per cent the result obtained bears positive sign whereas in the case of loss per cent the result obtained bears negative sign.

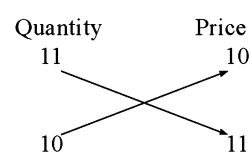
How to remember:



1. Cross-multiply the numbers connected by the arrows (xw and zy)
2. Mark the directon of the arrows for cross-multiplicaton. The arrow going down forms the numerator while the arrow going up forms the denominator $\left(\dfrac{xw}{zy}\right)$.

Illustration 7 If 11 oranges are bought for Rs. 10 and sold at 10 for Rs. 11, what is the gain or loss%?

Solution



$$\% \text{ profit} = \left(\frac{xw}{zy} - 1\right) \times 100\%$$

$$= \left(\frac{11 \times 11}{10 \times 10} - 1\right) \times 100\%$$

$$= \frac{21}{100} \times 100\% = 21\%.$$

Illustration 8 A fruit seller buys apples at the rate of Rs. 12 per dozen and sells them at the rate of 15 for Rs. 12. Find his percentage gain or loss.

Solution

Quantity	Price
12	12
12	12

$$\% \text{ gain or loss} = \left(\frac{xw}{zy} - 1\right) \times 100\%$$

$$= \left(\frac{12 \times 12}{15 \times 12} - 1\right) \times 100\%$$

$$= -\frac{36}{144} \times 100\% = -25\%.$$

Since the sign is –ve, there is a loss of 25%.

2. If the cost price of m articles is equal to the selling price of n articles, then

$$\% \text{ gain or loss} = \left(\frac{m-n}{n}\right) \times 100$$

[If $m > n$, it is % gain and if $m < n$, it is % loss]

Explanation

Let the C.P. of one article be Re. 1

$\therefore$ C.P. of m articles = Rs. $m \times 1$ = Rs. m

$\because$ S.P. of n articles = Rs. m

$\therefore$ S.P. of 1 article = Rs. $\frac{m}{n}$

$\therefore$ Profit on 1 article = Rs. $\left(\frac{m}{n} - 1\right)$

i.e. Rs. $\left(\frac{m-n}{n}\right)$

$$\therefore \% \text{ profit} = \frac{m-n}{n} \times \frac{100}{1} \text{ i.e. } \left(\frac{m-n}{n}\right) \times 100.$$

Illustration 9 A shopkeeper professes to sell his goods on cost price but uses 800 gm, instead of 1kg. What is his gain %?

Solution Here, cost price of 1000 gm is equal to selling price of 800 gm,

$$\therefore \% \text{ gain} = \left(\frac{m-n}{n}\right) \times 100$$

$$= \left(\frac{1000-800}{800}\right) \times 100$$

$$= \frac{200}{800} \times 100 = 25\%.$$

Illustraton 10 If the selling price of 12 articles is equal to the cost price of 18 articles, what is the profit %?

Solution Here, $m = 18$, $n = 12$

$$\therefore \text{ Profit\%} = \left(\frac{m-n}{n}\right) \times 100$$

$$= \left(\frac{18-12}{12}\right) \times 100$$

$$= \frac{6}{12} \times 100 = 50\%.$$

3. If an article is sold at a price $S.P_1$., then % gain or % loss is x and if it is sold at a price $S.P_2$., then % gain or % loss is y. If the cost price of the article is C.P., then

$$\frac{S.P_1}{100+x} = \frac{S.P_2}{100+y} = \frac{C.P.}{100} = \frac{S.P_1 - S.P_2}{x-y},$$

where x or y is –ve, if it indicates a loss, otherwise it is +ve.

Illustration 11 By selling a radio for Rs. 1536, Suresh lost 20%. What per cent shall he gain or lose by selling it for Rs. 2000?

Solution Here, $S.P_1 = 1536$, $x = -20$

(–ve sign indicates loss)

$S.P_2$ = Rs. 2000, y = ?

Using the formula,

$$\frac{S.P_1}{100+x} = \frac{S.P_2}{100+y}$$

we get $$\frac{1536}{100-20} = \frac{2000}{100+y}$$

$$\Rightarrow \quad 100 + y = \frac{2000 \times 80}{1536} = 104\frac{1}{6}$$

$$\Rightarrow \quad y = 4\frac{1}{6}\%.$$

Thus, Suresh has a gain of $4\frac{1}{6}\%$ by selling it for Rs. 2000.

4. If 'A' sells an article to 'B' at a gain/loss of m% and 'B' sells it to 'C' at a gain/loss of n%. If 'C' pays Rs. z for it to 'B' then the cost price for 'A' is

$$\left[\frac{100^2 z}{(100+m)(100+n)}\right],$$

where m or n is –ve, of it indicates a loss, otherwise it is +ve.

Illustration 12 Mohit sells a bicycle to Rohit at a gain of 10% and Rohit again sells it to Jyoti at a profit of 5%. If Jyoti pays Rs. 462 to Rohit, what is the cost price of the bicycle for Mohit?

Solution Here, $m = 10$, $n = 5$, z = Rs. 462.

Using the formula,

$$\text{C.P.} = \left[\frac{100^2 z}{(100+m)(100+n)}\right],$$

we get, $$\text{C.P. for Mohit} = \left[\frac{100^2 \times 462}{(100+10)(100+5)}\right]$$

$$= \frac{462 \times 10000}{110 \times 105} = \text{Rs. } 400.$$

Illustration 13 'A' sells a DVD to 'B' at a gain of 17% and 'B' again sells it to 'C' at a loss of 25%. If 'C' pays Rs. 1053 to 'B', what is the cost price of the DVD to 'A'?

Solution We have, $m = 17$, $n = -25$, z = Rs. 1053.

$\therefore$ Cost price of DVD to 'A'

$$= \left[\frac{100^2 z}{(100+m)(100+n)}\right]$$

$$= \frac{100 \times 100 \times 1053}{(100+17)(100-25)}$$

$$= \frac{100 \times 100 \times 1053}{117 \times 75} = \text{Rs. } 1200.$$

5. If 'A' sells an article to 'B' at a gain/loss of m% and 'B' sells it to 'C' at a gain/loss of n%, then the resultant profit/loss per cent is given by

$$\left(m + n + \frac{mn}{100}\right) \qquad \text{...(1)}$$

where m or n is –ve, if it indicates a loss, otherwise it is +ve.

Note: The expression given by (1) represents resultant profit or loss according as it is +ve or –ve.

Illustration 14 'A' sells a horse to 'B' at a profit of 5% and 'B' sells it to 'C' at a profit of 10%. Find the resultant profit per cent.

Solution We have, $m = 5$ and $n = 10$

$$\therefore \text{ Resultant profit\%} = \left(m + n + \frac{mn}{100}\right)$$

$$= \left(5 + 10 + \frac{5 \times 10}{100}\right)$$

$$= \frac{31}{2}\% \text{ or } 15\frac{1}{2}\%.$$

Illustraton 15 Manoj sells a shirt to Yogesh at a profit of 15% and Yogesh sells it to Suresh at a loss of 10%. Find the resultant profit or loss.

Solution Here, $m = 15$, $n = -10$

$$\therefore \text{ Resultant profit/loss\%} = \left(m + n + \frac{mn}{100}\right)$$

$$= \left(15 - 10 + \frac{15 \times -10}{100}\right) = \left(15 - 10 - \frac{150}{100}\right)$$

$$= 7/2\% \text{ or } 3\frac{1}{2}\%,$$

which represents profit as the sign is +ve.

6. When two different articles are sold at the same selling price, getting gain/loss of x% on the first and gain/loss of y% on the second, then the overall% gain or % loss in the transaction is given by

$$\left[\frac{100(x+y) + 2xy}{(100+x) + (100+y)}\right]\%$$

The above expression represent overall gain or loss according as its sign is +ve or –ve.

7. When two different articles are sold at the same selling price getting a gain of x% on the first and loss of x% on the second, then the overall% loss in the transaction is given by

$$\left(\frac{x}{10}\right)^2\%.$$

Note that in such questions there is always a loss.

Explanation

Let each article be sold at Rs. z.

Since gain/loss of x% is made on the first, cost price of the first article

$$= \text{Rs. } z\left(\frac{100}{100+x}\right).$$

Also, gain/loss of y% is made on the second, therefore cost price of the second article

$$= \text{Rs. } z\left(\frac{100}{100+y}\right).$$

$$\therefore \text{ Total C.P.} = z\left(\frac{100}{100+x}\right) + z\left(\frac{100}{100+y}\right)$$

$$= z\left[\frac{100(100+y) + 100(100+x)}{(100+x)(100+y)}\right]$$

Total S.P. = $2z$.

$$\therefore \text{ Overall \% gain or loss} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$

$$= \frac{2z - \dfrac{100z[100+x+100+y]}{(100+x)(100+y)}}{\dfrac{100z[100+x+100+y]}{(100+x)(100+y)}} \times 100$$

$$= \frac{2(100+x)(100+y) - 100(200+x+y)}{100(200+x+y)} \times 100$$

$$= \frac{100x + 100y + 2xy}{(100+x)+(100+y)}\%$$

$$= \left[\frac{100(x+y)+2xy}{(100+x)+(100+y)}\right]\%.$$

Note: In case $y = -x$, we have

Overall% gain or loss $= -\frac{x^2}{100}\%$

Since the sign is –ve, there is always a loss.

Illustration 16 Mahesh sold two scooters, each for Rs. 24000. If he makes 20% profit on the first and 15% loss on the second, what is his gain or loss per cent in the transaction?

Solution Here, $x = 20$ and $y = -15$.

$\therefore$ Over all gain/loss%

$$= \left[\frac{100(x+y)+2xy}{(100+x)+(100+y)}\right]\%$$

$$= \left[\frac{100(20-15)+2\times 20\times -15}{(100+20)+(100-15)}\right]\%$$

$$= -\frac{100}{205}\% = -\frac{20}{41}\%,$$

which represents loss, being a –ve expression.

Illustration 17 Rajesh sold two horses for Rs. 990 each; gaining 10% on the one and losing 10% on the other. Find his total gain or loss per cent.

Solution Here, $x = 10$.

$$\therefore \text{ Overall loss\%} = \left(\frac{x}{10}\right)^2\% = \left(\frac{10}{10}\right)^2\% = 1\%.$$

8. A merchant uses faulty measure and sells his goods at gain/loss of x%. The overall % gain/loss(g) is given by

$$\frac{100+g}{100+x} = \frac{\text{True measure}}{\text{Faulty measure}}$$

Note: If the merchant sells his goods at cost price, then $x = 0$.

9. A merchant uses y% less weight/length and sells his goods at gain/loss of x%. The overall % gain/loss is given by

$$\left[\left(\frac{y+x}{100-y}\right)\times 100\right]\%.$$

Illustration 18 A dis-honest shopkeeper professes to sell cloth at the cost price but he uses faulty metre rod. His metre rod measures 95 cms only. Find his gain per cent.

Solution Here, True measure = 100 cms

False measure = 95 cms.

Since the shopkeeper sells the cloth at cost price, $\therefore$ $x = 0$.

$\therefore$ Over all gain% is given by

$$\frac{100+g}{100+x} = \frac{\text{True measure}}{\text{Faulty measure}}$$

$$\Rightarrow \frac{100+g}{100} = \frac{100}{95} \Rightarrow 100 + g = \frac{100\times 100}{95}$$

$$\Rightarrow g = \frac{10000}{95} - 100 = 5\frac{5}{19}\%.$$

Illustration 19 A dis-honest shopkeeper professes to sell his goods at cost price but he uses a weight of 800 g for the kg weight. Find his gain per cent.

Solution True measure = 1000 g

False measure = 800 g. Also, $x = 0$.

$\therefore$ Overall gain% is given by

$$\frac{100+g}{100+x} = \frac{\text{True measure}}{\text{False measure}}$$

$$\Rightarrow \frac{100+g}{100} = \frac{1000}{800} \Rightarrow 100 + g = \frac{1000\times 100}{800}$$

$$\Rightarrow g = \frac{1000}{8} - 100 = 25\%.$$

Illustration 20 A shopkeeper sells the goods at 44% loss on cost price but uses 30% less weight. What is his percentage profit or loss?

Solution Here, $x = -44$ and $y = 30$.

$$\therefore \text{ Overall gain/loss\%} = \left(\frac{y+x}{100-y}\right)\times 100\%$$

$$= \left(\frac{30-44}{100-30}\times 100\right)\%$$

$$= \left(\frac{-14}{70}\times 100\right)\% = -20\%,$$

which represents loss being a negative expression.

10. A person buys two items for Rs. A and sells one at a loss of l% and other at a gain of g%. If each item was sold at the same price, then

(a) The cost price of the item sold at loss

$$= \frac{A(100+\%\text{gain})}{(100-\%\text{loss})+(100+\%\text{gain})}$$

(b) The cost price of the item sold at gain

$$= \frac{A(100-\%\text{loss})}{(100-\%\text{loss})+(100+\%\text{gain})}$$

Illustration 21 Ramesh buys two books for Rs. 410 and sells one at a loss of 20% and the other at a gain of 25%. If both the books are sold at the same price, find the cost price of two books.

Solution Cost price of the book sold at a loss of 20%

$$= \frac{410\,(100+25)}{(100-20)+(100+25)}$$

$$= \frac{410 \times 125}{80 + 125} = \text{Rs. } 250.$$

Cost price of the book sold at a profit of 25%

$$= \frac{410\,(100 - 20)}{(100 - 20) + (100 + 25)} = \frac{410 \times 80}{80 + 125}$$

$$= \text{Rs. } 160.$$

11. If two successive discounts on an article are m% and n% respectively, then a single discount equivalent to the two successive discounts will be

$$\left(m + n - \frac{mn}{100}\right)\%.$$

Explanation

Let the marked price of the article be Rs. 100.

$\therefore$ S.P. after the first discount = Rs. $(100 - m)$ and discount at n% on Rs. $(100 - m)$ = Rs. $\frac{(100 - m) \times n}{100}$.

$\therefore$ Single equivalent discount

$$= \left[m + \frac{(100 - m) \times n}{100}\right]\%$$

$$= \left(\frac{100m + 100n - mn}{100}\right)\%$$

$$= \left(m + n - \frac{mn}{100}\right)\%.$$

12. If three successive discounts on an article are l%, m% and n% respectively, then a single discount equivalent to the three successive discounts will be

$$\left[l + m + n - \frac{(lm + ln + mn)}{100} + \frac{lmn}{100^2}\right]\%.$$

Explanation

Let the marked price of the article be Rs. 100

$\therefore$ S.P. after the first discount = Rs. $(100 - l)$.

Second discount at m% on Rs. $(100 - l)$

$$= \text{Rs. } \frac{(100 - l) \times m}{100}.$$

$\therefore$ S.P. after second discount

$$= \text{Rs. } (100 - l) - \frac{(100 - l)m}{100}$$

$$= \text{Rs. } \frac{100\,(100 - l) - (100 - l)\,m}{100}$$

$$= \text{Rs. } \frac{(100 - l) \cdot (100 - m)}{100}$$

Third discount at n% on Rs. $\frac{(100 - l)\,(100 - m)}{100}$

$$= \text{Rs. } \frac{(100 - l)\,(100 - m)n}{100 \times 100}$$

$\therefore$ S.P. after third discount

$$= \text{Rs. } \frac{(100 - l)\,(100 - m)}{100} - \frac{(100 - l)\,(100 - m)n}{100 \times 100}$$

$$= \text{Rs. } \frac{(100 - l)\,(100 - m)\,(100 - n)}{100 \times 100}$$

$$= \left(l + m + n - \frac{(lm + ln + mn)}{100} + \frac{lmn}{(100)^2}\right)$$

$\therefore$ Single equivalent discount

$$= \left(l + m + n - \frac{(lm + ln + mn)}{100} + \frac{lmn}{(100)^2}\right)\%.$$

Illustration 22 Find a single discount equivalent to two successive discounts of 10% and 20%.

Solution The equivalent single discount is given by

$$\left(10 + 20 - \frac{10 \times 20}{100}\right)\% \text{ i.e., } 28\%.$$

Illustration 23 Find a single discount equivalent to three successive discounts of 10%, 20% and 30%.

Solution The equivalent single discount is given by

$$\left(10 + 20 + 30 - \frac{(10 \times 20 + 10 \times 30 + 20 \times 30)}{100} + \frac{10 \times 20 \times 30}{100^2}\right)\%$$

i.e. $\left(60 - 11 + \frac{6}{10}\right)\% = \frac{496}{10}\%$ or 49.6%.

Illustration 24 Two shopkeepers sell machines at the same list price. The first allows two successive discounts of 30% and 16% and the second 20% and 26%. Which discount series is more advantageous to the purchaser?

Solution A single discount equivalent to the two successive discounts of 30% and 16% is

$$\left(30 + 16 - \frac{30 \times 16}{100}\right)\%$$

or, $\left(46 - \frac{24}{5}\right)\%$ or $41\frac{1}{5}\%$

Also, a single discount equivalent to the two successive discounts of 20% and 26% is

$$\left(20 + 26 - \frac{20 \times 26}{100}\right)\%$$

or, $\left(46 - \frac{26}{5}\right)\%$ or $40\frac{4}{5}\%$.

Clearly, the discount series being offered by the first shopkeeper is more advantageous to the purchaser.

13. A shopkeeper sells an item at Rs. z after giving a discount of d% on labelled price. Had he not given the discount, he would have earned a profit of p% on the cost price.

The cost price of each item is given by

$$\text{C.P.} = \left[\frac{100^2 z}{(100-d)(100+p)}\right].$$

Illustration 25 A shopkeeper sold sarees at Rs. 266 each after giving 5% discount on labelled price. Had he not given the discount, he would have earned a profit of 12% on the cost price. What was the cost price of each saree?

Solution We have, labelled price z = Rs. 266, discount d = 5% and profit p = 12%.

Using the formula

$$\text{C.P.} = \left[\frac{100^2 z}{(100-d)(100+p)}\right]$$

we get the cost price of each saree

$$= \left[\frac{100 \times 100 \times 266}{(100-5)(100+12)}\right]$$

$$= \frac{100 \times 100 \times 266}{95 \times 112} = \text{Rs. } 250.$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Mohan buys a watch for Rs. 350 and sells it for Rs. 392. Find his percentage of profit.
(a) 9% (b) 12%
(c) 14% (d) None of these

2. Ramesh purchased a bicycle for Rs. 5200 and spent Rs. 800 on its repairs. He had to sell it for Rs. 5500. Find his profit or loss per cent.
(a) $8\frac{1}{3}$% loss (b) $7\frac{1}{2}$% gain
(c) 9% (d) None of these

3. A man buys 10 articles for Rs. 8 and sells them at the rate of Rs. 1.25 per article. His gain per cent is
(a) 55% (b) $56\frac{1}{4}$%
(c) 40% (d) None of these

4. A tooth paste marked at Rs. 80 is sold for Rs. 68. The rate of discount is
(a) 12% (b) 14%
(c) 15% (d) None of these

5. Sardar Singh bought 200 dozen oranges at Rs. 10 a dozen. He spent Rs. 500 on transportation. He sold them at Re. 1 each. What was his profit or loss percent?
(a) 4% (b) 6%
(c) 5% (d) None of these

6. Mr. Verma sold his scooter for Rs. 10500 at a gain of 5%. Find the cost price of the scooter.
(a) Rs. 10300 (b) Rs. 10700
(c) Rs. 10000 (d) None of these

7. Suresh buys a camera for Rs. 1800 and sells it at 10% loss. Find its selling price.
(a) Rs. 1620 (b) Rs. 1730
(c) Rs. 1650 (d) None of these.

8. Hemant purchased 120 rims of paper at Rs. 80 per rim. He spent Rs. 280 on transportation, paid octroi at the rate of 40 paise per rim and paid Rs. 72 to the coolie. If he wants to have a gain of 8%, the selling price per rim must be:
(a) Rs. 89 (b) Rs. 90
(c) Rs. 95 (d) None of these

9. A shopkeeper loses 7 % by selling a cricket ball for Rs. 31. For how much should he sell the ball so as to gain 5 %?
(a) Rs. 50 (b) Rs. 65
(c) Rs. 35 (d) None of these

10. A shopkeeper sold some articles at Rs. 35 per article and gained 40% on it. What would be the selling price of each article to get 60% profit?
(a) Rs. 40 (b) Rs. 45
(c) Rs. 50 (d) None of these

11. A man bought apples at the rate of 6 for Rs. 20 and sold them at 4 for Rs. 16. His estimated profit % is
(a) 23% (b) 18%
(c) 20% (d) None of these

12. A fruitseller buys 10 bananas for Rs. 14 and sells them at 12 for Rs. 15. Find his percentage gain or loss.
(a) $10\frac{5}{7}$% loss (b) $10\frac{5}{9}$% gain
(c) 9% gain (d) None of these

13. If eggs are bought 12 for Rs. 10 and sold at 10 for Rs. 12. What is the gain or loss %.
(a) 40% loss (b) 44% gain
(c) 44% loss (d) None of these

14. If the cost price of 21 watches is equal to the selling price of 18 of them, what will be the gain per cent in this transaction?
(a) $6\frac{1}{2}$% (b) 7%
(c) $6\frac{2}{3}$% (d) None of these

15. A shopkeeper gains the cost of 8 metres of thread by selling 40 metres of thread. Find his gain per cent.

(a) 19% (b) 20%
(c) 22% (d) None of these

16. If the selling price of 2/3 rd of a certain quantity of milk be equal to the cost price of whole milk, what will be the gain per cent in this transaction?
(a) 50% (b) 48%
(c) 53% (d) None of these

17. A shopkeeper sells 20 pencils for the same money as he paid for 25. What is his gain per cent?
(a) 20% (b) 25%
(c) 24% (d) None of these

18. Mohit lost 18% by selling a bicycle for Rs. 1230. What per cent shall he gain or loss by selling it for Rs. 1600?
(a) $6\frac{2}{3}$% loss (b) 4% gain
(c) $6\frac{2}{3}$% gain (d) None of these

19. A shopkeeper sells an article at a gain of 10%. Had he sold it at a loss of 20%, its selling price would have been Rs. 180 less. What is the cost price of the article?
(a) Rs. 630 (b) Rs. 600
(c) Rs. 580 (d) None of these

20. A person sells 36 oranges per rupee and suffers a loss of 4%. Find how many oranges per rupee to be sold to have a gain of 8 %?
(a) 1/32 (b) 5
(c) 1/16 (d) None of these

21. A person sells a colour TV at 10% below cost price. Had he received Rs. 1494 more, he would have made a profit of $12\frac{1}{2}$%. What was the cost price of colour TV ?
(a) Rs. 6400 (b) Rs. 7200
(c) Rs. 6640 (d) None of these

22. Vijay sold a watch at a gain of 5 %. Had he sold it for Rs. 72 more, he would have gained 13 %. Find the cost price of the watch.
(a) Rs. 900 (b) Rs. 910
(c) Rs. 870 (d) None of these

23. Sita sells a calculator to Gita at a gain of 17% and Gita sells it to Anu at a loss of 25%. If Anu pays Rs. 1842.75 for it, what did Sita pay for it?
(a) Rs. 2080 (b) Rs. 2100
(c) Rs. 2110 (d) None of these

24. '*A*' buys an article and sells it to '*B*' at a profit of 10%, '*B*' sells it to '*C*' gaining 20 %. If '*C*' gives Rs. 924, what did '*A*' give?
(a) Rs. 700 (b) Rs. 724
(c) Rs. 780 (d) None of these

25. '*A*' sells an article to '*B*' at a gain of 20% and '*B*' sells it to '*C*' at a gain of 10% and '*C*' sells it to '*D*' at a gain of $12\frac{1}{2}$%. If '*D*' pays Rs. 29.70 what did it cost '*A*'?
(a) Rs. 20 (b) Rs. 24
(c) Rs. 18 (d) None of these

26. Rajesh sells taperecorder to Mihir at a loss of 10 % and Mihir sells it to Shiv at a loss of 20 %. If Shiv pays Rs. 1440 for it, at what price did Rajesh buy?
(a) Rs. 1920 (b) Rs. 2000
(c) Rs. 1800 (d) None of these

27. A man sells a scooter to his friend at 10 % loss. If the friend sells it for Rs. 54000 and gains 20 %, find the original cost price of the scooter.
(a) Rs. 50000 (b) Rs. 45000
(c) Rs. 40000 (d) None of these

28. '*A*' sells a good to '*B*' at a profit of 10 % and *B* sells it to '*C*' at a profit of 20 %. Find the resultant profit.
(a) Rs. 35% (b) 20%
(c) Rs. 32% (d) None of these

29. A manufacturer sells an article to a wholesale dealer at a profit of 20 % and the wholesale dealer sells it to a retail merchant at a loss of 5 %. Find the resultant profit or loss.
(a) 14% loss (b) 14% gain
(c) 12% gain (d) None of these

30. A man sold two watches for Rs. 3750 each; on one he gained 5 % and on the other he lost 5 %. What was his total gain or loss as a percentage ?
(a) $1\frac{1}{4}$% (b) $\frac{1}{2}$%
(c) $\frac{1}{4}$% (d) None of these

31. A men sells two houses at the rate of Rs. 1.995 lakh each. On one he gains 20 % and on the other he loses 20%. His gain or loss per cent in the whole transaction is:
(a) 5% loss (b) 4%
(c) 4% loss (d) None of these

32. A shopkeeper sold two bicycles for Rs. 1500 each. On one he gains 25 % and on the other he loses 20 %. His gain or loss per cent in the whole transaction is:
(a) $2\frac{18}{41}$% loss (b) $2\frac{18}{41}$% gain
(c) 2% gain (d) None of these

33. A man sells two articles, each for the same price of Rs. 640. He earns 20 % profit on the first and 40 % profit on the second. Find his overall per cent profit.
(a) $29\frac{1}{2}$% (b) $28\frac{1}{2}$%
(c) $29\frac{3}{13}$% (d) None of these

34. A person sells two articles, each for the same price Rs. 1040. He incurs 20 % loss on the first and 10 % loss on the second. Find his overall per cent loss.
(a) $12\frac{5}{17}\%$ (b) $15\frac{5}{17}$
(c) $13\frac{3}{4}\%$ (d) None of these

35. A grocer sells rice at a profit of 20 % and uses a weight which is 25 % less. Find his overall percentage gain.
(a) 60% (b) 65%
(c) 58% (d) None of these

36. A shopkeeper sells the goods at 10% loss on cost price but uses 20% less weight. What is his percentage profit or loss?
(a) 2% gain (b) $2\frac{1}{2}$% loss
(c) $2\frac{1}{2}$% gain (d) None of these.

37. A cloth merchant says that due to slump in the market, he sells the cloth at 10 % loss but he uses a false metre scale and actually gains 15 %. Find the actual length of the scale.
(a) 72.4 cm (b) 71.34 cm
(c) 78.25 cm (d) None of these

38. A cloth dealer professes to sell cotton at cost price, but uses a meter having a length of 80 cm only and charges for the meter. Find his gain per cent.
(a) 25% (b) 30%
(c) 40% (d) None of these

39. Sudeep buys two *CD*s for Rs. 380 and sells one at a loss of 22 % and the other at a gain of 12 %. If both the *CD*s are sold at the same price, then the cost price of two *CD*s is
(a) Rs. 196, Rs. 225 (b) Rs. 230, Rs. 140
(c) Rs. 224, Rs. 156 (d) None of these

40. An article is listed at Rs. 65. A customer bought this article for Rs. 56.16 and got two successive discounts of which one is 10 %. Find the other discount of this discount scheme that was allowed by the shopkeeper.
(a) 4% (b) 3%
(c) 6% (d) None of these

41. A cash payment that will settle a bill for 250 chairs at Rs. 50 per chair less 20 % and 15 % with a further discount of 5 % on cash payment is
(a) Rs. 7025 (b) Rs. 8075
(c) Rs. 8500 (d) None of these

42. A person sells taperecorders at Rs. 1134 each after giving a discount of 19 % on the marked price. Had he not given the discount, he would have earned a profit of 40 % on the cost price. The cost price of each taperecorder is
(a) Rs. 1000 (b) Rs. 1200
(c) Rs. 1400 (d) None of these

EXPLANATORY ANSWERS

1. Ans. (b)

Solution Here, C.P. = Rs. 350, S.P. = Rs. 392

Profit = S.P. – C.P. = 392 – 350 = Rs. 42.

$$\therefore \text{ Profit \%} = \frac{\text{Profit} \times 100}{\text{C.P.}} = \frac{42 \times 100}{350} = 12\%.$$

2. Ans. (a)

Solution C.P. of the bicycle = 5200 + 800 = Rs. 6000. S.P. = Rs. 5500.

Since S.P. < C.P.,

$\therefore$ loss = C.P. – S.P. = 6000 – 5500 = Rs. 500.

$$\therefore \text{ Loss\%} = \frac{\text{Loss} \times 100}{\text{C.P.}} = \frac{500 \times 100}{6000} = \frac{25}{3}\% \text{ or } 8\frac{1}{3}\%.$$

3. Ans. (b)

Solution Cost price of 10 articles (C.P.) = Rs. 8

Selling price of 10 articles (S.P.) = 1.25 × 10 = Rs. 12.50

Profit = S.P. – C.P. = 12.50 – 8 = Rs. 4.50.

$$\therefore \text{ Gain\%} = \frac{\text{Gain} \times 100}{\text{C.P}} = \frac{4.50 \times 100}{8} = 56\frac{1}{4}\%.$$

4. Ans. (c)

Solution Marked price (M.P.) = Rs. 80.

Selling price (S.P.) = Rs. 68.

Discount = M.P. – S.P. = 80 – 68 = Rs. 12.

$$\therefore \text{ Rate of discount} = \frac{\text{Discount} \times 100}{\text{Marked Price}} = \frac{12 \times 100}{80} = 15\%.$$

5. Ans. (a)

Solution Cost price of 200 dozen oranges = 200 × 10 = Rs. 2000

Transportation cost = Rs. 500.

$\therefore$ Cost Price (C.P) = 2000 + 500 = Rs. 2500.

Selling Price (S.P.) = 200 × 12 × 1 = Rs. 2400

Loss = C.P. – S.P. = 2500 – 2400 = Rs. 100.

$$\therefore \text{ Loss\%} = \frac{\text{Loss} \times 100}{\text{C.P.}} = \frac{100 \times 100}{2500} = 4\%.$$

6. Ans. (c)

Solution We have, S.P. = Rs. 10500, gain % = 5 %

$$\therefore \quad \text{C.P.} = \left(\frac{100}{100 + \text{Gain\%}}\right) \times \text{S.P.}$$

$$= \left(\frac{100}{100 + 5}\right) \times 10500$$

$$= \frac{100}{105} \times 10500 = \text{Rs. } 10000.$$

7. Ans. (a)

Solution Here, C.P. = Rs. 1800, loss% = 10%.

$$\therefore \quad \text{S.P.} = \left(\frac{100 - \text{Loss\%}}{100}\right) \times \text{C.P.}$$

$$= \left(\frac{100 - 10}{100}\right) \times 1800$$

$$= \frac{90}{100} \times 1800 = \text{Rs. } 1620.$$

8. Ans. (b)

Solution We have,

Cost price (C.P.) $= (120 \times 80 + 280 + 72 + 120 \times 40)$
$= \text{Rs. } 10000.$

Gain% = 8%.

$\therefore$ Selling Price (S.P.) of 120 rims

$$= \left(\frac{100 + \text{Gain\%}}{100}\right) \times \text{C.P.}$$

$$= \left(\frac{100 + 8}{100}\right) \times 10000$$

$$= \frac{108}{100} \times 10000 = \text{Rs. } 10800.$$

Thus, selling price per rim $= \dfrac{10800}{120} = \text{Rs. } 90.$

9. Ans. (c)

Solution In the first case, S.P. = Rs. 31 and loss% = 7 %.

$$\therefore \text{C.P.} = \left(\frac{100}{100 - \text{Loss\%}}\right) \times \text{S.P.} = \left(\frac{100}{100 - 7}\right) \times 31$$

$$= \frac{100}{93} \times 31 = \text{Rs. } 33\frac{1}{3}.$$

In the second case, C.P. = Rs. $33\frac{1}{3}$ and gain% = 5%

$$\therefore \text{S.P.} = \left(\frac{100 + \text{Gain\%}}{100}\right) \times \text{C.P.}$$

$$= \left(\frac{100 + 5}{100}\right) \times \frac{100}{3} = \frac{105}{100} \times \frac{100}{3} = \text{Rs. } 35.$$

10. Ans. (a)

Solution In the first case, we have, S.P. = Rs. 35 and gain % = 40 %

$$\therefore \quad \text{C.P.} = \left(\frac{100}{100 + \text{Gain\%}}\right) \times \text{S.P.}$$

$$= \left(\frac{100}{100 + 40}\right) \times 35$$

$$= \frac{100}{140} \times 35 = \text{Rs. } 25.$$

In the second case, C.P. = Rs. 25 and gain% = 60 %.

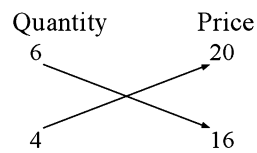
$$\therefore \quad \text{S.P.} = \left(\frac{100 + \text{Gain\%}}{100}\right) \times \text{C.P.}$$

$$= \left(\frac{100 + 60}{100}\right) \times 25$$

$$= \frac{160}{100} \times 25 = \text{Rs. } 40.$$

11. Ans. (c)

Solution



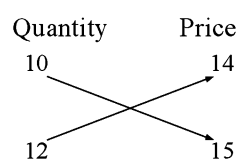
$$\text{\% profit} = \left(\frac{xw}{zy} - 1\right) \times 100\% = \left(\frac{6 \times 16}{4 \times 20}\right) \times 100\%$$

$$= \frac{16}{80} \times 100\%$$

$$= 20\ \%.$$

12. Ans. (a)

Solution



$$\text{\% gain or loss} = \left(\frac{xw}{zy} - 1\right) \times 100\%$$

$$= \left(\frac{10 \times 15}{12 \times 14} - 1\right) \times 100\%$$

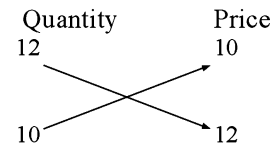
$$= \left(\frac{150}{168} - 1\right) \times 100\%$$

$$= -\frac{18}{168} \times 100\% = -10\frac{5}{7}\%.$$

Since the sign is – ve, since there is a loss of $10\frac{5}{7}\%$.

13. Ans. (b)

Solution



$$\text{\% gain or loss} = \left(\frac{xw}{zy} - 1\right) \times 100\%$$

$$= \left(\frac{12\times 12}{10\times 10}-1\right)\times 100\% = \left(\frac{144}{100}-1\right)\times 100\%$$

$$= \frac{44}{100}\times 100\% = 44\%.$$

Since the sign is + ve, therefore there is a profit of 44 %.

14. Ans. (c)

Solution Here, $m = 21$, $n = 18$.

$$\therefore \text{ gain } \% = \left(\frac{m-n}{n}\right)\times 100 = \left(\frac{21-18}{18}\right)\times 100$$

$$= \left(\frac{3}{18}\times 100\right)\% = 16\frac{2}{3}\%.$$

15. Ans. (b)

Solution Here, $m = 48$, $n = 40$.

$$\therefore \text{ gain\% } = \left(\frac{m-n}{n}\right)\times 100 = \left(\frac{48-40}{40}\right)\times 100$$

$$= \frac{8}{40}\times 100 = 20\ \%.$$

16. Ans. (a)

Solution Let the quantity of milk be x litres.

Then, $m = x$, $n = \frac{2}{3}x$.

$$\therefore \text{ gain\% } = \left(\frac{m-n}{n}\right)\times 100 = \left(\frac{x-\frac{2}{3}x}{\frac{2}{3}x}\right)\times 100$$

$$= \frac{1}{2}\times 100 = 50\%.$$

17. Ans. (b)

Solution Here, $m = 25$, $n = 20$.

$$\therefore \text{ \% gain } = \left(\frac{m-n}{n}\right)\times 100 = \left(\frac{25-20}{20}\right)\times 100$$

$$= 25\%.$$

18. Ans. (c)

Solution Here, $S.P_1$ = Rs. 1230, $x = -18$,
$S.P_2$ = Rs. 1600, y = ?

Using the formula,

$$\frac{S.P_1}{100+x} = \frac{S.P_2}{100+y}$$

we get $\frac{1230}{100-18} = \frac{1600}{100+y}$

$$\Rightarrow 100 + y = \frac{1600\times 82}{1230} = 106\frac{2}{3} \Rightarrow y = 6\frac{2}{3}\%$$

Thus, Mohit has a gain of $6\frac{2}{3}\%$ by selling it for Rs. 1600.

19. Ans. (b)

Solution Here, $S.P_1 - S.P_2$ = Rs. 180, $x = 10$, $y = -20$, C.P. = ?

Using the formula, $\frac{C.P}{100} = \frac{S.P_1 - S.P_2}{x-y}$

we get C.P. $= \left(\frac{180}{10+20}\right)\times 100 = \frac{180}{30}\times 100$ = Rs. 600.

20. Ans. (a)

Solution We have,

$S.P_1$ = Selling price of one orange = Rs. $\frac{1}{36}$,

$x = -4$, $y = 8$.

Using the formula, $\frac{S.P_1}{100+x} = \frac{S.P_2}{100+y}$

we get $\frac{1/36}{100-4} = \frac{S.P_2}{100+8} \Rightarrow S.P_2 = \frac{108}{96}\times\frac{1}{36} = \frac{1}{32}$.

21. Ans. (c)

Solution Here, $S.P_1 - S.P_2 = -1494$, $x = -10$, $y = \frac{25}{2}$.

Using the formula,

$$\frac{C.P.}{100} = \frac{S.P_1 - S.P_2}{x-y}$$

we get $\frac{C.P.}{100} = \frac{-1494}{-10-25/2} = \frac{1494}{45/2}$

$$\Rightarrow \text{C.P.} = \frac{1494\times 2\times 100}{45} = \text{Rs. } 6640.$$

22. Ans. (a)

Solution Here, $S.P_1 - S.P_2 = -72$, $x = 5$, $y = 13$.

Using the formula,

$$\frac{C.P}{100} = \left(\frac{S.P_1 - S.P_2}{x-y}\right)$$

we get $\frac{C.P.}{100} = \left(\frac{-72}{5-13}\right)$

$$\Rightarrow \text{C.P.} = \frac{72}{8}\times 100 = \text{Rs. } 900.$$

23. Ans. (b)

Solution Here, $m = 17$, $n = -25$, z = Rs. 1842.75.

$\therefore$ Cost price of calculator to Sita

$$= \left[\frac{100^2 z}{(100+m)(100+n)}\right]$$

$$= \frac{100\times 100\times 1842.75}{(100+17)(100-25)}$$

$$= \frac{100\times 100\times 1842.75}{117\times 75} = \text{Rs. } 2100.$$

24. Ans. (a)

Solution We have, $m = 10$, $n = 20$, z = Rs. 924.

$\therefore$ Cost price of A

$$= \left[\frac{100^2 z}{(100+m)(100+n)}\right]$$

$$= \left[\frac{100 \times 100 \times 924}{(100+10)(100+20)}\right] = \frac{100 \times 100 \times 924}{110 \times 120}$$

= Rs. 700.

25. Ans. (a)

Solution Here, $m = 20$, $n = 10$, $p = \frac{25}{2}$,

z = Rs. 29.70.

$\therefore$ Cost price of A

$$= \left[\frac{100^3 z}{(100+m)(100+n)(100+p)}\right]$$

$$= \left(\frac{100 \times 100 \times 100 \times 29.70 \times 2}{120 \times 110 \times 225}\right)$$

$$= \left(\frac{100 \times 100 \times 100 \times 29.70 \times 2}{120 \times 110 \times 225}\right) = \text{Rs. } 20.$$

26. Ans. (b)

Solution We have, $m = -10$, $n = -20$, z = Rs. 1440

$\therefore$ Cost price of taperecorder for Rajesh

$$= \left[\frac{100^2 z}{(100+m)(100+n)}\right]$$

$$= \left[\frac{100 \times 100 \times 1440}{(100-10)(100-20)}\right] =$$

= Rs. 2000.

27. Ans. (a)

Solution Here, $m = -10$, $n = 20$, z = Rs. 54000.

$\therefore$ Original cost price of the scooter

$$= \left[\frac{100^2 z}{(100+m)(100+n)}\right]$$

$$= \left[\frac{100 \times 100 \times 54000}{(100-10)(100+20)}\right]$$

$$= \left(\frac{100 \times 100 \times 54000}{90 \times 120}\right) = \text{Rs. } 50000.$$

28. Ans. (c)

Solution We have, $m = 10$, $n = 20$.

$$\therefore \text{ Resultant profit } \% = \left(m + n + \frac{mn}{100}\right)$$

$$= \left(10 + 20 + \frac{10 \times 20}{100}\right)$$

$$= 32\ \%.$$

29. Ans. (b)

Solution Here, $m = 20$, $n = -5$

$$\therefore \text{ Resultant profit or loss } \% = \left(m + n + \frac{mn}{100}\right)$$

$$= \left(20 - 5 + \frac{20 \times -5}{100}\right) = 14\ \%,$$

which represents profit as the sign is + ve.

30. Ans. (c)

Solution Here, $x = 5$

$$\therefore \text{ Over all loss } \% = \left(\frac{x}{10}\right)^2 \% = \left(\frac{5}{10}\right)^2 \% = \frac{1}{4}\%.$$

31. Ans. (c)

Solution Here, $x = 20$

$$\therefore \text{ over all loss } \% = \left(\frac{x}{10}\right)^2 \% = \left(\frac{20}{10}\right)^2 \% = 4\ \%.$$

32. Ans. (a)

Solution Here $x = 25$ and $y = -20$.

$\therefore$ Overall gain/loss %

$$= \left[\frac{100(x+y) + 2xy}{(100+x) + (100+y)}\right]\%$$

$$= \left[\frac{100(25-20) + 2 \times 25 \times -20}{(100+25) + (100-20)}\right]\%$$

$$= -\frac{100}{41}\% \text{ or } -2\frac{18}{41}\%,$$

which represents loss being a –ve expression.

33. Ans. (c)

Solution Here, $x = 20$ and $y = 40$.

$\therefore$ Over all gain %

$$= \left[\frac{100(x+y) + 2xy}{(100+x) + (100+y)}\right]\%$$

$$= \left[\frac{100(20+40) + 2 \times 20 \times 40}{(100+20) + (100+40)}\right]\%$$

$$= \frac{7600}{260}\% = 29\frac{3}{13}\%.$$

34. Ans. (b)

Solution We have, $x = -20$ and $y = -10$.

$\therefore$ Over all gain/loss

$$= \left[\frac{100(-20-10) + 2 \times -20 \times -10}{(100-20) + (100-10)}\right]\%$$

$$= -\frac{2600}{170}\%$$

$$= -15\frac{5}{17}\%,$$

which represents loss being a negative expression.

35. Ans. (a)

Solution We have, $x = 20$ and $y = 25$

$$\therefore \text{ Overall gain } \% = \left[\frac{y+x}{100-y} \times 100\right]\%$$

$$= \left[\frac{25+20}{100-25} \times 100\right]\% = 60\%.$$

36. Ans. (c)

Solution Here, $x = -10$ and $y = 20$.

$$\therefore \text{ Overall gain/loss } \% = \left[\frac{y+x}{100-y} \times 100\right]\%$$

$$= \left[\frac{20-10}{100-20} \times 100\right]\%$$

$$= 12\frac{1}{2}\%,$$

which represents gain being a positive expression.

37. Ans. (c)

Solution Here, $x = -10$ and $g = 15$.

Let false scale length = y cm.

Using the formula, $\frac{100+g}{100+x} = \frac{\text{True measure}}{\text{False measure}}$,

we get $\frac{100+15}{100-10} = \frac{100}{y} \Rightarrow y = \frac{100 \times 90}{115} = 78.25$ cm.

$\therefore$ Actual length of the scale is 78.25 cm instead of 1 m.

38. Ans. (a)

Solution True measure = 100 cm.

False measure = 80 cm. Also, $x = 0$.

$\therefore$ Overall gain % is given by

$$\frac{100+g}{100+x} = \frac{\text{True measure}}{\text{False measure}}$$

$$\Rightarrow \frac{100+g}{100} = \frac{100}{80} \Rightarrow 100 + g = \frac{100 \times 100}{80}$$

$$\Rightarrow g = \frac{1000}{8} - 100 = 25\ \%.$$

39. Ans. (c)

Solution Cost price of the CD sold at a loss of 22 %

$$= \frac{A(100 + \%\text{ gain})}{(100 - \%\text{loss}) + (100 + \%\text{ gain})}$$

$$= \frac{380(100+12)}{(100-22)+(100+12)} = \frac{380 \times 112}{78+112}$$

= Rs. 224.

Cost price of the CD sold at a gain of 12 %

$$= \frac{A(100 - \%\text{ loss})}{(100 - \%\text{ loss}) + (100 + \%\text{ gain})}$$

$$= \frac{380(100-22)}{(100-22)+(100+12)} = \frac{380 \times 78}{78+112}$$

= Rs. 156.

40. Ans. (a)

Solution List price of the article = Rs. 65.

Selling price of the article = Rs. 56.16.

$\therefore$ Discount = 65 – 56.16 = Rs. 8.84.

$$\therefore \text{ Discount } \% = \frac{8.84 \times 100}{65} = 13.6\ \%.$$

The first discount of the given discount series is 10 %

Let the second discount of the series be m %

$$\therefore \text{ Single discount } \% = \left(m + n - \frac{mn}{100}\right)\%$$

or, $13.6\ \% = \left(m + 10 - \frac{10m}{100}\right)\%$

or, $13.6 = m + 10 - \frac{m}{10}$

or, $136 = 100 + 10m - m$ or, $9m = 36$

or, $m = 4\ \%$.

41. Ans. (b)

Solution A single discount equivalent to three given successive discounts of 20 %, 15 % and 5 % is given by

$$= \left(20+15+5-\frac{(20\times15+5\times15+5\times20)}{100}+\frac{20\times15\times5}{100^2}\right)\%$$

i.e. $(40 - 4.75 + 0.15)\ \% = 35.4\ \%$.

Marked price of 250 chairs = 250 × 50 = Rs. 12500.

$$\therefore \text{ Cash payment} = 12500 - 12500 \times \frac{35.4}{100}$$

= Rs. 8075.

42. Ans. (a)

Solution Using the formula

$$\text{C.P.} = \left[\frac{100^2 z}{(100-d)(100+p)}\right],$$

the cost price of each taperecorder is given by

$$\text{C.P.} = \left[\frac{100 \times 100 \times 1134}{(100-19)(100+40)}\right]$$

$$= \frac{100 \times 100 \times 1134}{81 \times 140} = \text{Rs. } 1000.$$

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. Jagdeep purchased 25 kg of rice at the rate of Rs. 16.50 per kg and 35 kg of rice at the rate of Rs. 24.50 per kg. He mixed the two and sold the mixture. Approximately, at what price per kg did he sell the mixture to make 25 per cent profit?

(a) Rs. 26.50 (b) Rs. 27.50
(c) Rs. 28.50 (d) Rs 30.30
(e) Rs. 29.00 **[Based on BSRB Mumbai PO 1998]**

2. Jeevan bought an article with 30 per cent discount on the labelled price. He sold the article with 12 per cent profit on the labelled price. What was his per cent profit on the price he bought?
(a) 40 (b) 50
(c) 60 (d) Data inadequate
(e) None of these **[Based onBank of Baroda PO 1999]**

3. A shopkeeper sold sarees at Rs. 266 each after giving 5% discount on labelled price. Had he not given the discount, he would have earned a profit of 12% on the cost price. What was the cost price of each saree?
(a) Rs. 280 (b) Rs. 260
(c) Rs. 240 (d) Data inadequate
(e) None of these
[Based on SBI Associates PO 1999]

4. A garment company declared 15% discount for wholesale buyers. Mr Sachdev bought garments from the company for Rs 25000 after getting discount. He fixed up the selling price of garments in such a way that he earned a profit of 8% on original company price. What is the approximate total selling price?
(a) Rs. 28000 (b) Rs. 29000
(c) Rs. 31000 (d) Rs. 28500
(e) Rs. 29500 **[Based on Guwahati PO 1999]**

5. A shopkeeper sold an article for Rs. 720 after giving 10% discount on the labelled price and made 20% profit on the cost price. What would have been the percentage profit, had he not given the discount?
(a) 25% (b) 30%
(c) 23% (d) 28%
(e) None of these **[Based on Guwahati PO 1999]**

6. A shopkeeper labels the price of article 20% above the cost price. If he allows Rs. 31.20 off on a bill of Rs. 312, find his profit per cent on the article?
(a) 8 (b) $12\frac{1}{3}$
(c) $11\frac{2}{3}$ (d) $8\frac{1}{3}$
(e) None of these
[Based on BSRB Mumbai PO 1999]

7. Mohan purchases 30 kg of wheat at the rate of Rs. 11.50 per kg and 20 kg of wheat at the rate of Rs. 14.25 per kg. He mixed the two and sold the mixture. Approximately at what price per kg should he sell the mixture to make 30% profit?
(a) Rs. 16.30 (b) Rs. 18.20
(c) Rs. 15.60 (d) Rs. 14.80
(e) Rs. 15.40 **[Based on BSRB Calcutta PO 1999]**

8. A shopkeeper bought 150 calculators at the rate of Rs. 250 per calculator. He spent Rs. 2500 on transportation and packing. If the market price of calculator is Rs. 320 per calculator and the shopkeeper gives a discount of 5% on the marked price then what will be the percentage profit gained by the shopkeeper?
(a) 20% (b) 14%
(c) 15% (d) 16%
(e) None of these
[Based on BSRB Hyderabad PO 1999]

9. If a dealer wants to earn 5% profit on an article after offering 10% discount to the customer, by what percentage should he increase his marked price to arrive at the label price?
(a) 15 (b) $16\frac{2}{3}$
(c) $15\frac{1}{2}$ (d) $16\frac{1}{2}$
(e) None of these **[Based on NABARD 1999]**

10. The ratio between the sale price and cost price of an article is 7 : 5. What is the ratio between the profit and the cost price of that article?
(a) 5 : 2 (b) 7 : 2
(c) 2 : 7 (d) Data inadequate
(e) None of these
[Based on BSRB Chennai PO 2000]

11. Ram purchased twenty dozen of toys at the rate of Rs. 375 per dozen. He sold each one of them at the rate of Rs. 33. What was his percentage profit?
(a) 6.5 (b) 5.6
(c) 3.5 (d) 4.5
(e) None of these
[Based on BSRB Chennai PO 2000]

12. The profit earned by selling an article for Rs. 832 is equal to the loss incurred when the same article is sold for Rs. 448. What should be the sale price of the article for making 50 per cent profit?
(a) Rs. 960 (b) Rs. 1060
(c) Rs. 1200 (d) Rs 920
(e) None of these
[Based on BSRB Chennai PO 2000]

13. Ram purchased 30 kg of rice at the rate of Rs. 17.50 per kg and another 30 kg rice at a certain rate. He mixed the two and sold the entire quantity at the rate of Rs. 18.60 per kg and made 20 per cent overall profit. At what price per kg did he purchase the lot of another 30 kg rice?
(a) Rs. 14.50 (b) Rs. 12.50
(c) Rs. 15.50 (d) Rs. 13.50
(e) None of these
[Based on BSRB Chennai PO 2000]

14. What per cent of selling price would be 34% of cost price if gross profit is 26% of the selling price?
(a) 17.16 (b) 74.00
(c) 25.16 (d) 88.40
(e) None of these
[Based on BSRB Bangalore PO 2000]

15. Mr. Gupta, Mr. Shastri and Mr Saxena together earned Rs. 19800. The ratio of earnings between Mr. Gupta and Mr. Shastri is 2 : 1 while that between Mr. Shastri and Mr. Saxena is 3 : 2. How much did Mr. Shastri earn?
(a) Rs. 3600 (b) Rs. 5400
(c) Rs. 1800 (d) Rs. 6300
(e) None of these
[Based on BSBR Bangalore PO 2000]

16. A grocer purchased 20 kg of wheat at the rate of Rs. 15 per kg and 30 kg of wheat at the rate of Rs. 13 per kg. At what price per kg should he sell the mixture to earn $33\frac{1}{3}$% profit on the cost price?
(a) Rs. 28.00 (b) Rs. 20.00
(c) Rs. 18.40 (d) Rs. 17.40
(e) None of these **[Based on BSRB Delhi PO 2000]**

17. By selling an article for Rs. 96, double the profit is obtained than the profit that would have been obtained by selling it for Rs. 84. What is the cost price of the article?
(a) Rs. 72.00 (b) Rs. 75.00
(c) Rs. 70.00 (d) Rs. 68.00
(e) None of these **[Based on BSRB Delhi PO 2000]**

18. A shopkeeper sells a T.V. set for Rs. 16560 at 10% discount on its marked price and earns 15% profit. If no discount is offered then what will be his per cent profit?
(a) $27\frac{7}{9}$ (b) $22\frac{7}{9}$
(c) $25\frac{7}{9}$ (d) Data inadequate
(e) None of these **[Based on BSRB Patna PO 2001]**

19. A shopkeeper sold an article offering a discount of 5% and earned a profit of 23.5%. What would have been the percentage of profit earned if no discount had been offered?
(a) 28.5 (b) 24.675
(c) 30 (d) Data inadequate
(e) None of these **[Based on Andhra Bank SO 2002]**

20. A shopkeeper sold a TV set for Rs. 17940, at a discount of 8% and gained 19.6%. If no discount is allowed, what will be his gain per cent?
(a) 25% (b) 36.4%
(c) 24.8% (d) Cannot be determined
(e) None of these
[Based on RBI Grade 'B' Officers 2002]

21. A man sold two watches for Rs. 240 each; on one he gained 20% and on the other he lost 20%. The gain or loss per cent in the transaction is
(a) 1% gain (b) 1% loss
(c) 4% gain (d) 4% loss
[Based on SI of Police Rec. Exam 1997]

22. Mohan sells two taperecorders at the same price. On one he gains 10% and on the other he loses 10%. The total gain or loss in transaction is
(a) 1% gain (b) 1% loss
(c) No loss or no gain (d) 2% loss
[Based on SI of Police Rec. Exam 1997]

23. A man bought a car for Rs. 60000 and spent 10% of the cost of the car for purchase of new tyres. At what price should he sell the car to make a gain of 15%?
(a) Rs. 79,500 (b) Rs. 74,500
(c) Rs. 75900 (d) Rs. 73500
[Based on SI of Police Rec. Exam 1997]

24. A merchant bought 60 sheeps at Rs. 120 per sheep. He sold 40 of them at Rs. 150 each. 10 of them died. What should be the selling price of the remaining sheeps if he wants a profit of Rs. 800?
(a) Rs. 150 (b) Rs. 200
(c) Rs. 250 (d) Rs. 180
[Based on SI of Police Rec. Exam 1997]

25. A merchant allows 10% for each payment on the marked price of an article and still gains at the rate of 10%. The cost price of an article which is marked as Rs. 77 is
(a) Rs. 70 (b) Rs. 60
(c) Rs. 63 (d) Rs. 62.37
[Based on SI of Police Rec. Exam 1997]

26. The price of a jewel, passing through three hands, rises on the whole to 65%. If the first and the second sellers earned 20% and 25% profit respectively, the profit earned by the third seller is
(a) 20% (b) 15%
(c) 10% (d) 5%
[Based on SI of Police Rec. Exam 1997]

27. A shopkeeper professes to sell his goods at cost price but uses a weight of 800 g instead of kilogram weight. Thus, he makes a profit of
(a) 2% (b) 8%
(c) 20% (d) 25%
[Based on SI of Police Rec. Exam 1997]

28. If selling price of an article is $\frac{4}{3}$ of its cost price, the profit in transaction is
(a) $\frac{1}{3}$% (b) $20\frac{1}{2}$%
(c) $33\frac{1}{3}$% (d) $25\frac{1}{2}$%
[Based on SI Rec. Exam (D.P.) 1997]

29. A grocer mixes 26 kg of tea which costs Rs. 20 a kg with 30 kg of tea which costs Rs. 36 a kg and sells the mixture at Rs. 30 a kg. His profit per cent is
(a) 5% (b) 6%
(c) 7% (d) 8%
[Based on SI Rec. Exam (D.P.) 1997]

30. If the cost price of 15 tables be equal to the selling price of 20 tables, the loss per cent in the transaction is
(a) 18% (b) 20%
(c) 25% (d) 24%
[Based on SI Rec. Exam (D.P.) 1997]

31. If 5% more is gained by selling an article for Rs. 350 then by selling it for Rs. 340, the cost of the article (in rupees) is
(a) 50 (b) 160
(c) 200 (d) 225
[Based on SI Rec. Exam (D.P.) 1997]

32. On selling an article for Rs. 700, the loss is 20%. To make a profit of 20%, the article must be sold at
(a) Rs. 1050 (b) Rs. 850
(c) Rs. 1075 (d) Rs. 875
[Based on SI Rec. Exam (D.P.) 1997]

33. A man purchased a box full of pencils at the rate of 7 for Rs. 9 and sold all of them at the rate of 8 for Rs. 11. In this transaction, he gained Rs. 10. How many pencils did the box contain?
(a) 100 (b) 112
(c) 114 (d) 115
[Based on SI Rec. Exam (D.P.) 1997]

34. The cost price of 16 articles is equal to selling price of 12 of them. The gain or loss per cent in the transaction is
(a) $33\frac{1}{3}$% loss (b) $23\frac{1}{3}$% loss
(c) $33\frac{1}{3}$% gain (d) $23\frac{1}{3}$% gain
[Based on SI Rec. Exam (D.P.) 1997]

35. A businessman marks his goods in such a way that even after allowing 12.5% discount on cash purchase, he gains 20%. If the cost price of the goods is Rs. 140, the marked price is
(a) Rs. 162 (b) Rs. 172
(c) Rs. 192 (d) Rs. 198
[Based on SI Rec. Exam (D.P.) 1997]

36. A watch is sold for Rs. 880 at a loss of 20%. For how much should it be sold to gain 10%?
(a) Rs. 1000 (b) Rs. 1100
(c) Rs. 1210 (d) Rs. 1400
[Based on SI Rec. Exam (D.P.) 1997]

37. Applied to a bill for Rs. 15000, the difference (in Rs.) between a discount of 50% and two successive discount of 30% and 20% is
(a) 0 (b) 450
(c) 900 (d) 1000
[Based on SI Rec. Exam (D.P.) 1997]

38. A sum of Rs. 2430 was divided among three persons X, Y, Z such that if their shares were diminished by Rs. 5, would be in the ratio 3 : 4 : 5. In this division, X got
(a) Rs. 1015 (b) Rs. 810
(c) Rs. 605 (d) Rs. 595
[Based on Assistant's Grade Exam 1997]

39. The ratio between the annual incomes of A and B is 4 : 3 and between their annual expenditures is 3 : 2. If at the end of a year both save Rs. 600 each, find the difference in the income.
(a) Rs. 450 (b) Rs. 500
(c) Rs. 600 (d) Rs. 750
[Based on Assistant's Grade Exam 1997]

40. If 3 toys are sold at the cost price of 4 toys of the same kind, the profit will be
(a) 25% (b) $33\frac{1}{3}$%
(c) $66\frac{2}{3}$% (d) 50%
[Based on SSC (GL) Prel. Exam 2000]

41. A sells a bicycle to B at a profit of 20%. B sells it to C at a profit of 25%. If C pays Rs. 225 for it, the cost price of the bicycle for A is
(a) Rs. 110 (b) Rs. 125
(c) Rs. 120 (d) Rs. 150
[Based on SSC (GL) Prel. Exam2000 (MS)]

42. A retailer buys 40 pens at the marked price of 36 pens from a whole-saler. If he sells these pens giving a discount of 10%, what is the profit per cent?
(a) 9% (b) 10%
(c) $10\frac{1}{9}$% (d) 11%
[Based on SSC (GL) Prel. Exam 2000]

43. 12 copies of a book were sold for Rs. 1800, thereby gaining cost price of 3 copies. The cost price of a copy is
(a) Rs. 120 (b) Rs. 150
(c) Rs. 1200 (d) Rs. 1500
[Based on SSC (GL) Prel. Exam 2000]

44. A shopkeeper marks his sharees at 20% above the cost price and allows a purchaser a discount of 10% for cash buying. What profit per cent does he make?
(a) 18 (b) 12
(c) 10 (d) 8
[Based on SSC (GL) Prel. Exam 2000]

45. A shopkeeper purchased a chair marked at Rs. 800, at two successive discounts of 10% and 15%, respectively. He spent Rs. 28 on transportation and sold the chair for Rs. 800. His gain per cent is
(a) 40 (b) 30
(c) 25 (d) 14
[Based on SSC (GL) Prel. Exam 2000]

46. The selling price of 5 articles is same as the cost price of 3 articles. The gain or loss is
(a) 20% gain (b) 25% gain

(c) 33.33% loss (d) 40% loss

[Based on SSC (GL) Prel. Exam 2000]

47. By selling an article for Rs. 480 *A* lost 20%. For what should he sell it to make a profit of 20%?
(a) Rs. 800 (b) Rs. 760
(c) Rs. 720 (d) Rs. 680

[Based on SSC (GL) Prel. Exam 2000]

48. A house worth Rs. 150000 is sold by *X* at 5% profit to *Y*. *Y* sells the house back to *X* at a 2% loss. Then, in the entire transaction
(a) *X* gains Rs. 4350 (b) *X* loses Rs. 4350
(c) *X* gains Rs. 3150 (d) *X* loses Rs. 3150

[Based on SSC (GL) Prel. Exam 2000]

49. A shopkeeper purchases 12 balloons for Rs. 10 and sells them at 10 balloons for Rs. 12. Thus, he earns a profit of
(a) 35% (b) 36%
(c) 44% (d) 45%

[Based on SSC (GL) Prel. Exam 2000]

50. Anu sold 2 books at Rs. 1.40 each. Her profit on one was 20% and her loss on the other was 20%. Then, she
(a) made no gain no loss
(b) Gained 20 paise
(c) Lost 20 paise (d) lost 12 paise

[Based on SSC (GL) Prel. Exam 2000]

51. If *I* had purchased 11 articles for Rs. 10 and sold all articles at the rate of 10 for Rs. 11, the profit per cent would have been
(a) 10% (b) 11%
(c) 21% (d) 100%

[Based on SSC (GL) Prel. Exam 2002]

52. By selling an article for Rs. 72, there is a loss of 10%. In order to gain 5%, its selling price should be
(a) Rs. 87 (b) Rs. 85
(c) Rs. 80 (d) Rs. 84

[Based on SSC (GL) Prel. Exam 2002]

53. An article is sold at a loss of 10%. Had it been sold for Rs. 9 more, there would have been a gain of $12\frac{1}{2}$% on it. The C.P. of the article is
(a) Rs. 40 (b) Rs. 45
(c) Rs. 50 (d) Rs. 35

[Based on SSC (GL) Prel. Exam 2002]

54. If C.P. is Rs. 80, overhead is Rs. 20 and S.P. is Rs. 120, then profit per cent is
(a) 20% (b) 50%
(c) 40% (d) 30%

[Based on SSC (GL) Prel. Exam 2002]

55. If S.P. of an article is $\frac{8}{5}$ times its C.P., the profit per cent on it is
(a) 120% (b) 160%
(c) 40% (d) 60%

[Based on SSC (GL) Prel. Exam 2002]

56. A man sells two articles at Rs. 99 each. On one he gains 10% and on the other he loses 10%. What is his gain or loss per cent in the whole transaction?
(a) Loss, 1% (b) Loss, 1.5%
(b) Profit, 1% (d) Profit, 1.5%

[Based on SSC (GL) Prel. Exam 2002]

57. The cost price of 18 articles is equal to S.P. of 15 articles. The gain per cent is
(a) 15% (b) 20%
(c) 25% (d) 18%

[Based on SSC (GL) Prel. Exam 2002]

58. If a person sells a sari for Rs. 5200, making a profit of 30%, then the cost price of sari is
(a) Rs. 4420 (b) Rs. 4000
(c) Rs. 3900 (d) Rs. 3800

[Based on SSC (GL) Prel. Exam 2002]

59. The ratio of the cost price and the selling price is 4 : 5. The profit per cent is
(a) 10% (b) 20%
(c) 25% (d) 30%

[Based on SSC (GL) Prel. Exam 2002]

60. Two bicycles were sold for Rs. 3990 each, gaining 5% on one and losing 5% on the other. The gain or loss per cent on the whole transaction is
(a) neither gain nor loss
(b) 2.5% gain
(c) 2.5% loss
(d) 0.25% loss

[Based on SSC (GL) Prel. Exam 2002]

61. If the S.P. of 40 articles is equal to the C.P. of 50 articles, then the loss or gain per cent is
(a) 25% loss (b) 20% loss
(c) 25% gain (d) 20% gain

[Based on SSC (GL) Prel. Exam 2003]

62. A man bought a second hand machine for Rs. 1200 and spent Rs. 200 on its repairs. He sold it for Rs. 1680. His profit per cent is
(a) 20% (b) 10%
(c) 8% (d) 16%

[Based on SSC (GL) Prel. Exam 2003]

63. An item costing Rs. 840 was sold by a shop-keeper at a gain of 10% and it was again sold by the new buyer at a loss of 5%. Find S.P. of the item
(a) Rs. 877.80 (b) Rs. 798
(c) Rs. 924 (d) Rs. 37.80

[Based on SSC (GL) Prel. Exam 2003]

64. 100 oranges are bought for Rs. 350 and sold at the rate of Rs. 48 per dozen. The per cent of profit or loss is
(a) 12% loss (b) 15% gain

(c) $14\frac{2}{7}$% loss (d) $14\frac{2}{7}$% profit

[Based on SSC (GL) Prel. Exam 2003]

65. A merchant fixes the sale price of his goods at 15% above the cost price. He sells his goods at 12% less than the fixed price. His percentage of profit is

(a) $2\frac{1}{2}$ (b) $1\frac{1}{5}$

(c) $1\frac{1}{2}$ (d) 2

[Based on SSC (GL) Prel. Exam 2003]

66. If the ratio of C.P. and S.P. is 5 : 6. The gain % is

(a) 20% (b) $33\frac{1}{3}$%

(c) 25% (d) 30%

[Based on SSC (GL) Prel. Exam 2003]

67. Ramesh bought a gas stove and paid 10% less than its original price. He sold it at 30% profit on the price he had paid what percentage of profit did Ramesh earn on the original price?

(a) 32% (b) 11%

(c) 20% (d) 17%

[Based on IBPS Jr. Executive 2002]

68. A builder purchased a plot of land for Rs. 80 lakh and constructed a five storey building inclusive of ground floor on it. How much should he charge for each flat to make 25% profit on his investment on land, if there are five flats on each storey?

(a) Rs. 50000 (b) Rs. 100000

(c) Rs. 500000 (d) Rs. 2000000

(e) None of these

[Based on IBPS JR. Executive 2002]

EXPLANATORY ANSWERS

1. Ans (a)

Solution C.P. = 25 × 16.50 + 35 × 24.50 = Rs. 1270

S.P. = $1270 \times \frac{125}{100}$ = Rs. 1587.50

Price per kg = $\frac{1587 \cdot 50}{60}$ = Rs. 26.50 (approx.)

2. Ans. (c)

Solution Let the labelled price of the article = Rs. 100

Then C.P. = Rs. 70 and S.P. = Rs. 112

∴ Required profit per cent = $\frac{112-70}{70} \times 100$

$= \frac{42}{7} \times 10$

= 60.

3. Ans. (e)

Solution Marked price = $266 \times \frac{100}{95}$ = Rs. 280

cost price = $280 \times \frac{88}{100}$ = Rs. 246.40

4. Ans. (c)

5. Ans. (e)

6. Ans. (a)

Solution Let the C.P. = Rs. 100

Marked price = Rs. 120

Discount = $\frac{312}{31 \cdot 2}$ = 10% = Rs. 12

S.P. = 120 – 12 = Rs. 108. Profit % = 108 – 100 = 8%.

7. Ans. (a)

Solution S.P. = $630 \times \frac{130}{100} \times \frac{1}{50}$ = Rs. 16.30.

8. Ans. (b)

Solution C.P. of 150 calculators = Rs. 150 × 250

= Rs. 37500

∴ Total C.P. = 37500 + 2500 = Rs. 40000

Marked price of 150 calculators = 150 × 320

= Rs. 48000

Selling price after discount = $\frac{95}{100} \times 48000$

= Rs. 45600

∴ Percentage profit = $\frac{45600-40000}{40000} \times 100$

= 14%.

9. Ans. (b)

Solution Required % increase

$= \left(\frac{100+5}{100-10}\right) \times 100 - 100$

$= \frac{105}{90} \times 100 - 100 = 100\left[\frac{105}{90} - 1\right]$

$= 100 \times \frac{15}{90} = \frac{100}{6} = 16\frac{2}{3}$%.

10. Ans. (e)

11. Ans. (b)

Solution Cost price of 20 dozen toys = 20 × 375

= Rs. 7500

Selling price of 20 dozen toys = 20 × 33 × 12

= Rs. 7920

$$\text{Profit percentage} = \frac{7920-7500}{7500} \times 100$$
$$= 5.6\%$$

12. Ans. (a)

Solution Let the profit or loss be Rs. x. Then, $832 - x = 448 + x$

or, $x = \frac{384}{2}$ = Rs. 192.

$\therefore$ Cost price of the article = $832 - x = 448 + x$ = Rs. 640

$\therefore$ S.P. of the article = $640 \times \frac{150}{100}$ = Rs. 960.

13. Ans. (d)

14. Ans. (c)

Solution Let the S.P. of the article be Rs. 100

$\therefore$ Profit = Rs. 26

$\therefore$ Cost price of the article = 100 – 26 = Rs. 74

$\therefore$ Required percentage = $\frac{34 \times 74}{100}$ = 25.16%.

15. Ans. (b)

Solution Ratio of investment = 6 : 3 : 2

$\therefore$ Share of Mr. Shastri = $\frac{3}{11} \times 19800$
= Rs. 5400

16. Ans. (c)

Solution C.P. = 20 × 15 + 30 × 13 = Rs. 690

$\therefore$ S.P. = $\frac{4}{3}$ of $690 \times \frac{1}{50}$ = Rs. 18.40.

17. Ans. (a)

Solution Let the cost price of the article be Rs x

Then, $2(84 - x) = 96 - x$

or, $168 - 2x = 96 - x$

$\therefore$ x = Rs. 72.

18. Ans. (a)

19. Ans. (c)

Solution Giving no discount to customer implies selling the product on printed price. Suppose the cost price of the article is Rs. 100

$$\text{Then, printed price} = \frac{100 \times (100+23.5)}{(100-5)}$$
$$= \frac{100 \times 247}{190}$$
$$= \text{Rs. } 130.$$

20. Ans. (e)

Solution If no price given, selling price of T.V.

$$= 17940 \times \frac{100}{92}$$
$$= \text{Rs. } 19500$$

$$\text{Cost price of T.V.} = 17940 \times \frac{100}{119.60}$$
$$= \text{Rs. } 15000$$

$$\text{Gain \%} = \frac{19500-15000}{15000} \times 100$$
$$= 30\%.$$

21. Ans. (d)

Solution S.P. of both the watches = Rs. 480

S.P. of each watch = Rs. 240

Gain = 20%

$\therefore$ C.P. = $\frac{240 \times 100}{100+20}$ = 200

If loss = 20%, then C.P. = $\frac{240 \times 100}{100-20}$ = 300

$\therefore$ C.P. of both the watches = Rs. 500

$\therefore$ Loss % = $\frac{20}{500} \times 100$ = 4.

22. Ans. (b)

Solution Suppose S.P. of each recorder = Rs K

If Gain = 10%, then

$$\text{C.P.} = \frac{K \times 100}{100+10} = \frac{100K}{110} = \frac{10K}{11}$$

If loss = 10%, then

$$\text{C.P.} = \frac{K \times 100}{100-10} = \frac{100K}{90} = \frac{10K}{9}$$

$\therefore$ C.P. of both the taperecorders

$$= \frac{10K}{11} + \frac{10K}{9} = \frac{200K}{99}$$

S.P. of both the taperecorders = $2K$

$\therefore$ Loss % = $\frac{2K}{200K} \times 100$ = 1.

23. Ans. (c)

24. Ans. (b)

Solution Let S.P. of remaining 10 sheep = K each

$\therefore$ $40 \times 150 + 10K - 60 \times 120 = 800$

$\Rightarrow$ $K = 200$.

25. Ans. (c)

Solution Suppose C.P. = Rs. K

Marked price = Rs. 77

$\therefore$ S.P. = 77 – 10% = 69.30

$\therefore$ $69.30 - K = 10\%$ of K

$\Rightarrow$ $11K = 693$

$\Rightarrow$ $K = 63.$

26. Ans. (c)

27. Ans. (d)

Solution Suppose C.P. of 1000 gm = Rs. 1000

S.P. of 800 gm = Rs. 1000

$\therefore$ S.P. of 1000 gm = $\frac{1000}{800} \times 1000$ = Rs. 1250

$\therefore$ Profit % = $\frac{250}{1000} \times 100 = 25\%.$

28. Ans. (c)

Solution S.P. = $\frac{4}{3}$ C.P.

Gain = $\frac{1}{3}$ C.P. = $\frac{100}{3}$% of C.P.

$\therefore$ Gain % = $33\frac{1}{3}$.

29. Ans. (a)

Solution C.P. = $26 \times 20 + 30 \times 36 = 520 + 1080$
$= 1600$

S.P. = $56 \times 30 = 1680$

$\therefore$ Profit % = 5.

30. Ans. (c)

Solution Let S.P. of 20 tables = Rs. 20

$\therefore$ C.P. of 15 tables = Rs. 20

$\therefore$ C.P. of 20 tables = $\frac{20}{15} \times 20 = \frac{80}{3}$

$\therefore$ Loss % = $\frac{\frac{80}{3} - 20}{\frac{80}{3}} \times 100 = 25.$

31. Ans. (c)

32. Ans. (a)

Solution S.P. = Rs. 700, Loss = 20%

$\therefore$ C.P. = $\frac{\text{S.P.} \times 100}{100 - \text{loss\%}} = \frac{700 \times 100}{80} = 875$

If gain is 20%, then

S.P. = 875 + 20% of 875

= 875 + 175 = Rs. 1050

33. Ans. (b)

Solution C.P. of 7 pencils = Rs. 9

C.P. of 1 pencil = Rs. $\frac{9}{7}$

S.P. of 8 pencils = Rs. 11

S.P. of 1 pencil = Rs. $\frac{11}{8}$

$\therefore$ Gain on 1 pencil = $\frac{11}{8} - \frac{9}{7}$

$= \frac{77 - 72}{56} = \frac{5}{56}$

If gain is Rs. 10, then number of pencils purchased $= \frac{56}{5} \times 10 = 112.$

34. Ans. (c)

Solution Let C.P. of 16 articles = Rs. 16

S.P. of 12 articles = Rs. 16

$\therefore$ S.P. of 16 articles = $\frac{16}{12} \times 16$ = Rs. $\frac{256}{12}$

= Rs. $\frac{64}{3}$

$\therefore$ Gain on Rs. 16 = $\frac{64}{3} - 16 = \frac{16}{3}$

Gain % = $\frac{\frac{16}{3}}{16} \times 100 = 33\frac{1}{3}.$

35. Ans. (c)

Solution Suppose marked price = Rs. K

$\therefore$ $K - 12.5\%$ of $K = 140 + 20\%$ of 140

$\Rightarrow$ 87.5% of $K = 140 + 28 = 168$

$\therefore$ $K = \frac{16800}{87.50} = 192.$

36. Ans. (c)

Solution S.P. = Rs. 880, Loss = 20%

$\therefore$ C.P. = $\frac{\text{S.P.} \times 100}{100 - \text{Loss\%}}$

$= \frac{880 \times 100}{80} = 1100$

If gain is to be 10%, then S.P. should be Rs. 1210.

37. Ans. (c)

Solution 50% of 15000 = 7500

30% of 15000 = 4500

Balance = 10500

20% of 10500 = 2100

Balance = 8400

$\therefore$ Difference between a single discount of 50% and two successive discounts of 30% and 20% is Rs. 900.

38. Ans. (c)

Solution Rs. 2400 are divided in the ratio of 3 : 4 : 5 between X, Y and Z

$\therefore$ X's share $= \frac{3}{12} \times 2400 =$ Rs. 600

Since his share has been diminished by Rs. 5, X got Rs. 605.

39. Ans. (c)

Solution Annual income of A and B are $4K$ and $3K$, say

$\therefore$ $4K - 3L = 600$

$3K - 2L = 600$

$K = 600, L = 600$

$\therefore$ Difference in incomes of A and B

$= 4K - 3K = K = 600.$

40. Ans. (b)

Solution Let C.P. of 4 toys = Rs. x

$\therefore$ S.P. of 3 toys = Rs. x

S.P of 4 toys $= \frac{4x}{3}$

Profit % $= \frac{x/3}{x} \times 100 = 33\frac{1}{3}$.

41. Ans. (d)

Solution Let C.P. of the bicycle for A = Rs. x

C.P. of $B = x + 20\%$ of $x = \frac{6x}{5}$

C.P. of $C = \frac{6x}{5} + 25\%$ of $\frac{6x}{5}$

$= \frac{6x}{5} + \frac{6x}{20} = \frac{3}{2}x$

$\therefore$ $\frac{3}{2}x = 225 \Rightarrow x = 150.$

42. Ans. (b)

Solution Marked price of 36 pens = Rs. 36, say

$\therefore$ C.P. of 40 pens = Rs. 36

C.P. of 36 pens = Rs. $\frac{36}{40} \times 36$ = Rs. $\frac{9}{10} \times 36$

$= \frac{324}{10} = \frac{3240}{100}$

S.P. of 36 pens = Rs. 36 – 10% of 36

$= 36 - \frac{36}{100} = \frac{3564}{100}$

$\therefore$ Profit % $= \frac{3564 - 3240}{3240} \times 100 = 10.$

43. Ans. (a)

Solution Let C.P. of a book = Rs. x

$\therefore$ C.P. of 12 books = Rs. $12x$

S.P. of 12 books = Rs. 1800

Profit = Rs. $3x$

$\therefore$ $15x = 1800 \Rightarrow x = 120.$

44. Ans. (d)

45. Ans. (c)

Solution Cost price of chair

$= 800\left(1 - \frac{10}{100}\right)\left(1 - \frac{15}{100}\right) = 800 \times \frac{9}{10} \times \frac{17}{20} =$ Rs. 612

Total cost price after transportion = 612 + 28

= 640

$\therefore$ Gain % $= \frac{800 - 640}{640} \times 100 = \frac{160}{640} \times 100 = 25\%$

46. Ans. (d)

Solution Loss percentage $= \frac{5-3}{5} \times 100 = \frac{2}{5} \times 100$

$= 40\%.$

47. Ans. (c)

Solution New S.P. $= 480 \times \frac{100}{80} \times \frac{120}{100}$ = Rs. 720.

48. Ans. (a)

Solution X's gain $= \left(5 - 2 - \frac{5 \times 2}{100}\right)\%$ of 1,50,000

= Rs. 4350.

49. Ans. (c)

Solution Suppose number of balloons purchased = 120

$\therefore$ C.P. of 120 balloons = Rs. 100

S.P. of 120 balloons = Rs. 144

$\therefore$ Profit = 44%.

50. Ans. (d)

Solution S.P. of 2 books = Rs. 2.80

Over all percentage change

$= 20 + (-20) + \frac{20 \times (-20)}{100} = -4 = 4\%$ loss

$\therefore$ C.P. $= \frac{2.80 \times 100}{100 - 4} = 2.92$

$\Rightarrow$ Loss = Re. 0.12.

51. Ans. (a)

Solution Percentage profit or loss is given by

$= \frac{11 \times 11 - 10 \times 10}{10 \times 10} \times 100 = 21\%$

Since the sign is +ve, there is a gain of 21%.

52. Ans. (d)

Solution $(100 - 10)\%$ of $x = 72$

$\therefore$ 105% of $x = \frac{72}{90} \times 105$ = Rs. 84.

53. Ans. (a)

Solution $\left(10 + 12\frac{1}{2}\right)\%$ of $x = 9$

$\therefore$ 100% of $x = \frac{9 \times 100}{22 \cdot 5}$ = Rs. 40.

54. Ans. (a)

Solution % profit $= \frac{120 - (80 + 20)}{(80 + 20)} \times 100 = 20\%.$

55. Ans. (d)

Solution % profit = $\frac{8-5}{5} \times 100 = 60\%$.

56. Ans. (a)

Solution S.P. of two articles = Rs. 2 × 99 = Rs. 198

Ove rall % change

$= 10 + (-10) + \frac{10\times(-10)}{100} = -1 = 1\%$ loss.

57. Ans. (b)

Solution Gain per cent = $\frac{18-15}{15} \times 100 = 20\%$.

58. Ans. (b)

Solution C.P. = $\frac{5200\times100}{100+30} = \frac{5200\times100}{130}$ = Rs. 4000

59. Ans. (c)

60. Ans. (d)

Solution S.P. = 3990, gain = 5%

$\therefore$ C.P. = $\frac{3990\times100}{100+5} = 3800$

S.P. = Rs. 3990, Loss = 5%

$\therefore$ C.P. = $\frac{3990\times100}{100-5} = 4200$

$\therefore$ Total C.P. = Rs. 8000

Total S.P. = Rs. 7980

$\therefore$ Loss% = $\frac{20\times100}{8000} = \frac{1}{4} = 0.25$.

61. Ans. (c)

Solution C.P. of 50 articles = Rs. 50, say

$\therefore$ S.P. of 40 articles = Rs. 50

$\therefore$ S.P. of 50 articles = Rs. 62.50

$\therefore$ Gain % = $\frac{12\cdot50}{50} \times 100 = 25$.

62. Ans. (a)

Solution C.P. of machine = 1200 + 200 = Rs. 1400

% profit = $\frac{1680-1400}{1400} \times 100 = 20\%$.

63. Ans. (a)

Solution S.P. = $840 \times \frac{110}{100}\times\frac{95}{100}$ = Rs. 877.80.

64. Ans. (d)

Solution C.P. of 1 orange = Rs. 3.50

S.P. of 1 orange = Rs. 4

% profit = $\frac{0.50}{3.50} \times 100 = 14\frac{2}{7}\%$

65. Ans. (b)

Solution % profit = $15 - 12 - \frac{15\times12}{100}$

$= 3 - 1.80 = 1.20 = 1\frac{1}{5}\%$.

66. Ans. (a)

Solution Gain % = $\frac{1}{5} \times 100 = 20\%$.

67. Ans. (d)

Solution % profit which Ramesh gets

$= -10 + 30 - \frac{10\times30}{100} = +17\%$.

68. Ans. (d)

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. A fruitseller buys oranges at 6 for Re. 1 and an equal number at 4 for Re. 1. He mixes them and sells at 20 paise per orange. Find his gain or loss per cent.
(a) 3 % profit (b) 5 % loss
(c) 4 % loss (d) 10 % profit

2. Mohan purchased 7 oranges for Rs. 6 and sold the whole lot at the rate of 3 for Rs. 4. In the process he gained Rs. 40. He purchased oranges:
(a) 63 (b) 80
(c) 84 (d) 90

3. *A* purchased an article for Rs. 500. He sold it to *B* at a profit of 10 %. *B* sold it back to *A* at a loss of 10 %. In the whole process how much did *A* gain?
(a) Rs. 40 (b) Rs. 4
(c) Rs. 55 (d) Rs. 24
(e) None of these

4. A pressure cooker was sold for Rs. 120. $\frac{1}{5}$th part of S.P. was sales tax and $\frac{1}{3}$rd part of the remaining was profit. What is C.P. of the pressure cooker?
(a) Rs. 75 (b) Rs. 64
(c) Rs. 80 (d) Rs. 72
(e) None of these

5. By selling tea at Rs. 120 per kg, a shopkeeper makes a profit of 20 %. By how much per cent should he increase his S.P. so as to gain 30 %?
(a) $9\frac{1}{3}\%$ (b) $8\frac{2}{3}\%$

(c) $8\frac{1}{3}\%$ (d) $7\frac{1}{3}\%$

6. A man sold a watch for Rs. 600 at a loss. Had he sold it for Rs. 750, his gain would have been double the former loss. The cost price of the watch is
(a) Rs. 800 (b) Rs. 700
(c) Rs. 670 (d) Rs. 650

7. Sita buys a fridge at $\frac{15}{16}$th of its value and sells it for 10 % more than its value. Then, gain % is
(a) 11.67 (b) 15.55
(c) 16.67 (d) None of these

8. A shopkeeper earns 15 % profit on a shirt even after allowing 31 % discount on the list price. If list price is Rs. 125, then cost price of shirt is
(a) Rs. 87 (b) Rs. 80
(c) Rs. 75 (d) Rs. 69
(e) None of these

9. A man bought 70 kg rice worth Rs. 175. If he sells it at Rs. 2.75 per kg, his total gain or loss per cent is:
(a) 10% profit (b) 5% loss
(c) 8% profit (d) 4% loss

10. Tarun bought a T.V. with 20 % discount on the labelled price. Had he bought it with 25 % discount, he would have saved Rs. 500. At what price did he buy T.V.?
(a) Rs. 5000 (b) 10000
(c) Rs. 12000 (d) 16000

Test II

[Time: 10 min]

11. A trader sells his articles 20 % above C.P. and allows a discount of 10% on cash payment, his gain per cent is:
(a) 10% (b) 8%
(c) 6% (d) 5%

12. A fruit vendor purchased 88 dozens of bananas at Rs. 10 per dozen. Out of this, 12 dozens got rotten and he sold them at Rs. 6 per dozen. Remaining bananas were sold by him at Rs. 14 per dozen. What profit did the make in this transaction ?
(a) 26.5 % (b) 29.1 %
(c) 31% (d) 33.3 %

13. If an article is successively sold at a profit of 8% and 11%, difference in two selling prices is Rs. 15. What is the cost price of this article ?
(a) Rs. 490 (b) Rs. 480
(c) Rs. 510 (d) Rs. 500

14. A person bought some oranges at Rs 10 per dozen and bought the same number of oranges at Rs. 8 per dozen. He sold these oranges at Rs. 11 per dozen and gained Rs. 120. The total number of oranges bought by him was
(a) 30 dozens (b) 40 dozens
(c) 50 dozens (d) 60 dozens

15. A tradesman marks his goods 30% more than the cost price. If he allows a discount of $6\frac{1}{4}$, then his gain per cent is:
(a) $22\frac{3}{4}\%$ (b) 22%
(c) $21\frac{7}{8}\%$ (d) none of these

16. 20 percent profit is made on goods when a 10 % discount is given on the list price. When 20 % discount is given on the list price, the profit will be
(a) $6\frac{1}{2}\%$ (b) 15%
(c) $6\frac{2}{3}$ (d) 10 %

17. If a shopkeeper buys almonds at Rs. 250 per kg and sells at Rs. 10 per 50 gm, then he
(a) loses 20% (b) Loses 25 %
(c) Gains 20 % (d) gains 25 %
(e) None of these

18. A trader bought some knives at the rate of 8 knives for Rs. 34 and sold them at 12 knives for Rs. 57. How many knives must be sold by him to gain Rs. 900 ?
(a) 1400 (b) 1600
(c) 1800 (d) 200
(e) None of these

19. A shopkeeper sold an article at a profit of 17.5%. If he had bought it at 8% less and sold it at 30% profit, he would have earned Rs. 11.55 more as profit. Cost price of the article is:
(a) Rs. 550 (b) Rs. 675
(c) Rs. 750 (d) Rs. 475

20. Ajay sold an article at a loss of 5%. Had he sold it at a gain of 5%, he would have gained Rs. 67.50 more. Find the cost price of the article:
(a) Rs. 670 (b) Rs. 575
(c) Rs. 675 (d) Rs. 560

Answers

1. (c)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (b)
11. (b)	**12.** (b)	**13.** (d)	**14.** (d)	**15.** (c)	**16.** (c)	**17.** (a)	**18.** (c)	**19.** (a)	**20.** (c)

13

TIME AND WORK, WORK AND WAGES

INTRODUCTION

In our daily life, we come across situations where we need to complete a particular job in a reasonable time. We have to complete the project earlier or later depending upon the needs. Accordingly, the men on duty have to be increased or decreased, i.e. the time allowed and the men engaged for a project are inversely proportional to each other, i.e. the more the number of men involved, the lesser is the time required to finish a job. We also come across situations where time and work or men and work are in direct proportion to each other.

For solving problems on time and work, we follow the following general rules:

1. If 'A' can do a piece of work in n days, then at a uniform rate of working 'A' will finish $\frac{1}{n}$th work in one day.
2. If $\frac{1}{n}$ of a work is done by 'A' in one day, then 'A' will take n days to complete the full work.
3. If 'A' does $\frac{1}{n}$th of a work in one hour then to complete the full work, 'A' will take $\frac{n}{m}$ hours.
4. If 'A' does three times faster work than 'B', then ratio of work done by A and B is 3 : 1 and ratio of time taken by A and B is 1 : 3.
5. A, B and C can do a piece of work in T_1, T_2 and T_3 days, respectively. If they have worked for D_1, D_2 and D_3 days respectively, then

   Amount of work done by $A = \frac{D_1}{T_1}$

   Amount of work done by $B = \frac{D_2}{T_2}$

   and, Amount of work done by $C = \frac{D_3}{T_3}$

   Also, the amount of work done by A, B and C together

   $$= \frac{D_1}{T_1} + \frac{D_2}{T_2} + \frac{D_3}{T_3}.$$

   which will be equal to 1, if the work is complete.

SOME USEFUL SHORT-CUT METHODS

1. If A can do a piece of work in X days and B can do the same work in Y days, then both of them working together will do the same work in $\frac{XY}{X+Y}$ days.

Explanation

A's 1 day's work $= \frac{1}{X}$

B's 1 day's work $= \frac{1}{Y}$

Then, $(A + B)$'s 1 day's work $= \frac{1}{X} + \frac{1}{Y} = \frac{X+Y}{XY}$

$\therefore$ A and B together can complete the work in

$$= \frac{XY}{X+Y} \text{ days.}$$

Illustration 1 A can finish a piece of work by working alone in 6 days and B, while working alone, can finish the same work in 12 days. If both of them work together, then in how many days, the work will be finished?

Solution Here, $X = 6$ and $Y = 12$.

$\therefore$ Working together, A and B will complete the work in $= \frac{XY}{X+Y}$ days $= \frac{6 \times 12}{6+12}$ days, i.e. 4 days.

2. If A, B and C, while working alone, can complete a work in X, Y and Z days respectively, then they will together complete the work in $\dfrac{XYZ}{XY + YZ + ZX}$ days.

Explanation

A's 1 day's work $= \dfrac{1}{X}$

B's 1 day's work $= \dfrac{1}{Y}$

C's 1 day's work $= \dfrac{1}{Z}$

$\therefore$ $(A + B + C)$'s 1 day's work

$$= \frac{1}{X} + \frac{1}{Y} + \frac{1}{Z} = \frac{XY + YZ + ZX}{XYZ}.$$

So, A, B and C together can complete the work in

$$= \left(\frac{XYZ}{XY + YZ + ZX}\right) \text{ days.}$$

Illustration 2 A, B and C can complete a piece of work in 10, 15 and 18 days, respectively. In how many days would all of them complete the same work working together?

Solution Here, $X = 10$, $Y = 15$ and $Z = 18$.

Therefore, the work will be completed in

$$= \frac{XYZ}{XY + YZ + ZX} \text{days}$$

$$= \frac{10 \times 15 \times 18}{10 \times 15 + 15 \times 18 + 18 \times 10} \text{days,}$$

i.e. $\dfrac{2700}{600}$ or, $4\dfrac{1}{2}$ days.

3. Two persons A and B, working together, can complete a piece of work in X days. If A, working alone, can complete the work in Y days, then B, working alone, will complete the work in $\dfrac{XY}{Y - X}$ days.

Explanation

A and B together can complete the work in X days.

$\therefore$ $(A + B)$'s 1 day's work $= \dfrac{1}{X}$

Similarly, A's 1 day's work $= \dfrac{1}{Y}$

Therefore, B's 1 day's work $= \dfrac{1}{X} - \dfrac{1}{Y} = \dfrac{Y - X}{XY}$.

$\therefore$ B alone can complete the work in $\left(\dfrac{XY}{Y - X}\right)$ days.

$\therefore$ B alone will complete the work in

$$= \frac{XY}{Y - X} \text{ days} = \frac{15 \times 20}{20 - 15}, \text{ i.e. 60 days.}$$

Illustration 3 A and B working together take 15 days to complete a piece of work. If A alone can do this work in 20 days, how long would B take to complete the same work?

Solution Here, $X = 15$ and $Y = 20$.

4. If A and B, working together, can finish a piece of work in X days, B and C in Y days, C and A in Z days, then

(a) A, B and C working together, will finish the job in $\left(\dfrac{2XYZ}{XY + YZ + ZX}\right)$ days.

(b) A alone will finish the job in $\left(\dfrac{2XYZ}{XY + YZ - ZX}\right)$ days.

(c) B alone will finish the job in $\left(\dfrac{2XYZ}{ZX + XY - YZ}\right)$ days.

Explanation

$(A + B)$'s 1 day's work $= \dfrac{1}{X}$

$(B + C)$'s 1 day's work $= \dfrac{1}{Y}$

$(C + A)$'s 1 day's work $= \dfrac{1}{Z}$.

So, $[(A + B) + (B + C) + (C + A)]$'s 1 day's work $= \dfrac{1}{X} + \dfrac{1}{Y} + \dfrac{1}{Z}$.

or, $2\,(A + B + C)$'s 1 day's work $= \left(\dfrac{1}{X} + \dfrac{1}{Y} + \dfrac{1}{Z}\right)$

or, $(A + B + C)$'s 1 day's work $= \dfrac{1}{2}\left(\dfrac{1}{X} + \dfrac{1}{Y} + \dfrac{1}{Z}\right)$

i.e. $\left(\dfrac{XY + YZ + ZX}{2XYZ}\right)$

$\therefore$ A, B and C working together, will complete the work in $\left(\dfrac{2XYZ}{XY + YZ + ZX}\right)$ days.

Also, A's 1 day's work = $(A + B + C)$'s 1 day's work $- (B + C)$'s 1 day's work

$$= \frac{1}{2}\left(\frac{1}{X} + \frac{1}{Y} + \frac{1}{Z}\right) - \frac{1}{Y}$$

$$= \frac{1}{2}\left(\frac{1}{X} - \frac{1}{Y} + \frac{1}{Z}\right)$$

$$= \frac{XY + YZ - ZX}{2XYZ}.$$

So, A alone can do the work in $\left(\frac{2XYZ}{XY + YZ - ZX}\right)$ days.

Similarly, B alone can do the work in $\left(\frac{2XYZ}{YZ + ZX - XY}\right)$ days and C alone can do the work in $\left(\frac{2XYZ}{ZX + XY - YZ}\right)$ days.

Illustration 4 A and B can do a piece of work in 12 days, B and C in 15 days, C and A in 20 days. How long would each take separately to do the same work?

Solution Here, $X = 12$, $Y = 15$ and $Z = 20$.

$\therefore$ A alone can do the work in

$$= \frac{2XYZ}{XY + YZ - ZX}$$

$$= \frac{2 \times 12 \times 15 \times 20}{12 \times 15 + 15 \times 20 - 20 \times 12} \text{ days}$$

or, $\frac{7200}{240}$, i.e. 30 days.

B alone can do the work in

$$= \frac{2XYZ}{YZ + ZX - XY} \text{ days}$$

$$= \frac{2 \times 12 \times 15 \times 20}{15 \times 20 + 20 \times 12 - 12 \times 15} \text{ days}$$

or, $\frac{7200}{360}$, i.e. 20 days.

C alone can do the work in

$$= \frac{2XYZ}{ZX + XY - YZ} \text{ days}$$

$$= \frac{2 \times 12 \times 15 \times 20}{20 \times 12 + 12 \times 15 - 15 \times 20} \text{ days}$$

or, $\frac{7200}{120}$, i.e. 60 days.

5. (a) If A can finish a work in X days and B is k times efficient than A, then the time taken by both A and B working together to complete the work is $\frac{x}{1+k}$.

   (b) If A and B working together can finish a work in X days and B is k times efficient than A, then the time taken by

   (i) A, working alone, to complete the work is $(k + 1)\,X$.

   (ii) B, working alone, to complete the work is $\left(\frac{k+1}{k}\right)X$.

Illustration 5 Harbans Lal can do a piece of work in 24 days. If Bansi Lal works twice as fast as Harbans Lal, how long would they take to finish the work working together?

Solution Here, $X = 24$ and $k = 2$.

$\therefore$ Time taken by Harbans Lal and Bansi Lal, working together, to complete the work

$$= \left(\frac{X}{1+k}\right) \text{ days.}$$

$$= \left(\frac{24}{1+2}\right) \text{ days, i.e. 8 days.}$$

Illustration 6 A and B together can do a piece of work in 3 days. If A does thrice as much work as B in a given time, find how long A alone would take to do the work?

Solution Here, $X = 3$ and $k = 3$.

$\therefore$ Time taken by A, working alone, to complete the work

$$= \left(\frac{k+1}{k}\right)X = \left(\frac{3+1}{3}\right)3 = 4 \text{ days.}$$

6. If A working alone takes a days more than A and B working alone takes b days more than A and B together, then the number of days taken by A and B, working together, to finish a job is given by $\sqrt{ab}$.

Illustration 7 A alone would take 8 hours more to complete the job than if both A and B worked together. If B worked alone, he took $4\frac{1}{2}$ hours more to complete the job than A and B worked together. What time would they take if both A and B worked together?

Solution Here, $a = 8$ and $b = \frac{9}{2}$.

$\therefore$ Time taken by A and B, working together, to complete the job

$$= \sqrt{ab} \text{ days}$$

$$= \sqrt{8 \times \frac{9}{2}} \quad \text{or,} \quad 6 \text{ days.}$$

7. If A is k times more efficient than B and is therefore able to finish a work in l days less than B, then
   (a) A and B, working together, can finish the work in $\frac{kl}{k^2-1}$ days.
   (b) A, working alone, can finish the work in $\frac{l}{k-1}$ days.
   (c) B, working alone, can finish the work in $\frac{kl}{k-1}$ days.

Illustration 8 A is thrice as good a workman as B and takes 10 days less to do a piece of work than B takes. Find the time in which B alone can complete the work.

Solution Here, $k = 3$ and $l = 10$.

$\therefore$ Time taken by B, working alone, to complete the work

$$= \frac{kl}{k-1} \text{ days}$$

$$= \frac{3 \times 10}{3-1} \text{ days, i.e. 15 days.}$$

8. If A can complete $\frac{a}{b}$ part of work in X days, then $\frac{c}{d}$ part of the work will be done in $\frac{b \times c \times X}{a \times d}$ days.

Illustration 9 A can do $\frac{3}{4}$ of a work in 12 days. In how many days can he finish $\frac{1}{8}$ of the work?

Solution Here, $a = 3$, $b = 4$, X = 12, $c = 1$ and $d = 8$.

Therefore, number of days required to finish $\frac{1}{8}$ of the work

$$= \frac{b \times c \times X}{a \times d} = \frac{4 \times 1 \times 12}{3 \times 8} = 2 \text{ days.}$$

9. (a) There are two groups of people with same efficiency. In one M_1 persons can do W_1 works in D_1 time and in the other M_2 persons can do W_2 works in D_2 time. The relationship between the two groups is given by

$$M_1D_1W_2 = M_2D_2W_1$$

   (b) There are two groups of people with same efficiency. In one M_1 persons can do W_1 works in D_1 time working t_1 hours a day and M_2 persons can do W_2 works in D_2 time working t_2 hours a day. The relationship between the two groups is given by

$$M_1D_1t_1W_2 = M_2D_2t_2W_1.$$

Illustration 10 If 10 persons can complete 2/5th of a work in 8 days, then find the number of persons required to complete the remaining work in 12 days.

Solution We have, $M_1 = 10$, $W_2 = \frac{2}{5}$, $D_1 = 8$

$M_2 = ?$, $W_2 = \frac{3}{5}$, $D_2 = 12$.

$$\therefore \quad M_1D_1W_2 = M_2D_2W_1$$

$$\Rightarrow \quad 10 \times 8 \times \frac{23}{5} = M_2 \times 12 \times \frac{2}{5}$$

$$\Rightarrow \quad M_2 = 10.$$

Illustration 11 If 10 persons can cut 20 trees in 3 days working 12 hours a day. Then, in how many days can 24 persons cut 32 trees working 4 hours a day?

Solution We have, $M_1 = 10$, $W_1 = 20$, $D_1 = 3$, $t_1 = 12$

$M_2 = 24$, $W_2 = 32$, $D_2 = ?$, $t_2 = 4$

$$\therefore \quad M_1D_1t_1W_2 = M_2D_2t_2W_1$$

$$\Rightarrow \quad 10 \times 3 \times 12 \times 32 = 24 \times D_2 \times 4 \times 20$$

$$\Rightarrow \quad D_2 = 6 \text{ days.}$$

10. If a men and b women can do a piece of work in n days, then c men and d women can do the work in

$$\left(\frac{nab}{bc + ad}\right) \text{ days}$$

Illustration 12 12 men or 15 women can do a work in 14 days. In how many days, 7 men and 5 women would complete the work?

Solution Here, $a = 12$, $b = 15$, $n = 14$, $c = 7$ and $d = 5$.

Required number of days

$$= \frac{nab}{bc + ad} = \left(\frac{14 \times 12 \times 15}{15 \times 7 + 12 \times 5}\right) \text{ days}$$

$$= \frac{168}{11} \text{ days or, } 15\frac{3}{11} \text{ days.}$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. 10 men can complete a piece of work in 15 days and 15 women can complete the same work in 12 days. If all the 10 men and 15 women work together, in how many days will the work get completed?

(a) $6\frac{2}{3}$ days (b) $8\frac{1}{3}$ days

(c) $7\frac{2}{3}$ days (d) None of these

2. A can do a piece of work in 30 days while B can do it in 40 days. A and B working together can do it in

(a) $15\frac{2}{7}$ days (b) $17\frac{1}{7}$ days

(c) $18\frac{3}{7}$ days (d) None of these

3. A can do 1/3 of a work in 5 days and B can do 2/5 of the work in 10 days. In how many days both A and B together can do the work?

(a) $13\frac{2}{3}$days (b) $9\frac{3}{8}$ days

(c) $18\frac{5}{8}$ days (d) None of these

4. A, B and C can do a piece of work in 6, 12 and 24 days, respectively. They altogether will complete the work in

(a) $5\frac{2}{7}$ days (b) $4\frac{3}{7}$ days

(c) $3\frac{3}{7}$ days (d) None of these.

5. A is thrice as good as B and is therefore able to finish a piece of work in 60 days less than B. Find the time in which they can do it, working together.

(a) $22\frac{3}{4}$ days (b) $22\frac{1}{2}$ days

(c) 24 days (d) None of these

6. Ramesh takes twice as much time as Mahesh and thrice as much time as Suresh to complete a job. If working together, they can complete the job in 4 days, then the time taken by each of them separately to complete the work is

(a) 36, 24 and 16 days (b) 20, 16 and 12 days

(c) 24, 42 and 18 days (d) None of these

7. Sita takes twice as much time as Gita to complete a work and Rita does it in the same time as Sita and Gita together. If all three working together can finish the work in 6 days, then the time taken by each of them to finish the work is

(a) 18, 36, and 12 days (b) 20, 38 and 14 days

(c) 24, 42 and 18 days (d) None of these

8. 5 men can complete a work in 2 days, 4 women can complete the same work in 3 days and 5 children can do it in 3 days. 1 man, 1 woman and 1 child, working together, can complete the work in

(a) 6 days (b) 4 days

(c) 8 days (d) None of these

9. A and B can do a piece of work in 6 days and A alone can do it in 9 days. The time taken by B alone to do the work is

(a) 20 days (b) 18 days

(c) 24 days (d) None of these

10. A and B can finish a piece of work in 30 days, B and C in 40 days while C and A in 60 days. A, B, C together can do the work in

(a) $24\frac{3}{4}$ days (b) $28\frac{2}{3}$ days

(c) $26\frac{2}{3}$ days (d) None of these

11. A and B can do a piece of work in 18 days; B and C in 24 days; C and A in 36 days. A alone can do the work in

(a) 48 days (b) 56 days

(c) 40 days (d) None of these

12. Ajay and Sunil can do a piece of work in 10 days, Sunil and Sanjay in 15 days and Sanjay and Ajay in 20 days. They together work at it for 6 days and then Ajay leaves and Sunil and Sanjay go on together for 4 days more. If Sunil then leaves, how long will Sanjay take to complete the work?

(a) 12 days (b) 10 days

(c) 16 days (d) None of these

13. Anu can complete a work in 10 days. Manu is 25% more efficient than Anu and Sonu is 60% more efficient than Manu. Working together, how long would they take to finish the job?

(a) $3\frac{5}{8}$ days (b) $5\frac{6}{7}$ days

(c) $2\frac{6}{17}$ days (d) None of these

14. A and B finish a job in 12 days while A, B and C can finish it in 8 days. C alone will finish the job in

(a) 24 days (b) 36 days

(c) 28 days (d) None of these.

15. Bansal, Gupta and Singhal together can complete a work in 4 days. If Bansal and Gupta together can complete the work in $4\frac{4}{5}$ days, Gupta and Singhal togehter can do it in 8 days, then Gupta alone can complete the work in
(a) 16 days (b) 12 days
(c) 20 days (d) None of these.

16. Nikita, Nishita and Kavita can complete a work in $2\frac{2}{3}$ days. If Nishita and Kavita can do it in 4 days and Nishita alone can do it in 6 days, then Nikita and Nishita can complete the work in
(a) $5\frac{4}{7}$ days (b) $4\frac{2}{7}$ days
(c) $3\frac{3}{7}$ days (d) None of these

17. A is twice as good a workman as B and together they finish a piece of work in 1 day. A alone can finish the work in
(a) 28 days (b) 21 days
(c) 24 days (d) None of these

18. Bindal can finish a work in 10 days. Jindal is twice as efficient as Bindal. If they work together, in how many days, the work will be finished?
(a) $3\frac{1}{3}$ days (b) $5\frac{2}{3}$ days
(c) $4\frac{1}{3}$ days (d) None of these

19. A alone would take 27 days more to complete the job than if both A and B would together. If B worked alone, he took 3 days more to complete the job than A and B worked together. What time would they take if both A and B worked together?
(a) 7 days (b) 9 days
(c) 11 days (d) None of these.

20. A is 4 times as fast as B and is therefore able to finish a work in 45 days less than B. A and B, working together, can complete the work in
(a) 12 days (b) 16 days
(c) 8 days (d) None of these.

21. If A can complete a work in 16 days, then in how many days can he complete $\frac{3}{4}$th of the work?
(a) 16 days (b) 20 days
(c) 12 days (d) None of these

22. Working 7 hours daily 24 men can complete a piece of work in 27 days. In how many days would 14 men complete the same piece of work working 9 hours daily?
(a) 36 days (b) 30 days
(c) 32 days (d) None of these

23. 10 men can cut 15 trees in 2 hours. If 2 men leave the job, how many trees will be cut in 3 hours?
(a) 20 trees (b) 18 trees
(c) 24 trees (d) None of these

24. 45 men complete a piece of work in 30 days working 12 hours a day. In how many days will 60 men complete the work working 10 hours a day?
(a) 27 days (b) 30 days
(c) 24 days (d) None of these

25. The work done by a woman in 8 hours is equal to the work done by a man in 6 hours and by a boy in 12 hours. If working 6 hours per day 9 men can complete a work in 6 days then in how many days can 12 men, 12 women and 12 boys together finish the same work working 8 hours per day?
(a) $2\frac{1}{2}$ days (b) $1\frac{1}{2}$ days
(c) $3\frac{1}{2}$ days (d) None of these

26. 4 men or 6 women can finish a piece of work in 20 days. In how many days can 6 men and 11 women finish the same work?
(a) 9 days (b) 6 days
(c) 7 days (d) None of these

27. 10 men can finish a piece of work in 10 days whereas it takes 12 women to finish it in 10 days. If 15 men and 6 women undertake to complete the work, how many days will they take to complete it?
(a) 7 days (b) 5 days
(c) 9 days (d) None of these

28. A can do a piece of work in 10 days, while B alone can do it in 15 days. They work together for 5 days and the rest of the work is done by C in 2 days. If they get Rs. 450 for the whole work, how should they divide the money?
(a) Rs. 250, Rs. 100, Rs. 100
(b) Rs. 225, Rs. 150, Rs. 75
(c) Rs. 200, Rs. 150, Rs. 100
(d) Rs. 175, Rs. 175, Rs. 100

29. Two men undertake to do a piece of work for Rs. 1400. First man alone can do this work in 7 days while the second man alone can do this work in 8 days. If they working together complete this work in 3 days with the help of a boy, how should money be divided?
(a) Rs. 600, Rs. 500, Rs. 300
(b) Rs. 600, Rs. 525, Rs. 275
(c) Rs. 600, Rs. 550, Rs. 250
(d) Rs. 500, Rs. 525, Rs. 375

30. A does half as much work as B in three-fourths of the time. If together they take 18 days to complete a work, how much time shall B take to do it?

(a) 30 days (b) 35 days
(c) 40 days (d) None of these

31. Two men A and B working together complete a piece of work which it would have taken them respectively 30 and 40 days to complete if they worked separately. If they received a payment of Rs. 2100, B's share is
(a) Rs. 900 (b) Rs. 1200
(c) Rs. 800 (d) Rs. 1300

32. Two men undertake to do a piece of work for Rs. 600. One alone could do it in 6 days and the other in 8 days. With the assistance of a boy they finish it in 3 days. Boy's share should be
(a) Rs. 300 (b) Rs. 225
(c) Rs. 75 (d) Rs. 100

33. A can do a piece of work in 8 days. A undertook to do it for Rs. 320. With the help of B, he finishes the work in 6 days. B's share is
(a) Rs. 80 (b) Rs. 240
(c) Rs. 100 (d) Rs. 120

34. 5 men and 2 boys working together can do four times as much work per hour as a man and a boy together. The work done by a man and a boy should be in the ratio
(a) 1 : 2 (b) 2 : 1
(c) 1 : 3 (d) 4 : 1

35. A, B and C can do a piece of work in 16, 32 and 48 days, respectively. They started working together but C left after working 4 days and B 2 days before the completion of work. Total number of days taken for completion of work was
(a) 8 days (b) $9\frac{1}{9}$ days
(c) 11 days (d) $10\frac{4}{9}$ days

36. A and B working separately can do a piece of work in 9 and 12 days, respectively. If they work for a day alternately, A beginning, in how many days the work will be completed?
(a) $10\frac{1}{2}$ days (b) $10\frac{1}{4}$ days
(c) $10\frac{2}{3}$ days (d) $10\frac{1}{2}$ days

37. A and B can do a piece of work in 45 and 40 days, respectively. They began the work together, but A leaves after some days and B finished the remaining work in 23 days. After how many days did A leave?
(a) 6 days (b) 8 days
(c) 9 days (d) 12 days

38. A and B working together can complete a piece of work in 12 days and B and C working together can complete the same work in 16 days. A worked at it for 5 days and B worked at it for 7 days. C finished the remaining work in 13 days. How many days would C alone take to complete it?
(a) 10 days (b) 24 days
(c) 32 days (d) 40 days

39. A can do a piece of work in 90 days. He starts working, but having some other engagements he drops out after 5 days. Thereafter B completes this work in 21 days. How many days would A and B take to complete this work working together?
(a) 15 days (b) 16 days
(c) 17 days (d) 11 days

40. A can do a piece of work in 30 days, B in 50 days and C in 40 days. If A is assisted by B on one day and by C on the next day alternately work will be completed in
(a) $17\frac{32}{35}$ days (*b*) $19\frac{2}{3}$ days
(c) $16\frac{31}{37}$ days (d) $18\frac{1}{3}$ days

EXPLANATORY ANSWERS

1. Ans. (a)

Solution Here, $X = 15$ and $Y = 12$

$\therefore$ Working together, 10 men and 15 women will complete the work in

$$= \frac{XY}{X+Y} \text{ days}$$

$$= \frac{15 \times 12}{15+12}, \text{ i.e. } \frac{20}{3} \text{ or, } 6\frac{2}{3} \text{ days.}$$

2. Ans. (b)

Solution Here, $X = 30$ and $Y = 40$.

$\therefore$ A and B working together can do the work in

$$= \frac{XY}{X+Y} \text{ days.}$$

$$= \frac{30 \times 40}{30+40} \text{ days, i.e. } \frac{120}{7} \text{ or, } 17\frac{1}{7} \text{ days.}$$

3. Ans. (b)

Solution A can do the complete work in $5 \times 3 = 15$ days.

B can do the complete work in $10 \times \frac{5}{2} = 25$ days. Here, $X = 15$ and $Y = 25$.

$\therefore$ A and B, working together, can complete the work in

$$= \frac{XY}{X+Y} \text{ days}$$

$$= \frac{15 \times 25}{15+25} \text{ days, i.e. } \frac{75}{8} \text{ or, } 9\frac{3}{8} \text{ days.}$$

4. Ans. (c)

Solution Here, $X = 6$, $Y = 12$ and $Z = 24$.

$\therefore$ Working together, A, B and C will complete the work in

$$= \frac{XYZ}{XY + YZ + ZX} \text{ days}$$

$$= \frac{6 \times 12 \times 24}{6 \times 12 + 12 \times 24 + 24 \times 6} \text{ days,}$$

i.e. $\frac{24}{7}$ days or, $3\frac{3}{7}$ days.

5. Ans. (b)

Solution Let B take x days to do the work. Then, A takes $(x - 60)$ days to complete the work.

Since ratio of work done by A and B is 3 : 1, ratio of time taken by A and B is 1 : 3.

We have, $\frac{x-60}{x} = \frac{1}{3}$

$\Rightarrow 3(x - 60) = x$ or, $x = 90$.

$\therefore$ Time taken by B to finish the work = 90 days and time taken by A to finish the work = $\frac{90}{3}$ = 30 days.

$\therefore$ A and B, working together, will complete the work in

$$= \frac{XY}{X+Y} \text{ days}$$

$$= \frac{90 \times 30}{90+30} \text{ days, i.e. } \frac{45}{2} \text{ or, } 22\frac{1}{2} \text{ days.}$$

6. Ans. (c)

Solution Let Ramesh take x days to finish the work.

Then, Mahesh takes $\frac{x}{2}$ and Suresh takes $\frac{x}{3}$ days to finish the same work.

$\therefore$ Ramesh, Mahesh and Suresh, working together, will complete the work in

$$= \frac{XYZ}{XY + YZ + ZX} \text{ days}$$

$$= \frac{x + \frac{x}{2} \times \frac{x}{3}}{x \times \frac{x}{2} + \frac{x}{2} \times \frac{x}{3} + x \times \frac{x}{3}} \text{ days}$$

i.e. $\frac{x^3/6}{x^2}$ or, $\frac{x}{6}$ days.

Given : $\frac{x}{6} = 4$.

$\therefore$ $x = 24$

$\therefore$ Ramesh takes 24 days, Mahesh takes $\frac{24}{2}$ or 12 days and Suresh takes $\frac{24}{3}$ or 8 days to finish the work.

7. Ans. (a)

Solution Let Gita take x days to complete the work. Then, Sita takes $2x$ days to complete the same work.

$\therefore$ Time taken by Rita to complete the work

$$= \frac{XY}{X+Y} \text{ days}$$

$$= \frac{x \times 2x}{x + 2x} \text{ days or, } \frac{2x}{3} \text{ days.}$$

$\therefore$ Working together, Gita, Sita and Rita will finish the work in

$$= \frac{XYZ}{XY + YZ + ZX} \text{ days}$$

$$= \frac{x \times 2x \times \frac{2x}{3}}{x \times 2x + 2x \times \frac{2x}{3} + \frac{2x}{3} \times x} \text{ days or, } \frac{x}{3} \text{ days.}$$

Given: $\frac{x}{3} = 6$. $\therefore$ $x = 18$ days.

$\therefore$ Gita takes 18 days, Sita takes 36 days and Rita takes 12 days to finish the work.

8. Ans. (b)

Solution 1 Man can complete the work in $5 \times 2 = 10$ days,

1 woman can complete the work in $4 \times 3 = 12$ days and 1 child can complete the work in $5 \times 3 = 15$ days.

$\therefore$ 1 man, 1 woman and 1 child, working together, can complete the work in

$$= \frac{XYZ}{XY + YZ + ZX} \text{ days}$$

$$= \frac{10 \times 12 \times 15}{10 \times 12 + 12 \times 15 + 15 \times 10} = 4 \text{ days.}$$

9. Ans. (b)

Solution Here $X = 6$ and $Y = 9$.

$\therefore$ B alone will complete the work in = $\frac{XY}{Y - X}$ days

$$= \frac{6 \times 9}{9 - 6}, \text{ i.e. 18 days.}$$

10. Ans. (c)

Solution Here, $X = 30$, $Y = 40$ and $Z = 60$.

$\therefore$ A, B and C together will finish the work in

$$= \left(\frac{2XYZ}{XY + YZ + ZX}\right) \text{ days}$$

$$= \left(\frac{2 \times 30 \times 40 \times 60}{30 \times 40 + 40 \times 60 + 60 \times 30}\right) \text{ days}$$

or, $\frac{144000}{5400}$, i.e. $26\frac{2}{3}$ days.

11. Ans. (a)

Solution Here, $X = 18$, $Y = 24$ and $Z = 36$.

$\therefore$ A alone can do the work in

$$= \left(\frac{2XYZ}{XY + YZ - ZX}\right) \text{ days}$$

$$= \left(\frac{2 \times 18 \times 24 \times 36}{18 \times 24 + 24 \times 36 - 36 \times 18}\right) \text{ days}$$

or, $\frac{31104}{648}$, i.e. 48 days.

12. Ans. (b)

Solution Ajay, Sunil and Sanjay, working together, can complete the work in

$$= \left(\frac{2XYZ}{XY + YZ + ZX}\right) \text{ days}$$

$$= \left(\frac{2 \times 10 \times 15 \times 20}{10 \times 15 + 15 \times 20 + 20 \times 10}\right) \text{ days}$$

or, $\frac{6000}{650}$, i.e. $\frac{120}{13}$ days.

$\therefore$ Work done by all of them together in 6 days

$$= 6 \times \frac{13}{120}, \text{ i.e. } \frac{13}{20}.$$

Also, work done by Sunil and Sanjay in 4 days $= \frac{4}{15}$.

$\therefore$ Remaining work $= 1 - \left(\frac{13}{20} + \frac{4}{15}\right) = \frac{1}{12}$, which is to be done by Sanjay.

Now, Ajay, Sunil and Sanjay, can complete the work in $\frac{120}{13}$ days and Ajay and Sunil can complete the work in 10 days.

$\therefore$ Sanjay alone can complete the work in

$$= \frac{\frac{120}{13} \times 10}{10 - \frac{120}{13}} \text{ days} = 120 \text{ days.}$$

$\therefore$ $\frac{1}{12}$ of the work is done by Sanjay in $\frac{120}{12} = 10$ days.

13. Ans. (c)

Solution Since Anu can complete the work in 10 days, Manu can complete the work in $= 10 \times \frac{100}{125} = 8$ days. Also, Sonu can complete the work in $= 8 \times \frac{100}{160} = 5$ days.

$\therefore$ Anu, Manu and Sonu will complete the work in

$$= \frac{XYZ}{XY + YZ + ZX} \text{ days}$$

$$= \left(\frac{10 \times 8 \times 5}{10 \times 8 + 8 \times 5 + 5 + 10}\right) \text{ days}$$

or, $\frac{400}{170}$, i.e. $2\frac{6}{17}$ days.

14. Ans. (a)

Solution C alone will finish the job in

$$= \frac{XY}{Y - X} \text{ days} = \frac{8 \times 12}{12 - 8} \text{ days}$$

[Here, $X = 8$ and $Y = 12$]

$= 24$ days.

15. Ans. (b)

Solution Bansal, Gupta and Singhal together can finish the work in 4 days.

Bansal and Gupta together can do it in $\frac{24}{5}$ days.

Gupta and Singhal together can do it in 8 days. Therefore, Bansal alone can complete the work in

$$= \frac{XY}{Y - X} \text{ days}$$

$$= \left(\frac{8 \times 4}{8 - 4}\right) \text{ days}$$

[Here, $X = 4$ and $Y = 8$]

$= 8$ days.

Also, Singhal alone can complete the work in

$$= \left(\frac{XY}{Y - X}\right) \text{ days} = \left(\frac{\frac{24}{5} \times 4}{\frac{24}{5} - 4}\right) \text{ days}$$

$\left[\text{Here, } X = 4 \text{ and } Y = \frac{24}{5}\right]$

$= 24$ days.

$\therefore$ Bansal and Singhal can complete the work in

$$= \left(\frac{XY}{X + Y}\right) \text{ days} = \left(\frac{24 \times 8}{24 + 8}\right) \text{ days}$$

[Here, $X = 24$ and $Y = 8$]

$= 6$ days.

∴ Gupta alone can complete the work in

$$= \left(\frac{XY}{Y-X}\right) \text{ days} = \left(\frac{4\times 6}{6-4}\right) \text{ days}$$

[Here, $X = 4$ and $Y = 6$]

= 12 days.

16. Ans. (c)

Solution Nishita and Kavita complete the work in 4 days and Nishita alone does it in 6 days.

∴ Kavita alone can do the work in

$$= \left(\frac{XY}{Y-X}\right) \text{ days} = \left(\frac{4\times 6}{6-4}\right) \text{ days}$$

[Here, $X = 4$ and $Y = 6$]

= 12 days.

∴ Nikita and Nishita can complete the work in

$$= \left(\frac{XY}{Y-X}\right) \text{ days} = \left(\frac{\frac{8}{3}\times 12}{12-\frac{8}{3}}\right) \text{ days}$$

$$\left[\text{Here, } X = \frac{8}{3} \text{ and } Y = 12\right]$$

$$= \frac{96}{28} = 3\frac{3}{7} \text{ days.}$$

17. Ans. (b)

Solution Here, $X = 14$ and $K = 2$

∴ A alone can finish the work in

$$= \left(\frac{K+1}{K}\right)X \text{ days} = \left(\frac{2+1}{2}\right) 14 \text{ days, i.e. 21 days.}$$

18. Ans. (a)

Solution Here, $X = 10$ and $K = 2$.

∴ The time taken by Bindal and Jindal, working together, to complete the work

$$= \frac{X}{1+K} = \frac{10}{1+2}, \text{ i.e. } \frac{10}{3} \text{ or, } 3\frac{1}{3} \text{ days.}$$

19. Ans. (b)

Solution Here, $a = 27$ and $b = 3$.

∴ Time taken by A and B, working together, to complete the job

$$= \sqrt{ab} \text{ days} = \sqrt{27\times 3} \text{ or, 9 days.}$$

20. Ans. (a)

Solution Here, $k = 4$ and $l = 45$.

Therefore, A and B, working together, can complete the work in

$$= \frac{kl}{k^2-1} \text{ days} = \frac{4\times 45}{16-1} \text{ days, i.e. 12 days.}$$

21. Ans. (c)

Solution Here, $a = 1$, $b = 1$, $X = 16$, $c = 3$ and $d = 4$.

Therefore, number of days required to finish $\frac{3}{4}$ of the work

$$= \frac{b\times c\times X}{a\times d} = \frac{1\times 3\times 16}{1\times 4} = 12 \text{ days.}$$

22. Ans. (a)

Solution We have, $M_1 = 24$, $D_1 = 27$, $W_1 = 1$, $t_1 = 7$

$M_2 = 14$, $D_2 = ?$, $W_2 = 1$, $t_2 = 9$

∴ $M_1D_1t_1W_2 = M_2D_2t_2W_1$

⇒ $24 \times 27 \times 7 \times 1 = 14 \times D_2 \times 9 \times 1$

⇒ $D_2 = 36$ days.

23. Ans. (b)

Solution We have, $M_1 = 10$, $D_1 = 2$, $W_1 = 15$

$M_2 = 10 - 2 = 18$, $D_2 = 3$, $W_2 = ?$

∴ $M_1D_1W_2 = M_2D_2W_1$

⇒ $10 \times 2 \times W_2 = 8 \times 3 \times 15$

⇒ $W_2 = 18$ trees.

Thus, 18 trees will be cut in 3 hours.

24. Ans. (a)

Solution We have, $M_1 = 45$, $D_1 = 30$, $t_1 = 12$, $W_1 = 1$

$M_2 = 60$, $D_2 = ?$, $t_2 = 10$, $W_2 = 1$

∴ $M_1D_1t_1W_2 = M_2D_2t_2W_1$

⇒ $45 \times 30 \times 12 \times 1 = 60 \times D_2 \times 10 \times 1$

⇒ $D_2 = 27$ days.

25. Ans. (b)

Solution Given: 8 women = 6 Men = 12 Boys

∴ 12 Men + 12 Women + 12 Boys

= 12 Men + 9 Men + 6 Men = 27 Men

We have, $M_1 = 9$, $D_1 = 6$, $t_1 = 6$, $W_1 = 1$

$M_2 = 27$, $D_2 = ?$, $t_2 = 8$, $W_2 = 1$

∴ $M_1D_1t_1W_2 = M_2D_2t_2W_1$

⇒ $9 \times 6 \times 6 \times 1 = 27 \times D_2 \times 8 \times 1$

⇒ $D_2 = \frac{3}{2}$ days or, $1\frac{1}{2}$ days.

26. Ans. (b)

Solution Here, $a = 4$, $b = 6$, $n = 20$, $c = 6$ and $d = 11$.

∴ Required number of days

$$= \left(\frac{nab}{bc+ad}\right) \text{ days} = \left(\frac{20\times 4\times 6}{6\times 6+4\times 11}\right) \text{ days}$$

= 6 days.

27. Ans. (b)

Solution Here, $a = 10$, $b = 12$, $n = 10$, $c = 15$ and $d = 6$.

∴ Required number of days

$$= \left(\frac{nab}{bc + ad}\right) \text{ days}$$

$$= \left(\frac{10 \times 10 \times 12}{12 \times 15 + 10 \times 6}\right) \text{ days}$$

= 5 days.

28. Ans. (b)

Solution $(A + B)$'s 5 day's work $= 5\left(\frac{1}{10} + \frac{1}{15}\right) = \frac{5}{6}$

Remaining work $= 1 - \frac{5}{6} = \frac{1}{6}$

∴ C's 2 days' work $= \frac{1}{6}$

Now, A's 5 days' work : B's 5 days' work

∴ C's 2 days' work $= \frac{5}{10} : \frac{5}{15} : \frac{1}{6} = 3 : 2 : 1$

∴ A's share = Rs. $\left(450 \times \frac{3}{6}\right)$ = Rs. 225

B's share = Rs. $\left(450 \times \frac{2}{6}\right)$ = Rs. 150

C's share = Rs. [450 – (225 + 150)] = Rs. 75

29. Ans. (b)

Solution Wages of the first man for 3 days

= work done by him in 3 days × Rs. 1400

$= \frac{3}{7} \times 1400$ = Rs. 600

Wages of second man for 3 days

= work done by him in 3 days × Rs. 1400

$= \frac{3}{8} \times 1400$ = Rs. 525

∴ Wages of the boy for 3 days

= Rs. 1400 – Rs. (600 + 525) = Rs. 275

∴ Their shares will be Rs. 600, Rs. 525 and Rs. 275, respectively.

30. Ans. (a)

Solution Suppose, B takes x days to do the work

∴ A takes $\left(2 \times \frac{3}{4}x\right)$, i.e. $\frac{3x}{2}$ days to do it.

Now, $(A + B)$'s 1 day's work $= \frac{1}{18}$

∴ $\frac{1}{x} + \frac{2}{3x} = \frac{1}{18}$ or, $x = 30$.

31. Ans. (a)

Solution A's 1 days' work $= \frac{1}{30}$

B's 1 days' work $= \frac{1}{40}$

∴ Share of A and B should be in the ratio

$\frac{1}{30} : \frac{1}{40} = 4 : 3$

∴ B's share = Rs. $\left(\frac{3}{7} \times 2100\right)$ = Rs. 900.

32. Ans. (c)

Solution 1st man's 3 days' work $= \frac{3}{6} = \frac{1}{2}$

2nd man's 3 days' work $= \frac{3}{8}$

Boy's 3 days' work $= 1 - \left(\frac{1}{2} + \frac{3}{8}\right) = \frac{1}{8}$

∴ They should get money in the ratio

$\frac{1}{2} : \frac{3}{8} : \frac{1}{8}$ i.e. 4 : 3 : 1

∴ Boy's share = Rs. $\frac{1}{8} \times 600$ = Rs. 75.

33. Ans. (a)

Solution $(A + B)$'s 1 day's work $= \frac{1}{6}$

A's 1 days' work $= \frac{1}{8}$

∴ B's 1 days' work $= \frac{1}{6} - \frac{1}{8} = \frac{1}{24}$

∴ Money should be divided in the ratio

$\frac{1}{8} : \frac{1}{24} = 3 : 1$

∴ B gets = Rs. $\left(\frac{1}{4} \times 320\right)$ = Rs. 80.

34. Ans. (b)

Solution Obviously,

$(5M + 2B) = 4(1M + 1B)$

∴ $M = 2B$

∴ Work done by a man and a boy are in the ratio 2 : 1.

35. Ans. (d)

Solution $(A + B + C)$'s 1 day's work

$$= \frac{1}{16} + \frac{1}{32} + \frac{1}{48} = \frac{11}{96}$$

Work done in first 4 days $= \frac{11}{96} \times 4 = \frac{11}{24}$

For last two days, A works alone

∴ Work done in last 2 days $= \frac{1}{16} \times 2 = \frac{1}{8}$

Rest of the work $\left(\text{i.e. } 1 - \frac{11}{24} - \frac{1}{8} = \frac{5}{12}\right)$ is done by A and B together.

$(A + B)$'s 1 day's work $= \frac{1}{16} + \frac{1}{32} = \frac{3}{32}$

$\therefore \frac{5}{12}$ work is finished in $\frac{32}{3} \times \frac{5}{12} = \frac{40}{9}$ days

$\therefore$ Total number of days in which whole work is finished $= 4 + 2 + \frac{40}{9} = 10\frac{4}{9}$ days.

36. Ans. (b)

Solution $(A + B)$'s 2 days' work $= \frac{1}{9} + \frac{1}{12} = \frac{7}{36}$

Evidently, the work done by A and B during 5 pairs of days

$$= 5 \times \frac{7}{36} = \frac{35}{36}$$

Remaining work $= 1 - \frac{35}{36} = \frac{1}{36}$

Now, on 11th day, it is A's turn

Now, $\frac{1}{9}$ work is done by A in 1 day

$\therefore \frac{1}{36}$ work will be done by A in $9 \times \frac{1}{36} = \frac{1}{4}$ day

So, total time taken $= 10\frac{1}{4}$ days.

37. Ans. (c)

Solution B's 23 days' work $= \frac{23}{40}$

Remaining work $= 1 - \frac{23}{40} = \frac{17}{40}$

Now, $(A + B)$'s 1 day's work $= \frac{1}{45} + \frac{1}{40} = \frac{17}{360}$

$\frac{17}{360}$ work is done by A and B in 1 day

$\therefore \frac{17}{40}$ work is done by A and B in $\frac{360 \times 17}{17 \times 40} = 9$ days

So, A left after 9 days.

38. Ans. (b)

Solution Suppose, C alone can do this work in x days

$\therefore C$ will do $\frac{1}{x}$ work in 1 day

Now, work done by $(B + C)$ in 1 day $= \frac{1}{16}$

$\therefore$ Work done by B in 1day $= \left(\frac{1}{16} - \frac{1}{x}\right)$

And, work done by $(A + B)$ in 1 day $= \frac{1}{12}$

$\therefore$ Work done by A in 1 day $= \frac{1}{12} - \left(\frac{1}{16} - \frac{1}{x}\right)$

$$= \frac{1}{48} + \frac{1}{x}$$

According to question,

Work done by A in 5 days + work done by B in 7 days + work done by C in 13 days = whole work

$$\therefore \quad 5\left(\frac{1}{48} + \frac{1}{x}\right) + 7\left(\frac{1}{16} - \frac{1}{x}\right) + \frac{13}{x} = 1$$

or, $\frac{5}{48} + \frac{5}{x} + \frac{7}{16} - \frac{7}{x} + \frac{13}{x} = 1$

or, $\frac{26}{48} + \frac{11}{x} = 1$ or, $\frac{11}{x} = 1 - \frac{26}{48}$

or, $\frac{11}{x} = \frac{22}{48}$ or, $x = 24$

$\therefore C$ alone would do this work in 24 days.

39. Ans. (a)

Solution Work done by A in 5 days $= \frac{5}{40} = \frac{1}{8}$

$\therefore$ Remaining work $= 1 - \frac{1}{8} = \frac{7}{8}$

$\therefore B$ completes $\frac{7}{8}$ work in 21 days

$\therefore B$ would complete 1 work in $\frac{21 \times 8}{7} = 24$ days

Here, $x = 40$, $y = 24$

$\therefore$ Working together, A and B would complete this work in

$$= \frac{xy}{x + y} = \frac{40 \times 24}{40 + 24} = \frac{40 \times 24}{64} = 15 \text{ days.}$$

40. Ans. (a)

Solution $(A + B)$'s 1 day's work $= \frac{1}{30} + \frac{1}{50} = \frac{8}{150}$

$\therefore (A + C)$'s 1 day's work $= \frac{1}{30} + \frac{1}{40} = \frac{7}{120}$

$\therefore$ Work done in first 2 days $= \frac{8}{150} + \frac{7}{120} = \frac{67}{600}$

Work done in $8 \times 2 = 16$ days $= \frac{67 \times 8}{600} = \frac{67}{75}$

Work left $= 1 - \frac{67}{75} = \frac{8}{75}$

On 17th day $(A + B)$ will work and they will finish $\frac{8}{150}$ work

$\therefore$ Work left $= \frac{8}{75} - \frac{8}{150} = \frac{8}{150} = \frac{4}{75}$

On 18th day $(A + C)$ will work and they will finish it in $\frac{120}{7} \times \frac{4}{75} = \frac{32}{35}$ days

$\therefore$ Whole work will be done in $17\frac{32}{35}$ days.

QUESTIONS FROM COMPETITIVE EXAMS

1. A man, a woman and a boy can complete a job in 3, 4 and 12 days, respectively. How many boys must assist 1 man and 1 woman to complete the job in $\frac{1}{4}$ of a day?

(a) 1 (b) 4
(c) 19 (d) 41

[Based on SSC (GL) Prel. Exam. 2000]

2. A contractor undertakes to make a road in 40 days and employs 25 men. After 24 days, he finds that only one-third of the road is made. How many extra men should he employ so that he is able to complete the work 4 days earlier?

(a) 100 (b) 60
(c) 75 (d) None of these

[Based on SSC (GL) Prel. Exam. 2000]

3. *A* can do a work in 15 days and *B* in 20 days. If they together work on it for 4 days, then the fraction of the work that is left is

(a) $\frac{8}{15}$ (b) $\frac{7}{15}$
(c) $\frac{1}{4}$ (d) $\frac{1}{10}$

[Based on SSC (GL) Prel. Exam. 2000]

4. *A* alone can complete a work in 12 days while *A* and *B* together can complete the same work in 8 days. The number of days that *B* will take to complete the work alone is

(a) 10 (b) 24
(c) 20 (d) 9

[Based on SSC (GL) Prel. Exam. 2000]

5. *A* work could be completed in 100 days. However, due to the absence of 10 workers, it was completed in 110 days. The original number of workers was

(a) 100 (b) 110
(c) 55 (d) 50

[Based on SSC (GL) Prel. Exam. 2000]

6. *A* can do a job in 15 days, *B* in 10 days and *C* in 30 days. If *A* is helped by *B* and *C* on every third day, then the job will be completed in

(a) $6\frac{1}{3}$ days (b) $8\frac{1}{3}$ days
(c) 8 days (d) 9 days

[Based on SSC (GL) Prel. Exam. 2000]

7. *A* works twice as fast as *B*. If both of them can together finish a work in 12 days, *B* alone can do it in

(a) 48 days (b) 36 days
(c) 27 days (d) 24 days

[Based on SSC (GL) Prel. Exam. 2000]

8. A contractor undertook to finish a certain work in 124 days and employed 120 men on it. After 64 days, he found that he had already done $\frac{2}{3}$ of the work. How many men he can discharge now so that the work may finish in time?

(a) 24 (b) 56
(c) 64 (d) 80

[Based on SSC (GL) Prel. Exam. 2000]

9. A particular job can be completed by a team of 10 men in 12 days. The same job can be completed by a team of 10 women in 6 days. How many days are needed to complete the job if the two teams work together?

(a) 4 (b) 6
(c) 9 (d) 18

[Based on SSC (GL) Prel. Exam. 2000]

10. *A* can do a work in 6 days and *B* in 9 days. How many days will both take together to complete the work?

(a) 7.5 (b) 5.4
(c) 3.6 (d) 3

[Based on SSC (GL) Prel. Exam. 2000]

11. *A* certain number of men can complete a job in 30 days. If there were 5 men more, it could be completed in 10 days less. How many men were there in the beginning?

(a) 10 (b) 15
(c) 20 (d) 25

[Based on SSC (GL) Prel. Exam. 2000]

12. *A* and *B* can do a job in 6 and 12 days, respectively. They began the work together but *A* leaves after 3 days.Then the total number of days needed for the completion of the work is

(a) 4 (b) 5
(c) 6 (d) 9

[Based on SSC (GL) Prel. Exam. 2000]

13. A man is paid Rs. 20 for each day he works, and forfeits Rs. 3 for each day he is idle. At the end of 60 days he gets Rs. 280. Then, he was idle for

(a) 20 days (b) 25 days
(c) 30 days (d) 40 days

[Based on SSC (GL) Prel. Exam. 2000]

14. If the wages of 6 men for 15 days be Rs. 700, then the wages of 9 men for 12 days will be

(a) Rs. 700 (b) Rs. 840
(c) Rs. 1050 (d) Rs. 900

[Based on SSC (GL) Prel. Exam. 2000]

15. *A* and *B* together complete a piece of work in

10 days. If A alone takes 30 days, then how many days will B alone take to complete the same work?
(a) 12 (b) 15
(c) 20 (d) 25

[Based on SSC (GL) Prel. Exam. 2000]

16. A man, a woman and a boy can together complete a piece of work in 3 days. If a man alone can do it in 6 days and a boy alone can do it in 18 days, how long will a woman alone take to complete the work?
(a) 9 days (b) 21 days
(c) 24 days (d) 27 days

[Based on SSC (GL) Prel. Exam. 2000]

17. A and B can do a piece of work in 10 days, B and C in 15 days and A and C in 20 days. C alone can do the work in
(a) 60 days (b) 120 days
(c) 80 days (d) 30 days

[Based on SSC (GL) Prel. Exam. 2002]

18. A can cultivate $\frac{2}{5}$ of a land in 6 days and B can cultivate $\frac{1}{3}$ of the same land in 10 days. Working together, A and B can cultivate $\frac{4}{5}$ of the land in
(a) 4 days (b) 5 days
(c) 8 days (d) 10 days

[Based on SSC (GL) Prel. Exam. 2002]

19. A does half as much work as B in one-sixth of the time. If together they take 10 days to complete a work, how much money shall B take to do it alone?
(a) 70 days (b) 30 days
(c) 40 days (d) 50 days

[Based on SSC (GL) Prel. Exam. 2002]

20. A can do a piece of work in 4 hours, B and C can do it in 3 hours; A and C can do it in 2 hours. How long will B alone take to do it?
(a) 10 hours (b) 12 hours
(c) 8 hours (d) 24 hours

[Based on SSC (GL) Prel. Exam. 2002]

21. A does $\frac{4}{5}$ of a piece of work in 20 days; he then calls in B and they finish the remaining work in 3 days. How long will B alone take to do the whole work?
(a) $37\frac{1}{2}$ days (b) 37 days
(c) 40 days (d) 23 days

[Based on SSC (GL) Prel. Exam. 2002]

22. A can finish a work in 18 days and B can do the same work in half the time taken by A. Then, working together what part of the same work can they finish in a day?
(a) $\frac{1}{6}$ (*b*) $\frac{2}{5}$
(c) $\frac{1}{9}$ (d) $\frac{2}{7}$

[Based on SSC (GL) Prel. Exam. 2002]

23. A and B can finish a piece of work in 30 days, B and C can finish it in 15 days while C and A can finish it in 10 days. Time taken by them together to do this work is
(a) 5 days (b) $2\frac{1}{2}$ days
(c) $7\frac{1}{2}$ days (d) 10 days

[Based on SSC (GL) Prel. Exam. 2002]

24. A can do $\frac{1}{4}$th part of a work in 10 days, B can do 40% of the work in 40 days and C can do $\frac{1}{3}$ of the work in 13 days. Who will complete the work first?
(a) A (b) B
(c) C (d) A and C both

[Based on SSC (GL) Prel. Exam. 2002]

25. A takes twice as much time as B or thrice as much time as C to finish a piece of work. Working together, they can finish the work in 2 days. B can do the work alone in
(a) 12 days (b) 4 days
(c) 8 days (d) 6 days

[Based on SSC (GL) Prel. Exam. 2002]

26. A and B can together finish a work in 30 days. They worked together for 20 days and then B left. After another 20 days A finished the remaining work. In how many days can A alone finish the job?
(a) 50 days (b) 60 days
(c) 40 days (d) 65 days

[Based on SSC (GL) Prel. Exam. 2003]

27. If 72 men can build a wall 280 m long in 21 days, how many men will take 18 days to build a similar type of wall of length 100 m?
(a) 30 (b) 10
(c) 18 (d) 28

[Based on SSC (GL) Prel. Exam. 2003]

28. Babu and Asha can do a job together in 7 days. Asha is $1\frac{3}{4}$ times as efficient as Babu. The same job can be done by Asha alone in
(a) $\frac{49}{4}$ days (b) $\frac{49}{3}$ days
(c) 11 days (d) $\frac{28}{3}$ days

[Based on SSC (GL) Prel. Exam. 2003]

29. If 3 men or 4 women can plough a field in 43 days, how long will 7 men and 5 women take to plough it?

(a) 10 days (b) 11 days
(c) 9 days (d) 12 days

[Based on SSC (GL) Prel. Exam. 2003]

30. 8 men can do a work in 12 days. After 6 days of work, 4 more men were engaged to finish the work. In how many days would the remaining work be completed?
(a) 2 (b) 3
(c) 4 (d) 5

[Based on SSC (GL) Prel. Exam. 2003]

31. *A* can finish a work in 24 days, *B* in 9 days and *C* in 12 days. *B* and *C* start the work but are forced to leave after 3 days. The remaining work is done by *A* in
(a) 5 days (b) 6 days
(c) 10 days (d) $10\frac{1}{2}$ days

[Based on SSC (GL) Prel. Exam. 2003]

32. *A* and *B* undertook to do a piece of work for Rs. 4500. *A* alone could do it in 8 days and *B* alone in 12 days. With the assistance of *C* they finished the work in 4 days. Then *C*'s share of the money is
(a) Rs. 2250 (b) Rs. 1500
(c) Rs. 750 (d) Rs. 375

[Based on SSC (GL) Prel. Exam. 2003]

33. If 6 persons working 8 hours a day earn Rs. 8400 per week, then 9 persons working 6 hours a day will earn per week
(a) Rs. 8400 (b) Rs. 16800
(c) Rs. 9450 (d) Rs. 16200

[Based on SSC (GL) Prel. Exam. 2003]

34. *A* can do a certain work in the same time in which *B* and *C* together can do it. If *A* and *B* together could do it in 10 days and *C* alone in 50 days, then *B* alone could do the work in
(a) 15 days (b) 20 days
(c) 25 days (d) 30 days

[Based on SSC (GL) Prel. Exam. 2003]

35. Twenty-four men can complete a work in sixteen days. Thirty-two women can complete the same work in twenty-four days. Sixteen men and sixteen women started working and worked for twelve days. How many more men are to be added to complete the remaining work in 2 days?
(a) 48 (b) 24
(c) 36 (d) 16

[Based on Bank of Baroda PO 1999]

36. 10 men can complete a piece of work in 15 days and 15 women can complete the same work in 12 days. If all the 10 men and 15 women work together, in how many days will the work get completed?
(a) 6 (b) $7\frac{2}{3}$
(c) $6\frac{2}{3}$ (d) None of these

[Based on SBI Associates PO 1999]

37. 25 men and 15 women can complete a piece of work in 12 days. All of them start working together and after working for 8 days the women stopped working. 25 men completed the remaining work in 6 days. How many days will it take for completing the entire job if only 15 women are put on the job?
(a) 60 days (b) 88 days
(c) 94 days (d) 50 days
(e) None of these **[Based on Guwahati PO 1999]**

38. '*A*' completes a work in 12 days. '*B*' completes the same work in 15 days. '*A*' started working alone and after 3 days *B* joined him. How many days will they now take together to complete the remaining work?
(a) 5 (b) 8
(c) 6 (d) 4
(e) None of these

[Based on BSRB Calcutta PO 1999]

39. 10 men and 15 women finish a work in 6 days. One man alone finishes that work in 100 days. In how many days will a woman finish the work?
(a) 125 days (b) 150 days
(c) 90 days (d) 225 days
(e) None of these

[Based on BSRB Hyderabad PO 1999]

40. The work done by a woman in 8 hours is equal to the work done by a man in 6 hours and by a boy in 21 hours. If working 6 hours per day 9 men can complete a work in 6 days then in how many days can 12 men, 12 women and 12 boys together finish the same work working 8 hours per day?
(a) $1\frac{1}{3}$ days (b) $3\frac{2}{3}$ days
(c) 3 days (d) $1\frac{1}{2}$ days
(e) None of these **[Based on BSRB Patna PO 2001]**

41. Seven men and four boys can complete a work in 6 days. A man completes double the work than a boy. In how many days will 5 men and 4 boys complete the work ?
(a) 5 (b) 4
(c) 6 (d) Cannot be determined
(e) None of these **[IBPS Bank PO 2002]**

42. Vinod can complete a job in 15 hours. Vinay alone can complete the same job in 10 hours. Vinod works for 9 hours and then the remaining job is completed by Vinay. How many hours will it take Vinay to complete the remaining job alone?
(a) 4 (b) 5
(c) 6 (d) 2

[Based on IBPS Jr. Executive 2002]

EXPLANATORY ANSWERS

1. Ans. (d)

Solution $(1\,M + 1\,W + 1\,B)$'s one day's job

$$= \frac{1}{3} + \frac{1}{4} + \frac{1}{12} = \frac{2}{3}$$

$(1\,M + 1\,W)$'s one day's job $= \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

$(1\,M + 1\,W + 1B)$'s $\frac{1}{4}$ day's job $= \frac{1}{6} = \frac{8}{48}$

$(1\,M + 1\,W)$'s $\frac{1}{4}$ day's job $= \frac{7}{48}$

$\therefore$ Remaining $\frac{41}{48}$ of the job is to be finished by boys only so that the work is finished in $\frac{1}{4}$th day only.

One boy's one day's job $= \frac{2}{3} - \frac{7}{12} = \frac{1}{12}$

One boy's $\frac{1}{4}$th day's job $= \frac{1}{48}$

Thus, 41 boys must assist one man and one woman so that the whole job is finished in $\frac{1}{4}$th of a day.

2. Ans. (c)

Solution 25 men's 24 days' job $= \frac{1}{3}$

Remaining $\frac{2}{3}$ of the work is to be finished in 12 days, which can be done by 100 men. Therefore, 75 extra men should be employed.

3. Ans. (a)

Solution $(A + B)$'s four days' work $= 4\left(\frac{1}{15} + \frac{1}{20}\right)$

$$= 4 \times \frac{7}{60} = \frac{7}{15}$$

$\therefore$ Fraction of the work left $= 1 - \frac{7}{15} = \frac{8}{15}$.

4. Ans. (b)

Solution B's one day's work $= \frac{1}{8} - \frac{1}{12} = \frac{1}{24}$.

5. Ans. (b)

Solution Suppose, original number of workers was x

$\therefore\ 100\,x = (x - 10) \times 110$

$\Rightarrow\ 10x = 110 \times 10$

$\Rightarrow\ x = 110.$

6. Ans. (d)

Solution $(A + B + C)$'s 3 days' work

$$= 3 \times \frac{1}{15} + \frac{1}{10} + \frac{1}{30} = \frac{6+3+1}{30} = \frac{1}{3}$$

7. Ans. (b)

8. Ans. (b)

Solution Remaining $\frac{1}{3}$ of the work can be done by 120 men in 32 days.

Same work can be finished in 60 days by only

$$\frac{32 \times 120}{60} = 64 \text{ men}$$

$\therefore$ 56 men can be discharged after $\frac{2}{3}$ of the work is over.

9. Ans. (a)

Solution No. of days required to complete the job

$$= \frac{12 \times 6}{18} = 4 \text{ days.}$$

10. Ans. (c)

Solution Work done by A and B together $= \frac{6 \times 9}{15}$

$= 3.6$ days.

11. Ans. (a)

Solution No. of men in the beginning

$$= \frac{\text{No. of more men} \times \text{No. of days taken by second group}}{\text{No. of less days}}$$

$$= \frac{5 \times (30 - 10)}{10} = \frac{5 \times 20}{10} = 10 \text{ men.}$$

12. Ans. (c)

Solution Work done by A and B in 3 days

$= 3\left(\frac{1}{6} + \frac{1}{12}\right) = \frac{3}{4}$, remaining work $= \frac{1}{4}$

$\therefore$ Time taken by B to finish the remaining work

$$= \frac{1}{4} \times 12 = 3 \text{ days}$$

$\therefore$ Total no. of days needed for the completion of the work $= 3 + 3 = 6$ days.

13. Ans. (d)

Solution Suppose, the man was idle for x days

$\therefore\ 20\,(60 - x) - 3x = 280 \Rightarrow 23x = 920 \Rightarrow\ x = 40.$

14. Ans. (b)

Solution $\frac{700}{6\times15}\times9\times12 = 840.$

15. Ans. (b)

Solution B's one day's work $= \frac{1}{10}-\frac{1}{30} = \frac{1}{15}.$

16. Ans. (a)

Solution Woman's one day's work

$$= \frac{1}{3}-\frac{1}{6}-\frac{1}{18} = \frac{6-3-1}{18} = \frac{1}{9}.$$

17. Ans. (b)

Solution $(A + B + C)$ will do the same piece of work in

$$\frac{10\times15\times20\times2}{10\times15+15\times20+20\times10} = \frac{120}{13} \text{ days.}$$

18. Ans. (c)

Solution 'A' can cultivate full part of the land in $\frac{6\times5}{2}$ days.

'B' can cultivate full part of the land in $(10 \times 3) = 30$ days.

Hence, the required time to do $\frac{4}{5}$ part when 'A' and 'B' work together $= \left(\frac{15\times30}{30+15}\right)\times\frac{4}{5} = 8$ days.

19. Ans. (c)

Solution Efficiency of A is three times that of B. Hence, time required to finish the work by

B (alone) $= (3 + 1) \times 10 = 40$ days.

20. Ans. (b)

Solution A can do a piece of work in 4 hours

A and C can do it in 2 hours.

$\therefore$ C alone can do it in $\left(\frac{4\times2}{4-2}=4\right)$ hours

B and C can do it in 3 hours.

$\therefore$ B alone can do it in $\left(\frac{4\times3}{4-3}=12\right)$ hours.

21. Ans. (a)

Solution A does the whole work in $\frac{5}{4}\times20 = 25$ days.

Remaining work $\left(1-\frac{4}{5}\right) = \frac{1}{5}$, is done by A and B in 3 days.

$\therefore$ Whole work is done by A and B in $(5 \times 3 = 15)$ days.

$\therefore$ B alone can do the whole work in

$$\left(\frac{25\times15}{25-15}\right) = 37\frac{1}{2} \text{ days.}$$

22. Ans. (a)

Solution $\frac{18+9}{18\times9} = \frac{1}{6}.$

23. Ans. (d)

Solution $(A + B)$'s one day's work $= \frac{1}{30}$

$(B + C)$'s one day's work $= \frac{1}{15}$

$(C + A)$'s one day's work $= \frac{1}{10}$

$\therefore$ $2(A + B + C)$'s one day's work

$$= \frac{1}{30}+\frac{1}{15}+\frac{1}{10} = \frac{1+2+3}{30} = \frac{1}{5}$$

$\therefore$ $(A + B + C)$'s one day's work $= \frac{1}{10}.$

24. Ans. (c)

Solution A can finish the work in 40 days.

B can finish the work in 100 days.

C can finish the work in 39 days.

25. Ans. (d)

Solution $(A + B + C)$'s one day's work $= \frac{1}{2}$

Let C can finish the work in x days.

$\therefore$ A can finish the work in $3x$ days.

$\Rightarrow$ B can finish the work in $\frac{3x}{2}$ days.

$\therefore$ $(A + B + C)$'s one day's work $= \frac{1}{3x}+\frac{2}{3x}+\frac{1}{x}$

$$\therefore \quad \frac{1}{3x}+\frac{2}{3x}+\frac{1}{x} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{3x}{2} = 6$$

$$\Rightarrow \quad x = 4$$

$\therefore$ B alone can do the whole work in 6 days.

26. Ans. (b)

Solution A and B together do $\frac{20}{30} = \frac{2}{3}$ part of the work in 20 days.

Remaining work $= 1 - \frac{2}{3} = \frac{1}{3}$

$\therefore$ A alone can do the work in $20 \times 3 = 60$ days.

27. Ans. (a)

Solution Required number of men

$$= 72 \times \frac{21}{18} \times \frac{100}{280} = 30.$$

28. Ans. (c)

Solution Ratio of the efficiencies of

Babu	:	Asha
4	:	7

Ratio of the no. of days taken by Babu and Asha to complete the job = 7 : 4

Also, $\frac{28x^2}{11x} = 7$ or, $x = \frac{11}{4}$

$\therefore$ Asha alone can do the work in $4 \times \frac{11}{4} = 11$ days.

29. Ans. (d)

Solution $3M = 4W$

$$7M + 5W = 7M + \frac{5 \times 3M}{4} = 7M + \frac{15M}{4} = \frac{43M}{4}$$

$\because$ 3 men plough a field in 43 days

$\therefore$ $\frac{43}{4}$ men plough a field in $\frac{43 \times 3 \times 4}{43} = 12$ days.

30. Ans. (c)

Solution Part of the work done by 8 men in 6 days

$$= \frac{6}{12} = \frac{1}{2}$$

Remaining work $= 1 - \frac{1}{2} = \frac{1}{2}$

Required number of days $= \frac{1}{2} \times 8 = 4$ days.

31. Ans. (c)

Solution Part of work done by B and C in 3 days

$$= 3\left(\frac{1}{9} + \frac{1}{12}\right) = \frac{7}{12}$$

Remaining work $= 1 - \frac{7}{12} = \frac{5}{12}$

Remaining work is done by A in $= \frac{5}{12} \times 24 = 10$ days.

32. Ans. (c)

Solution C alone can do the work in

$$\frac{1}{4} - \left(\frac{1}{8} + \frac{1}{12}\right) = \frac{1}{4} - \frac{5}{24} = \frac{6-5}{24} = \frac{1}{24}$$

i.e. 24 days

Ratio of their wages $= \frac{1}{8} : \frac{1}{12} : \frac{1}{24} = 3 : 2 : 1$

C's share $= \frac{1}{6} \times 4500 =$ Rs. 750.

33. Ans. (c)

Solution Required sum $= 8400 \times \frac{6}{8} \times \frac{9}{6} =$ Rs. 9450.

34. Ans. (c)

Solution A, B and C together can do the work in

$$= \frac{10 \times 50}{60} = \frac{25}{3} \text{ days.}$$

Suppose, A alone could do it in 'x' days.

Then, $(B + C)$ together can also do it in 'x' days

i.e. $\frac{x \times x}{2x} = \frac{25}{3} \Rightarrow x = \frac{50}{3}$ days

$\therefore$ B alone can do it in $\frac{1}{10} - \frac{3}{50} = \frac{5-3}{50} = \frac{1}{25}$

i.e. 25 days.

35. Ans. (b)

Solution 24 men complete the work in 16 days.

$\therefore$ 16 men complete $\left(\frac{16}{24} \times \frac{12}{16}\right) = \frac{1}{2}$ part of work in 12 days.

32 women complete the work in 24 days.

$\therefore$ 16 women complete $\frac{16}{32} \times \frac{14}{24} = \frac{7}{24}$ part of work in (12 + 2) = 14 days.

So, the remaining part of the work which is done by sixteen men + sixteen women and the required additional no. of men in 2 days

$$= 1 - \left(\frac{1}{2} + \frac{7}{24}\right) = \frac{1}{2} - \frac{7}{24} = \frac{5}{24} \text{ (Part)}$$

Now, in 2 days $\frac{5}{24}$ part of work is done by

$$24 \times \frac{16}{2} \times \frac{5}{24} = 40 \text{ men}$$

Hence, the required no. of men = 40 – 16 = 24.

36. Ans. (c)

Solution 10 men + 15 women in 1 day do

$$\frac{1}{15} + \frac{1}{12} = \frac{9}{60} \text{ work}$$

$\therefore$ Time taken $= \frac{60}{9}$ days $= 6\frac{2}{3}$ days.

37. Ans. (e)

38. Ans. (a)

Solution Work done by 'A' in 3 days $= \frac{1}{12} \times 3 = \frac{1}{4}$

$\therefore$ Remaining work $= 1 - \frac{1}{4} = \frac{3}{4}$

Work done by A and B together $= \frac{12 \times 15}{27} = \frac{20}{3}$ days

$\therefore$ Remaining work is done by A and B together in

$= \frac{3}{4} \times \frac{20}{3} = 5$ days.

39. Ans. (d)

40. Ans. (d)

Solution $\because$ $8W = 6M = 12B$

$\therefore$ $12M + 12W + 12B \Rightarrow 12M + 9M + 6M = 27M$

$\because$ 9 men can complete the work working 1 hour per day in 6×6 days

$\therefore$ 27 men working 8 hours per day

$= \frac{6 \times 6 \times 9}{27 \times 8} = 1\frac{1}{2}$ days.

41. Ans. (e)

Solution $\because$ $M = 2B$

$\therefore$ $7M + 4B = 14B + 4B = 18B$

and $5M + 4B = 10B + 4B = 14B$

$\because$ 18 boys complete the work in 6 days

$\therefore$ 14 boys complete the work in $= \frac{6 \times 18}{14} = 7\frac{5}{7}$ days.

42. Ans. (a)

Solution The part of job that Vinod completes in 9 hours $= \frac{9}{15} = \frac{3}{5}$

Remaining job $= 1 - \frac{3}{5} = \frac{2}{5}$

Remaining job can be done by Vinay in $\frac{2}{5} \times 10 = 4$ hours.

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. Two men undertake to do a piece of work for Rs. 400. One alone can do it in 6 days, the other in 8 days. With the help of a boy, they finish it in 3 days. The boy's share is

(a) Rs. 40 (b) Rs. 50
(c) Rs. 60 (d) Rs. 80

2. 2 men and 3 women can finish a piece of work in 10 days, while 4 men can do it in 10 days. In how many days will 3 men and 3 women finish it?

(a) 6 days (b) $5\frac{2}{3}$ days
(c) 8 days (d) $8\frac{1}{3}$ days

3. A, B and C can complete a work in 6, 8 and 12 days, respectively. A and B start the work together but B drops out after 2 days and C replaces him. How many days in whole they take to finish the job?

(a) $3\frac{2}{3}$ days (b) $2\frac{2}{3}$ days
(c) $3\frac{1}{3}$ days (d) $3\frac{2}{3}$ days

4. 8 children and 12 men complete a certain piece of work in 9 days. If each child takes twice the time taken by a man to finish the work, 12 men will finish the same work in

(a) 15 days (b) 12 days
(c) 14 days (d) 8 days

5. A, B and C can do a piece of work in 12, 18 and 24 days, respectively. They started work at it together. A stops the work after 4 days and B is called off 2 days before the work is done. The work was finished in

(a) 6 days (b) 10 days
(c) 8 days (d) 9 days

6. One man, 3 women and 4 boys can do a piece of work in 96 hours; 2 men and 8 boys can do it in 80 hours; 2 men and 3 women can do it in 120 hours. 5 men and 12 boys can do it in

(a) $42\frac{7}{11}$ hours (b) $43\frac{7}{11}$ hours
(c) 44 hours (d) $39\frac{1}{11}$ hours

7. 8 children and 12 men complete a certain piece of work in 9 days. If each child takes twice the time taken by a man to finish the work, in how many days will 12 men finish the same work?

(a) 8 (b) 15
(c) 9 (d) 12

8. A is twice as good a workman as B and thrice as good a workman as C. If C alone can do a piece of work in 24 days, how long would the three persons take to finish the work working together?

(a) $3\frac{3}{11}$ days (b) $4\frac{4}{7}$ days
(c) $4\frac{4}{11}$ days (d) $3\frac{4}{11}$ days

9. Two persons A and B can complete a piece of work in 8 hours and 16 hours, respectively. If they work at it alternately for an hour, A starting first, in how many hours will the work be finished?
(a) $9\frac{1}{3}$ hours (b) $10\frac{1}{2}$ hours
(c) $11\frac{1}{2}$ hours (d) $8\frac{1}{2}$ hours

10. A can do a certain job in 12 days. B is 60% more efficient than A. The number of days, it takes B to do the same piece of work is
(a) 6 (b) $6\frac{1}{4}$
(c) $7\frac{1}{2}$ (d) 8

Test I

[Time: 10 min]

11. 15 men can complete a work in 210 days. They started the work but at the end of 10 days 15 additional men, with double efficiency, were inducted. How many days, in whole, did they take to finish the work?
(a) $76\frac{2}{3}$ days (b) $84\frac{3}{4}$ days
(c) $72\frac{1}{2}$ days (d) 70 days

12. A, B and C can do a work in 6, 8 and 12 days' respectively. Doing that work together they get an amount of Rs. 1350. What is the share of B in that amount?
(a) Rs. 450 (b) Rs. 500
(c) Rs. 550 (d) Rs. 700

13. A and B undertake to do a work for Rs. 56. A can do it alone in 7 days and B in 8 days. If with the assistance of a boy they finish the work in 3 days, then the boy gets
(a) Rs. 11 (b) Rs. 12
(c) Rs. 15 (d) Rs. 18

14. Efficiencies of two men are in the ratio 2 : 3. If the first man takes 30 days to do a piece of work, the time required when both of them work together on a job twice as big as the first, will be
(a) 30 days (b) 18 days
(c) 24 days (d) 72 days

15. If 4 men and 6 boys can finish a piece of work in 24 days, in what time will 8 men and 9 boys finish it, supposing that 8 men can do as much work in 16 hours as 24 boys can do in 12 hours
(a) 12 days (b) $13\frac{1}{3}$ days
(c) 15 days (d) $12\frac{1}{2}$ days

16. A, B and C contract a work for Rs. 550. Together A and B are supposed to do $\frac{7}{11}$ of the work. How much does C get?
(a) Rs. 200 (b) Rs. 300
(c) Rs. 250 (d) Rs. 350

17. A can complete a job in 9 days, B in 10 days and C in 15 days. B and C start the work and are forced to leave after 2 days. The time taken to complete the remaining work is
(a) 6 days (b) 9 days
(c) 10 days (d) 13 days

18. 12 men can complete a work in 8 days. Three days after they started the work, 3 more men joined them. In how many days will all of them together complete the remaining work?
(a) 2 days (b) 4 days
(c) 5 days (d) 6 days

19. Sumit and Amit can complete a work in 10 days and 15 days, respectively. Amit starts the work and after 5 days Sumit also joins him. In all, the work would be completed in
(a) 7 days (b) 9 days
(c) 11 days (d) None of these

20. A and B working separately can do a piece of work in 8 and 10 hrs, respectively. If they work for an hour alternately, A beginning, work will be completed in
(a) $8\frac{4}{5}$ hours (b) 9 hours
(c) $6\frac{4}{5}$ hours (d) None of these

Answers

1. (b)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (c)	**6.** (b)	**7.** (d)	**8.** (c)	**9.** (b)	**10.** (c)
11. (a)	**12.** (a)	**13.** (a)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (a)	**18.** (b)	**19.** (b)	**20.** (a)

14

PIPES AND CISTERNS

INTRODUCTION

Pipes are connected to a *tank* or *cistern* and are used to fill or empty the tank.

Inlet A pipe connected with a tank or a cistern that fills it is known as *inlet*.

Outlet A pipe connected with a tank or cistern emptying it is known as *outlet*.

Pipes and Cistern problems are similar to those on Time and Work. The only difference here is, the work done is in terms of filling or emptying a cistern and the time taken is the time taken by a pipe or leak (crack) to fill or empty a cistern, respectively.

Generally, the time taken to fill a cistern is taken as positive and the time taken to empty a cistern is taken as negative. The amount of work done, i.e. filling or emptying a cistern is generally taken as unity, unless otherwise specified.

SOME BASIC FORMULAE

1. If an inlet can completely fill the empty tank in X hours, the part of the tank filled in 1 hour = $\frac{1}{X}$.
2. If an outlet can empty the full tank in Y hours, the part of the tank emptied in 1 hour = $\frac{1}{Y}$.
3. If both inlet and outlet are open, net part of the tank filled in 1 hour = $\frac{1}{X}-\frac{1}{Y}$.

Illustration 1 A pipe can fill a tank in 5 hours. Find the part of tank filled in one hour.

Solution The part of the tank filled in 1 hour = $\frac{1}{5}$.

Illustration 2 A pipe can fill a tank in 28 minutes. Find the time in which $\frac{1}{7}$ part of the tank will be filled.

Solution We have, $\frac{1}{28}$ part of the tank is filled in 1 minute.

$\therefore$ $\frac{1}{7}$ part of the tank is filled in $\frac{28}{7}$ minutes

= 4 minutes.

Illustration 3 A pipe can empty a cistern in 40 minutes. Find the time in which $\frac{3}{4}$ part of the cistern will be emptied.

Solution We have, $\frac{1}{40}$ part of the cistern is emptied in = 1 minute.

$\therefore$ $\frac{3}{4}$ part of the cistern is emptied in

$$= 40 \times \frac{3}{4} = 30 \text{ minutes.}$$

Illustration 4 A pipe can empty a cistern in 12 hours. Find the part of the cistern emptied in 4 hours.

Solution We have, part of the cistern emptied in 1 hour = $\frac{1}{12}$,

$\therefore$ part of the cistern emptied in 4 hours = $\frac{1}{12}\times 4 = \frac{1}{3}$.

Illustration 5 A tap can fill a cistern is 8 hours and another can empty it in 16 hours. If both the taps are opened simultaneously, find the time (in hours) to fill the cistern.

Solution Here, $X = 8$ and $Y = 16$.

$\therefore$ Part of the cistern filled in 1 hour

$$= \frac{1}{X}-\frac{1}{Y}$$

$$= \frac{1}{8}-\frac{1}{16} = \frac{1}{16}$$

$\therefore$ Total time taken to fill the cistern = 16 hours.

SOME USEFUL SHORT-CUT METHODS

1. Two pipes A and B can fill (or empty) a cistern in X and Y hours respectively, while working alone. If both the pipes are opened together, then the time taken to fill (or empty) the cistern is given by

$$\left(\frac{XY}{X+Y}\right)\text{hours.}$$

Explanation

Part of the cistern filled (or emptied) by pipe A alone in 1 hour $= \frac{1}{X}$

Part of the cistern filled (or emptied) by pipe B alone in 1 hour $= \frac{1}{Y}$.

$\therefore$ Part filled (or emptied) by $(A + B)$ in 1 hour

$$= \frac{1}{X}+\frac{1}{Y} = \frac{X+Y}{XY}.$$

Therefore, both the pipes A and B together will fill (or empty) the cistern in $\left(\frac{XY}{X+Y}\right)$hours.

Illustration 6 Two pipes A and B can fill a cistern in 20 and 30 minutes, respectively. If both the pipes are opened simultaneously, how long will it take to fill the cistern?

Solution Here, $X = 20$ and $Y = 30$.

$\therefore$ Part of the cistern filled by $(A + B)$ in 1 minute

$$= \frac{1}{X}+\frac{1}{Y} = \frac{1}{20}+\frac{1}{30} = \frac{5}{60} = \frac{1}{12}.$$

$\therefore$ Both the pipes A and B together will fill the cistern in 12 minutes.

2. Three pipes A, B and C can fill a cistern in X, Y and Z hours respectively, while working alone. If all the three pipes are opened together, the time taken to fill the cistern is given by

$$\left(\frac{X\times Y\times Z}{XY+YZ+ZX}\right)\text{hours.}$$

Explanation

Part of the cistern filled by A alone in 1 hour $= \frac{1}{X}$

Part filled by B alone in 1 hour $= \frac{1}{Y}$

Part filled by C alone in 1 hour $= \frac{1}{Z}$

All the three pipes are opened.

$\therefore$ Part filled in 1 hour $= \frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$

$$= \frac{XY+YZ+ZX}{XYZ}$$

$\therefore$ Cistern will be filled in $\frac{XYZ}{XY+YZ+ZX}$ hours.

Note: We can generate more formulae like above by replacing negative sign wherever a pipe starts emptying a cistern instead of the standard positive sign.

Illustration 7 Two pipes A and B can separately fill a cistern is 8 hours and 12 hours respectively, while a third pipe C can empty it in 6 hours. In what time will the cistern be full, if all the pipes are opened together?

Solution Here, $X = 8$, $Y = 12$ and $Z = -6$.

$\therefore$ The cistern will be full in

$$= \left(\frac{8\times12\times-6}{8\times12-12\times6-6\times8}\right)\text{hours}$$

$$= \left(\frac{576}{24}\right)\text{hours or 24 hours.}$$

3. Two pipes A and B can fill a cistern in X hours and Y hours, respectively. There is also an outlet C. If all the three pipes are opened together, the tank is full in Z hours. The time taken by C to empty the full tank is given by

$$\left(\frac{XYZ}{XZ+YZ-XY}\right)\text{hours.}$$

Explanation

Part of the tank emptied by C in 1 hour

$$= \left(\frac{1}{X}+\frac{1}{Y}-\frac{1}{Z}\right)$$

$\therefore$ C can empty the full tank in $\left(\frac{XYZ}{XZ+YZ-XY}\right)$ hours.

Illustration 8 Two taps A and B can fill a cistern in 30 minutes and 60 minutes, respectively. There is third exhaust tap C at the bottom of the tank. If all taps are opened at the same time, the cistern will be full in 45 minutes. In what time can exhaust tap C empty the cistern when full?

Solution Here, $X = 30$, $Y = 60$ and $Z = 45$.

$\therefore$ Exhaust tap C can empty the cistern in

$$= \left(\frac{XYZ}{XZ+YZ-XY}\right)\text{ minutes}$$

$$= \left(\frac{30\times60\times45}{30\times45\times60\times45-30\times60}\right)\text{ minutes}$$

$= 36$ minutes.

4. A tank takes X hours to be filled by a pipe. But due to a leak, it is filled in Y hours. The amount of time in which the leak can empty the full tank

$$= \left(\frac{XY}{Y-X}\right)\text{hours.}$$

Illustration 9 A pipe can fill a tank in 12 hours. Due to leakage in the bottom, it is filled in 24 hours. If the tank is full, how much time will the leak take to empty it?

Solution Here, $X = 12$ and $Y = 24$.

$\therefore$ The time taken by the leak to empty the full tank

$$= \left(\frac{XY}{Y-X}\right)\text{hours}$$

$$= \left(\frac{12\times 24}{24-12}\right)\text{hours or 24 hours.}$$

5. A cistern has a leak which can empty it in X hours. A pipe which admits Y litres of water per hour into the cistern is turned on and now the cistern is emptied in Z hours. The capacity of the cistern is

$$\left(\frac{XYZ}{Z-X}\right)\text{ litres.}$$

Illustration 10 A leak in the bottom of a tank can empty the full tank in 6 hours. An inlet pipe fills water at the rate of 4 litres per minute. When the tank is full, the inlet is opened and due to leak, the tank is empty in 8 hours. Find the capacity of the tank.

Solution Here, $X = 6$, $Y = 4 \times 60 = 240$ and $Z = 8$.

$\therefore$ The capacity of the tank is

$$= \left(\frac{XYZ}{Z-X}\right)\text{ litres} = \left(\frac{6\times 240\times 8}{8-6}\right)\text{ litres}$$

$$= 5760 \text{ litres.}$$

6. One fill pipe A is k times faster than the other fill pipe B.
   (a) If B can fill a cistern in x hours, then the time in which the cistern will be full, if both the fill pipes are opened together, is $\left(\frac{x}{k+1}\right)$ hours.
   (b) If A can fill a cistern in y hours, then the time in which the cistern will be full, if both the fill pipes are opened together, is $\left(\frac{k}{k+1}\right)y$ hours.

Illustration 11 One fill pipe A is 10 times faster than second fill pipe B. If B can fill a cistern in 55 minutes, then find the time when the cistern will be full if both fill pipes are opened together.

Solution Here, $k = 10$ and $x = 55$.

$\therefore$ Cistern will be full in

$$= \left(\frac{x}{k+1}\right)\text{ minutes}$$

$$= \left(\frac{55}{10+1}\right)\text{ minutes or 5 minutes.}$$

Illustration 12 One fill pipe A is 4 times faster than second fill pipe B. If A can fill a cistern in 15 minutes, then find the time when the cistern will be full if both fill pipes are opened together.

Solution Here, $k = 4$ and $y = 15$.

$\therefore$ Cistern will be full in $= \left(\frac{k}{k+1}\right)y$ minutes

$$= \left(\frac{4}{4+1}\right)15 \text{ minutes}$$

$$= 12 \text{ minutes.}$$

7. If one fill pipe A is k times faster and takes x minutes less time than the other fill pipe B, then
   (a) the time taken to fill a cistern, if both the pipes are opened together is $\left(\frac{kx}{(k-1)^2}\right)$ minutes.
   (b) A will fill the cistern in $\left(\frac{x}{k-1}\right)$ minutes.
   (c) B will fill the cistern in $\left(\frac{kx}{k-1}\right)$ minutes.

Illustration 13 One fill pipe A is 5 times faster than second fill pipe B and takes 32 minutes less than the fill pipe B. When will the cistern be full if both fill pipes are opened together?

Solution Here, $k = 5$ and $x = 32$.

$\therefore$ Cistern will be full in $= \frac{kx}{(k-1)^2}$ minutes

$$= \frac{5\times 32}{(5-1)^2} \text{ minutes}$$

$$= 10 \text{ minutes.}$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. One tap can fill a cistern in 2 hours and another can empty the cistern in 3 hours. How long will they take to fill the cistern if both the taps are opened?
(a) 6 hours (b) 7 hours
(c) 6.30 hours (d) None of these

2. A tap can fill a tank in 25 minutes and another can empty it in 50 minutes. Find whether the tank will be filled up or emptied and in how many minutes?
(a) Tank is filled up in 50 minutes
(b) Tank is emptied in 25 minutes
(c) Tank is filled up in 25 minutes
(d) None of these

3. A water tank is $\frac{2}{5}$th full. Pipe A can fill the tank in 10 minutes and the pipe B can empty it in 6 minutes. If both the pipes are open, how long will it take to empty or fill the tank completely?
(a) 6 minutes to fill (b) 6 minutes to empty
(c) 8 minutes to fill (d) None of these

4. Two taps A and B can fill a tank in 10 hours and 15 hours, respectively. If both the taps are opened together the tank will be full in
(a) 8 hours (b) 6 hours
(c) 5 hours (d) None of these

5. Two pipes A and B can separately empty a cistern in 12 hours and 15 hours, respectively. In what time will the cistern be emptied, if both the pipes are opened together?
(a) 5 hours 30 minutes (b) 7 hours
(c) 6 hours 40 minutes (d) None of these

6. Two pipes can fill a tank in 10 hours and 12 hours, respectively. While a third pipe emptied the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time the tank will be filled?
(a) 7 hours 30 minutes (b) 6 hours 40 minutes
(c) 8 hours 30 mintues (d) None of these

7. Three pipes A, B and C can fill a cistern in 10, 12 and 15 hours, respectively, while working alone. If all the three pipes are opened together, the time taken to fill the cistern will be
(a) 4 hours (b) 6 hours
(c) 7 hours (d) None of these

8. Two pipes A and B can fill a cistern in 24 minutes and 30 minutes, respectively. There is also an outlet C. If all the three pipes are opened together, the cistern is full in 20 minutes. How much time will be taken by C to empty the full cistern?
(a) 30 minutes (b) 40 mintues
(c) 45 mintues (d) None of these

9. A cistern is normally filled in 8 hours but takes 2 hours longer to fill because of a leak in its bottom. If the cistern is full, the leak will empty it in
(a) 35 hours (b) 45 hours
(c) 40 hours (d) None of these

10. A cistern has a leak which would empty in 8 hours. A tap is turned on which admits 6 litres a minute into the cistern and it is now emptied in 12 hours. The cistern can hold
(a) 6840 litres (b) 7860 litres
(c) 8640 litres (d) None of these

11. If two pipes function simultaneously, the reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does the faster pipe take to fill the reservoir?
(a) 35 hours (b) 30 hours
(c) 40 hours (d) None of these

12. One fill pipe A is 3 times faster than second fill pipe B and takes 32 minutes less than the fill pipe B. When will the cistern be full if both pipes are opened together?
(a) 28 mintues (b) 24 minutes.
(c) 30 minutes (d) Data inadequate.

13. Two pipes A and B can fill a cistern in 4 minutes and 6 minutes, respectively. If these pipes are turned on alternately for 1 minute each how long will it take for the cistern to fill?
(a) 4 min 40 sec (b) 3 min 20 sec
(c) 4 min 50 sec (d) 3 min 30 sec.

14. There are two taps to fill a tank while a third to empty it. When the third tap is closed, they can fill the tank in 10 minutes and 12 minutes, respectively. If all the three taps be opened, the tank is filled in 15 minutes. If the first two taps are closed, in what time can the third tap empty the tank when it is full?
(a) 7 min (b) 9 min and 32 sec
(c) 8 min and 34 sec (d) 6 min.

15. Two pipes A and B can separately fill a cistern in 15 minutes and 18 minutes, respectively, while a third pipe C can empty it in 6 minutes. Two pipes A and B are kept open for 6 minutes in the beginning and then the third pipe is also opened. In what time will the cistern be emptied?

(a) $16\frac{1}{2}$ minutes (b) 15 minutes
(c) $15\frac{1}{2}$ minutes (d) 16 minutes

16. A reservoir is provided by two pipes A and B. A can fill the reservoir 5 hours faster than B. If both together fill the reservoir in 6 hours, the reservoir will be filled by A alone in
(a) 10 hours (b) 8 hours
(c) 12 hours (d) 11 hours

17. A cistern is provided by two taps A and B. A can fill it in 20 minutes and B in 25 minutes. Both the taps are kept open for 5 minutes and then the second is turned off. The cistern will be completely filled in another
(a) 11 minutes (b) 10 minutes
(c) 15 minutes (d) 12 minutes

18. Two pipes A and B can separately fill a tank in 6 hours and 8 hours, respectively. Both the pipes are opened together, but $1\frac{1}{2}$ hours after the start, the pipe A is turned off. How much time will it take to fill the tank?
(a) 5 hours (b) 6 hours
(c) $4\frac{1}{2}$ hours (d) $5\frac{1}{2}$ hours

19. A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the pipes are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty a full cistern?
(a) 8 minutes (b) 10 minutes
(c) 12 minutes (d) 16 minutes

20. Two taps can separately fill a cistern in 10 minutes and 15 minutes, respectively and when the waste pipe is open, they can together fill it in 18 minutes. The waste pipe can empty the full cistern in
(a) 7 min (b) 9 min
(c) 13 min (d) 23 min

EXPLANATORY ANSWERS

1. Ans. (a)

Solution Here, $X = 2$ and $Y = 3$.

$\therefore$ Part of the cistern filled in 1 hour

$$= \frac{1}{X} - \frac{1}{Y}$$

$$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$

$\therefore$ Total time taken to fill the cistern = 6 hours.

2. Ans. (a)

Solution Here, $X = 25$ and $Y = 50$.

$\therefore$ Part of the tank filled or emptied in 1 minute

$$= \frac{1}{X} - \frac{1}{Y} = \frac{1}{25} - \frac{1}{50} = \frac{1}{50},$$

which is positive, therefore the tank will be filled.

$\therefore$ Total time taken to fill the tank = 50 minutes.

3. Ans. (b)

Solution Here, $X = 10$ and $Y = 6$.

$\therefore$ Part of the tank filled or emptied in 1 minute

$$= \frac{1}{X} - \frac{1}{Y} = \frac{1}{10} - \frac{1}{6} = -\frac{1}{15}$$

which is negative, therefore tank will be emptied.

Thus, $\frac{2}{5}$th full of the tank will be emptied in

$$= 15 \times \frac{2}{5} = 6 \text{ minutes.}$$

4. Ans. (b)

Solution Here, $X = 10$ and $Y = 15$.

$\therefore$ The tank will be full in

$$= \left(\frac{XY}{X+Y}\right) \text{ hours}$$

$$= \left(\frac{10 \times 15}{10+15}\right) \text{ hours or 6 hours.}$$

5. Ans. (c)

Solution Here, $X = 12$ and $Y = 15$.

$\therefore$ The cistern will be empty in

$$= \left(\frac{XY}{X+Y}\right) \text{ hours}$$

$$= \left(\frac{12 \times 15}{12+15}\right) \text{ hours}$$

$$= \frac{20}{3} \text{ hours or, 6 hours 40 minutes.}$$

6. Ans. (a)

Solution Here, $X = 10$, $Y = 12$ and $Z = -20$.

$\therefore$ The tank will be full in

$$= \left(\frac{X \times Y \times -Z}{XY - YZ - ZX}\right) \text{ hours}$$

$$= \left(\frac{10 \times 12 \times -20}{10 \times 12 - 12 \times 20 - 20 \times 10}\right) \text{ hours}$$

$$= \left(\frac{15}{2}\right) \text{ hours or, 7 hours 30 minutes.}$$

7. Ans. (a)

Solution Here, $X = 10$, $Y = 12$ and $Z = 15$.

$\therefore$ The total time taken to fill the cistern

$$= \left(\frac{XYZ}{XY + YZ + ZX}\right) \text{ hours}$$

$$= \left(\frac{10\times12\times15}{10\times12+12\times15+10\times15}\right) \text{ hours}$$

$= 4$ hours.

8. Ans. (b)

Solution Here, $X = 24$, $Y = 30$ and $Z = 20$.

$\therefore$ The total time taken by C to empty the full cistern

$$= \left(\frac{XYZ}{XZ + YZ - XY}\right) \text{ minutes}$$

$$= \left(\frac{24\times30\times20}{24\times20+30\times20-24\times30}\right) \text{ minutes}$$

$= 40$ minutes.

9. Ans. (c)

Solution Here, $X = 8$ and $Y = 8 + 2 = 10$.

$\therefore$ The leak will empty the cistern in

$$= \left(\frac{XY}{Y-X}\right) \text{ hours}$$

$$= \left(\frac{8\times10}{10-8}\right) \text{ hours or, 40 hours.}$$

10. Ans. (c)

Solution Here, $X = 8$, $Y = 6 \times 60 = 360$ and $Z = 12$.

$\therefore$ The capacity of the cistern is

$$= \left(\frac{XYZ}{Z-X}\right) \text{ litres}$$

$$= \left(\frac{8\times360\times12}{12-8}\right) \text{ litres}$$

$= 8640$ litres.

11. Ans. (b)

Solution Let one pipe take x hours to fill the reservoir. Then, another pipe takes $(x - 10)$ hours.

$$\therefore \frac{1}{x}+\frac{1}{x-10} = \frac{1}{12} \Rightarrow x(x - 10) = 12(x + x - 10)$$

$$\Rightarrow x^2 - 34x + 120 = 0$$

or, $(x - 30)(x - 4) = 0$

$\therefore$ $x = 30$ or $x = 4$.

$\therefore$ The faster pipe takes 30 hours to fill the reservoir.

12. Ans. (b)

Solution Here, $k = 3$ and $x = 32$

$\therefore$ Cistern will be full in $= \dfrac{kx}{(k-1)^2}$ minutes

$$= \frac{3\times32}{(3-1)^2} \text{ minutes}$$

$= 24$ minutes.

13. Ans. (a)

Solution As the pipes are operating alternately, thus their 2 minutes job is $\frac{1}{4}+\frac{1}{6} = \frac{5}{12}$.

In the next 2 minutes the pipes can fill another $\frac{5}{12}$ part of cistern. Therefore, in 4 minutes the two pipes which are operating alternately will fill $\frac{5}{12}+\frac{5}{12} = \frac{10}{12} = \frac{5}{6}$ part.

The part of the cistern left unfilled $= 1 - \frac{5}{6} = \frac{1}{6}$

Pipe A can fill $\frac{1}{4}$ of the cistern in 1 minute

Pipe A can fill $\frac{1}{6}$ of the cistern in $4 \times \frac{1}{6} = \frac{2}{3}$ minutes.

Total time taken to fill the cistern $4 + \frac{2}{3} = 4\frac{2}{3}$ minutes or, 4 minutes 40 seconds.

14. Ans. (c)

Solution Part emptied by the third pipe in 1 minute

$$= \left(\frac{1}{10}+\frac{1}{12}\right)-\frac{1}{15} = \frac{7}{60}$$

So, the full tank will be emptied by third pipe in $\left(\frac{60}{7}\right)$ minute $= 8$ minute 34 seconds

15. Ans. (a)

Solution Pipe $(A + B)$'s 6 minutes job $= 6\left(\frac{1}{15}+\frac{1}{18}\right)$

$$= \frac{11}{15}$$

Net work done by the three pipes $(A + B + C)$ in 1 minute $= \left(\frac{1}{15}+\frac{1}{18}\right)-\frac{1}{6} = \frac{-4}{90} = \frac{-2}{45}$

Net $\frac{2}{45}$ part of the tank is emptied by the pipe C in 1 minute

Net $\frac{11}{15}$ part of the tank is emptied by the pipe C in $\frac{45}{2}\times\frac{11}{15} = \frac{33}{2}$ minutes $= 16\frac{1}{2}$ minutes.

16. Ans. (a)

Solution Let A alone can fill the reservoir in x hours.

Then, B can fill the reservoir in $(x + 5)$ hours

$$\therefore \frac{1}{x}+\frac{1}{x+5} = \frac{1}{6}$$

$\therefore \quad 6(2x + 5) = x(x + 5)$

or, $x^2 - 7x - 30 = 0$

or, $(x - 10)(x + 3) = 0$

or, $x = 10$ hours.

17. Ans. (a)

Solution Part filled in 1 minute $= \frac{1}{20} + \frac{1}{25} = \frac{9}{100}$

Part filled in 5 minutes $= \frac{9}{100} \times 5 = \frac{9}{20}$

Unfilled part $= 1 - \frac{9}{20} = \frac{11}{20}$

This is to be filled by A alone and hence will be filled in $20 \times \frac{11}{20} = 11$ minutes .

18. Ans. (b)

Solution Pipe $(A + B)$'s $1\frac{1}{2}$ hours job $= \frac{3}{2}\left(\frac{1}{6} + \frac{1}{8}\right) = \frac{7}{16}$

Part unfilled $= 1 - \frac{7}{16} = \frac{9}{16}$

B can fill $\frac{1}{8}$ of the tank in = 1 hour

B can fill $\frac{9}{16}$th of the tank in $= 8 \times \frac{9}{16} = \frac{9}{2}$ hours

Total time taken to fill the tank $= \left(\frac{3}{2} + \frac{9}{2}\right)$ hours

$= 6$ hours.

19. Ans. (b)

Solution Work done by waste pipe in 1 minute

$$= \left(\frac{1}{12} + \frac{1}{15}\right) - \frac{1}{20}$$

$$= \left(\frac{3}{20} - \frac{1}{20}\right) = \frac{1}{10}$$

$\therefore$ Waste pipe can empty the cistern in 10 minutes

20. Ans. (b)

Solution Work done by waste pipe in 1 minute

$$= \left(\frac{1}{10} + \frac{1}{15}\right) - \frac{1}{18}$$

$$= \left(\frac{1}{6} - \frac{1}{18}\right) = \frac{1}{9}$$

$\therefore$ Waste pipe can empty the cistern in 9 minutes

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. A water tank is $\frac{2}{5}$th full. Pipe A can fill the tank in 10 minutes and pipe B can empty it in 6 minutes. If both the pipes are open, how long will it take to empty or fill the tank completely?

(a) 6 minutes to empty (b) 6 minutes to fill
(c) 9 minutes to empty (d) 9 minutes to fill
(e) None of these

[Based on BSRB Mumbai PO 1998]

2. Tap 'A' can fill a water tank in 25 minutes, tap 'B' can fill the same tank in 40 minutes and tap 'C' can empty that tank in 30 minutes. If all the three taps are opened together, in how many minutes will the tank be completely filled up or emptied?

(a) $3\frac{2}{13}$ (b) $15\frac{5}{13}$
(c) $8\frac{2}{13}$ (d) $31\frac{11}{19}$
(e) None of these **[Based on BSRB Patna PO 2001]**

3. Taps A, B and C are connected to a water tank and the rate of flow of water is 42 litres/hr, 56 litres/hr and 48 litres/hr, respectively. Taps A and B fill the tank while tap C empties the tank. If all the three taps are opened simultaneously, the tank gets completely filled up in 16 hours. What is the capacity of the tank?

(a) 960 litres (b) 2346 litres
(c) 1600 litres (d) 800 litres

[Based on SBI PO 2001]

4. 20 buckets of water fill a tank when the capacity of each bucket is 13.5 litres. How many buckets will be required to fill the same tank if the capacity of each bucket is 9 litres?

(a) 30 (b) 32
(c) Data inadequate (d) None of these

[Based on IBPS Bank PO 2002]

5. A pipe of diameter d can drain a certain water tank in 40 minutes. The time taken by a pipe of diameter $2d$ for doing the same job is

(a) 5 minutes (b) 10 minutes
(c) 20 minutes (d) 50 minutes

[Based on SSC (GL) Prel. Exam 2000]

6. A tap can empty a tank in one hour. A second tap can empty it in 30 minutes. If both the taps operate simultaneously, how much time is needed to empty the tank?

(a) 20 minutes (b) 30 minutes
(c) 40 minutes (d) 45 minutes

[Based on SSC (GL) Prel. Exam 2000]

7. Two taps can fill a tank in 4 hours and 6 hours, respectively. A third tap can empty the tank in 3 hours. If all the three taps are opened how much time will it take to fill the tank?

(a) 3 hours (b) 9 hours
(c) 12 hours (d) 24 hours

[Based on SSC (GL) Prel. Exam 2000]

8. A cistern can be filled with water by a pipe in 5 hours and it can be emptied by a second pipe in 4 hours. If both the pipes are opened when the cistern is full, the time in which it will be emptied is
(a) 9 hours (b) 18 hours
(c) 20 hours (d) $20\frac{1}{2}$ hours

[Based on SSC (GL) Prel. Exam 2002]

9. A pump can fill a tank with water in 2 hours. Because of a leak in the tank it was taking $2\frac{1}{3}$ hours to fill the tank. The leak can drain all the water off the tank in
(a) 8 hours (b) 7 hours
(c) $4\frac{1}{3}$ hours (d) 14 hours

[Based on SSC (GL) Prel. Exam 2002]

10. A cistern is normally filled with water in 10 hours but takes 5 hours longer to fill because of a leak in its bottom. If the cistern is full, then the leak will empty the cistern in
(a) 20 hours (b) 40 hours
(c) 50 hours (d) 30 hours

[Based on SSC (GL) Prel. Exam 2002]

11. Two pipes A and B can separately fill a cistern in 60 minutes and 75 minute respectively. There is a third pipe at the bottom of the cistern to empty it. If all the three pipes are simultaneously opened, then the cistern is full in 50 minutes. In how much time can the third pipe alone empty the cistern?
(a) 110 minutes (b) 100 minutes
(c) 120 minutes (d) 90 minutes

[Based on SSC (GL) Prel. Exam 2003]

12. A tap can fill a tank in 6 hours. After half the tank is filled, three more similar taps are opened. What is the total time taken to fill the tank completely?
(a) 4 hours (b) 4 hours 15 minutes
(c) 3 hours 15 minutes (d) 3 hours 45 minutes

[Based on SSC (GL) Prel. Exam 2003]

EXPLANATORY ANSWERS

1. Ans. (a)

Solution Pipe A in 1 minute fills $\frac{1}{10}$ part and pipe B in 1 minute empties $\frac{1}{6}$ part

$\therefore$ Pipe $A + B$ in 1 minute fill $\frac{1}{10} - \frac{1}{6} = \frac{-1}{15}$ part

$\because$ $\frac{1}{15}$ part gets emptied in 1 minute

$\therefore$ $\frac{2}{5}$ part is emptied in $15 \times \frac{2}{5}$ minute = 6 minute

2. Ans. (d)

3. Ans. (d)

Solution Capacity of the tank $= (42 + 56 - 48) \times 16$
$= 800$ litres

4. Ans. (a)

Solution Capacity of the tank $= 20 \times 13.5$
$= 270$ litres

When the capacity of each bucket = 9 litres then the required no. of buckets

$= \frac{270}{9} = 30.$

5. Ans. (b)

Solution Ratio of volumes $= \frac{\pi d^2 h}{\pi\left(\frac{d}{2}\right)^2 h} = \frac{4}{1}$

$\therefore$ A pipe of radius $\frac{d}{2}$ can drain a water tank in 40 minutes.

$\Rightarrow$ A pipe of radius d can drain the same water tank in 10 minutes.

6. Ans. (a)

Solution In one minute $\frac{1}{60} + \frac{1}{30} = \frac{1}{20}$ of the tank will be empty.

7. Ans. (c)

Solution In one hour, $\frac{1}{4} + \frac{1}{6} - \frac{1}{3} = \frac{3+2-4}{12}$

$= \frac{1}{12}$ of the tank will be filled.

8. Ans. (c)

Solution Required time $= \frac{5 \times 4}{5-4} = 20$ hours.

9. Ans. (d)

Solution Required time = $\dfrac{2\times\frac{7}{3}}{\frac{7}{3}-2}$ = 14 hours.

10. Ans. (d)

Solution In one hour, $\frac{1}{10}-\frac{1}{15}=\frac{1}{30}$ of the cistern will be empty.

11. Ans. (b)

Solution Let the third pipe empty the cistern in K minutes,

i.e. $\frac{1}{60}+\frac{1}{75}-\frac{1}{K}=\frac{1}{50}$

or, $\frac{1}{60}+\frac{1}{75}-\frac{1}{50}=\frac{1}{K}$ or, K = 100 minutes.

12. Ans. (d)

Solution Half of the tank is filled in $\frac{1}{2}\times 6 = 3$ hours.

Now, we have four taps and each tap can fill the tank in 6 hours.

When all the four taps are opened, then they can fill $\frac{1}{2}$ of the tank in $\frac{6}{4}\times\frac{1}{2}=\frac{3}{2}$ hours = 45 minutes

∴ Total time = 3 hours 45 minutes.

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. Two pipes A and B can separately fill a cistern in 15 and 10 minutes, respectively and a waste pipe C can carry off 7 litres per minute. If all the pipes are opened when the cistern is full, it is emptied in 2 hours. How many litres does the cistern hold?
(a) 30 litres (b) 40 litres
(c) 35 litres (d) None of these

2. If two pipes function simultaneously, the reservoir is filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does the faster pipe take to fill the reservoir?
(a) 18 hours (b) 19 hours
(c) 20 hours (d) 17 hours

3. Three pipes P, Q and R together can fill a tank in 12 hours. After working at it together for 3 hours, P is closed and Q and R fill it in 15 hours. The time (in hours) taken by P alone to fill the tank is
(a) 30 (b) 32
(c) 25 (d) 28

4. Two pipes A and B can fill a tank in 36 minutes and 45 minutes, respectively. A waste pipe C can empty the tank in 30 minutes. First A and B are opened. After 7 minutes, C is also opened. In how much time, the tank is full?
(a) 60 minutes (b) 46 minutes
(c) 42 minutes (d) 48 minutes

5. If three taps are opened together, a tank is filled in 12 hours. One of the taps can fill it in 10 hours and another in 15 hours How does the third tap work?
(a) 12 hours (b) 14 hours
(c) 18 hours (d) 16 hours

6. One pipe can fill a tank three times as fast as another pipe. If together the two pipes can fill the tank in 36 minutes, the slower pipe alone will be able to fill the tank in
(a) 81 minutes (b) 108 minutes
(c) 144 minutes (d) 192 minutes

7. Two pipes A and B can separately fill a cistern in $7\frac{1}{2}$ and 5 minutes, respectively and a waste pipe C can carry off 14 litres per minute. If all the pipes are opened when the cistern is full, it is emptied in 1 hour. The capacity of the cistern is
(a) 40 litres (b) 35 litres
(c) 25 litres (d) 32 litres

8. A tap can fill a tank in 16 minutes and another can empty it in 8 minutes. If the tank is already half full, and both the taps are opened together, the tank will be
(a) filled in 12 minutes (b) emptied in 12 minutes
(c) filled in 8 minutes (d) emptied in 8 minutes

9. A cistern can be filled by two pipes in 20 and 30 minutes, respectively. Both pipes being opened, when must the first pipe be turned off so that the cistern may be filled in 10 minutes more?
(a) after 10 minutes (b) after 12 minutes
(c) after 8 minutes (d) after 6 minutes

10. There is a leak in the bottom of a cistern. When the cistern is in thorough repair, it would be filled in $3\frac{1}{2}$ hours. It now takes half an hour longer. If the cistern in full, time (in hours) taken by the leak to empty it is
(a) 28 (b) 32
(c) 24 (d) 30

Test II **[Time: 10 min]**

11. A cistern can be filled by one pipe in 6 hours and by another in 5 hours. There is a waste pipe also and if this is open when the second pipe is working the cistern takes $7\frac{1}{2}$ hours to fill. If all the three pipes are open, time (in hours) taken by the cistern to be full is:
(a) $4\frac{2}{3}$ hours (b) 2 hours
(c) $3\frac{1}{3}$ hours (d) $5\frac{1}{2}$ hours

12. Taps A and B can fill a bucket in 12 minutes, respectively. If both taps are opened and tap A is closed after 3 minutes, how much further time would it take for tap B to fill the bucket?
(a) 8 minutes 5 seconds
(b) 8 minutes 15 seconds
(c) 7 minutes 45 seconds
(d) 7 minutes 15 seconds

13. Two pipes P and Q would fill a cistern in 24 and 32 minutes, respectively. Both pipes being opened, find when the first pipe must be turned off so that the cistern may be just filled in 16 mimutes.
(a) after 12 minutes (b) after 10 minutes
(c) after 14 minutes (d) None of these

14. A pipe can fill a tank in 12 minutes and another pipe in 15 minutes, but a third pipe can empty it in 6 minutes. The first two pipes are kept open for 5 minutes in the beginning and then the third pipe is also opened. In what time is the cistern emptied?
(a) 40 minutes (b) 45 minutes
(c) 55 minutes (d) 50 minutes

15. A cistern is filled by three taps A, B and C. A can fill it in 12 minutes, B in 15 minutes and C can empty in 1 hour. If A, B and C be opened successively for a minute each, the cistern will be filled in
(a) $22\frac{3}{5}$ minutes (b) 21 minutes
(c) $21\frac{4}{5}$ minutes (d) 20 minutes

16. Two fill pipes A and B can fill a cistern in 12 and 16 minutes, respectively. Both fill pipes are opened together, but 4 minutes before the cistern is full, one pipe A is closed. How much time will the cistern take to fill?
(a) $8\frac{1}{7}$ minutes (b) 8 minutes
(c) $9\frac{1}{7}$ minutes (d) 9 minutes

17. Three pipes A, B and C can fill a cistern in 6 hours. After working together for 2 hours, C is closed and A and B fill the cistern in 8 hours. Then, find the time in which the cistern can be filled by pipe C.
(a) 12 hours (b) 16 hours
(c) 18 hours (d) 10 hours

18. Two pipes A and B fill a tank in 15 hours and 20 hours, respectively while a third pipe C can empty the full tank in 25 hours. All the three pipes are opened in the beginning. After 10 hours, C is closed. The time to fill the tank is
(a) 12 hours (b) $13\frac{1}{2}$ hours
(c) 16 hours (d) 18 hours

19. A tap can fill a bath in 20 minutes and another tap can fill it in 30 minutes. Amit opens both the taps simultaneously. When the bath should have been full, he finds that the waste pipe was open. He then closes the waste pipe and in another 4 minutes the bath is full. In what time would the waste pipe empty it?
(a) 35 minutes (b) 38 minutes
(c) 36 minutes (d) 39 minutes

20. A cistern has a leak which would empty it in 8 hours. A tap is turned on which admits 6 litres a minute into the cistern, and it is now emptied in 12 hours. How many litres does the cistern hold?
(a) 7580 litres (b) 7960 litres
(c) 8290 litres (d) 8640 litres

Answers

1. (b)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (a)	**6.** (c)	**7.** (a)	**8.** (d)	**9.** (c)	**10.** (a)
11. (c)	**12.** (b)	**13.** (a)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (a)	**18.** (a)	**19.** (c)	**20.** (d)

15

TIME AND DISTANCE

INTRODUCTION

The terms '*Time*' and '*Distance*' are related to the speed of a moving object.

Speed: We define the *speed* of an object as the distance covered by it in a unit time interval. It is obtained by dividing the distance covered by the object, by the time it takes to cover that distance.

Thus, $$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}.$$

Notes:

1. If the time taken is constant, the distance travelled is proportional to the speed, that is, more the speed; more the distance travelled in the same time.
2. If the speed is constant, the distance travelled is proportional to the time taken, that is, more the distance travelled; more the time taken at the same speed.
3. If the distance travelled is constant, the speed is inversely proportional to the time taken, that is, more the speed; less the time taken for the same distance travelled.

SOME BASIC FORMULAE

1. $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$
2. $\text{Distance} = \text{Speed} \times \text{Time}$
3. $\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$

Units of Measurement

Generally, if the distance is measured in kilometre, we measure time in hours and speed in kilometre per hour and is written as km/hr and if the distance is measured in metre then time is taken in second and speed in metre per second and is written as m/sec.

Conversion of Units

$$\text{One kilometre/hour} = \frac{1000 \text{ metre}}{60 \times 60 \text{ seconds}} = \frac{5}{18} \text{ m/sec.}$$

$$\therefore \quad \text{One metre/second} = \frac{18}{5} \text{ km/hr.}$$

Thus, $$x \text{ km/hr} = \left(x \times \frac{5}{18}\right) \text{ m/sec.}$$

and, $$x \text{ m/sec.} = \left(x \times \frac{18}{5}\right) \text{ km/hr.}$$

Illustration 1 Calculate the speed of a train which covers a distance of 150 km in 3 hours.

Solution $$\text{Speed} = \frac{\text{Distance covered}}{\text{Time taken}} = \frac{150}{3} = 50 \text{ km/hr.}$$

Illustration 2 How long does a train 100 metres long running at the rate of 40 km/hr take to cross a telegraphic pole?

Solution In crossing the pole, the train must travel its own length.

$\therefore$ Distance travelled is 100 metres.

$$\text{Speed} = 40 \text{ km/hr.} = \frac{40 \times 1000}{60 \times 60} = \frac{100}{9} \text{m/sec.}$$

$$\therefore \quad \text{Time taken to cross the pole} = \frac{100}{100/9} = 9 \text{ seconds.}$$

Illustration 3 A train running at a speed of 90 km/hr passes a pole on the platform in 20 seconds. Find the length of the train in metres.

Solution $$\text{Speed of the train} = 90 \text{ km/hr} = 90 \times \frac{5}{18} = 15 \text{ m/sec.}$$

$\therefore$ Length of the train = Speed of the train × time taken in crossing the pole

$= 25 \times 20 = 500$ m.

SOME USEFUL SHORT-CUT METHODS

1. (a) If A covers a distance d_1 km at s_1 km/hr and then d_2 km at s_2 km/hr, then the average speed during the whole journey is given by

$$\text{Average speed} = \frac{s_1 s_2 (d_1 + d_2)}{s_1 d_2 + s_2 d_1} \text{km/hr.}$$

(b) If A goes from X to Y at s_1 km/hr. and comes back from Y to X at s_2 km/hr., then the average speed during the whole journey is given by

$$\text{Average speed} = \frac{2 s_1 s_2}{s_1 + s_2}$$

Explanation

(a) Time taken to travel d_1 km at s_1 km/hr is

$$t_1 = \frac{d_1}{s_1} \text{hr.}$$

Time taken to travel d_2 km at s_2 km/hr is

$$t_2 = \frac{d_2}{s_2} \text{hr.}$$

$$\text{Total time taken} = t_1 + t_2 = \left(\frac{d_1}{s_1} + \frac{d_2}{s_2}\right) \text{hr.}$$

$$= \left(\frac{s_1 d_2 + s_2 d_1}{s_1 s_2}\right) \text{hr.}$$

Total distance covered = $(d_1 + d_2)$ km. Therefore,

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{s_1 s_2 (d_1 + d_2)}{(s_1 d_2 + s_2 d_1)} \text{km/hr.} \quad \text{...(i)}$$

(b) Let the distance from X to Y be d km.
Take $d_1 = d_2 = d$ in (i), we get

$$\text{Average speed} = \frac{2 d s_1 s_2}{d(s_1 + s_2)} = \frac{2 s_1 s_2}{s_1 + s_2}.$$

Illustration 4 A ship sails to a certain city at the speed of 15 knots/hr and sails back to the same point at the rate of 30 knots/hr. What is the average speed for the whole journey?

Solution Here, $s_1 = 15$ and $s_2 = 30$.

$$\therefore \quad \text{Average speed} = \frac{2 s_1 s_2}{s_1 + s_2} = \frac{2 \times 15 \times 30}{15 + 30}$$

$$= 20 \text{ knots/hr.}$$

2. A person goes certain distance (A to B) at a speed of s_1 km/hr. and returns back (B to A) at a speed of s_2 km/hr. If he takes T hours in all, the distance between A and B is

$$T\left(\frac{s_1 s_2}{s_1 + s_2}\right).$$

Explanation

Let the distance between A and B be d km.

$$\text{Time taken during onward journey} = t_1 = \frac{d}{s_1} \text{hrs.}$$

$$\text{Time taken during return journey} = t_2 = \frac{d}{s_2} \text{hrs.}$$

$\therefore$ Total time taken during the entire journey is

$$T = t_1 + t_2 = \frac{d}{s_1} + \frac{d}{s_2} = \frac{d(s_1 + s_2)}{s_1 s_2}$$

$$\therefore \quad d = T\left(\frac{s_1 s_2}{s_1 + s_2}\right).$$

Thus, the distance between A and B is

$$= T\left(\frac{s_1 s_2}{s_1 + s_2}\right)$$

$$= \text{Total time taken} \times \frac{\text{Product of two speeds}}{\text{Sum of two speeds}}.$$

Illustration 5 A boy goes to school with the speed of 3 km an hour and returns with a speed of 2 km/hr. If he takes 5 hours in all, find the distance in km between the village and the school.

Solution Here, $s_1 = 3$, $s_2 = 2$ and $T = 5$.

$\therefore$ The distance between the village and the school

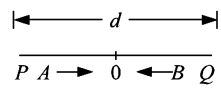
$$= T\left(\frac{s_1 s_2}{s_1 + s_2}\right) = 5\left(\frac{3 \times 2}{3 + 2}\right) = 6 \text{ km.}$$

3. If two persons A and B start at the same time from two points P and Q towards each other and after crossing they take T_1 and T_2 hours in reaching Q and P respectively, then

$$\frac{A'\text{s speed}}{B'\text{s speed}} = \frac{\sqrt{T_2}}{\sqrt{T_1}}.$$

Explanation

Let the total distance between P and Q be d km. Let the speed of A be s_1 km/hr and that of B be s_2 km/hr.



Since they are moving in opposite directions, their relative speed is $(s_1 + s_2)$ km/hr.

They will meet after $\left(\frac{d}{s_1+s_2}\right)$hr.

Distance travelled by A in $\left(\frac{d}{s_1+s_2}\right)$ hr.

$$= PO = \left(\frac{ds_1}{s_1+s_2}\right) \text{km.}$$

Distance travelled by B in $\left(\frac{d}{s_1+s_2}\right)$ hr.

$$= QO = \left(\frac{ds_2}{s_1+s_2}\right) \text{km.}$$

Time taken by A to travel QO

$$= \frac{\left(\frac{ds_2}{s_1+s_2}\right)}{s_1}$$

$$= T_1 \text{ (given).} \quad \ldots(1)$$

Time taken by B to travel PO

$$= \frac{\left(\frac{ds_1}{s_1+s_2}\right)}{s_2}$$

$$= T_2 \text{ (given)} \quad \ldots(2)$$

Dividing equation (2) by equation (1), we get

$$\frac{s_1/s_2}{s_2/s_1} = \frac{T_2}{T_1}$$

or, $$\left(\frac{s_1}{s_2}\right)^2 = \frac{T_2}{T_1} \text{ or, } \frac{s_1}{s_2} = \sqrt{\frac{T_2}{T_1}}.$$

$$\therefore \quad \frac{A'\text{s speed}}{B'\text{s speed}} = \sqrt{\frac{T_2}{T_1}}.$$

Illustration 6 Nikita starts her journey from Delhi to Bhopal and simultaneously Nishita starts from Bhopal to Delhi. After crossing each other they finish their remaining journey in $5\frac{4}{9}$ hours and 9 hours, respectively. What is Nishita's speed if Nikita's speed is 36 km/hr?

Solution $$\frac{\text{Nikita's speed}}{\text{Nishita's speed}} = \sqrt{\frac{T_2}{T_1}} = \frac{\sqrt{9}}{\sqrt{5\frac{4}{9}}} = \frac{\sqrt{9}}{\sqrt{\frac{49}{9}}}$$

$$= \sqrt{\frac{81}{49}} = \frac{9}{7}.$$

$\therefore$ Nishita's speed $= \frac{7}{9}$ Nikita's speed

$$= \frac{7}{9} \times 36 = 28 \text{ km/hr.}$$

4. If a body travels $d_1, d_2, d_3, \ldots, d_n$ metres with different speeds $s_1, s_2, s_3, \ldots, s_n$ m/sec. in time $T_1, T_2, T_3, \ldots, T_n$ seconds respectively, then the average speed of the body throughout the journey is given by

$$V_a = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{d_1+d_2+d_3+\ldots+d_n}{T_1+T_2+T_3+\ldots+T_n}$$

[If $d_1, d_2, \ldots, d_n$ and $T_1, T_2, \ldots, T_n$ are known]

and, $$V_a = \frac{s_1T_1+s_2T_2+s_3T_3+\ldots+s_nT_n}{T_1+T_2+T_3+\ldots T_n}$$

[If $d_1, d_2, \ldots, d_n$ and $s_1, s_2, \ldots, s_n$ are known]

Illustration 7 A car during its journey travels 40 minutes at a speed of 30 km/hr, another 50 minutes at a speed of 60 km/hr and 1 hour at a speed of 30 km/hr. Find the average speed of the car.

Solution Here, $T_1 = \frac{40}{60}$, $T_2 = \frac{50}{60}$, $T_3 = 1$, $s_1 = 30$, $s_2 = 60$, $s_3 = 30$.

$\therefore$ Average speed of the car

$$= \frac{s_1T_1+s_2T_2+s_3T_3}{T_1+T_2+T_3} = \frac{30\times\frac{40}{60}+60\times\frac{50}{60}+30\times 1}{\frac{40}{60}+\frac{50}{60}+1}$$

$$= 40 \text{ km/hr.}$$

5. If the new speed is $\frac{a}{b}$ of the original speed, then the change in time taken to cover the same distance is given by

Change in time $= \left(\frac{b}{a}-1\right) \times$ original time.

Illustration 8 By walking at $\frac{4}{5}$ of his usual speed, Mohan is 6 minutes late to his office. Find his usual time to cover the distance.

Solution Here, change in time = 6 and $\frac{a}{b} = \frac{4}{5}$.

We have, change in time $= \left(\frac{b}{a}-1\right) \times$ original time

$$\Rightarrow \text{original time} = \frac{\text{change in time}}{\left(\frac{b}{a}-1\right)}$$

$$= \frac{6}{\left(\frac{5}{4}-1\right)} = 24 \text{ minutes.}$$

6. A body covers a distance d in time T_1 with speed s_1, but when it travels with speed s_2 covers the same distance in time T_2.

   The following relations hold

$$\frac{\text{Product of speed}}{d} = \frac{s_1}{T_2} = \frac{s_2}{T_1} = \frac{\text{Difference of speed}}{\text{Difference of time}}$$

   Equating any two of the above, we can find the unknowns as per the given question.

Illustration 9 Two bicyclists do the same journey by travelling respectively, at the rates of 9 and 10 km an hour. Find the length of the journey when one takes 32 minutes longer than the other.

Solution Here, change in speed = 10 – 9 = 1; product of speed = 9 × 10 = 90 and difference of time = $\frac{32}{60}$.

We have, $\frac{\text{Product of speed}}{d} = \frac{\text{Difference of speed}}{\text{Difference of time}}$

$$\Rightarrow \quad d = \text{Product of speed} \times \left(\frac{\text{Difference of time}}{\text{Difference of speed}}\right)$$

$$= 90 \times \frac{32}{60} = 48 \text{ km.}$$

7. A train travels a certain distance at a speed of s_1 km/hr without stoppages and with stoppages, it covers the same distance at a speed of s_2 km/hr. The stoppage time per hour is given by

$$\left(\frac{s_1 - s_2}{s_1}\right) \text{hr.} \quad \text{or,} \quad \left(\frac{\text{Difference of speed}}{\text{Speed without stoppages}}\right)$$

Explanation

Let the distance travelled be d km.

∴ Time taken by the train without stopping any where

$$= \frac{d}{s_1} \text{hr.}$$

Also, time taken by the train with stoppages

$$= \frac{d}{s_2} \text{hr.}$$

$$\text{Total stoppage time} = \frac{d}{s_2} - \frac{d}{s_1} = \left(\frac{s_1 - s_2}{s_1 s_2}\right) d \text{ hr.}$$

$$\therefore \quad \text{Stoppage time per hour} = \frac{\left(\frac{s_1 - s_2}{s_1 s_2}\right) d}{\frac{d}{s_2}}$$

$$= \left(\frac{s_1 - s_2}{s_1}\right) \text{hr.}$$

Illustration 10 Without stoppages, a train travels certain distance with an average speed of 80 km/hr and with stoppages, it covers the same distance with an average speed of 60 km/hr. How many minutes per hour the train stops?

Solution Here, $s_1 = 80$ and $s_2 = 60$

∴ Stoppage time/hr.

$$= \frac{s_1 - s_2}{s_1} = \frac{80 - 60}{80} = \frac{1}{4} \text{hour} = 15 \text{ minutes.}$$

8. (a) If a train overtakes a pole or a man or a milestone, then the distance covered in overtaking is equal to the length of the train.

   (b) If a train overtakes a bridge or a tunnel or a platform or another train, then the distance covered is equal to the sum of the two lengths.

Illustration 11 A train 600 m long crosses a pole in 9 seconds. What is the speed of the train in km/hr?

Solution Speed of the train

$$= \frac{\text{Length of the train}}{\text{time taken in crossing the pole}}$$

$$= \frac{600}{9} \text{m/s} = \frac{600}{9} \times \frac{18}{5} = 240 \text{ km/hr.}$$

Illustration 12 A train 130 m long passes a bridge in 21 seconds moving with a speed of 90 km/hr. Find the length of the bridge.

Solution We have, speed of the train

$$= \frac{\text{length of the train} + \text{length of the bridge}}{\text{time taken in crossing the bridge}}$$

$$\Rightarrow \quad \frac{5}{18} \times 90 = \frac{130 + \text{length of the bridge}}{21}$$

∴ Length of the bridge = 525 – 130 = 395 m.

9. **Relative Speed**

   (a) If two trains of lengths L_1 km and L_2 km, respectively are travelling in the same direction at s_1 km/hr and s_2 km/hr respectively such that $s_1 > s_2$, then $s_1 - s_2$ is called their relative speed and the time taken by the faster train to cross the slower train is given by

$$\left(\frac{L_1 + L_2}{s_1 - s_2}\right) \text{hr.}$$

   (b) If two trains of length L_1 km and L_2 km, respectively are travelling in the opposite directions at s_1 km/hr and s_2 km/hr respectively, then $s_1 + s_2$ is called their relative speed and the time taken by the trains to cross each other is given by

$$\left(\frac{L_1 + L_2}{s_1 + s_2}\right) \text{hr.}$$

Illustrations 13 A train 135 metres long is running with a speed of 49 km/hr. In what time will it pass a man who is walking at 5 km/hr in the direction opposite to that of the train?

Solution Here, $L_1 = 135$, $L_2 = 0$, $s_1 = 49$ km/hr, $s_2 = 5$ km/hr.

$$\therefore \quad s_1 + s_2 = 49 + 5 = 54 \text{ km/hr.} = 54 \times \frac{5}{18} \text{m/sec.}$$

$$\therefore \quad \text{The time taken} = \frac{L_1 + L_2}{s_1 + s_2} = \frac{135}{54 \times \frac{5}{18}}$$

$$= \frac{135 \times 18}{54 \times 5} = 9 \text{ sec.}$$

Illustration 14 Two trains of length 110 metres and 90 metres are running on parallel lines in the same direction with a speed of 35 km/hr and 40 km/hr, respectively. In what time will they pass each other.

Solution Here, $L_1 = 110$ m, $L_2 = 90$ m, $s_1 = 35$ km/hr and $s_2 = 40$ km/hr

$$\therefore \quad s_2 - s_1 = 40 - 35 = 5 \text{ km/hr} = 5 \times \frac{5}{18} \text{m/s}$$

$$\therefore \quad \text{Time taken} = \frac{L_1 + L_2}{s_2 - s_1}$$

$$= \frac{110 + 90}{5 \times \frac{5}{18}} = \frac{200 \times 18}{5 \times 5} = 144 \text{ sec.}$$

10. Two trains of lengths L_1 m and L_2 m run on parallel tracks. When running in the same direction, the faster train passes the slower one in T_1 seconds, but when they are running in opposite directions with the same speeds as earlier, they pass each other in T_2 seconds.

    Then, the speed of the faster train

    $$= \frac{L_1 + L_2}{2}\left(\frac{1}{T_1} + \frac{1}{T_2}\right) \text{ m/s}$$

    and, the speed of the slower train

    $$= \frac{L_1 + L_2}{2}\left(\frac{1}{T_1} - \frac{1}{T_2}\right) \text{ m/s.}$$

Explanation

Let the speed of the faster train be s_1 m/s and that of the slower train be s_2 m/s.

Total distance covered when the two trains cross each other $= L_1 + L_2$.

When the two trains are running is the same direction, their relative speed $= (s_1 - s_2)$ m/s.

$$\therefore \quad (s_1 - s_2) = \frac{L_1 + L_2}{T_1} \quad \text{...(1)}$$

When the two trains are running in the opposite directions, their relative speed $= (s_1 + s_2)$ m/s.

$$\therefore \quad s_1 + s_2 = \frac{L_1 + L_2}{T_2} \quad \text{...(2)}$$

Adding equations (1) and (2), we get

$$2s_1 = \frac{L_1 + L_2}{T_1} + \frac{L_1 + L_2}{T_2} = (L_1 + L_2)\left(\frac{1}{T_1} + \frac{1}{T_2}\right)$$

$$\text{or,} \quad s_1 = \left(\frac{L_1 + L_2}{2}\right)\left(\frac{T_1 + T_2}{T_1 T_2}\right).$$

On subtracting equation (1) from (2), we get

$$2s_2 = (L_1 + L_2)\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$\text{or,} \quad s_2 = \left(\frac{L_1 + L_2}{2}\right)\left(\frac{T_1 - T_2}{T_1 T_2}\right) \text{m/s.}$$

Therefore,

speed of the faster train

$$= \left(\frac{L_1 + L_2}{2}\right)\left(\frac{T_1 + T_2}{T_1 T_2}\right) \text{m/sec.}$$

and,

speed of the slower train

$$= \left(\frac{L_1 + L_2}{2}\right)\left(\frac{T_1 - T_2}{T_1 T_2}\right) \text{m/sec.}$$

Note: If the two trains are of equal length that is, $L_1 = L_2 = L$ (say), then

$$s_1 = L\left(\frac{T_1 + T_2}{T_1 T_2}\right) \text{m/s and, } s_2 = L\left(\frac{T_1 - T_2}{T_1 T_2}\right) \text{m/s.}$$

Illustration 15 Two trains of lengths 200 metres and 175 metres run on parallel tracks. When running in the same direction the faster train crosses the slower one in $37\frac{1}{2}$ seconds. When running in opposite directions at speeds same as their earlier speeds, they pass each other completely in $7\frac{1}{2}$ seconds. Find the speed of each train.

Solution We have, $L_1 = 200$, $L_2 = 175$, $T_1 = \frac{75}{2}$ and $T_2 = \frac{15}{2}$.

Therefore, speed of the faster train

$$= \left(\frac{L_1 + L_2}{2}\right)\left(\frac{T_1 + T_2}{T_1 T_2}\right) = \left(\frac{200 + 175}{2}\right)\left(\frac{\frac{75}{2} + \frac{15}{2}}{\frac{75}{2} \times \frac{15}{2}}\right)$$

$$= \frac{375}{2} \times \frac{45 \times 4}{75 \times 15} = 30 \text{ m/s.}$$

Speed of slower train

$$= \left(\frac{L_1+L_2}{2}\right)\left(\frac{T_1-T_2}{T_1T_2}\right) = \left(\frac{200+175}{2}\right)\left(\frac{\frac{75}{2}-\frac{15}{2}}{\frac{75}{2}\times\frac{15}{2}}\right)$$

$$= \frac{375}{2}\times\frac{30\times4}{75\times15} = 20 \text{ m/s.}$$

11. (a) A train starts from a place at s_1 km/hr and another fast train starts from the same place after T hours at s_2 km/hr in the same direction. Then, the distance from the starting place at which both the trains will meet is given by

$$\left(\frac{s_1\times s_2\times T}{s_2-s_1}\right) \text{ km.}$$

Also, the time after which the two trains will meet is given by

$$\left(\frac{s_1T}{s_2-s_1}\right) \text{ hr.}$$

(b) The distance between two stations A and B is d km. A train starts from A to B at s_1 km/hr. T hours later another train starts from B to A at s_2 km/hr. Then, the distance from A, at which both the trains will meet is given by

$$s_1\left(\frac{d+s_2T}{s_1+s_2}\right) \text{ km.}$$

Also, the time after which the two trains will meet is given by

$$\left(\frac{d+s_2T}{s_1+s_2}\right) \text{ hr.}$$

Illustration 16 A train starts from Mumbai at 10 A.M. with a speed of 25 km/hr and another train starts from there on the same day at 3 P.M. in the same direction with a speed of 35 km/hr. Find at what distance from Mumbai both the trains will meet and find also the time of their meeting.

Solution Time from 10 A.M. to 3 P.M. = 5 hr.

Distance of meeting point from Mumbai

$$= \left(\frac{s_1\times s_2\times T}{s_2-s_1}\right) \text{ km.}$$

$$= \left(\frac{25\times35\times5}{35-25}\right) \text{ km} = 437\frac{1}{2}\text{km.}$$

Also, time of their meeting

$$= \left(\frac{s_1T}{s_2-s_1}\right) \text{ hr.} = \left(\frac{25\times5}{35-25}\right) \text{ hr.}$$

$$= \frac{125}{10} = 12\frac{1}{2}\text{hr. after 3 P.M.}$$

That is, 3.30 A.M. next day

Illustration 17 Chennai is at a distance of 560 km from Mumbai. A train starts from Mumbai to Chennai at 6 A.M. with a speed of 40 km/hr. Another train starts from Chennai to Mumbai at 7 A.M. with a speed of 60 km/hr. At what distance from Mumbai and at what time will the two trains be at the point of crossing?

Solution Time from 6 A.M. to 7 A.M. = 1 hr.

Therefore, distance of meeting point from Mumbai

$$= s_1\left(\frac{d+s_2T}{s_1+s_2}\right) \text{ km.}$$

$$= 40\left(\frac{560+60\times1}{40+60}\right) = 248 \text{ km.}$$

Also, time of their meeting

$$= \left(\frac{d+s_2T}{s_1+s_2}\right) \text{ hr}$$

$$= \left(\frac{560+60\times1}{40+60}\right) = \frac{31}{5} \text{ hr}$$

= 6 hr 12 min. after 6 A.M.

That is, at 12.12 noon.

12. Two trains start simultaneously from the stations A and B towards each other with speeds s_1 km/hr and s_2 km/hr, respectively. When they meet it is found that the second train had travelled d km more than the first. Then the distance between the two stations is given by

$$d\left(\frac{s_1+s_2}{s_2-s_1}\right) \text{ km.}$$

Explanation

Let the distance between the two stations be x km. If the first train travels y km then the second travels $y + d$ km.

$\therefore \quad x = y + y + d = 2y + d.$

Since the time taken by both the trains is same

$$\therefore \quad \frac{y+d}{s_2} = \frac{y}{s_1}$$

$$\Rightarrow \quad s_1y + s_1d = s_2y$$

$$\Rightarrow \quad (s_2 - s_1)y = s_1d \text{ or, } y = \frac{s_1d}{s_2-s_1}.$$

$$\therefore \quad x = 2\left(\frac{s_1d}{s_2-s_1}\right)+d = \frac{d(s_1+s_2)}{(s_2-s_1)}\text{km.}$$

Illustration 18 Two trains start at the same time from Delhi and Rohtak and proceed towards each other at the rate of 75 km and 65 km per hour, respectively. When they meet, it is found that one train has travelled 10 km more than the other. Find the distance between Delhi and Rohtak.

Solution Distance between Delhi and Rohtak

$$= d\left(\frac{s_1+s_2}{s_1-s_2}\right)\text{km}.$$

$$= 10\left(\frac{75+65}{75-65}\right)\text{km}$$

$$= 140 \text{ km}.$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Ramesh crosses a street 600 m long in 5 minutes. His speed in km/hr is
(a) 8.2 (b) 7.2
(c) 9.2 (d) None of these

2. Compare the speed of two trains, one moving at the speed of 80 km/hr and the other at 10 m/sec.
(a) 30 : 9 (b) 40 : 9
(c) 20 : 9 (d) None of these

3. Mohan covers 10.2 km in 3 hours, the distance covered by him in 5 hours is
(a) 15 km (b) 17 km
(c) 19 km (d) None of these

4. A train 100 metres long passes a bridge at the rate of 72 km/hr in 25 seconds. What is the length of the bridge?
(a) 400 m (b) 17 m
(c) 600 m (d) None of these

5. A train passes a railway bridge 150 m long in 18 seconds. If the train is running at a speed of 60 km/hr., then the length of the train in metres is
(a) 160 m (b) 150 m
(c) 180 m (d) None of these

6. Sound travels 330 metres a second. If the sound of a thunder-cloud follows the flash after 10 seconds, the thunder-cloud is at a distance of
(a) 3.7 km (b) 3.5 km
(c) 3.3 km (d) None of these

7. A train travels 92.4 km/hr. How many metres will it travel in 10 minutes?
(a) 14500 m (b) 15400 m
(c) 15200 m (d) None of these

8. The distance of the sun from the earth is one hundred and forty-three million four hundred thousand kilometres and light travels from the former to the latter in 7 minutes and fifty eight seconds. The velocity of light per second is
(a) 3×10^5 km/sec (b) 0.3×10^5 km/sec.
(c) 30×10^5 km/sec. (d) None of these

9. A train covers a distance in 50 minutes if it runs at a speed of 48 km/hr. The speed at which the train must run to reduce the time of journey to 40 minutes, will be
(a) 70 km/hr (b) 80 km/hr
(c) 60 km/hr (d) None of these

10. The wheel of an engine is $3\frac{3}{4}$ metres in circumference and makes 4 revolutions in 2 seconds. The speed of the train is
(a) 27 km/hr (b) 31 km/hr
(c) 35 km/hr (d) None of these

11. A person covers half of his journey at 30 km/hr and the remaining half at 20 km/hr. The average speed for the whole journey is
(a) 25 km/hr (b) 28 km/hr
(c) 32 km/hr (d) None of these

12. Rajesh covers a certain distance by bus at 16 km/hr and returns back at the starting point on a cycle at 9 km/hr. His average speed for the whole journey is
(a) 13.54 km/hr (b) 11.52 km/hr
(c) 15.52 km/hr (d) None of these

13. A and B are two towns. A car goes from A to B at a speed of 64 km/hr and returns to A at a slower speed. If its average speed for the whole journey is 56 km/hr, it returned with speed
(a) 52.54 km/hr (b) 47.74 km/hr
(c) 49.78 km/hr (d) None of these

14. A bicycle rider covers his onward journey from A to B at 10 km/hr and during the return journey from B to A he covers the same distance at 8 km/hr. If he finishes the onward and return journey in $4\frac{1}{2}$ hours, then the total distance covered by him during the entire journey is
(a) 30 km (b) 40 km
(c) 50 km (d) None of these

15. On a tour a man travels at the rate of 64 km an hour for the first 160 km, then travels the next 160 km at the rate of 80 km an hour. The average speed in km per hour for the first 320 km of the tour is
(a) 81.13 km/hr (b) 73.11 km/hr
(c) 71.11 km/hr (d) None of these

16. A car completes a journey in 6 hours. It covers half the distance at 50 km/hr and the rest at 70 km/hr

The length of the journey is
(a) 165 km (b) 175 km
(c) 185 km (d) None of these

17. Rakesh sets out to cycle from Delhi to Mathura and at the same time Suresh starts from Mathura to Delhi. After passing each other they complete their journeys in 9 and 16 hours, respectively. At what speed does Suresh cycle if Rakesh cycles at 16 km per hour?
(a) 12 km/hr (b) 16 km/hr
(c) 14 km/hr (d) None of these

18. A train travels 225 km in 3·5 hours and 370 km in 5 hours. Find the average speed of train.
(a) 80 km/hr (b) 60 km/hr
(c) 70 km/hr (d) None of these

19. A man walks 6 km at a speed of $1\frac{1}{2}$ km/hr, runs 8 km at a speed of 2 km/hr and goes by bus another 32 km. Speed of the bus is 8 km/hr. If the speed of the bus is considered as the speed of the man, find the average speed of the man.
(a) $4\frac{5}{6}$ km/hr (b) $3\frac{5}{6}$ km/hr
(c) $5\frac{7}{6}$ km/hr (d) None of these

20. A car during its journey travels 30 minutes at a speed of 40 km/hr, another 45 minutes at a speed of 60 km/hr, and 2 hours at a speed of 70 km/hr. The average speed of the car is
(a) 63 km/hr (b) 65 km/hr
(c) 70 km/hr (d) None of these

21. By walking at $\frac{3}{4}$ of his usual speed, a man reaches office 20 minutes later than usual. His usual time is
(a) 65 minutes (b) 60 minutes
(c) 70 minutes (d) None of these

22. Two men start together to walk a certain distance, one at 4 km/hr and another at 3 km/hr. The former arrives half an hour before the latter. Find the distance.
(a) 6 km (b) 9 km
(c) 8 km (d) None of these

23. A car starts from A for B travelling 20 km an hour. $1\frac{1}{2}$ hours later another car starts from A and travelling at the rate of 30 km an hour reaches B $2\frac{1}{2}$ hours before the first car. Find the distance from A to B.
(a) 280 km (b) 260 km
(c) 240 km (d) None of these

24. Mohan walks from Tilak Nagar to Moti Nagar and back in a certain time at the rate of $3\frac{1}{2}$ km/hr. But if he had walked from Tilak Nagar to Moti Nager at the rate of 3 km/hr and back from Moti Nagar to Tilak Nagar at the rate of 4 km/hr., he would have taken 10 minutes longer. The distance between Tilak Nagar and Moti Nagar is
(a) 28 km (b) 32 km
(c) 24 km (d) None of these

25. A train does a journey without stopping in 8 hours. If it had travelled 5 km an hour faster, it would have done the journey in 6 hours 40 min. What is its slower speed?
(a) 35 km/hr (b) 25 km/hr
(c) 40 km/hr (d) None of these

26. Without any stoppage a person travels a certain distance at an average speed of 42 km/hr and with stoppages he covers the same distance at an average speed of 28 km/hr. How many minutes per hour does he stop?
(a) 25 minutes (b) 30 minutes
(c) 20 minutes (d) None of these

27. A train is running at a uniform speed of 60 km/hr. It passes a railway platform in 15 seconds. If the length of the platform is 130 m, then the length of the train is
(a) 160 m (b) 120 m
(c) 140 m (d) None of these

28. A train passes through a telegraph post in 9 seconds moving with a speed of 54 km per hour. The length of the train is
(a) 135 metres (b) 145 metres
(c) 125 metres (d) None of these

29. A train 135 m long is running with a speed of 54 km per hour. In what time will it pass a telegraph post?
(a) 11 sec. (b) 9 sec.
(c) 7 sec. (d) None of these

30. A train 160 metres long passes a standing man in 18 seconds. The speed of the train is
(a) 35 km/hr (b) 45 km/hr
(c) 32 km/hr (d) None of these

31. A train 280 m long is moving at a speed of 60 km/hr. The time taken by the train to cross a platform 220 m long is
(a) 30 sec. (b) 40 sec.
(c) 60 sec. (d) 20 sec.

32. A train 50 m long passes a platform 100 m long in 10 seconds. The speed of the train in m/sec. is
(a) 25 sec. (b) 15 sec.
(c) 35 sec. (d) None of these

33. A train 300 metres long is running at a speed of 90 km/hr. How many seconds will it take cross a 200 metres long train running in the opposite direction at a speed of 60 km/hr?
(a) 70 sec. (b) 60 sec
(c) 50 sec. (d) None of these

34. Two trains are running in opposite directions with the same speed. If the length of each train is 135 metres and they cross each other in 18 seconds, the speed of each train is
(a) 29 km/hr (b) 35 km/hr
(c) 27 km/hr (d) None of these

35. A train 150 m long is running at 95 km/hr. How much time will it take to pass a man moving in the same direction at 5 km/hr?
(a) 9 sec. (b) 6 sec.
(c) 7 sec. (d) None of these

36. A train 100 metres long takes $3\frac{3}{5}$ seconds to cross a man walking at the rate of 6 km/hr in a direction opposite to that of the train. Find the speed of the train.
(a) 76 km/hr (b) 94 km/hr
(c) 86 km/hr (d) None of these

37. Two trains are moving in the same direction at 50 km/hr and 30 km/hr. The faster train crosses a man in the slower train in 18 seconds. Find the length of the faster train.
(a) 120 m (b) 110 m
(c) 100 m (d) None of these

38. Two trains, 130 m and 110 m long, while going in the same direction, the faster train takes one minute to pass the other completely. If they are moving in opposite direction, they pass each other completely in 3 seconds. Find the speed of each train.
(a) 42 m/sec., 38 m/sec. (b) 38 m/sec, 36 m/sec.
(c) 36 m/sec., 42 m/sec. (d) None of these

39. Two trains, each of length 90 metres, run on parallel tracks. When running in the same direction, the faster train passes the slower train completely in 18 seconds, but when they are running in opposite directions approaching each other at the same speeds as before they cross each other in 9 seconds. Find the speed of each train.
(a) 9 m/sec., 15 m/sec. (b) 7 m/sec., 5 m/sec.
(c) 15 m/sec., 5 m/sec. (d) None of these

40. A train leaves the station at 5 A.M. at 60 km/hr. Another train leaves the same station at 6.30 A.M. at 75 km/hr and travels in the direction of the first train. At what time and at what distance from the station will they meet?
(a) 12.30. A.M., 450 km (b) 1.30 P.M., 375 km
(c) 11.30 A.M., 425 km (d) None of these

41. Two stations A and B are 100 km apart on a straight line. One train starts from A at 7 A.M. and travels towards B at 20 km/hr speed. Another train starts from B at 8 A.M. and travels towards A at 25 km/hr speed. At what time will they meet?
(a) 10.30 A.M. (b) 11 A.M.
(c) 10 A.M. (d) None of these

42. A train starts from station A at 9 A.M. travels at 50 km/hr towards station B, 210 km away. Another train starts from station B at 11 A.M. and travels at 60 km/hr towards station A. At what distance from A, will they meet?
(a) 150 km (b) 200 km
(c) 250 km (d) None of these

43. Two trains start at the same time from Mumbai and Pune and proceed towards each other at the rate of 60 km and 40 km per hour, respectively. When they meet, it is found that one train has travelled 20 km more than the other. Find the distance between Mumbai and Pune.
(a) 150 km (b) 100 km
(c) 120 km (d) None of these

44. A car covers four successive three km stretches at speed of 10 km/hr, 20 km/hr, 30 km/hr and 60 km/hr. respectively. Its average speed over this distance is
(a) 10 km/hr (b) 20 km/hr
(c) 30 km/hr (d) 25 km/hr

45. Two men A and B walk from P to Q at a distance of 21 km at rates 3 and 4 km an hour, respectively. B reaches Q and returns immediately and meets A at R. The distance from P to R is
(a) 14 km (b) 20 km
(c) 16 km (d) 18 km

46. A boy takes as much time in running 12 metres as a car takes in covering 36 metres. The ratio of the speeds of the boy and the car is
(a) 1 : 3 (b) 1 : 2
(c) 2 : 3 (d) 2 : 5

47. A and B are two stations. A train goes from A to B at 64 km/hr and returns to A at a slower speed. If its average speed for the whole journey is 56 km/hr, at what speed did it return?
(a) 48 km/hr (b) 49.77 km/hr
(c) 30 km/hr (d) 47.46 km/hr

48. Excluding stoppages, the speed of a bus is 54 km/hr and including stoppages, it is 45 km/hr. For how many minutes does the bus stop per hour?
(a) 9 (b) 10
(c) 12 (d) 20

49. Two boys begin to together to write out a booklet containing 817 lines. The first boy starts with the first line, writing at the rate of 200 lines an hour

and the second boy starts with the last line, then writes line 816 and so on, backwards proceeding at the rate of 150 lines an hour. They will meet on
(a) 467th line (b) 466th line
(c) 460th line (d) 472th line

50. Ramesh sees a train passing over 1 km long bridge. The length of the train is half that of bridge. If the train clears the bridge in 2 minutes, the speed of the train is
(a) 45 km/hr (b) 43 km/hr
(c) 50 km/hr (d) None of these

51. A bullock cart has to cover a distance of 80 km in 10 hours. If it covers half of the journey in (3/5) the time, What should be its speed to cover the remaining distance in the time left?
(a) 8 km/hr (b) 20 km/hr
(c) 6.4 km/hr (d) 10 km/hr

52. Amit started cycling along the boundaries of a square field from cover point A. After half an hour, he reached the corner point C, diagonally opposite to A. If his speed was 8 km/hr, what is the area of the field in square km
(a) 64 (b) 8
(c) 4 (d) Cannot be determined

53. A train 100 metres long completely passes a man walking in the same direction at 6 km p.h. in 5 seconds and a car travelling in the same direction in 6 seconds. At what speed was the car travelling?
(a) 18 km p.h. (b) 48 km p.h.
(c) 24 km p.h. (d) 30 km p.h.

54. A motor cyclist goes from Mumbai to Pune, a distance of 192 kms, at an average speed of 32 km p.h. Another man starts from Mumbai by car, $2\frac{1}{2}$ hours after the first and reaches Pune half an hour earlier. What is the ratio of the speed of the motor cycle and the car?
(a) 1 : 2 (b) 1 : 3
(c) 10 : 27 (d) 5 : 4

55. Two trains are running in opposite directions with speed of 62 km p.h. and 40 km p.h, respectively. If the length of one train is 250 metres and they cross each other in 18 seconds, the length of the other train is
(a) 145 metres (b) 230 metres
(c) 260 metres (d) Cannot be determined

56. A train speeds past a pole in 15 seconds and speeds past a platform 100 metres long in 25 seconds. Its length in metres is
(a) 200 (b) 150
(c) 50 (d) Data inadequate

57. A train of length 150 metres takes 10 seconds to pass over another train 100 metres long coming from the opposite direction. If the speed of the first train be 30 km p.h., the speed of the second train is:
(a) 54 km/hr (b) 60 km/hr
(c) 72 km/hr (d) 36 km/hr

58. A person sets to cover a distance of 12 km in 45 minutes. If he covers $\frac{3}{4}$ of the distance in $\frac{2}{3}$rd time, what should be his speed to cover the remaining distance in the remaining time?
(a) 16 km/hr (b) 8 km/hr
(c) 12 km/hr (d) 14 km/hr

59. A train 110 metres in length passes a man walking at the speed of 6 km/hr, against it in 6 seconds. The speed of the train in km per hour is
(a) 60 km/hr (b) 45 km/hr
(c) 50 km/hr (d) 55 km/hr

60. If a train 110 metres in length passes a man walking at the rate of 6 km/hr against it in 6 seconds, it will pass another man walking at the same speed in the same direction in time of
(a) $9\frac{1}{3}$sec (b) $10\frac{2}{3}$sec
(c) 8 sec (d) $7\frac{1}{3}$sec

61. A man performs $\frac{2}{15}$ of the total journey by rail, $\frac{9}{20}$ by tonga and the remaining 10 km on foot. The total journey is :
(a) 15.6 km (b) 12.8 km
(c) 16.4 km (d) 24 km

EXPLANATORY ANSWERS

1. Ans. (b)

Solution Speed $= \dfrac{\text{Distance travelled}}{\text{Time taken}}$

$= \left(\dfrac{600}{5\times 60}\right)$ m/sec

$= \left(\dfrac{600}{5\times 60}\times\dfrac{18}{5}\right)$ km/hr

$=$ 7.2 km/hr.

2. Ans. (c)

Solution 80 km/hr means $\left(80\times\dfrac{5}{18}\right)$ m/sec.

$\therefore$ Required comparison is $80 \times \frac{5}{18} : 10$ or, 20 : 9.

3. Ans. (b)

Solution Mohan's speed = $\left(\frac{10.2}{3}\right)$ km/hr = 3.4 km/hr

$\therefore$ Distance covered by him in 5 hours

= (3.4 × 5)km = 17 km.

4. Ans. (a)

Solution Distance travelled by the train in 25 seconds at 72 km/hr.

$$= 72 \times \frac{5}{18} \times 25 = 500 \text{ m.}$$

$\therefore$ Length of the Bridge = 500 – length of train

= 500 – 100 = 400 m.

5. Ans. (b)

Solution Let the length of the train be x m.

$\therefore$ Total distance covered by the train = (x + 150) m.

Speed of the train = 60 km/hr = $60 \times \frac{5}{18} = \frac{50}{3}$ m/sec.

Since, Distance = Speed × time

$\therefore$ $x + 150 = \frac{50}{3} \times 18 = 300$

or, $x = 300 - 150 = 150$ m.

$\therefore$ Length of the train = 150 m.

6. Ans. (c)

Solution Distance of thunder-cloud

= distance travelled by sound in 10 seconds

= (330 × 10) metres

= 3.3 km.

7. Ans. (b)

Solution Speed of the train = 92.4 km/hr

$$= \left(92.4 \times \frac{5}{18}\right) \text{ m/sec}$$

$$= \frac{77}{3} \text{ m/sec.}$$

Distance covered in 10 minutes or 10 × 60 (= 600) seconds

$$= \frac{77}{3} \times 600 = 15400\text{m.}$$

8. Ans. (a)

Solution Distance of sun from earth

= $(143 \times 10^6 + 400 \times 10^3)$ km

= 1434×10^5 km.

Time taken by light to travel from sun to earth

= 7 × 60 + 58 = 478 seconds.

$\therefore$ Velocity of light = $\frac{1434 \times 10^5}{478}$

= 3×10^5 km/sec.

9. Ans. (c)

Solution Distance travelled = Speed × Time = $\left(48 \times \frac{50}{60}\right)$

= 40 km.

$\therefore$ Speed = $\frac{\text{Distance}}{\text{Time}} = \left(\frac{40 \times 60}{40}\right)$ km/hr

= 60 km/hr.

10. Ans. (a)

Solution Distance covered in 2 seconds = $\frac{15}{4} \times 4$

= 15 m.

$\therefore$ Speed = $\frac{\text{Distance}}{\text{Time}} = \frac{15}{2}$ m/sec.

$= \left(\frac{15}{2} \times \frac{18}{5}\right)$ km/hr = 27 km/hr.

11. Ans. (a)

Solution Here, $s_1 = 30$ and $s_2 = 20$.

$\therefore$ Average speed = $\frac{2s_1s_2}{s_1 + s_2}$

$= \frac{2 \times 30 \times 20}{30 + 20} = 25$ km/hr.

12. Ans. (b)

Solution Here, $s_1 = 16$ and $s_2 = 9$.

$\therefore$ Average speed = $\frac{2s_1s_2}{s_1 + s_2} = \frac{2 \times 16 \times 9}{16 + 9}$

= 11.52 km/hr.

13. Ans. (c)

Solution Let the speed on the return journey be x km/hr.

Then, $56 = \frac{2s_1s_1}{s_1 + s_2} = \frac{2 \times 64 \times x}{64 + x}$

$\Rightarrow$ $7(64 + x) = 16x$ or $9x = 448$

$\therefore$ $x = \frac{448}{9} = 49.78$ km/hr.

14. Ans. (b)

Solution Here, $s_1 = 10$ and $s_2 = 8$.

$\therefore$ Average speed = $\frac{2s_1s_2}{s_1 + s_2} = \frac{2 \times 10 \times 8}{10 + 8}$

$= \frac{80}{9}$ km/hr.

Total time taken for the entire journey = $\frac{9}{2}$ hours.

$\therefore$ Total distance covered

= Average speed × total time taken

$= \frac{80}{9} \times \frac{9}{2} = 40$ km.

15. Ans. (c)

Solution Here, $s_1 = 64$ and $s_2 = 80$.

$\therefore$ Average speed $= \frac{2s_1s_2}{s_1+s_2} = \frac{2\times 64\times 80}{64+80}$

$= 71.11$ km/hr.

16. Ans. (b)

Solution Here, $s_1 = 50$, $s_2 = 70$ and $T = 6$.

$\therefore$ The length of the journey is

$= T\left(\frac{s_1s_2}{s_1+s_2}\right) = 6\left(\frac{50\times 70}{50+70}\right) = 175$ km.

17. Ans. (a)

Solution $\frac{\text{Rakesh's speed}}{\text{Suresh's speed}} = \frac{\sqrt{T_2}}{\sqrt{T_1}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3}$.

$\therefore$ Suresh's speed $= \frac{3}{4}$ Rakesh's speed

$= \frac{3}{4}\times 16 = 12$ km/hr.

18. Ans. (c)

Solution Here, $x_1 = 225$, $x_2 = 370$, $T_1 = 3\cdot 5$ and $T_2 = 5$.

$\therefore$ Average speed of train $= \frac{x_1+x_2}{T_1+T_2} = \frac{225+370}{3\cdot 5+5}$

$= 70$ km/hr.

19. Ans. (b)

Solution Here, $x_1 = 6$, $x_2 = 8$, $x_3 = 32$, $s_1 = \frac{3}{2}$, $s_2 = 2$ and $s_3 = 8$.

$\therefore$ Average speed of the man

$= \frac{x_1+x_2+x_3}{\frac{x_1}{s_1}+\frac{x_2}{s_2}+\frac{x_3}{s_3}} = \frac{6+8+32}{\frac{6}{3/2}+\frac{8}{2}+\frac{32}{8}}$

$= \frac{46}{12} = 3\frac{5}{6}$ km/hr.

20. Ans. (a)

Solution Here, $T_1 = \frac{30}{60}$, $T_2 = \frac{45}{60}$, $T_3 = 2$, $s_1 = 40$, $s_2 = 60$ and $s_3 = 70$.

$\therefore$ The average speed of the car

$= \frac{s_1T_1+s_2T_2+s_3T_3}{T_1+T_2+T_3}$

$= \frac{40\times\frac{30}{60}+60\times\frac{45}{60}+70\times 2}{\frac{30}{60}+\frac{45}{60}+2}$

$= 63$ km/hr.

21. Ans. (b)

Solution Here, change in time = 20 and $\frac{a}{b} = \frac{3}{4}$.

We have, change in time $= \left(\frac{b}{a}-1\right)$ × original time

$\Rightarrow$ Original time $= \frac{\text{Change in time}}{\left(\frac{b}{a}-1\right)}$

$= \frac{20}{\left(\frac{4}{3}-1\right)} = 60$ minutes.

22. Ans. (a)

Solution Here, difference in speed = 4 – 3 = 1

difference in time $= \frac{1}{2}$

and, product of speed = 4 × 3 = 12.

We have,

$\frac{\text{Product of speed}}{d} = \frac{\text{Difference of speed}}{\text{Difference of time}}$

$\Rightarrow d = \text{Product of speed} \times \left(\frac{\text{Difference of time}}{\text{Difference of speed}}\right)$

$= 12 \times \frac{1/2}{1} = 6$ km.

23. Ans. (c)

Solution Here, difference in speed = 30 – 20 = 10.

difference in time $= 2\frac{1}{2} + 1\frac{1}{2} = 4$

and, product of speed = 20 × 30 = 600.

We have, $\frac{\text{Product of speed}}{d} = \frac{\text{Difference of speed}}{\text{Difference of time}}$

$\Rightarrow d = \text{Product of speed} \times \left(\frac{\text{Difference of time}}{\text{Difference of speed}}\right)$

$= 600 \times \frac{4}{10} = 240$ km.

24. Ans. (a)

Solution Mohan's speed in the first case $= \frac{7}{2}$ km/hr.

Mohan's average speed in the second case

$= \frac{2s_1s_2}{s_1+s_2} = \frac{2\times 3\times 4}{3+4} = \frac{24}{7}$ km/hr.

$\therefore$ Difference of speed $= \frac{7}{2} - \frac{24}{7} = \frac{1}{14}$,

Product of speed $= \frac{7}{2} \times \frac{24}{7} = 12$

and, difference of time $= \frac{10}{60}$.

We have,

d = product of speed $\times \left(\frac{\text{Difference of time}}{\text{Difference of speed}}\right)$

$= 12 \times \frac{10/60}{1/14} = 28$ km.

25. Ans. (b)

Solution Let the slower speed = s km/hr.

Since the distance travelled is same in both the cases, therefore,

$$\frac{s_1}{T_2} = \frac{s_2}{T_1} \Rightarrow s_1 \times T_1 = s_2 \times T_2$$

$$\Rightarrow s \times 8 = (s + 5) \times \frac{20}{3}$$

$$\Rightarrow 24\,s = 20(s + 5)$$

$$\Rightarrow s = 25 \text{ km/hr.}$$

26. Ans. (c)

Solution Here, $s_1 = 42$ and $s_2 = 28$.

$\therefore$ Stoppage time/hr $= \frac{s_1 - s_2}{s_1} = \frac{42-28}{42}$

$= \frac{1}{3}$ hour = 20 minutes

27. Ans. (b)

Solution We have, speed of the train

$$= \frac{\text{Length of the train + length of the platform}}{\text{Total time taken in crossing the platform}}$$

$\Rightarrow$ $60 \times \frac{5}{18} = \frac{\text{Length of the train + 130}}{15}$

$\Rightarrow$ Length of the train = 250 – 130 = 120 m.

28. Ans. (a)

Solution Speed of the train

$$= \frac{\text{Length of the train}}{\text{Time taken in crossing the post}}$$

$\Rightarrow$ Length of the train = Speed of the train × Time taken in crossing the post

$$= \left(54 \times \frac{5}{18}\right) \times 9 = 135 \text{ m.}$$

29. Ans. (b)

Solution Time taken in crossing the telegraph post

$$= \frac{\text{Length of the train}}{\text{Speed of the train}}$$

$$= \frac{135}{54 \times \frac{5}{18}} = \frac{135 \times 18}{54 \times 5} = 9 \text{ sec.}$$

30. Ans. (c)

Solution Speed of the train

$$= \frac{\text{Length of the train}}{\text{Time taken in crossing the man}}$$

$$= \frac{160}{18} \text{ or, } \frac{80}{9} \text{m/s} = \left(\frac{80}{9} \times \frac{18}{5}\right) \text{km/hr}$$

$= 32$ km/hr.

31. Ans. (a)

Solution Time taken in crossing the platform

$$= \frac{\text{Length of the train + Length of the platform}}{\text{Speed of the train}}$$

$$= \frac{280+220}{60 \times \frac{5}{18}} = \frac{500 \times 18}{60 \times 5} = 30 \text{ sec.}$$

32. Ans. (b)

Solution We have,

Speed of the train

$$= \frac{\text{Length of the train + Length of the platform}}{\text{Time taken in crossing the platform}}$$

$$= \frac{50 + 100}{10} = 15 \text{ sec.}$$

33. Ans. (b)

Solution Here, $L_1 = 300$ m, $L_2 = 200$ m,

$s_1 = 90$ km/hr and $s_2 = 60$ km/hr

$\therefore$ $s_1 - s_2 = 90 - 60 = 30$ km/hr $= 30 \times \frac{5}{18}$m/s

$\therefore$ Time taken $= \frac{L_1 + L_2}{s_1 - s_2} = \frac{300+200}{30 \times \frac{5}{18}}$

$$= \frac{500 \times 18}{30 \times 5} = 60 \text{ sec.}$$

34. Ans. (c)

Solution Let the speed of each train be x m/sec.

We have, $L_1 = L_2 = 135$ m and $s_1 = s_2 = x$ m/sec

Therefore, time taken $= \frac{L_1 + L_2}{s_1 + s_2}$

$\Rightarrow$ $18 = \frac{135+135}{x+x}$ or, $x = \frac{270}{2 \times 18}$m/s

$= \frac{270}{2 \times 18} \times \frac{18}{5}$km/hr $= 27$ km/hr.

35. Ans. (c)

Solution Here, $L_1 = 150$ m, $L_2 = 0$, $s_1 = 95$ km/hr and $s_2 = 5$ km/hr.

$\therefore$ $s_1 - s_2 = 90$ km/hr $= 90 \times \frac{5}{18}$m/s $= 25$ m/s.

$\therefore$ Time taken $= \dfrac{L_1 + L_2}{s_1 - s_2} = \dfrac{150}{25} = 6$ sec.

36. Ans. (b)

Solution Here, $L_1 = 100$ m, $L_2 = 0$ and $s_2 = 6$ km/hr.
Let $s_1 = x$ km/hr.

Then, $s_1 + s_2 = (x + 6)$ km/hr $= (x + 6)\dfrac{5}{18}$ m/sec.

$\therefore$ Time taken $= \dfrac{L_1 + L_2}{s_1 + s_2}$

$\Rightarrow \dfrac{18}{5} = \dfrac{100}{(x+6)\dfrac{5}{18}}$

$\therefore$ $x = 100 - 6 = 94$ km/hr.

37. Ans. (c)

Solution Relative speed = (50 – 30) km/hr = 20 km/hr

$$= \left(20 \times \frac{5}{18}\right) = \left(\frac{50}{9}\right) \text{m/sec.}$$

Distance covered in 18 sec at this speed

$$= \left(18 \times \frac{50}{9}\right) \text{m} = 100 \text{ m.}$$

$\therefore$ Length of faster train = 100 m.

38. Ans. (a)

Solution Speed of the faster train

$$= \left(\frac{L_1 + L_2}{2}\right)\left(\frac{T_1 + T_2}{T_1 T_2}\right) = \left(\frac{130+110}{2}\right)\left(\frac{60+3}{60\times 3}\right)$$

= 42 m/sec.

Speed of the slower train

$$= \left(\frac{L_1 + L_2}{2}\right)\left(\frac{T_1 - T_2}{T_1 T_2}\right) = \left(\frac{130+110}{2}\right)\left(\frac{60-3}{60\times 3}\right)$$

= 38 m/sec.

39. Ans. (c)

Solution Speed of faster train

$$= L\left(\frac{T_1 + T_2}{T_1 T_2}\right) = 90\left(\frac{18+9}{18\times 9}\right)$$

$$= \frac{90\times 27}{18\times 9} = 15 \text{ m/sec.}$$

Speed of the slower train

$$= L\left(\frac{T_1 - T_2}{T_1 T_2}\right) = 90\left(\frac{18-9}{18\times 9}\right)$$

$$= \frac{90\times 9}{18\times 9} = 5 \text{ m/sec.}$$

40. Ans. (a)

Solution Time from 5 A.M. to 6.30 A.M. $= 1\dfrac{1}{2}$ hours.

Therefore, distance of meeting from station

$$= \left(\frac{s_1 \times s_2 \times T}{s_2 - s_1}\right) \text{km}$$

$$= \left(\frac{60\times 75\times \frac{3}{2}}{75-60}\right) \text{km} = 450 \text{ km}$$

Also, time of their meeting

$$= \left(\frac{s_1 T}{s_2 - s_1}\right) \text{hr}$$

$$= \left(\frac{60\times \frac{3}{2}}{75-60}\right) \text{hr}$$

= 6 hr after 6.30 A.M.

That is, 12.30 P.M.

41. Ans. (c)

Solution Time from 7 A.M. to 8 A.M. = 1 hour.
Therefore, time of their meeting

$$= \left(\frac{d + s_1 T}{s_1 + s_2}\right) \text{hr}$$

$$= \left(\frac{100 + 25\times 1}{20+25}\right) \text{hr}$$

= 3 hr after 7 A.M
i.e. 10 A.M.

42. Ans. (a)

Solution Time from 9 A.M. to 11 A.M. = 2 hours.
Therefore, distance of meting point from station A

$$= s_1\left(\frac{d + s_2 T}{s_1 + s_2}\right) \text{km}$$

$$= 50\left(\frac{210 + 60\times 2}{50+60}\right) \text{km} = 150 \text{ km.}$$

43. Ans. (b)

Solution Distance between Mumbai and Pune

$$= d\left(\frac{s_1 + s_2}{s_1 - s_2}\right) \text{km}$$

$$= 20\left(\frac{60+40}{60-40}\right) \text{km} = 100 \text{ km.}$$

44. Ans. (b)

Solution Total time taken $= \left(\dfrac{3}{10} + \dfrac{3}{20} + \dfrac{3}{30} + \dfrac{3}{60}\right)$ hr

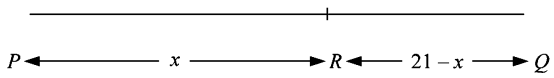
$= \dfrac{3}{5}$ hr

$\therefore$ Average speed $= \left\{\dfrac{12}{3/5}\right\}$ km/hr $= \dfrac{12\times 5}{3}$ km/hr

= 20 km/hr.

45. Ans. (d)

Solution Let the distance between P to R = x km



Then, $\frac{x}{3} = \frac{21}{4} + \frac{21-x}{4}$

$\therefore$ $7x = 2 \times 3 \times 21$

$\therefore$ $x = 18.$

46. Ans. (a)

Solution Let speeds of the boy and the car by x km/hr and y km/hr, respectively

Then, $\frac{12/1000}{x} = \frac{36/1000}{y}$

$\therefore$ $\frac{x}{y} = \frac{1}{3}.$

47. Ans. (b)

Solution Let the required speed be x km/hr.

Then, $\frac{2 \times 64 \times x}{64 + x} = 56$

$\Rightarrow$ $128x = 64 \times 56 + 56x$

$\therefore$ $x = \frac{64 \times 56}{72} = 49.77$ km/hr.

48. Ans. (b)

Solution Due to stoppages, it covers 9 km less per hour.

Time taken to cover 9 km $= \left(\frac{9}{54} \times 60\right)$ min

$= 10$ min.

So, the bus stops for 10 min. per hr.

49. Ans. (a)

Solution Let the boys meet after first has written x lines

Then 2nd has written $817 - (x + 1) = 816 - x$ lines

Then, $\frac{x}{200} = \frac{816 - x}{150}$

$\therefore$ $x = 466.28$

$\therefore$ Two boys will meet on 467th line.

50. Ans. (a)

Solution Distance travelled in 2 minutes

$= \left(1 + \frac{1}{2}\right)$ km i.e. $\frac{3}{2}$ kms.

Distance covered in 1 hr $= \left(\frac{3}{2} \times \frac{60}{2}\right)$ km = 45 km.

$\therefore$ Speed of the train = 45 km/hr.

51. Ans. (d)

52. Ans. (a)

53. Ans. (a)

Solution Let x km p.h. be the speed of the train

$\Rightarrow$ $(x - 6) \times \frac{5}{18} \times 5 = 100$

$\Rightarrow$ $x = 78$ km p.h.

Let the speed of the car be y km p.h.

$\Rightarrow$ $(78 - y) \times \frac{5}{18} \times 6 = 100 \Rightarrow y = 18.$

54. Ans. (a)

Solution Speed of the first man = 32 km/hr.

Time taken = 192 ÷ 32 = 6 hr

Second man covers 192 km in 3 hr

$\therefore$ Speed of the second man = 192 ÷ 3

= 64 km/hr

Ratio = 32 : 64 or 1 : 2

55. Ans. (c)

Solution Let the length of another train = x m

Their relative speed = (62 + 40) km/hr

$= \left(102 \times \frac{5}{18}\right)$ m/sec $= \frac{85}{3}$ m/sec

$\frac{250 + x}{\frac{85}{3}} = 18 \Rightarrow \frac{3(250 + x)}{85} = 18$

$\Rightarrow 250 + x = 510$

$\Rightarrow x = 260$

$\therefore$ Length of another train = 260 m.

56. Ans. (b)

Solution Let the length of the train be x metres and its speed by y metres/sec.

Then, $\frac{x}{y} = 15 \Rightarrow y = \frac{x}{15}$

Now, $\frac{x + 100}{25} = \frac{x}{15} \Rightarrow x = 150$ m.

57. Ans. (b)

Solution Relative speed $= \left(\frac{150 + 100}{10}\right)$ m/sec.

= 25 m/sec.

$= \left(25 \times \frac{18}{5}\right)$ km/hr

= 90 km/hr

$\therefore$ Speed of second train = (90 – 30) km/hr

= 60 km/hr.

58. Ans. (c)

Solution Distance already covered = $\frac{3}{4}\times 12$ = 9 km

Time spent = $\frac{2}{3}\times 45$ min = 30 min

Distance left = (12 – 9) km
= 3 km

Time left = (45 – 30) min = 15 min

∴ Required speed = $\frac{3}{15/60}$km/hr = 12 km/hr.

59. Ans. (a)

Solution Let the speed of the train in km/hr = x

Then, relative speed = $(x + 6)$ km/hr

$= (x + 6) \times \frac{5}{18}$m/sec.

∴ $(x + 6) \times \frac{5}{18} \times 6 = 110$

∴ $x = 60$

∴ Speed of the train = 60 km/hr.

60. Ans. (d)

Solution Let the speed of the train = x km/hr

Relative speed = $(x + 6)$ km/hr

$= (x + 6) \times \frac{5}{18}$m/sec.

∴ $(x + 6) \times \frac{5}{18} \times 6 = 110$

∴ $x = 60$

∴ Speed of train = 60 km/hr for 2nd person,

Relative speed = (60 – 6) km/hr

$= 54 \times \frac{5}{18}$m/sec = 15 m/sec.

∴ Time taken to cross 2nd person

$= \frac{110}{15} = \frac{22}{3} = 7\frac{1}{3}$sec.

61. Ans. (d)

Solution Let the total journey be x km. Then,

$\frac{2}{15}x+\frac{9}{20}x+10 = x$

⇒ $8x + 27x + 600 = 60x \Rightarrow x = 24$

∴ Total journey = 24 km.

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. How many seconds will a 500 m long train take to cross a man walking with a speed of 3 km/hr in the direction of the moving train if the speed of the train is 63 km/hr?
(a) 25 (b) 30
(c) 40 (d) 45
[Based on SSC (GL) Prel. 2000]

2. A man walking at the rate of 5 km/hr crosses a bridge in 15 minutes. The length of the bridge (in metres) is
(a) 600 (b) 750
(c) 1000 (d) 1250
[Based on SSC (GL) Prel. 2000]

3. If a man reduces his speed to 2/3, he takes 1 hr more in walking a certain distance. The time (in hr) to cover the distance with his normal speed is
(a) 2 (b) 1
(c) 3 (d) 1.5
[Based on SSC (GL) Prel. 2000]

4. I walk a certain distance and ride back taking a total time of 37 minutes. I could walk both ways in 55 minutes. How long it take me to ride both ways?
(a) 9.5 minutes (b) 19 minutes
(c) 18 minutes (d) 20 minutes
[Based on SSC (GL) Prel. 2000]

5. A 120 metre long train is travelling at a speed of 90 km per hr. It will cross a railway platform 230 m long in
(a) $4\frac{4}{5}$ sec (b) $9\frac{1}{5}$ sec
(c) 8 sec (d) 14 sec
[Based on SSC (GL) Prel. 2000]

6. A and B start at the same time with speeds of 40 km/hr and 50 km/hr, respectively. If in covering the journey A takes 15 minutes longer than B, the total distance of the journey is:
(a) 46 km (b) 48 km
(c) 50 km (d) 52 km

7. A can go round a circular path 8 times in 40 minutes. If the diameter of the circle is increased to 10 times the original diameter, the time required by A to go round the new part once, travelling at the same speed as before is
(a) 25 min (b) 20 min
(c) 50 min (d) 100 min
[Based on SSC (GL) Prel. 2000]

8. A train passes a 50 metre long platform in 14 seconds and a man standing on the platform in 10 seconds. The speed of the train is
(a) 24 km/hr (b) 26 km/hr
(c) 40 km/hr (d) 45 km/hr
[Based on SSC (GL) Prel. 2000]

9. Two trains start from P and Q respectively and travel towards each other at a speed of 50 km/hr

and 40 km/hr, respectively. By the time they meet, the first train has travelled 100 km more than the second. The distance between P and Q is

(a) 500 km (b) 630 km
(c) 660 km (d) 900 km

[SBased on SC (GL) Prel. 2000]

10. A train 110 m long takes 3 seconds to pass a standing man. How long is the platform if the train passes through it in 15 seconds moving with the same speed?

(a) 440 m (b) 400 m
(c) 550 m (d) 450 m

[Based on SSC (GL) Prel. 2000]

11. A train running at $\frac{7}{11}$ of its own speed reached a place in 22 hours. How much time could be saved if the train ran at its own speed?

(a) 14 hr (b) 7 hr
(d) 8 hr (d) 16 hr

[Based on SSC (GL) Prel. 2002]

12. A train is 125 m long. If the train takes 30 seconds to cross a tree by the railway line, then the speed of the train is

(a) 14 km/hr (b) 15 km/hr
(c) 16 km/hr (d) 12 km/hr

[Based on SSC (GL) Prel. 2000]

13. A train passes two bridges of length 800 m and 400 m in 100 seconds and 60 seconds, respectively. The length of the train is

(a) 80 m (b) 90 m
(c) 200 m (d) 150 m

[Based on SSC (GL) Prel. 2002]

14. A car goes 10 metres in a second. Find its speed in km/hr.

(a) 40 (b) 32
(c) 48 (d) 36

[Based on SSC (GL) Prel. 2002]

15. A man can reach a certain place in 30 hours. If he reduces his speed by $\frac{1}{15}$, he goes 10 km less in that time. Find his speed per hour.

(a) 6 km/hr (b) $5\frac{1}{2}$ km/hr
(c) 4 km/hr (d) 5 km/hr

[Based on SSC (GL) Prep. 2002]

16. A 120 m long train takes 10 seconds to cross a man standing on a platform. What is the speed of the train?

(a) 12 m/sec (b) 10 m/sec
(c) 15 m/sec (d) 20 m/sec

[Based on SSC (GL) Prel. 2002]

17. A train passes a man standing on a platform in 8 seconds and also crosses the platform which is 264 metres long in 20 seconds. The length of the train (in metres) is

(a) 188 (b) 176
(c) 175 (d) 96

[Based on SSC (GL) Prel. 2002]

18. A train moves with the speed of 180 km/hr. Its speed (in metre per second) is

(a) 5 (b) 40
(c) 30 (d) 50

[Based on SSC (GL) Prel. 2002]

19. A car travelling with $\frac{5}{7}$ of its actual speed covers 42 km is 1 hr 40 min 48 sec. Find the actual speed of the car.

(a) $17\frac{6}{7}$ km/hr (b) 35 km/hr
(c) 25 km/hr (d) 30 km/hr

[Based on SSC (GL) Prel. 2002]

20. A train, 120 metres long, passes a telegraph post in 6 seconds. Find the speed of the train.

(a) 60 km/hr (b) 72.5 km/hr
(c) 80 km/hr (d) 72 km/hr

[Based on SSC Prel. (GL) 2002]

21. A trains passes a telegraph post in 8 seconds and a 264 m long bridge in 20 seconds. What is the length of the train?

(a) 180 m (b) 176 m
(c) 164 m (d) 158 m

[Based on SSC (GL) Prel. 2002]

22. A train 800 metres long is running at the speed of 78 km/hr. If it crosses a tunnel in 1 minute, then the length of the tunnel (in metres) is

(a) 700 (b) 500
(c) 400 (d) 600

[Based on SSC (GL) Prel. 2003]

23. A man covered a certain distance at some speed. Had he moved 3 km per hour faster, he would have taken 40 minutes less. If he had moved 2 km per hour slower, he would have taken 40 minutes more. The distance (in km) is

(a) 20 (b) 35
(c) $36\frac{2}{3}$ (d) 40

[Based on SSC (GL) Prel. 2003]

24. Two trains one 160 m and other 140 m long are running in opposite directions on parallel rails, the first at 77 km an hour and the other at 67 km an hour. How long will they take to cross each other.

(a) 7 sec (b) $7\frac{1}{2}$ sec
(c) 6 sec (d) 10 sec

[Based on SSC (GL) Prel. 2003]

25. Two trains are running in opposite directions with the same speed. If the length of each train is 120 metres and they cross each other in 12 seconds,

the speed of each train (in km/hour) is
(a) 72 (b) 10
(c) 36 (d) 18
[Based on SSC (GL) Prel. 2003]

26. In what time will a train 100 metres long cross an electric pole if its speed be 144 km/hr?
(a) 2.5 seconds (b) 5 seconds
(c) 12.5 seconds (d) $3\frac{5}{4}$ seconds
[Based on SSC (GL) Prel. 2003]

27. A car travelling at speed of 40 km/hour can complete a journey in 9 hours. How long will it take to travel the same distance at 60 km/hr?
(a) 6 hours (b) 3 hours
(c) 4 hours (d) $4\frac{1}{2}$ hours
[Based on SSC (GL) Prel. 2003]

28. A train running at speed of 120 km/hr crosses a signal post in 15 seconds. What is the length of the train in metres?
(a) 300 (b) 200
(c) 500 (d) Cannot be determined
(e) None of these **[Based on BSRB Bhopal PO 2000]**

29. A car finishes a journey in ten hours at the speed of 80 km/hr. If the same distance is to be covered in eight hours how much more speed does the car have to gain?
(a) 8 km/hr (b) 10 km/hr
(c) 12 km/hr (d) 16 km/hr
(e) None of these **[Based on BSRB Delhi PO 2000]**

30. Train '*A*' leaves Mumbai Central for Lucknow at 11 am, running at the speed of 60 km/hr. Train '*B*' leaves Mumbai Central for Lucknow by the same route at 2 pm on the same day, running at the speed of 72 km/hr. At what time will the two trains meet each other?
(a) 2 am on the next day
(b) 5 am on the next day
(c) 5 pm on the next day
(d) 2 pm on the next day
(e) None of these **[Based on BSRB Patna PO 2001]**

31. Trains, *A* and *B* are travelling on the same route heading towards the same destination. Train *B* has already covered a distance of 220 km before train *A* started. The two trains meet each other 11 hours after the start of train *A*. Had the trains been travelling towards each other (from a distance of 220 km), they would have met after one hour. What is the speed of train *B* in km/hr?
(a) 100 (b) 180
(c) 116 (d) Data inadequate
(e) None of these **[Based on SBI PO 2001]**

32. The speed of a car increases by 2 km after every hour. If the distance travelled in the first hour was 35 km, what was the total distance travelled in 12 hours?
(a) 522 km (b) 456 km
(c) 556 km (d) 482 km
(e) None of these
[Based on Bank of Maharashtra PO 2003]

EXPLANATORY ANSWERS

1. Ans. (b)

Solution Relative speed of the train with respect to speed of the man = 63 – 3 = 60 km/hr.

∴ The time taken by the train in crossing the man = The time taken in moving a distance equivalent to the length of the train, i.e. 500 m at 650 km/hr.

$$S = 60 \text{ km/hr} = 60 \times \frac{5}{18} \text{m/sec}$$

$$T = \frac{500}{60} \times \frac{18}{5} = 30 \text{ sec.}$$

2. Ans. (d)

3. Ans. (a)

Solution Let the normal speed be x km/hr

∴ Reduced speed = $\frac{2}{3}x$ km/hr.

Suppose distance covered = y km

Then, $\frac{y}{x} + 1 = \frac{3y}{2x} \Rightarrow \frac{3y}{2x} - \frac{y}{x} = 1 \Rightarrow \frac{y}{x} = 2$

∴ Time taken to cover the distance of y km with normal speed = $\frac{y}{x}$ = 2 hours.

4. Ans. (b)

Solution Time taken in walking a certain distance from x to y = $27\frac{1}{2}$ minutes

∴ Time taken in riding the same distance

$$= 37 - 27\frac{1}{2} = 9\frac{1}{2} \text{ minutes}$$

∴ It will take 19 minutes to ride both ways.

5. Ans. (d)

Solution 90 km/hr = 90000 m/3600 sec
= 900 m/36 sec = 25 m/sec

$$\therefore \quad T = \frac{230 + 120}{25} = \frac{350}{25} = 14 \text{ sec.}$$

6. Ans. (c)

Solution Let x be the total distance of the journey.

According to the question,

$$\frac{x}{40}-\frac{x}{50}=\frac{15}{60}\Rightarrow\frac{x}{200}=\frac{1}{4}$$

$\therefore$ $x = 50$ km.

7. Ans. (c)

Solution Time taken to go round a circular path

$$=\frac{40}{8}=5 \text{ min}$$

$\therefore$ Time required by A to go round new path

$= 10 \times 5 = 50$ min.

8. Ans. (d)

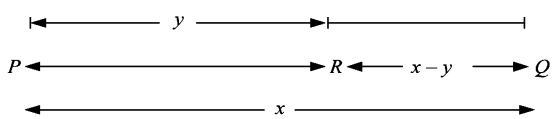
Solution Speed of the train

$$=\frac{\text{Length of platform}}{\text{Time difference}}=\frac{50}{4}\times\frac{18}{5}=45 \text{ km/hr.}$$

9. Ans. (d)

Solution Let distance between P and $Q = x$ km

$\therefore$ Suppose the two trains meet after y km.



$\therefore$ $y = (x - y) + 100$

$\Rightarrow$ $2y = x + 100$

$$\therefore \frac{y}{50}=\frac{x-y}{40}\Rightarrow 40y = 50x - 50y$$

$$\Rightarrow 90y = 50x$$

$$\Rightarrow 9y = 5x$$

$$\Rightarrow 5x=\frac{9(x+100)}{2}\Rightarrow x = 900.$$

10. Ans. (a)

Solution 110 metres are covered in 3 seconds

550 metres are covered in 15 seconds

$\therefore$ length of the platform = 440 metres.

11. Ans. (c)

Solution Saving in time $= 22-\frac{22\times7}{11}=8$ hours.

12. Ans. (b)

Solution Speed of train $=\frac{125}{30}\times\frac{18}{5}=15$ km/hr.

13. Ans. (c)

14. Ans. (d)

Solution Speed in km/hr $=\frac{10\times18}{5}=36$ km/hr.

15. Ans. (d)

Solution $30\times S-30\times\frac{14}{15}S=10$

or, $30\left(\frac{1}{15}S\right)=10$ or, $S=\frac{10\times15}{30}=5$

Hence, required speed = 5 km/hr.

16. Ans. (a)

Solution Speed of train $=\frac{120}{10}=12$ m/sec.

17. Ans. (b)

Solution Speed of train $=\frac{264}{20-8}=22$ m/sec.

Hence, length of train $= 22 \times 8 = 176$ m.

18. Ans. (d)

Solution Speed of the train = 180 km/hr

= 180000 m/3600 sec.

= 50 m/sec.

19. Ans. (b)

Solution Let the actual speed of the car be x km/hr.

If the speed of the car is $\frac{5}{7}x$ km/hr, then running at this speed, the distance of 42 km is covered in 1 hr 40 min 48 sec, i.e. 1 hour $40\frac{4}{5}$ minutes, i.e. $\frac{126}{75}$ hour

$\therefore$ $42\times\frac{75}{126}$ km. hr. i.e. 25 km/hr $=\frac{5x}{7}$

$\therefore$ $25=\frac{5x}{7}\Rightarrow x = 35$ km/hr.

20. Ans. (d)

Solution Speed of the train = 120 m/6 sec

= 7200 m/360 sec.

= 72000 m/3600 sec

= 72 km/hr.

21. Ans. (b)

Solution Let the length of the train be x metres. It passes a telegraph post in 8 seconds.

$\because$ A distance of $(x + 264)$ metres is covered by the x metres long train in 20 seconds

$\therefore$ $\frac{x}{8}=\frac{x+264}{20}\Rightarrow x = 176$ m.

22. Ans. (b)

Solution $78\times\frac{5}{18}=\frac{\text{Length of (train + tunnel)}}{60}$

or, $78\times\frac{5}{18}\times60=800+x$

or, $1300-800=x$ or, $x = 500$ m.

23. Ans. (d)

24. Ans. (b)

Solution Required time = $\dfrac{160+140}{(77+67)\times\dfrac{5}{18}} = \dfrac{300\times18}{144\times5}$

= 7.5 seconds.

25. Ans. (c)

Solution Suppose, the speed of the train is V m/sec

Then, $V + V = \dfrac{120+120}{12} = 20$

or, $2V = 20 \times \dfrac{18}{5} = 72$

or, $V = 36$ km/hr.

26. Ans. (a)

Solution Required time = $\dfrac{100}{144\times\dfrac{5}{18}} = \dfrac{100}{40} = 2.5$ sec.

27. Ans. (a)

Solution For same distance, we have

$V_1 t_1 = V_2 t_2$

or, $t_2 = \dfrac{V_1 t_1}{V_2} = \dfrac{40\times9}{60} = 6$ hours.

28. Ans. (c)

Solution Length of the train = $120 \times \dfrac{5}{18} \times 15$

= 500 metres.

29. Ans. (e)

Solution Distance travelled by the car = 80 × 10

= 800 km.

∴ Speed = $\dfrac{800}{8}$ = 100 km/hr

∴ Speed gain = 100 – 80 = 20 km/hr.

30. Ans. (b)

Solution Distance covered by train A before the train B leaves Mumbai Central = 60 × 3 = 180 km

∴ Time taken to cross each other

$= \dfrac{180}{12} = 15$ hours

∴ Required time = 2 p.m. + 15 hours

= 5 a.m. on the next day.

31. Ans. (a)

32. Ans. (e)

Solution This is the problem of arithmetic progression with the first term $a = 35$, common difference $d = 2$ and total number of terms $(n) = 12$. The sum of this series will be the total distance travelled.

Sum $S_n = \dfrac{n}{2}[2a+(n-1)d] = \dfrac{12}{2}[70+11\times2]$

$= \dfrac{12\times92}{2} = 552$ km.

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. Walking with $\dfrac{6}{5}$ of my usual speed, I reach my office 10 minutes too early. Find my usual time?
(a) 50 min (b) 60 min
(c) 45 min (d) 40 min

2. Two trains start from a place moving with 45 km/hr and 75 km/hr, respectively in the opposite direction. After how many hours they will be 1280 km away.
(a) $12\dfrac{2}{3}$ (b) $10\dfrac{2}{3}$
(c) $14\dfrac{1}{3}$ (d) $10\dfrac{1}{2}$

3. Two scooterists do the same journey by driving at the rates of 50 km/hr and 45 km/hr, respectively. Find the length of the journey when one takes 32 min longer than the other.
(a) 168 km (b) 240 km
(c) 210 km (d) 480 km

4. A man walking at 3 km/hr crosses a square field diagonally in 2 min. The area of the field is
(a) 2.5 km (b) 3 km
(c) 5 km (d) 6 km

5. A car travels a distance of 840 km at a uniform speed. If the speed of the car is 10 km/hr more, it takes two hours less to cover the same distance. The original speed of the car was
(a) 45 km/hr (b) 50 km/hr
(c) 60 km/hr (d) 75 km/hr

6. A certain passenger train travels at the rate of 42 km/hr and a goods train, half as long as the travelling train, travels at 33 km/hr. If, while travelling in the same direction, it takes 50 seconds for the passenger train to clear the goods train, while travelling in opposite direction they will cross each other completely in
(a) 6 sec. (b) 8 sec.
(c) 9 sec. (d) 12 sec.

7. Two persons are walking in the same direction at rates 3 km/hr and 6 km/hr. A train comes running from behind and passes them in 9 and 10 seconds. The speed of the train is
(a) 33 km/hr (b) 40 km/hr
(c) 22 km/hr (d) 35 km/hr

8. The poles on the rail road are 50 metres apart, how many will be passed by a train in one and a half hour if the speed of the train be 72 km/hr?
(a) 2160 (b) 2060
(c) 2100 (d) 2050

9. A train 120 m long takes 6 seconds to pass a man, who is running in the opposite direction at the speed of 12 km/hr. Find the speed of the train.
(a) 60 km/hr (b) 65 km/hr
(c) 55 km/hr (d) 50 km/hr

10. Two trains 121 metres and 99 metres in length respectively, are running in opposite direction, one at the rate of 40 km/hr and the other at the rate of 32 km/hr. From the moment trains meet, they will be completely clear of each other in
(a) 20 sec. (b) 16 sec.
(c) 11 sec. (d) 9 sec.

Test II

[Time: 10 min]

11. If a train runs at 40 km/hr, it reaches its destination late by 11 min. but if it runs at 50 km/hr, it is late by 5 min. only. The correct time for the train to cover its journey is
(a) 13 min (b) 15 min
(c) 21 min (d) 19 min

12. A train travelling at 54 km/hr completely crosses another train of half its length and travelling in opposite direction at 72 km/hr in 12 seconds. If it also passes a railway platform in one minute, the length of the platform is
(a) 620 metres (b) 840 metres
(c) 920 metres (d) 750 metres

13. Two trains start at the same time from Aligarh and Delhi and proceed towards each other at 16 km/hr and 21 km/hr, respectively. When they meet, it is found that one train has travelled 60 km more than the other. The distance between the two stations is
(a) 445 km (b) 444 km
(c) 440 km (d) 450 km

14. A man travels 600 km in 15 hours partly by rail and partly by car. If he had gone all the way by car he would have arrived at his destination 3 hours earlier, find the speed of the car.
(a) $33\frac{1}{3}$km/hr (b) 40 km/hr
(c) 50 km/hr (d) 45 km/hr

15. Two trains 108 m and 112 m in length are running towards each other on parallel lines at a speed of 45 km/hr and 54 km/hr, respectively. How long will it take then to cross each other after they meet?
(a) 5 seconds (b) 7 seconds
(c) 8 seconds (d) 6 seconds

16. Two buses start from a certain town at 9.00 a.m. and move in opposite direction. One goes towards east at an average speed of 44 km/hr and the other goes towards west at an average speed of 40 km/hr. After what time will they be 210 km apart from each other?
(a) 2 hours (b) $2\frac{1}{4}$ hours
(c) $2\frac{1}{2}$ hours (d) 3 hours

17. Two trains A and B running on parallel lines enter a tunnel 2 km long at the opposite ends at the same time. They meet in $1\frac{1}{2}$ minutes and A has then travelled in the tunnel 400 metres more than B. In $4\frac{1}{2}$ seconds they cross each other and 1 minute later A has completely passed through the tunnel. The length (in metres) of B is
(a) 40 (b) 60
(c) 50 (d) 55

18. Sound travels at 330 metres a second. How many kilometres is a thunder-cloud distance when the sound follows the flash after 10 seconds?
(a) 3.3 m (b) 3.8 m
(c) 4.2 m (d) None of these

19. Two places P and Q are 162 km apart. A train leaves P for Q and at the same time another train leaves Q for P. The two trains meet at the end of 6 hours. If the train travelling from P to Q travels 8 km an hour faster than the other, find the speed of the two trains.
(a) 16 km/hr (b) 17 km/hr
(c) $17\frac{1}{2}$km/hr (d) 18 km/hr

20. A man takes 8 hours to walk to a certain place and ride back. However, he could have gained 2 hours, if he had covered both ways riding. How long would he have taken to walk both ways?
(a) 10 hrs (b) 8 hrs
(c) 6 hrs (d) None of these

Answers

1. (b)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (a)	**7.** (a)	**8.** (a)	**9.** (a)	**10.** (c)
11. (d)	**12.** (a)	**13.** (b)	**14.** (c)	**15.** (c)	**16.** (c)	**17.** (a)	**18.** (a)	**19.** (c)	**20.** (a)

16

BOATS AND STREAMS

Some Important Terms

1. **Still Water** If the speed of the water in the river is zero, it is *still water*.
2. **Stream** If the water of the river is moving, it is called a *stream*.
3. **Upstream** If a boat (or a swimmer) moves against the stream, i.e. in the direction opposite to that of the stream, it is called *upstream*.
4. **Downstream** If a boat (or a swimmer) moves with the stream, i.e. along the direction of the stream, it is called *downstream*.

Note: When the speed of a boat or a swimmer is given, it usually means speed in the still water.

SOME BASIC FORMULAE

1. If the speed of a boat (or a swimmer) be x km/hr and the speed of the stream or the current be y km/hr, then
   (a) speed of the boat (or swimmer) downstream $= (x + y)$ km/hr.
   (b) speed of the boat (or swimmer) upstream $= (x - y)$ km/hr.

Illustration 1 The speed of a boat in still water is 20 km/hr. If the speed of the stream be 4 km/hr, find its downstream and upstream speeds.

Solution Speed of the boat $(x) = 20$ km/hr

Speed of the stream $(y) = 4$ km/hr

$\therefore$ Downstream speed $= x + y = (20 + 4) = 24$ km/hr

and, upstream speed $= x - y = (20 - 4) = 16$ km/hr.

2. (a) Speed of the boat (or swimmer) in still water

$$= \frac{1}{2}(\text{Downstream Speed} + \text{Upstream Speed})$$

(b) Speed of the stream

$$= \frac{1}{2}(\text{Downstream Speed} - \text{Upstream Speed})$$

Illustration 2 A boat is rowed down a river 40 km in 5 hr and up a river 21 km in 7 hr. Find the speed of the boat and the river.

Solution Speed of the boat downstream $= \frac{40}{5} = 8$ km/hr.

Speed of the boat upstream $= \frac{21}{7} = 3$ km/hr.

$\therefore$ Speed of the boat

$$= \frac{1}{2}(\text{Downstream Speed} + \text{Upstream Speed})$$

$$= \frac{1}{2}(8 + 3) = \frac{11}{2} = \text{or } 5 \cdot 5 \text{ km/hr.}$$

and, speed of the river

$$= \frac{1}{2}(\text{Downstream Speed} - \text{Upstream Speed})$$

$$= \frac{1}{2}(8 - 3) = \frac{5}{2} \text{ or } 2{\cdot}5 \text{ km/hr.}$$

SOME USEFUL SHORT-CUT METHODS

1. If a man capable of rowing at the speed of x km/hr in still water, rows the same distance up and down a stream which flows at a rate of y km/hr, then his average speed throughout the journey is

$$= \frac{\text{Uptream} \times \text{Downstream}}{\text{Man's rate in still water}}$$

$$= \frac{(x-y)(x+y)}{x}\text{km/hr.}$$

Illustration 3 A man rows at a speed of 8 km/hr in still water to a certain distance upstream and back to the starting point in a river which flows at 4 km/hr. Find his average speed for total journey.

Solution Average Speed

$$= \frac{\text{Upstream} \times \text{Downstream}}{\text{Man's rate in still water}}$$

$$= \frac{(8-4)(8+4)}{8} = 6 \text{ km/hr.}$$

2. A man can row a boat in still water at x km/hr. In a stream flowing at y km/hr, if it takes t hours more in upstream than to go downstream for the same distance, then the distance is given by

$$\frac{(x^2-y^2)t}{2y}\text{km}$$

Illustration 4 A man can row 7 km/hr in still water. If the river is running at 3 km/hr, it takes 6 hours more in upstream than to go downstream for the same distance. How far is the place?

Solution The required distance

$$= \frac{(x^2-y^2)t}{2y}$$

$$= \frac{(49-9)6}{2\times 3} = 40 \text{ km.}$$

3. A man rows a certain distance downstream in t_1 hours and returns the same distance upstream in t_2 hours. If the speed of the stream be y km/hr, then the speed of the man in still water is given by

$$y\left(\frac{t_2+t_1}{t_2-t_1}\right) \text{ km/hr.}$$

Explanation

Let the speed of the man in still water be x km/hr. Then, downstream speed = $(x + y)$ km/hr and upstream speed = $(x - y)$ km/hr.

Since the distance covered downstream and upstream are equal, we have

$$(x + y)t_1 = (x - y)t_2$$

or, $$xt_1 + yt_1 = xt_2 - yt_2$$

or, $$x(t_2 - t_1) = y(t_2 + t_1)$$

$$\therefore \quad x = y\left(\frac{t_2+t_1}{t_2-t_1}\right)\text{km/hr.}$$

Illustration 5 A motorboat covers a certain distance downstream in 6 hours but takes 8 hours to return upstream to the starting point. If the speed of the stream be 6 km/hr, find the speed of the motor boat in still water.

Solution Speed of the motorboat in still water

$$= y\left(\frac{t_2+t_1}{t_2-t_1}\right) \text{ km/hr}$$

$$= 6\left(\frac{8+6}{8-6}\right) = 42 \text{ km/hr.}$$

4. A man can row a boat in still water at x km/hr. In a stream flowing at y km/hr if it takes him t hours to row to a place and come back, then the distance between the two places is

$$\frac{t(x^2-y^2)}{2x}\text{km.}$$

Explanation

Downstream speed = $(x + y)$km/hr

Upstream speed = $(x - y)$km/hr.

Let the distance between the two places be d km. We have,

Total time = Sum of time taken downstream and upstream

$$\Rightarrow \quad t = \frac{d}{x+y} + \frac{d}{x-y}$$

$$= d\left[\frac{(x-y)+(x+y)}{(x-y)(x+y)}\right]$$

$$= d\left[\frac{2x}{x^2-y^2}\right]$$

$$\therefore \quad d = \frac{t(x^2-y^2)}{2x}\text{km.}$$

Illustration 6 A man can row 6 km/hr in the still water. If the river is running at 2 km/hr, it takes him 3 hours to row to a place and back. How far is the place?

Solution The required distance

$$= \frac{t(x^2 - y^2)}{2x} \text{km}$$

$$= \frac{3(36-4)}{2\times 6} = 8 \text{ km.}$$

5. A boat (or a swimmer) takes n times as long to row upstream as to row downstream the river. If the speed of boat (or swimmer) be x km/hr and the speed of stream be y km/hr, then

$$x = y\left(\frac{n+1}{n-1}\right).$$

Illustration 7 A man can row at the rate of 4 km/hr. in still water. If the time taken to row a certain distance upstream is 3 times as much as to row the same distance downstream, find the speed of the current.

Solution We have,

$$\text{Speed of the man} = \left(\frac{n+1}{n-1}\right) \text{speed of the current}$$

$$\Rightarrow \quad 4 = \left(\frac{3+1}{3-1}\right) \text{speed of the current.}$$

$\therefore$ Speed of the current = 2 km/hr.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. A boat goes 13 km upstream in 39 minutes. The speed of stream is 3 km/hr. The speed of boat in still water is:
(a) 23 km/hr (b) 27 km/hr
(c) 25 km/hr (d) None of these

2. The speed of a boat in still water is 8 km/hr. If its speed downstream be 15 km/hr, then speed of the stream is:
(a) 7.5 km/hr (b) 7 km/hr
(c) 9 km/hr (d) None of these

3. Speed of a man is 10 km/hr in still water. If the rate of current is 3 km/hr, then the effective speed of the man upstream is:
(a) 7 km/hr (b) 8.5 km/hr
(c) 9 km/hr (d) None of these

4. A man can row with the stream at 7 km/hr and against the stream at 3 km/hr. His speed in still water is:
(a) 6.5 km/hr (b) 7 km/hr
(c) 5 km/hr (d) None of these

5. A swimmer covers a distance of 28 km against the current and 40 km in the direction of the current. If in each case he takes 4 hours, then the speed of the current is
(a) 3.5 km/hr (b) 1.5 km/hr
(c) 2.5 km/hr (d) None of these

6. A boat moves downstream at the rate of one km in 10 minutes and upstream at the rate of 4 km an hour. What is the velocity of the current
(a) 5 km/hr (b) 3 km/hr
(c) 1 km/hr (d) None of these

7. If a man's rate with the current is 12 km/hr and the rate of the current is $1\frac{1}{2}$ km/hr, then his rate against the current is
(a) 13 km/hr (b) 7 km/hr
(c) 9 km/hr (d) None of these

8. A boatman can row 2 km against the stream in 20 minutes and return in 18 minutes. Find the rate of current.
(a) 1/3 km/hr (b) 2/3 km/hr
(c) 5/3 km/hr (d) None of these

9. A boatman can row 48 km downstream in 4 hr. If the speed of the current is 5 km/hr, then find in what time will he be able to cover 8 km upstream?
(a) 6 hr (b) 4 hr
(c) 8 hr (d) None of these

10. A man can row at a speed of 10 km/hr in still water to a certain upstream point and back to the starting point in a river which flows at 4 km/hr. Find his average speed for total journey.
(a) $9\frac{2}{5}$ km/hr (b) $8\frac{2}{5}$ km/hr
(c) $11\frac{2}{5}$ km/hr (d) None of these

11. A man can row 6 km/hr in still water. If the river is running at 2 km/hr, it takes 3 hours more in upstream than to go downstream for the same distance. How far is the place?
(a) 24 km (b) 28 km
(c) 36 km (d) None of these

12. A boat covers a certain distance downstream in 2 hours but takes 4 hours to return upstream to the starting point. If the speed of the stream be 3 km/hr, find the speed of the boat in still water.
(a) 11 km/hr (b) 13 km/hr
(c) 9 km/hr (d) None of these

13. In a river flowing at 2 km/hr, a boat travels 32 km

upstream and then returns downstream to the starting point. If its speed in still water be 6 km/hr, find the total journey time.
(a) 16 hours (b) 12 hours
(c) 12 hours (d) None of these

14. A boat travels upstream from B to A and downstream from A to B in 3 hrs. If the speed of the boat in still water is 9 km/hr and the speed of the current is 3 km/hr, the distance between A and B is:
(a) 8 km (b) 16 km
(c) 12 km (d) None of these

15. A boat travels 2 km upstream in a stream flowing at 3 km/hr and then returns downstream to the starting point in 30 minutes. The speed of the boat in still water is:
(a) 17 km/hr (b) 9 km/hr
(c) 13 km/hr (d) None of these

16. A man swimming in a stream which flows $1\frac{1}{2}$km/hr finds that in a given time he can swim twice as far with the stream as he can against it. At what rate does he swim?
(a) $4\frac{1}{2}$km/hr (b) $5\frac{1}{2}$km/hr
(c) $7\frac{1}{2}$km/hr (d) None of these

17. A boat travels upstream from B to A and downstream from A to B in 3 hours. If the speed of the boat in still water is 9 km/hour and the speed of the current is 3 km/hour, the distance between A and B is:
(a) 4 km (b) 6 km
(c) 8 km (d) 12 km

18. A man rows upstream 12 km and downstream 28 km taking 5 hours each time. The velocity of water current is
(a) $2\frac{1}{5}$km/hr (b) $2\frac{1}{2}$km/hr
(c) 3 km/hr (d) $1\frac{3}{5}$km/hr

19. Twice the speed downstream is equal to the thrice the speed upstream, the ratio of speed in still water to the speed of the current is:
(a) 1 : 5 (b) 5 : 1
(c) 1 : 3 (d) 2 : 3

20. A man can swim 3 km/hr in still water. If the velocity of the stream be 2 km/hr, the time taken by him to swim to a place 10 km upstream and back, is:
(a) $8\frac{1}{3}$hr (b) $9\frac{1}{5}$hr
(c) 10 hr (d) 12 hr

21. A boat covers 24 km upstream and 36 km downstream in 6 hr, while it covers 36 km upstream and 24 km downstream in $6\frac{1}{2}$hrs. The velocity of the current is:
(a) 1.5 km/hr (b) 1 km/hr
(c) 2 km/hr (d) 2.5 km/hr

22. A boatman goes 2 km against the current of the stream in 1 hr and goes 1 km along the current in 10 min. How long will he take to go 5 km in stationary water?
(a) 1 hr (b) 1 hr 15 min
(c) $1\frac{1}{2}$hr (d) 40 min

23. P, Q, R are three towns on a river which flows uniformly. Q is equidistant from P and R. A man rows from P to Q and back in 10 hr. He can row from P to R in 4 hr. The ratio of speed of man in still water to the speed of the current is:
(a) 5 : 3 (b) 3 : 5
(c) 2 : 5 (d) 1 : 2

24. In a stream running at 2 km/hr, a motor boat goes 10 km upstream and back again to the starting point in 55 min. Find the speed of the motorboat in still water.
(a) 20 km/hr (b) 21 km/hr
(c) 22 km/hr (d) 24 km/hr

25. A man can row 30 km upstream and 44 km downstream in 10 hr. Also, he can row 40 km upstream and 55 km downstream in 13 hours. Find the rate of the current and the speed of the man in still water
(a) 3 km/hr, 8 km/hr (b) 3·5 km/hr, 7·5 km/hr
(c) 4 km/hr, 7 km/hr (d) 4·5 km/hr, 6·5 km/hr

EXPLANATORY ANSWERS

1. Ans. (a)

Solution Speed of the boat upstream

$$= \frac{13 \times 60}{39} = 20 \text{ km/hr}$$

Speed of the stream = 3 km/hr

Let the speed of the boat in still water = x km/hr.

We have, $x - 3 = 20$

$\therefore \quad x = 20 + 3 = 23$ km/hr.

2. Ans. (b)

Solution Speed of the boat downstream = 15 km/hr.

Speed of the boat in still water = 8 km/hr.

Let the speed of the stream = y km/hr.

We have, $15 = 8 + y$

$\therefore$ $y = 15 - 8 = 7$ km/hr.

3. Ans. (a)

Solution Speed of man in still water = 10 km/hr

Speed of current = 3 km/hr

$\therefore$ Speed of man upstream = 10 – 3 = 7 km/hr.

4. Ans. (c)

Solution Speed of the man upstream = 7 km/hr.

Speed of the man downstream = 3 km/hr.

$\therefore$ Speed of the man in still water

$= \frac{1}{2}$ (Downstream Speed + Upstream Speed)

$= \frac{1}{2}(7 + 3) = 5$ km/hr.

5. Ans. (b)

Solution Speed of the swimmer upstream

$= \frac{28}{4} = 7$ km/hr.

Speed of the swimmer downstream

$= \frac{40}{4} = 10$ km/hr.

$\therefore$ Speed of the stream

$= \frac{1}{2}$ (Downstream Speed – Upstream Speed)

$= \frac{1}{2}(10 - 7) = \frac{3}{5} = 1{\cdot}5$ km/hr.

6. Ans. (c)

Solution Speed of the boat downstream

$= \frac{60}{10} = 6$ km/hr.

Speed of the boat upstream = 4 km/hr

$\therefore$ Velocity of the current

$= \frac{1}{2}$(Downstream speed – Upstream Speed)

$= \frac{1}{2}(6 - 4) = 1$ km/hr.

7. Ans. (c)

Solution Speed of the man downstream = 12 km/hr.

Speed of the stream $= \frac{3}{2}$km/hr.

Let the speed of the man upstream = x km/hr.

We have,

Speed of the stream

$= \frac{1}{2}$(Downstream Speed – Upstream Speed)

$\Rightarrow$ $\frac{3}{2} = \frac{1}{2}(12 - x)$.

$\therefore$ $x = 12 - 3 = 9$ km/hr.

8. Ans. (a)

Solution Speed of the boatman upstream

$= \frac{2}{20} \times 60 = 6$ km/hr.

Speed of the boatman downstream

$= \frac{2}{18} \times 60 = \frac{20}{3}$km/hr.

$\therefore$ Rate of current

$= \frac{1}{2}$(Downstream Speed – Upstream Speed)

$= \frac{1}{2}\left(\frac{20}{3} - 6\right) = \frac{1}{3}$km/hr.

9. Ans. (b)

Solution Speed of the boatman downstream

$= \frac{48}{4} = 12$ km/hr.

Speed of the current = 5 km/hr.

Let the boatman takes t hours to cover 8 km upstream.

Then, speed of the current

$= \frac{1}{2}$(Downstream Speed – Upstream Speed)

$\Rightarrow$ $5 = \frac{1}{2}\left(12 - \frac{8}{t}\right)$

$\therefore$ $t = 4$ hours.

10. Ans. (b)

Solution Average Speed

$= \frac{\text{Upstream} \times \text{Downstream}}{\text{Man's rate in still water}}$

$= \frac{(10-4)(10+4)}{10} = 8\frac{2}{5}$km/hr.

11. Ans. (a)

Solution The required distance

$= \frac{(x^2 - y^2)t}{2y} = \frac{(36-4)3}{2 \times 2} = 24$ km.

12. Ans. (c)

Solution Speed of the boat in still water

$= y\left(\frac{t_2 + t_1}{t_2 - t_1}\right)$ km/hr

$= 3\left(\frac{4+2}{4-2}\right) = 9$ km/hr.

13. Ans. (b)

Solution Let the total journey time be t hr.

Then, we have $d = \frac{t(x^2 - y^2)}{2x}$

$\Rightarrow$ $32 = \frac{t(36-4)}{2 \times 6}$

$\therefore$ $t = 12$ hr.

14. Ans. (c)

Solution The distance between A and B is

$$= \frac{t(x^2 - y^2)}{2x} \text{km}$$

$$= \frac{3(81-9)}{2\times 9} = 12 \text{ km.}$$

15. Ans. (b)

Solution Let the speed of the boat be x km/hr.

We have,

$$d = \frac{t(x^2 - y^2)}{2x}$$

$$\Rightarrow \quad 2 = \frac{1/2(x^2-9)}{2x}, \text{ i.e. } 2 = \frac{x^2-9}{4x}$$

or, $x^2 - 8x - 9 = 0$

or, $(x - 9)(x + 1) = 0$

or, $x = -1$ or 9.

Since the speed cannot be negative, we neglect -1.

Therefore, speed of the boat in still water = 9 km/hr.

16. Ans. (a)

Solution Speed of man

$$= \left(\frac{n+1}{n-1}\right) \text{ speed of stream}$$

$$= \left(\frac{2+1}{2-1}\right)\times\frac{3}{2} = \frac{9}{2} \text{ or } 4\frac{1}{2}\text{km/hr.}$$

17. Ans. (d)

Solution Speed downstream = (9 + 3) km/hr

= 12 km/hr

Speed upstream = (9 – 3) km/hr = 6 km/hr

Let the distance $AB = x$ km

Then, $\frac{x}{6}+\frac{x}{12} = 3 \Rightarrow 2x + x = 36$

$\Rightarrow x = 12$

$\therefore$ Distance AB = 12 km.

18. Ans. (d)

Solution Let man's rowing speed in still water = x km/hr and speed of current = y km/hr

Speed upstream = $(x - y)$ km/hr

and, speed downstream = $(x + y)$ km/hr

$\therefore$ $5(x - y) = 12$ and $5(x + y) = 28$

subtracting $10y = 16$

$\therefore$ $y = \frac{8}{5} = 1\frac{3}{5}$ km/hr.

19. Ans. (b)

Solution Let speed in still water = x km/hr

Speed of current = y km/hr

Speed downstream = $(x + y)$ km/hr

Speed upstream = $(x - y)$ km/hr

$\therefore$ $2(x + y) = 3(x - y)$

$\therefore$ $x = 5y$

or, $\frac{x}{y} = \frac{5}{1}$ or 5 : 1.

20. Ans. (d)

Solution Speed upstream = (3 – 2) km/hr = 1 km/hr

Speed downstream = (3 + 2) km/hr = 5 km/hr

Total time taken = $\left(\frac{10}{1}+\frac{10}{5}\right)$hr

= 12 hr.

21. Ans. (c)

Solution Let the speed upstream be x km/hr and the speed downstream by y km/hr. Then,

$$\frac{24}{x}+\frac{36}{y} = 6 \Rightarrow 24u + 36v = 6$$

where $\frac{1}{x} = u$ and $\frac{1}{y} = v$

And, $\frac{36}{x}+\frac{24}{y} = \frac{13}{2} \Rightarrow 36u + 24v = \frac{13}{2}$

Adding these equations, we get

$$60(u + v) = \frac{25}{2} \text{ or } u + v = \frac{5}{24}$$

Subtracting, we get $12(u - v) = \frac{1}{2}$ or $u - v = \frac{1}{24}$

Solving, $u + v = \frac{5}{24}$ and $u - v = \frac{1}{24}$, we get

$u = \frac{1}{8}$ and $v = \frac{1}{12}$

$\therefore$ $x = 8$ km/hr and $y = 12$ km/hr

$\therefore$ Velocity of current = $\frac{1}{2}(12 - 8) = 2$ km/hr.

22. Ans. (b)

Solution Upstream speed = 2 km/hr

Downstream speed = 6 km/hr

$\therefore$ Speed in still water = $\frac{2+6}{2} = 4$ km/hr

$\therefore$ Time required to go 5 km in still water

$= \frac{5}{4}$ hr = 1 hr 15 min.

23. Ans. (a)

Solution Let the speed of man in still water = x km/hr

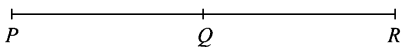
Speed of the current = y km/hr

Speed downstream = $(x + y)$ km/hr

Speed upstream = $(x - y)$ km/hr

Let river be flowing from P to R and $PQ = QR = a$

Then, $PR = 2a$



$\therefore \quad \dfrac{a}{x+y} + \dfrac{a}{x-y} = 10.$ ...(i)

and, $\dfrac{2a}{x+y} = 4$

$\therefore \quad \dfrac{a}{x+y} = 2$ ...(ii)

$\therefore \quad$ (i) $\Rightarrow \dfrac{a}{x-y} = 8$ ...(iii)

Dividing (ii) and (iii), we get

$$\frac{x-y}{x+y} = \frac{1}{4}$$

$\therefore \quad 4x - 4y = x + y$

or, $3x = 5y$

or, $\dfrac{x}{y} = \dfrac{5}{3}$ or 5 : 3.

24. Ans. (c)

Solution Let the speed of the motorboat in still water be x km/hr

$$\frac{10}{x+2} + \frac{10}{x-2} = \frac{55}{60}$$

or, $240x = 11x^2 - 44$

or, $11x^2 - 240x - 44 = 0$

$\therefore \quad (x - 22)(11x + 2) = 0$

So, $x = 22$ km/hr (neglecting the –ve value)

$\therefore$ Speed of the motorboat in still water

= 22 km/hr.

25. Ans. (a)

Solution Let, upstream speed = x km/hr and downstream speed = y km/hr

Then, $\dfrac{30}{x} + \dfrac{44}{y} = 10$, and $\dfrac{40}{x} + \dfrac{55}{y} = 13$

or, $30u + 44v = 10$, and $40u + 55v = 13$

where $u = \dfrac{1}{x}$ and $v = \dfrac{1}{y}$

Solving, we get $u = \dfrac{1}{5}$ and $v = \dfrac{1}{11}$

$\therefore \quad x = 5$ and $y = 11$

$\therefore$ Rate in still water $= \dfrac{5+11}{2} = 8$ km/hr

Rate of current $= \dfrac{11-5}{2} = 3$ km/hr.

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. In one hr a boat goes 11 km along the stream and 5 km against the stream. The speed of the boat in still water (in km/hr) is:
(a) 5 (b) 6
(c) 8 (d) 9

[Based on SSC (GL) Prel. Exam 2000]

2. A boat goes 8 km in one hr along the stream and 2 km in one hr against the stream. The speed of the stream (in km/hr) is
(a) 2 (b) 3
(c) 4 (d) 5

[Based on SSC (GL) Prel. Exam 2000]

3. A man rows a boat 18 km in 4 hr downstream and returns upstream in 12 hr. The speed of the stream (in km/hr)
(a) 1 (b) 1.5
(c) 2 (d) 1.75

[Based on SSC (GL) Prel. Exam 2003]

4. A boat takes 6 hours to travel from place M to N downstream and back from N to M upstream. If the speed of the boat in still water is 4 km/hr, what is the distance between the two places?
(a) 8 km (b) 12 km
(c) 6 km (d) Date inadequate
(e) None of these

[Based on SSC (GL) Prel. Exam 2000]

5. A person can swim at $7\frac{1}{2}$ km/hr in stagnant water. In a river with 1·5 km/hr current, the person swims to a certain distance and comes back within 50 min. What is the distance between the two points?
(a) 3 km (b) 4 km
(c) 1 km (d) 2 km

[Based on RRB Mahendraghat Patna Goods Guard Exam. 2002]

6. A boat takes 9 hr to travel a distance upstream and 3 hr to travel the same distance downstream. If its speed in still water is 4 km/hr, what is the velocity of the stream?
(a) 4 km/hr (b) 3 km/hr
(c) 6 km/hr (d) None of these

[Based on DMRC Exam 2002]

7. A boat goes 20 km downstream in one hr and the same distance upstream in two hr. The speed of the boat in still water is
(a) 15 km/hr (b) 10 km/hr
(c) 5 km/hr (d) 7.5 km/hr

[Based on SSC CPO (SI) Prel. Exam 2003]

8. A boat takes 4 hr for travelling downstream from point A to point B and coming back to point A upstream. If the velocity of the stream is 2 km/hr and the speed of the boat in still water is 4 km/hr, what is the distance between A and B?
(a) 8 km (b) 9 km
(c) 4 km (d) 6 km

[Based on RRB Allahabad ASM Exam 2002]

9. A boat takes 9 hr to travel a distance upstream and takes 3 hr to travel the same distance downstream. If the speed of the boat in still water is 4 km/hr, what is the velocity of the stream?
(a) 4 km/hr (b) 3 km/hr
(c) 6 km/hr (d) 2 km/hr
(e) None of these

[Based on RRB Bhubaneshwar ASM Exam 2002]

10. A boat running downstream covers a distance of 16 km in 2 hr while for covering the same distance upstream it takes 4 hr. What is the speed of the boat in still water?
(a) 4 km/hr (b) 6 km/hr
(c) 8 km/hr (d) Data inadequate
(e) None of these

[Based on SBI Associates Bank PO 2002]

11. A boat has to travel upstream 20 km distance from point X of a river to point Y. The total time taken by boat in travelling from point X to Y and Y to X is 41 min 40 sec. What is the speed of the boat?
(a) 66 km/hr (b) 72 km/hr
(c) 48 km/hr (d) Data inadequate
(e) None of these

[Based on BSRB Hyderabad PO 1999]

12. A boat takes 2 hr to travel from point A to B in still water. To find out its speed upstream, which of the following information is/are required?
A. Distance between point A and B.
B. Time taken to travel downstream from B to A.
C. Speed of the stream of water.
D. Effective speed of boat while travelling downstream from B to A.
(a) all are required
(b) even with all these, the answer cannot be determined
(c) only A, C and either B or D
(d) only A, C and D
(e) None of these

[Based on BSRB Bhopal Bank PO 2000]

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Let speed of the boat in still water = x km/hr
Let speed of the stream = y km/hr

$\therefore \quad x + y = 11$

$x - y = 5$

$\Rightarrow \quad x = 8, y = 3.$

2. Ans. (b)

Solution Let the speed of the stream be y km/hr and speed of the boat in still water be x km/hr.

$\therefore \quad x + y = 8$ and $x - y = 2$

$\Rightarrow \quad y = 3, x = 5.$

3. Ans. (b)

Solution Let x km/hr and y km/hr be respective speeds of boat and stream, then

$$x + y = \frac{18}{4} = 4.50 \quad \text{...(i)}$$

$$x - y = \frac{18}{12} = 1.50 \quad \text{...(ii)}$$

Solving, we get $x = 3$, $y = 1.5$.

4. Ans. (d)

Solution Total time = 6 hr

Speed of the boat in still water = 4 km/hr

Let the distance between M and N be D kms.

Let speed of the stream be x km/hr

$$D\left[\frac{1}{4+x}+\frac{1}{4-x}\right] = 6$$

or, $$D\left[\frac{4-x+4+x}{4^2-x^2}\right] = 6 \quad \text{or,} \quad \frac{8D}{16-x^2} = 6$$

or, $$D = \frac{6}{8}(16-x^2) = \frac{3}{4}(16-x^2)$$

Since, the speed of the stream (x) is not given, the distance D cannot be determined.

5. Ans. (a)

Solution Speed in still water, S = 7.5 km/hr
Speed of current, Z = 1.5 km/hr
Upstream speed = $S - Z$ = 6 km/hr
Downstream speed = $S + Z$ = 9 km/hr
Let the distance between the two points be x km

$\therefore$ Total journey time $= \frac{x}{6}+\frac{x}{9} = \frac{50}{60}$

or, $x\left(\frac{3+2}{18}\right) = \frac{5}{6}$ or $x = \frac{5}{6}\times\frac{18}{5} = 3$ km.

6. Ans. (d)

Solution Let the distance one way be x km and stream speed be Z km/hr.

Then, $x = 9(4 - Z) = 3(4 + Z)$

or, $3(4 - Z) = 4 + Z$ or, $12 - 3Z = 4 + Z$

or, $4Z = 8$ or, $Z = 2$ km/hr.

7. Ans. (a)

Solution Let the speed of the boat in still water $= x$ km/hr and the rate of stream $= y$ km/hr

$\therefore$ Downstream rate $= (x + y)$ km/hr

Upstream rate $= (x - y)$ km/hr

Now, $\frac{20}{x+y} = 1$

or, $x + y = 20$ ...(i)

and, $\frac{20}{x-y} = 2$ or, $x - y = 10$ ...(ii)

Solving (i) and (ii), we get $x = 15$ km/hr.

8. Ans. (b)

Solution Let the distance between A and B be x km

Rate upstream $= 4 - 2 = 2$ km/hr

Rate downstream $= 4 + 2 = 6$ km/hr

According to question,

$\frac{x}{2}+\frac{x}{6} = 4$

or, $\frac{3x+x}{6} = 6$ or, $x = 9$ km.

9. Ans. (d)

Solution Let the velocity of the stream be x km/hr

Then, the speed downstream $= (4 + x)$ km/hr and the speed upstream $= (4 - x)$ km/hr

$\because$ Distance covered downstream and upstream is equal

$\therefore$ $3(4 + x) = 9(4 - x)$

or, $12 + 3x = 36 - 9x$

or, $12x = 24$ or, $x = 2$.

10. Ans. (b)

Solution Rate Upstream $= \frac{16}{2}$km/hr $= 8$ km/hr

Rate downstream $= \frac{16}{4} = 4$ km/hr

$\therefore$ Rate in still water $= \frac{1}{2}(8 + 4) = 6$ km/hr

11. Ans. (d)

Solution Let x be the speed of the boat and y be the speed of the current

$\therefore \frac{20}{x-y}+\frac{20}{x+y} = \frac{25}{36}$

In the equation there are two variables, but only one equation, so, the value of 'x' cannot be determined.

12. Ans. (e)

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. A man can row three quarters of a km against the stream in $11\frac{1}{4}$ min and return in $7\frac{1}{2}$ min. The speed of the man in still water is:
(a) 2 km/hr (b) 3 km/hr
(c) 4 km/hr (d) 5 km/hr

2. The speed of a boat in still water is 12 km/hr and rate of flow of stream is 4 km/hr. If it travels upstream for 3 hr 15 min, find the distance in kms travelled by the boat during this journey.
(a) 24 km (b) 25 km
(c) 26 km (d) 27 km

3. A man can row $7\frac{1}{2}$km an hr in still water and a river is running at $1\frac{1}{2}$km an hr. How far is the place if it takes him 50 min to row to a place and back?
(a) 2 km (b) 2.5 km
(c) 3 km (d) 3.5 km

4. A boat's crew rowed down a stream from A to B and up again in $7\frac{1}{2}$ hr. If the stream flows at 3 km an hr and the crew can row in still water at 5 km an hr, the distance from A to B is
(a) 10 km (b) 9 km
(c) 15 km (d) 12 km

5. A man who can swim 48 m/min in still water swims 200 m against the current and 200 m with the current. If the difference between those two times is 10 min, find the speed of the current in m per min.
(a) 30 m/min (b) 31 m/min

(c) 29 m/min (d) 32 m/min

6. A boat travels upstream from Q to P and downstream from P to Q in 3 hr. If the distance between P to Q is 4 km and the speed of the current is 1 km/hr, then what is the speed of the boat in still water?
(a) 4.5 km/hr (b) 5.2 km/hr
(c) 2.5 km/hr (d) 3 km/hr
(e) None of these

7. A swimmer can swim a certain distance in the direction of current in 5 hr and return the same distance in 7 hr. If the stream flows at the rate of 1 km/hr, find the speed of the swimmer in still water.
(a) 4 km/hr (b) 5 km/hr
(c) 6 km/hr (d) 7 km/hr

8. A man rowed against a stream flowing $1\frac{1}{2}$ km/hr to a certain point and then turned back, stopping 2 km, short of the place where he originally started. If the whole time occupied in rowing be 2 hr 10 min and his uniform speed in still water be $4\frac{1}{2}$ km/hr, the man went up the stream a distance of:
(a) 7 km (b) 3.5 km
(c) 4 km (d) 5 km

9. A man rows to a place 48 km distant and back in 14 hr. He finds that he can row 4 km with the stream in the same time as 3 km against the stream. The rate of the stream is
(a) 0.5 km/hr (b) 1 km/hr
(c) 3.5 km/hr (d) 1.8 km/hr

10. On a river, B is intermediate to and equidistant from A and C. If a boat can go from A to B and back in 5 hr 15 min and from A to C in 7 hr, it would go from C to A in
(a) 3 hr 30 min (b) 3 hr 45 min
(c) 2 hr 20 min (d) 2 hr 55 min

Answers

1. (d)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (d)	**6.** (d)	**7.** (c)	**8.** (d)	**9.** (b)	**10.** (a)

17

ALLIGATION OR MIXTURE

INTRODUCTION

Alligation literally means "*linking*". It is a rule to find:

(a) the ratio in which two or more ingredients at their respective prices should be mixed to give a mixture at a given price.

(b) The mean or average price of a mixture when the prices of two or more ingredients which may be mixed together and the proportion in which they are mixed are given.

Here, cost price of a unit quantity of mixture is called the *mean price*.

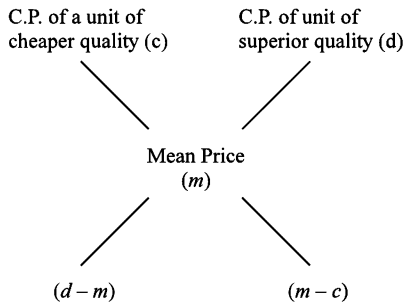
Alligation Rule

Suppose, Rs. d per unit be the price of first ingredient (superior quality) mixed with another ingredient (cheaper quality) of price Rs. c per unit to form a mixture whose mean price is Rs. m per unit, then the two ingredients must be mixed in the ratio:

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of superior}} = \frac{\text{C. P. superior} - \text{Mean price}}{\text{Mean price} - \text{C. P. of cheaper}}$$

i.e. the two ingredients are to be mixed in the inverse ratio of the differences of their prices and the mean price.

The above rule may be represented schematically as under:



$$\frac{\text{Quantity of cheaper quality}}{\text{Quantity of superior quality}} = \frac{d-m}{m-c}$$

Explanation

Suppose x kg of cheaper quality is mixed with y kg of superior quality.

Price of cheaper ingredient = Rs. cx

Price of superior ingredient = Rs. dy

$\therefore$ Price of mixture = Rs. $(cx + dy)$

and quantity of mixture = $(x + y)$ kg.

$\therefore$ Price of mixture/kg = Rs. $\left(\frac{cx+dy}{x+y}\right)$

$$\therefore \frac{cx + dy}{x + y} = m \Rightarrow cx + dy = mx + my$$

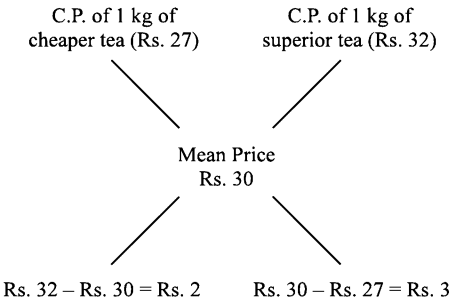
$$\Rightarrow dy - my = mx - cx$$

$$\Rightarrow y(d - m) = x(m - c)$$

$$\Rightarrow \frac{x}{y} = \frac{d-m}{m-c}.$$

Illustration 1 In what ratio two varieties of tea one costing Rs. 27 per kg and the other costing Rs. 32 per kg should be blended to produce a blended variety of tea worth Rs. 30 per kg. How much should be the quantity of second variety of tea, if the first variety is 60 kg.

Solution



The required ratio of the two varieties of tea is 2 : 3, i.e.

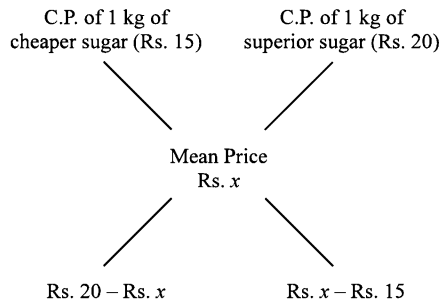
$$\frac{\text{Quantity of cheaper tea}}{\text{Quantity of superior tea}} = \frac{2}{3}$$

$\therefore$ Quantity of superior tea $= \frac{60\times 3}{2} = 90$ kg

Thus, the second variety of tea is 90 kg.

Illustration 2 Sugar at Rs. 15 per kg is mixed with sugar at Rs. 20 per kg in the ratio 2 : 3. Find the price per kg of the mixture.

Solution Let the mean price of the mixture be Rs. x



$$\frac{\text{Quantity of cheaper sugar}}{\text{Quantity of dearer sugar}} = \frac{20-x}{x-15}$$

$$\therefore \quad \frac{20-x}{x-15} = \frac{2}{3} \Rightarrow 60 - 3x = 2x - 30$$

$$\Rightarrow 5x = 90 \text{ or } x = 18.$$

Thus, the price per kg of the mixture is Rs. 18.

SOME USEFUL METHODS

1. A vessel, full of wine, contains 'a' litres of it of which 'b' litres are withdrawn. The vessel is then filled with water. Next, 'b' litres of the mixture are withdrawn and again the vessel is filled with water. This process is repeated n times. Then,

$$\frac{\text{Wine left in the vessel after } n\text{th operation}}{\text{Original quantity of wine in the vessel}} = \left(\frac{a-b}{a}\right)^n.$$

Explanation

Amount of wine after 1st operation

$$= a - b = \left(1-\frac{b}{a}\right)\times a$$

Ratio of wine and water after 1st operation is $(a - b) : b$.

$\therefore$ In 'b' litres of mixture withdrawn in 2nd operation,

amount of wine withdrawn

$$= \frac{a-b}{(a-b)+b}\times b = (a-b)\frac{b}{a}$$

$\therefore$ Amount of wine left after 2nd operation

$$= (a-b) - (a-b)\times\frac{b}{a} = \left(1-\frac{b}{a}\right)^2 a$$

In general, quantity of wine left after nth operation

$$= \left(1-\frac{b}{a}\right)^n a$$

$$\therefore \quad \frac{\text{Wine left after } n\text{th operation}}{\text{Original quantity of wine}} = \left(1-\frac{b}{a}\right)^n.$$

Illustration 3 A vessel contains 125 litres of wine. 25 litres of wine was taken out of the vessel and replaced by water. Then, 25 litres of mixture was withdrawn and again replaced by water. The operation was repeated for third time. How much wine is now left in the vessel?

Solution Amount of wine left in the vessel

$$= \left(1-\frac{25}{125}\right)^3 \times 125 = \frac{100\times100\times100\times125}{125\times125\times125} = 64 \text{ litres.}$$

2. There are n vessels of equal size filled with mixtures of liquids A and B in the ratio $a_1 : b_1 : a_2, b_2 \ldots, a_n : b_n$, respectively. If the contents of all the vessels are poured into a single large vessel, then

$$\frac{\text{Quantity of liquid } A}{\text{Quantity of liquid } B} = \frac{\left(\frac{a_1}{a_1+b_1}+\frac{a_2}{a_2+b_2}+\ldots+\frac{a_n}{a_n+b_n}\right)}{\left(\frac{b_1}{a_1+b_1}+\frac{b_2}{a_2+b_2}+\ldots+\frac{b_n}{a_n+b_n}\right)}$$

Explanation

Let the capacity of each vessel be c litres.

Amount of liquid A in different vessels

$$= \frac{a_1c}{a_1+b_1}, \frac{a_2c}{a_2+b_2}, \frac{a_3c}{a_3+b_3}, \ldots, \frac{a_nc}{a_n+b_n}$$

Amount of liquid B in different vessels

$$= \frac{b_1c}{a_1+b_1}, \frac{b_2c}{a_2+b_2}, \frac{b_3c}{a_3+b_3}, \ldots \frac{b_nc}{a_n+b_n}$$

So, in the resulting mixture, amount of liquid A

$$= \left(\frac{a_1}{a_1+b_1}+\frac{a_2}{a_2+b_2}+\ldots+\frac{a_n}{a_n+b_n}\right)\times c$$

Amount of liquid B

$$= \left(\frac{b_1}{a_1+b_1}+\frac{b_2}{a_2+b_2}+\ldots+\frac{b_n}{a_n+b_n}\right)\times c$$

$$\frac{\text{Quantity of liquid } A}{\text{Quantity of liquid } B} = \frac{\left(\frac{a_1}{a_1+b_1}+\frac{a_2}{a_2+b_2}+\ldots+\frac{a_n}{a_n+b_n}\right)}{\left(\frac{b_1}{a_1+b_1}+\frac{b_2}{a_2+b_2}+\ldots+\frac{b_n}{a_n+b_n}\right)}.$$

Illustration 4 Three equal glasses are filled with mixture of milk and water. The proportion of milk and water in each glass is as follows: In the first glass as 3 : 1, in the second glass as 5 : 3 and in the third as 9 : 7. The contents of the three glasses are emptied into a single vessel. What is the proportion of milk and water in it?

Solution

$$\frac{\text{Quantity of milk}}{\text{Quantity of water}} = \frac{\frac{3}{3+1}+\frac{5}{5+3}+\frac{9}{9+7}}{\frac{1}{3+1}+\frac{3}{5+3}+\frac{7}{9+7}} = \frac{31/16}{17/16}$$

$$= 31 : 17.$$

3. There are n vessels of sizes $c_1, c_2, \ldots, c_n$ filled with mixtures of liquids A and B in the ratio $a_1 : b_1, a_2 : b_2, \ldots, a_n : b_n$, respectively. If the contents of all the vessels are poured into a single large vessel, then

$$\frac{\text{Quantity of Liquid } A}{\text{Quantity of liquid } B} = \frac{\frac{a_1c_1}{a_1+b_1}+\frac{a_2c_2}{a_2+b_2}+\ldots+\frac{a_nc_n}{a_n+b_n}}{\frac{b_1c_1}{a_1+b_1}+\frac{b_2c_2}{a_2+b_2}+\ldots+\frac{b_nc_n}{a_n+b_n}}$$

Illustration 5 Three glasses of sizes 3 litres, 4 litres and 5 litres contain mixture of milk and water in the ratio 2 : 3, 3 : 7 and 4 : 11, respectively. The contents of all the three glasses are poured into a single vessel. Find the ratio of milk to water in the resulting mixture.

Solution

$$\frac{\text{Quantity of milk}}{\text{Quantity of water}} = \frac{\left(\frac{2\times3}{2+3}+\frac{3\times4}{3+7}+\frac{4\times5}{4+11}\right)}{\left(\frac{3\times3}{2+3}+\frac{7\times4}{3+7}+\frac{11\times5}{4+11}\right)}$$

$$= \frac{\frac{6}{5}+\frac{12}{10}+\frac{20}{15}}{\frac{9}{5}+\frac{28}{10}+\frac{55}{15}} = 56 : 124 \text{ or, } 14 : 31.$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. How many kg of tea worth Rs. 25 per kg must be blended with 30 kg of tea worth Rs. 30 per kg so that by selling the blended variety at Rs. 30 per kg there should be a gain of 10%?
(a) 36 kg (b) 40 kg
(c) 32 kg (c) 42 kg

2. How much water be added to 14 litres of milk worth Rs. 5.40 a litre so that the value of the mixture may be Rs. 4. 20 a litre?
(a) 7 litres (b) 6 litres
(c) 5 litres (d) 4 litres

3. In what ratio two varieties of tea, one costing Rs. 25 per kg and the other costing Rs. 30 per kg should be blended to produce blended variety of tea worth Rs. 28 per kg?
(a) 3 : 4 (b) 4 : 3
(c) 2 : 3 (d) 3 : 5

4. In an examination out of 80 students 85% of the girls and 70% of the boys passed. How many boys appeared in the examination if total pass percentage was 75%?
(a) 370 (b) 340
(c) 320 (d) 360

5. In what proportion must tea worth 75 paise per packet be mixed with tea worth Rs. 5.50 per packet so that the mixture may cost Rs. 4.50 per packet?
(a) 3 : 11 (b) 4 : 15
(c) 15 : 11 (d) 4 : 5

6. How many kg of sugar costing Rs. 5.50 per kg must be mixed with 60 kg of sugar costing Rs. 4.80 per kg so that the mixture is worth Rs. 5.25 per kg?
(a) 90 kg (b) 95 kg
(c) 108 kg (e) 106 kg

7. How many kg of sugar costing Rs. 5.75 per kg should be mixed with 75 kg of cheaper sugar costing Rs. 4.50 per kg so that the mixture is worth Rs. 5.50 per kg?
(a) 250 kg (b) 300 kg
(c) 350 kg (d) 325 kg

8. In what ratio must water be added to spirit to gain 10% by selling it at the cost price?
(a) 1 : 11 (b) 1: 5
(c) 1 : 10 (d) 1 : 9

9. 300 gm of salt solution has 40% salt in it. How much salt should be added to make it 50% in the solution?
(a) 40 gm (b) 60 gm
(c) 70 gm (d) 80 gm

10. A man buys two cows for Rs. 1350 and sells one so as to lose 6% and the other so as to gain 7.5% and on the whole he neither gains nor loses. What does each cow cost?

(a) 36, 39 (b) 42, 45
(c) 39, 36 (d) 45, 48

11. There are 65 students in a class, 39 rupees are distributed among them so that each boy gets 80 P and girl gets 30 P. Find the number of boys and girls in that class.
(a) 43, 40 (b) 36, 33
(c) 39, 26 (d) 45,42

12. A trader has 50 kg of sugar, a part of which he sells at 10 % profit and the rest at 5% loss. He gains 7% on the whole. What is the quantity sold at 10% gain and 5% loss ?
(a) 40 kg, 10 kg (b) 10 kg, 35 kg
(c) 25 kg, 15 kg (d) 30 kg, 20 kg

13. A person has Rs. 5000. He invests a part of it at 3% per annum and the remainder at 8% per annum simple interest. His total income in 3 years is Rs. 750. Find the sum invested at different rates of interest.
(a) Rs. 2000, Rs. 1000 (b) Rs. 3000, Rs. 1000
(c) Rs. 1000, Rs. 4000 (d) Rs. 3000, Rs. 2000

14. Some amount out of Rs. 7000 was lent at 6% p.a. and the remaining at 4% p.a. If the total simple interest from both the fractions in 5 years was Rs. 1600, the sum lent at 6% p.a. was
(a) Rs. 3000 (b) Rs. 4000
(c) Rs. 5000 (d) Rs. 2000

15. 729 ml of a mixture contains milk and water in the ratio 7 : 2. How much more water is to be added to get a new mixture containing milk and water in the ratio 7 : 3 ?
(a) 600 ml (b) 710 ml
(c) 520 ml (d) None of these

16. In what proportion water must be added to spirit to gain 20 % by selling it at the cost price ?
(a) 1 : 5 (b) 2 : 5
(c) 3 : 5 (d) 4 : 5

17. The average monthly salary of employees, consisting of officers and workers of an organisation is Rs. 3000. The average salary of an officer is 10000 while that of a worker is Rs. 2000 per month. If there are total 400 employees in the organisation, find the number of officers and workers separately.
(a) 50, 275 (b) 350, 450
(c) 50, 350 (d) 325, 350

18. A person covers a distance 100 kms in 10 hr, partly by walking at 7 km per hr and rest by running at 12 km per hr. Find the distance covered in each part.
(a) 28 km, 72 km (b) 32 km, 82 km
(c) 24 km, 68 km (d) 26 km, 70 km

19. The average weekly salary per head of all employees (supervisors and labourers) is Rs. 100. The average weekly salary per head of all the supervisors is Rs. 600 while the average weekly salary per head of all the labourers is Rs. 75. Find the number of supervisors in the factory if there are 840 labourers in it.
(a) 46 (b) 42
(c) 44 (d) 48

20. A person has a chemical of Rs. 25 per litre. In what ratio should water be mixed in that chemical so that after selling the mixture at Rs 20/litre he may get a profit of 25% ?
(a) 13 : 16 (b) 16 : 19
(c) 12 : 15 (d) 19 : 22

21. A person travels 285 km in 6 hr in two stages. In the first part of the journey, he travels by bus at the speed of 40 km per hr. In the second part of the journey, he travels by train at the speed of 55 km per hr. How much distance did he travel by train?
(a) 205 km (b) 145 km
(c) 165 km. (d) 185 km

22. A trader has 50 kg of pulses, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. What is the quantity sold at 18% profit?
(a) 30 kg (b) 25 kg
(c) 20 kg (d) 40 kg

23. A trader has 50 kg of rice, a part of which he sells at 10% profit and the rest at 5% loss. He gains 7% on the whole. What is the quantity sold at 10% gain and 5% loss?
(a) 30 kg, 10 kg (b) 40 kg, 15 kg
(c) 35 kg, 40 kg (d) 40 kg, 10 kg

24. Mira's expenditure and savings are in the ratio 3 : 2. Her income increases by 10%. Her expenditure also increases by 12%. By how many % does her saving increase?
(a) 7% (b) 10%
(c) 9% (d) 13%

EXPLANATORY ANSWERS

1. Ans. (a)

Solution S.P. = Rs. 30 per kg
Gain = 10 %

$$\text{C.P.} = \text{Rs. } \frac{30\times100}{110} = \text{Rs. } \frac{300}{11}$$

C.P. of 1 kg of cheaper tea (Rs. 25) | C.P. of 1 kg of superior tea (Rs. 30)

Mean Price Rs. $\frac{300}{11}$

$\text{Rs.}30 - \text{Rs.}\frac{300}{11} = \text{Rs.}\frac{30}{11}$ $\quad$ $\text{Rs.}\frac{300}{11} - \text{Rs.}25 = \text{Rs.}\frac{25}{11}$

The required ratio of the two varieties of tea

$$= \frac{30}{11} : \frac{25}{11} = 6 : 5$$

$$\frac{\text{Quantity of cheaper tea}}{\text{Quantity of superior tea}} = \frac{6}{5}$$

Quantity of cheaper tea $= \frac{6}{5} \times 30 = 36$ kg.

2. Ans. (d)

Solution

Water 0 | Milk 540

420

120 ⋮ 420

2 ⋮ 7

Water is free, its cost can be taken as 0 paisa per litre. By method of alligation the ratio of water and milk is 2 : 7, i.e. with 7 litres of milk, 2 litres of water is added, with 14 litres, water added is 4 litres.

3. Ans. (c)

Solution

C.P. of 1 kg of cheaper tea (Rs. 25) | C.P. of 1 kg of superior tea (Rs. 30)

Mean Price Rs. 28

Rs. 30 – Rs. 28 = Rs. 2 | Rs. 28 – Rs. 25 = Rs. 3

∴ The required ratio of two varieties of tea is 2 : 3.

4. Ans. (c)

Solution

Pass % of boys (70%) | Pass % of girls (85%)

Average 75%

10 | 5

By Alligation rule:

$$\frac{\text{Number of boys}}{\text{Number of girls}} = \frac{10}{5} = \frac{2}{1}$$

Total number of students = 480

Number of boys $= \frac{480 \times 2}{3} = 320.$

5. Ans. (b)

Solution By method of alligation the required ratio is 4 : 15

75 *P* | 550 *P*

450 *P*

550 – 450 ⋮ 450 – 75

100 ⋮ 375

4 ⋮ 15

6. Ans. (c)

Solution

4.80 | 5.50

5.25

Rs. (5.50 – 5.25) = Rs. 0.25 | Rs. (5.25 – 4.80) = Rs. 0.45

Now, $\frac{.25}{.45} = \frac{5}{9}$

$\therefore \frac{5}{9} = \frac{60}{\text{superior sugar}}$

$\therefore$ Quantity of superior sugar $= \frac{60 \times 9}{5} = 108$ kg.

7. Ans. (b)

Solution

C.P. of 1 kg of cheaper sugar (Rs. 4.50) | C.P. of 1 kg of superior sugar (Rs. 5.75)

Mean Price (Rs. 5.50)

Rs. 5.75 – Rs. 5.50 = Rs. 0.25 | Rs. 5.50 – Rs. 4.50 = Rs. 1.00

Quantity of cheaper sugar : Quantity of superior sugar = 0.25 : 1

or, $\frac{\text{Quantity of cheaper sugar}}{\text{Quantity of superior sugar}} = \frac{0.25}{1} = \frac{1}{4}$

$$\Rightarrow \quad \frac{75}{\text{Quantity of dearer sugar}} = \frac{1}{4}$$

$\therefore$ Quantity of dearer sugar = 75 × 4 = 300 kg.

8. Ans. (c)

Solution Let the C.P. of spirit = Rs. 10 per litre

S.P. of the mixture = Rs. 10 per litre

Profit = 10 %

$\therefore$ C.P. of the mixture $= \dfrac{10 \times 100}{110}$

$= \text{Rs. } \dfrac{100}{11}$ per litre

C.P. of water per litre Re. 0 | C.P. of pure spirit per litre (Rs. 10)

Mean Price Rs. $\frac{100}{11}$

Rs. $10 - \text{Rs. } \frac{100}{11} = \text{Rs. } \frac{10}{11}$ | Rs. $\frac{100}{11} - 0 = \text{Rs. } \frac{100}{11}$

$$\therefore \quad \frac{\text{Quantity of water}}{\text{Quantity of spirit}} = \frac{10/11}{100/11} = 1 : 10.$$

9. Ans. (b)

Solution The existing solution has 40% salt. More salt is to be mixed to make 100% salt solution. So, by alligation method:

40% | 100%

50%

50% | 10%

$\therefore$ The two mixtures should be added in the ratio 5 : 1

$\therefore$ Required salt $= \dfrac{300}{5} \times 1 = 60$ gm.

10. Ans. (b)

Solution 1st cow – 6% | 2nd cow 7.5%

0

7.5 | 6

Thus, we see that the ratio of the cost of two cows is 5 : 4

$\therefore$ Cost of 1st cow $= \dfrac{1350}{5+4} \times 5 = \text{Rs. } 750$

and, cost of 2nd cow $= \dfrac{1350}{5+4} \times 4 = \text{Rs. } 600.$

11. Ans. (c)

Solution Here, alligation is applicable for "money per boy or girl".

Mean value of money per student $= \dfrac{3900}{65} = 60\,P.$

Boys 80 | Girls 30

60

30 | 20

$\therefore$ Boys : Girl = 3 : 2

$\therefore$ Number of boys $= \dfrac{65}{3+2} \times 3 = 39$

and, number of girls = 65 – 39 = 26.

12. Ans. (a)

Solution I part 10 | II part (–) 5

7

12 | 3

$\therefore$ Ratio of quantities sold at 10% profit and 5% loss = 12 : 3 = 4 : 1

$\therefore$ The quantity sold at 10% profit $= \dfrac{50}{4+1} \times 4$

$= 40$ kg

and, the quantity sold at 5% loss = 50 – 40 = 10 kg.

13. Ans. (d)

Solution Average rate of interest

$$= \frac{100 \times 750}{5000 \times 3} = 5\% \text{ per annum}$$

3 | 8

5

3 | 2

Investment at 3% per annum

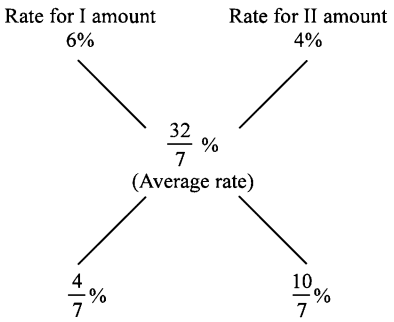
$$= \frac{3}{3+2} \times 5000 = \text{Rs. } 3000$$

Investment at 8% per annum

$$= \frac{2}{3+2} \times 5000 = \text{Rs. } 2000.$$

14. Ans. (d)

Solution Overall rate of interest $= \frac{1600 \times 100}{5 \times 7000} = \frac{32}{7}\%$



$\therefore$ Ratio of two amounts = 2 : 5

$\therefore$ Amount lent at 6% $= \frac{2}{7} \times 7000 = \text{Rs. } 2000.$

15. Ans. (d)

Solution Milk $= \left(729 \times \frac{7}{9}\right) = 567$ ml

Water = 729 – 567 = 162 ml

Now, $\frac{567}{162 + x} = \frac{7}{3} \Rightarrow x = 81$ ml.

16. Ans. (a)

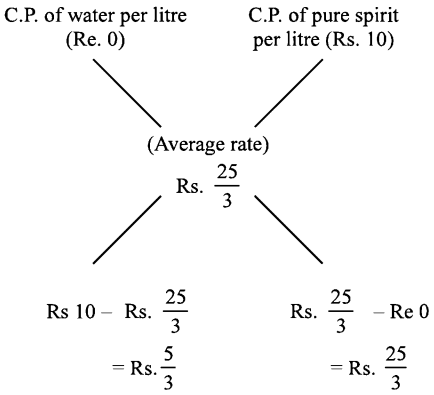
Solution Let the C. P. of the spirit = Rs. 10 per litre

S.P. of the mixture = Rs. 10 per litre

Profit = 20%

$\therefore$ C. P. of the mixture = Rs. $\frac{10 \times 100}{120}$

$= \text{Rs.} \frac{25}{3}$ per litre

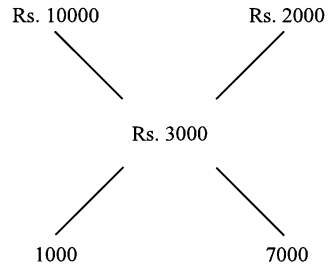


$$\frac{\text{Quantity of water}}{\text{Quantity of spirit}} = \frac{5/3}{25/3} = \frac{1}{5}$$

Ratio of water and spirit = 1 : 5.

17. Ans. (c)

Solution



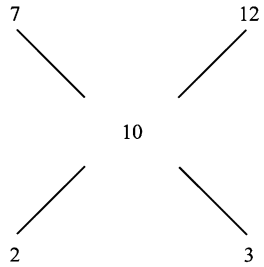
$$\frac{\text{Number of officers}}{\text{Number of workers}} = \frac{1000}{7000} = \frac{1}{7}$$

Number of officers $= \frac{1}{1 + 7} \times 400 = 50$

Numbers of worker = 400 – 50 = 350.

18. Ans. (a)

Solution Average speed $= \frac{100}{10} = 10$ kms/hr



Ratio of time taken at 7 kms per hr to 12 km per hr = 2 : 3

Time taken at 7 km per hr

$$= \frac{2}{2 + 3} \times 10 = 4 \text{ hr.}$$

Distance covered at 7 km per hr

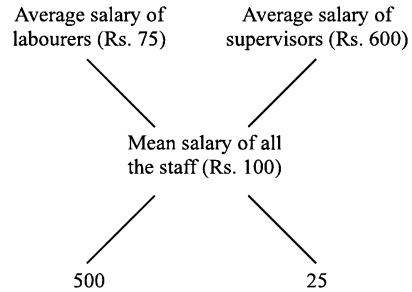
= 7 × 4 = 28 km.

Distance covered at 12 km per hr

= 100 – 28 = 72 km.

19. Ans. (b)

Solution



The required ratio is 500 : 25 or 20 : 1

$$\frac{\text{Number of labourers}}{\text{Number of supervisors}} = \frac{20}{1}$$

$$\Rightarrow \quad \frac{840}{\text{Number of supervisors}} = \frac{20}{1}$$

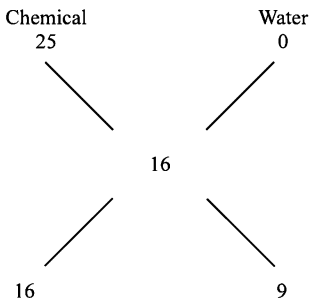
$$\therefore \quad \text{Number of supervisors} = \frac{840}{20} = 42.$$

20. Ans. (b)

Solution In this question the alligation method is applicable on prices, so we should get the average price of mixture.

S.P. of mixture = Rs. 20/litre, profit = 25%

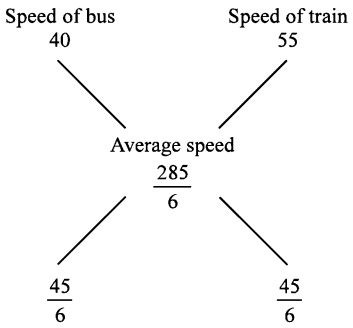
$$\therefore \quad \text{Average price} = 20 \times \frac{100}{125} = \text{Rs. } 16/\text{litre}$$



$\therefore$ Chemical : water = 16 : 9.

21. Ans. (c)

Solution In this question, the alligation method is applicable for the speed.



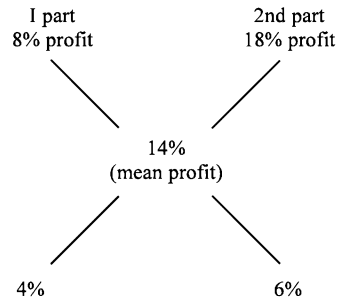
$\therefore$ Time spent in bus : time spent in train

$$= \frac{45}{6} : \frac{45}{6} = 1 : 1.$$

$\therefore$ Distance travelled by train = 55 × 3 = 165 km.

22. Ans. (a)

Solution



$\therefore$ Ratio of quantities sold at 8% profit and 18% profit

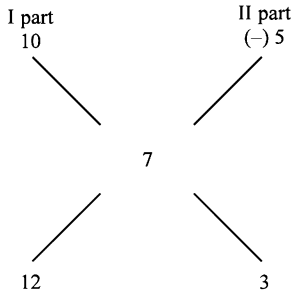
= 4 : 6 = 2 : 3

Therefore, the quantity sold at 18% profit

$$= \frac{50}{2 + 3} \times 3 = 30 \text{ kg.}$$

23. Ans. (d)

Solution



$\therefore$ Ratio of quantities sold at 10 % profit and 5% loss = 12 : 3 = 4 : 1

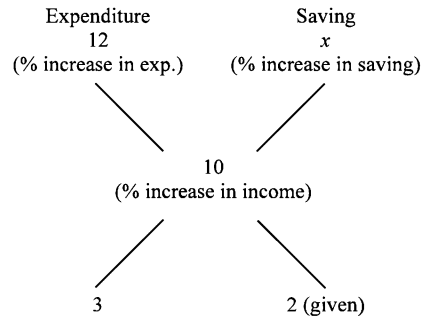
$\therefore$ The quantity sold at 10% profit

$$= \frac{50}{4 + 1} \times 4 = 40 \text{ kg}$$

and, the quantity sold at 5% loss = 50 – 40 = 10 kg.

24. Ans. (a)

Solution



We get two values of x, 7 and 13. But to get a viable answer, we must keep in mind that the central value (10) must lie between x and 12. Thus, the value of x should be 7 and not 13.

$\therefore$ Required % increase = 7%.

QUESTIONS FROM COMPETITIVE EXAMS

1. 40 litres of mixture of milk and water contains 10 % of water. The water to be added to make the water content 20% in the new mixture is
(a) 6 litres (b) 6.5 litres
(c) 5.5 litres (d) 5 litres
[Based on SSC (GL) Prel. Exam 2003]

2. A mixture contains spirit and water is the ratio 3 : 2. If it contains 3 litres more spirit than water, the quantity of spirit in the mixture is
(a) 10 litres (b) 12 litres
(c) 8 litres (d) 9 litres
[Based on SSC(GL) Prel. Exam 2003]

3. Two types of steel are available. Type A contains 5% of nickel and type B contains 40% of nickel. How much of each type be mixed to obtain 140 tonnes of steel containing 30% of nickel?
(a) 40, 100 tonnes (b) 50, 125 tonnes
(c) 30, 75 tonnes (d) 60, 180 tonnes
[Based on SSC (GL) Asstt. Grade Main Exam 2000]

4. Two alloys A and B contain silver and copper in the ratio 5 : 1 and 7 : 2, respectively. Find the ratio in which they be mixed such that there be 80% of silver.
(a) 3 : 2 (b) 2 : 3
(c) 1 : 3 (d) 3 : 4
[Based on SSC Asstt. Grade Main Exam 1998]

5. In what ratio water should be mixed with a liquid at Rs. 12 per litre so that by selling the mixture at Rs. 13.75 per litre, the seller gains 25% ?
(a) 1 : 11 (b) 1 : 15
(c) 1 : 17 (d) 1 : 13
[Based on SSC Assistant Grade Main Exam 1992]

6. A tea seller mixes two kinds of tea one at Rs. 15 kg and other at Rs. 20 per kg. In what ratio should he mix them to get the price of mixture at Rs. 16.50 per kg?
(a) 8 : 3 (b) 5 : 7
(c) 7 : 3 (d) 7 : 5
[Based on SSC UDC Exam 1996]

7. A tea producer mixes two categories of tea from two gardens is the ratio 5 : 3. The cost price of one category is Rs. 27 per kg and that of other is Rs. 30 per kg. The mixture is sold at Rs 30.25 a kg. Find his gain per cent.
(a) $8\frac{5}{3}$ (b) $7\frac{5}{9}$
(c) $9\frac{5}{9}$ (d) $11\frac{5}{9}$
[Based on SSC Auditor's Exam 1994]

8. A jar full of whisky contains 40% of alcohol. A part of this whisky is replaced by another containing 19% alcohol and now the percentage of alcohol was found to be 26. The quantity of whisky replaced is
(a) $\frac{2}{5}$ (b) $\frac{1}{3}$
(c) $\frac{2}{3}$ (d) $\frac{3}{5}$
[Based on Hotel Management 1991]

9. In a mixture of 60 litres, the ratio of milk and water is 2 : 1. If the ratio of the milk and water is to be 1 : 2, then the amount of water to be further added is
(a) 20 litres (b) 30 litres
(c) 40 litres (d) 60 litres
[Based on NDA Exam 1990]

10. A solution of sugar syrup has 15% sugar. Another solution has 5% sugar. How many litres of the second solution must be added to 20 litres of the first solution to make a solution of 10% sugar?
(a) 10 (b) 5
(c) 15 (d) 20
[Based on LIC AAO Exam 1988]

11. A part of sum of Rs. 10000 is lent at 8% and the remaining sum at 10% per annum. If the average rate of interest is 9.2%, then the two parts are
(a) Rs. 4000, Rs. 6000 (b) Rs. 5500, Rs. 4500
(c) Rs. 5000, Rs. 5000 (d) Rs. 5500, Rs. 4500
[Based on SSC (GL) Prel. Exam 1993]

12. Zinc and copper are in the ratio of 5 : 3 in 200 gm of an alloy. How much gm of copper be added to make the ratio as 3 : 5?
(a) $133\frac{1}{3}$ (b) $\frac{1}{200}$
(c) 72 (d) 66
[Based on SSC (GL) Prel. Exam 2002]

13. The wheat sold by a grocer contained 10% low quantity wheat. What quantity of good quality wheat should be added to 150 kg. of wheat so that the percentage of low quality wheat become 5%?
(a) 150 kg (b) 135 kg
(c) 50 kg (d) 85 kg
(e) None of these
[Based on SBI Associate Banks PO Exam 2002]

14. Tea at Rs. 126 per kg and at Rs. 135 per kg are mixed with a third variety in the ratio 1 : 1 : 2.
If the mixture is worth Rs. 153 per kg, the price of the third variety per kg is
(a) Rs. 169.50 (b) Rs. 175.50
(c) Rs. 175 (d) Rs. 185
[Based on SSC CPO (SI) Exam 2003]

15. A shopkeeper sells milk which contains 5% water. What quantity of pure milk should be added to 2 litres of milk (containing 5% water) so that proportions of water become 4%?

(a) 1 litre (b) 2 litres
(c) 0.5 litre (d) None of these

[Based on SBI Mumbai Bank PO Exam 2000]

16. What quantity of water should be added to 3 litres of 10% solution of salt, so that it becomes 5% salt solution?
(a) 1.5 litres (b) 2.7 litres
(c) 3 litres (d) Cannot be determined
(e) None of these

[Based on ECGC of India Ltd., Bank PO Exam 2001]

17. A container contains 10 litres mixture in which there is 10% sulphuric acid. How much sulphuric acid is to be added to make the solution to contain 25% sulphuric acid?
(a) 2 litres (b) 1 litre
(c) 4 litres (e) Data inadequate
(e) None of these

[Based on BSRB Baroda Bank Clerical Exam 2000]

18. A painter mixes blue paint with white paint so that the mixture contains 10% blue paint. In a mixture of 40 litres paint how many litres blue paint should be added so that the mixture contains 20% of blue paint?
(a) 2.5 litres (b) 4 litres
(c) 5 litres (d) 2 litres
(e) None of these

[Based on BSRB Lucknow Bank Clerical Exam 2000]

19. A mixture of 66 litres of milk and water are in the ratio of 5 : 1, water is added to make the ratio 3 :5. Find the quantity of water added.
(a) 20 litres (b) 18 litres
(c) 22 litres (d) 24 litres

[Based on LIC AAO Exam 1998]

20. A petrol pump owner mixed leaded and unleaded petrol in such a way that the mixture contains 10% unleaded petrol. What quantity of leaded petrol should be added to 1 litre mixture so that the percentage of unleaded petrol becomes 5%?
(a) 1000 ml (b) 900 ml
(c) 1900 ml (d) 1800 ml

[Based on SBI Associates PO 1999]

21. Kantilal mixes 80 kg of sugar worth of Rs. 6.75 per kg with 120 kg worth of Rs. 8 per kg. At what rate shall he sell the mixture to gain 20%?
(a) Rs. 7.50 (b) Rs. 9
(c) Rs. 8.20 (d) Rs. 8.85

[Based on BSRB Bhopal 1998]

22. Jaydeep purchased 25 kg of rice at the rate of Rs. 16.80 per kg and 35 kg of rice at the rate of Rs. 25.50 per kg. He mixed the two and sold the mixture. Approximately, at what price per kg did he sell the mixture to make 25 percent profit?
(a) Rs. 26.50 (b) Rs. 27.50
(c) Rs. 28.50 (d) Rs. 30.00

[Based on BSRB Mumbai PO 1998]

23. Jagtap purchases 30 kg of wheat at the rate of Rs. 11.50 per kg and 20 kg of wheat at the rate of Rs. 14.25 per kg. He mixed the two and sold the mixture. Approximately at what price per kg should he sell the mixture to make 30 percent profit?
(a) Rs. 16.30 (b) Rs. 18.20
(c) Rs. 15.60 (d) Rs. 14.80

[Based on BSRB Calcutta PO 1999]

24. Prabha purchased 30 kg of rice at the rate of Rs. 17.50 per kg and another 30 kg rice at a certain rate. He mixed the two and sold the entire quantity at the rate of Rs. 18.60 per kg and made 20 per cent overall profit. At what price per kg did he purchase the lot of another 30 kg. rice?
(a) Rs. 14.50 (b) Rs. 12.50
(c) Rs. 15.50 (d) Rs. 13.50

[Based on BSRB Chennai PO 2000]

25. A grocer purchased 20 kg of rice at the rate of Rs. 15 per kg and 30 kg of rice at the rate of 13 per kg. At what price per kg should he sell the mixture to earn $33\frac{1}{3}$% profit on the cost price?
(a) Rs. 28.00 (b) Rs. 20.00
(c) Rs. 18.40 (d) Rs. 17.40

[Based on BSRB Delhi PO 2000]

EXPLANATORY ANSWERS

1. Ans. (d)

Solution Amount of milk and water in the mixture is 36 litres and 4 litres, respectively.

Now, let x litres of water be added

20% of $(40 + x) = 4 + x$

or, $40 + x = 20 + 5x$

or, $4x = 20$ or, $x = 5$ litres

2. Ans. (d)

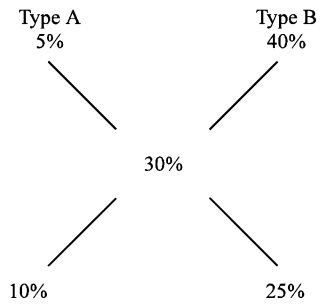
3. Ans. (a)

Solution Nickel in Type A = 5%

Nickel in Type B = 40%

Nickel in resulting steel = 30%.

By Alligation rule



Type A : Type B = 10 : 25 = 2 : 5

We have to obtain 140 tonnes of steel

$\therefore$ Steel of type $A = \frac{2}{7} \times 140 = 40$ tonnes

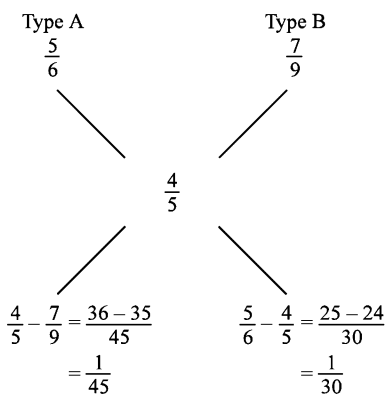
Steel of type $B = \frac{5}{7} \times 140 = 100$ tonnes.

4. Ans. (b)

Solution Resulting alloy has 80% of silver and hence, 20% of copper.

$\therefore$ Silver : Copper = 80 : 20 = 4 : 1

By Alligation rule,



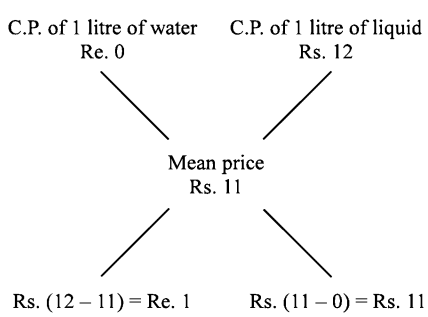
$\therefore$ Required ratio $= \frac{1}{45} : \frac{1}{30}$

i.e. Type A : Type $B = \frac{1}{3} : \frac{1}{2} = 2 : 3$.

5. Ans. (a)

Solution Gain = 25%, S.P. = Rs. 13.75 per litre

$\therefore$ C.P. of mixture $= \frac{100 \times 13.75}{100 + 25}$ = Rs. 11 per litre



C.P. of water may be assumed as Re 0 per litre

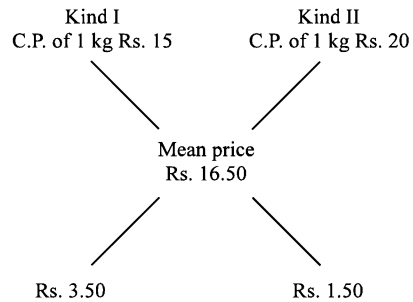
C.P.of liquid = Rs. 12 per litre

By the rule of alligation, we have

$\therefore$ Water : Liquid = 1 : 11.

6. Ans. (c)

Solution By Alligation Rule



$\therefore$ Kind I : Kind II = 3.5 : 1.5 = 7 : 3.

7. Ans. (b)

Solution Given ratio = 5 : 3

If the tea producer mixed $5x$ kg of 1st category of tea to that of $3x$ kg of 2nd category of tea.

C.P. of category I tea = Rs. 27 per kg

C.P. of $5x$ kg tea = $5x \times 27$ = Rs. $135x$

C.P. of category II tea = Rs. 30 per kg

$\therefore$ C.P. of $3x$ kg tea = $3x \times 30$ = Rs. $90x$

Total mixture = $5x + 3x = 8x$ kg

Total cost price = $135x + 90x$ = Rs. $225x$

Selling price of 1 kg of mixture = Rs. 30.25

Total S.P. = $8x \times 30.25$ = Rs. $242x$

Gain = Rs. $(242x - 225x)$ = Rs. $17x$

Gain % $= \frac{17x}{225x} \times 100 = \frac{68}{9} = 7\frac{5}{9}$ %.

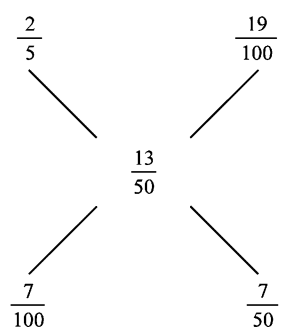
8. Ans. (c)

Solution Ratio of alcohol to whisky in the jar = 40 : 60 = 2 : 3.

Ratio of alcohol to whisky in the new mixture

= 26 : 74 = 13 : 37

Now, applying the given alligation method, we have



∴ Ratio of alcohol to whisky in the replaced mixture

$$= \frac{7}{100} : \frac{7}{50} = 1 : 2$$

∴ Quantity of whisky replaced $= \frac{2}{1+2} = \frac{2}{3}$.

9. Ans. (d)

Solution $60\left[\dfrac{\frac{200}{3}-\frac{100}{3}}{100-\frac{200}{3}}\right] = 60$ litres.

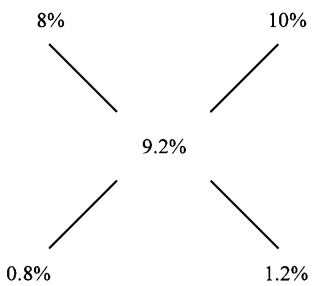
10. Ans. (d)

Solution $\dfrac{15\times20+5\times m}{20+m} = 10$

∴ $m = 20$ litres.

11. Ans. (a)

Solution By the rule of alligation



∴ First part $= \frac{0.8}{2} \times 1000 =$ Rs. 4000

Second part = (10000 – 4000) = Rs. 6000.

12. Ans. (a)

Solution Weight of zinc $= 200 \times \frac{5}{8} = 125$ gm

Weight of copper $= 200 \times \frac{3}{8} = 75$ gm

Let the ratio of 125 gm zinc and x gm copper be 3 : 5

∴ $\frac{125}{x} = \frac{3}{5}$

∴ $x = \frac{125\times5}{3} = \frac{625}{3}$ gm

∴ Addition of copper in mixture

$\frac{625}{3} - 75 = \frac{625-225}{3} = \frac{400}{3} = 133\frac{1}{3}$ gm.

13. Ans. (a)

Solution Good quality content in 150 kg of wheat = 90% of 150 = 135 kg

∴ 5% of the new mixture = 15 kg

∴ New mixture $= \frac{15\times100}{5} = 300$ kg

∴ Good quality of wheat added = (300 – 150) kg = 150 kg.

14. Ans. (b)

Solution Let the price of the third variety be Rs. x per kg

∴ $126 \times 1 + 135 \times 1 + 2 \times x = 153 \times 4$

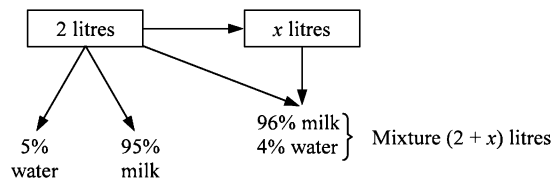
or, $261 + 2x = 612$ or, $2x = 612 - 261 = 351$

∴ $x = \frac{351}{2} = 175.50$

∴ The required price = Rs. 175.50 per kg.

15. Ans. (c)

Solution



∴ $\frac{95}{100} + 2 + x = \frac{96}{100}(2 + x)$

or, $190 + 100x = 192 + 96x$ or, $4x = 2$

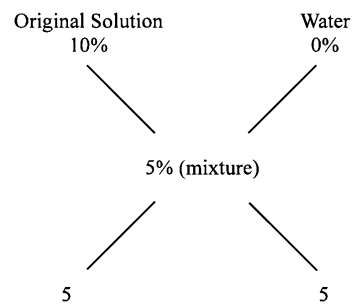
or, $x = 0.5$ litres.

16. Ans. (c)

Solution The given solution has 10% salt. Water which is to be added has 0% salt concentration.

Final concentration of solution is 5%.

Salt concentration:



So, water should be added to the given solution in the ratio 5 : 5 = 1 : 1.

∴ Quantity of water to be added $= \frac{1}{1} \times 3 = 3$ litres.

17. Ans. (a)

18. Ans. (c)

Solution Quantity of blue paint in the mixture = 10% of 40 = 4 litres

Quantity of white paint in the mixture

= 40 – 4 = 36 litres

Let x litres blue paint be added

∴ According to the question, in the final mixture

$$\frac{\text{Quantity of blue paint}}{\text{Quantity of white paint}} = \frac{4+x}{36} = \frac{20}{80}$$

$$4 + x = \frac{20 \times 36}{80}$$

$\therefore$ $x = 9 - 4 = 5$ litres.

19. Ans. (c)

20. Ans. (a)

Solution Here, we have to find the quantity of leaded petrol. Hence, we have to make certain changes in the given data.

% of leaded petrol in the mixture = 100 – 10 = 90%

After addition of leaded petrol (that has to be calculated) percentage of leaded petrol becomes (100 – 5) = 95%.

Required answer = $\left(\frac{95-90}{100-95}\right)$ 1000 ml = 1000 ml.

21. Ans. (b)

Solution $120\left[\frac{100Z-8(100+20)}{675(100+20)-100Z}\right] = 80$

$\therefore$ Z = Rs. 9 per kg.

22. Ans. (a)

Solution $35\left[\frac{100\times Z-24.50(100+25)}{16.50(100+25)-100\times Z}\right] = 25$

or, $\frac{100Z-3062.5}{2062.5-100Z} = \frac{5}{7}$

or, $700Z - 21437.5 = 10312 - 500Z$

or, $1200Z = 31750$

$\therefore$ Z = Rs. 26.458 per kg ≈ Rs.26.50 per kg.

23. Ans. (a)

Solution $20\left[\frac{100\times Z-14.25(100+30)}{11.50(100+30)-100\times Z}\right] = 30$

or, $\frac{100z-1852.3}{1495-100Z} = \frac{3}{2}$ or $500Z = 8190$

$\therefore$ $Z = \frac{8190}{500}$ = Rs 16.38 ≈ Rs.16.30.

24. Ans. (d)

Solution $30\left[\frac{100\times 18.60-y\times 120}{17.50\times 120-100\times 18.60}\right] = 30$

or, $1860 - 120y = 2100 - 1860 = 240$

or, $120y = 1620$

$\therefore$ $y = \frac{1620}{120}$ = Rs. 13.50.

25. Ans. (c)

Solution $\left[\frac{100\times Z-13\left(100+\frac{100}{3}\right)}{15\times\left(100\frac{100}{3}\right)-100Z}\right] \times 30 = 20$

$\therefore$ Z = Rs. 18.40.

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. 15 litres of a mixture contains 20% alcohol and the rest water. If 3 litres of water be mixed in it the percentage of alcohol in the mixture will be

(a) 17 (b) $16\frac{2}{3}$

(c) $18\frac{2}{3}$ (d) 15

2. In what proportion must water be mixed with spirit to gain $16\frac{2}{3}$% by selling it at cost price?

(a) 1 : 6 (b) 1 : 9

(c) 1 : 7 (d) 1 : 9

3. Two vessels A and B contain milk and water in the ratio of 5 : 2 and 4 : 1, respectively. Find the ratio in which these two qualities are to be mixed to get a new mixture of milk and water in the ratio 3 : 1.

(a) 9 : 6 (b) 7 : 5

(c) 8 : 5 (d) 6 : 5

4. 300 gm of sugar solution has 40% sugar in it. How much sugar should be added to make it 50% in the solution?

(a) 50 gm (b) 70 gm

(c) 60 gm (d) 80 gm

5. A person buys two watches for Rs. 1000. He sells one at a loss of 5% and the other at 20% gain and on the whole he gains Rs. 50. Find the cost price of each watch.

(a) Rs. 300 (b) Rs. 450

(c) Rs. 350 (d) Rs. 400

6. A man borrows a total sum of Rs. 10000 from two sources. To one he pays 10% and to the other 5% per annum simple interest. If the total interest paid by him every year is Rs. 700, how much did he borrow at each rate of interest?

(a) Rs. 4000, Rs. 6000 (b) Rs. 3000, Rs. 5000

(c) Rs. 3500, Rs. 5500 (d) Rs. 5000, Rs. 7000

7. One type of liquid contains 25% of milk, the other contains 30% of milk. A can is filled with 6 parts of the first liquid and 4 parts of the second liquid. Find the percentage of milk in the mixture.
(a) 29% (b) 31%
(c) 27% (d) 33%

8. A man lends Rs. 4000 for 3 years. A part he lends at the rate of 10% per annum at simple interest and the rest at the rate of 12% per annum at simple interest. If he receives Rs. 1350 in all as interest on both the sums, how much money does he lend at 12%?
(a) Rs. 2000 (b) Rs. 2250
(c) Rs. 2500 (d) Rs. 2750

9. A mixture of 45 litres of spirit and water, contains 20% of water in it. How much water must be added to it to make the water 25% in the new mixture?
(a) 5 litres (b) 4 litres
(c) 3 litres (d) 6 litres

10. How many kg of salt at 42 *P* per kg must a man mix with 25 kg of salt at 24 *P* per kg, so that he may, on selling the mixture at 40 kg, gain 25% on the outlay?
(a) .5 (b) 20
(c) 25 (d) 30

Test II

[Time: 9 min]

11. A mixture of a certain quantity of milk with 16 litres of water is worth 90 P per litre. If pure milk be worth 108 P per litre how much milk is there in the mixture?
(a) 80 litres (b) 60 litres
(c) 70 litres (d) 90 litres

12. Vimal mixes 80 kg of sugar worth Rs. 6.75 per kg with 120 kg of sugar worth Rs. 8 per kg. At what rate shall he sell the mixture to gain 20%?
(a) Rs. 7.50 (b) Rs. 9
(c) Rs. 8.20 (d) Rs. 8.85

13. How many kg of tea worth Rs. 29 per kg must be blended with 25 kg of tea worth Rs. 32 per kg so that by selling the blended variety at Rs. 33 per kg there should be 10% gain?
(a) 45 kg (b) 33 kg
(c) 25 kg (d) 50 kg

14. The expenditure and savings of an employee are in the ratio 3 : 1. His income increases by 16 % but at the same time his expenditure also increases by 20%. Find increase or decrease in his savings.
(a) 5.5% (b) 6%
(c) 4% (d) 4.5%

15. A vessel of 80 litres is filled with milk and water. 70% of milk and 30% of water is taken out of the vessel. It is found that the vessel is vacated by 55%. Find the initial quantity of milk and water.
(a) 50 litres, 30 litres (b) 60 litres, 40 litres
(c) 40 litres, 20 litres (d) 70 litres, 50 litres

16. In a zoo, there are rabbits and pigeons. If heads are counted, there are 200 and if legs are counted, these are 580. How many pigeons are there?
(a) 120 (b) 130
(c) 110 (d) 140

17. A person bought two tables for Rs. 2200. He sells one at 5% loss and the other at 6% profit and thus on the whole he neither gains nor loses. Find the cost price of each table.
(a) Rs. 1000, Rs. 1200 (b) Rs. 2000, Rs. 1200
(c) Rs. 1500, Rs. 1000 (d) Rs. 2500, Rs. 1200.

18. The average daily wages of staff, consisting of supervisors and labourers of a company is Rs. 50. The average wages of supervisors is Rs. 150 while that of labourers is Rs. 40 per day. If the number of supervisors is 15, find the number of labourers in the company.
(a) 350 (b) 150
(c) 200 (d) 250

19. A jar full of whisky contains 40% of alcohol. A part of this whisky is replaced by another containing 19% alcohol and the percentage of alcohol was found to be 26. The quantity of whisky replaced is
(a) $\frac{2}{5}$ (b) $\frac{1}{3}$
(c) $\frac{2}{3}$ (d) $\frac{3}{5}$

Answers

1. (b)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (a)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (b)
11. (a)	**12.** (b)	**13.** (d)	**14.** (c)	**15.** (a)	**16.** (c)	**17.** (a)	**18.** (c)	**19.** (c)	

18

RACES AND GAMES OF SKILL

INTRODUCTION

Race A contest of speed in running, driving, riding sailing or rowing over a specified distance is called *race*.

Race Course The ground or path on which contests are arranged is called a *race course*.

Starting Point The point from where a race begins is called the *starting point*.

Winning Post (or Goal) The point where the race finishes is called the *winning post* or *finishing point* or *goal*.

Dead-heat Race If all the persons contesting a race reach the finishing point exactly at the same time, then the race is called a *dead-heat race*.

Winner The person who first reaches the finishing point is the *winner*.

Suppose, A and B are two contestants in a race. We give below certain statements and their corresponding mathematical meanings, which are frequently used:

Statements	Mathematical Interpretation
1. A beats B by t seconds	A finishes the race t seconds before B finishes.
2. A gives B a start of t seconds	A starts t seconds after B starts from the same point.
3. A gives B a start of x metres	While A starts at the starting point, B starts x metres ahead from the starting point at the same time.
4. Game of 100	A game in which the participant scoring 100 points first is the winner.
5. In a game of 100, "A can give B 20 points"	While A scores 100 points, B scores only (100 – 20) or 80 points.

SOME USEFUL SHORT-CUT METHODS

1. If A is n times as fast as B and A gives B a start of x metres, then the length of the race course, so that both A and B reach the winning post at the same time, must be

$$x\left(\frac{n}{n-1}\right) \text{ metres.}$$

Illustration 1 A is $1\frac{2}{3}$ times as fast as B. If A gives B a start of 60 m, how long should the race course be so that both of them reach at the same time?

Solution Here, $n = \frac{5}{3}$ and $x = 60$.

$$\therefore \text{ Length of race course} = x\left(\frac{n}{n-1}\right)$$

$$= 60\left(\frac{5/3}{5/3-1}\right) = 60\left(\frac{5}{5-3}\right) = 150 \text{ m.}$$

2. If A can run xm race in t_1 seconds and B in t_2 seconds, where $t_1 < t_2$, then A beats B by a distance

$$\frac{x}{t_2} \times (t_2 - t_1)m.$$

Illustration 2 A can run 100 m in 27 sec and B in 30 sec. By what distance A beats B?

Solution A beasts B by a distance

$$= \frac{x}{t_2} \times (t_2 - t_1)$$

$$= \frac{100}{30}(30-27) = \frac{300}{30} = 10 \text{ m.}$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. In a km race A beats B by 25 m or 5 sec, find the time taken by A to complete the race.
(a) 3 min 15 sec (b) 4 min. 20 sec
(c) 2 min 30 sec (d) 5 min. 10 sec

2. In a race of 300 m A beats B by 15 m or 5 sec. A's time over the course is
(a) 100 sec (b) 95 sec
(c) 105 sec (d) 90 sec

3. A can run 500 m in 30 sec and B in 35 sec. How many m start can A give to B in a km race so that the race may end in a dead-heat?
(a) $139\frac{5}{7}$ (b) $138\frac{5}{7}$
(c) $142\frac{6}{7}$ (d) $140\frac{5}{7}$

4. A runs $1\frac{3}{8}$ times as fast as B. If A gives B a start of 120 m and they reach the goal at the same time, the goal is at a distance of
(a) 360 m (b) 440 m
(c) 460 m (d) 380 m

5. In a game of 100 points A can give B 20 points and C 28 points. Number of points B can give C in a game of 100 points is
(a) 10 (b) 90
(c) 15 (d) 85

6. A's speed is $1\frac{1}{2}$ times of B's. In a race A gives B a start of 300 m. How long should the race course be so that both reach the winning post simultaneously?
(a) 700 m (b) 900 m
(c) 800 m (d) 850 m

7. In a race of 600 m, A can beat B by 60 m and in a race of 500 m, B can beat C by 50 m. By how many m will A beat C in a race of 400 m?
(a) 364 m (b) 254 m
(c) 324 m (d) 354 m

8. In a 100 m race, A runs at 5 km/hr. A gives B a start of 8 m and still beats him by 8 sec. Find the speed of B.
(a) 6.14 km/hr (b) 4.14 km/hr
(c) 3.14 km/hr (d) 2.14 km/hr

9. In a game, A can give B 20 points, A can give C 32 points and B can give C 15 points. How many points make the game?
(a) 100 (b) 200
(c) 300 (d) 400

10. At a game of billiards A can give B 6 points in 50 and he can give C 13 in 65. In a game of 55, number of points B can give C is
(a) 3 (b) 4
(c) 5 (d) 8

11. In a km race A can beat B by 80 m and B can beat C by 60 m. In the same race A can beat C by
(a) 135.2 m (b) 130.5 m
(c) 142 m (d) 132.5 m

12. In a game of 90 points A can give B 15 points and C 30 points. How many points can B give C in a game of 100 points?
(a) 140 (b) 20
(c) 50 (d) 30

13. In a race of 600 m, A can beat B by 60 m and in a race of 500 m, B can beat C by 50 m. By how many m will A beat C in a race of 400 m?
(a) 78 m (b) 56 m
(c) 76 m (d) 86 m

14. In a game A can give B 25 points in 75 and C 18 points in 90. How many points can C give B in a game of 120?
(a) 20 (b) 30
(c) 40 (d) 50

15. A and B run a 5 km race on a round course of 400 m. If their speeds be in the ratio 5 : 4, how often does the winner pass the other?
(a) $4\frac{1}{2}$ times (b) $3\frac{1}{2}$ times
(c) $2\frac{3}{4}$ times (d) $2\frac{1}{2}$ times

16. In a 500 m race, the ratio of speeds of two contestants A and B is 3 : 4. A has a start of 140 m. Then, A wins by
(a) 60 m (b) 40 m
(c) 20 m (d) 10 m

17. In a km race A beats B by 100 m and C by 200 m, by how much can B beat C in a race of 1350 m?
(a) 150 m (b) 120 m
(c) 1200 m (d) 210 m

18. Two boys A and B run at $4\frac{1}{2}$ and 6 km an hour, respectively, A having 190 m start. The course being 1 km, B wins by a distance of
(a) 60 m (b) 65 m
(c) 45 m (d) 75 m

19. A and B run a km race. If A gives B a start of

50 m, A wins by 14 sec and if A gives B a start of 22 sec km, B wins by 20 m. The time taken by A to run a km is:

(a) 100 sec (b) 120 sec
(c) 105 sec (d) 125 sec

20. A and B take part in a 100 m race. A runs at 5 km per hr. A gives B a start of 8 m and still beat him by 8 sec. Speed of B is

(a) 5.15 km/hr (b) 4.14 km/hr
(c) 4.25 km/hr (d) 4.4 km/hr

EXPLANATORY ANSWERS

1. Ans. (a)

Solution B runs 25 m in 5 sec

$\therefore$ B's time to cover one km $= \frac{5}{25} \times 100$
$= 200$ sec

A's time to cover one km $= 200 - 5$
$= 195$ sec
$= 3$ min 15 sec.

2. Ans. (b)

Solution 15 m is covered by B in 5 sec

$\therefore$ 300 m is covered by B in $\frac{5}{15} \times 300 = 100$ sec

$\therefore$ A takes $100 - 5 = 95$ sec.

3. Ans. (c)

Solution Time taken by A to run 1 km
$= 30 \times 2 = 60$ sec.

Time taken by B to run 1 km $= 35 \times 2 = 70$ sec.

$\therefore$ A can give B a start of $(70 - 60) = 10$ sec.

In 35 sec B runs 500 m

$\therefore$ In 10 sec B runs $= \frac{500}{35} \times 10 = \frac{1000}{7}$
$= 142\frac{6}{7}$ m

So, A can give B a start of $142\frac{6}{7}$ metres in a km race.

4. Ans. (b)

Solution The speed of A and B are in the ratio 11 : 8.

Let speeds be $11s$ and $8s$ (in m/sec)

Let race be of x m

Then time taken by A to run xm is same as that of B to run $(x - 120)$ m.

$\therefore$ $\frac{x}{11s} = \frac{x-120}{8s}$

$\therefore$ $3x = 11 \times 120$

$\therefore$ $x = 440$.

5. Ans. (a)

Solution A scores 100 while B scores $100 - 20 = 80$ and C scores $100 - 28 = 72$.

$\therefore$ While B scores 80, C scores 72

$\therefore$ While B scores 100, C scores $\frac{72}{80} \times 100 = 90$

$\therefore$ B can give C $100 - 90 = 10$ points.

6. Ans. (b)

Solution A's speed : B's speed

$= 1\frac{1}{2} : 1 = \frac{3}{2} : 1 = 3 : 2$

It means that in a race of 3 m, A gains $(3 - 2)$
$= 1$ m over B

1 m is gained by A in a race of 3 m

$\therefore$ 300 m is gained by A in a race of

$= \frac{3}{1} \times 300 = 900$ m.

7. Ans. (c)

Solution Clearly, if A runs 600 metres, B runs = 540 m

$\therefore$ If A runs 400 m, B runs $= \left(\frac{540}{600} \times 400\right)$
$= 360$ metres

Again, when B runs 500 m, C runs = 450 m

$\therefore$ When B runs 360 m, C runs

$= \left(\frac{450}{500} \times 360\right)$ m $= 324$ m.

8. Ans. (b)

Solution Time taken by A to cover 100 m

$= 100 \div \left(5 \times \frac{5}{18}\right) = 72$ sec.

$\therefore$ B covers $(100 - 8)$ or, 92 m in $(72 + 8)$ or, 80 sec.

$\therefore$ Speed of $B = \frac{92}{80} \times \frac{16}{5} = 4.14$ km/hr.

9. Ans. (a)

Solution Suppose, x points make the game.

Clearly, when A scores x points, B scores $(x - 20)$ points and C scores $(x - 32)$ points.

Now, when B scores x points, C scores $(x - 15)$ points.

When B scores $(x - 20)$ points,

C scores $\left[\frac{(x-15)}{x} \times (x-20)\right]$ points

$\therefore \quad \frac{(x-15)(x-20)}{x} = x - 20$ or, $x = 100$

Hence, 100 points make the game.

10. Ans. (c)

Solution In a game of 50

While A scores 50, B scores 50 – 6 = 44 and in a game of 65,

While A scores 65, C scores 65 – 13 = 52.

$\therefore$ While A scores 50, C scores $\frac{52}{65} \times 50 = 40$

$\therefore$ While B scores 44, C's score = 40

While B scores 55, C's score = $\frac{40}{44} \times 55 = 50$

$\therefore$ In a game of 55,

B can give C 55 – 50 = 5 points.

11. Ans. (a)

Solution While A runs 1000 m, B runs 1000 – 80 = 920 m and while B runs 1000 m, C runs 1000 – 60 = 940 m.

$\therefore$ While B runs 920 m; C runs

$$\frac{940}{1000} \times 920 = \frac{4324}{5}\text{m}$$

$\therefore$ While B runs 920 m; C runs

$$\frac{940}{1000} \times 920 = \frac{4324}{5}\text{m}$$

$\therefore$ While A runs 1000 m, C runs $\frac{4324}{5}$m

$\therefore$ A can beat C by $1000 - \frac{4324}{5} = \frac{676}{5} = 135\frac{1}{5}$m.

12. Ans. (b)

Solution $A : B : C = 90 : 75 : 60$

$$B : C = \frac{75}{60} = \frac{70 \times \frac{100}{75}}{60 \times \frac{100}{75}} = \frac{100}{80}$$

Hence, in a game of 100 points, B can give C(100 – 80) = 20 points.

13. Ans. (c)

Solution If A runs 600 m, B runs 600 – 60 or, 540 m

If A runs 400 m, B runs = $\frac{540 \times 400}{600} = 360$ m

Now, when B runs 500 m, C runs 500 – 50 = 450 m

$\therefore$ When B runs 360 m, C runs

$$= \frac{450 \times 360}{500} = 324 \text{ m}$$

$\therefore$ A beats C by 400 – 324 = 76 m.

14. Ans. (a)

Solution When A scores 75 points, B scores 50 points

When A scores 90 points, C scores (90 – 18) = 72 points

$\therefore$ When A scores 75 points, C scores

$$= \frac{72}{90} \times 75 = 60 \text{ points}$$

$A : B : C = 75 : 50 : 60$

$C : B = \frac{60}{50} = \frac{120}{100}$

So, in a game of 120, C can give B

(120 – 100) = 20 points.

15. Ans. (d)

Solution When A makes 5 rounds, B makes 4 rounds.

In order to pass each other, the difference in number of rounds made by each must be one. Here, A passes B each time, when A makes 5 rounds.

Distance covered by A in 5 rounds

$$= \frac{5 \times 400}{100} = 2 \text{ km}$$

In covering 2 km A passes B 1 time

$\therefore$ In covering 5 km, A passes B

$$= \frac{5}{2} = 2\frac{1}{2} \text{ times.}$$

16. Ans. (c)

Solution To reach the winning post A will have to cover a distance of (500 – 140),

i.e. 360 m

While A covers 3 m, B covers 4 m

While A covers 360 m, B covers $\frac{4}{3} \times 360 = 480$ m

So, A reaches the winning post while B remains 20 m behind.

$\therefore$ A wins by 20 m.

17. Ans. (a)

Solution While A runs 1000 m, B runs 1000 – 100 = 900 m and C runs 1000 – 200 = 800 m.

$\therefore$ While B runs 900 m, C runs 800 m.

$\therefore$ While B runs 1350 m; C runs $\frac{800}{900} \times 1350$

= 1200 m

$\therefore$ B can beat C by 1350 – 1200 = 150 m.

18. Ans. (a)

Solution Speeds (in m/sec) of A and B are

$\frac{9}{2} \times \frac{5}{18} = \frac{5}{4}$ and $6 \times \frac{5}{18} = \frac{5}{3}$, respectively

A has a start of 190 m. So A has to run 1000 – 190 = 810 m while B 1000 m.

Time taken by B to cover 1000 m = $\frac{3}{5} \times 1000$

= 600 sec.

In this time A covers $\frac{5}{4} \times 600 = 750$ m.

So B reaches the winning post while A remains $810 - 750 = 60$ m behind.

$\therefore$ B wins by 60 m.

19. Ans. (a)

Solution Let times (in sec) taken by A and B to run a km, be x and y, respectively.

When B gets a start of 50 m, B runs

$1000 - 50 = 950$ m while A runs 1000 m.

$\therefore$ $\frac{950}{100}y - x = 14,$

i.e. $0.95y - x = 14$ ...(i)

and, When B gets a start of 22 sec, A runs for $(y - 22)$ sec, while B runs for y sec.

$\therefore$ $1000 - \frac{1000}{x}(y - 22) = 20$

i.e. $50y - 49x = 1100.$ ...(ii)

Multiplying (i) by 49 and subtract from (ii)

$3.45y = 414$

$\therefore$ $y = 120$ sec.

$\therefore$ (i) $\Rightarrow x = 0.95 \times 120 - 14 = 100$ sec.

20. Ans. (b)

Solution A's speed $= \left(5 \times \frac{5}{18}\right)$ m/sec $= \frac{25}{18}$ m/sec

Time taken by A to cover 100 m

$= \left(100 \times \frac{18}{25}\right)$sec $= 72$ sec.

$\therefore$ B covers 92 m in $72 + 8 = 80$ sec.

B's speed $= \left(\frac{92}{80} \times \frac{18}{5}\right)$km/hr $= 4.14$ km/hr.

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. A runner runs $1\frac{1}{4}$ laps of a 5 lap race. What fractional part of the race remains to be run?

(a) $\frac{15}{4}$ (b) $\frac{4}{5}$

(c) $\frac{5}{6}$ (d) $\frac{2}{3}$

[Based on SSC (GL) Prel. Exam 2000]

2. A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 sec, B in 308 sec and C in 198 sec, all starting at the same point. After what time will they meet next at the starting point again?

(a) 46 min 12 sec (b) 45 min

(c) 42 min 36 sec (d) 26 min 18 sec

[Based on SSC (GL) Prel. Exam 2003]

3. From a point on a circular tract 5 km long A, B and C started running in the same direction at the same time with speeds of $2\frac{1}{2}$km per hr, 3 km per hr and 2 km per hr respectively. Then on the starting point all three will meet again after

(a) 30 hr (b) 6 hr

(c) 10 hr (d) 15 hr

[Based on SSC (GL) Prel. Exam 2003]

4. A can run a distance of 1 km in 3 min 10 sec and B can run the same distance in 3 min 20 sec. If they start together, by what distance will A beat B?

(a) 50 m (b) 30 m

(c) 36 m (d) 60 m

[Based on SI of Police Rec. Exam 1997]

5. X and Y start from the same point and run around a circular stadium, whose circumference is 4200 m, at the rate of 500 m and 700 m per minute, respectively in the opposite directions. They will meet each other in

(a) 3.5 min (b) 6.0 min

(c) 8.4 min (d) 21 min

[Based on SSC (GL) Prel. Exam 1999]

EXPLANATORY ANSWERS

1. Ans. (a)

Solution $5 - \frac{5}{4} = \frac{15}{4}$.

2. Ans. (a)

Solution L.C.M. of 252, 308 and 198 = 2772 sec

$= 46 \times 60 + 12 = 46$ min 12 sec.

3. Ans. (c)

Solution A will reach at starting point in $\frac{5 \times 2}{5} = 2$ hr

B will reach at starting point in $\frac{5}{3}$ hr

C will reach at starting point in $\frac{5}{2}$ hr

Then, on the starting point all three will meet after the L.C.M. of 2, $\frac{5}{3}, \frac{5}{2} = \frac{10}{1} = 10$ hr.

4. Ans. (a)

5. Ans. (a)

■ EXERCISES FOR SELF-PRACTICE ■

Test II **[Time: 10 min]**

1. A can run a km in 3 min 10 sec and B in 3 min 20 sec. A can beat B by a distance of
(a) 50 m (b) 60 m
(c) 45 m (d) 65 m

2. A takes $7\frac{1}{2}$ hr and B 6 hr to walk round a circular track. If they start together in the same direction, they will next meet after time of
(a) 15 hr (b) 25 hr
(c) 22 hr (d) 30 hr

3. A and B start from the same point and travel in the same direction round a circular track 3 km in circumference. If their speeds be 200 and 150 m a min, they will be together again at the starting point after time of
(a) 60 min (b) 45 min
(c) 38 min (d) 42 min

4. A can give B 25 points, A can give C 40 points and B can give C 20 points. Number of points in the game is
(a) 80 (b) 75
(c) 100 (d) 90

5. A takes $7\frac{1}{2}$ hrs and B 6 hr to walk round a circular track. If they start together in opposite directions, they will meet after
(a) $3\frac{1}{3}$hr (b) $2\frac{2}{3}$hr
(c) $\frac{1}{2}$hr (d) $1\frac{1}{2}$hr

6. A, B and C run round a course 97 m in circumference, A in direction opposite to that of B and C. Their rates per minute are : A 30 m, B 26 m and C 18 m. They will be together for the first time after starting in time of
(a) $12\frac{1}{8}$ min (b) $\frac{97}{48}$ min
(c) $\frac{97}{56}$ min (d) $12\frac{1}{4}$ min

7. Three boys agree to start together and run, until all come together again, round a circular court 15 m in circumference. One runs at the rate of 6, the sec 7 and 3rd 8 kms per hr. The race will end after
(a) 50 sec (b) 42 sec
(c) 36 sec (d) 54 sec

8. In running a 2 km race on a course $\frac{1}{4}$ of a km round, A overlaps B at the middle of 6th round. Running at the same rate A will win by distance of
(a) $\frac{1}{11}$km (b) $\frac{9}{8}$km
(c) $\frac{18}{11}$km (d) $\frac{4}{11}$km

9. A can give B 20 points, A can give C 32 points and B can give C 15 points. Number of points that make the game are
(a) 100 (b) 60
(c) 85 (d) 90

10. In a km race A, B and C are the three participants. A can give B a start of 50 m and C a start of 69 m. How many m start can B give to C?
(a) 40 m (b) 25 m
(c) 20 m (d) 30 m

Test II **[Time: 10 min]**

11. A can give B 50 m start and C 80 m start in a km. race. In the same race B can give C a start of
(a) 30 m (b) $31\frac{11}{19}$m
(c) $32\frac{11}{19}$m (d) $29\frac{1}{19}$m

12. A can give B 10 m and C 20 m in a 100 m race. In the same race B can give C:
(a) $12\frac{2}{9}$m (b) $13\frac{1}{9}$m
(c) $11\frac{1}{9}$m (d) 10 m

13. In a race of 1000 m A can beat B by 100 m. In a race of 400 m B beats C by 40 m. By how many metres will A beat C in a race of 500 m?
(a) 85 m (b) 95 m
(c) 75 m (d) 105 m

14. Two men A and B run a 4 km race on a course $\frac{1}{4}$km round. If their rates be 5 : 4, how often does the winner pass the other?
(a) 2 (b) 4
(c) 5 (d) 3

15. In a game of billiards, A can give B 12 points in 60 and A can give C 10 in 90. How many can C give B in a game of 70?

(a) 63 points (b) 73 points
(c) 53 points (d) 83 points

16. A can give B 15 points, A can give C 22 points and B can give C 10 points. Number of points that make the game is
(a) 60 (b) 40
(c) 45 (d) 50

17. In a km race, A gives B a start of 100 m and still wins by 20 sec. But if A gives B a start of 25 sec, B wins by 50 m. Find the time taken by each to run a km.
(a) $\frac{500}{29}$ sec, $\frac{1200}{29}$ sec (b) $\frac{500}{29}$ sec, $\frac{700}{29}$ sec
(c) $\frac{700}{29}$ sec, $\frac{400}{29}$ sec (d) None of these

18. Two men A and B walk round a circle 1200 m in circumference. They both start at the same time from the same point and walk in the same direction if they walk at rates 150 and 80 m per min, they will be together again after time of
(a) $16\frac{2}{3}$ min (b) 15 min
(c) $19\frac{1}{7}$ min (d) $17\frac{1}{7}$ min

19. In a race A runs at 10 km/hr and B at 13 km/hr. A has a start of 100 m and also A sets off 3 min before B. How soon will B overtake A?
(a) 16 min (b) 12 min
(c) 13 min (d) 14 min

20. In a km race A can give B 50 m's start and B can give C 40 m's start. In a 2 km race A can give C a start of
(a) 195 m (b) 185 m
(c) 176 m (d) 188 m

Answers

1. (a)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (d)	**8.** (d)	**9.** (a)	**10.** (c)
11. (b)	**12.** (c)	**13.** (b)	**14.** (d)	**15.** (a)	**16.** (d)	**17.** (a)	**18.** (d)	**19.** (b)	**20.** (c)

19

STOCKS, SHARES AND DEBENTURES

INTRODUCTION

To start a big business or an industry, a large sum of money is required. It may not be possible for one or two persons to arrange for the requisite finance and expertise required for the project. So a number of individuals join hands to form a company called a **Joint Stock Company**. It is a registered body under the Companies Act. The persons who join together to form the company are called its **Promoters.** The total amount of money required by the company is called the **Capital.**

The promoters of the company issue a circular giving the details of the project, its benefits and drawbacks and invite the public to come forward and subscribe towards the capital of the company. The company divides the required capital into small units of equal amount. Each unit is called a **share.** Each person, who purchases one or more shares of the company is called a **shareholder** of the **company**. The company issues a **share certificate** to each of its shareholders stating the number of shares alloted to the person and the value of each share. The value of a share, as stated on the share certificate is called the **nominal value (or face value, or par value)** of the share.

When a company earns a profit during a financial year, a part of it is used in paying for working expenses, taxes, interest on loans and keeping some part of it as reserve fund for future expansion of the project, the remaining profit is distributed among the shareholders. The distributed profit is called the **dividend.**

Dividends are declared annually, semi-annually, quarterly as per regulations of the company. The dividend on a share is expressed as certain percentage of its face value which is printed on the share certificate. Sometimes it is also expressed as a specified amount per share. For example, we may say that dividend on a share is 12% of its face value or the dividend is Rs. 2 per share.

Illustration 1 Find the annual dividend paid in each of the following cases:

Sl. No.	*Par value of a share*	*Number of Common Shares*	*Rate of dividend declared on a Common Share*
(i)	Rs. 10	500	10% per annum
(ii)	Rs. 10	800	5% semi-annually
(iii)	Rs. 100	1500	5% quarterly
(iv)	Rs. 10	2500	2% per month

Solution

(i) Annual dividend on one share

$$= 10\% \text{ of Rs. } 10 = \text{Rs.}\left(\frac{10}{100}\times 10\right) = \text{Re. } 1$$

Annual dividend on 500 shares

= Rs. (500 × 1) = Rs. 500.

(ii) Annual dividend on one share

= Rs. (2 × 5)% of Rs. 10

$$= \text{Rs. }\left(\frac{10}{100}\times 10\right) = \text{Re.}1$$

∴ Annual dividend on 800 shares

= Rs. (800 × 1) = Rs. 800.

(iii) Annual dividend on one share

= (4 × 5)% of Rs. 100

$$= \text{Rs. }\left(\frac{20}{100}\times 100\right) = \text{Rs. } 20$$

∴ Annual dividend on 1500 shares

= Rs. (1500 × 20) = Rs. 30000.

(iv) Annual dividend on one share

= (12 × 2)% of Rs. 10

$$= \text{Rs. }\left(\frac{24}{100}\times 10\right) = \text{Rs. } 2.40$$

∴ Annual dividend on 2500 shares

= Rs. (2500 × 2.40) = Rs. 6000.

TYPES OF SHARES

The shares are generally of two types:

(i) Preferred shares and (ii) Common (ordinary) shares

(i) **Preferred shares** These shares get preference in terms of payment of dividend and return of capital over ordinary shares. The rate of dividend for these shares is decided when they are issued and dividend to preferred shareholders is paid before any dividend is paid to common shareholders.

(ii) **Ordinary shares** Ordinary shareholders are paid dividend only when profits are left after preferred shareholders have been paid dividend at specified rate. The rate of dividend on these shares is also not fixed and depends upon the amount of available profit.

FACE VALUE AND MARKET VALUE OF A SHARE

The price at which the shares are initially issued by the company to its shareholders is called the *face value* of a share (This is also called *nominal* or *par value* of a share). In fact, this is that value of a share which is mentioned in the share certificate issued by the company to its shareholders.

As other things, shares are also sold in (or purchased from) the market. The value of a share quoted in the market is called the *market value* of the share. The market value of a share keeps on changing according to its demand and supply changes.

If the market value of a share is equal to the par value of the share, the share is said to be *at par*. If the market value of a share is more than its face (or par) value, the share is said to be at *premium*. On other hand, if the market value of a share is less than its face value, the share is said to be *at discount* (or *below par*).

For example, if the market value of a Rs. 100 share is Rs. 130, it is said to be at 30% premium.

If the market value of a Rs. 100 share is Rs. 90, it is said to be at 10% discount. If Rs. 100 share is quoted at 45 premium then its market value is

Rs. (100 + 45) = Rs. 145.

Every company declares dividend on the face value of its shares irrespective of the market value of the share.

Note: The statement "32%, Rs. 100 shares at Rs. 125" means:

(i) Face value of each share is Rs. 100
(ii) Dividend on each share is 32% of the face value
(iii) Market value of each share is Rs. 125
(iv) An investment of Rs. 125 earns an annual income of Rs. 32.

Illustration 2 Find the cost of purchasing 150 shares of a company, each of par value Rs. 10, quoted at Rs. 16 each in the market, from the original shareholder. Also, find the gain to the new shareholder if he sells each share at a premium of Rs. 10.

Solution Market value of share = Rs. 16

∴ Market value of 150 shares

= Rs. (150 × 16) = Rs. 2400

Thus, the new shareholder spent Rs. 2400 for buying 150 shares. The new shareholder sold the shares at a premium of Rs. 10.

∴ Now, market value of a share

= Rs. (10 + 10) = Rs. 20

The selling price of 150 shares at the new market value = Rs. (150 × 20) = Rs. 3000

∴ Gain of the new shareholder in the transaction

= Rs. (3000 – 2400) = Rs. 600.

Illustration 3 Raja buys 200 shares, each of par value Rs. 10 of a company which pays annual dividend of 15% at such a price that he gets 12% on his investment. Find the market value of a share.

Solution Par value of 200 shares

= Rs. (200 × 10) = Rs. 2000.

Dividend received by Raja

$$= \text{Rs. } \left(\frac{2000\times 15}{100}\right) = \text{Rs. } 300.$$

Let the market value of 200 shares be Rs. x. We have to find x such that 12% of x = 300,

i.e. $\frac{12}{100} \times x = 300 \quad \therefore \quad x = \frac{100\times 300}{12} = 2500$

i.e. Market value of 200 shares = Rs. 2500

Hence, the market value of one share = Rs. 12.50.

STOCKS AND BROKERAGE

Stock

In the previous section, we have learnt about shares, which can be sold and purchased by the public. The nominal value or face value of shares is fixed (usually Rs. 10 or Rs. 100) but their market value varies.

Sometimes, joint stock companies or the government also raises loans from the market by issuing *bonds* or *promisory notes*. They promise to pay a fixed amount (called *redemption value*) on a future date and interest payments at fixed periods until that time. The money paid to company or government for buying such bonds is called *stock*.

The stocks are usually known by their rates of dividend. Thus, by 9% stock we mean that the dividend on a Rs. 100 stock is Rs. 9.

If the market value of Rs. 100 stock, which yields a dividend of Rs. 5, is Rs. 115, the stock is called 5% stock at 115. Similarly, 10% stock at 120 means that a stock of face value Rs. 100 gives a dividend of Rs. 10 and is available in the market of 120.

Note: There can be stocks in units different from Rs. 100, say Rs. 500, Rs. 1000, etc., but the phrase "8% stock at 90" can be used only in case of that stock whose face value is Rs. 100. Dividend on a stock is fixed (declared at the time of issue) whereas for a share it varies with time. Usually, the *date of maturity* of the stock is fixed. In case the holder of the stock requires money before the due date he can sell his stock to some other person, whereby his claim of interest is transferred to that person.

Brokerage

The sale and purchase of stock is generally executed through a stockbroker, who charges some money, called *Brokerage* from both the seller and purchaser. *The brokerage is charged either as some fixed amount on each unit of stock or as some percentage of the market value of unit of stock.*

Thus, the brokerage of Rs. x means that x rupees are to be added or subtracted from the market value of the stock. Similarly, brokerage 2% means that the brokerage equal to 2% of the market value of a unit of stock and be added to (or subtracted from) the market value of a unit of stock.

Notes:

(i) The brokerage is added to the market value when the stock is purchased.

(ii) The brokerage is subtracted from the market value when the stock is sold.

CALCULATION OF INCOME ON A STOCK

When the face value of the total stock is given, the income can be calculated on the assumption that the face value of each unit of stock is Rs. 100. On the contrary, if the market value of the total investment is given, the income can be calculated on the basis of the market value of a unit of stock.

Illustration 4 Find the income from Rs. 2875 of 4% stock.

Solution By 4% stock, we mean a stock of Rs. 100 will fetch a dividend of Rs. 4 p.a.

Hence, the income from Rs. 2875 of 4% stock

$$= \frac{2875\times 4}{100} = \text{Rs. } 115.$$

Illustration 5 Find the income on 10% stock of Rs. 25000 purchased at Rs. 120.

Solution Face value of stock = Rs. 25000

Income on Rs. 100 stock = Rs. 10

Income on Re. 1 stock = Rs. $\left(\frac{10}{100}\right)$

Income on Rs. 25000 stock = Rs. $\left(\frac{25000\times 10}{100}\right)$

= Rs. 2500.

COMPUTATION OF INVESTMENT OR MARKET VALUE OF A STOCK

If the face value of a stock is given, the market value of the stock can be found on the basis of market value of each unit of stock.

Illustration 6 Find the investment required to purchase Rs. 75000 of 10% stock at 95.

Solution Market value of Rs. 100 stock = Rs. 95

∴ Market value of Rs. 75000 stock

$$= \text{Rs. } \left(\frac{95}{100}\times 75000\right) = \text{Rs. } 71250$$

∴ An investment of Rs. 71250 is required to purchase Rs. 75000 of 10% stock at 95.

Illustration 7 Find the investment required to get an income of Rs. 4200 from $10\frac{1}{2}$% stock at 80 (Brokerage: 2%).

Solution Brokerage = 2% of Rs. 80 = Rs. $\left(\frac{2}{100}\times 80\right)$

= Rs. 1.60

∴ Investment needed to buy Rs. 100 stock

= Rs. 81.60 on which the income is Rs. $10\frac{1}{2}$

For income of Rs. $10\frac{1}{2}$, the investment = Rs. 81.60

For income of Rs. 4200, the investment

$$= \text{Rs. } \left(81.60\times\frac{2}{21}\times 4200\right) = \text{Rs. } 32640.$$

COMPUTATION OF GAIN OR LOSS IN THE SALE AND PURCHASE OF A STOCK

When the market is favourable to stock holders, i.e. they are likely to get better proceeds for their stock, they sell the stock and may reinvest the money so obtained in another stock which may give them more income.

Illustration 8 Ram bought Rs. 12000 of 8% stock at 92 and sold it when the price rose to 98. Find his total gain and gain per cent.

Solution Investment made by Ram in buying Rs. 12000 of 8% stock at 92

$$= \text{Rs. } \left[12000\times\frac{92}{100}\right] = \text{Rs. } 11040.$$

When the price rose to Rs. 98, Ram sold the stock, thus money realised from selling the stock

$$= \text{Rs.}\left[12000\times\frac{98}{100}\right] = \text{Rs. } 11760.$$

∴ Gain realised in the transaction

= Rs. (11760 – 11040) = Rs. 720

∴ Gain per cent = $\frac{(720\times 100)}{11040} = 6\frac{12}{23}$%.

CHANGE IN INCOME ON SALE OR REINVESTMENT

A person having one type of stock may sell it to buy another which gives higher income. In such problems, the income in two cases is calculated and change is found out.

Illustration 9 Ram invests Rs. 46500 in 6% stock at 93 and sells the stock when its price rose to Rs. 95. He invests the sale proceeds in 9% stock at 95. Find the change in Ram's income.

Solution Income from first stock = Rs. $\left(\frac{6}{93}\times 46500\right)$

= Rs. 3000

We have to find the amount realized on selling this stock.

Amount realised on selling Rs. 93 stock = Rs. 95

∴ Amount realised on selling Rs. 46500 stock

$$= \text{Rs. } \frac{95}{93} \times 4650 = \text{Rs. } 47500$$

This amount is invested in 9% stock at 95

∴ Income from 2nd stock

$$= \text{Rs. } \left(\frac{9}{95} \times 47500\right) = \text{Rs. } 4500$$

Hence, increase in income

$$= \text{Rs. } (4500 - 3000) = \text{Rs. } 1500.$$

DEBENTURES

Sometimes a running joint stock company may require more capital for its further expansion. The company borrows the required sum of money from the general public for a fixed period of time and at a fixed rate of interest by dividing the amount required into small parts. These small parts are called *debentures*.

The debenture holders are creditors of the company and do not have any right on the profits declared by the company. However, interest at fixed rate and fixed time is payable to debenture holders, irrespective of the fact whether the company is running in profits or losses.

Like shares, debentures can also be sold in or purchased from the market. The terms used in case of shares, are also used with the same meaning in case of debentures. Thus we use the terms '*debentures at premium*', '*debentures at discount*' etc. Furthermore, the rules for calculating the brokerage on debenture are also the same as those in case of shares.

DIFFERENCE BETWEEN SHARES AND DEBENTURES

Shares	*Debentures*
(a) Share money forms a part of the capital of the company.	(a) Debentures are a mere debt.
(b) Shareholders have right on the profit declared by the company.	(b) Debentureholders are creditors of the company and do not have any right on the profit declared by the company.
(c) Shareholders may receive different dividend according as profit is more or less.	(c) Debenture holders receive interest at a fixed rate.

Illustration 10 Find the income percent of a buyer on 8% debentures of face value Rs. 120 and available in the market at Rs. 180.

Solution The market value of a debenture is Rs. 180

∴ Income on Rs. 180 is Rs. 8

∴ Income on Rs. 120 is Rs. $\left(\frac{8}{180} \times 120\right) = \text{Rs. } 5\frac{1}{3}$

∴ Per cent income on the debenture is $5\frac{1}{3}\%$.

Illustration 11 Ram has 500 shares of par value Rs. 10 each of a company and 500 debentures of par value Rs. 100 each. The company pays a dividend of 8% on the shares and pays an interest of 10% on its debentures. Find the total annual income of Ram and the rate of return on her investments.

Solution Annual dividend on 500 shares

$$= \text{Rs. } \frac{(500 \times 10 \times 8)}{100}$$

$$= \text{Rs. } 400.$$

Annual interest on 500 debentures

$$= \text{Rs. } \frac{(500 \times 100 \times 10)}{100}$$

$$= \text{Rs. } 5000.$$

∴ Total annual income of Ram

$$= \text{Rs. } (5000 + 400)$$

$$= \text{Rs. } 5400.$$

Total investment of Ram

$$= \text{Rs. } (500 \times 10 + 500 \times 100)$$

$$= \text{Rs. } 55000.$$

Rate of return on Ram's investment

$$= \left(\frac{5400}{55000} \times 100\right)\%$$

$$= \frac{108}{11}\% \quad \text{or} \quad 9\frac{9}{11}\%.$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. A company declared an annual dividend of 10%. Find the annual dividend of Ram owning 1500 shares of the company of par value Rs. 10 each.

(a) Rs. 1400 (b) Rs. 1500
(c) Rs. 1700 (d) Rs. 1600

2. A company declared an annual dividend of 10%. Find the annual dividend received by Anu owning 4000 shares of the company having a par value of Rs. 100 each.
(a) Rs. 45000 (b) Rs. 40000
(c) Rs. 50000 (d) Rs. 60000

3. Jatin invested Rs. 27260 in buying Rs. 100 shares of a company at Rs. 116 each. If the company paid 16% dividend at the end of the year, find his income from the dividend.
(a) Rs. 3560 (b) Rs. 2760
(c) Rs. 3760 (d) Rs. 3660

4. A company issued 50000 shares of par value Rs. 10 each. If the total dividend declared by the company is Rs. 62500, find the rate of dividend paid by the company.
(a) $8\frac{1}{2}\%$ (b) $12\frac{1}{2}\%$
(c) 12% (d) $13\frac{3}{4}\%$

5. A company declared a semi-annual dividend of $7\frac{1}{2}\%$. Find the annual dividend of Chetan, owning 1250 shares of the company having a par value of Rs. 10 each.
(a) Rs. 1875 (b) Rs. 1757
(c) Rs. 1680 (d) Rs. 1575

6. A medicine company issued 125000 shares of par value Rs. 20 each. If the total dividend declared by the company is Rs. 375000, find the rate of dividend paid by the company.
(a) 15 % (b) 13%
(c) 10% (d) 14%

7. Seema had 50 preferred shares and 400 common shares of par value Rs. 100 each. If the dividend declared on preferred shares is 10% per annum and a semi-annual dividend of 7.5% is on common shares find the annual dividend received by Seema.
(a) Rs. 7500 (b) Rs. 6500
(c) Rs. 8500 (d) Rs. 5500

8. Find the annual dividend received by Sunil for his 200 preferred shares and 1000 common shares, both of par value Rs. 100 each if the dividend declared on a preferred share is 10% per annum and an annual dividend of $12\frac{1}{2}$ % on the common shares.
(a) Rs. 4500 (b) Rs. 550
(c) Rs. 4000 (d) Rs. 3500

9. A company issued 50000 shares of par value Rs. 100 each. If the total dividend declared by the company is Rs. 125000, out of which Rs. 50000 have been kept in reserve fund and the remaining is distributed as dividend, find the rate of dividend paid by the company.
(a) $2\frac{3}{4}\%$ (b) $1\frac{1}{2}\%$
(c) $1\frac{1}{4}\%$ (d) 2%

10. Find the annual dividend received by Nishita from 1200 preferred shares and 3000 common shares both of par value Rs. 50 each if the dividend paid on preferred shares is 10% and semi-annual dividend of $3\frac{1}{2}$ % is declared on common shares.
(a) Rs. 18500 (b) Rs. 16500
(c) Rs. 14500 (d) Rs. 14500

11. 12500 shares, of par value Rs. 20 each, are purchased from Ram by Mohan at a price of Rs. 25 each. Find the amount required to purchase the shares. If Mohan further sells the shares at a premium of Rs. 11 each, find his gain in the transaction.
(a) Rs. 75000 (b) Rs. 85000
(c) Rs. 70000 (d) Rs. 65000

12. Mac buys 200 shares of par value Rs. 10 each, of a company, which pays an annual dividend of 8% at such a price that he gets 10% on his investment. Find the market value of share.
(a) Rs. 8 (b) Rs. 10
(c) Rs. 6 (d) Rs. 12

13. Shyam purchased 12000 shares of a company, of par value Rs. 10 each, paying an annual dividend of 15% at such a price, that she gets 10% on her investment. Find the market value of a share.
(a) Rs. 25 (b) Rs. 15
(c) Rs. 20 (d) Rs. 14

14. The capital of a company is made up of 50000 preferred shares with dividend of 20% and 20000 common shares, the par value of each type of share being Rs. 10. The company had a total profit of Rs. 180000 out of which Rs. 30000 were kept in reserve fund and the remaining distributed to shareholders. Find the dividend percent to the common shareholders.
(a) 24% (b) 20%
(c) 25% (d) 30%

15. A company has issued 10000 preferred shares and 50000 common shares both of par value Rs. 100 each. The dividend on a preferred share and a common share is 12% and 17.6%, respectively. The company had a total profit of 15 lakh rupees out of which some amount was kept in reserve fund and the remaining distributed as dividend. Find the amount kept in reserve fund.
(a) Rs. 5 lakh (b) Rs. 6 lakh
(c) Rs. 6.5 lakh (d) Rs. 5.5 lakh

16. A man sells 5000 common shares of a Company *X* (each of par value Rs. 10), which pays a dividend of 20%, at Rs. 30 per share. He invests the sale

proceeds in ordinary shares of Company Y (each of par value Rs. 25) that pays a dividend of 15%. If the market value of a share of Company Y is Rs. 40, find the number of shares of Company Y purchased by the man

(a) 3850 (b) 3750
(c) 3700 (d) 3800

17. The shares of a company of par value Rs. 10 each, are available at 20% premium. Find the amount paid by the buyer who wants to buy 2500 shares. What would be the gain of the buyer if he sells those shares at the rate of Rs. 20 per share?

(a) Rs. 25000 (b) Rs. 30000
(c) Rs. 20000 (d) Rs. 22000

18. Find the income on 12% stock of Rs. 60000 purchased at Rs. 110.

(a) Rs. 7200 (b) Rs. 7500
(c) Rs. 7400 (d) Rs. 8200

19. Find the income on $7\frac{1}{2}$% stock of Rs. 20000 purchased at Rs. 120.

(a) Rs. 1550 (b) Rs. 1450
(c) Rs. 1500 (d) Rs. 1600

20. Find the income by investing Rs. 81000 in 9% stock at 135.

(a) Rs. 5500 (b) Rs. 6400
(c) Rs. 5400 (d) Rs. 6000

21. Find the income obtained by investing Rs. 90000 in $7\frac{1}{2}$% stock at $112\frac{1}{2}$.

(a) Rs. 6000 (b) Rs. 6500
(c) Rs. 7500 (d) Rs. 7000

22. A person buys $9\frac{1}{2}$% stock of Rs. 72000 at 144. Find his annual income.

(a) Rs. 6640 (b) Rs. 6840
(c) Rs. 6900 (d) Rs. 7240

23. Mr. Lal invested Rs. 92000 in $9\frac{1}{2}$% stock at 91 (Brokerage: Re. 1). Find the annual income of Mr. Lal from this investment.

(a) Rs. 9000 (b) Rs. 9500
(c) Rs. 10500 (d) Rs. 8000

24. Raja invested Rs. 99000 in $7\frac{1}{2}$% stocks at $81\frac{1}{2}$ (Brokerage: Re. 1). Find Ram's annual income from his investment.

(a) Rs. 9500 (b) Rs. 10000
(c) Rs. 10500 (d) Rs. 9000

25. Ram invested Rs. 88008 in $9\frac{1}{2}$% stock at 112 (Brokerage: Rs. 2) Find annual income of Ram from this investment.

(a) Rs. 6334 (b) Rs. 6874
(c) Rs. 7334 (d) Rs. 6534

26. Find the investment required to purchase Rs. 125000 of 8% stock at 92.

(a) Rs. 115000 (b) Rs. 120000
(c) Rs. 105000 (d) Rs. 125000

27. What investment will be required to purchase Rs. 90000 of 8% stock at 110?

(a) Rs. 88000 (b) Rs. 99000
(c) Rs. 88500 (d) Rs. 9950

28. Find the investment required to get an income of Rs. 1938 from $9\frac{1}{2}$% stock at 90 (Brokerage 1%).

(a) Rs. 19642.60 (b) Rs. 17543.00
(c) Rs. 18543.60 (d) Rs. 18600.60

29. A man bought Rs. 20000 of 5% stock at 90 and sold it when its price rose to Rs. $93\frac{3}{4}$. Find his gain per cent.

(a) $5\frac{1}{6}$% (b) $4\frac{1}{6}$%
(c) $5\frac{5}{6}$% (d) $4\frac{5}{6}$%

30. Meena bought Rs. 36000 of $7\frac{1}{2}$% stock at 92 and sold it when its price rose to Rs. $93\frac{3}{4}$. Find her gain per cent.

(a) 1.9% (b) 2.9%
(c) 2.3% (d) 1.4%

31. A man invests Rs. 27600 in 4% stock at 92. He sold Rs. 20000 stock when the price rose to Rs. 96, and sold the remaining stock when the market value fell to Rs. 90. How much does he gain or loss in the transaction?

(a) Gain = Rs. 600 (b) Loss = Rs. 600
(c) Loss = Rs. 650 (d) Gain = Rs. 650

32. A person invests Rs. 28500 in 5% stock at 95. He sold Rs. 15000 stock when the price rose to Rs. 98 and sold the remaining stock when the market value of the stock fell to Rs. 90. How much does he gain or loss in the transaction?

(a) Gain = Rs. 300 (b) Loss = Rs 300
(c) Gain = Rs. 400 (d) Loss = Rs. 400

33. Sushma invested Rs. 245000 in 7% stock at 98 and sold the stock when its price rose to Rs. 100. She invested the sale proceeds in 9% stock at 125. Find the change in income of Sushma.

(a) Rs. 600 (b) Rs. 400
(c) Rs. 500 (d) Rs. 650

34. Anu invested Rs. 32400 in 8% stock at 90. She sold out Rs. 18000 stock when the price rose to Rs. 95 and the remaining stock at Rs. 98. She invested the total sale proceeds in 10% stock at $96\frac{1}{2}$. Find the change in income of Anu.

(a) Rs. 750 (b) Rs. 720
(c) Rs. 760 (d) Rs. 740

35. A man invested Rs. 50490 in 5% stock at 99 and sold it when the price rose to Rs. 102. He invested the sale proceeds in 8% stock at 96. Find the change in man's income (Brokerage: Rs. 3)
(c) Rs. 1485 (b) Rs. 1585
(c) Rs. 1385 (d) Rs. 1685

36. A man invested Rs. 260000 in 5% stock at 104. He sold the stock when the price rose to Rs. 125 and invested the sale proceeds in 6% stock. By doing this his income increased by Rs. 2500. At what price did he purchase the second stock?
(a) Rs. 225 (b) Rs. 175
(c) Rs. 125 (d) Rs. 150

37. Find the income per cent of a buyer on 5% debentures of face value Rs. 95 and available in the market for Rs. 125.
(a) 4.8% (b) 5.8%
(c) 3.8% (d) 2.8%

38. Find the income per cent on 10% debentures of par value Rs. 120 available in the market for Rs. 150.
(a) 9% (b) 8%
(c) 7% (d) 6%

39. Brij has 800 shares of par value Rs. 50 each and 600 debentures of par value Rs. 100 each of the company. The company pays an annual dividend of 6% on the shares and interest of 12% on the debentures. Find the total annual income of Brij and rate of return on his investment.
(a) Rs. 9600, 9.6% (b) Rs. 8000, 8%
(c) Rs. 10600, 10.6% (d) Rs. 9000, 8.6%

40. A man bought 20 shares of Rs. 50 at 5 discount, the rate of discount being $4\frac{3}{4}\%$. The rate of interest obtained is
(a) $4\frac{3}{4}\%$ (b) $3\frac{1}{4}\%$
(c) 5.28% (d) 4.95%

EXPLANATORY ANSWERS

1. Ans. (b)

Solution Annual dividend on one share

= 10% of Rs. 10

$= \text{Rs. } \left(10\times\frac{10}{100}\right) = \text{Re. } 1$

Annual dividend of Ram owning 1500 shares

= (1500 ×1) = Rs. 1500.

Alternatively, we could have found the total par value of 1500 shares first and then find dividend at 10% of it as shown below:

Total par value of 1500 shares

= Rs. (1500 × 10)= Rs. 15000

∴ Total annual dividend of Ram

$= \left(15000\times\frac{10}{100}\right) = \text{Rs. } 1500.$

2. Ans. (b)

Solution Annual dividend on one share = 10% of Rs. 100

$= \text{Rs. } \left(\frac{10}{100}\times100\right) = \text{Rs. } 10$

∴ Annual dividend on 4000 shares

= Rs. (4000 × 10) = Rs. 40000.

3. Ans. (c)

Solution Number of shares purchased by Jatin

$= \frac{27260}{116} = 235.$

Face value of 235 shares

= Rs. (235 × 100) = Rs. 23500.

Annual income from 235 shares

= 16% of Rs. 23500

$= \text{Rs. } \left(\frac{16}{100}\times23500\right) = \text{Rs. } 3760.$

4. Ans.(b)

Solution Number of shares = 50000

Par value of a share = Rs. 10

∴ Total par value of 50000 shares = Rs. 500000

Total dividend = Rs. 62500

∴ Rate of dividend paid by the company

$= \left(\frac{62500}{500000}\times100\right)\% = 12\frac{1}{2}\%.$

5. Ans. (a)

Solution Annual dividend on one share $= \left(2\times7\frac{1}{2}\right)\%$

i.e. 15% of Rs. 10

$= \left(\frac{15}{100}\times10\right) = \text{Rs. } 1.50$

∴ Annual dividend on 1250 shares

= Rs. (1250 × 1.50) = Rs. 1875.

6. Ans. (a)

Solution Number of shares = 125000

Par value of a share = Rs. 20

∴ Total par value of 125000 shares

= Rs. (1250000 × 20) = Rs. 2500000

Total dividend = Rs. 375000

∴ Rate of dividend paid by the company

$$= \left(\frac{375000}{2500000}\times 100\right)\% = 15\%.$$

7. Ans. (b)

Solution Dividend on 50 preferred shares

$$= \text{Rs.}\left(50\times 100\times\frac{10}{100}\right) = \text{Rs. } 500$$

Dividend on 400 common shares

$$= \text{Rs.}\left(400\times\frac{100}{100}\times\frac{15}{2}\times 2\right) = \text{Rs. } 6000.$$

∴ Total dividend received by Seema

= Rs. (500 + 6000) = Rs. 6500.

8. Ans. (a)

Solution Dividend on 200 preferred shares

= 10% of Rs. (200 × 100)

$$= \text{Rs.}\left(\frac{10}{100}\times 20000\right) = \text{Rs. } 2000$$

Dividend on 1000 common shares

$$= 12\frac{1}{2}\% \text{ of Rs. } (1000 \times 100)$$

$$= \text{Rs.}\left(\frac{25/2}{100}\times 100000\right)$$

$$= \text{Rs.}\left(\frac{25}{2}\times 1000\right) = \text{Rs. } 12500$$

∴ Total dividend received

= Rs. (2000 + 12500) = Rs. 4500.

9. Ans. (b)

Solution The total dividend declared = Rs. 125000

Amount kept in reserve fund = Rs. 50000

Net amount paid as dividend to the shareholders

= Rs. (125000 – 50000) = Rs. 75000

Number of shares of par value Rs.100 each = 50000

Total par value of 50000 shares

= Rs. (50000 × 100) = Rs. 5000000

Rate of dividend paid by the company

$$= \left(\frac{75000}{5000000}\times 100\right)\% = \frac{3}{2}\% = 1\frac{1}{2}\%.$$

10. Ans. (b)

Solution Dividend on 1200 preferred shares

= 10% of Rs. (1200 × 50)

$$= \text{Rs.}\left(\frac{10}{100}\times 1200\times 50\right) = \text{Rs. } 6000$$

Dividend on 3000 common shares

$$= \left(3\frac{1}{2}\times 2\right)\% \text{ of Rs. } (3000 \times 50)$$

$$= \text{Rs.}\left(\frac{7}{100}\times 3000\times 50\right) = \text{Rs. } 10500$$

∴ Total dividend received by Nishita

= Rs. (6000 + 10500) = Rs. 16500.

11. Ans. (a)

Solution Market value of a share = Rs. 25

∴ Market value of 12500 shares

= Rs. (25 × 12500) = Rs. 312500

Thus, the amount required to purchase 12500 shares = Rs. 312500

Then, Mohan sells these shares at a premium of Rs. 11 each.

∴ New market rate per shares

= Rs. (20 + 11) = Rs. 31

∴ Selling price of these shares

= Rs. (31 × 12500) = Rs. 387500

∴ Gain = S.P. – C.P. = Rs. (387500 – 312500)

= Rs. 75000.

12. Ans. (a)

Solution Par value of 200 shares = Rs. (200 × 10)

= Rs. 2000

$$\text{Dividend received by Mac} = \text{Rs.}\left(\frac{8}{100}\times 2000\right)$$

= Rs. 160

Let the market value of 200 shares be Rs. x.

We have to find x such that 10% of x = 160

$$\Rightarrow \frac{10}{100}\times x = 160 \Rightarrow x = 160 \times 10 = 1600$$

i.e. Market value of 200 shares = Rs. 1600.

Hence, the market value of one share

$$= \text{Rs.}\left(\frac{1600}{200}\right) = \text{Rs. } 8.$$

13. Ans. (b)

Solution Par value of 12000 shares = Rs. (12000 × 10)

= Rs. 120000

$$\text{Dividend received by Shyam} = \text{Rs.}\left(\frac{15}{100}\times 120000\right)$$

= Rs. 18000

Let the market value of 12000 shares be Rs. x.

We have to find x such that 10% of x = 18000

$$\Rightarrow \frac{10}{100}\times x = 18000 \Rightarrow x = 18000 \times 10 = 180000$$

i.e. Market value of 12000 shares = Rs. 180000.

Hence, the market value of one share

$$= \text{Rs.}\left(\frac{180000}{12000}\right) = \text{Rs. } 15.$$

14. Ans. (c)

Solution The total profit of the company = Rs. 180000

Amount kept in reserve fund = Rs. 30000

∴ Net amount paid as dividend to shareholders

= Rs. (180000 – 30000) = Rs. 150000

Dividend paid by the company on 50000 preferred shares

$$= \text{Rs.}\left(50000\times\frac{10\times20}{10}\right) = \text{Rs. } 100000$$

∴ Dividend to be paid to common shareholders

= Rs. (150000 – 100000) = Rs. 50000

Thus, dividend paid on a common share

$$= \text{Rs.}\left(\frac{50000}{20000}\right) = \text{Rs. } 2.50$$

Hence, dividend percent paid on a common share

$$= \text{Rs.}\left(\frac{2.50}{10}\times100\right)\% = 25\%.$$

15. Ans. (a)

Solution = 12% of Rs. (10000 × 100)

$$= \text{Rs.}\left(\frac{12}{100}\times10000\times100\right) = \text{Rs. } 120000$$

Dividend on 50000 common shares

= 17.6% of Rs. (50000 × 100)

$$= \text{Rs.}\left(\frac{17.6}{100}\times50000\times100\right) = \text{Rs. } 880000$$

∴ Total dividend paid = Rs. (120000 + 880000)

= Rs. 1000000 = Rs. 10 lakh

∴ Amount kept in reserve fund

= Rs. 15 lakh – 10 lakh = Rs. 5 lakh.

16. Ans. (b)

Solution Income of the man from 5000 ordinary shares of Company X, which pays a dividend of 20%

$$= \text{Rs.}\left(\frac{5000\times10\times20}{100}\right) = \text{Rs. } 10000.$$

Selling price of a share of Company X = Rs. 30

∴ Selling price of 5000 shares of Company X

= Rs. (5000 × 30)

= Rs. 150000.

Now, the market value of a share of Company Y is given to be Rs. 40.

∴ Number of shares of Company Y purchased by the man from Rs. 150000

$$= \left(\frac{150000}{40}\right) = \text{Rs. } 3750.$$

17. Ans. (c)

Solution Par value of a share = Rs. 10

$$\text{Market value of a share} = \text{Rs.}\left(10\times\frac{120}{100}\right) = \text{Rs. } 12$$

The amount to be paid by the buyer to purchase 2500 shares = Rs. (2500 × 12) = Rs. 30000

Gain of the shareholder on selling one share

= Rs. (20 – 12) = Rs. 8

∴ Gain from selling 2500 shares

= Rs. (2500 × 8) = Rs. 20000.

18. Ans. (a)

Solution Face value of the stock = Rs. 60000

Income on Rs. 100 stock = Rs. 12

$$\text{Income on Re. 1 stock} = \text{Rs.}\left(\frac{12}{100}\right)$$

$$\text{Income on Rs. 60000} = \text{Rs.}\left(\frac{12}{100}\times60000\right) = \text{Rs. } 7200.$$

19. Ans. (c)

Solution Face value of the stock = Rs. 20000

$$\text{Income on Rs. 100 stock} = \text{Rs. } 7\frac{1}{2}$$

$$\text{Income on Re. 1 stock} = \text{Rs.}\left(\frac{15/2}{100}\right) = \text{Rs.}\left(\frac{15}{200}\right)$$

$$\text{Income on Rs. 20000 stock} = \text{Rs.}\left(\frac{15}{200}\times20000\right)$$

= Rs. 1500.

20. Ans. (c)

Solution Here, the market value of the stock

= Rs. 81000

By investing Rs. 135, stock of par value Rs. 100 is available

∴ Income on Rs. 135 is Rs. 9

$$\therefore \text{ Income on Rs. 81000 is Rs.}\left(\frac{9}{135}\times81000\right)$$

= Rs. 5400.

21. Ans. (a)

Solution Here, market value of the stock = Rs. 90000

By investing Rs. $112\frac{1}{2}$, stock of par value Rs. 100 is available.

$$\therefore \text{ Income on Rs. } 112\frac{1}{2} \text{ is } 7\frac{1}{2}\%$$

$$\therefore \text{ Income on Rs. 90000 is Rs.}\left(\frac{15}{2}\times\frac{2}{225}\times90000\right)$$

= Rs. 6000.

22. Ans. (b)

Solution Face value of the stock = Rs. 72000

$\therefore$ Income on stock = Rs. $\left(\frac{72000}{100}\times\frac{19}{2}\right)$ = Rs. 6840.

23. Ans. (b)

Solution Market value of Rs. 100 stock
= Rs. (91 + 1) = Rs. 92

Income on Rs. 92 = Rs. $9\frac{1}{2}$

$\therefore$ Income on Rs. 92000 = Rs. $\left(\frac{19}{2}\times\frac{19}{92}\times 92000\right)$
= Rs. 9500.

24. Ans. (d)

Solution Market value of Rs. 100 stock
= Rs. $\left(81\frac{1}{2}+1\right)$ = Rs. $82\frac{1}{2}$

Inceome on Rs. $82\frac{1}{2}$ = Rs. $7\frac{1}{2}$

$\therefore$ Income on Rs. 99000 = Rs. $\left(\frac{15}{2}\times\frac{2}{165}\times 99000\right)$
= Rs. 9000.

25. Ans. (c)

Solution Market value of Rs. 100 stock
= Rs. (112 + 2) = Rs. 114

Income on Rs. 114 = Rs. $9\frac{1}{2}$

$\therefore$ Income on Rs. 88008 = Rs. $\frac{19}{2}\times\frac{1}{114}\times 88008$
= Rs. 7334.

26. Ans. (a)

Solution Market value of Rs. 100 stock = Rs. 92

$\therefore$ Market value of Rs. 125000 stock

= Rs. $\left(\frac{92}{100}\times 125000\right)$ = Rs. 115000.

$\therefore$ An investment of Rs. 115000 is required to purchase Rs. 125000 of 8% stock at 92.

27. Ans. (b)

Solution Market value of Rs. 100 stock = Rs. 110

$\therefore$ Market value of Rs. 90000 stock

= Rs. $\left(\frac{110}{100}\times 90000\right)$ = Rs. 99000

$\therefore$ An investment of Rs. 99000 is required to purchase Rs. 90000 of 8% stock at 110.

28. Ans. (c)

Solution Brokerage = 1% Rs. 90 = Re. 0.90

$\therefore$ Investment needed to buy Rs. 100 stock
= Rs. 90.90 on which the income is Rs. 9%

For income of Rs. $9\frac{1}{2}$, the investment
= Rs. 90.90

For income of Rs. 1938, the investment

= Rs. $\left(\frac{90.90\times 2}{19}\times 1938\right)$ = Rs. 18543.60.

29. Ans. (b)

Solution Investment made by the man in buying Rs. 20000 of 5% stock at 90 = Rs. $\left(\frac{90}{100}\times 20000\right)$
= Rs. 18000.

When the price rose to Rs. $93\frac{3}{4}$, the man sold the stock. Thus, money realised from selling the stock

= Rs. $\left(\frac{375}{4}\times\frac{1}{100}\times 20000\right)$ = Rs. 18750

$\therefore$ Gain in the transaction
= Rs. (18750 – 18000) = Rs. 750

$\therefore$ Gain per cent = $\left(\frac{750}{18000}\times 100\right)\%$ = $4\frac{1}{6}\%$.

30. Ans. (a)

Solution Investment made by Meena in buying Rs. 36000 of $7\frac{1}{2}\%$ stock at 92 = Rs. $\left(\frac{92}{100}\times 36000\right)$
= Rs. 33120.

When the price rose to Rs. $93\frac{3}{4}$, Meena sold the stock. Thus, money realised from selling the stock

= Rs. $\left(\frac{375}{4}\times\frac{1}{100}\times 36000\right)$ = Rs. 33750

$\therefore$ Gain in the transaction
= Rs. (33750 – 33120) = Rs. 630

$\therefore$ Gain per cent = $\left(\frac{630}{33120}\times 100\right)\%$ = 1.9 (approx).

31. Ans. (a)

Solution Stock purchased by investing Rs. 27600 in 4% stock at 92

= Rs. $\left(\frac{27600\times 100}{92}\right)$ = Rs. 30000.

Money realised by selling Rs. 20000 stock at market value of Rs. 96

= Rs. $\left(\frac{20000\times 96}{100}\right)$ = Rs. 19200.

Remaining stock = Rs. (30000 – 20000) = Rs. 10000.

Money realised by selling Rs. 10000 stock at Rs. 90

= Rs. $\left(10000\times\frac{90}{100}\right)$ = Rs. 9000.

∴ Total money realised by selling the whole stock

= Rs. (19200 + 9000) = Rs. 28200.

Money invested = Rs. 27600

∴ Gain = Rs. (28200 – 27600) = Rs. 600.

32. Ans. (b)

Solution Stock purchased by investing Rs. 28500 in 5% stock at 95

$$= \text{Rs.}\left(\frac{100}{95}\times 28500\right) = \text{Rs. } 30000$$

Money realised by selling Rs. 15000 stock market value of Rs. 98.

$$= \text{Rs.}\left(\frac{98}{100}\times 15000\right) = \text{Rs. } 14700$$

Remaining stock = Rs. (30000 – 15000) = Rs. 15000

Money realised by selling Rs. 15000 stock at Rs. 90

$$= \text{Rs.}\left(\frac{90}{100}\times 15000\right) = \text{Rs. } 13500$$

∴ Total money realised

= Rs. (14700 + 13500) = Rs. 28200

Money invested = Rs. 28500

∴ Loss = Rs. (28500 – 28200) = Rs. 300.

33. Ans. (c)

Solution Income from first stock $= \text{Rs.}\left(\frac{7}{98}\times 245000\right)$

= Rs. 17500

We have to find the amount realised on selling this stock.

Amount realised on selling Rs. 98 stock = Rs. 100

∴ Amount realised on selling Rs. 245000 stock

$$= \text{Rs.}\left(\frac{100}{98}\times 245000\right) = \text{Rs. } 250000$$

This amount is invested in 9% stock at 125

∴ Income from second stock

$$= \text{Rs.}\left(\frac{9}{125}\times 250000\right) = \text{Rs. } 18000$$

Hence, increase in income

= Rs. (18000 – 17500) = Rs. 500.

34. Ans. (b)

Solution Income from first stock

$$= \text{Rs.}\left(\frac{8}{90}\times 32400\right) = \text{Rs. } 2880$$

Amount of stock purchased by Anu

$$= \text{Rs.}\left(\frac{100}{90}\times 32400\right) = \text{Rs. } 36000$$

Amount received by selling Rs. 18000 stock at 95

$$= \text{Rs.}\left(\frac{95}{100}\times 18000\right) = \text{Rs. } 17100$$

Amount received by selling the remaining Rs. 18000 stock at 98

$$= \text{Rs.}\left(\frac{98}{100}\times 18000\right) = \text{Rs. } 17640$$

∴ Total amount received

= Rs. (17100 + 17640) = Rs. 34740

The amount of Rs. 34740 is invested in 10% stock at $96\frac{1}{2}$

∴ Income from this stock

$$= \text{Rs.}\left(10\times\frac{2}{193}\times 34740\right) = \text{Rs. } 3600$$

Hence, change in income

= Rs. (3600 – 2880) = Rs. 720.

35. Ans. (a)

Solution Purchase price of first stock

= Rs. (99 + 3) = Rs. 102

∴ Income on first stock

$$= \text{Rs.}\left(\frac{5}{102}\times 50490\right) = \text{Rs. } 2475$$

Sale price of stock = Rs. (102 – 3) = Rs. 99

∴ Amount received by selling the first stock

$$= \text{Rs.}\left(\frac{99}{102}\times 50490\right) = \text{Rs. } 49005$$

Purchase price of the second stock

= Rs. (96 + 3) = Rs. 99

∴ Income on second stock

$$= \text{Rs.}\left(\frac{8}{99}\times 49005\right) = \text{Rs. } 3960$$

Hence, change in income

= Rs. (3960 – 2475) = Rs. 1485.

36. Ans. (c)

Solution Income on first stock $= \text{Rs.}\left(\frac{5}{104}\times 260000\right)$

= Rs. 12500

Money realised by selling the stock when price rose to Rs. 125

$$= \text{Rs.}\left(\frac{125}{104}\times 260000\right) = \text{Rs. } 312500$$

Income on second stock is Rs. 2500 more on the first stock

∴ Income on second stock

= Rs. (12500 + 2500) = Rs. 15000

Let Rs. x be the market value of the second stock

$$\therefore \frac{312500\times 6}{x} = 15000 \Rightarrow x = \frac{312500\times 6}{15000} = 125$$

i.e. The man purchased the stock at Rs. 125.

37. Ans. (c)

Solution The market value of a debenture = Rs. 125

∴ Income on Rs. 125 is Rs. 5

∴ Income on Rs. 95 is Rs. $\left(\frac{5}{125}\times 95\right)$ = Rs. $\frac{19}{5}$

∴ Per cent income on the debentures is 3.8%.

38. Ans. (b)

Solution The market value of a debenture = Rs. 150

∴ Income on Rs. 150 is Rs. 10

∴ Income on Rs. 120 = Rs. $\left(\frac{10}{150}\times 120\right)$ = Rs. 8

∴ Per cent income on the debentures = 8%.

39. Ans. (a)

Solution Annual dividend on 800 shares

= Rs. $\left(\frac{800\times 50\times 6}{100}\right)$ = Rs. 2400

Annual interest on 600 debentures

= Rs. $\left(\frac{600\times 100\times 12}{100}\right)$ = Rs. 7200

∴ Total annual income of Brij

= Rs. (2400 + 7200) = Rs. 9600

Total investment of Brij

= Rs. (800 × 50 + 600 × 100)

= Rs. (40000 + 60000)

= Rs. 100000

∴ Rate of return

= $\left(\frac{9600}{100000}\times 100\right)$% = 9.6%.

40. Ans. (c)

Solution Face value = Rs (50 × 20) = Rs. 1000

Dividend = Rs. $\left(\frac{1000\times 19}{4\times 100}\right)$ = Rs. $\left(\frac{95}{2}\right)$

Investment = Rs. (45 × 20) = Rs. 900

Rate = Rs. $\left(\frac{95\times 100}{2\times 900}\right)$ = 5.28%.

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. A man invests in a $4\frac{1}{2}$% stock at 96. The interest obtained by him is
(a) 4% (b) 4.5%
(c) 4.69% (d) $\frac{1}{2}$%

2. Find the purchase cost of 80 shares of Rs. 10 each at $\frac{3}{8}$ discount, brokerage being $\frac{1}{8}$ per share.
(a) Rs. 780 (b) Rs. 680
(c) Rs. 980 (d) Rs. 880

3. A man invests Rs. 2730 in 9% at 90 and sells out part of the stock when they have risen to $94\frac{1}{2}$ and the remainder when they have fallen to 86. If he loses Rs. 10 by the transaction, the amount of stock he sold at first (brokerage : 1%) is
(a) Rs. 1500 (b) Rs. 1600
(c) Rs. 2000 (d) Rs. 2200

4. Determine the increase in income obtained by transferring Rs. 4000 of $3\frac{1}{4}$% stock at 87 to $6\frac{1}{4}$% stock at 145.
(a) Rs. 10 increase (b) Rs. 20 increase
(c) Rs. 15 increase (d) None of these

5. Find what a purchaser would have to pay for 300 shares of Rs. 10 each quoted at Rs. 25. What would be the gain to the shareholder, if he had purchased the share at par?
(a) Rs. 5000, Rs. 4500 (b) Rs. 5500, Rs. 4500
(c) Rs. 7500, Rs. 4500 (d) Rs. 4000, Rs. 7500

6. A man invests a certain sum in $9\frac{1}{2}$% government paper at 104. The price falling to Rs. 101, he sells out and loses Rs 600 by the transaction, exclusive of brokerage. The sum invested was
(a) Rs. 31200 (b) Rs. 10400
(c) Rs. 20800 (d) Rs. 15600

7. Which one of the following is the best investment?
(a) 13% stock at 104 (b) 17% stock at 140
(c) 15% stock at 125 (d) 9% stock at 70

8. A man invests half of his money in $13\frac{1}{2}$% stock at 120 and the other half in $10\frac{1}{2}$% at 90. Had he invested his money in order to buy equal amounts of each stock, he would have got Rs. 4.5 less income. His total investment was
(a) Rs 14200 (b) Rs. 17240
(b) Rs. 15120 (d) Rs. 13010

9. Divide Rs. 5300 into two parts such that if one part is invested in $10\frac{1}{2}$% at 98 and the other in 12% at par, the resulting income are equal (neglect brokerage).

(a) Rs. 2800, Rs. 2500 (b) Rs. 3200, Rs. 2100
(c) Rs. 2500, Rs. 2800 (d) Rs. 2100, Rs. 3200

10. A man invests some money partly in 3% stock at 96 and partly in 4% stock at 120. To get equal dividends from both, he must invest the money in the ratio
(a) 16 : 15 (b) 3 : 4
(c) 4 : 5 (d) 3 : 5

Test II

[Time: 10 min]

11. Find the cash realised by selling Rs. 2400, $5\frac{1}{2}$% stock at 5 premium (brokerage $\frac{1}{4}$%).
(a) Rs. 2614 (b) Rs. 2400
(c) Rs. 2514 (d) Rs. 2714

12. Which one of the following is the best investment?
(a) $9\frac{1}{2}$% stock at 120 (b) 11% stock at 125
(c) 7% stock at 84 (d) 20% stock at 230

13. A man buys Rs. 20 shares paying 9% dividend. The man wants to have an interest of 12% on his money. The market value of each share must be
(a) Rs. 12 (b) Rs. 15
(c) Rs. 18 (d) Rs. 21

14. I invest Rs. 5400 in 3% stock at 90 and sell out at 85. With the proceeds I buy $4\frac{1}{2}$% stock at 102. How is my income effected?
(a) Rs. 35 increase (b) Rs. 45 increase
(c) Rs. 30 increase (d) None of these

15. By investing in a company paying a dividend of 30% one obtains a return of 12%. Company's Rs. 10 share is quoted at
(a) Rs. 35 (b) Rs. 20
(c) Rs. 25 (d) Rs. 45

16. I transfer Rs. 4000 from 3% stock at 92 to a 4% stock and thus gain Rs. 5 in income. Find the price of the 4% stock.
(a) Rs. 117.76 (b) Rs. 118.26
(c) Rs. 119.76 (d) Rs. 107.76

17. Having Rs. 8370 to invest a man puts part of it in 9% stock at 96 and the remainder in 12% stock at 120. If his dividend from each investment is same, amount invested in second stock was
(a) Rs. 4140 (b) Rs. 4320
(c) Rs. 4050 (d) Rs. 4230

18. I invested Rs. 10000 partly in 4% at 120 and partly in 5% at 130 and my total income was Rs. 360. How much did I invest at 5% stock?
(a) Rs. 5200 (b) Rs 5000
(c) Rs. 4800 (d) Rs. 4700

19. The sale proceeds of Rs. 4500, $6\frac{1}{2}$% stock at 95 is transferred to 16% stock and thereby income increases by Rs 11.50. The price of 16% stock is
(a) Rs. 110 (b) Rs. 150
(c) Rs. 175 (d) Rs. 225

20. By investing in $3\frac{3}{4}$% stock at 96, one earns Rs. 100. The investment made is
(a) Rs. 36000 (b) Rs. 3600
(c) Rs. 2560 (d) Rs. 4800

Answers

1. (c)	**2.** (a)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (a)
11. (c)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (c)	**16.** (a)	**17.** (c)	**18.** (a)	**19.** (d)	**20.** (c)

20

CLOCKS AND CALENDAR

CLOCKS

The circumference of a dial of a clock (or watch) is divided into 60 equal parts called **minute spaces.** The clock has two hands—the hour hand and the minute hand. The hour hand (or short hand) indicates time in hours and the minute hand (or long hand) indicates time in minutes. In an hour, the hour hand covers 5 minute spaces while the minute hand covers 60 minute spaces. Thus, in one hour or 60 minutes, the minute hand gains 55 minute spaces over the hour hand.

SOME BASIC FACTS

1. In every hour:
   (a) Both the hands coincide once
   (b) The hands are straight (point in opposite directions) once. In this position, the hands are 30 minute spaces apart.
   (c) The hands are twice at right angles. In this position, the hands are 15 minute spaces apart.
2. The minute hand moves through 6° in each minute whereas the hour hand moves through $\frac{1^\circ}{2}$ in each minute. Thus, in one minute, the minute hand gains $5\frac{1^\circ}{2}$ than the hour hand.
3. (a) When the hands are coincident, the angle between them is 0°.
   (b) When the hands point in opposite directions, the angle between them is 180°.
   (c) The hands are in the same straight line, when they are coincident or opposite to each other. So, the angle between the two hands is 0° or 180°.
3. The minute hand moves 12 times as fast as the hour hand.
4. If a clock indicates 6.10, when the correct time is 6, it is said to be 10 minutes **too fast**. And if it indicates 5.50, when the correct time is 6, it is said to be 10 minutes **too slow.**

SOME USEFUL SHORT-CUT METHODS

1. The two hands of the clock will be together between H and $(H + 1)$ O'clock at $\left(\frac{60H}{11}\right)$ minutes past H O'clock.

Explanation

At H O'clock the minute hand is $5H$ minute spaces behind the hour hand.

$\because$ the minute hand gains 55 minute spaces in 60 minute,

$\therefore$ the minute hand will gain $5H$ minute spaces in $\frac{60}{55} \times 5H = \frac{60H}{11}$ minutes. Thus, the two hands of clock will be together between H and $(H + 1)$ O'clock at $\left(\frac{60H}{11}\right)$ minutes past H O'clock.

Illustration 1 At what time between 5 and 6 O'clock are the hands of a clock together?

Solution Here, $H = 5$.

$$\therefore \quad \frac{60H}{11} = \frac{60}{11} \times 5 = \frac{300}{11} = 27\frac{3}{11}.$$

$\therefore$ Hands of a clock are together at $22\frac{3}{11}$ minutes past 5 O'clock.

2. The two hands of the clock will be at right angles between H and $(H + 1)$ O'clock at $(5H \pm 15)\frac{12}{11}$ minutes past H O'clock.

Explanation

At H O'clock, the minute hand will be $5H$ minute spaces behind the hour hand. When the two hands are at right angle, they are 15 minute spaces apart. So there are two cases:

Case I When the minute hand is 15 minute spaces behind the hour hand, to be in this position, the minute hand will have to gain $(5H - 15)$ minute spaces over the hour hand.

Case II When the minute hand is 15 minute spaces ahead of the hour hand, to be in this position, the minute hand will have to gain $(5H + 15)$ minute spaces over the hour hand. Combining the two cases, the minute hand will have to gain $(5H \mp 15)$ minute spaces over the hour hand.

Now, 55 minute spaces are gained in 60 minutes.

$\therefore$ $(5H \mp 15)$ minute spaces are gained in

$$\frac{60}{55}(5H \mp 15) = \frac{12}{11}(5H \mp 15) \text{ minutes.}$$

So, they are at right angle at $(5H \mp 15)\frac{12}{11}$ minutes past H O'clock.

Illustration 2 At what time between 5 and 6 O'clock will the hands of a clock be at right angle?

Solution Here, $H = 5$

$$\therefore \quad (5H \mp 15)\frac{12}{11} = (5 \times 5 \mp 15)\frac{12}{11} = 10\frac{10}{11} \text{ and } 43\frac{7}{11}$$

$\therefore$ Hands of a clock are at right angle at $10\frac{10}{11}$ minutes past 5 and $43\frac{7}{11}$ minutes past 5.

3. The two hands of the clock will be in the same straight line but not together between H and $(H + 1)$ O'clock at

$(5H - 30)\frac{12}{11}$ minutes past H, when $H > 6$

and, $(5H + 30)\frac{12}{11}$ minutes past H, when $H < 6$.

Illustration 3 Find at what time between 2 and 3 O'clock will the hands of a clock be in the same straight line but not together.

Solution Here, $H = 2 < 6$.

$$\therefore \quad (5H + 30)\frac{12}{11} = (5 \times 2 + 30)\frac{12}{11}$$
$$= \frac{480}{11}, \text{ i.e., } 43\frac{7}{11}.$$

So, the hands will be in the same straight line but not together at $43\frac{7}{11}$ minutes past 2 O'clock.

4. Between H and $(H + 1)$ O'clock, the two hands of a clock are M minutes apart at $(5H \mp M)\frac{12}{11}$ minutes past H O'clock.

Explanation

At H O'clock, the two hands are $5H$ minute spaces apart.

Case I Minute hand is M minute spaces behind the hour hand. In this case, the minute hand has to gain $(5H - M)$ minute spaces over the hour hand.

Case II Minute hand is M minute spaces ahead of the hour hand. In this case, the minute hand has to gain $(5H + M)$ minute spaces over the hour hand.

Combining the two cases, the minute hand has to gain $(5H \pm M)$ minute spaces over the hour hand.

Now, 55 minute spaces are gained in 60 minutes.

$\therefore$ $(5H \pm M)$ minute spaces are gained in

$$\frac{60}{55}(5H \pm M) = \frac{12}{11}(5H \pm M) \text{ minutes.}$$

$\therefore$ The hands will be M minutes apart at

$$\frac{12}{11}(5H \pm M) \text{ minutes past } H \text{ O'clock.}$$

Illustration 4 Find the time between 4 and 5 O'clock when the two hands of a clock are 4 minutes apart.

Solution Here, $H = 4$ and $M = 4$.

$$\therefore \quad \frac{12}{11}(5H \pm M) = \frac{12}{11}(5 \times 4 \pm 4)$$
$$= 26\frac{2}{11} \text{ and } 17\frac{5}{11}.$$

$\therefore$ The hands will be 4 minutes apart at $26\frac{2}{11}$ minutes past 4 and $17\frac{5}{11}$ minutes past 4 O'clock.

5. Angle between hands of a clock

(a) When the minute hand is behind the hour hand, the angle between the two hands at M minutes past H O'clock $= 30\left(H - \frac{M}{5}\right) + \frac{M}{2}$ degrees.

(b) When the minute hand is ahead of the hourhand, the angle between the two hands, at M minutes past H O'clock $= 30\left(\frac{M}{5} - H\right) - \frac{M}{2}$ degree.

Illustration 5 Find the angle between the two hands of a clock at 15 minutes past 4 O'clock.

Solution Here, $H = 4$ and $M = 15$.

$\therefore$ The required angle

$$= 30\left(H - \frac{M}{5}\right) + \frac{M}{2} \text{ degrees}$$

$$= 30\left(4 - \frac{15}{5}\right) + \frac{15}{2}$$

$$= \frac{75}{2}, \text{ i.e. } 37.5°.$$

6. The minute hand of a clock overtakes the hour hand at intervals of M minutes of correct time. The clock gains or loses in a day by

$$= \left(\frac{720}{11} - M\right)\left(\frac{60 \times 24}{M}\right) \text{ minutes.}$$

Illustration 6 The minute hand of a clock overtakes the hour hand at intervals of 65 minutes. How much a day does the clock gain or lose?

Solution Here, $M = 65$

$\therefore$ The clock gains or loses in a day by

$$= \left(\frac{720}{11} - M\right)\left(\frac{60 \times 24}{M}\right)$$

$$= \left(\frac{720}{11} - 65\right)\left(\frac{60 \times 24}{65}\right)$$

$$= \frac{5}{11} \times \frac{12 \times 24}{13} = \frac{1440}{143}$$

$$= 10\frac{10}{143} \text{ minutes.}$$

Since the sign is +ve, the clock gains by $10\frac{10}{143}$ minutes.

CALENDAR

In this section we shall mainly deal with finding the day of the week on a particular given date. The process of finding it depends upon the number of odd days, which are quite differrent from the odd numbers. So, we should be familiar with **odd days.**

Odd Days

The days more than the complete number of weeks in a given period are called odd days.

Ordinary Year

An ordinary year has 365 days.

Leap Year

That year (except century) which is divisible by 4 is called a leap year, whereas century is a leap year by itself when it is divisible by 400.

For example, 1964, 1968, 1972, 1984, and so on, are all leap years whereas 1986, 1990, 1994, 1998, and so on, are not leap years.

Further, the centuries 1200, 1600, 2000 and so on, are all leap years as they are divisible by 400 whereas 900, 1300, 1500 and so on, are not leap years.

SOME BASIC FACTS

1. An ordinary year has 365 days, i.e. 52 weeks and 1 odd day.
2. A leap year has 366 days, i.e. 52 weeks and 2 odd days.
3. A century has 76 ordinary years and 24 leap years.

   $\therefore$ 100 years = 76 ordinary years + 24 leap years

   = 76 odd days + 24 × 2 odd days

   = 124 odd days = 17 weeks + 5 days

   $\therefore$ 100 years contain 5 odd days.
4. 200 years contain 10 and therefore 3 odd days.
5. 300 years contain 15 and therefore 1 odd day.
6. 400 years contain (20 + 1) and therefore 1 odd day.
7. February in an ordinary year has no odd day, but in a leap year has one odd day.
8. Last day of a century cannot be either Tuesday, Thursday or Saturday.
9. The first day of a century must either be Monday, Tuesday, Thursday or Saturday.

Explanation

No. of odd days in first century = 5

$\therefore$ Last day of first century is Friday.

No. of odd days in two centuries = 3

$\therefore$ Wednesday is the last day.

No. of odd days in three centuries = 1

$\therefore$ Monday is the last day.

No. of odd days in four centuries = 0

$\therefore$ Last day is Sunday.

Since the order is continually kept in successive cyles, the last day of a century cannot be Tuesday, Thursday or Saturday. So, the last day of a century should be either Sunday, Monday, Wednesday or Friday. Therefore, the first day of a century must be either Monday, Tuesday, Thursday or Saturday.

SOME USEFUL SHORT-CUT METHODS

1. Working rule to find the day of the week on a particular date when reference day is given:

Step I Find the net number of odd days for the period between the reference date and the given date (Exclude the reference day but count the given date for counting the number of net odd days).

Step II The day of the week on the particular date is equal to the number of net odd days ahead of the reference day (if the reference day was before this date) but behind the reference day (if this date was behind the reference day).

Illustration 7 11 January 1997 was a Sunday. What day of the week was on 7 January 2000?

Solution Total number of days between 11 January 1997 and 7 January 2000

= (365 – 11) in 1997 + (365 days in 1998) + (365 days in 1999) + (7 days in 2000)

= (50 weeks + 4 odd days) + (52 weeks + 1 odd day) + (52 weeks + 1 odd day) + (7 odd days)

= 13 days = 1 week + 6 odd days.

Hence, 7 Jan. 2000 would be 6 days ahead of Sunday, i.e. it was on Saturday.

2. Working Rule to find the day of the week on a particular date when no reference day is given

Step I Count the net number of odd days on the given date

Step II Write:

Sunday for 0 odd day
Monday for 1 odd day
Tuesday for 2 odd days
⋮ ⋮ ⋮
Saturday for 6 odd days.

Illustration 8 What day of the week was on 5 June 1999?

Solution 5 June 1999 means 1998 years + first five months up to May of 1999 + 5 days of June.

1600 years have 0 odd day.
300 years have 1 odd day.
98 years have 24 leap years + 74 ordinary years
= (24 × 2) + (74 × 1) days
= 122 days = 17 weeks + 3 odd days
Thus, 1998 years have 4 odd days.
1 January 1999 to 31 May 1999 has
= (3 + 0 + 3 + 2 + 3 + 5) 2 odd days
= 16 days = 2 weeks + odd days
Total number of odd days on 5 June 1999
= (4 + 2) odd days = 6 odd days.

Hence, 5 June 1999 was Saturday.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. At what time between 3 and 4 O'clock are the hands of a clock together?
(a) $15\frac{7}{11}$ minutes past 4 (b) $16\frac{4}{11}$ minutes past 3.
(c) $16\frac{2}{11}$ minutes past 2 (d) None of these

2. At what time between 7 and 8 O'clock will the hands of a clock be at right angle?
(a) $19\frac{5}{11}$ minutes past 2 (b) $21\frac{9}{11}$ minutes past 7
(c) 18 minutes past 4 (d) None of these

3. Find at what time between 8 and 9 O'clock will the hands of a clock be in the same straight line but not together?
(a) $11\frac{9}{11}$ minutes past 5
(b) $9\frac{7}{11}$ minutes past 5
(c) $10\frac{10}{11}$ minutes past 8
(d) None of these

4. At what time between 5 and 6 O'clock are the hands of a clock 3 minutes apart?
(a) 24 minutes past 5 (b) 22 minutes past 3
(c) 26 minutes past 4 (d) None of these

5. Find the angle between the two hands of a clock at 30 minutes past 4 O'clock.
(a) 40° (b) 30°
(c) 45° (d) None of these

6. How much does a watch gain or lose per day, if its hands coincide every 64 minutes?

(a) $32\frac{8}{11}$ minutes gain (b) $34\frac{2}{11}$ minutes gain
(c) $32\frac{8}{11}$ minutes loss (d) None of these

7. How often between 11 O'clock and 12 O'clock are the hands of the clock an integral number of minutes apart?
(a) 55 times (b) 56 times
(c) 58 times (d) 60 times

8. Number of times the hands of a clock are in a straight line everyday is
(a) 44 (b) 24
(c) 42 (d) 22

9. My watch gains 5 seconds in 3 minutes was set right at 7 a.m. In the afternoon of the same day, when the watch indicated quarter past 4 O'clock, the true time is
(a) $59\frac{7}{12}$ min. past 3 (b) $12\frac{3}{11}$ min. past 3
(c) 4 p.m. (d) $7\frac{5}{12}$ min. past 4

10. My watch gains 5 min. every hour. How many degrees the second hand moves in every minute?
(a) 375° (b) 380°
(c) 390° (d) 365°

11. At what time between 4 : 30 and 5 will the hands of a clock be in a straight line?
(a) 50 mins. past 4 (b) 42 mins. past 4
(c) $54\frac{6}{11}$ mins. past 4 (d) 46 mins. past 4

12. Two clocks are set right at 10 a.m. One gains 20 seconds and the other loses 40 seconds in 24 hours. What will be the true time when the first clock indicates 4 p.m. on the following day?
(a) $3 : 59\frac{2521}{4321}$ p.m. (b) $3 : 31\frac{1}{471}$ p.m.
(c) $3 : 59\frac{7}{12}$ p.m. (d) $3 : 57\frac{2521}{4321}$ p.m.

13. A clock strikes 4 taking 9 seconds. In order to stride 12 at the same rate, the time taken is
(a) 27 seconds (b) 36 seconds
(c) 30 seconds (d) 33 seconds

14. How often are the hands of a clock at right angle everyday?
(a) 38 times (b) 44 times
(c) 40 times (d) 48 times

15. A clock is set right at 5 a.m. The clock loses 16 min. in 24 hours. What will be the true time when the clock indicates 10 p.m. on the 4th day?
(a) 9 a.m. (b) 11 p.m.
(c) 11 a.m. (d) 9 p.m.

16. My watch was 3 minutes slow at 5 p.m. Tuesday and it was 5 minutes fast at 11 p.m. Wednesday. When did it give correct time?
(a) Wednesday 4 : 15 a.m.
(b) Wednesday 7 : 30 a.m.
(c) Tuesday 3 : 45 p.m.
(d) None of these

17. How many times do the hands of a clock point towards each other in a day?
(a) 24 (b) 20
(c) 12 (d) 22

18. A man who went out between 3 and 4 and returned between 8 and 9, found that the hands of the watch had exactly changed places. He returned at
(a) 14 mins. past 8 (b) $21\frac{1}{13}$ mins. past 8
(c) $19\frac{2}{13}$ mins. past 8 (d) $18\frac{6}{13}$ mins. past 8

19. A clock gains 10 minutes in every 24 hours. It is set right on Monday at 8 a.m. What will be the correct time on the following Wednesday, when the watch indicates 6 p.m.?
(a) 5.36 p.m. (b) 5.40 p.m.
(c) 4.36 p.m. (d) None of these

20. If the hands of a clock coincide every 65 minutes (true time), in 24 hrs the clock will gain
(a) $10\frac{10}{143}$ mins. (b) $9\frac{12}{143}$ mins.
(c) $11\frac{12}{143}$ mins. (d) $12\frac{10}{143}$ mins.

21. The watch which gains uniformly is 2 mins. slow at noon on Sunday and is 4 min. 48 secs. fast at 2 p.m. on the following Sunday. The watch was correct at
(a) 2 p.m. on Tuesday
(b) 12 noon on Monday
(c) 1 : 30 p.m. on Tuesday
(d) 12 : 45 p.m. on Monday

22. A watch which gains uniformly is 6 minutes slow at 4 p.m. on a Sunday and $10\frac{2}{3}$ minutes fast on the following Sunday at 8 p.m. During this period (Day and Time) when was the watch correct?
(a) 2.36 a.m. (b) 1.36 a.m.
(c) 2.36 p.m. (d) 1.36 p.m.

23. If a clock takes 22 seconds to strike 12, how much time will it take to strike 6?
(a) 10 seconds (b) 12 seconds
(c) 14 seconds (d) None of these

24. Mahatma Gandhi was born on 2 October 1869. The day of the week was
(a) Sunday (b) Monday
(c) Saturday (d) Friday

25. 5 March 1999 was on Friday. What day of the week was on 5 March 2000?
(a) Monday (b) Sunday
(c) Friday (d) Tuesday

26. On what dates of August, 1988 did Fridays fall ?
(a) 5 (b) 4
(c) 14 (d) 17

27. India got independence on 15 August 1947. What was the day of the week?
(a) Monday (b) Friday
(c) Thursday (d) Sunday

28. 7 January 1992 was Tuesday. Find the day of the week on the same date after 5 years, i.e. on 7 January 1997?
(a) Tuesday (b) Wednesday
(c) Saturday (d) Friday

29. Number of times 29th day of the month occurs in 400 consecutive years is
(a) 4497 (b) 4800
(c) 4400 (d) None of these

30. The first Republic Day of India was celebrated on 26 January 1950. What was the day of the week on that date?
(a) Monday (b) Wednesday
(c) Saturday (d) Thursday

31. In an ordinary year? 'March' ? begin on the same day of the week
(a) Feb.; Nov. (b) Jan.; Nov.
(c) Feb.; Oct. (d) Jan.; Sept.

32. If 2 March 1994 was on Wednesday, 25 Jan. 1994 was on
(a) Wednesday (b) Thursday
(c) Tuesday (d) Monday

33. Calendar for 2000 will serve for also:
(a) 2003 (b) 2006
(c) 2007 (d) 2005

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Here, $H = 3$.

$$\therefore \quad \frac{60H}{11} = \frac{60}{11} \times 3 = \frac{180}{11} = 16\frac{4}{11}$$

So, the hands of a clock will coincide at $16\frac{4}{11}$ minutes past 3.

2. Ans. (b)

Solution Here, $H = 7$

$$\therefore \quad (5H \mp 15)\frac{12}{11} = (5 \times 7 \mp 15)\frac{12}{11}$$
$$= 21\frac{9}{11} \text{ and } 54\frac{6}{11}.$$

$\therefore$ The hands of a clock are at right angle at $21\frac{9}{11}$ minutes past 7 and $54\frac{6}{11}$ minutes past 7 O'clock.

3. Ans. (c)

Solution Here, $H = 8 > 6$.

$$\therefore \quad (5H - 30)\frac{12}{11} = (5 \times 8 - 30)\frac{12}{11} = \frac{120}{11} = 10\frac{10}{11}.$$

So, the hands will be in the same straight line but not together at $10\frac{10}{11}$ minutes past 8 O'clock.

4. Ans. (a)

Solution Here, $H = 5$ and $M = 3$.

$$\therefore \quad \frac{12}{11}(5H \pm M) = \frac{12}{11}(5 \times 5 \pm 3) = 31\frac{5}{11} \text{ and } 24.$$

$\therefore$ The hands will be 3 minutes apart at $31\frac{5}{11}$ minutes past 5 and 24 minutes past 5 O'clock.

5. Ans. (c)

Solution Here, $H = 4$ and $M = 30$.

$$\therefore \text{ The required angle} = 30\left(\frac{M}{5} - H\right) - \frac{M}{2} \text{ degrees}$$
$$= 30\left(\frac{30}{5} - 4\right) - \frac{30}{2}$$
$$= 60 - 15 = 45°.$$

6. Ans. (a)

Solution Here, $M = 64$

$\therefore$ The clock gains or loses in a day by

$$= \left(\frac{720}{11} - M\right)\left(\frac{60 \times 24}{M}\right)$$
$$= \left(\frac{720}{11} - 64\right)\left(\frac{60 \times 24}{64}\right)$$
$$= \left(65\frac{5}{11} - 64\right) \times \left(\frac{60 \times 24}{64}\right)$$
$$= \frac{16}{11} \times \frac{60 \times 24}{64} = \frac{360}{11} = 32\frac{8}{11} \text{ minutes.}$$

Since the sign is +ve, the clock gains by $32\frac{8}{11}$ minutes.

7. Ans. (b)

Solution At 11 O'clock, the hour hand is 4 spaces apart from the minute hand. Since there are 60 spaces in one hour, so (60 – 4) times, i.e. 56 times the hands of the clock are an integral number of minutes apart.

8. Ans. (a)

Solution We know that, any relative position of the hands of a clock is repeated 11 times in every 12 hrs.

∴ In every 12 hrs, hands coincide 11 times and are opposite to each other 11 times.

∴ In every 12 hrs, hands are in a straight line 11 + 11 = 22 times.

∴ In every 24 hrs hands are in a straight line 44 times.

9. Ans. (c)

Solution From 7 a.m. to 4 : 15, the time is 9 hours 15 min, i.e. 555 min.

Now, $\frac{37}{12}$ min of this watch = 3 minutes of correct watch.

⇒ 555 min. of this watch = $\left(\frac{3\times12}{37}\times555\right)$ on a correct watch = 540 min. or 9 hours of correct watch.

∴ correct time after 7 a.m. is 4 p.m.

10. Ans. (c)

Solution Since minute hand gains 5 minutes in every 60 minutes

⇒ second hand gains 5 seconds in every 60 seconds

∴ In every 60 seconds true time, it moves 65 seconds or 65 × 6° = 390°.

11. Ans. (c)

Solution At 4 O'clock hands are 20 min. spaces apart. At time between 4 : 30 and 5 the hands will be in straight line when they point in opposite directions and there is a space of 30 mins. between them. So, to be in this position min. hand has to gain 30 + 20 = 50 mins. spaces. Min. hand gains 50 min. in

$$\frac{60}{55}\times50=\frac{600}{11}=54\frac{6}{11}\text{ min.}$$

∴ Required time = $54\frac{6}{11}$ mins. past 4.

12. Ans. (a)

Solution From 10 a.m. to 4 p.m. on the following day = 30 hours

Now, 24 hrs 20 seconds of the first clock

= 24 hrs of the current clock.

∴ 1 hour of the first clock = $\frac{24\times180}{4321}$ hours

∴ 30 hours of the first clock = $\frac{24\times180\times30}{4321}$ hours

Now, $\frac{24\times180\times30}{4321}$ hours = 29 hours $59\frac{2521}{4321}$ min.

∴ When the first clock indicates 4 p.m. on the following day the true time will be 3 hrs $59\frac{2521}{4321}$ mins.

13. Ans. (d)

Solution There are 3 intervals when the clock strikes 4

Time taken in 3 intervals = 9 seconds

∴ Time taken for 1 interval = 3 seconds

In order to strike 12, there are 11 intervals, for which the time taken is 11 × 3 seconds = 33 seconds.

14. Ans. (b)

Solution In every hour there are two positions in which hands are at right angle. Each of these positions is repeated 11 times in every 12 hrs.

∴ In every 12 hrs, hands are at right angles 11 + 11 = 22 times and in a day hands are at right angles 22 + 22 = 44 times.

15. Ans. (b)

Solution From 5 a.m. on first day to 10 p.m. on 4th day is 89 hours. When a clock loses 16 min. then 23 hours 44 min. of this clock are the same as 24 hours of correct clock, i.e. $\frac{356}{15}$ hours of this clock = 24 hours of correct clock

∴ 89 hours of this clock

$$=\left(\frac{24\times15}{356}\times89\right)\text{ hours of correct clock}$$

= 90 hours of correct clock

∴ The correct time is 11 p.m.

16. Ans. (a)

Solution Time from 5 p.m. Tuesday to 11 p.m. Wednesday = 30 hours

Clock gains 8 minutes in 30 hours

∴ It gains 3 minutes in $\frac{30}{8}\times3$ hours

= 11 hrs. 15 min.

∴ Correct time is 11 hrs. 15 min. after 5 p.m.

= 4 : 15 a.m. on Wednesday.

17. Ans. (b)

Solution The hands of a clock point towards each other 11 times in every 12 hours. (because between 5 and 7, at 6 O'clock only they point towards each other)

So, in a day the hands point towards each other 22 times.

18. Ans. (d)

19. Ans. (a)

Solution Total number of hours from Monday at 8 a.m. to the following Wednesday at 6 p.m.

24 × 2 + 10 = 58 hours

24 hours 10 minutes of this clock are the same as 24 hours of a correct clock.

$\frac{145}{6}$ hours of the incorrect clock = 24 hours of correct clock.

58 hours of the incorrect clock = $\frac{24\times6}{145}\times58$ hours of correct clock = $57\frac{3}{5}$ hours of correct clock.

Thus, the correct time on the following Wednesday will be 5.36 p.m.

20. Ans. (a)

Solution The min. hand gains 60 mins in $\frac{60}{55}\times60$

$$=\frac{720}{11}=65\frac{5}{11}\text{ mins.}$$

∴ The hands of a correct clock coincide every $65\frac{5}{11}$ mins. But the hands of the clock in question coincide every 65 mins.

The clock in question gains $\frac{5}{11}$ mins. in 65 min.

∴ In 24 hrs = 24 × 60 mins. the clock gains

$$\frac{5}{11}\times\frac{1}{65}\times24\times60=\frac{1440}{143}\text{ mins.}=10\frac{10}{143}\text{ mins.}$$

21. Ans. (a)

Solution From Sunday noon to the following Sunday at 2 p.m., total time = 7 days + 2 hrs

= (7 × 24 + 2) hrs = 170 hrs.

In this period watch gains 2 + 4 mins. 48 seconds

$$=6\frac{48}{60}=6\frac{4}{5}\text{ mins.}$$

∴ Watch gains $6\frac{4}{5}$ mins in 170 hrs.

∴ Watch gains 2 mins. in $\frac{170}{34}\times5\times2=50$ hrs i.e. 2 days and 2 hrs.

∴ Watch will be correct at 2 p.m. on Tuesday.

22. Ans. (b)

Solution Total time in hours from Sunday at 4 p.m. to the following Sunday at 8 a.m.

= 6 × 24 + 16 = 160 hours

Thus, the watch gains $6+10\frac{2}{3}$

$$=16\frac{2}{3}\text{ minutes in 160 hours}$$

Now, $\frac{50}{3}$ minutes are gained in 160 hours.

∴ 6 minutes are gained in $160\times\frac{3}{50}\times6$

$$=\frac{288}{5}\text{ hours}=57\frac{3}{5}\text{ hours.}$$

or, the watch was correct on Wednesday at 1.36 a.m.

23. Ans. (a)

Solution In order to hear 12 strikes, there are 11 intervals (12 – 1) and time of each interval is uniform.

Hence, time to hear each strike is $\frac{22}{11}$ = 2 seconds

Now, to hear six strikes, there are 6 – 1, i.e. 5 × 2 = 10 seconds.

Hence, it will take 10 seconds for a clock to strike 6.

24. Ans. (c)

Solution 2 October 1869 means

1868 complete years + 9 months + 2 days

1600 years give 0 odd days

200 years give 3 odd days

No. of leap years in 68 years = largest integer less than $\frac{68}{4}$ = 17

∴ 68 years contain 17 leap years and 51 non-leap years

∴ 68 years have 2 × 17 + 51 = 85, i.e. 1 odd day

Also count no. of days from 1 Jan. 1869 to 2 Oct. 1869

Jan.	Feb.	March	April	May	June
31	+28	+31	+30	+31	+30
July	Aug.	Sept.	Oct.		
31	+31	+30	+2	= 275	

= 39 weeks + 2 days

∴ This gives 2 odd days

∴ Total no. of odd days = 0 + 3 + 1 + 2 = 6

∴ Day on 2 Oct. 1869 was Saturday.

25. Ans. (b)

Solution Year 2000 was a leap year

Number of days remaining in 1999

= 365 – [31 days Jan. + 28 days Feb. + 5 days March]

= 301 days = 43 weeks, i.e. 0 odd day

Number of days passed in 2000:

January 31 days have 3 odd days

February 29 days (being leap year) have 1 odd day

March 5 days have 5 odd days

∴ Total number of odd days = 0 + 3 + 1 + 5 + 9 days, i.e. 2 odd days

Therefore, 5 March 2000 would be two days beyond Friday, i.e. on Sunday.

26. Ans. (c)

Solution 1 August 1988 means

1987 years + 7 months

Number of odd days in 1987 years:

1600 years have 0 odd days

300 years have 1 odd day

87 years have 21 leap years and 66 ordinary years

So, there are 21 × 2 + 66 × 1 = 108 days, i.e. 15 weeks and 3 odd days

Number of days between 1 Jan. 1988 to 1 August 1989.

Jan.	Feb.	Mar.	April	May	June	July	Aug.	
31	+29	+31	+30	+31	+30	+31	+1	= 241 days

i.e. 30 weeks and 4 odd days

Total number of odd days = 0 + 1 + 3 + 4 = 8 odd days or 1 odd day

Thus, Friday falls on 5th, 12th, 19th and 26th in August 1988.

27. Ans. (b)

Solution 15 August 1947 = (1600 + 300 + 46) years + 1 Jan. to 15 Aug. of 1947

= (1600 + 300 + 46) years + 365 – 16 Aug to 31 Dec. 1947

= (1600 + 300 + 46) years + (365 – 138) days

No. of odd days = 0 + 1 + 1 (from 11 leap years and 35 ordinary years) + 3 = 5 odd days

∴ The day was Friday.

28. Ans. (a)

Solution During the interval we have two leap years as 1992 and 1996 and it contains February of both these years

∴ The interval has (5 + 2) = 7 odd days or 0 odd day.

Hence, 7 January 1997 was also Tuesday.

29. Ans. (a)

Solution 400 consecutive years contain 97 leap years.

∴ In 400 consecutive years Feb. has 29 days 97 times and the remaining 11 months have 29th day

400 × 11 = 4400 times

∴ 29th day of the month occurs 4400 + 97 = 4497 times.

30. Ans. (d)

Solution Total number of odd days = 1600 years have 0 odd day + 300 years have 1 odd day + 49 years (12 leap + 37 ordinary) have 5 odd days + 26 days of Jan. have 5 odd days = 0 + 1 + 5 + 5 = 4 odd days

So, the day was Thursday.

31. Ans. (a)

Solution In an ordinary year, Feb. has no odd day

∴ Feb. and March begin on same day of week.

Also we know that, November and March begin on same day of the week.

32. Ans. (c)

Solution No. of days from 25 Jan. 1994 to 2 march 1994 is

Jan.	Feb.	March	
6	+28	+2	= 36

∴ No. of odd days = 1

∴ Day on 25 Jan. 1994 is one day before the day on 2 March 1994.

But 2 March 1994 was on Wednesday.

∴ 25 Jan. 1994 was on Tuesday.

33. Ans. (b)

Solution Starting with 2000, count for number of odd days in successive years till the sum is divisible by 7.

2000	2001	2002	2003	2004	
2	+ 1	+ 1	+1	+ 2	= 7

∴ No. of odd days up to 2004 = 0

∴ Calendar for 2000 will serve for 2005 also.

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. A clock strikes once at 1 O'clock, twice at 2 O'clock, thrice at 3 O'clock and so on. How many times will it strike in 24 hours?

(a) 78 (b) 136
(c) 156 (d) 196

[Based on (GL) Prel. Exam 2002]

2. The minute hand of a clock is 7 cm long. The area swept by the minute hand in 15 minutes will be

(a) 25.6 sq. cm. (b) 38.5 sq. cm.
(d) 44.0 sq. cm. (d) 77.0 sq. cm.

[Based on SI Rec. Exam 1997]

3. If a clock strikes 6 times in five seconds, the number of strikes in 10 seconds is

(a) 10 (b) 11
(c) 9 (d) 8

[Based on Assstant's Grade Exam 1998]

EXPLANATORY ANSWERS

1. Ans. (c)

Solution $2(1 + 2 + 3 = \ldots \ldots + 12) = \frac{2 \times 12 \times (12+1)}{2}$

$= 12 \times 13 = 156$

2. Ans. (b)

Solution $\frac{\pi r^2}{4} = \frac{22}{7} \times \frac{1}{4} \times (7)^2 = \frac{154}{4} = 38.5$ sq. cm.

3. Ans. (b)

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. Two clocks begin to strike 12 together. One strikes its strokes in 33 seconds and the other in 22 seconds. What is the interval of time between 6th stroke of the first and 8th stroke of the second?
(a) 2 seconds (b) 1 second
(c) 3 seconds (d) None of these

2. At what time between 3 and 4 the minute hand is 4 minutes behind the hour hand?
(a) 12 min. past 3 (b) 8 min. past 3
(c) 15 min. past 3 (d) None of these.

3. The year next to 1988 having the same calendar as that of 1988 is
(a) 1990 (b) 1992
(c) 1993 (d) 1995

4. Two watches one of which gains 1 min. 54.6 sec. and the other loses 1 min. 55.8 secs. daily were set correctly at 10 a.m. on 1 January 2000. When did the watches next indicate the same time?
(a) July 3; midnight (b) July 6; 10 p.m.
(c) June 29; 9 p.m. (d) July 2; 8 p.m.

5. A watch which is set correctly at noon indicates 10 mins. after 9 the same evening when the true time is 10. The true time when the watch indicates 11 on the same evening is
(a) 11 : 30 (b) 12 : 15
(c) 12 : 00 (i.e. midnight)
(d) 11 : 45

6. The year next to 1991 having the same calendar as that of 1990 is
(a) 1998 (b) 2001
(c) 2002 (d) 2003

7. A clock is set right at 6 a.m. The clock gains 10 minutes in 24 hours. What will be the true time when the clock indicates 11 a.m. on the following day?
(a) 28 hrs. 48 min. (b) 29 hrs. 48 min.
(c) 27 hrs. 46 min. (d) None of these

8. Find what time between 8 and 9 O'clock will the hands of a clock be in the same straight line but not together.
(a) $8\frac{10}{11}$ min. past 4 (b) $10\frac{10}{11}$ min. past 8
(c) $12\frac{10}{11}$ min. past 6 (d) $9\frac{10}{11}$ min. past 6

9. On 2 July 1985, it was Wednesday. The day of the week on 2 July 1984 was
(a) Wednesday (b) Tuesday
(c) Monday (d) Thursday

10. The number of odd days in a leap year is
(a) 0 (b) 1
(c) 3 (d) 3

Test II **[Time: 10 min]**

11. When the time in the clock is 12 : 20, then the angle between the hands of the clock is $x°$. Find x.
(a) 120° (b) 130°
(c) 110° (d) 145°

12. 15 August 1992 was Saturday. What day of the week was on 15 August 1991?
(a) Friday (b) Thursday
(c) Tuesday (d) Saturday

13. The time between 3 and 4 when the min. hand is 7 minute spaces ahead of the hour hand is
(a) $8\frac{8}{11}$ mins. past 3 (b) $23\frac{1}{11}$ mins. past 3
(c) 24 mins. past 3 (d) $9\frac{2}{11}$ mins. past 3

14. Between 7 O'clock and 8 O'clock, what will be the time when the hands are 60° apart or 10 minutes apart?
(a) $21\frac{9}{4}$ min. past 7 and $54\frac{6}{11}$ min. past 7
(b) $23\frac{9}{11}$ min. past 6 and $53\frac{7}{11}$ min. past 9
(c) $25\frac{9}{11}$ min. past 7 and $55\frac{6}{11}$ min. past 7
(d) None of the above.

15. The number of odd days in 400 years is
(a) 0 (b) 1
(c) 2 (d) 5

16. Smt. Indira Gandhi died on 31 October 1984. The day of the week was
(a) Monday (b) Tuesday
(c) Wednesday (c) Friday

17. The time between 5 and 6 at which the hands of a clock are together is
(a) 25 mins. past 5 (b) $25\frac{1}{11}$ mins. past 5
(c) $27\frac{3}{11}$ mins. past 5 (d) $28\frac{2}{11}$ mins. past 5

18. My watch which gains uniformly, is 2 mins. slow at noon on Sunday, and is 4 min. 48 seconds fast at 2 p.m. on the following Sunday. When was it correct?
(a) 2 p.m. Tuesday (b) 2 a.m. Tuesday
(c) 3 p.m. Tuesday (d) 4 a.m. Tuesday

19. The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of correct time. How much in a day does the clock gain or lose?
(a) $12\frac{11}{143}$ minutes (b) $10\frac{11}{143}$ minutes
(c) $13\frac{11}{143}$ minutes (d) $9\frac{11}{143}$ minutes

20. The number of times the hands of a clock coincide in 12 hrs. is
(a) 11 (b) 12
(c) 13 (d) 10

Answers

1. (b)	**2.** (a)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (c)	**7.** (a)	**8.** (b)	**9.** (c)	**10.** (d)
11. (c)	**12.** (b)	**13.** (c)	**14.** (a)	**15.** (a)	**16.** (c)	**17.** (c)	**18.** (a)	**19.** (b)	**20.** (a)

21

PROBLEMS ON AGES

INTRODUCTION

Problems based on ages are generally asked in most of the competitive examinations. To solve these problems, the knowledge of linear equations is essential. In such problems, there may be three situations:

(i) Age some years ago
(ii) Present age (iii) Age some years hence

Two of these situations are given and it is required to find the third. The relation between the age of two persons may also be given. Simple linear equations are framed and their solutions are obtained. Sometimes, short cut methods given below are also helpful in solving such problems.

SOME USEFUL SHORT-CUT METHODS

1. If the age of A, t years ago, was n times the age of B and at present A's age is n_2 times that of B, then

$$A\text{'s present age} = \left(\frac{n_1 - 1}{n_1 - n_2}\right) n_2\, t \text{ years}$$

and, $$B\text{'s present age} = \left(\frac{n_1 - 1}{n_1 - n_2}\right) t \text{ years}$$

Explanation

Let the present age of B be x years.

Then, the present age of $A = n_2\, x$ years

Given, t years ago,

$n_1 (x - t) = n_2 x - t$ or, $(n_1 - n_2)\, x = (n_1 - 1)\, t$

or, $x = \left(\frac{n_1 - 1}{n_1 - n_2}\right) t.$

Therefore, B's present age $= \left(\frac{n_1 - 1}{n_1 - n_2}\right) t$ years

and, A's present age $= \left(\frac{n_1 - 1}{n_1 - n_2}\right) n_2\, t$ years .

Illustration 1 The age of father is 4 times the age of his son. If 5 years ago father's age was 7 times the age of his son at that time, what is father's present age?

Solution The father's present age

$= \left(\frac{n_1 - 1}{n_1 - n_2}\right) n_2 t$ [Here, $n_1 = 7$, $n_2 = 4$ and $t = 5$]

$= \left(\frac{7 - 1}{7 - 4}\right) 4 \times 5 = \frac{6 \times 4 \times 5}{3} = 40$ years.

2. The present age of A is n_1 times the present age of B. If t years hence, the age of A would be n_2 times that of B, then

$$A\text{'s present age} = \left(\frac{n_2 - 1}{n_1 - n_2}\right) n_2\, t \text{ years}$$

and $$B\text{'s present age} = \left(\frac{n_2 - 1}{n_1 - n_2}\right) t \text{ years}$$

Explanation

Let the present age of B be x years.

Then, the present age of $A = n_1 x$

Given, t years hence,

$(n_1 x + t) = n_2 (x + t)$

or, $(n_1 - n_2) x = (n_2 - 1) t$

or, $x = \left(\frac{n_2 - 1}{n_1 - n_2}\right) t$

Therefore, B's present age $= \left(\frac{n_2 - 1}{n_1 - n_2}\right) n_1 t$ years

and, A's present age $= \left(\frac{n_2 - 1}{n_1 - n_2}\right) n_1 t$ years.

Illustration 2 The age of Mr. Gupta is four times the age of his son. After ten years, the age of Mr. Gupta will be only twice the age of his son. Find the present age of Mr. Gupta's son.

Solution The present age of Mr. Gupta's son

$$= \left(\frac{n_2 - 1}{n_1 - n_2}\right)t$$

$$= \left(\frac{2-1}{4-2}\right)10$$

[Here, $n_1 = 4$, $n_2 = 2$ and $t = 10$]

= 5 years.

3. The age of A, t_1 years ago, was n_1 times the age of B. If t_2 years hence A's age would be n_2 times that of B, then,

$$A\text{'s present age} = \frac{n_1(t_1+t_2)(n_2-1)}{n_1-n_2} + t_1 \text{ years}$$

$$\text{and, } B\text{'s present age} = \frac{t_2(n_2-1)+t_1(n_1-1)}{n_1-n_2} \text{ years.}$$

Explanation

Let A's present age = x years and B's present age = y years.

Given : $x - t_1 = n_1 (y - t_1)$ and $x + t_2 = n_2 (y + t_2)$

i.e. $x - n_1 y = (1 - n_1) t_1$ ...(1)

and $x - n_2 y = (-1 + n_2) t_2$ ...(2)

Solving (1) and (2), we get

$$x = \frac{n_1(t_1+t_2)(n_2-1)}{n_1-n_2} + t_1$$

$$\text{and, } y = \frac{t_2(n_2-1)+t_1(n_1-1)}{n_1-n_2}.$$

Illustration 3 10 years ago Anu's mother was 4 times older than her daughter. After 10 years, the mother will be twice older than the daughter. Find the present age of Anu.

Solution Present age of Anu

$$= \frac{t_1(n_2-1)+t_1(n_1-1)}{n_1-n_2}$$

[Here, $n_1 = 4$, $n_2 = 2$, $t_1 = 10$ and $t_2 = 10$]

$$= \frac{10(2-1)+10(4-1)}{4-2} = \frac{10-30}{2} = 20 \text{ years.}$$

4. The sum of present ages of A and B is S years. If, t years ago, the age of A was n times the age of B, then

$$\text{Present age of } A = \frac{Sn-t(n-1)}{n-1} \text{ years,}$$

$$\text{and Present age of } B = \frac{S+t(n-1)}{n+1} \text{ years.}$$

Explanation

Let the present ages of A and B be x and y years respectively.

Given: $x + y = S$ ...(1)

and, $x - t = n (y - t)$

or, $x - ny = (1 - n) t$ ...(2)

Solving (1) and (2), we get

$$x = \frac{Sn-t(n-1)}{n+1}$$

$$\text{and, } y = \frac{S+t(n-1)}{n+1}.$$

Illustration 4 The sum of the ages of A and B is 42 years. 3 years back, the age of A was 5 times the age of B . Find the difference between the present ages of A and B.

Solution Here, $S = 42$, $n = 5$ and $t = 3$

∴ Present age of A

$$= \frac{Sn-t(n-1)}{n+1} = \frac{42\times5-3(5-1)}{5+1}$$

$$= \frac{198}{6} = 33 \text{ years}$$

and, present age of B

$$= \frac{5+t(n+1)}{n+1} = \frac{42+3(5-1)}{5+1}$$

$$= \frac{54}{6} = 9 \text{ years.}$$

∴ Difference between the present ages of A and B = 33 − 9 = 24 years.

Note: If, instead of sum (S), difference (D) of their ages is given, replace S by D and in the denominator (n + 1) by (n − 1) in the above formula.

5. The sum of present ages of A and B is S years. If, t years hence, the age of A would be n times the age of B, then

$$\text{present age of } A = \frac{Sn+t(n-1)}{n+1} \text{ years}$$

$$\text{and, present age of } B = \frac{S-t(n-1)}{n+1} \text{ years.}$$

Explanation

Let the present ages of A and B be x and y years, respectively

Given: $x + y = S$ ...(1)

and, $x + t = n (y + t)$

or, $x - ny = t (n - 1)$ ...(2)

Solving (1) and (2), we get

$$x = \frac{Sn+t(n-1)}{n+1}$$

$$\text{and, } y = \frac{S-t(n-1)}{n+1}.$$

Illustration 5 The sum of the ages of a son and father is 56 years. After four years, the age of the father will be three times that of the son. Find their respective ages.

Solution The age of father

$$= \frac{Sn+t(n-1)}{n+1} = \frac{56\times3+4(3-1)}{3+1}$$

[Here, $S = 56$, $t = 4$ and $n = 3$]

$$= \frac{176}{4} = 44 \text{ years.}$$

$$\text{The age of son} = \frac{S - t(n-1)}{n+1}$$

$$= \frac{56 - 4(3-1)}{3+1}$$

$$= \frac{48}{4} = 12 \text{ years.}$$

6. If the ratio of the present ages of A and B is $a : b$ and t years hence, it will be $c : d$, then

$$A\text{'s present age} = \frac{at(c-d)}{ad-bc}$$

$$\text{and, } B\text{'s present age} = \frac{bt(c-d)}{ad-bc}.$$

Illustration 6 The ratio of the age of father and son at present is 6 : 1. After 5 years, the ratio will become 7 : 2. Find the present age of the son.

Solution The present age of the son $= \frac{bt(c-d)}{ad-bc}$

[Here, $a = 6$, $b = 1$, $c = 7$, $d = 2$ and $t = 5$]

$$= \frac{1 \times 5(7-2)}{6 \times 2 - 1 \times 7} = 5 \text{ years.}$$

Note: If, with the ratio of present ages, the ratio of ages t years ago is given, then replace t by $(-t)$ in the above formula.

Illustration 7 6 years ago Mahesh was twice as old as Suresh. If the ratio of their present ages is 9 : 5 respectively, what is the difference between their present ages?

Solution Present age of Mahesh

$$= \frac{-at(c-d)}{ad-bc}$$

$$= \frac{-9 \times 6(2-1)}{1 \times 9 - 5 \times 2}$$

[Here, $a = 9$, $b = 5$, $c = 2$, $d = 1$ and $t = 6$]

= 54 years

Present age of Suresh

$$= \frac{-bt(c-d)}{ad-bc}$$

$$= \frac{-5 \times 6(2-1)}{1 \times 9 - 5 \times 2} = 30 \text{ years.}$$

$\therefore$ Difference of their ages = 54 – 30 = 24 years.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Ten years ago, Mohan was thrice as old as Ram was but 10 years hence, he will be only twice as old. Find Mohan's present age.
(a) 60 years (b) 80 years
(c) 70 years (d) 76 years

2. The ages of Ram and Shyam differ by 16 years. Six years ago, Mohan's age was thrice as that of Ram's, find their present ages.
(a) 14 years, 30 years (b) 12 years, 28 years
(c) 16 years, 34 years (d) 18 years, 38 years

3. 15 years hence, Rohit will be just four times as old as he was 15 years ago. How old is Rohit at present?
(a) 20 (b) 25
(c) 30 (d) 35

4. A man's age is 125% of what it was 10 years ago, but $83\frac{1}{3}$% of what it will be after ten 10 years. What is his present age?
(a) 45 years (b) 50 years
(c) 55 years (d) 60 years

5. If twice the son's age in years be added to the father's age, the sum is 70 and if twice the father's age is added to the son's age, the sum is 95. Father's age is
(a) 40 years (b) 35 years
(c) 42 years (d) 45 years

6. Three years ago, the average age of a family of 5 members was 17. A baby having been born the average age of the family is the same today? What is the age of the child?
(a) 3 years (b) 5 years
(c) 2 years (d) 1 year

7. The ratio of A's and B's ages is 4 : 5. If the difference between the present age of A and the age of B 5 years hence is 3, then what is the total of present ages of A and B?
(b) 68 years (b) 72 years
(c) 76 years (d) 64 years

8. The ages of A and B are in the ratio of 6 : 5 and sum of their ages is 44 years. The ratio of their ages after 8 years will be
(a) 4 : 5 (b) 3 : 4
(c) 3 : 7 (d) 8 : 7

9. One year ago the ratio between Samir and Ashok's age was 4 : 3. One year hence the ratio of their ages will be 5 : 4. What is the sum of their present ages in years?

(a) 12 years (b) 15 years
(c) 16 years (d) Cannot be determined

10. Ratio of Ashok's age to Pradeep's age is 4 : 3. Ashok will be 26 years old after 6 years. How old is Pradeep now?
(a) 18 years (b) 21 years
(c) 15 years (d) 24 years

11. Jayesh is as much younger to Anil as he is older to Prashant. If the sum of the ages of Anil and Prashant is 48 years, what is the age of Jayesh?
(a) 20 years (b) 24 years
(c) 30 years (d) Cannot be determined

12. Five years ago Mr. Sohanlal was thrice as old as his son and ten years hence he will be twice as old as his son. Mr. Sohanlal's present age (in years) is
(a) 35 (b) 45
(c) 50 (d) 55

13. Three times the present age of a father is equal to eight times the present age of his son. Eight years hence the father will be twice as old as his son at that time. What are their present ages?
(a) 35, 15 (b) 32, 12
(c) 40, 15 (d) 27, 8

14. The sum of ages of a father and son is 45 years. Five years ago, the product of their ages was four times the father's age at that time. The present age of the father is
(a) 39 years (b) 36 years
(c) 25 years (d) None of these

15. One year ago a father was four times as old as his son. In 6 years time his age exceeds twice his son's age by 9 years. Ratio of their ages is
(a) 13 : 4 (b) 12 : 5
(c) 11 : 3 (d) 9 : 2

16. The ages of A, B and C together total 185 years. B is twice as old as A and C is 17 years older than A. Then, the respective ages of A, B and C are
(a) 40, 86 and 59 years (b) 42, 84 and 59 years
(c) 40, 80 and 65 years (d) None of these

17. A father's age is three times the sum of the ages of his two children, but 20 years hence his age will be equal to the sum of their ages. Then the father's age is
(a) 30 years (b) 40 years
(c) 35 years (d) 45 years

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Let Mohan's present age be x years and Ram's present age be y years.

Then, according to the first condition,

$x - 10 = 3(y - 10)$

or, $x - 3y = -20$ ...(1)

Now, Mohan's age after 10 years

$= (x + 10)$ years

Ram's age after 10 years $= (y + 10)$

$\therefore$ $(x + 10) = 2(y + 10)$

or, $x - 2y = 10$ ...(2)

Solving (1) and (2), we get

$x = 70$ and $y = 30$

$\therefore$ Mohan's age = 70 years and Ram's age = 30 years.

2. Ans. (a)

Solution Let Ram's age = x years

So, Mohan's age = $(x + 16)$ years

Also, $3(x - 6) = x + 16 - 6$ or, $x = 14$

$\therefore$ Ram's age = 14 years

and, Mohan's age = 14 + 16 = 30 years.

3. Ans. (b)

Solution Let the present age of Rohit be x years

Then, given: $x + 15 = 4(x - 15) \Rightarrow x = 25$.

4. Ans. (b)

Solution Let the present age be x years.

Then, 125% of $(x - 10) = x$

and, $83\frac{1}{3}$% of $(x + 10) = x$

$\therefore$ 125 % of $(x - 10) = 83\frac{1}{3}$% of $(x + 10)$

or, $\frac{5}{4}(x - 10) = \frac{5}{6}(x + 10)$

or, $\frac{5}{4}x - \frac{5}{6}x = \frac{50}{6} + \frac{50}{4}$

or, $\frac{5x}{12} = \frac{250}{12}$ or, $x = 50$ years.

5. Ans. (a)

Solution Let son's age (in years) = x and father's age (in years) = y

Given: $2x + y = 70$ and, $x + 2y = 95$

Solving for y, we get $y = 40$.

6. Ans. (c)

Solution Present age of 5 members

$= 5 \times 17 + 3 \times 5 = 100$ years

Also, present ages of 5 members + Age of the baby

$= 6 \times 17 = 102$ years

$\therefore$ Age of the baby = 102 − 100 = 2 years.

7. Ans. (b)

Solution Given: $\frac{A}{B} = \frac{4}{5}$ or, $B = \frac{5}{4}A$

and, $B - (A + 5) = 3$ or, $B = A + 8$

$\therefore \quad \frac{5}{4}A = A + 8$

or, $A\left(\frac{5}{4} - 1\right) = 8$

$\therefore \quad A = 32$ years

and, $B = \frac{5}{4} \times 32 = 40$ years

$\therefore \quad A + B = 40 + 32 = 72$ years.

8. Ans. (d)

Solution Let present ages (in years) of A and B respectively, be $6x$ and $5x$

Given: $6x + 5x = 44 \Rightarrow x = 4$

Ratio of ages after 8 years will be

$6x + 8 : 5x + 8$

or, $32 : 28$ or, $8 : 7$.

9. Ans. (c)

Solution Let one year ago

Samir's age be $4x$ years

and, Ashok's age be $3x$ years

Present age of Samir = $(4x + 1)$ years

Present age of Ashok = $(3x + 1)$ years

One year hence

Samir's age = $(4x + 2)$ years

Ashok's age = $(3x + 2)$ years

According to question,

$\frac{4x+2}{3x+2} = \frac{5}{4} \Rightarrow 16 + 8 = 15x + 10$

or, $x = 2$.

$\therefore$ Sum of their present ages = $4x + 1 + 3x + 1$

$= 7x + 2$

$= 7 \times 2 + 2 = 16$ years.

10. Ans. (c)

Solution Let the present ages of Ashok and Pradeep be $4x$ and $3x$

So that $4x + 6 = 26 \Rightarrow x = 5$

$\therefore$ Present age of Pradeep is $3x = 3 \times 5$, i.e. 15 years

11. Ans. (b)

12. Ans. (c)

Solution Let Mr. Sohanlal's age (in years) = x and his son's age = y

Then, $x - 5 = 3(y - 5)$ i.e. $x - 3y + 10 = 0$

and, $x + 10 = 2(y + 10)$ i.e. $x - 2y - 10 = 0$

Solving the two equations, we get

$x = 50, y = 20$.

13. Ans. (b)

14. Ans. (b)

Solution Let father's present age = x years

Then, son's present age = $(45 - x)$ years

Given: $(x - 5)(45 - x - 5) = 4(x - 5)$

or, $x^2 - 41x + 180 = 0$ or, $(x - 36)(x - 5) = 0$

$\therefore \quad x = 36$ years.

15. Ans. (c)

Solution Let the present ages of father and son be x and y years, respectively

Then, $(x - 1) = 4(y - 1)$

or, $4y - x = 3$ ...(1)

and, $(x + 6) - 2(y + 6) = 9$

or, $-2y + x = 15$ ...(2)

Solving (1) and (2), we get, $x = 33, y = 9$

$\therefore$ Ratio of their ages = $33 : 9 = 11 : 3$.

16. Ans. (b)

Solution Let A's age be x years

B's age be $2x$ years

C's age = $(x + 17)$ years

According to the question,

$x + 2x + (x + 17) = 185$

$\therefore \quad 4x = 185 - 17 = 168 \quad \therefore \quad x = 42$

$\therefore$ A's age = 42 years

B's age = 84 years

C's age = $42 + 17 = 59$ years.

17. Ans. (a)

Solution Let the present age of father be x years and the present age of son be y years.

$\therefore \quad x = 3y$ ...(1)

Also, $(x + 20) = (y + 20 + 20)$ ...(2)

Solving (1) and (2), we get

$x = 30$ years.

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. A man was n years old x years ago. His age m years from now would be expressed as

(a) $n - x + m$ (b) $x + n - m$

(c) $x + m - n$ (d) $n + x + m$

(e) None of these **[Based on NABARD 1999]**

2. Six years ago Jagunath was twice as old as Badri. If the ratio of their present ages is 9 : 5 respectively, what is

the difference between their present ages in years?
(a) 24 (b) 30
(c) 5 (d) Cannot be determined
(e) None of these
[Based on BSRB Bhopal Bank PO 2000]

3. Jayesh is twice as old as Vijay and half as old as Suresh. If sum of Suresh's and Vijay's ages is 85 years, what is Jayesh's age in years?
(a) 34 (b) 36
(c) 68 (d) Cannot be determined
(e) None of these
[Based on BSRB Bhopal Bank PO 2000]

4. The ratio of the present ages of a son and his father is 1 : 5 and that of his mother and father is 4 : 5. After 2 years the ratio of the ages of the son to that of his mother becomes 3 : 10. What is the present age of the father?
(a) 37 years (b) 34 years
(c) 30 years (d) Data inadequate
(e) None of these
[Based on BSRB Patna Bank PO 2001]

5. The ratio of the present ages of Suresh and Anil is 7 : 8. If 4 years ago the ratio of their ages be 5 : 6, then find the present age of Anil.
(a) 16 years (b) 14 years
(c) 10 years (d) 12 years
(e) None of these
[Based on BSRB Baroda Bank Clerical Exam 2000]

6. Harsha is 40 years old and Rita is 60 years old. How many years ago was the ratio of their ages 3 : 5?
(a) 10 years (b) 20 years
(c) 37 years (d) 5 years
[Based on SSC (GL) Prel. Exam 2002]

7. One year ago the ratio of Yamini's and Gamini's ages was 6 : 7, respectively. Four years hence this ratio would become 7 : 8. How old is Gamini?
(a) 35 years (b) 30 years
(c) 31 years (d) Cannot be determined
(e) None of these
[Based on NABARD Assitant Manager's Exam 2002]

8. A father is 30 times older than his son. 18 years later, he will be only thrice as old as his son. Father's present age in years is
(a) 25 (b) 30
(c) 40 (d) 45
(e) None of these
[Based on RRB Bhubaneshwar ASM Exam 2002]

9. Ratio of present ages of P and Q are 7 : 3. After four years their ages will be in the ratio 2 : 1. What is the present age of P?
(a) 24 years (b) 28 years
(c) 32 years (d) Data inadequate
(e) None of these
[Based on BSRB Chennai Bank PO 2000]

10. Four years ago a mother was $2\frac{3}{4}$ times elder than her daughter in age. If the total age of the two then be 60 years, what is the present age of the daughter?
(a) 16 years (b) 20 years
(c) 12 years (d) 4 years
(e) None of these
[Based on BSRB Jaipur Bank Clerical Exam 2003]

11. Six years ago, Seema was half of that of Rupa in age. Four years hence the respective ratio of their ages would be 3 : 5. How old is Rupa at present?
(a) 32 years (b) 16 years
(c) 40 years (d) Cannot be determined
(e) None of these
[Based on BSRB Bhubaneshwar Bank Clerical Exam 2001]

12. Father is aged three times more than his son Ramu. After 8 years, he would be $2\frac{1}{2}$ times of Ramu's age. After further 8 years, how many times would he be of Ramu's age?
(a) 2 times (b) $2\frac{1}{2}$ times
(c) $2\frac{3}{4}$ times (d) 3 times
[Based on SI Rec. Exam CPO, 1998]

13. The present age of a father is 3 years more than three times the age of his son. Three years hence, father's age will be 10 years more than twice the age of the son. The father's present age is
(a) 33 years (b) 39 years
(c) 45 years (d) 40 years
[Based on SSC (GL) Prel. Exam 1999]

14. My grandfather was 8 times older than me 16 years ago. He would be 3 times my age 8 years from now. Eight years ago, what was the ratio of my age to that of my grandfather?
(a) 3 : 8 (b) 1 : 5
(c) 1 : 2 (d) None of these
[Based on SSC (GL) Prel. Exam 2003]

15. In a class, the average age of 40 boys is 13.5 years and that of the girls is 13 years. The average age of the whole class is 13.4 years. Find the number of girls in the class.
(a) 20 (b) 13
(c) 11 (d) 10
[Based on SSC (GL) Prel. Exam 2000]

16. The average age of 8 men is increased by 2 years when two of them whose ages are 21 and 23 years are replaced by two new men. The average age of the two new men is
(a) 22 years (b) 24 years
(c) 28 years (d) 30 years
[Based on SSC (GL) Prel. Exam 2000]

17. In a school with 600 students, the average age of the boys is 12 years and that of girls is 11 years. If the average age of the school is 11 years and 9 months, then the numbers of girls in the school is
(a) 450 (b) 250
(c) 150 (d) 350
[Based on SI Rec. Exam in CPO, 1998]

18. A father said to his son "I was as old as you are at present at the time of your birth". If the father's age is 38 years now, the son's age five years back was
(a) 14 years (b) 19 years
(c) 38 years (d) 33 years
[Based on Assistant's Grade Exam 1998]

19. The average age of a husband and wife was 23 years at the time of their marriage. After 5 years they have a one year old child. The average age of the family now is
(a) 28.5 years (b) 19 years
(c) 29.9 years (d) 23 years
[Based on Assistant's Grade Exam 1998]

20. Renu's mother was three times as old as Renu 5 years ago. After 5 years, she will be twice as old as Renu. Renu's present age in years is
(a) 35 (b) 10
(c) 20 (d) 15
[Based on Assistant's Grade Exam 1998]

21. The average age of a class of 20 students is 20 years. If the teacher's age is also included the average age increases by one year. The teacher's age is
(a) 24 years (b) 30 years
(c) 41 years (c) 44 years
[Based on Assistant's Grade Exam 1998]

22. The average age of x and y is 18. If z is equal to 9, the average of x, y and z is
(a) 3 (b) 9
(c) 12 (d) 15
[Based on SSC (GL) Prel. Exam 1999]

23. The average age of 12 players of a team is 25 years. If the captain's age is included, the average age increases by 1 year. The age of the captain is
(a) 25 years (b) 38 years
(c) 36 years (d) 26 years
[Based on SSC (GL) Prel. Exam 1999]

24. The average age of 24 students and the class teacher is 16 years. If the class teacher's age is excluded, the average reduces by one year. What is the age of the class teacher?
(a) 50 years (b) 45 years
(c) 40 years (d) Data inadequate
(e) None of these
[Based on SI Rec. Exam in Delhi Police 1997]

25. If the ages of P and R are added to twice the age of Q, the total becomes 59. If the ages of Q and R are added to thrice the age of P, the total become 68. And, if the age of P is added to thrice the age of Q and twice the age of R, the total becomes 108. What is the age of P?
(a) 15 years (b) 19 years
(c) 17 years (d) 12 years
(e) None of these **[Based on SBI PO 1999]**

26. The product of the ages of Harish and Seema is 240. If twice the age of Seema is more than Harish's age by 4 years, what is Seema's age in years?
(a) 12 (b) 20
(c) 10 (d) 14
(e) Data inadequate **[Based on SBI PO 1999]**

27. The average age of 34 boys in a class is 14 years If the teacher's age is included the average age of the boys and the teacher becomes 15. What is the teacher's age?
(a) 48 years (b) 46 years
(c) 49 years (d) 45 years
(e) None of these **[Based on SBI PO 2000]**

28. Q is as much younger than R as he is older than T. If the sum of the ages of R and T is 50 years, what is definitely the difference between R and Q's age?
(a) 25 years (b) 1 year
(c) 2 years (d) Data inadequate
(e) None of these **[Based on SBI PO 2000]**

EXPLANATORY ANSWERS

1. Ans. (b)

2. Ans. (a)

Solution $J - 6 = 2(B - 6)$

or, $J - 2B = -6$ ... (1)

and, $\frac{J}{B} = \frac{9}{5} \Rightarrow J = \frac{9}{5}B$ ... (2)

From (1) and, (2), we get

$B = 30$ years and $J = 54$ years

$\therefore$ Difference between their ages

$= 54 - 30 = 24$ years.

3. Ans. (a)

Solution Let the present age of Jayesh be x years.

Then, age of Vijay $= \frac{x}{2}$ and age of Suresh $= 2x$

Now, age of Suresh + Age of Vijay = 85

or, $2x + \frac{x}{2} = 85$

or, $4x + x = 85 \times 2$ or, $5x = 170$

$\therefore$ $x = 34$ years

Hence, age of Jayesh = 34 years.

4. Ans. (e)

Solution Let the present age of the son be x years

$\therefore$ Present age of the father = $5x$

and present age of the mother = $4x$

After two years,

Age of the son = $x + 2$

Age of the mother = $4x + 2$

According to the question,

$$\frac{x+2}{4x+2} = \frac{3}{10} \Rightarrow 10(x+2) = 3(4x+2)$$

$$\Rightarrow 10x + 20 = 12x + 6$$

$$\Rightarrow 2x = 14 \quad \text{or,} \quad x = 7$$

$$\therefore \quad 5x = 5 \times 7 = 35$$

$\therefore$ Present age of the father = 35 years.

5. Ans. (a)

Solution Ratios of present and past ages are given

$$n_2 = \frac{7}{8},\ n_1 = \frac{5}{6},\ t_1 = 4 \text{ years}$$

Then, Anil's present age

$$= \left(\frac{n_1 - 1}{n_1 - n_2}\right) t_1$$

$$= \left(\frac{\frac{5}{6} - 1}{\frac{5}{6} - \frac{7}{8}}\right) \times 4 = \left(\frac{\frac{-1}{6}}{\frac{-1}{24}}\right) \times 4 = 16 \text{ years.}$$

6. Ans. (a)

Solution Let x years ago the ratio of their ages was 3 : 5.

$\therefore$ According to the question,

$$\frac{40-x}{60-x} = \frac{3}{5}$$

or, $200 - 5x = 180 - 3x$ or, $2x = 20$

$\therefore$ $x = 10$ years.

7. Ans. (e)

Solution Let one year ago, the age of Yamini and Gamini be $6x$ and $7x$ years respectively.

$\therefore$ Their present ages will be $6x + 1$ and $7x + 1$ years, respectively.

Now, four years hence

$$\frac{6x+1+4}{7x+1+4} = \frac{7}{8}$$

or, $\frac{6x+5}{7x+5} = \frac{7}{8}$ or, $49x + 35 = 48x + 40$

or, $x = 40 - 35 = 5$

$\therefore$ Gamini's present age = $7x + 1 = 7 \times 5 + 1$

= 36 years.

8. Ans. (c)

Solution Let the age of the son = x years

$\therefore$ Father's age = $30x$ years

After 18 years,

$$30 + 18 = 3(x + 18)$$

or, $27x = 36$

or, $x = \frac{36}{27} = \frac{4}{3}$

$\therefore$ Father's present age = $30x$

$$= 30 \times \frac{4}{3} = 40 \text{ years.}$$

9. Ans. (b)

Solution Let the present ages of P and Q be $7x$ and $3x$, respectively.

According to the question,

$$\frac{7x+4}{3x+4} = \frac{2}{1}$$

or, $6x + 8 = 7x + 4$

or, $7x - 6x = 8 - 4$

or, $x = 4$ years

Therefore, the present age of P

$= 7x = 7 \times 4 = 28$ years.

10. Ans. (b)

Solution Let the age of the daughter 4 years ago be x

$\therefore$ Age of the mother 4 years ago = $2\frac{3}{4}x = \frac{11}{4}x$

$\therefore$ Total age of the daughter and mother 4 years ago

$$= x + \frac{11}{4}x = \frac{15}{4}x$$

According to question,

$$\frac{15}{4}x = 60$$

or, $x = \frac{60 \times 4}{15} = 16$

$\therefore$ Present age of the daughter = 16 + 4 = 20 years.

11. Ans. (e)

Solution Let Seema's age at present be x years and Rupa's age be y years.

Now, according to the question,

$$(y - 6) = 2(x - 6) \Rightarrow y - 6 = 2x - 12$$

or, $2x - y = 6$ ...(1)

Also, $\frac{x+4}{y+4} = \frac{3}{5}$

or, $5x + 20 = 3y + 12$

or, $5x - 3y = -8$ ...(2)

From equations (1) and (2), we get

$x = 26$ years and $y = 46$ years

12. Ans. (a)

Solution Given: $F = R + 3R = 4R$ ...(1)

and, $F + 8 = 2\frac{1}{2}(R + 8) = \frac{5}{2}(R + 8)$ ...(2)

$F + 16 = ? \times (R + 16)$

(1) and (2) gives $F = 32$ and $R = 8$

$\therefore$ $F + 16 = 48$, $R + 16 = 24$.

13. Ans. (a)

Solution Given: $F - 3S = 3$ ...(1)

and, $(F + 3) - 2(S + 3) = 10$

or, $F - 2S = 13$ ...(2)

Solving (1) and (2) , we get $F = 33$ years.

14. Ans. (b)

Solution Let the present age of grandfather and me be 'x' years and 'y' years, respectively

Given: $x - 16 = 8(y - 16) \Rightarrow 8y - x = 112$ ...(1)

and, $x + 8 = 3(y + 8) \Rightarrow x - 3y = 16$ ...(2)

Solving (1) and (2), we get

$x = \frac{464}{5}$ and $y = \frac{128}{5}$

$\therefore$ Ratio 8 years ago $= y - 8 : x - 8$

$= 88 : 424 = 11 : 53$.

15. Ans. (d)

Solution Suppose, number of girls = K

$40 \times 13.5 + 13K = 13.4(K + 40)$

$\Rightarrow$ $13K + 540 = 13.4K + 536$

$\Rightarrow$ $0.4K = 4 \Rightarrow K = 10$.

16. Ans. (d)

Solution Let average age of 8 men be K and average age of two new men be M

$\therefore$ $8K - 21 - 23 + 2M = 8(K + 2) \Rightarrow M = 30$.

17. Ans. (c)

Solution Let number of girls = G

$\therefore$ Number of boys = $600 - G$

Given: $12(600 - G) + 11G = 11\frac{3}{4} \times 600$

$\Rightarrow$ $7200 - 12G + 11G = 7050$

$\Rightarrow$ $G = 150$.

18. Ans. (b)

Solution At present, the son's age = 19 years

19. Ans. (b)

Solution We have, $H + W = 46$

and, $(H + 5) + (W + 5) + C = H + W + 10 + C$

$= 46 + 10 + 1 = 57$

$\therefore$ Average age of H, W and C now $= \frac{57}{3} = 19$.

20. Ans. (d)

Solution Given: $3(R - 5) = (M - 5)$

and, $2(R + 5) = (M + 5)$

$\therefore$ $M = 35, R = 15$.

21. Ans. (c)

Solution The teacher's age $= 21 \times 21 - 20 \times 20$

$= 441 - 400 = 41$ years.

22. Ans. (d)

Solution Given: $x + y = 36$ and $z = 9$

$\therefore$ $\frac{x + y + z}{3} = 15$.

23. Ans. (b)

Solution Age of the captain $= (25 + 1) \times 13 - 25 \times 12$

$= 338 - 300 = 38$ years.

24. Ans. (c)

Solution Age of the class teacher

$= 25 \times 16 - 24 \times 15$

$= 400 - 360 = 40$ years.

25. Ans. (d)

Solution $P + R + 2Q = 59$

$Q + R + 3P = 68$ and $P + 3Q + 2R = 108$

Solving these, we get

$P = 12$ years.

26. Ans. (a)

Solution Let the ages of Harish and Seema be x and y years, respectively.

According to question,

$x \times y = 240$ ...(1)

$2y - x = 4$ ...(2)

Solving equations (1) and (2), we get

$y = 12$ years

27. Ans. (c)

Solution Teacher's age $= 35 \times 15 - 34 \times 14$

$= 525 - 476 = 49$ years.

28. Ans. (d)

■ EXERCISES FOR SELF-PRACTICE ■

Test II **[Time: 10 min]**

1. Today Mohan is 20 years younger than his father. Ten years ago he was one-half as old as his father. How old will his father be 10 years hence?
(a) 40 years (b) 60 years
(c) 50 years (d) Data inadequate
(e) None of these

2. The difference between the ages of two persons is 10 years. 15 years ago, the elder one was twice as old as the younger one. The present age of the elder person is
(a) 25 years (b) 35 years
(c) 45 years (d) 55 years

3. The ratio of Laxmi's age to the age of her mother is 3 : 11. The difference of their ages is 24 years. The ratio of their ages after 3 years will be
(a) 1 : 3 (b) 2 : 3
(c) 3 : 5 (d) None of these

4. The age of a man is three times the sum of the ages of his three children. After three years his age will be twice the sum of their ages. Find the present age of the man.
(a) 40 years (b) 25 years
(c) 35 years (d) 45 years

5. If 6 years are subtracted from the present age of Randheer and the remainder is divided by 18, then the present age of his grandson Anup is obtained. If Anup is 2 years younger to Mahesh whose age is 5 years, then what is the age of Randheer?
(a) 96 years (b) 84 years
(c) 48 years (d) 60 years

6. The difference of present ages of A and B is 12 years. If 6 years back their ages were in the ratio 3 : 2, how old are A and B?
(a) 42 years, 30 years (b) 36 years, 24 years
(c) 48 years, 36 years (d) 44 years, 32 years

7. In 10 years, A will be twice as old as B was 10 years ago. If A is now 9 years older than B, the present age of B is
(a) 29 years (b) 39 years
(c) 19 years (d) 49 years

8. Ages of A and B are in the ratio 4 : 3. A will be 26 years old after 6 years. B's present age is:
(a) 15 years (b) 20 years
(c) 12 years (d) 18 years

9. A mother tells her daughter " I was half your present age when you were born". If the mother is 18 years older than her daughter, what is the present age of the mother?
(a) 54 (b) 63
(c) 36 (d) 72

10. Ten years ago A was half of B in age. If the ratio of their present ages is 3 : 4, what will be the total of their present ages?
(a) 8 years (b) 20 years
(c) 35 years (d) 45 years

Test I **[Time: 9 min]**

11. The sum of the ages of A and B is 60 years. After 10 years A will be twice as old as B. Find their present ages.
(a) 40 years, 8 years (b) 50 years, 10 years
(c) 60 years, 20 years (d) 70 years, 30 years

12. If Rahul as $\frac{1}{3}$rd as old as Vineet 5 years back and Rahul is 17 years old now, how old is Vineet now?
(a) 41 years (b) 36 years
(c) 9 years (d) 51 years
(e) None of these

13. After five years the age of a father will be twice the age of his son, whereas five years ago, he was seven times as old as his son was. What is father's present age?
(a) 35 years (b) 40 years
(c) 45 years (d) 50 years

14. 20 years ago my age was $\frac{1}{3}$ of what it is now. What is my present age?
(a) 30 years (b) 35 years
(c) 40 years (d) 45 years

15. The ratio of father's age to son's age is 3 : 1. The product of their ages is 243. The ratio of their ages 3 years hence will be
(a) 3 : 1 (b) 5 : 1
(c) 7 : 3 (d) 5 : 2

16. Vinay was twice as old as Dev 10 years back. How old is Dev today if Vinay will be 40 years old 10 years hence?
(a) 20 years (b) 10 years
(c) 30 years (d) 15 years

17. Neena was married 6 years ago. Today, her age is $1\frac{1}{4}$ times of her age at the time of marriage. The age

of her son is $\frac{1}{10}$ times of her present age. What is the age of her son?

(a) 3 years (b) 2 years
(c) 4 years (d) 4 years
(e) None of these

18. If $1\frac{1}{2}$ years age are added to $\frac{3}{7}$ age of Sohan, he will be half his age. What is his actual age?

(a) 20 years (b) 21 years
(c) 25 years (d) 9 years

19. Rakesh is as much younger to Ajay as he is older to Anil. If the sum of the ages of Ajay and Anil is 48 years, what is the age of Rakesh?

(a) 36 years (b) 24 years
(c) 30 years (d) 28 years

Answers

1. (b)	**2.** (b)	**3.** (a)	**4.** (d)	**5.** (d)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (a)	**10.** (c)
11. (b)	**12.** (a)	**13.** (b)	**14.** (a)	**15.** (d)	**16.** (a)	**17.** (a)	**18.** (b)	**19.** (b)	

22

MENSURATION I: AREA AND PERIMETER

INTRODUCTION

In this chapter, we shall be dealing with plane figures of various shapes finding their sides, perimeters and areas.

Area

The area of any figure is the amount of surface enclosed within its bounding lines. Area is always expressed in square units.

Units of Measuring Area

100 sq millimetres = 1 sq centimetre

100 sq centimetres = 1 sq decimetre

100 sq decimetres = 1 sq metre

100 sq metres = 1 sq decametre or arc

10,000 sq metres = 1 hectare

1,000,000 sq metres = 100 hectares

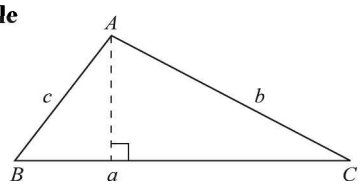
= 1 sq kilometre

Perimeter

The perimeter of a geometrical figure is the total length of the sides enclosing the figure.

SOME BASIC FORMULAE

1. Triangle



A *triangle* is a closed figure bounded by three sides. ABC is a triangle.

The sides AB, BC and AC are respectively denoted by c, a and b.

Area of a Triangle (A)

(a) $A = \frac{1}{2}$ (base × height) $= \frac{1}{2}ah$

(b) $A = \sqrt{s(s-a)(s-b)(s-c)}$,

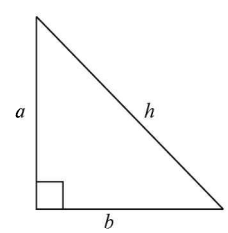
where $s = \frac{1}{2}(a + b + c)$ or semi-perimeter of the triangle

This formula is known as *Hero's formula*.

Perimeter (P) $= a + b + c = 2s$.

2. Right Angled Triangle

A triangle having one of its angles equal to 90° is called a *right-angled triangle*. The side opposite to the right angle is called the *hypotenuse*.



In a right angled triangle,

(Hypotenuse)^2 = sum of squares of sides

i.e. $h^2 = a^2 + b^2$.

Area (A) $= \frac{1}{2}$(product of the sides containing the right angle)

i.e. $A = \frac{1}{2}ab$.

Illustration 1 What is area of a triangle having sides 3 m, 4 m and 5 m?

Solution Let $a = 3$ m, $b = 4$ m, $c = 5$ m.

Then, $s = \frac{a+b+c}{2} = \frac{3+4+5}{2} = 6$ m.

$$\therefore \quad \text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{6(6-3)(6-4)(6-5)}$$
$$= \sqrt{6\times3\times2\times1} = \sqrt{36} = 6 \text{ sq m}$$

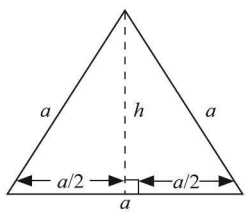
Illustration 2 Find the area of a triangle whose base is 4.6 m and height is 67 cm.

Solution Area of triangle $= \frac{1}{2}(\text{base} \times \text{height})$

$$= \frac{1}{2}(4.6 \times 100 \times 67)$$
$$= 15410 \text{ sq cm.}$$

3. Equilateral Triangle

A triangle whose all sides are equal is called an equilateral triangle.



Area (A) of an equilateral triangle

$$= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}a^2$$

Permeter (P) of an equilateral triangle

$$= 3 \times (\text{side}) = 3a$$

Altitude (h) of an equilateral triangle

$$= \frac{\sqrt{3}}{2} \times (\text{side}) = \frac{\sqrt{3}}{2}a.$$

In an equilateral triangle

$$\angle A = \angle B = \angle c = 60°.$$

Area (A) of an equilateral triangle

$$= \frac{(\text{altitude})^2}{\sqrt{3}} = \frac{h^2}{\sqrt{3}}.$$

Illustration 3 Find the area of an equilateral triangle each of whose sides measures 6 cm.

Solution Area of an equilateral triangle

$$= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4} \times 36 = 9\sqrt{3} \text{ sq cm.}$$

Illustration 4 Length of the side of an equilateral triangle is $\frac{4}{\sqrt{3}}$cm. Find its height.

Solution Height of the equilateral triangle

$$= \frac{\sqrt{3}}{2} \times (\text{side})$$
$$= \frac{\sqrt{3}}{2} \times \frac{4}{\sqrt{3}} = 2 \text{ cm.}$$

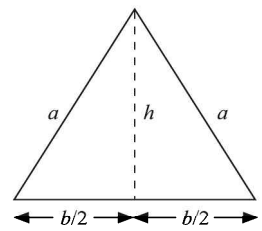
Illustration 5 Height of an equilateral triangle is $4\sqrt{3}$cm. Find its area.

Solution Area of equilateral triangle

$$= \frac{(\text{altitude})^2}{\sqrt{3}} = \frac{4\sqrt{3}\times4\sqrt{3}}{\sqrt{3}} = 16\sqrt{3} \text{ sq cm.}$$

4. Isosceles Triangle

A triangle whose two sides are equal is an *isosceles triangle*.



Area (A) of an isosceles triangle

$$= \frac{b}{4}\sqrt{4a^2 - b^2}$$

Perimeter (P) of an isosceles triangle

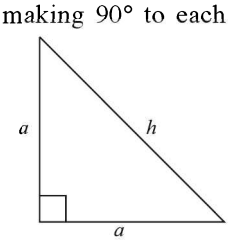
$$= (2a + b)$$

Height (h) of an isosceles triangle

$$= \frac{1}{2}\sqrt{4a^2 - b^2}.$$

5. Isosceles Right-angled Triangle

An *isosceles right-angled triangle* has two sides equal with equal sides making 90° to each other.



Hypotenuse (h) $= \sqrt{2}a$

Area (A) $= \frac{1}{2}a^2$

Perimeter (P) $= 2a + \sqrt{2}a = \sqrt{2}a(\sqrt{2}+1)$

$= h(\sqrt{2}+1)$.

If the perimeter of an isosceles triangle is P and the base is b, then the length of the equal sides is $\left(\frac{P-b}{2}\right)$.

If the perimeter of an isosceles triangle is P and the length of equal sides is a, then base is $(P - 2a)$.

Illustration 6 An isosceles right-angled triangle has two equal sides of length 6 m each. Find its area.

Solution Area $= \frac{1}{2}$ (equal side)$^2 = \frac{1}{2}(6)^2 = 18$ sq m.

Illustration 7 The perimeter of an isosceles triangle is 80 cm. If the length of the equal sides is given by 15 cm, find the length of the base.

Solution Length of the base $= P - 2a$

$= 80 - 2(15) = 50$ cm.

Illustration 8 The perimeter of an isosceles triangle is 42 cm. If the base is 16 cm, find the length of equal sides.

Solution The length of equal sides $= \dfrac{P-b}{2}$

$= \dfrac{42-16}{2} = \dfrac{26}{2} = 13$ cm.

Illustration 9 If the base of an isosceles triangle is 10 cm and the length of equal sides is 13 cm, find its area.

Solution Area of the isosceles triangle

$$= \frac{b}{4}\sqrt{4a^2 - b^2}$$

$$= \frac{10}{4}\sqrt{4\times(13)^2 - (10)^2}$$

$$= \frac{10}{4}\sqrt{676-100} = \frac{10}{4}\times 24 = 60 \text{ sq cm.}$$

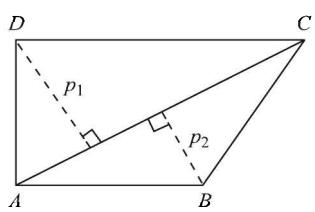
6. Quadrilateral

A closed figure bounded by four sides is called a *quadrilateral*.

It has four angles included in it.

The sum of these four angles is 360°.

i.e. $\angle A + \angle B + \angle C + \angle D = 360°$.



Area (A) of a quadrilateral

$= \dfrac{1}{2} \times$ one diagonal $\times$ (sum of perpendiculars to it from opposite vertices)

$= \dfrac{1}{2}d(p_1 + p_2)$

Note: If the lengths of four sides and one of its diagonals are known, then

A = Area of ΔADC + Area of ΔABC.

The special cases of quadrilateral are parallelogram, rectangle, square, rhombus, trapezium, etc., which are discussed below separately.

7. Parallelogram

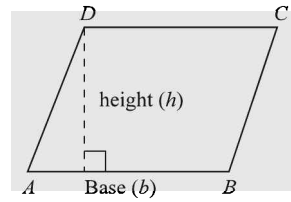
A quadrilateral in which opposite sides are equal and parallel is called a parallelogram.

The diagonals of a parallelogram bisect each other.

Area (A) of a parallelogram

= base × altitude corresponding to the base

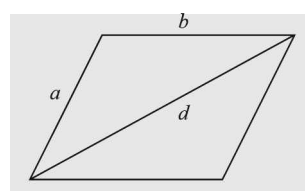
$= b \times h$



Area (A) of a parallelogram

$= 2\sqrt{s(s-a)(s-b)(s-d)}$

where a and b are adjacent sides, d is the length of the diagonal connecting the ends of the two sides and $s = \dfrac{a+b+d}{2}$.



In a parallelogram, the sum of the squares of the diagonals = 2 (the sum of the squares of the two adjacent sides),

i.e.

$d_1^2 + d_2^2 = 2(a^2 + b^2)$.

Perimeter (P) of a parallelogram

$= 2(a + b)$,

where a and b are adjacent sides of the parallelogram.

Illustration 10 One side of a parallelogram is 15 cm and the corresponding altitude is 5 cm. Find the area of the parallelogram.

Solution Area of parallelogram

= base × corresponding altitude

$= 15 \times 5 = 75$ sq cm.

Illustration 11 In a parallelogram, the lengths of adjacent sides are 11 cm and 13 cm, respectively. If the length of one diagonal is 20 cm, find the length of the other diagonal.

Solution We have,

$d_1^2 + d_2^2 = 2(a^2 + b^2)$

$\Rightarrow \quad (20)^2 + d_2^2 = 2(11^2 + 13^2)$

$\Rightarrow \quad d_2^2 = 2(121 + 169) - 400 = 180.$

$\therefore \quad d_2 = \sqrt{180} = 13.4$ m (approx.)

Illustration 12 Find the area of a quadrilateral of whose diagonal is 38 cm long and the lengths of perpendiculars from the other two vertices are 31 cm and 19 cm, respectively.

Solution Area of the quadrilateral

$$= \frac{1}{2} d(p_1 + p_2)$$
$$= \frac{1}{2} \times 38 \times (31 + 19)$$
$$= 19 \times 50 = 950 \text{ sq cm.}$$

Illustration 13 Find the area of a parallelogram whose two adjacent sides are 130 m and 140 m and one of the diagonals is 150 m long.

Solution Here, $a = 130$, $b = 140$ and $d = 150$.

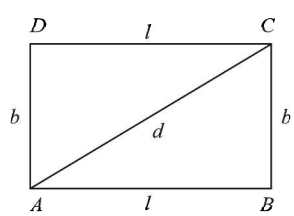
$$\therefore \quad s = \frac{a+b+d}{2} = \frac{130+140+150}{2} = \frac{420}{2} = 210.$$

$\therefore$ Area of the parallelogram

$$= 2\sqrt{s(s-a)(s-b)(s-a)}$$
$$= 2\sqrt{210(210-130)(210-140)(210-150)}$$
$$= 2\sqrt{210\times80\times70\times60} = 2 \times 8400 \text{ sq m.}$$

8. Rectangle

A *rectangle* is a quadrilateral with opposite sides equal and all the four angles equal to 90°.



The diagonals of a rectangle bisect each other and are equal.

(a) Area (A) of rectangle = length × breadth
$= l \times b$

OR

Area of rectangle = $(l \times \sqrt{d^2 - l^2})$,
if one side (l) and diagonal (d) are given.

OR

Area of rectangle = $\left(\frac{p^2}{8} - \frac{d^2}{2}\right)$,
if perimeter (P) and diagonal (d) are given.

(b) Perimeter (P) of rectangle
= 2(length + breadth)
$= 2(l + b)$.

OR

Perimeter of rectangle = $2(l + \sqrt{d^2 - l^2})$,
if one side (l) and diagonal (d) are given.

(c) Diagonal of a rectangle
$= \sqrt{(\text{length})^2 + (\text{breadth})^2}$
$= \sqrt{l^2 + b^2}$

(d) If area (A) and perimeter (P) of a rectangle are given, then

length of the rectangle = $\left(\sqrt{\frac{P^2}{16} - A} + \frac{P}{4}\right)$

and,

breadth of the rectangle = $\left(\frac{P}{4} - \sqrt{\frac{P^2}{16} - A}\right)$

Illustration 14 Find the diagonal of a rectangle whose sides are 8 cm and 6 cm.

Solution Diagonal of the rectangle

$$= \sqrt{l^2 + b^2}$$
$$= \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = 10 \text{ cm.}$$

Illustration 15 Find the perimeter of a rectangle of length 12 m and breadth 6 m.

Solution Perimeter of the rectangle

$$= 2(l + b)$$
$$= 2(12 + 6) = 36 \text{ m.}$$

Illustration 16 Calculate the area of a rectangular field whose length is 12.5 cm and breadth is 8 cm.

Solution Area of the rectangular field

$$= l \times b$$
$$= 12.5 \times 8 = 100 \text{ sq cm.}$$

Illustration 17 Calculate the area of a rectangular field whose one side is 16 cm and the diagonal is 20 cm.

Solution Area of the rectangular field

$$= (l \times \sqrt{d^2 - l^2})$$
$$= (16 \times \sqrt{20^2 - 16^2})$$
$$= 16 \times 12 = 192 \text{ sq cm.}$$

Illustration 18 A rectangular carpet has an area of 120 sq m and perimeter of 46 m. Find the length of its diagonal.

Solution We have,

Area of rectangle = $\left(\frac{P^2}{8} - \frac{d^2}{2}\right)$

$$\Rightarrow \quad 120 = \frac{46^2}{8} - \frac{d^2}{2}$$
$$\Rightarrow \quad 46^2 - 4d^2 = 120 \times 8$$
$$\Rightarrow \quad 4d^2 = 2116 - 960 = 1156$$
$$\therefore \quad d = \sqrt{289} = 17 \text{ m.}$$

Illustration 19 The perimeter of a rectangle is 82 cm and its area is 400 sq m. Find the length and breadth of the rectangle.

Solution Length of the rectangle

$$= \left(\sqrt{\frac{P^2}{16} - A} + \frac{P}{4}\right)$$

$$= \left(\sqrt{\frac{(82)^2}{16} - 400} + \frac{82}{4}\right)$$

$$= (4.5 + 20.5) = 25 \text{ m.}$$

Breadth of the rectangle

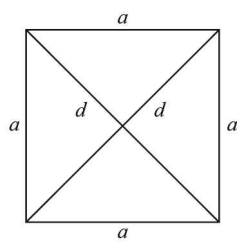
$$= \left(\frac{P}{4} - \sqrt{\frac{P^2}{16} - A}\right)$$

$$= \left(\frac{82}{4} - \sqrt{\frac{(82)^2}{16} - 400}\right)$$

$$= (20.5 - 4.5) = 16 \text{ m.}$$

9. Square

A square is a quadrilateral with all sides equal and all the four angles equal to 90°.



The diagonals of a square are equal and bisect each other at 90°.

(a) Area (A) of a square

$$= a^2, \text{ i.e. } (\text{side})^2$$

$$= \frac{d^2}{2}, \text{ i.e. } \frac{(\text{diagonal})^2}{2}$$

$$= \frac{P^2}{16}, \text{ i.e. } \frac{(\text{perimeter})^2}{16}$$

(b) Perimeter (P) of a square

$$= 4a, \text{ i.e. } 4 \times \text{side}$$

$$= \sqrt{16 \times \text{area}}$$

$$= 2\sqrt{2}d, \text{ i.e. } 2\sqrt{2} \times \text{diagonal}$$

(c) Length (d) of the diagonal of a square

$$= \sqrt{2}a, \text{ i.e. } \sqrt{2} \times \text{side}$$

$$= \sqrt{2 \times \text{area}}$$

$$= \frac{P}{2\sqrt{2}}, \text{ i.e. } \frac{\text{Perimeter}}{2\sqrt{2}}.$$

Illustration 20 If the area of a square field be 6050 sq m, find the length of its diagonal.

Solution Length of the diagonal of the square field

$$= \sqrt{2 \times \text{area}}$$

$$= \sqrt{2 \times 6050}$$

$$= \sqrt{12100}, \text{ i.e. } 110 \text{ m.}$$

Illustration 21 Find the area of a square with perimeter 48 m.

Solution Area of the square

$$= \frac{(\text{Perimeter})^2}{16}$$

$$= \frac{(48)^2}{16} = \frac{48 \times 48}{16} = 3 \times 48 = 144 \text{ sq m.}$$

Illustration 22 Find the diagonal of a square field whose side is of 6 m length.

Solution Length of the diagonal

$$= \sqrt{2} \times \text{side}$$

$$= 6\sqrt{2} \text{ m.}$$

Illustration 23 In order to fence a square Ramesh fixed 36 poles. If the distance between two poles is 6 metres, then find the area of the square so formed.

Solution Perimeter of the square

$$= 36 \times 6 = 216 \text{ m.}$$

$$\therefore \text{ Area of the square} = \left(\frac{\text{Perimeter}}{4}\right)^2$$

$$= \left(\frac{216}{4}\right)^2 = 54 \times 54$$

$$= 2916 \text{ sq m.}$$

Illustration 24 Perimeter of a square field is $16\sqrt{2}$ cm. Find the length of its diagonal.

Solution We have,

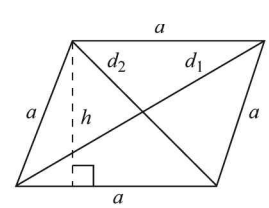
Perimeter of square field $= 2\sqrt{2} \times$ diagonal

$\Rightarrow$ $16\sqrt{2} = 2\sqrt{2} \times$ diagonal

$$\therefore \text{ Length of the diagonal} = \frac{16\sqrt{2}}{2\sqrt{2}} = 8 \text{ cm.}$$

10. Rhombus

A *rhombus* is a quadrilateral whose all sides are equal.



The diagonals of a rhombus bisect each other at 90°.

(a) Area (A) of a rhombus

$= a \times h$, i.e. base × height

$= \frac{1}{2} d_1 \times d_2$, i.e. $\frac{1}{2}$ × product of its diagonals

$= d_1 \times \sqrt{a^2 - \left(\frac{d_1}{2}\right)^2}$,

since $d_2^2 = 4\left[a^2 - \left(\frac{d_1}{2}\right)^2\right]$

$= d_1 \times \sqrt{\left(\frac{\text{Perimeter}}{4}\right)^2 - \left(\frac{d_1}{2}\right)^2}$,

since $d_2^2 = 4\left[\left(\frac{\text{Perimeter}}{4}\right)^2 - \left(\frac{d_1}{2}\right)^2\right]$

(b) Perimeter (P) of a rhombus

$= 4a$, i.e. 4 × side

$= 2\sqrt{d_1^2 + d_2^2}$,

where d_1 and d_2 are two diagonals.

(c) Side (a) of a rhombus

$= \frac{1}{2}\sqrt{d_1^2 + d_2^2}$.

Illustration 25 The area of a rhombus is 156 sq m. If one of its diagonals is 13 m, find the length of the other diagonal.

Solution Area of rhombus $= \frac{1}{2}(d_1 \times d_2)$

$\Rightarrow 156 = \frac{1}{2}(13 \times d_2)$

$\Rightarrow d_2 = \frac{2 \times 156}{13} = 24$ m.

Illustration 26 Find the area of a rhombus whose one side is 13 cm and one diagonal is 24 cm.

Solution Area of rhombus $= d_1 \times \sqrt{a^2 - \left(\frac{d_1}{2}\right)^2}$

$= 24 \times \sqrt{(13)^2 - \left(\frac{24}{2}\right)^2}$

$= 24 \times \sqrt{169 - 144}$

$= 24 \times 5 = 120$ sq cm.

Illustration 27 If the perimeter of a rhombus is 73 cm and one of its diagonals is 27.5 cm, find the other diagonal and the area of the rhombus.

Solution One side of rhombus (a) $= \frac{73}{4} = 18.25$ cm.

$\therefore$ Other diagonal (d_2) $= 2 \times \sqrt{a^2 - \left(\frac{d_1}{2}\right)^2}$

$= 2 \times \sqrt{(18.25)^2 - \left(\frac{27.5}{2}\right)^2}$

$= 24$ cm.

$\therefore$ Area of rhombus $= \frac{1}{2} \times d_1 \times d_2$

$= \frac{1}{2} \times 24 \times 27.5$

$= 330$ sq cm.

Illustration 28 In a rhombus, the lengths of two diagonals are 18 m and 24 m. Find its perimeter.

Solution Perimeter of the rhombus

$= 2 \times \sqrt{d_1^2 + d_2^2}$

$= 2 \times \sqrt{(18)^2 + (24)^2}$

$= 2 \times \sqrt{324 + 576}$

$= 2 \times \sqrt{900} = 60$ m.

Illustration 29 Find the side of a rhombus, one of whose diagonals measure 4 m and the other 3 m.

Solution Side of the rhombus

$= \frac{1}{2} \times \sqrt{d_1^2 + d_2^2}$

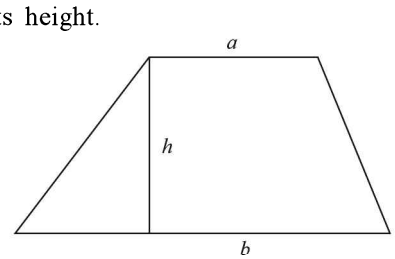
$= \frac{1}{2} \times \sqrt{(4)^2 + (3)^2}$

$= \frac{1}{2} \times 25$, i.e. 12.5 m.

11. Trapezium (Trapezoid)

A *trapezium* is a quadrilateral whose any two opposite sides are parallel.

Distance between parallel sides of a trapezium is called its height.



(a) Area (A) of a trapezium

$= \frac{1}{2}$ × (sum of parallel sides) × perpendicular distance between the parallel sides

i.e. $\frac{1}{2} \times (a + b) \times h$.

$= \frac{a+b}{l}\sqrt{s(s-l)(s-c)(s-d)}$,

where, $l = b - a$ if $b > a$

$= a - b$ if $a > b$

and, $s = \dfrac{c+d+l}{2}$

(b) Height (h) of the trapezium

$$= \frac{2}{l}\sqrt{s(s-l)(s-c)(s-d)}$$

$$= \left(\frac{2A}{a+b}\right)$$

Illustration 29 Find the area of a trapezium having parallel sides 65 m and 44 m and their separation being 20 m.

Solution Area of the trapezium

$$= \frac{1}{2} \times (a + b) \times h$$

$$= \frac{1}{2} \times (65 + 44) \times 20$$

$$= 1100 \text{ sq cm.}$$

Illustration 30 The parallel sides of a trapezium are 24 m and 52 m. If its other two sides are 26 m and 30 m, what is the area of the trapezium?

Solution Area of trapezium

$$= \frac{a+b}{l}\sqrt{s(s-l)(s-c)(s-d)}.$$

Here, $a = 24$, $b = 52$, $c = 26$, $d = 30$, $l = b - a = 28$,

$$s = \frac{c+d+l}{2} = \frac{26+30+28}{2} = 42.$$

$\therefore$ Area of trapezium

$$= \frac{24+52}{28}\sqrt{42(42-28)(42-26)(42-30)}$$

$$= \frac{76}{28} \times \sqrt{42 \times 14 \times 16 \times 12}$$

$$= \frac{76 \times 336}{28} = 912 \text{ sq m.}$$

Illustration 31 The two parallel sides of a trapezium of area 180 sq cm measure 28 cm and 12 cm. What is the height of the trapezium?

Solution Height of the trapezium

$$= \left(\frac{2A}{a+b}\right) = \left(\frac{2 \times 180}{28+12}\right) = \frac{360}{40} = 9 \text{ cm.}$$

12. Walls of a Room

Area of four walls of a room

$$= 2(\text{length} + \text{breadth}) \times \text{height}$$

Illustration 32 Find the cost of painting the walls of a room of 6 m long, 5 m broad and 4 m high at Rs. 7.50 per sq m.

Solution Area of four walls of room

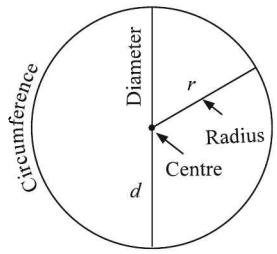
$$= 2(\text{length} + \text{breadth}) \times \text{height}$$

$$= 2(6 + 5) \times 4 = 88 \text{ sq m.}$$

$\therefore$ Cost of painting = 88 × 7.50 = Rs. 660.

12. Circle

A *circle* is the path travelled by a point which moves in such a way that its distance from a fixed point remains constant.



The fixed point is known as *centre* and the fixed distance is called the *radius*.

(a) Circumference or perimeter of circle

$$= 2\pi r = \pi d,$$

where r is radius and d is diameter of circle

(b) Area of circle

$$= \pi r^2, \ r \text{ is radius}$$

$$= \frac{\pi d^2}{4}, \ d \text{ is diameter}$$

$$= \frac{c^2}{4\pi}, \ c \text{ is circumference}$$

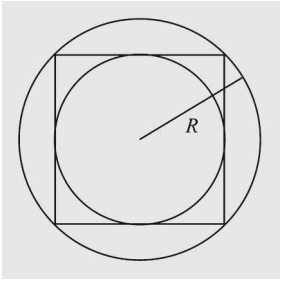
$$= \frac{1}{2} \times \text{circumference} \times \text{radius}$$

(c) Radius of circle $= \sqrt{\dfrac{\text{Area}}{\pi}}$

$$= \frac{\text{Perimeter or circumference}}{2\pi}$$

(d) Ratio of the areas of the two circles is

$$= \frac{\text{Area of circle circumscribing the square}}{\text{Area of circle inscribed in the square}} = \frac{2}{1}.$$



(e) Ratio of the area of the two squares is

$$= \frac{\text{Area of square circumscribing the circle}}{\text{Area of square inscribed in the circle}} = \frac{2}{1}.$$

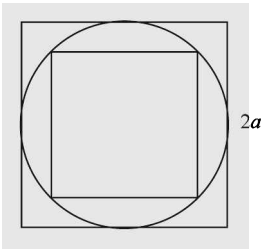


Illustration 33 What is the radius of a circular plot whose circumference is 176 m?

Solution $r = \dfrac{\text{Circumference}}{2\pi}$

$$= \frac{176}{2\times\frac{22}{7}} = \frac{176\times 7}{2\times 22} = 28 \text{ m.}$$

Illustration 34 A circular plot covers an area of 154 sq m. How much wire is required for fencing the plot?

Solution Area of the plot $= \pi r^2 = 154$

i.e. $r^2 = 154 \times \dfrac{7}{22} = 49.$

$\therefore$ $r = 7$ m.

$\therefore$ Length of the wire $= 2\pi r = 2 \times \dfrac{22}{7}\times 7 = 44$ m.

Illustration 35 Find the length of a rope by which a buffalo must be tethered in order that she may be able to graze an area of 9856 sq m.

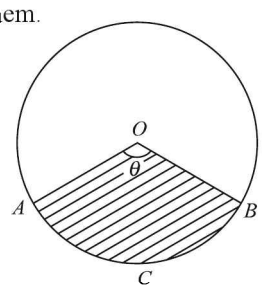
Solution The required length of rope

$$= r = \sqrt{\frac{\text{Area}}{\pi}}$$

$$= \sqrt{\frac{9856\times 7}{22}} = \sqrt{3136} = 56 \text{ m.}$$

Sector

A *sector* is a figure enclosed by two radii and an arc lying between them.



For sector AOB,

Arc $AB = \dfrac{2\pi r\theta}{360}$,

where r = radius and $\angle AOB = \theta$

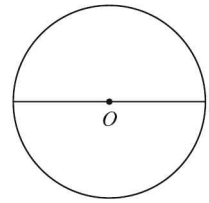
Area of sector $ACBO$

$$= \frac{1}{2} \times (\text{arc } AB) \times \text{radius}$$

$$= \frac{\pi(\text{radius})^2\,\theta}{360}.$$

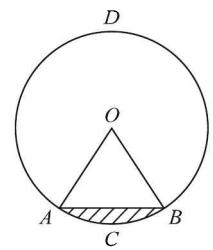
Semi-Circle

A *semi-circle* is a figure enclosed by a diameter and the part of the circumference cut off by it.



Segment

A *segment* of a circle is a figure enclosed by a chord and an arc which it cuts off.



Note: Any chord of a circle which is not a diameter (such as AB), divides the circle into two segments, one greater and one less than a semi-circle.

Area of segment ACB

= area of sector $ACBO$ – area of ΔOAB

and area of segment ADB

= area of circle – area of segment ACB

Illustration 36 If a piece of wire 20 cm long is bent into an arc of a circle subtending an angle of 60° at the centre, find the radius of the circle.

Solution Length of the arc $= \dfrac{2\pi r\theta}{360}$

$$\Rightarrow\ 20 = \frac{2\pi r\times 60}{360} \Rightarrow r = \frac{20\times 360}{60\times 2\times\pi} = \frac{60}{\pi}\text{cm.}$$

Illustration 37 Find the area of sector of a circle whose radius is 14 cm and the angle at the centre is 60°.

Solution Area of sector $= \dfrac{\pi(\text{radius})^2\,\theta}{360}$

$$= \frac{22\times 14\times 14\times 60}{7\times 360}$$

$$= \frac{22\times 2\times 14}{6} = 102\frac{2}{3} \text{ sq cm.}$$

Illustration 38 Find the area of sector of a circle whose radius is 10 cm and the length of the arc is 13 cm.

Solution Area of sector $= \dfrac{1}{2} \times (\text{length of arc}) \times \text{radius}$

$$= \frac{1}{2} \times 13 \times 10$$

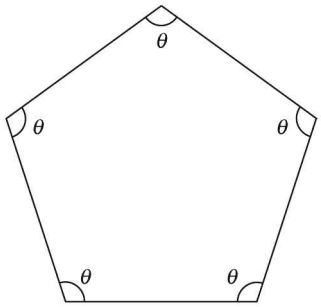
$$= 65 \text{ sq cm.}$$

Polygon

A *polygon* is a plane figure enclosed by four or more straight lines.

Regular Polygon

If all the sides of a polygon are equal, it is called a *regular polygon*



All the interior angles of a regular polygon are equal.

For a regular polygon:

Sum of exterior angles = 2π

Sum of interior angles = $(n - 2)\pi$

No. of diagonals in a polygon = $\frac{n(n-3)}{2}$

Perimeter $(P) = n \times a$,

where n = number of sides

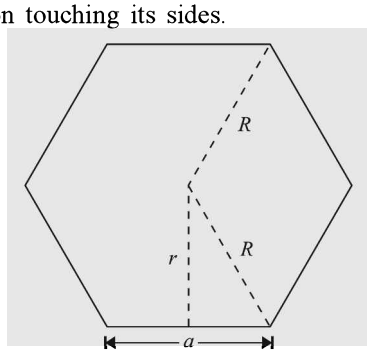
and, a = length of each side

Each interior angle = $\frac{n-2}{n} \times \pi$

Each exterior angle = $\frac{2\pi}{n}$

Area = $\frac{1}{2} \times P \times r = \frac{1}{2} \times n \times a \times r$,

where, r is radius of the circle drawn inside the polygon touching its sides.



$$= \frac{1}{2} \times n \times a \times \sqrt{R^2 - \left(\frac{a}{2}\right)^2},$$

where, R is radius of the circle drawn outside the polygon touching its sides.

$$= \frac{na^2}{4} \cot\left(\frac{\pi}{n}\right).$$

Area of a regular hexagon = $\frac{3\sqrt{3}}{2}(\text{side})^2$

Area of a regular octagon = $2(\sqrt{2}+1)\ (\text{side})^2$.

Illustration 39 Find the side of a regular hexagon whose area is $48\sqrt{3}$ sq cm.

Solution Area of a regular hexagon

$$= \frac{3\sqrt{3}}{2} \times (\text{side})^2$$

$$\Rightarrow 48\sqrt{3} = \frac{3\sqrt{3}}{2} \times (\text{side})^2$$

$$\Rightarrow (\text{side})^2 = 32$$

$\therefore$ Side of the hexagon = $4\sqrt{2}$ cm.

Illustration 40 Find the area of a regular octagon whose side measures $\sqrt{2}$ cm.

Solution Area of regular octagon

$$= 2(\sqrt{2}+1) \times (\text{side})^2$$

$$= 2(\sqrt{2}+1) \times (\sqrt{2})^2$$

$$= 4(\sqrt{2}+1) \text{ sq cm.}$$

Illustration 41 Find the sum of interior angles of a regular polygon of 10 sides. Also, find the value of each interior angle.

Solution Sum of interior angles $= (n - 2) \times \pi$

$= (10 - 2) \times \pi$

$= 8\pi$.

Also, value of each interior angle

$$= \left(\frac{n-2}{n}\right) \times \pi$$

$$= \left(\frac{10-2}{10}\right)\pi = \frac{4\pi}{5}.$$

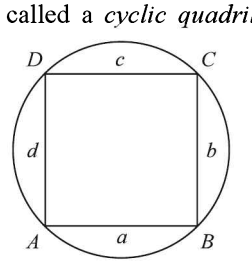
Illustration 42 Find the sum of all the exterior angles of a regular polygon of 12 sides. Also, find the value of each exterior angle.

Solution Sum of exterior angles = 2π

Also, value of each exterior angle = $\frac{2\pi}{n} = \frac{2\pi}{12} = \frac{\pi}{6}$.

Cyclic Quadrilateral

A quadrilateral whose vertices lie on the circumference of the circle is called a *cyclic quadrilateral*.



For a cyclic quadrilateral

- Area = $\sqrt{s(s-a)(s-b)(s-c)(s-d)}$, where $s = \dfrac{a+b+c+d}{2}$
- $\angle A + \angle B + \angle C + \angle D = 2\pi$
- $\angle A + \angle C = \angle B + \angle D = \pi$.

SOME USEFUL SHORT-CUT METHODS

1. If the length and the breadth of a rectangle are increased by x% and y% respectively, then the area of rectangle will increase by $\left(x+y+\dfrac{xy}{100}\right)$%.

Explanation

Area of the original rectangle is

$A = l \times b$.

Area of the new rectangle is

$$A' = l\left(\frac{100+x}{100}\right)\times b\left(\frac{100+y}{100}\right)$$

$$= lb\left(\frac{100+x}{100}\right)\left(\frac{100+y}{100}\right)$$

$$= A\left(\frac{100+x}{100}\right)\left(\frac{100+y}{100}\right)$$

$$\therefore \quad \frac{A'}{A} - 1 = \left(\frac{100+x}{100}\right)\left(\frac{100+y}{100}\right) - 1$$

or, $$\frac{A'-A}{A} = \frac{(100+x)(100+y)-(100)^2}{(100)^2}$$

or, $$\left(\frac{A'-A}{A}\right)\times 100$$

$$= \frac{(100)^2 + 100(x+y) + xy - (100)^2}{(100)}$$

$$\therefore \quad \text{\% increase in area} = \frac{100(x+y)+xy}{100}\%$$

or, $$\left(x+y+\frac{xy}{100}\right)\%.$$

Note: If any of x or y decreases, we put negative sign.

Illustration 1 The length and breadth of a rectangle are increased by 20% and 5%, respectively. Find the percentage increase in its area.

Solution % increase in the area of rectangle

$$= \left(x+y+\frac{xy}{100}\right)\%$$

$$= \left(20+5+\frac{20\times 5}{100}\right)\% = 20\%.$$

2. If the length of a rectangle is increased by x%, then its breadth will have to be decreased by $\left(\dfrac{100x}{100+x}\right)$% in order to maintain the same area of rectangle.

Explanation

% increase in area of rectangle

$$= \left(x+y+\frac{xy}{100}\right)\%$$

or, $$0 = \left(x+y+\frac{xy}{100}\right)$$

or, $$-x = y\left(1+\frac{x}{100}\right) \quad \text{or} \quad y = -\left(\frac{100x}{100+x}\right)$$

–ve sign indicates decrease.

Therefore, breadth must be decreased by $\left(\dfrac{100x}{100+x}\right)$% in order to maintain the same area.

Illustration 2 The length of a rectangle is increased by 25%. By what per cent should its breadth be decreased so as to maintain the same area?

Solution The breadth must be decreased by

$$= \left(\frac{100x}{100+x}\right)\% = \left(\frac{100\times 25}{100+25}\right)\%, \text{ i.e. } 20\%.$$

3. If each of the defining dimensions or sides of any two-dimensional figure (triangle, rectangle, square, circle, quadrilateral, pentagon, hexagon, etc.) is changed by x%, its area changes by $x\left(2+\dfrac{x}{100}\right)$%.

Illustration 3 If the radius of a circle is decreased by 10%, what is the percentage decrease in its area?

Solution Here, $x = -10$ (–ve sign indicates decrease)

$$\therefore \quad \text{\% change in area} = x\left(2+\frac{x}{100}\right)\%$$

$$= -10\left(2-\frac{10}{100}\right)\%$$

$$= (-10)\left(\frac{19}{10}\right)\% = -19\%.$$

$\therefore$ Area of the circle decreases by 19%.

4. If all the sides of a quadrilateral are increased (or decreased) by x%, its diagonals also increase (or decrease) by x%.

Illustration 4 The length and the two diagonals of a rectangle are decreased by 5% each. What is the percentage decrease in its breadth?

Solution Since the length and the two diagonals decrease by 5% each, the breadth also must decrease by 5%.

5. If each of the defining dimensions or sides of any two-dimensional figures are increased (or decreased) by x%, its perimeter also increases (or decreases) by x%.

Illustration 5 If all the sides and diagonals of a square are increased by 8% each, then find the percentage increase in its perimeter.

Solution The perimeter also increases by 8%.

6. If the ratio of the areas of two squares be $a : b$, then the ratio of their sides, ratio of their perimeters and the ratio of their diagonals, each will be in the ratio $\sqrt{a} : \sqrt{b}$.

Illustration 6 Ratio of the areas of two squares is 16 : 9. Find the ratio of their diagonals.

Solution The ratio of their diagonals

$$= \sqrt{a} : \sqrt{b}$$

$$= \sqrt{16} : \sqrt{9}, \text{ i.e. } 4 : 3.$$

7. If the diagonal of a square increases by x times, then the area of the square becomes x^2 times.

Illustration 7 The diagonal of a square is doubled. How many times will the area of the new square become?

Solution The area of the new square will become x^2 times, i.e. $(2)^2 = 4$ times.

8. Standard Properties of Diagonals of Quadrilaterals

Quadrilateral	*Meet at right angles*	*Bisect each other*	*Equal to each other*	*Bisect angle at vertex*
Square	√	√	√	√
Rectangle	×	√	√	×
Parallelogram	×	√	×	×
Rhombus	√	√	×	√
Trapezium	×	×	may or may not be	×

9. Carpeting the Floor of a Room

If the length and breadth of a room are l and b, respectively, and a carpet of width w is used to cover the floor, then the required length of the carpet

$$= \frac{l \times b}{w}.$$

Illustration 8 How many metres of a carpet 12 cm wide will be required to cover the floor of a room which is 600 cm long and 420 cm broad? Also, calculate the amount required in carpeting the floor if the cost of carpet is Rs. 15 per metre.

Solution Length of the carpet

$$= \frac{l \times b}{w}$$

$$= \frac{600 \times 420}{12}$$

$$= 21000 \text{ cm, i.e. } 210 \text{ m.}$$

The amount required for carpeting the floor

$$= 15 \times 210 = \text{Rs. } 3150.$$

10. Number of Square Tiles Required for Flooring

If the length and breadth of a room are l and b respectively, then the least number of square tiles required to cover the floor

$$= \frac{l \times b}{\text{H.C.F. } (l, b)}$$

Also, the size of the largest tile so that the tiles exactly fit

$$= \text{H.C.F. } (l, b).$$

Illustration 9 A hall of length 24 cm and breadth 20 m is to be paved with equal square tiles. What will be the size of the largest tile so that the tiles exactly fit and also find the number of tiles required.

Solution Size of the largest possible square tile

$$= \text{H.C.F. } (l, b)$$

$$= \text{H.C.F. } (24, 20) = 4 \text{ m.}$$

Number of tiles required

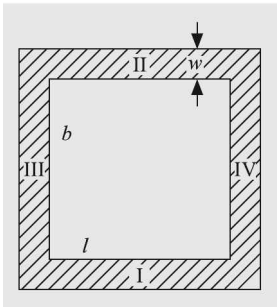
$$= \frac{l \times b}{\text{H.C.F. } (l, b)}$$

$$= \frac{24 \times 20}{4} = 120 \text{ tiles.}$$

11. Path around a Rectangular Space

(a) A rectangular garden l m long and b m broad is sorrounded by a path w m wide. The area of the path is given by

$$= 2w(l + b + 2w) \text{ sq m.}$$



Explanation

Area of part I = Area of part II

$= (l + 2w)w$ sq m

Area of part III = Area of part IV

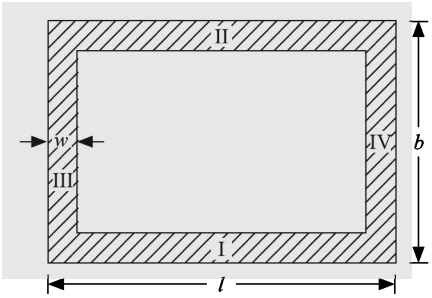
$= bw$ sq m.

$\therefore$ Total area of the path $= 2[(l + 2w)w + bw]$

$= 2w(l + b + 2w)$ sq m.

(b) A rectangular garden l m long and b m broad is sorrounded by a path w m wide constructed inside it along its boundary. The area of the path is given by

$= 2w(l + b - 2w)$ sq m.



Explanation

Area of part I = Area of part II

$= lw$ sq m

Area of part III = Area of part IV

$= (b - 2w)w$ sq m

$\therefore$ Total area of the part

$= 2[lw + (b - 2w)w]$

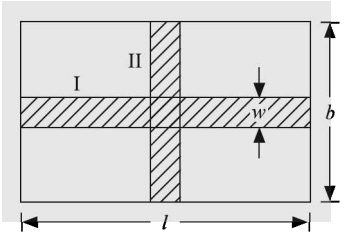
$= 2w(l + b - 2w)$ sq m.

(c) A rectangular park is l m long and b m broad. Two paths w m wide each are perpendicular to each other inside the park. The area of the paths

$= w(l + b - w)$ sq m.

Also, area of the park minus the paths

$= (l - w)\ (b - w)$ sq m.



Explanation

Total area of the path

= Area of part I + Area of path II – Area of common central part

$= lw + bw - w^2$

$= w(l + b - w)$ sq m.

$\therefore$ Area of the park minus the parts

$= [lb - w(l + b - w)]$

$= lb - lw - w(b - w)$

$= l(b - w) - w(b - w)$

$= (l - w)\ (b - w)$ sq m.

Notes:

1. Clearly, from the figure, the area of the paths does not change on shifting their location as long as they are perpendicular to each other.
2. For a square park, take $l = b$ in all the results derived above.

Illustration 10 A rectangular park 18 m × 12 m, is sorrounded by a path 4 m wide. Find the area of the path.

Solution The area of the path

$= 2w(l + b + 2w)$

$= 2 \times 4(18 + 12 + 2 \times 4) = 304$ sq m.

Illustration 11 A park is square in shape with side 18 m. Find the area of the pavement 3 m wide to be laid all around it on its inside.

Solution Area of the pavement

$= 2w(l + b - 2w)$

$= 2 \times 3(18 + 18 - 2 \times 3)$ (Here, $l = b = 18$)

$= 180$ sq m.

Illustration 12 A playground measures 27 m × 13 m. From the centre of each side a path 2 m wide goes across to the centre of the opposite side. Calculate the area of the path and the cost of constructing it at Rs. 4 per sq m.

Solution Area of the path

$= w(l + b - w) = 2(27 + 13 - 2)$

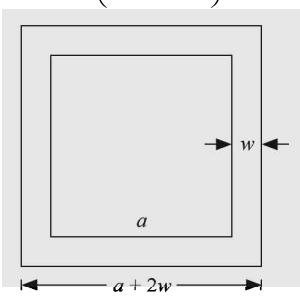
$= 76$ sq m.

$\therefore$ Cost $= 4 \times 76 =$ Rs. 304.

12. Square Room Surrounded by a Verandah

(a) A square room of side a is sorrounded by a verandah of width w on the outside of the square room. If the area of the verandah is A, then the area of the room is given by

$$\left(\frac{A-4w^2}{4w}\right)^2.$$



Explanation

Area of the room = a^2.

Area of the (room + verandah) = $(a + 2w)^2$.

$\therefore$ Area (A) of the verandah = $(a + 2w)^2 - a^2$

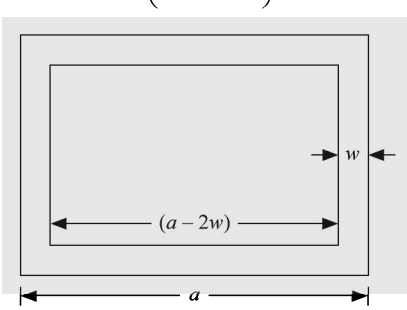
$= (4aw + 4w^2)$

or, $a = \dfrac{A-4w^2}{4w}$

$\therefore$ Area of the room = $a^2 = \left(\dfrac{A-4w^2}{4w}\right)^2$.

(b) A square room of side a is surrounded by a verandah of width w on its inside. If the area of the verandah is A, then the area of the room is given by

$$\left(\frac{A+4w^2}{4w}\right)^2.$$



Explanation

Area (A) of the verandah = $a^2 - (a - 2w)^2$

$= 4aw - 4w^2$

$= 4w(a - w)$

or, $a = \dfrac{A}{4\omega} + w = \dfrac{A+4w^2}{4w}$

$\therefore$ Area of the room = $a^2 = \left(\dfrac{A+4w^2}{4w}\right)^2$.

Illustration 13 A square field is surrounded by a path 2 m wide on its outside. The area of the path is 72 sq m. What is the area of the field?

Solution Area of the field

$$= \left(\frac{A-4w^2}{4w}\right)^2$$

$$= \left(\frac{72-4\times 2^2}{4\times 2}\right)^2 = 49 \text{ sq m.}$$

Illustration 14 A square room has a verandah of area 24 sq m and width 1 m all round it on its inside. Find the area of the room.

Solution Area of the room

$$= \left(\frac{A+4w^2}{4w}\right)^2$$

$$= \left(\frac{24\times 4\times 1^2}{4\times 1}\right)^2 = 49 \text{ sq m.}$$

13. (a) A circular ground of radius r has a pathway of width w around it on its outside. The area of circular pathway is given by

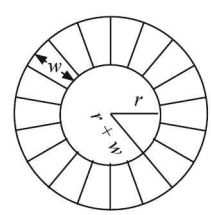
$= \pi w(2r + w)$.

Explanation

Area of circular ground = πr^2

Area of circular ground + pathway

$= \pi(r + w)^2 = \pi r^2 + 2\pi rw + \pi w^2$.



$\therefore$ Area of circular path way

$= (\pi r^2 + 2\pi rw + \pi w^2) - \pi r^2$

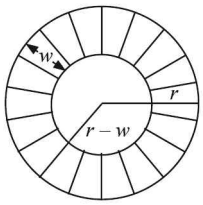
$= \pi w(2r + w)$.

(b) A circular ground of radius r has a pathway of width w around it on its inside. The area of the circular pathway is given by

$= \pi w(2r - w)$.

Explanation

Area of circular ground = πr^2



Area of circular ground
– pathway = $\pi (r - w)^2$
$= \pi r^2 - 2\pi rw + \pi w^2$.

$\therefore$ Area of circular pathway
$= \pi r^2 - (\pi r^2 - 2\pi rw + \pi w^2)$
$= \pi w(2r - w)$.

Illustration 15 A circular ground of radius 16 m has a path of width 2.8 m around it on its outside. Find the area of the path.

Solution The area of the circular path
$= \pi w(2r + w)$
$= \frac{22}{7} \times 2.8 \times (2 \times 16 + 2.8)$
$= 8.8 \times (32 + 2.8) = 306.2$ sq m.

Illustration 16 A circular park of radius 22 m has a path of width 1.4 m around it on its inside. Find the area of the path.

Solution The area of the circular path
$= \pi w(2r - w)$
$= \frac{22}{7} \times 1.4 \times (2 \times 22 - 1.4)$
$= 4.4 \times (44 - 1.4) = 187.45$ sq m.

14. If the area of a square is a sq cm, then the area of the circle formed by the same perimeter is $\left(\frac{4a}{\pi}\right)$ sq cm.

Explanation

Area of the square = a.

$\therefore$ Side of the square = $\sqrt{\text{Area}} = \sqrt{a}$.

$\therefore$ Perimeter of the square = $4\sqrt{a}$.

Given: Circumference of the circle = Perimeter of the square

$\Rightarrow$ $2\pi r = 4\sqrt{a}$

$\therefore$ Radius of circle (r) = $\frac{4\sqrt{a}}{2\pi} = \frac{2\sqrt{a}}{\pi}$.

$\therefore$ Area of circle = $\pi r^2 = \pi\left(\frac{2\sqrt{a}}{\pi}\right)^2$
$= \frac{4a}{\pi}$ sq cm.

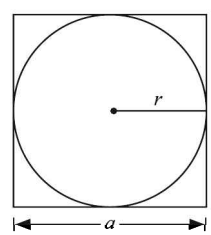
Illustration 17 If the area of a square is 33 sq cm, then find the area of the circle formed by the same perimeter.

Solution Required area of the circle
$= \frac{4a}{\pi}$
$= \frac{4 \times 33}{\pi} = \frac{4 \times 33 \times 7}{22} = 42$ sq cm.

15. The area of the largest circle that can be inscribed in a square of side a is $\frac{\pi a^2}{4}$.

Explanation

Clearly, from the figure, the diameter of the inscribed circle equals the side of the square i.e. $D = a$.



Area of the circle = $\frac{\pi D^2}{4}$

$\therefore$ Area of the inscribed circle = $\frac{\pi a^2}{4}$.

Illustration 18 Find the area of largest circle inscribed in a square of side 112 cm.

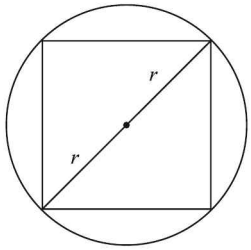
Solution The area of the largest circle
$= \frac{\pi a^2}{4}$
$= \frac{22 \times 112 \times 112}{7 \times 4} = 9856$ sq cm.

16. Area of a square inscribed in a circle of radius r is $2r^2$ and the side of the square is $\sqrt{2}\, r$.

Explanation

Clearly, from the figure, diagonal of the inscribed square is equal to the diameter of the circle, i.e. $2r$.

$\therefore$ Area of square = $\frac{1}{2}$ (diagonal)^2
$= \frac{1}{2}(2r)^2 = 2r^2$.



Also, side of the square = $\sqrt{\text{Area}} = \sqrt{2r^2} = \sqrt{2}\,r$.

Illustration 19 Find the side of the square inscribed in a circle whose circumference is 308 cm.

Solution Circumference of the circle $(2\pi r) = 308$

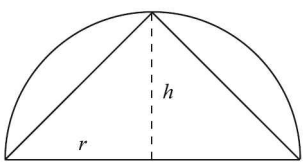
$\Rightarrow r = \dfrac{308}{2\pi} = \dfrac{308 \times 7}{2 \times 22} = 49$ cm.

$\therefore$ Side of the inscribed square = $\sqrt{2}\,r = 49\sqrt{2}$ cm.

17. The area of largest triangle inscribed in a semi-circle of radius r is r^2.

Explanation

Clearly, from the figure, the largest triangle inscribed in a semi-circle is an isosceles triangle with diameter as its base and radius as its height.



Area of the triangle = $\dfrac{1}{2} \times$ base $\times$ height

$= \dfrac{1}{2} \times 2r \times r = r^2$.

18. The number of revolutions made by a circular wheel of radius r in travelling distance d is given by

$$= \left(\frac{d}{2\pi r}\right).$$

Explanation

Circumference of the wheel = 2π r

In travelling a distance 2π r, the wheel makes 1 revolution.

$\therefore$ In travelling a distance d, the wheel makes $\left(\dfrac{d}{2\pi r}\right)$ revolutions.

Illustration 20 The diameter of a wheel is 2 cm. If it rolls forward covering 10 revolutions, find the distance travelled by it.

Solution Radius of the wheel = 1 cm.

The distance travelled by the wheel in 10 revolutions

$= 10 \times 2\pi r$

$= 10 \times 2 \times \dfrac{22}{7} \times 1 = 62.8$ cm.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. The area of a triangle whose sides are 15 m, 16 m and 17 m is
(a) $24\sqrt{4}$ sq m (b) $24\sqrt{3}$ sq m
(c) $24\sqrt{21}$ sq m (d) None of these

2. The area of right-angled triangle with base 6 m and hypotenuse 6.5 m is
(a) 7.5 sq m (b) 9.5 sq m
(c) 8.5 sq m (d) None of these

3. The length of each side of a triangle is 12 cm. The height of the triangle is
(a) $3\sqrt{2}$ m (b) $6\sqrt{3}$ m
(c) $6\sqrt{2}$ (d) None of these

4. The area of a triangular lawn is 1600 sq m. If one side is 64 m long and the other two sides are equal in length, the length of each equal side is
(a) 60.37 m (b) 59.36 m
(c) 60.36 cm (d) None of these

5. Three sides of a triangular field are 20 m, 21 m and 29 m long, respectively. The area of the field is
(a) 215 sq m (b) 230 sq m
(c) 210 sq m (d) None of these

6. The hypotenuse and the semi-perimeter of right triangle are 20 cm and 24 cm, respectively. The other two sides of the triangle are
(a) 16 cm, 12 cm (b) 12 cm, 16 cm
(c) 20 cm, 16 cmn (d) None of these

7. The sides of a triangle are in the ratio 3 : 4 : 5. If its perimeter is 36 cm, area of the triangle is
(a) 57 sq m (b) 54 sq m
(c) 56.5 sq m (d) None of these

8. For a triangle whose sides are 50 m, 78 m and 112 m, respectively, the length of perpendicular from the opposite angle on the side 112 m is
(a) 45 m (b) 35 m
(c) 30 m (d) None of these

9. A ladder is resting with one end in contact with the top of a wall of height 12 m and the other end on the ground is at a distance 5 m from the wall. The length of the ladder is
(a) 13 m (b) 17 m
(c) 16 m (d) None of these

10. A ladder is placed so as to reach a window 63 cm high. The ladder is then turned over to the opposite side of the street and is found to reach a point 56 cm high. If the ladder is 65 cm long, the width of the street is
(a) 59 cm (b) 39 cm
(c) 49 cm (d) None of these

11. If the area of a triangle with base x is equal to the area of a square with side x, then the altitude of the triangle is
(a) $\frac{x}{2}$ (b) x
(c) $2x$ (d) $3x$

12. If the area of a triangle is 150 sq m and base : height is 3 : 4, find its height.
(a) 25 m (b) 35 m
(c) 20 m (d) None of these

13. The base of a triangular field is three times its height. If the cost of cultivating the field at Rs. 36.72 per hectare is Rs. 495.72, find its base and height.
(a) 950 m, 350 m (b) 800 m, 500 m
(c) 900 m, 300 m (d) None of these

14. If the sides of a triangle are doubled, its area
(a) remains same (b) is doubled
(c) becomes 4 times (d) Can't say.

15. Two sides of a triangular field are 85 m and 154 m, respectively, and its perimeter is 324 m. The cost of ploughing the field at the rate of Rs. 10 per sq m is
(a) Rs. 27720 (b) Rs. 37620
(c) Rs. 26750 (d) None of these

16. The area of an equilateral triangle each of whose sides measures $2\sqrt{3}$ cm is
(a) $5\sqrt{3}$ sq m (b) $4\sqrt{3}$ sq m
(c) $3\sqrt{3}$ sq cm (d) None of these

17. If the height of an equilateral triangle is $2\sqrt{3}$ cm, then the length of its side is
(a) 4 cm (b) 6 cm
(c) 5 cm (d) None of these

18. If the perimeter of an equilateral triangle is 12 m, find its area.
(a) $3\sqrt{4}$ m (b) $4\sqrt{3}$ m
(c) $5\sqrt{3}$ m (d) None of these

19. The height of an equilateral triangle whose perimeter is 24 cm, is
(a) $4\sqrt{3}$ cm (b) $3\sqrt{4}$ cm
(c) $5\sqrt{3}$ cm (d) None of these

20. The perimeter of a right angled triangle is 90 cm and its hypotenuse is 39 cm. Find its other sides
(a) 30 cm, 10 cm (b) 36 cm, 15 cm
(c) 48 cm, 20 cm (d) None of these

21. The perimeter of an isosceles triangle is 306 m and each of the equal sides is $\frac{5}{8}$ of the base. Find the area.
(a) 3648 cm (b) 3468 sq m
(c) 3846 cm (d) None of these

22. Find the area of an isosceles right-angled triangle whose hypotenuse is 8 cm.
(a) 32 sq m (b) 24 sq cm
(c) 16 sq cm (d) None of these

23. The perimeter of an isosceles triangle is equal to 14 cm; the lateral side is to the base in the ratio 5 to 4. The area of the triangle is
(a) $3\sqrt{21}$ sq cm (b) $2\sqrt{21}$ sq cm
(c) $4\sqrt{21}$ sq cm (d) None of these

24. If all the sides of a triangle are increased by 200%, the area of the triangle will increase by
(a) 400% (b) 600%
(c) 800% (d) None of these

25. A plot of land is in the shape of a right-angled isosceles triangle. The length of hypotenuse is $50\sqrt{2}$ m. The cost of fencing is Rs. 3 per metre. The cost of fencing the plot will be
(a) less than Rs. 300 (b) less than Rs. 500
(c) more than Rs. 500 (d) None of these

26. If the area of an equilateral triangle is equal to the area of an isosceles triangle whose base and equal sides area 16 cm and 10 cm respectively, then the side of the equilateral triangle is
(a) 10.5 cm (b) 9.5 cm
(c) 12.5 cm (d) None of these

27. If the perimeter of a right-angled isosceles triangle is $4\sqrt{2}+4$ m, then the hypotenuse is
(a) 8 m (b) 6 m
(c) 4 m (d) None of these

28. The two adjacent sides of a parallelogram are 60 m and 40 m and one of the diagonals is 80 m long. The area of the parallelogram is
(a) $600\sqrt{15}$ sq m (b) $800\sqrt{25}$ sq m
(c) $700\sqrt{15}$ sq m (d) None of these

29. One side of the parallelogram is 14 cm. Its distance from the opposite side is 16 cm. The area of the parallelogram is
(a) 234 sq cm (b) 324 sq cm
(c) 224 sq cm (d) None of these

30. A field is in the shape of a parallelogram. Its adjacent sides and one diagonal are 65 m, 119 m and 156 m, respectively. Find the cost of gravelling it at the rate of Rs. 10 per sq m.
(a) Rs. 81400 (b) Rs. 71400
(c) Rs. 91400 (d) None of these

31. One side of a parallelogram is 10 m and the corresponding altitude is 7 m. The area of the parallelogram is
(a) 70 sq m (b) 60 sq m
(c) 80 sq m (d) None of these

32. The adjacent sides of a parallelogram are 8 m and 5 m. The distance between the longer sides is 4 m. The distance between the shorter sides is
(a) 4.6 m (b) 6.4 m
(c) 8.6 m (d) None of these

33. The area of a quadrilateral is 420 sq m and the perpendiculars drawn to one diagonal from the opposite vertices are 18 m and 12 m. Then, the length of the diagonal is
(a) 32 m (b) 24 m
(c) 28 m (d) None of these

34. The area of a parallelogram is 72 sq cm and its altitude is twice the corresponding base. The length of the base is
(a) 6 cm (b) 8 cm
(c) 4 cm (d) None of these

35. The area of a parallelogram is 240 sq cm and its height is 12 cm. The base of the parallelogram is
(a) 24 cm (b) 20 cm
(c) 28 cm (d) None of these

36. In a quadrilateral *ABCD*, the sides, *AB*, *BC*, *CD*, *DA* measure 20 m, 13 m, 17 m and 10 m, respectively and the diagonal *AC* is 21 m. The area of the quadrilateral is
(a) 210 sq m (b) 220 sq m
(c) 240 sq m (d) None of these

37. If the two diagonals of a parallelogram are 72 cm and 30 cm respectively, find its perimeter.
(a) 156 cm (b) 164 cm
(c) 172 cm (d) None of these

38. If the base of a parallelogram is $(x + 4)$, altitude to the base is $(x - 3)$ and the area is $(x^2 - 4)$, then the actual area is equal to
(a) 64 sq units (b) 48 sq units
(c) 60 sq units (d) None of these

39. In a parallelogram, the lengths of adjacent sides are 12 cm and 14 cm, respectively. If the length of one diagonal is 16 cm, find the length of the other diagonal.
(a) 24.8 cm (b) 20.6 cm
(c) 22.4 cm (d) None of these

40. Find the perimeter of a circular plot which occupies an area of 154 sq m.
(a) 54 m (b) 44 m
(c) 22 m (d) 11 m

41. The perimeter of a circle is equal to that of a square. Compare their areas.
(a) 14 : 11 (b) 25 : 12
(c) 24 : 7 (d) 22 : 7

42. The length of a rectangle is thrice its breadth and its perimeter is 96 m. The area of the rectangle is
(a) 288 sq m (b) 442 sq m
(c) 438 sq m (d) 432 sq m

43. A cow is tied by a rope at the corner of a rectangular field. If the length of the rope is 14 m, the area of the field which the cow could graze is
(a) 77 sq m (b) 308 sq m
(c) 23 sq m (d) 154 sq m

44. The wheel of a scooter has diameter 70 cm. How many revolutions per minute must the wheel make so that the speed of the scooter is kept 66 km per hour?
(a) 400 (b) 600
(c) 500 (d) 800

45. Two small circular parks of diameters 16 m, 12 m are to be replaced by a bigger circular park. What would be the radius of this new park, if the new park occupies same space as the two small parks?
(a) 10 (b) 15
(c) 20 (d) 25

46. A rectangular park is 65 m long and 50 m wide. Two cross paths each 2 m wide are to be constructed parallel to the sides. If these paths pass through the centre of the rectangle and cost of construction is 17.25 per sq m, find the total cost involved in the construction.
(a) Rs. 2265.59 (b) Rs. 1772.45
(c) Rs. 3898.50 (d) Rs. 8452.32

47. The area of a trapezium is 2500 sq m. One of its parallel sides is 75 m. If the distance between the two parallel sides is 40 m, find the other parallel side.
(a) 20 m (b) 30 m
(c) 40 m (d) 50 m

48. The length of a rectangle exceeds its breadth by 3 cm. If the numerical values of the area and the

perimeter of the rectangle are equal, the breadth is
(a) 1 cm (b) 2 cm
(c) 3 cm (d) 3.5 cm

49. If the ratio of the areas of two squares is 9 : 1, the ratio of their perimeters is
(a) 9 : 1 (b) 3 : 4
(c) 3 : 1 (d) 1 : 3

50. A square field with side 30 m is surrounded by a path of uniform width. If the area of the path is 256 m^2, its width is
(a) 16 m (b) 14 m
(c) 4 m (d) 2 m

51. A rope by which a calf is tied is increased from 12 m to 23 m. How much additional grassy ground shall it graze?
(a) 1120 m^2 (b) 1250 m^2
(c) 1210 m^2 (d) 1200 m^2

52. Four circular cardboard pieces, each of radius 7 cm are placed in such a way that each piece touches two other pieces. The area of the space enclosed by the four pieces is
(a) 21 cm^2 (b) 42 cm^2
(c) 84 cm^2 (d) 168 cm^2

53. The length and breadth of a rectangular field are in the ratio 5 : 3. If the cost of cultivating the field at 25 paise per square metre is Rs. 6000, find the dimensions of the field
(a) 250 m, 100 m (b) 50 m 30 m
(c) 200 m, 120 m (d) Cannot be determined

54. The cost of carpeting a room 5 m wide with carpet at Rs. 3.50 per m^2 is Rs. 105. The length of the room is
(a) 3.5 m (b) 5 m
(c) 6 m (d) 6.5 m

55. The length of a rectangular field is twice its breadth. If the rent of the field at Rs. 3500 a hectare is Rs. 28000, find the cost of surrounding it with fence at Rs. 5 per metre.
(a) Rs. 6000 (b) Rs. 7000
(c) Rs. 6500 (d) Rs. 8000

56. The area of a rectangular field is 27000 sq m. The ratio of its length and breadth is 6 : 5. The length and breadth of the field are respectively
(a) 180 m, 150 m (b) 200 m, 150 m
(c) 180 m, 120 m (d) 150 m, 100 m

57. The area of a sector of a circle of radius 5 cm, formed by an arc of length 3.5 cms, is
(a) 35 cm^2 (b) 17.5 cm^2
(c) 8.75 cm^2 (d) 55 cm^2

58. The length of a plot is double its width. If a square piece of land of area 150 m^2 occupies $\frac{1}{3}$ area of the plot, then what is the length of the plot?
(a) 15 m (b) 7.5 m
(c) 30 m (d) 10 m

59. A wire is in the form of a semi-circle of 7 cm radius. The length of the wire will be
(a) 25 cm (b) 36 cm
(c) 5 cm (d) 69 cm

60. A circular road runs round a circular ground. If the difference between the circumferences of the outer circle and the inner circle is 66 metres, the width of the road is
(a) 21 metres (b) 10.5 metres
(c) 7 metres (d) 5.25 metres

61. If the area of a square is 50 sq units, then the area of the circle drawn on its diagonal is
(a) 25 π sq units (b) 50 π sq units
(c) 100 π sq units (d) None of these

62. A rectangular sheet of cardboard is of 4 cm × 2 cm. If a circle of greatest possible area is cut from it, then the area of remaining portion is
(a) $(2 - \pi)$ cm^2 (b) $(4 - \pi)$ cm^2
(c) $(8 - \pi)$ cm^2 (d) $(16 - \pi)$ cm^2

63. What is the radius of a circle, to the nearest cm, whose area is equal to the sum of the areas of three circles of radii 22 cm, 19 cm and 8 cm, respectively?
(a) 17 cm (b) 30 cm
(c) 29 cm (d) 19 cm

64. The length of a rectangle is increased by 33.33%. By what per cent should the width be decreased to maintain the same area?
(a) 25% (b) 33.33%
(c) 22.5% (d) None of these

65. If the area of a square is equal to the area of a rectangle 6.4 m long and 2.5 m wide, then each side of the square measures
(a) 8 m (b) 5.4 m
(c) 3.8 m (d) 4 m

66. The diameters of two concentric circles are 8 cm and 10 cm. The area of the region between them is
(a) π cm^2 (b) 3π cm^2
(c) 6π cm^2 (d) 9π cm^2

67. The length of a rectangular room is 4 metres. If it can be partitioned into two equal square rooms, what is the length of the partition in metres?
(a) 1 (b) 2
(c) 4 (d) Data inadequate

68. A rectangular carpet has an area of 120 sq m and a perimeter of 46 m. The length of its diagonal is

(a) 15 m (b) 16 m
(c) 17 m (d) 20 m

69. The side of a square is 22 m. What is the radius of the circle whose circumference is equal to the perimeter of the square?
(a) 28 m (b) 3.5 m
(c) 14 m (d) 7 m

70. A piece of wire 132 cm long is bent successively in the shapes of an equilateral triangle, a square, a regular hexagon, and a circle. Then, which has the largest surface area?
(a) equilateral triangle (b) square
(c) circle (d) regular hexagon

71. If the radius of one circle is twelve times the radius of another, how many times does the area of the greater contain the area of the smaller?
(a) 12 (b) 72
(c) 144 (d) 96

72. If the circumference of a circle is equal to the perimeter of a square, what is the ratio of the area of the circle to the area of the square?
(a) 22 : 7 (b) 14 : 11
(c) 11 : 7 (d) 4 : 1

73. The length of a rectangular plot of land is three times as much as its breadth. A playground measuring 1200 sq. ft. occupies one-fourth of the total area of the plot. What is the length of the plot in feet?
(a) 40 (b) 360
(c) 120 (d) Data inadequate

74. There are two squares s_1 and s_2. The ratio of their areas are 4 : 25. If the side of s_1 is 6 cm, what is the side of s_2?
(a) 20 cm (b) 15 cm
(c) 5 cm (d) 12 cm

75. The radius of the wheel of a vehicle is 70 cm. The wheel makes 10 revolutions in 5 seconds. The speed of the vehicle is
(a) 29.46 km/hr (b) 31.68 km/hr
(c) 36.25 km/hr (d) 32.72 km/hr

76. A rectangular carpet has an area of 60 m^2. Its diagonal and longer side together equal 5 times the shorter side. The length of the carpet is
(a) 5 m (b) 13 m
(c) 14.5 m (d) 12 m

77. A playground has the shape of a rectangle with two semi-circles on its smaller sides as diameters, added outside. If the sides of the rectangle are 36 m and 24.5 m, then the area of the playground is $\left(\text{use } \pi = \frac{22}{7}\right)$
(a) 2259.529 sq m (b) 1353.625 sq m
(c) 1139.523 sq m (d) None of these

78. A man runs a circle of 50 m radius at a speed of 12 km per hour. Find the time taken by him for going around it ten times
(a) 10 min (b) 12.5 min
(c) 15.7 min (d) None of these

79. A room 5 m × 8 m is to be carpeted leaving a margin of 10 cm from each wall. If the cost of the carpet is Rs. 18 per m^2, the cost of carpeting the room will be
(a) Rs. 702.60 (b) Rs. 691.80
(c) Rs. 682.46 (d) Rs. 673.92

80. The area of a big rectangle is equal to the area of a small equal rectangle. If the length of the big rectangle is equal to the length of the small rectangle and the width of big rectangle is 2 m, what is the width of a small rectangle?
(a) $\frac{1}{3}$m (b) 1 m
(c) 2 m (d) Cannot be determined
(e) None of these

81. If the radius of a circle is reduced by 40%, its circumference is reduced by
(a) 60% (b) 40%
(c) 35% (d) 45%

82. A figure consists of a square of side 'a' m with semi-circles drawn on the outside of the square. The area (in m^2) of the figure so formed will be
(a) a^2 (b) $a^2 + 2\pi a^2$
(c) $4\pi a^2$ (d) $a^2 + \frac{\pi a^2}{2}$

83. The area of a square field is 6050 m^2. How much time it will take to reach from one of its corner to the opposite corner at the rate of 10 m in every 30 seconds?
(a) $5\frac{1}{2}$ min (b) 11 min
(c) 22 min (d) 110 min

84. If a regular hexagon is inscribed in a circle of radius r, then its perimeter is
(a) $3r$ (b) 6r
(c) $9r$ (d) $12r$

85. Two poles 15 m and 30 m high stand upright in a playground. If their feet be 36 m apart, find the distance between their tops.
(a) 36 m (b) 39 m
(c) 15 m (d) None of these

86. The length of a rectangular hall is $\frac{4}{3}$ of its width. If the area of the hall is 300 m^2, what is the difference between the length and breadth?
(a) 15 m (b) 20 m
(c) 3 m (d) 5 m
(e) 6 m

87. Each side of an equilateral triangle is increased by 1.5%. The percentage increase in its area is
(a) 1.5% (b) 3%
(c) 4.5% (d) 5.7%

88. A rope, by which a horse is tied, is increased from 12 to 23 m. How much additional ground will it be able to graze?
(a) 1315 m^2 (b) 765 m^2
(c) 1210 m^2 (d) 1012 m^2

89. If a diagonal of a square is doubled, how does the area of the square change?
(a) Becomes four-fold (b) Becomes three-fold
(c) Becomes two-fold (d) None of these

90. If the sides of a rectangle are increased by 20%, the percentage increase in its perimeter is
(a) 80 (b) 40
(c) 20 (d) None of these

91. A circle road runs around a circular garden. If the difference between the circumference of the outer circle and the inner circle is 44 m, find the width of the road.
(a) 4 m (b) 7 m
(c) 3.5 m (d) 7.5 m

92. The length of a rectangular field is double its width. Inside the field there is a square-shaped pond 8 m long. If the area of the pond is $\frac{1}{8}$ of the area of the field, what is the length of the field?
(a) 32 m (b) 64 m
(c) 16 m (d) 20 m
(e) None of these

93. A circular disc of area 0.49π sq. m rolls down a length of 1.76 km. The number of revolutions it makes is
(a) 300 (b) 400
(c) 600 (d) 4000

94. If area of a triangle whose base is 6 cm is equal to the area of a circle of radius 6 cm, find the height of this triangle
(a) 10 cm (b) 22 cm
(c) 12 cm (d) 18 cm

95. The length of a ladder exactly equals the height of a wall. If the ladder is placed on a 2-feet tall stool placed 10 feet away from the wall, its tip just touches the top of the wall. The height of the wall in feet is
(a) 15 (b) 26
(c) 28 (d) 32

96. If the diagonal of a square is doubled to make another square, the area of the new square will
(a) become four-fold (b) become two-fold
(c) become six-fold (d) become eight-fold

97. The area of a circle is 154 cm^2. The length of an arc of the circle which subtends an angle of 45° at the centre is
(a) 11 cm (b) 5.5 cm
(c) 7 cm (d) None of these

98. A lawn is in the form of a triangle having its base and height in the ratio 2 : 3. The area of the lawn is $\frac{1}{12}$ hectare. Find the base and height of the lawn.
(a) 55 m, 34 m (b) 50 m, $33\frac{1}{3}$m
(c) 50 m, 35 m (d) Data inadequate

99. A rectangular farm has to be fenced on one long side, one short side and the diagonal. If the cost of fencing is Rs. 10 per m, the area of farm is 1200 m^2 and the short side is 30 m long, how much would the job cost?
(a) Rs. 700 (b) Rs. 1200
(c) Rs. 1400 (d) Rs. 1500

100. The diameter of a circle is 105 cm less than the circumference. What is the diameter of circle?
(a) 44 cm (b) 46 cm
(c) 48 cm (d) 49 cm

101. A garden is 24 m long and 14 m wide. There is a path 1 m wide outside the garden along its sides. If the path is to be constructed with square marble tiles 20 cm × 20 cm, find the number of tiles required to cover the path
(a) 1800 (b) 200
(c) 2000 (d) 2150

102. If the length of the diagonal of a rhombus is 80% of the length of the other diagonal, the area of the rhombus is how many times the square of the length of the longer diagonal?
(a) $\frac{4}{5}$ (b) $\frac{2}{5}$
(c) $\frac{3}{4}$ (d) $\frac{1}{4}$

103. $ABCD$ is a trapezium in which $AB \parallel CD$ and $AB = 2CD$. If its diagonals intersect each other at O, the ratio of area of triangle AOB and COD is
(a) 1 : 2 (b) 2 : 1
(c) 1 : 4 (d) 4 : 1

104. The ratio of the corresponding sides of two similar triangles is 3 : 4. The ratio of their areas is
(a) 4 : 3 (b) 3 : 4
(c) 9 : 16 (d) $\sqrt{3}:2$

105. The area of the circle inscribed in an equilateral triangle of side 24 cm is
(a) 24π cm^2 (b) 36π cm^2
(c) 48π cm^2 (d) 18π cm^2

106. The radius of wheel is 1.4 decimetre. How many times does it revolve during a journey of 0.66 km?
(a) 375 (b) 750
(c) 1500 (d) 3000

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Let $a = 15$ m, $b = 16$ m, $c = 17$ m.

Then, $s = \frac{a+b+c}{2} = \frac{15+16+17}{2} = 24$ m.

$\therefore$ Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{24(24-15)(24-16)(24-17)}$

$= \sqrt{24\times9\times8\times7}$

$= \sqrt{12096} = 24\sqrt{21}$ sq m.

2. Ans. (a)

Solution Height of the triangle $= \sqrt{(6.5)^2 - 6^2}$

$= \sqrt{42.25-36}$

$= \sqrt{6.25} = 2.5$ m

$\therefore$ Area $= \frac{1}{2}$(base × height)

$= \frac{1}{2} \times 6 \times 2.5 = 7.5$ sq m.

3. Ans. (b)

Solution Each side = 12 cm

Then, $s = \frac{12+12+12}{2} = 18.$

Area $= \sqrt{18(18-12)(18-12)(18-12)}$

$= \sqrt{18\times6\times6\times6} = \frac{1}{2} \times 12 \times$ height

or, height $= \frac{36\sqrt{3}}{6} = 6\sqrt{3}$ cm.

4. Ans. (b)

Solution Let the length of equal sides be x.

Then, $s = \frac{x+x+64}{2} = x + 32.$

Area = 1600 sq m.

$= \sqrt{(x+32)(x+32-x)(x+32-x)(x+32-64)}$

$= \sqrt{(x+32)\times32\times32\times(x-32)}$

or, $1600 = 32\sqrt{x^2-32^2}$

$\Rightarrow \sqrt{x^2-32^2} = 50$

or, $x^2 = 32^2 + 50^2 = 1024 + 2500 = 3524.$

$\therefore x = 59.36$ m.

5. Ans. (c)

Solution Here, $s = \frac{20+21+29}{2} = \frac{70}{2} = 35$ m.

$\therefore$ Area $= \sqrt{35(35-20)(25-21)(35-29)}$

$= \sqrt{35\times15\times14\times6}$

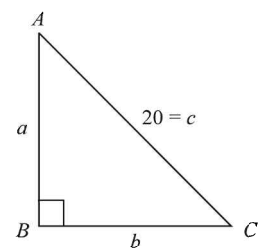
$= \sqrt{5^2\times7^2\times3^2\times2^2}$

$= 5 \times 7 \times 3 \times 2 = 210$ sq m.

6. Ans. (a)

Solution We have, $a + b + 20 = 2 \times 24$

$\therefore a + b = 28$ ...(1)



By Pythagores Theorem, $a^2 + b^2 = 400$

$\because (a + b)^2 = 28^2$

$\Rightarrow ab = \frac{28^2-(a^2+b^2)}{2} = \frac{28^2-400}{2} =$ 192

$\therefore a - b = \sqrt{(a+b)^2-4ab}$

$= \sqrt{784-4\times192} = 4$ ...(2)

Solving (1) and (2), we get

$a = 16$ cm and $b = 12$ cm.

7. Ans. (b)

Solution The sides of the triangle are

$a = \frac{3}{12} \times 36 = 9$ cm, $b = \frac{4}{12} \times 36 =$ 12 cm

$c = \frac{5}{12} \times 36 = 15$ cm

$s = \frac{a+b+c}{2} = \frac{36}{2} = 18$ cm

$\therefore$ Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18(18-9)(18-12)(18-15)}$

$= \sqrt{18\times9\times6\times3}$

$= \sqrt{9^2\times2^2\times3^2} = 9 \times 2 \times 3 = 54$ sq cm.

8. Ans. (c)

Solution We have, $2s = 50 + 78 + 112 = 240$

$\therefore s = 120.$

$\therefore$ Area $= \sqrt{120(120-50)(120-78)(120-112)}$

$= \sqrt{120\times70\times42\times8}$

$= \sqrt{2^3\times5\times3\times7\times5\times2\times2\times7\times3\times2^3}$

$= 2^4 \times 5 \times 3 \times 7 = 1680$ sq m

$\therefore \quad \frac{1}{2} \times 112 \times h = 1680,$

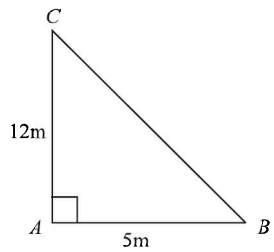
where h is the length of perpendicular

$\therefore \quad h = \frac{1680 \times 2}{112} = 30$ m.

9. Ans. (a)

Solution Let BC be the ladder.

Then, $BC = \sqrt{12^2 + 5^2} = 13$ m.



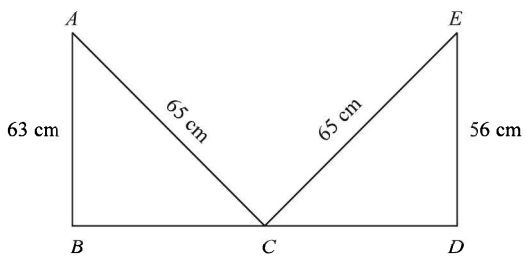
10. Ans. (c)

Solution $BC = \sqrt{(65)^2 - (63)^2} = 16$

$CD = \sqrt{(65)^2 - (56)^2} = 33.$

$\therefore$ Width of the street

$= 16 + 33 = 49$ cm.



11. Ans. (c)

Solution Given: $\frac{1}{2} \times x \times h = x^2$

$\therefore \quad h = 2x.$

12. Ans. (c)

Solution Let the base be $3x$.

Then, the height is $4x$.

Given: $\frac{1}{2} \times 3x \times 4x = 150 \Rightarrow x = 5.$

$\therefore$ Base $= 3 \times 5 = 15$ m and height $= 4 \times 5 = 20$ m.

13. Ans. (c)

Solution Area of field $= \frac{495.72}{36.72} \times 10000$

$= 135000$ sq m

Let the height be x.

Then, base $= 3x$.

We have, $\frac{1}{2} \times x \times 3x = 135000$

$\Rightarrow \quad x^2 = 90000$ or $x = 300.$

$\therefore$ Height = 300 m and base = 3 × 300 = 900 m.

14. Ans. (c)

Solution Let the original sides be a, b, c, then

$s = \frac{1}{2}(a + b + c)$

and area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

For the new triangle, the sides are $2a$, $2b$, $2c$.

Then, $S = \frac{1}{2}(2a + 2b + 2c) = a + b + c = 2s.$

$\therefore$ Area of new triangle

$= \sqrt{S(S-2a)(S-2b)(S-2c)}$

$= \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$

$= \sqrt{16s(s-a)(s-b)(s-c)}$

$= 4\sqrt{s(s-a)(s-b)(s-c)}$

$= 4 \times$ (area of original triangle).

15. Ans. (a)

Solution The third side of the triangle

$= 324 - (85 + 154) = 85$ m.

Also, $s = \frac{a+b+c}{2} = \frac{324}{2} = 162.$

$\therefore$ Area of the triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{162(162-85)(162-85)(162-154)}$

$= \sqrt{162 \times 77 \times 77 \times 8} = 2772$ sq m.

$\therefore$ The cost of ploughing the field

$= 2772 \times 10 =$ Rs. 27720.

16. Ans. (c)

Solution The area of the equilateral triangle

$= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4} \times (2\sqrt{3})^2$

$= 3\sqrt{3}$ sq cm.

17. Ans. (a)

Solution Height of the equilateral triangle $= \frac{\sqrt{3}}{2} \times$ (side)

$\Rightarrow \quad 2\sqrt{3} = \frac{\sqrt{3}}{2} \times$ (side)

$\therefore$ Side of the equilateral triangle = 4 cm.

18. Ans. (b)

Solution $3 \times$ (side) $= 12 \Rightarrow$ side $= 4$ m.

$\therefore$ Area of equilateral triangle $= \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4} \times 16 = 4\sqrt{3}$ m.

19. Ans. (a)

Solution $3 \times (\text{side}) = 24 \Rightarrow \text{side} = 8$ cm.

$\therefore$ Height of the equilateral triangle

$$= \frac{\sqrt{3}}{2} \times (\text{side}) = \frac{\sqrt{3}}{2} \times 8 = 4\sqrt{3}\text{ cm.}$$

20. Ans. (b)

Solution Let x and $(51 - x)$ be the other two sides of the triangle

Then, $x^2 + (51 - x)^2 = 39^2$

$\Rightarrow \quad x^2 + 2601 - 102x + x^2 = 1521$

$\Rightarrow \quad x = \dfrac{51 \pm \sqrt{441}}{2} = \dfrac{51 \pm 21}{2} = 36, 15.$

$\therefore$ The other two sides are 36 cm and 15 cm.

21. Ans. (b)

Solution Given: $a = \dfrac{5}{8}b.$

Now, perimeter of isosceles triangle $= 2a + b$

$\Rightarrow \quad 306 = 2 \times \dfrac{5}{8}b + b$ or, $b = 136.$

$\therefore \quad a = \dfrac{5}{8}b = \dfrac{5}{8} \times 136 = 85.$

$\therefore$ Area of isosceles triangle $= \dfrac{b}{4}\sqrt{4a^2 - b^2}$

$$= \frac{136}{4}\sqrt{4 \times (85)^2 - (136)^2}$$

$$= 34 \times 102 = 3468 \text{ sq m.}$$

22. Ans. (c)

Solution We have, hypotenuse $= \sqrt{2}\,a = 8 \Rightarrow a = 8/\sqrt{2}$.

$\therefore$ Area of isosceles triangle $= \dfrac{1}{2}a^2$

$$= \frac{1}{2} \times \frac{64}{2} = 16 \text{ sq cm.}$$

23. Ans. (b)

Solution Let the lateral side $= 5x$ and the base $= 4x$.

Then, $5x + 5x + 4x = 14 \Rightarrow x = 1.$

$\therefore$ The sides of the triangle are 5 cm, 5 cm and 4 cm.

$\therefore$ Area of the isosceles triangle

$$= \frac{b}{4}\sqrt{4a^2 - b^2}$$

$$= \frac{4}{4}\sqrt{4(5)^2 - (4)^2} = \sqrt{84} = 2\sqrt{21} \text{ sq cm.}$$

24. Ans. (c)

Solution Area $= \sqrt{s(s-a)(s-b)(s-c)} = A$

where $s = \dfrac{a+b+c}{2}$ and a, b, c are sides of the triangle.

When the sides are increased by 200%, the sides become $3a$, $3b$ and $3c$.

$$s_1 = \frac{3a + 3b + 3c}{2} = 3\frac{(a+b+c)}{2} = 3s.$$

$$A_1 = \sqrt{s_1(s_1 - 3a)(s_1 - 3b)(s_1 - 3c)}$$

$$= \sqrt{3s.3(s-a).3(s-b).3(s-c)}$$

$$= 9\sqrt{s(s-a)(s-b)(s-c)} = 9A.$$

$\therefore$ Increase in area $= 9A - A = 8A$ or, 800%.

25. Ans. (c)

Solution Let the length of each of equal sides of the triangle be xm.

Then, $x^2 + x^2 = (50\sqrt{2})^2 = 5000.$

$\Rightarrow \quad 2x^2 = 5000 \Rightarrow x = 50.$

$\therefore$ Perimeter of the triangle $= 50 + 50 + 50\sqrt{2}$

$= 100 + 50 \times 1.4146 = 170.73$ m.

$\therefore$ Cost of fencing = Rs. (170.73×3) = Rs. 512.19.

26. Ans. (a)

Solution Area of isosceles triangle $= \dfrac{b}{4}\sqrt{4a^2 - b^2}$

$$= \frac{16}{4}\sqrt{4 \times (10)^2 - (16)^2} = 4 \times 12 = 48 \text{ sq cm}$$

Given: Area of equilateral triangle = 48.

$\Rightarrow \quad \dfrac{\sqrt{3}}{4}(\text{side})^2 = 48$

$\Rightarrow$ Side of equilateral triangle = 10.5 cm.

27. Ans. (c)

Solution Perimeter of a right-angled isosceles triangle

$= (\sqrt{2} + 1) \times$ hypotenuse

$\Rightarrow \quad 4\sqrt{2} + 4 = (\sqrt{2} + 1) \times$ hypotenuse

$\Rightarrow$ hypotenuse = 4 m.

28. Ans. (a)

Solution Here, $a = 60$, $b = 40$ and $d = 80$

$\therefore \quad s = \dfrac{a+b+d}{2} = \dfrac{60+40+80}{2} = 90.$

$\therefore$ Area of the parallelogram

$$= 2\sqrt{s(s-a)(s-b)(s-d)}$$

$$= 2\sqrt{90(90-60)(90-40)(90-80)}$$

$$= 2\sqrt{90 \times 30 \times 50 \times 10}$$

$$= 600\sqrt{15} \text{ sq m.}$$

29. Ans. (c)

Solution Area of the parallelogram

$= 16 \times 14 = 224$ sq cm.

30. Ans. (b)

Solution We have, $s = \dfrac{a+b+d}{2}$

$= \dfrac{65+119+156}{2} = 170.$

$\therefore$ Area of parallelogram

$= 2\sqrt{s(s-a)(s-b)(s-d)}$

$= 2\sqrt{170(170-65)(170-119)(170-156)}$

$= 2\sqrt{170\times51\times105\times14}$

$= 2 \times 3570 = 7140$ sq m.

$\therefore$ Cost of gravelling = 7140 × 10 = Rs. 71400.

31. Ans. (a)

Solution The area of the parallelogram

= 10 × 7 = 70 sq m.

32. Ans. (b)

Solution Area of the parallelogram = 8 × 4 = 32 sq m.

Distance between the shorter sides = $\dfrac{32}{5}$ = 6.4 m.

33. Ans. (c)

Solution Area of quadrilateral = $\dfrac{1}{2}d(p_1+p_2)$

$\Rightarrow \quad 420 = \dfrac{1}{2}\times d\times(18+12)$

$\Rightarrow \quad d = \dfrac{420\times2}{30} = 28$ m.

34. Ans. (a)

Solution Let the base be x and altitude be $2x$.

Then, $x \times 2x = 72 \Rightarrow x^2 = 36 \Rightarrow x = 6$.

35. Ans. (b)

Solution Let the base be x.

Area of the parallelogram = base × altitude

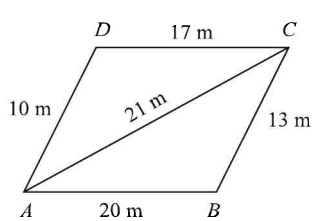
$\Rightarrow \quad 240 = x \times 12$

$\therefore \quad x = \dfrac{240}{12} = 20$ cm.

36. Ans. (a)

Solution Area of quadrilateral $ABCD$

= Area of ΔADC + Area of ΔABC, where,



Area of ΔABC

$= \sqrt{s(s-AB)(s-BC)(s-d)}$

$\left(s = \dfrac{AB+BC+d}{2} = \dfrac{20+13+21}{2} = 27\right)$

$= \sqrt{27(27-20)(27-13)(27-21)}$

$= \sqrt{27\times7\times14\times6} = 126$ sq m.

Area of ΔADC

$= \sqrt{s(s-AD)(s-DC)(s-d)}$

$\left(s = \dfrac{AD+DC+d}{2} = \dfrac{10+17+21}{2} = 24\right)$

$= \sqrt{24(24-10)(24-17)(24-21)}$

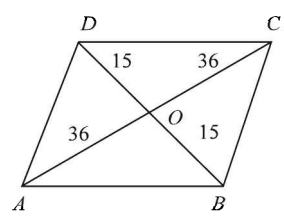
$= \sqrt{24\times14\times7\times3} = 84$ sq m.

Here, the area of quadrilateral $ABCD$

= 126 + 84 = 210 sq m.

37. Ans. (a)

Solution Since $AC = 72$ and $BD = 30$



$\therefore \quad OB = OD = \dfrac{BD}{2} = \dfrac{30}{2} = 15$

and, $\quad OA = OC = \dfrac{AC}{2} = \dfrac{72}{2} = 36$

$\therefore \quad AB = BC = CD = DA = \sqrt{36^2+15^2}$

$= \sqrt{1296+225}$

$= \sqrt{1521} = 39.$

$\therefore$ Perimeter of the parallelogram = 4 × 39

= 156 cm.

38. Ans. (c)

Solution Area of the parallelogram

= base × altitude

$\Rightarrow \quad (x^2 - 4) = (x + 4) \times (x - 3)$

$\Rightarrow \quad x^2 - 4 = x^2 + 4x - 3x - 12$

$\Rightarrow \quad x = 8.$

$\therefore$ Area of the parallelogram

$= x^2 - 4 = (8)^2 - 4 = 60$ sq units.

39. Ans. (b)

Solution We have,

$d_1^2 + d_2^2 = 2(a^2 + b^2)$

$\Rightarrow \quad (16)^2 + d_2^2 = 2(12^2 + 14^2)$

$\Rightarrow \quad d_2^2 = 2(144 + 196) - 256 = 424.$

$\therefore \quad d_2^2 = \sqrt{424} = 20.6$ cm.

40. Ans. (b)

Solution $\pi r^2 = 154 \text{ m}^2$

$\Rightarrow \quad r^2 = \frac{7}{22} \times 154 \text{ m}^2 = 49 \text{ m}^2$

$\therefore \quad r = 7 \text{ m}$

Perimeter $= 2 \times \frac{22}{7} \times 7 \text{ m} = 44 \text{ m}.$

41. Ans. (a)

Solution If x be the side of the square and r be the radius of the circle, then

$4x = 2\pi r$

or, $x = \frac{\pi r}{2}$

Now, $\pi r^2 : : x^2 = \pi r^2 : : \frac{\pi^2 r^2}{4}$ or, $4 : \pi$

$= 4 : : \frac{22}{7}$ or, $14 : 11.$

42. Ans. (d)

Solution Let the width of the rectangle be x m

Then length $= 3x$ m

Perimeter $= 2(x + 3x) = 96$

$\Rightarrow \quad 8x = 96$ or, $x = 12$

Area $= 12 \times 36 = 432$ sq m.

43. Ans. (d)

Solution As the cow is tied at the corner of a rectangular field, it will graze the area of the field enclosed between two sides of the rectangle.

$= \frac{1}{4}(\pi \times 14 \times 14)$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$

$= 154 \text{ m}^2.$

44. Ans. (c)

Solution The distance covered by the wheel in one m

$= \frac{66 \times 1000 \times 100}{60} = 110000 \text{ cm}$

The distance covered by the wheel in one revolution

= The circumference of the wheel

$= 2\pi r = 2 \times \frac{22}{7} \times \frac{70}{2} = 220 \text{ cm}$

$\therefore$ Number of revolutions of the wheel

$= \frac{110000}{220} = 500.$

45. Ans. (a)

Solution $\pi(8)^2 + \pi(6)^2 = \pi R^2$

$\therefore \quad R^2 = 64 + 36$

$R = 10$

46. Ans. (c)

Solution Area of two paths $= 2 \times (65 + 50 - 2) = 226 \text{ m}^2$

Cost of construction $= 226 \times 17.25$

= Rs. 3898.50.

47. Ans. (d)

Solution $\frac{1}{2}(75 + x) \times 40 = 2500$

$\Rightarrow \quad 75 + x = 125$

$\Rightarrow \quad x = 50$

$\therefore$ The other parallel side = 50 m.

48. Ans. (c)

Solution Let the breadth be x, then length $= x + 3$

Given: $2(x + x + 3) = x(x + 3)$

$\Rightarrow \quad 4x + 6 = x^2 + 3x$

$\Rightarrow \quad 4x + 6 = x^2 + 3x$

$\Rightarrow \quad x^2 - x - 6 = 0$

$\therefore \quad x = \frac{1 \pm \sqrt{1+24}}{2} = \frac{1 \pm 5}{2}$

$\therefore \quad x = 3, -2$

$\therefore$ Breadth = 3 cm.

49. Ans. (c)

Solution Let x, y be the sides of squares

$\frac{x^2}{y^2} = \frac{9}{1} \Rightarrow \frac{x}{y} = \frac{3}{1}$

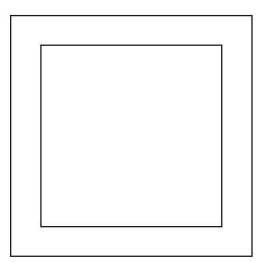
$\therefore$ The ratio of perimeters is $4x : 4y$

i.e. $x : y = 3 : 1.$

50. Ans. (d)

Solution Let x (in metres) be the width of the path

Side of outer square $= 30 + 2x$



$\therefore$ Area of path $= (30 + 2x)^2 - 30^2$

$\therefore \quad (30 + 2x)^2 - 30^2 = 256$

$\Rightarrow \quad 4x^2 + 12x - 256 = 0$

$\Rightarrow \quad x^2 + 30x - 64 = 0$

$\Rightarrow \quad (x - 2)(x + 32) = 0$

$\therefore \quad x = 2. \qquad (\because x \nless 0)$

51. Ans. (c)

Solution Additional grassy ground grazed

$= \pi(23^2 - 12^2)\ m^2$

$= \frac{22}{7} \times 35 \times 11$

$= 1210\ m^2.$

52. Ans. (b)

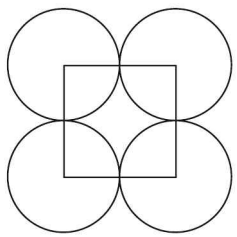
Solution Required area enclosed

$= 14 \times 14 - 4 \times$ (area of a quadrant)

$= \left[196 - 4 \times \frac{22}{7} \times 7 \times 7 \times \frac{90}{360}\right] cm^2$

$= (196 - 154)\ cm^2$

$= 42\ cm^2.$



53. Ans. (c)

Solution Area $= \frac{6000 \times 100}{25} = 24000\ m^2$

Let length be $5x$ and breadth be $3x$

$\therefore\ 5x \times 3x = 24000$

$\therefore\ x = \sqrt{1600} = 40$

$\therefore$ Length $= 5 \times 40 = 200$ m, breadth $= 3 \times 40 = 120$ m.

54. Ans. (c)

Solution Area of the carpet $= \frac{105}{3.50} = 30\ m^2$

Area of the room = 30 sq m

Width = 5 m

$\therefore$ Length $= \frac{30}{5} = 6$ m.

55. Ans. (a)

Solution Area of the rectangular field

$= \frac{28000}{3500} = 8$ hectares

$2x \times x = 80000$

$\therefore\ x = \sqrt{40000} = 200$

$\therefore$ Breadth = 200 m

Length = 400 m

Perimeter = 2(400 + 200) = 1200 m

$\therefore$ Cost of fencing = 1200 × 5 = Rs. 6000.

56. Ans. (a)

Solution Let the length and breadth be $6x$ and $5x$ m respectively. Then

$6x \times 5x = 27000$ or $30x^2 = 27000$

$\Rightarrow\ x^2 = 900$ or $x = 30$

Hence, length of the field = 180 m and width = 150 m.

57. Ans. (c)

Solution Area of sector $= \left(\frac{1}{2} \times \text{arc length} \times \text{radius}\right) cm^2$

$= \left(\frac{1}{2} \times 3.5 \times 5\right) cm^2$

$= 8.75\ cm^2.$

58. Ans. (c)

Solution Area of the plot $= 150 \times 3 = 450\ m^2$

If the length of the plot be x m then breadth

$= \frac{x}{2}$ m

$\therefore\ x \times \frac{x}{2} = 450$

or, $x^2 = 900$

or, $x = 30$ m.

59. Ans. (b)

Solution Perimeter of circle $= 2 \times \frac{22}{7} \times 7$

$= 44$ cm

Perimeter of half circle = 22 cm

The length of the wire = 22 + 14 = 36 cm.

60. Ans. (b)

Solution $2\pi R - 2\pi r = 66$

$2\pi(R - r) = 66$

$2 \times \frac{22}{7} \times (R - r) = 66$

or, $R - r = 66 \times \frac{7}{22} \times \frac{1}{2} = \frac{21}{2} = 10.5$ m.

61. Ans. (a)

Solution One side of the square $= \sqrt{50} = 5\sqrt{2}$

Length of diagonal $= 5\sqrt{2} \times \sqrt{2} = 10$

Radius of circle = 5

$\therefore$ Area of the circle $= 25\pi$ units.

62. Ans. (c)

Solution Area of remaining portion

= Area of rectangle − Area of circle

$= 4 \times 2 - \pi \times (1)^2 = (8 - \pi)\ cm^2.$

63. Ans. (b)

Solution $r = \sqrt{22^2 + 19^2 + 8^2}$ cm

$= \sqrt{484 + 361 + 64}$ cm

$= \sqrt{909}$ cm $= 30$ cm

64. Ans. (a)

Solution Area of rectangle = $l \times b$

Let the new width be b_1

Then, $l \times b = \frac{4}{3} l \times b_1$

$\therefore \quad b_1 = \frac{3}{4} b = 0.75b$

Thus, there should be a reduction of 25% in the width.

65. Ans. (d)

Solution Area of rectangle = $6.4 \times 2.5 = 16\,\text{m}^2$

According to question:

Area of square = Area of the rectangle

$\therefore$ Area of square = $16\,\text{m}^2$

$\therefore$ Side of the square = 4 m.

66. Ans. (d)

Solution $\pi \times 5 \times 5 - \pi \times 4 \times 4$

$= 25\pi - 16\pi = 9\pi\,\text{cm}^2$.

67. Ans. (b)

Solution The length of the room = 4 m

Since it can be partitioned into two equal square rooms, the two equal parts can only be of 2 m each.

68. Ans. (c)

Solution Let length = a m and breadth = b m

Then, $2(a + b) = 46$

or, $a + b = 23$ and $ab = 120$

$\therefore$ Diagonal $= \sqrt{a^2 + b^2} = \sqrt{(a+b)^2 - 2ab}$

$= \sqrt{(23)^2 - 2 \times 120} = \sqrt{289} = 17$ m.

69. Ans. (c)

Solution Perimeter of the square = 4×22 m = 88 m

$2\pi r = 88$ m

$\Rightarrow \quad r = 88 \times \frac{7}{22 \times 2}$ m = 14 m.

70. Ans. (c)

Solution Area of an equilateral $\Delta = \frac{\sqrt{3}}{4} a^2$

$[3a = 132 \therefore a = 44]$

$= \frac{\sqrt{3}}{4} \times 44 \times 44$

$= 838.312$ sq m.

Area of square $= a^2 = 33 \times 33$

$= 1089$ sq m. $\left[a = \frac{132}{4} = 33\right]$

Area of regular hexagon

$= \frac{3\sqrt{3}a^2}{2} = \frac{3\sqrt{3} \times 22 \times 22}{2}$

$\left[a = \frac{132}{6} = 22\right]$

Area of circle $= \pi r^2$

$= \frac{22}{7} \times 21 \times 21 \left[r = \frac{132}{2\pi} = 21\right]$

$\therefore$ Circle has largest surface area.

71. Ans. (c)

Solution Let r be the radius of smaller circle

Radius of larger circle = $12r$

$$\frac{\text{Area of larger circle}}{\text{Area of smaller circle}} = \frac{\pi(12r)^2}{\pi r^2} = \frac{144}{1}$$

$\therefore$ Area of large circle contains the area of smaller circle 144 times.

72. Ans. (b)

Solution If a be the side of the square and r be the radius of the circle, then

$2\pi r = 4a$

or, $r = \frac{4a}{2\pi} = \frac{2a}{\pi}$

$\therefore$ Area of the circle $= \pi r^2$ and,

area of the square $= a^2$

$\therefore$ Area of the circle : Area of the square

$= \pi r^2 : a^2$

$= \frac{\pi\left(\frac{2a}{\pi}\right)^2}{a^2} = \frac{\pi \frac{4a^2}{\pi^2}}{a^2}$

$= \frac{4}{\pi} = \frac{4 \times 7}{22} = 4 : 11.$

73. Ans. (c)

Solution Let the length and breadth of the plot be $3x$ and x feet, respectively.

Total area of the plot = $4 \times 1200 = 4800$ sq ft.

$\therefore \quad x \times 3x = 4800 \Rightarrow x = 40$ ft

$\therefore$ Length = $3 \times 40 = 120$ ft.

74. Ans. (b)

Solution Ratio of the areas = 4 : 25

Ratio of the sides = 2 : 5

If the side of s_1 is 2 cm, then side of s_2 is 5 cm.

If the side of s_1 is 6 cm, the side of s_2 is $\frac{5}{2} \times 6 = 15$ cm.

75. Ans. (b)

Solution Circumference = $\left(2\times\frac{22}{7}\times 70\right)$ cm = 440 cm

Distance travelled in 10 revolutions

= 4400 cm = 44 m

$\therefore$ Speed = $\frac{\text{Distance}}{\text{Time}} = \left(\frac{44}{5}\right)$ m/sec

$= \left(\frac{44}{5}\times\frac{18}{5}\right)$ km/hr = 31.68 km/hr.

76. Ans. (d)

Solution $lb = 60\text{ m}^2$

$d^2 = l^2 + b^2$

$d + l = 5b \Rightarrow d = 5b - l$

$\therefore\ d^2 = 25b^2 + l^2 - 10bl$

$\Rightarrow\ l^2 + b^2 = 25b^2 + l^2 - 10bl$

$\Rightarrow\ l^2 + b^2 = 25b^2 + l^2 - 10 \times 60$

$\Rightarrow\ 24b^2 = 600$ or $b = 5$

$\therefore\ l = 60 \div 5 = 12$ m.

77. Ans. (b)

78. Ans. (c)

Solution Distance = 2 × 3.14 × 50 × 10

= 3140 m

12 km/hr = $\frac{10}{3}$ m/sec

Time = $\frac{3140}{10/3}$ = 942 sec

$= \frac{942}{60}$ min

= 15.7 min.

79. Ans. (d)

Solution Length of the area to be carpeted = 8 – 0.2

= 7.8 m

Width = 5 – 0.2 = 4.8 m

$\therefore$ Area to be carpeted = 7.8 × 4.8 m^2

Total cost = 18 × 7.8 × 4.8 = Rs. 673.92.

80. Ans. (a)

Solution Let the length of the big rectangle be x m

$\therefore$ Area of the big rectangle = $x \times 2 = 2x\text{ m}^2$

$\therefore$ Area of the small rectangle = $\frac{1}{6} \times 2x = \frac{x}{3}\text{ m}^2$

$\therefore$ Breadth of the small rectangle = $\frac{x}{3} \div x = \frac{1}{3}$ m.

81. Ans. (b)

Solution Let original radius= r

Reduced radius = $r - 0.4r = 0.6r$

$\therefore$ Percentage reduction in circumference

$= \frac{2\pi r - 2\pi(0.6)r}{2\pi r} \times 100 = 40\%.$

82. Ans. (d)

Solution Required area = (Area of a square of side a).

$+ 4\left(\text{Area of semi-circle of radius } \frac{a}{2}\right)$

$= a^2 + 4 \times \frac{1}{2}\pi \times \left(\frac{1}{2}a\right)^2$

$= a^2 + \frac{\pi a^2}{2}.$

83. Ans. (a)

Solution $(\text{Diagonal})^2 = 2 \times 6050\text{ m}^2 = 12100\text{ m}^2$

Diagonal = 110 m

Time taken = $\frac{110}{10}\times\frac{1}{2}$ min = $5\frac{1}{2}$ min.

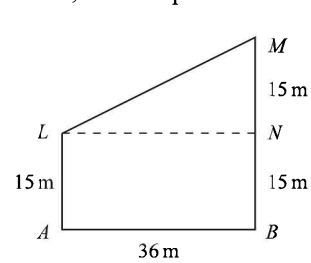
84. Ans. (b)

Solution Length of each side of hexagon = r

$\therefore$ Its perimeter = $6r$.

85. Ans. (b)

Solution Let AL, BM be poles with tops at L and M, respectively.



$LA = 15$ m, $MB = 30$ m

$AB = 36$ m.

Let LN be parallel to the ground

$LN = AB = 36$ m

and, $MN = 30 - 15 = 15$ m

$\therefore\ LM = \sqrt{36^2 + 15^2} = \sqrt{1296 + 225} = 39$ m.

86. Ans. (d)

Solution Let the breadth be x m

$\therefore$ Length = $\frac{4x}{3}$ m

$\therefore\ \frac{4x}{3}\times x = 300$

or, $x^2 = 300 \times \frac{3}{4} = 225$

$x = \sqrt{225} = 15$ m

and, length $= \frac{4}{3} \times 15 = 20$ m

length – breadth = 20 – 15 m = 5 m.

87. Ans. (a)

Solution Let the original length of each side = a

Then, area $= \frac{\sqrt{3}}{4}a^2 = A$

New area $= \frac{\sqrt{3}}{4}\left[\left(\frac{101.5}{100}a^2\right)\right]$

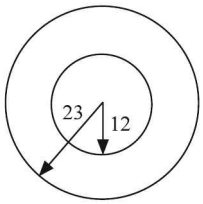
$= \frac{\sqrt{3}}{4}\left(\frac{20.3}{20}\right)a^2 = \frac{20.3}{20}A$

Increase in area $= \left(\frac{0.3}{20}A \times \frac{1}{A} \times 100\right)\%$

$= 1.5\%$.

88. Ans. (c)

Solution Shaded area of the two (in between) concentric circle of radius R and r is $\pi(R + r)(R - r)$



∴ Additional grassy area

$= \frac{22}{7} \times (23 + 12)(23 - 12)\text{ m}^2$

$= \frac{22}{7} \times 35 \times 11\text{ m}^2$

$= 1210\text{ m}^2$.

89. Ans. (a)

Solution Let originally, diagonal = x

∴ Original area $= \frac{1}{2}x^2$

After increase, diagonal = $2x$

New area $= \frac{1}{2}(2x)^2$

∴ $\frac{\text{original area}}{\text{New area}} = \frac{1}{4}$

∴ New area = 4 (original area).

90. Ans. (c)

Solution If length of the rectangle is L and its width is B, then its perimeter = $2(L + B)$

Increased length = $1.2L$,

Increased width = $1.2B$

Increase perimeter $= 2(1.2L + 1.2B)$

$= 2 \times 1.2(L + B)$

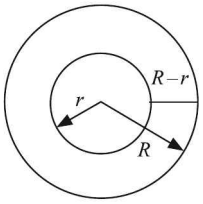
$= 2.4(L + B)$

Increase in perimeter $= 0.4(L + B)$

Percentage increase $= \frac{0.4(L+B)}{2(L+B)} \times 100 = 20\%$.

91. Ans. (b)

Solution $2\pi R - 2\pi r = 44$ m



∴ $2\pi(R - r) = 44$ m

or, $R - r = 44 \div 2\pi$

$= 44 \div \left(2 \times \frac{22}{7}\right) = 7$ m.

92. Ans. (a)

Solution Area of the square-shaped pond $= 8 \times 8$

$= 64\text{ m}^2$

∴ Area of field $= 8 \times 64 = 512\text{ m}^2$

If the length of the field be x m.

Then, the breadth of the field $= \frac{x}{2}$m

∴ $x \times \frac{x}{2} = 512$

or, $x^2 = 2 \times 512$.

or, $x^2 = 1024$

or, $x = \sqrt{1024} = 32$ m.

93. Ans. (b)

Solution $\pi r^2 = 0.49\pi \Rightarrow r = 0.7$ m

Number of revolutions $= \frac{1.76 \times 1000}{2 \times \frac{22}{7} \times 0.7} = 400$.

94. Ans. (c)

Solution Radius of the circle = 6 cm

Area of the circle = 36 sq cm.

According to question:

Area of the triangle of base 6 cm

= Area of the circle of radius 6 cm

∴ $\frac{1}{2} \times$ base $\times$ height = 36

$\Rightarrow \quad \frac{1}{2} \times 6 \times \text{height} = 36$

$\Rightarrow$ Height of the triangle $= \frac{36 \times 2}{6} = 12$ cm.

95. Ans. (b)

Solution Let the height of the wall be h metre when the ladder is placed at distance 10 m away from the wall on a stool of 2 m height, it will form a right triangle with sides 10 m, $(h - 2)$ m and taper side of length h m.

Hence, we have $h^2 = 10 + (h - 2)^2$

or, $h^2 - (h - 2)^2 = 100$

$\Rightarrow (h + h - 2) \times (h - h + 2) = 100$

or, $(2h - 2) \times 2 = 100$ or $4h - 4 = 100$

$\Rightarrow 4h - 4 = 100$

$\Rightarrow 4h = 104$ or, $h = 26$ m

96. Ans. (a)

Solution Suppose the side of the square $= x$ cm

Diagonal of the square $= \sqrt{2}x$ cm

Area of the square $= x^2$ cm, i.e. $\frac{(\text{Diagonal})^2}{2}$

According to question:

Diagonal of the new square $= 2 \times \sqrt{2}x$
$= 2\sqrt{2}x$ cm

$\therefore$ Area of the new square

$= \frac{(\text{Diagonal of the new square})^2}{2}$

$= \frac{(2\sqrt{2}x)^2}{2} = 4x^2$ sq cm.

97. Ans. (b)

Solution Area of the circle $= 154$ cm^2

Let its radius $= r$, then $154 = \frac{22}{7}r^2$

$\Rightarrow r^2 = \frac{154 \times 7}{22} = 49$ or, $r = 7$ cm

Circumference $= 2\pi r = 2 \times \frac{22}{7} \times 7 = 44$ cm

Length of arc $= \frac{44 \times 45}{360} = 5.5$ cm.

98. Ans. (b)

Solution Let the base be $2x$ m and height $3x$ m.

Then, $\frac{1}{2}(2x \times 3x) = \frac{1}{12} \times 10000$

[$\because$ 1 hectare = 10000 m^2]

or, $x = \sqrt{\frac{10000 \times 2}{6 \times 12}} = \frac{100}{6} = \frac{50}{3}$

$\therefore$ Base $= 2 \times \frac{50}{3} = 33\frac{1}{3}$ m

Height $= \frac{3 \times 50}{3} = 50$ m.

99. Ans. (b)

Solution Length of the longer side $= \frac{1200}{30} = 40$ m

and the length of the diagonal $= \sqrt{30^2 + 40^2} = 50$ m

$\therefore$ Length of the fence $= 30 + 40 + 50 = 120$ m

$\therefore$ The job cost $= 120 \times 10 =$ Rs. 1200.

100. Ans. (d)

Solution $\pi d - d = 105 \Rightarrow (\pi - 1)d = 105$

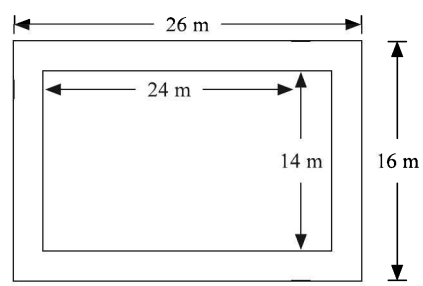
$\Rightarrow \left(\frac{22}{7} - 1\right)d = 105$

$\therefore \quad d = 105 \times \frac{7}{15} = 49$ cm.

101. Ans. (c)

Solution Area of garden $= 24 \times 14 = 336$ m^2

Area of the (garden + path) $= 26 \times 16$
$= 416$ m^2



$\therefore$ Area of the path $= 416 - 336 = 80$ m^2

Area of 1 tile $= 20 \times 20 = 400$ cm^2 $= 0.04$ m^2

$\therefore$ Number of tiles required $= \frac{80}{0.04} = 2000$.

102. Ans. (b)

Solution Let one diagonal $= x$ cm

Then, another diagonal

$= \left(\frac{80}{100}x\right)$ cm $= \left(\frac{4}{5}x\right)$ cm

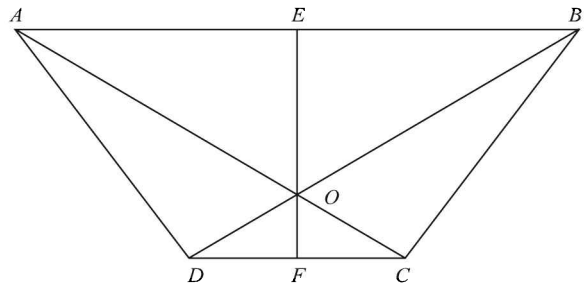
Area of rhombus $= \frac{1}{2}x \times \frac{4}{5}x = \frac{2}{5}x^2$

$= \frac{2}{5} \times$ (square of longer diagonal).

103. Ans. (d)

Solution $\Delta OAB = \frac{1}{2} \times AB \times OE$

$$= \frac{1}{2} \times 2CD \times OE$$
$$= CD \times OE$$



$$\Delta OCD = \frac{1}{2} \times CD \times OF$$

$$\therefore \quad \frac{\Delta AOB}{\Delta COD} = \frac{CD \times OE}{\frac{1}{2} \times CD \times OF} = \frac{CD \times 2 \times OF}{\frac{1}{2} \times CD \times OF} = \frac{4}{1}$$
$$= 4 : 1.$$

104. Ans. (c)

Solution Ratio of areas of similar triangles
= Ratio of the squares of corresponding sides

$$= \frac{(3x)^2}{4x^2} = \frac{9x^2}{16x^2} = \frac{9}{16} = 9 : 16.$$

105. Ans. (c)

Solution $\frac{1}{2} \times 24 \times h = \frac{\sqrt{3}}{4} \times 24 \times 24$

or, $h = 12\sqrt{3}$

$\therefore \quad 3r = 12\sqrt{3} \Rightarrow r = 4\sqrt{3}$ cm

Area of the circle $= \pi \times (4\sqrt{3})^2 \text{ cm}^2 = 48\pi \text{ cm}^2$.

106. Ans. (b)

Solution $r = 0.14$

Number of revolutions

$$= \left(\frac{0.66 \times 1000}{2} \times \frac{7}{22} \times \frac{1}{0.14}\right) = 750.$$

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. How many sides does a regular polygon have whose interior and exterior angles are in the ratio 2 : 1?
(a) 3 (b) 5
(c) 6 (d) 12

[Based on SSC (GL) Prel. Exam 2000]

2. The area (in sq cm) of the largest circle that can be drawn inside a square of side 28 cm is
(a) 17248 (b) 784
(c) 8624 (d) 616

[Based on SSC (GL) Prel. Exam 2000]

3. If the circumference of a circle increases from 4π to 8π, what change occurs in its area?
(a) It doubles (b) It triples
(c) It quadruples (d) It is halved

[Based on SSC (GL) Prel. Exam 2000]

4. The perimeter of the top of a rectangular table is 28 m whereas its area is 48 m^2. What is the length of its diagonal?
(a) 5 m (b) 10 m
(c) 12 m (d) 12.5 cm

[Based on SSC (GL) Prel. Exam 2000]

5. A rectangular paper, when folded into two congruent parts had a perimeter of 34 cm for each part folded along one set of sides and the same is 38 cm when folded along the other set of sides. What is the area of the paper?
(a) 140 cm^2 (b) 240 cm^2
(c) 560 cm^2 (d) None of these

[Based on SSC (GL) Prel. Exam 2000]

6. The length of a rectangle is three times of its width. If the length of a diagonal is $8\sqrt{10}$ cm, then the perimeter of the rectangle is
(a) $24\sqrt{10}$ cm (b) $16\sqrt{10}$ cm
(c) 64 cm (d) $15\sqrt{10}$ cm

[Based on SSC (GL) Prel. Exam 2000]

7. Three sides of a triangle are 5 cm, 12 cm and 13 cm respectively, then its area is
(a) 30 cm^2 (b) $10\sqrt{3}$ cm^2
(c) $10\sqrt{6}$ cm^2 (d) 20 cm^2

[Based on SSC (GL) Prel. Exam 2000]

8. The difference between the circumference and the diameter of a circle is 30 cm. The radius of the circle is
(a) 6 cm (b) 7 cm
(c) 5 cm (d) 8 cm

[Based on SSC (GL) Prel. Exam 2000]

9. The sides of a triangle are in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. If the perimeter of the triangle is 52 cm, the length of the smallest side is
(a) 24 cm (b) 18 cm
(c) 12 cm (d) 9 cm

[Based on SSC (GL) Prel. Exam 2000]

10. If the area of a triangle is 1176 cm^2 and base : corresponding altitude is 3 : 4, then the altitude of the triangle is

(a) 42 cm (b) 52 cm
(c) 54 cm (d) 56 cm

[Based on SSC (GL) Prel. Exam 2000]

11. If the length and breadth of a rectangle are in the ratio 3 : 2 with its perimeter as 20 cm, the area of the rectangle (in cm^2) must be
(a) 24 (b) 48
(c) 72 (d) 96

[Based on SSC (GL) Prel. Exam 2000]

12. The area of the largest circle, that can be drawn inside a rectangle with sides 18 cm by 14 cm, is
(a) 49 cm^2 (b) 154 cm^2
(c) 378 cm^2 (d) 1078 cm^2

[Based on SSC (GL) Prel. Exam 2000]

13. If the length of the diagonal of a square is 20 cm, then its perimeter must be
(a) $40\sqrt{2}$ cm (b) 40 cm
(c) 200 cm (d) $10\sqrt{2}$ cm

[Based on SSC (GL) Prel. Exam 2000]

14. If the sides of a triangle are 5 cm, 4 cm and $\sqrt{41}$ cm, then the area of the triangle is
(a) 20 cm^2 (b) $(5+4+\sqrt{41})cm^2$
(c) $\frac{5+4+\sqrt{41}}{2}cm^2$ (d) 10 cm^2

[Based on SSC (GL) Prel. Exam 2000]

15. The diagonal of a square is $4\sqrt{2}$ cm. The diagonal of another square is
(a) $8\sqrt{2}$ cm (b) 16 cm
(c) $\sqrt{32}$ cm (d) 8 cm

[Based on SSC (GL) Prel. Exam 2002]

16. The areas of a square and a rectangle are equal. The length of the rectangle is greater than the length of any side of the square by 5 cm and the breadth is less by 3 cm. Find the perimeter of the rectangle.
(a) 17 cm (b) 26 cm
(c) 30 cm (d) 34 cm

[Based on SSC (GL) Prel. Exam 2002]

17. In two triangles, the ratio of the areas is 4 : 3 and that of their heights is 3 : 4. Find the ratio of their bases.
(a) 16 : 9 (b) 9 : 16
(c) 9 : 12 (d) 16 : 12

[Based on SSC (GL) Prel. Exam 2002]

18. The diagonals of a rhombus are 24 cm and 10 cm. The perimeter of the rhombus (in cm) is
(a) 68 (b) 65
(c) 54 (d) 52

[Based on SSC (GL) Prel. Exam 2002]

19. Find the length of the longest rod that can be placed in a hall of 10 m length, 6 m breadth and 4 m height.
(a) $2\sqrt{38}$ m (b) $4\sqrt{38}$ m
(c) $2\sqrt{19}$ m (d) 19 m

[Based on SSC (GL) Prel. Exam 2002]

20. If a wire is bent into the shape of a square, the area of the square is 81 sq cm. When the wire is bent into a semi-circular shape, the area of the semi-circle $\left(\text{taking } \pi = \frac{22}{7}\right)$ is
(a) 154 cm^2 (b) 77 cm^2
(c) 44 cm^2 (d) 22 cm^2

[Based on SSC (GL) Prel. Exam 2002]

21. The perimeters of two squares are 24 cm and 32 cm. The perimeter (in cm) of a third square equal in area to the sum of the areas of there squares is
(a) 45 (b) 40
(c) 32 (d) 48

[Based on SSC (GL) Prel. Exam 2002]

22. The length of a rectangular garden is 12 m and its breadth is 5 m. Find the length of the diagonal of a square garden having the same area as that of the rectangular garden
(a) $2\sqrt{30}$ m (b) $\sqrt{13}$ m
(c) 13 m (d) $8\sqrt{15}$ m

[Based on SSC (GL) Prel. Exam 2002]

23. The base of a triangle is 15 cm and height is 12 cm. The height of another triangle of double the area having base 20 cm is
(a) 9 cm (b) 18 cm
(c) 8 cm (d) 12.5 cm

[Based on SSC (GL) Prel. Exam 2002]

24. The diagonals of a rhombus are 32 cm and 24 cm, respectively. The perimeter of the rhombus is
(a) 80 cm (b) 72 cm
(c) 68 cm (d) 86 cm

[Based on SSC (GL) Prel. Exam 2002]

25. A wire when bent in the form of a square encloses an area of 484 sq cm. What will be the enclosed area when the same wire is bent into the form of a circle? $\left(\text{take } \pi = \frac{22}{7}\right)$
(a) 462 cm^2 (b) 539 cm^2
(c) 616 cm^2 (d) 693 cm^2

[Based on SSC (GL) Prel. Exam 2002]

26. The perimeter of a square is 48 cm. The area of rectangle is 4 cm^2 less than the area of the square. If the length of the rectangle is 14 cm, then its perimeter is

(a) 24 cm (b) 48 cm
(c) 50 cm (d) 54 cm

[Based on SSC (GL) Prel. Exam 2002]

27. The side and one of the diagonals of a rhombus are 13 cm and 24 cm, respectively. The area of the rhombus (in sq cm) is
(a) 156 (b) 240
(c) 120 (d) 130

[Based on SSC (GL) Prel. Exam 2002]

28. A circular wire of radius 21 cm is cut and bent in the form of a rectangle whose sides are in the ratio of 6 : 5. Assuming $\pi = \frac{22}{7}$, the area enclosed by the rectangle is
(a) 540 cm^2 (b) 1080 cm^2
(c) 2160 cm^2 (d) 4320 cm^2

[Based on SSC (GL) Prel. Exam 2002]

29. The area (in m^2) of the square which has the same perimeter as a rectangle whose length is 48 m and is 3 times its breadth, is
(a) 1000 (b) 1024
(c) 1600 (d) 1042

[Based on SSC (GL) Prel. Exam 2002]

30. The area of an equilateral triangle is $400\sqrt{3}$ sq m. Its perimeter is
(a) 120 m (b) 150 m
(c) 90 m (d) 135 m

[Based on SSC (GL) Prel. Exam 2003]

31. *A* took 15 sec to cross a rectangular field diagonally walking at the rate of 52 m/min and *B* took the same time to cross the same field along its sides walking at the rate of 68 m/min. The area of the field is
(a) 30 m^2 (b) 40 m^2
(c) 50 m^2 (d) 60 m^2

[Based on SSC (GL) Prel. Exam 2003]

32. Diameter of a wheel is 3 m. The wheel revolves 28 times in a minute. To cover 5,280 km distance, the wheel will take $\left(\text{take } \pi = \frac{22}{7}\right)$
(a) 10 min (b) 20 min
(c) 30 min (d) 40 min

[Based on SSC (GL) Prel. Exam 2003]

33. The sides of a triangle are 3 cm, 4 cm and 5 cm. The area (in cm^2) of the triangle formed by joining the mid-points of this triangle is
(a) 6 (b) 3
(c) $\frac{3}{2}$ (d) $\frac{3}{4}$

[Based on SSC (GL) Prel. Exam 2003]

34. From a point in the interior of an equilateral triangle the perpendicular distance of the sides are $\sqrt{3}$ cm, $2\sqrt{3}$ cm and $5\sqrt{3}$ cm. The perimeter (in cm) of the triangle is
(a) 64 (b) 32
(c) 48 (d) 24

[Based on SSC (GL) Prel. Exam 2003]

35. Find the diameter of a wheel that makes 113 revolutions to go 2 km 26 decametres $\left(\text{take } \pi = \frac{22}{7}\right)$
(a) $4\frac{4}{13}$ m (b) $6\frac{4}{11}$ m
(c) $12\frac{4}{11}$ m (d) $12\frac{8}{11}$ m

[Based on SSC (GL) Prel. Exam 2003]

36. The difference of the areas of two squares drawn on two line segments of different lengths is 32 sq cm. Find the length of the greater line segment if one is longer than the other by 2 cm.
(a) 7 cm (b) 9 cm
(c) 11 cm (d) 16 cm

[Based on SSC (GL) Prel. Exam 2003]

37. The perimeters of two squares are 40 cm and 32 cm. The perimeter of a third square whose area is difference of the areas of the two squares is
(a) 24 cm (b) 42 cm
(c) 40 cm (d) 20 cm

[Based on SSC (GL) Prel. Exam 2003]

38. The perimeter of a rhombus is 40 cm. If the length of one of its diagonals be 12 cm, the length of the other diagonal is
(a) 14 cm (b) 15 cm
(c) 16 cm (d) 12 cm

[Based on SSC (GL) Prel. Exam 2003]

39. Three circles of radius 3.5 cm each are placed in such a way that each touches the other. The area of the portion enclosed by the circles is
(a) 1.975 cm^2 (b) 1.967 cm^2
(c) 19.67 cm^2 (d) 21.21 cm^2

[Based on SSC (GL) Prel. Exam 2003]

40. Four equal-sized maximum circular plates are cut off from a square paper sheet of area 784 sq cm. The circumference of each plate is
(a) 22 cm (b) 44 cm
(c) 66 cm (d) 88 cm

[Based on SSC (GL) Prel. Exam 2003]

41. The area of a right-angled triangle is two-thirds of the area of a rectangle. The base of the triangle is 80 per cent of the breadth of the rectangle. If the perimeter of the rectangle is 200 cm, what is the height of the triangle?

(a) 20 cm (b) 30 cm
(c) 15 cm (d) Data inadequate
(e) None of these

[Based on BSRB Chennai PO 2000]

42. When the length of a rectangular plot is increased by four times its perimeter becomes 480 metres and area 12800 sq m. What was its original length (in metre)?
(a) 160 (b) 40
(c) 20 (d) Cannot be determined

[Based on BSRB Bhopal PO 2000]

43. Four sheets of 50 cm × 5 cm are to be arranged in such a manner that a square could be formed. What will be the area of inner part of the square so formed?
(a) 2000 cm^2 (b) 1600 cm^2
(c) 1800 cm^2 (d) 2500 cm^2
(e) None of these

[Based on BSRB Bangalore PO 2000]

44. In order to fence a square Manish fixed 48 poles. If the distance between two poles is 5 m then what will be the area of the square so formed?
(a) Cannot be determined (b) 3025 cm^2
(c) 2500 cm^2 (d) 3025 cm^2
(e) None of these

[Based on BSRB Bangalore PO 2000]

45. What will be the area of semicircle of 14 metres diameter?
(a) 154 sq m (b) 77 sq m
(c) 308 sq m (d) 22 sq m
(e) None of these

[Based on NABARD Asst. Mamager Exam 2002]

46. The area of a rectangular field is 460 sq m. If the length is 15 per cent more than the breadth, what is the breadth of the rectangular field?
(a) 15 m (b) 26 m
(c) 34.5 m (d) Can't be determined
(e) None of these **[Based on Canara Bank PO 2003]**

47. The breadth of a rectangular hall is two-thirds of its length. If the area of the hall is 2400 sq m, what is the length in metres?
(a) 120 (b) 80
(c) 60 (d) 40
(e) None of these

[Based on IBPS Jr. Executive Exam 2002]

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Let the number of sides be n

$\therefore$ Interior angle $= 180 - \dfrac{360}{n}$

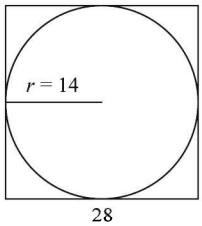
Exterior angle $= \dfrac{360}{n}$

$$\therefore \quad \frac{180-\frac{360}{n}}{\frac{360}{n}} = \frac{2}{1} \Rightarrow \frac{180-360}{360} = 2 \Rightarrow n = 6.$$

2. Ans. (d)

Solution Area of the circle of radius 14 cm.

$$= \frac{22}{7} \times 14 \times 14 = 616 \text{ cm}^2$$



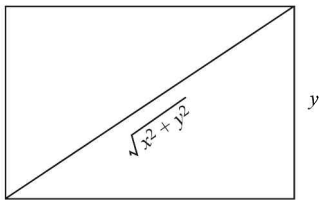
3. Ans. (c)

Solution

	1st Circle	2nd Circle
Circumference	4π	8π
Radius	2	4
Area	4π	16π

4. Ans. (b)

Solution $xy = 48$, $2x + 2y = 28 \Rightarrow x + y = 14$, $xy = 48$



$$\Rightarrow \quad x - y = \sqrt{(x+y)^2 - 4xy} = \sqrt{196-192} = 2$$

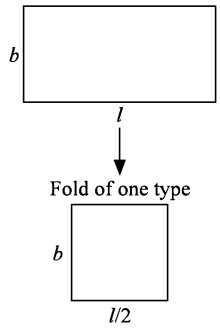
$$\therefore \quad x = \frac{x+y+x-y}{2} = \frac{14+2}{2} = \frac{16}{2} = 8$$

$$\text{and, } y = \frac{(x+y)-(x-y)}{2} = \frac{14+2}{2} = \frac{12}{2} = 6$$

$$\therefore \quad \text{Diagonal} = \sqrt{x^2+y^2} = \sqrt{8^2+6^2} = 10 \text{ m.}$$

5. Ans. (a)

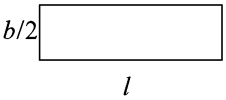
Solution



$\therefore \quad 2\left(\frac{l}{2}+b\right) = 34$

$\Rightarrow \quad l + 2b = 34$ ...(i)

Fold of another type



$2\left[l+\frac{b}{2}\right] = 38$

$\Rightarrow \quad 2l + b = 38$ ...(ii)

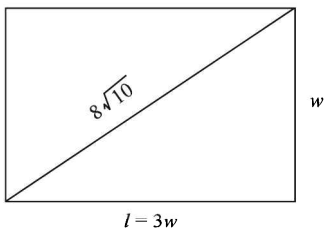
Solving (i) and (ii), we get

$l = 14,\ b = 10$

$\therefore$ Area = 140 cm^2.

6. Ans. (c)

Solution



$640 = 10w^2 \Rightarrow w = 8$

$\therefore \quad l = 24$ and perimeter = 64 cm.

7. Ans. (a)

Solution $s = \frac{5+12+13}{2} = 15$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{15(15-5)(15-12)(15-13)}$$
$$= \sqrt{15\times10\times3\times2}$$
$$= \sqrt{5\times3\times5\times2\times3\times2} = 30 \text{ cm}^2.$$

8. Ans. (b)

Solution $2\pi r - 2r = 30 \Rightarrow r\left[\frac{44}{7}-2\right] = 30$

$\Rightarrow r = \frac{30\times7}{30}$

$\therefore \quad r = 7$ cm.

9. Ans. (c)

Solution The sides of the triangle $= \frac{1}{2}:\frac{1}{3}:\frac{1}{4}$

$= 6 : 4 : 3$

$\therefore$ Length of the smallest side $= \frac{3}{13} \times 52$

$= 12$ cm.

10. Ans. (d)

Solution Area of triangle $= \frac{1}{2} \times$ base $\times$ corresponding altitude

$\therefore \quad 1176 = \frac{1}{2} \times 3x \times 4x$

$\therefore \quad 6x^2 = 1176$

$x^2 = 196$

$\therefore \quad x = 14$ cm

$\therefore$ Altitude of the triangle $= 4x = 56$ cm.

11. Ans. (a)

Solution $\frac{L}{3} = \frac{B}{2} = K$ (say)

$\therefore \quad L = 3K,\ B = 2K$

Perimeter $= 10K = 20 \Rightarrow K = 2$

$\therefore \quad L = 6,\ B = 4$

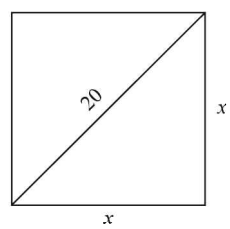
$\therefore$ Area = 24 cm^2

12. Ans. (b)

Solution Area $= \frac{22}{7} \times 7 \times 7 = 154$ cm^2.

13. Ans. (a)

Solution



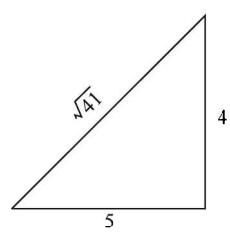
$x^2 + x^2 = (20)^2 \Rightarrow 2x^2 = 400$

$\Rightarrow x^2 = 200 \Rightarrow x = \sqrt{200}$

$\therefore$ Perimeter $= 4\sqrt{200} = 40\sqrt{2}$ cm.

14. Ans. (d)

Solution It is a right triangle



$\therefore$ Area $= \frac{1}{2} \times 5 \times 4 = 10$ cm^2.

15. Ans. (d)

Solution Area of the first square $= \left(\frac{4\sqrt{2}}{\sqrt{2}}\right)^2 = 16$ cm^2

Area of the second square $= 16 \times 2 = 32$ cm^2

Hence, diagonal of second square $= \sqrt{32}\times\sqrt{2}$

$= \sqrt{64}$

$= 8$ cm.

16. Ans. (d)

Solution Here, $(x + 5) \times (x - 3) = x^2$

or, $x^2 + 5x - 3x - 15 = x^2$

or, $2x = 15$

or, $x = 7.5$ cm

Hence, perimeter of the rectangle

$= 2(7.5 + 5 + 7.5 - 3) = 34$ cm.

17. Ans. (a)

Solution Suppose, ratio of their bases is $x : y$

$\therefore \quad \frac{x}{y} \times \frac{3}{4} = \frac{4}{3}$ or $\frac{x}{y} = \frac{4}{3} \times \frac{4}{3} = \frac{16}{9}$

Hence, required ratio = 16 : 9.

18. Ans. (d)

Solution One side of the rhombus

$$= \sqrt{\left(\frac{24}{2}\right)^2 + \left(\frac{10}{2}\right)^2} = 13 \text{ cm}$$

Hence, perimeter = 13 × 4 = 52 cm.

19. Ans. (a)

Solution Length of the longest rod

$$= \sqrt{10^2 + 6^2 + 4^2} = 2\sqrt{38} \text{ m.}$$

20. Ans. (b)

Solution Length of wire $= 4 \times \sqrt{81} = 36$ m

Perimeter of semi-circle = 36 m

or, $\pi r + 2r = 36$

or, $r(\pi + 2) = 36$

or, $r = \frac{36}{\pi + 2} = \frac{36}{22/7 + 2} = 7$ m

Therefore, area of the semi-circle

$$= \frac{1}{2}\pi r^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77 \text{ cm}^2.$$

21. Ans. (b)

Solution Required perimeter $= \sqrt{\left(\frac{24}{4}\right)^2 + \left(\frac{32}{4}\right)^2} \times 4$

$= 40.$

22. Ans. (a)

Solution Length of the diagonal $= (\sqrt{12 \times 5}) \times \sqrt{2}$

$= 2\sqrt{30}$ m.

23. Ans. (b)

Solution Required height $= \frac{2 \times 15 \times 12}{20}$

$= 18$ cm.

24. Ans. (a)

Solution Perimeter of rhombus $= 4 \times \sqrt{12^2 + 16^2}$

$= 80$cm.

25. Ans. (c)

Solution Enclosed area $= \left(\frac{\sqrt{484} \times 4}{2\pi}\right)^2 \times \pi$

$= 616$ sq cm.

26. Ans. (b)

Solution Area of the square = 144 sq cm

Area of the rectangle = 140 sq cm

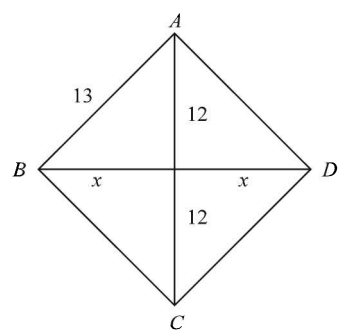
Length of the rectangle = 14 cm

$\Rightarrow$ Width of the rectangle = 10 cm

$\therefore$ Perimeter of the rectangle = 48 cm.

27. Ams. (c)

Solution $x = \sqrt{13^2 - 12^2} = 5$



$\therefore$ The diagonals of the rhombus are 10 and 24.

$\therefore$ Area of the rhombus $= \frac{1}{2} \times 10 \times 24$

$= 120$ sq cm.

28. Ans. (b)

Solution Let the sides of the rectangle be $6K$ and $5K$, respectively.

$\therefore \quad 2[6K + 5K] = 2 \times \frac{22}{7} \times 21 = 132 \Rightarrow K = 6$

$\therefore$ Area of the rectangle $= 30K^2 = 1080$ cm^2.

29. Ans. (b)

Solution Breadth of the rectangle $= \frac{48}{3} = 16$ m

Perimeter of the rectangle = 2(l + b) = 2 × 64 = 128 m

Side of the square $= \frac{128}{4} = 32$

Area of the square $= (32)^2 = 1024$ m^2.

30. Ans. (a)

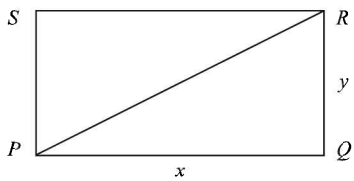
Solution Side of the equilateral triangle

$$= \sqrt{\frac{4}{\sqrt{3}} \times 400\sqrt{3}} = \sqrt{1600} = 40 \text{ m}$$

Perimeter = 40 × 3 = 120 m.

31. Ans. (d)

Solution Length of $PR = 52 \times \frac{15}{60} = 13$ m



$$PQ + RQ = 68 \times \frac{15}{60} = 17 \text{ m}$$

i.e. $x + y = 17$ and $x^2 + y^2 = 169$

Solving the above two, we get

$x = 12$ and $y = 5$

Area = 12 × 5 = 60 m^2.

32. Ans. (b)

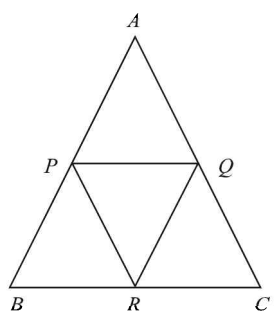
Solution Circumference of wheel = $2\pi r$

$$= 2 \times \frac{22}{7} \times \frac{3}{2} = \frac{66}{7} \text{m}$$

Wheel will take $= \frac{5280 \times 7}{28 \times 66} = 20$ m.

33. Ans. (c)

Solution ABC is a triangle. PQR is the triangle which is formed with the mid-points of the ΔABC. First of all, find the area of the ΔABC



$$s = \frac{a+b+c}{2} = \frac{3+4+5}{2} = \frac{12}{2} = 6 \text{ cm}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{6 \times 3 \times 2 \times 1} = 6 \text{ cm}^2$$

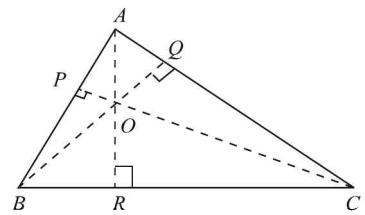
$$\therefore \quad (\Delta PQR) = \frac{1}{4} \text{ of } (\Delta ABC) = \frac{1}{4} \times 6 = \frac{3}{2} \text{ cm}^2.$$

34. Ans. (c)

Solution In ΔABC, O is the point from where three perpendicualar lines are drawn. And, also $OP = \sqrt{3}$ cm, $OQ = 2\sqrt{3}$ cm and $OR = 5\sqrt{3}$ cm

$$\text{Area of } \Delta AOB = \frac{1}{2} \times OP \times AB$$

$$\text{Area of } \Delta AOC = \frac{1}{2} \times OQ \times AC$$



$$\text{Area of } \Delta OBC = \frac{1}{2} \times OR \times BC$$

$$\frac{1}{2} \times \sqrt{3} \times x + \frac{1}{2} \times 2\sqrt{3} \times x + \frac{1}{2} \times 5\sqrt{3} \times x$$

$$= \frac{\sqrt{3}}{4}x^2$$

[x = Length of the side]

or, $\frac{\sqrt{3} + 2\sqrt{3} + 5\sqrt{3}}{2} = \frac{\sqrt{3}}{4}x$

or, $\frac{8\sqrt{3}}{2} = \frac{\sqrt{3}x}{4}$

or, $x = 16$ cm

$\therefore$ Perimeter of triangle = $3x = 3 \times 16 = 48$ cm.

35. Ans. (b)

Solution We know that 1 decametre = 10 m

$\therefore$ 2 km + 26 decametre = 2 × 1000 + 26 × 10

= 2260 m

Also, 113 revolutions = 2260 m

$\therefore$ 1 revolution $= \frac{2260}{113} = 20$ m

$\therefore$ diameter of wheel $= \frac{20 \times 7}{22} = \frac{70}{11} = 6\frac{4}{11}$m.

36. Ans. (b)

Solution Let the side of one of the squares be x cm. Then, side of longer square = $(x + 2)$ cm

Now, $(x + 2)^2 - x^2 = 32$

or, $x^2 + 4x + 4 - x^2 = 32$

or, $4x = 28$

or, $x = 7$ cm

Length of the side of the longer square = 7 + 2

= 9 cm.

37. Ans. (a)

Solution Side of two squares are $\frac{40}{4} = 10$ cm and $\frac{32}{4} = 8$ cm

Let the side of the third square be 'x' cm

Then, $x^2 = 10^2 - 8^2 = 100 - 64 = 36$

or, $x = 6$ cm

$\therefore$ Perimeter of the third square $= 6 \times 4$

$= 24$ cm.

38. Ans. (c)

Solution Side of the rhombus $= \frac{40}{4} = 10$ cm

Length of the other diagonal $= 2\sqrt{10^2 - 6^2}$

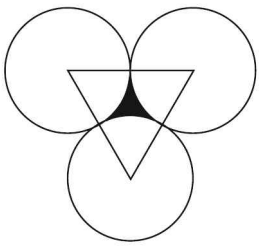
$= 2\sqrt{100 - 36}$

$= 2 \times 8$

$= 16$ cm.

39. Ans. (b)

Solution Required Area $= \frac{1}{2} \times \frac{\sqrt{3}}{2} \times 7 \times 7 - \frac{22}{7} \times 3.5 \times \frac{3.5}{2}$



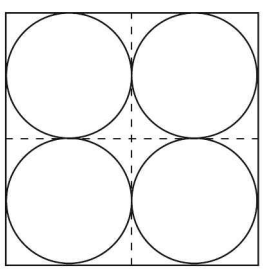
$= 3.5 \times 3.5\left(\sqrt{3} - \frac{11}{7}\right)$

$= 12.25 \times 0.1605 = 1.967$ cm^2.

40. Ans. (b)

Solution Side of square $= \sqrt{784} = 28$ cm

Radius of each plate $= 7$ cm



$\therefore$ Circumferencee $= 2 \times \frac{22}{7} \times 7$

$= 44$ cm.

41. Ans. (d)

Solution Let the base and height of triangle, and length and breadth of rectangle be L and h and L_1 and b_1, respectively.

Then, $\frac{1}{2} \times L \times h = \frac{2}{3} \times L_1 \times b_1$ ...(i)

$L = \frac{4}{5} b_1$ ...(ii) and $L_1 + b_1 = 100$ ...(iii)

In the above we have three equations and four unknowns. Hence, the value of 'h' cannot be determined.

42. Ans. (d)

Solution Let the length and breadth of the rectangle be l and b, respectively.

Then, $2(4l + b) = 480$

$\therefore$ $4l + b = 240$ ...(i)

and, $4lb = 12800$

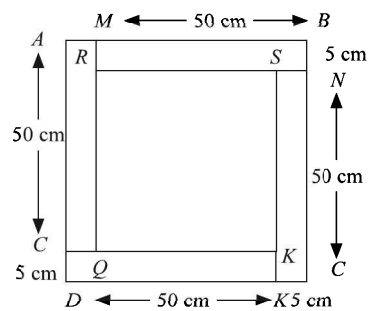
$\therefore$ $lb = 3200$ ...(ii)

Solving equations (i) and (ii), we get

$l = 40$ or, 20 m.

43. Ans. (e)

Solution The four sheets are BMRN, AMQL, NSKC and DLPK.



$\therefore$ Side of the new square sheet $= 50 + 5 = 55$ cm and the side of the inner part of the square $= 55 - 10 = 45$ cm

Hence, area $= (45)^2 = 2025$ sq cm.

44. Ans. (e)

Solution Let the side of the square be x m

$\therefore$ Perimeter of the square $= 48 \times 5 = 4x$

$\therefore$ $x = 60$ m

$\therefore$ Area $= (60)^2 = 3600$ m^2.

45. Ans. (b)

Solution Area of semi-circle $= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$= 77$ m^2.

46. Ans. (e)

Solution Let the length of the rectangular field be 'x' m. Then, length of the field will be

$x + \frac{x \times 15}{100} = \frac{23x}{20}$

Now, $x \times \frac{23x}{20} = 460$

or, $23x^2 = 460 \times 20$

or, $x^2 = 20 \times 20$ or, $x = 20$ m.

47. Ans. (c)

Solution Let the length of the rectangular hall be 'x' m, then the breadth of the rectangular hall $= \frac{2x}{3}$ m

$$\text{Area} = \frac{2x}{3} \times x = \frac{2x^2}{3}$$

or, $\frac{2x^2}{3} = 2400$ or, $x = 60$ m.

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. The length and the breadth of the floor of a room is 20 ft and 10 ft, respectively. Square tiles of 2 ft dimension having three different colours are placed on the floor. The first row of tiles on all sides is of black colour, out of the remaining one-third is of while colour and the remaining are of blue colour. How many blue-colour tiles are there?
(a) 16 (b) 32
(c) 48 (d) 24

2. The length and breadth of a rectangular piece of land are in the ratio of 5 : 3. The owner spent Rs. 3000 for surrounding it from all the sides at the rate of Rs. 7.50 per m. The difference between length and breadth is
(a) 50 m (b) 100 m
(c) 200 m (d) 150 m

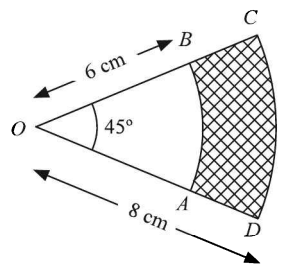
3. A circular park of radius 21 m has a road 7 m wide running around it. Find the cost of metalling the road at rate of Rs. 25.50 per sq m.
(a) Rs. 28749 (b) Rs. 29748
(c) Rs. 27489 (d) Rs. 28794

4. Two small circular parks of diameters 16 m and 12 m are to be replaced by a bigger circular park. What would be the radius of this new park, if the new park occupies same space as the two small parks?
(a) 15 m (b) 10 m
(c) 20 m (d) 20.5 m

5. Length of a rectangular blackboard is 8 cm more than that of its breadth. If its length is increased by 7 cm and its breadth is decreased by 4 cm, its area remains unchanged. Find the length and breadth of the rectangular blackboard.
(a) 28 cm, 20 cm (b) 20 cm 24 cm
(c) 24 cm 16 cm (d) 28 cm 16 cm

6. The driving wheel of a locomotive engine 2.1 m in radius makes 75 revolutions in one m. Find the speed of the train in km/hr
(a) 59.4 km/hr (b) 60 km/hr
(c) 61.5 km/hr (d) None of these

7. Find the area of the shaded portion



(a) 13 m^2 (b) 15 m^2
(c) 11 m^2 (d) 17 m^2

8. The cost of fencing a rectangular field at Rs. 3.50 per m is Rs. 595. If the length of the field be 60 m, find the cost of levelling it at 50 paise per sq m.
(a) Rs. 700 (b) Rs. 960
(c) Rs. 750 (d) Data inadequate

9. The area of a square and a rectangular field is 900 m^2. If the perimeter of the rectangular field is 2 m more than that of the square field, calculate the dimensions of the rectangular field.
(a) 24 m, 10 m (b) 30 m, 25 m
(c) 36 m, 25 m (d) 36 m, 24 m

10. The length of a room is 6.5 m. The cost of painting the walls at Rs. 56 m^2 is Rs. 4928 and the cost of carpeting the room at Rs. 224 m^2 is Rs. 6552. Find the height and width of the room.
(a) 4 m, 5 m (b) 4.5 m, 5.5 m
(c) 4 m, 4.5 m (d) Data inadequate

Test II **[Time: 10 min]**

11. The cross-section of a canal is a trapezium in shape. If the canal is 10 m wide at the top and 6 m wide at the bottom and the area of cross-section is 640 m^2, the length of canal is
(a) 40 m (b) 80 m
(c) 160 m (d) 384 m

12. The expenditure incurred in carpeting a hall 28 m long with a carpet $1\frac{1}{3}$m wide at Rs. 3.50 per m is Rs. 1176. Find the breadth of the hall.
(a) 16 m (b) 24 m
(c) 12 m (d) 36 m

13. The length of a rectangular floor is twice its width. If the length of a diagonal is $9\sqrt{5}$ m, then perimeter of the rectangle is
(a) 27 m (b) 54 m
(c) 81 m (d) 162 m

14. There is a regular polygon of 12 sides. Find the sum of interior angles and the value of each interior angle.
(a) $10\pi, \frac{5}{6}\pi$ (b) 12p, $\frac{8}{9}\pi$
(c) $6\pi, \frac{5}{6}\pi$ (d) Data inadequate

15. Length of a square is increased by 8 cm. Its area becomes 208 sq cm. Find its perimeter.
(a) 108 cm (b) 112 cm
(c) 104 cm (d) 116 cm

16. A path all around the inside of a rectangular park 37 m by 30 m occupies 570 sq m. Find the width of the path.
(a) 10 m (b) 15 m
(c) 20 m (d) 5 m

17. There is a rectangular field of area 60 cm^2. Sum of its diagonal and length is 5 times of its breadth. Find the breadth of the rectangular field
(a) 10 m (b) 5 cm
(c) 20 m (d) 15 m

18. A rectangular grassy plot is 112 m by 78 m. It has a gravel path 2.5 m wide all around it on the inside. Find the cost of constructing it at Rs. 2 per sq m?
(a) Rs. 1749 (b) Rs. 2065
(c) Rs. 1850 (d) Rs. 1959

19. Two square rooms, one a m longer each way than the other, are of equal height, and cost respectively Rs. 33600 and Rs. 35280 to paper the walls at Rs. 70 per sq m. Find the height
(a) 6 m (b) 8 m
(c) 5 m (d) 4 m

20. A table cloth 4 m × 2 m is spread on a table. Calculate the cost of polishing the top of the table at Rs. 2.25 per sq m. If 25 cm of the table cloth is hanging all round the table.
(a) Rs. 13.71 (b) Rs. 15.61
(c) Rs. 11.81 (d) Rs. 14.63

Answers

1. (a)	**2.** (a)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** (a)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (c)
11. (b)	**12.** (a)	**13.** (b)	**14.** (a)	**15.** (c)	**16.** (d)	**17.** (b)	**18.** (c)	**19.** (a)	**20.** (c)

23

MENSURATION II: VOLUME AND SURFACE AREA

INTRODUCTION

Solids

A *solid* is a figure bounded by one or more surfaces. It has three dimensions namely length, breath or width and thickness or height. The plane surfaces that bind it are called its *faces*.

The *volume* of any solid figure is the amount of space enclosed within its bounding faces. It is measured in cubic units, e.g., m^3, cm^3, etc.

The area of the plane surfaces that bind the solid is called its *surface area*.

For any regular solid,

Number of faces + Number of vertices

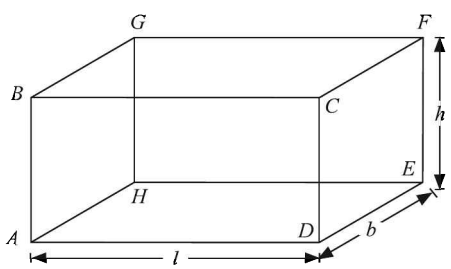
= Number of edges + 2.

We discuss below some important three dimensional figures and the formulae associated with them.

SOME BASIC FORMULAE

Cubic

It is a solid figure which has six rectangular faces. It is also called *rectangular parallelopiped.*



If l, b and h denote the length, breadth and height of the cuboid and d denotes the body diagonal (AF or BE or DG or CH), then

(i) Volume $= l \times b \times h = \sqrt{A_1 \times A_2 \times A_3}$,

where A_1 = area of base or top

A_2 = area of one side face, and

A_3 = area of other side face

(ii) Total Surface Area $= 2(lb + bh + lh)$

$= (l + b + h)^2 - d^2$

(iii) Diagonal of cuboid $= \sqrt{l^2 + b^2 + h^2}$

Note: (i) For painting the surface area of a box or to know how much tin sheet is required for making a box, we use formula (ii).

(ii) To find how much a box contains or how much space a box shall occupy, we use formula (i). To find the length of the longest pole to be placed in a room, we use formula (iii).

(iii) The rise or fall of liquid level in a container

$$= \frac{\text{Total volume of objects submerged or taken out}}{\text{Cross-sectional area of container}}.$$

Illustration 1 Find the volume and the total surface area of a cuboid whose dimensions are 25 m, 10 m and 2 m.

Solution Here, $l = 25$ m, $b = 10$ m and $h = 2$ m.

Volume of the cuboid $= l \times b \times h$

$= 25 \times 10 \times 2$

$= 500\ m^3$

Total surface area of the cuboid

$= 2(lb + bh + lh)$

$= 2(25 \times 10 + 10 \times 2 + 25 \times 2)$

$= 2(250 + 20 + 50)$

$= 640\ m^2$.

Illustration 2 Find the length of the longest bamboo that can be placed in a room 12 m long, 9 m broad and 8 m high.

Solution Length of the bamboo

= length of the diagonal of the room

$= \sqrt{12^2 + 9^2 + 8^2}$

$= \sqrt{289} = 17$ m.

Illustration 3 The area of a side of a box is 120 sq cm. The area of the other side of the box is 27 sq cm. If the area of the upper surface of the box is 60 sq cm, then find the volume of the box.

Solution Volume to the box

$= \sqrt{\text{area of base} \times \text{area of one face} \times \text{area of the other face}}$

$= \sqrt{60 \times 120 \times 72}$

$= \sqrt{518400} = 720 \text{ cm}^3$.

Illustration 4 The sum of length, breadth and height of a cuboid is 12 cm long. Find the total surface area of the cuboid.

Solution Total surface area

$= (\text{Sum of all three sides})^2 - (\text{Diagonal})^2$

$= 12^2 - 8^2 = 144 - 64 = 80$ sq cm.

Cube

It is a special type of cuboid in which each face is a square.

For a cube, length, breadth and height are equal and is called the edge of the cube.

If a be the edge of a cube, then

(i) Volume of the cube $= (\text{edge})^3 = a^3$

(ii) Total surface area of the cube $= 6\,(\text{edge})^2 = 6a^2$

(iii) Diagonal of the cube $= \sqrt{3}a$ (edge) $= \sqrt{3}a$

(iv) Volume of the cube $= \left(\dfrac{\text{diagonal}}{\sqrt{3}}\right)^3 = \left(\dfrac{d}{\sqrt{3}}\right)^2$

$= \left(\sqrt{\dfrac{\text{Surface area}}{6}}\right)^3$

(v) Total surface area of the cube

$= 2\,(\text{diagonal})^2 = 2d^2$

(vi) For two cubes

(a) Ratio of volumes = (ratio of sides)^3

(b) Ratio of surface areas = (Ratio of sides)^2

(c) (Ratio of surface areas)^3 = (Ratio of volumes)^2.

Illustration 5 Find the volume, surface area and the diagonal of a cube, each of whose sides measures 4 cm.

Solution Volume of the cube $= a^3 = (4)^3 = 64 \text{ cm}^3$.

Surface area of the cube $= 6a^2 = 6\,(4)^2 = 96 \text{ cm}^2$.

Diagonal of the cube $= \sqrt{3}a = 4\sqrt{3}$ cm.

Illustration 6 The surface area of a cube is 216 sq cm. Find its volume.

Solution Volume of the cube

$= \left(\sqrt{\dfrac{\text{Surface area}}{6}}\right)^3$

$= \left(\sqrt{\dfrac{216}{6}}\right)^3 = (6)^3 = 216 \text{ cm}^3$.

Illustration 7 The diagonal of a cube is $8\sqrt{3}$ cm. Find its total surface area and volume.

Solution We have,

Diagonal of cube $= \sqrt{3}$ (edge)

$\therefore$ Edge of cube $= \dfrac{\text{Diagonal of cube}}{\sqrt{3}}$

$= \dfrac{8\sqrt{3}}{\sqrt{3}} = 8$ cm.

Total surface area $= 6\,(\text{edge})^2 = 6(8)^2$

$= 384$ sq cm.

Volume of cube $= (\text{edge})^3 = (8)^3 = 512 \text{ cm}^3$.

Illustration 8 If the volumes of two cubical blocks are in the ratio of 8 : 1, what will be the ratio of their edges?

Solution We have,

Ratio of volumes = (Ratio of sides)^3

Since, ratio of volumes = 8 : 1, i.e. $2^3 : 1^3$

$\therefore$ ratio of sides = 2 : 1.

Illustration 9 Volumes of the two cubes are in the ratio of 1 : 9. Find the ratio of their surface areas.

Solution (Ratio of the surface areas)^3

= (Ratio of volumes)^2

$\therefore$ Ratio of surface areas $= \sqrt[3]{1:81} = 1 : 3\,(3)^{1/3}$.

Illustration 10 Sides of two cubes are in the ratio of 2 : 3. Find the ratio of their surface areas.

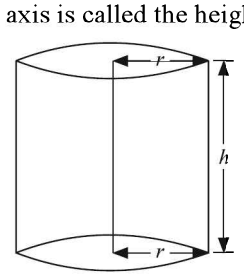
Solution Ratio of surface areas

= (Ratio of sides)^2

$= (2 : 3)^2 = 4 : 9$.

Right Circular Cylinder

A *right circular cylinder* is a solid with circular ends of equal radius and the line joining their centres perpendicular to them. This is called axis of the cylinder. The length of the axis is called the height of the cylinder.



Note: Take a rectangular sheet of paper and role it lengthwise or breadthwise in a round way, you will get a cylinder, i.e. a cylinder is generated by rotating a rectangle by fixing one of its sides.

If r is the radius of base and h is the height of the cylinder, then

(i) Volume of cylinder

= Area of the base × height

$= \pi r^2 \times h = \pi r^2 h$ cubic units

(ii) Area of the curved surface

= Circumference of the base × height

$= 2\pi r \times h = 2\pi rh$ sq units

(iii) Area of the total surface

= Area of the curved surface + Area of the two circular ends

$= 2\pi rh + 2\pi r^2$

$= 2\pi r\ (h + r)$ sq units.

(iv) For two cylinders,

When radii are equal

(a) Ratio of volumes = Ratio of heights

(b) Ratio of volumes = Ratio of curved surface areas

(c) Ratio of curved surface areas = Ratio of heights

When heights are equal

(a) Ratio of volumes = (Ratio of radii)^2

(b) Ratio of volumes = (Ratio of curved surface areas)^2

(c) Radii of curved surface areas = Ratio of radii

When volumes are equal

(a) Ratio of radii = $\sqrt{\text{Inverse ratio of heights}}$

(b) Ratio of curved surface areas = Inverse ratio of radii

(c) Ratio of curved surface areas = $\sqrt{\text{Ratio of heights}}$

When curved surface areas are equal

(a) Ratio of radii = Inverse ratio of heights

(b) Ratio of volumes = Inverse ratio of heights

(c) Ratio of volumes = Ratio of radii

(v) For a cylinder

(a) Ratio of radii = (Ratio of curved surfaces) × (Inverse ratio of heights)

(b) Ratio of heights = (Ratio of curved surfaces) × (Inverse ratio of radii)

(c) Ratio of curved surfaces = (Ratio of radii) × (Ratio of heights).

Illustration 11 The diameter of the base of a right circular cylinder is 28 cm and its height is 10 cm. Find the volume and area of the curved surface of the cylinder.

Solution Radius of the base $= \frac{28}{2} = 14$ cm.

Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 14 \times 14 \times 10$

$= 6160$ cm^3.

Area of the curved surface $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 14 \times 10$

$= 880$ sq cm.

Illustration 12 A cylinder of height 21 cm has base of radius 4 cm. Find the total surface area of the cylinder.

Solution Total surface area $= 2\pi r\ (h + r)$

$= 2 \times \frac{22}{7} \times 4 \times (21 + 4)$

$= \frac{4400}{7} = 628\ \frac{4}{7}$ sq cm.

Illustration 13 A rectangular piece of paper is 71 cm long and 10 cm wide. A cylinder is formed by rolling the paper along its breadth. Find the volume of the cylinder.

$\left[\text{Take } \pi = \frac{355}{113}\right]$

Solution Circumference of the paper = Breadth of the paper

$\Rightarrow\ 2\pi r = 10$

$\Rightarrow\ r = \frac{10}{2\pi} = \frac{10 \times 113}{2 \times 355} = \frac{113}{71}$ cm.

As the length of the paper becomes the height of the cylinder,

∴ Volume of the cylinder

$= \pi r^2 l$

$= \frac{355}{113} \times \frac{113}{71} \times \frac{113}{71} \times 71 = 565$ cm^3.

Illustration 14 Two circular cylinders of equal volume have their heights in the ratio of 9 : 16. Find the ratio of their radii.

Solution Ratio of radii $= \sqrt{\text{inverse ratio of heights}}$

$= \sqrt{16:9} = 4 : 3.$

Illustration 15 Two circular cylinders of equal volume have their heights in the ratio of 16 : 25. Find the ratio of their curved surface areas.

Solution Ratio of curved surface areas

$= \sqrt{\text{Ratio of heights}} = \sqrt{16:25} = 4 : 5.$

Illustration 16 Two circular cylinders of equal volume have their radii in the ratio of 4 : 9. Find the ratio of their curved surface areas.

Solution Ratio of curved surface areas

= inverse ratio of radii = 9 : 4.

Illustration 17 Two circular cylinders of equal heights have their radii in the ratio of 2 : 5. Find the ratio of their volumes.

Solution Ratio of volumes = (Ratio of radii)^2 = 4 : 25.

Illustration 18 Two circular cylinders of equal heights have their curved surface areas in the ratio of 3 : 5. Find the ratio of their volumes.

Solution Ratio of volumes
= (Ratio of curved surface areas)^2
= 9 : 25.

Illustration 19 Two circular cylinders of equal curved surface areas have their heights in the ratio of 4 : 7. Find the ratio of their volumes.

Solution Ratio of volumes = Inverse ratio of heights

$$= \frac{1}{4} : \frac{1}{7} = 7 : 4.$$

Illustration 20 Two circular cylinders of equal curved surface areas have their heights in the ratio of 4 : 5. Find the ratio of their volumes.

Solution Ratio of volumes = Inverse ratio of heights

$$= \frac{1}{4} : \frac{1}{5} = 5 : 4.$$

(vi) If the ratio of heights and the ratio of radii of two right circular cylinders are given, then
Ratio of curved surface areas = (ratio of radii) (ratio of heights).

Illustration 21 If the heights and the radii of two right circular cylinders are in the ratio 2 : 3 and 4 : 5, respectively. Find the ratio of their curved surface areas.

Solution Ratio of curved surface areas = (ratio of radii) (ratio of heights)
= (4 : 5) (2 : 3) = 8 : 15.

(vii) If the ratio of heights and the ratio of curved surface areas of two right circular cylinders are given, then
Ratio of radii = (ratio of curves surface areas) (inverse ratio of heights).

Illustration 22 The heights and curved surface areas of two right circular cylinders are in the ratio 3 : 4 and 5 : 8, respectively. Find the ratio of their radii.

Solution Ratio of radii = (ratio of curved surface areas) (inverse ratio of heights)

$$= (5 : 8)\left(\frac{1}{3} : \frac{1}{4}\right) = (5 : 8)\ (4 : 3) = 5 : 6.$$

(viii) If the ratio of radii and the ratio of curved surface areas of two right circular cylinders are given, then
Ratio of heights = (ratio of curved surface areas) (inverse ratio of radii)

Illustration 23 The radii of two right circular cylinders are in the ratio of 3 : 4 and their curved surface areas are in the ratio of 5 : 6. Find the ratio of their heights.

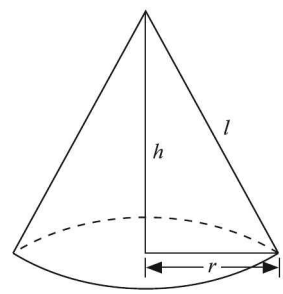
Solution Ratio of heights = (ratio of curved surface areas) (inverse ratio of radii)

$$= (5 : 6)\left(\frac{1}{3} : \frac{1}{4}\right)$$

$$= (5 : 6)\ (4 : 3) = 10 : 9.$$

Right Circular Cone

A *right circular cone* is a solid obtained by rotating a right-angled triangle around its height.



If r = radius of base; h = height,

l = slant height = $\sqrt{h^2 + r^2}$, then

(i) Volume of cone

$$= \frac{1}{3} \times \text{area of the base} \times \text{height}$$

$$= \frac{1}{3} \times \pi r^2 h \text{ cubic units}$$

(ii) Area of curved surface = πrl

$$= \pi r\sqrt{h^2 + r^2} \text{ sq. units}$$

(iii) Total surface area of cone
= Area of the base + area of the curved surface
= $\pi r^2 + \pi rl = \pi r\ (r + l)$ sq units.

(iv) For two cones

(a) When volumes are equal
Ratio of radii = $\sqrt{\text{inverse ratio of heights}}$

(b) When radii are equal
Ratio of volumes = Ratio of heights

(c) When heights are equal
Ratio of volumes = (ratio of radii)^2

(d) When curved surface areas are equal
Ratio of radii = inverse ratio of slant heights.

Illustration 24 Find the slant heights of a cone whose volume is 1232 cm^3 and radius of the base is 7 cm.

Solution Volume of the cone = $\frac{1}{3}\pi r^2 h = 1232$

$$\Rightarrow \quad h = \frac{1232 \times 3}{\pi r^2} = \frac{1232 \times 3 \times 7}{22 \times 7 \times 7} = 24 \text{ cm}.$$

Slant height l is given by the relation

$$l = \sqrt{h^2 + r^2}$$
$$= \sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49}$$
$$= \sqrt{625} = 25 \text{ cm.}$$

$\therefore$ Slant height of the cone is 25 cm.

Illustration 25 A tent is of diameter 12 m at the base and its height is 8 m.

(i) Find the slant height; and

(ii) The canvas required in sq metres.

How many persons can the tent accommodate, at the most, if each person requires 18 m^3 of air?

Solution Diameter of the base of a conical tent = 12 m.

$\therefore$ Radius $(r) = \frac{12}{2} = 6$ m and its height $(h) = 8$ m

(i) Slant height $(l) = \sqrt{r^2 + h^2} = \sqrt{6^2 + 8^2}$

$$= \sqrt{64 + 36} = \sqrt{100} = 10 \text{ m.}$$

(ii) Area of canvas required

$$= \pi \times r \times l$$
$$= \frac{22}{7} \times 6 \times 10 = 188.57 \text{ m}$$

(iii) Volume of conical portion

$$= \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 8 = 301.71 \text{ m}^3.$$

Space required for each person = 18 m^3.

$\therefore$ Number of persons that can be accommodated

$$= \frac{301.71}{18} = 16.$$

Illustration 26 The height of a cone is 21 cm and radius of its base is 28 cm. Find its total surface area.

Solution We have, $r = 28$ cm and $h = 21$ cm.

Slant height $(l) = \sqrt{r^2 + h^2} = \sqrt{(28)^2 + (21)^2}$

$$= \sqrt{1225} = 35 \text{ cm.}$$

Total surface area $= \pi r (l + r)$

$$= \frac{22}{7} \times 28 \times (35 + 28)$$
$$= \frac{22}{7} \times 28 \times 63$$
$$= 5544 \text{ sq cm.}$$

Illustration 27 Two right circular cones of equal curved surface areas have their slant heights in the ratio of 3 : 5. Find the ratio of their radii.

Solution Ratio of radii = inverse ratio of slant heights

$$= \frac{1}{3} : \frac{1}{5} = 5 : 3.$$

Illustration 28 Two right circular cones of equal volumes have their heights in the ratio of 4 : 9. Find the ratio of their radii.

Solution Ratio of radii $= \sqrt{\text{inverse ratio of heights}}$

$$= \sqrt{\frac{1}{4} : \frac{1}{9}} = \sqrt{9:4} = 3 : 2.$$

Illustration 29 Two right circular cones of equal heights have their radii in the ratio of 1 : 3. Find the ratio of their volumes.

Solution Ratio of volumes = (Ratio of radii)^2

$$= (1 : 3)^2 = 1 : 9.$$

(v) If the ratio of volumes and the ratio of heights of two right circular cones (or cylinders) are given, then

Ratio of radii

$$= \sqrt{(\text{ratio of volumes})\,(\text{inverse ratio of heights})}$$
$$= \sqrt{(3:2)\,(8:3)} : \sqrt{4:1} = 2 : 1.$$

Illustration 30 The volumes of two cones are in the ratio 3 : 2 and their heights in the ratio 3 : 8. Find the ratio of their radii.

Solution Ratio of radii

$$= \sqrt{(\text{ratio of volumes})\,(\text{inverse ratio of heights})}$$
$$= \sqrt{(3:2)\,(8:3)} = \sqrt{4:1} = 2 : 1.$$

(vi) If the ratio of heights and the ratio of diameters (or radii) of two right circular cones (or cylinders) are given, then

Ratio of volumes = (ratio of radii)^2 × (ratio of heights).

Illustration 31 The heights of two cones are in the ratio of 5 : 3 and their radii in the ratio 2 : 3. Find the ratio of their volumes.

Solution Ratio of volumes

$$= (\text{ratio of radii})^2 \times (\text{ratio of heights})$$
$$= (2 : 3)^2 \times (5 : 3)$$
$$= \frac{4}{9} \times \frac{5}{3} = 20 : 27.$$

(vii) If the ratio of radii (or diameter) and the ratio of volumes of two right circular cones are given, then

Ratio of heights

= (inverse ratio of radii)^2 (ratio of volumes).

Illustration 32 The volumes of two cones are in the ratio of 1 : 4 and their diameters are in the ratio of 4 : 5. Find the ratio of their heights.

Solution Ratio of heights

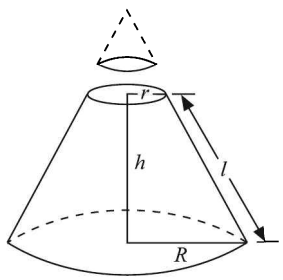
= (inverse ratio of diameters)^2 × (ratio of volumes)

$$= \left(\frac{1}{4} : \frac{1}{5}\right)^2 (1:4) = (5 : 4)^2 (1 : 4)$$

$$= \frac{25}{16} \times \frac{1}{4} = 25 : 64.$$

Frustum of a right circular cone

A cone with some of its top portion cut off is called the *frustum* of the original cone.



If R = Radius of the base of frustum
r = Radius of the top of the frustum
h = Height of the frustum
l = Slant height of the frustum, then

(a) Slant height = $\sqrt{h^2 + (R - r)^2}$ units

(b) Area of the curved surface = $\pi (R + r) l$ sq units

(c) Total surface area of the frustum
$= \pi [(R^2 + r^2) + l (R + r)]$ sq units

(d) Volume of the frustum = $\frac{\pi h}{3} (R^2 + r^2 + Rr)$ cu units.

Illustration 33 A reservoir is in the shape of a frustum of a right circular cone. It is 8 m across at the top and 4 m across the bottom. It is 6 m deep. Find the area of its curved surface, total surface area and also its volume.

Solution Here, $R = 4$, $r = 2$ and $h = 6$.

$\therefore$ Slant height $(l) = \sqrt{h^2 + (R - r)^2}$

$$= \sqrt{(6)^2 + (4 - 2)^2} = \sqrt{40}.$$

$\therefore$ Area of the curved surface

$$= \pi (R + r) l$$

$$= \frac{22}{7} (4 + 2) \sqrt{40}$$

$$= 18.8 \times 6.3 = 118.4 \text{ m}$$

Total surface area $= \pi [(R^2 + r^2) + l (R + r)]$

$$= \frac{22}{7} [(4^2 + 2^2) + \sqrt{40} (4 + 2)]$$

$$= \frac{22}{7} (20 + 6\sqrt{40}) = 181.6 \text{ sq m}$$

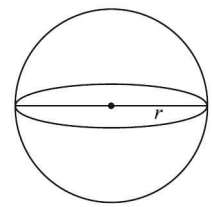
Volume of the frustum $= \frac{\pi h}{3} (R^2 + r^2 + Rr)$

$$= \frac{22}{7} \times \frac{6}{3} (4^2 + 2^2 + 4 \times 2)$$

$$= \frac{44}{7} (20 + 4 + 8) = 176 \text{ m}^3.$$

Sphere

A *sphere* is the solid figure formed by revolving a semi-circle on its diameter.

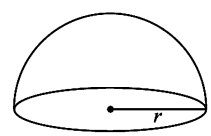


The mid-point of the diameter is called centre of the sphere and the radius of the semi-circle is called the radius of the sphere.

If r = radius of the spheres, then

(i) Volume of sphere = $\frac{4}{3} \pi r^3$ cubic units

(ii) Surface area = $4\pi r^2$ sq units.



(iii) Volume of hemisphere = $\frac{2}{3} \pi r^2$ cubic units

(iv) Area of curved surface = $2\pi r^2$ sq units of hemisphere

(v) Total surface area of hemisphere = $3\pi r^2$ sq units.

Illustration 34 Diameter of a sphere is 28 cm. Find its surface area and volume.

Solution Radius of the sphere $(r) = \frac{28}{2} = 14$ cm.

Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times 14 \times 14$

$= 2464$ sq cm.

Volume of sphere $= \frac{4}{3} \pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14$$

$$= 11498.6 \text{ cm}^3.$$

Illustration 35 Find the volume, curved surface area and total surface area of a hemisphere of radius 21 cm.

Solution Volume of the hemisphere

$$= \frac{2}{3} \pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$$

$$= 19494 \text{ cm}^3.$$

Curved surface area $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 21 \times 21$

$= 2772$ sq cm.

Total surface area $= 3\pi r^2$

$= 3 \times \frac{22}{7} \times 21 \times 21$

$= 4158$ sq cm.

(vi) For two spheres

(a) (Ratio of radii)^2 = Ratio of surface areas

(b) (Ratio of radii)^3 = Ratio of volumes

(c) (Ratio of surface areas)^3 = (Ratio of volumes)^2.

Illustration 36 The radii of two spheres are in the ratio of 2 : 3. What is the ratio of their surface areas?

Solution Ratio of surface areas = (ratio of radii)^2 = $(2 : 3)^2 = 4 : 9$.

Illustration 37 The surface areas of two spheres are in the ratio 1 : 2. Find the ratio of their volumes.

Solution We have,

(Ratio of surface areas)^3 = (Ratio of volumes)^2

$\Rightarrow (1 : 2)^3$ = (Ratio of volumes)^2

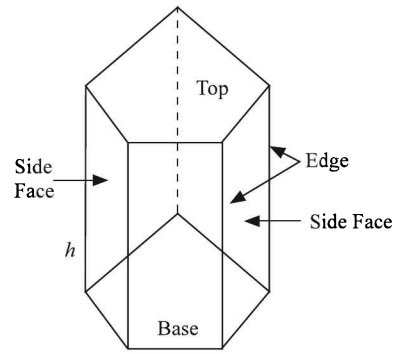
$\therefore$ Ratio of volumes $= \sqrt{1:8} = 1 : 2\sqrt{2}$.

Illustration 38 The radii of two spheres are in the ratio of 2 : 45. Find the ratio of their volumes.

Solution Ratio of volumes = (Ratio of radii)^3 = $(2 : 5)^3 = 8 : 125$.

Prism

A solid having top and bottom faces identical and side faces rectangular is a prism.



In a prism with a base of n sides,

Number of vertices = $2n$

and Number of faces = $n + 2$.

Volume of the prism = area of base × height

Lateral surface area = perimeter of base × height

Total surface area = 2 × Base area + Lateral Surface area.

Illustration 39 Find the volume and the total surface area of a triangular prism whose height is 30 m and the sides of whose base are 21 m, 20 m and 13 m, respectively.

Solution Perimeter of base = 21 + 20 + 13 = 54 m. height = 30 m

Area of base $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{27(27-21)(27-20)(27-13)}$

$= \sqrt{27 \times 6 \times 7 \times 14} = 126$ sq m.

$\therefore$ Volume of the prism = area of base × height

$= 126 \times 54 = 6804$ m^3.

Also, surface area of the prism

= 2 × Base area + lateral surface area

= 2 Base area + perimeter of base × height

$= 2 \times 126 + 54 \times 30 = 1872$ sq m.

SOLIDS INSCRIBED/CIRCUMSCRIBING OTHER SOLIDS

1. If a largest possible sphere is circumscribed by a cube of edge 'a' cm, then the radius of the sphere $= \frac{a}{2}$.

Illustration 40 Find the volume of largest possible sphere circumscribed by a cube of edge 8 cm.

Solution Radius of the sphere $= \frac{a}{2} = \frac{8}{2} = 4$ cm.

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 4 \times 4 \times 4$

$= 26.81$ cm^3.

2. If a largest possible cube is inscribed in a sphere of radius 'a' cm, then the edge of the cube $= \frac{2a}{\sqrt{3}}$.

Illustration 41 Find the surface area of largest possible cube inscribed in a sphere of radius 4 cm.

Solution Edge of the cube $\frac{2a}{\sqrt{3}} = \frac{2 \times 4}{\sqrt{3}} = \frac{8}{\sqrt{3}}$.

$\therefore$ Surface area of the cube = 6 (edge)^2

$= 6 \times \frac{64}{3}$

$= 128$ sq cm.

3. If a largest possible sphere is inscribed in a cylinder of radius 'a' cm and height 'h' cm, then

$$\text{radius of the sphere} = \begin{cases} a \text{ for } h > a \\ \dfrac{h}{2} \text{ for } a > h \end{cases}$$

Illustration 42 Find the surface area of largest possible sphere inscribed in a cylinder of radius 14 cm and height 17 cm.

Solution Radius of the sphere = 14 cm ($\because$ $h > a$)

$\therefore$ Surface area of sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times 14 \times 14$$

$$= 2464 \text{ sq cm.}$$

4. If a largest possible sphere is inscribed in a cone of radius 'a' cm and slant height equal to the diameter of the base, then radius of the sphere $= \dfrac{a}{\sqrt{3}}$.

Illustration 43 Find the surface area of largest possible sphere inscribed in a cone of radius 21 cm and slant height equal to the diameter of the base.

Solution Radius of the sphere $= \dfrac{a}{\sqrt{3}} = \dfrac{21}{\sqrt{3}}$ cm.

$\therefore$ Surface area of the sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times \frac{21}{\sqrt{3}} \times \frac{21}{\sqrt{3}}$$

$$= 1848 \text{ sq cm.}$$

5. If a largest possible cone is inscribed in a cylinder of radius 'a' cm and height 'h' cm, then radius of the cone = a and height = h.

Illustration 44 Find the volume of largest possible cone inscribed in a cylinder of radius 6 cm and height 14 cm.

Solution Radius of the cone (r) = 6 cm.

and height of the cone (h) = 14 cm.

$\therefore$ Volume of the cone $= \dfrac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 14$$

$$= 528 \text{ sq cm.}$$

6. If a largest possible cube is inscribed in a hemisphere of radius 'a' cm, then the edge of the cube $= a\sqrt{\dfrac{2}{3}}$.

Illustration 45 Find the length of the diagonal of largest possible cube inscribed in a hemisphere of radius $4\sqrt{2}$ cm.

Solution Edge of the cube $= a\sqrt{\dfrac{2}{3}} = 4\sqrt{2} \times \sqrt{\dfrac{2}{3}}$

$$= \frac{8}{\sqrt{3}} \text{ cm.}$$

$\therefore$ Diagonal of the cube $= \sqrt{3}$ (edge)

$$= \sqrt{3} \times \frac{8}{\sqrt{3}} = 8 \text{ cm.}$$

SOME USEFUL SHORT-CUT METHODS

1. If all three measuring dimensions of a sphere, cuboid, cube, cylinder or cone are increased or decreased by x%, y% and z% respectively, then the volume of the figure will increase or decrease by

$$\left(x + y + z + \frac{xy + yz + zx}{100} + \frac{xyz}{100^2}\right)\%.$$

For cuboid, the three measuring dimensions are length, breadth and height.

For cube, all three measuring dimensions are equal, i.e. $x = y = z$.

For sphere also, (or diameter) all three measuring dimensions are equal and is given by radius, i.e. $x = y = z = r$.

For cylinder or a cone two measuring dimensions are equal to radius and third measuring dimension is height

i.e. $x = y = r$ and $z = h$.

Illustration 46 The length, breadth and height of a cuboid are increased by 5%, 10% and 20%, respectively. Find the percentage increase in its volume.

Solution Here, $x = 5$, $y = 10$ and $z = 20$.

$\therefore$ Percentage increase in volume

$$= \left[x + y + z + \frac{xy + xz + yz}{100} + \frac{xyz}{(100)^2}\right]\%$$

$$= \left[5 + 10 + 20 + \frac{(5\times10) + (5\times20) + (10\times20)}{100} + \frac{5\times10\times20}{(100)^2}\right]\%$$

$$= \left(35 + \frac{350}{100} + \frac{1000}{(100)^2}\right)$$

$$= (35 + 3.5 + 0.1)\%$$

$$= 38.6\%.$$

Illustration 47 The sides of a cube are decreased by 10% each. Find the percentage change in its volume.

Solution Here, $x = y = z$.

∴ Percentage change in volume

$$= \left[3x + \frac{3x^2}{100} + \frac{x^3}{(100)^2}\right]\%$$

$$= \left[3(-10) + \frac{3(-10)^2}{100} + \frac{(-10)^3}{(100)^2}\right]\%$$

$$= (-30 + 3 - 0.1)\% = -27.1\%.$$

–ve sign indicates decrease in volume, that is, there is a decrease in volume by 27.1%.

Illustration 48 The diameter of a sphere is increased by 20%. What is the percentage increase in its volume?

Solution Percentage increase in volume

$$= \left[3x + \frac{3x^2}{100} + \frac{x^3}{(100)^2}\right]\% \qquad [\text{Here, } x = y = z]$$

$$= \left[3 \times 20 + \frac{3(20)^2}{100} + \frac{(20)^3}{(100)^2}\right]\%$$

$$= (60 + 12 + 0.8)\% = 72.8\%.$$

Illustration 49 The radius of a right circular cylinder is decreased by 5% but its height is increased by 10%. What is the percentage change in its volume?

Solution Here, $x = y = -15$ and $z = 10$.

∴ Percentage change in volume

$$= \left[-5 - 5 + 10 + \frac{(-5)(-5) + (-5)(10) + (-5)(10)}{100} + \frac{(-5)(-5)(10)}{(100)^2}\right]\%$$

$$= (0 - 0.75 + 0.025)\% = -0.725\%.$$

Therefore, volume decrease by 0.725%.

Illustration 50 Each of the radius and the height of a cone is increased by 25%. Find the percentage increase in volume.

Solution Here, $x = y = 25$ and $z = 25$.

∴ Percentage increase in volume

$$= \left[25 + 25 + 25 + \frac{25 \times 25 + 25 \times 25 \times 25 \times 25}{100} + \frac{25 \times 25 \times 25}{(100)^2}\right]\%$$

$$= (75 + 18.75 + 1.56)\% = 95.3\%.$$

2. If the two measuring dimensions which are included in the surface area of a sphere, cuboid, cube, cylinder or cone are increased or decreased by x% and y%, then the surface area of the figure will increase or decrease by

$$\left(x + y + \frac{xy}{100}\right)\%.$$

Note that in case of percentage increase, values of x, y and z are positive and in case of percentage decrease, values of x, y and z are negative.

Illustration 51 Each edge of a cube is increased by 20%. What is the percentage increase in its surface area?

Solution Here, $x = y = 20$.

∴ Percentage increase in surface area

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(20 + 20 + \frac{20 \times 20}{100}\right)\%$$

$$= (40 + 4)\% = 44\%.$$

Illustration 52 The radius of a hemisphere is decreased by 10%. Find the percentage change in its surface area.

Solution Here, $x = y = -10$.

∴ Percentage change in surface area

$$= \left(x + y + \frac{xy}{100}\right) \quad \%$$

$$= \left(-10 - 10 + \frac{(-10)(-10)}{100}\right)\%$$

$$= (-20 + 1)\% = -19\%.$$

Therefore, surface area of hemisphere decreases by 19%.

Illustration 53 The radius of a right circular cone is increased by 25% and slant height is decreased by 30%. Find the percentage change in curved surface area of the cone.

Solution Here, $x = 25$ and $y = -30$.

∴ Percentage change in curved surface area

$$= \left(x + y + \frac{xy}{100}\right) \quad \%$$

$$= \left[25 - 30 + \frac{(25)(-30)}{100}\right]\%$$

$$= (-5 - 7.5)\% = -12.5\%.$$

Therefore, curved surface area decreases by 12.5%.

Illustration 54 The radius and height of a cylinder are increased by 10% and 20%, respectively. Find the percentage increase in its surface area.

Solution Here, $x = 10$ and $y = 20$.

∴ Percentage increase in surface area

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(10 + 20 + \frac{10 \times 20}{100}\right)\% = 32\%.$$

3. If a sphere of radius R is melted to form smaller spheres each of radius r, then

The number of smaller spheres

$$= \frac{\text{Volume of the bigger sphere}}{\text{Volume of the smaller sphere}}$$

$$= \left(\frac{R}{r}\right)^3.$$

Illustration 55 Find the number of lead balls of radius 1 cm each that can be made from a sphere of radius 4 cm.

Solution Number of lead balls $= \left(\frac{R}{r}\right)^3 = \left(\frac{4}{1}\right)^3 = 64.$

4. If by melting n spheres, each of radius r, a big sphere is made, then

   Radius of the big sphere $= r.\sqrt[3]{n}$.

Illustration 56 If by melting 8 spheres, each of radius 5 cm, a big sphere is made, what will be the radius of the big sphere?

Solution Radius of the big sphere $= r.\sqrt[3]{n}$

$= 5.\sqrt[3]{8} = 5.2 = 19$ cm.

5. If a cylinder is melted to form smaller spheres each of radius r, then

   The number of small spheres $= \frac{\text{Volume of cylinder}}{\text{Volume of 1 sphere}}$.

Illustration 57 How many bullets can be made out of a loaded cylinder 24 cm high and 5 cm diameter, each bullet being 2 cm in diameter?

Solution Number of bullets $= \frac{\text{Volume of cylinder}}{\text{Volume of 1 sphere}}$

$$= \frac{\pi \times \frac{5}{2} \times \frac{5}{2} \times 24}{\frac{4}{3} \times \pi \times 1 \times 1 \times 1} = 450.$$

6. If a sphere of radius r is melted and a cone of height h is made, then

   Radius of the cone $= 2 \times \sqrt{\frac{r^3}{h}}$.

   or,

   If a cone of height h is melted and a sphere of radius r is made, then

   Radius of the cone $= 2 \times \sqrt{\frac{r^3}{h}}$.

Illustration 58 A solid cone of copper of height 3 cm is melted and a solid sphere of radius 3 cm is made. What is the diameter of the base of the cone?

Solution Radius of the base of the cone

$$= 2 \times \sqrt{\frac{r^3}{h}} = 2 \times \sqrt{\frac{3^3}{3}} = 6.$$

$\therefore$ Diameter of the base of the cone $= 2 \times 6$
$= 12$ cm.

Illustration 59 If a solid cone of copper of height 2 cm is melted and a solid sphere of radius 2 cm is made, what is the diameter of the base of the cone?

Solution Radius of the base of the cone

$$= 2 \times \sqrt{\frac{r^3}{h}} = 2 \times \sqrt{\frac{(2)^3}{2}}$$

$= 2 \times 2 = 4$ cm.

$\therefore$ Diameter of the base of the cone $= 2 \times 4 = 8$ cm.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. A tank 16 m long and 23 m wide contains water. How many cubic metres of water must be rushed into it to make the surface rise by $16\frac{2}{3}$ cm?
(a) 48 m^3 (b) 40 m^3
(c) 32 m^3 (d) 42 m^3

2. The outer dimensions of a closed box are 12 cm by 10 cm by 8 cm. If the box is made of wood 1 cm thick, find the capacity of the box.
(a) 360 cm^3 (b) 480 cm^3
(c) 240 cm^3 (d) 560 cm^3

3. A cistren of dimensions 2.4 m × 2.0 m × 1.5 m takes 2 hr 30 min to get filled with water. The rate at which water flows into the cistern is
(a) 0.48000 cu.m/hr (b) 800 cu.m/min
(c) 800 cu.m/sec (d) None of these

4. The area of three adjacent faces of a rectangular box are p, q and r square cm. The volume of the box is given by
(a) $(p + q + r)$ cm^3 (b) $\sqrt{pqr}$ cm^3
(c) $(pqr)^{1/3} cm^3$ (d) pqr cm^3

5. A reservoir 30 m long, 15 m broad is filled with water. How many gallons of water must be taken out to lower the level of water by 4 metres?
(a) 342000 (b) 364200
(c) 324000 (d) 386400

6. How many bricks each measuring 250 cm by 12.5 cm by 7.5 cm will be required to build a wall 5 m long, 3 m high and 20 cm thick?
(a) 1480 (b) 1280
(c) 1680 (d) 1480

7. Find the cost of the log of wood measuring $15\frac{1}{2}$ m by $2\frac{3}{4}$ m by $1\frac{1}{3}$ m at Rs. 45 per cu m.
(a) Rs. 4257.50 (b) Rs. 4005.00
(c) Rs. 4207.50 (d) Rs. 4357.50

8. How many bricks are required to build a wall 15 m long 3 m high and 50 cm thick, if each brick measures 25 cm by 12 cm by 6 cm.
(a) 16500 (b) 14500
(c) 12500 (d) 10500

9. Find the diagonal of a cuboid whose dimensions are 12 m by 10 m by 8 m.
(a) 18 m (b) 17.5 m
(c) 17 m (d) 16.5 m

10. The outer dimensions of a closed wooden box of 1 cm thick are 12 cm by 10 cm by 8 cm. Find the cost of the wood required to make the box if 1 cm^3 of wood costs Rs. 3.00
(a) Rs. 1440 (b) Rs. 1640
(c) Rs. 1840 (d) Rs. 2040

11. Three equal cubes are placed adjacently in a row. Find the ratio of the total surface area of the new cuboid to that of the sum of the surface areas of the three cubes
(a) 3 : 5 (b) 4 : 5
(c) 6 : 7 (d) 7 : 9

12. The diagonal of a cubical box is $\sqrt{300}$ cm. Find the surface area
(a) $600\sqrt{3}$ cm^2 (b) 600 cm^2
(c) 1200 cm^2 (d) $900\sqrt{3}\,cm^2$

13. An iron cube of side 10 cm is hammered into a rectangular sheet of thickness 0.5 cm. If the sides of the sheet be in the ratio 1 : 5, the sides (in cm) are
(a) 110, 50 (b) 20, 100
(c) 40, 200 (d) None of these

14. The length of a room is 12 metres, width 8 metres, height 6 metres. How many boxes will it hold if each is allowed 1.5 cubic metre of space?
(a) 864 (b) 506
(c) 384 (d) 436

15. A room 3.3 metres high is half as long again as it is wide and its volume is $123\frac{3}{4}$ cu m. Find its length and breadth.
(a) 7.5 m, 6 m (b) 8 m, 5 m
(c) 7.5 m, 5 m (d) 8.5 m, 5 m

16. A tank 3 m long, 2 m wide and 1.5 m deep is dug in a field 22 m long and 14 m wide. If the earth dug out is evenly spread out over the field, the level of the field will rise by nearly
(a) 0.299 cm (b) 0.29 mm
(c) 2.98 cm (d) 4.15 cm

17. A school room is to be built to accommodate 70 children, so as to allow 2.2 sq., metres of floor and 11 cu m of space for each child. If the room be 14 metres long, what must be its breadth and height?
(a) 12 metres, 5,5 metres
(b) 11 metres, 5 metres
(c) 13 metres, 6 metres
(d) 11 metres, 4 metres

18. If 210 cu m of sand be thrown into a tank 12 m long and 5 m wide, find how much the water will rise?
(a) 3.5 m (b) 4 m
(c) 7 m (d) Data inadequate

19. If the length, breadth and height of a rectangular parallelopiped are in the ratio 6 : 5 : 4 and if total surface area is 33,300 m^2, then the length, breadth and height of parallelopiped (in cm) respectively are
(a) 90, 85, 600 (b) 90, 75, 70
(c) 85, 75, 60 (d) 90, 75, 60

20. A cu m of metal weighing 90 kg is rolled into a square bar 9 metre long. An exact cube is cut off from the bar. How much does it weigh?
(a) $5\frac{2}{3}$ kg (b) $6\frac{1}{3}$ kg
(c) $3\frac{1}{3}$ kg (d) $4\frac{2}{3}$ kg

21. How many cubes each of surface 24 sq cm can be made out of a cube of edge measure one metre?
(a) 165000 (b) 125000
(c) 180000 (d) 155000

22. Three solid cubes whose edges are 6, 8 and 10 cm respectively, are melted and formed into a single cube. If there be no loss of metal in the process, find the edge of the new cube.
(a) 16 cm (b) 10 cm
(c) 14 cm (d) 12 cm

23. A cube of edge 6 cm is melted and smaller cubes of edge 2 cm each are formed; how many such cubes are possible?
(a) 39 (b) 24
(c) 27 (d) 21

24. How many small cubical blocks of side 5 cm can be cut from a cubical block whose each edge measures 20 cm?
(a) 56 (b) 48
(c) 64 (d) 52

25. Surface area of a cube is 600 sq. cm. Find the length of its diagonal.
(a) $15\sqrt{3}$ (b) $12\sqrt{3}$
(c) $10\sqrt{3}$ cm (d) None of these

26. A rectangular tank is 30 m long 20 m broad. Water is being flown into it through a square pipe of side 5 cm. What is the speed of water if the level of water in the tank rises by 1 m in 8 hours?
(a) 30 km/hr (b) 36 km/hr
(c) 40 km/hr (d) None of these

27. Calculate the number of bricks, each measuring 25 cm × 15 cm × 8 cm, required to construct a wall of dimension 19 m × 4 m × 5 m, when 10% of its volume is occupied by mortar.
(a) 4000 (b) 8000
(c) 7000 (d) 6000

28. Three cubes of metal whose edges are in the ratio 3 : 4 : 5 are melted into a single cube, the length of whose diagonal is $48\sqrt{3}$ m. Calculate the edges of the three cubes.
(a) 24 m, 32 m, 40 m (b) 40 m, 32 m, 24 m
(c) 30 m, 22 m, 18 m (d) 48 m, 36 m, 24 m

29. A cube of lead with edges measuring 6 cm each is melted and recasted into 27 equal cubes. The length of the edge of the new cube is
(a) 3 cm (b) 4 cm
(c) 2 cm (d) 1.5 cm

30. The volume of a cube is 729 cm^3. The total surface area of the cube is
(a) 216 cm^2 (b) 384 cm^2
(c) 486 cm^2 (d) 512 cm^2

31. Two cubes have volumes in the ratio 1 : 27. The ratio of the area of the face of one to that of the other is
(a) 1 : 2 (b) 1 : 3
(c) 1 : 6 (d) 1 : 9

32. Two cubes, each of side 12 cm, are joined end to end. The surface area of the resulting cuboid is
(a) 1240 sq cm (b) 1440 sq cm
(b) 2250 sq cm (d) 4252 sq cm

33. The perimeter of one face of a cube is 20 cm. Its volume is
(a) 1009 cm^3 (b) 525 cm^3
(c) 320 cm^3 (d) 125 cm^3

34. A cube of edge 3 cm of iron weighs 12 gm. What is the weight of a similar cube of iron whose edge is 12 cm?
(a) 768 gm (b) 678 gm
(c) 964 gm (d) 864 gm

35. The weight of the solid cube of iron of 1 cm edge is 17 gm. What should be the weight of a similar cube of edge 3 cm?
(a) 449 gm (b) 459 gm
(c) 469 gm (d) 4390 gm

36. A cubic metre of silver weighing 900 kg is rolled into a square bar 16 m long. Find the weight of an exact cube cut off from it.
(a) 14 kg $62\frac{1}{2}$ gm (b) 30 kg
(c) 10 kg (d) 7 kg 50 gm

37. A 4 cm cube is cut into 1 cm cubes. What is the ratio of surface area of small cubes to that of the large cube?
(a) 1 : 16 (b) 2 : 3
(c) 4 : 1 (d) 6 : 1

38. A large cube is formed from the material obtained by melting three smaller cubes of 3, 4 and 5 cms side. What is the ratio of the total surface areas of the smaller cubes and the large cube?
(a) 2 : 1 (b) 3 : 2
(c) 25 : 18 (d) 27 : 20

39. How many small cubes, each of 96 sq cms surface area, can be formed from the material obtained by melting a larger cube of 384 cm^2 surface area?
(a) 8 (b) 5
(c) 800 (d) 8000

40. A cubical metallic tank whose each edge measures 30 cm, is completely filled with water. If 2,7 litres water is taken out of it, what will be the depth of remaining water in tank?
(a) 37 cm (b) 27 cm
(c) 17 cm (d) None of these

41. Find the weight of a hollow cylindrical lead pipe 28 cm long and $\frac{1}{2}$ cm thick. Its internal diameter is 8 cm. $\left(\text{Weight of 1 cm}^3 \text{ of lead is 11.4 g, } \pi = \frac{22}{7}\right)$
(a) 3.762 kg (b) 4.562 kg
(c) 7.462 kg (d) 6.762 kg

42. Volume of the cylinder is 1650 cu m. whereas the surface area of its base is $78\frac{4}{7}m^2$. Find the height of the cylinder
(a) 2.1 m (b) 7.5 m
(c) 21 m (d) 14 m

43. 1496 cm^3 of a metal is used to cast a pipe of length 28 cm. If the internal radius of the pipe is 8 cm, the outer radius of the pipe is
(a) 7 cm (b) 9 cm
(c) 10 cm (d) 12 cm

44. The base of a solid cylinder of height 10 cm is a semi-circle of radius 7 cm. Its total surface (in sq cm) is $\left(\pi = \frac{22}{7}\right)$

(a) 154 (b) 176
(c) 514 (d) None of these

45. A sphere is melted to form a cylinder whose height is $4\frac{1}{2}$ times its radius; what is the ratio of radii of sphere to the cylinder?
(a) 3 : 2 (b) 4 : 3
(c) 3 : 5 (d) 2 : 3

46. The radius of a cylinder is the same as that of a sphere. Their volumes are equal. The height of the cylinder is
(a) 4/3 times its radius
(b) 2/3 times its radius
(c) equal to its radius
(d) equal to its diameter

47. A 12 m deep well with internal diameter 3.5 m is dug up. The earth from it is spread evenly to form a platform 10.5 m by 8.8 m. Determine the height of the platform
(a) 2.25 m (b) 3.25 m
(c) 1.25 m (d) 4.25 m

48. The curved surface of a well is 264 sq m and its capacity is 924 cu m. What is the diameter and the depth of the well?
(a) 8 m (b) 9 m
(c) 4.5 m (d) 6 m

49. The sum of the radius of the base and the height of a solid cylinder is 37 m. If the total surface area of the cylinder be 1628 sq. m, find the volume
(a) 4620 cu m (b) 4630 cu m
(c) 4520 cu m (d) 4830 cu m

50. A brick measures 20 cm by 10 cm by $7\frac{1}{2}$ cm. How many bricks will be required for a wall 25 m long, 2 m high and $\frac{3}{4}$ m thick?
(a) 25000 (b) 35000
(c) 20000 (d) 45000

51. The height of a right circular cylinder is 6 m. Three times the sum of the areas of its two circular faces is twice the area of its curved surface. The radius of the base is
(a) 4 m (b) 2 m
(c) 6 m (d) 1.5 m

52. The radius of the cylinder is made twice as large. How should the height be changed so that the volume remains the same?
(a) $\frac{1}{2}$ × height of two cylinders
(b) $\frac{1}{4}$ × height of original cylinder
(c) $\frac{1}{4}\pi r^2$
(d) None of these

53. A right cylinder and a right circular cone have the same radius and the same volume. The ratio of the height of the cylinder to that of the cone is
(a) 3 : 5 (b) 2 : 5
(c) 3 : 1 (d) 1 : 3

54. Two cans have the same height equal to 21 m. One can is cylindrical, the diameter of whose base is 10 cm. The other can has square base of side 10 cm. What is the difference in their capacities?
(a) 350 cm^2 (b) 450 cm^3
(c) 250 cm^2 (d) None of these

55. A roller is 120 cm long and has diameter 84 cm. If it takes 500 complete revolutions to level a play-ground, determine the cost of levelling at the rate of 30 paise per sq m. $\left(\text{Use } \pi = \frac{22}{7}\right)$
(a) Rs. 475.40 (b) Rs. 375.45
(c) Rs. 375.20 (d) Rs. 475.20

56. The circumference of one end of a frustum of a right circular cone is 48 cm and of the other end is 34 cm. If the height of the frustrum is 10 cm, its volume (in cu cm) is:
(a) 5400 (b) 1350
(c) 2700 (d) 4050

57. Find the amount of concrete required to erect a concrete pillar whose circular base will have a perimeter 8.8 m and whose curved surface is 17.6 m $\left(\pi = \frac{22}{7}\right)$
(a) $12\frac{4}{25}m^3$ (b) $12\frac{3}{25}m^3$
(c) $12\frac{1}{2}m^3$ (d) $12\frac{8}{25}m^3$

58. Sum of the length, width and depth of a cuboid is s and its diagonal is d. Its surface area is
(a) s^2 (b) d^2
(c) $s^2 - d^2$ (b) $s^2 + d^2$

59. A cylindrical tower is 5 m in diameter and 14 m high. The cost of white washing its curved surface at 50 paise per m^2 is
(a) Rs. 90 (b) Rs. 97
(c) Rs. 100 (d) Rs. 110

60. A solid piece of iron of dimensions 49 × 33 × 24 cm is moulded into a sphere the radius of the sphere is
(a) 35 cm (b) 21 cm
(c) 29 cm (d) None of these

61. How many coins 2 mm thick and 1.5 cm. in diameter should be melted in order to form a right circular

cylinder having base diameter 6 cm and height 8 cm?
(a) 640 (b) 540
(c) 740 (d) 840

62. A hemisphere of lead and of radius 6 cm, is cast into a right circular cone of height 75 cm. The radius of the base of the cone is
(a) 1.4 cm (b) 2.4 cm
(c) 1.6 cm (d) 3.2 cm

63. A solid cylinder has a total surface area of 231 sq cm. Its curved surface area is (2/3) of the total surface area. Find the volume of the cylinder
(a) 270 cu cm (b) 269.5 cu cm
(c) 256.5 cu cm (d) 289.5 cu cm

64. It is required to design a circular pipe such that water flowing through it at a speed of 7 metres per min fills a tank of capacity 440 cubic metres in 10 min. The inner radius of the pipe should be
(a) 2 m (b) $\sqrt{2}$ m
(c) $\frac{1}{2}$ m (d) $\frac{1}{\sqrt{2}}$ m

65. From a solid right circular cylinder with height 10 cm and radius of the base 6 cm; a right circular cone of the same height and base is removed. The volume (in cm^3) of the remaining solid is
(a) 377 (b) 754.3
(c) 1131 (d) None of these

66. The radii of two cylinders are in the ratio 2 : 3 and their height are in the ratio 5 : 3. Ratio of their volume is
(a) 20 : 27 (b) 10 : 9
(c) 18 : 13 (d) 9 : 20

67. The capacity of a tank in the form of a cylinder is 6160 cu m. If the diameter of its base is 28 m, find the cost of painting its inner curved surface at the rate of Rs. 2.8 per sq m. $\left(\text{Use } \pi = \frac{22}{7}\right)$
(a) 2464 (b) 2664
(c) 3064 (d) 2864

68. The sum of the radius of the base and the height of a solid cylinder is 37 m. If the total surface area of the solid cylinder is 1628 m^2, then the circumference of its base and the volume of the cylinder are
(a) 68 m; 7875 m^3 (b) 52 m; 5825 m^3
(c) 44 m; 4620 m^3 (d) 30 m; 3859 m^3

69. A rectangular piece of paper is 22 cm long and 10 cm wide. A cylinder is formed by rolling the paper along its length. The volume of the cylinder is
(a) 225 π cu cm (b) 385 cu cm
(c) 25 π cu cm (d) None of these

70. The ratio of total surface area to lateral surface area of a cylinder whose radius is 80 cm and height 20 cm, is
(a) 2 : 1 (b) 3 : 1
(c) 4 : 1 (d) 5 : 1

71. A right cylindrical vessel is full with water. How many right cones having the same diameter and height as those of right cylinder will be needed to store that water?
(a) 2 (b) 3
(c) 4 (d) 5

72. A cylindrical bucket is 72 cm high and 29 cm in diameter and is full of water. This water is emptied in a rectangular tank whose length and breadth are 66 cm and 28 cm, respectively. What will be the height of the water level in the tank?
(a) 36 cm (b) 48 cm
(c) 24 cm (d) 22 cm

73. A cylindrical iron rod is 70 cm long and the diameter of its end is 2 cm. What is its weight, reckoning a cubic cm of iron to weigh 10 grams?
(a) 4 kg (b) 4.2 kg
(c) 2.2 kg (d) Data inadequate

74. If the radius of a cylinder is doubled and the height is halved, what is the ratio between the new curved surface area and the previous curved surface area of the cylinder
(a) 1 : 1 (b) 2 : 1
(c) 3 : 2 (d) 2 : 3

75. A cylindrical jar of diameter 24 cm contains water to a height 30 cm. A spherical steel ball is dropped into the jar and the level of the water rises by 67.5 mm. The diameter of the ball is
(a) 16 cm (b) 15 cm
(c) 20 cm (d) 18 cm

76. The material of solid cone is converted into the shape of solid cylinder of equal radius. If the height of the cylinder is 5 cm, what is the height of the cone?
(a) 25 cm (b) 15 cm
(c) 20 cm (d) 10 cm

77. The volume of solid cylinder whose diameter of the base is 14 mm and length 25 mm is 3850 cu mm. If length of the cylinder is doubled but the diameter is halved, what will be the volume of the resulting cylinder?
(a) 1172 cu mm (b) 1925 cu mm
(c) 3850 cu mm (d) 7700 cu mm
(d) None of these

78. A monument has 50 cylindrical pillars each of diameter 50 cm and height 4 m. What will be the labour charges for getting these pillars cleaned at the rate of 50 paise per sq m? (Use π = 3.14)
(a) Rs. 237 (b) Rs. 257
(c) Rs. 157 (d) Rs. 353

79. The radius of a cylinder is made twice as large. How should the height be changed so that volume remains the same?
(a) $\frac{1}{4}$th of original (b) $\frac{1}{3}$rd of original
(c) $\frac{1}{2}$ of original (d) $\frac{1}{8}$th of original

80. A spherical ball of lead, 3 cm in diameter is melted and re-cast into three spherical balls. The diameter of two of these are 1.5 cm and 2 cm, respectively. The diameter of the third ball is
(a) 2.66 cm (b) 2.5 cm
(c) 3 cm (d) 3.5 cm

81. A cone and a cylinder having the same area of the base have also the same area of curved surfaces. If the height of cylinder be 2 m, find the slant height of the cone
(a) 3 m (b) 3.5 m
(c) 4.5 m (d) 4 m

82. The radii of a cylinder and a cone are equal. If the height of the cylinder is equal to the slant height of the cone then the ratio of the curved surfaces of the cylinder and the cone is
(a) 1 : 1 (b) 2 : 1
(c) 3 : 1 (d) 4 : 1

83. From a cubical block of wood of side 1 m, a cylinder of the largest possible volume is curved out. The volume (in cu m) of the remaining wood is
(a) $\frac{3}{14}$ (b) $\frac{5}{14}$
(c) $\frac{1}{2}$ (d) $\frac{2}{7}$

84. The radius of the base of a solid cylinder is r cm and its height is 3 cm. It is re-casted into a cone of same radius, the height of the cone will be
(a) 3 cm (b) 6 cm
(c) 9 cm (d) 27 cm

85. Two cm of rain has fallen on a sq km of land. Assuming that 50% of the raindrops could have been collected and contained in a pool having a 100 m × 10 m base, by what level would the water level in the pool have increased?
(a) 15 m (b) 20 m
(c) 10 m (d) 25 m

86. The perpendicular height of a conical tent is $4\frac{2}{3}$ m and the diameter of its base is 6 metres. If 11 persons can sleep in this tent, find how many average cu m of air each person gets?
(a) 2 cu m (b) 4 cu m
(c) 6 cu m (d) 8 cu m

87. The circumference of the base of a 9 m high conical tent is 44 m. The volume of the air contained in it is
(a) 426 m^3 (b) 452 m^3
(c) 472 m^3 (d) 512 m^3

88. A conical vessel of base radius 2 cm and height 3 cm is filled with kerosene. This liquid leaks through a hole in the bottom and collects in a cylindrical jar of radius 2 cm. The kerosene level in the jar is
(a) 1.5 cm (b) π cm
(c) 1 cm (d) 3 cm

89. A hollow cylinder of height 3 cm, is re-casted into a solid cylinder. If the external and internal radii of the hollow cylinder are 4.3 cm and 1.1 cm respectively. What will be the radius of the solid cylinder?
(a) 2.8 cm (b) 2.4 cm
(c) 3.2 cm (d) 4.8 cm

90. A solid consists of a circular cylinder with an exact fitting right circular cone placed on the top. The height of the cone is h. If the total volume of the solid is three times the volume of the cone, then the height of the cylinder is
(a) $2h$ (b) $4h$
(c) $\frac{2h}{3}$ (d) $\frac{3h}{2}$

91. A well of 11.2 m diameter is dug 8 m deep. The earth taken out has been spread all around it to a width of 7 cm to form a circular embankment. Find the height of this embankment.
(a) 304.8 m^2 (b) 400.4 m^2
(c) 408.4 m^2 (d) 412.4 m^2

92. The curved surface of a circular cylinder of height 'h' and the slant surface of the cone of slant height '$2h$' having the same circular base are in the ratio of
(a) 1 : 1 (b) 1 : 2
(c) 3 : 2 (d) 1 : 3

93. The material of a cone is converted into the shape of a cylinder of equal radius. If the height of the cylinder is 5 cm, the height of the cone is
(a) 10 cm (b) 15 cm
(c) 18 cm (d) 24 cm

94. A right circular cone is exactly fitted inside a cube in such a way that the edges of the base of the cone are touching the edges of one of the faces of the cube and the vertex is on the opposite face of the cube. If the volume of the cube is 343 c.c, what approximately is the volume of the cone?
(a) 90 c.c. (b) 75 c.c.
(c) 80 c.c. (d) 85 c.c.

95. A solid cone is 25 cm high and radius of its base is 50 cm. It is melted and re-cast into a solid sphere. Determine the surface area of the sphere.
(a) 8757.28 sq cm (b) 5877.42 sq cm
(c) 7857.14 sq m (d) None of these

96. The radius and height of right circular cone are in the ratio 5 : 12. If its volume is $314\frac{3}{7}$ cu m. find the radius of the cone.
(a) 5 m (b) 8 m
(c) 12 m (d) 6 m

97. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their volumes are
(a) 1 : 2 : 2 (b) 1 : 2 : 3
(c) 1 : 2 : 4 (d) 2 : 3 : 4

98. Find the length of the canvas 2 m in width required to make a conical tent 12 m in diameter and 6.3 m in slant height
(a) 118.8 m (b) 62.4 m
(c) 59.4 m (d) 112.4 m

99. The radius of a cylinder is doubled and the height is halved, what is the ratio between the new volume and the previous volume?
(a) 3 : 1 (b) 2 : 3
(c) 2 : 1 (d) 1 : 3

100. A circus tent is cylindrical to a height of 3 m and conical above it. If its diameter is 105 m and slant height of the conical portion is 53 m, calculate the length of the canvas 5 m wide to make the tent.
(a) 1857 m (b) 1647 m
(c) 1947 m (d) 1847 m

101. If base radius of a cone is increased by 20% and its slant height is made double, then by how much per cent will the area of its curved surface be increased?
(a) 140% (b) 160%
(c) 130% (d) 180%

102. The radius of the base of conical tent is 5 cm. If the tent is 12 m high then area of the canvas required in making the tent is
(a) $60\ \pi m^2$ (b) $300\ \pi m^2$
(c) $90\ \pi m^2$ (d) None of these

103. A cone of height 7 cm and base radius 3 cm is carved from a rectangular block of wood 10 cm × 5 cm × 2 cm. The percentage % wood wasted is
(a) 34% (b) 46%
(c) 54% (d) 66%

104. The diameter and slant height of a conical tomb are 28 m and 50 m, respectively. The cost of white washing its curved surface at the rate of 80 paise per sq m is
(a) Rs. 2640 (b) Rs. 1760
(c) Rs. 264 (d) Rs. 176
(e) None of these

105. A rectangular sheet of area 264 cm^2 and width 11 cm is rolled along its breadth to make a hollow cylinder. The volume of the cylinder is
(a) 231 c.c. (b) 230 c.c.
(c) 235 c.c. (d) 234 c.c.

106. A cylinder and a cone have their heights in the ratio 2 : 3 and the radii of their bases in the ratio 3 : 4. Find the ratio of their volumes.
(a) 1 : 9 (b) 2 : 9
(c) 9 : 8 (d) 1 : 8

107. If the height of the cone is doubled then its volume is increased by
(a) 100% (b) 200%
(c) 300% (d) 400%

108. Three cubes of side 3, 4 and 5, respectively are melted to form a new cube. The side of the new cube is
(a) 5 cm (b) 6 cm
(c) 6.5 cm (d) 7 cm

109. The height and base radius of a cone are each increased by 100%. The volume of the cone now becomes
(a) double the original
(b) four times the original
(c) three times the original
(d) eight times the original

110. If the radius of a sphere is doubled then its volume is increased by
(a) 100% (b) 200%
(c) 700% (d) 800%

111. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 0.2 cm. Find the length of the wire.
(a) 24 m (b) 28 m
(c) 36 m (d) 32 m

112. A cone-shaped circular tent is 9 m high and the circumference of its circular base is 44 m. How much is the air contained in the tent? $\left(\text{Use } \pi = \frac{22}{7}\right)$
(a) 362 cu m (b) 462 cu m
(c) 562 cu m (d) 662 cu m

113. If the radius of a sphere is doubled then its surface area is increased by
(a) 100% (b) 200%
(c) 300% (d) 50%

114. The height of a cylinder is decreased by 8%, keeping its radius unchanged. What is the percentage change in its volume?
(a) 8% increase (b) 12% decrease
(c) 8% decrease (d) None of these

115. The radius of a cylinder is increased by 20%, keeping its height unchanged. What is the percentage increase in its volume?
(a) 33% (b) 44%
(c) 22% (d) None of these

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Volume of water $= 16 \times 12 \times \dfrac{16\frac{2}{3}}{100}$ m^3

$= 16 \times 12 \times \dfrac{50}{3 \times 100}$ m^3

$= 32$ m^3.

2. Ans. (b)

Solution Since the wood is 1 cm thick, the inner measurements are

$l = 12$ cm $- 2$ cm $= 10$ cm

$b = 10$ cm $- 2$ cm $= 8$ cm

$h = 8$ cm $- 2$ cm $= 6$ cm

Capacity of the box $= l \times b \times h$

$= 10$ cm $\times 8$ cm $\times 6$ cm

$= 480$ cm^3.

3. Ans. (c)

Solution Volume of the cistern $= 2.4 \times 2 \times 1.5$ m^3

$= 7.2$ m^3

Time taken $= 2.5$ hr

$\therefore$ Rate of water flow $= \dfrac{7.2}{2.5}$ m^3/hr

$= \dfrac{7.2 \times 100 \times 100 \times 100}{2.5 \times 60 \times 60}$

$= 800$ cu m/sec.

4. Ans. (b)

Solution Let the sides of the box be x, y and z

Then, $p = xy$, $q = yz$, $r = zx$

Volume of the box $= xyz$

We have, $p \times q \times r = xy \times yz \times zx = x^2y^2z^2$

or, $xyz = \sqrt{pqr}$.

5. Ans. (c)

Solution Volume of water lowered

$= 30 \times 15 \times 4$ m^3 $= 1800$ m^3

$\therefore$ Number of gallons taken out

$= 1800 \times 180$ gallons

$= 324000$ gallons

[1 m^3 of water = 180 gallons]

6. Ans. (b)

Solution Number of bricks

$= \dfrac{\text{Volume of the wall}}{\text{Volume of the brick}}$

$= \dfrac{(5 \times 100) \times (3 \times 100) \times 20}{25 \times 12.5 \times 7.5}$

$= 1280.$

7. Ans. (c)

Solution Volume of the wood $= \dfrac{31}{2} \times \dfrac{11}{4} \times \dfrac{4}{3}$ m^3

$= 93.5$ m^3

Cost of the wood = Rs. 45×93.5 = Rs. 4207.50.

8. Ans. (a)

Solution Number of bricks

$= \dfrac{\text{Volume of the wall}}{\text{Volume of the brick}}$

$= \dfrac{1500 \text{ cm} \times 300 \text{ cm} \times 50 \text{ cm}}{12 \text{ cm} \times 12 \text{ cm} \times 6 \text{ cm}}$

$= 125000$ cm.

9. Ans. (b)

Solution Length of the diagonal

$= \sqrt{12^2 + 10^2 + 8^2}$ m

$= \sqrt{308}$ m

$= 17.5$ m.

10. Ans. (a)

Solution External dimensions

$l = 12$ cm, $b = 10$ cm, $h = 8$ cm

External volume $= 12$ cm $\times 10$ cm $\times 8$ cm

$= 960$ cm^3

Internal dimensions

$l = 12 - 2 = 10$ cm

$b = 10 - 2 = 8$ cm

$h = 8 - 2 = 6$ cm

Internal volume $= 10$ cm $\times 8$ cm $\times 6$ cm

$= 480$ cm^3

Volume of the wood $= 960$ cm$^3 - 480$ cm^3

$= 480$ cm^3

Cost of the wood = Rs. (480×3) = Rs. 1440.

11. Ans. (d)

Solution Let side of the cube be x units

$\therefore$ Surface area of cube $= 6x^2$

$\therefore$ Sum of the areas of three cubes $= 18x^2$

For the cuboid, length $= 3x$ units

Breadth $= x$ units

height $= x$ units

$\therefore$ Surface area $= 2\,[lb + bh + hl]$

$= 2\ [3x \times x + x \times x + x \times 3x]$
$= 2\ (7x^2) = 14x^2$

$\therefore \dfrac{\text{Surface area of new cuboid}}{\text{Total surface area of three cubes}}$

$= \dfrac{14x^2}{18x^2} = \dfrac{7}{9}.$

12. Ans. (b)

13. Ans. (b)

Solution Let the sides be x, $5x$

Cubic cm of iron sheet $= 0.5 \times x \times 5x = 2.5x^2$

This should be the volume of the cube

$\therefore\ 2.5x^2 = 10 \times 10 \times 10$

$x^2 = \dfrac{1000 \times 10}{25}$

$\therefore\ x = 20 \quad \therefore$ Sides are 20, 100 cm

14. Ans. (c)

Solution Capacity of room $= 12 \times 8 \times 6$ cu m

Required number of boxes $= \dfrac{12 \times 8 \times 6}{1.5} = 384.$

15. Ans. (c)

Solution Length $= \dfrac{3}{2} \times$ breadth; height $= 3\dfrac{3}{10}$ metres

$\dfrac{3}{2} \times$ breadth $\times$ breadth $\times \dfrac{33}{10} = 123\dfrac{3}{4}$ cu metres or,

$(\text{breadth})^2 = \dfrac{445}{5} \times \dfrac{2}{3} \times \dfrac{10}{33} = 25$ sq m.

$\therefore$ Breadth $= \sqrt{25} = 5$ m

$\therefore$ Length $= \dfrac{3}{5} \times 5 = 7.5$ m.

16. Ans. (c)

Solution Volume of earth dug out $= (3 \times 2 \times 1.5)\ \text{m}^3 = 9\ \text{m}^3$

Area over which earth is spread

$= [(22 \times 141) - (3 \times 2)]\ \text{m}^2 = 302\ \text{m}^2$

$\therefore$ Increase in level $= \dfrac{9}{302}$ m $= \dfrac{9 \times 100}{302}$ cm

$= 2.98$ cm.

17. Ans. (b)

Solution $14 \times b = 70 \times 2.2$

$\therefore\ b = \dfrac{70 \times 2.2}{14} = 11$ metres

and $l \times b \times h = 70 \times 11$

$\therefore\ h = \dfrac{70 \times 11}{l \times b} = \dfrac{70 \times 11}{70 \times 2.2} = 5$ metres.

18. Ans. (a)

Solution Let the initial height be h and the height after sand is thrown be H metres.

We have to find $H - h$.

According to question,

$12 \times 5 \times (H - h) = 210$

$\therefore\ H - h = \dfrac{210}{60} = \dfrac{7}{2} = 3.5$ metres.

19. Ans. (d)

Solution Let the length, breadth and height be $6x$, $5x$ and $4x$ metres, respectively.

Then, $2 \times [6x \times 5x + 4x \times 5x + 4x \times 6x] = 33300$

$\therefore\ 148x^2 = 33300.$

or, $x^2 = 225$ or, $x = 15$

So, length = 90 m

breadth = 75 m

height = 60 m.

20. Ans. (c)

Solution (Area of the square end) $\times$ 9 = Volume

$= 1\ \text{m}^3$

$\therefore$ Side of the square end $= \sqrt{\dfrac{1}{9}}$ m $= \dfrac{1}{3}$ m

$\therefore$ Volume of this cube $= \left(\dfrac{1}{3}\right)^3 \text{m}^3 = \dfrac{1}{27}\ \text{m}^3$

$\therefore$ Weight of this cube $= \dfrac{1}{27} \times 90$ kg $= 3\dfrac{1}{3}$ kg.

21. Ans. (b)

Solution $6e^2 = 24\ \text{cm}^2$, so that e (edge) = 2 cm

$\therefore$ Number of such cubes out of a cube of edge 1 metres (or 100 cm)

$= \left(\dfrac{100}{2}\right)^3 = 125000.$

22. Ans. (d)

Solution Side of the new cube

$= \sqrt[3]{\text{Sum of the cubes of sides of all the cubes}}$

$\therefore$ Side $= \sqrt[3]{6^3 + 8^3 + 10^3} = \sqrt[3]{216 + 512 + 1000}$

$= \sqrt[3]{1728} = 12$ cm.

23. Ans. (c)

Solution Number of cubes

$= \left(\dfrac{\text{Original length of edge}}{\text{New length of edge}}\right)^3$

$\therefore$ Number of cubes $= \left(\dfrac{6}{2}\right)^3 = 27.$

24. Ans. (c)

Solution Volume of the cube of side 20 cm

$= (20)^3 = 8000 \text{ cm}^3$

Volume of the cube of side 5 cm = 125 cm^3

$\therefore$ Number of smaller cubes of side 5 cm

$= \frac{8000}{125} = 64.$

25. Ans. (c)

Solution Surface area of a cube = $6 \times (\text{side})^2$

$\therefore \quad 6 \times (\text{side})^2 = 600$

$\Rightarrow (\text{Sides})^2 = 100$

$\Rightarrow \text{Side} = \sqrt{100} = 10 \text{ cm}$

$\therefore$ Diagonal of the cube $= \sqrt{3} \times \text{side}$

$= \sqrt{3} \times 10 = 10\sqrt{3}$ cm.

26. Ans. (a)

Solution Volume of water collected in the tank in 8 hours

$= 30 \text{ m} \times 20 \text{ m} \times 1 \text{ m}$

$= 600$ cu m.

$\therefore$ Volume of water collected in the tank in 1 hour

$= \frac{600}{8} = 75$ cu m.

Water comes through a pipe of cross-section

$= 5 \text{ cm} \times 5 \text{ cm} = \frac{25}{10000}$ sq m.

The speed of water = Distance travelled by the water in the pipe in one hour

$= \frac{75 \times 10000}{25}$ m = 30 km/hr.

27. Ans. (d)

Solution Volume of the wall $= 10 \times \frac{4}{10} \times 5$

$= 20$ cu m.

Volume of the mortar $= \frac{10}{100} \times 20 = 2$ cu m.

Hence, the volume occupied by the bricks

$= 20 - 2 = 18$ cu m

Volume of each brick $= \frac{25}{100} \times \frac{15}{100} \times \frac{8}{100}$ cu m.

$= \frac{3}{1000}$ cu m.

Therefore, the required number of bricks

$= 18 \div \frac{3}{1000} = 6000.$

28. Ans. (a)

Solution Since the edges of the cubes are in the ratio 3 : 4 : 5, let these be $3k$, $4k$, $5k$ metres, respectively.

Their volumes are $27k^3$, $64k^3$, $125k^3$ cu m.

Thus, the volume of the single cube

$= (27 + 64 + 125)\, k^3$ cu m

$= 216\, k^3 = (6k)^3$ cu m

We know that the length of the diagonal of a cube of side x is $\sqrt{3}x$. Therefore, the length of the diagonal of the single cube mentioned above is equal to $6k\sqrt{3}$. But the length of the diagonal of this cube is given to be $48\sqrt{3}$, hence

$6k\sqrt{3} = 48\sqrt{3}$ or, $k = 8$.

Therefore, the length of the edges of the three cubes are 3×8, 4×8, 5×8 m, that is 24 m, 32 m, 40 m.

29. Ans. (c)

Solution Edge of the cube = 6 cm

$\therefore$ Volume of lead $= 6^3 \text{ cm}^3 = = 216 \text{ cm}^3$

Let the edge of the new cube be x cm

Then $27x^3 = 216$

$\Rightarrow x^3 = 8$ or $x = 2$ cm.

30. Ans. (c)

Solution Length of the cube $= (729)^{1/3} = 9$

Total surface area $= 6 \times (9)^2 = 486 \text{ cm}^2$.

31. Ans. (d)

Solution $a_1^3 : a_2^3 = 1 : 27$

$\Rightarrow a_1 : a_2 = 1 : 3$

$\therefore$ Required ratio is $1^2 : 3^2 = 1 : 9$.

32. Ans. (b)

Solution $4(24 \times 12) + 2(12 \times 12) = 1440$ sq cm.

33. Ans. (d)

Solution $4a = 20 \Rightarrow a = 5$

$\therefore$ Volume $= a^3 = 5^3 = 125 \text{ cm}^3$.

34. Ans. (a)

Solution Ratio of the edge of cubes

$= 3 : 12 = 1 : 4$

Ratio of their volumes $= 1^3 : 4^3 = 1 : 64$.

Because volume of the new cube is 64 times the volume of the first cube, the weight of the new cube is also 64 times the weight of the first cube.

Weight of the new cube

$= 64 \times 12$ gm $= 768$ gm

35. Ans. (b)

Solution When edge is increased 3 times, the volume or weight is increased 3^3, i.e. 27 times.

$\therefore$ The weight of the other cube

$= 27 \times 17 = 459$ gm.

36. Ans. (a)

Solution Let x be the side of the base of the square bar 16 m long.

$\therefore \quad 16 \times x^2 = 1$

or, $x^2 = \frac{1}{16}$ or, $x = \frac{1}{4}$m

Volume of the cube of edge $\frac{1}{4}$m $= \frac{1}{16}$ m^3

Now, 1 m^3 weighs 900 kg

$\therefore \quad \frac{1}{16}$ m^3 weighs $\frac{900}{64}$ kg = 14 kg $62\frac{1}{2}$ gm.

37. Ans. (a)

38. Ans. (c)

Solution Sum of the surface areas of three smaller cubes

$= 6 \times 3^2 + 6 \times 4^2 + 6 \times 5^2$

$= 300$ cm^2.

Volume of large cube $= 3^3 + 4^3 + 5^3 = 216$ cm^3.

$\therefore$ Edge of large cube = 6 cm

$\therefore$ Surface area of large cube $= 6 \times 6^2 = 216$ cm^2

Total S.A. of smaller cubes: S.A. of large cube

$= 300 : 216 = 25 : 18$.

39. Ans. (d)

Solution Edge of the small cube $= \sqrt{\frac{96}{6}} = 4$ m

Edge of the large cube $= \sqrt{\frac{384}{6}} = 8$ cm

No. of small cubes $= \frac{8 \times 8 \times 8}{\frac{4}{10} \times \frac{4}{10} \times \frac{4}{10}} = 8000$.

40. Ans. (b)

Solution Volume of the cubical metallic tank

$= l \times b \times h$

$= 30 \times 30 \times 30$

$= 27000$ cm^3

$\therefore$ Volume of water in the tank $= \frac{27000}{1000}$

$= 27$ litre [$\because$ 1 litre = 100 cm^3]

$\therefore$ Volume of remaining water

$= 24.3$ litre $= 243000$ cm^3

Now, $l \times b \times h = 243000$

$\Rightarrow 30 \times 30 \times h = 243000 \Rightarrow h = 27$ cm.

41. Ans. (a)

Solution External radius of the pipe $= \frac{8}{4} = 4$ cm

External volume $= \pi r^2 h$

$= \frac{22}{7} \times 4 \times 4 \times 28$

$= 1408$ cu cm

Internal diameter $= 8 - 1 = 7$ cm

Internal radius $= \frac{7}{2} = 3.5$ cm

Internal volume $= \pi r^2 h$

$= \frac{22}{7} \times 3.5 \times 3.5 \times 28$

$= 1078$ cu cm.

Weight of lead $= 330 \times 11.4 = 3762$ g

$= 3.762$ kg.

42. Ans. (c)

Solution Height of cylinder

$= \frac{\text{Volume of cylinder}}{\text{Base Area}}$

$= \frac{1650}{78\frac{4}{7}} = 21$ m.

43. Ans. (b)

Solution Let the outer radius be x cm

Then, we have

$1496 = \pi \times (28) \times (x^2 - 8^2)$

$\Rightarrow x^2 - 8^2 = \frac{1496 \times 7}{22 \times 28} = 17$

or, $x^2 = 17 + 64 = 81$

$\therefore \quad x = 9$ cm.

44. Ans. (c)

Solution Curved surface area $= \pi rh$

$= \frac{22}{7} \times 7 \times 10 = 220$ sq cm.

Total area of plane faces

$= \frac{\pi r^2}{2} + \frac{\pi r^2}{2} + 2r \times h$

$= \frac{22}{7} \times 7^2 + 2 \times 7 \times 10$

$= 154 + 140 = 294$ sq cm.

Total surface area

$= 220 + 294 = 514$ sq cm.

45. Ans. (a)

Solution Let the radius of the sphere and cylinder be 'R' and 'r', respectively

Volume of the cylinder

$= \pi r^2 h = \pi r^2 \left(\frac{9}{2} r\right) \qquad \left(\because h = \frac{9}{2} r\right)$

$= \frac{9}{2} \pi r^3$

Volume of the sphere = $\frac{4}{3}\pi R^3$

According to question:

Volume of the sphere = Volume of the cylinder

or, $\frac{4}{3}\pi R^3 = \frac{9}{2}\pi r^3$

or, $\left(\frac{R}{r}\right)^3 = \frac{27}{8}$

$\therefore \frac{R}{r} = \frac{3}{2}$.

46. Ans. (a)

Solution $\frac{4}{3}\pi r^3 = \pi r^2 h \Rightarrow h = \frac{4}{3}r$.

47. Ans. (c)

Solution Volume of the earth dug

= Volume of the well

$= \pi r^2 h = \frac{22}{7}\times\frac{3.5}{2}\times\frac{2.5}{2}\times 12$

$= \frac{231}{2}$ cu m

Area of the platform

= 10.5 m × 8.8 m

= 92.4 sq m

Height of the platform = $\frac{231}{2}\times\frac{1}{92.4}$ = 1.25 m.

48. Ans. (d)

Solution Let r and h be the radius and depth of the well respectively, then

Curved surface = $2\pi rh$ = 264 sq m ...(1)

Volume = $\pi r^2 h$ = 924 cu m.

$\Rightarrow \frac{\pi r^2 h}{2\pi rh} = \frac{924}{264}$

or, $r = \frac{2\times 924}{264}$ = 7 m

$\therefore$ Diameter = 14 m

$\therefore$ from equation (1), we get

$2\pi \times 7 \times h = 264$

or. $h = \frac{264}{14\times\pi} = \frac{264\times 7}{14\times 22}$ = 6 m.

49. Ans. (a)

Solution $r + h = 37$ and $2\pi r\,(r + h) = 1628$

or, $\pi r = \frac{1628}{74} = 22$

$\therefore$ r = 7 cm and h = 37 – 7 = 30 m

$\therefore$ Volume = $\pi r^2 h = \frac{22}{7}\times 7\times 7\times 30$

= 4620 cu m.

50. Ans. (a)

Solution Volume of wall = 25 × 2 × $\frac{3}{4}$ cu m.

Volume of one brick = $\frac{20}{100}\times\frac{10}{100}\times\frac{15}{200}$

$= \frac{3}{2000}$ cu m.

Required number of bricks

$= \left(25\times 2\times\frac{3}{4}\right)\div\frac{3}{2000} = 25000.$

51. Ans. (a)

Solution $3 \times 2\pi r^2 = 2 \times 2\pi r \times 6$

$\therefore$ r = 4 m.

52. Ans. (b)

Solution $r \to 2r$, $h \to \frac{1}{4}h$, then

Volume = $\pi(2r)^2 \times \frac{1}{4}h = \pi r^2 h$.

53. Ans. (d)

Solution Let the height of the cylinder = h

Then, $\pi r^2 h = \frac{1}{3}\pi r^2 H$

or, $\frac{h}{H} = \frac{1}{3}$ = 1 : 3.

54. Ans. (b)

Solution Volume of cylindrical can = $\pi r^2 h$

$= \frac{22}{7}\times\left(\frac{10}{2}\right)^2\times 21\text{ cm}^3$

= 1650 cm^3

Volume of the square can = (side)^2 × h

= (10)^2 × 21 cm^3 = 2100 cm^3

Difference in the capacities of the two cans = (2100 – 1650) cm^3 = 450 cm^3.

55. Ans. (d)

Solution $r = \frac{84}{2}$ cm $= \frac{21}{50}$m,

h = 120 cm $= \frac{120}{100}$m $= \frac{6}{5}$m

The levelled area in one revolution of the roller

= curved surface = $2\pi rh$

$= 2 \times \frac{22}{7}\times\frac{21}{50}\times\frac{6}{5}$

$= \frac{396}{125}$ sq m.

The levelled area in 500 revolutions

$= \frac{396}{125}\times 500$ = 1584 sq m.

Required cost of levelling $= \frac{30}{100} \times 1584$
$= \text{Rs. } 475.20.$

56. Ans. (b)

Solution The radii of ends of the frustum are

$$\frac{48}{2\pi}, \frac{34}{2\pi}$$

$$\therefore \text{ Volume} = \frac{\pi}{3} \times 10 \left\{ \frac{48 \times 48}{(2\pi)^2} + \frac{34 \times 34}{(2\pi)^2} + \frac{48 \times 34}{(2\pi)^2} \right\}$$

$$= \frac{10\pi}{3} \{2304 + 1156 + 1632\}$$

$$= \frac{10 \times 7 \times 5092}{22 \times 12}$$

$$= 1350.$$

57. Ans. (d)

Solution $2\pi r = 8.8$ m

$2\pi rh = 17.6$ m

$\therefore \quad h = 2m, \; r = 1.4$

Amount of concrete = volume

$$= \pi r^2 h$$

$$= \frac{22}{7} \times \frac{14}{10} \times \frac{14}{10} \times 2$$

$$= 12 \frac{8}{25} \text{ m}^3.$$

58. Ans. (c)

Solution $l + b + h = s$ and $\sqrt{l^2 + b^2 + h^2} = d$

So, $l^2 + b^2 + h^2 = d^2$

$\therefore \quad (l + b + h)^2 = s^2$

$\Rightarrow l^2 + b^2 + h^2 + 2(lb + bh + hl) = s^2$

$\Rightarrow d^2 + 2(lb + bh + hl) = s^2$

$\Rightarrow 2(lb + bh + hl) = s^2 - d^2$

$\therefore$ Surface area $= s^2 - d^2$.

59. Ans. (d)

Solution Curved surface $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{5}{2} \times 14$$

$$= 220 \text{ m}^2$$

Cost of white washing = Rs. $\left(220 \times \frac{1}{2}\right)$ = Rs. 110.

60. Ans. (b)

Solution Let r be the radius

$$\frac{4}{3}\pi r^3 = 49 \times 33 \times 24$$

$$r^3 = \frac{3 \times 7^2 \times 3 \times 11 \times 2^3 \times 3 \times 7}{2^2 \times 2 \times 11} = 3^3 \times 7^3$$

$\therefore \quad r = 3 \times 7 = 21$ cm.

61. Ans. (a)

Solution Radius of the coin $= r = \frac{1.5}{2} = \frac{3}{4}$ cm

Thickness of the coin $= h = 2$ mm $= \frac{1}{5}$ cm

$\therefore$ Volume of one cone $= \pi r^2 h = \pi \times \left(\frac{3}{4}\right)^2 \times \frac{1}{5}$

$$= \frac{9\pi}{80} \text{ cu m}$$

Radius of the cylinder $= R = 3$ cm

Height of the cylinder $= H = 8$ cm

$\therefore$ Volume of the cylinder $= \pi R^2 H = 72\pi$ cu cm.

Number of coins $= \frac{\text{Volume of the cylinder}}{\text{Volume of the coin}}$

$$= \frac{72\pi}{9\pi/80} = 72 \times \frac{80}{9} = 640.$$

62. Ans. (b)

Solution Let r cm be the radius of base of cone

$$\frac{2}{3}\pi(6)^3 = \frac{1}{3}\pi r^2 \times 75$$

$$\therefore \quad r^2 = \frac{2 \times 216}{75} = \frac{2 \times 72}{25}$$

$$\therefore \quad r = \frac{12}{5} = 2.4 \text{ cm.}$$

63. Ans. (b)

Solution $2\pi rh + 2\pi r^2 = 231$

and $2\pi rh = \frac{2}{3} \times 231 = 154$

or, $2\pi r^2 = 77$ or, $\pi r^2 = \frac{22}{7}$

$$\therefore \quad r = \sqrt{\frac{77}{2} \times \frac{7}{22}} = \frac{7}{2} \text{ cm}$$

Now, $2 \times \frac{22}{7} \times \frac{7}{2} \times h = 154$

$\therefore \quad h = 7$ cm.

$\therefore$ Volume $= \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 7$

$= 269.5$ cu cm.

64. Ans. (b)

Solution Volume of the water to pass through pipe in 1 min $= \frac{440}{10} = 44$ cu m.

As the speed of the water is 7 m per min.

Volume of the water per min is

$V = \pi r^2 \times 7$, where r is the inner radius of the pipe.

$\therefore \quad 44 = \frac{22}{7} \times r^2 \times 7 \Rightarrow r^2 = 2$ or, $r = \sqrt{2}$ m.

65. Ans. (b)

Solution Required volume

= Volume of cylinder – Volume of cone

$= \pi \times 6^2 \times 10 - \frac{1}{3} \times \pi \times 6^2 \times 10$

$= \frac{2}{3} \times \pi \times 36 \times 10$

$= \frac{2}{3} \times \frac{22}{7} \times 360 = \frac{5280}{7}$

= 754.3 cu cm.

66. Ans. (a)

Solution $r = 2x,\ r_1 = 3x$

$h = 5y,\ h_1 = 3y$

Volume ratio $= \pi r^2 h : \pi r_1^2 h_1$

$= \pi \times (2x)^2 \times 5y : \pi \times (3x)^2 \times 3y$

$= 20\,\pi x^2 y : 27\pi x^2 y$

$= 20 : 27.$

67. Ans. (a)

Solution Radius of the base of the cylinder

$= r = 14$ m.

h = depth of the tank

Capacity = Volume of the tank

$= \pi r^2 h = 6160$ cu m.

or, $\frac{22}{7} \times 14 \times 14 \times h = 6160 \quad \therefore \quad h = 10$ m

Curved surface area $= 2\pi rh = 2 \times \frac{22}{7} \times 14 \times 10$

= 880 sq m

$\therefore$ Cost of painting this curved surface

= 880 × 2.80 = Rs. 2464.

68. Ans. (c)

Solution $(h + r) = 37,\ 2\pi r(h + r) = 1628$

$\Rightarrow 2\pi r\,(37) = 1628 \Rightarrow r = \frac{1628 \times 7}{2 \times 22 \times 37} = 7$

$\Rightarrow r = 7$ and $h = 30$

$\therefore$ Circumference $= 2\pi r = 2 \times \frac{22}{7} \times 7 = 44$ m.

69. Ans. (b)

Solution Rolled along with its length, then $h = 10$ cm, and the other side = 22 cm

$\therefore \quad V = \frac{10 \times (22)^2 \times 7}{4 \times 22} = 385\ \text{cm}^2.$

70. Ans. (d)

Solution $\frac{\text{Total surface area}}{\text{Lateral surface area}} = \frac{2\pi r h + 2\pi r^2}{2\pi rh}$

$= \frac{2\pi r\,(h + r)}{2\pi rh}$

$= \frac{h + r}{h} = \frac{20 + 80}{20}$

$= \frac{5}{1} = 5 : 1.$

71. Ans. (b)

Solution Let x cones be needed

Then, $\frac{1}{3}\pi r^2 h \times x = \pi r^2 h$ or, $x = 3$.

72. Ans. (c)

Solution Radius of the bucket $= r = 14$ cm

Height of the bucket $= h = 72$ cm

Volume of the water = Volume of the bucket

$= : \pi r^2 h = \frac{22}{7} \times 14 \times 14 \times 72$

$\therefore \quad 66 \times 28 \times H = \frac{22}{7} \times 14 \times 14 \times 72$

$\therefore \quad H = \frac{22 \times 72}{66} = 24$ cm.

73. Ans. (c)

Solution Volume of the iron rod $= \frac{22}{7} \times 1 \times 1 \times 70$

= 220 cu cm.

$\therefore$ Weight of the cylinder $= \frac{220 \times 10}{1000} = 2.2$ kg.

74. Ans. (a)

Solution Let the initial radius and height of the cylinder be r cm and h cm, respectively.

Then, curved surface area of the original cylinder $= 2\pi rh$ and curved surface area of the new cylinder

$= 2\pi(2r) \times \frac{h}{2} = 2\pi rh$

$\therefore$ Required ratio

$= \frac{\text{New curved surface area}}{\text{Previous curved surface area}}$

$= \frac{2\pi rh}{2\pi rh} = 1 : 1.$

75. Ans. (d)

Solution Volume of the water in jar $= \pi \times 12^2 \times 30\ \text{cm}^3$

When the ball is dropped into the jar, volume of water + ball

$= \pi \times 12^2 \times (30 + 6.75)$

Increase in volume

$= \pi \times 12^2 \times (30 + 6.75 - 30)\ \text{cm}^3$

$= \pi \times 144 \times 6.75\ \text{cm}^3$

It r is the radius of the ball, then

$\frac{4}{3}\pi r^3 = \pi \times 144 \times 6.75$

$\Rightarrow r^3 = \dfrac{144 \times 6.75 \times 3}{4} = 729$

$\Rightarrow r^3 = 9^3$

$\therefore \quad r = 9$

Thus, diameter of ball = $2r$ = 18 cm.

76. Ans. (b)

Solution Volume of the cylinder = 3 times volume of the cone. This is valid if base and height is the same. Radius is the same, so the height of cone is 3 times the height of the cylinder

$\therefore$ Height of the cone = 3 × 5 cm = 15 m.

77. Ans. (b)

Solution $\dfrac{\text{First volume of cylinder}}{\text{Second volume of cylinder}}$

$= \dfrac{(\text{First radius})^2}{(\text{Second radius})^2} \times \dfrac{\text{First height}}{\text{Second height}}$

or, $\dfrac{3850}{\text{Second volume}} = \left(\dfrac{2}{1}\right)^2 \times \dfrac{1}{2}$

$\therefore$ Second volume = $\dfrac{1}{2} \times 3850 = 1925$ sq mm.

78. Ans. (c)

Solution Radius of each pillar = 25 cm = $\dfrac{1}{4}$m

curved surface of one pillar = $2\pi rh$

$= 2 \times 3.14 \times \dfrac{1}{4} \times 4 = 6.28$ sq m

$\therefore$ Curved surface of the 50 pillars = 314 sq m

Required cost of cleaning these pillars

$= 314 \times \dfrac{50}{100} =$ Rs. 157.

79. Ans. (a)

Solution Let r be the radius and h be the height

$\pi(2r)^2 \times H = \pi r^2 h$

$\therefore \quad H = \dfrac{1}{4} \cdot h.$

80. Ans. (b)

Solution $\dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi \times \left\{\left(\dfrac{3}{2}\right)^3 - \left[\left(\dfrac{3}{4}\right)^3 + 1^3\right]\right\}$

$\therefore \quad r^3 = \dfrac{125}{64}$

$\therefore \quad r^3 = \left(\dfrac{5}{4}\right)^3$

$\therefore \quad r = \dfrac{5}{4}$

$\therefore$ Diameter = $2r = 2 \times \dfrac{5}{4} = \dfrac{5}{2} = 2.5$ cm.

81. Ans. (d)

Solution $\pi rl = 2\pi r \cdot b$ where $b = 2$ m $\Rightarrow l = 4$ m.

82. Ans. (b)

Solution Let the height and radius of the cylinder be h and r, respectively.

Curved surface of the cylinder = $2\pi rh$

and, curved surface of the cone = $\pi rl = \pi rh$ ($h = l$)

$\therefore$ Required ratio = 2 : 1.

83. Ans. (a)

Solution Cylinder of largest possible volume is of base with diameter 1 m. and height 1 m.

$\therefore$ Volume of this cylinder = $\pi \times \left(\dfrac{1}{2}\right)^2 \times 1$

$= \dfrac{\pi}{4}$ cu m.

84. Ans. (c)

Solution Volume of cylinder = $\pi r^2 \times 3$ cm^3

and volume of cone = $\dfrac{1}{3}\pi r^3 h$ cm^3

$\therefore \quad \dfrac{1}{3}\pi r^2 h = 3\pi r^2 \quad$ or, $\quad h = 9$ cm.

85. Ans. (c)

Solution Volume of rain water

= Area × height = (1 km)^2 × 2 cm

= (1000 m)^2 × 0.02 m = 20000 m^3

Quantity of collected water = 50% of 20000 m^3

$= \dfrac{1}{2} \times 20000$

= 10000 m^3

$\therefore$ Increased level in pool

$= \dfrac{\text{Volume collected}}{\text{Base area of pool}}$

$= \dfrac{10000}{10 \times 100} = 10$ m

$\therefore$ The water level would be increased by 10 m.

86. Ans. (b)

Solution $h = \dfrac{14}{3}$ m, $r = 3$ m

Volume = $\dfrac{22}{7 \times 3} \times 3 \times 3 \times \dfrac{14}{3} = 44$ cm^3

Average of air/person = $\dfrac{44}{11} = 4$ cu m.

87. Ans. (a)

Solution $2 \times \dfrac{22}{7} \times r = 44$ m

So, $r = 7$ m

Volume of conical tent $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 \times 9 \text{ m}^3$

$= 462 \text{ m}^3$.

88. Ans. (c)

Solution Volume of conical vessel

$= \frac{1}{3} \times \frac{22}{7} \times 2 \times 2 \times 3$

$= \frac{88}{7}$ c.c.

If the level of kerosene in the jar is x cm.

$\frac{88}{7} = \pi \times 2 \times 2 \times h = \frac{22}{7} \times 4 \times h$

$\therefore \quad h = 1$ cm.

89. Ans. (b)

Solution Let the radius of the solid cylinder be r cm

$\therefore \quad \pi r^2 \times 9 = \pi [(4.3)^2 - (1.1)^2] \times 3$

$\Rightarrow \quad r^2 = \frac{3}{9} (4.3 + 1.1) (4.3 - 1.1)$

$= \frac{1}{3} \times 5.4 \times 3.2 = 5.76$

$r = \sqrt{5.76} = 2.4$ cm.

90. Ans. (c)

Solution Let the height of the cylinder be H and its radius $= r$

Then, $\pi r^2 H + \frac{1}{3}\pi r^2 h = 3 \times \frac{1}{3}\pi r^2 h$

$\therefore \quad \pi r^2 H = \frac{2}{3}\pi r^2 h$ or, $H = \frac{2}{3}h$.

91. Ans (b)

Solution Volume of earth dug out

$\pi r^2 h = \frac{22}{7} \times \left(\frac{11.2}{2}\right)^2 \times 8$

$= \frac{22}{7} \times 5.6 \times 5.6 \times 8 = 788.48 \text{ m}^3$

Area of embankment

$= \pi (5.6 + 7)^2 - \pi (5.6)^2$

$= \pi [(5.6 + 7)^2 - (5.6)^2]$

$= \pi [(5.6 + 7 + 5.6) (5.6 + 7 - 5.6)]$

$= \frac{22}{7} [18.2 \times 7]$

$= 400.4 \text{ m}^2$.

92. Ans. (a)

Solution Curved surface area of cylinder $= 2\pi rh$

Slant surface area of the cone

$= \pi rl = \pi \times r \times 2h = 2\pi rh$

$\therefore$ Ratio of the two surface areas

$= 2\pi rh : 2\pi rh = 1 : 1$.

93. Ans. (b)

Solution $\frac{1}{3}\pi r^2 \times h = \pi r^2 \times 5$ or, $h = 15$ cm.

94. Ans. (a)

Solution Edge of the cube $= \sqrt[3]{343} = 7$ cm

$\therefore$ Radius of cone $= 3.5$ cm and height $= 7$ cm

$\therefore$ Volume of the cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 7$

$= \frac{1}{3} \times 22 \times 12.25$

$= 90$ c.c.

95. Ans. (c)

Solution Volume of the cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 50 \times 50 \times 25$ c.c.

Volume of the sphere $= \frac{4}{3}\pi R^3$

Since the sphere is made from the cone, their volumes will be equal

$\therefore \quad \frac{4}{3}\pi R^3 = \frac{1}{3} \times \frac{22}{7} \times 50 \times 50 \times 25$

$\Rightarrow \quad R^3 = \frac{1}{3} \times \frac{22}{7} \times 50 \times 50 \times 25 \times \frac{3}{4} \times \frac{7}{22}$

or, $\quad R^3 = 353$

or, $\quad R = 25$ cm

Surface area of the sphere

$= 4\pi R^2$

$= 4 \times \frac{22}{7} \times 25 \times 25 = \frac{55000}{7}$

$= 7857.14$ sq cm.

96. Ans. (a)

Solution $\frac{1}{3} \times \pi \times (5x)^2 \times 12x = 314\frac{3}{7}$

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times 25 \times 12x^3 = \frac{2200}{7} \Rightarrow x^3 = 1$

i.e. $x = 1$

$\therefore$ Radius $= 5x = 5 \times 1 = 5$ m.

97. Ans. (b)

Solution $\frac{1}{3}\pi \times r^2 \times r : \frac{2}{3}\pi r^3 : \pi r^2 \times r$

[height = radius of base]

or, $1 : 2 : 3$.

98. Ans. (c)

Solution Curved surface of the tent = πrl

$$= \frac{22}{7} \times 6 \times 6.3 \text{ m}^2$$
$$= 118.8 \text{ m}^2$$

$\therefore$ Length of the canvas = $\frac{118.8}{2}$ m = 59.4 m.

99. Ans. (c)

Solution Let the initial radius and height of the cylinder be r cm and h cm, respectively.

Then $V_1 = \pi r^2 h$ and $V_2 = \pi (2r)^2 \frac{h}{2} = 2\pi r^2 h$

$$\frac{\text{New volume}}{\text{Previous volume}} = \frac{2\pi r^2 h}{\pi r^2 h} = \frac{2}{1} = 2 : 1.$$

100. Ans. (c)

Solution Curved surface area of cylindrical portion

$$= 2\pi rh$$
$$= 2 \times \frac{22}{7} \times \frac{105}{2} \times 3$$
$$= 990 \text{ m}^2$$

Lateral surface area of the conical portion

$$= \pi rl = \frac{22}{7} \times \frac{105}{2} \times 53$$
$$= 8745 \text{ m}^2$$

Total surface area = 990 + 8745 = 9735 m^2

Width of the canvas = 5 m

$\therefore$ Length of the canvas = 9735 ÷ 5 = 1947 m.

101. Ans. (a)

Solution Radius of the cone

$$= r + 20\% \text{ of } r = 1.2\ r \text{ cm}$$

and, slant height = 2l cm

$\therefore$ Surface area of the new cone

$$= 2\pi \times 1.2\ r \times 2l$$
$$= 2\pi \times 2.4\ rl \text{ cm}^2$$

Increase in surface area

$$= 2\pi \times 2.4\ rl - 2\pi rl$$
$$= 2\pi \times 1.4\ rl \text{ cm}^2$$

Percentage increase

$$= \frac{2\pi \times 1.4\ rl}{2\pi rl} \times 100 = 140\%$$

Therefore, surface area of the cone will be increased by 140%.

102. Ans. (d)

Solution Area of canvas = $\pi rl = \pi \times 5 \times 13 = 65\pi$

$$[l = \sqrt{5^2 + 12^2} = \sqrt{169} = 13]$$

103. Ans. (a)

Solution Total volume = $(10 \times 5 \times 2)$ cm^3

$$= 100 \text{ cm}^3$$

Volume carved = $\left(\frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 7\right)$ cm^3

$$= 66 \text{ cm}^3.$$

Wood wasted = (100 – 66)% = 34%.

104. Ans. (b)

Solution Curved surface of the tomb

$$= \pi rl = \frac{22}{7} \times 14 \times 50 = 22000 \text{ m}^2$$

$\therefore$ Cost of white washing

$$= 22000 \times 0.80 = \text{Rs. } 1760.$$

105. Ans. (a)

Solution Length of the sheet = $\frac{264}{\pi}$ = 24 cm

When the sheet is rolled along its breadth, the width of the sheet will be equal to the circumference of the cylinder and the length of the sheet will be height of the cylinder.

$\therefore$ Radius of cylinder formed

$$= \frac{11}{2\pi} = \frac{11}{2} \times \frac{22}{7} = \frac{7}{4} \text{ cm}$$

Volume of the cylinder

$$= \pi r^2 h = \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 24 = 231 \text{ c.c.}$$

106. Ans. (c)

Solution $h_1 : h_2 = 2 : 3$,

$r_1 : r_2 = 3 : 4$

Ratio of volumes $V_1 : V_2 = \pi (3)^2 \times 2 : \frac{1}{3}\pi \times 4^2 \times 3$

$$= 9 : 8.$$

107. Ans. (a)

Solution Original volume = $\frac{1}{3}\pi r^2 h$

New volume = $\frac{2}{3}\pi r^2 h$

$$\text{Increase \%} = \left(\frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi r^2 h} \times 100\right)\% = 100\%.$$

108. Ans. (b)

Solution Volume of the cubes with sides 3, 4 and 5 cm are 3^3, 4^3 and 5^3, respectively

$\therefore$ Total volume = $3^3 + 4^3 + 5^3$ cm^3

$$= 27 + 64 + 125$$
$$= 216 \text{ cm}^3.$$

109. Ans. (d)

Solution Let radius of the cone = r, height = h

Then, volume of the cone = $\frac{1}{3}\pi r^2 h$

Increased radius = $2r$, height = $2h$

$\therefore$ Increased volume

$$= \frac{1}{3}\pi(2r)^2(2h)$$

$$= \frac{1}{3}\pi \times 8r^2h = 8\left(\frac{1}{3}\pi r^2 h\right)$$

= 8 times the original volume.

110. Ans. (c)

Solution Original volume = $\frac{4}{3}\pi r^3$

New volume = $\frac{4}{3}\pi(2r)^3 = \frac{32}{3}\pi r^3$

Increase % = $\left(\frac{28}{3}\pi r^3 \times \frac{3}{4\pi r^3} \times 100\right)\% = 700\%$.

111. Ans. (c)

Solution Radius of the sphere = 3 cm

Volume of the sphere = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \pi \times 3 \times 3 \times 3$$

$$= 36\pi \text{ cm}^3 \quad ....(1)$$

Radius of the wire = 0.1 m

Volume of the wire of length l cm and radius 1.0 m

$$= \pi r^2 l = \pi \times 0.1 \times 0.1 \times 1 \quad ...(2)$$

Now, $36\pi = \pi \times 0.1 \times 0.1 \times 1$

$$\Rightarrow \quad 1 = \frac{36\pi}{\pi \times 0.1 \times 0.1} = 3600 \text{ cm} = 36 \text{ m}.$$

112. Ans. (b)

Solution If r is the radius of the base then the circumference of the base = $2\pi r$ = 44 m

$$\therefore \quad r = \frac{44 \times 7}{2 \times 22} = 7 \text{ cm}$$

The height of the cone = h

Then, the volume of the air in the tent

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 9$$

= 462 cu m.

113. Ans. (c)

Solution Original area = $2\pi r^2$

New area = $4\pi(2r)^2 = 16\pi r^2$

Increase % = $\left(\frac{12\pi r^2}{4\pi r^2} \times 100\right)\% = 300\%$.

114. Ans. (c)

Solution Here, $x = y = 0$ and $z = -8$.

$\therefore$ Percentage change in volume

$$= \left[x + y + z + \frac{xy + yz + zx}{100} + \frac{xyz}{(100)^2}\right]\%$$

$$= \left[0 + 0 - 8 + \frac{0 \times 0 + 0 \times .8 + 0 \times .8}{(100)^2}\right]\%$$

$= -8\%$.

$\therefore$ Volume of the cylinder decreases by 8%.

115. Ans. (b)

Solution Here, $x = y = 20$ and $z = 0$.

$\therefore$ Percentage increase in volume

$$= \left[x + y + z + \frac{xy + yz + zx}{100} + \frac{xyz}{(100)^2}\right]\%$$

$$= \left[20 + 20 + 0 + \frac{20 \times 20 + 20 \times 0 \times 20 \times 0}{100} + \frac{20 + 20 \times 0}{(100)^2}\right]\%$$

$= (40 + 4)\% = 44\%$.

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. The circumference of the base of a circular cylinder is 6π cm. The height of the cylinder is equal to the diameter of the base. How many litres of water can it hold?

(a) 54 π c.c. (b) 36 π c.c.
(c) 0.054 π c.c. (d) 0.54 π c.c.

[Based on SSC (GL) Prel. Exam 2000]

2. The volume of a right circular cylinder whose height is 40 cm and circumference of its base is 66 cm is

(a) 55440 cm^3 (b) 3465 cm^3
(c) 7720 cm^3 (d) 13860 cm^3

[Based on SSC (GL) Prel. Exam 2000]

3. 12 spheres of the same size are made from melting a solid cylinder of 16 cm diameter and 2 cm height. The diameter of each sphere is

(a) 2 cm (b) 4 cm
(c) 3 cm (d) $\sqrt{3}$ cm

4. If the height of a right circular cone is increased by 200% and the radius of the base is reduced by 50%, then the volume of the cone

(a) increases by 25% (b) increases by 50%
(c) Remains unaltered (d) decreases by 25%

[Based on SSC (GL) Prel. Exam 2000]

5. A right triangle with sides 3 cm, 4 cm and 5 cm is rotated about the side of 3 cm to form a cone. The volume of the cone so formed is
(a) 16π (b) 12π
(c) 15π (d) 20π

[Based on SSC (GL) Prel. Exam 2000]

6. A rectangular paper 11 cm by 8 cm can be exactly wrapped to cover the curved surface of a cylinder of height 8 cm. The volume of the cylinder is
(a) 6 cm^3 (b) 77 cm^3
(c) 88 cm^3 (d) 121 cm^3

[Based on SSC (GL) Prel. Exam 2000]

7. The volumes of two spheres are in the ratio 8 : 27. The ratio of their surface areas is
(a) 4 : 9 (b) 2 : 3
(c) 4 : 5 (d) 5 : 6

[Based on SSC (GL) Prel. Exam 2000]

8. How many cubes, each of edge 3 cm, can be cut to form a cube of edge 15 cm?
(a) 25 (b) 27
(c) 125 (d) 144

[Based on SSC (GL) Prel. Exam 2000]

9. If the volumes of two cubes are in the ratio 27 : 64, then the ratio of their total surface areas is
(a) 27 : 64 (b) 3 : 4
(c) 9 : 16 (d) 3 : 8

[Based on SSC (GL) Prel. Exam 2000]

10. The base radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. The ratio of their volumes is
(a) 27 : 20 (b) 20 : 27
(c) 9 : 4 (d) 4 : 9

[Based on SSC (GL) Prel. Exam 2000]

11. The slant height of a conical mountain is 2.5 km and the area of its base is 1.54 km^2. Find the height.
(a) 2.2 km (b) 2.4 km
(c) 3 km (d) 3.11 km

[Based on SSC (GL) Prel. Exam 2000]

12. A hemisphere and a cone have equal bases. If their heights are also equal, the ratio of their curved surfaces will be
(a) $1:\sqrt{2}$ (b) $\sqrt{2}:1$
(c) 1 : 2 (d) 2 : 1

[Based on SSC (GL) Prel. Exam 2000]

13. Three solid metallic spheres of diameters 6 cm, 8 cm and 10 cm are melted and recast into a new solid sphere. The diameter of the new sphere is
(a) 4 cm (b) 6 cm
(c) 8 cm (d) 12 cm

[Based on SSC (GL) Prel. Exam 2000]

14. The base of a conical tent is 19.2 m in diameter and the height of its vertex is 2.8 m. The area of the canvas required to put up such a tent (in sq m) $\left(\text{taking } \pi = \frac{22}{7}\right)$ is nearly
(a) 3017.1 (b) 3170
(c) 301.7 (d) 30.17

[Based on SSC (GL) Prel. Exam 2000]

15. A cone of height 7 cm and base radius 1 cm is carved from a cuboidal block of wood 10 cm × 5 cm × 2 cm. Assuming $\pi = \frac{22}{7}$, the percentage wood wasted in the process is
(a) $92\frac{2}{3}\%$ (b) $46\frac{1}{3}\%$
(c) $53\frac{2}{3}\%$ (d) $7\frac{1}{3}\%$

[Based on SSC (GL) Prel. Exam 2000]

16. Three solid metallic balls of radii 3 cm, 4 cm and 5 cm are melted and mounted into a single solid ball. The radius of the new ball is
(a) 2 cm (b) 3 cm
(c) 4 cm (d) 6 cm

[Based on SSC (GL) Prel. Exam 2000]

17. The volume of a cuboid is twice the volume of a cube. If the dimensions of the cuboid are 9 cm, 8 cm and 6 cm the total surface area of the cube is
(a) 72 cm^2 (b) 216 cm^2
(c) 432 cm^2 (d) 108 cm^2

[Based on SSC (GL) Prel. Exam 2002]

18. The curved surface of a cylindrical pillar is 264 m^2 and its volume is 924 m^2. Taking $\pi = \frac{22}{7}$, find the ratio of its diameter to its height
(a) 7 : 6 (b) 6 : 7
(c) 3 : 7 (d) 7 : 3

[Based on SSC (GL) Prel. Exam 2002]

19. A room is 6 m long, 5 m broad and 4 m high. The maximum length of the rod that can be kept in the room is
(a) $\sqrt{61}$ m (b) $\sqrt{16}$ m
(c) $\sqrt{36}$ m (d) $\sqrt{77}$ m

20. The number of coins of radius 0.75 cm and thickness 0.2 cm to be melted to make a right circular cylinder of height 8 cm and base radius 3 cm is
(a) 640 (b) 460
(c) 500 (d) 600

[Based on SSC (GL) Prel. Exam 2002]

21. The area of the base of a right circular cone is 154 cm^2 and its height is 14 cm. Taking $\pi = \frac{22}{7}$,

the curved surface of the cone is

(a) $(154 \times \sqrt{5})$ cm^2 (b) $(154 \times \sqrt{7})$ cm^2
(c) 11 cm^2 (d) 5324 cm^2

[Based on SSC (GL) Prel. Exam 2002]

22. If the areas of the three adjacent faces of a cuboidal box are 120 cm^2, 72 cm^2 and 60 cm^2, respectively, then find the volume of box

(a) 7200 cm^3 (b) 720 cm^3
(c) 864 cm^3 (d) $(72)^2$ cm^3

[Based on SSC (GL) Prel. Exam 2002]

23. If a right circular cone of height 24 cm has a volume of 1232 cm^3, then the area of its curved surface is $\left(\text{Use } \pi = \frac{22}{7}\right)$

(a) 1254 cm^2 (b) 794 cm^2
(c) 550 cm^2 (d) 154 cm^2

[Based on SC (GL) Prel. Exam 2002]

24. Spheres A and B have their radii 40 cm and 10 cm, respectively. Ratio of surface area of A to the surface area of B is

(a) 1 : 16 (b) 4: 1
(c) 1 : 4 (d) 16 : 1

25. A cylindrical tube open at both ends is made of metal. The internal diameter of the tube is 11.2 cm and its length is 21 cm. The metal everywhere is 0.4 cm thick. The volume of the metal is $\left(\text{Use } \pi = \frac{22}{7}\right)$

(a) 316 cm^3 (b) 310 cm^3
(c) 306.24 cm^3 (d) 280.52 cm^3

[Based on SSC (GL) Prel. Exam 2003]

26. A wooden box measures 20 cm by 12 cm by 10 cm. Thickness of wood is 1 cm. Volume of wood to make the box (in cubic cm) is

(a) 960 (b) 519
(c) 2400 (d) 1120

[Based on SSC (GL) Prel. Exam 2003]

27. A hollow cylindrical tube 20 cm long is made of iron and its external and internal diameters are 8 cm and 6 cm, respectively. The volume of iron used in making the tube is $\left(\text{Use } \pi = \frac{22}{7}\right)$

(a) 1760 cu cm. (b) 880 cu cm.
(c) 440 cu cm. (d) 220 cu cm.

[Based on SSC (GL) Prel. Exam 2003]

28. The area of a side of a box is 120 sq cm. The area of the other side of the box is 72 sq cm. If the area of the upper surface of the box is 60 sq cm., find the volume of the box.

(a) 259200 cm^3 (b) 86400 cm^3
(c) 720 cm^3 (d) Cannot be determined
(d) None of these

[Based on BSRB Bangalor PO Exam 2000]

EXPLANATORY ANSWERS

1. Ans. (a)

Solution Radius of the base of the circular cylinder $= \frac{\text{Circumference}}{2\pi} = 3$ cm

$\therefore$ Height = 6 cm

$\therefore$ Volume of the cylinder

$= \pi \times (3)^2 \times 6 = 54\,\pi$ c.c.

2. Ans. (d)

Solution Circumference of the base of cylinder

$= 2\pi r = 66$

where r is the radius of the base.

$\therefore \quad r = \frac{66 \times 7}{44} = \frac{21}{2}$

$\therefore$ Volume of the cylinder

$= \pi r^2 h = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 40 = 13860$ cm^3.

3. Ans. (b)

Solution Let $2r$ be the diameter of each sphere.

$\therefore \quad \frac{\pi}{\frac{4}{3} \times \pi \times (r)^3} = 12 \Rightarrow r^3 = 8$

$\therefore \quad r = 2.$

4. Ans. (d)

Solution Let height = h and radius = $2r$

then, volume $= \frac{1}{3} \times \pi \times (2r)^2 \times h = \frac{4}{3}\pi r^2 h$

New height $= h + 2h = 3h$

New radius $= 2r - r = r$

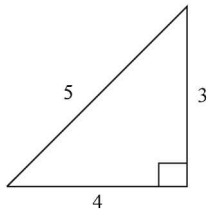
Then, volume $= \frac{1}{3}\pi r^2 (3h) = \pi r^2 h$

Decrease in volume $= \frac{4}{3}\pi r^2 h - \pi r^2 h = \frac{1}{3}\pi r^2 h$

% decrease in volume $= \frac{1/2\,\pi r^2 h}{4/3\,\pi r^2 h} \times 100 = 25.$

5. Ans. (b)

Solution Volume of the cone formed



$$= \frac{1}{3} \times \pi \times (3)^2 \times 4 = 12\,\pi.$$

6. Ans. (b)

Solution Area of the curved surface $= 2\pi rh = 88$

$\because h = 8$, therefore $2\pi r = 11$

$\Rightarrow r = \frac{7}{4}$

$\therefore$ Volume $= \pi r^2 h = \frac{22}{7} \times \frac{49}{16} \times 8 = 77 \text{ cm}^3$.

7. Ans. (b)

8. Ans. (c)

Solution Required no. of cubes

$$= \left(\frac{15}{3}\right)^3 = 5 \times 5 \times 5 = 125.$$

9. Ans. (c)

Solution $\left(\sqrt[3]{\frac{27}{64}}\right)^2 = \frac{9}{16}$.

10. Ans. (b)

Solution $\frac{2 \times 2 \times 5}{3 \times 3 \times 3} = 20 : 27$.

11. Ans. (b)

Solution Here, $\pi r^2 = 1.54 \text{ km}^2$

$\therefore$ $r = 0.7$ km, $l = 2.5$ km (given)

Therefore, $h = \sqrt{l^2 - r^2}$

$= \sqrt{(2.5)^2 - (0.7)^2}$

$= 2.4$ km.

12. Ans. (b)

Solution $\frac{2\pi r^2}{\pi r\sqrt{r^2 + r^2}} = \frac{2}{\sqrt{2}} = \frac{\sqrt{2}}{1}$.

13. Ans. (d)

Solution $\sqrt[3]{6^3 + 8^3 + 10^3} = 12$ cm.

14. Ans. (c)

Solution Here, radius (r) of the cone $= \frac{19.2}{2} = 9.6$ m

Slant height (l) of the cone $= \sqrt{(9.6)^2 + (2.8)^2} = 10$ m.

Hence, area of the canvas required

$= \pi rl = \frac{22}{7} \times 9.6 \times 10$

$= 301.7$ sq m.

15. Ans. (a)

Solution Volume of cuboidal block

$= 10 \times 5 \times 2 = 100 \text{ cm}^3$

Volume of cone carved from the cuboidal block

$= \frac{1}{3} \times \frac{22}{7} \times 1 \times 1 \times 7$

$= \frac{22}{7} \text{ cm}^3$

Wood wasted $= 100 - \frac{22}{7} = \frac{278}{3} = 92\frac{2}{3} \text{ cm}^3$

Hence, required % $= 92\frac{2}{3}$ %.

16. Ans. (d)

Solution Total volume of three solid metallic balls

$= 3 \times 3 \times 3 + 4 \times 4 \times 4 + 5 \times 5 \times 5$

$= 27 + 64 + 125$

$= 216$ = volume of single solid ball

Hence, required radius $= \sqrt[3]{216} = 6$ cm.

17. Ans. (b)

Solution Volume of the cuboid

$= 9 \times 8 \times 6 \text{ cm}^3 = 432 \text{ cm}^3$

Volume of the cube $= 216 \text{ cm}^3$

Side of the cube = 6 cm

The total surface area of the cube

$= 6 \times \text{side}^2 = 6 \times 6^2 = 216 \text{ cm}^2$.

18. Ans. (d)

Solution Curved surface of a pillar $= 264 \text{ m}^2$

or, $2\pi rh = 264 \text{ m}^2$

or, $rh = \frac{264 \times 7}{2 \times 22}$

$\therefore$ $rh = 42 \text{ m}^2$ ...(i)

Again, $\pi r^2 h = 924\ m^3$

$r^2 h = \frac{924}{22} \times 7$

$\therefore$ $r^2 h = 294$ ...(ii)

Dividing (ii) by (i)

$r = 7\ m \quad \therefore \quad h = \frac{42}{7} = 6$ m

Hence, required ratio $= 7 \times 2 : 6 = 7 : 3$.

19. Ans. (d)

Solution $\sqrt{6^2 + 5^2 + 4^2} = \sqrt{77}$.

20. Ans. (a)

Solution $\dfrac{\pi \times (3)^2 \times 8}{\pi (0.75)^2 \times 0.2} = 640.$

21. Ans. (a)

Solution $\pi r^2 = 154 \Rightarrow r^2 = \dfrac{154 \times 7}{77} \Rightarrow r = 7$

$\therefore$ Curved surface of the cone

$$= \pi rl = \frac{22}{7} \times 7 \times \sqrt{7^2 + 14^2}$$

$$= 22 \times \sqrt{245} = 154\sqrt{5} \text{ cm}^2.$$

22. Ans. (a)

Solution Let $l \times b = 120$, $l \times h = 72$ and $b \times h = 60$

$\therefore \dfrac{b}{h} = \dfrac{120}{72} = \dfrac{5}{3} \Rightarrow h = 6$

$\therefore l = 12, b = 10, h = 6$

$\therefore$ Volume of the box = 720 cu cm.

23. Ans. (c)

Solution Volume of circular cone $= \dfrac{1}{3}\pi r^2 h$

or, $\dfrac{1}{3} \times \dfrac{22}{7} \times 24 \times r^2 = 1232$

or, $r^2 = \dfrac{1232 \times 3 \times 7}{22 \times 24} = 22 \times 25 = 550 \text{ cm}^2.$

24. Ans. (d)

Solution Required ratio $= \dfrac{4\pi \times 40 \times 40}{4\pi \times 10 \times 10} = 16 : 1.$

25. Ans. (c)

Solution Volume of the metal

$$= \frac{22}{7} \times 21 \left[\left(\frac{11.2 + 2 \times 0.4}{2} \right)^2 - \left(\frac{11.2}{2} \right)^2 \right]$$

$$= 66\,[6^2 - 5.6^2] = 66 \times 11.6 \times 0.4$$

$$= 306.24 \text{ cm}^3.$$

26. Ans. (a)

Solution Volume of the wood

$$= [20 \times 12 \times 10] - [(20 - 2)(12 - 2)(10 - 2)]$$

$$= 2400 - 1400 = 960 \text{ cm}^3.$$

27. Ans. (c)

Solution Volume of iron

$$= \frac{22}{7} \times 20\,(4^2 - 3^2)$$

$$= \frac{22}{7} \times 20 \times 7 = 440 \text{ cm}^3.$$

28. Ans. (c)

Solution Volume of the box $= \sqrt{120 \times 72 \times 60}$

$$= 720 \text{ cm}^3.$$

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. A cistern is constructed to hold 200 litres. The base of the cistern is a square metre. What is the depth of the cistern? *A* cubic metre is 1000 litres
(a) 50 cm (b) 20 cm
(c) 25 cm (d) 40 cm

2. A certain cube of wood was bought for Rs. 768. If the wood costs Rs. 1500 per cu m, find the length of each edge of the cube
(a) 70 cm (b) 80 cm
(d) 90 cm (d) Cannot be determined

3. A cu m of copper weighing 9000 kg is rolled into a square bar 9 m long. An exact cube is cut off from the bar. How much does it weigh?
(a) $333\frac{1}{3}$ kg (b) $233\frac{1}{3}$ kg
(c) $334\frac{1}{3}$ kg (d) $333\frac{1}{2}$ kg

4. A cone, whose height is half of its radius, is melted to form a sphere. Find the ratio of radius of the sphere to that of the cone
(a) 2 : 1 (b) 1 : 2
(c) 3 : 2 (d) 1 : 1

5. Two cubes each of edge 10 cm are joined to form a single cuboid. What is the surface area of the new cuboid so formed?
(a) 1500 cm^2 (b) 1200 cm^2
(c) 1000 cm^2 (d) 1400 cm^2

6. A solid consists of a circular cylinder with exact fitting right circular cone placed on the top. The height of the cone is *h*. If total volume of the solid is 3 times the volume of cone, then the height of circular cylinder is
(a) $2h$ (b) $4h$
(c) $\dfrac{3h}{2}$ (d) $\dfrac{2h}{3}$

7. The number of solid spheres, each of diameter 5 cm, that could be moulded to form a solid metal cylinder of height 45 cm and diameter 4 cm; is
(a) 3 (b) 4
(c) 5 (d) 6

8. From a lead cube of edge 11 cm, how many spherical balls of diameter 0.5 cm, will be made?
(a) 2541 (b) 2415
(c) 2641 (d) 2741

9. Let S_1 and S_2 be the whole surface of a sphere and the curved surface of the circumscribed cylinder. Then, S_1 is equal to
(a) $2S_2$ (b) S_2
(c) $\frac{1}{2}S_2$ (d) $\frac{2}{3}S_2$

10. A solid spherical ball is prepared by melting a cone and a cylinder having the same height and same base diameter equal $2r$. Find the radius of the sphere
(a) $2^{1/3}r$ (b) $2^{1/3}r^2$
(c) $4^{1/3}r$ (d) $18^{1/3}r$

Test II

[Time: 10 min]

11. Three spherical metal balls of radii 6 cm, 8 cm and R cm are melted into a solid sphere of radius 12 cm. The value of R is
(a) 8 cm (b) 10 cm
(c) 14 cm (d) 18 cm

12. If 70 millilitre of paint is required for printing a surface area 100 sq m, how much paint is needed for painting a closed cylindrical drum with radius 5 m and height 21 m?
(a) 572 ml (b) 562 ml
(c) 532 ml (d) 642 ml

13. The water from a roof, 7 sq m in area, flows down into a cylindrical container of 700 sq. cm base. To what height will the water rise in the cylinder when there is a rainfall of 0.1 mm?
(a) 0.1 mm (b) 4 mm
(c) 1 cm (d) 10 cm

14. A metal sheet 27 cm long, 8 cm broad and 2 cm thick is melted into a cube. The difference between the surface area of two solids, is
(a) 284 cm^2 (b) 296 cm^2
(c) 286 cm^2 (d) 300 cm^2

15. A solid cylinder of glass, the diameter of whose base is 1.5 m and height 1 m, is melted and turned into a sphere. Find the diameter of the sphere so formed
(a) 2.5 m (b) 1.5 m
(c) 3.5 m (d) 4.5

16. How many lead shots each 0.3 cm in diameter can be made from a cuboid of dimensions 9 cm × 11 cm × 12 cm ?
(a) 42000 (b) 63000
(c) 55000 (d) 84000

17. A cylinder 6 cms in diameter is partially filled with water. A sphere 3 cm in diameter is gently dropped into the cylinder. To what further height will the water in the cylinder rise?
(a) 6 cm (b) 3 cm
(c) 2 cm (d) $\frac{1}{2}$ cm

18. A cone is 8.4 cm high and the radius of the base is 2.1 cm. It is melted and re-cast into a sphere. Determine the radius of the sphere.
(a) 1.2 cm (b) 2.1 cm
(c) 2.7 cm (d) 2.9 cm

19. The material of solid cone is converted into the shape of solid cylinder of equal radius. If the height of the cylinder is 5 cm, what is the height of the cone?
(a) 10 cm (b) 30 cm
(c) 15 cm (d) 5 cm

20. A sphere has the same curved surface as a cone of height 12 cm and base radius 5 cm. Find the radius to the nearest cm.
(a) 3 cm (b) 2 cm
(c) 4 cm (d) 5 cm

Answers

1. (b)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (a)
11. (b)	**12.** (a)	**13.** (d)	**14.** (c)	**15.** (b)	**16.** (d)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (c)

24

PERMUTATIONS AND COMBINATIONS

INTRODUCTION

We often come across questions such as the following:

1. In how many ways can four bottles be arranged in a row?
2. In how many ways can five students be seated at a round table?
3. In how many ways can a group of five people be selected out of a gathering of ten people?
4. In how many ways can 5 maps be selected out of 8 and displayed in a row?

Answers to these questions and many other important and more difficult ones can often be given without actually writing down all the different possibilities. In the present chapter we shall study some basic principles of the art of counting without counting which will enable us to answer such questions in an elegant manner.

FACTORIAL NOTATION

The continued product of first n natural numbers is called n factorial or factorial n and is denoted by $\lfloor n$ or $n!$

Thus, $\lfloor n$ or $n! = 1 \cdot 2 \cdot 3 \cdot 4 \ldots\ldots (n-1)n$

$= n \cdot (n-1) \cdot (n-2) \ldots\ldots 3 \cdot 2 \cdot 1$

(in reverse order)

Notes:

1. When n is a negative integer or a fraction, $n!$ is not defined. Thus, $n!$ is defined only for positive integers.
2. According to the above definition, $0! = 1$.
3. $n! = n(n-1)!$
4. $(2n)! = 2^n \cdot n!\ [1 \cdot 3 \cdot 5 \cdot 7 \ldots\ldots (2n-1)]$.

Illustration 1 Evaluate

(i) $\dfrac{30!}{28!}$ (ii) $\dfrac{9!}{5!3!}$

(iii) $\dfrac{12!-10!}{9!}$ (iv) $\dfrac{1}{3!}+\dfrac{1}{4!}+\dfrac{1}{5!}$.

Solution (i) $\dfrac{30!}{28!} = \dfrac{30 \times 29 \times 28!}{28!} = 30 \times 29 = 870.$

(ii) $\dfrac{9!}{5!\,3!} = \dfrac{9 \times 8 \times 7 \times 6 \times 5!}{5! \times 3 \times 2} = 504.$

(iii) $\dfrac{12!-10!}{9!} = \dfrac{12 \times 11 \times 10! - 10!}{9!}$

$= \dfrac{10!}{9!}[132-1]$

$= 10 \times 131$

$= 1310.$

(iv) $\dfrac{1}{3!}+\dfrac{1}{4!}+\dfrac{1}{5!} = \dfrac{4 \times 5}{3! \times 4 \times 5}+\dfrac{5}{4! \times 5}+\dfrac{1}{5!}$

$= \dfrac{20}{5!}+\dfrac{5}{5!}+\dfrac{1}{5!}$

$= \dfrac{26}{5!} = \dfrac{13}{60}.$

Illustration 2 Convert into factorials:

(i) 4.5.6.7.8.9.10.11. (ii) 2.4.6.8.10.

Solution (i) $4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 \cdot 11$

$= \dfrac{1.2.3.4.5.6.7.8.9.10.11}{1.2.3.}$

$= \dfrac{11!}{3!}.$

(ii) $2 \cdot 4 \cdot 6 \cdot 8 \cdot 10$

$= (2 \cdot 1) \cdot (2 \cdot 2) \cdot (2 \cdot 3) \cdot (2 \cdot 4) \cdot (2.5)$

$= (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) \cdot (1 \cdot 2 \cdot 3 \cdot 4 \cdot 5)$

$= 2^5 \cdot 5!.$

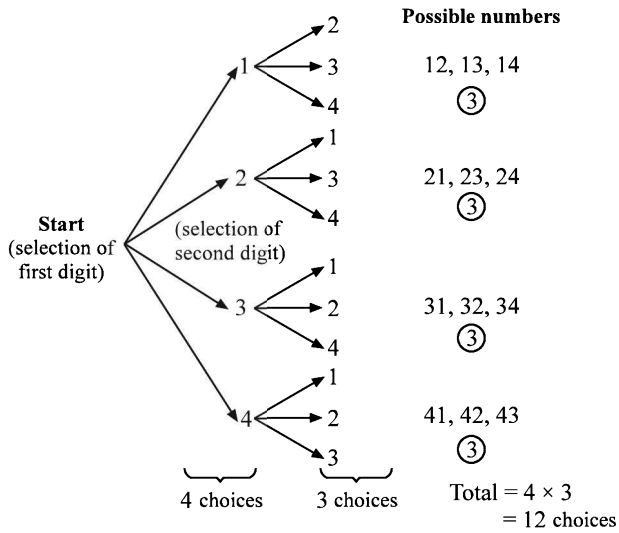
Fundamental Principle of Counting

Multiplication Principle If an operation can be performed in 'm' different ways; following which a second operation can be performed in 'n' different ways, then the two operations in succession can be performed in $m \times n$ different ways.

Illustration 3 How many numbers of two digits can be formed out of the digits 1, 2, 3, 4, no digit being repeated?

Solution The first digit can be any one of the four digits 1, 2, 3, 4, that is the first digit can be chosen in four ways. Having chosen the first digit, we are left with three digits from which the second digit can be chosen. Therefore, the possible ways of choosing the two digits are

Since the first digit can be chosen in four ways and for each choice of the first digit there are three ways of choosing the second digit, therefore, there are 4 × 3



ways, that is, 12 ways of choosing both the digits. Thus, 12 numbers can be formed.

Illustration 4 Anu wishes to buy a birthday card for her brother Manu and send it by post. Five different types of cards are available at the card-shop, and four different types of postage stamps are available at the post-office. In how many ways can she choose the card and the stamp?

Solution She can choose the card in five ways. For each choice of the card she has four choices for the stamp. Therefore, there are 5 × 4 ways, that is, 20 ways of choosing the card and the stamp.

Illustration 5 Mohan wishes to go from Agra to Chennai by train and return from Chennai to Delhi by air. There are six different trains from Agra to Chennai and five different flights from Chennai to Agra. In how many ways can he perform the journey?

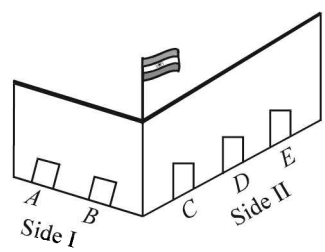
Solution Since he can choose any one of the six trains for going to Chennai, and for each such choice he has five choices for returning to Agra, he can perform the journey in 6 × 5 ways, that is, 30 ways.

Addition Principle

If an operation can be performed in 'm' different ways and another operation, which is independent of the first operation, can be performed in 'n' different ways, then either of the two operations can be performed in $(m + n)$ ways.

Note: The above two principles can be extended for any finite number of operations.

Illustration 6 Suppose there are 5 gates to a stadium, 2 on one side and 3 on the other. Sohan has to go out of the stadium. He can go out from any one of the 5 gates. Thus, the number of ways in which he can go out is 5. Hence, the work of going out through the gates on one side will be done in 2 ways and the work of going out through the gates on other side will be done in 3 ways. The work of going out will be done when Sohan goes out from side I or side II. Thus the work of going out can be done in (2 + 3) = 5 ways.



Note: Addition theorem of counting is also true for more than two opeartions.

Permutation

Each of the different arrangements which can be made by taking some or all of given number of things or objects at a time is called a *permutation.*

Note: Permutation of things means arrangement of things. The word arrangement is used if order of things is taken into account. Thus, if order of different things changes, then their arrangement also changes.

Notation

Let r and n be positive integers such that $1 \le r \le n$. Then, the number of permutations of n different things, taken r at a time, is denoted by the symbol ${}^nP_r$ or $P(n, r)$.

SOME BASIC RESULTS

1. ${}^nP_r$ or $P(n, r) = \dfrac{n!}{(n-r)!} = n(n-1)(n-2) \ldots [n-(r+1)],\ 0 \le r \le n.$

Illustration 7 Evaluate the following:

(i) $P(6, 4)$, (ii) $P(15, 3)$, (iii) $P(30, 2)$.

Solution (i) We have

$$P(6, 4) = \frac{6!}{(6-4)!} = \frac{6.5.4.3.2.1}{2.1} = 360.$$

(ii) We have

$$P(15, 3) = \frac{15!}{(15-3)!} = \frac{15!}{12!} = \frac{(15 \cdot 14 \cdot 13)(12!)}{12!} = 2730.$$

(iii) $P(30, 2) = \dfrac{30!}{(30-2)!} = \dfrac{30!}{28!} = \dfrac{(30.29)(28!)}{28!}$

$= 870.$

2. The number of permutations of n things, taken all at a time, out of which p are alike and are of one type, q are alike and are of second type and rest are all different $= \dfrac{n!}{p!\,q!}$.

Illustration 8 There are 5 red, 4 white and 3 blue marbles in a bag. They are drawn one by one and arranged in a row. Assuming that all the 12 marbles ae drawn, determine the number of different arrangements.

Solution Here, $n = 12$, $p_1 = 5$, $p_2 = 4$ and $p_3 = 3$

$\therefore$ The required number of different arrangements

$$= \frac{n!}{p_1!\,p_2!\,p_3!} = \frac{12!}{5!4!3!}$$

$$= \frac{12\cdot11\cdot10\cdot9\cdot8\cdot7\cdot6}{4\cdot3\cdot2\cdot1\cdot3\cdot2\cdot1} = \frac{11\cdot10\cdot9\cdot8\cdot7}{2}$$

$$= 990 \times 4 \times 7 = 27720.$$

3. The number of permutations of n different things taken r at a time when each thing may be repeated any number of times is n^r.

Illustration 9 In how many ways can 5 apples be distributed among 4 boys, there being no restriction to the number of apples each boy may get?

Solution The required number of ways $= 4^5$.

4. **Permutations under Restrictions**
   (a) Number of permutations of n different things, taken r at a time, when a particular thing is to be always included in each arrangement, is
   $$r \cdot {}^{n-1}P_{r-1}.$$
   (b) Number of permutations of n different things, taken r at a time, when s particular things are to be always included in each arrangement, is
   $$s!\,(r-(s-1)) \cdot {}^{n-s}P_{r-s}.$$
   (c) Number of permutations of n different things, taken r at a time, when a particular thing is never taken in each arrangement, is
   $${}^{n-1}P_r.$$
   (d) Number of permutations of n different things, taken all at a time, when m specified things always come together, is $m! \times (n - m + 1)!$
   (e) Number of permutations of n different things, taken all at a time, when m specified things never come together, is $n! - m! \times (n - m + 1)!$.

5. **Circular Permutations**
   (a) Number of circular arrangements (permutations) of n different things $= (n - 1)!$.

Illustration 10 In how many ways can eight people be seated at a round table?

Solution Required number of ways $= (8 - 1)!$
$= 7! = 5040.$

   (b) Number of circular arrangements (permutations) of n different things when clockwise and anticlockwise arrangements are not different, that is, when observation can be made from both sides $= \dfrac{1}{2}(n-1)!$.

Illustration 11 Find the number of ways in which n different beads can be arranged to form a necklace.

Solution Required number of arrangements

$$= \frac{1}{2}(5-1)! = \frac{1}{2} \times 4! = 12.$$

Combination

Each of the different groups or selections which can be made by taking some or all of a number of things (irrespective of order) is called a *combination*.

Note: Combination of things means selection of things. Obviously, in selection of things order of things has no importance. Thus, with the change of order of things selection of things does not change.

Notations The number of combinations of n different things taken r at a time is denoted by ${}^nC_r$ or $C(n, r)$.

Thus, $${}^nC_r = \frac{n!}{r!\,(n-r)!} \qquad (0 \le r \le n)$$

$$= \frac{{}^nP_r}{r!}$$

$$= \frac{n(n-1)(n-2)\ldots(n-r+1)}{r(r-1)(r-2)\ldots3\cdot2\cdot1}$$

If $r > n$, then ${}^nC_r = 0$.

Illustration 12 Evaluate:

(i) ${}^{11}C_3$ (ii) ${}^{10}C_8$ (iii) ${}^{100}C_{98}$

Solution (i) $${}^{11}C_3 = \frac{11!}{3!\,(11-3)!} = \frac{11!}{3!\,8!}$$

$$= \frac{11\times10\times9\times8!}{3\times2\times8!} = 165.$$

(ii) $${}^{10}C_8 = \frac{10!}{8!\,2!} = \frac{10\times9\times8!}{8!\times2} = 45.$$

(iii) $${}^{100}C_{98} = \frac{100!}{98!\,2!} = \frac{100\times99\times98!}{98!\times2} = 4950.$$

Some Important Results

1. ${}^nC_r = {}^nC_{n-r}$
2. ${}^nC_0 = {}^nC_n = 1$, ${}^nC_1 = n$
3. If ${}^nC_x = {}^nC_y$ then either $x = y$ or $y = n - x$ that is, $x + y = n$.
4. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
5. $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n-r+1}{r}$.

6. If n is even then the greatest value of ${}^nC_r$ is ${}^nC_{n/2}$.
7. If n is odd then the greatest value of ${}^nC_r$ is

$$ {}^nC_{\frac{n+1}{2}} \quad \text{or} \quad {}^nC_{\frac{n-1}{2}}. $$

8. $${}^nC_r = \frac{r \text{ decreasing numbers starting with } n}{r \text{ increasing numbers starting with } 1}$$
$$= \frac{n(n-1)(n-2)\ldots.(n-r+1)}{1\cdot 2\cdot 3\ldots..r}.$$
9. ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$.
10. ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots = 2^{n-1}$.
11. Number of combinations of n different things taken r at a time
    (a) when p particular things are always included $= {}^{n-p}C_{r-p}$.
    (b) when p particular things are never included $= {}^{n-p}C_r$
    (c) when p particular things are not together in any selection $= {}^nC_r - {}^{n-p}C_{r-p}$.

Illustration 13 In how many ways can 5 members forming a committee out of 10 be selected so that
(i) two particular members must be included.
(ii) two particular members must not be included.

Solution (i) When two particular members are included, then, we have to select $5 - 2 = 3$ members out of $10 - 2 = 8$.
$\therefore$ The required no. of ways

$$= C(8, 3) = \frac{8!}{3!\,5!} = \frac{8\times 7\times 6}{6} = 56.$$

(ii) When 2 particular members are not included, then, we have to select 5 members out of $10 - 2 = 8$.
$\therefore$ The required no. of ways

$$= C(8, 5) = \frac{8!}{5!\,3!}$$
$$= \frac{8\times 7\times 6}{6} = 56.$$

12. (a) Number of selections of r consecutive things out of n things in a row $= n - r + 1$.
    (b) Number of selections of r consecutive things out of n things along a circle
    $$\begin{cases} n, \text{ when } r < n \\ 1, \text{ when } r = n \end{cases}.$$
13. (a) Number of selections of zero or more things out of n different things
    ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$
    (b) Number of combinations of n different things selecting at least one of them is
    ${}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n - 1$.
    (c) Number of selections of r things ($r \le n$) out of n identical things is 1.
    (d) Number of selections of zero or more things out of n identical things $= n + 1$.
    (e) Number of selections of one or more things out of n identical things $= n$.
    (f) If out of $(p + q + r + t)$ things, p are alike of one kind, q are alike of second kind, r are alike of third kind and t are different, then the total number of selections is
    $(p + 1)(q + 1)(r + 1)\,2^t - 1$.
    (g) The number of ways of selecting some or all out of $p + q + r$ items where p are alike of one kind, q are alike of second kind and rest are alike of third kind is $[(p + 1)(q + 1)(r + 1)] - 1$.
14. (a) Number of ways of dividing $m + n$ different things in two groups containing m and n things, respectively ($m \ne n$):
    $${}^{m+n}C_m = \frac{(m+n)!}{m!\,n!}.$$
    (b) Number of ways of dividing $m + n + p$ different things in three groups containing m, n and p things, respectively ($m \ne n \ne p$):
    $$\frac{(m+n+p)!}{m!\,n!\,p!}.$$

SOME USEFUL SHORT-CUT METHODS

1. The number of triangles which can be formed by joining the angular points of a polygon of n sides as vertices are $\dfrac{n(n-1)(n-2)}{6}$.

Illustration 14 Find the number of triangles formed by joining the vertices of an octagon.

Solution The required number of triangles

$$= \frac{n(n-1)(n-2)}{6}$$
$$= \frac{8(8-1)(8-2)}{6} = \frac{8\times 7\times 6}{6} = 56.$$

2. The number of diagonals which can be formed by joining the vertices of a polygon of n sides are $\dfrac{n(n-3)}{2}$.

Illustration 15 How many diagonals are there in a decagon?

Solution The required number of diagonals

$$= \frac{n(n-3)}{2} = \frac{10(10-3)}{2}$$
$$= \frac{10\times 7}{2} = 35.$$

3. If there are 'm' horizontal lines and 'n' vertical lines then the number of different rectangles formed are given by $({}^mC_2 \times {}^nC_2)$.

Illustration 16 In a chess board there are 9 vertical and 9 horizontal lines. Find the number of rectangles formed in the chess board.

Solution The require number of rectangles.
$$= {}^9C_2 \times {}^9C_2 = 36 \times 36 = 1296.$$

4. These are 'n' points in a plane out of which 'm' points are collinear. The number of triangles formed by the points as vertices are given by ${}^nC_3 - {}^mC_3$.

Illustration 17 There are 14 points in a plane out of which 4 are collinear. Fine the number of triangles formed by the points as vertices.

Solution The required number of triangles
$$= {}^{14}C_3 - {}^4C_3 = 364 - 4 = 360.$$

5. There are 'n' points in a plane out of which 'm' points are collinear. The number of straight lines formed by joining them are given by
$$({}^nC_2 - {}^mC_2 + 1).$$

Illustration 18 There are 10 points in a plane out of which 5 are collinear. Find the number of straight lines formed by joining them.

Solution The required number of straight lines
$$= {}^nC_2 - {}^mC_2 + 1$$
$$= {}^{10}C_2 - {}^5C_2 + 1 = 45 - 10 + 1 = 36.$$

6. If there are 'n' points in a plane and no three points are collinear, then the number of triangles formed with 'n' points are given by $\frac{n(n-1)(n-2)}{6}$.

Illustration 19 Find the number of triangles that can be formed with 14 points in a plane of which no three points are collinear.

Solution The required number of triangles
$$= \frac{n(n-1)(n-2)}{6} = \frac{14\times 13\times 12}{6} = 364.$$

7. The number of quadrilaterals that can be formed by joining the vertices of a polygon of n sides are given by $\frac{n(n-1)(n-2)(n-3)}{24}$, where $n > 3$.

Illustration 20 Find the number of quadrilaterals that can be formed by joining the vertices of a septagon.

Solution The required number of quadrilaterals
$$= \frac{n(n-1)(n-2)(n-3)}{24}$$
$$= \frac{7(7-1)(7-2)(7-3)}{24}$$
$$= \frac{7\times 6\times 5\times 4}{24} = 35.$$

8. There are n points in a plane and no points are collinear, then the number of straight lines that can be drawn using these 'n' points are given by $\frac{n(n-1)}{2}$.

Illustration 21 How many straight lines can be drawn with 18 points on a plane of which no points are collinear?

Solution The required number of straight lines
$$= \frac{n(n-1)}{2} = \frac{18(18-1)}{2} = \frac{18\times 17}{2} = 153.$$

9. In a party every person shakes hands with every other person. If there was a total of H handshakes in the party, then the number of persons 'n' who were present in the party can be calculated from the equation:
$$\frac{n(n-2)}{2} = H.$$

Illustration 22 In a party every person shakes hands with every other person. If there was a total of 105 handshakes in the party, find the number of persons who were present in the party.

Solution Let 'n' be the number of persons present in the party.

We have, the equation
$$\frac{n(n-1)}{2} = H \Rightarrow \frac{n(n-1)}{2} = 105$$
$$\Rightarrow n(n-1) = 15 \times (15-1) \Rightarrow n = 15.$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. There are 6 candidates for 3 posts. In how many ways can the posts be filled?
(a) 120 (b) 130
(c) 100 (d) 110

2. From among the 36 teachers in a school, one principal and one vice-principal are to be appointed. In how many ways can this be done?
(a) 1360 (b) 1260
(c) 1060 (d) 1160

3. There are 15 buses running between Delhi and Mumbai. In how many ways can a man go to Mumbai and return by a different bus?
(a) 280 (b) 310
(c) 240 (d) 210

4. A teacher of a class wants to set one question from each of two exercises in a book. If there are 15 and 12 questions in the two exercises respectively, then in how many ways can the two questions be selected?
(a) 160 (b) 140
(c) 180 (d) 120

5. The students in a class are seated according to their marks in the previous examination. Once, it so happens that four of the students got equal marks and therefore the same rank. To decide their seating arrangement, the teacher wants to write down all possible arrangements one in each of separate bits of paper in order to choose one of these by lots. How many bits of paper are required?
(a) 24 (b) 12
(c) 48 (d) 36

6. For a set of five true-or-false questions, no student has written all the correct answers, and no two students have given the same sequence of answers. What is the maximum number of students in the class, for this to be possible?
(a) 31 (b) 21
(c) 51 (d) 41

7. A code word is to consist of two English alphabets followed by two distinct numbers between 1 and 9. For example, $CA23$ is a code word. How many such code words are there?
(a) 615800 (b) 46800
(c) 719500 (d) 410800

8. There are 6 multiple choice questions on an examination. How many sequences of answers are possible, if the first three questions have 4 choices each and the next three have 5 each?
(a) 6000 (b) 5000
(c) 4000 (d) 8000

9. There are six multiple choice questions in an examination. How many sequences of answers are possible, if the first two questions have 3 choices each, the next two have 4 choices each and the last two have 5 choices each?
(a) 3450 (b) 3300
(c) 3600 (d) 3400

10. Each section in the first year of plus two course has exactly 40 students. If there are 5 sections, in how many ways can a set of 4 student representatives be selected, one from each section?
(a) 2560000 (b) 246500
(c) 2240000 (d) 2360000

11. There are 5 letters and 5 directed envelopes. Find the number of ways in which the letters can be put into the envelopes so that all are not put in directed envelopes?
(a) 129 (b) 119
(c) 109 (d) 139

12. There horses H_1, H_2, H_3 entered a field which has seven portions marked P_1, P_2, P_3, P_4, P_5, P_6 and P_7. If no two horses are allowed to enter the same portion of the field, in how many ways can the horses graze the grass of the field?
(a) 195 (b) 205
(c) 185 (d) 210

13. How many different numbers of two digits can be formed with the digits 1, 2, 3, 4, 5, 6; no digits being repeated?
(a) 40 (b) 30
(c) 35 (d) 45

14. How many three-digit odd numbers can be formed from the digits 1, 2, 3, 4, 5, 6 when
(i) repitition of digits is not allowed
(ii) repitition of digits is allowed?
(a) (i) 60, (ii) 108 (b) (i) 50, (ii) 98
(c) (i) 70, (ii) 118 (d) (i) 80, (ii) 128

15. How many two-digit odd numbers can be formed from the digits 1, 2, 3, 4 5 and 8, if repitition of digits is allowed?
(a) 5 (b) 15
(c) 35 (d) 25

16. How many odd numbers less than 1000 can be formed using the digits 0, 2, 5, 7? (repetition of digits is allowed).
(a) 52 (b) 32
(c) 22 (d) 42

17. How many 3-digit numbers each less than 600 can be formed from the digits 1, 2, 3, 4, 5 and 9, if repetition of digits is allowed?
(a) 180 (b) 160
(c) 165 (d) 185

18. How many words (with or without meaning) of three distinct English alphabets are there?
(a) 15600 (b) 14650
(c) 12800 (d) 13700

19. How many numbers are there between 100 and 1000 in which all the digits are distinct?
(a) 548 (b) 648
(c) 748 (d) 756

20. How many integers between 1000 and 10000 have no digits other than 4, 5 or 6?
(a) 91 (b) 51
(c) 81 (d) 71

21. A number lock on a suitcase has 3 wheels each labelled with 10 digits from 0 to 9. If opening of the lock is a particular sequence of three digits with no repeats, how many such sequences will be possible?
(a) 720 (b) 760
(c) 680 (d) 780

22. A customer forgets a four-digit code for an Automatic Teller Machine (A.T.M.) in a bank. However, he remembers that this code consists of digits 3, 5, 6 and 9. Find the largest possible number of trials necessary to obtain the correct code.
(a) 12 (b) 24
(c) 48 (d) 36

23. If $(n + 2)! = 2550\,(n!)$, find n
(a) 38 (b) 35
(c) 49 (d) 43

24. If $(n + 1)! = 6[(n - 1)!]$, find n
(a) 6 (b) 4
(c) 8 (d) 2

25. If $\frac{n!}{2!(n-2)!}$ and $\frac{n!}{4!(n-4)!}$ are in the ratio 2 : 1, find the value of n.
(a) 0 (b) 1
(c) 2 (d) 3

26. Find n if ${}^{n}P_4 = 18.\ {}^{n-1}P_2$.
(a) 4 (b) 8
(c) 6 (d) 12

27. If $P(56, r + 6) : P(54, r + 3) = 30800 : 1$, find r.
(a) 51 (b) 41
(c) 31 (d) 43

28. In how many ways can 10 people line up at a ticket window of a cinema hall?
(a) 3628800 (b) 3482800
(c) 344800 (d) 3328800

29. How many words, with or without meaning, can be formed using all letters of the word EQUATION, using each letter exactly once?
(a) 38320 (b) 39320
(c) 40320 (d) 38400

30. Ten students are participating in a race. In how many ways can the first three prizes be won?
(a) 920 (b) 680
(c) 820 (d) 720

31. It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?
(a) 2880 (b) 2480
(c) 3680 (d) 3280

32. Four books, one each in Chemistry, Physics, Biology and Mathematics are to be arranged in a shelf. In how many ways can this be done?
(a) 12 (b) 36
(c) 24 (d) 48

33. There are three different rings to be worn in four fingers with at most one in each finger. In how many ways can this be done?
(a) 36 (b) 28
(c) 24 (d) 32

34. In an examination hall, there are four rows of chairs. Each row has 8 chairs one behind the other. There are two classes sitting for the examination with 16 students in each class. It is desired that in each row all students belong to the same class and that no two adjacent rows are allotted to the same class. In how many ways can these 32 students be seated?
(a) $2 \times 16! \times 16!$ (b) $2 \times 15! \times 15!$
(c) $2 \times 16! \times 15!$ (d) None of these

35. How many numbers lying between 1000 and 10000. can be formed by using the digits 1, 3, 5, 6, 7, 8, 9, no digits being repeated?
(a) 940 (b) 640
(c) 840 (d) 740

36. How many different numbers of six digits can be formed with the numbers 3, 1, 7, 0, 9, 5?
(a) 500 (b) 400
(c) 400 (d) 600

37. How many three-digit numbers are there, with no digits repeated?
(a) 648 (b) 548
(c) 848 (d) 748

38. If there are six periods in each working day of a school, in how many ways can one arrange 5 subjects such that each subject is allowed at least one period?
(a) 3500 (b) 3600
(c) 3550 (d) 3650

39. Four alphabets *E*, *K*, *S* and V, one in each, were purchased from a plastic warehouse. How many ordered pairs of alphabets, to be used as initials, can be formed from them?
(a) 18 (b) 12
(c) 14 (d) 16

40. There are 8 students appearing in an examination of which 3 have to appear in a Mathematics paper and the remaining 5 in different subjects. In how many ways can they be made to sit in a row if the candidates in Mathematics cannot sit next to each other?
(a) 14400 (b) 16400
(c) 15400 (d) 17400

41. Find how many words can be formed out of the letters of the word 'ORIENTAL' so that vowels always occupy the odd places.
(a) 576 (b) 578
(c) 676 (d) None of these

42. The number of different six-digit numbers that are divisible by 10, which can be formed using the digits 1, 2, 7, 0, 9, 5?
(a) 100 (b) 120
(c) 140 (d) 160

43. In how many ways can the letters of the word 'UNIVERSAL' be arranged? In how many of these will *E*, *R*, *S* always occur together?
(a) 32240 (b) 30240
(c) 30240 (d) 31240

44. The principal wants to arrange 5 students on the platform such that the boy SUNIL occupies the second position and such that the girl GITA is always adjacent to the girl NITA. How many such arrangements are possible?
(a) 12 (b) 8
(c) 14 (d) 16

45. In how many different ways, the letters of the work ALGEBRA can be arranged in a row if
(i) The two As are together?
(ii) The two As are not together?
(a) (i) 720, (ii) 1800 (b) (i) 620, (ii) 1600
(c) (i) 780, (ii) 1860 (d) (i) 720, (ii) 1600

46. In how many ways can 6 apples be distributed among 3 boys, there being no restriction to the number of apples each boy may get?
(a) 729 (b) 739
(c) 759 (d) 749

47. In how many different ways can the letters of the word 'KURUKSHETRA' be arranged?
(a) 4497600 (b) 4979600
(c) 4989600 (d) 4789600

48. In how many different ways can the letters of the word 'ALLAHABAD' be permuted?
(a) 7560 (b) 7840
(c) 7460 (d) 7650

49. How many three-digit numbers can be formed by using the digits 1, 3, 6 and 8, when the digits may be repeated any no. of times?
(a) 48 (b) 64
(c) 80 (d) 32

50. How many three-digit numbers can be formed by using the digits 0, 2, 3, 6, 8 when the digits may be repeated any number of times?
(a) 110 (b) 120
(c) 100 (d) None of these

51. How many different words can be formed with the letters of the word 'BHARAT'?
In how many of these *B* and *H* are never together?
(a) 240, 180 (b) 360, 240
(c) 320, 200 (d) 380, 260

52. How many arrangements can be made of the letters of the word 'ARRANGEMENT'?
(a) 2492800 (b) 249300
(c) 2494800 (d) 2491800

53. If the different permutations of the word EXAMINATION are listed as in a dictionary, how many items are there in this list before the first word starting with *E*?
(a) 906200 (b) 907200
(c) 908200 (d) 905200

54. How many 5-digit even numbers can be formed using the digits 1, 2, 5, 5, 4?
(a) 16 (b) 36
(c) 24 (d) 48

55. How many numbers greater than a million can be formed with the digits 2, 3, 0, 3, 4, 2, 3?
(a) 360 (b) 240
(c) 480 (d) None of these

56. Find the number of arrangements of the letters of the word 'ALGEBRA' without altering the relative position of the vowels and the consonants.
(a) 80 (b) 48
(c) 64 (d) 72

57. In how many ways can the letters of the word BALLOON be arranged so that two Ls do not come together?
(a) 900 (b) 1200
(c) 800 (d) 600

58. How many different signals can be transmitted by arranging 3 red, 2 yellow and 2 green flags on a pole? [Assume that all the 7 flags are used to transmit a signal.]
(a) 220 (b) 240
(c) 200 (d) 210

59. How many numbers can be formed with the digits 1, 2, 3, 4, 3, 2, 1, so that odd digits always occupy the odd places?
(a) 36 (b) 24
(c) 18 (d) 12

60. There are 5 gentlemen and four ladies to dine at a round table. In how many ways can they seat themselves so that no two ladies are together?
(a) 3280 (b) 2880
(c) 2080 (d) 2480

61. Three boys and three girls are to be seated around a table in a circle. Among the boys, X does not want any girl neighbour and the girl Y does not want any boy neighbour. How many such arrangements are possible?
(a) 6 (b) 4
(c) 8 (d) 2

62. How many different necklaces can be formed with 6 white and 5 red beads?
(a) 18 (b) 24
(c) 21 (d) 27

63. The Chief Ministers of 11 States of India meet to discuss the language problem. In how many ways can they seat themselves at a round table so that the Punjab and Madras Chief Ministers sit together?
(a) 725760 (b) 625760
(c) 925760 (d) 825760

64. If $C(n, 7) = C(n, 5)$, find n
(a) 15 (b) 12
(c) 18 (d) 2

65. If $C(n, 8) = C(n, 6)$, find $C(n, 2)$
(a) 91 (b) 81
(c) 61 (d) 71

66. If the ratio $C(2n, 3) : C(n, 3)$ is equal to 11 : 1, find n.
(a) 6 (b) 9
(c) 12 (d) 18

67. If ${}^{2n}C_r = {}^{2n}C_{r+2}$; find r.
(a) $n - 1$ (b) $n - 2$
(c) $n - 4$ (d) $n - 3$

68. If ${}^{18}C_r = {}^{18}C_{r+2}$; find ${}^{r}C_5$.
(a) 56 (b) 63
(c) 49 (d) 42

69. If $12\ {}^{n}C_2 = {}^{2n}C_3$: find n.
(a) 7 (b) 5
(c) 9 (d) 3

70. Find $\sum_{r=1}^{5} C(5, r)$
(a) 41 (b) 31
(c) 51 (d) 61

71. In how many ways can 5 sportsmen be selected from a group of 10?
(a) 272 (b) 282
(c) 252 (d) 242

72. In how many ways can a cricket team of 11 players be selected out of 16 players, if two particular player are always to be included?
(a) 2006 (b) 2004
(c) 2008 (d) 2002

73. In how many ways can a cricket team of 11 players be selected out of 16 players if one particular players is to be excluded?
(a) 1565 (b) 1365
(c) 1165 (d) 1265

74. In how many ways can a cricket team of 11 players be selected out of 16 players if two particular players are to be included and one particular player is to be rejected?
(a) 715 (b) 615
(c) 915 (d) 515

75. A question paper has two parts, part A and part B, each containing 10 questions. If the student has to choose 8 from part A and 5 from part B, in how many ways can he choose the question?
(a) 11240 (b) 12240
(c) 13240 (d) 11340

76. In how many ways can a football team of 11 players be selected from 15 players? In how many cases a particular player be included?
(a) 1101 (b) 1011
(c) 1001 (d) 1111

77. How many words, each of 3 vowels and 2 consonants, can be formed from the letters of the word 'INVOLUTE'?
(a) 2280 (b) 2480
(c) 2880 (d) 2680

78. How many lines can be drawn through 21 points on a circle?
(a) 310 (b) 210
(c) 410 (d) 570

79. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls, if each selection consists of 3 balls of each colour.
(a) 3000 (b) 1000
(c) 2000 (d) 4000

80. In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 courses are compulsory for every student?
(a) 45 ways (b) 35 ways
(c) 55 ways (d) 65 ways

81. In an examination, Yamini has to select 4 questions from each part. There are 6, 7 and 8 questions in Part I, Part II and Part III, respectively. What is the number of possible combinations in which she can choose the questions?
(a) 39650 (b) 37650
(c) 36750 (d) 38750

82. We wish to select 6 persons from 8, but if the person A is chosen, then B must be chosen too. In how many ways can the selection be made?

(a) 24 (b) 32
(c) 16 (d) 40

EXPLANATORY ANSWERS

1. Ans. (a)

Solution The 1st post can be filled up in 6 ways.
The 2nd post can be filled up in 5 way.
and the 3rd post can be filled up in 4 ways.

$\therefore$ By the principle of association, the three posts can be filled up in $6 \times 5 \times 4 = 120$ ways.

2. Ans. (c)

Solution There are 36 teachers and every one has equal chance of being selected as a principal. Hence, the principal can be appointed in 36 ways. When one person is appointed as principal, we are left with 35 teachers. Out of these 35 teachers, we can select one vice-principal. So, a vice-principal can be selected in 35 ways. Hence, the number of ways in which a principal and vice-principal can be selected $= 36 \times 35 = 1260$.

3. Ans. (d)

Solution The first event of going from Delhi to Mumbai can be performed in 15 ways as he can go by any of the 15 buses. But the event of coming back from Mumbai can be performed in 14 ways (a different bus is to be taken).

Hence, both the events can be performed in

$$15 \times 14 = 210 \text{ ways.}$$

4. Ans. (c)

Solution Since the first exercise contains 15 questions, the number of ways of choosing the first question is 15. Since the second exercise contains 12 questions, the number of ways of choosing the second question is 12. Hence, by the fundamental principle, two questions can be selected in $= 15 \times 12 = 180$ ways.

5. Ans. (a)

Solution We are given that four students got equal marks. On one bit of paper, one arrangement of rank is to be written. Let the students be named as P, Q, R and S.

Now, P can be treated as having rank I in 4 ways
Q can be treated as having rank II in 3 ways
R can be treated as having rank III in 2 ways
S can be treated as having rank IV in 1 way.

$\therefore$ Total number of bits of paper required for all arrangements $= 4 \times 3 \times 2 \times 1 = 24$.

6. Ans. (a)

Solution Question I can be answered in 2 ways.

Question II can be answered in 2 ways.

Similarly questions III, IV, V each can be answered in 2 ways. Hence, total number of possible different answers $= 2 \times 2 \times 2 \times 2 \times 2 = 32$.

There is only one sequence of all correct answers

Thus, the total number of sequences are $32 - 1 = 31$

[Since no student has written all correct answers]

Now, as no two students have given the same sequence of answers, hence the maximum number of students in the class = 31.

7. Ans. (b)

Solution (i) There are in all 26 English alphabets.

We have to choose 2 distinct alphabets.

First alphabet can be selected in 26 ways.

Second alphabet can be selected in 25 ways.

Again, out of 9 digits (1 to 9), first digit can be selected in 9 ways. Second digit can be selected in 8 ways

Thus, the number of distinct codes

$$= 26 \times 25 \times 9 \times 8$$
$$= 46800.$$

8. Ans. (d)

Solution Each of the first three questions can be answered in 4 ways.

Each of the last three questions can be answered in 5 ways.

$\therefore$ By the fundamental principle of counting, sequences of answers are

$$4 \times 4 \times 4 \times 5 \times 5 \times 5 = 64 \times 125 = 8000.$$

9. Ans. (c)

Solution First question can be answered in 3 ways.

Second question can be answered in 3 ways.

Third question can be answered in 4 ways.

Fourth question can be answered in 4 ways.

Fifth question can be answered in 5 ways.

Sixth question can be answered in 5 ways.

Hence, by fundamental principle of counting, the required number of sequences of answers

$$= 3 \times 3 \times 4 \times 4 \times 5 \times 5 = 3600.$$

10. Ans. (a)

Solution One student representative can be selected from section I in 40 ways.

One student representative can be selected from section II in 40 ways.

One student representative can be selected from section III in 40 ways.

One student representative can be selected from section IV in 40 ways

Hence, the number of ways in which a set of 4 student representatives can be selected

$$= 40 \times 40 \times 40 \times 40$$
$$= 2560000.$$

11. Ans. (b)

Solution Here, the first letter can be put in any one of the 5 envelopes in 5 ways. Second letter can be put in any one of the 4 remaining envelopes in 4 ways. Continuing in this way, we get the total number of ways in which 5 letters can be put into 5 envelopes $= 5 \times 4 \times 3 \times 2 \times 1 = 120$.

Since out of the 120 ways, there is only one way for putting each letter in the correct envelope. Hence, the number of ways of putting letters all not in directed envelopes $= 120 - 1 = 119$ ways.

12. Ans. (d)

Solution H_1 can graze the grass of either P_1 or P_2 or P_3 ..., or P_7, that is, in 7 ways. After H_1 entered the field, there are six portions left for H_2 as no two can enter into the same portion of the field. After first two entered the field, H_3 can enter the field in 5 ways.

$\therefore$ By the fundamental principle of counting, the three horses can graze the grass of the field in $7 \times 6 \times 5 = 210$ ways.

13. Ans. (b)

Solution We have to fill up two places (since numbers are of two digits.

The first place can be filled up in 6 ways, as any one of the six digits can be placed in the first place. The 2nd place can be filled up in 5 ways as no digit is to be repeated. Hence, both places can be filled up in $6 \times 5 = 30$ ways.

14. Ans. (a)

Solution (i) When repetition of digits is not allowed: Since we have to form a three-digit odd number, thus the digit at unit's place must be odd. Hence, the unit's place can be filled up by 1, 3 or 5, that is, in 3 ways.

Now, the ten's digit can be filled up by any of the remaining 5 digits in 5 ways and then the hundred's place can be filled up by the remaining 4 digits in 4 ways.

Hence, the number of three-digit odd numbers that can be formed $= 3 \times 4 \times 5 = 60$.

(ii) When repetition of digits is allowed: Again, the unit's place can be filled up by 1, 3, 5, that is, in 3 ways. But the ten's and hundred's place can be filled up by any of the six given digits in 6 ways each. (Since repetition is allowed)

Hence, the number of three-digit odd numbers that can be formed $= 3 \times 6 \times 6 = 108$.

15. Ans. (b)

Solution The number is odd if 1 or 3 or 5 appears in the unit's place. Therefore, there are three ways of filling the unit's place.

Since repetition of digits is allowed, the ten's place can be filled by any of the five digits 1, 3, 4, 5 and 8. Hence, number of two-digit odd numbers $= 3 \times 5 = 15$.

16. Ans. (b)

Solution Since the required numbers are less than 1000, they are 1-digit, 2-digit or 3-digit numbers.

(i) Only 2 one-digit odd numbers are possible, namely 5 and 7.

(ii) For two-digit odd numbers, the unit's place can be filled up by 5 or 7 in 2 ways and ten's place can be filled up by 2, 5 or 7 (not 0) in 3 ways.

$\therefore$ Possible 2-digit odd numbers $= 2 \times 3 = 6$

(iii) For three-digit odd numbers, the unit's place can be filled up by 5 or 7 in 2 ways. The ten's place can be filled up by any one of the given four digits in 4 ways. The hundred's place can be filled up by 2, 5 or 7 (not 0) in 3 ways.

$\therefore$ Possible 3-digit odd number $= 2 \times 4 \times 3 = 24$

$\therefore$ Required number of numbers $= 2 + 6 + 24 = 32$.

17. Ans. (a)

Solution Unit's place can be filled in 6 ways by any one of the digits 1, 2, 3, 4, 5 or 9. Also, ten's place can be filled in 6 ways. But hundred's place can be filled only is 5 ways using either 1, 2, 3, 4 or 5 ; 9 cannot be filled in hundred's place as the required number is < 600.

$\therefore$ Required number of numbers $= 6 \times 6 \times 5 = 180$.

18. Ans. (a)

Solution There are 26 distinct English alphabets. First alphabet can be chosen in 26 ways. Second alphabet can be chosen in 25 ways. Third alphabet can be chosen in 24 ways

Number of three letter words $= 26 \times 25 \times 24$
$= 15600.$

19. Ans. (b)

Solution Any number between 100 and 1000 is of three digits. Since the numbers should have distinct digits, repetition of digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 is not allowed.

Also 0 cannot be placed on the extreme left place. Hundredth place can be filled in 9 ways. Tenth place can be filled in 9 ways. Unit's place can be filled in 8 ways.

$\therefore$ The total three-digit numbers

$= 9 \times 9 \times 8 = 648.$

20. Ans. (c)

Solution Any number between 1000 and 10000 is of 4 digits. The unit's place can be filled up by 4 or 5 or 6, that is, in 3 ways.

Similarly, the ten's place can be filled up by 4 or 5 or 6, that is, in 3 ways. The hundred's place can be filled up by 4 or 5 or 6, that is in 3 ways and the thousand's place can be filled up by 4 or 5 or 6, that is, in 3 ways.

Hence, the required number of numbers

$= 3 \times 3 \times 3 \times 3 = 81.$

21. Ans. (a)

Solution On first wheel there can be 10 digits. On the second wheel there will be one of the 9 digits and on the third wheel there will be 8 digits. Therefore, the number of numbers is $10 \times 9 \times 8 = 720$.

22. Ans. (b)

Solution At the first place he can try any of the four digits hence in first trial he tries four digits. In the second place he will try three remaining digits. Similarly, he will try two and one digit at the third and fourth places.

Thus, the number of trials is $= 4 \times 3 \times 2 \times 1 = 24$.

23. Ans. (c)

Solution $(n + 2)! = 2550\ (n!)$

$\Rightarrow (n + 2)\ (n + 1)\ (n!) = 2550(n!)$

$\Rightarrow (n + 2)\ (n + 1) = 2550$

$\Rightarrow n^2 + 3n + 2 - 2550 = 0$

$\Rightarrow n^2 + 3n - 2548 = 0$

$\Rightarrow n^2 + 52n - 49n - 2548 = 0$

$\Rightarrow n(n + 52) - 49\ (n + 52) = 0$

$\Rightarrow (n - 49)\ (n + 52) = 0$

$\Rightarrow n = 49$ or, $n = -52$

$\Rightarrow n = 49$ as $n = -52$ is rejected being $n \varepsilon N$

$\therefore\ n = 49.$

24. Ans. (d)

Solution $(n + 1)\ ! = 6[(n - 1)!]$

$\Rightarrow (n + 1).\ n.\ [(n - 1)!] = 6[(n - 1)!]$

$\Rightarrow n^2 + n = 6 \Rightarrow n^2 + n - 6 = 0$

$\Rightarrow (n - 2)\ (n + 3) = 0$

$\therefore$ Either $n - 2 = 0$ or $n + 3 = 0$

$\Rightarrow n = 2$ or $n = -3$

n being natural number, so $n \neq -3$, $\therefore n = 2$.

25. Ans. (c)

Solution $\dfrac{n!}{2!(n-2)!} : \dfrac{n!}{4!(n-4)!} = 2 : 1$

$\Rightarrow \dfrac{n!}{2!(n-2)!} \times \dfrac{4!(n-4)!}{n!} = \dfrac{2}{1} \Rightarrow \dfrac{4!(n-4)!}{2!(n-2)!} = 2$

$\Rightarrow \dfrac{4 \times 3 \times 2!}{2!} \times \dfrac{(n-4)!}{(n-2)(n-3)(n-4)!} = 2$

$\Rightarrow 12 = 2(n - 2)(n - 3)$

$\Rightarrow 6 = n^2 - 5n + 6 \Rightarrow n^2 - 5n = 0$

$\Rightarrow n(n - 5) = 0 \Rightarrow n = 0$ or 5.

26. Ans. (c)

Solution ${}^nP_4 = 18 \cdot {}^{n-1}P_2$

$\Rightarrow \dfrac{n!}{(n-4)!} = 18 \cdot \dfrac{(n-1)!}{(n-1-2)!}$

$\Rightarrow \dfrac{n!}{(n-4)!} = 18 \cdot \dfrac{(n-1)!}{(n-3)!}$

$\Rightarrow \dfrac{n(n-1)!}{(n-4)!} = 18 \cdot \dfrac{(n-1)!}{(n-3)(n-4)!}$

$\therefore\ n = \dfrac{18}{n-3}$

i.e. $n^2 - 3n - 18 = 0 \Rightarrow (n - 6)(n + 3) = 0$

$\Rightarrow n = 6, -3$

But n cannot be negative

$\therefore\ n = 6.$

27. Ans. (b)

Solution $P\ (56,\ r + 6) : P\ (54,\ r + 3) = 30800 : 1$

$\Rightarrow \dfrac{56!}{(50-r)!} : \dfrac{54!}{(51-r)!} = 30800 : 1$

$\Rightarrow \dfrac{54!.55.56}{(50-r)!} : \dfrac{54!}{(50-r)!.(51-r)} = 30800 : 1$

$\Rightarrow \dfrac{3080}{1} : \dfrac{1}{51-r} = 30800 : 1$

$\Rightarrow 3080\ (51 - r) = \dfrac{30800}{1}$

$\Rightarrow 51 - r = 10 \Rightarrow r = 51 - 10$

$\therefore\ r = 41.$

28. Ans. (a)

Solution The required number of ways = number of permutations of 10 people taking all 10 at a time.

$= P(10,\ 10) = 10!$

$= 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 3628800.$

29. Ans. (c)

Solution The word EQUATION has exactly 8 letters which are all different.

$\therefore$ Number of words that can be formed = number of permutations of 8 letters taken all at a time

$= P\ (8,\ 8) = 8!$

$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 40320.$

30. Ans. (d)

Solution Out of 10 students, the first three prizes can be won in

$$^{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!}$$
$$= 10 \times 9 \times 8$$
$$= 720 \text{ ways.}$$

31. Ans. (a)

Solution Total number of candidates = 5 + 4 = 9. In the row of 9 positions, the even places are 2nd, 4th, 6th and 8th. Now, number of even places = 4. Number of women to occupy the even places = 4.

∴ Even places can be filled = $P(4, 4)$ ways

Number of men = 5

∴ The remaining 5 places can be filled by 5 men

$= P(5, 5)$ ways

∴ By the fundamental principle of counting:

∴ The required number of seating arrangements

$$= P(4, 4) \times P(5, 5) = 4! \times 5!$$
$$= 24 \times 120 = 2880.$$

32. Ans. (c)

Solution 4 different books can be arranged among themselves, in a shelf, in $P(4, 4)$

$$= 4 \times 3 \times 2 \times 1 = 24 \text{ ways.}$$

33. Ans. (c)

Solution Wearing 3 different rings in four fingers with at most one in each finger is equivalent to arranging 3 different objects in 4 places.

This can be done in $P(4, 3) = 4 \times 3 \times 2 = 24$ ways.

34. Ans. (a)

Solution There are four rows of chairs (say I, II, III, IV) consisting of 8 chairs each. It is desired that in each row, all students belong to the same class and no two adjacent rows are allotted to same class.

Therefore, one class can be seated in either I and III or in II and IV, that is in 2 ways.

Now, 16 students of this class can be arranged in 16 chairs in $^{16}P_{16} = 16!$ ways.

16 students of other class can be arranged in remaining 16 chairs in $^{16}P_{16} = 16!$ ways

∴ Total number of ways = $2 \times 16! \times 16!$.

35. Ans. (c)

Solution A number lying between 1000 and 10000 has four places which can be filled up out of 7 digits in $^7P_4 = 7 \times 6 \times 5 \times 4 = 840$ ways.

36. Ans. (d)

Solution The numbers that can be formed, by taking all six digits together

$$= {}^6P_6 = 6!$$

But we have to neglect the numbers which begin with zero. Now, the numbers in which zero comes in the 1st place = 5!

Hence, the required number $= 6! - 5!$
$$= 720 - 120$$
$$= 600.$$

37. Ans. (a)

Solution The required number of three-digit numbers = The permutations of the 10 objects 0, 1, 2, 3, 4, 5, 6,7, 8, 9, take 3 at a time, with the condition that 0 is not in the hundred's place.

$$= P(10, 3) - P(9, 2)$$
$$= \frac{10!}{7!} - \frac{9!}{7!}$$
$$= \frac{10 \times 9 \times 8 \times 7!}{7!} - \frac{9 \times 8 \times 7!}{7!}$$
$$= 10 \times 9 \times 8 - 9 \times 8$$
$$= 720 - 72 = 648.$$

38. Ans. (b)

Solution Six periods can be arranged for 5 subjects in $P(6, 5)$ ways

$$= 6 \times 5 \times 4 \times 3 \times 2$$
$$= 720$$

One period is left, which can be arranged for any of the five subjects.

∴ One left period can be arranged in 5 ways.

∴ The required number of arrangements

$$= 720 \times 5 = 3600.$$

39. Ans. (b)

Solution The required number of ordered pairs of alphabets, to be used as initials, can be formed = $P(4, 2)$

$$= \frac{4!}{2!} = 4 \times 3 = 12.$$

40. Ans. (a)

Solution Total number of candidates = 8.

5 different subjects candidates can be seated in $P(5, 5) = 5!$ ways.

In between 5 candidates there are 6 places for 8 Mathematics candidates.

∴ The Mathematics candidates can be seated in $P(6, 3)$ ways

∴ By fundamental principle of counting:

The required number of ways = $5! \times P(6, 3)$

$$= 120 \times \frac{6!}{3!}$$
$$= 120 \times 6 \times 5 \times 4 = 14400.$$

41. Ans. (a)

Solution The vowels in the word 'ORIENTAL' are: O, I, E and A

Total number of letters in the word 'ORIENTAL' = 8.

No. of vowels = 4

O	R	I	E	N	T	A	L
1	2	3	4	5	6	7	8
×		×		×		×	

$\because$ Vowels occupy odd places, that is 1, 3, 5 and 7. No. of odd places = 4

$\therefore$ 4 vowels can be arranged in 4 'X' marked places = $P(4, 4)$ ways = 4! ways

No. of consonants = 4.

$\therefore$ 4 consonants can be arranged in 4 places

$= P(4, 4)$ ways = 4! ways

$\therefore$ The required no. of words

$= 4! \times 4! = 24 \times 24 = 576.$

42. Ans. (b)

Solution The numbers are divisible by 10 if 0 is in the unit's place.

× × × × × 0

$\therefore$ The required numbers which are divisible by $10 = P(5, 5) = 5! = 120.$

43. Ans. (b)

Solution (i) The word 'UNIVERSAL' has 9 letters which are all different. These 9 letters can be arranged amongst themselves in 9! ways.

(ii) Three letters E, R, S can be taken to form one block. Thus, 7 letters U, N, I, V, ERS, A, L can be arranged in 7! ways.

Also, 3 letters E, R, S can be arranged amongst themselves in 3! ways.

$\therefore$ The total number of arrangements $= 7! \times 3!$

$= 30240.$

44. Ans. (a)

Solution 5 students are to be arranged on a platform. One boy SUNIL is fixed at second position.

$\therefore$ We have to arrange only 4 students. But GITA is always adjacent to NITA. Considering the two girls (NITA and GITA) as one, the three students can be arranged in 3! ways. NITA and GITA themselves can be arranged in 2! ways.

$\therefore$ The required number of arrangements

$= 2! \times 3! = (2 \times 1) \times (3 \times 2 \times 1) = 12.$

45. Ans. (a)

Solution ALGEBRA has seven letters where $2 - A$, $1 - L$, $1 - G$, $1 - E$, $1 - B$ and $1 - R$.

(i) Since two A's are always together, we take both the A's as one letter.

If P is the number of arrangements, then

$P = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720.$

(ii) Total number of permutations

$$q = \frac{7!}{2!} = 7 \times \frac{7\times6\times5\times4\times3\times2\times1}{2\times1} = 2520$$

In these permutations, in some permutations, two A's are together while in the rest they are not together.

Hence, the number of permutations in which two A's are not together is

$q - p = 2520 - 720 = 1800.$

46. Ans. (a)

Solution Each apple can be given to any one of the 3 boys and this can be done in 3 ways.

$\therefore$ The required number of ways $= 3^6 = 729.$

47. Ans. (c)

Solution Number of letters in the word 'KURUKSHETRA' is 11 of which 2 are K's, 2 are U's, 2 are R's and remaining are different.

$\therefore$ Required number of permutations $= \dfrac{11!}{2!\,2!\,2!}$

$= 4989600.$

48. Ans. (a)

Solution The word ALLAHABAD has 9 letters in all. The letter 'A' occurs 4 times, the letter 'L' occurs 2 times and the remaining three letters H, B, D each occur once.

$\therefore$ The require number of permutations

$$= \frac{9!}{4!\,2!\,1!\,1!} = \frac{9\times8\times7\times6\times5\times4!}{4!\times2}$$

$= 9 \times 8 \times 7 \times 3 \times 5 = 7560.$

49. Ans. (b)

Solution There are three places to be filled up to form a three-digit number. Since any digit may be repeated any number of times, each one of three places can be filled up by any of the given 4 digits in 4 ways.

Hence, the number of words that can be formed

$= 4^3 = 64.$

50. Ans. (c)

Solution There are in all 5 digits. Now, 0 cannot be placed at the hundredth place as in that case the number will not be 3-digited. Thus, hundredth place can be filled up by any of remaining 4 digits in 4 ways.

Since the digits may be repeated any number of times, each of the remaining two places can be filled up by any of the 5 digits in 5 ways each. Thus, the total number of such arrangements $= 5^2 = 25.$

Hence, the total number of words that can be formed

$= 4 \times 25 = 100.$

51. Ans. (b)

Solution Out of letters in the word 'BHARAT' two letters, that is, A's are alike.

$\therefore$ Number of permutations $= \frac{6!}{2!} = 360.$

Number of words in which B and H are never together

= Total number of words – number of words in which B and H are together

$= 360 - \frac{5!}{2!}.\ 2 = 360 - 120 = 240.$

52. Ans. (d)

Solution The given word consists of 11 letters out of which A occurs 2 times, R occurs 2 times, N occurs 2 times and E occurs 2 times and remaining three are different.

$\therefore$ Number of arrangements

$$= \frac{11!}{2!\,2!\,2!\,2!} = 2491800.$$

53. Ans. (b)

Solution Starting with A and arranging the other ten letters $A, E, I, I, M, N, N, O, T, X$ (not all distinct, I occurs twice, N occurs twice), there are

$$\frac{10!}{2!\,2!} = \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{2\times2}$$

= 907200 words.

$\therefore$ The number of items in the list before the first word starting with E is 907200.

54. Ans. (c)

Solution The 5-digit even numbers can be formed out of 1, 2, 5, 5, 4 by using either 2 or 4 in the unit's place. This can be done in 2 ways.

Corresponding to each such arrangement, the remaining 4 places can be filled up by any of the remaining four digits in $\frac{4!}{2!} = 12$ ways. [$\because$ 50 occurs twice]

Hence, the total number of words = 2 × 12 = 24.

55. Ans. (a)

Solution A number greater than a million has 7 places, and thus all the 7 given digits are to be used. But 2 is repeated twice and 3 is repeated thrice.

$\therefore$ Total number of ways of arranging these 7 digits amongst themselves

$$= \frac{7!}{2!.3!} = 420$$

But numbers beginning with zero are no more seven digited numbers, hence we have to reject those numbers which begin with zero, and such numbers are

$$= \frac{6!}{2!.3!} = 60.$$

Hence, the required number of arrangements,

= 420 – 60 = 360.

56. Ans. (d)

Solution There are 7 letters, of which two are A's and the rest are all different. The vowels A, A, E occupy 1st, 4th and 7th places. The number of ways in which they can be arranged in these places is $\frac{3!}{2!} = 3.$

The consonants L, G, B, R are all different. The number of ways in which they can be arranged in the remaining places in 4!. Since each way of arranging the vowels can be *associated* with each way of arranging the consonants, we find that the total number of arrangements = 3 × 4!, = 3 × 24 = 72.

57. Ans. (a)

Solution There are in all seven letters in the word BALLOON in which L occurs 2 times and O occurs 2 times.

$\therefore$ The number of arrangements of the seven letters of the word $= \frac{7!}{2!\times2!} = 1260.$

If two L's always come together, taking them as one letter, we have to arrange 6 letters in which O occurs 2 times.

$\therefore$ The no. of arrangements in which the two L's come together

$$= \frac{6!}{2!} = 6 \times 5 \times 4 \times 3 = 360.$$

Hence, the required no. of ways in which the two L's do not come together

= 1260 – 360 = 900.

58. Ans. (d)

Solution Here, $n = 3 + 2 + 2 = 7$

$p_1 = 3,\ p_2 = 2$ and $p_3 = 2$

$\therefore$ The required number of different signals

$$= \frac{n!}{p_1!\,p_2!\,p_3!} = \frac{7!}{3!\,2!\,2!} = \frac{7.6.5.4}{2.2}$$

$= 7.6.5 = 210.$

59. Ans. (c)

Solution The given digits are: 1, 2, 3, 4, 3, 2, 1. Out of these 1, 3, 3, 1 are odd digits.

The odd digits occupy the odd places, that is, they occupy the 1st, 3rd, 5th and 7th place.

$\therefore$ 4 odd places can be filled with 4 odd digits

$$= \frac{4!}{2!\,2!} = \frac{4.3}{2} = 6 \text{ ways.}$$

The three even places to be filled with the even digits 2, 4, 2.

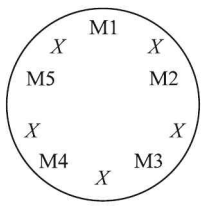
These places can be filled = $\frac{3!}{2!\,1!} = \frac{3.2!}{2!} = 3$ ways

Hence, the required number of numbers

$= 6 \times 3 = 18.$

60. Ans. (b)

Solution Refer to Fig. Let us first seat 5 gentlemen on the round table and this can be done in $(5 - 1)! = 24$ ways.



Since no two ladies are to sit together, they can occupy the places marked as 'X'

Number of 'X' signs = 5

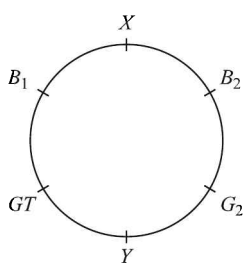
$\therefore$ 4 ladies can be arranged in five places in

$${}^5P_4 = \frac{5!}{1!} = 120 \text{ ways.}$$

$\therefore$ Required number of ways $= 24 \times 120 = 2880.$

61. Ans. (b)

Solution Let B_1, B_2 and X be three boys and G_1, G_2, Y be three girls. Since X does not want any girl neighbour, B_1, B_2 can be their neightbours.



Similarly, the girl Y does not want any boy neighbour, therefore G_1, G_2 are the only neighbours of Y. Now, B_1, B_2 can arrange themselves in 2! ways and G_1, G_2 can also arrange themselves in 2! ways.

Hence, the required number of permutations

$= 2! \times 2! = 4.$

62. Ans. (c)

Solution n = Total no. of beads = 6 + 5 = 11

$P_1 = 6,\ P_2 = 5$

$\therefore$ No. of different necklaces

$$= \frac{1}{2}\frac{(11-1)!}{6!\,5!} = \frac{10!}{2.6!\,5!}$$

$$= \frac{10.9.8.7.6!}{2.6!.5.4.3.2.1} = 3 \times 7 = 21.$$

63. Ans. (a)

Solution Treat the Punjab and Madras Chief Ministers as one then we have (P, M) + 9 others.

$\therefore$ We have to arrange 10 persons round a table. This can be done in $(10 - 1)! = 9!$ ways.

Corresponding to each of these 9! ways, the Punjab and Madras Chief Ministers can interchange their places in 2! ways. Associating the two operations, total number of ways

$= 9!\ 2! = (9 . 8 . 7 . 6 . 4 . 3 . 2 . 1)\ (2 . 1) = 725760.$

64. Ans. (b)

Solution We know that $C(n, r) = \frac{n!}{r!(n-r)!}$.

Now, $C(n, 7) = C(n, 5)$

$\Rightarrow \frac{n!}{7!(n-7)!} = \frac{n!}{5!(n-5)!}$

$\Rightarrow 5!(n - 5)! = 7!(n - 7)!$

$\Rightarrow [5!] . (n - 5)(n - 6)[(n - 7)!] = 7 . 6 . [5!](n - 7)!$

$\Rightarrow n^2 - 11n + 30 = 42$

$\Rightarrow n^2 - 11n + 30 - 42 = 0$

$\Rightarrow n^2 - 11n - 12 = 0$

$\Rightarrow (n - 12)\ (n + 1) = 0$

$\Rightarrow n - 12 = 0$ or, $n + 1 = 0$

$\Rightarrow n = 12$ or, $n = -1$.

But $n = -1$ is rejected as n is a non-negative integer,

$\therefore$ $n = 12$.

65. Ans. (a)

Solution $C(n, 8) = C(n, 6)$

$\Rightarrow \frac{n!}{8!(n-8)!} = \frac{n!}{6!(n-6)!}$

$\Rightarrow 6!(n - 6)! = 8!(n - 8)!$

$\Rightarrow 6!(n - 6)(n - 7)[(n - 8)!] = 8.7.6!(n - 8)!$

$\Rightarrow n^2 - 13n + 42 = 56$

$\Rightarrow n^2 - 13n + 42 - 56 = 0$

$\Rightarrow n^2 - 13n - 14 = 0$

$\Rightarrow n^2 - 13n - 14 = 0$

$\Rightarrow (n - 14)(n + 1) = 0$

$\Rightarrow n - 14 = 0$ or $n + 1 = 0$

$\Rightarrow$ $n = 14$ or $n = -1$.

But $n = -1$ is rejected as n is a non-negative integer

$\Rightarrow$ $n = 14$

$$\therefore \quad C(n, 2) = C(14, 2) = \frac{14!}{2!\,12!} = \frac{14.13.12!}{2.12!}$$

$= 91.$

66. Ans. (a)

Solution $C(2n, 3) : (C, 3) = 11 : 1$

$\Rightarrow \dfrac{(2n)!}{3!(2n-3)!} : \dfrac{n!}{3!(n-3)!} = 11 : 1$

$\Rightarrow (2n)(2n-1)(2n-2) : n(n-1)(n-2) = 11 : 1$

$\Rightarrow 2 \times 2(2n-1)(n-1) : (n-1)(n-2) = 11 : 1$

$\Rightarrow 4(2n-1) : n-2 = 11 : 1$

$\Rightarrow \dfrac{8n-4}{n-2} = \dfrac{11}{1}$

$\Rightarrow 11n - 22 = 8n - 4$

$\Rightarrow 11n - 8n = 22 - 4$

$\Rightarrow 3n = 18 \quad \therefore \quad n = 6.$

67. Ans. (a)

Solution ${}^{2n}C_r = {}^{2n}C_{2n-r}$ $\quad [\because {}^nC_r = {}^nC_{n-r}]$

But ${}^{2n}C_r = {}^{2n}C_{r+2}$ [Given]

$\therefore \quad {}^{2n}C_{2n-r} = {}^{2n}C_{r+2}$

$\therefore \quad 2n - r = r + 2 \Rightarrow r = n - 1.$

68. Ans. (a)

Solution ${}^{18}C_r = {}^{18}C_{18-r}$ $\quad [\because {}^nC_r = {}^nC_{n-r}]$

But ${}^{18}C_r = {}^{18}C_{r+r}$ [Given]

$\therefore \quad {}^{18}C_{18-r} = {}^{18}C_{r+2}$ or, $18 - r = r + 2 \Rightarrow r = 8.$

$\therefore \quad {}^rC_5 = {}^8C_5 = \dfrac{8!}{5!\,3!} = \dfrac{8\times7\times6\times5!}{3\times2\times1\times5!} = 56.$

69. Ans. (b)

Solution Here, $12 \cdot {}^nC_2 = {}^{2n}C_3$

$\therefore 12 \cdot \dfrac{n!}{2!(n-2)!} = \dfrac{(2n)!}{3!(2n-3)!} \quad \left[\because {}^nC_r = \dfrac{n!}{r!(n-r)!}\right]$

or, $12 \cdot \dfrac{n(n-1)(n-2)!}{2!.(n-2)!} = \dfrac{(2n)(2n-1)(2n-2)(2n-3)!}{6(2n-3)!}$

or, $\quad 6 \cdot n(n-1) = \dfrac{2n(2n-1)(2n-2)}{6}$

or, $\quad 18 \cdot (n-1) = (2n-1)(2n-2)$

or, $\quad 9 \cdot (n-1) = (2n-1)(n-1)$

i.e. $\quad 9 = 2n - 1 \Rightarrow n = 5.$

70. Ans. (b)

Solution $\sum_{r=1}^{5} C(5, r) = C(5, 1) + C(5, 2) + C(5, 3) + C(5, 4) + C(5, 5)$

$= \dfrac{5!}{1!\,4!} + \dfrac{5!}{2!\,3!} + \dfrac{5!}{3!\,2!} + \dfrac{5!}{4!\,1!} + \dfrac{5!}{5!\,0!}$

$= \dfrac{5}{1} + \dfrac{5.4}{2} + \dfrac{5.4}{2} + \dfrac{5}{1} + 1$

$= 5 + 10 + 10 + 5 + 1 = 31.$

71. Ans. (c)

Solution The require number of ways $= C(10, 5)$

$= \dfrac{10!}{5!\,5!} = \dfrac{10.9.8.7.6}{5.4.3.2} = 3.2.42 = 252.$

72. Ans. (d)

Solution 11 players can be selected out of 16 players in ${}^{16}C_{11}$ ways $= \dfrac{16!}{11!\,5!} = 4368$ ways.

When two particular players are always to be included, then 9 more players are to be selected out of remaining 14 players, which can be done in

${}^{14}C_9$ ways $= \dfrac{14!}{9!\,5!} = 2002$ ways.

73. Ans. (b)

Solution 11 players can be selected out of 16 players in ${}^{16}C_{11}$ ways $= \dfrac{16!}{11!\,5!} = 4368$ ways.

If one particular player is to be excluded, then selection is to be made of 11 players out of 15 players and this can be done in ${}^{15}C_{11}$ ways

$= \dfrac{15!}{11!\,4!} = 1365$ ways.

74. Ans. (a)

Solution 11 players can be selected out of 16 players in ${}^{16}C_{11}$ ways $= \dfrac{16!}{11!\,5!} = 4368$ ways

If two particular players are to be included and one particular player is to be rejected, then we have to select 9 more out of 13 in ${}^{13}C_9$ ways

$= \dfrac{13!}{9!\,4!} = 715$ ways.

75. Ans. (d)

Solution The required number of ways

$= C(10, 8) . C(10, 5)$

$= \dfrac{10!}{8!\,2!} \times \dfrac{10!}{5!\,5!} = \dfrac{10\times9}{2} \times \dfrac{10\times9\times8\times7\times6}{5\times4\times3\times2}$

$= 5 \times 9 \times 3 \times 2 \times 7 \times 6 = 11340.$

76. Ans. (c)

Solution Here, we want to find the number of ways of selecting 11 players out of 15.

$\therefore$ The required no. of ways

$= C(15, 11) = \dfrac{15!}{11!\,4!} = \dfrac{15\times14\times13\times12}{4\times3\times2\times1}$

$= 15 \times 7 \times 13 = 1365.$

A particular player to be included

So, we have to select 10 players out of 14.

$\therefore$ The required no. of ways

$= C(14, 10) = \dfrac{14!}{10!\,4!} = \dfrac{14.13.12.11}{4.3.2.1}$

$= 7.13.11 = 1001.$

77. Ans. (c)

Solution No. of letters in the word = 8

No. of vowels in the word = 4

(*I*, *O*, *U*, *E*)

No. of consonants in the word = 4

(*N*, *V*, *L*, *T*)

Out of 4 vowels, we have to select 3.

Out of 4 consonants, we have to select 2.

Also, we have to arrange 3 vowels and 2 consonants.

∴ The required no. of words

$$= C(4, 3) \,.\, C(4, 2) \,.5! = 4 \times \frac{4\times 3}{2} = 120 = 2880.$$

78. Ans. (b)

Solution We get a line by joining two points. If p is the number of lines from 21 points, then,

$$p = C(21, 2) = \frac{21!}{2!(21-2)!} = \frac{21\times 20(19!)}{2\times 1(19!)}$$

$$= 21 \times 10 = 210 \text{ lines.}$$

79. Ans. (c)

Solution If p is the required number of ways, then

$$p = C(6, 3) \times C(5, 3) \times C(5, 3)$$

$$= \frac{6!}{3!(6-3)!}\times\frac{5!}{3!(5-3)!}\times\frac{5!}{3!(5-3)!}$$

$$= \frac{6\times5\times4\times3!}{3\times2\times1\times3!}\times\frac{5\times4\times3!}{2\times1\times3!}\times\frac{5\times4\times3!}{2\times1\times3!}$$

$$= 5 \times 4 \times 5 \times 2 \times 5\times 2 = 2000.$$

80. Ans. (b)

Solution Out of available nine courses, two are compulsory. Hence, the student is free to select 3 courses out of 7 remaining courses: If p is the number of ways of selecting 3 courses out of 7 courses, then

$$p = C(7, 3) = \frac{7!}{3!(7-3)!} = \frac{7!}{3!\,4!} = \frac{7\times6\times5\times4!}{3\times2\times1\times4!}$$

$$= 7 \times 5 = 35 \text{ ways.}$$

81. Ans. (c)

Solution If p is the number of ways of selection, then

$$p = C(6, 4) \times C(7, 4) \times (8, 4) = \frac{6!}{4!\,2!}\times\frac{7!}{4!\,3!}\times\frac{8!}{4!\,4!}$$

$$= \frac{6\times5\times4!}{2\times1\times4!}\times\frac{7\times6\times5\times4!}{3\times2\times1\times4!}\times\frac{8\times7\times6\times5\times4!}{4\times3\times2\times1\times4!}$$

$$= 15 \times 35 \times 70 = 36750 \text{ ways.}$$

82. Ans. (c)

Solution If A and B both are not selected, then the number of permutations = $C(6, 6) = 1$.

If A and B both are selected, then we are to select 4 persons out of six persons.

The number of ways in which this can be done

$$= C(6, 4) = \frac{6!}{4!(6-2)!} = \frac{6\times5\times4!}{4!\times2\times1}$$

$$= 15$$

Hence, total number of permutations = 1 + 15 = 16.

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. If $1 \times 2 \times 3 \times \ldots \times n$ is denoted by $n!$, $(8! - 7! - 6!)$ is equal to

(a) $6 \times 8 \times 6!$ (b) $7 \times 8 \times 6!$
(c) $6 \times 7 \times 8!$ (d) $7 \times 8 \times 7!$

[Based on SSC (GL) Prel. Exam 2000]

2. If $1 \times 2 \times 3 \times 4 \ \ldots \times n = n!$, then $14! - 13! - 12!$ is equal to

(a) $14 \times 12 \times 12!$ (b) $14 \times 12 \times 13!$
(c) $13 \times 12 \times 12!$ (d) $14 \times 13 \times 13!$

[Based on SSC (GL) Prel. Exam 2000]

3. Letters of the word DIRECTOR are arranged in such a way that all the vowels come together. Find out the total number of ways for making such arrangements

(a) 4320 (b) 2720
(c) 2160 (d) 1120
(e) None of these **[Based on SBR PO 1999]**

4. On the occasion of a certain meeting each member gave shakehands to the remaining members. If the total shkehands were 28, how many members were present for the meeting?

(a) 14 (b) 7
(c) 9 (d) 8
(e) None of these **[Based on NABARD 1999]**

5. 8 men entered a lounge simultaneously. If each person shook hands with the other then find the total number of handshakes.

(a) 16 (b) 36
(c) 56 (d) 28

[Based on BSRB Bangalore PO 2000]

6. In how many different ways can the letter of the word JUDGE be arranged so that the vowels always come together?

(a) 48 (b) 24
(c) 120 (d) 60

[Based on SBI PO 2001]

7. From a group of 6 men and 4 women a committee of 4 persons is to be formed. In how many different ways can it be done so that the committee has at least one woman?

(a) 210 (b) 225

(c) 195 (d) 185
(e) None of these **[Based on SBI PO 2001]**

8. From a group of 6 men and 4 women a committee of 4 persons is to be formed. In how many different ways can it be done so that the committee has at least 2 men?
(a) 210 (b) 225
(c) 195 (d) 185
(e) None of these **[Based on SBI PO 2001]**

9. In how many different ways can the letters of the word LEADING be arranged so that the vowels always come together?
(a) 39600 (b) 360
(c) 40320 (d) 720
(e) None of these
[Based on Corporation Bank PO 2002]

10. In how many different ways can the letters of the word BOOKLET be arranged such that *B* and *T* always come together?
(a) 360 (b) 720
(c) 480 (d) 5040
(e) None of these **[Based on IBPS Bank PO 2002]**

11. In how many different ways can the letters of the word DETAIL be arranged in such a way that the vowels occupy only the odd positions?
(a) 120 (b) 60
(c) 48 (d) 32
(e) None of these
[Based on Andhra Bank (SO) Exam 2002]

12. In how many different ways can the letters of the word SOFTWARE be arranged in such a way that the vowels always come together?
(a) 13440 (b) 1440
(c) 360 (d) 120
(e) None of these
[Based on CANARA Bank PO 2003]

13. In how many different ways can a group of 4 men and 4 women be formed out of 7 men and 8 women?
(a) 2450 (b) 105
(c) 1170 (d) Cannot be determined
(e) None of these
[Based on CANARA Bank PO 2003]

14. In how many different ways can the letters of the word RUMOUR be arranged?
(a) 180 (b) 720
(c) 30 (d) 90
(e) None of these
[Based on PNB Management Trainne Exam 2003]

15. In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?
(a) 63 (b) 45
(c) 126 (d) 90
(e) None of these **[Based on SBI PO 2003]**

16. In how many different ways can the letters of the word CORPORATION be arranged in such a way that the vowels always come together?
(a) 840 (b) 86400
(c) 8400 (d) 1440
(e) None of these **[Based on SBI PO 2003]**

17. In a box carrying one dozen oranges, one-third have become bad. If 3 oranges are taken out from the box at random, what is the probability that at least one orange out of the three oranges picked up is good?
(a) $\frac{1}{55}$ (b) $\frac{54}{55}$
(c) $\frac{45}{55}$ (d) $\frac{3}{55}$
(e) None of these
[Based on Bank of Maharashtra PO 2003]

18. Out of a box containing 5 green, 4 yellow and 3 white marbles, 3 marbles are drawn at random. What is the probability that they are not of the same colour?
(a) $\frac{13}{44}$ (b) $\frac{41}{44}$
(c) $\frac{13}{55}$ (d) $\frac{52}{55}$
(e) None of these **[Based on IBPS PO 2002]**

19. Out of 15 students studying in a class, 7 are from Maharashtra, 5 are from Karnataka and 3 are from Goa. Four students are to be selected at random. What are the chances that at least one is from Karantaka?
(a) $\frac{12}{13}$ (b) $\frac{11}{13}$
(c) $\frac{10}{15}$ (d) $\frac{1}{15}$
(e) None of these **[Based on BSRB Patna PO 2001]**

20. The probability that a teacher will give one surprise test during any class meeting in a week is $\frac{1}{5}$. If a student is absent twice, what is the probability that he will miss at least one test?
(a) $\frac{4}{15}$ (b) $\frac{1}{15}$
(c) $\frac{91}{25}$ (d) $\frac{16}{125}$
(e) None of these
[Based on BSRB Bangalore PO 2000]

21. *A* speaks truth in 75% and *B* in 80% cases. In what percentage of cases are they likely to contradict each other when narrating the same incident?
(a) 35 (b) 30
(c) 25 (d) 20
(e) None of these
[Based on BSRB Chennai PO 2000]

22. In a box there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green?

(a) $\frac{7}{19}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{9}{21}$

(e) None of these

EXPLANATORY ANSWERS

1. Ans. (a)

Solution $8! - 7! - 6! = 8 \times 7 \times 6! - 7 \times 6! - 6!$
$= 6!(56 - 7 - 1) = 48 \times 6!.$

2. Ans. (a)

Solution $14! - 13! - 12! = 14 \times 13 \times 12! - 13 \times 12! - 12!$
$= 12!\ (14 \times 13 - 13 - 1)$
$= 12!\ (14 \times 13 - 14)$
$= 14 \times 12 \times 12!.$

3. Ans. (c)

Solution Taking all vowels (IEO) as a single letter (since they come together) there are six letters with 2 R's.

Hence, no. of arrangements $= \frac{6!}{2!} \times 3! = 2160$

[Three vowels can be arranged in 3! ways among themselves, hence multiplied by 3!].

4. Ans. (d)

Solution Let the total number of members in the meeting be x.

According to question, $x(x - 1) = 28 \times 2$

$\therefore \quad x = 8.$

5. Ans. (d)

Solution Total number of hand shakes

$$= {}^8C_2 = \frac{8 \times 7}{2} = 28.$$

6. Ans. (a)

Solution Required number $= 4! \times 2! = 24 \times 2 = 48.$

7. Ans. (c)

Solution Required number of ways

$$= {}^4C_1 \times {}^6C_3 + {}^4C_2 \times {}^6C_2 + {}^4C_3 \times {}^6C_1 + {}^4C_4$$

$$= 4 \times \frac{6 \times 5 \times 4}{1 \times 2 \times 3} + \frac{4 \times 3}{1 \times 2} \times \frac{6 \times 5}{1 \times 2} + \frac{4 \times 3 \times 2}{1 \times 2 \times 3} \times 6 + 1$$

$$= 80 + 90 + 24 + 1 = 195.$$

8. Ans. (d)

Solution Required number of ways

$$= {}^6C_2 \times {}^4C_2 + {}^6C_3 \times {}^4C_1 + {}^6C_4$$

$$= \frac{6 \times 5}{1 \times 2} \times \frac{4 \times 3}{2 \times 1} + \frac{6 \times 5 \times 4}{1 \times 2 \times 3} \times 4 + \frac{6 \times 5 \times 4 \times 3}{1 \times 2 \times 3 \times 4}$$

$$= 90 + 80 + 15 = 185.$$

9. Ans. (d)

Solution Here, no. of vowels = 3

No. of consonants = 4

Total no. of letters (when all the vowels and treated as one letter) = 5

Hence, these 5 letters can be arranged in 5!. Again, the three vowels can be arranged in 3! ways. Hence, total no. of words in which vowels occur together

$$= 5! \times 3! = 720.$$

10. Ans. (b)

Solution Treat B and T as a single letter

Then, the remaining letters (5 + 1 = 6) can be arranged in 6! ways. Since O is repeated twice, we have to divide by 2 and the B and T letters can be arranged in 2! ways.

$\therefore$ Total number of ways $= \frac{6! \times 2!}{2} = 720.$

11. Ans. (e)

Solution 3 vowels can be arranged in three odd places in 3! ways. Similarly, 3 consonants can be arranged in three even places in 3! ways. Hence, the total number of words in which vowels occupy odd positions

$$= 3! \times 3! = 6 \times 6 = 36 \text{ ways.}$$

12. Ans. (e)

Solution

O, A, E	S	F	T	W	R

When the vowels are always together, then treat all the vowels as a single letter and then all the letters can be arranged in 6! ways and also all three vowels can be arranged in 3! ways. Hence, required number of arrangements $= 6! \times 3! = 4320.$

13. Ans. (a)

Solution Required number of ways

$$= {}^7C_4 \times {}^8C_4$$

$$= \frac{7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 4} \times \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4}$$

$$= 35 \times 70 = 2450.$$

14. Ans. (a)

Solution Required Number

$$= \frac{6!}{2!\,2!} = \frac{6 \times 5 \times 4 \times 3}{1 \times 2} = 180.$$

15. Ans. (a)

Solution The group consists of 7 men and 3 women. We have to select 5 out of 7 men, that is, ${}^7C_5$, and 2 out of 3 women, that is, ${}^3C_2$.

Hence, required number of ways

$$= {}^7C_5 \times {}^3C_2 = \frac{7\times6}{2\times1}\times\frac{3\times2}{2\times1} = 63.$$

16. Ans. (e)

Solution There are eleven letters in the word CORPORATION. Among these letters, vowels are O, O, A, I and O. Among these vowels O occur thrice. Similarly, there are six consonants C, R, P, R, T and N. Among these consonants R occurs twice. According to the given condition, consider all the vowels as one letter. In this way 7 letters can be arranged in $\frac{7!}{2!}$ ways and the 5 vowels can be arranged in $\frac{5!}{3!}$ ways. Hence, the number of arrangements in which vowels are always together $= \frac{7!}{2!}\times\frac{5!}{3!} = 50400$.

17. Ans. (b)

Solution $n(S) = {}^{12}C_3 = \frac{12\times11\times10}{3\times2}$

$= 2 \times 11 \times 10 = 220$

Number of selection of 3 oranges out of the total 12 oranges $= {}^{12}C_3 = 2 \times 11 \times 10 = 220$.

Number of selection of 3 bad oranges out of the total 4 bad oranges $= {}^4C_3 = 4$.

$\therefore$ $n(E)$ = no. of desired selection of oranges

$= 220 - 4 = 216$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{216}{220} = \frac{54}{55}.$$

18. Ans. (b)

Solution Total number of ways of drawing 3 marbles

$$= {}^{12}C_3 = \frac{12\times11\times10}{1\times2\times3} = 220$$

Total number of ways of drawing marbles which are of same colour

$$= {}^5C_3 \times {}^4C_3 \times {}^3C_3 = 10 + 4 + 1 = 15$$

$\therefore$ Probability of same colour $= \frac{15}{220} = \frac{3}{44}$

$\therefore$ Probability of not same colour $= 1 - \frac{3}{44} = \frac{41}{44}$.

19. Ans. (b)

Solution Total possible ways of selecting 4 students out of 15 students $= {}^{15}C_4$

$$= \frac{15\times14\times13\times12}{1\times2\times3\times4} = 1365$$

The no. of ways of selecting 4 students in which no student belongs to Karnataka $= {}^{10}C_4$.

Hence, no. of ways of selecting at least one student from Karnataka $= {}^{15}C_4 - {}^{10}C_4 = 1155$

$$\therefore \quad \text{Probability} = \frac{1155}{1365} = \frac{77}{91} = \frac{11}{13}.$$

20. Ans. (b)

Solution The probability that the student is absent in the class $= \frac{2}{6} = \frac{1}{3}$

$\therefore$ Probability of missing his test will be

$$= \frac{1}{5}\times\frac{1}{3} = \frac{1}{15}.$$

21. Ans. (a)

Solution Let the probability of truth spoken by A and B be p_1 and p_2, respectively

$$\therefore \quad p_1 = \frac{3}{4} \text{ and } p_2 = \frac{4}{5}$$

They will contradict each other only when one speaks truth and the other is lying

i.e. $\frac{3}{4}\times\frac{1}{5}+\frac{4}{5}\times\frac{1}{4} = \frac{3}{20}+\frac{4}{20} = \frac{7}{20} = \frac{35}{100}$, i.e. 35%.

22. Ans. (e)

Solution If the ball drawn is neither red nor green, then it must be blue, which can be picked in ${}^7C_1 = 7$ ways. One ball can be picked from the total $(8 + 7 + 6 = 21)$ is ${}^{21}C_1 =$ 21 ways.

$$\therefore \quad \text{Required probability} = \frac{7}{21} = \frac{1}{3}.$$

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. In a monthly test, the teacher decides that there will be three questions one from each of Exercises 7, 8 and 9 of the text book. If there are 12 questions in Exercise 7, 18 in Exercise 8 and 9 in Exercise 9, in how many ways can the three questions be selected?

(a) 1894 (b) 2094
(c) 1944 (d) 2194

2. It has been decided that the flag of a newly formed forum will be in the form of three blocks, □□□ each coloured differently. If there are five different colours on the whole to choose from, how many such designs are possible?

(a) 60 (b) 50
(a) 80 (b) 70

3. A lady wants to select one cotton saree and one chiffon saree from a textile shop. If there are 10 cotton varieties and 12 chiffon varieties, in how many ways can she choose the two sarees?
(a) 124 (b) 120
(c) 132 (d) 128

4. A three-digit number is formed using the digits 0, 1, 2,..., 9. Find the number of odd numbers out of these three digit numbers.
(a) 380 (b) 650
(c) 450 (d) 550

5. How many numbers are there between 100 and 1000 in which all the digits are distinct?
(a) 548 (b) 648
(c) 748 (d) 848

6. How many numbers are there between 1000 and 10000 such that 9 is in the units place?
(a) 900 (b) 700
(c) 800 (d) 600

7. If $\frac{1}{9!}+\frac{1}{10!} = \frac{x}{11!}$, find x.
(a) 111 (b) 121
(c) 101 (d) 131

8. Find the value of n if $\frac{(2n)!}{3!(2n-3)!} : \frac{n!}{2!(n-2)!}$ = 44 : 3.
(a) 6 (b) 8
(c) 12 (d) 10

9. Find the value (s) of n such that
$30\ P(n, 6) = P(n + 2, 7);\ n \geq 6.$
(a) 21 (b) 25
(c) 23 (d) 19

10. How many signals can be given with 5 flags of different colours?
(a) 425 (b) 325
(c) 275 (d) 225

Test I

[Time: 10 min]

11. Find the number of different 8 letter words formed from the letters of the word EQUATION, if each word is to start with a vowel.
(a) 25200 (b) 24200
(c) 26200 (d) 27200

12. How many 6-digited telephone numbers can be constructed with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. If each number starts with 35 and no digits appears more than once?
(a) 178 (b) 1880
(c) 1680 (d) 1480

13. How many different words can be formed of the letters of the word 'MALENKOV' so that no two vowels are together?
(a) 15400 (b) 14400
(c) 16400 (d) 17400

14. In how many ways can the letters of the word MODESTY be arranged so that D and E are always together?
(a) 1640 (b) 1740
(c) 1440 (d) 1840

15. Ten guests are to be seated in a row. Three of them are to be seated together. Of the remaining 7, two do not wish to sit side by side. Find the number of possible arrangements.
(a) 191400 (b) 181440
(c) 171400 (d) 161400

16. In how many ways can the letters of the word ''STRANGE'' be arranged so that the vowels may appear in the odd places.
(a) 1640 (b) 1440
(c) 1740 (d) 1840

17. In how many ways can 6 different rings be worn on the four fingers of hand?
(a) 4^8 (b) 4^7
(c) 4^6 (d) 4^4

18. In how many ways can 7 beads of different colours form necklace?
(a) 360 (b) 380
(c) 320 (d) 340

19. If there are 12 persons in a party, and if each two of them shake hands with each other, how many handshakes happen in the party?
(a) 44 (b) 66
(c) 77 (d) 55

20. A man has 7 friends. In how many ways can he invite one or more of them to a party?
(a) 127 (b) 137
(c) 227 (d) 327

Answers

1. (c)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (b)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (d)	**10.** (b)
11. (a)	**12.** (c)	**13.** (b)	**14.** (c)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (a)	**19.** (b)	**20.** (a)

25

PROBABILITY

INTRODUCTION

The word *probability* or *chance* is very frequently used in day-to-day life. For example, we generally say, 'He may come today' or 'probably it may rain tomorrow' or 'most probably he will get through the examination'. All these phrases involve an element of uncertainty and probability is a concept which measures this uncertainties. The probability when defined in simplest way is the chance of occurring of a certain event when expressed quantitatively, i.e. probability is a quantitative measure of the certainty.

The probability has its origin in the problems dealing with games of chance such as gambling, coin tossing, die throwing and playing cards. In all these cases the outcome of a trial is uncertain. These days probability is widely used in business and economics in the field of predictions for future.

The following remarks may be important for learning this chapter on probability.

1. **Die:** A die is a small cube used in games of chance. On its six faces dots are marked as

   . .. ... : : : . : ⋮ ⋮

   Plural of die is dice. The outcome of throwing (or tossing) a die is the number of dots on its uppermost face. An ace on a die means one dot.
2. **Cards:** A pack (or deck) of playing cards has 52 cards, divided into four suits:

   (i) Spades हुकम (♠) (ii) Clubs चिड़ा (♣)

   (iii) Hearts पान (♥) (iv) Diamonds ईंट (♦)

   Each suit has 13 cards, nine cards numbered 2 to 10, an Ace (इक्का), a King (बादशाह), Queen (बेगम) and a Jack or Knave (गुलाम). Spades and Clubs are black-faced cards while Hearts and Diamonds are red-faced cards. The Aces, Kings, Queens and Jacks are called *face cards* and other cards are called *number cards*. The Kings, Queens and Jacks are called *court cards*.
3. The number of combinations of n objects taken r at a time ($r \le n$) is denoted by $C(n, r)$ or ${}^nC_r$ and is defined as

$$ {}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)\ldots \text{ to } r \text{ factors}}{1\cdot 2\cdot 3\ldots r.} $$

Illustration 1 ${}^5C_3 = \dfrac{5\cdot 4\cdot 3}{1\cdot 2\cdot 3} = 10$, ${}^nC_0 = 1$ and ${}^nC_n = 1$.

If $r > \dfrac{n}{2}$, then it is better to simplify ${}^nC_r$ as ${}^nC_{n-r}$.

Illustration 2 ${}^{52}C_{50} = {}^{52}C_{52-50} = {}^{52}C_2 = \dfrac{52.51}{2.1}$

$= 26.51 = 1326.$

When, $r > n$, ${}^nC_r = 0$.

Some Important Terms and Concepts

Random Experiment or Trial: The performance of an experiment is called a *trial*. An experiment is characterised by the property that its observations under a given set of circumstances do not always lead to the same observed outcome but rather to the different outcomes. If in an experiment all the possible outcomes are known in advance and none of the outcomes can be predicted with certainity, then such an experiment is called a *random experiment*.

For example, tossing a coin or throwing a die are random experiments.

Event: The possible outcomes of a trial are called *events*. Events are generally denoted by capital letters A, B, C and so on.

Illustration 3 (i) When a coin is tossed the outcome of getting a head or a tail is an event.

(ii) When a die is thrown the outcome of getting 1 or 2 or 3 or 4 or 5 or 6 is an event.

Sample Space: The set of all possible outcomes of an experiment is called a *sample space*. We generally denote it by S.

Illustration 4 (i) When a coin is tossed, $S = \{H, T\}$ where H = head, T = tail.

(ii) When a die is thrown, $S = \{1, 2, 3, 4, 5, 6\}$.

(iii) When two coins are tossed simultaneously,

$S = \{HH, HT, TH, TT\}$

Equally Likely Events: Events are said to be *equally likely* if there is no reason to expect any one in preference to other. Thus, equally likely events mean outcome is as likely to occur as any other outcome.

Illustration 5 In throwing a die, all the six faces (1, 2, 3, 4, 5, 6) are equally likely to occur.

Simple and Compound Events

In the case of *simple events* we consider the probability of happening or non-happening of single events.

Illustration 6 We might be interested in finding out the probability of drawing an ace from a pack of cards.

In the case of *compound events* we consider the joint occurrence of two or more events.

Illustration 7 If from a bag, containing 8 red and 5 green balls, two successive draws of 2 balls are made, we shall be finding out the probability of getting 2 red balls in the first draw and 2 green balls in the second draw. We are thus dealing with a compound event.

Exhaustive Events: It is the total number of all possible outcomes of any trial.

Illustration 8 (i) When a coin is tossed, either head or tail may turn up and therefore, there are two exhaustive cases.

(ii) There are six exhaustive cases or events in throwing a die.

(iii) If two dice are thrown simultaneously, the possible outcomes are

(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

Thus, in this case, there are $36(=6^2)$ ordered pairs. Hence, the number of exhaustive cases in the simultaneous throw of two dice is 36.

(iv) Three dice are thrown, the number of exhaustive cases is 6^3, i.e. 216.

Algebra of Events

If A and B are two events associated with sample space S, then

(i) $A \cup B$ is the event that either A or B or both occur.

(ii) $A \cap B$ is the event that A and B both occur simultaneously.

(iii) $\overline{A}$ is the event that A does not occur.

(iv) $\overline{A} \cap \overline{B}$ is an event of non-occurrence of both A and B, i.e. none of the events A and B occurs.

Illustration 9 In a single throw of a die, let A be the event of getting an even number and B be the event of getting a number greater than 2. Then,

$$A = \{1, 3, 5\}, B = \{3, 4, 5, 6\}$$

$$\therefore \quad A \cup B = \{1, 3, 4, 5, 6\}$$

$A \cup B$ is the event of getting an odd number or a number greater than 2.

$$A \cap B = \{3, 5\}.$$

$A \cap B$ is the event of getting an odd number greater than 2.

$\overline{A} = \{2, 4, 6\}$ [Those elements of S which are not in A.]

$\overline{A}$ is the event of not getting an odd number, i.e. getting an even number.

$$\overline{B} = \{1, 2\}.$$

$\overline{B}$ is the event of not getting a number greater that 2, i.e. getting a number less than or equal to 2.

$$\overline{A} \cap \overline{B} = \{2\}.$$

$\overline{A} \cap \overline{B}$ is the event of neither getting an odd number nor a number greater than 2.

Mutually Exclusive Events

In an experiment, if the occurrence of an event precludes or rules out the happening of all the other events in the same experiment.

Illustration 10 (i) When a coin is tossed either head or tail will appear. Head and tail cannot appear simultaneously. Therefore, occurrence of a head or a tail are two mutually exclusive events.

(ii) In throwing a die all the 6 faces numbered 1 to 6 are mutually exclusive since if any one of these faces comes, the possibility of others in the same trial, is ruled out.

Note: A and B are mutually exclusive events $\Leftrightarrow A \cap B = \phi$, i.e. A and B are disjoint sets.

Illustration 11 (i) If the random experiment is '*a* die is thrown' and A, B are the events, A : the number is less than 3; B : the number is more than 4, then $A = \{1, 2\}$, $B = \{5, 6\}$.

$A \cap B = \phi$, thus A and B are mutually exclusive events.

(ii) If the random experiment is '*a* card is drawn from a well-shuffled pack of cards' and A, B are the events A: the card is Black; B: the card is an ace.

Since a black card can be an ace, $A \cap B \neq \phi$, thus A and B are not mutually exclusive events.

Mutually Exclusive and Exhaustive Events

Events $E_1, E_2, \ldots, E_n$ are called mutually exclusive and exhaustive if $E_1 \cup E_2 \cup \ldots \cup E_n = S$, i.e. $\bigcup_{i=1}^{n} E_i = S$ and $E_i \cap E_j = \phi$ for all $i \neq j$.

For example, in a single throw of a die, let A be the event of getting an even number and B be event of getting odd numbers, then

$$A = \{2, 4, 6\}, B = \{1, 3, 5\}$$

$$A \cap B = \phi, A \cup B = \{1, 2, 3, 4, 5, 6\} = S$$

$\therefore$ A and B are mutually exclusive and exhaustive events.

Illustration 12 Two dice are thrown and the sum of the numbers which come up on the dice noted. Let us consider the following events:

A : "the sum is even"

B : "the sum is a multiple of 3"

C : "the sum is less than 4"

D : "the sum is greater than 11"

Which pairs of these events are mutually exclusive?

Solution There are $6 \times 6 = 36$ elements in the sample space (Refer to Example 2).

A is the event "the sum is even". It means we have to consider those ordered pairs (x, y) in which $x + y$ is even. Thus,

$A = [(1, 1), (2, 2)\ (1, 3), (1, 5), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6)$
$(5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)]$.

Similarly,

$B = [(1, 2), (2, 1), (1, 5), (5, 1), (3, 3), (2, 4), (4, 2), (3, 6), (6, 3), (4, 5), (5, 4), (6, 6)]$

$C = [(1, 1), (2, 1), (1, 2)]$ $D = [(6, 6)]$.

We find that $A \cap B = [(1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 6)] \neq \phi$

Thus, A and B are not mutually exclusive.

Similarly, $A \cap C \neq \phi$, $A \cap D \neq \phi$, $B \cap C \neq \phi$, $B \cap D \neq \phi$, $C \cap D = \phi$. Thus, C and D are mutually exclusive.

PROBABILITY OF AN EVENT

The probability of an event is defined in the following two ways:

(i) Mathematical (or *a priori*) definition

(ii) Statistical (or empirical) definition.

Mathematical Definition of Probability: Probability of an event A, denoted as $P(A)$, is defined as

$$P(A) = \frac{\text{Number of cases favourable to } A}{\text{Number of possible outcomes}}$$

Thus, if an event A can happen in m ways and fails (does not happen) in n ways and each of $m + n$ ways is equally likely to occur then the probability of happening of the event A (also called success of A) is given by

$$P(A) = \frac{m}{m+n}$$

and that the probability of non-occurrence of the A (also called its failure) is given by

$$P\,(\text{not } A) \text{ or } P(\overline{A}) = \frac{n}{m+n}$$

If the probability of the happening of a certain event is denoted by p and that of not happening by q, then

$$p + q = \frac{m}{m+n} + \frac{n}{m+n} = 1.$$

Here, p, q are non-negative and cannot exceed unity, i.e. $0 \leq p \leq 1$ and $0 \leq q \leq 1$.

When $p = 1$, then the event is certain to occur.

When $p = 0$, then the event is impossible. For example, the probability of throwing eight with a single die is zero.

Probability as defined above is sometimes called **Priori Probability,** i.e. it is determined before hand, that is, before the actual trials are made.

Illustration 13 A coin is tossed once. What are all possible outcomes? What is the probability of the coin coming up "tails"?

Solution The coin can come up either "heads" (H) or "tails" (T). Thus, the set S of all possible outcomes is $S = \{H, T\}$

$$\therefore \qquad P(T) = \frac{1}{2}$$

Illustration 14 What is the probability of getting an even number in a single throw of a die?

Solution Clearly, a die can fall with any of its faces upper most. The number on each of the faces is, therefore, a possible outcome. Thus, there are total 6 outcomes. Since there are 3 even numbers on the die, namely, 2, 4 and 6,

$$P(\text{even number}) = \frac{3}{6} = \frac{1}{2}.$$

Illustration 15 What is the probability of drawing a 'king' from a well-shuffled deck of 52 cards?

Solution Well-shuffled ensures equally-likely outcomes. There are 4 kings in a deck. Thus,

$$P(\text{a king}) = \frac{4}{52} = \frac{1}{13}.$$

Odds of an Event

Suppose, there are m outcomes favourable to a certain event and n outcomes unfavourable to the event in a sample space, then

odds in favour of the event

$$= \frac{\text{Number of favourable outcomes}}{\text{Number of unfavourable outcomes}} = \frac{m}{n}$$

and odds against the event

$$= \frac{\text{Number of unfavourable outcomes}}{\text{Number of favourable outcome}} = \frac{n}{m}.$$

If odds in favour of an event A are $a : b$, then the probability of happening of event $A = P(A) = \dfrac{a}{a+b}$ and probability of not happening of event $A = P(\overline{A}) = \dfrac{b}{a+b}$.

If odds against happening of an event A are $a : b$, then probability of happening of event $A = P(A) = \dfrac{b}{a+b}$

and probability of not happening of event

$$A = P(\overline{A}) = \frac{a}{a+b}.$$

Illustration 16 What are the odds in favour of getting a '3' in a throw of a die? What are the odds against getting a '3'?

Solution There is only one outcome favourable to the event "getting" a 3, the other five outcomes, namely, 1, 2, 4, 5, 6 are unfavourable. Thus,

Odds in favour of getting a '3'

$$= \frac{\text{Number of favourable outcomes}}{\text{Number of unfavourable outcomes}}$$

$$= \frac{1}{5} \text{ or 1 to 5.}$$

Odd against getting a '3'

$$= \frac{\text{Number of unfavourable outcomes}}{\text{Number of favourable outcomes}}$$

$$= \frac{5}{1} \text{ or 5 to 1.}$$

Illustration 17 If the odds in favour of an event are 4 to 5, find the probability that it will occur.

Solution The odds in favour of the event are $\frac{4}{5}$. Thus,

$$\frac{P(A)}{1-P(A)} = \frac{4}{5}, \text{ i.e. } 4[1-P(A)] = 5P(A),$$

i.e. $$P(A) = \frac{4}{9}.$$

The probability that it will occur $= \frac{4}{9}$.

FUNDAMENTAL THEOREMS ON PROBABILITY

Theorem 1 In a random experiment, if S is the sample space and E is an event, then

(i) $P(E) \geq 0$ (ii) $P(\phi) = 0$ (iii) $P(S) = 1$.

Remarks: It follows from above results that

(i) probability of occurrence of an event is always non-negative;

(ii) probability of occurrence of an impossible event is 0;

(iii) probability of occurrence of a sure event is 1.

Theorem 2 If E and F are mutually exclusive events, then

(i) $P(E \cap F) = 0$ and

(ii) $P(E \cup F) = P(E) + P(F)$.

Notes: **1.** For mutually exclusive events E and F, we have

$$P(E \text{ or } F) = P(E \cup F) = P(E) + P(F).$$

2. If $E_1, E_2, \ldots, E_k$ are mutually exclusive events, then

$$P(E_1 \cup E_1 \cup, \ldots, \cup E_k) = P(E_1) + P(E_2) + \ldots + P(E_k).$$

Theorem 3 If E and F are two mutually exclusive and exhaustive events, then $P(E) + P(F) = 1$.

Theorem 4 Let E be any event and $\overline{E}$ be its complementary event, then $P(\overline{E}) = 1 - P(E)$.

Theorem 5 For any two events E and F,

$$P(E - F) = P(E) - P(E \cap F).$$

Theorem 6 (Addition Theorem). For any two events E and F,

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Notes: **1.** We may express the above results as

$$P(E \text{ or } F) = P(E) + (F) - P(E \text{ and } F)$$

2. If E and F are mutually exclusive, then

$$P(E \cap F) = 0 \text{ and so } P(E \cup F) = P(E) + P(F).$$

Theorem 7 If E_1 and E_2 be two events such that $E_1 \subseteq E_2$, then prove that $P(E_1) \leq P(E_2)$.

Theorem 8 If E is an event associated with a random experiment, then $0 \leq P(E) \leq 1$.

Theorem 9. For any three events E, F, G

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(F \cap G) - P(E \cap G) + P(E \cap F \cap G)$$

Illustration 18 A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability of getting

(i) a jack or a queen or a king,

(ii) a two of heart or diamond.

Solution (i) In a pack of 52 cards, we have:

4 jacks, 4 queens and 4 kings.

Now, clearly a jack and a queen and a king are mutually exclusive events.

$$\text{Also, } P(\text{a jack}) = \frac{{}^4C_1}{{}^{52}C_1} = \frac{4}{52} = \frac{1}{13}$$

$$P(\text{a queen}) = \frac{{}^4C_1}{{}^{52}C_1} = \frac{4}{52} = \frac{1}{13}$$

$$P(\text{a king}) = \frac{{}^4C_1}{{}^{52}C_1} = \frac{4}{52} = \frac{1}{13}.$$

$\therefore$ By the addition theorem of Probability,

P(a jack or a queen or a king) = P(a jack) + P(a queen) + P(a king)

$$= \frac{1}{13} + \frac{1}{13} + \frac{1}{13} = \frac{3}{13}.$$

(ii) P(two of heart or two of diamond)

= P(two of heart) + P(two of diamond)

$$= \frac{1}{52} + \frac{1}{52} = \frac{2}{52} = \frac{1}{26}.$$

Illustration 19 Find the probability of getting a sum of 7 or 11 in a simultaneous throw of two dice.

Solution When two dice are thrown we have observed that there are 36 possible outcomes. Now, we can have a sum of 7 as

$$1 + 6 = 7,\ 2 + 5 = 7,\ 3 + 4 = 7,\ 4 + 3 = 7,\ 5 + 2 = 7,\ 6 + 1 = 7$$

Thus, the six favourable cases are (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

$$\therefore\ P(\text{a sum of 7}) = \frac{6}{36} = \frac{1}{6}.$$

Again, the favourable cases of getting a sum of 11 are (5, 6), (6, 5)

$$\therefore\ P(\text{a sum of 11}) = \frac{2}{36} = \frac{1}{18}.$$

Since the events of getting 'a sum of 7' or 'a sum of 11' are mutually exclusive:

$$\therefore\ P(\text{a sum of 7 or 11}) = P(\text{a sum of 7}) + P(\text{a sum of 11}) = \frac{1}{6} + \frac{1}{18} = \frac{4}{18} = \frac{2}{9}.$$

Illustration 20 From a well-shuffled pack of 52 cards, a card is drawn at random, find the probability that it is either a heart or a queen.

Solution A : Getting a heart card B : Getting a queen card

$$P(A) = \frac{13}{52},\ P(B) = \frac{4}{52},\ P(A \cap B) = \frac{1}{52}$$

$$\text{Required probability} = P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}.$$

INDEPENDENT EVENTS

Two event A and B are said to be independent if the occurrence (or non-occurrence of one does not affect the probability of the occurrence (and hence non-occurrence) of the other.

Illustration 21 In the simultaneous throw of two coins, 'getting a head' on first coin and 'getting a tail on the second coin are independent events.

Illustration 22 When a card is drawn from a pack of well-shuffled cards and replaced before the second card is drawn, the result of second draw is independent of first draw. We now state, without proof, the theorem which gives the probabilities of simultaneous occurrence of the independent events.

Theorem 10 If A and B are two independent events, then

$$P(A \text{ and } B) = P(A).\ P(B)$$

Illustration 23 Two dice are thrown. Find dthe probability of getting an odd number on the one die and a multiple of three on the other.

Solution Since the events of 'getting an odd number' on one die and the event of getting a multiple of three on the other are independent events,

$$P(A \text{ and } B) = P(A) \times P(B) \quad \text{...(i)}$$

Now, $P(A) = P(\text{an odd number}) = \frac{3}{6} = \frac{1}{2}$ [There are three odd numbers 1, 3, 5] and $P(B) = P(\text{a multiple of 3}) = \frac{2}{6} = \frac{1}{3}$ [Multiples of 3 are 3 and 6]

$$\therefore\ \text{From (i), required probability} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}.$$

Illustration 24 Arun and Tarun appear for an interview for two vacancies. The probability of Arun's selection is 1/3 and that of Tarun's selection is 1/5. Find the probability that

(i) only one of them will be selected,

(ii) none of them be selected.

Solution Let A : Arun is s selected B : Tarun is selected.

Then, $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{5}$.

Clearly, 'A' and 'not B' are independent also 'not A' and 'not B' are independent, 'B' and 'not A' are independent.

(i) P (only one of them will be selected)

$$= P\ (A \text{ and not } B \text{ or } B \text{ and not } A)$$
$$= P\ (A)\ P\ (\text{not } B) + P\ (B)\ P\ (\text{not } A)$$
$$= \frac{1}{3}\left(1 - \frac{1}{5}\right) + \frac{1}{5}\left(1 - \frac{1}{3}\right)$$
$$= \frac{1}{3} \times \frac{4}{5} \times \frac{1}{5} \times \frac{2}{3} = \frac{4}{15} + \frac{2}{12}$$
$$= \frac{6}{15} = \frac{2}{5}.$$

(ii) P (onle one of them be selected)

$$= P\ (\text{not } A \text{ and not } B)$$
$$= P(\text{not } A) \times P(\text{not } B)$$
$$= \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{5}\right)$$
$$= \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}.$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. In a simultaneous toss of two coins, find the probability of 2 tails.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{1}{3}$

2. In a simultaneous toss of two coins, find the probability of exactly 1 tail.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) None of these.

3. In a simultaneous toss of two coins, find the probability of no tail.

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) None of these.

4. Three coins are tossed. Find the probability of heads.

(a) $\frac{1}{6}$ (b)1/8

(c) $\frac{1}{4}$ (d) None of these.

5. Three coins are tossed. Find the probability of exactly 2 heads.

(b) $\frac{3}{8}$ (b) $\frac{1}{2}$

(c) $\frac{1}{8}$ (d) None of these.

6. Three coins are tossed. Find the probability of at least 2 heads.

(a) $\frac{1}{2}$ (b) $\frac{3}{8}$

(c) $\frac{1}{8}$ (d) None of these.

7. Three coins are tossed. Find the probability of atmost 2 heads.

(a) $\frac{3}{8}$ (b) $\frac{1}{2}$

(c) $\frac{7}{8}$ (d) None of these.

8. Three coins are tossed. Find the probability of no heads.

(a) $\frac{3}{8}$ (b) $\frac{1}{8}$

(c) $\frac{1}{2}$ (d) None of these.

9. Three coins are tossed. Find the probability of at least one head and one tail.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) None of these.

10. A coin is tossed three times. Find the chance that head and tail show alternately.

(a) $\frac{3}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) None of these.

11. Four coins are tossed once. Find the probability of 4 tails.

(a) $\frac{1}{16}$ (b) $\frac{5}{16}$

(c) $\frac{9}{16}$ (d) None of these.

12. Four coins are tossed once. Find the operability of exactly 3 tails.

(a) $\frac{1}{16}$ (b) $\frac{1}{4}$

(c) $\frac{5}{16}$ (d) None of these.

13. Four coins ore tossed once. Find the probability of exactly 2 tails.

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$

(c) $\frac{3}{8}$ (d) $\frac{5}{16}$

14. Four coins are tossed once. Find the probability by of at least 1 tail.

(a) $\frac{1}{16}$ (b) $\frac{15}{16}$

(c) $\frac{3}{16}$ (d) None of these.

15. In a single throw of two dice, find the probability of getting a total of 3 or 5.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{16}$ (d) $\frac{5}{6}$

16. In a single throw of two dice, find the probability of getting a total of 12.

(a) $\frac{1}{36}$ (b) $\frac{1}{9}$

(c) $\frac{1}{18}$ (d) $\frac{35}{36}$

17. In a single throw of two dice, find the probability of getting a total of 11.

(a) $\frac{1}{9}$ (b) $\frac{1}{18}$
(c) $\frac{1}{12}$ (d) $\frac{35}{36}$

18. In a single throw of two dice, find the probability of getting a total of 8.

(a) $\frac{5}{36}$ (b) $\frac{1}{18}$
(c) $\frac{1}{12}$ (d) $\frac{31}{36}$

19. In a single throw of two dice, the probability of getting a total of 7.

(a) $\frac{5}{36}$ (b) $\frac{1}{18}$
(c) $\frac{1}{6}$ (d) $\frac{5}{6}$

20. In a single throw of two dice, what is the probability of a doublet?

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$
(c) $\frac{1}{9}$ (d) $\frac{1}{18}$

21. In a single throw of two dice, what is the probability of a multiple of 2 on one and a multiple of 3 on the other?

(a) $\frac{5}{36}$ (b) $\frac{25}{36}$
(c) $\frac{11}{36}$ (d) $\frac{1}{9}$

22. Two dice are thrown. Find the probability of getting an odd number on one and a multiple of three on the other.

(a) $\frac{5}{36}$ (b) $\frac{25}{36}$
(c) $\frac{11}{36}$ (d) $\frac{1}{9}$

23. Doublet of even numbers.

(a) $\frac{1}{36}$ (b) $\frac{1}{18}$
(c) $\frac{1}{12}$ (d) $\frac{1}{9}$

24. A sum less than 6.

(a) $\frac{7}{18}$ (b) $\frac{5}{18}$
(c) $\frac{1}{3}$ (d) $\frac{4}{9}$

25. A sum more than 7.

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$
(c) $\frac{1}{4}$ (d) $\frac{5}{12}$

26. A sum greater than 10.

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$
(c) $\frac{1}{4}$ (d) $\frac{5}{12}$

27. A sum at least 10.

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$
(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

28. An odd number as the sum.

(a) $\frac{1}{36}$ (b) $\frac{1}{4}$
(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

29. An even number as the sum.

(a) $\frac{1}{36}$ (b) $\frac{1}{4}$
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

30. Six as the product.

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$
(c) $\frac{1}{3}$ (d) $\frac{4}{9}$

31. A multiple of 3 as the sum.

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{1}{9}$ (d) $\frac{5}{36}$

32. The product a perfect square (square of a natural number)

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$
(c) $\frac{1}{3}$ (d) $\frac{4}{9}$

33. At least one of the two numbers as 4.

(a) $\frac{1}{4}$ (b) $\frac{5}{18}$
(c) $\frac{11}{36}$ (d) $\frac{1}{3}$

34. Sum as a prime number

(a) $\frac{5}{12}$ (b) $\frac{1}{2}$
(c) $\frac{7}{12}$ (d) $\frac{3}{4}$

35. In a single throw of three dice, find the probability of getting a total of 17 or 18.

(a) $\frac{1}{54}$ (b) $\frac{1}{27}$

(c) $\frac{1}{18}$ (d) None of these.

(Q.Nos. 36 – 38) In a single throw of three dice, find the probability of getting

36. A total of 5.

(a) $\frac{1}{4}$ (b) $\frac{1}{18}$

(c) $\frac{1}{36}$ (d) $\frac{1}{9}$

37. A total of at most 5.

(a) $\frac{5}{108}$ (b) $\frac{103}{108}$

(c) $\frac{1}{18}$ (d) None of these.

38. A total of at least 5.

(a) $\frac{7}{54}$ (b) $\frac{1}{54}$

(c) $\frac{53}{54}$ (d) None of these.

39. What is the chance that a leap year, selected at random will contain 53 Sundays?

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$

(c) $\frac{3}{7}$ (d) $\frac{4}{7}$

40. A card is drawn from a pack of 100 cards numbered 1 to 100. Find the probability of drawing a number which is a square.

(a) $\frac{1}{10}$ (b) $\frac{9}{10}$

(c) $\frac{1}{5}$ (d) $\frac{2}{5}$

41. The letters of word "SOCIETY" are placed in a row. What is the probability that three come together?

(a) $\frac{3}{7}$ (b) $\frac{2}{7}$

(c) $\frac{1}{7}$ (d) None of these.

42. Find the probability that in a random arrangement of letters of the words "UNIVERSITY" two '*I*'s do not come together.

(a) $\frac{4}{5}$ (b) $\frac{1}{5}$

(c) $\frac{3}{5}$ (d) $\frac{2}{3}$

43. If letters of the word PENCIL are arranged in random order, what is the probability that N is always next to E?

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

44. Two dice are thrown. Find the odds in favour of getting the sum 4.

(a) 1 : 11 (b) 11 : 1

(c) 4 : 11 (d) 11 : 4

45. Two dice are thrown. Find the odds in favour of getting the sum 5.

(a) 8 : 1 (b) 1 : 8

(c) 7 : 8 (d) 8 : 7

46. Two dice are thrown. Find the odds against getting the sum 6.

(a) 5 : 31 (b) 6 : 31

(c) 31 : 5 (d) 31 : 6

47. What is the probability that one card drawn at random from the pack of playing cards may be either a queen or an ace?

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$

(c) $\frac{3}{13}$ (d) None of these.

48. In a class of 25 students with roll numbers 1 to 25, a student is picked up at random to answer a question. Find the probability that the roll number of the selected student is either multiple of 5 or 7.

(a) $\frac{6}{25}$ (b) $\frac{4}{25}$

(c) $\frac{8}{25}$ (d) $\frac{7}{25}$

49. An integer is chosen at random from first two hundred natural numbers. What is the probability that the integer chosen is divisible by 6 or 8?

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) None of these.

50. Two dice are together. What is the probability that the sum of two numbers is divisible by 3 or by 4?

(a) $\frac{4}{9}$ (b) $\frac{2}{9}$

(c) $\frac{5}{9}$ (d) $\frac{1}{3}$

51. In a simultaneous throw of two dice, find P (A or B) if A denotes the event '*a* total of 11 and B denotes the event' 'an odd number on each die'.

(a) $\frac{11}{36}$ (b) $\frac{1}{4}$

(c) $\frac{5}{18}$ (d) $\frac{1}{6}$

52. A box contains 36 tickets numbered 1 to 36, one ticket drawn at random. Find the probability that the number on the ticket is either divisible by 3 or is a perfect square.

(a) $\frac{2}{9}$ (b) $\frac{4}{9}$
(c) $\frac{5}{9}$ (d) $\frac{1}{3}$

53. A drawer contain 50 bolts and 150 nuts. Half of the bolts and half of the nuts are rusted. If one item is chosen at random, what is the probability that it is rusted or is a bolt?
(a) $\frac{3}{8}$ (b) $\frac{1}{2}$
(c) $\frac{5}{8}$ (d) None of these.

54. Two unbiased dice are thrown. Find the probability that neither a doublet nor a total of 10 will appear.
(a) $\frac{7}{9}$ (b) $\frac{4}{9}$
(c) $\frac{2}{9}$ (d) $\frac{1}{3}$

55. Two dice are thrown together. What is the probability that the sum of the number on two faces is neither 9 nor 11?
(a) $\frac{1}{6}$ (b) $\frac{5}{6}$
(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

56. A card is drawn from a pack of 52 cards, find the probability of getting spade or ace or red card.
(a) $\frac{9}{13}$ (b) $\frac{4}{13}$
(c) $\frac{11}{13}$ (d) $\frac{10}{13}$

57. A ticket is drawn from two hundred tickets numbered from 1 to 200, find the probability that the number is divisible by 2 or 3 or 5.
(a) $\frac{73}{100}$ (b) $\frac{27}{100}$
(c) $\frac{63}{100}$ (d) None of these.

58. A and B are mutually exclusive events of an experiment. If $P(\text{'not } A') = 0.65$, $P(A \cup B) = 0.65$ and $P(B) = p$, find the value of p.
(a) 0.70 (b) 0.30
(c) 0.63 (d) 0.35

59. The probability of an event A occurring is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, find the probability that neither A nor B occurs.
(a) 0.2 (b) 0.8
(c) 0.6 (d) None of these.

60. The probabilities that a student will receive an A, B, C or D grade are 0.30, 0.38, 0.22 and 0.01, respectively. What is the probability that the student will receive at least B grade?
(a) 0.38 (b) 0.42
(c) 0.68 (d) None of these

61. The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will not get an electric contract is 5/9. If the probability of getting at least one contract is 4/5, what is the probability that he will get both?
(a) $\frac{8}{45}$ (b) $\frac{31}{45}$
(c) $\frac{14}{45}$ (d) None of these.

62. A card is drawn from an ordinary pack and a gambler bets that it is a spade or an ace. What are the odds against his winning the bet?
(a) 9 : 4 (b) 4 : 9
(c) 5 : 9 (d) 9 : 5

63. In a race the odds in favour of horses A, B, C and D are 1 : 3, 1 : 4, 1 : 5 and 1 : 6, respectively. Find the probability that one of them wins the race.
(a) $\frac{221}{420}$ (b) $\frac{391}{420}$
(c) $\frac{331}{420}$ (d) None of these

64. A Chartered Accountant applies for a job in two firms X and Y. The ability of his being selected in firm X is 0.7, and being rejected at Y is 0.5 and the probability of at least one of his applications being rejected is 0.6. What is the probability that he will be selected in one of the firms?
(a) 0.2 (b) 0.8
(c) 0.4 (d) 0.7

65. There are three events A, B, C one of which must and only one can happen, the odds are 8 to 3 against A, 5 to 2 against B, find the odds against C.
(a) 43 : 34 (b) 34 : 43
(c) 43 : 77 (d) 77 : 43

66. A problem in Statistics is given to four students A, B, C and D. Their chances of solving it are $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$, respectively. What is the probability that the problem will be solved?
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{4}{5}$ (d) None of these.

(Q.Nos. 67 – 69) One bag contains 4 white and 2 black balls. Another contains 3 white and 5 black balls. One ball is drawn from each bag.

67. Find the probability that both are white.
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

68. Find the probability that both are black.

(a) $\frac{5}{24}$ (b) $\frac{19}{24}$

(c) $\frac{11}{24}$ (d) $\frac{1}{24}$

69. Find the probability that one is white and one is black.

(a) $\frac{11}{24}$ (b) $\frac{13}{24}$

(c) $\frac{1}{2}$ (d) None of these

(Q. Nos. 70 – 77) An urn contains 25 balls numbered 1 to 25. Suppose an odd number is considered a 'success'. Two balls are drawn from the urn with replacement.

70. Find the probability of getting two successes.

(a) $\frac{169}{625}$ (b) $\frac{312}{625}$

(c) $\frac{481}{625}$ (d) $\frac{144}{625}$

71. Find the probability of getting exactly one success.

(a) $\frac{169}{625}$ (b) $\frac{312}{625}$

(c) $\frac{481}{625}$ (d) $\frac{144}{625}$

72. Find the probability of getting at least one success.

(a) $\frac{169}{625}$ (b) $\frac{312}{625}$

(c) $\frac{481}{625}$ (d) $\frac{144}{625}$

73. Find the probability of getting no success.

(a) $\frac{169}{625}$ (b) $\frac{312}{625}$

(c) $\frac{481}{625}$ (d) $\frac{144}{625}$

74. Find the probability of getting 3 successes.

(a) $\frac{1}{27}$ (b) $\frac{2}{9}$

(c) $\frac{26}{27}$ (d) $\frac{7}{27}$

75. Find the probability of getting exactly 2 succeses.

(a) $\frac{1}{27}$ (b) $\frac{2}{9}$

(c) $\frac{26}{27}$ (d) $\frac{7}{27}$

76. Find the probability of getting at most 2 successes.

(a) $\frac{1}{27}$ (b) $\frac{2}{9}$

(c) $\frac{26}{27}$ (d) $\frac{7}{27}$

77. Find the probability of getting at least 2 successes.

(a) $\frac{1}{27}$ (b) $\frac{2}{9}$

(c) $\frac{26}{27}$ (d) $\frac{7}{27}$

78. From a pack of cards, two are drawn, the first being replaced before the second is drawn. Find the probability that the first is a diamond and the second is a king.

(a) $\frac{3}{52}$ (b) $\frac{1}{26}$

(c) $\frac{1}{52}$ (d) $\frac{1}{4}$

(Q.Nos. 79 – 82) A husband and wife appear in an interview for two vacancies in the same post. The probability of husband's selection is 1/7 and that of wife's is 1/7.

79. What is the probability that only one of them will be selected?

(a) $\frac{2}{7}$ (b) $\frac{1}{35}$

(c) $\frac{24}{35}$ (d) $\frac{11}{35}$

80. What is the probability that both of them will be selected?

(a) $\frac{2}{7}$ (b) $\frac{1}{35}$

(c) $\frac{24}{35}$ (d) $\frac{11}{35}$

81. What is the probability that none of them will be selected?

(a) $\frac{2}{7}$ (b) $\frac{1}{35}$

(c) $\frac{24}{35}$ (d) $\frac{11}{35}$

82. What is the probability that at least one of them will be selected?

(a) $\frac{2}{7}$ (b) $\frac{1}{35}$

(c) $\frac{24}{35}$ (d) $\frac{11}{35}$

(Q.Nos. 83 – 86) Probability that a man will be alive 25 years hence is 0.3 and the probability that his wife will be alive after 25 years hence is 0.4. Find the probability that 25 years hence.

83. Both will be alive.

(a) 0.12 (b) 0.18

(c) 0.28 (d) 0.58

84. Only the man will be alive.

(a) 0.12 (b) 0.18

(c) 0.28 (c) 0.58

85. Only the woman will be alive.
(a) 0.12 (b) 0.18
(c) 0.28 (d) 0.58

86. At least one of them will be alive.
(a) 0.12 (b) 0.18
(c) 0.28 (d) 0.58

87. A man speaks truth in 80% of the cases and another in 90% of the cases. While stating the same fact, what is the probability that they contradict?
(a) $\frac{37}{50}$ (b) $\frac{13}{50}$
(c) $\frac{16}{50}$ (d) None of these.

88. There are 3 urns A, B and C, A contains 4 red balls and 3 black balls. Urn B contains 5 red balls and 4 black balls. Urn C contains 4 red balls and 4 black balls. One ball is drawn from each of these urns. What is the probability that the 3 balls drawn consist of 2 red balls and a black ball?
(a) $\frac{17}{42}$ (b) $\frac{25}{42}$
(c) $\frac{19}{42}$ (d) $\frac{23}{42}$

89. An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probability of hitting the plane at the first, second third and fourth shots are 0.4, 0.3, 0.2 and 0.1, respectively. What is the probability that the gun hits the plane?
(a) 0.4379 (b) 0.6872
(c) 0.6976 (d) None of these.

90. A can solve 90% of the problems given in a book and B solve 70%. What is the probability that at least one of them will solve a problem selected at random from the book?
(a) $\frac{3}{100}$ (b) $\frac{97}{100}$
(c) $\frac{83}{100}$ (d) $\frac{17}{100}$

91. A and B throw a coin alternately till one of them gets a head and wins the game. If A starts the game, find the probability of winning of A.
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) 1 (d) None of these.

92. Two persons A and B throw a die alternately till one of them gets a 'six' and wins the game. Find the probability of winning of B.
(a) $\frac{5}{11}$ (b) $\frac{6}{11}$
(c) $\frac{4}{11}$ (d) $\frac{3}{11}$

93. The letters of the word 'SOCIETY' are placed at row. What is probability that the three vowels come together?
(a) $\frac{4}{7}$ (b) $\frac{3}{7}$
(c) $\frac{2}{7}$ (d) $\frac{1}{7}$

94. Find the probability that in a random arrangement of the letters of the word DAUGHTER, the letter D occupies the first place.
(a) $\frac{1}{8}$ (b) $\frac{1}{4}$
(c) $\frac{3}{8}$ (d) $\frac{1}{2}$

EXPLANATORY ANSWERS

1. Ans. (b)

Solution Sample space S = {HH, HT, TH, TT}
Number of exhaustive cases = 4
There is only one favourable case TT.

$\therefore$ P (2 tails) = $\frac{1}{4}$.

2. Ans. (a)

Solution Sample space S = {HH, HT, TH, TT}
Number of exhaustive cases = 4
There are two favourable cases HT, TH.

$\therefore$ P (exactly 1 tail) = $\frac{2}{4} = \frac{1}{2}$.

3. Ans. (c)

Solution Sample space S = {HH, HT, TH, TT}
Number of exhaustive cases = 4.
There is only one favourable case HH.

$\therefore$ P (no tails) = $\frac{1}{4}$.

4. Ans. (b)

Solution Sample space S = {HHH, HHT, HTH, HTT, THT, TTH, THH, TTT}
Number of exhaustive cases = 8
There is only one favourable case HHH.

$\therefore$ P (all heads) = $\frac{1}{8}$.

5. Ans. (b)

Solution Sample space S = {HHH, HHT, HTH, HTT, THT, TTH, THH, TTT}

Number of exhaustive cases = 8.

There are three favourable cases HHT, HTH, THH.

$\therefore$ P (exactly 2 heads) $= \frac{3}{8}$

6. Ans. (a)

Solution Sample space S = {HHH, HHT, HTH, HTT, TTH, THH, TTT}

Number of exhaustive cases = 8

There are four favourable cases HHT, HTH, THH, HHH.

$\therefore$ P (at least 2 heads) $= \frac{4}{8} = \frac{1}{2}$.

7. Ans. (c)

Solution Sample space S = {HHH, HHT, HTH, HTT, TTH, THH, TTT}

Number of exhaustive cases = 8.

P (atmost 2 heads) = P (not 3 heads)

$= 1 - P$ (3 heads) $= 1 - \frac{1}{8} = \frac{7}{8}$.

8. Ans. (b)

Solution Sample space S = {HHH, HHT, HTH, HTT, THT, TTH, THH, TTT}

Number of exhaustive cases = 8

P (no heads) = P (all tails) $= \frac{1}{8}$.

($\because$ there is only favourable case TTT).

9. Ans. (c)

Solution There are 6 favourable cases HHT, HTH, HTT, THT, TTH, THH.

Required probability $= \frac{6}{8} = \frac{3}{4}$.

10. Ans. (b)

Solution Sample space S = {HHH, HHT, HTT, HTT, THH, THT, TTH, TTT}

Number of exhaustive cases = 8.

Favourable cases are HTH, THT

Number of favourable cases = 2.

$\therefore$ Required probability $= \frac{2}{8} = \frac{1}{4}$.

11. Ans. (a)

Solution There are $2^4 = 16$ possible outcomes.

Sample space S = {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THTH, TTHH, TTHH,TTTH, TTHT, THTT, HTTT, TTTT}

There is only one favourable case TTTT.

P (4 tails) $= \frac{1}{16}$.

12. Ans. (b)

Solution There are 4 favourable cases TTTH, TTHT, THTT, HTTT,

$\therefore$ P (exactly 3 tails) $= \frac{4}{16} = \frac{1}{4}$.

13. Ans. (c)

Solution There are 6 favourable cases HHTT, HTHT, HTTH,THHT, THTH, TTHH.

$\therefore$ P (exactly 2 tails) $= \frac{6}{16} = \frac{3}{8}$

14. Ans. (b)

Solution P (at least 1 tail) = P (not all heads)

$= 1 - P$ (all heads)

$= 1 - \frac{1}{16} = \frac{15}{16}$.

15. Ans. (c)

Solution A total of 3 or 5 may be obtained in 6 ways, viz,

(1, 2), (2, 1), (1, 4), (2, 3), (3, 2), (4, 1).

No. of exhaustive cases = 6 × 6 = 36.

$\therefore$ Probability of getting a total of 3 or 5 $= \frac{6}{36}$

$= \frac{1}{6}$.

16. Ans. (a)

Solution A total of 12 may be obtained in 1 way, viz, (6, 6).

$\therefore$ Required probability $= \frac{1}{36}$.

17. Ans. (b)

Solution A total of 11 may be obtained in 2 ways, viz, (5, 6), (6, 5).

$\therefore$ Required probability $= \frac{2}{36} = \frac{1}{18}$.

18. Ans. (a)

Solution A total of 8 may be obtained in 5 ways, viz, (2, 6), (3, 5), (4, 4), (5, 3), 6, 2).

$\therefore$ Required probability $= \frac{5}{36}$.

19. Ans. (c)

Solution A total of 7 may be obtained in 6 ways, viz, (1, 6), (2, 5), (3, 4), (5, 2), (6, 1).

$\therefore$ Required probability $= \frac{6}{36} = \frac{1}{6}$.

20. Ans. (a)

Solution A 'doublet' means that both the dice show the same number on the upper most faces. Therefore, the outcomes, favourable to this event are

(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

Thus, the number of favourable cases = 6.

Hence, P (doublet) = $\frac{6}{36} = \frac{1}{6}$.

21. Ans. (c)

Solution In this case, the favourable cases are

(2, 3), (2, 6), (4, 3), (4, 6), (6, 3), (6, 6), (3, 2), (3, 4), (3, 6), (6, 2), (6, 4).

Thus, the number of favourable cases = 11.

$\therefore$ Required probability = $\frac{11}{36}$.

22. Ans. (c)

Solution Favourable cases are

(1, 3), (1, 6), (3, 3), (3, 6), (5, 3), (5, 6), (3, 1), (6, 1), (6, 3), (3, 5), (6, 6)

Total No. of exhaustive cases = 6 × 6 = 36

$\therefore$ Required probability = $\frac{11}{36} = \frac{11}{36}$.

23. Ans. (c)

Solution A : Getting doublet of even number A =[(2, 2), (4, 4), (6, 6)]

$n(A) = 3$ $n(S) = 36$

$\therefore$ Required probability = $P(A) = \frac{n(A)}{n(S)} = \frac{3}{36}$

$= \frac{1}{12}$.

24. Ans. (b)

Solution A : Getting total less than 6

A = [(1, 1), (1, 2), (2, 1), (2, 2), (3, 1), (1, 3), (4, 1), (1, 4), (3, 2), (2, 3)]

$n(A) = 10$, $n(A)$, $n(S) = 36$

$\therefore$ Required probability = $P(A) = \frac{n(A)}{n(S)}$

$= \frac{10}{36} = \frac{5}{18}$.

25. Ans. (d)

Solution A : Getting total more than 7

[(5, 3), (3, 5), (6, 2), (2, 6), (4, 4), (6, 3), (3, 6), (5, 4), (4, 5), (6, 4), (4, 6), (5, 5), (6, 5), (5, 6), (6, 6)]

$n(A) = 15$, $n(S) = 36$

$\therefore$ Required probability = $P(A) = \frac{n(A)}{n(S)}$

$= \frac{15}{36} = \frac{5}{12}$.

26. Ans. (a)

Solution A : Sum greater than 10.

A = [(6, 5), (5, 6), (6, 6)] $n(A) = 3$, $n(S) = 36$

$\therefore$ Required probability = $P(A) = \frac{n(A)}{n(S)}$

$= \frac{3}{36} = \frac{1}{12}$.

27. Ans. (b)

Solution A : a sum of at least 10 (i.e. 10 or more than 10)

A = [(6, 4), (4, 6), (5, 5), (6, 5), (5, 6), (6, 6)]

$n(A) = 6$, $n(S) = 36$

$\therefore$ Required probability = $P(A) = \frac{n(A)}{n(S)}$

$= \frac{6}{36} = \frac{1}{6}$.

28. Ans. (d)

Solution A : Getting sum as an odd number

A = [(1, 2), (2, 1), (1, 4), (4, 1), (2, 3), (3, 2), (4, 3), (3, 4), (6, 1), (1, 6), (5, 2), (2, 5), (5, 4) (4, 5), (6, 3), (6, 5), (5, 6)]

$n(A) = 18$, $n(S) = 36$

$\therefore$ Required probability = $P(A) = \frac{n(A)}{n(S)}$

$= \frac{18}{36} = \frac{1}{2}$.

29. Ans. (c)

Solution A : Getting even number as the sum

A = [(1, 1), (1, 3), (3, 1), (2, 2), (3, 3), (4, 2), (2, 4), (5, 1), (1, 5), (6, 2), (2, 6), (5, 3), (3, 5), (4, 4), (5, 5), (6, 4), (4, 6), (6, 6)]

$n(A) = 18$, $n(S) = 36$

$\therefore$ Required probability = $P(A) = \frac{n(A)}{n(S)}$

$= \frac{18}{36} = \frac{1}{2}$.

30. Ans. (a)

Solution A : Getting 6 as the product

A = [(1, 6), (6, 1), (2, 3), (3, 2)]

$n(A) = 4$, $n(S) = 36$

$\therefore$ Required probability = $P(A) = \frac{n(A)}{n(S)}$

$= \frac{4}{36} = \frac{1}{9}$.

31. Ans. (b)

Solution A : Getting a multiple of 3 as the sum

A = [(1, 2), (2, 1), (3, 3), (5, 1), (1, 5), (4, 2), (2, 4), (6, 3), (3, 6), (4, 5), (5, 4), (6, 6)]

$$n(A) = 12,\ n(S) = 36$$

$\therefore$ Required probability $= P(A) = \dfrac{n(A)}{n(S)}$

$$= \frac{12}{36} = \frac{1}{3}.$$

32. Ans. (b)

Solution A : Getting the product of two numbers to be a perfect square

$A = [(1, 1), (1, 4), (2, 2), (4, 1), (3, 3), (4, 4), (5, 5), (6, 6)]$

$$n(A) = 8,\ n(S) = 36$$

$\therefore$ Required probability $= P(A) = \dfrac{n(A)}{n(S)}$

$$= \frac{8}{36} = \frac{2}{9}.$$

33. Ans. (c)

Solution A : Getting at least one of the two number as 4.

$A = [(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6)]$

$$n(A) = 11,\ n(S) = 36$$

$\therefore$ Required probability $= P(A) = \dfrac{n(A)}{n(S)} = \dfrac{11}{36}.$

34. Ans. (a)

Solution A : Getting sum as a prime number

$A = [(1, 1), (1, 2), (2, 1), (2, 3), (3, 2), (4, 1), (1, 4), (4, 3), (3, 4), (6, 1), (1, 6), (2, 5), (5, 2), (6, 5), (5, 6)]$

$$n(A) = 11,\ n(S) = 36$$

$\therefore$ Required probability $= P(A) = \dfrac{n(A)}{n(S)}$

$$= \frac{15}{36} = \frac{5}{12}.$$

35. Ans (a)

Solution Number of exhaustive cases in a single throw of three dice

$= 6 \times 6 \times 6 = 216$

Cases favourable to a total of 17 are (5, 6, 6), (6, 5, 6), (6, 6, 5)

Number of cases favourable to a total of 17 or 18 is 4.

$\therefore$ P(a total of 17 or 18) $= \dfrac{4}{216} = \dfrac{1}{54}.$

36. Ans. (c)

Solution Number of exhaustive cases in a single throw of three dice $= 6 \times 6 \times 6 = 216$.

Cases favourable to a total of 5 are (1, 2, 2), (2, 1, 2), (2, 2, 1), (1, 1, 3), (1, 3, 1), (3, 1, 1).

$\therefore$ P(a total of 5) $= \dfrac{6}{216} = \dfrac{1}{36}.$

37. Ans. (a)

Solution A total of at most 5 means a total 3, 4 or 5.

Cases favourable to a total of 3 are (1, 1, 1)

Cases favourable to a total of 4 are (1, 1, 2), (1, 2, 1), (2, 1, 1)

Cases favourable to a total of 5 are (1, 2, 2), (2, 1, 2), (2, 2, 1), (1, 1, 3), (1, 3, 1), (3, 1, 11).

Number of cases favourable to a total of 3 or 4 or 5 is 10.

$\therefore$ P (a total of at most 5) $= \dfrac{10}{216} = \dfrac{5}{108}.$

38. Ans. (c)

Solution A total of at least 5 means not a total of 3 or 4.

Number of cases favourable to a total of 3 or 4 is 4.

P(a total of 3 or 4) $= \dfrac{4}{216} = \dfrac{1}{54}$

$\therefore$ P(a total of at least 5) $= P$(not a total of 3 or 4)

$= 1 - P$(a total of 3 or 4)

$$= 1 - \frac{1}{54} = \frac{53}{54}.$$

39. Ans. (b)

Solution We know that a leap year has 366 days and thus a leap year has 52 weeks and 2 days over.

The two over (successive days have the following likely cases:

(i) Sunday and Monday
(ii) Monday and Tuesday
(iii) Tuesday and Wednesday
(iv) Wednesday and Thursday
(v) Thursday and Friday
(vi) Firday and Saturday
(vii) Saturday and Sunday.

$\therefore$ Number of exhaustive cases 'n' = 7.

Out of these, the favourable cases are (i) and (vii)

$\therefore$ Number of favourable cases 'm' = 2

$\therefore$ Probability of having 53 Sundays $= \dfrac{2}{7}.$

40. Ans. (a)

Solution A : Getting a number which is a square

$A = (1, 4, 9, 16, 25, 36, 49, 64, 81, 100)$

$$n(A) = 10,\ n(S) = 100$$

$\therefore$ Required probability $= P(A) = \dfrac{n(A)}{n(S)}$

$$= \frac{10}{100} = \frac{1}{10}.$$

41. Ans. (c)

Solution There are 7 letters in the word 'SOCIETY' which can be arranged in 71 ways. Considering the three vowels in the word 'SOCIETY' as one letter, we can

arrange 5 letters in a row in 5! ways. Also, three vowels can themselves be arranged in 3! ways.

∴ The total number of arrangements in which three vowels come together are 5! × 3!

Hence, the required probability $= \frac{5! \times 3!}{7!}$

$= \frac{3 \times 2 \times 1}{7 \times 6} = \frac{1}{7}$.

42. Ans. (a)

Solution Out of the letters in the word 'UNIVERSITY' two letters '*I*' are alike.

∴ Number of permutations $= \frac{10!}{2}$ ...(i)

Number of words in which two '*I*' are never together

= Total number of words – Number of words in which two '*I*' are together

$= \frac{10!}{2} - 9! = \frac{10! - 2.9!}{2} = \frac{9![10-2]}{2} = \frac{9!.8}{2} = 9!.4$

∴ Required probability $= \frac{9.4}{10!/2} = \frac{9!.8}{10!}$

$= \frac{8}{10} = \frac{4}{5}$.

43. Ans. (a)

Solution Number of ways in which 6 letters of the word PENCIL can be arranged is $P(6, 6) = 6!$.

If *N* is next to *E*, they can be considered as one and the 5 letters can be arranged in $P(5, 5) = 5!$ ways.

∴ The required probability $= \frac{5!}{6!} = \frac{1}{6}$.

44. Ans. (a)

Solution Let *A* be the event of "getting the sum 4".

Then, $A = [(1, 32), (3, 1), (2, 2)]$.

Therefore, 3 favourable outcomes and (36 –3) = 33 outcomes are unfavourable.

∴ Odds in favour of sum of 4 $= \frac{3}{33} = \frac{1}{11}$.

45. Ans. (b)

Solution Let *A* be the event of "getting a sum of 5".

Then, $A = [(1, 4), (4, 1), (2, 3), (3, 2)]$.

There are 4 favourable outcomes and (36 – 4) = 32 outcomes are unfavourable.

∴ Odds in favour of sum 5 $= \frac{4}{32} = \frac{1}{8}$.

46. Ans. (c)

Solution Let *A* be the event "getting the sum 6".

Then, $A = [(1, 5), (5, 1), (2, 4), (4, 2), (3, 3)]$

There are 5 favourable outcomes and (36 – 5) = 31 outcomes are unfavourable.

∴ Odds against getting the sum 6 $= \frac{31}{5}$.

47. Ans. (b)

Solution *A* : Getting a queen *B* : Getting an ace

$P(A) = \frac{4}{52}, P(B) = \frac{4}{52}, P(A \cap B) = \frac{0}{52} = 0$

∴ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{4}{52} + \frac{4}{52} - 0 = \frac{8}{52} = \frac{2}{13}$.

48. Ans. (c)

Solution *A* : Roll number is multiple of 5 *B* : Roll number is multiple of 7

$A = (5, 10, 15, 20, 25)$ $B = (7, 14, 21)$

$P(A) = \frac{5}{25}, P(B) = \frac{3}{25}, P(A \cap B) = \frac{0}{25} = 0$

∴ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{5}{25} + \frac{3}{25} - 0 = \frac{8}{25}$.

49. Ans. (a)

Solution *A* : Integer chosen is divisible by 6*B* : Integer chosen is divisible by 8

$n(A) = 33, n(B) = 25, n(A \cap B) = 8, n(S) = 200$

$P(A) = \frac{33}{200}, P(B) = \frac{25}{200}$

$P(A \cap B) = P(A) + A(B) - P(A \cap B)$

$= \frac{33}{200} + \frac{33}{200} - \frac{8}{200} = \frac{50}{200} = \frac{1}{4}$.

50. Ans. (c)

Solution *A* : Sum of two numbers is divisible by 3

B : Sum of two numbers is divisible by 4

$A = [(1, 2), (2, 1), (3, 3), (5, 1), (1, 5), (2, 4), (4, 2), (5, 4), (4, 5), (6, 3), (3, 6), 6, 6)]$

$B = [(2, 2), (3, 1), (1, 3), (5, 3), (3, 5), (4, 4), (6, 2), (2, 6), (6, 6)]$

$P(A) = \frac{12}{36}, P(B) = \frac{9}{36}, P(A \cap B) = \frac{1}{36}$

∴ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{12}{36} + \frac{9}{36} - \frac{1}{36} = \frac{20}{36} = \frac{5}{9}$.

51. Ans. (a)

Solution *A* : Getting total of 11 *B* : Getting odd number one each die

$A = [(6, 5), (5, 6)]$

$B = [1, 1). (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)]$

$P(A) = \frac{2}{36}, P(B), P(B) = \frac{9}{36}, P(A \cap B) = \frac{9}{36} = 0$

$\therefore$ Required probability $= P(A) + P(B) - P(A \cap B)$

$$= \frac{2}{36} + \frac{9}{36} - 0 = \frac{11}{36}.$$

52. Ans. (b)

Solution A : Number divisible by 3

B : Number is a perfect square

$A = (3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36)$

$B = (1, 4, 9, 16, 25, 36)$

$$P(A) = \frac{12}{36}, P(B) = \frac{6}{36}, P(A \cap B) = \frac{2}{36}$$

$\therefore$ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B).$

$$= \frac{12}{36} + \frac{6}{36} - \frac{2}{36} = \frac{16}{36} = \frac{4}{9}.$$

53. Ans. (c)

Solution A : Getting rusted item B : Getting bolt

$$P(A) = \frac{100}{200}, P(B) = \frac{50}{200}, P(A \cap B) = \frac{125}{200}$$

$\therefore$ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$$= \frac{100}{200} + \frac{50}{200} - \frac{25}{200}$$

$$= \frac{125}{200} = \frac{5}{8}.$$

54. Ans. (a)

Solution A : Getting, a doublet, B : Getting a total of 10

$A = [(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)]$

$B = [(6, 4), (4, 6), (5, 5)]$

$$P(A) = \frac{6}{36}, P(B) = \frac{3}{36}, P(A \cap B) = \frac{1}{36}$$

$\therefore$ Required probability

$= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$$= 1 - \left(\frac{6}{36} + \frac{3}{36} - \frac{1}{36}\right)$$

$$= 1 - \frac{8}{36} = \frac{28}{36} = \frac{7}{9}.$$

55. Ans. (b)

Solution A : Getting a total of 9, B : Getting a total of 11

$A = [(5, 4), (4, 5), (6, 3), (3, 6)]$ $B = [(6, 5), (5, 6)]$

$$P(A) = \frac{4}{36}, P(B) = \frac{2}{36}, P(A \cap B) = \frac{0}{36}$$

$\therefore$ Required probability $= 1 - P(A \cup B)$

$= [P(A) + P(B) - P(A \cap B)]$

$$= 1 - \left(\frac{4}{36} + \frac{2}{36} - 0\right)$$

$$= 1 - \frac{1}{6} = \frac{5}{6}.$$

56. Ans. (d)

Solution A: Getting spade card B: Getting ace card C: Getting red card

$$P(A) = \frac{13}{52}, P(B) = \frac{4}{52}, P(C) = \frac{26}{52}, P(A \cap B) = \frac{1}{52},$$

$$P(B \cap C) = \frac{2}{52}, P(C \cap A) = \frac{0}{52} = 0,$$

$$P(A \cap B \cap C) = \frac{0}{52} = 0$$

$\therefore$ Required probability

$= P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$

$- P(C \cap A) + P(A \cap B \cap C)$

$$= \frac{13}{52} + \frac{4}{52} + \frac{26}{52} - \frac{1}{52} - \frac{2}{52} - 0 = \frac{40}{52} = \frac{10}{13}.$$

57. Ans. (a)

Solution A: Number is divisible by 2 B : Number is divisible buy 3 C : Number is divisible by 5

$$P(A) = \frac{100}{200}, P(B) = \frac{66}{200}, P(C) = \frac{40}{200},$$

$$P(B \cap C) \frac{33}{200} = P(B \cap C) = \frac{13}{200},$$

$$P(C \cap A) = \frac{20}{200}, P(A \cap B \cap C) = \frac{6}{200}$$

$\therefore$ Required probability

$= P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$

$- P(C \cap A) + P(A \cap B \cap C)$

$$= \frac{100}{200} + \frac{66}{200} + \frac{40}{200} - \frac{33}{200} - \frac{13}{200} - \frac{20}{200} + \frac{6}{200}$$

$$= \frac{146}{200} = \frac{73}{200}.$$

58. Ans. (b)

Solution We know $P(A) = 1 - P(\overline{A})$

$= 1 - 0.65 = 0.35$

and, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.65 = 0.35 + p - 0$

[$\because$ A and B are mutually exclusive events]

$\Rightarrow p = 0.65 - 0.35 = 0.30.$

59. Ans. (a)

Solution Required probability $= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)$

$= 1 - [0.5 + 0.3 - 0]$

[$\because$ A and B are mutually exclusive events]

$= 1 - 0.8 = 0.2.$

60. Ans. (c)

Solution P (at least B grade) $= P$ (B grade) $+ P$ (A grade)

$= 0.38 + 0.30 = 0.68$

61. Ans. (c)

Solution A : Contractor will get a plumbing contract

B : Contractor will get an electric contract

$P(A) = \frac{2}{3}$, $P(\overline{B}) = \frac{5}{9}$, $P(A \cup B) = \frac{4}{5}$

We know, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{4}{5} = \frac{2}{3} + [1 - P(\overline{B})] - P(A \cap B)$

$\Rightarrow \frac{4}{5} = \frac{2}{3} + 1 - \frac{5}{9} - P(A \cap B)$

$\Rightarrow (A \cap B) = \frac{2}{3} + 1 - \frac{5}{9} - \frac{4}{5}$

$= \frac{30+45-25-26}{45} = \frac{14}{45}$.

62. Ans. (a)

Solution Let A : a spade is drawn and B : an ace is drawn

Probability of winning the bet = $P(A$ or $B)$

$= P(A) + P(B) - P(A$ and $B)$

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{52}$

Probability of losing the bet = $1 - \frac{4}{13} = \frac{9}{13}$

Odds against winning the bet = $\frac{9}{13} : \frac{4}{13} = 9 : 4$.

63. Ans. (b)

Solution A : A wins the race B : B wins the race C: C wins the race

$P(A) = \frac{1}{1+3} = \frac{1}{4}$, $P(B) = \frac{1}{1+4} = \frac{1}{5}$,

$P(C) = \frac{1}{1+5} = \frac{1}{6}$, $P(D) = \frac{1}{1+6} = \frac{1}{7}$

$\therefore$ Required probability $= P(A) + P(B) + P(C) + P(D)$

$= \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} = \frac{319}{420}$.

64. Ans. (b)

Solution Let A and B denote the events that the chartered accountant is selected in firms X and Y, respectively. Then,

$P(A) = 0.7$, $P(\overline{B}) = 0.5$ and $P(\overline{A} \cup \overline{B}) = 0.6$

Now, $P(\overline{A}) = 1 - P(A) = 1 - 0.7 = 0.3$

$P(B) = 1 - P(\overline{B}) = 1 - 0.5 = 0.5$

Again, $\overline{A \cap B} = \overline{A \cup B}$ (By De Morgan's law)

$\therefore \quad P(A \cap B) = 1 - P\overline{A \cap B} = 1 - P(\overline{A} \cup \overline{B})$

$\Rightarrow P(A \cap B) = 1 - 0.6 = 0.4$

$\therefore \quad P(A \cup B) = 0.7 + 0.5 - 0.4 = 0.8$

Hence, the probability that the chartered accountant will be selected in one of the two firms X or Y is 0.8.

65. Ans. (a)

Solution Since odds against the event A are 8 : 3, the probability of the happening of the event A is given by $P(A) = \frac{3}{8+3} = \frac{3}{11}$.

Similarly, odds against the event B are 5 : 2, so we have

$$P(B) = \frac{2}{5+2} = \frac{2}{7}.$$

Since the events A, B, C are such that one of them is a must and only one can happen, so the events A, B, C are mutually exclusive and exhaustive and consequently the sum of their probability must be 1.

$\therefore \quad P(A) + P(B) + P(C) = 1$ or $\frac{3}{11} + \frac{2}{7} + P(C) = 1$

$\Rightarrow \quad P(C) = 1 - \frac{3}{11} - \frac{2}{7} = \frac{34}{77}$.

$\therefore \quad P(\overline{C}) = 1 - P(C) = 1 - \frac{34}{77} = \frac{43}{77}$.

$\therefore$ The odds against C are 43 : 34.

66. Ans. (b)

Solution Probability that A fails to solve the problem is $1 - \frac{1}{3} = \frac{2}{3}$

Probability that B fails to solve the problem is

$$1 - \frac{1}{4} = \frac{3}{4}$$

Probability that C fails to solve the problem is

$$1 - \frac{1}{5} = \frac{4}{5}.$$

Probability that D fails to solve the problem is

$$1 - \frac{1}{6} = \frac{5}{6}.$$

Since the events are independent, the probability that all the four students fail to solve the problem is

$$\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} = \frac{1}{3}.$$

$\therefore$ The probability that the problem will be sollved

$$= 1 - \frac{1}{3} = \frac{2}{3}.$$

67. Ans. (c)

Solution Probability of drawing a white ball from the first bag $= \frac{4}{5} = \frac{2}{3}$. Probability of drawing a white ball from the second bag $= \frac{3}{8}$.

Since the events are independent, the probability that both the balls are white $= \frac{2}{3} \times \frac{3}{8} = \frac{1}{4}$.

68. Ans. (a)

Solution Probability of drawing a black ball from first bag $= \frac{2}{6} = \frac{1}{3}$

Probability of drawing a black ball from the second bag $= \frac{5}{8}$.

$\therefore$ Probability that both balls are black

$$= \frac{1}{3} \times \frac{5}{8} = \frac{5}{24}.$$

69. Ans. (c)

Solution The event 'one is white and one is black' is the same as the event 'either the first is white and the second is black or the first is black and the second is white',

$\therefore$ The probability that one is white and one is black

$$= \frac{2}{3} \times \frac{5}{8} \times \frac{1}{3} \times \frac{3}{8} = \frac{12}{24}.$$

70. Ans. (a)

Solution Success: Getting odd number $p = \frac{13}{25}$

$$\Rightarrow q = 1 - p = 1 - \frac{13}{25} = \frac{12}{25}$$

$$P \text{ (two successes)} = pp = \frac{13}{25} \times \frac{13}{25} = \frac{169}{625}.$$

71. Ans. (b)

Solution P (exactly one success) $= pq + qp$

$$= \frac{13}{25} \times \frac{12}{25} \times \frac{12}{25} \times \frac{13}{25} = \frac{156+156}{625} = \frac{312}{625}.$$

72. Ans. (c)

Solution P (at one success) = 1 P (no success)

$$= 1 - qq = 1 - \left(\frac{12}{25}\right)\left(\frac{12}{25}\right) = 1 - \frac{144}{625}$$

$$= \frac{625-144}{625} = \frac{481}{625}.$$

73. Ans. (d)

Solution P (no success) $= qq = \frac{12}{25}\left(\frac{12}{25}\right) = \frac{144}{625}$.

74. Ans. (a)

Solution Success: getting 5 or 6 $p = \frac{2}{6} = \frac{1}{3}$

$$\Rightarrow q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(3 \text{ successes}) = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}.$$

75. Ans. (b)

Solution P(exactly 2 successes) $= ppq + pqp + qpp$

$$= \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3}$$

$$= \frac{2+2+2}{27} = \frac{6}{27} = \frac{2}{9}.$$

76. Ans. (c)

Solution P (at most 2 successes) = P(no success) + P(1 success) + P (2 successes)

$$= 1 - P(3 \text{ successes}) = 1 - pp$$

$$= 1 - \frac{1}{3}\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = 1 - \frac{1}{27} = \frac{26}{27}.$$

77. Ans. (d)

Solution P(at least 2 successes) = + P(3 successes)

$$= ppq + pqp + qpp + ppp$$

$$= \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$$

$$= \frac{2+2+2+1}{27} = \frac{7}{27}.$$

78. Ans. (c)

Solution A : First card is diamond card B : Second card is king card

$$P(A) = \frac{13}{52} = \frac{1}{4},\ P(B) = \frac{4}{52} = \frac{1}{13}$$

$\therefore$ Required probability $= P(A)\,P(B) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$.

79. Ans. (a)

Solution A : Husband selected; B : Wife selected

$$P(A) = \frac{1}{7} \Rightarrow (\overline{A}) = 1 - P(A) = 1 - \frac{1}{7} = \frac{6}{7}$$

$$P(B) = \frac{6}{7} \Rightarrow P(\overline{B}) = 1 - P(B) = 1 - \frac{1}{5} = \frac{4}{5}$$

P(only one of them will be selected)

$$= P(A)\,P(\overline{B}) + P(B)\,P(\overline{A})$$

$$= \frac{1}{7}\left(\frac{4}{5}\right) + \frac{1}{5}\left(\frac{6}{7}\right) = \frac{4+6}{35} = \frac{10}{35} = \frac{2}{7}.$$

80. Ans. (b)

Solution P(both of them will be selected)

$$= P(A) \times P(B) = \frac{1}{7} \times \frac{1}{5} = \frac{1}{35}.$$

81. Ans. (c)

Solution P(none of them will be selected)

$$= P(\overline{A})\,P(\overline{B}) = \frac{6}{7} \times \frac{4}{5} = \frac{24}{35}.$$

82. Ans. (d)

Solution P(at least one of them will be selected)

$$= 1 - P(\overline{A}) \times P(\overline{B})$$

$$= 1 - \frac{6}{7} \times \frac{4}{5} = 1 - \frac{24}{35} = \frac{11}{35}.$$

83. Ans. (3)

Solution A : Husband will be alive 25 years hence

B : Wife will be alive 25 years hence

$P(A) = 0.3 \Rightarrow P(\overline{A}) = 1 - P(A) = 1.03 = 0.7$

$P(B) = 0.4 \Rightarrow P(\overline{B}) = 1 - P(B) = 1 - 04 = 0.6$

Required probability $= P(A)\,P(B) = (0.3)\,(0.4) = 0.12.$

84. Ans. (b)

Solution Required probability $= P(A)\,P(\overline{B})$
$= (0.3)\,(0.6) = 0.18.$

85. Ans. (c)

Solution Required probability $= P(B)\,P(A)$
$= (0.4)\,(0.7) = 0.28.$

86. Ans. (d)

Solution Required probability $= 1 - P(\overline{A})\,P(\overline{B})$
$= 1 - (0.7)\,(0.6)$
$= 1 - 0.42 = 0.58.$

87. Ans. (b)

Solution Let the two men be A and B. A : A speaks truth; B : B speaks truth

$$P(A) = \frac{80}{100} \Rightarrow P(\overline{A}) = 1 - P(A) = 1 - \frac{80}{100} = \frac{20}{100}$$

$$P(B) = \frac{90}{100} \Rightarrow P(\overline{B}) = 1 - P(B) = 1 - \frac{90}{100} = \frac{10}{100}$$

$\therefore$ Required probability $= P(A)\,P(\overline{B}) + P(B)\,P(\overline{A})$

$$= \frac{80}{100} \times \frac{10}{100} + \frac{90}{100} \times \frac{20}{100}$$

$$= \frac{8+18}{100} = \frac{26}{100} = \frac{13}{50}.$$

88. Ans. (a)

Solution A : Getting a red ball from urn A
B : Getting a black ball from urn A
C : Getting a red ball from urn B
D : Getting a black ball fron urn C
E : Getting a red ball from urn C
F : Getting a black ball from urn C

$$P(A) = \frac{4}{7},\ P(B) = \frac{3}{7},\ P(C) = \frac{5}{9},$$

$$P(D) = \frac{4}{9},\ P(E) = \frac{4}{8}.\ P(F) = \frac{4}{8}$$

Required probability $= P(A)\,P(C)\,P(F) + P(A)\,P(D)\,P(E) + P(B)\,P(C)\,P(E)$

$$= \frac{4}{7} \times \frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{4}{8} + \frac{3}{7} \times \frac{5}{9} \times \frac{4}{8}$$

$$= \frac{80+64+60}{504} = \frac{204}{504} = \frac{17}{42}.$$

89. Ans. (c)

Solution A : Plane is hit by the first shot
B : Plane is hit by the second shot
C : Plane is hit by the third shot
D : Plane is hit by the fourth shot

$P(A) = 0.4 \Rightarrow P(\overline{A}) = 1 - P(A) = 1 - 0.4 = 0.6$

$P(B) = 0.3 \Rightarrow P(\overline{B}) = 1 - P(B) = 1 - 0.3 = 0.7$

$P(C) = 0.2 \Rightarrow P(\overline{C}) = 1 - P(C) = 1 - 0.2 = 0.8$

$P(D) = 0.1 \Rightarrow P(\overline{D}) = 1 - P(D) = 1 - 0.1 = 0.9$

$\therefore$ Required probability
$= 1 - P(\overline{A})\,P(\overline{B})\,P(\overline{C})\,P(\overline{D})$
$= 1 - (0.6)\,(0.7)\,(0.8)\,(0.9)$
$= 1 - 0.3024 = 0.6976.$

90. Ans. (b)

Solution A : A solves the problem; B : B solves the problem.

$$P(A) = \frac{90}{100} \Rightarrow P(\overline{A}) = 1 - P(A) = 1 - \frac{90}{100} = \frac{10}{100}$$

$$P(B) = \frac{70}{100} \Rightarrow P(\overline{B}) = 1 - P(B) = 1 - \frac{70}{100} = \frac{30}{100}$$

Required probability $= 1 - P(\overline{A})\,P(\overline{B})$

$$= 1 - \frac{10}{100} \times \frac{30}{100} = 1 - \frac{3}{100}$$

$$= \frac{97}{100}.$$

91. Ans. (b)

Solution Success : Getting head $\therefore\ p = \frac{1}{2}$

$$\Rightarrow q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}.$$

Let A start the game. A can win the game in 1st, IIIrd, Vth, ... throws. Then

$P(A \text{ winning}) = p + qqp + qqqqp + \ldots$

$$= p[1 + q^2 + q^4 + \ldots]$$

$$= p\frac{1}{1-q^2} = \frac{1}{2}\frac{1}{1-(1/2)^2}$$

$$= \frac{4}{2(3)} = \frac{2}{3}.$$

92. Ans. (a)

Solution Success : Getting 6

$$p = \frac{1}{6} \Rightarrow q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}.$$

A can win the game in 1st, IIIrd, Vth ... throws

$P(A \text{ winning}) = p + qqp + qqqqp + \ldots$

$$= p[1 + q^2 + q^4 + \ldots] = p\frac{1}{1-q^2}$$

$$= \frac{1}{6}\frac{1}{1-(5/6)^2} = \frac{1}{6}\frac{36}{36-25} = \frac{6}{11}$$

$$\therefore\ P(B \text{ winning}) = 1 - \frac{6}{11} = \frac{5}{11}.$$

93. Ans (d)

Solution A : Three vowels come together

$n(A) = 5!3!, \quad n(S) = 7!$

$\therefore$ Required probability $= P(A) = \dfrac{n(A)}{n(S)} = \dfrac{5!\,3!}{7!}$

$= \dfrac{3\times 2}{7\times 6} = \dfrac{1}{7}.$

94. Ans. (a)

Solution A : D occupies the first place

$n(A) = 7!, n(S) = 8!$

$\therefore$ Required probability $= P(A) = \dfrac{n(A)}{n(S)} = \dfrac{7!}{8!} = \dfrac{1}{8}.$

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. In a box carrying one dozen of oranges, one-third have become bad. If 3 oranges are taken out from the box at random, what is the probability that at least one orange out of the three oranges picked up is good?

(a) $\dfrac{1}{55}$ (b) $\dfrac{54}{55}$

(c) $\dfrac{45}{55}$ (d) $\dfrac{3}{55}$

(e) None of these. **[Based on SBI PO 1999]**

2. A box contains, 5 green, 4 yellow and 3 white marbles. 3 marbles are drawn at random. What is the probability that they are not of the same colour?

(a) $\dfrac{13}{44}$ (b) $\dfrac{41}{44}$

(c) $\dfrac{13}{55}$ (d) $\dfrac{52}{55}$

(e) None of these. **[Based on SBI Associates PO 1990]**

3. Out of 15 students studying in a class, 7 are from Maharashtra, 5 are from Karnataka and 3 are from Goa. Four students are to be selected at random. What are the chances that at least one is from Karnataka?

(a) $\dfrac{12}{13}$ (b) $\dfrac{11}{13}$

(c) $\dfrac{10}{15}$ (d) $\dfrac{1}{15}$

(e) None of these. **[Based on Guwahati PO 1999]**

4. The probability that a teacher will give one surprise test during any class meeting in a week is $\dfrac{1}{5}$. If a student is absent twice, what is the probability that he will miss at least one test?

(a) $\dfrac{4}{15}$ (b) $\dfrac{1}{15}$

(c) $\dfrac{91}{25}$ (d) $\dfrac{16}{125}$

(e) None of these. **[Based on BSRB Mumbai PO 1999]**

5. A speaks truth in 75% and B in 80% cases. In what percentage of case are they likely to contradict each other when narrating the same incident?

(a) 35 (b) 30

(c) 25 (d) 20

(e) None of these. **[Based on BSRB Chennai PO 2000]**

6. In a box there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor gtreen?

(a) $\dfrac{7}{19}$ (b) $\dfrac{2}{3}$

(c) $\dfrac{3}{4}$ (d) $\dfrac{9}{21}$

(e) None of these. **[Based on IBPS Bank PO Exam 2002]**

EXPLANATORY ANSWERS

1. Ans. (b)

Solution $n(S) = {}^{12}C_3 = \dfrac{12\times 11\times 10}{3\times 2} = 2 \times 11 \times 10 = 220.$

No. of selection of 3 oranges out of the total 12 oranges $= {}^{12}C_3 = 2 \times 11 \times 10 = 220.$

No. of selections of 3 bad oranges out of the total 4 bad oranges $= {}^{4}C_3 = 4.$

$\therefore \quad n(E) =$ No. of desired selection of oranges

$= 220 - 4 = 216.$

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{216}{220} = \dfrac{54}{55}.$

2. Ans. (b)

Solution Total no. of ways of drawing 3 marbles

$= {}^{12}C_3 = \dfrac{12\times 11\times 10}{1\times 2\times 3} = 220.$

Total no. of ways of drawing marbles, which are of same colour $= {}^{5}C_3 \times {}^{4}C_3 \times {}^{3}C_3 = 10 + 4 + 1 = 15.$

$\therefore$ Probability of same colour $= \frac{15}{220} = \frac{3}{44}$.

$\therefore$ Probability of not same colour $= 1 - \frac{3}{44} = \frac{41}{44}$.

3. Ans. (b)

Solution No. of ways of selecting 4 students out of 15 students $= {}^{15}C_4 = \frac{15\times14\times13\times12}{1\times2\times3\times4} = 1365$.

The no. of ways of selecting 4 students in which no student belongs to Karnataka $= {}^{10}C_4$

$\therefore$ No of ways of selecting at least one student from Karnataka $= {}^{15}C_4 - {}^{10}C_4 = 1155$.

$\therefore$ Required probability $= \frac{1155}{1365} = \frac{77}{91} = \frac{11}{13}$.

4. Ans. (b)

Solution The probabilty that a student is absent in the class $= \frac{2}{6} = \frac{1}{3}$.

$\therefore$ The probability of missing his test will be

$$= \frac{1}{5} \times \frac{1}{3} = \frac{1}{15}.$$

5. Ans. (a)

6. Ans. (e)

Solution Clearly, the ball picked up must be blue, which can be picked in ${}^{7}C_1 = 7$ ways.

One ball can be picked from total $(8 + 7 + 6 = 21)$ in ${}^{21}C_1$ ways.

$\therefore$ Required probability $= \frac{7}{21} = \frac{1}{3}$.

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. What is the probability that a number selected from the numbers 1, 2... 25 is a prime number? You may assume that each of the 25 numbers is equally likely to be selected.

(a) $\frac{4}{25}$ (b) $\frac{16}{25}$
(c) $\frac{9}{25}$ (d) None of these.

2. The letters of the word 'ARTICLE' are arranged at random. Find the probability that the vowels may occupy the even places.

(a) $\frac{1}{35}$ (b) $\frac{7}{35}$
(c) $\frac{6}{35}$ (d) $\frac{8}{35}$

3. Find the probability of getting an even number on the first die or a total of 8 in a single throw of two dice.

(a) $\frac{5}{9}$ (b) $\frac{4}{9}$
(c) $\frac{1}{3}$ (d) $\frac{2}{9}$

4. Two unbiased dice are thrown. Find the probability that the sum of the numbers obtained on the two dice is neither a multiple of 2, nor a multiple of 3.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{5}{6}$ (d) None of these.

5. A card is drawn from a pack of playing cards. Find the probability of getting a king or a heart or a red card.

(a) $\frac{6}{13}$ (b) $\frac{7}{13}$
(c) $\frac{4}{13}$ (d) $\frac{9}{13}$

6. The probabilities of happening of two events A and B are 0.25 and 0.50, respectively. If the probability of happening of A and B together is 0.14, find the probability that neither A nor B happens.

(a) 0.39 (b) 0.61
(c) 0.36 (d) 0.64

7. Five letters are placed at random in five addressed envelopes. Find the probability that all the letters are not dispatched in the right envelopes.

(a) $\frac{1}{120}$ (b) $\frac{7}{120}$
(c) $\frac{119}{120}$ (d) None of these.

8. Two cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Find the probability of drawing two aces.

(a) $\frac{1}{169}$ (b) $\frac{19}{169}$
(c) $\frac{1}{13}$ (d) $\frac{168}{169}$

9. An unbiased dice is tossed twice. Find the probability of getting a 4, 5 or 6 on the first toss and a 1, 2, 3 or 4 on the second toss.

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

10. The probability of hitting a target by three marksmen is $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, respectively. Find the probability that one and only one of them will hit the target when they fire simultaneously.

(a) $\frac{9}{24}$ (b) $\frac{7}{24}$ (c) $\frac{11}{24}$ (d) $\frac{9}{24}$

Test II **[Time: 10 min]**

11. A party of n men is to be seated round a circular table. Find the probability that two particular men sit together.

(a) $\frac{1}{n-1}$ (b) $\frac{2}{n-1}$
(c) $\frac{3}{n-1}$ (d) $\frac{n}{n-1}$

12. In a single throw of two coins, find the probability of getting both heads or both tails.

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$
(c) $\frac{1}{2}$ (d) None of these.

13. A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability that it is neither an ace nor a king.

(a) $\frac{3}{13}$ (b) $\frac{4}{13}$
(c) $\frac{2}{13}$ (d) $\frac{11}{13}$

14. A candidate is selected for interview for three posts. For the first post there are 5 candidates, for the second there are 8 and for the third there are 7. What are the chances for his getting at least one post?

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$
(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

15. A student takes his examination in four subjects α, β, γ, δ. He estimates his chance of passing in α as 4/5, in β as 3/4, in γ as 5/6, and in δ as 2/3. To qualify he must pass in α and at least two other subjects. What is the probability that he qualifies?

(a) $\frac{61}{90}$ (b) $\frac{29}{90}$
(c) $\frac{57}{90}$ (d) None of these.

16. A class consists of 80 students, 25 of them are girls and 55 boys, 10 of them are rich and the remaining poor, 20 of them are fair complexioned. What is the probability of selecting a fair complexioned rich girl?

(a) $\frac{507}{512}$ (b) $\frac{5}{512}$
(c) $\frac{503}{512}$ (d) None of these.

17. A speaks truth in 60% of the cases and B in 90% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?

(a) 38% (b) 40%
(c) 42% (d) 47%

18. A bag contains 4 white and 3 black balls. Four balls are successively drawn out with replacement. Find the probability that they are alternately of different colours.

(a) $\frac{173}{2401}$ (b) $\frac{248}{2401}$
(c) $\frac{346}{2401}$ (d) $\frac{288}{2401}$

19. The odds that A speaks the truth are 3:2 and the odds that B speaks the truth are 5:3. In what percentage of cases are they likely to contradict each other on an identical point?

(a) 66% (b) 47%
(c) 47.5% (d) None of these.

20. In two successive throws of a pair of dice, determine the probability of getting a total of 8, each time.

(a) $\frac{17}{1296}$ (b) $\frac{25}{1296}$
(c) $\frac{48}{1296}$ (d) None of these.

Answers

1. (c)	**2.** (a)	**3.** (a)	**4.** (a)	**5.** (b)	**6.** (a)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (c)
11. (b)	**12.** (c)	**13.** (d)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (c)	**18.** (d)	**19.** (c)	**20.** (b)

26

SERIES

INTRODUCTION

Now-a-days questions on series are asked in almost every competitive examination. These questions may involve numbers only, letters (A, B,...) only, or a combination of both.

SERIES

A series is a sequence of numbers. These numbers are called *terms* of the sequence. All the terms of the sequence are arranged according to a certain predefined rule. After carefully studying the given series and finding the specific pattern in which the terms are changing, it is possible to find out the next term of the series.

NUMBER SERIES

1. **Arithmetic Series** An arithmetic series is one in which the difference between any two consecutive terms is always the same and is called the common difference, that is, each successive number is obtained by adding (or subtracting) a fixed number to the previous number.

Illustration 1 Consider the series: 1, 3, 5, 7, 9, ...

Here, 2nd term – Ist term = 3rd term – 2nd term

= 4th term – 3rd term = ... = 2.

Hence, 1, 3, 5, 7, ... is an arithmetic series.

2. **Geometric Series** A geometric series is one in which the ratio of any two consecutive terms is always the same and is called the common ratio, that is, each successive number is obtained by multiplying (or dividing) a fixed number by the previous number.

Illustration 2 The series given below:

(a) 2, 4, 8, 16, 32, ... (b) 3, –6, 12, –24, 48,...

(c) $\frac{1}{4}, \frac{1}{12}, \frac{1}{36}, \frac{1}{100}, \ldots$

(d) $\frac{1}{5}, \frac{1}{30}, \frac{1}{180}, \frac{1}{1080}, \ldots$

(e) $x, x^2, x^3, x^4, \ldots$ (where x is any fixed real number), are all geometric series. The ratio of any term in (a) to the preceding term is 2. The corresponding ratios in (b), (c), (d) and (e) are $-2, \frac{1}{3}, \frac{1}{6}$ and x, respectively.

3. **Series of Squares, Cubes and so on.** Simple powers of natural numbers (squares, cubes, etc.) or their combinations are sometimes used to form some series.

Illustrations 3

(a) 4, 9, 16, 25, 36, ...

Each term in this series is a perfect square. The square roots of the terms are 2, 3, 4, 5, 6, ... Clearly, the square roots of the terms of the given series are forming an arithmetic series with common difference 1. So, the next term of the series will be $(6 + 1)^2$, that is, 49.

(b) 1, 27, 125, 343, ...

Each term in this series is a perfect cube. The cube roots of its terms are 1, 3, 5, 7, ... clearly, the cube roots of the terms of the given series are forming an arithmetic series with common difference 2.

So, the next term of the series will be 9^3, that is, 729.

(c) $\frac{1}{8}, \frac{4}{27}, \frac{9}{64}, \frac{16}{125}, \ldots$

In the above series, the numerators are squares of natural number (n), while the denominators are cubes of $(n + 1)$.

So, the next term of the series will be $\frac{25}{216}$.

4. **Arithmetic Series of Second Order** We know that in an arithmetic series, the difference of any two consecutive terms is always the same. This is arithmetic series of first order.

A series in which the difference between successive terms themselves form an arithmetic series is called an arithmetic series of second order.

Illustration 4 Consider the series 1, 3, 7, 13, ...

The difference between successive terms of the above series are 2, 4, 6, ... which form an arithmetic series with common difference 2.

So, the next term of the series will be (13 + 8), that is, 21.

5. **Arithmetic Series of Third Order** A series in which the difference between successive terms themselves form an arithmetic series of second order, is called an arithmetic series of third order.

Illustration 5 Consider the series : 2, 9, 17, 28, ...

The difference of successive terms of the above series are 7, 8, 11, 16, ...

The difference of successive terms of the above series are 1, 3, 5, ... which forms an arithmetic series with common difference 2.

So, the next term of the series will be (28 + 16), i.e., 44.

In this manner, we can construct arithmetic series of higher order.

6. **Arithmetico-Geometric Series** In this series each successive term is obtained by first adding a fixed number to the previous term and then multiplying it by another fixed number.

Illustration 6 The series: 1, 9, 33, 105, ... is an arithmetico-geometric series as each successive term is obtained by first adding 2 to the previous term and multiplying it by 3.

So, the next term of the series will be (105 + 2) × 3, that is, 321.

It is important to note that the differences of successive numbers in the above series are 8, 24, 72, ... which are forming a geometric series.

7. **Geometrico-Arithmetic Series** In this series each successive term is obtained by first multiplying (or dividing) the previous term by a fixed number and then adding (or subtracting) another fixed number.

Illustration 7 The series : 2, 5, 17, 65, ... is a geometrico-arithmetic series as each successive term is obtained by first multiplying the previous term by 4 and then subtracting 3 from it.

So, the next term of the series will be (65 × 4) –3, that is, 257.

Again, note that the differences of successive numbers in the above series are 3, 12, 48, ... which are forming a geometric series.

8. **Double Series** It consists of two series combined into a single series. The alternating terms of this series form an independent series.

Illustration 8 Consider the series:

1, 2, 4, 6, 7, 18, 10, 54, ...

Terms at odd places of the series: 1, 4, 7, 10, ... is an arithmetic series.

Terms at even places of the series: 2, 6, 18, 54, ... is a geometric series.

So, the next term of the series will be (10 + 3), that is, 13.

Finding the Wrong Term in a Series

In such questions, a number series is given of which all others except one are similar in some respect. The one term of the sequence does not follow the same pattern as is followed by the others. This one is the wrong term in the series. To find the wrong term in a given series we must study the given series carefully and find the pattern/rule in which the terms are changing. After that, we should find which of the terms is not changing according that pattern/rule. Thus, the wrong term is found.

Illustration 9 Find the wrong term in the given series: 5, 10, 17, 24, 37, 50, 65.

Solution The terms of the series are in the following order:

$2^2 + 1, 3^2 + 1, 4^2 + 1, 5^2 + 1, 6^2 + 1, 7^2 + 1, 8^2 + 1$

Clearly, fourth term of the series, that is, 24 should be replaced by 26 so that all the terms of the series follow a particular pattern. Thus, 24 is the wrong term in the given series.

Finding the Missing Term of the Series

In such questions, a number series is given in which a blank space or question mark is provided in place of any one term of the series. The term at the blank space follow the same pattern as followed by other terms. We are required to find the missing term to replace the blank space or question mark.

Illustration 10 Find the missing term in the given series: 49, 56, 64, 72, ?, 90, 100

Solution The terms of the series are in the following order

$7^2, 7^2 + 7, 8^2, 8^2 + 8, 9^2, 9^2 + 9, 10^2$

Clearly, fifth term in place of question mark will be 9^2, that is, 81.

SOME SPECIAL SERIES

1. **Series of Date or Time**

(a) Consider the series,

3 – 2 – 2004, 13 – 2 – 2004, 23 – 2 – 2004, 5 – 3 – 2004,

Here, each successive date differs by 10 days. Since 2004 is a leap year, 5 – 3 – 2004 should be replaced by 4 – 3 – 2004.

(b) Consider the series,

3.35, 5.00, 6.25, 7.40, 9.15, 10.40

Here, each successive time differs by 1 hr 25 min. Therefore, 7.40 should be replaced by 7.50.

2. **Numbers followed by their L.C.M. or H.C.F.**

(a) Consider the series,

1, 2, 3, 6, 4, 5, 6, 60, 5, 6, 7, ...?

1st part	2nd part	3rd part
1, 2, 3, 6	4, 5, 6, 60	5, 6, 7 ?

Here, in each part fourth number is L.C.M. of first three numbers. Thus, the number in place of question mark will be 210 (L.C.M. of 5, 6, 7).

(b) Consider the series,

8, 4, 4, 7, 8, 1, 3, 9, 3, 2, 1, ?

1st part	2nd part	3rd part	4th part
8, 4, 4	7, 8, 1	3, 9, 3	2, 1 ?

Here, in each part third number is H.C.F. of first two numbers. Thus , the number in place of question mark will be 1 (H.C.F. of 2, 1,).

3. Numbers Followed by their Product

Consider the series,

1, 3, 3, 9, 27, 243, ?

Here, $1 \times 3 = 3$

$3 \times 3 = 9$

$3 \times 9 = 27$

$9 \times 27 = 243$

$27 \times 243 = 6561$

Thus, the number in place of question mark will be 27×243, that is, 6561.

4. By Use of Digit Sum

Consider the series,

11, 13, 17, 25, 32, ?

Here, $13 = 11 + (1 + 1)$

$17 = 13 + (1 + 3)$

$25 = 17 + (1 + 7)$

$32 = 25 + (2 + 5)$

That is, next number = previous number + digit sum of pervious number.

Thus, the number in place of question mark will be $32 + (3 + 2) = 37$.

Alpha – Numeric Series

Such series involve the use of both the letters of the alphabet as well as the numbers. It is a two-line series. One line is a number series while the other line is an alphabet series. The terms of both the series follow the same pattern/rule. One of these two series is completely known. We have to find the required number of the incomplete series.

Illustration 11 2, 7, 17, 37, 77,

3, *a, b, c, d,*

2 → 7 → 17 → 37 → 77

$\times 2 + 3$ $\times 2 + 3$ $\times 2 + 3$ $\times 2 + 3$

$\therefore$ $a = 3 \times 2 + 3 = 9$

$b = 9 \times 2 + 3 = 21$

$c = 21 \times 2 + 3 = 45$

$d = 45 \times 2 + 3 = 93$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Insert the missing number

5, 8, 12, 17, 23, –, 38

(a) 29 (b) 30
(c) 32 (d) 25

2. Insert the missing number

4, 9, 20, 43, 90, —

(a) 185 (b) 172
(c) 179 (d) 165

3. Insert the missing number

1, 1, 4, 8, 9, 27, 16,

(a) 25 (b) 36
(c) 125 (d) 64

4. Fill in the missing number

2, 6, 3, 4, 20, 5, 6, ?, 7

(a) 25 (b) 42
(c) 24 (d) 18

5. Fill in the missing number

1, 5, 11, 19, 29, ?

(a) 47 (b) 41
(c) 39 (d) 55

6. Fill in the missing number

3, 6, 21, 28, 55, 66, ?, 120

(a) 106 (b) 108
(c) 105 (d) 102

7. Fill in the missing number

5, 13, 25, 41, ?, 85, 113, 145

(a) 42 (b) 64
(c) 63 (d) 61

8. Fill in the missing number

4, 5, 9, 18, 34, ?

(a) 42 (b) 59
(c) 38 (d) None of these

9. Fill in the missing number

1799, 899, 449, ?

(a) 333 (b) 114
(c) 111 (d) 224

10. Fill in the missing number
2, 1, 2, 4, 4, 5, 6, 8, 8, 10, 11, ?
(a) 12 (b) 8
(c) 10 (d) 9

11. Fill in the missing number
5, 11, 19, 29, ?
(a) 31 (b) 52
(c) 41 (d) 51

12. Fill in the missing number
0, 3, 12, 30, ?, 105, 168
(a) 61 (b) 62
(c) 60 (d) 63

13. Fill in the missing number
15, 20, 30, ?
(a) 45 (b) 40
(c) 48 (d) 50

14. Fill in the missing number
11, 10, ?, 100, 1001, 1000, 1001
(a) 110 (b) 111
(c) 101 (d) None of these

15. Fill in the missing number
99, 95, 86, 70, ?
(a) 45 (b) 62
(c) 65 (d) 55

16. Fill in the missing number
5, 18, 10, 12, 15, ?
(a) 4 (b) 8
(c) 6 (d) 10

17. Fill in the missing number
12, 8, 14, 6, 16, ?
(a) 18 (b) 4
(c) 32 (d) 10

18. Fill in the missing number
13, 21, 29, 34, 43, 92, 12, ?
(a) 84 (b) 31
(c) 92 (d) 12

19. Fill in the missing number
3, 15, 35, ... , 99, 143
(a) 68 (b) 58
(c) 63 (d) 45

20. Fill in the missing number
4, 7, 11, 18, 29, 47, ?, 123, 199
(a) 71 (b) 82
(c) 86 (d) 76

In the following number series a wrong number is given. Find out the wrong number.

21. 455, 445, 465, 435, 485, 415, 475
(a) 475 (b) 465
(c) 435 (d) 455
(e) 445

22. 3, 10, 24, 54, 108, 220, 444
(a) 108 (b) 10
(c) 24 (d) 54
(e) 220

23. 8, 18, 40, 86, 178, 370, 752
(a) 86 (b) 178
(c) 40 (d) 370
(e) 752

24. 1, 2, 6, 21, 84, 445, 2676
(a) 6 (b) 21
(c) 2676 (d) 84
(e) 445

25. 1, 16, 9, 64, 25, 216, 49
(a) 64 (b) 216
(c) 16 (d) 49
(e) 9

26. 864, 420, 200, 96, 40, 16, 6
(a) 864 (b) 200
(c) 96 (d) 16
(e) 40

27. 9, 13, 21, 37, 69, 132, 261
(a) 9 (b) 13
(c) 261 (d) 261
(e) 132

28. 2, 5, 18, 19, 24, 29, 34
(a) 18 (b) 2
(c) 19 (d) 29
(e) 34

29. 1, 5, 11, 19, 29, 55
(a) 29 (b) 55
(c) 11 (d) 5

30. 2, 4, 4, 16, 8, 256, 64
(a) 8 (b) 16
(c) 64 (d) 256
(e) 4

Directions (31–40): In each of the questions below a number series has been given followed by five alternatives. One term of the given number series is wrong. Find out that worng term and spot out a number from the alternatives which will replace the wrong term of the series.

31. 2, 9, 28, 65, 126, 216, 344
(a) 38 (b) 217
(c) 356 (d) 66

32. 58, 57, 54, 50, 42, 33, 22
(a) 48 (b) 49
(c) 52 (d) 30
(e) 18

33. 0, 9, 64, 169, 576, 1225
(a) 225 (b) 360
(c) 444 (d) 556
(e) 630

34. 1, 3, 7, 19, 42, 89, 184
(a) 8 (b) 9
(c) 24 (d) 30
(e) 182

35. 169, 121, 80, 49, 25, 9, 1
(a) 100 (b) 81
(c) 36 (d) 16
(e) 4

36. 7, 9, 17, 42, 91, 172, 293
(a) 16 (b) 25
(c) 36 (d) 8
(e) 49

37. 8, 14, 26, 48, 98, 194, 386
(a) 60 (b) 50
(c) 72 (d) 96
(e) 108

38. 95, 86, 73, 62, 47, 30, 11
(a) 90 (b) 75
(c) 64 (d) 35
(e) 15

39. 7, 14, 56, 168, 336, 1344, 2688, 8064
(a) 3032 (b) 5032
(c) 4032 (d) 2680
(e) 332

40. 11, 15, 17, 19, 23, 25
(a) 1 (b) 18
(c) 21 (d) 10
(e) 13

Direction (41–49): In each of the following questions a number series is given. After the series a number is given followed by (A), (B), (C), (D) and (E). Complete the series starting with the number given following the sequence of the given series. Then, answer the question given below each:

41. 1 9 65 393
2 (A) (B) (C) (D) (E)
Which of the following numbers will come in place of (C)?
(a) 490 (b) 729
(c) 854 (d) 734
(e) None of these

42. 616, 496, 397, 317, 254,
838 (A), (B), (C), (D), (E),
Which of the following numbers will come in place of (E)?
(a) 428 (b) 608
(c) 426 (d) 529
(e) 712

43. 434, 353, 417, 368, 404, 379,
108 (A), (B), (C), (D), (E)
Which of the following numbers will come in place of (E)?
(a) 27 (b) 91
(c) 42 (d) 53
(e) 78

44. 4, 16, 48, 120, 272,
124, (A), (B), (C), (D), (E)
Which of the following numbers will come in place of (C)?
(a) 4424 (b) 256
(c) 528 (d) 1080
(e) 2192

45. 1, 9, 65, 393,
2, (A), (B), (C), (D), (E)
Which of the following numbers will come in place of (C)?
(a) 490 (b) 729
(c) 854 (d) 734
(e) None of these

46. 848, 420, 206, 99, 45.5,
664, (A), (B), (C), (D), (E)
Which of the following numbers will come in place of (D)?
(a) 32 (b) 34
(c) 160 (d) 328
(e) 13

47. 8, 8, 12, 24,
36 (A), (B), (C), (D), (E)
Which of the following numbers will come in place of (E)?
(a) 108 (b) 36
(c) 810 (d) 54
(e) None these

48. 6, 14, 35, 111, 449,
3 (A), (B), (C), (D), (E)
Which of the following numbers will come in place of (B)?
(a) 93 (b) 377
(c) 1892 (c) 11
(e) 29

49. 8, 49, 288, 1435, 5736,
5 (A), (B), (C), (D), (E)
Which of the following numbers will come in place of (E)?
(a) 162 (b) 805
(c) 9645 (d) 3216
(e) 28

EXPLANATORY ANSWERS

1. Ans. (b)

Solution $8 - 5 = 3$, $12 - 8 = 4$, $17 - 12 = 5$, $23 - 17 = 6$

$\therefore$ $? - 23 = 7$

i.e. $? = 23 + 7 = 30$

With this, $38 - 30 = 8$.

2. Ans. (a)

Solution The pattern is

$9 = 2 \times 4 + 1$

$20 = 2 \times 9 + 2$

$43 = 2 \times 20 + 3$

$90 = 2 \times 43 + 4$

$\therefore$? should be $2 \times 90 + 5 = 185$.

3. Ans. (d)

Solution The first alternate series is 1, 4, 9, 16

i.e. $1^2, 2^2, 3^2, 4^2$ and the second one is

1, 8, 27, ?

i.e. $1^3, 2^3, 3^3, 4^3$.

4. Ans. (b)

Solution $2 \times 3 = 6$

$4 \times 5 = 20$

$6 \times 7 = 42$.

5. Ans. (b)

Solution $5 - 1 = 4$, $11 - 5 = 6$, $19 - 11 = 8$

$29 - 19 = 10$

$\therefore$ $? - 29 = 12$

$\therefore$ $? = 41$.

6. Ans. (c)

Solution Difference between successive terms are

3, 15, 7, 27, 11, ? – 66, 120 – ?

Here, odd places terms form a series

3, 7, 11, 120 – ?

which is an A.P. with common difference 4 and even places terms form a series

15, 27, ? – 66

$\therefore$ $120 - ? = 11 + 4 = 15$

$\therefore$ $? = 120 - 15 = 105$.

7. Ans. (d)

Solution $13 = 5 + 4 \times 2$, $25 = 13 + 4 \times 3$, $41 = 25 + 4 \times 4$

$? = 41 + 4 \times 5$, $85 = ? + 4 \times 6$

$113 = 85 + 4 \times 7$, $145 = 113 + 4 \times 8$

$\therefore$ $? = 41 \times 4 \times 5 = 61$. With this choice

$85 = ? + 4 \times 6$

$= 61 + 24$, which follows the pattern.

8. Ans. (b)

Solution $5 = 4 + 1^2$, $9 = 5 + 2^2$, $18 = 9 + 3^2$,

$34 = 18 + 4^2$,

$\therefore$ $? = 34 + 5^2 = 59$.

9. Ans. (d)

Solution $1799 - 899 = 900$

$899 - 449 = 450 \left(= \frac{1}{2} \times 900\right)$

$\therefore$ $449 - ? = \frac{1}{2} \times 450 = 225$

$\therefore$ $? = 449 - 225 = 224$.

10. Ans. (c)

Solution 1st, 4th, 7th, 10th, and 13th terms are:

2, 4, 6, 8, ?

which is an A.P. with common difference 2

$\therefore$ $? = 8 + 2 = 10$.

11. Ans. (c)

Solution $11 - 5 = 6$, $19 - 11 = 8$, $29 - 19 = 10$

$\therefore$ $? - 29 = 12$

$\therefore$ $? = 12 + 29 = 41$.

12. Ans. (c)

Solution 0, 3, 12, 30, ?, 105, 168

3, 9, 18, ? – 30, 105 – ?, 63

6, 9, ? – 48, 135 – 2?, ? – 42

If we take ? – 48 = 12, then ? = 60, with this choice

$135 - 2? = 135 - 120 = 15$

and, $? - 42 = 18$

$\therefore$ 3rd row becomes

6, 9, 12, 15, 18,

which is an A.P. of common difference 3

$\therefore$? should be 60.

13. Ans. (a)

Solution $20 = 15 + 5 \times 1$;

$30 = 20 + 5 \times 2$

$\therefore$? should be $30 + 5 \times 3 = 45$.

14. Ans. (c)

Solution 1st, 3rd, 5th, 7th, terms are I : 11, ?, 1001 10001 and 2nd, 4th, 6th, terms are

II: 10, 100, 1000

In I:

(i) at first place there is no zero between 1's

(ii) at 3rd place there are two zeros between 1's

(iii) at 4th place there are three zeros between 1's

∴ According to this pattern there should be one zero between 1's at 2nd place

∴ ? be 101.

15. Ans. (a)

Solution

$99 - 95 = 4 = 2^2$, $95 - 86 = 9 = 3^2$

$86 - 70 = 16 = 4^2$

∴ $70 - ? = 5^2$

∴ $70 - 25 = ?$

∴ $? = 45$.

16. Ans. (c)

Solution Numbers at even places form series

I : 18, 12, ?

and numbers at odd places form series

II : 5, 10, 15

Keeping the pattern in I

? should be $12 - 6 = 6$.

17. Ans. (b)

Solution In the first alternate series, namely, 12, 14, 16 each term is increased by 2 and in the second, namely, 8, 6 each term is decreased by 2, missing figure is of 2nd series and hence should by $6 - 2 = 4$.

18. Ans. (b)

Solution Making pairs taking first number from right and first from left, 2nd number from right and 2nd number from left and so on.

(13, ?); (21, 12); (29, 92); (34, 43)

In each pair numbers have their digits reversed keeping this pattern, ? should be 31.

19. Ans. (c)

Solution

$3 = 2^2 - 1$; $15 = 4^2 - 1$; $35 = 6^2 - 1$

$99 = 10^2 - 1$; $143 = 12^2 - 1$

∴ Missing figure should be $8^2 - 1 = 63$.

20. Ans. (d)

Solution The pattern is

$4 + 7 = 11$

$18 + 29 = 47$

$? + 123 = 199$

∴ $? = 199 - 123 = 76$.

21. Ans. (a)

Solution Series formed by numbers at odd and even places respectively are:

455, 465, 485, 475 ...(I)

and, 445, 435, 415 ...(II)

The difference between successive terms of I are:

10, 20, –10

and of II are

–10, – 20

–10 in (I) is abnormal. It should be 30

∴ 475 is wrong and should be replaced by 485 + 30 = 515.

22. Ans. (d)

Solution

$10 = 2 \times 3 + 4$

$24 = 2 \times 10 + 4$

$54 = 2 \times 24 + 6$

$108 = 2 \times 54 + 0$

$220 = 2 \times 108 + 4$

$444 = 2 \times 220 + 4$

Pattern is disturbed at 3rd and 4th stages.

∴ 54 is wrong and should be replaced by $2 \times 24 + 4 = 52$. With this choice at 4th stage, $108 = 2 \times 52 + 4$ which follows pattern.

23. Ans. (b)

Solution

$18 = 8 \times 2 + 2 \times 1$; $40 = 18 \times 2 + 2 \times 2$

$86 = 40 \times 2 + 2 \times 3$; $178 \neq 86 \times 2 + 2 \times 4$

$370 \neq 178 \times 2 + 2 \times 5$; $752 = 370 \times 2 + 2 \times 6$

∴ 178 is wrong and should be replaced by

$2 \times 86 + 2 \times 4 = 180$

With this choice 5th place

$2 \times 180 + 2 \times 5 = 370$

which is according to the pattern.

24. Ans. (d)

Solution

$2676 = 6 \times 445 + 6$; $445 \neq 5 \times 84 + 5$

$84 \neq 4 \times 21 + 4$; $21 = 3 \times 6 + 3$

$6 = 2 \times 2 + 2$; $2 = 1 \times 1 + 1$

Obviously, 84 is wrong and should be replaced by $4 \times 21 + 4 = 88$

With this, $445 = 5 \times 88 + 5$.

25. Ans. (b)

Solution Numbers at even places are

$16 = 4^2$; $64 = 8^2$; $216 \neq 12^2$

and numbers at odd places are

$1 = 1^2$; $9 = 3^2$; $25 = 5^2$; $49 = 7^2$

∴ 216 is wrong.

26. Ans. (c)

Solution

$864 = 2 \times 420 + 4 \times 6$; $420 = 2 \times 200 + 4 \times 5$

$200 \neq 2 \times 96 + 4 \times 4$; $96 \neq 2 \times 40 + 4 \times 3$

$40 = 2 \times 16 + 4 \times 2$; $16 = 2 \times 6 + 4 \times 1$

∴ 96 is wrong and should be replaced by $2 \times 40 + 4 \times 3 = 92$. With this choice at 3rd stage

$$200 = 2 \times 92 + 4 \times 4.$$

27. Ans. (e)

Solution $13 - 9 = 4$; $21 - 13 = 8$; $37 - 21 = 16$

$69 - 37 = 32$; $132 - 69 = 63$; $261 - 132 = 129$

Pattern is disturbed at last two stages.

∴ 132 is wrong and should be replaced by $69 + 64 = 133$. With this choice at last stage

$$261 - 133 = 128.$$

28. Ans (b)

Solution Series formed by numbers at odd places and even places respectively are

I: 2, 18, 24, 34

II: 5, 19, 29

Successive terms in I and II have difference

16, 6, 10

and 14, 10, respectively.

Abnormality is at 16. It should be replaced by 2

∴ 2 is wrong and should be replaced by 16.

29. Ans. (b)

Solution

$5 - 1 = 4$; $11 - 5 = 6$; $19 - 11 = 8$

$29 - 19 = 10$; $55 - 29 = 26$

Pattern gets disturbed at last stage

∴ 55 is wrong

It should be $29 + 12 = 41$.

30. Ans. (c)

Solution Nos. at odd and even places form respective series

	I:	2,	4,	8,	64
i.e.		2^1,	2^2,	2^3,	2^6
and,	II:	4,	16,	256	
i.e.		2^2,	2^4,	2^8	

Obviously, $2^6 = 64$ is wrong and should be replaced by $2^4 = 16$.

31. Ans. (b)

Solution If 216 is replaced by 217 the terms of the series will get arranged in the order of

$1 \times (1)^2 + 1$, $2 \times (2)^2 + 1$, $3 \times (3)^2 + 1$, $4 \times (4)^2 + 1$ .... and so on.

Therefore, alternative (b) is the correct answer.

32. Ans. (b)

Solution A careful scrutiny of the series reveals that if 50 is replaced by 49, then difference between successive terms will in the order of 1, 3, 5, 7, 9, 11. Therefore, alternative (b) is the correct answer.

33. Ans. (a)

Solution If 169 is replaced by 225 the terms will get arranged in a particular series, that is, $(1^2 - 1)^2$, $(2^2 - 1)^2$, $(3^2 - 1)^2$, $(4^2 - 1)^2$, $(5^2 - 1)^2$ and $(6^2 - 1)^2$.

Therefore, alternative (a) is the correct answer.

34. Ans. (a)

Solution If 7 is replaced by 8 these terms of the series will get arranged in order of

$1 \times 2 + 1$, $3 \times 2 + 2$, $8 \times 2 + 3$,

$19 \times 2 + 4$, $42 \times 2 + 5$, $89 \times 2 + 6$

Therefore, alternative (a) is the correct answer.

35. Ans. (b)

Solution A careful scrutiny of the series reveals that if 80 is replaced by 81 then the series will be arranged in the order of

$13^2, 11^2, 9^2, 7^2, 5^2, 3^2, 1^2$. Therefore, (b) is the correct alternative.

36. Ans. (d)

Solution It is obvious from the given series that if 9 is replaced by 8 then difference between successive terms will be in the order of 1^2, 3^2, 5^2, 7^2, 9^2 and 11^2

Therefore, alternative (d) is the correct answer.

37. Ans. (b)

Solution If 48 is replaced by 50 each term of the series is obtained by subtracting 2 from twice its previous term. Therefore, alternative (b) is the correct answer.

38. Ans. (b)

Solution If 73 is replaced by 75, difference between successive terms of the series will be in the order of 9, 11, 13, 15, 17 and 19.

Therefore, alternative (b) is the correct alternative.

39. Ans. (c)

Solution A careful scrutiny of the given series reveals that second term is two times the first, third term is four times the second and fourth term is three times the third. The same pattern is being followed by the remaining terms of the series. Therefore, 2688 should be replaced by 4032. Hence, alternative (c) is the correct alternative.

40. Ans. (e)

Solution If 15 is replaced by 13 the difference between successive terms will be in the order of 2, 4, 2, 4, ... and so on. Therefore, alternative (e) is the correct alternative.

41. Ans. (b)

Solution The pattern followed by the numbers of the given series is:

$$9 = 8 \times 1 + 1; \quad 65 = 7 \times 9 + 2$$

$$393 = 6 \times 65 + 3$$

$\therefore$ (A) $= 8 \times 2 + 1 = 17$
(B) $= 7 \times 17 + 2 = 121$
(C) $= 6 \times 121 + 3 = 729$
(D) $= 5 \times 729 + 4 = 3649$
(E) $= 4 \times 3649 + 5 = 14601$.

42. Ans. (a)

Solution The pattern is

$616 - 496 = 120 = 12 \times 10$
$469 - 397 = 99 = 11 \times 9$
$397 - 317 = 80 = 10 \times 8$
$317 - 254 = 63 = 9 \times 7$
$\therefore$ (A) $= 838 - 120 = 718$
(B) $= 718 - 99 = 619$
(C) $= 619 - 80 = 539$
(D) $= 539 - 63 = 476$
(E) $= 476 - 48\ (= 8 \times 6) = 428$.

43. Ans. (d)

Solution

$434 - 353 = 9^2$
$353 - 417 = 8^2$
$417 - 368 = 7^2$
$368 - 404 = 6^2$
$404 - 379 = 5^2$
$\therefore$ $108 - (A) = 9^2 \Rightarrow (A) = 108 - 81 = 27$
$27 - (B) = -8^2 \Rightarrow (B) = 27 + 64 = 91$
$91 - (C) = 7^2 \Rightarrow (C) = 91 - 49 = 42$
$42 - (D) = -6^2 \Rightarrow (D) = 42 + 36 = 78$
$78 - (E) = 5^2 \Rightarrow (E) = 78 - 25 = 53$.

44. Ans. (d)

Solution The rule followed is:

$272 = 2 \times 120 + 8 \times 4$
$120 = 2 \times 48 + 8 \times 3$
$48 = 2 \times 16 + 8 \times 2$
$16 = 2 \times 4 + 8 \times 1$
$\therefore$ (A) $= 2 \times 124 + 8 \times 1 = 256$
(B) $= 2 \times 256 + 8 \times 2 = 528$
(C) $= 2 \times 528 + 8 \times 3 = 1080$
(D) $= 2 \times 1080 + 8 \times 4 = 2192$
(E) $= 2 \times 2192 + 8 \times 5 = 4424$.

45. Ans. (b)

Solution The pattern followed by the numbers of given series is:

$9 = 8 \times 1 + 1$; $65 = 7 \times 9 + 2$; $393 = 6 \times 65 + 3$
$\therefore$ (A) $= 8 \times 2 + 1 = 17$
(B) $= 7 \times 17 + 2 = 121$
(C) $= 6 \times 121 + 3 = 729$
(D) $= 5 \times 729 + 4 = 3649$
(E) $= 4 \times 3649 + 5 = 14601$.

46. Ans. (b)

Solution

$848 = 2 \times 420 + 8$ $\therefore$ (A) $= \frac{664-8}{2} = 328$
$420 = 2 \times 206 + 8$ (B) $= \frac{328-8}{2} = 160$
$206 = 2 \times 99 + 8$ (C) $= \frac{160-8}{2} = 76$
$99 = 2 \times 45.5 + 8$ (D) $= \frac{76-8}{2} = 34$
(E) $= \frac{34-8}{2} = 13$.

47. Ans. (c)

Solution The rule is

$8 = \frac{8}{2} \times 2$ $\therefore$ (A) $= \frac{36}{2} \times 2 = 36$
$12 = \frac{8}{2} \times 3$ (B) $= \frac{36}{2} \times 3 = 54$
$24 = \frac{12}{2} \times 4$ (C) $= \frac{54}{2} \times 4 = 108$
(D) $= \frac{108}{2} \times 5 = 270$
(E) $= \frac{270}{2} \times 6 = 810$.

48. Ans. (e)

Solution

$449 = 4 \times 111 + 5$ $\therefore$ (A) $= 1 \times 3 + 8 = 11$
$111 = 3 \times 35 + 6$ (B) $= 2 \times 11 + 7 = 29$
$35 = 2 \times 14 + 7$ (C) $= 3 \times 29 + 6 = 93$
$14 = 1 \times 6 + 8$ (C) $= 3 \times 29 + 6 = 93$
(E) $= 5 \times 377 + 4 = 1889$.

49. Ans. (c)

Solution The pattern is

$5736 = 4 \times 1435 - 4$
$1435 = 5 \times 288 - 5$
$288 = 6 \times 49 - 6$
$49 = 7 \times 8 - 7$
$\therefore$ (A) $= 7 \times 5 - 7 = 28$
(B) $= 6 \times 28 - 6 = 162$
(C) $= 5 \times 162 - 5 = 805$
(D) $= 4 \times 805 - 4 = 3216$
(E) $= 3 \times 3216 - 3 = 9645$.

QUESTIONS FROM COMPETITIVE EXAMS

1. The next term in the series
2, 10, 26, 50, 82, ... will be
(a) 118 (b) 120
(c) 122 (d) 124
[Based on Assistant's Grade Exam 1998]

2. The next number of the sequence
3, 5, 9, 17, 33, ... is
(a) 65 (b) 60
(c) 50 (d) 49
[Based on SSC (GL) Prel. Exam 2000]

3. The next number of the sequence 2, 5, 10, 14, 18, 23, 26, 32, ... is
(a) 33 (b) 34
(c) 36 (d) 37
[Based on SSC (GL) Prel. Exam 2000]

4. The missing number in the sequence 1, 5, 14, 30, ..., 91 is
(a) 45 (b) 55
(c) 56 (d) 51
[Based on SSC (GL) Prel. Exam 2000]

5. Find the missing number in the sequence 0, 2, 8, 18, —, 50
(a) 28 (b) 30
(c) 32 (d) 36
[Based on SSC (GL) Prel. Exam 2000]

6. The next number of the sequence
0, 3, 8, 15, 24, 25, ... is
(a) 46 (b) 47
(c) 48 (d) 50
[Based on SSC (GL) Prel. Exam 2000]

7. The next term of the sequence 1, 2, 5, 26, ... is
(a) 677 (b) 47
(c) 50 (d) 152
[Based on SSC (GL) Prel. Exam 2000]

8. The next number of the sequence
$\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}$, ... is
(a) $\frac{10}{24}$ (b) $\frac{11}{32}$
(c) $\frac{9}{24}$ (d) $\frac{9}{32}$
[Based on SSC (GL) Prel. Exam 2000]

9. The next number of the sequence
2, 16, 54, ? is
(a) 96 (b) 116
(c) 128 (d) 134
[Based on SSC (GL) Prel. Exam 2000]

10. 11, 15, 38, 126
7 (a) (b) (c) (d) (e)
Which of the following will come in place of (c)?
(a) 102 (b) 30
(c) 2140 (d) 80
(e) 424 **[Based on BSRB Mumbai Bank PO 1998]**

11. 2 3 8 27
5 (a) (b) (c) (d) (e)
Which of the following will come in place of (e)?
(a) 184 (b) 6
(c) 925 (d) 45
(e) 14 **[Based on BSRB Mumbai Bank PO 1998]**

12. 2 3 9 40.5
4 (a) (b) (c) (d) (e)
Which of the following will come in place of (b)?
(a) 486 (b) 81
(c) 3645 (d) 18
(e) 16 **[Based on BSRB Mumbai PO 1998]**

13. 12 28 64 140
137 (a) (b) (c) (d) (e)
Which of the following will come in place of (e)?
(a) 1412 (b) 164
(c) 696 (d) 78
(e) 340 **[Based on BSRB Mumbai Bank PO 1998]**

14. 5 12 60 340
7 (a) (b) (c) (d) (e)
Which of the following will come in place of (d)?
(a) 172 (b) 5044
(c) 1012 (d) 20164
(e) 28 **[Based on BSRB Mumbai Bank PO 1998]**

15. In the following number series, a wrong number is given. Find out the wrong number
11 24 66 164 462 1086 2414
(a) 462 (b) 164
(c) 324 (d) 1086
(e) 66 **[Based on BSRB Mumbai Bank PO 1998]**

Directions (16–20): One number is wrong in each of the number series given in each of the following questions. You have to identify that number and assuming that a new series starts with that number following the same logic as in the given series, which of the numbers given in (a), (b), (c), (d) and (e) given below each series will be the third number in the new series?

16. 3, 5, 12, 38, 154, 914, 4634
(a) 1636 (b) 1222
(c) 1834 (d) 3312
(e) 1488 **[Based on SBI PO 1999]**

17. 3 4 10 34 136 685 4116
(a) 22 (b) 276
(c) 72 (d) 1374
(e) 12 **[Based on SBI PO 1999]**

18. 214 18 162 62 143 90 106
(a) –34 (b) 110
(c) 10 (d) 91
(e) 38 **[Based on SBI PO 1999]**

19. 160 80 120 180 1050 4725 25987.5
(a) 60 (b) 90
(c) 3564 (d) 787.5
(e) 135 **[Based on SBI PO 1999]**

20. 2 3 7 13 26 26 47 78
(a) 11 (b) 13
(c) 15 (d) 18
(e) 20 **[Based on SBI PO 1999]**

Directions (21–25): In each of the questions given below there is a mathematical series. After the series a number is being given followed by (a), (b), (c), (d) and (e). You have to create another series after understanding the sequence of the given series which starts with the given number. Then, answer the question given below.

21. 1 9 65 393
2 (a) (b) (c) (d) (e)
Out of the following numbers which would come in the place of (c)?
(a) 490 (b) 853
(c) 731 (d) 729
(e) None of these **[Based on Bank of Baroda PO 1999]**

22. 8 8 12 24
36 (a) (b) (c) (d) (e)
Out of the following numbers which would come in the place of (e)
(a) 810 (b) 36
(c) 54 (d) 108
(e) None of these
[Based on Bank of Baroda PO 1999]

23. 424 208 100 46
888 (a) (b) (c) (d) (e)
What number would come in place of (b)?
(a) 20 (b) 440
(c) 216 (d) 56
(e) None of these
[Based on Bank of Baroda PO 1999]

24. 4 5 9.75 23.5
7 (a) (b) (c) (d) (e)
What number would come in the place of (d)?
(a) 32.5 (b) 271.5
(c) 8 (d) 14.25
(e) None of these
[Based on Bank of Baroda PO 1999]

25. 5 294 69 238
13 (a) (b) (c) (d) (e)
Which of the following numbers would come in the place of (e)?
(a) 246 (b) 206
(c) 125 (d) 302
(e) None of these
[Based on Bank of Baroda PO 1999]

Directions (26–30): In each of the following questions a number series is given. Only one number is wrong in each series. Find out that wrong number, and taking this wrong number as the first term of the second series formed following the same logic, find out the third term of the second series.

26. 1 2 8 21 88 445
(a) 24.5 (b) 25
(c) 25.5 (d) 24
(e) None of these
[Based on SBI Associates PO 1999]

27. 6 7 18 63 265 1365
(a) 530 (b) 534
(c) 526 (d) 562
(e) None of these
[Based on SBI Associates PO 1999]

28. 7 23 58 127 269 555
(a) 263 (b) 261
(c) 299 (d) 286
(e) None of these

29. 2 7 28 146 877 6140
(a) 242 (b) 246
(c) 252 (d) 341
(e) None of these
[Based on SBI Associates PO 1999]

30. 1 2 6 33 148 765 4626
(a) 46 (b) 124
(c) 18 (d) 82
(e) None of these
[Based on SBI Associates PO 1999]

EXPLANATORY ANSWERS

1. Ans. (c)

Solution The sequence in the given series is
$+ 8, + 16, + 24, + 32, + 40$
$\therefore$ Next term is $82 + 40 = 122$.

2. Ans. (a)

Solution The sequence in the given series is
$+ 2, + 2^2, + 2^3, + 2^4, + 2^5$.

3. Ans. (b)

Solution Numbers at odd and even places form respective series
I: 2, 10, 18, 26, ... is
and, II: 5, 14, 23, 32, ...
I sequence is an A.P. with common difference 8. Therefore, next term will be $26 + 8 = 34$.

4. Ans. (b)

Solution The sequence in the given series is
$+ 2^2, + 3^2, + 4^2, + 5^2, + 6^2.$

5. Ans. (c)

Solution The sequence in the given series is
$+ 2, + 6, + 10, + 14, + 18.$

6. Ans. (c)

Solution The series $+ 3, + 5, + 7, + 9, + 11, + 13.$

7. Ans. (a)

Solution The series is $1^2 + 1 = 2$, $2^2 + 1 = 5$, $5^2 + 1$
$\therefore$ Next term is $26^2 + 1 = 676 + 1 = 677.$

8. Ans. (d)

Solution The sequence in the numerator form is + 2
The sequence in the denominator = × 2.

9. Ans. (c)

Solution The terms of the given series are
$2 \times 1^3, 2 \times 2^3, 2 \times 3^3, 2 \times 4^3$

10. Ans. (a)

Solution The series is
$\times 1 + 4$, $\times 2 + 8$, $\times 3 + 12$, ...

11. Ans. (c)

Solution The series is
$\times 1 + 1$, $\times 2 + 2$, $\times 3 + 3$, ...

12. Ans. (d)

Solution The series is
$\times 1.5$, $\times 3$, $\times 4.5$, ...

13. Ans. (a)

Solution The series is
$\times 2 + 4$, $\times 2 + 8$, $\times 2 + 12$, ...

14. Ans. (b)

Solution The series is
$\times 8 - 28$, $\times 7 - 24$, $\times 6 - 20$, ...

15. Ans. (b)

Solution The series is
$\times 2 + (2 \times 1^2), \times 2 + (2 \times 3^2), \times 2 + (2 \times 5^2), \times 2 + (2 \times 7^2)$, ...
So, 164 should be replaced by 182.

16. Ans. (c)

Solution The series is
$\times 1 + 2$, $\times 2 + 2$, $\times 3 + 2$, $\times 4 + 2$, $\times 5 + 2$, $\times 6 + 2$.
914 is incorrect. It should be 772.
The new series begins with 914.

17. Ans. (c)

Solution The series is
$\times 1 + 1$, $\times 2 + 2$, $\times 3 + 3$, $\times 4 + 4$, $\times 5 + 5$, $\times 6 + 6$
34 should be 33 and thus the new series starts with 34.

18. Ans. (d)

Solution The series is
$- (14)^2, + (12)^2, - (10)^2, + (8)^2, - (16)^2$ and so on.
143 should be 126 and thus the new series starts with 143.

19. Ans (e)

Solution The series is
$\times \frac{1}{2}, \times \frac{3}{2}, \times \frac{5}{2}, \times \frac{7}{2}, \times \frac{9}{2}, \times \frac{11}{2}$ and so on.
So, 180 is incorrect.

20. Ans. (a)

Solution The series is $+ 1^2 - 0, + 2^2 - 1, + 3^2 - 2,$
$+ 4^2 - 3, + 5^2 - 4, + 6^2 - 5.$
So, 7 is incorrect.

21. Ans. (d)

Solution The series is $\times 8 + 1$, $\times 7 + 2$, $\times 6 + 3$,
$\therefore$ $a = 2 \times 8 + 1 = 17$, $b = 17 \times 7 + 2 = 121$
$c = 121 \times 6 + 3 = 729.$

22. Ans. (a)

Solution The series is $\times 1$, $\times 1.5$, $\times 2$
$\therefore$ $a = 36 \times 1 = 36$, $b = 36 \times 1.5 = 54$, $c = 54 \times 2$
$= 108$, $d = 108 \times 2.5 = 270$, $e = 270 \times 3 = 810.$

23. Ans. (c)

Solution The series is $\div 2 - 4$
$\therefore$ $a = 888 \div 2 - 4 = 440$
and, $b = 440 \div 2 - 4 = 216.$

24. Ans. (e)

Solution The series is $\times 1 + 1$, $\times 1.5 + 2.25$, $\times 2 + 4$, $\times 2.5 + 6.25$, $\times 3 + 9$, ...
$\therefore$ $a = 7 \times 1 + 1 = 8,$
$b = 8 \times 1.5 + 2.25 = 14.25$
$c = 14.25 \times 2 + 4 = 32.5,$
$d = 32.5 \times 2.5 + 6.25 = 81.25 + 6.25 = 87.25.$

25. Ans. (b)

Solution The series is
$+ (17)^2, - (15)^2, + (13)^2, - (11)^2, + (9)^2$
$\therefore$ $a = 13 + (17)^2 = 302$
$b = 302 - (15)^2 = 302 - 225 = 77$
$c = 77 + (13)^2 = 77 + 169 = 246$
$d = 246 - (11)^2 = 246 - 121 = 125$
$e = 125 + (9)^2 = 125 + 81 = 206.$

26. Ans. (e)

Solution The series is $\times 1 + 1$, $\times 2 + 2$, $\times 3 + 3$, ... So 8 is wrong.
Beginning with 8 we get 20 as third term.

27. Ans. (b)

Solution The series is $\times 1 + 1^2, \times 2 + 2^2, \times 3 + 3^2, \ldots$

So, 265 is incorrect.

28. Ans. (b)

Solution The series is $\times 2 + 9, \times 2 + 11, \times 2 + 13, \ldots$

So, 58 is incorrect.

29. Ans. (d)

Solution The series is

$\times 3 + 1, \times 4 + 1, \times 5 + 1, \ldots$

So, 28 is incorrect.

30. Ans. (c)

Solution The series is

$\times 1 + 1^2, \times 2 + 2^2, \times 3 + 3^2, \times 4 + 4^2 \ldots$

So, 6 is incorrect.

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. Complete the series

2, 5, 9, 19, 37, ...

(a) 76 (b) 74
(c) 75 (d) None of these

2. Find out the wrong number in the series

5, 15, 30, 135, 405, 1215, 3645

(a) 3645 (b) 1215
(c) 405 (d) 30
(e) 15

3. Find out the wrong number in the series

190, 166, 145, 128, 112, 100, 91

(a) 100 (b) 166
(c) 145 (d) 128
(e) 112

4. Find out the wrong number in the series

125, 106, 88, 76, 65, 58, 53

(a) 125 (b) 106
(c) 88 (d) 76
(e) 65

5. Insert the missing number

5, 10, 13, 26, 29, 58, 61, (?)

(a) 122 (b) 64
(c) 125 (d) 128

6. Insert the missing number

8, 24, 12, 36, 18, 54, (?)

(a) 27 (b) 108
(c) 68 (d) 72

7. Insert the missing number

7, 26, 124, 215, 342, (?)

(a) 481 (b) 511
(c) 391 (d) 421

8.

–4	–2	1	8	31
–1	*a*	*b*	*c*	*d*

Find the value of *b*.

(a) 19 (b) 17
(c) 22 (d) 21

9.

27	44	71	108	155	
34	*a*	*b*	*c*	*d*	*e*

What value should replace (e)?

(a) 319 (b) 217
(c) 219 (e) 419

10.

108	52	24	10	3	
64	*a*	*b*	*c*	*d*	*e*

What is the value of (*c*)?

(a) 5 (b) 4.5
(c) 4 (d) None of these

Test I **[Time: 10 min]**

11. Find out the wrong number in the series

3, 7, 15, 27, 63, 127, 255

(a) 7 (c) 15
(d) 27 (d) 63
(e) 127

12. Find out the wrong number in the series

19, 26, 33, 46, 59, 74, 91

(a) 26 (b) 33
(c) 46 (d) 59
(e) 74

13. Find out the wrong number in the series

445, 221, 109, 46, 25, 11, 4

(a) 221 (b) 109
(c) 46 (d) 25
(e) 11

14. Find out the wrong number in the series

3, 7, 15, 39, 63, 127, 255, 511

(a) 7 (b) 15
(c) 39 (d) 63
(e) 127

15. Insert the missing number

13, 31, 63, 127, 255, (?)

(a) 513 (b) 511
(c) 517 (d) 523

16. Insert the missing number

2, 4, 12, 48, 240, (?)

(a) 960 (b) 1440
(c) 1080 (d) 1920

17. Insert the missing number

10, 5, 13, 10, 16, 20 19, (?)

(a) 22 (b) 40
(c) 38 (d) 23

18.

1296	864	576	384	256	
1080	*a*	*b*	*c*	*d*	*e*

Find the value of *c*.

(a) 380 (b) 340
(c) 320 (d) 360

19.

7	13	78	83	415	
3	*a*	*b*	*c*	*d*	*e*

Find the value of *b*

(a) 54 (b) 56
(c) 58 (d) 52

20.

3240	540	108	27	9	
3720	*a*	*b*	*c*	*d*	*e*

Find the value of *d*

(a) 11.33 (b) 10.33
(c) 12.33 (d) 9.33

Answers

1. (c)	**2.** (d)	**3.** (d)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (c)	**10.** (b)
11. (c)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (b)	**17.** (b)	**18.** (c)	**19.** (a)	**20.** (b)

27

PROGRESSION

INTRODUCTION

In this chapter, we shall be concerned with the study of sequences, i.e. special types of functions whose domain is the set N of natural numbers. We shall study particular types of sequences called *arithmetic* sequences, *geometric* sequences and *harmonic* sequences and also their corresponding series.

Premiums on life insurance, fixed deposits in a bank, loan instalments payments, disintegration or decay of radioactive materials and the like are some of the examples where the concept of sequence and series is used.

SEQUENCE

A *sequence* is a function whose domain is the set N of natural numbers and range, a subset of real numbers or complex numbers.

A sequence whose range is a subset of real numbers is called a *real sequence*. Since we shall be dealing with real sequences only, we shall use the term 'sequence' to denote a 'real sequence'.

Notation

The different terms of a sequence are usually denoted by $a_1, a_2, a_3, \ldots$ or by $t_1, t_2, t_3, \ldots$ The subscript (always a natural number) denotes the position of the term in the sequence. The number occurring at the nth place of a sequence, i.e. t_n is called the *general term* of the sequence.

Note : A sequence is said to be *finite* or *infinite* (according as finite or infinite number of terms it has.)

PROGRESSIONS

If the terms of a sequence follow certain pattern, then the sequence is called a *progression*.

Illustration 1 Consider the following sequences:

(i) 3, 5, 7, 9, ..., 21. (ii) 8, 5, 2, – 1, – 4, ...

(iii) 2, 6, 18, 54, ..., 1458.

(iv) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ (v) 1, 4, 9, 16, ...

We observe that each term (except the first) in (i) is formed by adding 2 to the preceding term; each term in (ii) is formed by subtracting 3 from the preceding term; each term in (iii) is formed by multiplying the preceding term by 3; each term in (iv) is formed by dividing the preceding term by 2; each term in (v) is formed by squaring the next natural number. Thus, each of (i) to (v) is a progression. Moreover, (i) and (iii) are finite sequences, whereas (ii), (iv) and (v) are infinite sequences.

However, to define a sequence we need not always have an explicit formula for the nth term. For example, for the infinite sequence 2, 3, 5, 7, 11, 13, 17, ... of all positive prime numbers, we may not be able to give an explicit formula for the nth term.

SERIES

By adding or subtracting the terms of a sequence, we obtain a *series*. A series is finite or infinite according as the number of terms in the corresponding sequence is finite or infinite.

Illustration 2 The following:

(i) 3 + 5 + 7 + 9 + ... + 21.

(ii) 8 + 5 + 2 + (– 1) + ...

(iii) 2 + 6 + 18 + 54 + ... + 1458.

(iv) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

(v) 1 + 4 + 9 + 16 + ...

are the series corresponding to the above sequences, (i) to (v).

ARITHMETIC PROGRESSION (A.P.)

A sequence whose terms increase or decrease by a fixed number is called an *arithmetic progression*. The fixed number is called the *common difference* of the A.P.

In an A.P., we usually denote the first term by a, the common difference by d and the nth term by t_n. Clearly, $d = t_n - t_{n-1}$. Thus, an A.P. can be written as

$$a,\ a + d,\ a + 2d,\ \ldots,\ a + (n - 1)\,d,\ \ldots$$

Illustration 3 Consider the series: 1, 3, 5, 7, 9, ...

Here, 2nd term – 1st term = 3rd term – 2nd term = 4th term – 3rd term = ... = 2.

Hence 1, 3, 5, 7, ... are in A.P. whose first term is 1 and common difference is 2.

Illustration 4 The series: 5, 3, 1, –1, –3, –5, –7, ... is in A.P. whose first term is 5 and common difference is –2.

Notes:

- A sequence $t_1, t_2, t_3, t_4, \ldots$ will be in A.P. if $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \ldots$, i.e. $t_n - t_{n-1}$ = constant, for $n \geq 2$.
- Three numbers a, b, c are in A.P. if and only if $b - a = c - b$, i.e. if and only if $a + c = 2b$.
- Any three numbers in an A.P. can be taken as $a - d, a, a + d$. Any four numbers in an A.P. can be taken as $a - 3d, a - d, a + d$ and $a + 3d$.

  Similarly, five numbers in an A.P. can be taken as $a - 2d, a - d, a, a + d$ and $a + 2d$.

GENERAL TERM OF AN A.P.

Let a be the first term and d be the common difference of an A.P. Then, the A.P. is $a, a + d, a + 2d, a + 3d, \ldots$. We also observe that,

t_1, the first term, is $a = a + (1 - 1)\,d$;

t_2, the second term, is $a + d = a + (2 - 1)\,d$;

t_3, the third term, is $a + 2d = a + (3 - 1)\,d$;

t_4, the fourth term, is $a + 3d = a + (4 - 1)\,d$;

t_n, the nth term, is $a + (n - 1)\,d$.

Thus, the formula, $t_n = a + (n - 1)\,d$ gives the general term of an A.P.

Notes:

- If an A.P. has n terms, then the nth term is called the *last term* of A.P. and it is denoted by l. Therefore, $l = a + (n - 1)\,d$.
- If a is the first term and d the common difference of an A.P. having m terms, then nth term from the end is $(m - n + 1)$th term from the beginning.

  $\therefore$ nth term from the end $= a + (m - n)\,d$.

Illustration 5 A sequence $< t_n >$ is given by the formula $t_n = 10 - 3n$. Prove that it is an A.P.

Solution We have,

$$t_n = 10 - 3n \Rightarrow t_{n+1} = 10 - 3(n + 1) = 7 - 3n.$$

$$\therefore \quad t_{n+1} - t_n = (7 - 3n) - (10 - 3n) = -3,$$

which is independent of n and hence a constant. Therefore, the given sequence $< t_n >$ is an A.P.

Illustration 6 Find the nth term and 19th term of the sequence $5, 2, -1, -4, \ldots$

Solution Clearly, the given sequence is an A.P. with $a = 5$ and $d = -3$.

$$\therefore \quad t_n = a + (n - 1)\,d = 5 + (n - 1)(-3) = -3n + 8.$$

For the 19th term, putting $n = 19$, we get $t_{19} = -3.19 + 8 = -49$.

Sum of n terms of an A.P.

The sum of n terms of an A.P. with first term 'a' and common difference 'd' is given by

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Note:

- If S_n is the sum of n terms of an A.P. whose first term is 'a' and last term is l, then

$$S_n = \frac{n}{2}(a + l).$$

- If common difference d, number of terms n and the last term l, are given then

$$S_n = \frac{n}{2}[2l - (n - 1)\,d]$$

- $t_n = S_n - S_{n-1}$.

Illustration 7 Find the sum of the series $.5 + .51 + .52 + \ldots$ to 100 terms.

Solution The given series is an A.P. with first term, $a = .5$ and common difference, $d = .51 - .5 = .01$.

$\therefore$ Sum of 100 terms

$$= \frac{100}{2}[2 \times .5 + (100 - 1) \times .01]$$

$$= 50\,(1 + 99 \times .01) = 50\,(1 + .99)$$

$$= 50 \times 1.99 = 99.5.$$

Illustration 8 Find the sum of 20 terms of an A.P., whose first term is 3 and the last term is 57.

Solution We have, $a = 3$, $l = 57$, $n = 20$.

$$\therefore \quad S_n = \frac{n}{2}(a + l), \quad \therefore \; S_{20} = \frac{20}{2}(3 + 57) = 600.$$

Hence, the sum of 20 terms is 600.

GEOMETRIC PROGRESSION

A sequence (finite or infinite) of non-zero numbers in which every term, except the first one, bears a constant ratio with its preceding term, is called a *geometric* progression, abbreviated as G.P.

Illustration 9 The sequences given below:

(i) $2, 4, ,8, 16, 32, \ldots$

(ii) $3, -6, 12, -24, 48, \ldots$

(iii) $\frac{1}{4}, \frac{1}{12}, \frac{1}{36}, \frac{1}{108}, \frac{1}{324}, \ldots$

(iv) $\frac{1}{5}, \frac{1}{30}, \frac{1}{180}, \frac{1}{1080}, \frac{1}{6480}, \ldots$

(v) $x, x^2, x^3, x^4, x^5, \ldots$ (where x is any fixed real number),

are all geometric progressions. The ratio of any term in (i) to the preceding term is 2. The corresponding ratios in (ii), (iii), (iv) and (v) are $-2, \frac{1}{3}, \frac{1}{6}$, and x respectively. The ratio of any term of a G.P. to the preceding term is called the *common ratio* of the G.P. Thus, in the above examples, the common ratios are $2, -2, \frac{1}{3}, \frac{1}{6}$ and x respectively.

Note: In a G.P., any term may be obtained by multiplying the preceding term by the common ratio of the G.P. Therefore, if any one term and the common ratio of a G.P. be known, any term can be written out, i.e. the G.P. is then completely known.

In particular, if the first term and the common ratio are known, the G.P. is completely known. The first term and the common ratio of a G.P. are generally denoted by a and r, respectively.

GENERAL TERM OF A G.P.

Let a be the first term and $r\,(\neq 0)$ be the common ratio of a G.P. Let $t_1, t_2, t_3, \ldots, t_n$ denote Ist, 2nd, 3rd, ... , nth terms, respectively. Then, we have

$$t_2 = t_1 r;\ t_3 = t_2 r,\ t_4 = t_3 r,\ \ldots,\ t_n = t_{n-1} r.$$

On multiplying these, we get

$$t_2 t_3 t_4 \ldots t_n = t_1 t_2 t_3 \ldots t_{n-1} r^{n-1} \Rightarrow t_n = t_1 r^{n-1}; \text{ but } t_1 = a.$$

$\therefore$ General term $= t_n = ar^{n-1}$.

Thus, if a is the first term and r the common ratio of a G.P. then the G.P. is $a, ar, ar^2, \ldots, ar^{n-1}$ or $a, ar, ar^2, \ldots$ according as it is fnite or infinite.

Cor. If the last term of a G.P. consisting of n terms is denoted by l, then $l = ar^{n-1}$.

Notes:

- If a is the first term and r the common ratio of a finite G.P. consisting of m terms, then the nth term from the end is given by ar^{m-n}.
- The nth term from the end of a G.P. with the last term l and common ratio r is l/r^{n-1}.
- Three numbers in G.P. can be taken as $a/r, a, ar$; four numbers in G.P. can be taken as $a/r^3, a/r, ar, ar^3$; five numbers in G.P. can be taken as $a/r^2, a/r, a, ar, ar^2$, and so on...
- Three numbers a, b, c are in G.P. if and only if $b/a = c/b$, i.e. if and only if $b^2 = ac$.

Illustration 10 Find the nth term and 12th term of the sequence –6, 18, –54, ...

Solution The given sequence is a G.P. with $a = -6$ and $r = -3$.

$\therefore\ t_n = ar^{n-1} = (-6)(-3)^{n-1} = (-1)^n \cdot 6 \cdot 3^{n-1}$

For the 12th term, putting $n = 12$, we get

$$t_{12} = (-1)^{12} \cdot 6 \cdot 3^{11} = 2 \cdot 3^{12}.$$

Sum of *n* terms of a G.P.

The sum of first n terms of a G.P. with first term a and common ratio r is given by

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Notes:

(i) When $r = 1$

$$S_n = a + a + \ldots \text{ up to } n \text{ terms} = na$$

(ii) It l is the last term of the G.P., then

$$S_n = \frac{lr - a}{r - a},\ r \neq 1$$

Sum of an infinite G.P. when $|r| < 1$

The sum of an infinite G.P. with first term a and common ratio r is $S_\infty = \dfrac{a}{1-r}$; when $|r| < 1$, i.e. $-1 < r < 1$.

Illustration 11 Find the sum of 8 terms and n terms of the sequence 9, –3, 1, –1/3, ...

Solution The given sequence is a G.P. with $a = 9$ and $r = -1/3$.

We know that

$$S_8 = 9\frac{1-(-1/3)^8}{1-(-1/3)} = 9\frac{1-1/3^8}{4/3} = \frac{27}{4}\left(1 - \frac{1}{3^8}\right)$$

$$= \frac{27}{4}\,\frac{3^8 - 1}{3^8} = \frac{1}{4}\,\frac{6561 - 1}{3^5} = \frac{6560}{4 \times 243} = \frac{1640}{243}.$$

Also, $$S_n = 9\,\frac{1-(-1/3)^n}{1-(-1/3)} = 9\,\frac{1-(-1)^n/3^n}{4/3}$$

$$= \frac{27}{4}\,\frac{3^n - (-1)^n}{3^n} = \frac{3^n - (-1)^n}{4 \cdot 3^{n-3}}.$$

Illustration 12 Find the sum of the infinite sequence $7, -1, \frac{1}{7}, -\frac{1}{49}, \ldots$

Solution The given sequence is a G.P. with $a = 7$ and $r = -\frac{1}{7}$, so $|r| = \left|-\frac{1}{7}\right| = < 1$.

$$\therefore\ S = \frac{7}{1-(-1/7)} = \frac{7}{8/7} = \frac{49}{8}. \quad \left(\because S = \frac{a}{1-r}\right)$$

HARMONIC PROGRESSION

A sequence of non-zero numbers $a_1, a_2, a_3, \ldots$ is said to be a *harmonic* progression (abbreviated as H.P.) if the sequence $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots$ is an A.P.

Illustration 13 The sequence $1, \frac{1}{4}, \frac{1}{7}, \frac{1}{10}, \ldots$ is a H.P. The sequence obtained by taking reciprocals of its corresponding terms, i.e. 1, 4, 7, 10, ... is an A.P.

A general H.P. is $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots$

nth Term of an H.P.

nth term of H.P.

$$= \frac{1}{n\text{th term of the corresponding A.P.}}$$

Notes:

- Three numbers a, b, c are in H.P. if and only if $\frac{1}{a}$, $\frac{1}{b}$, $\frac{1}{c}$ are in A.P., i.e.

$$\frac{1}{a} + \frac{1}{c} = 2 \cdot \frac{1}{b} \text{ or } b = \frac{2ac}{a+c}.$$

- No term of H.P. can be zero.
- There is no general formula for finding the sum to n terms of H.P.
- Reciprocals of terms of H.P. are in A.P. and then properties of A.P. can be used.

Illustration 14 Find the 100th of the squence

$$1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$$

Solution The sequence $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$ is an H.P.

Corresponding A.P. is 1, 3, 5, 7, ...

Now, for the corresponding A.P., first term $a = 1$, $d = 2$.

$\therefore$ 100th term of the corresponding A.P.

$$= a + (100 - 1)\, d$$
$$= 1 + (100 - 1)\, 2 = 199$$

Hence, the 100th term of the given sequence $= \frac{1}{199}$.

Some Special Sequences

1. The sum of first n natural numbers

$$\Sigma n = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

2. The sum of squares of first n natural numbers

$$\Sigma n^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

3. The sum of cubes of first n natural numbers

$$\Sigma n^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

Note: If nth term of a sequences is

$$T_n = an^3 + bn^2 + cn + d$$

then the sum of n terms is given by

$$S_n = \Sigma T_n = a\Sigma n^3 + b\Sigma n^2 + c\Sigma n + \Sigma d,$$

which can be evaluated using the above results.

Illustration 15 Find $2^2 + 4^2 + 6^2 + \ldots + (2n)^2$.

Solution nth term of the given series is $(2n)^2$. Then, $T_n = 4n^2$.

$$\therefore \quad S_n = 4\,\Sigma\, n^2 = \frac{4n(n+1)(2n+1)}{6}$$

$$\therefore \quad S_n = \frac{2n(n+1)(2n+1)}{3}.$$

Illustration 16 Sum the series $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots$ to n terms.

Solution Here, $T_n = (1^2 + 2^2 + 3^2 + \ldots n^2)$

$$= \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(2n^2 + 3n + 1)}{6}$$

$$= \frac{1}{3} n^3 + \frac{1}{2} n^2 + \frac{1}{6} n.$$

$$\therefore \quad S_n = \Sigma\left(\frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n\right)$$

$$= \frac{1}{3}\Sigma n^3 + \frac{1}{2}\Sigma n^2 + \frac{1}{6}\Sigma n$$

$$= \frac{1}{3}\,\frac{n^2(n+1)^2}{4} + \frac{1}{2}\,\frac{n(n+1)(2n+1)}{6} + \frac{1}{6}\,\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{12}\,[n(n+1) + 2n + 1 + 1]$$

$$= \frac{n(n+1)}{12}(n^2 + 3n + 2)$$

$$= \frac{n(n+1)}{12}\,(n+1)(n+2)$$

$$= \frac{n}{12}\,(n+1)^2\,(n+2).$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Determine 25th term of an A.P. whose 9th term is –6 and common difference is $\frac{5}{4}$

(a) 16 (b) 18
(c) 12 (d) 14

2. Which term of the A.P. 5, 13, 21, ... is 181?

(a) 21st (b) 22nd
(c) 23rd (d) 24th

3. Find the nth term of the series

$$\frac{1}{n} + \frac{n+1}{n} + \frac{2n+1}{n} + \ldots$$

(a) $\frac{3+n^2+n}{n}$ (b) $\frac{1+n^2-n}{n}$
(c) $\frac{2+n^2-n}{n}$ (d) None of these

4. If the pth term of an A.P. is q and the qth term is p, then its rth term is
(a) $p + q - r$ (b) $p - q - r$
(c) $r + q + p$ (d) None of these

5. Determine k so that $\frac{2}{3}$, k and $\frac{5}{8}k$ are the three consecutive terms of an A.P.
(a) 16/33 (b) 14/33
(c) 12/33 (d) 18/33

6. Determine k, so that $k + 2$, $4k - 6$ and $3k - 2$ are three consecutive terms of an A.P.
(a) 5 (b) 7
(c) 9 (d) 3

7. The ratio of the 7th to the 3rd term of an A.P. is 12 : 5. Find the ratio of 13th to the 4th term.
(a) 8 : 5 (b) 9 : 4
(c) 7 : 3 (d) 10 : 3

8. If 7 times the 7th term of an A.P. is equal to 11 times its 11th term, then the 18th term of the A.P. is
(a) 1 (b) 2
(c) 0 (d) 3

9. The 4th term of an A.P. is equal to 3 times the first term and the seventh term exceeds twice the third term by 1. Find the first term and the common difference.
(a) 3 (b) 5
(c) 7 (d) 9

10. If the 9th term of an A.P. is 99 and 99th term is 9, find 108th term.
(a) 0 (b) 2
(c) 4 (d) 6

11. If the pth, qth and rth terms of an A.P. are a, b, c, respectively, find the value of

$$a(q - r) + b(r - p) + c(p - q)$$

(a) 2 (b) 1
(c) 0 (d) 3

12. A body falls 16 metres in the first second of its motion, 48 metres in the second, 80 metres in the third, 112 metres in the fourth and so on. How far does it fall during the 11th second of its motion?
(a) 338 metres (b) 340 metres
(c) 334 metres (d) 336 metres

13. A ball rolling up an incline covers 36 meters during the first second, 32 metres during the second, 28 metres during the next and so on. How much distance will it travel during the 8th second?
(a) 8 metres (b) 6 metres
(c) 7 metres (d) 9 metres

14. Determine the sum of the first 35 terms of an A.P. If $t_2 = 2$ and $t_7 = 22$.
(a) 2510 (b) 2310
(c) 2710 (d) 2910

15. If the 5th and the 12th terms of an A.P. are 30 and 65, respectively, what is the sum of the first 20 terms?
(a) 1175 (b) 1250
(c) 1150 (d) 1350

16. If the 12th term of an A.P. is –13 and the sum of the first four terms is 24, what is the sum of the first 10 terms?
(a) 0 (b) 2
(c) 1 (d) 4

17. The sum of a series in A.P. is 525. Its first term is 3 and last term is 39. Find the common difference.
(a) 3/2 (b) 3/3
(c) 2/3 (d) 1/3

18. Find the common difference of an A.P. whose first term is 100 and the sum of whose first six terms is five times the sum of the next six terms.
(a) –15 (b) –10
(c) –20 (d) –5

19. How many terms are there in an A.P. whose first and fifth terms are –14 and 2, respectively and the sum of terms is 40?
(a) 15 (b) 5
(c) 10 (d) 20

20. Sum the series

$$51 + 50 + 49 + \ldots + 21$$

(a) 1116 (b) 1122
(c) 1128 (d) 1124

21. The sum of p terms of an A.P. is $3p^2 + 4p$. Find the nth term.
(a) $5n + 2$ (b) $6n + 1$
(c) $8n + 3$ (d) $7n + 3$

22. How many terms of the A.P. 1, 4, 7, ... are needed to give the sum 715?
(a) 33 (b) 22
(c) 24 (d) 27

23. Find the sum of the first hundred even natural numbers divisible by 5.
(a) 50575 (b) 50560
(c) 50500 (d) 50505

24. Find the sum of all integers between 50 and 500 which are divisible by 7.
(a) 17966 (b) 1177996
(c) 17766 (d) 17696

25. Find the sum of the numbers of three digits divisible by 7.
(a) 70334 (b) 70338
(c) 70336 (d) 70332

26. Find the sum of all odd numbers of four digits which are divisible by 9.
(a) 2754000 (b) 2754004
(c) 2754008 (d) 2754000

27. Which term of the geometric sequence
$$\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots \text{ is } \frac{1}{19683}?$$
(a) 9 (b) 7
(c) 11 (d) 13

28. Find the 10th term of the geometric series 5 + 25 + 125 + ...,
(a) 5^{10} (b) 5^9
(c) 5^{11} (d) 5^8

29. Write down the 20th term of the G.P. 1, –1, 1, –1, ...
(a) 1 (b) –1
(c) + 1 (d) None of these

30. Write down the 5th term of the series $\frac{1}{4}-\frac{1}{2}+1$ ...
(a) 6 (b) 8
(c) 4 (d) 10

31. The 5th term of a G.P. is 2, find the product of first 9 terms.
(a) 508 (b) 512
(c) 504 (d) 516

32. What term of progression
$$18, -12, 8, \ldots \text{ is } \frac{512}{729}?$$
(a) 15 (b) 18
(c) 9 (d) 12

33. The 3rd term of a G.P. is the square of the first term. If the second term is 8, determine the 6th term.
(a) 136 (b) 132
(c) 128 (d) 124

34. If 4th and 8th terms of a G.P. are 24 and 384, respectively, then find out first term and common ratio.
(a) 2, 3 (b) 5, 3
(c) 3, 2 (d) None of these

35. The first term of a G.P. is 1. The sum of the third and fifth terms is 90. Find the common ratio of the G.P.
(a) 2, –3 (b) 3, –3
(c) 1, –3 (d) 5, –3

36. For what value of x, the numbers $-\frac{2}{7}, x, -\frac{7}{2}$ are in G.P.?
(a) 1, –2 (b) 1, –3
(c) 1, –5 (d) 1, –1

37. A person has two parents (father and mother), four grandparents, eight great grandparents and so on. Find the number of ancestors the person has up to the 10th generation.
(a) 1028 (b) 1024
(c) 1030 (d) 1026

38. In a G.P, the first term is 7, the last term 448 and the sum 889. Find the common ratio.
(a) 4 (b) 6
(c) 8 (d) 2

39. The sum of first three terms of a G.P. is to the sum of first six terms is 125 : 152. Find the common ratio of G.P.
(a) $\frac{2}{5}$ (b) $\frac{4}{5}$
(c) $\frac{3}{5}$ (d) $\frac{1}{5}$

40. Evaluate $\sum_{j=1}^{11}(2+3^j)$
(a) $22 + \frac{3}{2}(3^{11}-1)$ (b) $11 + \frac{3}{2}(3^{11} - 1)$
(c) $22 + \frac{3}{2}(3^{10} - 1)$ (d) None of these

41. The sum of the first two terms of a G.P. is 36 and the product of the first and the third terms is 9 times the second term; find the sum of the first 8 terms.
(a) $\frac{3480}{81}$ (b) $\frac{3280}{81}$
(c) $\frac{3680}{81}$ (d) $\frac{3880}{81}$

42. The common ratio of a G.P. is $-\frac{4}{5}$ and the sum to infinity is $\frac{80}{9}$. Find the first term.
(a) 14 (b) 16
(c) 14 (d) 10

43. Sum the series to infinity
$$\frac{3}{4}-\frac{5}{4^2}+\frac{3}{4^3}-\frac{5}{4^4}+\frac{3}{4^5}-\frac{5}{45}+\ldots$$
(a) $\frac{8}{15}$ (b) $\frac{7}{17}$
(c) $\frac{7}{15}$ (d) $\frac{8}{17}$

44. The product $(32)\,(32)^{1/6}\,(32)^{1/36} \ldots \infty$ is equal to
(a) 16 (b) 64
(c) 32 (d) 0

45. Find the 9th term of the H.P. 6, 4, 3, ...
(a) 7/5 (b) 6/5
(c) 5/6 (d) None of these

46. Find the nth term of the H.P. whose first two terms are 6 and 3, respectively.
(a) $6/n$ (b) $7/n$
(c) $5/n$ (d) $8/n$

47. If $x > 1$, $y > 1$, $z > 1$ are in G.P., then $\frac{1}{1+\log x}, \frac{1}{1+\log y}, \frac{1}{1+\log z}$ are in
(a) A.P. (b) H.P.
(c) G.P. (d) None of these

48. $\frac{2}{5}+\frac{3}{5^2}+\frac{2}{5^3}+\frac{3}{5^4}+\ldots\infty$
(a) 17/24 (b) 15/24
(c) 13/24 (d) 11/24

49. If the first term of a G.P. is 729 and 7th term is 64, determine S_7.
(a) 2259 (b) 3059
(c) 2059 (d) 2459

50. If a be the first term of a G.P., l the nth term and P the product of first n terms then P =
(a) $(al)^{n/2}$ (b) $(a - l)^{n/2}$
(c) $(a + l)^{n/2}$ (d) None of these

EXPLANATORY ANSWERS

1. Ans. (d)

Solution Let a be the first term and d the common difference of an A.P.

Then, $a_n = a + (n - 1)\, d$

$$a_9 = a + (9 - 1)\left(\frac{5}{4}\right)$$

$\Rightarrow \quad a_9 = a + 10$

$\Rightarrow \quad -6 = a + 10$

$\Rightarrow \quad a = -6 - 10 = -16$

$\therefore \quad a_{25} = -16 + (25 - 1)\frac{5}{4} = -16 + 30 = 14.$

2. Ans. (c)

Solution Here, first term $a = 5$

Common difference $d = 8$

Let 181 be the nth, i.e. $a_n = 181$

$\therefore \quad 181 = 5 + (n - 1)8$ or, $176 = (n - 1)8$

$\therefore \quad n - 1 = 176 \div 8 = 22$

$\therefore \quad n = 23$

Hence, 181 is the 23rd term.

3. Ans. (b)

Solution Here $a_1 = \frac{1}{n}$, $d = \frac{n+1}{n} - \frac{1}{n} = 1$

$\therefore \quad a_n = a + (n - 1)\, d$

$\therefore \quad a_n = \frac{1}{n} + (n-1)(1) = \frac{1}{n} + n - 1 = \frac{1+n^2-n}{n}.$

4. Ans. (a)

Solution Let 'a' be the first term and d, the common difference

$\therefore \quad a_p = q \Rightarrow a + (p - 1)\, d = q$ ...(i)

$a_q = p \Rightarrow a + (q - 1)\, d = p$ ...(ii)

Subltracting (ii) from (i)

$(p - q)\, d = q - p = -(p - q)$

$\therefore \quad d = -1$

$\therefore$ From (i), $a + (p - 1)(-1) = q$

i.e. $\quad a - p + 1 = q$

$\therefore \quad a = p + q - 1$

$\therefore \quad a_r = a + (r - 1)d$

$= (p + q - 1) + (r - 1)(-1)$

$= p + q - 1 - r + 1$

$= p + q - r.$

5. Ans. (a)

Solution $\because \frac{2}{3}, k, \frac{5}{8}k$ are in A.P.

$\therefore \quad k - \frac{2}{3} = \frac{5}{8}k - k \Rightarrow \frac{5k}{8} - 2k = \frac{-2}{3}$

$\Rightarrow \frac{-11k}{8} = \frac{-2}{3} \Rightarrow k = \frac{16}{33}.$

6. Ans. (d)

Solution $\because k + 2, 4k - 6, 3k - 2$ are in A.P.

$\therefore \quad (4k - 6) - (k + 2) = (3k - 2) - (4k - 6)$

$\Rightarrow \quad 3k - 8 = -k + 4$

$\therefore \quad 4k = 12 \therefore k = 3.$

7. Ans. (d)

Solution Let a be the first term and d the common difference of the A.P. Then,

$$\frac{a+6d}{a+2d} = \frac{12}{5} \Rightarrow 5a + 30d = 12a + 24d$$

$$\Rightarrow -7a + 6d = 0$$

$$\Rightarrow a = \frac{6}{7}\, d$$

$$\therefore \quad \frac{13th \text{ term}}{4th \text{ term}} = \frac{a+12d}{a+3d} = \frac{\frac{6}{7}d+12d}{\frac{6}{7}d+3d}$$

$$= \frac{90}{27} = \frac{10}{3}.$$

8. Ans. (c)

Solution Let a be the first term and d, the common difference of an A.P.

$\therefore \quad a_7 = a + 6d$

$a_{11} = a + 10d \quad \because \quad 7a_7 = 11a_{11}$

$\therefore \quad 7(a + 6d) = 11(a + 10d)$

$\Rightarrow \quad 7a + 42d = 11a + 110d \Rightarrow -4a = 68\ d$

$\therefore \quad a = -17\ d$ ...(i)

Now, $a_{18} = a + 17d = -17d + 17d$ [Using (i)]

$= 0.$

9. Ans. (a)

Solution Let a be the first term and d the common difference of A.P.

$\because \quad a_4 = 3a, \quad \therefore \quad a + 3d = 3a$

$\Rightarrow \quad 2a = 3d, \quad \therefore \quad a = \frac{3}{2}d$ ...(i)

Also, $a_7 = 2a_3 + 1$

$\Rightarrow \quad a + 6d = 2\ (a + 2d) + 1$

$\Rightarrow \quad a + 6d = 2a + 4d + 1$

$\therefore \quad a = 2d - 1$ ...(ii)

From (i) and (ii), $\frac{3}{2}\ d = 2d - 1 \Rightarrow 2d - \frac{3}{2}\ d = 1$

$\therefore \quad \frac{d}{2} = 1 \Rightarrow d = 2 \quad \therefore\ a = \frac{3}{2}\ d = \frac{3}{2} \times 2 = 3.$

10. Ans. (a)

Solution Let a be the first term and d the common difference of A.P.

$\because \quad a_9 = 99$

$\therefore \quad a + 8d = 99$ ...(i)

Also, $a_{99} = 9 \ \therefore\ a + 98d = 9$ ...(ii)

Subtracting (ii) from (i)

$-90\ d = 90$

$\therefore \quad d = -1$ ...(iii)

Substituting this value of d in (i)

$a + 8(-1) = 99 \Rightarrow a = 99 + 8 = 107$ ...(iv)

$\therefore \quad a_{108} = a + (108 - 1)\ d = 107 + 107\ (-1) = 0.$

11. Ans (c)

Solution Let A be the first term and D, the common difference of A.P.

$a_p = a, \quad \therefore \quad A + (p - 1)\ D = a$ ...(i)

$a_q = b, \quad \therefore \quad A + (q - 1)\ D = b$ ...(ii)

$a_r = c, \quad \therefore \quad A + (r - 1)\ D = c$ ...(iii)

$\therefore\ a(q - r) + b(r - p) + c(p - q)$

$= [A + (p - 1)\ D]\ (q - r) + [A + (q - 1)\ D]\ (r - p)$

$+ [A + (r - 1)\ D]\ (p - q)$

$= (q - r + r - p + p - q)\ A + [(p - 1)\ (q - r)$

$+ (q - 1)\ (r - p) + (r - 1)\ (p - q)]D$

$= OA + OD = O.$

12. Ans. (d)

Solution The distances through which the body falls in first, second, third, fourth, ... seconds form an A.P.

$16 + 48 + 80 + 112 + ...$

Here, $a = 16,\ d = 32$

Distance through which it falls in 11th second

= 11 the term of the A.P.

$= a + 10d = 16 + 10\ (32)$

$= 16 + 320 = 336$ metres.

13. Ans. (a)

Solution Distance covered during the first second 36 m

Distance covered during the 2nd second = 32m

Distance covered during the 3rd second = 28m

The distance covered form an A.P.

$= 36 + 32 + 28$ ..... in which

$a = 36,\ d = -4$

$\therefore$ Distances covered in 8th second

= 8th term of the A.P.

$= a + 7d = 36 + 7(-4)$

$= 36 - 28 = 8$ metres.

14. Ans. (b)

Solution Let a and d be the first term and the common difference, respectively.

$\because \quad t_n = a + (n - 1)d,$

$\therefore \quad t_2 = a + (2 - 1)\ d = 2 \Rightarrow a + d = 2$ ...(i)

and, $t_7 = a + (7 - 1)\ d = 22 \Rightarrow a + 6d = 22$ ...(ii)

Subtracting (i) from (ii), we get

$5d = 20 \Rightarrow d = 4$

$\therefore \quad a + 4 = 2 \Rightarrow a = 2 - 4 = -2$ [Using (i)]

Now, $S_n = \frac{n}{2}\ [2a + (n - 1)d]$

$\therefore \quad S_{35} = \frac{35}{2}\ [-4 + (35 - 1)4]$

$= \frac{35}{2} \times 132, = 35 \times 66 = 2310.$

15. Ans. (c)

Solution Let a be the first term and d the common difference of an A.P., then

$a_5 = a + 4d = 30$ ...(i)

$a_{12} = a + 11d = 65$ ...(ii)

Subtracting (i) from (ii), we get

$7d = 35 \Rightarrow d = 5$

$\therefore$ From (i)

$a + 4\ (5) = 30 \Rightarrow a = 30 - 20 = 10$

Now, $S_n = \frac{n}{2}\ [2a + (n - 1)d]$

$\therefore \quad S_{20} = \frac{20}{2}[2 \times 10 + (20 - 1)5]$

$= 10\ [20 + 95] = 1150.$

16. Ans. (a)

Solution Let a be the first term and d the common difference of the A.P.

Then, nth term $= a + (n - 1)d$

$\therefore \quad a_{12} = a + 11d = -13$ ...(i)

Again, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad S_4 = 2(2a + 3d)$

But $\quad S_4 = 24$

$\therefore \quad 2(2a + 3d) = 24$

$\Rightarrow \quad 2a + 3d = 12$ ...(ii)

Multiplying (i) by 2, we get

$2a + 22d = -26$ ...(iii)

Subtracting (ii) from (iii)

$19d = -38 \Rightarrow d = -2$

Substituting $d = -2$ in (i), we get

$a + 11 \times (-2) = -13 \Rightarrow a = -13 + 22 = 9$

$\therefore \quad S_{10} = \frac{10}{2}[2 \times 9 + (10 - 1) \times -2]$

$= 5(18 - 18) = 0.$

17. Ans. (a)

Solution If n be the number of terms, then

$$a_n = a + (n - 1)d,$$

where a is the first term and d the common difference.

$\therefore \quad 39 = 3 + (n - 1)d$

or, $(n - 1)d = 36$ ...(i)

Also, $S_n = \frac{n}{2}[a_1 + a_n]$

$\therefore \quad 525 = \frac{n}{2}[3 + 39] \Rightarrow 1050 = n(42)$

or, $\quad n = \frac{1050}{42} = 25$

Putting $n = 25$ in (i), we get

$(25 - 1)d = 36 \Rightarrow d = 36 \div 24 = \frac{3}{2} = 1\frac{1}{2}.$

18. Ans. (b)

Solution Here, $a = 100$

Let d be the common difference

Now, $a_1 + a_2 + a_3 + a_4 + a_5 + a_6$

$= 5(a_7 + a_8 + a_9 + a_{10} + a_{11} + a_{12})$

$\Rightarrow \quad 6\left(\frac{a_1 + a_6}{2}\right) = 5 \times 6\left(\frac{a_7 + a_{12}}{2}\right)$

$\Rightarrow \quad a_1 + a_6 = 5(a_7 + a_{12})$

$\Rightarrow \quad a + a + 5d = 5[a + 6d + a + 11d]$

$\Rightarrow \quad 2a + 5d = 10a + 85d$

$\Rightarrow \quad 80d = -8a$ or, $d = \frac{-a}{10}$

$\therefore \quad d = \frac{-100}{10} = -10.$

19. Ans. (c)

Solution Here, $a = -14$

Let d be the common difference

$a_5 = 2 \Rightarrow a + 4d = 2 \Rightarrow -14 + 4d = 2$

$\therefore \quad d = 4$

Let 40 be the sum of n terms of this A.P.

$\because \quad S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore 40 = \frac{n}{2}[2(-14) + (n - 1)4] \Rightarrow 80 = n(4n - 32)$

or, $\quad 4n^2 - 32n - 80 = 0 \Rightarrow n^2 - 8n - 20 = 0$

$\Rightarrow (n + 2)(n - 10) = 0$

$\therefore \quad n = 10$ or, -2. But $n \neq -2$.

Hence, the required number of terms are 10.

20. Ans. (a)

Solution Here, $a = 51$, $d = -1$, $a_n = 21$

Now, $a_n = a_1 + (n - 1)d$

or, $\quad 21 = 51 - n + 1 \quad \therefore \quad n = 52 - 21 = 31$

Now, $S_n = \frac{n}{2}(a_1 + a_n) = \frac{31}{2}(51 + 21)$

$= \frac{31}{2} \times 72 = 31 \times 36 = 1116.$

21. Ans (b)

Solution Here, $S_p = 3p^2 + 4p$

Putting $p = n$, we have

$S_n = 3n^2 + 4n$

Changing n to $(n - 1)$, we get

$S_{n-1} = 3(n - 1)^2 + 4(n - 1)$

$= 3(n^2 - 2n + 1) + 4n - 4$

$= 3n^2 - 2n - 1$

$\therefore \quad a_n = S_n - S_{n-1}$

$= 3n^2 + 4n - 3n^2 + 2n + 1 = 6n + 1.$

22. Ans. (b)

Solution Here, $a = 1$, $d = 3$,

Let 715 be the sum of n terms of this A.P.

That is, $S_n = 715$

$\therefore \quad \frac{n}{2}[2a + (n - 1)d] = 715$

Putting values, of a and d

$\frac{n}{2}[2 \times 1 + (n - 1) \times 3] = 715$

$\Rightarrow \quad \frac{n}{2}[2 + 3n - 3] = 715$

$\Rightarrow \quad 3n^2 - n - 1430 = 0$

$\therefore \quad n = \frac{1 \pm \sqrt{1 - 4(3)(-1430)}}{2 \times 3}$

$= \frac{1 \pm \sqrt{17161}}{6} = \frac{1 \pm 131}{6}$

$\therefore \quad n = \frac{1+131}{6} = 22, \; n = \frac{1-131}{6} = \frac{-65}{3}$

But $n \neq \frac{-65}{3} \; \therefore \; n = 22.$

23. Ans. (c)

Solution Even natural numbers which are divisible by 5 are 10, 20, 30, 40,....

They form an A.P. with $a = 10$, $d = 10$

$$S_{100} = \frac{100}{2}[2 \times 10 + (100 - 1) \times 10]$$
$$= 50(20 + 990) = 50\ (1010) = 50500.$$

24. Ans. (d)

Solution The first integer, after 50 which is divisible by 7 is 56 and the last integer before 500 which is divisible by 7 is 497.

$\therefore$ The sequence of integers between 50 and 500 which are divisible by 7 is

56, 63, 70, ..., 497

It is an A.P. with

$a = 56, \; d = 7$

$a_n = 497 = a + (n - 1)\ d$

$\therefore \quad 497 = 56 + (n - 1) \times 7$

$\therefore \quad 7n = 497 + 7 - 56$

or, $\quad 7n = 448$

or, $\quad n = 488 \div 7 = 64$

$$\text{Required sum} = \frac{n}{2}(a_1 + a_n) = \frac{64}{2}(56 + 497)$$
$$= 32 \times (553) = 17696.$$

25. Ans (c)

Solution The least and the greatest number of three digits divisible by 7 are 105 and 994, respectively.

$\therefore$ It is required to find the sum of

105 + 112 + 119 + ... + 994

Here, $a = 105$, $d = 7$, $a_n = 994$

Then, $n = ?$, $S_n = ?$

Now, $a_n = a + (n - 1)\ d$

$\Rightarrow 994 = 105 + (n - 1) \times 7$

$\Rightarrow 994 - 105 = 7\ (n - 1)$

$\Rightarrow 889 = 7\ (n - 1)$ or, $(n - 1) = 127 \Rightarrow n = 128$

$$\text{Also, Sum} = \frac{n}{2}[2a + (n - 1)\ d]$$
$$= \frac{128}{2}[2 \times 105 + (128 - 1) \times 7]$$
$$= 64[210 + 889]$$
$$= 64 \times 1099 = 70336.$$

26. Ans. (a)

Solution The odd numbers of four digits which are divisible by 9 are 1017, 1035, ..., 9999

These are in A.P. with common difference 18.

Hence, nth term $= a_n = a + (n - 1)\ d$

$a = 1017, \; d = 18, \; l = 9999$

$\therefore \quad 9999 = 1017 + (n - 1) \times 18$

$\Rightarrow \quad 18n = 9999 - 999 = 9000$

$\therefore \quad n = 9000 \div 18 = 500$

$$\therefore \quad S_n = \frac{n}{2}(a_1 + a_n) = \frac{500}{2}(1017 + 9999)$$
$$= 250 \times 11016 = 2754000.$$

27. Ans. (a)

Solution Let nth term of the given sequence be $\frac{1}{19683}$.

Then,

$$a_n = ar^{n-1} \Rightarrow \frac{1}{19683} = \frac{1}{3}\left(\frac{1}{3}\right)^{n-1}$$
$$\Rightarrow \left(\frac{1}{3}\right)^8 = \left(\frac{1}{3}\right)^{n-1}$$
$$\Rightarrow n - 1 = 8 \Rightarrow n = 9.$$

28. Ans. (a)

Solution The given geometric series is

5 + 25 + 125 + ...

a = 1st term = 5, r = common ratio = 5

$n = 10$

$\therefore$ 10th term $= a_{10} = ar^{10-1}$

$= (5)\ (5)^9 = 5^{10}.$

29. Ans. (b)

Solution Here, $a = 1, \quad r = -1$

Since $a_n = ar^{\ n-1}$

$\therefore \quad a_{20} = ar^{19} = 1 \times (-1)^{19} = -1.$

30. Ans. (c)

Solution Here, $a = \frac{1}{4}$, $r = -2$. $\because \; a_n = ar^{n-1}$

$\therefore \quad a_5 = \frac{1}{4} \times (-2)^4 = \frac{1}{4} \times 16 = 4.$

31. Ans. (b)

Solution Let a be the first term and r the common ratio

$\therefore \quad a_5 = 2 \Rightarrow ar^4 = 2$ ....(i)

Now, product of first 9 terms

$$= a \times ar \times ar^2 \times \ldots \times ar^8$$
$$= a^9\ r^{1+\ 2\ +\ \ldots +\ 8} = a^9\ r^{36}$$
$$= (ar^4)^9 = 2^9 = 512.$$

32. Ans. (c)

Solution Here $a = 18$, $r = \frac{-2}{3}$

Let $\frac{512}{729}$ be the nth term so that $a_n = \frac{512}{729}$

Since $a_n = ar^{n-1}$

$\therefore \quad \frac{512}{729} = 18\left(\frac{-2}{3}\right)^{n-1}$

$\Rightarrow \left(\frac{-2}{3}\right)^{n-1} = \frac{512}{729\times 18} = \frac{256}{9\times 729}$

$= \frac{2^8}{3^2\times 3^6} = \left(\frac{-2}{3}\right)^8$

$\therefore \quad n - 1 = 8 \Rightarrow n = 9$

Hence, $\frac{512}{729}$ is the 9th term of progression.

33. Ans. (c)

Solution Let a be the first term and r be the common ratio of G.P.

We have $a_3 = (a_1)^2 \Rightarrow ar^2 = a^2$

$\Rightarrow r^2 = a$ ...(i)

Also, $a_2 = 8 \Rightarrow ar = 8$ ...(ii)

Multiplying (i) and (ii), we get

$ar^3 = 8 \times a \therefore r^3 = 8 \Rightarrow r = 2$

From (i) $a = (2)^2 = 4$ $[\because a = r^2]$

Hence, $a_6 = ar^5 = (4)\ (2)^5 = 4 \times 32 = 128$.

34. Ans. (c)

Solution Let a be the first term and r the common ratio. Then,

$\because$ 4th term $= 24 \Rightarrow ar^3 = 24$ ...(i)

and, 8th term $= 384 \Rightarrow ar^7 = 384$ ....(ii)

Dividing (ii) by (i), we get

$\frac{ar^7}{ar^3} = \frac{384}{24} \Rightarrow r^4 = 16 = (2)^4 \Rightarrow r = 2$

Substituting $r = 2$ in (i), we get

$a(2)^3 = 24 \Rightarrow a = 24 \div 8 = 3$

Hence, first term = 3 and common ratio = 2.

35. Ans. (b)

Solution Let r be the common ratio of G.P.

First term, $a = 1$

Now, $a_3 = ar^2 = r^2$ $(\because a = 1)$

and, $a_5 = ar^4 = r^4$ But $a_3 + a_5 = 90$

$\Rightarrow r^2 + r^4 = 90 \Rightarrow r^4 + r^2 - 90 = 0$

$\Rightarrow (r^2 + 10)\ (r^2 - 9) = 0$

$\therefore \quad r^2 - 9 = 0$ $[\because r^2 + 10 \neq 0]$

$\therefore \quad r = \pm 3$.

36. Ans. (d)

Solution $\frac{-2}{7}, x, \frac{-7}{2}$ are in G.P.

$\Rightarrow \frac{x}{-2/7} = \frac{-7/2}{x} \Rightarrow x^2 = \frac{-7}{2} \times \frac{-2}{7}$

$\therefore \quad x^2 = 1 \Rightarrow x = \pm 1$.

37. Ans. (b)

Solution We have 2, 4, 8, ... 10 terms which are in G.P.

Here, $a = 2$, $r = 2$

$\therefore \quad a_n = ar^{n-1}$

$\Rightarrow \quad a_{10} = 2(2)^{10-1} = 2^{10} = 1024$.

Hence, the number of ancestors the person has up to 10th generation = 1024.

38. Ans. (d)

Solution Here, $a = 7$, $l = a_n = 448$, $S_n = 889$

Let r be the common ratio

$S_n = \frac{a(1-r^n)}{1-r} = \frac{a-lr}{1-r}$

$\therefore \quad 889 = \frac{7-448r}{1-r} \Rightarrow 889 - 889\ r = 7 - 448r$

$\Rightarrow 889 - 7 = 889r - 448r$

$\Rightarrow 882 = 441r \Rightarrow r = 2$.

39. Ans. (c)

Solution Here, $\frac{S_3}{S_6} = \frac{125}{152}, \frac{a(r^3-1)(r-1)}{a(r^6-1)(r-1)} = \frac{125}{152}$

$\Rightarrow \frac{r^3-1}{r^6-1} = \frac{125}{152}, \therefore \frac{r^3-1}{(r^3-1)(r^3+1)} = \frac{125}{152}$

$\Rightarrow \frac{1}{r^3+1} = \frac{125}{152}, \therefore 152 = 125r^3 + 125$

$\Rightarrow 125r^3 = 27, \Rightarrow r^3 = \frac{27}{125}$.

or, $r^3 = \left(\frac{3}{5}\right)^3, \therefore r = \frac{3}{5}$.

Hence, the common ratio of G.P. is $\frac{3}{5}$.

40. Ans. (a)

Solution $(2 + 3^1) + (2 + 3^2) + (2 + 3^3) + ... + (2 + 3^{11})$

$= (2 + 2 + 2 + ...$ up to 11 terms)

$+ (3 + 3^2 + 3^3 + ...$ up to 11 terms)

$= 11 \times 2 + \frac{3(3^{11}-1)}{3-1} = 22 + \frac{3}{2}\ (3^{11} - 1)$.

41. Ans. (b)

Solution Let a be the first term and r the common ratio of G.P.

Given: $a_1 + a_2 = 36 \Rightarrow a + a\ r = 36$

$\Rightarrow a\ (1 + r) = 36$ ...(i)

Also, $a_1\ a_3 = 9a_2 \Rightarrow a\ ar^2 = 9 \times ar$

$\Rightarrow ar = 9$ ...(ii)

Subtracting (ii) from (i), $a = 27$

From (ii), $27\ r = 9 \Rightarrow r = \frac{1}{3}$

$$\therefore \quad S_8 = \frac{27\left[1-\left(\frac{1}{3}\right)^8\right]}{1-1/3} = \frac{3\times 27}{2}\left(1-\frac{1}{6561}\right)$$

$$= \frac{81}{2} \times \frac{6560}{6561} = \frac{3280}{81}.$$

42. Ans. (b)

Solution $S_\infty = \frac{a}{1-r} \Rightarrow \frac{80}{9} = \frac{a}{1-\left(\frac{-4}{5}\right)} \Rightarrow \frac{80}{9} = \frac{a}{a/5}$

$$\Rightarrow a = \frac{80}{9} \times \frac{9}{5} = 16$$

Hence, the first term is 16.

43. Ans (c)

Solution $\left(\frac{3}{4}+\frac{3}{4^3}+\frac{3}{4^5}+\ldots \text{ to } \infty\right)$

$$-\left(\frac{5}{4^2}+\frac{5}{4^4}+\frac{5}{4^6}+\ldots \text{ to } \infty\right)$$

$$= \frac{3/4}{1-\left(\frac{1}{4}\right)^2} - \frac{5/4^2}{1-\left(\frac{1}{4}\right)^2}$$

$$= \frac{3}{4} \times \frac{16}{15} - \frac{5}{16} \times \frac{16}{15} = \frac{4}{5} - \frac{1}{3}$$

$$= \frac{12-5}{15} = \frac{7}{15}.$$

44. Ans. (b)

Solution $(32)\ (32)^{1/6}\ (32)^{1/36} \ldots \infty$

$$= (32)^{1+\frac{1}{6}+\frac{1}{36}+\ldots+\infty} = (32)^x,$$

where $x = 1+\frac{1}{6}+\frac{1}{36}+\ldots\infty = \frac{a}{1-r}$

$$= \frac{1}{1-\frac{1}{6}} = \frac{6}{5}$$

$\therefore$ Product $= (32)^x = (32)^{6/5} = (2^5)^{6/5} = 2^6 = 64.$

45. Ans. (b)

Solution The given sequence is 6,4,3, ... which is a H.P.

The sequence of reciprocals of its terms is

$\frac{1}{6}, \frac{1}{4}, \frac{1}{3}$ ... which is an A.P.

Here, $a = \frac{1}{6}$, $d = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

$\therefore$ a_9 of A.P. $= a + 8d$

$$= \frac{1}{6} + 8 \times \frac{1}{12} = \frac{1}{6} + \frac{4}{6} = \frac{5}{6}$$

$\therefore$ 9th term of H.P. is $\frac{6}{5}$.

46. Ans. (a)

Solution First term of H.P. = 6 and second term of H.P. = 3

$\therefore$ First and second terms of corresponding A.P. are $\frac{1}{6}$ and $\frac{1}{3}$.

$\therefore$ $a = \frac{1}{6}$ and $d = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

nth term of A.P. $= \frac{1}{6} + (n-1)\frac{1}{6}$

$$= \frac{1+n-1}{6} = \frac{n}{6}$$

Hence, nth term of H.P. $= \frac{n}{6}$.

47. Ans. (b)

Solution $\because$ x, y, z are in G.P.

$\therefore$ $y^2 = xz$

Taking log on both sides

$2 \log y = \log x + \log z$

$\Rightarrow$ $2 + 2 \log y = (1 + \log x) + (1 + \log z)$

$\Rightarrow$ $2(1 + \log y) = (1 + \log x) + (1 + \log z)$

$\Rightarrow$ $1 + \log x, 1 + \log y, 1 + \log z$, are in A.P.

$\Rightarrow$ $\frac{1}{1+\log x}, \frac{1}{1+\log y}, \frac{1}{1+\log z}$ are in H.P.

48. Ans. (c)

Solution $\frac{2}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \ldots \infty$

$$= \left(\frac{2}{5}+\frac{2}{5^3}+\frac{2}{5^5}+\ldots\infty\right) + \left(\frac{3}{5^2}+\frac{3}{5^4}+\ldots\infty\right)$$

$$= \frac{2/5}{1-\frac{1}{5^2}} + \frac{3/5^2}{1-\frac{1}{5^2}} = \frac{25}{24}\left(\frac{2}{5}+\frac{3}{25}\right)$$

$$= \frac{25}{24} \times \frac{13}{25} = \frac{13}{24}.$$

49. Ans. (c)

Solution $a = 729$. Let r be the common ratio

Now, $a_7 = 64 \Rightarrow ar^6 = 64 \Rightarrow 729 \times r^6 = 64$

$\therefore$ $r^6 = \frac{64}{729} = \left(\frac{2}{3}\right)^6$ $\therefore$ $r = \frac{2}{3}$

$$\therefore \quad S_7 = \frac{a(1-r^7)}{1-r} = \frac{729\left[1-\left(\frac{2}{3}\right)^7\right]}{1-2/3}$$

$$= \frac{3\times 729}{1}\left[1-\frac{128}{2187}\right]$$

$$= 2187 \times \frac{2059}{2187} = 2059.$$

50. Ans. (a)

Solution If r is the common ratio of G.P., then

$$l = a\,r^{n-1} \quad \text{...(i)}$$

The first n terms of the G.P. are

$a, ar, ar^2, ar^3, ..., ar^{n-1}$

$$P = a \times ar \times ar^2 \times ar^3 \times ... \times ar^{n-1}$$
$$= a^n \times r^{1+2+3+....+(n-1)}$$
$$= a^n \times r^{(n-1)\times\frac{n}{2}} = (a^2)^{n/2} \times (r^{n-1})^{n/2}$$
$$= (a^2\, r^{n-1})^{n/2} = (a.\ ar^{n-1})^{n/2} = (al)^{n/2}.$$

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. The next term of the sequence

$$\frac{1}{2},\ 3\frac{1}{4},\ 6,\ 8\frac{3}{4},\ ...\ \text{is}$$

(a) $10\frac{1}{4}$ (b) $10\frac{3}{4}$
(c) $11\frac{1}{4}$ (d) $11\frac{1}{2}$

[Based on SSC (GL) Prel. Exam 2000]

2. By adding the same constant to each of 31, 7, –1 a geometric progression results. The common ratio is

(a) 3 (b) $\frac{1}{3}$
(c) –2 (d) –4

[Based on SSC (GL) Prel. Exam 2000]

3. Which term of the series 72 + 63 + 54 + ... is zero?

(a) 11th (b) 10th
(c) 9th (d) 8th

[Based on SSC (GL) Prel. Exam 2000]

4. What is the 507th term of the sequence 1, –1, 2, –2, 1, –1, 2, –2, 1, ... ?

(a) –1 (b) 1
(c) –2 (d) 2

[Based on SSC (GL) Prel. Exam 2000]

5. If the 4th term of an A.P. is 14 and the 12 term is 70, then the first term is

(a) –10 (b) –7
(c) 7 (d) 10

[Based on SSC (GL) Prel. Exam 2000]

6. $[14^2 + 15^2 + ... + 30^2]$ is equal to

(a) 6836 (b) 8336
(c) 8336 (d) 8636

[Based on SSC (GL) Prel. Exam 2000]

7. If $1^3 + 2^3 + ...+ ..10^3 = 3025$, then 4 + 32 + 108 +... + 4000 is equal to

(a) 12000 (b) 12100
(c) 12200 (d) 12400

[Based on SSC (GL) Prel. Exam 2002]

8. If $1^2 + 2^2 + 3^2 + ... + 10^2 = 385$, then $3^2 + 6^2 + 9^2 + ... + 30^2$ is equal to

(a) 3465 (b) 2310
(c) 1155 (d) 770

[Based on SSC (GL) Prel. Exam 2002]

9. If $1^2 + 2^2 + 3^2 + ... + x^2 = \frac{x(x+1)(2x+1)}{6}$, then $1^2 + 3^2 + 5^2 + ... + 19^2$ is equal to

(a) 1330 (b) 2100
(c) 2485 (d) 2500

10. If $1^3 + 2^3 + ... + 9^3 = 2025$, then the value of $(0.11)^3 + (0.22)^3 + ... + (0.99)^3$ is close to

(a) 0.2695 (b) 2.695
(c) 3.695 (d) 0.3695

[Based on SSC (GL) Prel. Exam 2002]

EXPLANATORY ANSWERS

1. Ans. (d)

Solution It is an A.P. with first term $\frac{1}{2}$ and common difference $2\frac{3}{4}$

$$\therefore\ \text{Next term} = 8\frac{3}{4} + 2\frac{3}{4} = 11\frac{1}{2}.$$

2. Ans. (b)

Solution Let x be the constant which is added to each term

So, $\frac{7+x}{31+x} = \frac{-1+x}{7+x}$

or, $(7 + x)^2 = (31 + x)(-1 + x)$

or, $14x + 49 + x^2 = -31 + 31x - x + x^2$

or, $14x + 49 = 30x - 31$

$\therefore\ -16x = -80$

$\therefore\ x = 5$

$$\therefore\ \text{common ratio} = \frac{7+x}{31+x} = \frac{7+5}{31+5} = \frac{12}{36} = \frac{1}{3}.$$

3. Ans. (c)

Solution The given series is an A.P.

nth term of A.P. = $a_n = a + (n - 1)\,d$

where a = 1st term, d = common difference

So, $72 + (n - 1)(-9) = 0$

or, $72 - 9n + 9 = 0 \Rightarrow 9n = 81 \Rightarrow n = 9$.

4. Ans. (d)

5. Ans. (b)

Solution Let the first term of the A.P. be a and the common difference be d

According to question,

$a + 3d = 14$ ...(i)

and, $a + 11d = 70$ ...(ii)

On solving (i) and (ii), we get

$d = 7,\ a = -7$.

6. Ans. (d)

Solution $14^2 + 15^2 + ... + 30^2$

$= (1^2 + 2^2 + 3^2 + ... + 30^2) - (1^2 + 2^2 + ... + 13^2)$

$= \dfrac{30(30+1)(2\times30+1)}{6} - \dfrac{13(13+1)(2\times13+1)}{6}$

$= \dfrac{30\times31\times61}{6} - \dfrac{13\times14\times27}{6} = 8636.$

7. Ans. (b)

Solution $4 + 32 + 108 + ... + 4000$

$= 4\,(1 + 8 + 27 + ... + 1000)$

$= 4\,(1^3 + 2^3 + 3^3 + ... + 10^3)$

$= 4 \times 3025 = 12100.$

8. Ans. (a)

Solution $3^2 + 6^2 + 9^2 + ... + 30^2$

$= 3^2[1^2 + 2^2 + 3^2 + ... + 10^2]$

$= 9 \times 385 = 3465.$

9. Ans. (a)

Solution $(1^2 + 2^2 + 3^2 + .. + 19^2)$

$- (2^2 + 4^2 + 6^2 + ... + 18^2)$

$= \dfrac{19\times20\times39}{6} - 2^2(1^2 + 2^2 + 3^2 + ... + 9^2)$

$= \dfrac{19\times20\times39}{6} - 2^2 \times \dfrac{9\times10\times19}{6}$

$= \dfrac{19\times20\times39 - 4\times9\times10\times19}{6}$

$= \dfrac{14820 - 6840}{6} = 1330.$

10. Ans. (b)

Solution $1^3 + 2^3 + 3^3 + ... + 9^3 = 2025.$

Now, $(0.11)^3 + (0.22)^3 + (0.33)^3 + ... + (0.99)^3$

$= (0.11)^3\,[1^3 + 2^3 + 3^3 + ... + 9^3]$

$= (0.11)^3 \times 2025.$

$= 2025 \times 0.001331 = 2.695.$

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. Determine the 2nd term of an A.P. whose 6th term is 12 and 8th term is 22.
(a) –8 (b) –10
(c) –6 (d) –12

2. For the A.P. a_1, a_2, a_3 .... if $\dfrac{a_4}{a_7} = \dfrac{2}{3}$, find $\dfrac{a_6}{a_8}$
(a) $\dfrac{3}{5}$ (b) $\dfrac{4}{5}$
(c) $\dfrac{2}{5}$ (d) $\dfrac{4}{5}$

3. If m times the mth term of an A.P. is equal to n times its nth term; Then the $(m + n)$th term of the A.P. is
(a) 0 (b) 1
(c) 2 (d) 3

4. The mth term of an A.P. is $\dfrac{1}{n}$ and nth term is $\dfrac{1}{m}$. Find (mn)th term.
(a) 3 (b) 1
(c) 4 (d) 2

5. The first and the last term of an A.P. are –4 and 146 and the sum of the A.P. is 7171. Find the number of terms of the A.P. and the common difference.
(a) 101, 1.5 (b) 101, 2.5
(c) 101, 4.5 (d) 101, 3.5

6. How many terms of the sequence 18, 16, 14, ... should be taken so that their sum is zero?
(a) 17 (b) 19
(c) 21 (d) 23

7. Find the sum of 11 terms of an A.P. whose middle term is 30.
(a) 360 (b) 390
(c) 330 (d) 330

8. Find the sum of all natural numbers between 100 and 1000 which are multiples of 5.
(a) 98450 (b) 93450
(c) 95450 (d) None of these

9. Obtain the sum of all natural numbers in the first 1000 positive integers which are neither divisible by 5 nor by 2.

(a) 200000 (b) 300000
(c) 100000 (d) None of these

10. If the pth term of an A.P. is $1/q$ and qth term is $1/p$, then the sum of pq terms is

(a) $\frac{1}{2}(pq - 1)$ (b) $\frac{1}{2}(pq + 1)$
(c) $\frac{1}{2}(pq + 1)$ (d) $\frac{1}{3}(pq + 1)$

Test I

[Time: 10 min]

11. In a G.P. the product of the first three terms is $\frac{27}{8}$. Find the middle term (assume that the terms are positive).

(a) $\frac{4}{3}$ (b) $\frac{2}{3}$
(c) $\frac{3}{2}$ (d) $\frac{1}{3}$

12. The 5th, 8th and 11th terms of a G.P. are p, q and s, respectively, then

(a) $q^2 = ps$ (b) $q^3 = ps$
(c) $q = ps$ (d) $q^5 = ps$

13. Find the 4th term of the G.P. whose 5th term is 32 and whose 8th term is 256.

(a) 20 (b) 18
(c) 16 (d) 22

14. Find the sum of n terms of the sequence 1, –1, 1, –1, ..., $(-1)^{n+1}$, ... (n is odd)

(a) 2 (b) 1
(c) 3 (d) 0

15. The sum of the first two terms of a G.P. is 36 and the product of the first and the third terms is 9 times the second term, find the sum of the first 8 terms.

(a) $\frac{3380}{81}$ (b) $\frac{3480}{81}$
(c) $\frac{3080}{81}$ (d) $\frac{3280}{81}$

16. The first term of a G.P. is 2 and the sum to infinity is 6. Find the common ratio.

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{4}{3}$ (d) $\frac{3}{2}$

17. Find the first term of a G.P. whose second term is 2 and the sum to infinity is 8.

(a) 6 (b) 2
(c) 4 (d) 0

18. Evaluate $9^{1/3}.\ 9^{1/9}.\ 9^{1/27}$... to infinity

(a) 2 (b) 5
(c) 3 (d) 7

19. The third and sixth terms of an H.P. are $\frac{5}{16}$ and $\frac{1}{5}$ respectively. Find the 10th term.

(a) 9/37 (b) 3/37
(c) 5/37 (d) 7/37

20. The third term of an H.P. is 3 and the 9th term is $\frac{6}{5}$, find its 20th term.

(a) $\frac{3}{7}$ (b) $\frac{2}{7}$
(c) $\frac{5}{7}$ (d) $\frac{4}{7}$

Answers

1. (d)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (c)	**8.** (a)	**9.** (a)	**10.** (b)
11. (c)	**12.** (a)	**13.** (c)	**14.** (b)	**15.** (d)	**16.** (a)	**17.** (c)	**18.** (c)	**19.** (c)	**20.** (d)

28

HEIGHTS AND DISTANCES

INTRODUCTION

Solution of triangles has enormous applications to surveying, navigation, and so on. We shall now consider some simple ones from among them. For this purpose, we need to explain certain terms that are generally used in practical problems.

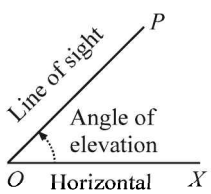


Fig. (a)

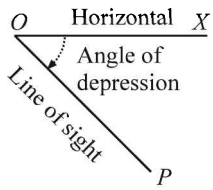


Fig. (b)

1. If OX be a horizontal line through O, the eye of the observer and P be an object in the vertical plane through OX, then $\angle XOP$ is called:

(i) **the angle of elevation,** if P is *above* OX as in Fig. (a); and

(ii) the **angle of depression,** if P is *below* OX as in Fig. (b).

The straight line OP (joining the eye of the observer to the object) is called the *line of sight* of the observer.

2. Values of the trigonometric ratios for some useful angles

angle (θ) / t-ratio	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

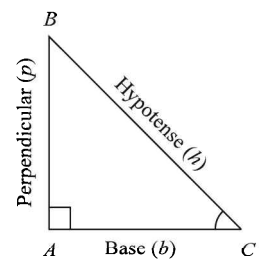
The values of cot θ, sec θ and cosec θ can be found from the above Table by using the relations $\cot\theta = \frac{\cos\theta}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$ and $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$.

3. Pythagoras Theorem

In a right-angled triangle the square of its hypoteuse is equal to the sum of the squares of its legs (i.e. perpendicular and base).

In other words,

$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$



or, $(BC)^2 = (AB)^2 + (AC)^2$

or, $h^2 = p^2 + b^2$.

4. Few important values to memorise:

$\sqrt{2} = 1.414$

$\sqrt{3} = 1.732$

$\sqrt{5} = 2.236$.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. The ratio of the length of a rod and its shadow is $1 : \sqrt{3}$. The angle of elevation of the sum is
(a) 30° (b) 45°
(c) 60° (d) 90°

2. The angle of elevation of a moon when the length of the shadow of a pole is equal to its height, is
(a) 30° (b) 45°
(c) 60° (d) None of these

3. A tower stands on a horizontal plane. A man on the ground 100 m from the base of the tower finds the angle of elevation of the top of the tower to be 30°. What is the height of the tower?
(a) 100 m (b) $100\sqrt{3}$
(c) $100/\sqrt{3}$ (d) None of these

4. When the sun is 30° above the horizontal, the length of shadow cast by a building 50 m high is
(a) $\frac{50}{\sqrt{3}}$ m (b) $50\sqrt{3}$ m
(c) 25 m (d) $25\sqrt{3}$ m

5. A pole being broken by the wind, the top struck the ground at an angle of 30° and at a distance of 21 m from the foot of the pole. Find the total height of the pole.
(a) 21 m (b) $21\sqrt{3}$ m
(c) $21/\sqrt{3}$ (d) None of these

6. The upper part of a tree broken by the wind makes an angle of 30° with the ground and the distance from the root to the point where the top of the tree touches the ground is 10 m. What was the height of the tree?
(a) $10\sqrt{3}$ (b) $10/\sqrt{3}$
(c) $20\sqrt{3}$ (d) None of these

7. A tower stands at the end of a straight road. The angles of elevation of the top of the tower from two points on the road 500 m apart are 45° and 60°, respectively. Find the height of the tower.
(a) $\frac{500\sqrt{3}}{\sqrt{3}-1}$ (b) $5000\sqrt{3}$
(c) $\frac{500\sqrt{3}}{\sqrt{3}+1}$ (d) None of these

8. The shadow of a tower standing on a level plane is found to be 50 m longer when sun's altitude is 30° than when it is 60°. Find the height of the tower.
(a) $20\sqrt{3}$ m (b) $25/\sqrt{3}$ m
(c) $25\sqrt{3}$ m (d) $20\sqrt{3}$ m

9. In a rectangle, if the angle between a diagonal and a side is 30° and the length of diagonal is 6 cm, the area of the rectangle is
(a) 9 cm^2 (b) $9\sqrt{3}$ cm^2
(c) 27 cm^2 (d) 36 cm^2

10. The height of a tower is 100 m. When the angle of elevation of the sun changes from 30° to 45°, the shadow of the tower becomes x metres less. The value of x is
(a) 100 m (b) $100\sqrt{3}$ m
(c) $100(\sqrt{3} - 1)$ m (d) $\frac{100}{\sqrt{3}}$ m

11. An electric pole 20 m high stands upright on the ground with the help of steel wire to its top and affixed on the ground. If the steel wire makes 60° with the horizontal ground, find the length of the steel wire.
(a) $40/\sqrt{3}$ m (b) $40\sqrt{3}$ m
(c) $20/\sqrt{3}$ m (d) $20\sqrt{3}$ m

12. From the top of a light house 50 m above the sea, the angle of depression of an incoming boat is 30°. How far is the boat from the light house?
(a) $25\sqrt{3}$ m (b) $25/\sqrt{3}$ m
(c) $50/\sqrt{3}$ m (d) $50/\sqrt{3}$ m

13. From the top of a cliff 25 m high the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. Find the height of the tower.
(a) 40 m (b) 48 m
(c) 50 m (d) 52 m

14. When the length of the shadow of a pole is equal to the height of the pole, then the elevation of source of light is
(a) 30° (b) 45°
(c) 60° (d) 75°

15. From the top of a light house 60 m high with its base at sea level, the angle of depression of a boat is 15°. The distance of the boat from the light house is
(a) $60\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$m (b) $60\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$m
(c) $30\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$m (d) $30\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$m

16. On the level ground, the angle of elevation of the top of the tower is 30°. On moving 20 m nearer,

the angle of elevation is 60°. The height of the tower is

(a) $20\sqrt{3}$ m (b) $10\sqrt{3}$ m

(c) $10(\sqrt{3} - 1)$ m (d) None of these

17. The angle of elevation of the top of a tower from a point 20 m away from its base is 45°. The height of the tower is

(a) 10 m (b) 20 m

(c) 40 m (d) $20\sqrt{3}$ m

18. At a point 15 m away from the base of a 15 m high house, the angle of elevation of the top is

(a) 45° (b) 30°

(c) 60° (d) 90°

19. Angle of depression from the top of a light house of two boats are 45° and 30° due east which are 60 m apart. The height of the light house is

(a) $60\sqrt{3}$ (b) $30(\sqrt{3}-1)$

(c) $30(\sqrt{3} + 1)$ (d) None of these

20. The angle of elevation of the top of a hill from each of the vertices *A, B, C* of a horizontal triangle is α. The height of the hill is

(a) $b \tan \alpha \operatorname{cosec} B$ (b) $\frac{a}{2} \tan \alpha \operatorname{cosec} A$

(c) $\frac{c}{2} \tan \alpha \operatorname{cosec} C$ (d) None of these

21. A tower subtends an angle of 30° at a point on the same level as the foot of the tower. At a second point h m above the first, the depression of the foot of the tower is 60°. The horizontal distance of the tower from the point is

(a) $h \cot 60°$ (b) $h \cot 30°$

(c) $\frac{h}{3} \cot 60°$ (d) $\frac{h}{3} \cot 30°$

22. If a flag staff of 6 m height placed on the top of a tower throws a shadow of $2\sqrt{3}$ m along the ground, then the angle that the sun makes with the ground is

(a) 60° (b) 30°

(c) 45° (d) None of these

23. A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°. When he retires 40 m from the bank, he finds the angle to be 30°. The breadth of the river is

(a) 40 m (b) 60 m

(c) 20 m (d) 30 m

24. The angle of elevation of the sun when the length of the shadow of a pole is $\sqrt{3}$ times the height of the pole will be

(a) 30° (b) 60°

(c) 90° (d) 45°

25. The angle of elevation of the top of a TV tower from three points *A, B, C* in a straight line through the foot of the tower are α, 2α, 3α, respectively. If $AB = a$, the height of the tower is

(a) $a \tan \alpha$ (b) $a \sin \alpha$

(c) $a \sin 2\alpha$ (d) $a \sin 3\alpha$

26. The angle of elevation of the top of an unfinished tower at a point distant 120 m from its base is 45°. If the elevation of the top at the same point is to be 60°, the tower must be raised to a height

(a) $120(\sqrt{3} + 1)$ m (b) $120(\sqrt{3} - 1)$ m

(c) $10(\sqrt{3} + 1)$ m (d) None of these

27. A person walking along a straight road towards a hill observes at two points, distance $\sqrt{3}$ km, the angles of elevation of the hill to be 30° and 60°. The height of the hill is

(a) $\frac{3}{2}$ km (b) $\sqrt{\frac{2}{3}}$ km

(c) $\frac{\sqrt{3}+1}{2}$ km (d) $\sqrt{3}$ km

28. The tops of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle 30° with the horizontal, then the length of the wire is

(a) 12 m (b) 10 m

(c) 8 m (d) None of these

29. A man is standing on the 8 m long shadow of a 6 m long pole. If the length of the shadow is 2.4 m, then the height of the man is

(a) 1.4 m (b) 1.6 m

(c) 1.8 m (d) 2.0 m.

30. The angle of elevation of the top of a tower at a point *G* on the ground is 30°. On walking 20 m towards the tower the angle of elevation becomes 60°. The height of the tower is equal to

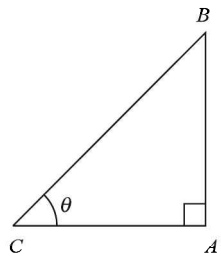
(a) $\frac{10}{\sqrt{3}}$ m (b) $20\sqrt{3}$ m

(b) $\frac{20}{\sqrt{3}}$ m (d) $10\sqrt{3}$ m

EXPLANATORY ANSWERS

1. Ans. (a)

Solution Let AB be the rod and AC be its shadow. $\angle ACB = \theta$. Let $AB = x$.



Then, $AC = \sqrt{3}x$.

$\therefore \quad \tan\theta = \dfrac{AB}{AC} = \dfrac{x}{\sqrt{3}x} = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = 30°.$

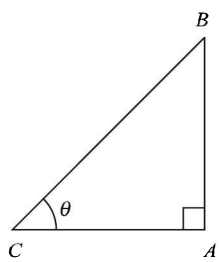
2. Ans. (b)

Solution Let $AB = x$.

Then, $AC = x$.

$\therefore \quad \tan\theta = \dfrac{AB}{AC} = 1$

$\Rightarrow \theta = 45°.$

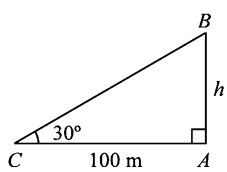


3. Ans. (c)

Solution $\tan 30° = \dfrac{h}{100}$

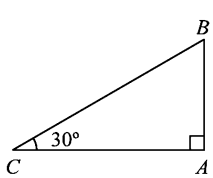
or, $\dfrac{1}{\sqrt{3}} = \dfrac{h}{100}$

or, $h = \dfrac{100}{\sqrt{3}}$ m.



4. Ans. (b)

Solution Let AB be the building and AC be its shadow. Then, $AB = 50$ m and $\theta = 30°$.

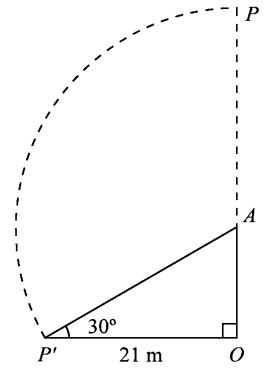


$\therefore \quad \dfrac{AC}{AB} = \cot 30° = \sqrt{3} \Rightarrow \dfrac{AC}{50} = \sqrt{3}$

$\Rightarrow AC = 50\sqrt{3}$ m.

5. Ans. (b)

Solution Let OAP be the pole. When broken by wind at A, let its top P strike the ground at P' so that $OP' = 21$ m, $\angle OP'A = 30°$, $AP = AP'$.



We have, $\dfrac{OA}{OP'} = \tan 30° \Rightarrow OA = \dfrac{21}{\sqrt{3}}$.

$\therefore \quad OA = 7\sqrt{3}$.

Also, $\dfrac{AP'}{OP'} = \sec 30°$ or, $\dfrac{AP}{21} = \dfrac{2}{\sqrt{3}}$

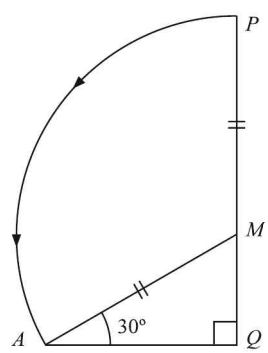
$\therefore \quad AP = \dfrac{42}{\sqrt{3}} = 14\sqrt{3}$.

Height of the pole $= OP = OA + AP$

$= 7\sqrt{3} + 14\sqrt{3} = 21\sqrt{3}$.

6. Ans. (a)

Solution Let QMP be the tree. When broken by the wind its top P strikes the ground at A such that $\angle QAM = 30°$, $AQ = 10$ m and $MA = MP$.



$\dfrac{MQ}{AQ} = \tan 30° \Rightarrow MQ = \dfrac{10}{\sqrt{3}}$ m

and, $\dfrac{AM}{AQ} = \sec 30° \Rightarrow AM = 10\left(\dfrac{2}{\sqrt{3}}\right) = \dfrac{20}{\sqrt{3}}$.

$\therefore$ Height of the tree

$= QM + MP = QM + AM = \dfrac{10}{\sqrt{3}} + \dfrac{20}{\sqrt{3}}$

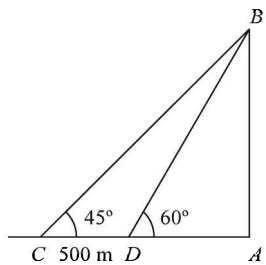
$= \dfrac{30}{\sqrt{3}} = 10\sqrt{3}$ m.

7. Ans. (a)

Solution $CD = AB\ (\cot 45° - \cot 60°)$

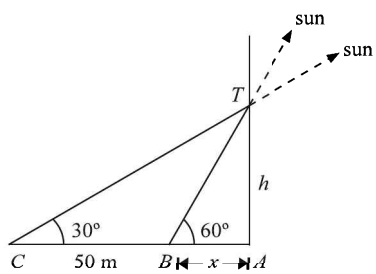
or, $$AB = \frac{CD}{\cot 45° - \cot 60°}$$

$$= \frac{500}{1 - \frac{1}{\sqrt{3}}} = \frac{500\sqrt{3}}{\sqrt{3}-1}\text{m.}$$



8. Ans. (c)

Solution Let T be the top of the tower AT. Let $AT = h$m. Let AB and AC be the shadows of the tower when the sun's altitude is 60° and 30°, respectively



Then, $BC = 50$ m.

Let $AB = x$ m. $\frac{x}{h} = \cot 60° \Rightarrow x = \frac{h}{\sqrt{3}}$ ...(1)

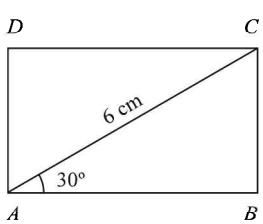
Also, $\frac{x+50}{h} = \cot 30° \Rightarrow x + 50 = \sqrt{3}h$ ...(2)

Subtracting (1) from (2),

$$50 = \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)h \Rightarrow h = 25\sqrt{3}\text{m.}$$

9. Ans. (b)

Solution Let $ABCD$ be the rectangle in which $\angle BAC = 30°$ and $AC = 6$ cm.



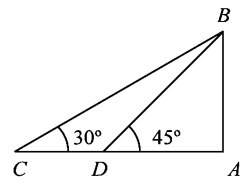
$$\frac{AB}{AC} = \cos 30° = \frac{\sqrt{3}}{2} \Rightarrow AB = 3\sqrt{3}\text{cm.}$$

$$\frac{BC}{AC} = \sin 30° = \frac{1}{2} \Rightarrow BC = 3\text{cm.}$$

$\therefore$ Area of the rectangle $= AB \times BC = 9\sqrt{3}\text{cm}^2$.

10. Ans. (c)

Solution Let AB be the tower and AC and AD be its shadows.



Then, $AB = 100$ m.

$$\frac{AD}{AB} = \cot 45° = 1 \Rightarrow \frac{AD}{100} = 1$$

$$\Rightarrow AD = 100 \text{ m.}$$

$$\frac{AC}{AB} = \cot 30° = \sqrt{3} \Rightarrow \frac{AC}{100} = \sqrt{3}$$

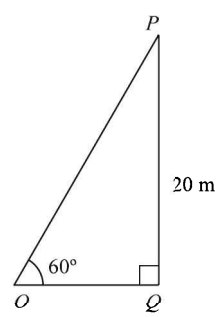
$$\Rightarrow AC = 100\sqrt{3}\text{m.}$$

$\therefore$ $x = AC - AD = 100(\sqrt{3} - 1)$m.

11. Ans. (a)

Solution $\sin 60° = \frac{PQ}{OP} \Rightarrow \frac{\sqrt{3}}{2} = \frac{20}{OP}$

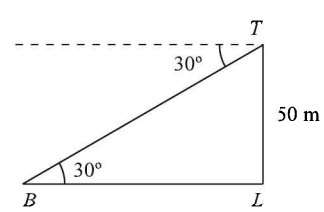
$$\Rightarrow OP = 20 \times \frac{2}{\sqrt{2}} = \frac{40}{\sqrt{3}}\text{m.}$$



12. Ans. (c)

Solution $\tan 30° = \frac{TL}{BL} \Rightarrow \frac{1}{\sqrt{3}} = \frac{50}{BL}$

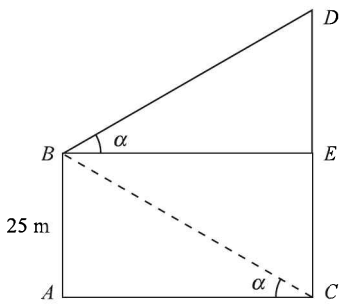
$\therefore$ $BL = 50\sqrt{3}$ m.



13. Ans. (c)

Solution Let AB be the cliff and CD be the tower.

From B, draw $BE \perp CD$.



$$\frac{DE}{BE} = \tan\alpha \text{ and } \frac{AB}{AC} = \tan\alpha$$

$\therefore \quad \dfrac{DE}{BE} = \dfrac{AB}{AC}$

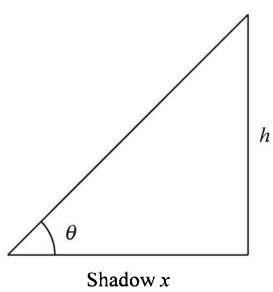
$\therefore \quad DE = AB \qquad (\because BE = AC)$

$\therefore \quad CD = CE + DE = AB + AB = 2AB = 50$ m.

14. Ans. (b)

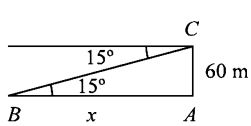
Solution Since $\dfrac{h}{x} = \tan\theta$ and $h = x$.

$\therefore \quad \tan\theta = 1 \Rightarrow \theta = 45°$.



15. Ans. (b)

Solution Here, B is the position of boat and AC is light house.



Now, $\dfrac{AC}{x} = \tan 15° = \tan(45° - 30°)$

$$= \frac{1-\tan 30°}{1+\tan 30°} = \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$$

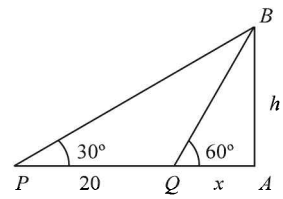
$\therefore \quad x = 60\left(\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right)$m.

16. Ans. (b)

Solution $\dfrac{h}{x} = \tan 60° = \sqrt{3}$

$\therefore \quad h = \sqrt{3}x.$

$\dfrac{h}{20+x} = \tan 30° = \dfrac{1}{\sqrt{3}}$



$\therefore \quad \sqrt{3}h = 20 + x$

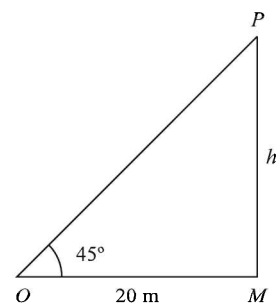
$\therefore \quad \sqrt{3}(\sqrt{3}x) = 20 + x$ or, $3x = 20 + x$

$\therefore \quad x = 10. \quad \therefore \quad h = 10\sqrt{3}$m.

17. Ans. (b)

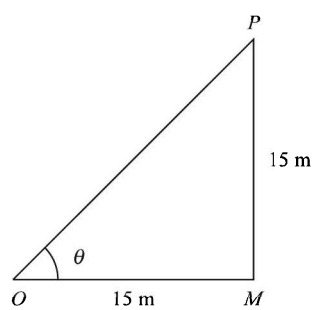
Solution Clearly, $\dfrac{h}{20} = \tan 45° = 1$

$\therefore \quad h = 20$ m.



18. Ans. (a)

Solution Let MP denote the house. Let θ be the angle of elevation.



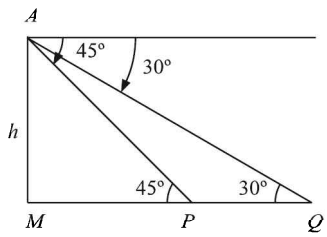
$\therefore \quad \tan\theta = \dfrac{MP}{OM} = \dfrac{15}{15} = 1. \quad \therefore \quad \theta = 45°.$

19. Ans. (c)

Solution Let the boats be at P, Q

So that $PQ = 60$ m.

Let MA be the light house.



Let $h = MA$

Then, $\dfrac{h}{MP} = \tan 45° = 1.$ $\therefore$ $h = MP$

Again, $\dfrac{h}{MP+60} = \tan 30° = \dfrac{1}{\sqrt{3}}.$

$\therefore$ $MP + 60 = \sqrt{3}h$ or, $h + 60 = \sqrt{3}h$

$\therefore$ $(\sqrt{3} - 1)h = 60$

$\therefore$ $h = \dfrac{60}{\sqrt{3}-1} = \dfrac{60(\sqrt{3}+1)}{2} = 30(\sqrt{3} + 1)$ m

20. Ans. (b)

Solution The distance of the foot from each vertex $= h\cot\alpha$.

$\therefore$ The foot is at the circumcentre of the triangle.

$\therefore$ $R = h\cot\alpha$

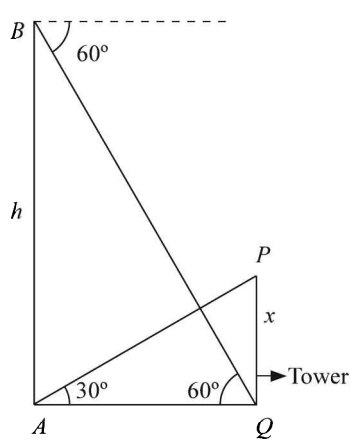
$\therefore$ $h = R\tan\alpha = \dfrac{a}{2\sin\alpha}\tan\alpha = \dfrac{a}{2}\tan\alpha\cdot\operatorname{cosec}\alpha.$

21. Ans. (a)

Solution Let $PQ = x$ m denote the tower so that $\angle PAQ = 30°$. Let $BA = h$ m

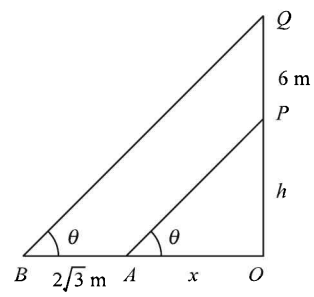
$\therefore$ $\angle BQA = 60°.$

Now, $\dfrac{h}{AQ} = \tan 60° = \sqrt{3}.$ $\therefore AQ = \dfrac{h}{\sqrt{3}} = h\cot 60°.$



22. Ans. (a)

Solution Let OP be the tower of height h m and PQ be the flagstaff of height 6 m. Let the sun make an angle θ with the ground. Let $OA = x$ and $AB = 2\sqrt{3}$ be the shadows of the tower and the flagstaff, respectively.



Now, $\tan\theta = \dfrac{h}{x}.$

Also, $\dfrac{h+6}{x+2\sqrt{3}} = \tan\theta$

$\therefore$ $\dfrac{h}{x} = \dfrac{h+6}{x+2\sqrt{3}} \Rightarrow hx + 2\sqrt{3}h = hx + 6x$

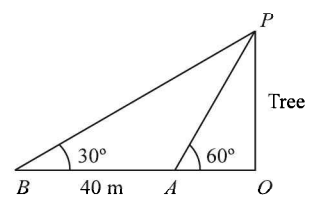
$\Rightarrow 2\sqrt{3}h = 6x$

$\Rightarrow \dfrac{h}{x} = \dfrac{6}{2\sqrt{3}} = \sqrt{3}$

$\Rightarrow \tan\theta = \sqrt{3}.$ $\therefore$ $\theta = 60°.$

23. Ans. (c)

Solution Let OA denote the breadth of the river.



$\dfrac{OP}{OA} = \tan 60° = \sqrt{3}$

$\therefore$ $OP = \sqrt{3}OA.$

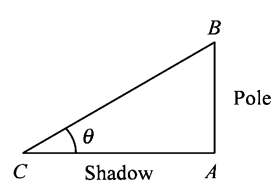
Also, $\dfrac{OP}{OA+40} = \tan 30° = \dfrac{1}{\sqrt{3}}$

$\therefore$ $OA + 40 = \sqrt{3}OP = \sqrt{3}(\sqrt{3}OA) = 3\,OA.$

$\therefore$ $2OA = 40 \Rightarrow OA = 20$ m.

24. Ans. (a)

Solution Given, $AC = \sqrt{3}AB.$

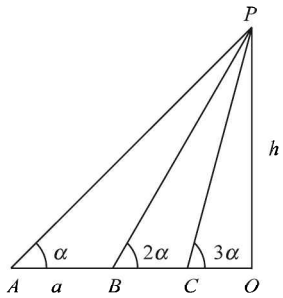


$\therefore \cot\theta = \dfrac{AC}{AB} = \sqrt{3} \Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30°.$

25. Ans. (c)

Solution Let OP be a vertical tower. The elevation of top P from A, B, C are α, 2α, 3α, respectively. $\angle APB = 2\alpha - \alpha = \angle PAB$.



$$\frac{OP}{BP} = \sin 2\alpha$$

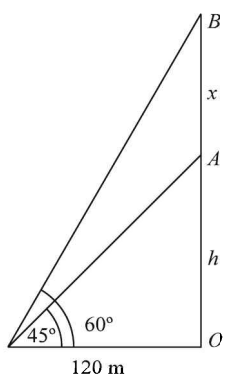
$\therefore \quad OP = BP \sin 2\alpha = a \sin 2\alpha.$

Thus, height of the tower $= a \sin 2\alpha$.

26. Ans. (b)

Solution $\dfrac{h+x}{120} = \tan 60° = \sqrt{3}$

$h + x = \sqrt{3}(120).$



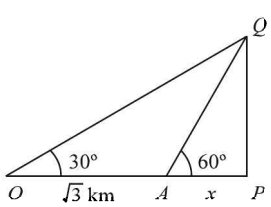
Also, $\dfrac{h}{120} = \tan 45° = 1.$

$\therefore \quad h = 120 \text{ m} \quad \therefore \quad 120 + x = 120\sqrt{3}$

$\therefore \quad x = 120(\sqrt{3} - 1) \text{ m}.$

27. Ans. (a)

Solution $\dfrac{h}{x} = \tan 60 = \sqrt{3}. \quad \therefore \quad h = \sqrt{3}x.$



Also, $\dfrac{h}{\sqrt{3}+x} = \tan 30° = \dfrac{1}{\sqrt{3}}$

$\therefore \quad \sqrt{3}h = \sqrt{3} + x$

$\therefore \quad \sqrt{3}(\sqrt{3}x) = \sqrt{3} + x$ or $3x - x = \sqrt{3}$

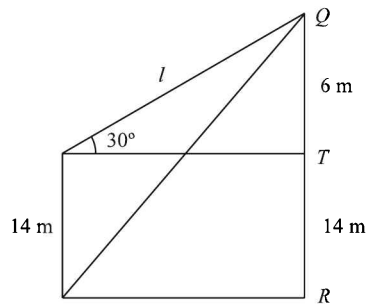
$\therefore \; 2x = \sqrt{3} \qquad \therefore \quad x = \dfrac{\sqrt{3}}{2}$

$\therefore \quad h = \sqrt{3}.\dfrac{\sqrt{3}}{2} = \dfrac{3}{2}\text{km}.$

28. Ans. (a)

Solution $\dfrac{6}{l} = \sin 30° = \dfrac{1}{2}$

$\therefore \quad l = 12 \text{ m}.$



29. Ans. (c)

Solution Let h be the height of the man.

$\therefore \quad \dfrac{6}{8} = \dfrac{h}{2.4} \Rightarrow h = \dfrac{3}{4}(2.4) = 1.8 \text{ m}.$

30. Ans. (d)

Solution Let $AB = h$ be the height of the tower.

Let $GA = x$.

Then, $\dfrac{h}{x} = \tan 30° = \dfrac{1}{\sqrt{3}}.$

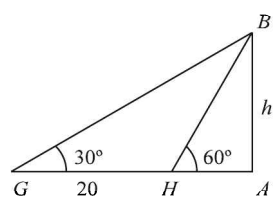
$\therefore \quad h = \dfrac{x}{\sqrt{3}}.$

Also, $\dfrac{h}{x-20} = \tan 60° = \sqrt{3}.$

$\therefore \quad h = \sqrt{3}(x - 20)$

$\therefore \quad \dfrac{x}{\sqrt{3}} = \sqrt{3}(x - 20) \Rightarrow x = 3(x - 20) = 3x - 60$

$\Rightarrow 2x = 60 \Rightarrow x = 30.$



$\therefore \quad h = \dfrac{30}{\sqrt{3}} = 10\sqrt{3}\text{m}.$

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. The top of a 15 m high tower makes an angle of elevation of 60° with the bottom of an electric pole and an angle of elevation of 30° with the top of the pole. What is the height of the electric pole?
(a) 5 m (b) 8 m
(c) 10 m (d) 12 m
(e) None of these **[Based on SB1 PO 1999]**

2. A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at a distance of 60 m from the tower. After 5 sec, the angle of depression becomes 30°. What is the approximate speed of the boat, assuming it is running in still water?
(a) 32 kmph (b) 42 kmph
(c) 38 kmph (d) 36 kmph
(e) 40 kmph **[Based on SB1 Associates PO 1999]**

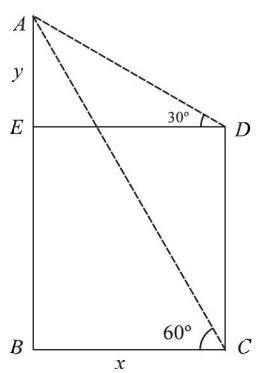
3. From a point P on a level ground, the angle of elevation of the top of the tower is 30°. If the tower is 100 m high, the distance of the point P from the foot of the tower is
(a) $100/\sqrt{3}$ m (b) $100\sqrt{3}$ m
(c) $50\sqrt{3}$ m (d) $50/\sqrt{3}$ m
[Based on RRB Gorakhpur ASM Exam 2002]

4. From the bridge on the river, 15 m high, the angle of depression of a boat is 30°. If the speed of the boat be 6 km/hr, then the time taken by the boat to ;reach just below the bridge will be
(a) $9\sqrt{3}$ sec. (b) $19/\sqrt{3}$ sec.
(c) $3\sqrt{3}$ sec. (d) None of these
[Based on RRB Gorakhpur ASM Exam 2002]

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Let AB be the tower and CD be the electric pole.



Let $BC = DE = x$

Now, $\frac{AB}{BC} = \tan 60° \Rightarrow \frac{15}{x} = \sqrt{3} \Rightarrow x = \frac{15}{\sqrt{3}}$.

Also, $\frac{AE}{DE} = \tan 30° \Rightarrow \frac{y}{15/\sqrt{3}} = \frac{1}{\sqrt{3}}$

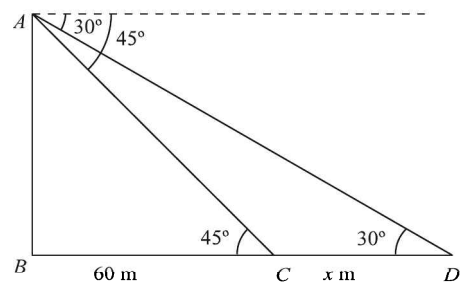
$\Rightarrow 3y = 15$ or, $y = 5$.

$\therefore$ $CD = BE = AB - AE = 15 - 5 = 10$ m.

2. Ans. (a)

Solution $\tan 45° = \frac{AB}{60} \Rightarrow AB = 60$ m ...(1)

$\tan 30° = \frac{AB}{60+x}$ or, $AB = \frac{1}{\sqrt{3}}(60+x)$ ...(2)



From (1) and (2), $60 + x = 60\sqrt{3}$

$\Rightarrow$ $x = 60(\sqrt{3} - 1) = 43.92$ m ($\because \sqrt{3} = 1.732$)

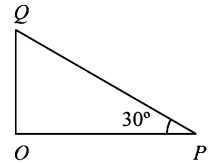
Speed of the boat $= \frac{43.92}{5}$ m/sec $= \frac{43.92}{5} \times \frac{18}{5}$

$= 32$kmph.

3. Ans. (b)

Solution Let OQ be the tower.

Then, $OQ = 100$ m and $\angle OPQ = 30°$.

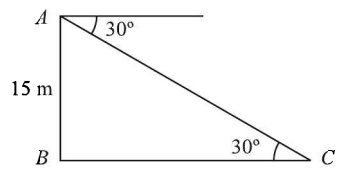


In ΔOPQ, $\tan 30° = \frac{OQ}{OP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{OP}$

$\Rightarrow OP = 100\sqrt{3}$ m.

4. Ans. (a)

Solution Let C be the initial position of the boat and A be the point on the top of the bridge from where the angle of depression of the boat is 30°.



$$AB = 15 \text{ m}, \; BC = \frac{AB}{\tan 30°} = \frac{15}{1/\sqrt{3}} = 15\sqrt{3}\text{m}.$$

$$\text{Speed of the boat} = 6 \text{ km/hr} = 6 \times \frac{5}{18}\text{m/s} = \frac{5}{3}\text{m/s}$$

$$\therefore \quad \text{Time required} = \frac{\text{Distance}}{\text{Speed}} = \frac{15\sqrt{3}}{5/3} = 9\sqrt{3}\text{ sec.}$$

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. Two posts are 25 m and 15 m high and the line joining their tops makes an angle of 45° with the horizontal. The distance between these posts is

(a) 5 m (b) $\frac{10}{\sqrt{2}}$ m
(c) 10 m (d) $10\sqrt{2}$ m

2. The angle of depression of the top of a tower at a point 70 m from the base is 45°. The height of the tower is

(a) 70 m (b) $70\sqrt{2}$ m
(c) $\frac{70}{\sqrt{2}}$ m (d) 35 m

3. The angle of elevation of the top of a tower from the top and bottom of a building of height 'a' are 30° and 45°, respectively. If the tower and the building stand at the same level, the height of the tower is

(a) $\sqrt{3}a$ (b) $a\,(\sqrt{3-1})$
(c) $\frac{(3+\sqrt{3})}{2}a$ (b) $a\,(\sqrt{3+1})$

4. The angle of elevation of the top of a tower at a distance of 500 m from its foot is 30°. The height of the tower is

(a) $\frac{500\sqrt{3}}{3}$m (b) $\frac{500(\sqrt{3-1})}{3}$m
(c) $\frac{500(\sqrt{3+1})}{3}$m (d) 500 m.

5. From the top of a house 32 m high, the angle of elevation of the top of a river is 45° and the angle of depression of the foot of the tower is 30°. The distance of the tower from the house is

(a) $30\sqrt{3}$ m (b) $32\sqrt{3}$ m
(c) $35\sqrt{3}$ m (d) None of these

6. If from a point 100 m above the ground the angles of depression of two objects due to south on the ground are 60° and 45°, then the distance between the objects is

(a) $\frac{100}{\sqrt{3}}(\sqrt{3+1})$ m (b) $\frac{100}{\sqrt{3}}(\sqrt{3-1})$ m
(c) $100\sqrt{3}$ m (d) 100 m

7. Length of the shadow of a person is x when the angle of elevation of the sun is 45°. If the length of the shadow increases by $(\sqrt{3}-1)x$, then the angle of elevation of the sun should become

(a) 15° (b) 18°
(c) 25° (d) 30°

8. A balloon is connected to a mateorological station by a cable of length 200 m, inclined at 60° to the horizontal. Find the height of the balloon from the ground.

(a) $100\sqrt{3}$ (b) $100/\sqrt{3}$
(c) $50\sqrt{3}$ (d) $50/\sqrt{3}$

9. The distance between two multistoreyed buildings is 60 m. The angle of depression of the top of the first building as seen from the top of the second building which is 150 m high is 30°. The height of the first building is

(a) $(150+20\sqrt{3})$m (b) $(150-20\sqrt{3})$
(c) $(150+10\sqrt{3})$m (d) None of these

10. A boot is being rowed away from a cliff 150 m high. At the top of the cliff the angle of depression of the boat changes from 60° to 45° in 2 minutes. The speed of the boat is

(a) 2 km/hr (b) 1.9 km/hr
(c) 3.4 km/hr (d) 3 km/hr

Test I **[Time: 10 min]**

11. The angle of elevation of an aeroplane from a point on the ground is 45°. After 15 seconds flight, the elevation changes to 30°. If the aeroplane is flying at a height of 3000 m, the speed of the plane in km

per hour is
(a) 306.72 (b) 208.34
(c) 402.56 (d) 527

12. A balloon of radius r makes an angle α at the eye of an observer and the angle of elevation of its centre is β. The height of its centre from the ground level is given by
(a) $r \cos \frac{\beta}{2} \sec \alpha$ (b) $r \cos \beta \sec \frac{\alpha}{2}$
(c) $r \sin \frac{\beta}{2} \operatorname{cosec} \alpha$ (d) $r \sin \beta \operatorname{cosec} \frac{\alpha}{2}$

13. A landmark on a river bank is observed from two points X and Y on the opposite bank of the river. The lines of sight make equal angles with the bank of the river. If XY = 1 km, then the width of the river is
(a) $\frac{3}{2}$km (b) $\frac{1}{2}$km
(c) $\frac{3\sqrt{2}}{2}$km (d) $\frac{3\sqrt{2}}{2}$km

14. Two men are on the opposite sides of a tower. They measure the angle of elevation of the top of the tower as 30° and 60°, respectively. If the height of the tower is 30 m, find the distance between the men.
(a) $20\sqrt{3}$m (b) $20/\sqrt{3}$m
(c) $40\sqrt{3}$m (d) $40/\sqrt{3}$m

15. A person standing on the bank of a river finds that the angle of elevation of the top of a tower on the opposite bank is 45°, then
(a) breadth of the river is twice the height of the tower
(b) height of the river is twice the breadth of the river
(c) both are equal (d) none of the above.

16. An artist climbs a rope stretched from the top of a pole and fixed at the ground. The height of the pole is 10 m and the angle made by the rope with the ground is 30°. Calculate the length of the rope.
(a) 10 m (b) 20 m
(c) 30 m (d) 40 m

17. The length of a string between a kite and a point on the ground is 90 m. The string makes an angle of 60° with the level ground. Assuming that there is no slack in the string, then the height of the kite is
(a) $45\sqrt{3}$m (b) $45/\sqrt{3}$m
(c) $50\sqrt{3}$m (d) $50/\sqrt{3}$m

18. A ladder leaning against a vertical wall makes an angle of 30° with the ground. The foot of the ladder is 5 m from the wall. Determine the length of the ladder.
(a) $10\sqrt{3}$m (b) $10/\sqrt{3}$m
(c) $20\sqrt{3}$m (d) $20/\sqrt{3}$m

19. The angles of elevation of an aeroplane flying vertically above the ground as observed from two consecutive stones 1 km apart are 45° and 60°. The height of the aeroplane above the ground in km is
(a) $\frac{\sqrt{3}+1}{2}$ (b) $\frac{3+\sqrt{3}}{2}$
(c) $3+\sqrt{3}$ (d) $\sqrt{3}+1$

20. From the top of a light house, the angles of depression of two ships on the opposite sides of it are observed to be α and β. If the height of the light house be h m and the line joining the ships passes through the foot of the light house, the distance between the ships is
(a) $\frac{h(\cot\alpha+\cot\beta)}{\cot\alpha\cot\beta}$ (b) $\frac{h(\tan\alpha+\tan\beta)}{\tan\alpha\tan\beta}$
(c) $h(\tan\alpha + \tan\beta)$ (d) $\frac{h\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$

Answers

1. (c)	**2.** (a)	**3.** (c)	**4.** (a)	**5.** (b)	**6.** (b)	**7.** (d)	**8.** (a)	**9.** (b)	**10.** (b)
11. (d)	**12.** (d)	**13.** (b)	**14.** (c)	**15.** (c)	**16.** (b)	**17.** (a)	**18.** (b)	**19.** (b)	**20.** (b)

29

LOGARITHMS

INTRODUCTION

Logarithm, in mathematics, is the 'exponent' or 'power' to which a stated number called the *base*, is raised to yield a specific number. For example, in the expression $10^2 = 100$, the logarithm of 100 to the **base** 10 is 2. This is written as $\log_{10} 100 = 2$. Logarithms were originally invented to help simplify the arithmetical processes of multiplication, division, expansion to a power and extraction of a 'root', but they are nowadays used for a variety of purposes in pure and applied mathematics.

Logarithm

If for a positive real number ($a \neq 1$), $a^m = b$, then the index m is called the logarithm of b to the base a. We write this as

$$\log_a b = m$$

'log' being the abbreviation of the word 'logarithm'. Thus,

$$a^m = b \Leftrightarrow \log_a b = m$$

where, $a^m = b$ is called the *exponential form* and $\log_a b = m$ is called the *logarithmic form*.

Illustration 1 Refer to the following Table

Exponential form	logarithmic form
$3^5 = 243$	$\log_3 243 = 5$
$2^4 = 16$	$\log_2 16 = 4$
$3^o = 1$	$\log_3 1 = 0$
$8^{1/3} = 2$	$\log_8 2 = \frac{1}{3}$

LAWS OF LOGARITHMS

1. **Product formula**
   The logarithm of the product of two numbers is equal to the sum of their logarithms.
   i.e. $\log_a (mn) = \log_a m + \log_a n$.
   Generalisation: In general, we have
   $\log_a (mnpq...) = \log_a m + \log_a n + \log_a p + \log_a q + ...$
2. **Quotient formula**
   The logarithm of the quotient of two numbers is equal to the difference of their logarithms.
   i.e. $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$,
   where, a, m, n are positive and $a \neq 1$.
3. **Power formula**
   The logarithm of a number raised to a power is equal to the power multiplied by logarithm of the number.
   i.e. $\log_a (m^n) = n \log_a m$,
   where, a, m are positive and $a \neq 1$.
3. **Base changing formula**
   $\log_n m = \frac{\log_a m}{\log_a n}$. So, $\log_n m = \frac{\log m}{\log n}$.
   where, m, n, a are positived and $n \neq 1$, $a \neq 1$.
4. **Reciprocal relation**
   $\log_b a \times \log_a b = 1$,
   where, a, b are positive and not equal to 1.
5. $\log_b a = \frac{1}{\log_a b}$
6. ${}_a\log_a x = x$, where, a and x are positive, $a \neq 1$.
7. If $a > 1$ and $x > 1$, then $\log_a x > 0$.
8. If $0 < a < 1$ and $0 < x < 1$, then $\log_a x > 0$.
9. If $0 < a < 1$ and $x > 1$, then $\log_a x > 0$.
10. If $a > 1$ and $0 < x < 1$, then $\log_a x < 0$.

SOME USEFUL FORMULAE

1. Logarithm of 1 to any base is equal to zero.
   i.e. $\log_a 1 = 0$, where $a > 0$, $a \neq 1$.
2. Logarithm of any number to the same base is 1.
   i.e. $\log_a a = 1$, where $a > 0$, $a \neq 1$.

Common Logarithms

There are two bases of logarithms that are extensively used these days. One is base e (e = 2. 71828 approx.) and the other is base 10. The logarithms to base e are called natural logarithms. The logarithms to base 10 are called the common logarithms.

$\log_{10}10 = 1$, since $10^1 = 10$.
$\log_{10}100 = 2$, since $10^2 = 100$.
$\log_{10}10000 = 4$, since $10^4 = 10000$.
$\log_{10}0.01 = -2$, since $10^{-2} = 0.01$.
$\log_{10}0.001 = -3$, since $10^{-3} = 0.001$
and, $\log_{10}1 = 0$, since $10° = 1$.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Find $\log_{3/2} 3.375$.
(a) 2 (b) 3
(c) 5/2 (d) 17/2

2. If $x = \log_{2a} a$, $y = \log_{3a} 2a$ and $z = \log_{4a} 3a$, find $yz\ (2 - x)$.
(a) 1 (b) –1
(c) 2 (d) –2

3. $\dfrac{\log x}{l+m-2m} = \dfrac{\log y}{m+n-2l} = \dfrac{\log z}{n+l-2m}$, find $x^2y^2z^2$.
(a) 2 (b) –1
(c) 4 (d) 1

4. If $\log \dfrac{x+y}{5} = \dfrac{1}{2}(\log x + \log y)$, then $\dfrac{x}{y} + \dfrac{y}{x} =$
(a) 20 (b) 23
(c) 22 (d) 21

5. If $\log (x + y) = \log\left(\dfrac{3x-3y}{2}\right)$, then $\log x - \log y =$
(a) log 2 (b) log 3
(c) log 5 (d) log 6

6. If $\log_2 x + \log_4 x + \log_{16} x = 21/4$, then $x =$
(a) 8 (b) 4
(c) 2 (d) 16

7. $7 \log \dfrac{16}{15} + 5 \log \dfrac{25}{24} + 3 \log \dfrac{81}{80} =$
(a) log 2 (b) log 3
(c) log 5 (d) None of these

8. If $0 < a \le x$, the minimum value of $\log_a x + \log_x a$ is
(a) 1 (b) 2
(c) 3 (d) 5

9. If $\dfrac{\log x}{b-c} = \dfrac{\log y}{c-a} = \dfrac{\log z}{a-b}$, then $xyz = x^a \cdot y^b \cdot z^c$
$= x^{b+c} \cdot y^{c+a} \cdot z^{a+b} =$
(a) 1 (b) 0
(c) 2 (d) None of these

10. $x^{\log y - \log z} \cdot y^{\log z - \log x} \cdot z^{\log x - \log y} =$
(a) 0 (b) 2
(c) 1 (d) None of these

11. If $\log_{10}[98 + \sqrt{x^2 - 12x + 36}] = 2$, then $x =$
(a) 4 (b) 8
(c) 12 (d) 4,8

12. If $x = \log_a bc$, $y = \log_b ca$, $z = \log_c ab$, then
(a) $xyz = x + y + z + 2$
(b) $xyz = x + y + z + 1$
(c) $x + y + z = 1$
(d) $xyz = 1$.

13. If $a^x = b^y = c^z = d^w$, then $\log_a (bcd) =$
(a) $\dfrac{1}{x}\left(\dfrac{1}{y}+\dfrac{1}{z}+\dfrac{1}{w}\right)$ (b) $x\left(\dfrac{1}{y}+\dfrac{1}{z}+\dfrac{1}{w}\right)$
(c) $\dfrac{y+z+w}{x}$ (d) None of these

14. If $\log_{10} 2 = 0.3010$, then $\log_{10} (1/2) =$
(a) –0.3010 (b) 0.6990
(c) $\bar{1}.6990$ (d) $\bar{1}.3010$

15. If $\log_2 (3^{2x-2} + 7) = 2 + \log_2 (3^{x-1} + 1)$, then $x =$
(a) 0 (b) 1
(c) 2 (d) 1 or 2

16. If $\log_a b = \log_b c = \log_c a$, then
(a) $a > b \ge c$ (b) $a < b < c$
(c) $a = b = c$ (b) $a < b \le c$.

17. If $\dfrac{1}{\log_x 10} = \dfrac{2}{\log_a 10} - 2$, then $x =$
(a) $a/2$ (b) $a/100$
(c) $a^2/10$ (d) $a^2/100$

18. If $a^2 + b^2 = c^2$, then $\dfrac{1}{\log_{c+a} b} + \dfrac{1}{\log_{c-a} b} =$
(a) 1 (b) 2
(c) –1 (d) –2

19. If $\log_{10} 87.5 = 1.9421$, then the number of digits in $(875)^{10}$ is

(a) 30 (b) 29
(c) 20 (d) 19

20. If $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$, then the number of zeros between the decimal point and the first significant figure in $(0.0432)^{10}$ is
(a) 10 (b) 13
(c) 14 (d) 15

21. If $(4.2)^x = (0.42)^y = 100$, then $\frac{1}{x} - \frac{1}{y} =$
(a) 1 (b) 2
(c) 1/2 (d) –1

22. $\frac{\log_9 11}{\log_5 13} - \frac{\log_3 11}{\log_{\sqrt{5}} 13} =$
(a) 1 (b) –1
(c) 0 (d) None of these

23. If $\frac{\log x}{2} = \frac{\log y}{3} = \frac{\log z}{5}$, then yz in terms of x is
(a) x (b) x^2
(c) x^3 (d) x^4

24. If $4^x + 2^{2x-1} = 3^{x+\frac{1}{2}} + 3^{x-\frac{1}{2}}$, then $x =$
(a) 1/2 (b) 3/2
(c) 5/2 (d) 1

25. $\frac{\log 49\sqrt{7} + \log 25\sqrt{5} - \log 4\sqrt{2}}{\log 17.5} =$
(a) 5 (b) 2
(c) 5/2 (d) 3/2

26. $\log_{10} \tan 40° \cdot \log_{10} 41° \ldots \log_{10} \tan 50° =$
(a) 1 (b) 0
(c) –1 (d) None of these

27. If $\log_8 p = 2.5$, $\log_2 q = 5$, then p in terms of q is
(a) $q\sqrt{q}$ (b) $2q$
(c) q (d) $q/2$

28. If $y = \frac{1}{a^{1-\log_a x}}$, $z = \frac{1}{a^{1-\log_a y}}$ and $x = a^k$, then $k=$
(a) $\frac{1}{a^{1-\log_a z}}$ (b) $\frac{1}{1-\log_a z}$
(c) $\frac{1}{1+\log_z a}$ (d) $\frac{1}{1-\log_z a}$

29. If $\log_e 2 \cdot \log_b 625 = \log_{10}{}^{16} \cdot \log_e 10$, then $b =$
(a) 4 (b) 5
(c) 1 (d) e

30. $5^{\sqrt{\log_5 7}} - 7^{\sqrt{\log_7 5}}$
(a) log 2 (b) 1
(c) 0 (d) None of these

31. $2^{\log_3^7} - 7^{\log_3^2}$
(a) $\log_2 7$ (b) log 7
(c) log 2 (d) 0

32. If $\log_{30} 3 = a$, $\log_{30} 5 = b$, then $\log_{30} 8 =$
(a) $3(1 - a - b)$ (b) $a - b + 1$
(c) $1 - a - b$ (d) $1(a - b + 1)$

33. If $0 < a < 1$, $0 < x < 1$ and $x < a$, then $\log_a x$
(a) < 1 (b) > 1
(c) < 0 (d) ≤ 1

34. $\log_5 2$ is
(a) an integer
(b) a rational number
(c) an irrational number
(d) a prime number

35. $\log_5\left(1+\frac{1}{5}\right)+\log_5\left(1+\frac{1}{6}\right)+\log_5\left(1+\frac{1}{7}\right) + \ldots + \log_5\left(1+\frac{1}{624}\right)$
(a) 5 (b) 4
(c) 3 (d) 2

36. If $\log_{10} 2986 = 3.4751$, then $\log_{10} 0.02986 =$
(a) $\bar{1}.2986$ (b) $\bar{2}.4751$
(c) 0.34751 (d) None of these

37. If $\log (2a - 3b) = \log a - \log b$, then $a =$
(a) $\frac{3b^2}{2b-1}$ (b) $\frac{3b}{2b-1}$
(c) $\frac{b^2}{2b+1}$ (d) $\frac{3b^2}{2b+1}$

38. If $\log(x - y) - \log 5 - \frac{1}{2}\log x - \frac{1}{2}\log y = 0$, then $\frac{x}{y}+\frac{y}{x} =$
(a) 25 (b) 26
(c) 27 (d) 28

39. If $\log x : 3 = \log y : 4 = \log z : 5$, then $zx =$
(a) $2y$ (b) y^2
(c) $8y$ (d) $4y$

40. If $3 + \log_5 x = 2 \log_{25} y$, then $x =$
(a) $y/125$ (b) $y/25$
(c) $y^2/625$ (d) $3 - y^2/25$

41. If $\frac{\log_2 a}{3} = \frac{\log_3 b}{3} = \frac{\log_4 c}{4}$ and $a^{1/2} \cdot b^{1/3} \cdot c^{1/4} = 24$, then
(a) $a = 24$ (b) $b = 81$
(c) $c = 64$ (d) $c = 256$

42. If $\frac{\log_2 x}{3} = \frac{\log_2 y}{4} = \frac{\log_2 z}{5k}$ and $\frac{z}{x^3y^4} = 1$, then $k =$
(a) 3 (b) 4
(c) 5 (d) –5.

43. $\frac{3+\log_{10} 343}{2+\frac{1}{2}\log\left(\frac{49}{4}\right)+\frac{1}{3}\log\left(\frac{1}{125}\right)} =$

(a) 3 (b) 3/2
(c) 2 (d) 1

44. If $\frac{\log x}{a^2+ab+b^2} = \frac{\log y}{b^2+bc+c^2} = \frac{\log z}{c^2+ca+a^2}$, then $x^{a-b} \cdot y^{b-c} \cdot z^{c-a} =$
(a) 0 (b) –1
(c) 1 (d) 2

45. If $3^{x-2} = 5$ and $\log_{10} 2 = 0.20103$, $\log_{10} 3 = 0.4771$, then $x =$
(a) $1\frac{22187}{47710}$ (b) $2\frac{22187}{47710}$
(c) $3\frac{22187}{47710}$ (d) None of these

46. If $\log_2 = 0.30103$ and $\log_3 = 0.4771$, then number of digits in $(648)^5$ is
(a) 12 (b) 13
(c) 14 (d) 15

47. If $\log x = \frac{\log y}{2} = \frac{\log z}{5}$, then $x^4 \cdot y^3 \cdot z^{-2} =$
(a) 2 (b) 10
(c) 1 (d) 0

48. $\frac{\log\sqrt{27}+\log\sqrt{1000}+\log 8}{\log 120}$
(a) 1/2 (b) 1
(c) 3/2 (d) 2

49. For $x > 0$, if $y = \frac{{}_{10}\log_{10}^{x}}{x^2}$ and $x = y^a$, then $a =$
(a) 1 (b) –1
(c) 0 (d) 2

50. If $x = 100_{4/3}(1/2)$, $y = \log_{1/2}(1/3)$, then
(a) $x > y$ (b) $x < y$
(c) $x = y$ (d) $x \geq y$

EXPLANATORY ANSWERS

1. Ans. (b)

Solution $\log_{3/2} 3.375 = x \Rightarrow \left(\frac{3}{2}\right)^x = 3.375$

$\Rightarrow (1.5)^x = (1.5)^3 \Rightarrow x = 3.$

2. Ans. (a)

Solution $yz(2 - x) = 2yz - xyz = 2\log_{4a} 2a - \log_{4a} a$

$= \log_{4a}\left(\frac{4a^2}{a}\right) = 1.$

3. Ans. (d)

Solution Each is equal to k

$\Rightarrow \log x = k(l + m - 2n),$

$\log y = k(m + n - 2l), \log z = k(n + l - 2m).$

$\Rightarrow \log xyz = k(0) \Rightarrow xyz = e^0 = 1 \Rightarrow x^2y^2z^2 = 1.$

4. Ans. (b)

Solution $\log\left(\frac{x+y}{5}\right) = \frac{1}{2}[\log x + \log y]$

$\Rightarrow x + y = 5\sqrt{xy} \Rightarrow x^2 + y^2 = 23xy$

$\Rightarrow \frac{x}{y} + \frac{y}{x} = 23.$

5. Ans. (c)

Solution $x + y = \frac{3x-3y}{2} \Rightarrow x = 5y \Rightarrow \frac{x}{y} = 5$

$\Rightarrow \log x - \log y = \log 5.$

6. Ans. (a)

Solution $\log_2 x + \frac{1}{2}\log_2 x + \frac{1}{4}\log_2 x = \frac{21}{4}$

$\Rightarrow \log_2 x\left(1+\frac{1}{2}+\frac{1}{4}\right) = \frac{21}{4} \Rightarrow \log_2 x = 3 \Rightarrow x = 8.$

7. Ans. (a)

Solution $7\log\left(\frac{2^4}{5\times 3}\right) + 5\log\left(\frac{5^2}{2^3\times 3}\right) + 3\log\left(\frac{3^4}{2^4\times 5}\right)$

$= 28\log 2 - 7\log 5 - 7\log 3 + 10\log 5 - 15\log 2$

$-5\log 3 + 12\log 3 - 12\log 2 - 3\log 5 = \log 2.$

8. Ans. (b)

Solution $0 < a \leq x$; Min. value of $\log_a x + \log_x a$ is 2 when we put $x = a$.

9. Ans. (a)

Solution $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b} = k$ (say)

$\Rightarrow \log x = k(b-c), \log y = k(c-a), \log z = k(a-b)$

$\Rightarrow \log x + \log y + \log z = 0 \Rightarrow xyz = 1.$

Also, $a\log x + b\log y + c\log z = 0 \Rightarrow x^a . y^b . z^c = 1.$

Again $(b+c)\log x + (c+a)\log y + (a+b)\log z = 0.$

$\Rightarrow x^{b+c} \cdot y^{c+a} \cdot z^{a+b} = 1.$

$\therefore \quad xyz = x^a \cdot y^b \cdot z^c = x^{b+c} \cdot y^{c+a} \cdot z^{a+b} = 1.$

10. Ans. (c)

Solution $x^{\log y - \log z} \cdot y^{\log z - \log x} \cdot z^{\log x - \log y} = k$ (say)

$\Rightarrow$ $(\log y - \log z)\log x + (\log z - \log x)\log y$
$+ (\log x - \log y)\log z = \log k = 0$

$\Rightarrow k = 1.$

11. Ans. (d)

Solution $98 + \sqrt{x^2 - 12x + 36} = 100$

$\Rightarrow \sqrt{x^2 - 12x + 36} = 2$

$\Rightarrow x^2 - 12x + 32 = 0$

$\Rightarrow x = 8, 4.$

12. Ans. (a)

Solution $x = \log_a bc \Rightarrow a^x = bc \Rightarrow a^{x+1} = abc$

$\Rightarrow a = (abc)^{1/x+1}.$

Similarly, $b = (abc)^{1/y+1}$ and $c = (abc)^{1/z+1}$

$\therefore \quad abc = (abc)^{\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}}$

$\Rightarrow \quad 1 = \frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}$

$\Rightarrow \quad (x+1)(y+1)(z+1) = (y+1)(z+1)$
$+ (x+1)(z+1) + (x+1)(y+1)$

$\Rightarrow \quad xyz = x + y + z + 2.$

13. Ans. (b)

Solution $b^y = a^x \Rightarrow b = a^{\frac{x}{y}},\ c = a^{\frac{x}{z}},\ d = a^{\frac{x}{w}}$

$\log_a(bcd) = \log_a\left(a^{\frac{x}{y}}.a^{\frac{x}{z}}.a^{\frac{x}{w}}\right) = \frac{x}{y}+\frac{x}{z}+\frac{x}{w}$

$= x\left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right).$

14. Ans. (c)

Solution $\log_{10}\left(\frac{1}{2}\right) = -\log_{10} 2 = -0.3010$

$= 1 - 0.3010 - 1 = \bar{1}.6990.$

15. Ans. (d)

Solution $\log_2(3^{2x-2} + 7) = \log_2 4 + \log_2(3^{x-1} + 1)$
$[\because 2 = 2\log_2 2 = \log_2 2^2]$

$\Rightarrow 3^{2x-2} + 7 = 4(3^{x-1} + 1)$

$\Rightarrow t^2 + 7 = 4(t + 1)$, where, $3^{x-1} = t$

$\Rightarrow t^2 - 4t + 3 = 0 \Rightarrow t = 1, 3$

When $t = 1 \Rightarrow 3^{x-1} = 1 \Rightarrow x = 1$

When $t = 3 \Rightarrow 3^{x-1} = 3^1 \Rightarrow x = 2.$

16. Ans. (c)

Solution $\log_a b = \log_b c = \log_c a = k$ (say)

$\Rightarrow b = a^k,\ c = b^k,\ a = c^k$

$\Rightarrow c = (a^k)^k = a^{k^2} = (c^{k^2})^k = c^{k^3}$

$\Rightarrow k^3 = 1 \Rightarrow k = 1. \quad \therefore \quad a = b = c.$

17. Ans. (d)

Solution $\log_{10} x = 2\log_{10} a - 2$

$\Rightarrow \log_{10} x = 2(\log_{10} a - 1)$

$\Rightarrow \log_{10} x = 2\log_{10}\left(\frac{a}{10}\right) \Rightarrow x = \frac{a^2}{100}.$

18. Ans. (b)

Solution $\log_b(c + a) + \log_b(c - a)$
$= \log_b(c^2 - a^2) = \log_b b^2 = 2.$

19. Ans. (a)

Solution $x = (875)^{10} = (87.5 \times 10)^{10}$

$\therefore \quad \log_{10} x = 10(\log_{10} 87.5 + 1)$

$= 10(1.9421 + 1)$

$= 10(2.9421) = 29.421.$

$\therefore \quad x = \text{Antilog } (29.421).$

$\therefore$ No. of digits in $x = 30.$

20. Ans. (b)

Solution $x = (0.0432)^{10} = \left(\frac{432}{10000}\right)^{10} = \left(\frac{3^3.2^4}{10^4}\right)^{10}$

$\therefore \quad \log_{10} x = 10(3\log_{10} 3 + 4\log_{10} 2 - 4)$

$= 10(1.4313 + 1.2040 - 4)$

$= 10(-1.3647) = -13.647$

$= \overline{14}.353$

$\therefore \quad x = \text{Antilog } (\overline{14}.053)$

$\therefore$ No. of zeros between the decimal and the first significant figure = 13.

21. Ans. (c)

Solution $(4.2)^x = 100 \Rightarrow (42)^x = 10^{2+x}$

$\Rightarrow 42 = 10^{\frac{2}{x}+1}$ ...(1)

$\left(\frac{42}{100}\right)^y = 100 \Rightarrow (42)^y = 10^{2+2y}$

$\Rightarrow 42 = 10^{\frac{2}{y}+2}$ ...(2)

From (1) and (2), $\frac{2}{x} - \frac{2}{y} = 1 \Rightarrow \frac{1}{x} - \frac{1}{y} = \frac{1}{2}.$

22. Ans. (c)

Solution $\frac{\log_9 11}{\log_5 13} - \frac{\log_3 11}{\log_{\sqrt{5}} 13} = \frac{\log_3 11}{2.\log_5 13} - \frac{\log_3 11}{2.\log_5 13}$

$= 0.$

23. Ans. (d)

Solution $\frac{\log x}{2} = \frac{\log y}{3} = \frac{\log z}{5} = k$ (say)

$\Rightarrow \quad \log x = 2k,\ \log y = 3k,\ \log z = 5k$

$\Rightarrow \quad \log yz = 3k + 5k = 8k;\ \log x^4 = 8k$

$\therefore \quad \log yz = \log x^4 \Rightarrow yz = x^4.$

24. Ans. (b)

Solution $4^x + \frac{4^x}{2} = \frac{3^x}{\sqrt{3}} + 3^x.\sqrt{3}$

$\Rightarrow 4^x \cdot \frac{3}{2} = 3^x \cdot \frac{4}{\sqrt{3}} \Rightarrow \left(\frac{4}{3}\right)^x = \frac{8}{3\sqrt{3}}$

$\Rightarrow \left(\frac{4}{3}\right)^x = \left(\frac{4}{3}\right)^{3/2} \Rightarrow x = \frac{3}{2}.$

25. Ans. (c)

Solution $\frac{\log 7^{5/2} + \log 5^{5/2} - \log 2^{5/2}}{\log 17.5}$

$= \frac{5(\log 7 + \log 5 - \log 2)}{2\log\left(\frac{35}{2}\right)} = \frac{5}{2}.$

26. Ans. (b)

Solution $\log_{10}\tan 40° \cdot \log_{10}\tan 41° \ldots \log_{10}\tan 50°$
$= 0$, since $\log_{10}\tan 45 = 0$.

27. Ans. (a)

Solution $\log_8 p = \frac{5}{2} \Rightarrow p = (8)^{5/2} = 2^{\frac{15}{2}} = (2^5)^{3/2}$

$\log_2 q = 5 \Rightarrow q = 2^5.$

$\therefore \quad p = q^{3/2}.$

28. Ans. (b)

Solution $\log_a y = \frac{1}{1-\log_a x}$, $\log_a z = \frac{1}{1-\log_a y}$

$\therefore \quad \log_a z = \frac{1}{1-\left(\frac{1}{1-\log_a x}\right)} = \frac{1-\log_a x}{-\log_a x}$

$\Rightarrow \quad -\log_a z = -1 + \frac{1}{\log_a x}$

$\Rightarrow \quad \frac{1}{\log_a x} = 1 - \log_a z$

$\therefore \quad \log_a x = \frac{1}{1-\log_a z} \Rightarrow x = \frac{1}{a^{1-\log_a z}} = a^k$ (given)

$\therefore \quad k = \frac{1}{1-\log_a z}.$

29. Ans. (b)

Solution $\log_e 2 \cdot 4\log_b 5 = 4 \cdot \log_{10} 2 \cdot \log_e 10 = 4\log_e 2$

$\Rightarrow \log_b 5 = 1 \Rightarrow b = 5.$

30. Ans. (c)

Solution $5^{\sqrt{\log_5 7}} - (7^{\log_7 5})^{\frac{1}{\sqrt{\log_7 5}}}$

$= 5^{\sqrt{\log_5 7}} - \frac{1}{5^{\sqrt{\log_7 5}}}$

$\Rightarrow 5^{\sqrt{\log_5 7}} - 5^{\sqrt{\log_5 7}} = 0.$

31. Ans. (d)

Solution $2^{\log_3 7} - 7^{\log_3 2} = 2^{\log_2 7 \,.\, \log_3 2} - 7^{\log_3 2}$
$= 7^{\log_3 2} - 7^{\log_3 2} = 0.$

32. Ans. (a)

Solution $a + b = \log_{30} 15 = \log_{30}\left(\frac{30}{2}\right) = 1 - \log_{30} 2$

$\Rightarrow \quad \log_{30} 2 = 1 - a - b.$

$\therefore \quad \log_{30} 8 = 3(1 - a - b).$

33. Ans. (b)

Solution $0 < a < 1$, $0 < x < 1$ and $x < a$

$\Rightarrow \quad \log_a x > \log_a a \Rightarrow \log_a x > 1.$

34. Ans. (c)

Solution $\log_5 2 = \frac{p}{q} \Rightarrow 2 = 5^{p/q} = 2^q = 5^p$

$\Rightarrow$ even no. = odd no.,

which is a contradiction.

$\therefore \quad \log_5 2$ is an irrational number.

35. Ans. (b)

Solution $\log_5\frac{6}{5} + \log_5\frac{7}{6} + \log_5\frac{8}{7} + \ldots + \log_5\frac{625}{624}$

$= \log_5\left(\frac{6}{5}\cdot\frac{7}{6}\cdot\frac{8}{7}\ldots\frac{625}{624}\right) = \log_5\left(\frac{625}{5}\right) = 4.$

36. Ans. (b)

Solution $\log_{10}(0.02986) = \log_{10}\left(\frac{2986}{100000}\right)$

$= 3.4751 - 5 = -1.5249$

$= \bar{2}.4751.$

37. Ans. (a)

Solution $2a - 3b = \frac{a}{b} \Rightarrow 2ab - 3b^2 = a$

$\Rightarrow 3b^2 = a(2b - 1)$

$\Rightarrow a = \frac{3b^2}{2b-1}.$

38. Ans. (c)

Solution $(x - y)^2 = 25xy \Rightarrow x^2 + y^2 = 27xy$

$\Rightarrow \frac{x}{y} + \frac{y}{x} = 27.$

39. Ans. (b)

Solution $\frac{\log x}{3} = \frac{\log y}{4} = \frac{\log z}{5} = k$

$\Rightarrow \log x = 3k;\ \log y = 4k;\ \log z = 5k.$

$\Rightarrow \log(zx) = \log z + \log x = 8k = 2\log y$

$\therefore \quad zx = y^2.$

40. Ans. (a)

Solution $3 + \log_5 x = \log_5 y \Rightarrow \log_5(125x) = \log_5 y$

$$\Rightarrow x = \frac{y}{125}.$$

41. Ans. (d)

Solution $\frac{\log_2 a}{3} = \frac{\log_3 b}{3} = \frac{\log_4 c}{4} = k$

$\Rightarrow \quad a = 2^{2k},\ b = 3^{3k},\ c = 4^{4k}$ and

$a^{1/2} \cdot b^{1/3} \cdot c^{1/4} = 2^k \cdot 3^k \cdot 4^k = 24$

$\Rightarrow \quad 24^k = 24^1 \Rightarrow k = 1.$

$\therefore \quad a = 4,\ b = 27,\ c = 256.$

42. Ans. (c)

Solution $\frac{z}{x^3y^4} = 1 \Rightarrow \log_2 z - 3\log_2 x - 4\log_2 y = 0$

$\Rightarrow \log_2 z$

$- \frac{3.3}{5k} \cdot \log_2 z - 4 \cdot \frac{4}{5k} \cdot \log_2 z = 0$

$\Rightarrow 1 - \frac{9}{5k} - \frac{16}{5k} = 0$

$\Rightarrow 5k - 25 = 0 \Rightarrow k = 5.$

43. Ans. (a)

Solution $\frac{3(1+\log_{10} 7)}{2+\log\frac{7}{2}+\log\frac{1}{5}} = \frac{3(1+\log_{10} 7)}{2+\log\left(\frac{7}{10}\right)}$

$$= \frac{3(1+\log_{10} 7)}{1+\log_{10} 7} = 3.$$

44. Ans. (c)

Solution Each ratio $= k \Rightarrow \log x = k(a^2 + ab + b^2)$

$\Rightarrow \quad (a - b)\log x = k(a^3 - b^3)$

$\Rightarrow \quad \log x^{a-b} = k(a^3 - b^3) \Rightarrow x^{a-b} = e^{k(a^3-b^3)}$

Similarly, $y^{b-c} = e^{k(b^3-c^3)},\ z^{c-a} = e^{k(c^3-a^3)}.$

$\therefore \quad x^{a-b} . y^{b-c} . z^{c-a} = e^0 = 1.$

45. Ans. (c)

Solution $3^{x-2} = 5 \Rightarrow 3^x = 45 = \left(\frac{90}{2}\right)$

$\Rightarrow x\log_{10}3 = \log_{10}90 - \log_{10}2$

$= 2\log_{10}3 + 1 - \log_{10}2$

$\Rightarrow x(0.4771) = 1.65317$

$\Rightarrow x = \frac{165317}{47710} = 3\frac{22187}{47710}.$

46. Ans. (d)

Solution $\log(648)^5 = 5\log(81 \times 8) = 20\log3 + 15\log2$

$= 20(0.4771) + 15(0.30103)$

$= 14.05745.$

$\therefore$ Number of digits in $(648)^5$ is 15.

47. Ans. (c)

Solution $\frac{\log x}{1} = \frac{\log y}{2} = \frac{\log z}{5} = k$

$\Rightarrow \quad \log x = k,\ \log y = 2k,\ \log z = 5k.$

$\therefore \quad \log(x^4 . y^3 . z^{-2}) = 4\log x + 3\log y - 2\log z = 0$

$\Rightarrow \quad x^4 . y^3 . z^{-2} = 1.$

48. Ans. (c)

Solution $\frac{\log\sqrt{27} + \log\sqrt{1000} + \log 8}{\log 120}$

$$= \frac{\frac{3}{2}(\log 3 + \log 10 + \log 4)}{\log 3 + \log 10 + \log 4} = \frac{3}{2}.$$

49. Ans. (b)

Solution $y = \frac{10^{\log_{10} x}}{x^2} = \frac{1}{x} = \frac{1}{y^a} = y^{-a} \Rightarrow a = -1.$

50. Ans. (b)

Solution $x = \log_{4/3}(1/2) = -\log_{4/3} 2 < 0$

and, $y = \log_{1/2}(1/3) = \log_2 3 > 0 \Rightarrow y > x.$

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. If $\log 2 = 0.30103$, the number of digits in 2^{64} is
(a) 18 (b) 19
(c) 20 (d) 21

[Based on SI of Police Rec. Exam 1997]

2. If $\log_{10} 2 = 0.301$, then the value of $\log_{10}(50)$ is
(a) 0.699 (b) 1.301
(c) 1.699 (d) 2.301

[Based on SI Rec. (Delhi Police) Exam 1997]

3. Given that $\log_{10} 2 = 0.3010$, the $\log_2 10$ is equal to
(a) 0.3010 (b) 0.6990
(c) $\frac{1000}{301}$ (d) $\frac{699}{301}$

[Based on Assistant's Grade Exam 1997]

4. If $\log 2 = x$, $\log 3 = y$ and $\log 7 = z$, then the value of $\log(4.\sqrt[3]{63})$ is
(a) $-2x + \frac{2}{3}y + \frac{1}{3}z$ (b) $2x + \frac{2}{3}y + \frac{1}{3}z$
(c) $2x + \frac{2}{3}y - \frac{1}{3}z$ (d) $2x - \frac{2}{3}y + \frac{1}{3}z$

[Based on Assistant's Grade Exam 1998]

5. If $\log_{12} 27 = a$, then $\log_6 16$ is
(a) $\frac{4(3-a)}{3+a}$ (b) $\frac{4(3+a)}{3-a}$
(c) $\frac{3+a}{4(3-a)}$ (d) $\frac{3-a}{4(3+a)}$

[Based on Assistant's Grade Exam 1998]

6. If $\log_x 4 = 0.4$, then the value of x is
(a) 4 (b) 16
(c) 1 (d) 32
[Based on Assistant's Grade Exam 1998]

7. If $\log_x y = 100$ and $\log_2 x = 10$, then the value of y is
(a) 2^{10} (b) 2^{1000}
(c) 2^{100} (d) 2^{10000}
[Based on SSC (GL) Prel. Exam 1999]

8. If $\log_{10} 2 = 0.3010$ and $\log_{10} 7 = 0.8451$, then the value of $\log_{10} 2.8$ is
(a) 0.4471 (b) 1.4471
(c) 2.4471 (d) $\bar{1}.4471$
[Based on SSC (GL) Prel. Exam 1999]

9. If $\log(0.57) = \bar{1}.756$, then the value of
$\log 57 + \log(0.57)^3 + \log\sqrt{0.57}$ is
(a) 0.902 (b) 1.902
(c) $\bar{1}.146$ (d) $\bar{2}.146$
[Based on SSC (GL) Prel. Exam 1999]

10. If $\log_{10} 2 = 0.3010$ is given, then $\log_2 10$ is equal to
(a) 0.3010 (b) 0.6990
(c) $\frac{1000}{301}$ (d) $\frac{699}{301}$
[Based on SSC (GL) Prel. Exam 2000]

11. If $\log 3 = 0.477$ and $(1000)^x = 3$, then x equals
(a) 0.159 (b) 10
(c) 0.0477 (d) 0.0159.
[Based on SSC (GL) Prel. Exam 2000]

12. If $\log 2 = 0.3010$, then $\log 5$ equals
(a) 0.3010 (b) 0.699
(c) 0.7525
(d) Given $\log_2$, it is not possible to calculate $\log_5$.
[Based on SSC (GL) Prel. Exam 2000]

13. If $\log 90 = 1.9542$ then $\log 3$ equals
(a) 0.9771 (b) 0.6514
(c) 0.4771 (d) 0.3181
[Based on SSC (GL) Prel. Exam 2000]

14. The number of digits in 8^{10} is (when $\log_2 = 0.30103$)
(a) 19 (b) 10
(c) 17 (d) 16
[Based on RRB, Kolkata Supervisor (P.Way) Exam 2000]

15. If $\log_{10}(x^2 - 6x + 45) = 2$, then the values of x are
(a) 10, 5 (b) 11, –5
(c) 6, 9 (d) 9, –5
[Based on RRB Allahabad ASM Exam 2002]

16. If $\log_{10} 2 = 0.30$, then $\log_2 10$ is
(a) 3.3220 (b) 5
(c) 0.3322 (d) 3.2320
[Based on RRB Allahabd ASM Exam 2002]

EXPLANATORY ANSWERS

1. Ans. (c)

Solution $\log 2^{64} = 64 \log 2 = 64 \times .30103 = 19.26592.$

$\therefore$ Number of digits in $2^{64} = 19 + 1 = 20.$

2. Ans. (c)

Solution $\log_{10} 50 = \log_{10}\left(\frac{50\times 2}{2}\right) = \log_{10}\frac{100}{2}$

$= \log_{10} 100 - \log_{10} 2 = 2 - .301$

$= 1.699.$

3. Ans. (c)

Solution $\log_2 10 = \frac{\log 10}{\log 2} = \frac{1}{\log 2} = \frac{1.0000}{.3010}$

$= \frac{10000}{3010} = \frac{1000}{301}$

4. Ans. (b)

Solution $\log(4 \times \sqrt[3]{63}) = \log(2^2 \times (3 \times 3 \times 7)^{1/3})$

$= \log 2^2 + \log(3 \times 3 \times 7)^{1/3}$

$= 2\log 2 + \frac{1}{3}\log(3^2 \times 7)$

$= 2\log 2 + \frac{1}{3}(\log 3^2 + \log 7)$

$= 2\log 2 + \frac{2}{3}\log 3 + \frac{1}{3}\log 7$

$= 2x + \frac{2}{3}y + \frac{1}{3}z.$

5. Ans. (a)

Solution $\log_{12} 27 = a \Rightarrow \frac{\log 27}{\log 12} = a$

$\Rightarrow a \log 12 = \log 3^3$

$\Rightarrow a \log(4 \times 3) = 3 \log 3$

$\Rightarrow a(\log 4 + \log 3) = 3 \log 3$

$\Rightarrow a \log 4 + a \log 3 = 3 \log 3$

$\Rightarrow a \log 2^2 = (3 - a) \log 3$

$\Rightarrow 2a \log 2 = (3 - a) \log 3$

$\Rightarrow \frac{\log 2}{\log 3} = \frac{3-a}{2a}.$

Now, $\log_6 16 = \frac{\log 16}{\log 6} = \frac{\log 2^4}{\log(2\times 3)}$

$$= \frac{4\log 2}{\log 2+\log 3} = \frac{4\dfrac{\log 2}{\log 3}}{\dfrac{\log 2}{\log 3}+1}$$

$$= \frac{4\left(\dfrac{3-a}{2a}\right)}{\dfrac{3-a}{2a}+1} = \frac{4(3-a)}{3+a}.$$

6. Ans. (d)

Solution $\dfrac{\log 4}{\log x} = \dfrac{2}{5} \Rightarrow \dfrac{2\log 2}{\log x} = \dfrac{2}{5}$

$\Rightarrow \log x = 5\log 2 = \log 2^5 = \log 32$

$\Rightarrow x = 32.$

7. Ans. (b)

Solution $\log_x y = 100,\ \log_2 x = 10$

$\Rightarrow \dfrac{\log y}{\log x} = 100,\ \dfrac{\log x}{\log 2} = 10$

$\Rightarrow \dfrac{\log y}{\log 2} = 1000 \Rightarrow \log_2 y = 1000$

$\Rightarrow y = 2^{1000}.$

8. Ans. (a)

Solution $\log_{10} 2.8 = \log_{10}\dfrac{28}{10} = \log 28 - \log 10$

$= \log(7 \times 4) - \log 10$

$= \log 7 + 2\log 2 - \log 10$

$= 0.8451 + 2 \times 0.3010 - 1$

$= 0.8451 + 0.6020 - 1 = 0.4471.$

9. Ans. (a)

Solution $\log\left(\dfrac{57\times 100}{100}\right) + 3\log(0.57) + \dfrac{1}{2}\log(0.57)$

$= \log(0.57) + \log 10^2 + 3\log(0.57) + \dfrac{1}{2}\log(0.57)$

$= \left(1+3+\dfrac{1}{2}\right)\log(0.57) + 2 = (4.5\times \bar{1}.756) + 2$

$= 4.5 \times (-1 + .756) + 2 = 0.902.$

10. Ans. (c)

Solution $\log_2 10 = \dfrac{\log 10}{\log 2} = \dfrac{1}{.3010} = \dfrac{1000}{301}.$

11. Ans (a)

Solution $x\log 1000 = \log 3 \Rightarrow 3x = \log 3$

$\Rightarrow x = \dfrac{\log 3}{3} = \dfrac{.477}{3} = .159.$

12. Ans. (b)

Solution $\log 5 = \log\dfrac{10}{2} = \log 10 - \log 2$

$= 1 - 0.3010 = 0.\ 6990.$

13. Ans. (c)

Solution $\log 90 = 1.\ 9542$

$\Rightarrow \log(3^2 \times 10) = 1.9542$

$\Rightarrow 2\log 3 + \log 10 = 1.9542$

$\Rightarrow \log 3 = \dfrac{.9542}{2} = .4771.$

14. Ans. (b)

Solution $8^{10} = (2^3)^{10} = 2^{30}.$

Let $y = 2^{30} \Rightarrow \log y = \log(2)^{30} = 30\log 2$

$= 30 \times 0.30103 = 9.0309$

$\therefore\ y = (10)^{9.0309}$, which contains 10 digits.

15. Ans. (b)

Solution $\log_{10}(x^2 - 6x + 45) = 2$

$\Rightarrow x^2 - 6x + 45 = 10^2$

$\Rightarrow x^2 - 6x + 45 - 100 = 0$

$\Rightarrow x^2 - 6x - 55 = 0$

$\Rightarrow (x+5)(x-11) = 0$

$\Rightarrow x = 11$ or $-5.$

16. Ans. (a)

Solution $\log_{10} 2 = 0.3010$

$\therefore\ \log_2 10 = \dfrac{1}{\log_{10} 2} = \dfrac{1}{0.3010} = 3.3220$

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. If $10^x = 1.73$ and $\log_{10} 1730 = 3.2380$, then $x =$
(a) 1.2380 (b) 0.2380
(c) 2.380 (d) 2.2380

2. The value of $3^{-\frac{1}{2}\log_3 9}$ is
(a) -1 (b) $-\dfrac{1}{3}$
(c) $\dfrac{1}{3}$ (d) $-\dfrac{3}{2}$

3. If $\log_4 x + \log_2 x = 6$, then the value of x is
(a) 2 (b) 4
(c) 8 (d) 16

4. The value of $\left(\frac{1}{\log_{(q/p)} x}+\frac{1}{\log_{(q/r)} x}+\frac{1}{\log_{(r/p)} x}\right)$ is
(a) 3 (b) 2
(c) 1 (d) 0

5. $\log_5 5 \cdot \log_4 9 \cdot \log_3 2 =$
(a) 2 (b) 1
(c) 5 (d) 3/2

6. If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, then the value of $a^a b^b c^a$ is
(a) abc (b) $\frac{1}{abc}$
(c) 1 (d) $\log (abc)$

7. Given $\log_2 = 0.30103$, the number of digits in 5^{20} is
(a) 14 (b) 16
(c) 18 (d) 25

8. The value of x satisfying $\log_{32} x = 0.8$ is
(a) 25.6 (b) 16
(c) 10 (d) 12.8

9. If $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, then the value of $\log_{100} (.72)$ is
(a) $\bar{1}.9286$ (b) $\bar{1}.8573$
(c) 1.8572 (d) .9286

10. If $\log (x + y) = \log x + \log y$ and $x = 1.15683$, then the value of y is
(a) $\bar{7}.736$ (b) 7.377
(c) 3.456 (d) 1.234

Test I

[Time: 10 min]

11. If $\log_{10} 2 = 0.3010$, the value of $\log_{10} 80$ is
(a) 1.9030 (b) 1.6020
(c) 3.9030 (d) None of these

12. $\log\left(\frac{a^2}{bc}\right)+\log\left(\frac{b^2}{ac}\right)+\log\left(\frac{c^2}{ab}\right)$ is
(a) 1 (b) 0
(c) 2 (d) abc

13. $\frac{1}{\log_a bc+1}+\frac{1}{\log_b ca+1}+\frac{1}{\log_c ab+1} =$
(a) 1 (b) 2
(c) 3 (d) $\frac{3}{2}$

14. If $\log 2 = 0.3010$ and $5^x = 400$, then $x =$
(a) 2.40 (b) 3.72
(c) 4.36 (d) 1

15. $\log_{10} 10 + \log_{10} 100 + \log_{10} 1000 + \log_{10} 10000 + \log_{10} 100000$ is
(a) 15 (b) $\log_{10} 11111$
(c) $\log_{10} 1111$ (d) $14\log_{10} 100$

16. The value of $\log_2 (\log_2 \log_2 \log_2 (65536)$ is
(a) 0 (b) 1
(c) 2 (d) 4

17. If $\log_2 (\log_3(\log_2 x)) = 1$, then $x =$
(a) 512 (b) 128
(c) 12 (d) 0

18. If a, b, c are three consecutive integers, then $\log (ac + 1)$ has the value
(a) $(\log b)^2$ (b) $\log b$
(c) $2 \log b$ (d) $\log 2b$

19. If $\log\frac{m}{n}+\log\frac{n}{m} = \log(m + n)$, then
(a) $m + n = 1$ (b) $\frac{m}{n} = 1$
(c) $m - n = 1$ (d) $m^2 - n^2 = 1$

20. If $\log_{10} x = 1.9665$, then $\log_{10} (1000x)$ is
(a) 19.675 (b) 4.9675
(c) 1.9675×3 (d) 1967.5

Answers

1. (d)	**2.** (c)	**3.** (d)	**4.** (d)	**5.** (b)	**6.** (c)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (b)
11. (a)	**12.** (b)	**13.** (a)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (a)	**18.** (c)	**19.** (a)	**20.** (b)

30

DATA INTERPRETATION

INTRODUCTION

Data interpretation deals with careful reading, understanding, organising and interpreting the data provided so as to derive meaningful conclusions. It is an important section today in all competitive examinations. The data representation can be broadly classified as tables, graphs and charts.

Types of Data Representation

1. Tables It is the systematic and scientific presentation of numerical data. It helps the person to make comparisons and draw quick conclusions. Tabular presentation makes complicated information easier to understand. In a table, data is arranged systematically in columns and rows.

Illustration 1 In the Table below are given daily wages of male and female workers (in rupees) of two different factories:

YEAR	*FACTORY A*		*FACTORY B*	
	Male	*Female*	*Male*	*Female*
1997	120	120	240	200
1998	100	108	170	160
1999	80	110	120	130
2000	90	108	140	160
2001	170	130	240	260
2002	120	170	250	270

2. Line Graph A line graph depicts the variation of a quantity with respect to the two parameters calibrated on the x and y axes, respectively. In most of the cases the quantity is measured as a function of time, that is, the variation in the quantity as time changes.

In the line graph:

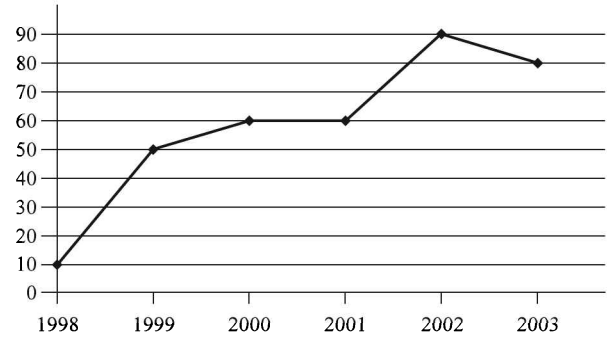
(a) x-axis represents the time parameter (may be year or month) and y-axis represents any other variable parameter which have different values with respect to time.

(b) the line going up indicates increase in the quantity with time.

(c) the line going down indicates decrease in the quantity with time.

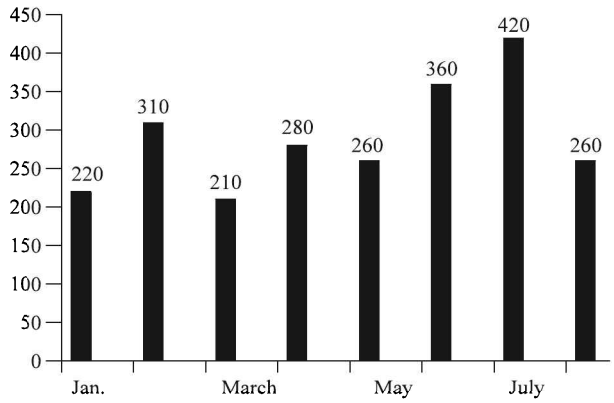
(d) a horizontal line indicates no change in the quantity over that period.

Illustration 2 The following line graph shows the number of children suffering from liver disorders in a State (in thousands).



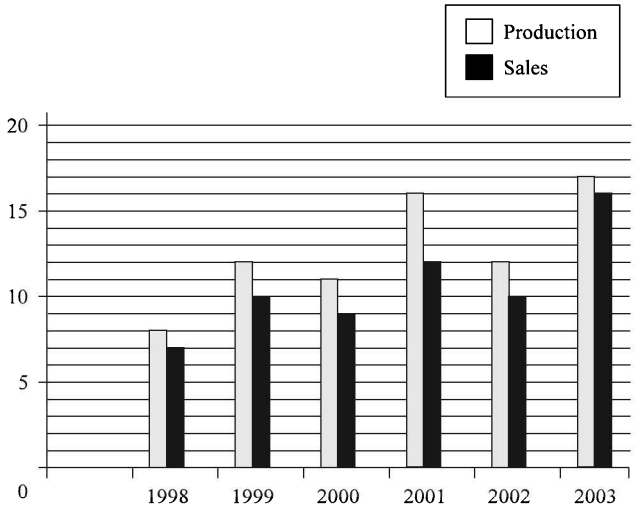
3. Bar Graph A bar is a thick line whose width is shown merely for attention. A bar graph consists of bars. The height of the bar is a measure of the quantity that it represents. Therefore, quantities can be compared by the height of bars in the graph. Bars may be horizontal or vertical. They may be placed adjacent to each other or may be separated from each other by spaces depending upon the problem.

Illustration 3 The graph given below shows the share price (in rupees) of a company during first eight months of a year.



SHARE VALUE IN RUPEES

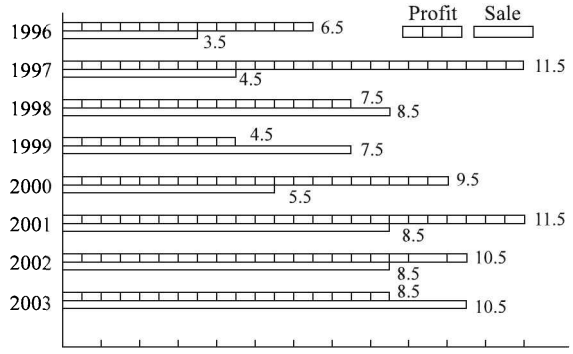
Illustration 4 The graph given below shows the yearly production and sale of a company in lakh tons.



Yearly Production and Sales (in lakh tons)

Illustration 5 The graph given below gives information about the sale and profit details of a departmental store during the years from 1996 to 2003.

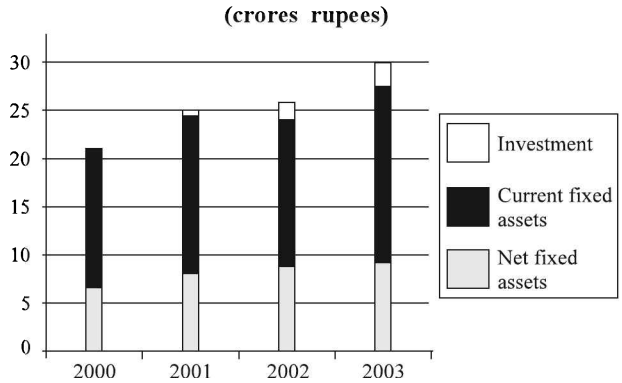
Sale and Profit details (1996–2003)



Sale: *(Rs. in lakhs)*
Profit: *Rs. in Ten thousands)*

4. Cumulative Bar Graph In a cumulative bar graph, the length of the bar is divided proportionately among various quantities represented in the graph. Thus, it may be conveniently used for making comparisons.

Illustration 6



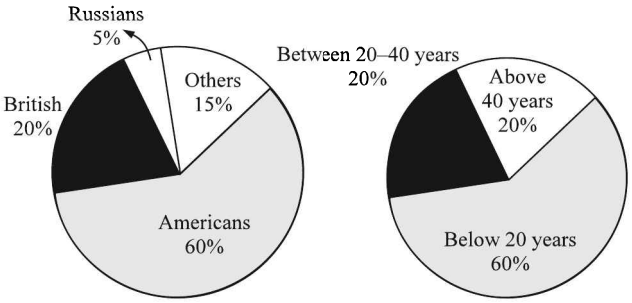
5. Pie Charts In a pie chart, the given data is distributed over a circle. Each part of the data makes a certain central angle.

For example, if from all the questions asked in a Bank PO exam, 25% are on data interpretation, then central angle made by this item

$$= \left(\frac{25}{100} \times 360\right) = 90^\circ.$$

Pie charts are useful for representing percentages or proportions of various elements with respect to the total quantity. They also represent shares of various parts of a particular quantity.

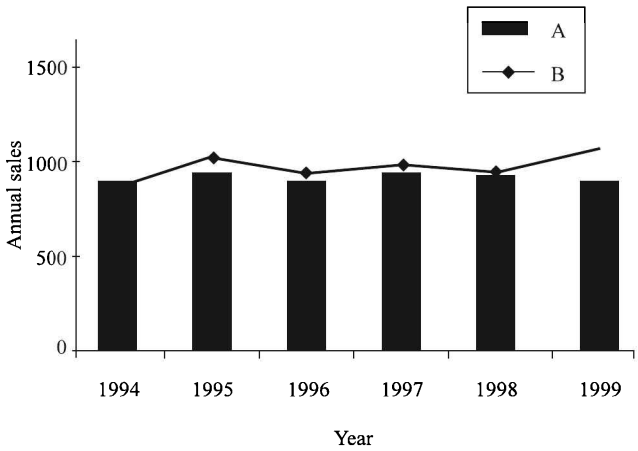
Illustration 7 The following pie charts describe the characteristics of foreign tourists visiting India in a particular year.



6. Combination graph Quite often, data interpretation question is based not on a single graph but on a combination of two or more graphs. It can be line and bar, line and pie, bar and pie, cumulative and pie charts.

Illustration 8 The following graph shows annual sales of companies *A* and *B* (Rs. in lakh) for 1994–1999.

Annual sales of companies
A and B (Rs. in lakh) for 1994-99



Line and Bar chart

Some Useful Instructions

1. Do not waste time memorising a Table. You can refer to it as many times as you want during the examination.

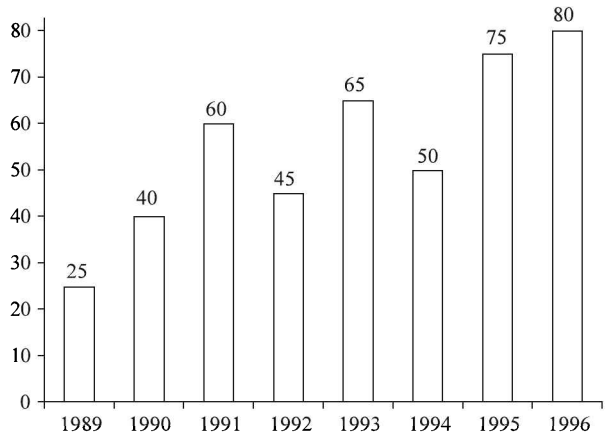
2. Your answer should be accurate and based on precise data given in the Table.
3. Do not include your own information in answering questions, however, accurate you may be. Stick to the data presented to you.
4. Be careful about minor details. Students often miss them and give wrong answers.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Directions: Study the following graph carefully and answer the questions given below it.

Production of foodgrains by a State over the years (1000 tons)



(i) The average production of 1990 and 1991 was exactly equal to the average production of which of the following pairs of years?
(a) 1991 and 1992 (b) 1992 and 1994
(c) 1993 and 1994 (c) 1994 and 1995
(e) None of these.

(ii) What was the difference in the production of foodgrains between 1991 and 1994?
(a) 10000 tons (b) 15000 tons
(c) 500 tons (d) 5000 tons
(e) None of these.

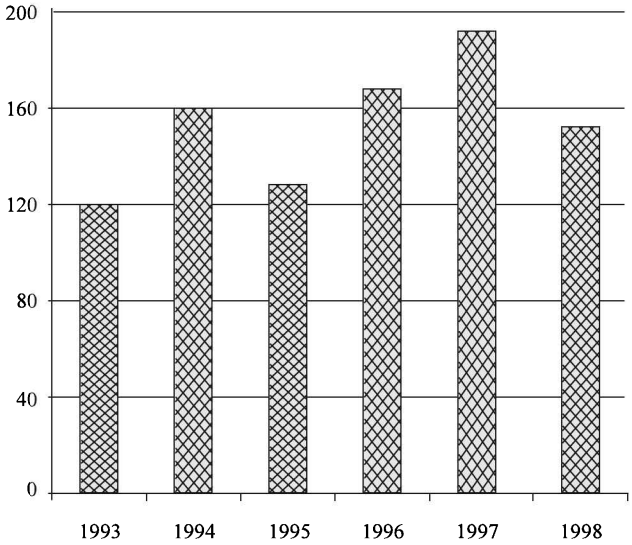
(iii) In which of the following years was the percentage increase in production from the previous year the maximum among the given years?
(a) 1991 (b) 1993
(c) 1995 (d) 1990
(e) None of these.

(iv) In how many of the given years was the production of foodgrains more than average production of the given years?
(a) 2 (b) 3
(c) 4 (d) 1
(e) None of these.

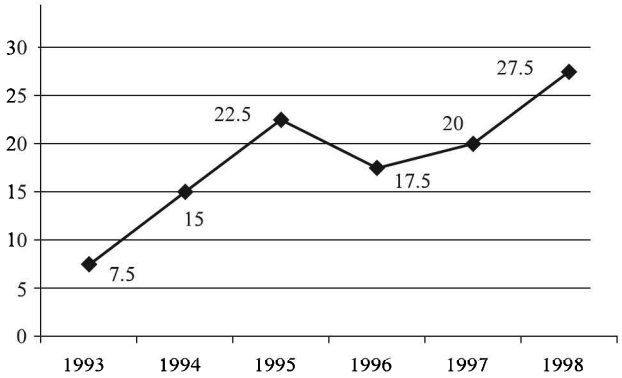
(v) What was the percentage drop in the production of foodgrains from 1991 to 1992?
(a) 15 (b) 20
(c) 25 (d) 30
(e) None of these.

2. Directions: Study the following graphs carefully and answer the questions given below:

INCOME OF A COMPANY (IN Rs. LAKHS)



PERCENTAGE PROFIT OVER THE YEARS



(i) In which of the following years was the amount of profit the maximum?
(a) 1997 (b) 1994
(c) 1993 (d) 1995
(d) None of these.

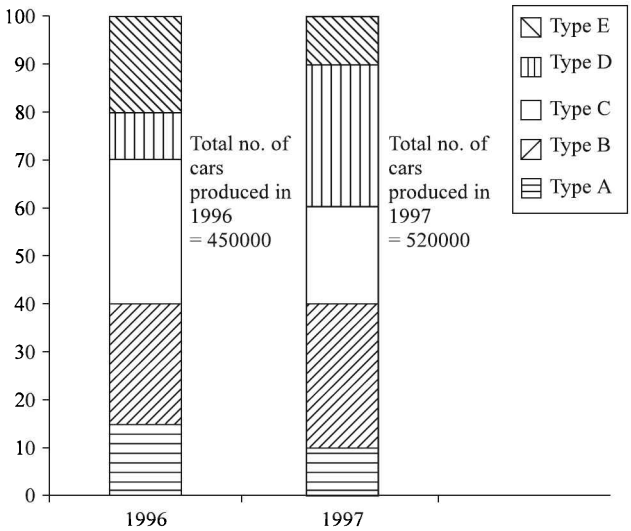
(ii) **Approximately** what was the average expenditure of the given years?
(a) Rs. 110 lakh (b) Rs. 130 lakh
(c) Rs. 120 lakh (d) Rs. 140 lakh
(e) Data inadequate.

(iii) In which of the following years was the increase/decrease in per cent profit from the previous year the minimum?
(a) 1994 (b) 1996
(c) 1997 (d) 1995
(e) None of these.

(iv) **Approximately** what was the expenditure in 1994?
(a) Rs. 120 lakh (b) Rs. 160 lakh
(c) Rs. 140 lakh (d) Rs. 180 lakh
(e) Data inadequate.

(v) If the profit percentage in 1997 was 25, what would have been the expenditure in that year?
(a) Rs. 130 lakh (b) Rs. 148 lakh
(c) Rs. 120 lakh (d) Rs. 152 lakh
(e) None of these.

3. Directions: Study the following graph carefully and then answer the questions based on it. The percentage of five different types of cars produced by a company during two years is given below.



(i) What was the difference in the production of *C* type cars between 1996 and 1997?
(a) 5000 (b) 7500
(c) 10000 (d) 2500
(e) None of these.

(ii) If 85% of *E* type cars produced during 1996 and 1997 are being sold by the company, then how many *E* type cars are left unsold by the company?
(a) 142800 (b) 21825
(c) 29100 (d) 25200
(e) None of these.

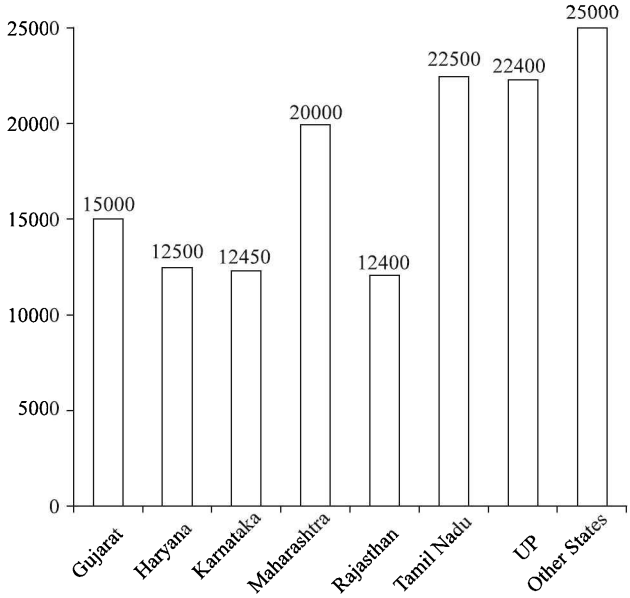
(iii) If the number of *A* type cars manufactured in 1997 was the same as that of 1996, what would have been its **approximate** percentage share in the total production of 1997?
(a) 11 (b) 13
(c) 15 (d) 9
(e) None of these.

(iv) In the case of which of the following types of cars was the percentage increase from 1996 to 1997 the maximum?
(a) A (b) E
(c) D (d) B
(e) C

(v) If the percentage production of *B* type cars in 1997 was the same as that of 1996, what would have been the number of cars produced in 1997?
(a) 112500 (b) 120000
(c) 130000 (d) Data inadequate.
(e) None of these.

4. Directions: Study the following graph carefully and answer the questions given below it.

Rose production



(i) Which of the following State(s) contribute(s) less than 10 per cent in the total rose production?
(a) Only Rajasthan
(b) Rajasthan, Karnataka
(c) Rajasthan, Karnataka, Haryana
(d) Rajasthan, Karnataka, Haryana and Gujarat
(e) None of these.

(ii) By what percentage rose production of other States is more than that of the Maharashtra?
(a) 25 (b) 30
(c) 20 (d) 15
(e) None of these.

(iii) What is the **approximate** average production of roses (in thousands) across all the states?
(a) 21 (b) 20

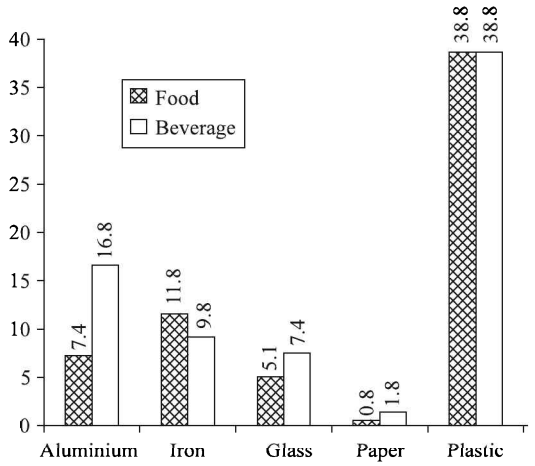
(c) 19 (d) 18
(e) 17

(iv) Approximately what percentage of the total rose production is shared by the other States?
(a) 10 (b) 20
(c) 30 (d) 40
(e) 35

(v) If total percentage contribution of the States having production of roses below twenty thousand is considered, which of the following statements is true?
(a) It is little above 40%
(b) It is exactly 35%
(c) It is below 35%
(d) It is little below 30%
(e) None of these.

5. Directions: Study the following graph carefully and answer the questions given below it.

Packaging Materials Used (In tonnes)



(i) What per cent of the total glass packaging material was used for packaging food items?
(a) 40.8 (b) 41.8
(c) 40.7 (d) 41.0
(e) None of these.

(ii) **Approximately** how much per cent more plastic was used than iron for packaging food items?
(a) 32 (b) 320
(c) 33 (d) 325
(e) 225.

(iii) In the case of which one of the following packaging materials used for packing food items and beverages respectively the ratio is 4 : 9?
(a) Glass (b) Paper
(c) Aluminium (d) Iron
(e) None of these.

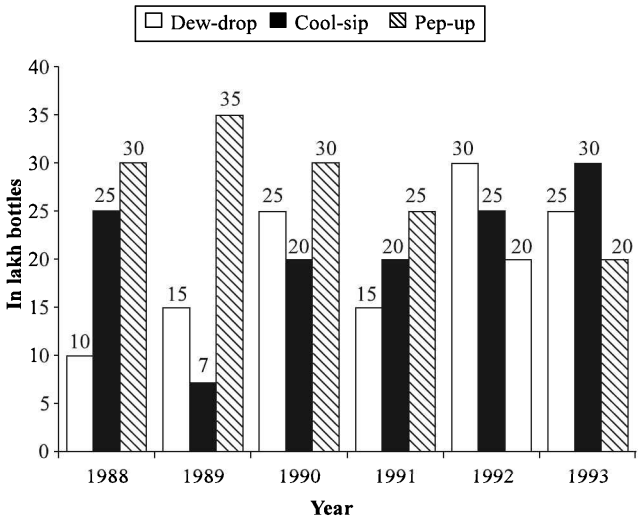
(iv) What is the ratio between the glass and aluminium packaging used for packing beverages?
(a) 17 : 56 (b) 56 : 17
(c) 84 : 37 (d) 37 : 84
(e) None of these.

(v) **Approximately** what per cent of all the packaging materials used for packing food items was contributed by plastic?
(a) 60 (b) 65
(c) 70 (d) 55
(e) 50.

(vi) **Approximately** what per cent of all the packaging materials used for packing food items and beverages was contributed by plastic and aluminium together?
(a) 60 (b) 70
(c) 80 (d) 65
(e) 75.

(vii) What per cent of all the packaging materials used for packing beverages was contributed by paper (Find the answer up to two decimal places).
(a) 2.42 (b) 3.41
(c) 2.41 (d) 3.42
(e) None of these.

6. Directions: Study the following graph carefully and answer the questions given below.



(i) In which year was the sale of 'Pep-up' the maximum?
(a) 1990 (b) 1991
(c) 1992 (d) 1993
(e) None of these.

(ii) In the case of which soft drink was the average annual sale maximum in the given period?
(a) Pep-up only
(b) Cool-sip only
(c) Dew-drop only
(d) Cool-sip and Dew-drop
(e) Pep-up and Dew-drop.

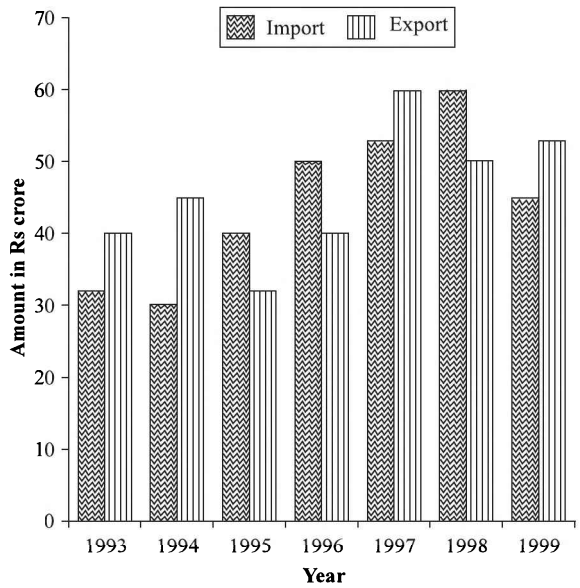
(iii) In the case of Cool-sip drink, what was the approximate per cent increase in sale in 1992 over its sale in 1991?
(a) Less than 20 (b) 20–25
(c) 25 (d) 31–35
(e) 36–40.

(iv) In the year 1990, what was the difference between the number of 'Pep-up' and 'Cool-sip' bottles sold?
(a) 5000000 (b) 500000
(c) 50000 (d) 5000
(e) 1000000.

(v) What was the approximate per cent drop in sale of Pep-up in 1990 over its sale in 1989?
(a) 5 (b) 12
(c) 14 (c) 20
(e) 28.

7. Directions: Study the following graph carefully to answer the questions given below it.

Import and Export of spare parts by an automobile company over the given years



(i) During which year the percentage rise/fall in imports from the previous year is the lowest?
(a) 1994 (b) 1998
(c) 1997 (d) 1995
(e) None of these.

(ii) What is the ratio of total imports to total exports for all the given years together?
(a) 31 : 35 (b) 35 : 31
(c) 65 : 63 (d) 63 : 65
(e) None of these.

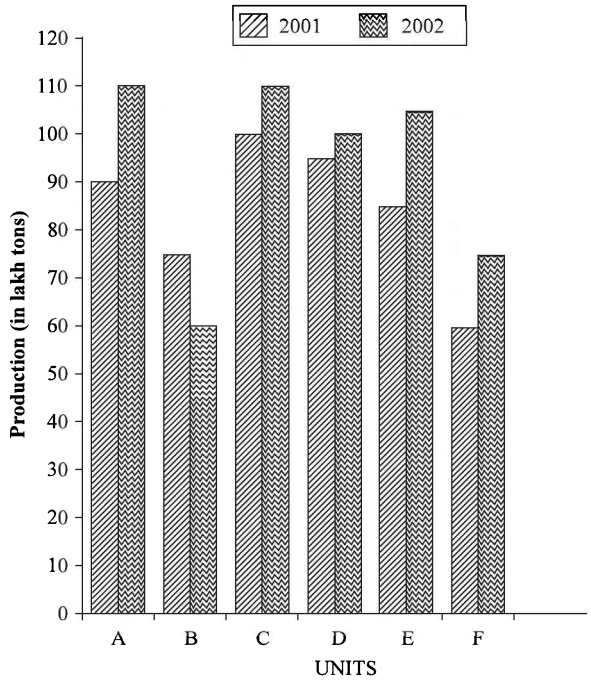
(iii) In which of the following pairs of years the total import is equal to total export in the same pair of years?
(a) 1996–1997 (b) 1993–1998
(c) 1998–1999 (d) 1995–1996
(e) None of these.

(iv) The total exports in the years 1995, 1996 and 1999 together are what per cent of the total import during the same period? (up to two decimal places)
(a) 107.41 (b) 107.14
(c) 93.33 (d) 93.67
(d) None of these.

(v) Which of the following pairs of years and the per cent increase in the export over the previous year is correctly matched?
(a) 1996–14.29 (b) 1997–10
(c) 1995–33.33 (d) 1994–11.11
(e) None of these.

8. Directions: Study the following graphs carefully to answer these questions.

Production (in lakh tons) of six units of a company 2001 and 2002



(i) What is the average production of all the units (in lakh tons) for the year 2002?
(a) 89 (b) 92
(c) 87 (d) 95
(e) None of these.

(ii) Average production of three units *A*, *B* and *C* in 2001 is what per cent of the average production of units *D*, *E* and *F* in 2002? (rounded off to two digits after decimal)
(a) 109.43 (b) 90.37
(c) 91.38 (d) 106.43
(e) None of these.

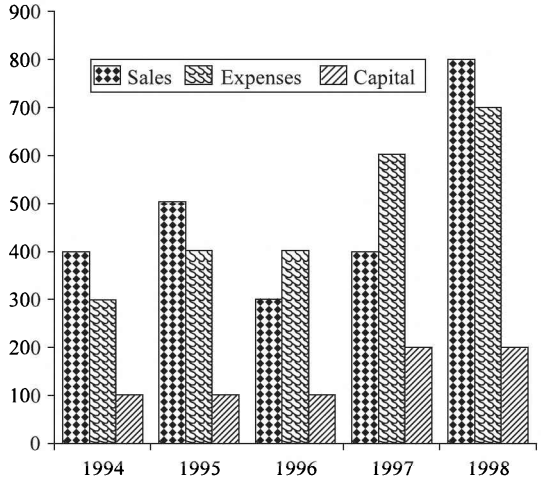
(iii) What is the ratio of total production for two years together for unit *B* to that for *C*?
(a) 17 : 13 (b) 13 :17
(c) 11 : 13 (d) 19 : 13
(e) None of these.

(iv) Total production for two years together by unit *F* is what per cent of the total production of the two years together by unit *D*? (rounded off to two digits after decimal)

(a) 79.49 (b) 78.49
(c) 78.47 (d) 79.29
(e) None of these.

(v) What is the total production of units C, D, and E together for both the years? (in lakh tons)
(a) 495 (b) 595
(c) 545 (d) 515
(e) None of these.

9. The following graph gives Sales, Expense and Capital of a company for a period of five years–1994 to 1998. Read the graph and answer questions 140 to 144.



(i) What has been the simple average growth rate per annum of expense between 1994 and 1998?
(a) 25% (b) $33\frac{1}{3}\%$
(c) 40% (d) 130%

(ii) In which year was the sales-to-expense ratio the lowest?
(a) 1994 (b) 1996
(c) 1997 (d) 1998

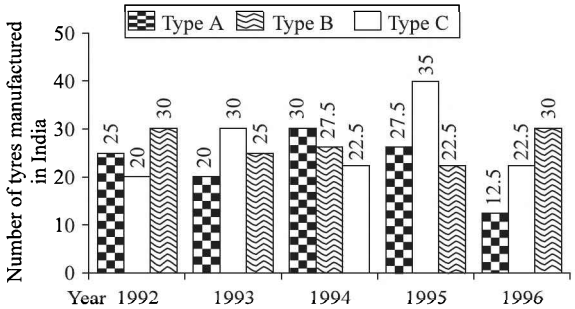
(iii) What was the average per annum increase in sales (in Rs cr) from 1994 to 1998?
(a) 50 (b) 60
(c) 80 (d) 100

(iv) In which year was the ratio of profits to capital the highest?
(a) 1998 (b) 1995
(c) 1996 (d) 1997

(v) In which year was the ratio of sales to capital the lowest?
(a) 1998 (b) 1997
(c) 1996 (d) 1995

10. Directions: Study the following graph carefully and answer the questions given below.

Production of three types of tyres by a company over the year (in lakh)



(i) What was the percentage drop in the number of C type tyres manufactured from 1993 to 1994?
(a) 25 (b) 10
(c) 15 (d) 25
(e) None of these

(ii) What was the difference between the number of B type tyres manufactured in 1994 and 1995?
(a) 100000 (b) 2000000
(c) 1000000 (d) 1500000
(e) None of these

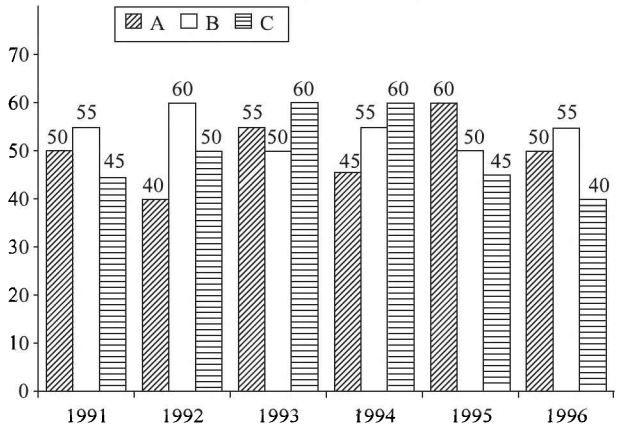
(iii) The total number of all the three types of tyres manufactured was the least in which of the following years?
(a) 1995 (b) 1996
(c) 1992 (d) 1994
(d) 1993

(iv) In which of the following years was the percentage production of B type to C type tyres the maximum?
(a) 1994 (b) 1992
(c) 1996 (d) 1993
(e) 1995

(v) The total production of C type tyres in 1992 and 1993 together was what percentage of production of B type tyres in 1994?
(a) 50 (b) 100
(c) 150 (d) 200
(e) None of these.

11. Directions: Study the following graph carefully to answer the questions given below it.

Production of paper (in lakh tonnes) by 3 different companies A, B, and C over the years



(i) What is the difference between the production of company *C* in 1991 and the production of company *A* in 1996?
(a) 50000 tonnes (b) 50000000 tonnes
(c) 5000000 tonnes (d) 500000 tonnes
(e) None of these.

(ii) What is the percentage increase in production of company *A* from 1992 to 1993?
(a) 37.5 (b) 38.25
(c) 35 (d) 36
(e) None of these.

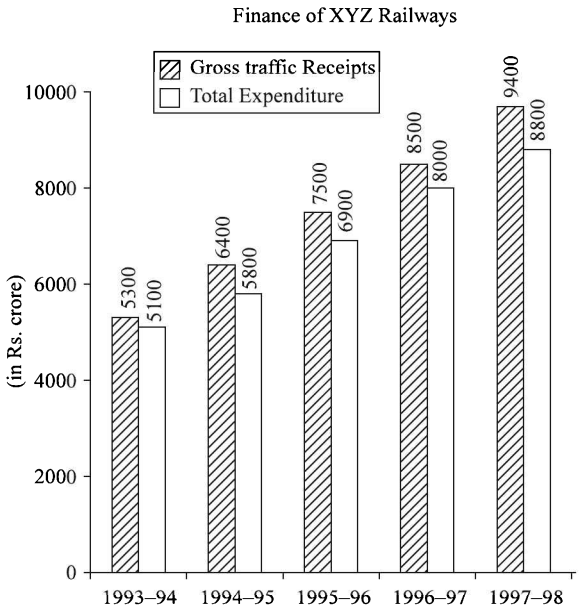
(iii) For which of the following years the percentage of rise/fall in production from the previous year the maximum for company *B*?
(a) 1992 (b) 1993
(c) 1994 (d) 1995
(e) 1996.

(iv) The total production of company *C* in 1993 and 1994 is what percentage of the total production of company *A* in 1991 and 1992?
(a) 95 (b) 90
(c) 110 (d) 115
(e) None of these.

(v) What is the difference between the average production per year of the company with highest average production and that of the company with lowest average production in lakh tonnes?
(a) 3.17 (b) 4.33
(c) 4.17 (d) 3.33
(e) None of these.

12. Directions: These questions are based on the following bar graph. Read the graph and answer the questions.

Finances of XYZ Railways



(i) What is the percentage increase in the gross traffic receipts in 1995–96 as compared to 1993–94?
(a) 33.9% (b) 41.5%
(c) 20.7% (d) 17%

(ii) If profit = gross traffic receipts – total expenditure, then in 1996–97, what percentage of gross traffic receipts is the profit made?
(a) 5.9% (b) 6.4%
(c) 7.2% (d) 8%

(iii) In which year was the profit as a percentage of gross traffic receipts the highest?
(a) 1997–98 (b) 1996–97
(c) 1995–96 (d) 1994–95

(iv) In order to make a profit of 10%, what should have been the gross traffic receipts (in Rs. crore) in 1994–95, total expenditure remaining the same?
(a) 5667 (b) 5876
(c) 6444 (d) 7667

(v) By what amount (in Rs. crore) has the expenditure increased over the period 1993–94 to 1997–98?
(a) 4100 (b) 3900
(c) 3850 (d) 3700

13. Directions: Study the following table carefully and answer the question given below it.

Number of candidates from different locations appeared and passed in a competitive examination over the years

Year	*Rural*		*Semi-urban*		*State capitals*		*Metropolises*	
	App.	*Passed*	*App.*	*Passed*	*App.*	*Passed*	*App.*	*Passed*
1990	1652	208	7894	2513	5054	1468	9538	3214
1991	1839	317	8562	2933	7164	3248	10158	4018
1992	2153	932	8139	2468	8258	3159	9695	3038
1993	5032	1798	9432	3528	8529	3628	11247	5158
1994	4915	1668	9784	4015	9015	4311	12518	6328
1995	5628	2392	9969	4263	1725	4526	13624	6419

(i) For the candidates from which of the following locations was there continuous increase both in appeared and passed?
(a) Semi-urban
(b) State capital
(c) State capital and Rural
(d) Metropolises
(e) None of these.

(ii) In which of the following years was the percentage passed to appeared candidates from semi-urban area the least?
(a) 1991 (b) 1993
(c) 1990 (d) 1992
(e) None of these.

(iii) What approximate value was the percentage drop in the number of semi-urban candidates appeared from 1991 to 1992?
(a) 5 (b) 10

(c) 15 (d) 8
(e) 12.

(iv) In 1993 percentage of candidates passed to appeared was **approximately** 35 from which location?
(a) Rural
(b) Rural and Metropolises
(c) Semi-urban and Metropolises
(d) Rural and Semi-urban
(e) None of these.

(v) The total number of candidates passed from Rural in 1993 and Semi-urban in 1990 was exactly equal to the total number of candidates passed from State capital in which of the following years?
(a) 1990 (b) 1993
(c) 1994 (d) 1992
(e) None of these.

14. Directions: Study the following table carefully and answer the questions given below it.

Fare in rupees for three different types of vehicles			
Fare for distance up to	*Vehicle*		
	Type *A*	Type *B*	Type *C*
2 km	Rs. 5.00	Rs. 7.50	Rs. 10.00
4 km	Rs. 900	Rs. 14.50	Rs. 19.00
7 km	Rs. 13.50	Rs. 24.25	Rs. 31.00
10 km	Rs. 17.25	Rs. 33.25	Rs. 41.50
15 km	Rs. 22.25	Rs. 45.75	Rs. 56.50
20 km	Rs. 26.00	Rs. 55.75	Rs. 69.00
Note: Fare per km for intermittent distance is the same.			

(i) Shiv Kumar has to travel a distance of 15 km in all. He decides to travel equal distance by each of the three types of vehicles. How much money is to be spent as fare?
(a) Rs. 51.75 (b) Rs. 47.50
(c) Rs. 47.25 (d) Rs. 51.25
(e) None of these.

(ii) Ajit Singh wants to travel a distance of 15 km. He starts his journey by Type *A* vehicle. After travelling 6 km, he changes the vehicle to Type *B* for the remaining distance. How much money will he be spending in all?
(a) Rs. 42.25 (b) Rs. 36.75
(c) Rs. 40.25 (d) Rs. 42.75
(e) None of these.

(iii) Mr. *X* wants to travel a distance of 8 km by Type *A* vehicle. How much more money will be required to be spent if he decides to travel by Type *B* vehicle instead of Type *A*?
(a) Rs. 16 (b) Rs. 12.50
(c) Rs. 14 (d) Rs. 13.50
(e) None of these.

(iv) Rita hired a Type *B* vehicle for travelling a distance of 18 km. After travelling 5 km, she changed the vehicle to type A. Again, after travelling 9 km by Type A vehicle, she changed the vehicle to Type *C* and completed her journey. How much money did she spend in all?
(a) Rs. 50 (b) Rs. 45.50
(c) Rs. 55 (d) Rs. 50.50
(e) None of these.

(v) Fare for 14th km by Type *C* vehicle is equal to the fare for which of the following?
(a) Type *B*—11th km (b) Type *B*—9th km
(c) Type *A*—4th km (d) Type *C*—8th km
(e) None of these.

15. Directions: Read the following table carefully and answer the questions given below it.

Details of leading openers' performance in 20 one-day cricket matches					
Openers	Total Runs	Highest Runs	No. of matches with runs		
			100 or more	50–99	0's
A	994	141	5	3	1
B	751	130	1	8	2
C	414	52	—	2	2
D	653	94	—	4	1
E	772	85	—	7	—

(i) What is the difference between the **average runs** of top two openers in terms of **highest runs**, if matches having 0's were ignored?
(a) 4.7 (b) 13.7
(c) 11.1 (d) 16.62
(e) None of these.

(ii) If matches having zero runs and the one with highest runs is ignored, what will be the average runs for opener *C*?
(a) 21.29 (b) 21.79
(c) 20.7 (d) 21.17
(e) 20.19.

(iii) By how much the difference between the two highest total runs differs from the difference between the two lowest total runs?
(a) Lower by 18 (b) More by 18
(c) Lower by 4 (d) More by 4
(e) None of these.

(iv) Which of the given pairs of openers have ratio 3 : 2 in their highest runs?
(a) *B* and *D* (b) *B* and *C*
(c) *A* and *D* (d) *D* and *C*
(e) None of these.

(v) Excluding the match with the highest runs and matches with 50–99 runs, what will be the **approximate** average runs for opener *B*?

(a) 25 (b) 15
(c) 10 (d) 30
(e) None of these.

16. Directions: Read the following Table carefully and answer the questions given below.

Highest marks and average marks obtained by students in subjects over the years.

The maximum marks in each subjects is 100.

	Subjects									
	English		Hindi		Maths		Science		History	
	High	Avg	High	Avg	High	Avg	High	Avg	High	Avg
1992	85	62	75	52	98	65	88	72	72	46
1993	80	70	80	53	94	60	89	70	65	55
1994	82	65	77	54	85	62	95	64	66	58
1995	71	56	84	64	92	68	97	68	68	49
1996	75	52	82	66	91	64	92	75	70	58
1997	82	66	81	57	89	66	98	72	74	62

(i) What was the grand average marks of the five subjects in 1996?
(a) 63 (b) 64
(c) 65 (d) 68
(e) None of these.

(ii) The difference in the average marks in History between 1994 and 1995 was exactly equal to the difference in the highest marks in Hindi between which of the following pairs of years?
(a) 1992 and 1995 (b) 1993 and 1995
(c) 1992 and 1996 (d) 1993 and 1997
(e) None of these.

(iii) What was the **approximate** percentage increase in average marks in History from 1992 and 1993?
(a) 20 (b) 25
(c) 24 (d) 16
(e) 18.

(iv) The average highest marks in English in 1992, 1993 and 1996 was exactly equal to the highest marks in Hindi in which of the following years?
(a) 1996 (b) 1997
(c) 1994 (d) 1996
(e) 1993.

(v) The difference between the highest marks and the average marks in Hindi was maximum in which of the following years?
(a) 1994 (b) 1997
(c) 1995 (d) 1996
(e) 1993.

(vi) The highest marks in Hindi in 1993 was what per cent of the average marks in Mathematics in 1996?
(a) 135 (a) 130
(c) 125 (d) 140
(e) None of these.

(vii) If there were 50 students in 1993, what was the total marks obtained by them in Mathematics?
(a) 2400 (b) 3000
(c) 2500 (d) 3200
(e) None of these.

(viii) The difference between the highest marks in Science was maximum between which of the following pairs of years among the given years?
(a) 1992 and 1993 (b) 1992 and 1996
(c) 1996 and 1997 (d) 1992 and 1995
(e) None of these.

17. Directions: Read the following information carefully and answer the questions based on it.

In 6 educational years, number of students taking admission and leaving from the 5 different schools which are founded in 1990 are given below.

School	*A*		*B*		*C*		*D*		*E*	
	Ad	L	Ad	L	Ad	L	Ad	L	Ad	L
1990	1025	—	950	—	1100	—	1500	—	1450	—
1991	230	120	350	150	320	130	340	150	250	125
1992	190	110	225	115	300	150	300	160	280	130
1993	245	100	185	110	260	125	295	120	310	120
1994	280	150	200	90	240	140	320	125	340	110
1995	250	130	240	120	310	180	360	140	325	115

In the above table shown Ad = Admitted, L = Left

(i) What is the average number of students studying in all the five schools in 1992?
(a) 1494 (b) 1294
(c) 1590 (d) 1640
(e) None of these.

(ii) What was the number of students studying in school *B* in 1994?
(a) 2030 (b) 1060
(c) 1445 (d) 1150
(e) None of these.

(iii) Number of students leaving school *C* from the year 1990 to 1995 is approximately what percentage of number of students taking admission in the same school and in the same year?
(a) 50% (b) 25%
(c) 48% (d) 36%
(e) 29%.

(iv) What is the difference in the number of students taking admission between the years 1991 and 1995 in school *D* and *B*?
(a) 514 (b) 1065
(c) 965 (d) 415
(e) None of these.

(v) In which of the following schools, percentage increase in the number of students from the year 1990 to 1995 is maximum?

(a) *A* (b) *B*
(c) *C* (d) *D*
(e) *E*.

18. Directions: Study the following Table and answer the following questions carefully.

Following Table shows the percentage population of six States below poverty line and the proportion of male and female.

State	Percentage population below poverty line	Proportion of male and female	
		Below poverty line	Above poverty line
		M : F	M : F
A	12	3 : 2	4 : 3
B	15	5 : 7	3 : 4
C	25	4 : 5	2 : 3
D	26	1 : 2	5 : 6
E	10	6 : 5	3 : 2
F	32	2 : 3	4 : 5

(i) The total population of state *A* is 3000, then what is the approximate no. of females above poverty line in state *A*?
(a) 1200 (b) 2112
(c) 1800 (d) 1950
(e) 2025.

(ii) If the total population of *C* and *D* together is 18000, then what is the total no. of females below poverty line in the above stated states?
(a) 5000 (b) 5500
(c) 4800 (d) Data inadequate
(e) None of these.

(iii) If the population of males below poverty line in State *A* is 3000 and that in State *E* is 6000, then what is the ratio of the total population of State *A* and *E*?
(a) 3 : 4 (b) 4 : 5
(c) 1 : 2 (d) 2 : 3
(e) None of these.

(iv) If the population of males below poverty line in State *B* is 500 then what is the total population of that State?
(a) 14400 (b) 6000
(c) 8000 (d) 7600
(e) None of these.

(v) If in State *E* population of females above poverty line is 19800 then what is the population of males below poverty line in that State?
(a) 55000 (b) 30000
(c) 29700 (d) Date inadequate
(e) None of these.

19. Directions: Study the following Table carefully and answer the questions given below.

Production of main crops in India (in million tonnes)

Crops	**91–92**	**92–93**	**93–94**	**94–95**	**95–96**	**96–97**
Pulses	20.5	22.4	24.6	23.5	27.8	28.2
Oilseeds	32.4	34.6	40.8	42.4	46.8	52.4
Rice	80.5	86.4	88.2	92.6	94.2	90.8
Sugercane	140.8	150.2	152.2	160.3	156.4	172.5
Wheat	130.2	138.4	146.8	141.6	152.2	158.4
Coarse grain	45.6	52.8	60.4	62.4	58.2	62.8
Sum	**450**	**484.8**	**513.2**	**522.8**	**535.6**	**565.1**

(i) Production of sugarcane in 1993–94 was approximately what percentage of production of rice in 1992–93?
(a) 50 (b) 75
(c) 150 (d) 125
(e) 175.

(ii) Production of what type of crop was going to increase in each year in the given years?
(a) Rice (b) Pulse
(c) Sugarcane (d) Oilseeds
(e) None of these.

(iii) What was the average production of pulse in the given years?
(a) 26.8 million tonnes (b) 20.5 million tonnes
(c) 24.5 million tonnes (d) 22.5 million tonnes
(e) None of these.

(iv) Production of oilseeds was what percentage of the total crops produced in the year 1991–92?
(a) 7.2 (b) 8.4
(c) 2.7 (d) 6.4
(e) None of these.

(v) In which of the following years the total production of oilseeds in the years 1994–95, 1995–96 and 1996–97 was equal to the production of wheat?
(a) 1993–94 (b) 1994–95
(c) 1996–97 (d) 1992–93
(e) None of these.

20. Directions: Study the following Tables carefully and answer the questions given below.

Number of Cars (in thousands) of Different Models and Colours sold in two Metro Cities in a year

	Metro M				
Type	**Colour**				
	Black	Red	Blue	White	Silver
A	40	25	55	75	15
B	20	35	60	80	20
C	35	30	50	90	35
D	45	40	45	85	40
E	50	35	35	60	30
F	55	42	40	65	52

	Metro H				
Type	**Colour**				
	Black	Red	Blue	White	Silver
A	45	32	40	60	20
B	30	37	39	81	35
C	40	42	41	6	37
D	35	39	37	90	42
E	50	44	43	77	22
F	47	34	45	87	17

(i) The difference between the white-coloured cars sold in the two metros of which of the following models is the **minimum**?
(a) *A* (b) *C*
(c) *D* (d) *F*
(e) None of these.

(ii) The total number of blue-coloured cars of Model *E* and *D* sold in Metro *H* is exactly equal to the number of white-coloured cars of which model in Metro *M*?
(a) *B* (b) *F*
(c) *C* (d) *A*
(e) None of these.

(iii) What is the difference between the number of blue-colour cars of model '*C*' sold in Metro *M* and number of red colour cars of model '*F*' sold in Metro *H*?
(a) 8,000 (b) 10,000
(c) 12,000 (d) 15,000
(e) None of these.

(iv) The total number of silver-coloured cars sold in Metro *H* is approximately what percentage of that in Metro *M*?
(a) 130 (b) 140
(c) 90 (d) 100
(e) 110.

(v) In Metro *M* the number of cars sold was maximum for which of the colour-model combinations?
(a) White-*C* (b) Blue-*B*
(c) Silver-*B* (d) White-*D*
(e) None of these.

21. Directions: Study the following Table to answer the given questions.
Number of students of different classes of a school playing different games

Class → Games ↓	XII	XI	X	IX	VIII	VII	VI
Chess	11	12	5	4	2	2	1
Cricket	38	40	12	17	25	18	20
Basketball	11	9	7	6	0	0	0
Table Tennis	9	9	21	19	11	9	0
Football	40	27	18	19	12	16	14
Carrom	16	15	8	19	12	16	14
Tennis	8	9	11	5	6	0	0
Badminton	47	39	33	21	19	0	0

(i) **Approximately** what per cent of Class VIII students play Cricket out of the total students playing Cricket?
(a) 13 (b) 4
(c) 25 (d) 15
(e) 17.

(ii) What is the ratio of the students playing Football in Class XI to those in Class *X*?
(a) 1 : 2 (b) 2 : 5
(c) 2 : 3 (d) 3 : 2
(e) None of these.

(iii) Which game is the most popular?
(a) Badminton (b) Football
(c) Carrom (d) Table Tennis
(e) Cricket.

(iv) **Approximately** what per cent of Class *X* students play Table Tennis out of the total Class *X* students playing the different given games?
(a) 20 (b) 21
(c) 27 (d) 26
(e) 18.

(v) Which game has ascending number of students from class IX to XII?
(a) Basketball Only
(b) Badminton Only
(c) Chess and Badminton
(d) No game
(e) None of these.

22. Directions: A Table showing the percentages of the total population of a State by age groups for the year 1991 is given below. Answer the questions given below it.

Age Group (in years)	*Per cent*
up to 15	30.00
16–25	17.75
26–35	17.25
36–45	14.50
46–55	14.25
56–65	5.12
66 and above	1.13
Total	100.0

(i) Which age group accounts for the maximum population in the State?
(a) 16–25 (b) 26–35
(c) 36–45 (d) 56–65

(ii) Out of every 4200 persons, the number of persons below 26 years is
(a) 2006 approx. (b) 1260 approx.
(c) 746 approx. (d) 515 approx.

(iii) There are 200 million people below 36 years. How many million (approx) people are in the age group 56–65?
(a) 30.07 (b) 15.75
(c) 12.72 (d) 59.30

(iv) If there are 10 million people in the age group 56 years and above, what is the difference between the number of people in the age groups 16–25 and 46–55?
(a) 6.8 million (b) 5.6 million
(c) 28.4 million (d) 34.7 million

(v) If the difference between the number of people in the age groups 46–55 and 26–35 is 11.75 million, then total population of State is approximately
(a) 360.23 million (b) 391.67 million
(c) 400 million (d) 460.67 million

23. Directions: Study the following table to answer these questions based on it

***XYZ* Co. (Pvt.) Ltd. (in lakh of Rs.)**

Year	*Total Sales*	*Gross Profit*	*Net Profit*
1990	351.6	155.5	54.2
1991	407.9	134.3	42.6
1992	380.1	149.9	38.9
1993	439.7	160.5	50.3
1994	485.9	203.3	65.8

(i) In which year the difference between the total sales and the gross profit is the least?
(a) 1990 (b) 1991
(c) 1992 (d) 1993

(ii) The total sales in 1993 is approximately what per cent of the total sales in 1990?
(a) 75 (b) 85
(c) 110 (d) 125

(iii) Which years show increase in all the categories simultaneously, that is total sales, gross profit and net profit as compared to the previous year?
(a) 1993 and 1994 both (b) 1994 and 1992 both
(c) 1992 and 1993 both (d) 1990 and 1991 both.

(iv) The net profit in 1991 is **approximately** what per cent of the total sales in 1993?
(a) 6.5 (b) 7
(c) 8 (d) 9.7

(v) The per cent increase in the gross profit was the largest in which year as compared to the previous one?
(a) 1991 (b) 1992
(c) 1993 (d) 1994

24. Directions: Refer to the following Table. Read the Table and answer the questions.

Foodgrain Production in a Country in 1999 (in lakh tons)

State	*Rice*	*Wheat*	*Jowar*	*Pulses*	*Others*
P	45	103	—	27	29
Q	48	86	73	19	15
R	59	32	67	14	31
S	41	37	59	21	15
T	37	22	41	13	11
U	68	15	12	—	18
V	57	8	7	12	10
W	38	28	31	22	45

(i) Which State had the highest grain production?
(a) *P* (b) *Q*
(d) *R* (d) *S*

(ii) What was the proportion of rice production to wheat production in the country?
(a) 1 : 1 (b) 0.8 : 1
(c) 1.2 : 1 (d) 2 : 1

(iii) Jowar was the most important foodgrain in the State/States:
(a) *Q, R, S* (b) *Q*
(c) *R, S* (d) *R, S, T*

(iv) States *P* alone accounted for **approximately** what percentage of wheat production in the country?
(a) 73% (b) 50%
(c) 41% (d) 30%

(v) If the average per hectare yield of rice in the country was 30 tons, then the area (approx.) under rice cultivation during the year was (in lakh hectares)
(a) 1.5 (b) 8
(c) 13 (d) 40

25. Directions: Following Table gives the population of a locality from 1988 to 1992. Read the Table and answer the questions.

Year	*Men*	*Women*	*Children*	*Total*	*Increase (+) or decrease (–) over preceding year*
1988	65104	60387	—	146947	—
1989	70391	62516	—	—	+ (11630)
1990	—	63143	20314	152922	—
1991	69395	—	21560	—	– (5337)
1992	71274	65935	23789	160998	—

(i) The number of children in 1988 is
(a) 31236 (b) 125491
(c) 14546 (d) 21456

(ii) The number of children in 1989 is
(a) 144537 (b) 158577
(c) 146947 (d) 149637

(iii) The number of women in 1991 is
(a) 25670 (b) 14040
(c) 13970 (d) 15702

(iv) The number of women in 1991 is
(a) 57630 (b) 56740
(c) 52297 (d) 62957

(v) Increase or decrease of population in 1992 over 1991 is
(a) – (12413) (b) + (12413)
(c) + 155661 (d) + 7086

26. Directions: The Table given below shows production of five types of cars by a company in the years 1989 to 1994. Study the Table and answer questions.

Production of cars by a company

Year → Type ↓	1989	1990	1991	1992	1993	1994	Total
P	8	20	16	17	21	6	88
Q	16	10	14	12	12	14	78
R	21	17	16	15	13	8	90
S	4	6	10	16	20	31	87
T	25	18	19	30	14	27	133
Total	74	71	75	90	80	86	476

(i) In which year the production of cars of all types taken together was approximately equal to the average of the total production during the period?
(a) 1989 (b) 1991
(c) 1993 (d) 1994

(ii) In which year the total production of cars of types *P* and *Q* together was equal to the total production of cars of types *R* and *S* together?
(a) 1990 (b) 1991
(c) 1994 (c) None of the above.

(iii) During the period 1989–94, in which type of cars was a continuous increase in production?
(a) *P* (b) *Q*
(c) *R* (d) *S*

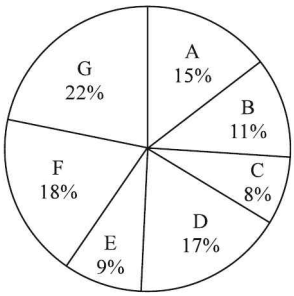
(iv) The production of which type of cars was 25% of the total production of all types of cars during 1993?
(a) *S* (b) *R*
(c) *Q* (d) *P*

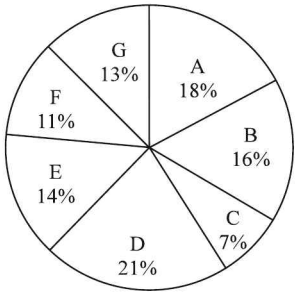
(v) The per cent increase in total production of all types of cars in 1993 to that in 1991 was?
(a) 15 (c) 20
(c) 25 (d) 30

27. Directions: These questions are based on the following graphs:

Classification of appeared candidates in a competitive test from different States and qualified candidates from those States.

Appeared candidates = 45000.
Qualified candidates = 9000.





(i) What is the ratio of the number of appeared candidates from States *C* and *E* together to that of the appeared candidates from States *A* and *F* together?
(a) 17 : 33 (b) 11 : 13
(c) 13 : 27 (d) 17 : 27
(d) None of these.

(ii) In which State, the percentage of qualified candidates with respect to that of appeared candidates is minimum?
(a) *C* (b) *F*
(c) *D* (d) *E*
(e) *G*.

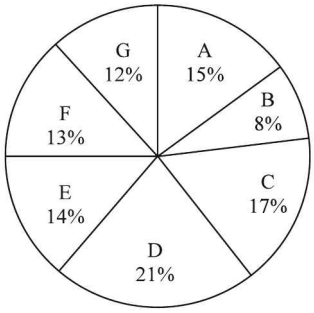
(iii) What is the difference between the number of qualified candidates of States *D* and those of *G*?
(a) 690 (b) 670
(c) 780 (d) 720
(e) None of these.

(iv) What is the percentage of qualified candidates with respect to appeared candidates from States *B* and *C* taken together? (rounded to two decimal places)
(a) 23.11 (b) 24.21
(c) 21.24 (d) 23
(e) None of these.

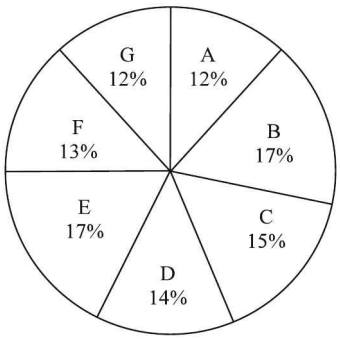
(v) What is the ratio between the number of candidates qualified from States *B* and *D* together to the number of candidates appeared from State 'C', respectively?
(a) 8 : 37 (b) 11 :12
(c) 37 : 40 (d) 7 : 37
(e) None of these.

28. Directions: These questions are based on the following graphs:

Distribution of candidates studying Arts and Commerce from seven different institutes *A*, *B*, *C*, *D*, *E*, *F* and *G*. Total Number of Students Studying Arts = 3800



Total Number of Students Studying Commerce = 4200



(i) What is the ratio between the number of students studying Arts from Institute *E* and the number of students studying *C*ommerce from Institute *D*, respectively?
(a) 17 : 19 (b) 19 : 27
(c) 14 : 19 (d) 19 : 21
(e) None of these.

(ii) What is the total number of students studying Arts from Institutes *A* and *G* together?
(a) 1102 (b) 918
(c) 966 (d) 1130
(e) None of these.

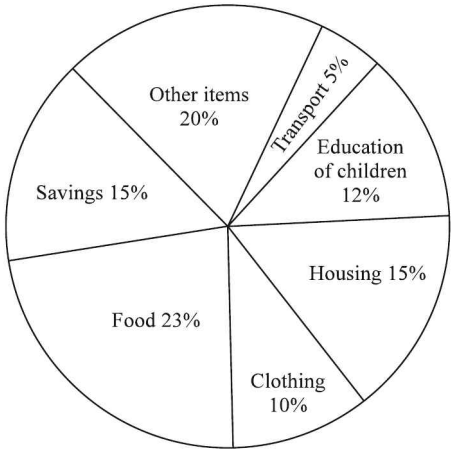
(iii) How many students are studying Commerce from Institutes *B* and *D* together?
(a) 1158 (b) 1302
(b) 1232 (d) 1272
(e) None of these.

(iv) How many students are studying Art and Commerce from Institute '*B*'?
(a) 1418 (b) 2000
(c) 1018 (d) 1208
(e) None of these.

(v) What is the ratio between the number of students studying Arts and Commerce, respectively from Institute '*E*'?
(A) 19 : 27 (b) 17 : 29
(c) 19 : 29 (d) 17 : 27
(e) None of these

29. Directions: The pie-chart drawn below shows the expenses of a family on various items and its savings during the year 2001. Study the graph and answer the questions given below:

Percentage of Money Spent on Various Items and Savings by a Family during 2001



(i) Maximum expenditure of the family was on
(a) Food (b) Housing
(c) Education of Children (d) Other items

(ii) The total savings of the family for the year were equal to the expenditure on
(a) Food (b) Clothing
(c) Housing
(d) Other items including transport.

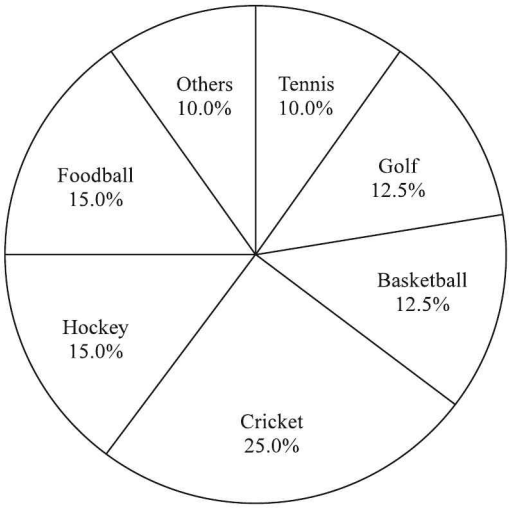
(iii) What per cent of the income was spent on transport and other items together?
(a) 25% (b) 20%
(c) 30% (d) 32%

(iv) If the total income of the family was Rs. 100000, how much money was spent on the eduction of children?
(a) Rs. 10000 (b) Rs. 12000
(c) Rs. 15000 (d) Rs. 23000

(v) If the total income for the year was Rs. 100000, the difference of the expenses (in rupees) between housing and transport was
(a) 15000 (b) 12000
(c) 7000 (d) 10000

30. Directions: The circle graph given here shows the spendings of a country on various sports during a year. Study the graph carefully to answer these questions.

Percent of Money Spent on Various Sports for One Year



(i) If the total amount spent on sports during the year was Rs. 15000000, then the amount spent on Cricket and Hockey together was
(a) Rs. 6000000 (b) Rs. 5000000
(c) Rs. 3750000 (d) Rs. 7500000

(ii) If the total amount spent during the year was Rs. 1,20,00,000 how much was spent on basketball?
(a) Rs. 12,50,000 (b) Rs. 10,00,000
(c) Rs. 12,00,000 (d) Rs. 15,00,000

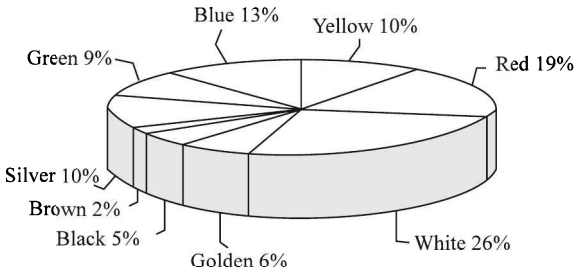
(iii) The ratio of the total amount spent on football to that spent on Hockey was
(a) 1 : 15 (b) 1 : 1
(c) 15 : 1 (d) 3 : 2

(iv) The graph shows that the most popular game is
(a) Hockey (b) Football
(c) Cricket (d) Basketball

(v) The country spent the same amount of money on
(a) Hockey and Tennis (b) Golf and Basketball
(c) Cricket and Football (d) Hockey and Golf.

31. Directions: Study the chart and give the answers of following questions.

Selling of cars in UK according to their colours



(i) 50% of all the cars consisted of which colours of car?
(a) Black, Golden, Blue, Red,
(b) Blue, Black, Red, Silver
(c) White, Golden, Blue, Black
(d) White, Blue, Green, Black
(e) None of these.

(ii) Cars of which colour are 20% less popular than white coloured cars?
(a) Black (b) Golden
(c) Red (d) Blue
(e) None of these.

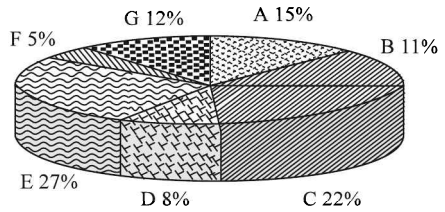
(iii) Cars of which colour are 13% less popular than white coloured cars?
(a) Blue (b) Green
(c) Silver (d) Yellow
(e) None of these.

(iv) Cars of which colour when increased by two percent and then combined with that of red cars will make 30 per cent of the total?
(a) Golden (b) Blue
(c) Black (d) Yellow
(e) None of these.

(v) If in a certain period the total production of all cars was 95400 then how many more blue cars were sold than green?
(a) 2580 (b) 3618
(c) 2850 (d) 3816
(e) None of these.

32. Directions: Seven companies *A, B, C, D, E, F* and *G* are engaged in production of two items I and II. The comparative data about production of these items by the seven companies is given in the following graph and Table. Study them carefully and answer the questions given below.

Percentage of the total production produced by the seven companies



Cost of the total production (both items together) by seven companies = Rs. 25 crores

Ratio of production between items I and II and the per cent profit earned for the two items.

Company	*Ratio of Production*		*Per cent profit earned*	
	Item I	*Item* II	*Item* I	*Item* II
A	2	3	25	20
B	3	2	32	35
C	4	1	20	22
D	3	5	15	25
E	5	3	28	30
F	1	4	35	25
G	1	2	30	24

(i) What is the total cost of the production of item I by companies *A* and *C* together in Rs crore?
(a) 9.25 (b) 5.9
(c) 4.1625 (d) 4.9
(e) None of these.

(ii) What is the amount of profit earned by company *D* on item II?
(a) Rs. 3.125 cr (b) Rs. 31.25 cr
(c) Rs. 3.125 lakh (d) Rs. 31.25 lakh
(e) None of these.

(iii) Cost of production of item I by company *F* is what per cent of the cost of production of item II by company *D*?
(a) 16% (b) 33.33%
(c) 66.67% (d) 12.5%
(e) None of these.

(iv) What is total profit earned by company *G* for items I and II together ?
(a) Rs. 78 lakh (b) Rs. 1.62 cr
(c) Rs. 7.8 cr (d) Rs. 16.2 lakhs
(e) None of these.

(v) What is the ratio of the cost of production of item I by company *A* to the cost of production of item I by company *D*?
(a) 3 : 5 (b) 1 : 2
(c) 2 : 1 (d) 2 : 3
(e) None of these.

(vi) What is the total of the profit earned by company *B* on production of item I and the profit earned by company *A* on production of item II?

(a) Rs. 9.78 cr (b) Rs. 97.8 lakhs
(c) Rs. 52.8 lakhs (d) Rs. 5.28 cr
(e) None of these.

(vii) The cost of production of both items together by company *E* is equal to the total cost of production of both items together by which of the two companies?
(a) *C* and *D* (b) *B* and *G*
(c) *A* and *D* (d) *C* and *F*
(e) *A* and *B*.

(viii) What is the total of the cost of production of item I by company *A* and the cost of production of item II by company *B*?
(a) Rs. 2.6 cr (b) Rs. 26 lakh
(c) Rs. 3. 35 cr (d) Rs. 33. 65 lakh
(e) None of these.

33. Directions: Study the following information to answer the given questions.
Percentage of students in various courses (*A*, *B*, *C*, *D*, *E*, *F*) in pie chart I and percentage of girls in pie chart II.
Total students: 1200 (800 girls + 4000 boys)

Chart I

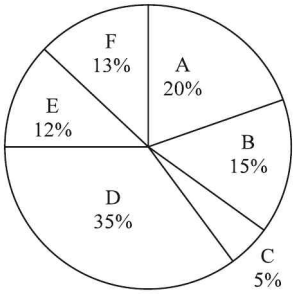
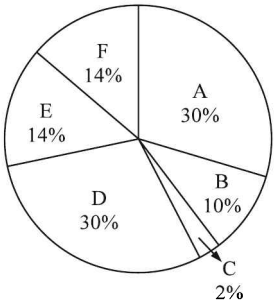


Chart II



(i) For course *D*, what is the respective ratio of boys and girls?
(a) 3 : 4 (b) 4 : 5
(c) 3 : 5 (d) 5 : 6
(e) None of these.

(ii) For which pair of courses is the number of boys the same?
(a) *E* and *F* (b) *A* and *D*
(c) *C* and *F* (d) *B* and *D*
(e) None of these.

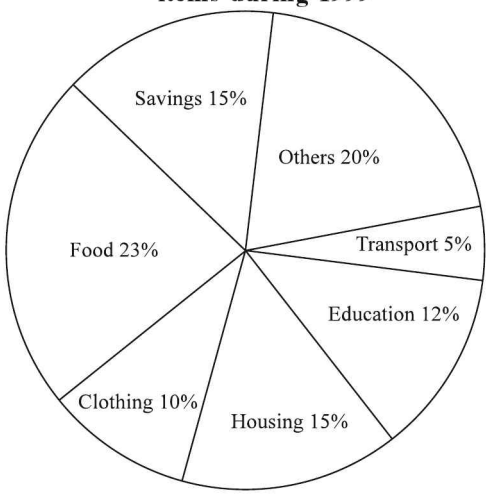
(iii) For course *E*, the number of girls is how much per cent more than the number of boys for course *E*?
(a) 250 (b) 350
(c) 150 (d) 80
(e) None of these.

(iv) For which course is the number of boys the minimum?
(a) *E* (b) *F*
(c) *C* (d) *A*
(e) None of these.

(v) How many girls are there in course *C*?
(a) 44 (b) 16
(d) 40 (e) 160
(e) None of these.

34. Directions: The circle graph given here shows the spendings by a family on various items during the year 1999. Study the graph and answer these questions.

Per cent of money spent by a family on various items during 1999



(i) If the total amount spent during the year 1999 was Rs. 46000, the amount spent on food was
(a) Rs. 2000 (b) Rs. 10580
(c) Rs. 23000 (d) Rs. 2300

(ii) If the total amount spent was Rs 46000, how much money was spent on clothing and housing together?
(a) Rs. 11500 (b) Rs. 1150
(c) Rs. 10000 (d) Rs. 15000

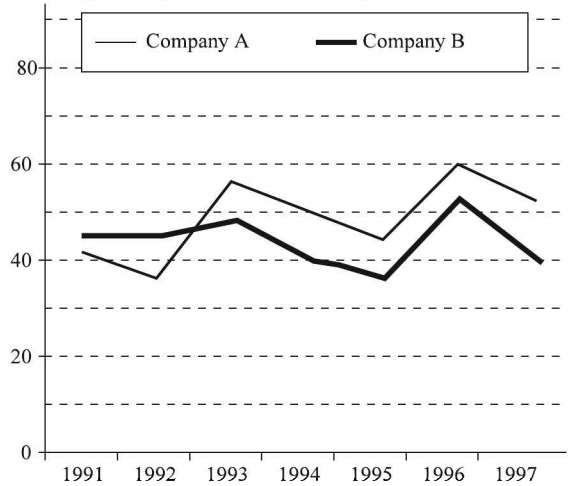
(iii) The ratio of the total amount of money spent on housing to that spent on education was
(a) 5 : 2 (b) 2 : 5
(c) 4 : 5 (d) 5 : 4

(iv) Graph shows that the maximum amount was spent on
(a) Food (b) Housing
(c) Clothing (d) Others

(v) If the total expenditure of the family for the year 1999 was Rs. 46000, the family saved during the year
(a) Rs. 1500 (b) Rs. 15000
(c) Rs. 6900 (d) Rs. 3067 approx.

35. Directions: Study the following graph carefully and answer the questions given below.

Percentage net profit of two companies over the years



(i) If the total income in 1992 for Company *B* was 140 crores, what was the total expenditure in that year?
(a) 100 cr (b) 110 cr
(c) 98 cr (d) Data inadequate
(e) None of these.

(ii) If the total expenditure of 1993 and 1994 together of Company *B* was Rs. 279 crore, what was the total income in these years?
(a) Rs. 121.5 cr (b) Rs. 135 cr
(c) Rs. 140 cr (d) Data inadequate
(e) None of these.

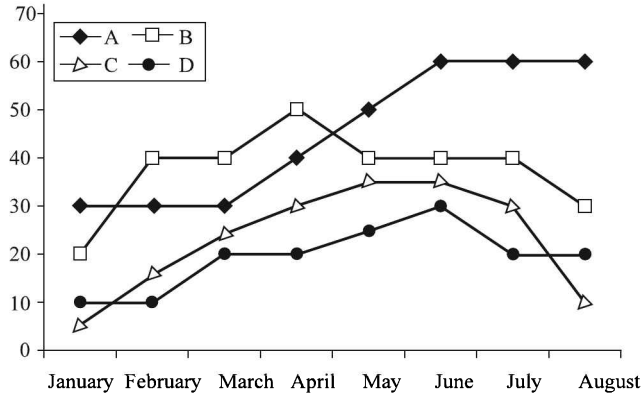
(iii) In how many of the given years the percentage of expenditure to the income of Company *A* was less than fifty?
(a) One (b) Two
(c) Three (d) Four
(c) None of these.

(iv) If the total expenditue of Company *B* in 1994 was Rs. 200 crore, what was the total income?
(a) Rs. 160 cr (b) Rs. 240 cr
(c) Rs. 260 cr (d) Data inadequate
(e) None of these.

(v) In which of the following years was the total income more than double the total expenditure in that year for Company *B*?
(a) 1995 (b) 1993
(c) 1997 (d) 1992
(e) None of these.

36. Directions: Study the following graph carefully and answer the questions given below

Percentage increase in the sale of four commodities *A*, *B*, *C* and *D* for the given months



(i) For which months did the sale of commodities *A* and *D* show increase?
(a) April only
(b) May and June only
(c) May, June and July only
(d) April, May and June only
(e) None of these.

(ii) In which month is the average percentage increase for the four commodities the lowest? (Those months where decrease took place to be ignored)
(a) May (b) March
(c) January (d) June
(e) None of these.

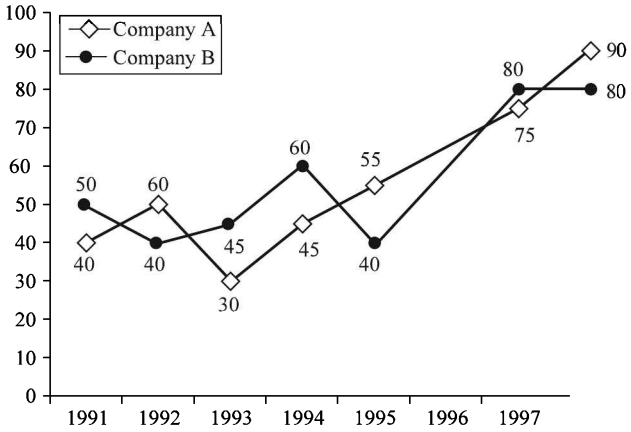
(iii) In which month(s) did all the commodities show decline or no increase from the previous months?
(a) July only
(b) August only
(c) April and July only
(d) July and August only
(e) None of these.

(iv) If the sale of *C* was 100 in May, what was its sale in July?
(a) 195 (b) 100
(c) 90 (d) Cannot be determined
(e) None of these.

(v) For which commodity is the per cent increase in sale the highest in May from January?
(a) *C* (b) *A*
(c) *B* (d) *A* and *B*
(e) None of these.

37. Directions: Study the following graph carefully and answer the questions given below it.

Per cent profit earned by two companies *A* and *B* over the years 1991 to 1997



(i) Investment of company '*B*' in 1997 is more by 40% than that in the previous year. Income in 1997 was what per cent of the investment in 1996?
(a) 280% (b) 252%
(c) 242% (d) 52%
(e) None of these.

(ii) Average investment of company '*A*' over the years was Rs. 26 lakh. What was its average income over the years?
(a) Rs. 40.56 lakhs (b) Rs. 41.60 lakhs
(c) Rs. 50.26 lakhs (d) Data inadequate
(e) None of these.

(iii) Income of company '*A*' in 1995 was Rs 21.7 lakh. What was the investment?

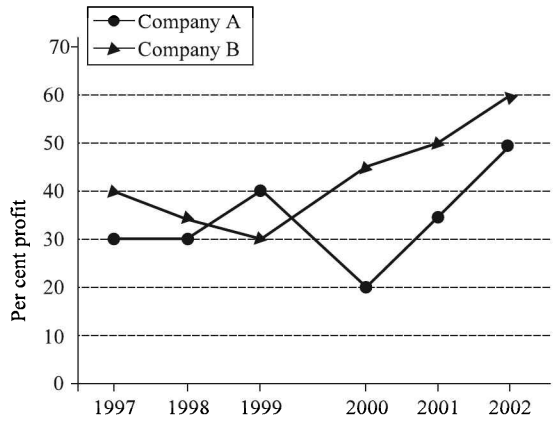
(a) Rs. 14.5 lakh (b) Rs. 15.4 lakh
(c) Rs. 15.8 lakh (d) Rs. 14.6 lakh
(e) None of these.

(iv) Income of company '*A*' in 1995 is equal to the investment of the company '*B*' in 1996. What is the ratio of the investment of company '*A*' in 1995 to the investment of company '*B*' in 1996?
(a) 31 : 36 (b) 31 : 20
(c) 20 : 31 (d) Data inadequate
(e) None of these.

(v) Investment of company '*B*' in 1993 was Rs. 1540000. What was its income in that year?
(a) Rs. 23. 33 lakh (b) Rs. 22.33 lakh
(d) Rs. 22. 23 lakh (d) Rs. 23.23 lakh
(e) None of these.

38. Directions: Study the following graph to answer the given questions.

Percent profit earned by two companies over the given years



(i) If the expenditure of Company *B* in 2000 was Rs 200 crore, what was its income?
(a) Rs. 240 crore (b) Rs. 220 crore
(c) Rs. 160 crore (d) Cannot be determined
(e) None of these.

(ii) If the income of Company *A* in 2002 was Rs 600 crore, what was its expenditure?
(a) Rs. 360 crore (b) Rs. 480 crore
(c) Rs. 375 crore (d) Cannot be determined
(e) None of these.

(iii) If the income of Company *B* in 1998 was Rs. 200 crores, what was its profit in 1999?
(a) Rs. 21.5 corer (b) Rs. 153 crore
(c) Rs. 46.15 crore (d) Cannot be determined
(e) None of these.

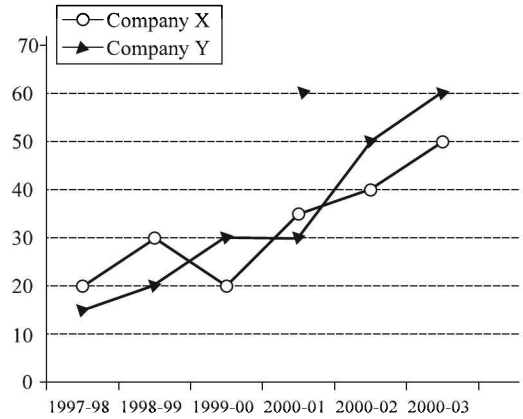
(iv) If the income of the two companies in 1998 were equal, what was the ratio of their expenditures?
(a) 1 : 2 (b) 26 : 27
(c) 100 : 67 (d) Cannot be determined
(e) None of these.

(v) What is the percent increase in profit for company *B* from year 2000 to 2001?
(a) 75 (b) 175
(c) 42.86 (d) Cannot be determined
(e) None of these.

39. Directions: Study the following graph to answer the given questions.

Per cent profit earned by two companies over the given years

$$\textbf{\% profit} = \frac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$$



(i) If the income of Company *X* in 1998–99 was equal to the expenditure of Company *Y* in 2001–2002, what was the ratio of their respective profits?
(a) 13 : 15 (b) 15 : 26
(c) 13 : 26 (d) Cannot be determined
(e) None of these.

(ii) For Company *X*, its income in 2001–2002 was equal to its expenditure in 2002–2003, what was the ratio of its respective incomes in these two years?
(a) 4 : 5 (b) 3 : 4
(c) 2 : 3 (d) Cannot be determined
(e) None of these.

(iii) For Company *Y*, in which year is the per cent of increase in per cent profit over that of previous year the highest?
(a) 2002–03 (b) 1999–2000
(c) 2001–02 (d) Cannot be determined
(e) None of these.

(iv) In 1997-98, the expenditure of Company *X* was Rs. 40 crores. What was its income in that year?
(a) Rs. 50 crore (b) Rs. 48 crore
(c) Rs. 46 crore (d) Cannot be determined
(e) None of these.

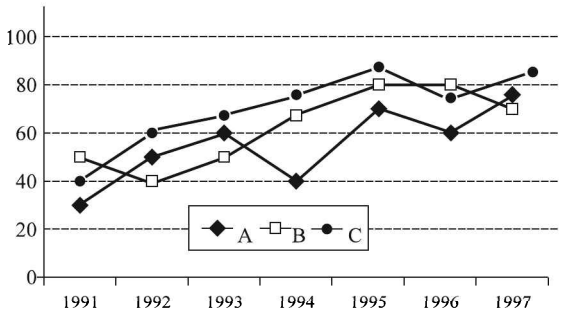
(v) What was the difference in the expenditures of the two companies in 1999–2000?
(a) 10 (b) 100
(c) 1000 (d) Cannot be determined
(e) None of these.

(vi) In 2002–03 the income of Company *Y* was Rs. 128 crores. What was its expenditure in that year?

(a) Rs. 76.8 crore (b) Rs. 64 crore
(c) Rs. 48 crore (d) Cannot be determined
(e) None of these.

40. Directions: Study the following graph carefully and answer the questions given below it.

Imports of 3 companies over the years
(Rs in crore)



(i) In which of the following years, the imports made by Company *A* was exactly equal to average imports made by it over the given years?
(a) 1992 (b) 1993
(c) 1994 (d) 1995
(e) None of these.

(ii) In which of the following years was the difference between the imports made by Company *B* and *C* the maximum?
(a) 1995 (b) 1994
(c) 1991 (d) 1992
(e) None of these.

(iii) In which of the following years was the imports made by Company *A* exactly half of the total imports made by Company *B* and *C* together in that year?
(a) 1992 only (b) 1993 only
(c) 1992 and 1993 (d) 1995 only
(e) None of these.

(iv) What was the percentage increase in imports by Company *B* from 1992 to 1993?
(a) 10 (b) 25
(c) 40 (d) 20
(e) None of these.

(v) In which of the following years was the total imports made by all the three companies together the maximum?
(a) 1996 only (b) 1997 only
(c) 1995 only (d) 1995 and 1997 only
(e) None of these.

EXPLANATORY ANSWERS

1. (i) (e)

(ii) (a) Required difference

$$= 60 - 50 = 10{,}000 \text{ tonne.}$$

(iii) (d) Percentage increase in production

$$= \frac{15}{25} \times 100 = 60\%$$

(iv) (c) Average production

$$= \frac{25+40+60+45+65+50+75+80}{8}$$

$$= \frac{440}{8} = 55.$$

(v) (c) Required percentage drop

$$= \frac{60-45}{60} \times 100 = 25\%.$$

2. (i) (e)

We can use the direct formula for

$$\text{Profit} = \text{Income}\left[1 - \frac{100}{100 + \%\ \text{Profit}}\right]$$

We see that the profit is maximum in 1998.

(ii) (b) Total expenditure

$$= 120 \times \frac{100}{107.5} + 160 \times \frac{100}{115} + 130 \times \frac{100}{122.5}$$

$$+ 170 \times \frac{100}{117.5} + 190 \times \frac{100}{120} + 150 \times \frac{100}{127.5}$$

= Rs. 777.51 lakh

$$\therefore \quad \text{Average} = \frac{777.51}{6} \approx \text{Rs 130 lakh.}$$

(iii) (a) Per cent profit increase/decrease from the previous year

1994	1995	1996	1997	1998
100	50	(–) 22.22	14.28	37.5

(iv) (c) Expenditure in 1994 $= 160 \times \frac{100}{115} \approx 140$ lakh.

(v) (d) Expenditure in 1997 $= 110 \times \frac{100}{125} =$ Rs. 152 lakh.

3. (i) (a)

Production of *C* type cars in 1996

= (70 – 40)% of 450000

= 30% of 450000 = 135000

Production of *C* type cars in 1997

= (65 – 40)% of 520000

= 25% of 520000 = 130000

∴ Required difference = 5000.

(ii) (e) Production of *E* type cars in 1996

= (100 – 80)% of 450000

= 20% of 450000 = 90000

And in 1997 = 10% of 520000 = 52000

∴ Total production = 90000 + 52000 = 142000

∴ Required no. of cars = 15% of 142000 = 21300

(iii) (b) Production of A type cars in 1997 = production of A type cars in 1996 (given) = (100 – 85) 15% of 450000 = 67500

∴ Reqd. percentage = $\frac{67500}{520000} \times 100 \approx 13.$

(iv) (c) Clearly, by visual inspection D is the desired option.

(v) (c) Percentage production of B type cars in 1997 = that in 1996 (given)

= (40 – 15) 25% of 520000 = 130000.

4. (i) (c)

Total rose production

= (15 + 12.5 + 12.45 + 20 + 12.4 + 22.5 + 22.4 + 25) × 1000

= 142250

Percentage production of rose in the States (the lowest four states)

Rajasthan	Karnataka	Haryana	Gujarat
8.71	8.75	8.78	10.54

(ii) (a) Required percentage

$$= \frac{25-20}{20} \times 100 = 25\% \text{ (more)}$$

(iii) (d) Total production of rose by all the States = 142250

∴ Average = $\frac{142250}{8\times1000} \approx 18$ thousand.

(iv) (b) Required percentage

$$= \frac{25}{142.25} \times 100 \approx 20\%.$$

(v) (e) It is 36.8% approximately.

5. (i) (a)

Reqd. % = $\frac{5.1}{12.5} \times 100 = 40.8\%.$

(ii) (e) Percentage increase

$$= \frac{38.8-11.8}{11.8} \times 100 \approx 225\%.$$

(iii) (b)

(iv) (d) Reqd. ratio = $\frac{7.4}{16.8}$ = 37 : 84.

(v) (a) Reqd. % = $\frac{38.8}{63.9} \times 100 \approx 60\%.$

(vi) (e) Reqd. % = $\frac{101.80}{138.50} \times 100 \approx 75\%.$

(vii) (c) Reqd. % = $\frac{1.8}{74.6} \times 100 = 2.41\%.$

6. (i) (e)

Sale of Pep-up was the maximum in the year 1989.

(ii) (a) Avg. annual sale of Dew-drop

$$= \frac{10+15+25+15+30+25}{6} = 20 \text{ lakh}$$

Avg. annual sale of Cool-sip

$$= \frac{25+7+20+20+25+30}{6} = 21.16 \text{ lakh}$$

Avg. annual sale of Pep-up

$$= \frac{30+35+30+25+20+20}{6}$$

= 26.66 lakh.

(iii) (c) Reqd. % = $\frac{25-20}{20} \times 100 = 25\%.$

(iv) (e) Reqd. no. = 30 – 20 = 1000000.

(v) (c) Reqd. % drop = $\frac{35-30}{35} \times 100 \approx 14\%.$

7. (i) (b)

Obvious from the chart.

(ii) (d) Total imports in the given years

= 35 + 30 + 40 + 50 + 55 + 60 + 45

= 315 crore

Total exports in the given years

= 40 + 45 + 35 + 40 + 60 + 50 + 55

= 325 crore

Hence, required ratio = $\frac{315}{325} = \frac{63}{65}.$

(iii) (c) Obvious from the chart.

(iv) (e) Total exports in the years 1995,1996 and 1999

= 35 + 40 + 55 = 130 crore

Total imports in the years 1995, 1996 and 1999

= 40 + 50 + 45 = 135 crore

Now required % = $\frac{130\times100}{135} = 96.29\%.$

(v) (a) If you calculate approximate value you reject (b), (c) and (d). Now check (a).

In 1996,% increase in export

$$= \frac{5}{35}\,100 = \frac{100}{7} = 14.29\%.$$

8. (i) (d)

$$\frac{(110+60+110+100+105+85) = 570}{6}$$

= 95 lakh tons

(ii) (c) Average production of units A, B and C in 2001 [use white bars]

$$= \frac{(90+75+100) = 265}{3}$$

Average production of units D, E and F in 2002

$$= \frac{(100+105+85)=290}{3}$$

$\therefore$ Required answer $= \frac{265\times3}{3\times290} \times 100 = 91.38.$

(iii) (e) Total production by unit B in 2001 and 2002 together

= (75 + 60) = 135 lakh tons

Total production by unit C in 2001 and 2002 together

= (100 + 110) = 210 lakh tons

$\therefore$ Required ratio = (135 : 210) = 9 : 14.

(iv) (a) Total production by unit F in year 2001 and 2002 together

= (70 + 85) = 155 lakh tons

Total production by unit D in year 2001 and 2002 together

= (95 + 100) = 195 lakh tons

$\therefore$ Required percentage $= \left(\frac{155}{195}\times100\right)$

$= 79.487 \approx 79.49.$

(v) (b) Required total production

= (100 + 110 + 95 + 100 + 85 + 105)

= 595 lakh tons.

9. (i) (a)

% growth of expense from 1994 to 1995

$$= \frac{400-300}{300} \times 100 = 33\frac{1}{3}\%$$

% growth of expense from 1995 to 1996 = zero

% growth of expense from 1996 to 1997

$$= \frac{600-400}{400} \times 100 = 50\%$$

% growth of expense from 1997 to 1998

$$= \frac{700-600}{600} \times 100 = 16\frac{2}{3}\%$$

$\therefore$ average $= \frac{100}{4} = 25\%.$

(ii) (c)

(iii) (d) Average $= \frac{100-200+100+400}{4}$ = Rs. 100 cr.

(iv) (b) Ratio of profit to capital

1998	1995	1996	1997
0.50	1	–1	–1

(v) (b)

10. (i) (b)

Required percentage drop

$$= \frac{25-22.5}{25} \times 100 = 10\%$$

(ii) (e) Required difference = (35 – 22.5) lakh = 1250000.

(iii) (b) Only by visual observation you can find the answer. You do not need to calculate the values.

(iv) (e) Production of B type cars is more than the production of C type cars only in 1993, 1994 and 1995. We see the largest difference exists in 1995. So, the answer is 1995.

(v) (d) Total production of C type tyres in 1992 and 1993 together

= (30 + 25) = 55 lakh and that of B in 1994 = 27.5 lakh.

$\therefore$ Reqd. percentage $= \frac{55}{27.5} \times 100 = 200.$

11. (i) (d)

Difference of production of C in 1991 and A in 1996 = 500000 tonnes.

(ii) (a) Percentage increase of A from 92 to 93

$$= \frac{55-40}{40} \times 100 = 37.5\%.$$

(iii) (b) Percentage rise/fall in production for B

1992	1993	1994	1995	1996
9%	–16.6%	10%	–8%	10%

(iv) (e) Percentage production $= \frac{120}{90} \times 100$

$= 133.3\%.$

(v) (c) Average production of A = 50

Average production of B = 54.17

Average production of C = 50

Difference of production = 54.17 – 50 = 4.17.

12. (i) (b)

$$\frac{(7500-5300=)2200\times100}{705300} = 41.5\%$$

(ii) (a)

(iii) (d)

(iv) (c) Let GTR be Rs. x

$\therefore$ $x - x \times 10\% = 5800$

$\therefore$ $x = \frac{5800\times10}{9}$ = Rs. 6444.4.

(v) (d) 8800 – 5100 = 3700.

13. (i) (d)

(ii) (c)

Our intelligent observation says that the required year cannot be 1993, 1994, 1995. Why? Because see the following conclusions:

% passed to appear $= \frac{\text{Passed}}{\text{Appeared}} \times 100$

% of passed to appear is least when $\frac{\text{Passed}}{\text{Appeared}}$ is the least

or, $\frac{\text{Appeared}}{\text{Passed}}$ is the most. Now, we do the further calculations mentally. See the following conclusions:

For 1990: $\frac{7894}{2513} \Rightarrow$ Quotient = 3 and Remainder ≈ 300

For 1991: $\frac{8562}{2933} \Rightarrow Q = 3$ and $R \approx 400$

For 1992: $\frac{8139}{2468} \Rightarrow Q = 3$ and $R \approx 800$

Similarly, for 1993, 1994, 1995, Q is 2.

So, 1992 gives the highest value.

(iii) (a) $\frac{8562-8139}{8562} \times 100 = \frac{423}{8562} \times 100 \approx \frac{42}{84} \times 10 = 5$

(iv) We do not need to calculate the values for each year. Follow as:

For Rural area: 35% of 5032 ≈ 35 × 50 ≈ 1750 ≈ 1798

For Semi-urban area: 35% of 9500 ≈ 35 × 95 ≈ 3300

Which cannot be approximated to 3500.

For State capitals: 35 × 85 ≈ 3000

For Metropolises: 35 × 110 ≈ 3850

(v) (c) 1798 + 2513 = 4311.

14. (i) (d)

Distance to be travelled by each type of vehicle

$$= \frac{15}{3} = 5 \text{ km}$$

Since, to travel 5 km by vehicle A, he will pay Rs. 9 for 4 km and for the next 1 km he will have to pay Rs. $\frac{13.5-9.00}{(7-4)} \times 1$.

Similarly, for other cases.

Fare by A = Rs. $9 + \frac{13.50-9}{7-4} = 9 + 1.50$ = Rs. 10.50

Fare by $B = 14.50 + \frac{24.25-14.50}{7-4}$

$= 14.50 + 3.25 = 17.75$

Fare by $C = 19 + \frac{31-19}{3} = 19 + 4 = 23$

Total fare = 10.50 + 17.75 + 23 = Rs. 51.25.

(ii) (a) Fare by $A = 9 + \frac{4.50}{3} \times 2$ = Rs. 12

Fare by $B = 24.25 + \frac{33.25-24.25}{3} \times 2$ = Rs. 30.25

Total fare = 30.25 + 12 = Rs. 42.25.

(iii) (b) Fare, for 8 km by $A = 13.50 + \frac{17.25-13.50}{10-7}$

$= 13.50 + \frac{3.75}{3}$ = Rs. 14.75

Fare by $B = 24.25 + \frac{33.25-24.25}{3}$ = Rs. 27.25

Difference = 27.25 – 14.75 = Rs. 12.50.

(iv) (e) Fare by B for 5 km = 14.50 + 3.25 = Rs. 17.75

Fare by A for 8 km $= 13.50 + \frac{17.25-13.50}{3}$

= Rs. 14.75

Fare by C for 5 km $= 19 + \frac{31-19}{3}$ = Rs. 23

Total fare = 17.75 + 14.75 + 23 = 55.50.

(v) (b) Fare for 14th km by $C = \frac{56.50-41.50}{15-10}$ = Rs. 3

Fare for 9th km by $B = \frac{33.25-24.25}{10-7}$ = Rs. 3.

15. (i) (e)

Avg. runs of $A = \frac{994}{19} = 52.31$

Avg. runs of $B = \frac{751}{18} = 41.72$

Difference = 52.31 – 41.72 = 10.59.

(ii) (a) Avg. runs of $C = \frac{414-52}{17} = 21.29$.

(iii) (e) Difference between two highest runs

= 994 – 772 = 222

Difference between two lowest runs

= 653 – 414 = 239

Difference = 239 – 222 = 17.

(iv) (c) Ratio of A and D = 141 : 94 = 3 : 2.

(v) (e) Without knowing the individual runs of 8 openers, we cannot find the average runs of remaining batsmen.

16. (i) (a)

Average $= \frac{52+66+64+75+58}{5} = \frac{315}{5} = 63$.

(ii) (a) The difference is 9.

(iii) (a) Percentage increase $= \frac{55-46}{46} \times 100 \approx 20\%$.

(iv) (e) Average highest marks $= \frac{85+80+75}{3} = \frac{240}{3} = 80$.

(v) (e)

(vi) (c) Required percentage $= \frac{80}{64} \times 100 = 125\%$.

(vii) (b) Marks obtained by students = 50 × 60 = 3000.

(viii) (e) The maximum difference is in the years 1992 and 1997, since the least value is in 1992 and the highest value is in 1997.

17. (i) (a)

Total no. of students studying in all schools in 1992

= (1025 + 230 + 190 + 950 + 350 + 225 + 1100 + 320 + 300 + 1500 + 340 + 300 + 1450 + 250 + 280) – (120 + 110 + 150 + 115 + 130 + 150 + 150 + 160 + 125 + 130)

= 8810 – 1340 = 7470

$\therefore$ Average $= \frac{7470}{5} = 1494$.

(ii) (c) Number of students studying in school B in 1994

$= 950 + (350-150) + (225-115) + (185-110) + (200-90)$

= 950 + 200 + 110 + 75 + 110 = 1445.

(iii) (e) Number of students leaving school C from 1990 to 1995

$$= 130 + 150 + 125 + 140 + 180 = 725$$

Number of students admitted during the period

$$= 1100 + 320 + 300 + 260 + 240 + 310$$
$$= 2530$$

$\therefore$ Required percentage $= \frac{725}{2530} \times 100 \approx 29\%$.

(iv) (d) Required difference

$= (340+300+295+320+360)-(350+225+185+200+240)$

$= 1615 - 1200 = 415$.

(v) (b) Increase in no. of students in school $A = (230-120) + (190-110) + (245-100) + (280-150) + (250-130) = 585$

$\therefore$ % increase from 1990 (1025) to 1995

$$= \frac{585}{1025} \times 100 = 57.07\%.$$

Similarly, we can calculate for other schools.

18. (i) (a)

No. of females above poverty line in State A

$$= 3000 \times (100 - 12)\% \times \frac{3}{7} \approx 1200.$$

(ii) (d) Since we cannot find the population of States C and D separately, we cannot find the required value.

(iii) (e) Population of State A below poverty line

$$= 3000 \times \frac{5}{3} = 5000$$

$\therefore$ Total population of State $A = \frac{5000}{12} \times 100$

and the population of State E below poverty line

$$= 6000 \times \frac{11}{6} = 11000$$

$\therefore$ Total population of State $E = \frac{11000}{10} \times 100$

$\therefore$ Required ratio $= \frac{5}{12} \times \frac{10}{11} = \frac{25}{66}$.

(iv) (c) Total population of State B

$$= 500\left(\frac{12}{5}\right)\left(\frac{100}{15}\right) = 8000.$$

(v) (b) Population of State E

$$= 19800\left(\frac{5}{2}\right)\left(\frac{100}{100-10}\right) = 55000$$

$\therefore$ Population of males below poverty line.

$$= \frac{6}{11} \times 55000 = 30000.$$

19. (i) (e)

Required per cent $= \frac{152.2}{86.4} \times 100 \approx 175\%$.

(ii) (d)

(iii) (c) Average production of pulse

$$= \frac{20.5+22.4+24.6+23.5+27.8+28.2}{6}$$
$$= \frac{147.0}{6}$$

= 24.5 million tonne.

(iv) (a) Required percentage $= \frac{32.4}{450} \times 100 = 7.2\%$.

(v) (b) Total production of oilseeds in the given years

$$= 42.4 + 46.8 + 52.4 = 141.6$$

which is equal to the production of wheat in 1994–95.

20. (i) (e)

The difference between the white-coloured cars sold is the minimum in B type model.

(ii) (a) Blue $(E + D) = 37 + 43 = 80 =$ White (B).

(iii) (e) Reqd. difference $= (50 - 34) \times 1000 = 16000$.

(iv) (c) Reqd. percentage $= \frac{173}{192} \times 100 \approx 90\%$.

(v) (a) While-C Colour-model combination of car in Metro M.

21. (i) (d)

Total no. of students who play cricket

$$= 38 + 40 + 12 + 17 + 25 + 18 + 20 = 170$$

Reqd.% $= \frac{25}{170} \times 100 \approx 15\%$.

(ii) (d) Reqd. ratio = 27 : 18 = 3 : 2.

(iii) (e)

(iv) (e) Total Class X students who play different games = 115

Reqd. % $= \frac{21}{115} \times 100 \approx 18\%$.

(v) (e) Basketball and Badminton are the two games which satisfy the conditions.

22. (i) (a)

Maximum population shows the age group up to 15 years, but among the options it is not there. Hence, among the given options age group 16–25 is responsible for maximum population.

(ii) (a) Reqd. no. = 47.75% of 4200 ≈ 2006.

(iii) (b) No. of people in the age group 56–65

$$= \frac{200}{65} \times 5.12 \approx 15.75.$$

(iv) (a) Reqd. difference in the population

$$= \frac{10}{5.12} \times 3.50 \approx 6.8 \text{ million}.$$

(v) (b) Total population

$$= \frac{11.75}{3} \times 100 \approx 391.67 \text{ million}.$$

23. (i) (a)

Total Sales – Gross Profit in

1990 = 351.6 – 155.5 = 196.1;
1991 = 407.9 – 134.3 = 273.6
1992 = 380.1 – 149.9 = 230.2;
1993 = 439.7 – 160.5 = 279.2
1994 = 485.9 – 203.3 = 282.6

(ii) (d) Let 439.7 = K% of 351.6 ⇒ K = 125.

(iii) (a)

(iv) (d) Let 42.6 = K% of 439.7 ⇒ $K = \frac{4260}{439.7} = 9.7$.

(v) (d) Percent increase in the gross profit:

From 1991 to 1992: $\frac{15.6}{134.3} \times 100 = 100 = 11.6\%$

From 1992 to 1993: $\frac{10.6}{149.9} \times 100 = 7.07\%$

From 1993 to 1994: $\frac{42.8}{160.5} \times 100$ 26.67%.

24. (i) (b)

(ii) (c) Total production of rice
= 45 + 48 + 59 + 41 + 37 + 68 + 57 + 38
= 393 lakh tons

Total production of wheat
= 103 + 86 + 32 + 37 + 22 + 15 + 8 + 28
= 331 lakh tons

∴ Required proportion = 393 : 331 ≈ 12 : 1.

(iii) (d)

(iv) (d) Total wheat production in the country = 331 lakh tons and state P alone produces 103 lakh tons of wheat

∴ Required percentage = $\frac{103}{331} \times 100 \approx 30\%$.

(v) (c) Total rice production in the country = 393 lakh tons

Yield per hectare = 30 tons

∴ Area under rice cultivation = $\frac{393}{30} \approx 13$ lakh hectares.

25. (i) (d)

146947 – (65104 + 60387) = 21456.

(ii) (b) 146947 + 11630 = 158577.

(iii) (a) 158577 – (70391 + 62516) = 25670.

(iv) (a) 153922 – (5337 +21560 + 69395) = 57630.

(v) (b) 160998 – (153922 – 5337) = 12413.

26. (i) (c)

Ave. of the total prod. during the period = $\frac{476}{6} \approx 80$.

(ii) (d) Answer will be 1993.

(iii) (d)

(iv) (d) 25% of 80 = 20 = production of S's car in 1993.

(v) (b) Required per cent increase = $\frac{90-75}{75} \times 100 = 20\%$.

27. (i) (a)

Reqd. ratio = $\frac{8+9}{15+18} = 17 : 33$.

(ii) (e) Here, do not find the ratio of the no. of qualified candidates to that of the appeared. Simply check the ratio of % qualified candidates with respect to the appeared is the least for which state. Ans. = G.

(iii) (d) Reqd. difference = (21 – 13) % of 9000 = 720.

(iv) (b) Reqd. % = $\frac{(16+7)\% \text{ of } 9000}{(11+8)\% \text{ of } 45000} \times 100 = 24.21\%$.

(v) (c) Reqd. ratio = $\frac{(16+21)\% \text{ of } 9000}{8\% \text{ of } 45000}$.

28. (i) (d)

Reqd. ratio = $\frac{14\% \text{ of } 3800}{14\% \text{ of } 4200} = 19 : 21$.

(ii) (e) Reqd. no = 27% of 3800 = 1026.

(iii) (b) Reqd. no. = 31% of 4200 = 1302.

(iv) (c) Reqd. no. = 8% of 3800 + 17% of 4200
= 304 + 714 = 1018.

(v) (e) Reqd. ratio = $\frac{14\% \text{ of } 3800}{17\% \text{ of } 4200} = 19 : 17$.

29. (i) (c)
(ii) (a)
(iii) (d)
(iv) (a)
(v) (d)

30. (i) (a)

40% of 15000000 = 6000000.

(ii) (d) 12.5% of 12000000 = 1500000.

(iii) (b) 15 : 15 = 1 : 1.

(iv) (c)

(v) (b) 12.5% in each.

31. (i) (c)
(ii) (b)
(iii) (a)
(iv) (e)
(v) (d)

32. (i) (b)

Total cost = $\frac{2}{5} \times \frac{15}{100} \times 25 + \frac{4}{5} \times \frac{22}{100} \times 25$
= 1.5 + 4.4 = 5.9 cr.

(ii) (d) Amount of profit earned by company D on item II

$= \frac{5}{8} \times \frac{8}{100} \times 25 \times \frac{25}{100} = 31.25$ lakh

(iii) (e) Cost of production of item I by company F

$$= \frac{1}{5} \times \frac{5}{100} \times 25 = 0.25 \text{ cr}$$

Cost of production of item II by company D

$$= \frac{5}{8} \times \frac{8}{100} \times 25 = 1.25 \text{ cr}$$

$$\therefore \quad \text{Reqd. } \% = \frac{0.25}{1.25} \times 100 = 20\%.$$

(iv) (a) Total profit earned by company G

$$= \frac{1}{3} \times \frac{12}{100} \times 25 \times \frac{30}{100} + \frac{2}{3} \times \frac{12}{100} \times 25 \times \frac{24}{100}$$

$= 0.3 + 0.48 =$ Rs. 78 lakh.

(v) (c) Reqd. ratio $= \dfrac{\frac{2}{5} \times \frac{15}{100} \times 25}{\frac{3}{8} \times \frac{8}{100} \times 25} = 2 : 1.$

(vi) (b) Reqd. profit

$$= \frac{3}{5} \times \frac{11}{100} \times 25 \times \frac{32}{100} + \frac{3}{5} \times \frac{15}{100} \times 25 \times \frac{20}{100}$$

$= 0.528 + 0.450 =$ Rs. 97.8 lakh.

(vii) (d)

(viii) (a).

33. (i) (a)

Total no. of students for course D = 35% of 1200 = 420

No. of girl students for course D = 30% of 800 = 240

No. of boy students for course D = 420 – 240 = 180

Reqd. ratio = 180 : 240 = 3 : 4.

(ii) (c) No. of boys for difference courses are

$A = 0$; $B = 100$; $C = 44$; $D = 180$; $E = 32$; $F = 44$.

(iii) (a) No. of girls for course E = 14% of 800 = 112

No. of boys for course E = 32

$$\text{Reqd. more } \% = \frac{112 - 32}{32} \times 100 = 250\%.$$

(iv) (d) Using the information given in Q. No. (ii)

(v) (b) No. of girls in course C = 2 % of 800 = 16.

34. (i) (b)

23% of 46000 = Rs. 10580.

(ii) (a) 25% of 46000 = Rs. 11500.

(iii) (d) Required ratio = 15 : 12 = 5 : 4.

(iv) (a) 23%.

(v) (c) 15% of 46000 = Rs. 6900.

35. (i) (e)

$$\% \text{ Profit} = \frac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$$

or, $$45 = \frac{140 - E}{E} \times 100$$

or, $$\frac{140}{E} = \frac{45}{100} + 1 = \frac{9}{20} + 1 = \frac{29}{20}$$

$$\therefore \quad E = 140 \left(\frac{20}{29}\right) = 96.6 \text{ cr.}$$

(ii) (d) $I_{93} = E_{94} \left(\dfrac{100+50}{100}\right) = \dfrac{3}{2} E_{93}$

$$I_{94} = E_{94} \left[\frac{100+40}{100}\right] = \frac{7}{5} E_{94}$$

$$E_{93} + E_{94} = 279$$

But we cannot find $\dfrac{3}{2} E_{93} + \dfrac{7}{5} E_{94}$.

Hence, we cannot solve it.

(iii) (e) $E = I \left(\dfrac{100}{100+P}\right)$

or, $$\frac{E}{I} = \frac{100}{100+P} \qquad ....(1)$$

We require $\dfrac{E}{I} \le 50\%$ or, $\dfrac{E}{I} \le \dfrac{1}{2}$

Now, from (1), $\dfrac{100}{100+P} \le \dfrac{1}{2}$

So, the value of P should be more than 100, which is not correct for any of the given years.

(iv) (b) $I = E \left[\dfrac{100 + \% \text{ Profit}}{100}\right]$

$$= 200 \left(\frac{100+40}{100}\right) \text{ cr} = 280 \text{ cr.}$$

(v) (e) $I > 2E$

$\Rightarrow$ Profit % is more than 100, which is not correct for any of the given years.

36. (i) (b)

(ii) (c) Average percentage increase of the commodities

Jan	Feb	March	April	June
16.25%	23.75%	28.75%	35%	41.25%

(iii) (d)

(iv) (e) Sale of commodity C in July $= 100 \times \dfrac{95}{100} = 95.$

(v) (e)

37. (i) (b)

Let the investment of company B in 1996 be Rs x lakh.

$\therefore$ Investment of company B in 1997 = Rs. $\dfrac{7}{5}x$

Income of company B in 1997 $= \dfrac{9}{5} \times \dfrac{7}{5}x = \dfrac{63}{25}x$

(ii) (d) $\therefore$ Reqd. % $= \dfrac{63}{25} \times 100 = 252\%$

(iii) (e) Investment of company A in 1995

$$= 21.7 \times \frac{100}{155} = 14 \text{ lakh}$$

(iv) (c) Let $^{i}95_{(A)} = {}^{e}96_{(B)}$ = Rs. x lakh

$$\therefore \quad \text{Reqd. ratio} = \frac{x \times \frac{100}{155}}{x} = 20 : 31.$$

(v) (b) Income of company B in 1993

$$= 1540000 \times \frac{145}{100} = \text{Rs. } 22.33 \text{ lakhs.}$$

38. (i) (a)

Income of Company B in 2000

$$= 200 \times \frac{120}{100} = \text{Rs. } 240 \text{ cr.}$$

(ii) (c) Expenditure of Company A in 2002

$$= 600 \times \frac{100}{160} = \text{Rs } 375 \text{ cr.}$$

(iii) (d) We can find out the amount of profit in 1998, we do not know the income and expenditure of A and B, therefore option (d) is the correct choice.

(iv) (b) Ratio of their expenditures

$$= \frac{100}{135} \times \frac{130}{100} = 26 : 27.$$

(v) (a) Reqd. % increase $= \frac{35-20}{20} \times 100 = 75\%$.

39. (i) (e)

Suppose in the year 1998-99 expenditure of company X = Rs. a

Then profit earned by company X in this year

= Rs. (30% of a)

Hence, income of company X

= Rs. (130% of a)

Again, expenditure of company Y in 2001–02

$$= \text{Rs. } \frac{a \times 130}{100}$$

Hence, profit earned by company Y in 2001–02

$$= \text{Rs. } \frac{a \times 130}{100} \times \frac{50}{100}$$

Thus, required ratio

$$= \frac{\frac{30}{100} \times a}{\frac{a \times 130}{100} \times \frac{50}{100}} = \frac{30}{100} \times \frac{100 \times 100}{130 \times 50} = \frac{30}{65} = 6:13.$$

(ii) (c) $I_{x\ 2001\text{–}02} = E_{x2002\text{–}03} = \frac{I_{x2002\text{-}03}}{1.5}$

$$I_{x\ 2001\text{–}02} : 1_{x\ 2002\text{–}03} = \frac{2}{3} = 2 : 3.$$

(iii) (c) Per cent of increase in per cent profit over that of the previous year for the given years is as follows:

Year

1998–99 : $\frac{(20-15)}{15} \times 100 = 33.33\ \%$

1999–00 : $\frac{(30-20)}{20} \times 100 = 50\%$

2000–01 : $= 0\%$

2001–02 : $\frac{(50-30)}{30} \times 100 = 66\frac{2}{3}\%$

2002–03 : $\frac{(60-50)}{50} \times 100 = 20\%$

You do not need to do any rough work. See the graph and search for steep rise in the line joining the two Δ's.

(iv) (b) Required income

= 120% of Rs. 40 crore = Rs. 48 crore

(v) (d) The given graph depicts only the per cent profit earned by the two companies over the given years. Hence, these information are insufficient to answer the question.

(vi) (e) In 2002–03 profit earned by company Y was 60% Therefore, 160% of expenditure = Rs. 128 crore.

$$\text{Thus, required expenditure} = \frac{128}{160} \times 100$$

= Rs. 80 crores.

40. (i) (e)

Average imports made by company A

$$= \frac{30+50+60+40+70+60+75}{7} = \frac{385}{7} = 55$$

In none of the given years the imports is exactly equal to 55 (crore). Hence, the answer is (e).

(ii) (d) By visual inspection it is clear that 1992 is the desired year (as the distance between two points is the maximum in 1992).

(iii) (a) By mental observation $\left(\text{as } 50 = \frac{40+60}{2}\right)$, 1992 only is the desired year. You do not need any calculation. See the year where the point A lies exactly in the middle of points of B and C.

(iv) (b) Reqd percentage increase $= \frac{50-40}{40} \times 25\%$.

(v) (c) The total imports (in crore) made by all the three companies together: From the heights of the points we observe that the total heights of three points is the maximum either in 1995 or 1997. If you observe carefully, our clear answer is 1995, but to be sure we find actual values for the two years.

In 1995 = 70 + 80 + 85 = 235.

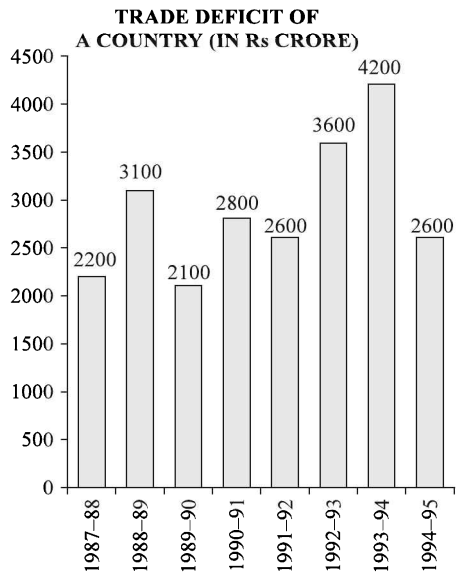
In 1997 = 75 + 70 + 85 = 230.

Clearly, 1995 is the desired year.

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. Directions: Examine the given graph and answer these questions.



(i) The deficit in 1993–94 was roughly how many times the deficit in 1990–91
(a) 1.4 (b) 1.5
(c) 2.5 (d) 0.5

(ii) The increase in deficit in 1993-94 was how much per cent of the deficit in 1989–90?
(a) 200 (b) 150
(c) 100 (d) 210

(iii) In which of the following years, the per cent increase of deficit was highest over its preceding year?
(a) 1992–93 (b) 1990–91
(c) 1993–94 (d) 1988–89

(iv) The ratio of the number of years, in which the trade deficit is above the average deficit, to those in which the trade deficit is below the average deficit, is
(a) 3 : 5 (b) 5 : 3
(c) 4 : 4 (d) 3 : 4

(v) The deficit in 1992–93 was approximately how much per cent of the average deficit?
(a) 150 (b) 140
(c) 125 (d) 90

2. Directions: Study the following table carefully and answer the questions given below.
Marks (out of 50) obtained by five students *P*, *Q*, *R*, *S* and *T* in Five periodical examination of each subject

Subject	Students														
	P					*Q*					*R*				
	Periodicals														
	I	II	III	IV	V	I	II	III	IV	V	I	II	III	IV	V
Maths	40	30	45	20	35	30	20	35	45	40	30	35	40	45	40
Science	30	40	25	30	20	25	45	30	37	28	48	46	31	40	80
History	35	25	15	30	40	33	27	40	34	26	35	45	40	30	35
Geography	45	47	32	39	37	42	43	30	40	25	23	35	48	37	25
English	24	28	36	39	43	30	28	37	34	31	26	28	31	30	40

Sub	Students									
	S					*T*				
	Periodicals									
	I	II	III	IV	V	I	II	III	IV	V
Maths	25	35	40	45	30	29	31	39	41	40
Science	31	34	38	27	30	44	36	40	30	40
History	34	40	36	42	48	37	43	35	45	40
Geography	39	37	44	40	30	38	39	33	40	40
English	31	34	35	45	40	30	30	35	45	40

(i) What was the average marks of the five subjects of student Q in the 1st periodical?
(a) 32 (b) 34
(c) 40 (d) 30
(e) None of these.

(ii) What was the total of marks of student T in Science in all the periodicals together?
(a) 160 (b) 180
(c) 190 (d) 140
(e) None of these

(iii) The average percentage of marks obtained by student P in Maths in the five periodicals was exactly equal to the average percentage of marks obtained by student R in the five periodicals in which of the following subjects?
(a) English
(b) Geography
(c) Science and Geography
(d) Maths
(e) None of these.

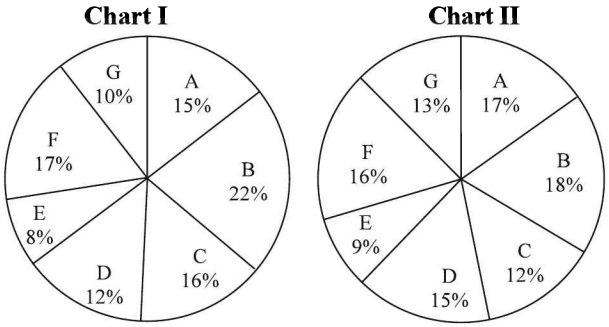
(iv) In which of the following subjects was the average percentage of marks obtained by student S the highest?
(a) Maths (b) Science
(c) History (d) Geography
(e) English

(v) In which of the periodicals the students P obtained highest percentage of marks in Geography?
(a) I (b) II
(c) III (d) IV
(e) V

3. Directions: Study the following graph carefully and answer the questions given below it.

Distribution of candidates enrolled for MBA (Chart I) and the candidates who have successfully completed the course (Chart II) from different institutes

Chart I **Chart II**



Candidates enrolled = 2500 Candidates who have successfully completed the course = 1900

(i) Which institute has the highest percentage of students successful to enrolled?
(a) D (b) E
(c) A (d) B
(e) None of these.

(ii) What per cent of candidates from institute D have come out to be successful out of the total candidates enrolled in this institute?
(a) 115 (b) 95
(c) 80 (d) 85
(e) None of these.

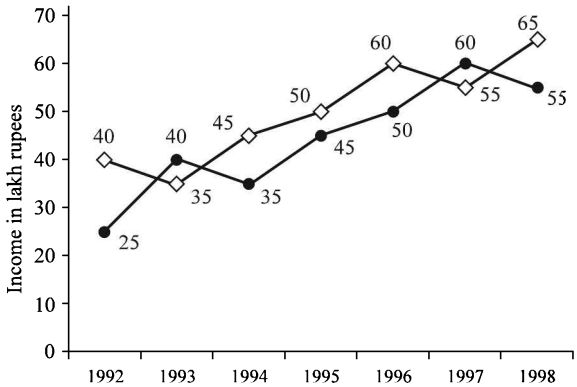
(iii) What is the approximate percentage of students successful to enrolled from institute A?
(a) 92 (b) 89
(c) 86 (d) 96
(e) 88.

(iv) What is ratio of candidates successful to enrolled from the institutes B and C together?
(a) 55 : 12 (b) 5 : 7
(c) 135 : 271 (d) 3 : 5
(e) None of these.

(v) What is the percentage of students successful from institute E and F together, over the students enrolled in these institutes together?
(a) 76 (b) 105
(c) 80 (d) 74
(e) None of these.

4. Directions: Study the following graph carefully and answer the questions given below.

Income of two plastic manufacturing Companies A and B over the years (in lakh rupees)



(i) If the per cent profit earned by both the companies A and B in 1997 is equal and the expenditure of company B in 1997 is Rs. 50 lakh, what approximately is the amount of profit earned by company A in 1997?
(a) Rs. 4.5 lakh (b) Rs. 5 lakh
(c) Rs. 6.2 lakh (d) Data inadequate
(e) Rs. 5.5 lakh.

(ii) For which of the following combinations of company and year is the percentage increase in income from the previous year the maximum among all such given combinations?
(a) Company B — 1994 (b) Company B — 1996
(c) Company A — 1993 (d) Company A — 1994
(e) Company A — 1997

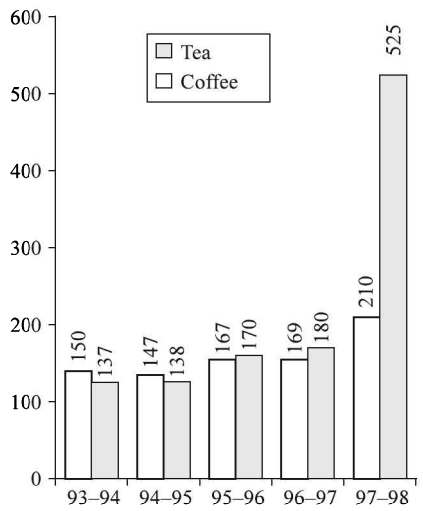
(iii) If company *A* had a loss of 15% in the year 1992, what approximately was its expenditure in that year?
(a) Rs. 22 lakh (b) Rs. 29 lakh
(c) Rs. 21 lakh (d) Rs. 28 lakh
(e) Rs. 23 lakh

(iv) Average income of company *A* per year **approximately** what percentage of the average income of company *B* per year?
(a) 80% (b) 110%
(c) 115% (d) 90%
(e) 75%.

(v) Income of company *B* in 1994 is what percent of income of company *A* in 1997?
(a) 75% (b) 133.33%
(c) 63.64% (d) 150%
(e) None of these.

5. Directions: Study the following graph carefully and answer the questions given below it.

Export ot Tea and Coffee (in million kg)



(i) By what **approximate percentage** the export of coffee increased from 1996 to 1998?
(a) 205 (b) 185
(c) 195 (d) 200
(e) 190.

(ii) What is the ratio of the export of coffee in 1994-95 to that in 1996-97?
(a) 69 : 85 (b) 30 : 23
(c) 85 : 69 (d) 23 : 30
(e) None of these.

(iii) What was the percentage increase in the export of tea in 1997–98 from that in 1993–94?
(a) 40 (b) 90
(c) 20 (d) 35
(e) None of these.

(iv) By what per cent did the export of tea fall in 1994–95 from that in the previous duration?
(a) 1 (b) 2
(c) 3 (d) 4
(e) None of these.

(v) What was the ratio between export of coffee to tea in 1997–98?
(a) 5 : 14 (b) 2 : 5
(c) 5 : 2 (d) 14 : 5
(e) 14 : 35.

Test II

[Time: 10 min]

6. Directions: Read the following Table carefully and answer the questions given below it.

Average marks obtained by 20 boys and 20 girls in five subject from five different schools

Subject	Max. Marks	*P*		*Q*		*R*		*S*		*T*	
		B	*G*	*B*	*G*	*B*	*G*	*B*	*G*	*B*	*G*
English	200	85	90	80	75	100	110	65	60	105	110
History	100	40	55	45	50	50	55	40	45	65	60
Geography	100	50	40	40	45	60	55	50	55	60	65
Mathematics	200	120	110	95	85	135	130	75	80	130	135
Science	200	105	125	110	120	125	115	85	90	140	135

(i) What was the total marks obtained by boys in History from school *Q*?
(a) 900 (b) 1000
(c) 800 (d) 1300
(e) None of these.

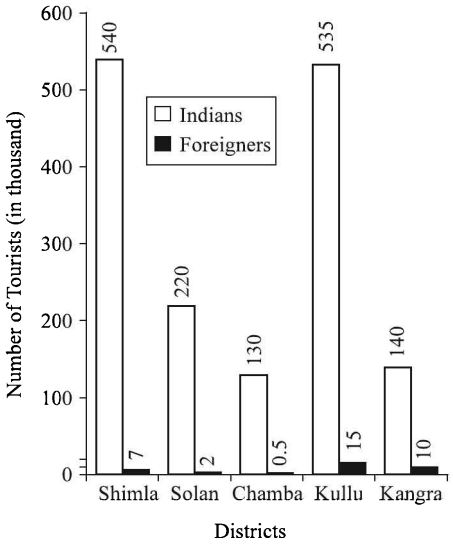
(ii) In which of the following subjects did the girls have highest average percentage of marks from all the schools?
(a) Science (b) Geography
(c) English (d) History
(e) Mathematics.

(iii) The pooled average marks of both boys and girls in all the subjects was minimum from which of the following schools?
(a) *Q* (b) *P*
(c) *T* (d) *S*
(e) *R*.

(iv) In the case of which of the following schools was total marks obtained by girls in Mathematics 100% more than the total marks obtained by boys in History?
(a) *R* (b) *S*
(c) *P* (d) *Q*
(e) *T*.

(v) What was the difference between the total marks obtained in Mathematics by boys from school *R* and the girls from school *S*?
(a) Nil (b) 1100
(c) 100 (d) 1200
(e) None of these.

7. Directions: Study the following graph carefully and answer the question given below it.



(i) How many districts in Himachal Pradesh were visited by more than 10% of the total Indian tourists?
(a) 5 (b) 3
(c) 4 (d) 2
(e) None of these.

(ii) By what percentage the Indian tourists visiting Chamba were less than those visiting Shimla?
(a) 50 (b) 55
(c) 60 (d) 70
(e) 75.

(iii) **Approximately** what percentage was shared by total foreign tourists among all the tourists visiting Himachal Pradesh?
(a) 2 (b) 8
(c) 4 (d) 5
(e) 6.

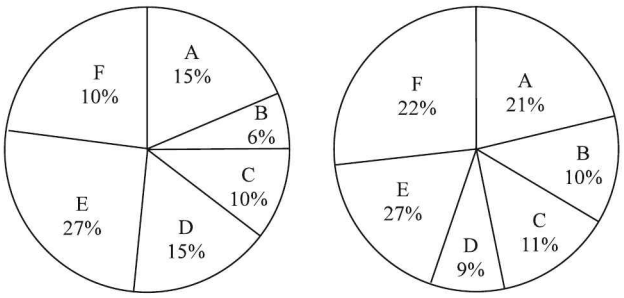
(iv) What was the ratio between the Indian tourists and the foreign tourists visiting Kullu?
(a) 105 : 3 (b) 70 : 3
(c) 107 : 3 (d) 35 : 1
(e) None of these.

(v) Which of the following districts were visited by less than 10% of the total foreign tourists who visited Himachal Pradesh?
(a) Chamba, Kullu
(b) Solan, Kangra
(c) Solan, Chamba, Shimla
(d) Solan, Chamba
(e) None of these.

8. Directions: Study the following graph carefully and answer the questions given below.

Percentage of Different Types of Employees in a Company in two successive years

Total No. of Employees in 1997 and 1998 is 42980 and 48640, respectively



(i) The number of *A* type employees in 1998 was approximately what percentage of *A* type employees in 1997?
(a) 115 (b) 140
(c) 14 (d) 130
(e) 95.

(ii) If there were 5000 *D* type employees in 1998 what would have been its approximate percentage in the company?
(a) 8 (b) 12
(c) 14 (d) 16
(e) 10.

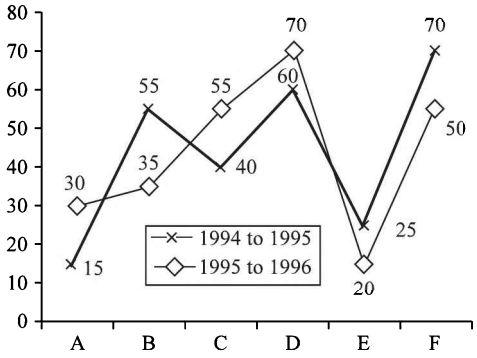
(iii) In the case of which of the following types of employees was there maximum change in the number from 1997 to 1998?
(a) *B* (b) *D*
(c) *C* (d) *A*
(e) None of these.

(iv) **Approximately**, what was the difference in the number of *B* type employees between 1997 and 1998?
(a) 2285 (b) 2325
(c) 2085 (d) 4620
(e) 1825.

(v) The total number of which of the following pairs of types of employees in 1997 was approximately equal to *A* type employees in 1998?
(a) *B* and *C* (b) *A* and *C*
(c) *D* and *E* (d) *C* and *D*
(e) *C* and *F*.

9. Directions: Study the following graph carefully and answer the questions given below.

Percentage growth in population of six States *A. B, C, D, E* and F from 1994 to 1995 and 1995 to 1996



(i) Population of State *F* in 1995 was **approximately** what per cent of its population in 1996?
(a) 60 (b) 67
(c) 75 (d) 55
(e) Data inadequate.

(ii) If the population of state *B* in the year 1994 was 5 lakh, what was **approximately** its population in the year 1996?
(a) 9.50 lakh (b) 8 lakh
(c) 10.50 lakh (d) 14.50 lakh
(e) 11 lakh

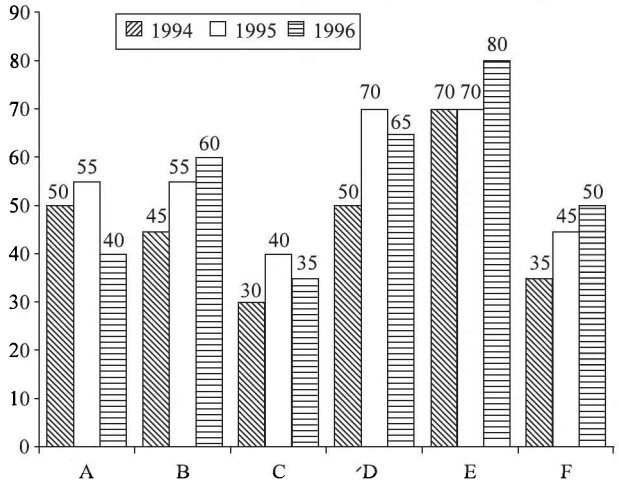
(iii) If the population of States *C* and *D* in 1995 are in the ratio of 2 : 3, respectively and the population of state *C* in 1994 was 2.5 lakh, what was the population of state *D* in 1995?
(a) 5.25 lakhs (b) 4.75 lakhs
(c) 3.50 lakhs (d) 6 lakhs
(e) None of these.

(iv) In 1994 the population of States *B* and *D* are equal and the population of State *B* in 1996 is 4 lakh, what **approximately** was the population of state *D* in 1996?
(a) 3 lakh (b) 3.50 lakh
(c) 6 lakh (d) 5 lakh
(e) 4.50 lakh

(v) Population of state *E* in 1994 was what fraction of its population in 1996?
(a) $\frac{4}{5}$ (b) $\frac{3}{2}$
(c) $\frac{5}{8}$ (d) $\frac{3}{4}$
(e) $\frac{2}{3}$

10. Directions: Study the following graph carefully and answer the questions that follow.

Production of steel by different companies in three consecutive years (in lakh tonne)



(i) What is the difference between average production of the six companies in 1995 and average production of the same companies in 1994?
(a) 7,05,000 tonnes (b) 7,50,000 tonnes
(c) 75,000 tonnes (d) 75,00,000 tonnes
(e) None of these.

(ii) What is the percentage decline in production by company *C* from 1995 to 1996?
(a) 5% (b) 20%
(c) 15% (d) 12.50%
(e) None of these.

(iii) Which of the following companies recorded the minimum percentage growth from 1994 to 1995?
(a) *A* (b) *B*
(c) *C* (d) *D*
(e) *E*.

(iv) Production of company *C* in 1995 and production of company *F* in 1994 together is what per cent of production of *B* in 1996?
(a) 80% (b) 75%
(c) 133% (d) 120%
(e) None of these.

(v) If which of the following pairs of companies the difference between average production for the three years in maximum?
(a) *E* and *F* (b) *D* and *C*
(c) *E* and *C* (d) *A* and *E*
(e) None of these.

EXPLANATORY ANSWERS

1. (i) (b)

$4200 = k \times 2800 \Rightarrow k = \frac{4200}{2800} = \frac{3}{2} = 1.5$

(ii) (a) $4200 = k\%$ of $2100 \Rightarrow k = 200$.

(iii) (d) Per cent increase in deficit in 1992–93

$$= \frac{1000}{2600} \times 100 = \frac{500}{13} = 38\frac{6}{3}\%$$

In 1990–91, $\frac{700}{2100} \times 100 = 33\frac{1}{3}\%$

In 1993–94, $\frac{600}{3600} \times 100 = 16\frac{2}{3}\%$

In 1988–89 , $\frac{900}{2200} \times 100 = 40\frac{10}{11}\%$.

(iv) (a) Average deficit

$$= \frac{2200+3100+2100+2800}{2600+3600+4200+2600}$$

$$= \frac{23200}{8} = 2900$$

$\therefore$ Required ratio = 3 : 5.

(v) (c) $3600 = k\%$ of 2900, say

$$\Rightarrow k = \frac{3600}{29} = 124.13 \approx 125.$$

2. (i) (a)

Average marks of Q in 1st periodical

$$= \frac{30+25+33+42+30}{5} = \frac{160}{5} = 32.$$

(ii) (c) Total marks of T in Science

$$= 44 + 36 + 40 + 30 + 40 = 190$$

(iii) (b) Average percentage of marks obtained by P in Maths

$$= \frac{80+60+90+40+70}{5} = 68\%$$

= percentage of marks obtained by student R in Geography.

(iv) (c) Our observation finds two options which are close to each other. These are History and Geography. When we find the actual value, we find that our answer is History.

(v) (b)

3. (i) (e) G (98.8%).

(ii) (b)

(iii) (c)

(iv) (d) $\frac{(342+228)}{(550+400)} = \frac{570}{950} = \frac{3}{5}$.

(v) (a) $\frac{(171+304)}{(200+425)} \times 100 = \frac{475\times100}{625} = 76\%$.

4. (i) (e)

According to the question,

$$\frac{I_{97(A)} - E_{97(A)}}{E_{97(A)}} = \frac{I_{97(B)} - E_{97(B)}}{E_{97(B)}}$$

or, $$\frac{I_{97(A)}}{E_{97(A)}} - 1 = \frac{I_{97(B)}}{E_{97(B)}} - 1$$

or, $$\frac{I_{97(A)}}{E_{97(A)}} = \frac{I_{97(B)}}{E_{97(B)}}$$

$\therefore$ $E_{97(A)} = \frac{60\times50}{55} = 54.50$ lakh

$\therefore$ Profit earned by company A in 1997

$I_{97(A)} - E_{97(A)} = 60 - 54.50 = 5.50$ lakh

(ii) (c) Percentage increase in income

B-94	B-96	A-93	A-94	A-97
28.5%	20%	60%	(–) 12.5%	20%

(iii) (b) Expenditure of A in 1992

$$= 25 \times \frac{100}{85} \approx 29 \text{ lakh}$$

(iv) (d) Required percentage = $\frac{310}{350} \times 100 \approx 90\%$.

(v) (a) Required percentage = $\frac{45}{60} \times 100 = 75\%$.

5. (i) (e)

Percentage increase in export of coffee in 1997–98

$$= \frac{525-180}{180} \times 100 \approx 190\%.$$

(ii) (d) Ratio of export of coffee in '94–95 and 96–97 = 138 : 180 = 23 : 30.

(iii) (a) Percentage increase in export of tea in 97–98

$$= \frac{210-150}{150} \times 100 = 40\%$$

(iv) (b) Fall in export of tea in 94–95

$$= \frac{150-147}{150} \times 100 = 2\%$$

(v) (c) Ratio of coffee to tea in 92 – 98

$$= 525 : 210 = 5 : 2$$

6. (i) (a)

Average marks obtained by 20 boys in History from school Q = 45

$\therefore$ Total marks = 20 × 45 = 900.

(ii) (a) From visual inspection it is clear that Science is the desired subject.

(iii) (d) Total marks obtained by boys and girls in all the subjects:

For school P = (85 + 40 + 50 + 120 + 105)
+ (90 + 50 + 40 + 110 + 125) = 815

Similarly, for Q = 745, for R = 935, for S = 645 and for T = 1005.

645 is the minimum, so S is the desired school.

(iv) (b) As the no. of boys and girls in the different schools are the same, so for the desired purpose we have to select a certain school in which the average marks of girls in Mathematics be exactly double the average marks of boys in History. By visual inspection (as 80 = 2 × 40) we get that S is the desired school.

(v) (b) In Mathematics total marks obtained by boys from school R = 135 × 20

By girls from school S = 80 × 20

∴ Reqd. difference = (135 – 80) = 55 × 20 = 1100.

7. (i) (b)

Total Indian tourists

= 540 + 220 + 130 + 535 + 140

= 1565 thousand

10% of Indian tourists = $\frac{1565\times10}{100}$ = 156.5 thousand

(ii) (e) Required % = $\frac{540-130}{540}\times100\approx75\%$

(iii) (a) Percentage share of foreign tourists visiting HP

$= \frac{34.5}{1600}\times100\approx2\%.$

(iv) (c) Ratio = 535 : 15 = 107 : 3

(v) (d) 10% of foreign tourists = $\frac{10}{100}\times34.5 = 3.45.$

8. (i) (c)

Reqd. percentage = $\frac{48,640\times22}{42,980\times20}\times100 = 125\%.$

(ii) (e) % of D-type of employees in 1998

$= \frac{5,000}{48,640}\times100\approx10\%.$

(iii) (a)

(iv) (a) Reqd. difference

$= 48,640\times\frac{10}{100} - 42,980\times\frac{6}{100}$

= 4864 – 2579 = 2285.

9. (v) (d)

Let the population of State F in 1995 = 100

Then, population of State F in 1996 = 150 (Since growth is 50%)

∴ Required % = $\frac{100}{150}\times100\approx67\%.$

(ii) (c) Population of State B in the year 1996

$= 5\times\frac{155}{100}\times\frac{135}{100}\approx10.50$ lakh.

(iii) (a) $\frac{C_{95}}{D_{95}} = \frac{2}{3}$ and C_{94} = 2.50 lakh

∴ $C_{95} = 2.5\times\frac{140}{100} = 3.50$ lakh

∴ The population of D in 1995

$= \frac{3\times3.50}{2} = 5.25$ lakh.

(iv) (d) Population of state B in 1994

$= \frac{4\times100\times100}{155\times135}$ = Population of D in 1994

∴ Population of state D in 1996

$= \frac{4\times100\times100\times160\times170}{100\times100\times155\times135}\approx5$ lakh.

(v) (e) Suppose population of State E in 1994 = 100

Then, population of State E in 1995 = 125 (since 25% growth)

and population of State E in 1996 = $125\left(\frac{120}{100}\right)$

= 150 (since 20% growth)

∴ Required ratio = $\frac{100}{150} = \frac{2}{3}.$

10. (i) (e)

Required difference

$= \frac{335-280}{6} = 916666$ tonne.

(ii) (d) Percentage decline = $\frac{40-35}{40}\times100 = 12.50\%.$

(iii) (a) It is clear from the graph. But see the Table which shows the percentage growth of six companies from 1994 to 1995.

A	*B*	*C*	*D*	*F*
10%	22.22%	33.33%	40%	28.57%

(iv) (e) Required percentage = $\frac{40+35}{60}\times100 = 12.5\%.$

(v) (c) It is clear that the highest average production is in E and lowest average production is in C. So, the maximum difference would be in E and C.

31

DATA SUFFICIENCY

INTRODUCTION

Data sufficiency problems as the name suggests test the ability of the students to judge whether the data given in the form of statements is sufficient to answer the question asked. There is no need to solve the problem. All you need to determine is whether it would be possible to answer the question. As soon as you can tell that an answer would be obtainable, you can stop working.

In each of these problems, a question is asked followed by two or three statements. You have to study the question and all the statements given and decide whether any information provided in the statement (s) is/are redundant and can be dispensed with while answering the questions. You have to decide whether the question can be answered with any one or two of the statements or all the three statements are required to answer the question. The answer number bearing the combination of statements or single statement which is necessary to answer the question is your answer.

Illustration 1 What is the perimeter of a retangular garden?

A. The area of the garden is 2400 sq. m.
B. The diagonal of the garden is 50 m.
C. The ratio between the length and the breadth of the garden is 3 : 2.

(a) All A, B and C together are required
(b) Any two of A, B and C are sufficient
(c) Only A and B are required
(d) Only B and C are required
(e) None of these.

Ans. (b)

Solution Let the length and breadth of rectangle be l and b, respectively.

$A \rightarrow l \times b = 2400.$

$B \rightarrow l^2 + b^2 = 2500$

$C \rightarrow l = \frac{3}{2}b.$

Solving any two of the above equations, we get the values of l and b.

Illustration 2 Two friends Anu and Manu earned profit in a business. Find out their shares.

A. Anu had invested her capital for 9 months and Manu for 1 year
B. The ratio of their capitals was 4 : 3
C. The total profit was Rs. 27500

(a) Only A and B together are sufficient
(b) Only B and C together are sufficient
(c) All together are necessary
(d) Either B or A and C together are sufficient
(e) All even together are not sufficient

Ans. (c)

Solution From (A) and (B), we have

Ratio of profits = 9 × 4 : 12 × 3 = 36 : 36 = 1 : 1.

Now, with the help of (C), share of each of them

$$= \frac{27500}{1+1} \times 1 = \text{Rs. } 13750.$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

Directions (1–50): Each of the questions below consists of a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statements are sufficient to answer the question. Read both the statements and...

Give answer (a) If the data in statement I alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.

Give answer (b) If the data in statement II alone

are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.

Give answer (c) If the data either in statement I alone or in statement II alone are sufficient to answer the question.

Give answer (d) If the data even in both statements I and II together are not sufficient to answer the question.

1. What will be the cost of the second necklace?

I. The cost of the first necklace is $\frac{1}{5}$ more than the second and the cost of the third necklace is $\frac{2}{5}$ more than the second. The total cost of all the three necklaces is Rs.120000.

II. The cost of the first necklace is $\frac{2}{5}$ more than the second. The cost of the third necklace is the least and total cost of all the three necklaces is Rs. 120000.

2. How many items did the distributor purchase?

I. The distributor purchased all the items for Rs 4500.

II. If the distributor had given Rs 5 more for each item he would have purchased 10 items less.

3. How long will it take to fill a tank?

I. One pipe can fill the tank completely in 3 hours.

II. Second pipe can empty that tank in 2 hours.

4. What will be the area of a plot in sq. metres?

I. The length of that plot is $1\frac{2}{3}$ times the breadth of that plot.

II. The diagonal of that plot is 30 metres.

5. How much minimum marks will be required to pass an examination?

I. Student *A* secured 32% marks in that examination and he failed by 1 mark. Student *B* secured 36% marks in the same examination and his marks were 1 more than the minimum pass marks.

II. Student *A* secured 30% of full marks in the examination and he failed by 2 marks. If he had secured 5 more marks his percentage of marks would have been 40%.

6. What will be the cost of painting of the inner wall of a room if the rate of painting is Rs 20 per sq.m?

I. Perimeter of the floor is 44 feet.

II. Height of the wall of the room is 12 feet.

7. What is the ratio of the number of boys and girls in a school?

I. Number of boys is 40 more than the girls.

II. Number of girls is 80 per cent of the number of boys.

8. What is the difference between two numbers?

I. First number is 60 per cent of the other number.

II. 50 per cent of the sum of first and second numbers is 24.

9. What was the speed of the running train?

I. Length of the train was 120 m.

II. The train crossed the other train whose length was 180 m in 4 sec.

10. What will be the compound interest after 3 years?

I. Rate of interest is 5 per cent.

II. The difference between the total simple interest and the total compound interest after two years is Rs 20.

11. How my boys are there in the class?

I. The class has total 45 children and ratio of boys to girls is 4 : 5.

II. The ratio of girls to boys is 4 : 5 and boys are nine more than the girls.

12. What is the average monthly income per family member?

I. Each male earns Rs 1250 a month and each female earns Rs 1050 a month.

II. Ratio of males to females in the family is 2 : 1.

13. What is the value of $m - n \div 37$?

I. m is the largest possible six-digit number and n is the smallest possible six-digit number.

II. The difference between m and n is known.

14. What selling price should be marked on the article?

I. Discount of 5% is to be given and profit percentage should be double the discount. Purchase cost is in the range of Rs 300 Rs 400.

II. 10% discount is to be allowed and 15% profit is to be obtained on the purchase cost of Rs 200 of the article.

15. What is the cost of polishing the rectangular floor?

I. Room is 9 m long and 7 m wide.

II. Cost of polishing the floor of 10 m by 5 m is Rs 112.50.

16. How many marks did Prakash obtain in Mathematics?

I. Prakash secured on an average 55 per cent marks in Mathematics, Physics and Chemistry together.

II. Prakash secured 10 per cent more than the average in Mathematics.

17. What is the rate of compound interest on a sum of money?

I. The total compound interest at the end of two years is Rs 820.

II. The total simple interest at the same rate on

Rs 5000 at the end of three years is Rs 750.

18. Which is the smaller of the two numbers?

I. The difference between these two numbers is one-third of the largest number.

II. The sum of these two numbers is 30.

19. What is the height of a right-angled triangle?

I. The area of the right-angled triangle is equal to the area of a rectangle whose breadth is 12 cm.

II. The length of the rectangle is 18 cm.

20. What is the speed of a running train which takes 9 sec to cross a signal post?

I. The length of the train is 90 m.

II. The train takes 27 sec to cross a platform of 180 m.

21. What was the ratio between the ages of P and Q four years ago?

I. The ratio between the present ages of P and Q is 3 : 4.

II. The ratio between the present ages of Q and R is 4 : 5.

22. What was the cost price of the suitcase purchased by Samir?

I. Samir got 20 per cent concession on the labelled price.

II. Samir sold the suitcase for Rs 2000 with 25 per cent profit on the labelled price.

23. What is the height of a triangle?

I. The area of the triangle is 20 times its base.

II. The perimeter of the triangle is equal to the perimeter of a square of 10 cm side.

24. What percentage rate of simple interest per annum did Ashok pay to Sudhir?

I. Asholk borrowed Rs 8000 from Sudhir for four years.

II. Ashok returned Rs. 8800 to Sudhir at the end of two years and settled the loan.

25. What is the speed of a running train?

I. The train crosses a signal post in 6 sec.

II. The train crosses another train running in the opposite direction in 15 sec.

26. Train 'A' running at a certain speed crosses another train 'B' running at a certain speed in the opposite direction in 12 sec. What is the length of train 'B'?

I. The length of both the trains together is 450 m.

II. Train 'A' is slower than train 'B'.

27. The area of a rectangle is equal to the area of a right-angled triangle. What is the length of the rectangle?

I. The base of the triangle is 40 cm.

II. The height of the triangle is 50 cm.

28. What was the total compound interest on a sum after three years?

I. The interest after one year was Rs 100 and the sum was Rs 1000.

II. The difference between simple and compound interest on a sum of Rs 1000 at the end of two years was Rs 10.

29. What is the two-digit number where the digit at the unit's place is smaller?

I. The difference between the digits is 5.

II. The sum of the two digits is 7.

30. What is the speed of the boat in still water?

I. It takes 2 hours to cover the distance between A and B downstream.

II. It takes 4 hours to cover the distance between A and B upstream.

31. What is the rate of simple interest per annum?

I. The sum triples in 20 years at simple interest.

II. The difference between the sum and the simple interest earned after 10 years is Rs 1000.

32. What is the sum which earned interest?

I. The total simple interest was Rs 7000 after 7 years.

II. The total of sum and simple interest was double of the sum after 5 years.

33. A train crosses a signal post in X sec. What is the length of the train?

I. The train crosses a platform of 100 m in Y sec.

II. The train is running at the speed of 80 km/hr.

34. What is the area of a circle?

I. The circumference of the circle is 308 m.

II. The radius of the circle is 28 m.

35. A, B and C are integers. Is B an even number?

I. $(A + B)$ is an odd number.

II. $(C + B)$ is an odd number.

36. How many children are there in the class?

I. Numbers of boys and girls are in the respective ratio of 3 : 4.

II. Number of girls is more than the number of boys by 18.

37. What was the population of State 'A' in 1999?

I. Population of the State increases every year by 20% and its population is 1997 was 120000.

II. Population of State A in 1997 was twice that of State B in the same year.

38. What is the cost of laying carpet in a rectangular hall?

I. Cost of the carpet is Rs 450 per sq. m.

II. Perimeter of the hall is 50 metre.

39. What is the rate of interest p.c.p.a.?

I. Difference between compound interest and simple interest on an amount of Rs 10,000 for two years is Rs 225.

II. The amount doubles itself on simple interest in 6 $\frac{2}{3}$ years.

40. What is a two-digit number?

I. The number obtained by interchanging the digits is smaller than the original number by 63.

II. Sum of the digits is 11.

41. By selling a product for Rs 100 how much profit was earned?

I. 20% profit would have been earned if it had been sold for Rs 90.

II. The profit was one-third of the purchase price.

42. A train crosses another train running in the opposite direction in x seconds. What is the speed of the train?

I. Both the trains are running at the same speed.

II. The first train is y cm long.

43. The difference between the two digits of a number is 6. What is the number?

I. The digit at the units place is bigger than the other digit.

II. The sum of the two digits is 12.

44. X, Y and Z are integers, Is X an odd number?

I. An odd number is obtained when X is divided by 5.

II. $(X + Y)$ is an odd number.

45. What is the capacity of a cylindrical tank?

I. Radius of the base is half of its height, which is 28 m.

II. Area of the base is 616 sq. m. and height is 28 m.

46. What will be the compounded amount?

I. Rs 200 were borrowed for 192 months at 6% compounded monthly.

II. Rs 200 were borrowed for 16 years at 6%.

47. What would have been the selling price per kg of rice?

I. 50 kg of rice was purchased for Rs 3350 and Rs 150 was spent on transport.

II. Profit earned was 5%.

48. What will be ratio of men to women and children in the town?

I. Population of the town is 93280 of which 56100 are men.

II. The ratio of men to children is 5 : 2 and women are double in number than the children.

49. What will be the average weight of the remaining class?

I. Average weight of 30 children out of total 46 in the class is 22.5 kg and that of the remaining children is 29.125 kg. A child having weight more than 40 kg is excluded.

II. Average weight of a class of 46 children is 23.5 kg. A child weighing 46 kg is dropped out.

50. What will be the number?

I. One-fifth of a number is equal to 20% of that number.

II. Thirty-five per cent of a number is $\frac{7}{20}$ of that number.

Directions (51–70): Each of the questions below consists of a question and three statements numbered I, II and III given below it. You have to study the questions and decide the data in which of the statements are sufficient to answer the questions.

51. What is the speed of the train 'A'?

I. Train A crosses 200-metre-long train B running in opposite direction in 20 sec.

II. Speed of the train B is 60 kmph.

III. Length of train A is twice that of train B.

(a) I and II only (b) II and III only
(c) I and III only (d) All, I II and III
(e) Question cannot be answered even with information in all three statements.

52. What is the area of the isosceles triangle?

I. Perimeter of the triangle is 14 m.

II. Base of the triangle is 14 m.

III. Height of the triangle is 5 m.

(a) I and II only
(b) II and III only
(c) I and II only or, II and III only
(d) I and III only
(e) All I, II and III

53. Who earns most among M, N, P, Q and R?

I. M earns less than P but not less than R.

II. Q earns more that M but not equal to N.

III. N earns more than M and R.

(a) Question cannot be answered even with information in all three statements
(b) I and II only
(c) Only I and II or only I and III
(d) Only I and III
(e) All the three statements I, II III together are necessary for answering the question

54. What is the price of 1 dozen oranges?

I. Price of 2 dozen oranges and 1 dozen banana is Rs 110.

II. Price of 3 dozen apples and 1 dozen banana is Rs 170.

III. Price of 1 dozen oranges and 1 dozen apples is Rs 95.

(a) Only I and II or, only I and III
(b) Only I and III or, only II and III
(c) Only I and II or, only II and III
(d) Only II and III
(e) All the three statements I, II and III necessary for answering the question.

55. The cost of carpeting a rectangular hall will be how much?

I. Perimeter of a rectangle is 60 m.
II. Angle between width and hypotenuse is 60°.
III. The cost of carpeting the surface floor is Rs 125 per sq. m.

(a) Only I and II
(b) Only II and III
(c) Only I and III or, only II and III
(d) Question cannot be answered even with information in all three.
(e) All the three statements I, II and III together are necessary for answering the question.

56. What is Sudha's present salary?

I. The salary increases every year by 15%.
II. Her salary at the time of joining was Rs 10000.
III. She had joined exactly 5 years ago.

(a) II and III only (b) I and II only
(c) All I, II and III (d) I and III only
(e) None of these

57. What was the amount of profit earned?

I. 10% discount was offered on the labelled price.
II. Had there been not discount, profit would have been 30%.
III. Selling price was more than the cost price by 20%.

(a) I and either II or III
(b) Any two of the three
(c) All I, II and III
(d) Either I or II and III
(e) Question cannot be answered even with the information in all three statements.

58. How many students are there in all in the institute of Arts, Commerce and Science?

I. 20% of the students study Science.
II. The numbers of students studying Arts and Commerce are in the ratio of 3 : 5.
III. The number of students studying Commerce is more than that studying Science by 375.

(a) II and III only
(b) III and either I or II only
(c) Any two of the three
(d) All I, II and III
(e) Question cannot be answered even with the information in all three statements.

59. What is the cost of flooring a rectangular hall?

I. Perimeter of the hall is 76 m
II. Area of the hall is 336 m^2.
III. Cost of flooring per square metre is Rs 550.

(a) I and III only
(b) II and III only
(c) Any two of the three
(d) All I, II and III
(e) None of these.

60. In how many days can a work be completed by *A* and *B* together?

I. A alone can complete the work in 8 days.
II. If *A* alone works for 5 days and *B* alone works for 6 days, the work gets completed.
III. *B* alone can complete the work in 16 days.

(a) Any two of the three
(b) II and either I or III
(c) I and II only
(d) II and III only (e) None of these.

61. What is the cost of flooring a rectangular hall?

I. The length and the breadth of the hall are in the ratio of 3 : 2.
II. The length of the hall is 48 m and the cost of flooring is Rs 850 per sq. m.
III. The perimeter of the hall is 160 m and the cost of flooring is Rs 850 per sq. m..

(a) Only I and II
(b) Only I and III
(c) Only III
(d) Only I and either II or III
(e) Any two of the three.

62. What is the rate of interest p.c.p.a.?

I. The amount doubles itself in 5 yrs on simple interest.
II. Difference between the compound interest and the simple interest earned on this amount in two years is Rs 400.
III. Simple interest earned per annum is Rs 2000.

(a) Only I
(b) Only II and III
(c) Any two of the three
(d) All I, II and III
(e) Only I or only II and III

63. What is a two-digit number?

I. The difference between the two-digit number and the number formed by interchanging the digits is 27.
II. The difference between the two digits is 3.
III. The digit at unit's place is less than that at ten's place by 3.

(a) Only I and II

(b) Only I and either II or III
(c) Only I and III
(d) All I, II and III
(e) Even with all the three statements the answer cannot be given.

64. What is the present age of Subir?
I. The present age of Subir is half that of his father.
II. After 5 yrs the ratio of Subir's age to his father's will be 6 : 11.
III. Subir is 5 yrs younger than his brother.
(a) Only I and II (b) Only I and III
(c) Only II and III (d) All I, II and III
(e) Even with all the three statements answer cannot be given.

65. In how many days can 10 women finish a work?
I. 10 men can complete the work in 6 days.
II. 10 men and 10 women together can complete the work in $3\frac{3}{7}$ days.
III. If 10 men work for 3 days and thereafter 10 women replace them, the remaining work is completed in 4 days.
(a) Only I and II
(b) Any two of the three
(c) Only I and III
(d) Only II and III
(e) None of these.

66. In how many days can a work be completed by *A*, *B* and *C* together?
I. *A* and *B* together can complete the work in 6 days.
II. *B* and *C* together can complete the work in 3¾ days.
III. *A* and *C* together can complete the work in $3\frac{1}{3}$ days.
(a) Only I (b) Only II
(c) Only III (d) Any one of the three
(e) Information in all the three statements is necessary to answer the question.

67. What is the cost of panting the two adjacent walls of a hall which has no windows or doors?
I. The area of the hall is 24 sq. m.
II. The breadth, length and the height of the hall are in the ratio of 4 : 6 : 5.
III. Area of one wall is 30 sq. m.
(a) Only I (b) Only II
(c) Only III (d) Either I or III
(e) Data inadequate.

68. What is the total compound interest earned at the end of three years?
I. Simple interest earned on that amount at the same rate and for the same period is Rs 4500.
II. The rate of interest is 10 p.c.p.a.
III. Compound interest for three years is more than the simple interest for that period by Rs 465.
(a) Only I and II (b) Only II and III
(c) Only I and III (d) Any one of the three
(e) Either II or III only.

69. What is the per cent profit earned by a shopkeeper on selling the articles in his shop?
I. Labelled price of the articles sold was 130% of the cost price.
II. Cost price of each article was Rs 550.
III. A discount of 10% on labelled price was offered.
(a) Only I (b) Only II
(c) Only III
(d) All the three are required.
(e) Question cannot be answered even with the information in all the three statements.

70. What is the average salary of 15 employees?
I. Average salary of 7 clerical cadre (out of the 15 employees) employees is Rs 8500.
II. Average salary of 5 officer cadre (out of the 15 employees) employees is Rs 10000.
III. Average salary of the 3 sub-staff employes (out of the 15 employees) is Rs 2500.
(a) None (b) Only I
(c) Only II (d) Only III
(e) Question cannot be answered even with information in all three statements.

Directions (71–85): In each of the following questions a question is asked followed by three statements. You have to study the questions and all the three statements given and decide whether any information provided in the statement(s) is/are redundant and can be dispensed with while answering the questions.

71. What will be the cost of fencing a circular plot? $\left(\pi = \frac{22}{7}\right)$ ...
A. Area of the plot is 616 sq .m.
B. Cost of fencing a rectangular plot whose perimeter is 120 m is Rs 780.
C. Area of a square plot with side equal to the radius of the circular plot is 196 sq. m.
(a) *A* only (b) *C* only
(c) *A* or *C* only (d) *B* only
(e) Question cannot be answered even with information in all three statements.

72. What will be the sum of the ages of father and the son after five years?
A. Father's present age is twice son's present age.
B. After ten years the ratio of father's age to the son's age will become 12 : 7.

C. Five years ago the difference between the father's age and son's age was equal to the son's present age.

(a) *A* or *B* only (b) *B* or *C* only
(c) *A* or *C* only (d) *C* only
(e) *A* or *B* or *C* only

73. The difference between the compound interest and the simple interest at the same rate on a certain amount at the end of two years is Rs. 12.50. What is the rate of interest?

A. Simple interest for two years is Rs 500.
B. Compound interest for two years is Rs 512.50.
C. Amount on simple interest after two years becomes Rs 5500.

(a) *A* or *B* only (b) *A* or *C* only
(c) *C* only (d) *C* and either *A* or *B*
(e) Any two of (*A*), (*B*) and (*C*)

74. 12 men and 8 women can complete a piece of work in 10 days. How many days will it take for 15 men and 4 women to complete the same work?

A. 15 men can complete the work in 12 days.
B. 15 women can complete the work in 16 days.
C. The amount of work done by a woman is three-fourth of the work done by a man in one day.

(a) *A* or *B* or *C* only (b) *B* or *C* only
(c) *C* only (d) Any two of the three
(e) *B* only

75. *P*, *Q* and *R* together invested an amount of Rs 20000 in the ratio of 5 : 3 : 2. What was the per cent profit earned by them at the end of one year?

A. *Q*'s share in the profit is Rs 2400.
B. The amount of profit received by *P* is equal to the amount of profit received by *Q* and *R* together.
C. The amount of profit received by *Q* and *R* together is Rs 4000.

(a) *B* and *A* or *C* only (b) *A* or *C* only
(c) *A* and *B* both (d) *B* and *C* both
(e) Information in all the three statements is required to answer the question.

76. 8 men and 14 women are working together in a filed. After working for 3 days, 5 men and 8 women leave the work. How many more days will be required to complete the work?

A. 19 men and 12 women together can complete the work in 18 days.
B. 16 men can complete two-third of the work in 16 days.
C. In a day, the work done by three men is equal to the work done by four women.

(a) Any two of the three together are sufficient
(b) *B* and *C* only (c) *C* only
(d) *B* or *C* only (d) *A* or *B* or *C*

77. Which is the area of the given right-angled triangle?

A. Length of the diagonal is 5 cm.
B. Perimeter of the triangle is four times its base.
C. One of the angles of the triangle is of 60°.

(a) *C* only (b) *A* and *C* only
(c) *B* or *C* only (d) *B* and *C* both
(e) *A* or *B* or *C*

78. Three friends *X, Y* and *Z* started a partnership business investing money in the ratio of 5 : 4 : 2, respectively for a period of 3 years. What is the amount received by *X* as the share in the total profit?

A. Total amount invested in the business is Rs 22000/-,
B. Profit was distributed after a period of 2 years.
C. The average amount of profit earned per year is Rs 2750.

(a) *A* only (b) *B* only
(c) *C* only (d) *A* or *C* only
(e) *A* or *B* or *C*

79. How much time will the train '*X*' take to cross another train '*Y*' running in opposite direction?

A. Train '*X*' crosses a signal pole in 6 sec.
B. Ratio of the speeds of trains '*X*' and '*Y*' is 3 : 2.
C. Length of the two trains together is 500 m.

(a) *A* only (b) *B* only
(c) *C* only (d) *A* and *B* only
(e) The question cannot be answered even with the information in all the three statements.

80. What will be the cost of painting the four walls of a room with length, width and height 5 m, 3 m and 8 m respectively? The room has one door and one window.

A. Cost of painting per sq. m is Rs 25.00
B. Area of window of 2.25 sq. m is half of the area of the door.
C. Area of the room is 15 sq. m.

(a) *A* only (b) B and *C* together
(c) *A* and *B* together (d) *C* only
(e) All are required to answer the question.

81. What is *R*'s share of profit in a joint venture?

A. *Q* started a business investing Rs 80000/-.
B. *R* joined him after 3 months.
C. *P* joined after 4 months with a capital of Rs 120000 and got Rs. 6000 as his share of profit.

(a) Only *A* and *C* are required
(b) Only *B* and *C* are required
(c) All *A, B* and *C* together are required
(d) Even with all *A, B* and *C* the answer cannot be arrived at.
(e) None of these

82. What is the area of a right-angled triangle?

A. The perimeter of the triangle is 30 cm.

B. The ratio between the base and the height of the triangle is 5 : 12.
C. The area of the triangle is equal to the area of rectangle of length 10 cm.
(a) Only B and C together are required
(b) Only A and B together are required
(c) Only either A or B and C together are required
(d) Only A and C together are required
(e) None of these.

83. What is the sum of two numbers?
A. The bigger of these two numbers is 6 more than the smaller number.
B. 40% of the smaller number is equal to 30% of the bigger number.
C. The ratio between half of the bigger number and one-third of the smaller number is 2 : 1.
(a) Only B and C together are required
(b) Only A and B together are required
(c) Any two of A, B and C together are required
(d) All A, B and C together are required
(e) None of these.

84. How many marks did Arun get in English?
A. Arun secured an average of 60 marks in four subjects including English.
B. He secured a total of 170 in English and Mathematics together.
C. He secured a total of 180 in Mathematics and Science together.
(a) All A, B the C together are required
(b) Only A and B together are required
(c) Only B and C together are required
(d) Only A and C together are required
(e) None of these.

85. What was the profit earned on the cost price by Mahesh by selling an article?
A. He got 15% concession on labelled price in buying that article.
B. He sold it for Rs 3060/-
C. He earned a profit of 2% on the labelled price.
(a) Only A and B together are required
(b) Only B and C together are required
(c) Only either A or C and B together are required
(d) Even with all A, B and C the answer cannot be arrived at.
(e) All A, B and C together are required.

EXPLANATORY ANSWERS

1. Ans. (a)

Solution From statement I,
Ratio of the costs of first, second and third necklace is 6 : 5 : 7. Hence, the price of second necklace can be calculated.

2. Ans. (e)

Solution From Statement I,

$$\text{Rate of an item} = \frac{4500}{x} \quad \text{...(i)}$$

Here, x = total number of items
Combining statement II and (i), we have

$$\left(\frac{4500}{x}+5\right)(x - 10) = 4500$$

or, $x^2 - 10x - 9000 = 0 \quad \therefore \quad x = 100$
Hence, both statements together are sufficient

3. Ans. (d)

4. Ans. (e)

Solution Combining statements (I) and (II),
$l^2 + b^2 = 9000$

$\frac{25}{9}b^2 + b^2 = 900 \quad \therefore \; b = 15\text{ m and } l = 25\text{ m}$

$\therefore$ Area = 375 m^2.

5. Ans. (c)

Solution From statement I,
32% + 1 = 36% – 1 = Minimum pass marks
$\therefore$ Minimum pass marks = 17
From statement II,
Minimum pass marks = 30% + 2 and
(40 – 30)% = 5 $\quad \therefore$ 30% = 15
$\therefore$ Minimum pass marks = 15 + 2 = 17
Hence, either A or B alone is sufficient.

6. Ans. (d)

Solution From the statement I, we will get the sum of length and breadth, but we need individual values of length and breadth.

7. Ans. (b)

Solution I $\rightarrow B - G = 40$

II $\rightarrow G = 80\%$ of $B = \frac{4}{5}B$

$\therefore \quad B : G = 5 : 4$

8. Ans. (e)

Solution I $\rightarrow a = 60\%$ of b,
where a and b are the first and second numbers, respectively.

$a = \frac{6}{10}b$, II $\rightarrow$ $(a + b)$ 50% = 24 $\therefore$ $a + b = 48$

After combining these two statements we get the difference between two numbers as 12.

9. Ans. (e)

Solution Combining both the statements, we get the speed of

$$\text{train} = \frac{180+120}{4} \times \frac{18}{5} = 270 \text{ km/hr.}$$

10. Ans. (e)

Solution I $\rightarrow$ $R = 5\%$

II $\rightarrow$ (CI – SI) for two yrs = Rs 20

Combining I and II and using

$$\text{Sum} = \frac{\text{Diff} \times 100 \times 100}{\text{Rate} \times \text{Rate}} = \frac{20\times100\times100}{25}$$

$$= \text{Rs } 8000$$

$$\text{So, CI} = 8000\left(1+\frac{5}{100}\right)^3 - 8000 = \text{Rs } 1261.$$

11. Ans. (c)

Solution I $\rightarrow$ No. of boys in the class $= \frac{4}{9} \times 45 = 20$

II $\rightarrow$ $\frac{B}{G} = \frac{5}{4}$ and $B - G = 9$

Solving the above two, we get $B = 45$.

12. Ans. (e)

Solution I $\rightarrow$ $M = 1{,}250$ and $F = 1050$

II $\rightarrow$ $M : F = 2 : 1$

On combining both the statements,

$$\text{Average} = \frac{2K\times1250+K\times1050}{3K} = \frac{2500+1050}{3}$$

$$= 1183.33.$$

13. Ans. (a)

Solution I $\rightarrow$ $m = 999999$, $n = 1000000$

$\therefore$ $m - n \div 37 = 999999 - 1000000 \div 37$

$= 999999 - 2702.70 = 997296.30$

II $\rightarrow$ $m - n$ = known, but neither the value of 'm' is known nor the value of 'n' is known. So, we cannot find the value of $m - n \div 37$ by this statement.

14. Ans. (b)

Solution I $\rightarrow$ The fixed value of CP is not given, so SP of the article cannot be determined.

II $\rightarrow$ Let x be SP of an article

$$x \times \frac{90}{100} = \frac{200\times115}{100} \therefore x = \frac{200\times115}{90} = \text{Rs } 255.55.$$

15. Ans. (e)

Solution I $\rightarrow$ Area of the room = 9 × 7 = 63 m^2

$$\text{II} \rightarrow \text{Rate} = \frac{\text{Cost of polishing the floor}}{\text{Area of floor}}$$

$$= \frac{112.50}{10\times5} = \text{Rs } 2.25 \text{ per m}^2$$

$\therefore$ Combining both the statements, cost of polishing the rectangular floor = 63 × 2.25 = Rs 141.75.

16. Ans. (d)

Solution I. $M + ph + ch = 165\%$

II. $Pr \rightarrow M + 10\%$ (average)

17. Ans. (b)

Solution I. CI in two years = Rs 820

II. Rate $= \frac{250}{5000} \times 100 = 5\%$.

18. Ans. (e)

Solution I. $x - y = \frac{1}{3}x$, or, $2x - 3y = 0$

II. $x + y = 30$

By combining I and II, we get $y = 12$.

19. Ans. (d)

Solution I. Area of right-angled triangle = 12 × L

II. L = 18 cm.

$\therefore$ By combing I and II we can find the area of right-angled triangle, but the height cannot be determined in absence of the base of the triangle.

20. Ans. (c)

Solution

21. Ans. (d)

Solution For solving this question, we want two equations in terms of P and Q.

22. Ans. (e)

Solution Combing both the statements together,

Let the labelled price be Rs 100.

Now, SP of the suitcase = 125% of 100 = Rs 125

$\therefore$ Labelled price $= \frac{2000}{125} \times 100 = \text{Rs } 1600$

$\therefore$ CP of the suitcase $= 1600 \times \frac{3}{4} = \text{Rs } 1200.$

23. Ans. (d)

Solution Here, we do not know the type of triangle. If the triangle is right-angled, then the height can be determined with the help of statement I alone.

24. Ans. (e)

Solution Combining both the statements together,

$$\text{Rate of interest} = \frac{800}{2\times8000} \times 100 = 5\%.$$

25. Ans. (d)

26. Ans. (d)

Solution Here, neither the speed of the train nor the individual length of the train is given. Hence, (d) is the correct answer.

27. Ans. (d)

Solution When we combine statements I and II together, we can find the area of the triangle, which is also the area of the rectangle. But without knowing the breadth of the rectangle, length of the rectangles cannot be determined.

28. Ans. (c)

Solution I. $\rightarrow$ Rate of interest $= \frac{100}{1000} \times 100 = 10\%$

$\therefore$ CI at the end of 3 yrs $= 1000 \times \frac{33.1}{100} =$ Rs 331

II. $\rightarrow$ Rate of interest $= \sqrt{\frac{10 \times 100 \times 100}{1000}} = 10\%$

$\therefore$ CI at the end of 3 yrs $= 1000 \times \frac{33.1}{100} =$ Rs 331.

29. Ans. (e)

Solution Let the two-digit no. be $10x + y$ $(y < x)$

I. $x - y = 5$

II. $x + y = 7$

When we combine both statements together value of 'x' and 'y' can be determined. Hence, both statements together are sufficient to answer the question.

30. Ans. (d)

Solution Let the distance between A and B be D km and the speed of the boat and current in still water be x km/hr and y km/hr, respectively.

I. $\rightarrow D = (x - y)2$

II. $\rightarrow D = (x - y)4$

Even if we combine both statements, we cannot find out the answer, because we have two equations and three variables.

31. Ans. (a)

Solution I. $R = (3 - 1) \times \frac{100}{20} = 10\%$.

II. Here, the sum is not given. Therefore, statement I alone is sufficient.

32. Ans. (e)

Solution From I, we can calculate the SI after 5 yrs. When we combine with II, we can get the value of the sum.

i.e. $(P + 5000) = 2P$ or, $P =$ Rs 5000.

33. Ans. (c)

Solution Let the length of the train be 'd' m.

Speed of the train $= \frac{d}{X}$

I. We know that when a train crosses a platform, it crosses not only its length but also the length of the platform.

i.e. $\frac{d}{X} = \frac{d+100}{Y}$ or, $d = \frac{100X}{Y - X}$

II. Length of the train $(d) = 80 \times \frac{5}{18}X = \frac{200X}{9}$

Therefore, either I alone or II alone is sufficient to answer the question.

34. Ans. (c)

Solution I. Radius of circle $= \frac{308 \times 7}{2 \times 22} = 49$ m

Area of circle $= \frac{22}{7} \times 49 \times 49 = 7546\ \text{m}^2$

II. Area of cicle $= \frac{22}{7} \times 28 \times 28 = 2464\ \text{m}^2$

Hence, either I alone or II alone is sufficient for answering the question.

35. Ans. (d)

Solution I. $A + B$ is odd $\Rightarrow$ If A is an even no. then B will be an odd no. and vice versa.

II. $C + B$ is odd $\Rightarrow$ if B is an even no. then C will be an odd no. and vice versa.

So, even by combining the two statements together, we are not able to say that B is an even integer.

36. Ans. (e)

Solution **From I:** Ratio of boys and girls $= 3k : 4k$

From II: No. of girls – No of boys = 18

From I and II: $4k - 3k = 18 \therefore k = 18$

$\therefore$ $4k + 3k = 18 \times 7 = 126$

37. Ans. (a)

Solution Only I alone is sufficient.

38. Ans. (d)

Solution To find out the cost of laying carpet we need (i) cost of carpet per sq. m and, (ii) Area of the floor to be carpeted. Both the information even together are not sufficient to fulfil our need.

39. Ans. (c)

Solution We know, if we have been given difference of CI and SI during two years, then this difference (D) is equal to $\frac{P \times r^2}{100^2}$

where, P = Principal, r = rate of interest

From I: We get the value of P and D. Hence, I alone is sufficient.

Again, we known SI = $\frac{P \times r \times T}{100}$

where, P = principal

r = rate of interest

T = time period

From II: We get value of $P = x$(say).

Then, SI = x and $T = 6\frac{2}{3}$ years

Hence, using the above formula we can get rate of interest from II alone also.

40. Ans. (e)

Solution Suppose units place of number is occupied by y and tens place by x.

From I: $(10x + y) - (10y + x) = 63$

$\Rightarrow 9x - 9y = 63$

$\Rightarrow x - y = 7$ ...(i)

From II: $x + y = 11$ ...(ii)

From equations (i) and (ii), $x = 9$, $y = 2$

Hence, required number = 92

Basically, to answer these type of questions we need the following information:

A. Difference of units and tens digits of the two-digit number.

B. Sum of units and tens digits.

C. Comparison of the value of units and tens digits.

With the help of the information in statements I and II together we get the required information.

41. Ans. (c)

Solution I. CP = $90 \times \frac{100}{120}$ = Rs 75

Profit = 100 – 75 = Rs 25

II. SP = CP + Profit

or, $x + \frac{x}{3} = 100$ or, $x = \frac{100 \times 3}{4} = 75$

$\therefore$ Profit = 75/3 = Rs 25

Therefore, either statement I or II alone is sufficient to answer the question.

42. Ans. (d)

Solution The length of the other train is not given in any of the statements.

43. Ans. (e)

Solution Let the digits be x and y. We are given $x - y = 6$ (Assume $x > y$)

From statement I: x occupies units place

From statement II: $x + y = 12$

With the help of information in the question part and in statement II, we can find the value of x and y easily because there are two equations to know about two unknowns. But to determine the number we need the help of statement I.

44. Ans. (a)

Solution Statement I alone is sufficient to answer the question. We know that whenever any odd number is divided by any odd number, it gives an odd number.

45. Ans. (c)

Solution We know the capacity of a cylindrical tank can be found out by using the following formula:

Area of the base of cylinder × height of cylinder

or, $\pi r^2 \times h$, where r = radius of cylinder

h = height of cylinder

Now,

Statement I gives the value of r and h. Hence, statement I alone is sufficient. Again, statement II gives information about area of the base and height. Hence, statement II alone is also sufficient.

46. Ans. (c)

47. Ans. (e)

Solution SP = $\frac{(3350+150) \times \frac{105}{100}}{50}$ per kg.

48. Ans. (b)

Solution Ratio of men : women : children = 5 : 4 : 2

49. Ans. (b)

50. Ans. (d)

51. Ans. (d)

Solution I. $V_A + V_B = \frac{D_A + 200}{20}$

II. V_B = 60 km/hr

III. $D_A = 2 \times 200 = 400$ m.

Putting the values of II and III in I, we get the value of V_A = 48 km/hr.

52. Ans. (b)

Solution I. $a + b + c = 14$ where, $b = c$

II. a = 14 cm

III. h = 5 cm

So, by combining I and II, we are getting the values of b and c as zero, which is not possible. So, only by combing II and III we get the value of the area, which is equal to 35 cm^2.

53. Ans (a)

Solution I. $P > M$, $M > R$ or, $M = R$

II. $Q > M$, $Q > N$ or, $Q < N$

III. $N > M/R$

So, by combining any one with the other or even by combining all, we cannot reach any conclusion about who earns most.

54. Ans. (e)

Solution Let the price of 1 dozen oranges, 1 dozen banana and 1 dozen apples be x, y and z, respectively.

I. $2x + y = 110$

II. $3z + y = 170$

III. $x + z = 95$

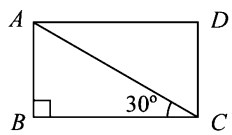
So, by combining all, we get the values of $x = 45$.

55. Ans. (e)

Solution I. $2(L + b) = 60$

$\therefore L + b = 30$ ...(i)

II. In ΔABC



$\tan 30° = \frac{b}{L}$ $\therefore L : b = \sqrt{3} : 1$

Combining I and II, we get the value of $L \approx 19$ and $b \approx 11$ m

$\therefore$ Area of rectangle $= 19 \times 11 = 209\,\text{m}^2$

III. Cost = Rs 125 per sq. m

$\therefore$ total cost of carpeting = 125 × 209 = Rs 26125.

56. Ans. (c)

Solution By combining all the three statements together,

Sudha's present salary $= 10000\left(1+\frac{15}{100}\right)^5$

57. Ans. (e)

Solution None of the statements give the amount of labelled price or SP. So, even by combining all the statements together, question cannot be answered.

58. Ans. (d)

Solution Statements I and II give the percentage number of students studying in different disciplines. Combining this with (iii), the total no. of students can be determined.

59. Ans. (b)

Solution Combining (ii) and (iii),

total cost = 336 × 550 = Rs 184800.

60. Ans. (a)

Solution With the help of I and II, part of work done by A in 5 days $= \frac{5}{8}$, Remaining $= \frac{3}{8}$

Therefore, B alone can do the work in $6 \times \frac{8}{3}$ = 16 days

$\therefore A + B = \frac{8\times16}{24} = \frac{16}{3}$ days

Similarly, by combining any two of the three, the required no. of days can be determined.

61. Ans. (e)

Solution With the help of any two statements, the value of length and breadth can be calculated. And, combing this with the cost per sq. m, we get the total cost of flooring the rectangular hall.

62. Ans. (e)

Solution **From I alone,**

Rate of interest $= \frac{(2-1)\times100}{5} = 20\%$

From II and III,

Rate of interest $= \frac{2\times\text{diff. in CI and SI}}{\text{SI}}$

[For 2 yrs only]

$= \frac{2\times400}{4000} \times 100 = 20\%.$

Hence, either I alone or II and III together are sufficient.

63. Ans. (e)

Solution Let the two-digit no. be $10x + y$.

I. $\Rightarrow |10x + y - 10y - x| = 27$ or, $|x - y| = 3$

II. $\Rightarrow |x - y| = 3$

III. $\Rightarrow x - y = 3$

Here, by taking any two, the values of x and y cannot be determined. So, choice (e) is the correct answer.

64. Ans. (a)

Solution Let the present age of Subir and his father be S and F, respectively.

I. $S = F/2$

II. $\frac{S+5}{1+5} = \frac{6}{11}$ or, $6F - 11S = 25$

III. $B - S = 5$[B = age of Subir's brother]

Now, with the help of I and II together, the value of S and F can be determined.

65. Ans. (b)

66. Ans. (e)

67. Ans. (e)

Solution Cost of painting is not given, hence data inadequate.

68. Ans. (d)

Solution Let the sum be Rs x

From the statements, I and II

$\frac{x\times10\times3}{100} = 4500 \Rightarrow x$ = Rs 15000

$\therefore$ CI = $15000\left(1+\frac{10}{100}\right)^3 - 15000$

= 19965 – 15000 = Rs 4965

From the statements, I and III,

CI – SI = 465

$\therefore$ CI = 465 + 4500 = Rs 4965

From the statements, II and III,

CI = Rs 4965

Hence, any of them can be dispensed with.

69. Ans. (b)

Solution **From the statements, I and II,**

Let the cost price of the article be Rs 100.

$\therefore$ Labelled price = 130

$\therefore$ SP = $130 \times \frac{90}{100}$ = Rs. 117

$\therefore$ % profit = 17%

Hence, II can be dispensed with.

70. Ans. (b)

71. Ans. (c)

Solution (B) is necessary because only this statement gives the rate of fencing. Any one of (A) or (C) gives the value of radius, which enables us to find the circumference. Hence, either (A) or (C) can be dispensed with.

72. Ans. (e)

Solution Any two of the three statements are sufficient to answer the question (As to find the two unknowns we need two equations). Hence, any one of the statements can be dispensed with.

73. Ans. (e)

Solution Any one of the three statements is alone sufficient to answer the question. So, any two can be dispensed with.

From (A) alone:

Rate = $\frac{\text{Diff.} \times 2}{\text{SI}} \times 100 = \frac{25}{500} \times 100 = 5\%$

From (B) alone:

CI = Rs 512.5

$\therefore$ SI = Rs 512.5 – Rs 12.5 = Rs 500

Again, Rate = $\frac{\text{Diff.} \times 2}{\text{SI}} \times 100$

$= \frac{25}{500} \times 100 = 5\%.$

From (C) alone:

Suppose Principal = P and Rate of Interest = $r\%$

Then, $P\left(1+\frac{r}{100}\right)^2 = 5500 + 12.5 = 5512.5$...(1)

and, $P + \frac{2rP}{100} = 5500$

or, $P\left[1+\frac{2r}{100}\right] = 5500$ ...(2)

Dividing (1) by (2) we have

$$\frac{\left(1+\frac{r}{100}\right)^2}{1+\frac{2r}{100}} = \frac{5512.5}{5500} \quad ...(3)$$

This is a quadratic equation which has only one variable, r. It can be solved. Hence, value of r can be obtained.

Note: is satisfied with the value $r = 5$.

So, it confirms that equation is solvable.

74. Ans. (d)

75. Ans. (a)

Solution Statement (B) is useless because it is the same as the given statement. [Profit is distributed in the same ratio as their investment. Since their investments are in ratio 5 : 3 : 2, the profit of P (=5) is equal to the profit of Q and R together (3 + 2 = 5)]

Statement (A) alone is sufficient to answer.

Q's share = Rs 2400

Total profit of $P + Q + R = \frac{2400}{3} \times (5 + 3 + 2)$

= Rs 8000

$\therefore$ % profit = $\frac{8000}{20000} \times 100 = 40\%$

Similarly, statement (C) alone is sufficient to answer the question.

Hence, (B) and (A) or (C) can be dispensed with.

76. Ans. (e)

77. Ans. (b)

Solution Let the length and breadth of the rectangle be l and b, respectively.

$A \rightarrow l^2 + b^2 = 25$

$B \rightarrow l + b + h = 4b$ or, $l = 3b - 5$

$C \rightarrow$ Ratio between l and b is given. After combining any of the above two statements, we get the values of l and b. Hence, any of them can be dispensed with.

78. Ans. (d)

Solution When investment ratio is given, the amount of profit can be found out with the help of C only.

79. Ans. (e)

80. Ans. (d)

Solution The area of four walls can be easily determined with the help of the data given in the question. Now, the area of the windows and door with the help of (B) can be subtracted in the calculated area and then multiplied with the cost given in (A).

81. Ans. (d)

Solution Investment ratio or amount is not given, hence even all statements together are not sufficient.

82. Ans. (b)

Solution Let the base, height and hypotenuse of a right-angled triangle be b, p and h, respectively.

From A, $b + p + h = 30$ ...(i)

From B, $b : p = 5 : 12$ ...(ii)

We know that

$$h^2 = p^2 + b^2 = 25x^2 + 144x^2 = 169x^2$$

$\therefore$ $h = 13x$ ...(iii)

Combining equations (i), (ii) and (iii), we get

$$5x + 12x + 13x = 30 \Rightarrow x = 1$$

$\therefore$ Area of triangle $= \frac{1}{2} \times 5 \times 12 = 30 \text{ cm}^2$

Hence, only A and B together are sufficient.

83. Ans. (e)

Solution Let the bigger and smaller nos. be x and y, respectively.

From A, $x - y = 6$ ...(i)

From B, 40% of y = 30% of x or, $4y = 3x$...(ii)

From C, $\frac{x}{3} : \frac{y}{3} = 2 : 1$ or, $3x = 4y$ ...(iii)

We see that equations (ii) and (iii) are same. Hence, A and either B or C is required.

84. Ans. (e)

85. Ans. (e)

Solution **From A and B,**

Labelled price $= \frac{3{,}060 \times 100}{85}$ = Rs 36000 ...(i)

Combining (i) and Statement C,

Profit $= 36000 \times \frac{2}{100}$ = Rs 72 ...(ii)

Combining (ii) and Statement B,

CP = 3,060 – 72 = Rs 2988

$\therefore$ % profit $= \frac{72}{2988} \times 100 = 2.40\%$

Hence, all statements are required.

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

Directions (1–15): Each of the questions below consists of a question and two statements numbered (I) and (II) given below it. You have to decide whether the data provided in the statements are sufficient to answer the questions. Read both the statements and

Give answer (a): If the data in statement (I) alone are sufficient to answer the question while the data in statement (II) alone are not sufficient to answer the question.

Give answer (b): If the data in statement (II) alone are sufficient to answer the question while the data in statement (I) alone are not sufficient to answer the question.

Give answer (c): If the data either in statement (I) alone or in statement (II) alone are sufficient to answer the question.

Give answer (d): If the data even in both the statements (I) and (II) together are not sufficient to answer the question.

Give answer (e): If the data in both the statements (I) and (II) together are necessary to answer the question.

1. What is the per cent rate of interest per annum on an investment of Rs 12500?

I. The compound interest for 2 years is more than the simple interest for the same period b y Rs. 500.

II. The income from simple interest is Rs 5000.

2. What is the length of the train which crosses a signal pole in 20 sec?

I. The speed of the train is 54 kmph.

II. The train crosses a 150 m long platform in 30 sec.

3. What is the depth of a cylindrical pipe?

I. The area of the base is 616 cm^2.

II. The perimeter of the base is 88 cm.

4. 48 children of a class were asked to sit in rows and columns. How many children are seated in each row?

I. The number of columns is more than the number of rows.

II. The number of rows is ¾ of the number of

columns.

5. What is the height of the triangle *ABC*?
   I. *AB* is the base and the sum of the sides is 25 cm.
   II. The ratio of the sides *AB*, *BC* and *CA* is 2 : 2 : 1.
6. What is the original number?
   I. Sum of two digits of a number is 10. The ratio between the two digits is 1 : 4.
   II. Product of two digits of a number is 16. Quotient of the two digits is 4.
7. What is the rate of the compound interest?
   I. A certain amount invested at the compound interest rate amounts to Rs 1331.
   II. The amount was invested for a period of three years.
8. What is the present age of the mother?
   I. Father's age is eight years more than the mother's age. Father got married at the age of 28 years.
   II. Present age of the father is 30 years. Four years back the ratio of mother's age to father's age was 12 : 13.
9. How many children are there in the group?
   I. Average age of this group of children is 16 years. The total of ages of all the children in the group is 240 years.
   II. The total of ages of all the children in the group and the teacher is 262 years. The teacher's age is six years more than the average age of the children.
10. What is the percentage profit earned?
   I. A shopkeeper invested Rs 14000 and purchased a certain number of articles.
   II. All the articles were sold at Rs 15000.
11. What is the height of a circular cone?
   I. The area of that cone is equal to the area of a rectangle whose length is 33 cm.
   II. The area of the base of that cone is 154 sq cm.
12. What is the price of a table?
   I. The total price of 3 chairs and 5 tables is Rs 18800.
   II. The total price of 6 chairs and 4 tables is Rs 20800.
13. What was the speed of a running train *A*?
   I. The relative speed of train *A* and another train B running in opposite direction is 160 kmph.
   II. The train *B* crosses a signal post in 9 seconds.
14. What is the difference between the two digits in a two-digit number?
   I. The sum of the two digits is 8.
   II. 1/5 of that number is 15 less than 1/2 of 44.
15. What is the monthly income of *Q*?
   I. *Q* earns Rs 6000 more than *R*, who earns Rs 3000 less than *P*.

Test II

[Time: 10 min]

Directions (16–26): Each of the questions below consists of a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statements are sufficient to answer the questions. Read both the statement and – Given answer

(a) If the data in statement I alone are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
(b) If the data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.
(c) If the data either in statement I alone or in statement II alone are sufficient to answer the question.
(d) If the data even in both statements I and II together are not sufficient to answer the question.
(e) If the data in both statements I and II together are necessary to answer the question.

16. By selling a product at 20% profit, how much profit was earned?
   I. The difference between cost and selling price is Rs 40.
   II. The selling price is 120 per cent of the cost price.
17. A train crosses another train running in the opposite direction in *x* sec. What is the speed of the train?
   I. Both the trains have the same length and are running at the same speed.
   II. One train crosses a pole in 5 sec.
18. What is a two-digit number?
   I. The difference between the two digits is 9.
   II. The sum of the digits is equal to the difference between the two digits.
19. A spherical ball of radius *x* cm in melted and made into a right circular cylinder. What is the height of the cylinder?
   I. The volume of the cylinder is equal to the volume of the ball.
   II. The area of the base of the cylinder is given.

20. The area of a square is equal to that of a circle. What is the circumference of the circle?

I. The diagonal of the square is x inches.

II. The side of the square is y inches.

21. A, B and C are positive integers. Is their product an even number?

I. A is an even number.

II. The product of A and B is an even number and that of A and C is also an even number.

22. What is the speed of a boat?

I. The boat covers a distance of 48 km in 6 hours while running upstream.

II. It covers the same distance in 4 hours while running downstream.

23. What was the population of State 'A' in 1999?

I. Population of State 'A' increases every year by 20%.

II. Population of State 'A' in 1999 was 172.8% of its population in 1996.

24. What is a two-digit number?

I. Sum of the digits is equal to the difference between the digits.

II. Difference between the digits is 9.

25. What is Sudha's present age?

I. Sudha's present age is five times her son's present age.

II. Five years ago her age was twenty-five times her son's age that time.

26. What is the average age of the children in a class?

I. The age of the teacher is as many years as the number of children.

II. The average age increases by 1 year if teacher's age is also included.

Directions (27–31): In each of the following questions a question is followed by three statements. Study the question and the statements and decide the information given in which of the statements is/are necessary to answer the question?

27. What is the capacity of a cylindrical tank?

I. The radius of the base is half of its height.

II. The area of the base is 616 sq. m.

III. The height of the cylinder is 28 m.

(a) Only I and II (b) Only II and III

(c) Only I and III (d) All I, II and III

(e) Any two of the three.

28. What is the speed of a train?

I. The train crosses a signal pole in 18 sec.

II. The train crosses a platform of equal length in 36 sec.

III. Length of the train is 330 m.

(a) I and III only

(b) II and III only

(c) I and II only

(d) III and either I or II only

(e) Any two of the three.

29. What is the staff strength of Company 'X'?

I. Male and female employees are in the ratio of 2 : 3, respectively.

II. Of the officer employees 80% are males.

III. Total number of officers is 132.

(a) I and III only

(b) II and either III or I only

(c) All I, II and III

(d) Any two of the three

(e) Question cannot be answered even with the information in all the three statements.

30. What is this two-digit number?

I. The number obtained by interchanging the digits is more than the original number by 9.

II. Sum of the digits is 7.

III. Difference between the digits is 1.

(a) I and III only (b) I and II only

(c) II and III only (d) All I, II and III

(e) Question cannot be answered even with the information in all the three statements.

31. How many articles were sold?

I. Total profit earned was Rs 1596.

II. Cost price per article was Rs 632.

III. Selling price per article was Rs 765.

(a) II and III only (b) I and II only

(c) All I, II and III (d) Any two of the three

(e) Question cannot be answered even with the information in all the three statements.

EXPLANATORY ANSWERS

1. Ans. (a)

2. Ans. (c)

Solution **From statement (I),**

Length of the train $= 54 \times \frac{5}{18} \times 20 = 300$ m

From statement (II),

Let the length of train be d m.

$$\frac{d}{20} = \frac{d+150}{30} \Rightarrow d = 300$$

Hence, either (I) or (II) is sufficient.

3. Ans. (d)

Solution From both the statements we get the value of radius only. Hence, even after combining together, depth cannot be determined.

4. Ans. (b)

Solution **From statement (II) alone,**

Let the number of columns be C.

$\therefore$ No. of rows $= \frac{3}{4}C$

$\therefore$ No. of children $= \frac{3}{4}C^2$

$\therefore$ $\frac{3}{4}C^2 = 48 \Rightarrow C = 8$

The number of children seated in each row is equal to the number of columns = 8

Hence, statement (II) alone is sufficient.

5. Ans. (e)

Solution Combining statements (I) and (II),

$2x + 2x + x = 25 \Rightarrow x = 5$

$\therefore$ Area of the triangle $= \sqrt{12.5\times2.5\times2.5\times7.5}$
$= 24.20 \text{ cm}^2$

$\therefore$ Height of the triangle $= \frac{2\times24.20}{10} = 4.84$ cm.

6. Ans. (d)

Solution Let the original number be $10x + y$.

I. Case (A): $x + y = 10$ and $x : y = 1 : 4$

$\therefore$ $x = \frac{1}{5} \times 10$ and $y = 8$

$\therefore$ Number $= 10 \times 2 + 8 = 28$

Case (B): $x + y = 10$ and $y : x = 1 : 4$

$\therefore$ $x = \frac{4}{5} \times 10 = 8$ and $y = 2$

$\therefore$ Number $= 10 \times 8 + 2 = 82$

II. Case (A): $xy = 16$ and $\frac{x}{y} = 4$

$x = \frac{16}{y}$ and $x = 4y$

$\therefore$ $4y = \frac{16}{y} \Rightarrow y = 2$ and $x = 8$

$\therefore$ Number $= 10 \times 8 + 2 = 82$

Case (B): $xy = 16$ and $\frac{y}{x} = 4$

$y = \frac{16}{x}$ and $y = 4x$

$\therefore$ $4x = \frac{16}{x} \Rightarrow x = 2$ and $y = 8$

$\therefore$ Number $= 10 \times 2 + 8 = 28$

From both the statements, we get two numbers 28 and 82. Hence the original number cannot be determined.

7. Ans (d)

Solution **I.** $1331 = P\left(1+\frac{\text{rate}}{100}\right)^{\text{time}}$

II. $1331 = P\left(1+\frac{\text{rate}}{100}\right)^{3}$ [using statement I]

In the statement II, we have two variables but one equation, hence cannot be determined.

8. Ans. (b)

Solution From statement I we can determine the ages of father and mother at the time of marriage only.

II. $\frac{M-4}{F-4} = \frac{12}{13}$

or, $13M - 52 = 12F - 48$

or, $13M = 12 \times 30 - 48 + 52 = 364$

$\therefore$ $M = 28$ years

Hence, only statement II alone is sufficient.

9. Ans. (a)

Solution **I.** No. of children $= \frac{240}{16} = 15$

10. Ans. (e)

Solution **Combining both the statements**

Percentage profit $= \frac{15000-14000}{14000}\times100 = 7\frac{1}{7}\%$.

11. Ans. (d)

Solution Total surface area of a right circular cone $= \pi r^2 + \pi r h$, h = height and r = radius of the base of the cone.

I $\Rightarrow$ $\pi r^2 + \pi r h$ = area of a rectangle whose length = 33 cm

II $\Rightarrow$ $\pi r^2 = 154$ sq cm. So, r can be obtained but as the breadth of the rectangle has not been given,

so, even after combining the two statements, h cannot be obtained.

12. Ans. (e)

Solution I $\Rightarrow$ $3C + 5T = 18800$

II $\Rightarrow$ $6C + 4T = 20800$

By combining the two statements clearly, C and T can be obtained. Hence, the answer is.

13. Ans. (d)

Solution I $\Rightarrow$ speed of train A + speed of train B = 160 kmph

$$\text{II} \Rightarrow \frac{\text{length of train } B}{\text{speed of train } B} = 9 \text{ sec.}$$

As the length of train B has not been given, so the speed of train B and consequently the speed of train A cannot be obtained.

14. Ans. (b)

Solution Let, the two-digit no. be xy, that is, $10x + y$.

$\text{I} \Rightarrow x + y = 8;$

$$\text{II} \Rightarrow \frac{1}{5}(10x + y) = \frac{44}{2} - 15 = 22 - 15 = 7$$

$\therefore$ The no., $10x + y = 7 \times 5 = 35$ and so, the required difference $= 5 - 3 = 2$.

15. Ans. (d)

Solution $\text{I} \Rightarrow Q = R + 6000,\ R = P - 3000;$

$\text{II} \Rightarrow P + Q = 27000$

$\text{I} + \text{II} \Rightarrow (R + 3000) + (R + 6000) = 27000$

$\therefore R = 9000$ and $Q = R + 6000 + 15000$.

16. Ans. (a)

Solution To answer the question we need one of the following:

(a) Cost price of the product

(b) Selling price of the product

(c) Difference of the selling price and the cost price

From I: We get that the required profit is Rs 40

[$\because$ Profit = Selling price – Cost price]

From II: It is restatement. Because when profit earned is 20% then obviously selling price will be 120% of the cost price.

Thus, only statement I alone is sufficient.

17. Ans. (d)

Solution We know sum of the speeds of the trains

$$= \frac{\text{Sum of the length of the trains}}{\text{Time taken to cross each other when the trains are running in opposite direction}}$$

From I: $S_1 = S_2$ and $L_1 = L_2$

where, S_1 and S_2 are speeds of the trains and L_1 and L_2 are the lengths of the trains.

These information do not lead us to answer.

Hence, not sufficient.

Again, **From II:** We get $\frac{L_1}{S_1} = 5$

Hence, II alone is also not sufficient.

From I and II: We still lack information regarding the length and the speed of the train. Hence, statement I and II even together are not sufficient.

18. Ans. (a)

Solution **From I:** Digit at ten's place – digit at unit's place = 9. This can be possible only in the case of 90. Hence, statement I alone is sufficient.

From II: The sum of two digits is equal to the difference between the two digits. This implies that unit's place digit is 0. And digits at ten's place may be any of the remaining nine digits. Hence, statement II alone is not sufficient.

19. Ans. (b)

Solution **From II:** Radius of the sphere in question part will give us the volume of the sphere as well as that of the right circular cylinder. We know height of right circular cylinder

$$= \frac{\text{Volume of the cylinder}}{\text{Area of the base of the cylinder}}$$

20. Ans. (c)

Solution **From I:** We will get the answer in following way:

Diagonal of the square $\rightarrow$ Side of the square

$\rightarrow$ Area of the square $\rightarrow$ Area of the circle

$\rightarrow$ Radius of the circle $\rightarrow$ Circumference of the circle.

Hence, statement I alone is sufficient. Similarly, we will get the answer also with the help of statement II alone.

21. Ans. (c)

22. Ans. (c)

Solution Here, two data values are important, the speed of the boat (V_B) and that of water flow (V_W). So, we must need two equations to find the value of any of them. So,

From I: $V_B - V_W = \frac{48}{6} = 8$ ...(i)

From II: $V_B + V_W = \frac{48}{4} = 12$ ...(ii)

Solving (i) and (ii), V_B can be determined.

23. Ans. (d)

Solution The population of State A for a given year is not given in any of the statements. Hence options 1, 2 and 3 can be rejected. When we start with statement I, we will get statement II. Hence no benefit.

24. Ans. (b)

Solution **From I:** This is possible only when one digit is zero. So, there are so many possible answers, for example, 10, 20, 30, ...

From II: This is possible only when one digit is 9 and another is zero (0). As the required no. is a two-digit no., 90 is the only solution.

25. Ans. (e)

Solution I $\Rightarrow M = 5S$ ...(1)

II $\Rightarrow M - 5 = 25(S - 5)$ ...(2)

Solving the two equations, we get value of M. Thus, both statements are required.

26. Ans. (d)

Solution **From I:** Let there be n children in the class. Then, the age of the teacher = n years. Also, assume average age of children = x years.

From II: Using 1, we get

$$\frac{nx+n}{n+1} = x + 1$$

Hence, no answer can be determined.

27. Ans. (e)

Solution To find the capacity of a cylindrical tank we need either radius of the tank or area of the base and height of the cylinder. Any two of the three fulfil our requirement.

28. Ans. (d)

29. Ans. (e)

30. Ans. (b)

Solution Note that with the help of II and III you can find only the two digits and not the two-digit number.

31. Ans. (c)

32

BINARY NUMBER SYSTEM

INTRODUCTION

A number system is nothing more than a code. For each distinct quantity there is an assigned symbol. The most familiar number system is the decimal system which uses 10 digits, that is, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The main advantage of this system is its simplicity and long use. Most of the ancient societies used this system. Even in our everyday life we use this system and is sometimes being taken as the natural way to count. Since this system uses 10 digits it is called a system to base 10.

A *binary number system* is a code that uses only two basic symbols, that is, 0 and 1. This system is very useful in computers. Since, in this system, only two symbols are there, it can be used in electronic industry using 'on' and 'off' positions of a switch denoted by the two digits 0 and 1.

Decimal Number System

Decimal number system used 10 digits, 0 through 9, that is, the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Binary Number System

Binary means two. The binary number system uses only two digits, 0 and 1.

Base or Radix

The *base* or *radix* of a number system is equal to the number of digits or symbols used in that number system. For example, decimal system uses 10 digits, so that base of decimal system (that is, decimal numbers) is 10. Binary numbers have base 2.

A subscript attached to a number indicates the base of the number. For example, 100_2 means binary 100. 100_{10} stands for decimal 100.

Weights

In any number to a given base, each digit, depending on its position in the number has a weight in powers of the base.

Illustration 1 In the number $(5342)_x$.

The weight of 2 is x^0

The weight of 4 is x^1

The weight of 3 is x^2

The weight of 5 is x^3.

The sum of all the digits multiplied by their respective weights is equal to the decimal equivalent of that number and gives the total amount represented by that number.

$$(5342)_x = (5x^3 + 3x^2 + 4x + 2x^0)_{10}$$

Illustration 2

5	7	0	3	4	Number to the base 10,
10^4	10^3	10^2	10^1	10^0	that is, decimal number weights

$\therefore\ 5 \times 10^4 + 7 \times 10^3 + 0 \times 10^2 + 3 \times 10 + 4 \times 10^0$

= Value represented or decimal equivalent

Illustration 3

1	1	0	0	1	Number to the base 2
2^4	2^3	2^2	2^1	2^0	that is, binary number weights

$\therefore\ 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 16 + 8 + 1 = 25$

= Decimal equivalent or value represented by 11001_2.

Decimal to Binary Conversion

Step 1: Divide the number by 2.

Step 2: Divide Quotient of Step 1 by 2

Continue the process till we get quotient = 0 and remainder as 1.

Then, the remainders from down upwards written from left to right give the binary number.

Illustration 4 Convert decimal 23 to binary.

Solution

		Remainders
2	23	
2	11	1
2	5	1
2	2	1
	1	0
	0	1

Reading the remainders upwards and writing from left to right we get the binary equivalent of decimal 23 as 10111.

That is, Binary 10111 is equivalent to decimal 23 or we can write $10111_2 = 23_{10}$.

Binary to Decimal Conversion

Following steps are involved to convert a binary number to its decimal equivalent

Step 1: Write the binary number.

Step 2: Write the weights $2^0, 2^1, 2^2, 2^3, \ldots$ under the binary digits starting from extreme right.

Step 3: Cross out any weight under a zero, that is, weights under zeros in the binary number should be deleted.

Step 4: Add the remaining weights.

Illustration 5 Convert binary 1101 to its decimal equivalent.

Solution

1	1	0	1	Binary number
2^3	2^2	2^1	2^0	weights

The weight 2^1 is under 0 so it can be deleted. Sum of the remaining weights

$$= 2^3 + 2^2 + 2^0 = 8 + 4 = 1 = 13.$$

$\therefore$ Decimal equivalent of binary 1101 = 13 that is, $1101_2 = 13_{10}$.

Binary Addition

In binary number system there are only 2 digits, that is, 0 and 1. In decimal system we carry 1 for every 10 whereas in binary system we carry 1 for every 2. Hence, rules of addition are as under:

$$0 + 0 = 0$$
$$0 + 1 = 1$$
$$1 + 0 = 1$$
$$1 + 1 = 10$$

Illustration 6 Add 1010 to 10100

Solution

$$\begin{array}{r} 10100 \\ +\,1010 \\ \hline 11110 \end{array}$$

Binary Subtraction

1. $0 - 0 = 0$
2. $1 - 0 = 1$
3. $1 - 1 = 0$
4. $10 - 1 = 1$
5. $0 - 1 = -1$

[Complement of a binary number is the exact reverse of the given number]

Complement of 0 = 1

Complement of 1 = 0

For subtraction of binary number the following method known as one's complement method is used.

Subtraction of a lower number from a higher number. To determine which binary number is lower and which is higher, it is advisable to find their decimal equivalents.

Step I: Make the number of digits equal in both the numbers.

Step II: Take the complement of the second number, that is, take the complement of the number to be subtracted.

Step III: Add the complement obtained in Step II to the first number. The carry over obtained from this addition indicates that the answer shall be positive.

Step IV: This carry over is taken out and added to the first digit on the right, that is, extreme right digit.

Step IV: The digits so obtained is the final answer.

Illustration 7 Subtract 11 from 101.

Solution Now, $101_2 = 4 + 1 = 5_{10}$, $11_2 = 2 + 1 = 3_{10}$. Clearly, 11 is smaller than 101. Making the number of digits equal, we write 11 as 011.

Complement of 011 = 100.

Adding 100 to 101, we get

$$\begin{array}{r} 101 \\ 100 \\ \hline (1)\,001 \end{array} \qquad \text{[Carry over is 1]}$$

Taking out the carry over and adding to extreme right digit, we get

$$\begin{array}{r} 001 \\ 1 \\ \hline 010 \end{array}$$

$\therefore$ The answer is 010 or 10.

Subtraction of a higher number from a lower number.

Step I: Take the complement of the second number.

Step II: Add the complement obtained in Step I to the first number. In this case there is no carry over indicating that the answer is negative.

Step III: Recomplement the digits obtained after adding the complement of the second number to the first number.

Step IV: Put a negative sign before the result obtained in Step IV.

Illustration 8 Subtract 1110 from 1001.

Solution Now, $1110_2 = 8 + 4 + 2 = 14_{10}$;

$1001 = 8 + 2 = 10_{10}$.

Clearly, $1110_2 > 1001_2$.

Complement of 1110 = 0001.

Adding 0001 to 1001, we get

$$\begin{array}{r} 1001 \\ 0001 \\ \hline 1010 \end{array} \qquad \text{[There is no carry over]}$$

Complement of 1010 = 0101.

$\therefore$ The answer is –0101 or – 101.

Binary Multiplication

Rules: $1 \times 1 = 1$, $1 \times 0 = 0$.

Illustration 9 Multiply 1111 by 11.

Solution

```
  1111
    11
 ------
  1111
 1111
 ------
101101
```

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Find the binary equivalent of decimal 117.
(a) 1010101 (b) 1110101
(c) 1111101 (d) None of these

2. Find the binary equivalent of decimal 52.
(a) 110100 (b) 111100
(c) Remainder (d) None of these

3. Find the decimal equivalent of binary 1110101.
(a) 110_{10} (b) 111_{10}
(c) 117_{10} (d) None of these

4. Find the binary equivalent of decimal 235.
(a) 1010111_2 (b) 1010111_2
(c) 11101011_2 (d) None of these

5. Find the binary equivalent of decimal 701.
(a) 1010111101_2 (b) 1011101101_2
(c) 1110111101_2 (d) None of these

6. Find the decimal equivalent of binary 101001.
(a) 31 (b) 41
(c) 51 (d) None of these

7. Find the decimal equivalent of binary 10000010011.
(a) 1043 (b) 1023
(c) 1033 (d) None of these

8. Find the decimal equivalent of binary 111011.
(a) 69 (b) 49
(c) 59 (d) None of these

9. Add 1001 to 0101
(a) 1111 (b) 1110
(c) 1010 (d) None of these

10. Add 11010 to 11100
(a) 110110 (b) 111110
(c) 110111 (d) None of these

11. $11111_2 + 10001_2 + 1011_2 =$
(a) 110111 (b) 111001
(c) 111011 (d) None of these

12. $11001_2 + 11011_2 + 11111_2 =$
(a) 1010011 (b) 111011
(c) 1110011 (d) None of these

13. $11_2 + 111_2 + 1111_2 + 11111_2 =$
(a) 101010 (b) 111000
(c) 101100 (d) None of these

14. $111_2 + 101_2 =$
(a) 1111 (b) 10111
(c) 1100 (d) None of these

15. $1000_2 + 1101_2 + 1111_2 =$
(a) 100100 (b) 111100
(c) 101010 (d) None of these

16. $111_2 + 101_2 + 011_2 =$
(a) 1011 (b) 1111
(c) 1101 (d) None of these

17. $111000_2 - 11001_2 =$
(a) 11111 (b) 10111
(c) 11011 (d) None of these

18. $10001_2 - 1111_2 =$
(a) 101 (b) 11
(c) 10 (d) None of these

19. $111101_2 - 10111_2 =$
(a) 111110 (b) 100110
(c) 101110 (d) None of these

20. $11111_2 - 10001_2 =$
(a) 1010 (b) 1111
(c) 1110 (d) None of these

21. $100001_2 - 11110_2 =$
(a) 11 (b) 111
(c) 10 (d) None of these

22. Multiply 1111 by 11
(a) 110101 (b) 101101
(c) 110100 (d) None of these

23. Multiply 101 by 11
(a) 1111 (b) 1011
(c) 1110 (d) None of these

24. Multiply 101101 by 1101
(a) 1111001001 (b) 1001101001
(c) 1001001001 (d) None of these

25. Multiply 11001 by 101
(a) 1111101 (b) 1110101
(c) 1011101 (d) None of these

EXPLANATORY ANSWERS

1. Ans. (b)

Solution

		Remainder
2	117	
2	58	1
2	29	0
2	14	1
2	7	0
2	3	1
	1	1
	0	1

∴ The binary equivalent of decimal 117 is 1110101.

2. Ans. (a)

Solution

		Remainder
2	52	
2	26	0
2	13	0
`2	6	1
2	3	0
	1	1
	0	1

∴ The binary equivalent of decimal 52 is 110100.

3. Ans. (c)

Solution

1	1	1	0	1	0	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

Delete the weights 2^3 and 2^1.

Adding the remaining weights, we get

$$2^6 + 2^5 + 2^4 + 2^2 + 2^0 = 64 + 32 + 16 + 4 + 1$$
$$= 117$$

i.e., $1110101_2 = 117_{10}$.

4. Ans. (c)

Solution

		Remainder
2	235	
2	117	1
2	58	1
2	29	0
2	14	1
2	7	0
2	3	1
	1	1
	0	1

∴ $235_{10} = 11101011_2$

5. Ans. (a)

Solution

		Remainder
2	701	
2	350	1
2	175	0
2	87	1
2	43	1
2	21	1
2	10	1
2	5	0
2	2	1
	1	0
	0	1

∴ $(701)_{10} = 1010111101_2$.

6. Ans. (b)

Solution

1	0	1	0	0	1
2^5	2^4	2^3	2^2	2^1	2^0

Decimal equivalent

$= 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 + 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 2^5 + 2^3 + 1 \times 2^0 = 32 = 32 + 8 + 1 = 41.$

7. Ans. (a)

Solution

1	0	0	0	0	0	1	0	0	1	1
2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

Decimal equivalent

$= 1 \times 2^{10} + 0 \times 2^9 + 0 \times 2^8 + 0 \times 2^7$
$+ 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3$
$+ 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 2^{10} + 2^4 + 2^1 + 2^0 = 1043.$

8. Ans. (c)

Solution

1	1	1	0	1	1
2^5	2^4	2^3	2^2	2^1	2^0

Decimal equivalent

$= 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 2^5 + 2^4 + 2^3 + 2^1 + 2^0 = 59.$

9. Ans. (b)

Solution

$$\begin{array}{r} 0101 \\ +1001 \\ \hline 1110 \end{array}$$

10. Ans. (a)

Solution

$$\begin{array}{r} 11100 \\ +\ 11010 \\ \hline 110110 \end{array}$$

11. Ans. (c)

Solution

$$\begin{array}{r} 11111 \\ 10001 \\ 1011 \\ \hline 111011 \end{array}$$

12. Ans. (a)

Solution

$$\begin{array}{r} 11001 \\ 11011 \\ 11111 \\ \hline 1010011 \end{array}$$

Column 1: $1 + 1 + 1 = 3$; $\frac{3}{2}$ = Quotient 1, Remainder 1

Column 2: $0 + 1 + 1 + 1$ (carry from first column)

$= 3$; $\frac{3}{2}$ = Quotient 1 and Remainder 1

Column 3: $0 + 0 + 1 + 1$ (carry from second column)

$= 2$; $\frac{2}{2}$ = Quotient 1 and Remainder 0

Column 4: $1 + 1 + 1$ (carry from column 3)

$= 4$; $\frac{4}{2}$ = Quotient 2 and Remainder 0

Column 5: $1 + 1 + 1 + 2$ (carry from column 4)

$= 5$, $5_{10} = 101_2$.

Note: Quotient in any column is carry for next column.

13. Ans. (b)

Solution

$$\begin{array}{r} 11 \\ 111 \\ 1111 \\ 11111 \\ \hline 111000 \end{array}$$

14. Ans. (c)

Solution

$$\begin{array}{r} 111 \\ 101 \\ \hline 1100 \end{array}$$

15. Ans. (a)

Solution

$$\begin{array}{r} 1000 \\ 1101 \\ 1111 \\ \hline 100100 \end{array}$$

16. Ans. (b)

Solution

$$\begin{array}{r} 111 \\ 101 \\ 011 \\ \hline 1111 \end{array}$$

17. Ans. (a)

Solution $111000_2 = 32 + 16 + 8 = 56$

$11001_2 = 16 + 8 + 1 = 25$

Since $11001_2 < 111000_2$, so we are to subtract a lower number from a higher number.

Making the digits equal in the number to be subtracted, we get

011001.

Complement of 011001 = 100110.

Adding 100110 to 111 000, we get

$$\begin{array}{r} 111000 \\ 100110 \\ \hline [1]011110 \end{array}$$

[1 in the □ is the 1 carried over]

Adding 1 to the extreme right digit in 011 110, we get

$$\begin{array}{r} 011110 \\ 1 \\ \hline 11111 \end{array}$$

$\therefore$ $111000_2 - 11001_2 = 11111$.

18. Ans. (c)

Solution $10001_2 = 2^4 + 1 = 17$,

$1111_2 = 2^3 + 2^2 + 2^1 + 1 = 15$.

Since $1111_2 < 10001_2$, we are to subtract a lower number from a higher number.

Making the digits equal in the number to be subtracted, we get

01111.

Complement of 01111 is 10000.

Adding 10000 to 10001, we get

$$\begin{array}{r} 10001 \\ 10000 \\ \hline [1]00001 \end{array}$$

Adding 1 to 1 in 00001, we get

$$\begin{array}{r} 00001 \\ 1 \\ \hline 00010 \end{array}$$

$\therefore$ $10001_2 - 1111_2 = 10$.

19. Ans. (b)

Solution Complement of $010111_2 = 101000$

Now,

$$\begin{array}{r} 111101 \\ +\,101000 \\ \hline [1]100101 \end{array}$$

Adding 1 to the extreme right digit in 100101, we get

$$\begin{array}{r} 100101 \\ +\quad 1 \\ \hline 100110 \end{array}$$

$\therefore$ $111101_2 - 10111_2 = 100110$.

20. Ans. (c)

Solution Complement of 10001_2 = 01110.

Now,
$$\begin{array}{r} 11111 \\ +\,01110 \\ \hline [1]01101 \end{array}$$
$$\begin{array}{r} 01101 \\ +\quad 1 \\ \hline 01110 \end{array}$$

$\therefore$ $11\ 111_2 - 10001_2 = 1110$.

21. Ans. (a)

Solution Complement of 011110 = 100001

Now,
$$\begin{array}{r} 100001 \\ +\,100001 \\ \hline [1]000010 \end{array}$$
$$\begin{array}{r} 000010 \\ +\quad 1 \\ \hline 000011 \end{array}$$

$\therefore$ $100001_2 - 11110_2 = 11$.

22. Ans. (b)

Solution
$$\begin{array}{r} 1111 \\ 11 \\ \hline 11\ 11 \\ 1\ 11\ 1 \\ \hline 10\ 1101 \end{array}$$

23. Ans. (a)

Solution
$$\begin{array}{r} 101 \\ 11 \\ \hline 101 \\ 101 \\ \hline 1111 \end{array}$$

24. Ans. (c)

Solution
$$\begin{array}{r} 101101 \\ 1101 \\ \hline 101101 \\ 000000 \\ 101101 \\ 101101 \\ \hline 1001001001 \end{array}$$

25. Ans. (a)

Solution
$$\begin{array}{r} 11001 \\ 101 \\ \hline 11001 \\ 00000 \\ 11001 \\ \hline 1111101 \end{array}$$

■ EXERCISES FOR SELF-PRACTICE ■

Test II **[Time: 10 min]**

1. Find the binary equivalent of decimal 1025.
(a) 1000000011 (b) 10000000001
(c) 1000100011 (d) None of these

2. Find the binary equivalent of decimal 381.
(a) 111111011 (b) 101101101
(c) 101111101 (d) None of these

3. Find the binary equivalent of decimal 610.
(a) 1110100010 (b) 1001100010
(c) 1011100010 (d) None of these

4. Find the decimal equivalent of binary 110011.
(a) 41 (b) 51
(c) 51 (d) None of these

5. Find the decimal equivalent of binary 1001111.
(a) 79 (b) 69
(c) 89 (d) None of these

6. $111101_2 + 11101_2 + 100011_2 =$
(a) 1011101 (b) 1111101
(c) 1001101 (d) None of these

7. $110110_2 + 101010_2 =$
(a) 11000011 (b) 1011000
(c) 1100000 (d) None of these

8. $1010_2 - 101_2 =$
(a) 101_2 (b) 111_2
(c) 100_2 (d) None of these

9. $1101_2 - 1001_2 =$
(a) 111_2 (b) 101_2
(c) 100_2 (d) None of these

10. Multiply 10111 by 1011
(a) 11100101 (b) 11111101
(c) 11101010 (d) None of these

EXPLANATORY ANSWERS

1. Ans. (b)

Solution

2	1025	Remainder
2	512	1
2	256	0
2	128	0
2	64	0
2	32	0
2	16	0
2	8	0
2	4	0
2	2	0
	1	0
	0	1

$\therefore \quad 1025_{10} = 10000000001_2.$

2. Ans. (c)

Solution

2	381	Remainder
2	190	1
2	95	0
2	47	1
2	23	1
2	11	1
2	5	1
2	2	1
	1	0
	0	1

$\therefore \quad (381)_{10} = 101111101_2.$

3. Ans. (b)

Solution

2	610	Remainder
2	305	0
2	152	1
2	76	0
2	38	0
2	19	0
2	9	1
2	4	1
2	2	0
2	1	0
	0	1

$\therefore \quad 610_{10} = 1001100010_2.$

4. Ans. (c)

Solution

$$\begin{array}{cccccc} 1 & 1 & 0 & 0 & 1 & 1 \\ 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$$\therefore \quad 110011 = 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

$$= 32 + 16 + 2 + 1 = 51.$$

5. Ans. (a)

Solution

$$\begin{array}{ccccccc} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$$\therefore \quad 1001111 = 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

$$= 64 + 8 + 4 + 2 + 1 = 79.$$

6. Ans. (b)

Solution

$$\begin{array}{r} 111101 \\ 11101 \\ \underline{100011} \\ 111110 \end{array}$$

7. Ans. (c)

Solution

$$\begin{array}{r} 110110 \\ \underline{101010} \\ \underline{1100000} \end{array}$$

8. Ans. (a)

Solution Complement of 0101 is 1010.

$$\begin{array}{r} 101 \\ \underline{+1010} \\ [1]0100 \end{array}$$

Adding 1 to the first digit on the extreme right, we get

$$\begin{array}{r} 0100 \\ \underline{+1} \\ 0101 \end{array}$$

$\therefore \quad 1010_2 - 101_2 = 101_2.$

9. Ans. (c)

Solution Complement of 1001 = 0110.

$$\begin{array}{r} 1101 \\ \underline{+0110} \\ [1]0011 \end{array}$$

Adding 1 to the first digit on the extreme right, we get

$$\begin{array}{r} 0011 \\ \underline{+1} \\ 0100 \end{array}$$

$\therefore \quad 1101_2 - 1001_2 = 100_2.$

10. Ans. (b)

Solution

$$\begin{array}{r} 10111 \\ \underline{1011} \\ 10111 \\ 10111 \\ 00000 \\ \underline{10111} \\ 11111101 \end{array}$$

33

MISCELLANEOUS TOPICS

TYPE I Simultaneous linear equations in two unknowns and their solutions

An equation of the form $ap + bq + c = 0$, where a, b, c are real numbers is called a linear equation in two variables p and q.

In such problems, we are given two linear equations in two variable, p and q.

$$a_1p + b_1q + c_1 = 0$$
$$a_2p + b_2q + c_2 = 0$$

We are required to find the values of p and q by solving the two equations. The algebraic methods given below achieve this purpose.

Method I Elimination by substitution

Step 1: Find the value of one variable, say, q in terms of the other, that is, p from either equation.

Step 2: Substitute the value of q so obtained in the other equation. Thus, get an equation in only one variable p.

Step 3: Solve this equation for p.

Step 4: Substitute the value of p, thus obtained, in the expression for q in terms of p found in Step 1 and find the value of q.

Illustration 1 Solve : $p + q = 7$; $3p - 2q = 11$.

Solution We have, $p + q = 7$ ...(1)

$3p - 2q = 11$ ...(2)

From equation (1), $q = 7 - p$

Putting this value of q in (2), we get

$3p - 2(7 - p) = 11 \Rightarrow 3p - 14 + 2p = 11$

$\Rightarrow 5p = 11 + 14 = 25 \Rightarrow p = \dfrac{25}{5} = 5.$

Substituting this value of p in $q = 7 - p$, we get

$q = 7 - 5 = 2.$

Hence, the solution is $p = 5$, $q = 2$.

Method II Elimination by equating the coefficients

Step 1. We multiply both equations by suitable non-zero constants so that the coefficients of the variable to be eliminated become numerically equal.

Step 2. Add the two equations if the equal coefficients are of opposite signs and subtract if they are of same sign. Thus, the variable with numerically equal coefficients gets eliminated.

Step 3. Solve the resulting equation in the remaining variable and find the value of this variable.

Step 4. Substitute the value of the variable obtained in Step 3 in either of the two original equations and get the value of the other variable.

Illustration 2 Solve : $7p - 2q = 1$; $3p + 4q = 15$.

Solution We have, $7p - 2q = 1$ ...(1)

$3p + 4q = 15$ ...(2)

Multiplying (1) by 2, we get $14p - 4q = 2$ ...(3)

Adding (2) and (3), we get $17p = 17 \Rightarrow p = 1$.

Putting $p = 1$ in (1), wet get $7.1 - 2q = 1$

$\Rightarrow -2q = 1 - 7 = -6 \Rightarrow q = \dfrac{-6}{-2} = 3.$

Hence the solution is $p = 1$, $q = 3$.

Method III By Cross-multiplication (Short-cut method):

The solution of the equations

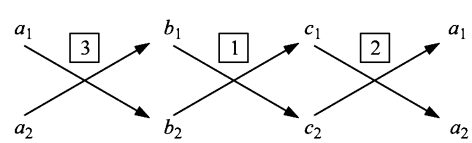
$$a_1p + b_1q + c_1 = 0$$
$$a_2p + b_2q + c_2 = 0$$

can be directly obtained by cross-multiplying the coefficients as shown below:

$$\frac{p}{b_1c_2 - b_2c_1} = \frac{q}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow p = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \text{ and } q = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$$

How to remember:



$\boxed{1} \Rightarrow b_1c_2 - b_2c_1$ Follow arrow diagram

$\boxed{2} \Rightarrow c_1a_2 - c_2a_1$ ——— do ———

$\boxed{3} \Rightarrow a_1b_2 - a_2b_1$ ——— do ———

$\therefore \quad p = \frac{\boxed{1}}{\boxed{3}}$ and $q = \frac{\boxed{1}}{\boxed{3}}$

Illustration 3 Solve $p + q - 7 = 0$; $5p + 12q - 7 = 0$

Solution By cross-multiplication, we have

$$\frac{p}{(1\times -7)-(12\times -7)} = \frac{q}{(-7\times 5)-(1\times -7)}$$
$$= \frac{1}{(1\times 12)-(5\times 1)}$$
$$\Rightarrow \frac{p}{77} = \frac{q}{-28} = \frac{1}{7}$$
$$\Rightarrow p = \frac{77}{7},\ q = -\frac{28}{7} \text{ or, } p = 11 \text{ and } q = -4.$$

Illustration 4 In the following problem, two equations are given. On the basis of the given equations, find the relationship between p and q.

I. $2p + 7q - 11 = 0$ **II.** $-3p + 5q + 1 = 0$

(a) $p = q$ (b) $p > q$

(c) $q > p$ (d) $p \geq q$

(e) $q \geq p$

Solution Solving the two equations

$2p + 7q - 11 = 0$

and, $-3p + 5q + 1 = 0$

By cross-multiplication, we get

$$\frac{p}{(7\times 1)-(5\times -11)} = \frac{q}{(-11\times -3)-(2\times 1)}$$
$$= \frac{1}{(2\times 5)-(-3\times 7)}$$
$$\Rightarrow \frac{p}{62} = \frac{q}{31} = \frac{1}{31}$$
$$\Rightarrow p = \frac{62}{31} \text{ and } q = \frac{31}{31} \text{ or, } p = 2 \text{ and } q = 1.$$

Clearly, $p > q$, $\therefore$ (b) is the correct alternative.

TYPE II Quadratic Equations

The general form of a quadratic equation is:

$ap^2 + bp + c = 0$, where a, b, c are real numbers and $a \neq 0$.

The values of p satisfying the given equation are called its roots or zeros.

To find the roots of a given quadratic equation, the following two methods are useful.

Method I: If α and β are the roots of $ap^2 + bp + c = 0$, then

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}.$$

Illustration 5 Solve the equation:

$$6p^2 - 5p - 21 = 0$$

Solution $6p^2 - 5p - 21 = 0$.

Compare with $ap^2 + bp + c = 0$, we get

$a = 6$, $b = -5$, $c = -21$.

Therefore, the roots of the given equation are

$$p = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$= \frac{5\pm\sqrt{25-4(6)(-21)}}{2(6)}$$
$$= \frac{5\pm\sqrt{529}}{12} = \frac{5\pm 23}{12}.$$

Hence, the roots are $\frac{7}{3}$ and $-\frac{3}{2}$.

Method II By factorisation

Step 1: Bring all the terms on the left side of the equation so that the right side is zero.

Step 2: Factorise the expression on the left side into linear expressions.

Step 3: Equate each linear factor to zero and solve.

Illustration 6 Solve: $p^2 + 3p = 18$

Solution We have,

$$p^2 + 3p - 18 = 0 \Rightarrow p^2 + 6p - 3p - 18 = 0$$
$$\Rightarrow p(p + 6) - 3(p + 6) = 0$$
$$\Rightarrow (p + 6)(p - 3) = 0$$
$$\Rightarrow p + 6 = 0 \text{ or, } p - 3 = 0$$
$$\therefore \quad p = -6 \text{ or, } p = 3.$$

Illustration 7 In the following problem, two equations are given. On the basis of the given equations, find the relationship between p and q.

I. $6p^2 - p - 2 = 0$ **II.** $9q^2 - 3q - 2 = 0$.

(a) $p = q$ (b) $p > q$

(c) $q > p$ (d) $p \geq q$

(e) $q \geq p$.

Solution Solving the equation I by factors method, we have

$$6p^2 - p - 2 = 0 \Rightarrow 6p^2 - 4p + 3p - 2 = 0$$
$$\Rightarrow 2p(3p - 2) + 1(3p - 2) = 0$$
$$\Rightarrow (2p + 1)(3p - 2) = 0$$
$$\Rightarrow 2p + 1 = 0 \text{ or, } 3p - 2 = 0$$
$$\Rightarrow p = -\frac{1}{2} \text{ or, } p = \frac{2}{3}.$$

Solving the equation II by factor method, we have

$$9q^2 - 3q - 2 = 0 \Rightarrow 9q^2 - 6q + 3q - 2 = 0$$
$$\Rightarrow 3q(3q - 2) + 1(3q - 2) = 0$$
$$\Rightarrow (3q + 1)(3q - 2) = 0$$
$$\Rightarrow 3q + 1 = 0 \text{ or, } 3q - 2 = 0$$
$$\Rightarrow q = -\frac{1}{3} \text{ or, } q = \frac{2}{3}$$

Therefore, $q \geq p$.

Thus, (e) is the correct alternative.

Type III. Inequalities

When two mathematical quantities 'a' and 'b' are not equal, then either a is greater than b or a is less than b. These relations are denoted in the symbolic form $a > b$ and $a < b$, respectively. The relation "a is not equal to b" is denoted as $a \neq b$ and is called an *inequality*.

Basis Rules of Inequalities

1. If $a > b$, then $a = b + k$, where $k > 0$.
2. If $a < b$, then $a = b - k$, where $k > 0$.
3. If $a > b$, then $a + k > b + k$, where k is a real number.
4. If $a > b$, then $ka > kb$ and $\frac{a}{k} > \frac{b}{k}$, where $k > 0$.
5. If $a > b$ and $k < 0$, then $ka < kb$ and $\frac{a}{k} < \frac{b}{k}$.
6. If $a > b$ and $a > 0$, $b > 0$ then $\frac{1}{a} < \frac{1}{b}$.
7. If $a > b$ and $a < 0$, $b < 0$ then $\frac{1}{a} < \frac{1}{b}$.

Solution of an Inequality

1. If $(x - a)(x - b) < 0$ and $a < b$, then the solution is $a < x < b$.
2. If $(x - a)(x - b) \leq 0$ and $a < b$, then the solution in $a \leq x \leq b$.
3. If $(x - a)(x - b) > 0$ and $a < b$, then the solution is $x < a$ or $x > b$, that is, $x < a \cup x > b$.
4. If $(x - a)(x - b) \geq 0$ and $a < b$, then the solution is $x \leq a$ or $x \geq b$ that is, $x \leq a \cup x \geq b$.

Illustration 8 Solve : $(4x - 3)(x - 4) \geq 0$.

Solution The given inequality can be written as

$$\left(x - \frac{3}{4}\right)(x - 4) \geq 0$$

[In the term $4x$, to get x, we divide by 4]

$\therefore$ The solution is $x \leq \frac{3}{4} \cup x \geq 4$.

Illustration 9 Solve: $(4 - x)(x - 3) > 0$.

Solution The given inequality can be written as

$$(x - 3)(x - 4) < 0$$

[In order to write $(4 - x)$ as $(x - 4)$, we multiply both sides by (-1) and so '>' sign will change to <]

$\therefore$ The solution is $3 < x < 4$.

Illustration 10 Solve: $3x^2 - 8x + 4 > 0$

Solution We have,

$$\begin{aligned} 3x^2 - 8x + 4 > 0 &\Rightarrow 3x^2 - 6x - 2x + 4 > 0 \\ &\Rightarrow 3x(x - 2) - 2(x - 2) > 0 \\ &\Rightarrow (3x - 2)(x - 2) > 0 \\ &\Rightarrow \left(x - \frac{2}{3}\right)(x - 2) > 0 \end{aligned}$$

$\therefore$ The solution is $x < \frac{2}{3}$ or, $x > 2$.

Illustration 11 $4x^2 + 13x + 3 \leq 0$.

Solution We have,

$$\begin{aligned} 4x^2 + 13x + 3 \leq 0 &\Rightarrow 4x^2 + 12x + x + 3 \leq 0 \\ &\Rightarrow 4x(x + 3) + 1(x + 3) \leq 0 \\ &\Rightarrow (4x + 1)(x + 3) \leq 0. \\ &\Rightarrow \left(x + \frac{1}{4}\right)(x + 3) \leq 0 \end{aligned}$$

$\therefore$ The solution is $-3 \leq x \leq -\frac{1}{4}$.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Which of the following values of x satisfy the inequality $2x^2 - 3x + 1 > 0$?

(a) $-1 < x < -\frac{1}{2}$ (b) $\frac{1}{2} < x < 1$

(c) $x > 1$ or $x < \frac{1}{2}$ (d) $-\frac{1}{2} < x < 1$

2. Which of the following values of x will satisfy the inequality $2x^2 - 7x < 15$?

(a) $-\frac{3}{2} < x < 5$ (b) $x > 5$ or $x < -\frac{3}{2}$

(c) $x < 5$ and $x < -\frac{3}{2}$ (d) $x > -\frac{3}{2}$ and $x > 5$.

(e) None of these

3. Which value of x does satisfy the inequality $2x^2 + x - 3 < 0$?

(a) $-\frac{3}{2} < x < 1$ (b) $-1 < x < \frac{3}{2}$

(c) $x > 1$ (d) $x < -\frac{2}{5}$

4 Which of the following values of x will satisfy the inequality $x^2 - x - 6 > 0$?

(a) $x < -2$ or $x > 3$ (b) $-2 < x < 3$

(c) $-3 < x < 2$ (d) $x > -2$ or $x < 3$

(e) None of these

5. Which values of x satisfy the inequality

$$x^2 - 3x + 2 > 2x - 4?$$

(a) $2 < x < 3$ (b) $x > 3$ or $x < 2$
(c) $x \geq 3$ (d) $x \leq 2$
(e) None of these

Directions: Four of the following five parts numbered (a), (b), (c), (d), and (e) are exactly equal. The number of the part which is not equal to the other four parts is your answer.

6. (a) $75 \times 8 \div 6$
(b) $98 \div 2.5 + 15.2 \times 4$
(c) $\sqrt{225} \times 23 - 5 \times 22$
(d) $76 \times 1.5 - 5.5 \times 2.6$
(e) $48 \times 1.2 + 127.2 \div 3$

7. (a) $2x^2 - 3xy + y^2$ (b) $(2x + y)(x - y)$
(c) $(x - y)^2 + x(x - y)$ (d) $(2x - y)^2 - x(2x - y)$
(e) $(2x - y)^2 - 2x^2 + xy$

8. (a) $24^2 - 12^2 + 112 \div 14$
(b) $17 \times 12 + 59 \times 4$
(c) $15 \times 28 + 20$
(d) $27 \times 16 + 56 \div 8$
(e) $185 \times 6 \div 2 - 23 \times 5$

9. (a) $85 \div 17 \times 110$
(b) $45 \times 6 + 75 \times 4$
(b) $175 \div 25 \times 75 + 5^2$
(d) $36 \times 4 + 21^2 - 7 \times 5$
(e) $65 \times 12 - 46 \times 5$

10. (a) $a^2 + b^2 - 4ab$
(b) $(a + b)^2 - 6ab$
(c) $(a - b)^2 - 2ab$
(d) $a(a + b) + b(b - 2a) - 3ab$
(e) $(a - b)a - b(a - b) - ab$

11. (a) $30 \times 14 \div 7 \times 5$ (b) $10^3 - 100 \times 7$
(c) $5 \times \sqrt{3600}$
(d) $450 \div 50 \times 50 - 5 \times 30$
(e) $10 \times 3 + 120 \times 2$

12. (a) $(2x + 3y)^2$
(b) $(2y - x)^2 + y(16x + 5y)$
(c) $4x(x + 2y) + y(4x + 9y)$
(d) $(2x + 2y)^2 + y(4x + 5y)$
(e) $(2x - y)^2 + 8y(2x + y)$

13. (a) $10.36 + 69.802 + 24.938$
(b) $2207.1 \div 21$
(c) $16\frac{2}{3}\%$ of 630.6
(d) 32.84375×3.2
(e) $\frac{1}{5}$ of $\frac{1}{9}$ of 4729.4

14. (a) $371.587 + 46.32 - 217.907$
(b) $4 \times 125 - 75 \times 4$
(c) $58.25 \times 4.5 - 65.875$
(d) $25 \times 12 - 2 \times 5 \times 10$
(e) $121 \times 3.5 - 2 \times 111.75$

15. (a) $\sqrt[3]{729} + \sqrt[2]{625}$ (b) $\sqrt{324} + \sqrt{256}$
(c) $\sqrt[3]{216} \times \sqrt[2]{81} - 40$ of $\frac{1}{2}$
(d) $\sqrt[2]{441} + \sqrt[3]{2197}$
(e) $\sqrt[3]{5832} + \sqrt[3]{2744}$

16. (a) $a^2 + 5ab + 6b^2$
(b) $(a + 3b)(a + 2b)$
(c) $a(a + 2b) + 2b(a + 3b)$
(d) $(3a + b)(a + 2b)$
(e) $a(a + 2b) + 3b(a + 2b)$

17. (a) $8362.64 + 768.3 - 190.57$
(b) $593.38 + 604.7 + 7742.29$
(c) 2235.925×4
(d) $9931.04 - 990.67$
(e) $17880.74 \div 2$

18. (a) $275 \times 12 - 15^2 + 5^3$
(b) $128 \times 5 - 5 \times 4^3 + 4^2 \times 3^2 \times 20$
(c) $350 \times 8 + 5^2 \times 4^2$
(d) $175 \times 14 + 17 \times 7 + 90 \times 7$
(e) $182.5 \times 16 + 7 \times 2^3 \times 5$

19. (a) $(2x + 3)(5x - 4)$
(b) $5x(2x + 1) + 2(x - 6)$
(c) $2(x - 2)(5x + 3)$
(d) $7x(x + 1) + 3(x - 2)(x + 2)$
(e) $10(x^2 - 1) + (7x - 2)$

20. (a) 45% of 1600 + $\frac{2}{3}$ of 270
(b) 80% of 1000 + 100% of 100
(c) 140% of 500 + 150% of 160
(d) 60% of 1200 + $\frac{1}{4}$ of 720
(e) $6\frac{1}{2}$ of 200 − $\frac{1}{3}$ of 1200

21. (a) $(x + 2y)(3x + 2y)$
(b) $(x + 2y)^2 + 2x(x + 2y)$
(c) $(3x + 2y)^2 - 2x(3x + 2y)$
(d) $3x(x + 2y) + 2y(x + 2y)$
(e) $x(3x + 2y) - 2y(3x - 2y)$

22. (a) $(a - b)^2 + 4ab$
(b) $a(a + b) + b(a + b)$
(c) $(a^2 - b^2) + 2b(b + a)$
(d) $(a - b)b + (a + b)2b$
(e) $(a + b)^2 - 2a(a - b) + a(2a - 2b)$

23. (a) $xy^2 - x^2y + 2x^2y^2$
(b) $xy^2(1 - 2x) + x^2y(2y - 1)$
(c) $xy^2(1 + x) - x^2y(1 - y)$
(d) $xy[y(1 + x) - x(1 - y)]$
(e) $xy[(x + y)^2 + y(1 - y) - x(1 + x)]$

24. If $3x - 5y = 5$ and $\frac{x}{x+y} = \frac{5}{7}$, what is the value of $x - y$?
(a) 9 (b) 6
(c) 4 (d) None of these.

25. Which of the following expressions are equal in value?
(1) $(2x + 3y)^2$
(2) $(2x + y)^2 + 8y(x + y)$
(3) $(2x - y)^2 - 8y(x + y)$
(4) $2^2(x + y)^2 + 4xy + 5y^2$
(a) 1 and 2 only (b) 2 and 3 only
(c) 1, 2 and 4 only (d) 2 and 4 only
(e) All are equal

26. If $\frac{x^2-1}{x+1} = 2$, then $x =$
(a) 1 (b) 0
(c) 2 (d) Cannot be determined
(e) None of these.

27. Which of the following equations are equivalent?
(1) $4p^2 - 9q^2$ (1) $(2p - 3q)^2$
(3) $(2p - 3q)(2p + 3q)$
(4) $(2p - 3q)^2 + 12pq - 18q^2$
(a) All except 2 (b) 1 and 3
(c) 2 and 3 (d) All
(e) None of these.

28. What will be the value of
$n^4 - 10n^3 + 36n^2 - 49n + 24$ if $n = 1$?
(a) 21 (b) 2
(c) 1 (d) 22
(e) None of these.

29. Which of the following equations are equivalent?
(1) $\left(\frac{1}{2}m+\frac{2}{3}n\right)^2$
(2) $\frac{4}{9}n^2 + \frac{1}{4}m^2 + \frac{2}{3}mn$
(3) $\left(\frac{1}{2}m+\frac{2}{3}n\right)\left(\frac{1}{2}m-\frac{2}{3}n\right)$
(4) $\frac{1}{4}\left(m+\frac{4}{3}n\right)^2$
(a) Only 1 and 2 (b) Only 2 and 3
(c) Only 1 and 3 (d) Only 1 and 4
(e) Only 1, 2 and 4.

30. If $2x + 3y = 26$, $2y + z = 19$ and $x + 2z = 29$, what is the value of $x + y + z$?
(a) 18 (b) 32
(c) 26 (d) 22
(e) None of these.

31. If $2x + y = 15$, $2y + z = 25$ and $2z + x = 26$, what is the value of z?
(a) 4 (b) 7
(c) 9 (d) 12
(e) None of these.

Directions (32 to 82): In each question below one or more equation(s) is/are provided. On the bases of these, you have to find out relation between p and q.
Give answer (a) if $p = q$,
Give answer (b) if $p > q$,
Give answer (c) if $q > p$,
Give answer (d) if $p \geq q$ and,
Give answer (e) if $q \geq p$.

32. **I.** $4q^2 + 8q = 4q + 8$ **II.** $p^2 + 9p - 12$
33. **I.** $2p^2 + 40 = 18p$ **II.** $q^2 = 13q - 42$
34. **I.** $6q^2 + \frac{1}{2} = \frac{7}{2}q$ **II.** $12p^2 + 2 = 10p$
35. $\frac{5}{28}\times\frac{9}{8}p = \frac{15}{14}\times\frac{13}{16}q$
36. (i) $p - 7 = 0$ (ii) $3q^2 - 10q + 7 = 0$
37. (i) $4p^2 = 16$ (ii) $q^2 - 10q + 25 = 0$
38. (i) $4p^2 - 5p + 1 = 0$ (ii) $q^2 - 2q + 1 = 0$
39. (i) $q^2 - 11q + 30 = 0$ (ii) $2p^2 - 7p + 6 = 0$
40. **I.** $6p^2 + 5p + 1 = 0$ **II.** $20q^2 + 9q = -1$
41. **I.** $3p^2 + 2p - 1 = 0$ **II.** $2q^2 + 7q + 6 = 0$
42. **I.** $3p^2 + 15p = -18$ **II.** $q^2 + 7q + 12 = 0$
43. **I.** $p = \frac{\sqrt{4}}{\sqrt{9}}$ **II.** $9q^2 - 12q + 4 = 0$
44. **I.** $p^2 + 13p + 42 = 0$ **II.** $q^2 = 36$
45. **I.** $p^2 - 6p = 7$ **II.** $2q^2 + 13q + 15 = 0$
46. **I.** $3p^2 - 7p + 2 = 0$ **II.** $2q^2 - 11q + 15 = 0$
47. **I.** $10p^2 - 7p + 1 = 0$ **II.** $35q^2 - 12q + 1 = 0$
48. **I.** $4p^2 = 25$ **II.** $2q^2 - 13q + 21 = 0$
49. **I.** $3p^2 + 7p = 6$ **II.** $6(2q^2 + 1) = 17q$
50. **I.** $p^2 + 5p + 6 = 0$ **II.** $q^2 + 3q + 2 = 0$
51. **I.** $p^2 = 4$ **II.** $q^2 + 4q = -4$
52. **I.** $p^2 + p = 56$ **II.** $q^2 - 17q + 72 = 0$
53. **I.** $3p + 2q - 58 = 0$ **II.** $4q + 4p = 92$
54. **I.** $3p^2 + 17p + 10 = 0$ **II.** $10q^2 + 9q + 2 = 0$
55. **I.** $q^2 - 6q + 9 = 0$ **II.** $p^2 + 2p - 3 = 0$
56. **I.** $p^2 - 5p + 6 = 0$ **II.** $2q^2 + 3q - 5 = 0$
57. **I.** $p = \sqrt{256}$ **II.** $y = (-4)^2$
58. **I.** $p^2 - 6p + 5 = 0$ **II.** $q^2 - 13q + 42 = 0$
59. **I.** $p^2 + 3p + 2 = 0$ **II.** $q^2 - 4q + 1 = 0$
60. **I.** $p^2 + 3p + 2 = 0$ **II.** $2q^2 = 5q$
61. **I.** $2p^2 + 5p + 2 = 0$ **II.** $4y^2 = 1$
62. **I.** $q^2 + 2q - 3 = 0$ **II.** $2p^2 - 7p + 6 = 0$
63. **I.** $p^2 + 2p - 8 = 0$ **II.** $q^2 - 2 = 7$
64. **I.** $p^2 - 5p + 6 = 0$ **II.** $q^2 + q - 6 = 0$
65. **I.** $p^2 - 7p = -12$ **II.** $q^2 - 3q + 2 = 0$
66. **I.** $12p^2 - 7p = -1$ **II.** $6q^2 - 7q + 2 = 0$
67. **I.** $p^2 + 12p + 35 = 0$ **II.** $2q^2 + 22q + 56 = 0$
68. **I.** $p^2 - 8p + 15 = 0$ **II.** $q^2 - 5q = -6$
69. **I.** $2p^2 + 20p + 50 = 0$ **II.** $q^2 = 25$

70. **I.** $2p^2 = 23p - 63$ **II.** $2q\ (q^{-8}) = q^{-36}$

71. **I.** $p\ (p^{-1}) = (p^{-1})$ **II.** $q^2 = 4q^{-1}$

72. **I.** $2p\ (.p - 4) = 8\ (p - 5)$
II. $q^2 + 4 = 7q^{-8}$

73. **I.** $pq + 30 = 6p + 5q$

74. **I.** $2p^2 + 12p + 16 = 0$ **II.** $2q^2 + 14q + 24 = 0$

75. **I.** $2p^2 + 48 = 20p$ **II.** $2q^2 + 18 = 12q$

76. **I.** $q^2 + q = 2$ **II.** $p^2 + 7p + 10 = 0$

77. **I.** $p^2 + 36 = 12p$ **II.** $4q^2 + 144 = 48q$

78. **I.** $\frac{4p}{5} - \frac{8}{15} = 0$ **II.** $9q^2 = 12q - 4$

79. **I.** $\frac{3p}{4} + \frac{2p}{5} = 2\frac{3}{10}$ **II.** $\frac{1}{2}q = \frac{1}{4}q - \frac{1}{6}$

80. **I.** $q^2 - 15q + 56 = 0$ **II.** $2p^2 - 10p + 12 = 0$

81. **I.** $18p^2 + 3p = 3$ **II.** $14q^2 + 9q + 1 = 0$

82. **I.** $p^2 - 12p + 36 = 0$ **II.** $q^2 + 48 = 14q$

EXPLANATORY ANSWERS

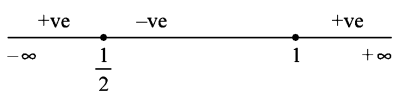
1. Ans. (c)

Solution $2x^2 - 3x + 1 = 0 \Rightarrow (2x - 1)\ (x - 1) = 0$

Hence, $x = \frac{1}{2}$ and $x = 1$.

Now, the given inequality is $2x^2 - 3x + 1 > 0$

Hence, sign scheme will be as follows:



$\therefore\ x < \frac{1}{2}$ or $x > 1$.

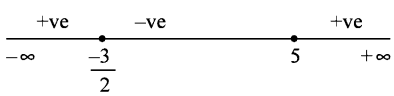
2. Ans. (a)

Solution $2x^2 - 7x < 15$ or, $2x^2 - 7x - 15 < 0$

or, $2x^2 - 10 + 3x - 15 < 0$

or, $2x\,(x - 5) + 3\,(x - 5) < 0$

or $(x - 5)\ (2x + 3) < 0$



$\therefore\ -\frac{3}{2} < x < 5$.

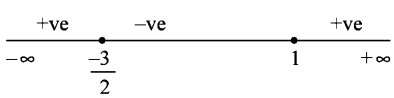
3. Ans. (a)

Solution $2x^2 + x - 3 < 0$

or, $2x^2 + 3x - 2x - 3 < 0$

or, $x\ (2x + 3) - 1\ (2x + 3) < 0$

or, $(x - 1)\ (2x + 3) < 0$



$\therefore\ -\frac{3}{2} < x < 1$.

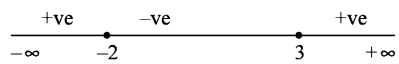
4. Ans. (a)

Solution $x^2 - x - 6 > 0$

or, $x^2 - 3x + 2x - 6 > 0$

or, $x\,(x - 3) + 2\ (x - 3) > 0$

or, $(x - 3)\ (x + 2) > 0$.



$\therefore\ x < -2$ or $x > 3$.

5. Ans. (b)

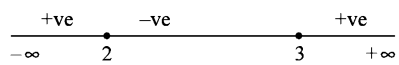
Solution $x^2 - 3x + 2 > 2x - 4$

or, $x^2 - 5x + 6 > 0$

or, $x^2 - 3x - 2x + 6 > 0$

or, $x\,(x - 3) - 2\ (x - 3) > 0$

or, $(x - 2)\ (x - 3) > 0$



$\therefore\ x < 2$ or $x > 3$.

6. Ans. (d)

Solution Check all options one by one.

(a) $\frac{75 \times 8}{6} = 100$

(b) $\frac{98}{2.5} + (15.2 \times 4) = 100$

(c) $(15 \times 8) - (5 \times 4) = 100$

(d) $(76 \times 1.5) - (5.5 \times 2.6) = 99.7$

(e) $(48 \times 1.2) + \frac{127.2}{3} = 100$.

7. Ans. (b)

Solution Other parts are equal to $2x^2 - 3xy + y^2$.

8. Ans. (d)

Solution Other parts are equal to 440.

9. Ans. (b)

Solution Other parts are equal to 550.

10. Ans. (e)

Solution Other parts are equal to $a^2 - 4ab + b^2$.

11. Ans. (e)

Solution Other parts are equal to 300.

12. Ans. (b)

13. Ans. (e)

Solution The other parts are equal to 105.10.

14. Ans. (c)

Solution The other parts are equal to 200.

15. Ans. (e)

Solution The other parts are equal to 34.

16. Ans. (d)

Solution The other parts are equal to $a^2 + 5ab + 6b^2$.

17. Ans. (c)

Solution The other parts are equal to 8940.37.

18. Ans. (d)

Solution Others are equal to 3200.

19. Ans. (c)

Solution The other parts are equal to $10x^2 + 7x - 12$.

20. Ans. (c)

Solution Others are equal to 900.

21. Ans. (e)

Solution Others are equal to $3x^2 + 8xy + 4y^2$.

22. Ans. (d)

Solution The other parts are equal to $(a + b)^2$.

23. Ans. (b)

Solution The other parts are equal to
$xy^2 - x^2y + 2x^2y^2$.

24. Ans. (d)

Solution $3x - 5y = 5$ ...(i)

and $\frac{x}{x+y} = \frac{5}{7} \Rightarrow 7x = 5x + 5y$

$\Rightarrow 2x = 5y$ ...(ii)

From (i) and (ii), $x = 5$ and $y = 2$

$\therefore\ x - y = 3$.

25. Ans. (c)

Solution All others are equal except (c).

26. Ans. (e)

Solution $\frac{x^2 - 1}{x + 1} = 2 \Rightarrow x^2 - 1 = 2x + 2$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow (x - 3)(x + 1) = 0$

$\Rightarrow x = 3, -1$.

27. Ans. (a)

28. Ans. (b)

Solution For $n = 1$, $n^4 - 10n^3 + 36n^2 - 49n + 24$
$= 1 - 10 + 36 - 49 + 24 = 2$.

29. Ans. (e)

30. Ans. (d)

31. Ans. (e)

Solution $2x + y = 15 \Rightarrow y = 15 - 2x$ ...(i)

$2y + z = 25 \Rightarrow 2(15 - 2x) + z = 25$

$\Rightarrow 4x - z = 5$ ...(ii)

and, $2z + x = 26$ ...(iii)

Solving (ii) and (iii), we get $z = 11$.

32. Ans. (c)

Solution **I.** $4q^2 + 8q = 4q + 8$ **II.** $p^2 + 9p = 2p - 12$

or, $q^2 + q - 2 = 0$ or, $p^2 + 7p + 12 = 0$

or, $(q - 1)(q + 2) = 0$ or, $(p + 4)(p + 3) = 0$

$\therefore\ q = 1$ or -2 $\therefore\ p = -3$ or -4

Hence, $q > p$.

33. Ans. (c)

Solution **I.** $2p^2 + 40 = 18p$ **II.** $q^2 = 13q - 42$

or, $p^2 - 9p + 20 = 0$ or, $q^2 - 13q + 42 = 0$

or, $(p - 4)(p - 5) = 0$ or, $(q - 7)(q - 6) = 0$

$\therefore\ p = 4$ or 5 $\therefore\ q = 6$ or 7

Hence, $q > p$.

34. Ans. (d)

Solution **I.** $6q^2 + \frac{1}{2} = \frac{7}{2}q$ **II.** $12p^2 + 2 = 10p$

or, $12q^2 - 7q + 1 = 0$ or, $6p^2 - 5p + 1 = 0$

or, $\left(q - \frac{1}{4}\right)\left(q - \frac{1}{3}\right) = 0$ or, $\left(p - \frac{1}{3}\right)\left(q - \frac{1}{2}\right) = 0$

$\therefore\ q = \frac{1}{4}$ or $\frac{1}{3}$ $\therefore\ p = \frac{1}{3}$ or $\frac{1}{2}$

Hence, $p \geq q$.

35. Ans. (b)

Solution $\frac{5}{28} \times \frac{9}{8}p = \frac{15}{14} \times \frac{13}{16}q$

or, $\frac{45p}{224} = \frac{195q}{224}$ or, $3p = 13q$

$\therefore\ p > q$.

36. Ans. (b)

Solution

(i) $p - 7 = 0$ or, $p = 7$

(ii) $3q^2 - 10q + 7 = 0$

or, $3q^2 - 3q - 7q + 7 = 0$

or, $3q(q - 1) - 7(q - 1) = 0$

or, $(3q - 7)(q - 1)$

or, $q = 1$ or, $\frac{7}{3}$

$\therefore\ p > q$.

37. Ans. (c)

Solution (i) $4p^2 = 16 \Rightarrow p = \sqrt{4} = 2$

(ii) $q^2 - 10q + 25 = 0$

$\Rightarrow (q - 5)(q - 5) = 0$ or, $q = 5 \therefore q > p$.

38. Ans. (e)

Solution (i) $4p^2 - 5p + 1 = 0$

or, $4p^2 - 4p - p + 1 = 0$

or, $4p(p - 1) - 1(p - 1) = 0$

or, $(4p - 1)(p - 1) = 0$

or, $p = 1$ and $p = \frac{1}{4}$

(ii) $q^2 - 2q + 1 = 0 \Rightarrow (q - 1)(q - 1) = 0$

or, $q = 1 \quad \therefore \quad q \geq p$.

39. Ans. (c)

40. Ans. (c)

Solution **I.** $6p^2 + 5p + 1 = 0$

or, $6p^2 + 3p + 2p + 1 = 0$

or, $3p(2p + 1) + 1(2p + 1) = 0$

or, $(3p + 1)(2p + 1) = 0$

Hence, $p = \frac{-1}{3}, \frac{-1}{2}$

II. $20q^2 + 9q + 1 = 0$

or, $20q^2 + 5q + 4q + 1 = 0$

or, $5q(4q + 1) + 1(4q + 1) = 0$

or, $(5q + 1)(4q + 1) = 0$

Hence, $q = \frac{-1}{5}, \frac{-1}{4}$ Thus $p < q$.

41. Ans. (b)

Solution **I.** $3p^2 + 2p - 1 = 0$

or, $3p^2 + 3p - p - 1 = 0$

or, $3p(p + 1) - 1(p + 1) = 0$

or, $(3p - 1)(p + 1) = 0$

Therefore, $p = \frac{1}{3}, -1$

II. $2q^2 + 7q + 6 = 0$

or, $2q^2 + 4q + 3q + 6 = 0$

or, $2q(q + 2) + 3(q + 2) = 0$

or, $(2q + 3)(q + 2) = 0$.

Therefore, $q = -\frac{3}{2}, -2$. Thus, $p > q$

42. Ans. (d)

Solution **I.** $3p^2 + 15p + 18 = 0$

or, $3p^2 + 9p + 6p + 18 = 0$

or, $3p(p + 3) + 6(p + 3) = 0$

or, $(3p + 6)(p + 3) = 0$

or, $p = \frac{-6}{3}, -3 = -2, -3$.

II. $q^2 + 7q + 12 = 0$

or, $q^2 + 4q + 3q + 12 = 0$

or, $q(q + 4) + 3(q + 4) = 0$

or, $(q + 3)(q + 4) = 0$ or, $q = -3, -4$

Therefore, $p \geq q$.

43. Ans. (a)

Solution **I.** $p = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$

II. $9q^2 - 12q + 4 = 0$

or, $9q^2 - 6q - 6q + 4 = 0$

or, $3q(3q - 2) - 2(3q - 2) = 0$

or, $(3q - 2)(3q - 2) = 0$ or, $q = \frac{2}{3}$

Therefore, $p = q$.

44. Ans. (e)

Solution **I.** $p^2 + 13p + 42 = 0$

or, $p^2 + 7p + 6p + 42 = 0$

or, $p(p + 7) + 6(p + 7) = 0$

or, $(p + 6) + (p + 7) = 0$

or, $p = -6, -7$

II. $q^2 = 36 \Rightarrow q = \sqrt{36}$

$\therefore \quad q = +6, -6$.

Therefore $q \geq p$.

45. Ans. (b)

Solution **I.** $p^2 - 6p = 7$ or, $p^2 - 6p - 7 = 0$

or, $(p - 7)(p + 1) = 0$ or, $p = 7, -1$

II. $2q^2 + 13q + 15 = 0$

or, $(2q + 3)(q + 5) = 0$ or, $q = \frac{-3}{2}, -5$

Hence, $p > q$.

46. Ans. (c)

Solution **I.** $3p^2 - 7p + 2 = 0$

or, $3p^2 - 6p - p + 2 = 0$

or, $(p - 2)(3p - 1) = 0$ or, $p = 2, 1/3$

II. $2q^2 - 11q + 15 = 0$

or, $2q^2 - 6q - 5q + 15 = 0$

or, $(2q - 5)(q - 3) = 0$ or, $q = 5/2, 3$

Hence, $q > x$.

47. Ans. (d)

Solution **I.** $10p^2 - 7p + 1 = 0$

or, $10p^2 - 5p - 2p + 1 = 0$

or, $(2p - 1)(5p - 1) = 0$ or, $p = 1/2, 1/5$

II. $35q^2 - 12q + 1 = 0$

or, $35q^2 - 7q - 5q + 1 = 0$

or, $(5q - 1)(7q - 1) = 0$ or, $q = 1/5, 1/7$

Hence, $p \geq q$.

48. Ans. (c)

Solution **I.** $4p^2 = 25$ or, $p = \pm \frac{5}{2}$

II. $2q^2 - 13q + 21 = 0$

or, $2q^2 - 6q - 7q + 21 = 0$

or, $(q - 3)(2q - 7) = 0$ or, $q = 3, 7/2$

Hence, $q > p$.

49. Ans. (e)

Solution **I.** $3p^2 + 7p - 6 = 0$

or, $3p^2 + 9p - 2p - 6 = 0$

or, $(p + 3)(3p - 2) = 0$ or, $p = -3, 2/3$

II. $6(2q^2 + 1) = 17q$

or, $12q^2 + 6 - 17q = 0$

or, $12q^2 - 9q - 8q + 6 = 0$

or, $(4q - 3)(3q - 2) = 0$ or, $q = \frac{3}{4}, \frac{2}{3}$

Hence, $q \geq p$.

50. Ans. (e)

Solution **I.** $p^2 + 5p + 6 = 0$

or, $(p + 3)(p + 2) = 0$ or, $p = -3$ or, -2

II. $q^2 + 3q + 2 = 0$

or, $(q + 2)(q + 1) = 0$ or, $q = -1$ or, -2

Hence, $q \geq p$.

51. Ans. (d)

Solution **I.** $p^2 = 4$ or, $p = \pm 2$

II. $q^2 + 4q = -4$

or, $q^2 + 4q + 4 = 0$ or, $(q + 2)^2 = 0$

or, $q + 2 = 0$ or, $q = -2$

Hence, $p \geq q$.

52. Ans. (c)

Solution **I.** $p^2 + p = 56$

or, $p^2 + p - 56 = 0$

or, $p^2 + 8p - 7p - 56 = 0$

or, $(p - 7)(p + 8) = 0$ or, $p = -8$ or, 7

II. $q^2 - 17q + 72 = 0$

or, $(q - 8)(q - 9) = 0$ or, $q = 8$ or, 9

Hence, $q > p$.

53. Ans. (b)

Solution **I.** $3p + 2q - 58 = 0$ or, $3p + 2q = 58$

II. $4q + 4p = 92$ or, $q + p = 23$

or, $p = 23 - q$

or, $3(23 - q) + 2q = 58$ [From I]

or, $69 - 3q + 2q = 58$ or, $q = 11$

Also, $p = 23 - q = 23 - 11 = 12$

Hence, $p > q$.

54. Ans. (c)

Solution **I.** $3p^2 + 17p + 10 = 0$

$3p^2 + 15p + 2p + 10 = 0$

or, $(p + 5)(3p + 2) = 0$ or, $p = -5$ or, $-2/3$

II. $10q^2 + 9q + 2 = 0$

or, $10q^2 + 5q + 4q + 2 = 0$

or, $5q(2q + 1) + 2(2q + 1) = 0$

or, $(5q + 2)(2q + 1) = 0$

or, $q = -2/5$ or, $-1/2$.

Hence, $q > p$.

55. Ans. (c)

Solution **I.** $q^2 - 6q + 9 = 0$ or, $(q - 3)^2 = 0$ or, $q = 3$

II. $p^2 + 2p - 3 = 0$ or, $p = 1, -3$

Hence, $q > p$.

56. Ans. (b)

Solution **I.** $p^2 - 5p + 6 = 0$

or, $(p - 3)(p - 2) = 0$ or, $p = 2$ or, 3

II. $2q^2 + 3q - 5 = 0$ or, $q = 1, -5/2$

Hence, $p > q$.

57. Ans. (a)

Solution **I.** $p = \sqrt{256} = 16$

II. $q = (-4)^2 = 16$

Hence, $p = q$.

58. Ans. (c)

Solution **I.** $p^2 - 6p + 5 = 0$ or, $p = 1, 5$

II. $q^2 - 13q + 42 = 0$

or, $(q - 7)(q - 6) = 0$ or, $q = 6, 7$

Hence, $q > p$.

59. Ans. (c)

Solution **I.** $p^2 + 3p + 2 = 0$

or, $(p + 2)(p + 1) = 0$ or, $p = -2$ or, -1

II. $q^2 - 4q + 1 = 0$ or, $q = 2 \pm \sqrt{3}$

Hence, $q > p$.

60. Ans. (c)

Solution **I.** $p^2 + 3p + 2 = 0$

or, $p^2 + 2p + p + 2 = 0$

or, $(p + 2)(p + 1) = 0$ or, $p = -2, -1$

II. $2q^2 = 5q$ or, $2q^2 - 5q = 0$

or, $q(2q - 5) = 0$ or, $q = 0, 5/2$

Hence, $q > p$.

61. Ans. (e)

Solution **I.** $2p^2 + 5p + 2 = 0$

or, $2p^2 + 4p + p + 2 = 0$

or, $(p + 2)(2p + 1) = 0$ or, $p = -2, -1/2$

II. $4q^2 = 1$ or, $q^2 = 1/4$ or, $q = \pm \frac{1}{2}$.

Hence, $q \geq p$.

62. Ans. (b)

Solution **I.** $q^2 + 2q - 3 = 0$ or, $q = 1, -3$

II. $2p^2 - 7p + 6 = 0$

or, $2p^2 - 4p - 3p + 6 = 0$

or, $(2p - 3)(p - 2) = 0$ or, $p = 2, 3/2$

Hence, $p > q$.

63. Ans. (e)

Solution **I.** $p^2 + 2p - 8 = 0$

or, $p^2 + 4p - 2p - 8 = 0$

or, $p(p + 4) - 2(p + 4) = 0$

or, $(p + 4)(p - 2) = 0$ or, $p = 2, -4$

II. $q^2 - 2 = 7$ or, $q^2 = 9$ or, $q = \pm 3$

Here, we are unable to find out the relationship between p and q.

64. Ans. (d)

Solution **I.** $p^2 - 5p + 6 = 0$

or, $p^2 - 3p - 2p + 6 = 0$

or, $p(p - 3) - 2(p - 3) = 0$ or, $(p - 3)(p - 2)$

or, $p = 2, 3$

II. $q^2 + q - 6 = 0$

or, $q^2 + 3q - 2q - 6 = 0$

or, $q(q + 3) - 2(q + 3) = 0$

or, $(q + 3)(q - 2) = 0$ or, $q = 2, -3$

Hence, $p \geq q$.

65. Ans. (b)

Solution **I.** $p^2 - 7p = -12$

or, $p^2 - 7p + 12 = 0$

or, $(p - 3)(p - 4) = 0$ or, $p = 3, 4$

II. $q^2 - 3q + 2 = 0$

or, $(q - 2)(q - 1) = 0$ or, $q = 1, 2$

Hence, $p > q$.

66. Ans. (c)

Solution **I.** $12p^2 - 7p = -1$

or, $12p^2 - 7p + 1 = 0$

or, $(3p - 1)(4p - 1) = 0$ or, $p = 1/2, 1/3$

II. $6q^2 - 7q + 2 = 0$

or, $(3q - 2)(2q - 1) = 0$ or, $q = 1/2, 2/3$

Hence, $q > p$.

67. Ans. (e)

Solution **I.** $p^2 + 12p + 35 = 0$

or, $(p + 7)(p + 5) = 0$ or, $p = -7, -5$

II. $2q^2 + 22q + 56 = 0$

or, $q^2 + 11q + 28 = 0$

or, $(q + 7)(q + 4) = 0$ or, $q = -7$ or, -4

Form the above values, we are unable to find the relationship between p and q.

68. Ans. (d)

Solution **I.** $p^2 - 8p + 15 = 0$

or, $(p - 3)(q - 5) = 0$ or, $p = 3, 5$

II. $q^2 - 5q + 6 = 0$

or, $(q - 2)(q - 3) = 0$ or, $q = 2, 3$

Hence, $p \geq q$.

69. Ans. (e)

Solution **I.** $2p^2 + 20p + 50 = 0$

or, $p^2 + 10p + 25 = 0$ or, $(p + 5)^2 = 0$

or, $p = -5$

II. $q^2 = 25$ or, $q = \pm 5$

Hence, $p \leq q$.

70. Ans. (b)

Solution **I.** $2p^2 = 23p - 63$

or, $2p^2 - 23p + 63 = 0$

or, $(2p - 9)(p - 7) = 0$

$\therefore$ $p = \frac{9}{2}, 7$

II. $2q(q^{-8}) = q^{-36}$ or, $q^{-7} \times q^{36} = \frac{1}{2}$ or, $q^{29} = \frac{1}{2}$

$\therefore$ $q = \left(\frac{1}{2}\right)^{1/29}$

Hence, $p > q$.

71. Ans. (c)

Solution **I.** $p(p^{-1}) = p^{-1}$

or, $p \times \frac{1}{p} = \frac{1}{p}$ $\therefore$ $p = 1$

II. $q^2 = 4(q^{-1})$ or, $q^3 = 4$ $\therefore$ $q = (4)^{1/3} > 1$

Hence, $q > p$.

72. Ans. (e)

Solution **I.** $2p(p - 4) = 8(p - 5)$

or, $2p^2 - 8p = 8p - 40$ or, $p^2 - 8p + 20 = 0$

$\therefore$ $p = 4 \pm 2i$

II. $q^2 + 4 = 7q^{-8}$ or, $q^{10} + 4q^8 - 7 = 0$

The solution of q will be either imaginary or real number.

Hence, relation between p and q cannot be established.

73. Ans. (c)

Solution **I.** $pq + 30 = 6p + 5q$

or, $(6p - 30) + (5q - pq) = 0$

or, $6(p - 5) - q(p - 5) = 0$

or, $(p - 5)(6 - q) = 0$ Þ $p = 5, q = 6$

Hence, $q > p$.

74. Ans (e)

Solution **I.** $2q^2 + 12p + 16 = 0$

or, $p^2 + 6p + 8 = 0$

or, $(p + 4)(p + 2) = 0$

$\therefore$ $p = -4$ or, -2

II. $2q^2 + 14q + 24 = 0$

or, $q^2 + 7q + 12 = 0$

or, $(q + 4)(q + 3) = 0$

$\therefore$ $q = -3$ or, -4

When $p = -2$, $q = -3$, then $p > q$

When $p = -4$, $q = -4$, then $p = q$

When $p = -4$, $q = -3$, then $p < q$

Thus, no particular relationship can be established between p and q.

75. Ans. (b)

76. Ans. (e)

Solution **I.** $q^2 + q = 2$

or, $q^2 + q - 2 = 0$

or, $(q + 2)(q - 1) = 0$

$\therefore$ $q = -2$ or, 1.

II. $p^2 + 7p + 10 = 0$

or, $p^2 + 5p + 2p + 10 = 0$

or, $(p + 5)(q + 2) = 0$

$\therefore$ $p = -5$ or, -2

Hence, $q \geq p$.

77. Ans. (a)

Solution **I.** $p^2 + 36 = 12p$

or, $p^2 - 12p + 36 = 0$

or, $(p - 6)^2 = 0$ $\therefore$ $p = 6$

II. $4q^2 + 144 = 48q$

or, $q^2 - 12q + 36 = 0$ or, $(q - 6)^2 = 0$

$\therefore$ $q = 6$

Hence, $p = q$.

78. Ans. (a)

Solution **I.** $\frac{4p}{5} - \frac{8}{15} = 0$ $\therefore$ $p = \frac{2}{3}$

II. $9q^2 = 12q - 4$ or, $(3q - 2)^2 = 0$ $\therefore$ $q = \frac{2}{3}$

Hence, $p = q$.

79. Ans. (b)

Solution **I.** $\frac{3p}{4} + \frac{2p}{5} = 2\frac{3}{10}$ or, $\frac{23}{20}p = \frac{23}{10}$

$\therefore$ $p = 2$

II. $\frac{1}{2}q = \frac{1}{4}q - \frac{1}{6}$ or, $\frac{q}{4} = -\frac{1}{6}$ $\therefore$ $q = -\frac{2}{3}$

Hence, $p > q$.

80. Ans. (c)

Solution **I.** $q^2 - 15q + 56 = 0$

or, $(q - 8)(q - 7) = 0$ $\therefore$ $q = 7$ or, 8

II. $2p^2 - 10p + 12 = 0$

or, $p^2 - 5p + 6 = 0$ $\therefore$ $p = 2$ or, 3

Hence, $q > p$.

81. Ans. (d)

Solution **I.** $18p^2 + 3p = 3$ or, $6p^2 + p - 1 = 0$

$\therefore$ $p = \frac{-1}{2}$ or, $\frac{1}{3}$

II. $14q^2 + 9q + 1 = 0$ or, $\left(q + \frac{1}{2}\right)\left(q + \frac{1}{7}\right) = 0$

$\therefore$ $q = \frac{-1}{7}$ or, $\frac{-1}{2}$

Hence, $p \geq q$.

82. Ans. (e)

Solution **I.** $p^2 - 12p + 36 = 0$ or, $(p - 6)^2 = 0$

$\therefore$ $p = 6$

II. $q^2 + 48 = 14q$

or, $q^2 - 14q + 48 = 0$ $\therefore$ $q = 8$ or, 6

Hence, $q \geq p$.

MODEL TEST PAPER I

BASED ON CDS EXAMINATION, FEBRUARY 2007

1. If $A = \{(2^{2n} - 3n - 1) \mid n \in N\}$ and $B = \{9(n-1) \mid n \in N\}$, then which one of the following is correct?
(a) $A \subset B$
(b) $B \subset A$
(c) $A = B$
(d) Neither A is a subset of B nor B is a subset of A

2. What least number should be subtracted from 26492518 so that the resulting number is divisible by 3 but not by 9 ?
(a) 1 (b) 3
(c) 4 (d) 7

3. A bell rings every 5 seconds. A second bell rings every 6 seconds and a third one rings every 8 seconds. If all the three ring at the same time at 8:00 a.m., at what time will they all ring together next?
(a) 1 minute past 8:00 a.m.
(b) 2 minutes past 8:00 a.m.
(c) 3 minutes past 8:00 a.m.
(d) 4 minutes past 8:00 a.m.

4. What is the unit digit of the product of all prime numbers between 1 and 100 ?
(a) 0 (b) 1
(c) 2 (d) 3

5. A scooterist completes a certain journey in 10 hours. He covers half the distance at 30 kmph and the rest at 70 kmph. What is the total distance of the journey?
(a) 210 km (b) 400 km
(c) 420 km (d) 500 km

6. 42 men take 25 days to dig a pond. If the pond would have to be dug in 14 days, then what is the number of men to be employed?
(a) 67 (b) 75
(c) 81 (d) 84

7. At what rate percent per annum calculated in simple interest will a sum of money double in 10 years?
(a) 10% (b) 12%
(c) 12.5% (d) 13.5%

8. An amount at compound interest doubles itself in 4 years. In how many years will the amount become 8 times itself?
(a) 8 years (b) 12 years
(c) 16 years (d) 24 years

9. A trader marked a watch 40% above the cost price and then gave a discount of 100%. He made a net profit of Rs 468 after paying a tax of 10% on the gross profit. What is the cost price of the watch?
(a) Rs 1,200 (b) Rs 1,800
(c) Rs 2,000 (d) Rs 2,340

10. By giving 25% discount a trader earns 25% profit. If he sells the item at 10% discount, what is his profit?
(a) 10% (b) 40%
(c) 45% (d) 50%

11. A person can do a job as fast as his two sons working together. If one son does the job in 6 days and the other in 12 days, how many days does it take the father to do the job?
(a) 9 days (b) 6 days
(c) 4 days (d) 3 days

12. A sum of Rs 24,000 is borrowed for $1\frac{1}{2}$ years at the rate of interest 10% per annum compounded semiannually. What is the compound interest (x)?
(a) $x <$ Rs 3,000
(b) Rs 3,000 $< x <$ Rs 4,000
(c) Rs 4,000 $< x <$ Rs 5,000
(d) $x >$ Rs 5,000

13. Given $3\sin\theta + 4\cos\theta = 5$, then what is $3\cos\theta - 4\sin\theta$ equal to?
(a) 0 (b) 3
(c) 4 (d) 5

14. If $\sec\theta = 13/5$, then what is the value of $\dfrac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$?
(a) 1 (b) 2
(c) 3 (d) 4

15. If $0° \le x \le 90°$ and $\sin x + \sqrt{3}\cos x = 1$, then what is the value of x ?
(a) 30° (b) 45°
(c) 60° (d) 90°

16. Two houses are collinear with the base of a tower and are at distances 3 m and 12 m from the base of the tower. The angles of elevation from these two houses of the top of the tower are complementary. What is the height of the tower?
(a) 4 m (b) 6 m
(c) 7.5 m (d) 36 m

17. What is the value of $\cos 0° + \cos 1° + \ldots + \cos 180°$?
(a) 2 (b) 1
(c) 0 (d) –1

18. If $0° < x < 90°$ and $2\sin x + 15\cos^2 x = 7$, then what is the value of $\tan x$?
(a) $\dfrac{2}{3}$ (b) $\dfrac{3}{2}$
(c) $\dfrac{3}{4}$ (d) $\dfrac{4}{3}$

19. Which of the following equations is/are correct if $1 + \cos x \cos y + \sin x \sin y = 0$?

1. $\cos x + \cos y = 0$ 2. $\sin x + \sin y = 0$
3. $\sin x + \cos y = 0$

Select the correct answer using the codes given below :

Codes:

(a) 1 only (b) 2 only
(c) 3 only (d) 1 and 2

20. The angle of elevation from the bank of a river of the top of a tree standing on the opposite bank is 60°. The angle of elevation becomes 30° when observed from a point 40 metres backwards in a direction perpendicular to the length of the river. What is the width of the river?

(a) 10 m (b) 20 m
(c) 30 m (d) 40 m

21. What is the simplest value of

$$\frac{(1-\sin A\cos A)(\sin^2 A-\cos^2 A)}{\cos A(\sec A-\operatorname{cosec} A)(\sin^3 A+\cos^3 A)}?$$

(a) $\sin A$ (b) $\cos A$
(c) $\sec A$ (d) $\operatorname{cosec} A$

22. What is the angle which equals one-fifth of its supplement?

(a) 15° (b) 30°
(c) 36° (d) 75°

23. Triangles ABC and DEF are similar triangles such that $\frac{AB}{DE}=\frac{BC}{EF}$. The area of ΔABC is 16 cm^2 and that of ΔDEF is 49 cm^2. If BC = $2\sqrt{2}$ cm, then what is EF equal to?

(a) 3.5 cm (b) $(3.5)\sqrt{2}$ cm
(c) $(3.5)\sqrt{3}$ cm (d) 7.0 cm

24. If the distance from the vertex to the centroid of an equilateral triangle is 6 cm, then what is the area of the triangle?

(a) 24 cm^2 (b) $27\sqrt{3}$ cm^2
(c) 12 cm^2 (d) $12\sqrt{3}$ cm^2

25. Consider the following statements :

If two parallel lines are intersected by a transversal, then each pair of corresponding angles are equal each pair of alternate angles are unequal.

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

26. In ΔABC and ΔDEF, it is given that AB = 5 cm, BC = 4 cm and CA = 4·2 cm; DE = 10 cm, EF = 8 cm and FD = 8·4 cm. If AL is perpendicular to BC, and DM is perpendicular to EF, then what is the ratio of AL to DM?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) 1

27. Which one of the following numbers is a composite number?

(a) 589 (b) 571
(c) 569 (d) 563

28. For what value of k, will the expression $3x^3 - kx^2 + 4x + 16$ be divisible by $\left(x-\frac{k}{2}\right)$?

(a) 4 (b) – 4
(c) 2 (d) 0

29. Which one of the following is the factor of $x^4 + xy^3 + xz^3 + x^3y + y^4 + yz^3$?

(a) $x + y + z$ (b) $x^2 + y^2 + z^2$
(c) $x^3 + y^3 + z^3$ (d) $x^2 + y^2$

30. If $x^2 - kx - 21 = 0$ and $x^2 - 3kx + 35 = 0$ have one common root, then what is the value of k?

(a) +4 only (b) – 4 only
(c) ±4 (d) ± 1

31. What is the HCF of the polynomials $x^3 + 3x^2y + 2xy^2$ and $x^4 + 6x^3y + 8x^2y^2$?

(a) $x(x + 2y)$ (b) $x(x + 3y)$
(c) $x + 2y$ (d) None of the above

32. What is the value of

$$\left(\log_{\frac{1}{2}} 2\right)\left(\log_{\frac{1}{3}} 3\right)\left(\log_{\frac{1}{4}} 4\right)\ldots\left(\log_{\frac{1}{1000}} 1000\right)?$$

(a) 1 (b) –1
(c) 1 or –1 (d) 0

33. If $pqr = 1$, what is the value of the expression

$$\frac{1}{1+p+q^{-1}}+\frac{1}{1+q+r^{-1}}+\frac{1}{1+r+p^{-1}}?$$

(a) 1 (b) –1
(c) 0 (d) $\frac{1}{3}$

34. If $x + y + z = 2s$, then what is

$(s-x)^3+(s-y)^3+3(s-x)(s-y)z$ equal to?

(a) z^3 (b) $-z^3$
(c) x^3 (d) y^3

35. If $x^2 = y + z$, $y^2 = z + x$, $z^2 = x + y$, then what is the value of

$$\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}?$$

(a) 1 (b) 0
(c) –1 (d) 2

36. Consider the following decimal numbers:

1. 1·6666666… 2. 1·181181118…
3. 2·010010001... 4. 1·454545...

Which of the above numbers represent(s) rational number(s)?

(a) 4 only (b) 2 and 3
(c) 1 and 4 (d) None of the above

37. Let p, q, r be such that $p + q = r$ and $pqr = 30$. What is the value of $p^3 + q^3 - r^3$?

(a) 0 (b) 90
(c) –90
(d) Cannot be determined as the data is insufficient

Directions: The following **four (4)** items consist of two statements: one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answers to these items using the codes given below:

Codes:

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

38. **Assertion (A):** The equations $2x - 3y = 5$ and $6y - 4x = 11$ cannot be solved graphically.
Reason (R): The equations given above represent parallel lines.

39. **Assertion (A):** If $\tan\theta + \cot\theta = 2$, then $\tan^n\theta + \cot^n\theta = 2$ for all $n \in N$.
Reason (R): $\tan\theta + \cot\theta = \tan^n\theta + \cot^n\theta$ for all $n \in N$.

40. **Assertion (A):** The mean of first seven prime numbers is greater than their median.
Reason (R): Mean is always greater than median.

41. **Assertion (A):** When eight drops of water combine to form a single drop, the surface area of all the eight drops is greater than the surface area of big drop.
Reason (R): Square of volume of a spherical body is directly proportional to cube of its surface area.

42. Which one of the following is correct?
(a) $\{\Phi\} \subset \{\{\Phi\}, \{\{\Phi\}\}\}$
(b) $\{\Phi\} \in \{\{\Phi\}, \{\{\Phi\}\}\}$
(c) $\Phi \in \{\{\Phi\}, \{\{\Phi\}\}\}$
(d) $\Phi = \{\{\Phi\}, \{\{\Phi\}\}\}$

43. The cumulative frequency (*c.f.*) distribution of a variate X is given as follows:

X :	1	2	3	4	5
c.f. :	2	7	13	k	31

What can be the frequency ($f \neq 0$) of the variate $X = 4$?
(a) $13 < f < 31$ (b) $13 \leq f \leq 31$
(c) $0 < f < 18$ (d) $0 < f \leq 18$

44. When rectangles are drawn with the areas proportional to the frequencies of the respective class intervals, what is the diagram called?
(a) Ogive (b) Frequency polygon
(c) Pie diagram (d) Histogram

45. The heights of 50 students are tabulated as below:

Class interval (in cm)	*Frequency*
145 – 146	1
147 – 148	2
149 – 150	4
151 – 152	5
153 – 154	8
155 – 156	15
157 – 158	9
159 – 160	6

Which one of the following is correct about the distribution?
(a) It is positively skewed
(b) It is negatively skewed
(c) It is symmetric
(d) It is extremely asymmetric

46. If the mean of the variable X is M, what is the mean of variable $Y = (X - 4)/5$?
(a) $(M - 4)/5$ (b) M
(c) $M - 5$ (d) $5M - 4$

47. A batsman scores 80 runs in his sixth innings and thus increases his average by 5. What is his average after six innings?
(a) 50 (b) 55
(c) 60 (d) 65

48. What type of classification is needed to enumerate the female population of India?
(a) Geographical (b) Chronological
(c) Qualitative (d) Quantitative

49. What is the geometrical mean of the variate which takes values 210, 201, 102, 20, 12, 10, 2, 1 and 0?
(a) 3 (b) 2
(c) 1 (d) 0

50. The vertical height of a right circular cone is twice its diameter and its volume is 36π cm^3. What is its vertical height?
(a) 4 cm (b) 8 cm
(c) 12 cm (d) 16 cm

51. A wax cube of edge 6 cm is melted and formed into three smaller cubes. If the edges of two smaller cubes are 3 cm and 4 cm, what is the edge of the third smaller cube?
(a) 5 cm (b) 4.5 cm
(c) 4 cm (d) 3.5 cm

52. A right circular cylinder and a right circular cone have equal bases and equal volumes. But the lateral surface area of the right circular cone is 15/8 times the lateral surface area of the right circular cylinder. What is the ratio of radius to height of the cylinder?
(a) 3 : 4 (b) 9 : 4
(c) 15 : 8 (d) 8 : 15

53. Water is distributed to a town of 50000 inhabitants from a rectangular reservoir consisting of 3 equal compartments. Each compartment has length and breadth 200 m, 100 m respectively and 12 m depth of water in the beginning. The allowance is 20 litres per head per day. For how many days will the supply of water hold out?
(a) 240 days (b) 720 days
(c) 800 days (d) 900 days

54. A swimming pool is 24 m long and 15 m broad. When x number of men dive into the pool, the height of the water rises by 1 cm. If the average amount of water displaced by one man is 0.1 m^3, then what is the value of x ?
(a) 36 (b) 72
(c) 108 (d) 360

55. A man borrowed Rs 40,000 at 8% simple interest per year. At the end of second year he paid back certain amount and at the end of fifth year he paid back Rs 35,960 and cleared the debt. What is the amount did he pay back after the second year?
(a) Rs 16,200 (b) Rs 17,400
(c) Rs 18,600 (d) None of the above

56. A ten-digit number is divisible by 4 as well as by 5. What could be the possible digit at the ten's place in the given number?
(a) 0, 1, 2, 4 or 6 (b) 1, 2, 4, 6 or 8
(c) 2, 3, 4, 6 or 8 (d) 0, 2, 4, 6 or 8

57. Six years hence a father's age will be three times his son's age and three years ago father was nine times as old as his son. What is the present age of father?
(a) 48 years (b) 42 years
(c) 36 years (d) 30 years

58. By adding x to 1254934, the resulting number becomes divisible by 11, while adding y to 1254934 makes the resulting number divisible by 3. Which one of the following is the set of values for x and y ?
(a) $x = 1, y = 1$ (b) $x = 1, y = -1$
(c) $x = -1, y = 1$ (d) $x = -1, y = -1$

59. x varies inversely as the square of y in such a way that if $x = 1$, then $y = 6$. If $y = 3$, then what is the value of x ?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) 2 (d) 4

60. LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 80, then what is the other number?

(a) 40 (b) 60
(c) 160 (d) 280

61. Two numbers have 16 as their HCF and 146 as their LCM. How many such pairs of numbers are there?

(a) Zero (b) Only 1
(c) Only 2 (d) Many

62. What is the least number which when divided by 7, 9 and 12 leaves the same remainder 1 in each case?

(a) 253 (b) 352
(c) 505 (d) 523

63. Which one of the following numbers is an integer?

(a) $\left[\left(\sqrt{2}+\sqrt{3}\right)/\left(\sqrt{3}-\sqrt{2}\right)\right]+\sqrt{6}$

(b) $\left[\left(\sqrt{2}+\sqrt{3}\right)/\left(\sqrt{3}-\sqrt{2}\right)\right]+2\sqrt{6}$

(c) $\left[\left(\sqrt{2}+\sqrt{3}\right)/\left(\sqrt{2}-\sqrt{3}\right)\right]+2\sqrt{6}$

(d) $\left[\left(\sqrt{2}+\sqrt{3}\right)/\left(\sqrt{2}-\sqrt{3}\right)\right]+\sqrt{6}$

64. Which one of the following is true for some value of θ, where $0° \le \theta \le 90°$?

(a) $\sin\theta = \sqrt{2}$ (b) $\sin\theta + \cos\theta = 2$
(c) $\sin\theta + \cos\theta = 0$ (d) $\sin\theta - \cos\theta = 1$

65. If $\sin\theta + \operatorname{cosec}\theta = 2$, then what is the value of $\sin^4\theta + \cos^4\theta$?

(a) 2 (b) 2^2
(c) 2^3 (d) 1

66. Let $\theta \in R$ be such that $\sec\theta > 0$ and $2\sec^2\theta + \sec\theta - 6 = 0$. What is the value of $\operatorname{cosec}\theta$?

(a) $\sqrt{5}$ (b) $\sqrt{3}/2$
(c) $3/\sqrt{5}$ (d) $2/\sqrt{3}$

67. Consider a circle C of radius 6 cm with centre at O. What is the difference in the area of the circle C and the area of the sector of C subtending an angle of 80° at O?

(a) 26π cm^2 (b) 16π cm^2
(c) 28π cm^2 (d) 30π cm^2

68. If $\pi = 22/7$, then what is the approximate value of 1 radian?

(a) 57°16′22″ (b) 57°15′22″
(c) 57°16′20″ (d) 57°15′20″

69. Under which one of the following conditions is the trigonometrical identity $\sin x/(1 + \cos x) = (1 - \cos x)/\sin x$ true?

(a) x is not a multiple of 360°
(b) x is not an odd multiple of 180°
(c) x is not a multiple of 180°
(d) None of the above

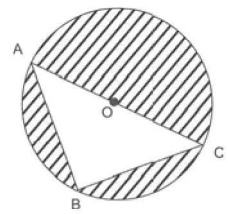
70. The volume of a cuboid whose sides are in the ratio of 1 : 2 : 4 is same as that of a cube. What is the ratio of length of diagonal of cuboid to that of cube?

(a) $\sqrt{1.25}$ (b) $\sqrt{1.75}$
(c) $\sqrt{2}$ (d) $\sqrt{3.5}$

71. If the length of a rectangle is increased by 10% and the area is unchanged, then by how much percent does the breadth decrease?

(a) 100/11% (b) 100/9%
(c) 9% (d) 10%

72.



In the figure given above, the area of the shaded portion is $(\pi - 1)R^2$, where R is the radius of the circle with centre at O. What is one of the acute angles of the triangle ABC?

(a) 30° (b) 45°
(c) 60° (d) Cannot be determined

73. ΔABC is an isosceles right-angled triangle. If r is its in-radius and R is its circum-radius, then what is R/r equal to?

(a) $\sqrt{2}+1$ (b) $3+\sqrt{2}$
(c) $\frac{5}{2}$ (d) $\sqrt{3}+\sqrt{2}$

74. If the diameter of a wire is decreased by 10%, by how much percent (approximately) will the length be increased to keep the volume constant?

(a) 5% (b) 17%
(c) 20% (d) 23%

75. The diameter of a solid metallic right circular cylinder is equal to its height. After cutting out the largest possible solid sphere S from this cylinder, the remaining material is recast to form a solid sphere S_1. What is the ratio of the radius of sphere S to that of sphere S_1?

(a) $1 : 2^{\frac{1}{3}}$ (b) $2^{\frac{1}{3}} : 1$
(c) $2^{\frac{1}{3}} : 3^{\frac{1}{3}}$ (d) $3^{\frac{1}{2}} : 2^{\frac{1}{2}}$

76. Two solid spheres of gold having diameters 3 cm and 4 cm are molten and then cast into one big sphere of gold. If the radius of this sphere is x, then what is the value of x^3 ?

(a) 125 cm^3 (b) 15.625 cm^3
(c) 11.375 cm^3 (d) 9.875 cm^3

77. A square has its side equal to the radius of a sphere. The square revolves round a side to generate a surface of total area S. If A be the surface area of the sphere, which one of the following is correct?

(a) $A = 3S$ (b) $A = 2S$
(c) $A = S$ (d) $A < S$

78. What is the area of a right triangle of hypotenuse 313 cm and base 312 cm?

(a) 40 cm^2 (b) 39 cm^2
(c) 25 cm^2 (d) 51 cm^2

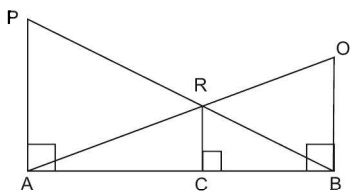
79. The diameter of a bus wheel is 140 cm. The speed of the bus is 66 km per hour. What is the number of revolutions of the wheel per minute?
(a) 200 (b) 250
(c) 300 (d) 350

80. A wire bent in the form of a circle has a radius of 16 cm. What will be the area of a square which is formed out of the full length of such wire?
(a) $\pi^2/4$ cm^2 (b) $4\pi^2$ cm^2
(c) $16\pi^2$ cm^2 (d) $64\pi^2$ cm^2

81. In ΔPQR, $PQ = 4$cm, $QR = 3$cm and $RP = 3.5$ cm. ΔDEF is similar to ΔPQR. If $EF = 9$ cm, then what is the perimeter of ΔDEF ?
(a) 10.5 cm (b) 21 cm
(c) 31.5 cm
(d) Cannot be determined as data is insufficient

82. AD is the angular bisector of a ΔABC such that $BD : DC = 2 : 3$. If $AB = 7$ cm, what is $AC : BC$?
(a) 2 : 3 (b) 3 : 2
(c) 21 : 10 (d) Data is insufficient

83.

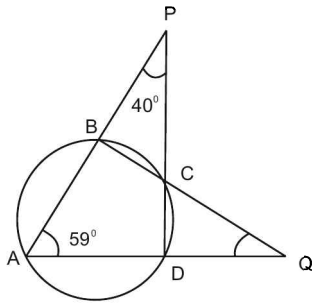


In the figure given above, if $PA = x$, $RC = y$, $QB = z$, then which one of the following is correct?
(a) $2y = x + z$ (b) $4y = x + z$
(c) $xy + yz = xz$ (d) $xy + xz = yz$

84. C is a point on the minor arc AB of a circle with centre O. If $\angle AOB = 100°$, what is $\angle ACB$?
(a) 80° (b) 90°
(c) 100° (d) 130°

85. In a ΔABC, $AB = AC$. A circle through B touches AC at D and intersects AB at P. If D is the midpoint of AC, which one of the following is correct?
(a) $AB = 2AP$ (b) $AB = 3AP$
(c) $AB = 4AP$ (d) $2AB = 5AP$

86.



In the figure given above, if $\angle PAQ = 59°$, $\angle APD = 40°$, then what is $\angle AQB$?
(a) 19° (b) 20°
(c) 22° (d) 27°

87. An obtuse angle made by a side of a parallelogram $PQRS$ with other pair of parallel sides is 150°. If the perpendicular distance between these parallel sides (PQ and SR) is 20 cm, what is the length of the side RQ?
(a) 40 cm (b) 50 cm
(c) 60 cm (d) 70 cm

88. What is the area of the region of the circle which is situated outside the inscribed square of side x?
(a) $(\pi - 2)x^2$ (b) $(\pi - 2)x^2/2$
(c) $2(\pi - 2)x^2$ (d) $(\pi - 2)x^2/4$

89. The area of a right-angled isosceles triangle is 72 cm^2. What is the length of the hypotenuse?
(a) $6\sqrt{2}$ cm (b) 12 cm
(c) $12\sqrt{2}$ cm (d) 24 cm

90. A rectangular field is half as wide as it is long and is completely enclosed by x m of fencing. What is the area of the field?
(a) $x^2/2$ m^2 (b) $2x^2$ m^2
(c) $2x^2/9$ m^2 (d) $x^2/18$ m^2

91. What is $((A \cup B)' \cap A) - (A - B)$ equal to?
(a) Φ (b) A
(c) B (d) B'

92. Which one of the following is the equation whose roots are respectively three times the roots of the equation $ax^2 + bx + c = 0$?
(a) $ax^2 + 3bx + c = 0$ (b) $ax^2 + 3bx + 9c = 0$
(c) $ax^2 - 3bx + 9c = 0$ (d) $ax^2 + bx + 3c = 0$

93. If $x + y + z = 0$, then what is $[(y - z - x)/2]^3 + [(z - x - y)/2]^3 + [(x - y - z)/2]^3$ equal to?
(a) $24xyz$ (b) $-24xyz$
(c) $3xyz$ (d) xyz

94. If α, β are the roots of $x^2 - 5x + k = 0$, then what is the value of k such that $a - p = 1$?
(a) 1 (b) 3
(c) 4 (d) 6

95. If $x + (1/x) = p$, then what is $x^6 + (1/x^6)$ equal to ?
(a) $p^6 + 6p$ (b) $p^6 - 6p$
(c) $p^6 + 6p^4 + 9p^2 + 2$ (d) $p^6 - 6p^4 + 9p^2 - 2$

96. If $2x - 2^{x-1} = 4$, then what is the value of $2x + 2^{x-1}$?
(a) 8 (b) 10
(c) 12 (d) 16

97. What is $\dfrac{x^8 + 4}{x^4 + 2x^2 + 2}$ equal to?
(a) $x^4 + 2x^2 - 2$ (b) $x^4 - 2x^2 + 2$
(c) $x^4 - 2x^2 - 2$ (d) Cannot be simplified

98. If sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of their squares, then which one of the following is correct?
(a) $a^2 + b^2 = c^2$ (b) $a^2 + b^2 = a + b$
(c) $2ac = ab + b^2$ (d) $2c + b = 0$

99. What is the square root of $\left(\dfrac{x^5 - 1}{x - 1}\right) = (x^3 + 2x^2 + x)$?
(a) $x^2 + x + 1$ (b) $x^2 - x + 1$
(c) $x^2 - x - 1$ (d) $x^2 + x - 1$

100. How many real values of x satisfy the equation
$x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 = 0$?
(a) Only 1 value (b) 2 values
(c) 3 values (d) No value

Answers

1. (d)	**2.** (d)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (b)	**9.** (a)	**10.** (d)
11. (c)	**12.** (b)	**13.** (a)	**14.** (c)	**15.** (d)	**16.** (b)	**17.** (c)	**18.** (d)	**19.** (a)	**20.** (b)
21. (a)	**22.** (b)	**23.** (b)	**24.** (b)	**25.** (a)	**26.** (a)	**27.** (b)	**28.** (b)	**29.** (c)	**30.** (a)
31. (d)	**32.** (a)	**33.** (a)	**34.** (a)	**35.** (a)	**36.** (a)	**37.** (b)	**38.** (d)	**39.** (b)	**40.** (b)
41. (b)	**42.** (d)	**43.** (b)	**44.** (d)	**45.** (d)	**46.** (a)	**47.** (a)	**48.** (d)	**49.** (c)	**50.** (c)
51. (a)	**52.** (b)	**53.** (b)	**54.** (a)	**55.** (b)	**56.** (d)	**57.** (d)	**58.** (b)	**59.** (d)	**60.** (d)
61. (d)	**62.** (a)	**63.** (c)	**64.** (b)	**65.** (d)	**66.** (c)	**67.** (c)	**68.** (a)	**69.** (d)	**70.** (c)
71. (c)	**72.** (d)	**73.** (b)	**74.** (d)	**75.** (d)	**76.** (c)	**77.** (b)	**78.** (b)	**79.** (b)	**80.** (d)
81. (c)	**82.** (d)	**83.** (b)	**84.** (d)	**85.** (c)	**86.** (c)	**87.** (a)	**88.** (b)	**89.** (c)	**90.** (d)
91. (a)	**92.** (b)	**93.** (c)	**94.** (d)	**95.** (d)	**96.** (c)	**97.** (b)	**98.** (c)	**99.** (a)	**100.** (a)

MODEL TEST PAPER II

BASED ON ANDHRA BANK PO EXAMINATION, JANUARY 2007

Directions (Q. 1–5): In the following question what will come in place of question mark (?)

1. $25 + 125 \times 2 = ?$
(a) 375 (b) 30
(c) 6250 (d) 152
(e) None of these

2. 150% of 15 + 75% of 75 = ?
(a) 78.75 (b) 75.75
(c) 281.25 (d) 135
(e) None of these

3. $\left(\left((2)^4\right)^{\frac{1}{2}}\right)^? = 256$
(a) 8 (b) 6
(c) 4 (d) 1
(e) None of these

4. $\sqrt[4]{10,000} = (100)?$
(a) $\frac{1}{2}$ (b) 2
(c) $\frac{1}{4}$ (d) $\frac{1}{8}$
(e) None of these

5. $-20 \times 40 \times \frac{1}{5} + \frac{30}{25}?$
(a) $-\frac{20}{3}$ (b) $\frac{400}{3}$
(c) $-\frac{400}{9}$ (d) – 192
(e) None of these

Directions (Q. 6–10): In the following series what will come in place of the question mark (?)

6. 12 22 69 272 1365 ?
(a) 8196 (b) 8184
(c) 8195 (d) 6830
(e) None of these

7. 1 ? 27 64 125
(a) 14 (b) 4
(c) 9 (d) 8
(e) None of these

8. 104 153 189 214 ?
(a) 239 (b) 225
(c) 264 (d) 235
(e) None of these

9. 15 17 32 49 81 130 ?
(a) 179 (b) 211
(c) 194 (d) 226
(e) None of these

10. 15 17 21 29 45 77 ?
(a) 109 (b) 125
(c) 141 (d) 173
(e) None of these

Directions (Q. 11–15): From the following, different committee are to be made as per the requirement given in each question. In how many different ways can it be done?

10 men and 8 women out of which 5 men are teachers, 3 men are doctors and 2 are businessmen. Among the women 3 are teachers, 2 doctors, 2 researchers and 1 social worker.

11. A committee of 5 in which 3 men and 2 women are there.
(a) 3360 (b) 8568
(c) 4284 (d) 1680
(e) None of these

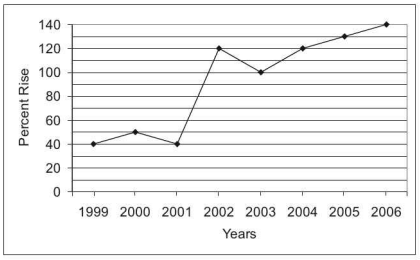
12. A committee of 4 in which at least 2 women are there.
(a) 1260 (b) 1820
(c) 3060 (d) 1890
(e) None of these

13. A committee of 5 in which 2 men teachers, 2 women techers and 1 doctors are there.
(a) 75 (b) 150
(c) 214 (d) 20
(e) None of these

14. A committee of 7 members.
(a) 31824 (b) 1200
(c) 9600 (d) 15912
(e) None of these

15. A committee of 3 in which there is no teacher and no doctor.
(a) 100 (b) 120
(c) 10 (d) 12
(e) None of these

Directions (Q. 16–20): Study the following graph to answer the given questions.



16. For how many years is the per cent rise more than 100?
(a) One (b) Two
(c) Five (d) Three
(e) None of these

17. What is the average of the percent rise over the given years?
(a) 100 (b) 105
(c) 100.5 (d) 92.5

18. For how many years is the per cent rise lower than the average of the percent rise over the given years?
(a) Two (b) One
(c) Five (d) Three
(e) None of these

19. For which of the given years is the per cent rise (from the previous year) in the per cent rise the least? (Years 2001 and 2003 not to be considered)
(a) 2000 (b) 2004
(c) 2006 (d) Cannot be determined
(e) None of these

20. If the production in 1998 was 10,000 units, then how much is the production in 2002?
(a) 35,280 units (b) 64,680 units
(c) 46,200 units (d) Cannot be determined
(e) None of these

Directions *(Q. 21–25)* Study the following table to answer the given questions.

Number of Books of Different Prices Bought in Different Months

Price↓ / Month→	Jan.	March	May	July	Sept.	Nov.
More than Rs. 5,000	50	106	2	30	25	75
Rs. 4,000-Rs. 5,000	105	1000	40	105	400	375
Rs. 3,000-Rs. 3,999	70	1000	80	115	200	240
Rs. 2,000-Rs. 2,999	300	500	100	216	135	300
Rs. 1,000-Rs. 1,999	140	370	200	225	175	470
Rs. 500-Rs. 999	200	700	15	400	75	530
Less than Rs. 500	65	135	111	188	25	65

21. In which month was *maximum* number of books bought ?
(a) March (b) November
(c) July (d) September
(d) None of these

22. For which price range was maximum number of books bought for the given month together?
(a) Rs 500 – Rs 999
(b) Rs 2,000 – Rs 2,999
(c) Rs 1,000 – Rs 1,999
(d) Rs 3,000 – Rs 3,999
(e) None of these

23. The books of price range Rs 1,000 – Rs 1,999 bought in January is what per cent of that of May.
(a) 30 (b) 70
(c) 142.86 (d) 60
(c) None of these

24. How much is the difference in the number of books bought in September and November?
(a) 244 (b) 776
(c) 1020 (d) 1310
(e) None of these

25. What is the respective ratio of books of Rs 4000 – Rs 5000 bought in January to that of March?
(a) 1 : 10 (b) 15 : 100
(c) 930 : 2911 (d) 21 : 200
(e) None of these

Answers

1. (e)	**2.** (a)	**3.** (c)	**4.** (e)	**5.** (e)	**6.** (b)	**7.** (d)	**8.** (e)	**9.** (b)	**10.** (c)
11. (a)	**12.** (d)	**13.** (b)	**14.** (a)	**15.** (c)	**16.** (e)	**17.** (d)	**18.** (d)	**19.** (c)	**20.** (e)
21. (a)	**22.** (e)	**23.** (b)	**24.** (e)	**25.** (d)					

MODEL TEST PAPER III

BASED ON IDBI BANK OFFICERS' EXAMINATION, JUNE 2007

Directions (1–5): Study the following table carefully and answer the questions given below :

Number of Students Passed and Failed in Five Classes of a School Over the Years

Years	CLASSES									
	VI		VII		VIII		IX		X	
	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail
2001	50	42	76	14	58	18	65	17	48	23
2002	60	19	95	22	71	30	75	12	76	28
2003	45	13	61	19	49	15	48	08	74	20
2004	58	21	75	25	80	28	60	11	84	14
2005	55	18	66	29	59	26	70	13	65	17
2006	68	31	54	38	77	34	82	21	55	14

1. What is the average number of failed students from class VII for the given years.
(a) 27.5 (b) 28
(c) 26.5 (d) 26
(e) 24.5

2. What is the ratio between total number of passed students and total number of failed students for the year 2006?
(a) 3:1 (b) 56:53
(c) 67:13 (d) 68:35
(e) 339:137

3. Which of the following classes has the maximum number of passed students, as compared to the total number of students of that class over the years?
(a) VI (b) VII
(c) VIII (d) IX
(e) X

4. What is the number of passed students for all the classes together in the year 2003?
(a) 277 (b) 298
(c) 316 (d) 354
(e) 373

5. What is the total percentage of passed students of class VI from all the years together?
(a) 52 (b) 87
(c) 70 (d) 78
(e) 95

Directions (6–10): Study the following table carefully and answer the questions given below :

Percentage of Marks Obtained by Different Students in Different Subjects

Students	Subjects (Maximum Marks)						
	Hindi (150)	English (150)	Math (150)	S. St. (150)	Science (75)	Marathi (50)	Phy. Edu. (75)
A	88	85	86	74	78	80	85
B	92	80	79	82	70	70	97
C	75	89	85	90	83	91	85
D	63	66	69	71	85	64	62
E	80	76	89	95	79	70	73
F	69	81	86	76	69	85	76

6. How many marks did E get in all the subjects together?
(a) 659 (b) 600
(c) 625 (d) 708
(e) 617

7. What are the average marks obtained by all the students together in English ?
(a) 110.27 (b) 113.76
(c) 121.52 (d) 119.25
(e) 123.74

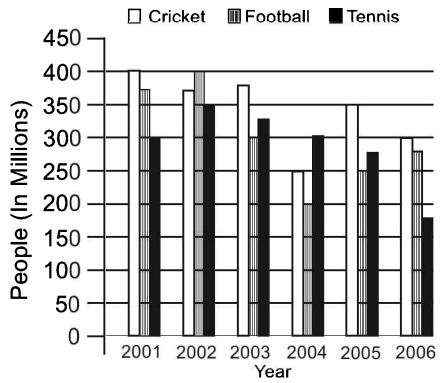
8. How many students have scored the highest marks in more than one subjects?
(a) one (b) two
(c) three (d) four
(e) five

9. Who has scored the highest marks in all the subjects together?
(a) E (b) C
(c) F (d) A
(e) D

10. Marks obtained by F in S.St are what per cent of marks obtained by E in the same subject ?
(a) 74 (b) 85
(c) 76 (d) 83
(e) 80

Directions (11–15): Study the following Graph carefully and answer the question given below:

Preference of People in Playing Different Games Over the Years



11. In the year 2006, the people preferring to play Tennis is what per cent of the people preferring to play Cricket, Football and Tennis together in that year?
(a) $25\frac{1}{3}$ (b) $24\frac{3}{7}$
(c) $21\frac{1}{3}$ (d) $22\frac{2}{9}$
(e) $23\frac{1}{3}$

12. From 2001 to 2006, the total number of people who preferred to play Football was what (in millions)?
(a) 1525 (b) 1620
(c) 1730 (d) 1800
(e) 1925

13. The number of people preferring to play Tennis in 2006, is how many millions fewer than the number of people preferring to play Tennis in 2005?
(a) 115 (b) 120
(c) 100 (d) 97
(e) 95

14. What is the respective ratio of the number of people preferring to play Cricket to the number of people, preferring to play Tennis in the year 2003?
(a) 14:15 (b) 15:13
(c) 15:17 (d) 13:15
(e) 17:14

15. How many people (in millions) have preferred to play Cricket in all the years together?
(a) 2050 (b) 2000
(c) 1850 (d) 1750
(e) 1600

Directions(16–25): What should come in place of question marks (?) in the following questions?

16. $\frac{4}{7} \times 2\frac{4}{5} \div \frac{2}{5} = ?$
(a) $\frac{6}{7}$ (b) 9
(c) $\frac{3}{5}$ (d) 4
(e) None of these

17. $(19.89)^2 - \sqrt{676} = ?$
(a) 369.6121 (b) 395.2177
(c) 359.2165 (d) 395.6161
(e) None of these

18. $-59 \times 61 + 89 = ?$
(a) – 3237 (b) 3327
(c) – 3327 (d) 3416
(e) None of these

19. $3625 \times ? = 1450$
(a) $\frac{1}{3}$ (b) $\frac{2}{5}$
(c) $\frac{1}{5}$ (d) $\frac{4}{5}$
(e) None of these

20. (0.85% of 405) + (2.25% of 550) =?
(a) 16.7143 (b) 15.7143
(c) 13.8143 (d) 14.7143
(e) None of these

21. $916 \times ? \times 3 = 214344$
(a) 78 (b) 58
(c) 84 (d) 66
(e) None of these

22. $(54679 + 5982 + 32614) - (312 \times 69) = ?$
(a) 71528 (b) 77147
(c) 71747 (d) 61757
(e) None of these

23. $\sqrt{?} = (88 \times 42) \div 16$
(a) 3696 (b) 39660
(b) 43163 (d) 53361
(e) None of these

24. (42% of 4240) – (39% of 3854) = ?
(a) 277.20 (b) 277.74
(c) 3283.86 (d) 3284.40
(e) None of these

25. $41 + 4.11 + 0.41 + 411 + 41.11 = ?$
(a) 456.63 (b) 487.63
(c) 456.52 (d) 479.63
(e) None of these

Directions (26–30): What should come in place of the question mark (?) in the following number series?

26. 1 4 27 256 3125 46656 ?
(a) 117649 (b) 279936
(c) 705894 (d) 16807
(e) 823543

27. 30 46 78 126 190 270 ?
(a) 356 (b) 366
(c) 382 (d) 398
(e) 414

28. 380 465 557 656 762 875 ?
(a) 995 (b) 1015
(c) 975 (d) 995
(e) 1025

29. 1250 500 200 80 32 12.8 ?
(a) 5.12 (b) 6.4
(c) 4.3 (d) 6.02
(e) 5.16

30. 23 26 24 27 25 28 ?
(a) 27 (b) 29
(c) 26 (d) 24
(e) 21

Directions (31–35): What **approximate** value should come in place of question marks(?) in the following questions ? (You are not expected to calculate the exact value)

31. $(415.63)^2 = ?$
(a) 172850 (b) 172700
(c) 172750 (d) 172800
(e) 17225

32. $543911 \times 85 = ? \times 46465$
(a) 995 (b) 975
(c) 1005 (d) 955
(e) 1015

33. $\frac{5}{6} \times \frac{2}{9} \times \frac{4}{9} + \frac{6}{7} = ?$
(a) 0.44 (b) 0.32
(c) 0.49 (d) 0.36
(e) 0.40

34. $754.002 + 3.00024 \times 88.976 = ?$
(a) 66616 (b) 1021
(c) 1000 (d) 66600
(e) 1035

35. $\sqrt{727.9995} + (5.1961)^2 = ? + \frac{2}{10.7960}$
(a) 53 (b) 44
(c) 5 (d) 15
(e) 10

36. If 5 workers collect 60 kg wheat in 3 day. How many kilograms of wheat will 8 worker collect in 5 days?
(a) 80 kg (b) 100 kg
(c) 120 kg (d) 160 kg
(e) None of these

37. By how much is $\frac{3}{4}$th of 52 lesser than $\frac{2}{3}$rd of 99?
(a) 27 (b) 33
(c) 39 (d) 66
(e) None of these

38. Which number should replace both the question marks in following equation?
$\frac{?}{432} = \frac{48}{?}$
(a) 141 (b) 143
(c) 145 (d) 147
(e) None of these

39. The average age of A, B and C is 26 years. If the average age of A and C is 29 years, what is the age of B in years?
(a) 26 (b) 20
(c) 29 (d) 23
(e) None of these

40. A sum of money is divided among A, B, C and D in the ratio of 3 : 5 : 8 : 9 respectively. If the share of D is Rs 1,872 more than the share of A, then what is the total amount of money of B and C together?
(a) Rs 4,156 (b) Rs 4,165
(c) Rs 4,056 (d) Rs 4,065
(e) None of these

41. Arun started a business investing Rs 38,000. After 5 months Bakul joined him with a capital of Rs 55,000. At the end of the year the total profit was Rs 22,000. What is the approximate difference between the share of profits of Arun and Bakul?
(a) Rs 1,192 (b) Rs 1,856
(c) Rs 1,007 (d) Rs 1,928
(e) Rs 1,568

42. What is the 40% of 50% of $\frac{3}{4}$ th of 3200?
(a) 480 (b) 560
(c) 420 (d) 600
(e) None of these

43. One fourth of a number is 17. What will 45% of that number be?
(a) 51 (b) 45.4
(c) 37.4 (d) 30.6
(e) None of these

44. Ishan spent Rs 35,645 on buying a bike, Rs 24,355 on buying a television and the remaining 20% of the total amount he had as cash with him. What was the total amount?
(a) Rs 60,000 (b) Rs 72,000
(c) Rs 75,000 (d) Rs 80,000
(e) None of these

45. In how many different ways can the letters of the word 'PROBLEM' be arranged?
(a) 20 (b) 40
(c) 80 (d) 125
(e) None of these

46. In how many ways can a committee of 4 people be chosen out of 8 people?
(a) 118 (b) 36
(c) 70 (d) 126
(e) None of these

47. In how many ways can 6 people be seated on a sofa if there are only 3 seats available?
(a) 18 (b) 120
(c) 2 (d) 216
(e) None of these

48. The sum of three consecutive even numbers is 44 more than the average of these numbers. Which of the following is the third (largest) of these numbers?
(a) 16 (b) 18
(c) 24 (d) Cannot be determined
(e) None of these

49. In a class of 65 students, each student got sweets that are 20% of the total number of students. How many sweets were there?

(a) 635 (b) 845
(c) 955 (d) Cannot be determined
(e) None of these

50. If the numerator of a fraction is increased by 200% and the denominator is increased by 300% the resultant fraction is $\frac{6}{11}$. What was the original fraction?

(a) $\frac{8}{11}$ (b) $\frac{5}{13}$
(c) $\frac{6}{17}$ (d) $\frac{17}{9}$
(e) None of these

Answers

1. (e)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (a)	**7.** (d)	**8.** (c)	**9.** (b)	**10.** (e)
11. (e)	**12.** (d)	**13.** (c)	**14.** (b)	**15.** (a)	**16.** (d)	**17.** (a)	**18.** (c)	**19.** (b)	**20.** (e)
21. (a)	**22.** (c)	**23.** (d)	**24.** (b)	**25.** (e)	**26.** (e)	**27.** (b)	**28.** (d)	**29.** (a)	**30.** (c)
31. (c)	**32.** (a)	**33.** (c)	**34.** (b)	**35.** (e)	**36.** (d)	**37.** (a)	**38.** (e)	**39.** (b)	**40.** (c)
41. (b)	**42.** (a)	**43.** (d)	**44.** (c)	**45.** (e)	**46.** (c)	**47.** (b)	**48.** (c)	**49.** (b)	**50.** (a)

MODEL TEST PAPER IV

BASED ON BANK CLERICAL EXAMINATION, APRIL 2007

Directions (Qs. 1–25): What should come in place of the question mark (?) in the following questions?

1. $\sqrt{7921} \times \sqrt{?} = 6942$
(a) 6241 (b) 5929
(c) 6084 (d) 5776
(e) None of these

2. $856 \times 567 = ?$
(a) 487064 (b) 484218
(c) 482784 (d) 485352
(e) None of these

3. $(?)^3 \times (5)^5 \div 225 = 3000$
(a) 6 (b) 8
(c) 5 (d) 9
(e) None of these

4. 8% of $800 \times ? = 224$
(a) 4.5 (b) 3.5
(c) 4 (d) 3
(e) None of these

5. $\frac{4}{7} \times \frac{2}{3} \times ? = 1008$
(a) 2688 (b) 2604
(c) 2667 (d) 2646
(e) None of these

6. $7.8 + 5.4 \times 8.2 = ?$
(a) 52.08 (b) 108.24
(c) 48.05 (d) 102.05
(e) None of these

7. $\sqrt[3]{185193} = ?$
(a) 51 (b) 59
(c) 57 (d) 53
(e) None of these

8. ?% of 1400 – 18% of 750 = 159
(a) 27 (b) 29
(c) 17 (d) 21
(e) None of these

9. $(? - 968) \div 79 \times 4 = 512$
(a) 11080 (b) 10190
(c) 11075 (d) 10185
(e) None of these

10. $94736 + 43693 + 25638 = ?$
(a) 160546 (b) 164076
(c) 165046 (d) 160467
(e) None of these

11. $-948 + 147 - ? = -1432$
(a) 683 (b) 631
(c) 647 (d) 676
(e) None of these

12. $\frac{7}{439} \div ? + 354 = 865$
(a) 32056 (b) 32047
(c) 33056 (d) 31025
(e) None of these

13. $\frac{5.5 \times 6.4}{0.4} = ?$
(a) 77 (b) 74
(c) 88 (d) 84
(e) None of these

14. $87693 - ? - 4725 - 169 = 20410$
(a) 62475 (b) 62379
(c) 62480 (d) 62489
(e) None of these

15. $3.7 \times 0.04 + 5.9 = ?$
(a) 6.48 (b) 6.032
(c) 21.978 (d) 21.98
(e) None of these

16. $(?)^2 \div 4 + \sqrt{1521} = 1063$
(a) 62 (b) 68
(c) 58 (d) 66
(e) None of these

17. $(764 \times ?) \div 250 = 382$
(a) 115 (b) 145
(c) 135 (d) 125
(e) None of these

18. $6156 \div \sqrt{?} \times 53 = 4028$
(a) 6889 (b) 6241
(c) 5929 (d) 6561
(e) None of these

19. $18\frac{2}{3} + 7\frac{1}{2} = ?$
(a) $26\frac{1}{3}$ (b) $19\frac{1}{2}$
(c) $26\frac{1}{6}$ (d) $25\frac{2}{3}$
(e) None of these

20. $\frac{1}{4} \times (4856 \times 0.5) \times 12 = ?$
(a) 7284 (b) 7462
(c) 7262 (d) 7414
(e) None of these

21. $(38)^2 + (63)^2 + (?)^2 = 6089$
(a) 26 (b) 24
(c) 28 (d) 32
(e) None of these

22. $-224 + (-314) \times (-9) = ?$
(a) –547 (b) 2602
(c) +547 (d) –2602
(e) None of these

23. $853 + ? \div 17 = 1000$
(a) 2516 (b) 2482
(c) 2499 (d) 16147
(e) None of these

24. $13.15 \times 7.4 + 76.12 = ?$
(a) 164.53 (b) 94.58
(c) 96.67 (d) 173.43
(e) None of these

25. $9643 - 7750 + ? = 4990$
(a) 3079 (b) 3097
(c) 3090 (d) 4010
(e) None of these

26. What should come in place of the question mark (?) in the following question?
6 13 28 59 ? 249
(a) 124 (b) 122
(c) 120 (d) 118
(e) None of these

27. The average age of a man and his son is 29 years. The ratio of their ages 4 years ago was 21 : 4 respectively. What is the present age of the son?
(a) 12 years (b) 8 years
(c) 10 years (d) 14 years
(e) None of these

28. 16% of a number is 216. What is 27% of that number?
(a) 274 (b) 367.20
(c) 279 (d) 364.50
(e) None of these

29. At what rate of simple interest per annum can an amount of Rs 1,553.40 be obtained on the principal amount of Rs 8,630.00 after 3 years?
(a) 8 p.c.p.a. (b) 4 p.c.p.a.
(c) 5 p.c.p.a. (d) 7 p.c.p.a.
(e) None of these

30. The product of two successive numbers is 6972. Which is the greater number?
(a) 85 (b) 83
(c) 82 (d) 86
(e) None of these

31. What is 742 times 124?
(a) 93492 (b) 92018
(c) 93429 (d) 92008
(e) None of these

32. 15% of 25% of $\frac{5}{6}$th of a number is 330. What is the number?
(a) 10280 (b) 10660
(c) 10560 (d) 9900
(e) None of these

33. There are 8 dozen candles kept in a box. If there are 14 such boxes, how many candles are there in all the boxes together?
(a) 1382 (b) 1334
(c) 1356 (d) 1344
(e) None of these

34. A shopkeeper sells 43 kgs of rice for Rs 946/-. What would be the price if he sells 137 kgs of rice at the same rate?
(a) Rs 3,014/- (b) Rs 3,121/-
(c) Rs 3,041/- (d) Rs 3,112/-
(e) None of these

35. Find the average of the following set of scores.
844, 621, 736, 142, 324, 273
(a) 490 (b) 348
(c) 520 (d) 450
(e) None of these

36. The area of a circular plot is 3850 sq. metres. What is the circumference of the plot?
(a) 240 metres (b) 210 metres
(c) 220 metres (d) 260 metres
(e) None of these

37. 'A' can complete a piece of work in 12 days and 'B' can complete the same piece of work in 16 days. In how many days can 'A' and 'B' together complete the same piece of work?
(a) $5\frac{1}{3}$ days (b) $6\frac{3}{7}$ days
(c) $5\frac{2}{3}$ days (d) $6\frac{6}{7}$ days
(e) None of these

38. The cost of 7 desks and 9 chairs is Rs 7,212/-. What is the cost of 21 chairs and 27 desks?
(a) Rs 21,665/- (b) Rs 20,976/-
(c) Rs 21,710/- (d) Cannot be determined
(e) None of these

39. If an amount of Rs 1,39,567/- is distributed equally amongst 75 people, approximately how much amount would each person get?
(a) Rs 1,829/- (b) Rs 1,734/-
(c) Rs 1,861/- (d) Rs 1,795/-
(e) Rs 1,898/-

40. What approximate value should come in place of the question mark (?)?
$\sqrt{8789} \div 8 + 178 = ?$
(a) 225 (b) 142
(c) 190 (d) 120
(e) 238

41. The ratio of length and breadth of a rectangular plot is 71 : 61 respectively. The area of the plot is 17324 sq. metres. What is the perimeter of the plot?
(a) 284 metres (b) 528 metres
(c) 264 metres (d) 614 metres
(e) None of these

42. If $56a + 56b = 1008$, what is the average of a and b?
(a) 6 (b) 9
(c) 12 (d) 18
(e) None of these

43. Manish, Manoj and Mitesh begin to jog around a circular stadium. They complete their revolutions in 42, 32 and 48 seconds respectively. After how many seconds will they be together at the starting point?
(a) 672 seconds (b) 721 seconds
(c) 678 seconds (d) 714 seconds
(e) None of these

44. The population of a town is 11200. It increases by 25% in the first year and decreases by 15% in the second year. What is the population of the town at the end of 2 years?
(a) 14000 (b) 11900
(c) 16100 (d) 13500
(e) None of these

45. A, B, C, D and E are 5 consecutive odd numbers. The average of the five numbers is 25. What is the product of C and E?
(a) 675 (b) 725
(c) 575 (d) 621
(e) None of these

46. The cost of 11 kgs of wheat is Rs 209/-. The cost of 8 kgs of sugar is Rs 136/- and the cost of 14 kgs of rice is Rs 294/-. What is the total cost of 38 kgs of wheat, 43 kgs of sugar and 46 kgs of rice?
(a) Rs 2,419/- (b) Rs 2,525/-
(c) Rs 2,396/- (d) Rs 2,441/-
(e) None of these

47. If $(28)^2$ is added to the square of a number, the answer so obtained is 1808. What is the number?
(a) 34 (b) 26
(c) 36 (d) 32
(e) None of these

48. If $x + y = 22$ and $xy = 105$, what is the value of $(x)^2 + (y)^2$?
(a) 210 (b) 289
(c) 245 (d) Cannot be determined
(e) None of these

49. A car covers a distance from Town I to Town II at the speed of 56 kms/hr and from Town II to Town I at the speed of 53 kms/hr. What is the average speed of the car?
(a) 55 kms/hr (b) 55.5 kms/hr
(c) 54 kms/hr (d) 53.5 kms/hr
(e) None of these

50. What amount of compound interest can be obtained on an amount of Rs 4,800/- @ 6 p.c.p.a. at the end of 2 years?
(a) Rs 544.96 (b) Rs 576/-
(c) Rs 593.28 (d) Rs 588/-
(e) None of these

Answers

1. (c)	**2.** (d)	**3.** (a)	**4.** (b)	**5.** (d)	**6.** (a)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (e)
11. (b)	**12.** (e)	**13.** (c)	**14.** (e)	**15.** (e)	**16.** (e)	**17.** (d)	**18.** (d)	**19.** (c)	**20.** (a)
21. (a)	**22.** (b)	**23.** (c)	**24.** (d)	**25.** (b)	**26.** (b)	**27.** (a)	**28.** (d)	**29.** (e)	**30.** (e)
31. (d)	**32.** (c)	**33.** (d)	**34.** (a)	**35.** (a)	**36.** (c)	**37.** (d)	**38.** (d)	**39.** (c)	**40.** (c)
41. (b)	**42.** (b)	**43.** (a)	**44.** (b)	**45.** (b)	**46.** (a)	**47.** (d)	**48.** (e)	**49.** (e)	**50.** (c)

MODEL TEST PAPER V

BASED ON MAT EXAMINATION, MAY 2007

1. The ratio between the number of passenger travelling by I and II class between the two railway stations is 1:50, whereas the ratio of I and II class fares between the same stations is 3:1. If on a particular day, Rs 1325 was collected from the passengers travelling between these stations, then what was the amount collected from the II class passengers?
   (a) Rs 1000 (b) Rs 850
   (c) Rs 750 (d) Rs 1250
2. A and B enter into a partnership with Rs 50,000 and Rs 60,000 respectively, C joins them after x months, contributing Rs 70,000, and B leaves x months before the end of the year. If they share the profit in the ratio of 20:18:21, then find the value of x.
   (a) 6 (b) 3
   (c) 9 (d) 8
3. Rahul started a business with a capital of Rs 8,000. After six months, Sanjay joined him with an investment of some capital. If at the end of the year each of them gets equal amount as profit, how much did Sanjay invest in the business?
   (a) Rs 16,000 (b) Rs 17,500
   (c) Rs 18,000 (d) Rs 16,500
4. A manufacturer of certain item can sell all he can produce at the selling price of Rs 60 each. It cost him Rs 40 in materials and labour to produce each item and he has overhead expenses of Rs 3000 per week in order to operate that plant. The number of units he should produce and sell in order to make a profit of at least Rs 1000 per week is.
   (a) 250 (b) 300
   (c) 400 (d) 200
5. If the selling price of a product in increased by Rs 162, then the business would make a profit of 17% instead of a loss of 19%. What is the cost price of the product?
   (a) 250 (b) 300
   (c) 360 (d) 600
6. Two men undertake to do a piece of work for Rs 1,400. The first man alone can do this work in 7 days while the second man alone can do this work in 8 days. If they working together complete this work in 3 day with the help of a boy, how should the money be divided?
   (a) Rs 600, Rs 550, Rs 250
   (b) Rs 600 Rs 525, Rs 275
   (c) Rs 600 Rs 500 Rs 300
   (d) Rs 500 Rs 525 Rs 375
7. Tow men undertake to do a piece of work for Rs 600. One alone could do it in 6 days and the other in 8 days. With the assistance of a boy they finish it in 3 days. The boy's share should be
   (a) Rs 75 (b) Rs 225
   (c) Rs 300 (d) Rs 100
8. 15 men can complete a work in 210 days. They started the work but at the end of 10 days 15 additional men, with double efficiency, were inducted. How many days in all did they take to finish the work?
   (a) $72\frac{1}{2}$ days (b) $84\frac{3}{4}$ days
   (c) $76\frac{2}{3}$ days (d) 70 days
9. There are two taps to fill a tank and a third to empty it. When the third tap is closed, they can fill the tank in 10 minutes and 12 minutes, respectively. If all the three taps be opened, the tank is filled in 15 minutes. If the first two taps are closed, in what time can the third tap empty the tank when it is full?
   (a) 8 min and 34 sec (b) 9 min and 32 sec
   (c) 7 min (d) 6 min
10. A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the pipes are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty a full cistern?
    (a) 12 minutes (b) 10 minutes
    (c) 8 minutes (d) 16 minutes
11. Two taps can fill a tank in 20 minutes and 30 minutes respectively. There is an outlet tap at exactly half level of that rectangular tank which can pump out 100 liters of water per minute. If the outlet tap is open, then it takes 24 minutes to fill an empty tank. What is the volume of the tank?
    (a) 1800 litres (b) 1500 litres
    (c) 1200 litres (d) 240 litre
12. Excluding stoppages, the speed of a bus is 54 km/hr and including stoppages, it is 45 km/hr. For how many minutes does the bus stop per hour?
    (a) 12 (b) 10
    (c) 9 (d) 20
13. Rampur is 100 km from Sitapur. At 3 pm Bharat Express leaves Rampur for Sitapur and travels at a constant speed of 30 kmph. One hour later, Laxman Mail leaves Sitapur for Rampur and travels at a constant speed of 40 kmph. Each train makes one stop only at a station 10 km from its starting point and remains there for 15 min.
    Which train is nearer to Rampur when they meet?
    (a) Both are equidistant (b) Laxman Mail
    (c) Bharat Express (d) None of these
14. A car start running with the initial speed of 40 kmph, with its speed increasing every hour by 5 kmph. How many hours will it take to cover a distance of 385 km?
    (a) 9 hrs (b) $9\frac{1}{2}$ hrs
    (c) $8\frac{1}{2}$ hrs (d) 7 hrs
15. How many kg of tea worth Rs 25 per kg must be blended with 30kg of tea worth Rs 30 per kg so that by selling the

blended variety at Rs 30 per kg there should be a gain of 10%?

(a) 32 kg (b) 40 kg
(c) 36 kg (d) 42 kg

16. How many kg of sugar costing Rs 5.75 per kg should be mixed with 75 kg of cheaper sugar costing Rs 4.50 per kg so that the mixture is worth Rs 5.50 per kg?
(a) 350 kg (b) 300 kg
(c) 250 kg (d) 325 kg
17. The average monthly salary of employees, consisting of officer and workers of an organization, is Rs 3000. The average salary of an officer is Rs 10,000 while that of a worker is Rs 2,000 per month. If there are total 400 employees in the organization, find the number of officers and workers separately.
(a) 50, 350 (b) 350, 450
(c) 50, 275 (d) 325, 350
18. A person travels 285 km in 6 hrs in two stages. In the first part of the journey, he travels by bus at the speed of 40 km per hr. In the second part of the journey, he travels by train at the speed of 55 km per hr. How much distance does he travel by train?
(a) 165 km (b) 145 km
(c) 205 km (d) 185 km
19. How many kg of pure salt must be added to 30 kg of a 2% solution of salt and water to increase it to a 10% solution?
(a) $2\frac{2}{3}$ kg (b) 15 kg
(c) 3 kg (d) $14\frac{1}{3}$ kg
20. Two persons are walking in the same direction at rates 3 km/hr and 6 km/hr. A train comes running from behind and passes them in 9 and 10 seconds. The speed of the train is
(a) 22 km/hr (b) 40 km/hr
(c) 33 km/hr (d) 35 km/hr

Directions (Q. 21–24): Each of the question below is followed by two statements labelled (A) and (B). Decide whether the data given in the statements is sufficient for answering the questions.

Mark answer as

1. If both statement (A) and (B) together are sufficient to answer the question asked, but neither statement alone is sufficient.
2. If statement (B) alone is sufficient but statement (A) alone is not sufficient to answer the question asked.
3. If statement (A) alone is sufficient but statement (B) alone is not sufficient to answer the question asked.
4. If each statement is sufficient by itself to answer the question asked.

21. How much did the salesman earn from the sale of 3 cars?
(A) Each car sold for Rs 3,40,000
(B) He received a 2% commission on each sale.
22. What does WXY equal?
(A) W = X +Y
(B) WXYZ = 6Z where Z = 0
23. Which number is the greatest, C, D or E?
(A) 2D > 2E > 2C
(B) C + 2 = D ≥ E
24. If Sanjay can paint a house in 15 hours working alone, how long will it take to paint the house if Mohit helps him?
(A) Mohit can paint the house in 20 hours working alone.
(B) Working together with Sanjay, Mohit does 3/7 of the total work.

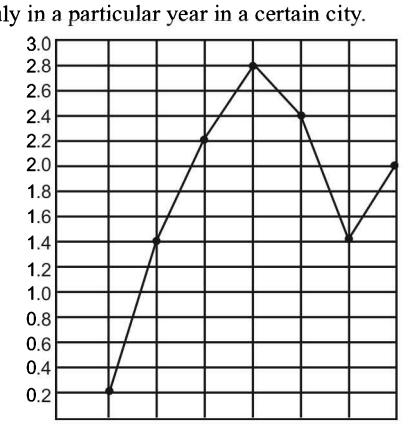
Directions (Q. 25–28): In each of these questions two quantities are given, one in column A and one in column B. Compare the two quantities.

Mark answer as

(a) if the two quantities are equal
(b) if the quantity in column B is greater
(c) if the quantity in column A is greater
(d) if the relationship cannot be determined from the information given

	Given Information	Column A	Column B
25.	$x = -2$	$3x^2 + 2x - 1$	$x^3 + 2x^2 + 1$
26.	$\frac{a}{a+b} = \frac{c}{c+d}$	cb	ad
27.	An audio cassette price at Rs 47.25 includes a 5% mark-up	Rs 44.89	The original price before mark-up
28.	25% of the 300 girls in our school wear spectacles	The ratio of girls wearing spectacles to those who do not	1 : 3

Directions (Q. 29–32): Refer to the following line graph which represents the rainfall in inches during the months of January through July in a particular year in a certain city.



In each of these questions two quantities are given, one in column A and one in column B. Compare the two quantities.

Mark answer as

(a) if the two quantities are equal
(b) if the quantity in column B is greater
(c) if the quantity in column A is greater
(d) if the relationship cannot be determined from the information given

	Column A	Column B
29.	The ratio of rainfall Between the month February and July	2 : 3
30.	The average monthly rainfall recorded for the 7 months	1.9 inches
31.	$12\frac{1}{2}$ % of all the rain that fell during the 7-month period	The amount of rainfall during rainfall in April and February
32.	The ratio between the average rainfall in May and January	The ratio between the average Rainfall in April and February

Directions (Q. 33–36): Study the table to answer these questions.

Table – Number of Cancer Cases over Two Years for Selected Countries

Country	2006 (Cases in '00)	2006 (Rate per 10,000)	2007 (Cases in '00)
A	53	0.1	46
B	345	2.1	145
C	87	1.1	39
D	81	33.9	26
E	84	0.8	23
F	1365	0.9	209
G	661	13.0	239
H	516	1.9	236
J	36	0.2	16
K	95	1.8	23
L	262	3.9	156
M	19	0.0	18
N	1862	3.3	563
P	47	56.2	11
Q	49	0.5	18
R	337	5.0	235
S	61	1.2	35
T	17	0.3	12
U	896	1.5	235
V	39	1.4	14
W	31	0.0	5
X	501	0.6	12
Y	217	1.4	73
Z	31	0.9	22
AA	39	0.8	13
AB	46	0.4	35
AC	48	0.1	21
AD	71	0.8	32
AE	162	2.4	83
AF	655	1.1	241
AG	21,861	8.9	6445
AH	869	1.4	219
AJ	19	0.0	13

All countries that have reported more than five hundred cases to the WHO in 2007 are listed here. The left column gives the total number of cases reported by each county for 2006, the middle column gives the 2006 rate (cancer cases per 10,000 population) and the last column shows the number of cases reported in early 2007

Most of the 2007 report were for only the first quarter of the year. Owing to reporting delays of six month or more cases reported in 2007 actully were diagnosed in 2006.

33. What is the population of AD on the basis of the reported cases of cancer in 2006 (in thousand) ?
(a) 825,000 (b) 812,500
(c) 810,000 (d) None of these

34. Which country has reported the second highest number of cancer cases to WHO during 2006?
(a) N (b) AG
(c) F (d) U

35. The countries which have reported less than 2000 cases both in 2006 and early 2007 are
(a) M, J and P (b) V, AJ and W
(c) W, M and T (d) M, T and AJ

36. Which of the following are true from the table?
I. The reported cancer cases of M, W and AJ as compared to their population are negligible.
II. The 2006 rate is highest for P though the reported cases are only 4700.
III. The population of R is 664,000 in 2006.
IV. P reported more than 20,000 cases of cancer in early 2007.
(a) I, II and III (b) II and III
(c) I and II (d) I, II and IV

Directions (Q. 37–40): Refer to the following Tables (A) and (B) to answer these questions.

Table (A) : Production of Inorganic Chemicals

Description	Production	
	2003–04	2003–04
Bleaching Powder	60,043	51,434
Aluminium Chloride	31,908	31,176
Sodium Tripoly Phosphate	60,639	60,041
Sodium Bicarbonate	61,615	72,895
Calcium Carbonate	143,980	142,125
Chlorine and Liquid Chlorine	717220	718,530
Calcium Carbide	83,445	83,388
Titanium Oxide	30,422	19,624
Ethylene Glycol	327,387	454,371
Formaldehyde	140,384	145,590
Fatty Acids	99,320	113,851
Aniline	44,273	39,340
Acetone	45,657	44,500
Acetic Acid	208,921	212,319
Pthalic Anhydride	127,318	125,959
Benzene	329,000	373,000
Xylenes	147,288	156,000
Phenol	69,281	66,210

37. The ratio of the number of organic chemicals for which the production has fallen to the number of inorganic chemicals for which production has fallen is.
    (a) 2 : 3 (b) 3 : 2
    (c) 6 : 5 (d) 1 : 1
38. Of all chemicals shown, the highest percentage increase in production has been exhibited for
    (a) Aniline (b) Ethylene glycol
    (c) Fatty acids (d) Sodium bicarbonate
39. The chemical, whose production in 2003-04 as a proportion of its total production for the two-year period was lowest is
    (a) Aniline (b) Ethylene glycol
    (c) Fatty acids (d) None of these
40. If productive volatility is defined as the percentage change in production, and it is known that any chemical whose productive volatility is greater than 5% is classified as highly volatile, then how many chemicals are not highly volatile (amongst organic chemicals)?
    (a) 6 (b) 5
    (c) 4 (d) None of these

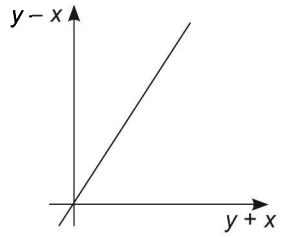
Answers

1. (d)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (a)	**10.** (b)
11. (a)	**12.** (b)	**13.** (a)	**14.** (d)	**15.** (c)	**16.** (b)	**17.** (a)	**18.** (a)	**19.** (a)	**20.** (c)
21. (a)	**22.** (b)	**23.** (c)	**24.** (d)	**25.** (c)	**26.** (a)	**27.** (b)	**28.** (a)	**29.** (c)	**30.** (b)
31. (c)	**32.** (c)	**33.** (d)	**34.** (a)	**35.** (d)	**36.** (c)	**37.** (d)	**38.** (d)	**39.** (d)	**40.** (b)

MODEL TEST PAPER VI

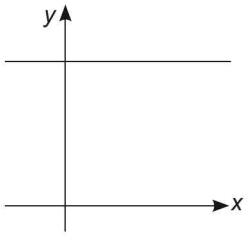
BASED ON COMMON ADMISSION TEST, NOVEMBER 2006

1. Which among $2^{1/2}$, $3^{1/3}$, $4^{1/4}$, $6^{1/6}$ and $12^{1/12}$ is the largest?
   (a) $2^{1/2}$ (b) $3^{1/3}$
   (c) $4^{1/4}$ (d) $6^{1/6}$
   (e) $12^{1/12}$
2. Consider a sequence where the nth term, $t_n = n/(n+2)$, $n = 1, 2, \ldots$ The value of $t_3 \times t_4 \times t_5 \times \ldots \times t_{53}$ equals:
   (a) 2/495 (b) 2/477
   (c) 12/55 (d) 1/1485
   (e) 1/2970
3. The length, breadth and height of a room are in the ratio 3:2:1. If the breadth and height are halved while the length is doubled, then the total area of the four walls of the room will
   (a) Remain the same (b) Decrease by 13.64%
   (c) Decrease by 15% (d) Decrease by 18.75%
   (e) Decrease by 30%
4. If $a/b = 1/3$, $b/c = 2$, $c/d = 1/2$, $d/e = 3$ and $e/f = 1/4$, then what is the value of abc/def?
   (a) 3/8 (b) 27/8
   (c) 3/4 (d) 27/4
   (f) 1/4
5. If $x = -0.5$, then which of the following has the smallest value?
   (a) $2^{1/x}$ (b) $1/x$
   (c) $1/x^2$ (d) 2
   (e) $1/\sqrt{-x}$
6. The sum of four consecutive two digit odd numbers, when divided by 10, becomes a perfect square. Which of the following can possibly be one of these four numbers.
   (a) 21 (b) 25
   (c) 41 (d) 67
   (e) 73
7. When you reverse the digits of the number 13, the number increases by 18. How many other two-digit numbers increase by 18 when their digits are reversed?
   (a) 5 (b) 6
   (c) 7 (d) 8
   (e) 10
8. Arun, Barun and Kiranmala start from the same place and travel in the same direction at speeds of 30, 40 and 60 km per hour, respectively. Barun starts two hours after Arun. If Barun and Kiranmala overtake Arun at the same instant, how many hours after Arun did Kiranmala start?
   (a) 3 (b) 3.5
   (c) 4 (d) 4.5
   (e) 5
9. What are the values of x and y that satisfy both the equations?
   $2^{0.7x}, 3^{-1.25y} = 8\sqrt{6}/27$
   $4^{0.3x}, 9^{0.2y} = 8(81)^{1/5}$
   (a) $x = 2$ $y = 5$
   (b) $x = 2.5$, $y = 6$
   (c) $x = 3$, $y = 5$
   (d) $x = 3$, $y = 4$
   (e) $x = 5$, $y = 2$
10. The number of solutions of the equation $2x + y = 40$ where both x and y are positive integers and $x \le y$ is :
    (a) 7 (b) 13
    (c) 14 (d) 18
    (e) 20
11. Consider the set $S = \{1, 2, 3, \ldots, 1000\}$. How many arithmetic progression can be formed from the elements of S that start with 1 and end with 1000 and have at least 3 elements?
    (a) 3 (b) 4
    (c) 6 (d) 7
    (e) 8
12. What values of x satisfy $x^{2/3} + x^{1/3} - 2 < 0$?
    (a) $-8 \le x \le 1$ (b) $-1 \le x \le 8$
    (c) $1 < x < 8$ (d) $1 \le x \le 8$
    (e) $-8 \le x \le 8$
13. The graph of $y - x$ against $y + x$ is as show below. (All graphs in this question are drawn to scale and the same has been used on each axis)

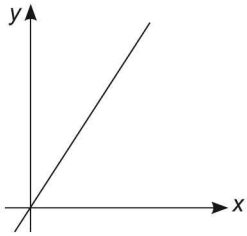


Which of the following shows the graph of y against x ?

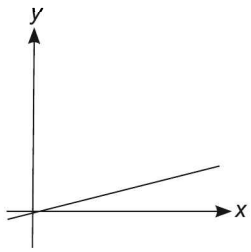
(a)



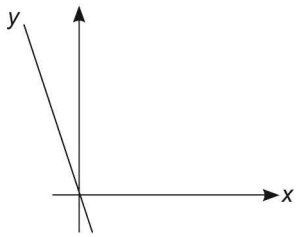
(b)



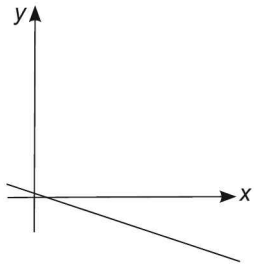
(c)



(d)

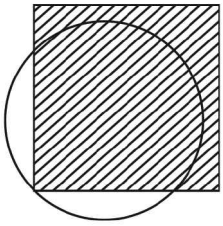


(e)



Directions (Qs. 14 and 15): Answer these question on the basis of the information given below:

A punching machine is used to punch a circular hole of diameter two units from a square sheet of aluminum of width 2 units, as shown below. The hole is punched such that the circular hole touches one corner P of the square sheet and the diameter of the hole originating at P is in line with a diagonal of the square.



14. The proportion of the sheet area that remains after punching is
    (a) $(\pi + 2)/8$ (b) $(6 - \pi)/8$
    (c) $(4 - \pi)/4$ (d) $(\pi - 2)/4$
    (e) $(14 - 3\pi)/6$
15. Find the area of the part of the circle (round punch) failing outside the square sheet.
    (a) $\pi/4$ (b) $(\pi - 1)/2$
    (c) $(\pi - 1)/4$ (d) $(\pi - 2)/2$
    (e) $(\pi - 2)/4$
16. A semi-circle is drawn with AB as its diameter. From C, a point on AB, a line perpendicular to AB is drawn meeting the circumference of the semicircle at D. Given that AC = 2 cm and CD= 6 cm, the area of the semicircle (in sq.cm) will be:
    (a) 32π (b) 50π
    (c) 40.5π (d) 81π
    (e) Undeterminable
17. The number of employees in Obelix Menhir co. is a prime number and is less than 300, The ratio of the number of employees who are graduates and above, to that of employees who are not, can possibly be:
    (a) 101:88 (b) 87:100
    (c) 110:111 (d) 85:98
    (e) 97:84
18. If log, $x = (a \log_z y) = (b. \log_x z) = ab$, then which of the following pairs of values for (a, b) is not possible ?
    (a) (–2, 1/2) (b) (1, 1)
    (c) (0.4, 2.5) (d) $(\pi, 1/\pi)$
    (e) (2, 2)

Directions (Qs. 19 and 20): Answer these questions on the basis of the information given below:

An airline has a certain free luggage allowance and charges for excess luggage at a fixed rate per kg. Two passengers, Raja and Praja have 60 kg of luggage between them and are charged Rs 1200 and Rs 2400, respectively for excess luggage. Had the entire luggage belonged to one them the excess luggage charge would have been Rs 5400.

19. What is the weight of Praja's luggage?
    (a) 20 kg (b) 25 kg
    (c) 30 kg (d) 35 kg
    (e) 40 kg
20. What is the free luggage allowance ?
    (a) 10 kg (b) 5 kg
    (c) 20 kg (d) 25 kg
    (d) 30 kg
21. A group of 630 children is arranged in rows for a group photograph session. Each row for a group photograph session. Each row contains three fewer children than the row in front of it. What number of rows is not possible?
    (a) 3 (b) 4
    (c) 5 (d) 6
    (e) 7
22. An equilateral triangle BPC is drawn inside a square ABCD. What is the value of the angle APD in degrees?
    (a) 75 (b) 90
    (c) 120 (d) 135
    (e) 150
23. A survey was conducted of 100 people to find out whether they had read recent issues of Golmal, a monthly magazine. The summarized information regarding readership in 3 months is given below.
    Only September : 18
    September but not August : 23
    September and July : 28
    September : 28
    July : 48
    July and August : 10
    None of the three months : 24
    What is the number of surveyed people who have read exactly two consecutive issues (out of the three)?
    (a) 7 (b) 9
    (c) 12 (d) 14
    (e) 17
24. Let $f(x) = \max(2x + 1, 3 - 4x)$, where x is any real number. Then the minimum possible value of $f(x)$ is
    (a) 1/3 (b) 1/2
    (c) 2/3 (d) 4/3
    (e) 5/3

25. There are 6 tasks and 6 persons. Task 1 cannot be assigned either to person 1 or to person 2; task 2 must be assigned to either person 3 or person 4. Every person is to be assigned one task. In how many ways can the assignment be done?
    (a) 144 (b) 180 (c) 192 (d) 360 (e) 716

Answers

1. (b)	**2.** (a)	**3.** (e)	**4.** (a)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (c)	**9.** (e)	**10.** (b)
11. (d)	**12.** (a)	**13.** (d)	**14.** (b)	**15.** (d)	**16.** (b)	**17.** (e)	**18.** (e)	**19.** (d)	**20.** (d)
21. (d)	**22.** (e)	**23.** (b)	**24.** (e)	**25.** (a)					