Intervention analysis

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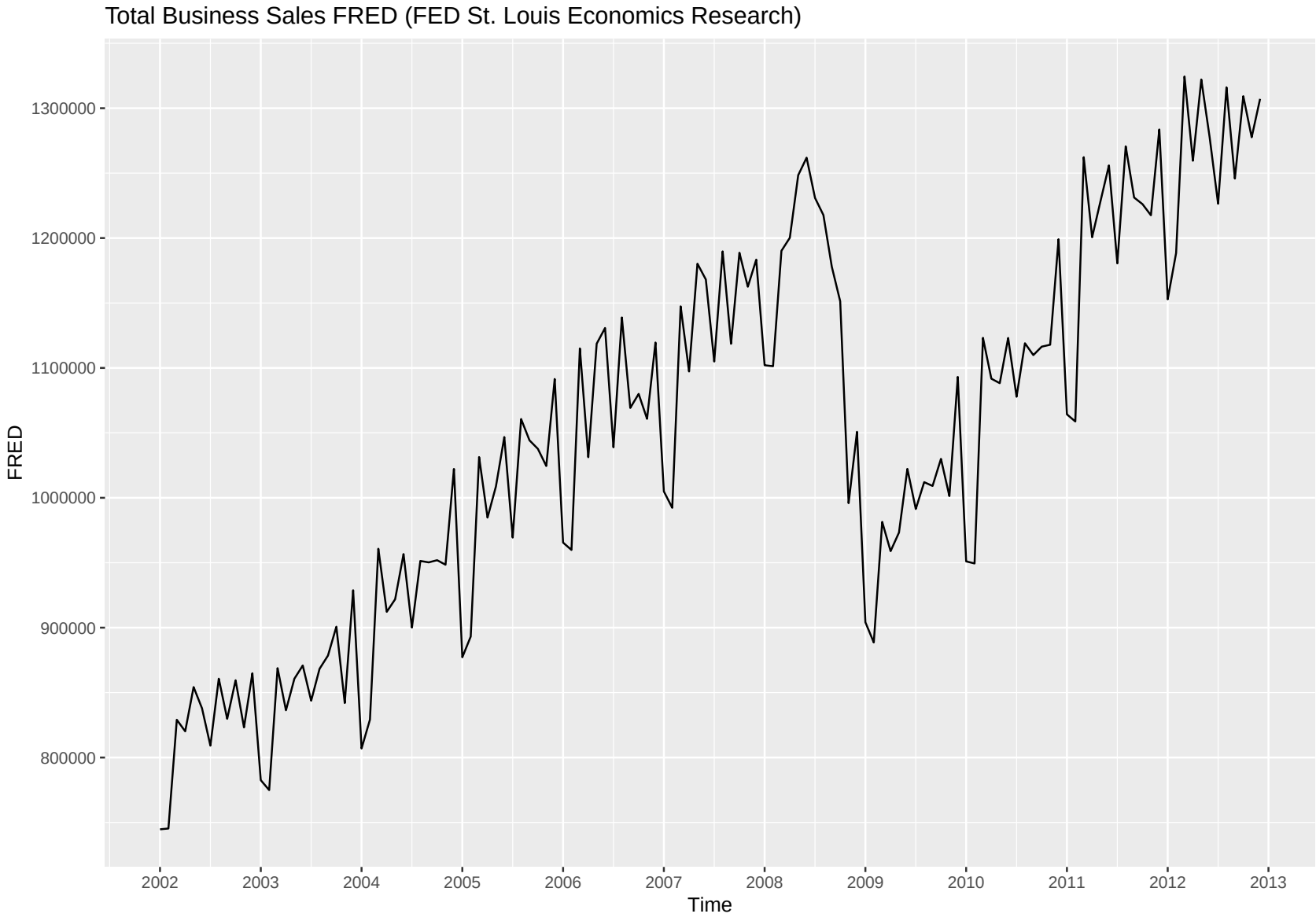
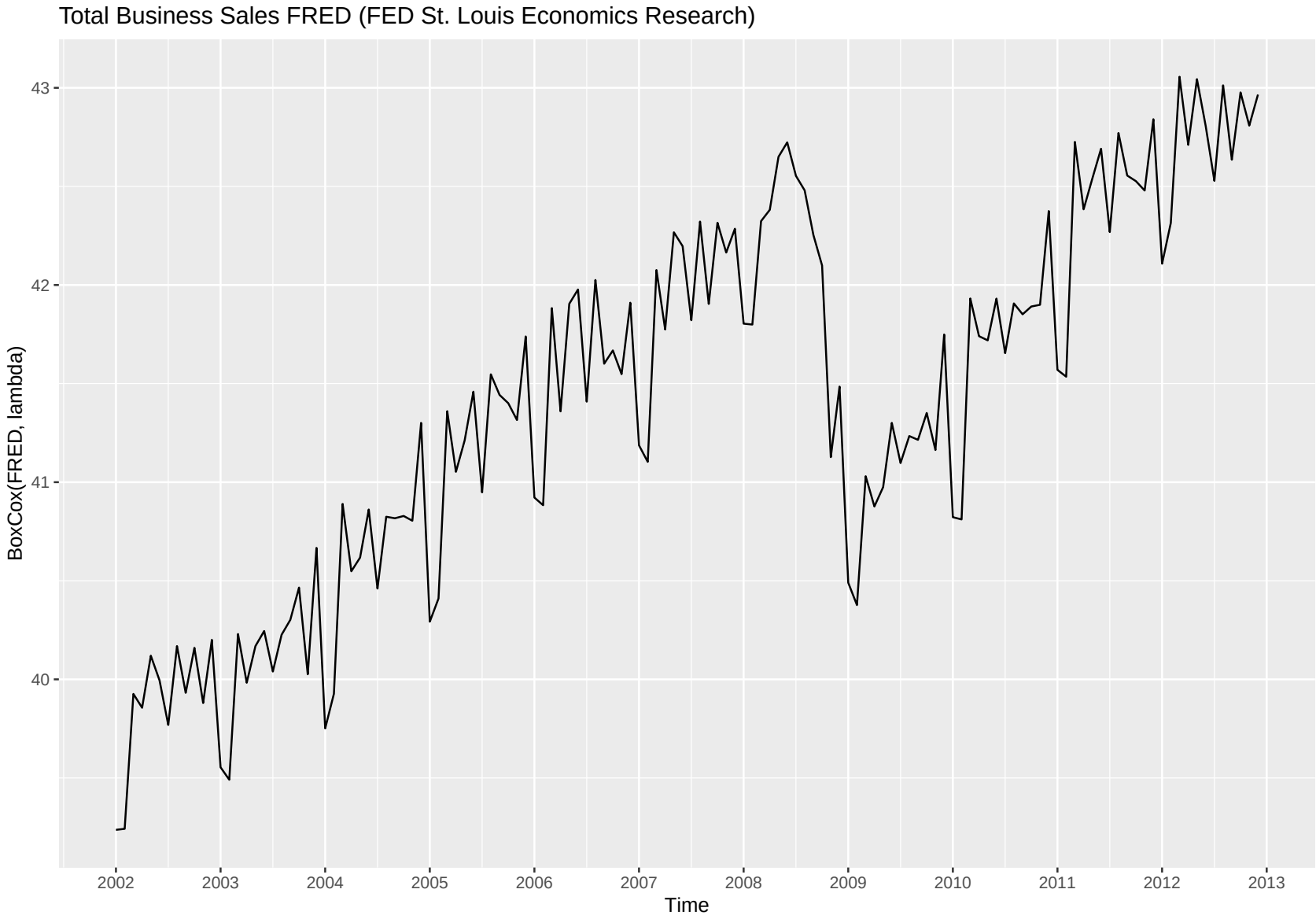
Total Business Sales FRED

- Data from 01/2002 to 12/2012, intervention on 07/2008. The period of 01/2013 to 12/2014 will be used as model validation.
- In 2008 we have a international banking crisis, strongly affecting sales on the US.


Total Business Sales FRED (FED St. Louis Economics Research)
FRED
1300000
1200000
1100000
1000000
900000
800000
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
Time

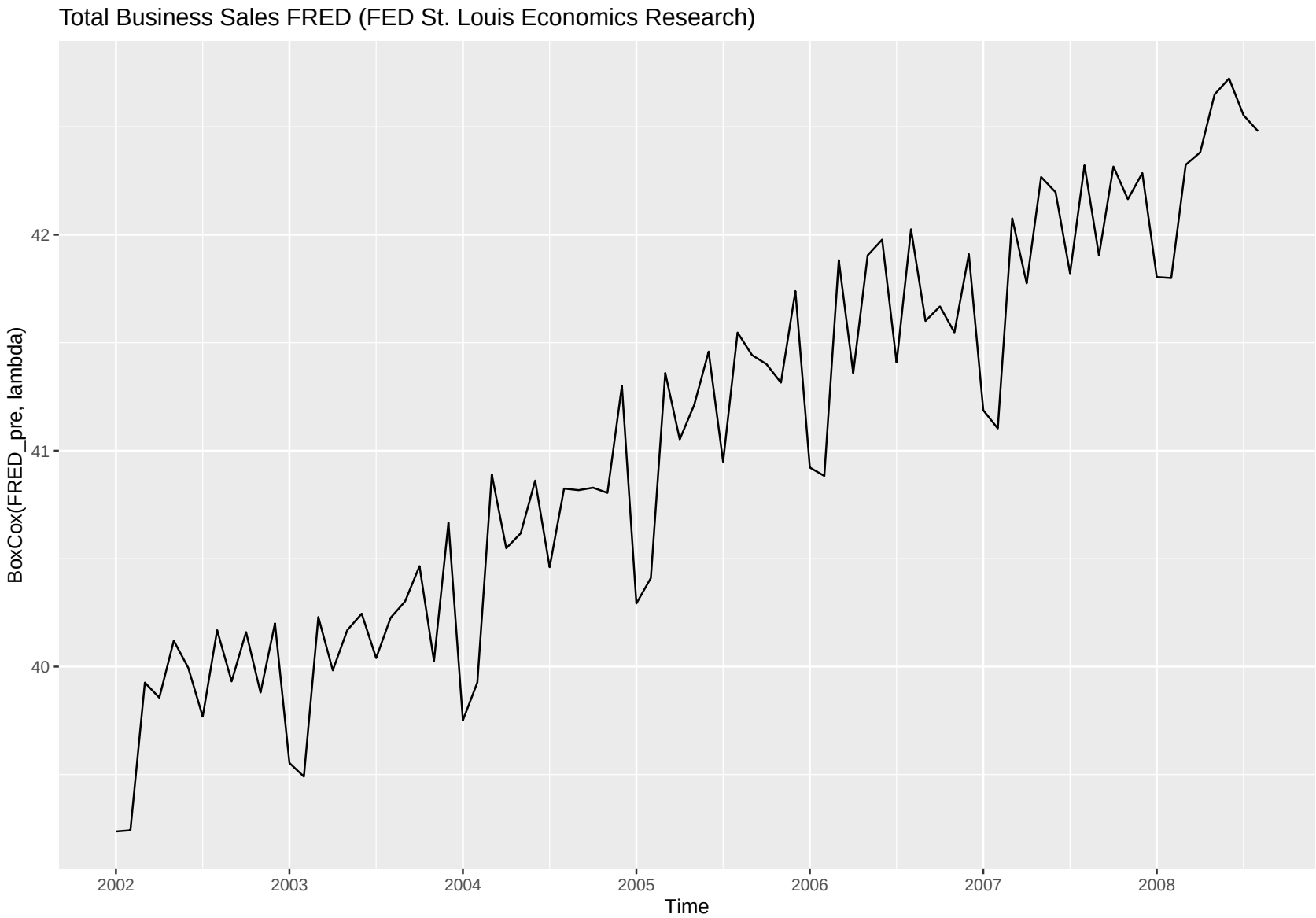
Variance stabilization

- With BoxCox transformation, the lambda is 0.1370143.



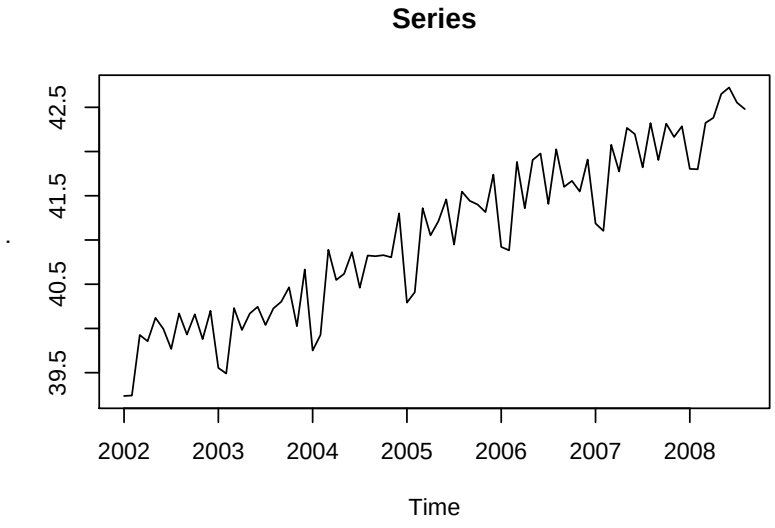
SARIMA pre-intervention

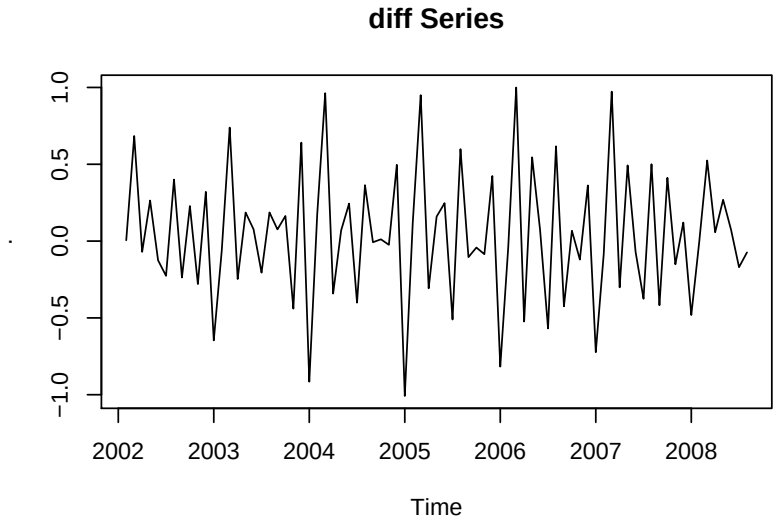
- Model with data until 07/2008. The plots and the models will use the transformed series.

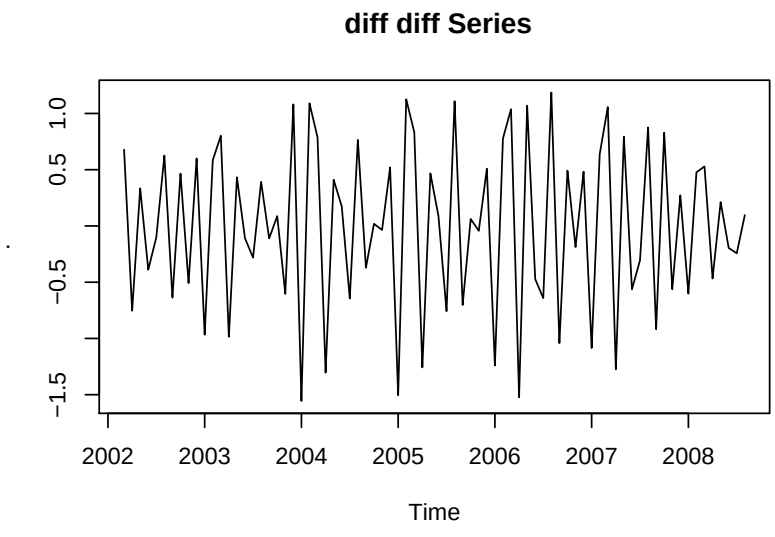


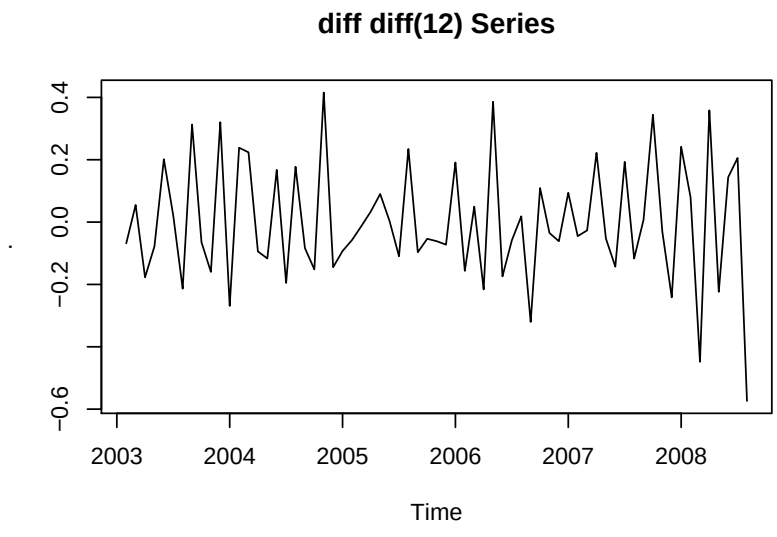
Check differences

- To identify parameters d and D for the SARIMA model.





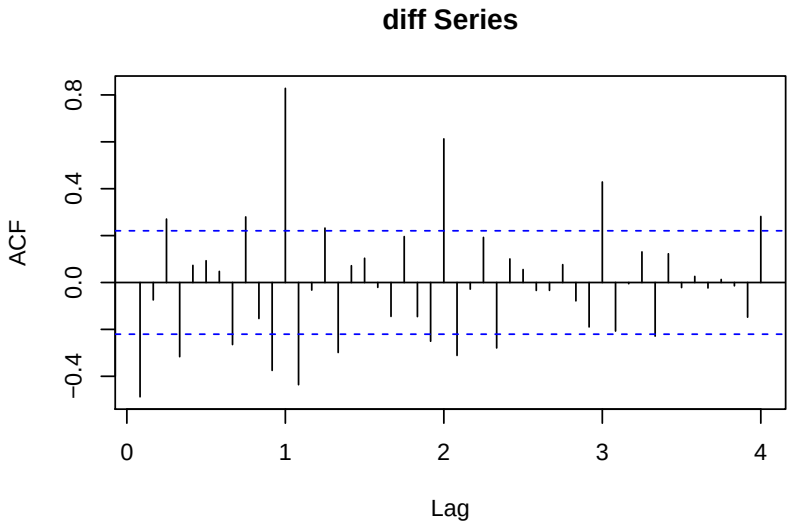


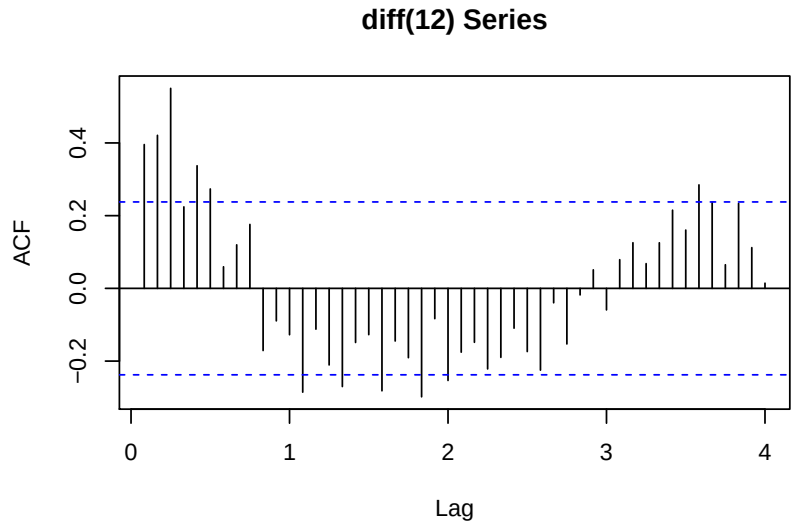


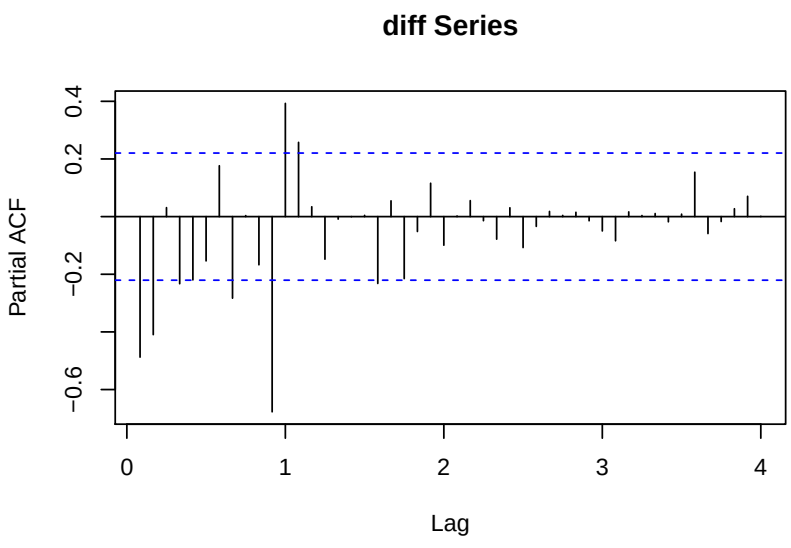
- Use Augmented Dickey Fuller test to verify if there is trend.
- ADF test p-values:
  - Original series: 0.01
  - Diff() series: 0.0206
  - Diff() Diff() series: 0.01
  - Diff() Diff(12) series: 0.01
- We will be using $d = 1$ and $D = 0$ or $D = 1$.

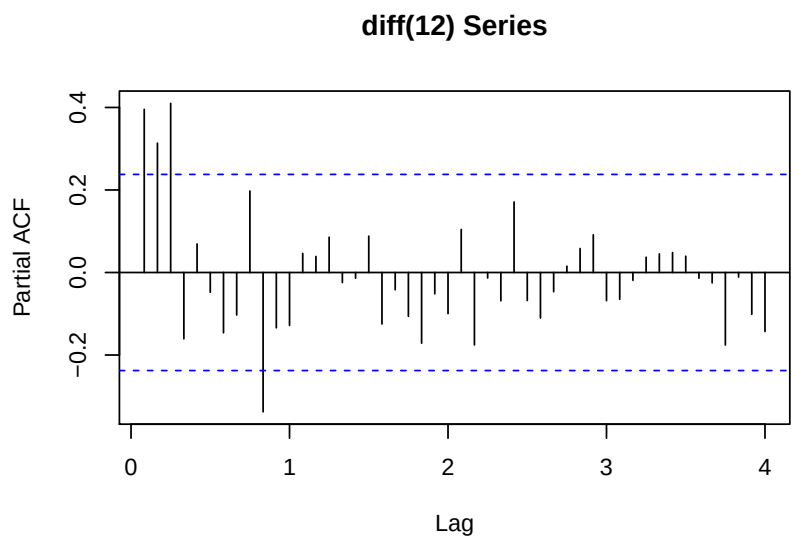
ACF and PACF

- Plot of ACF and PACF for model to identify parameters p, q, P, Q.





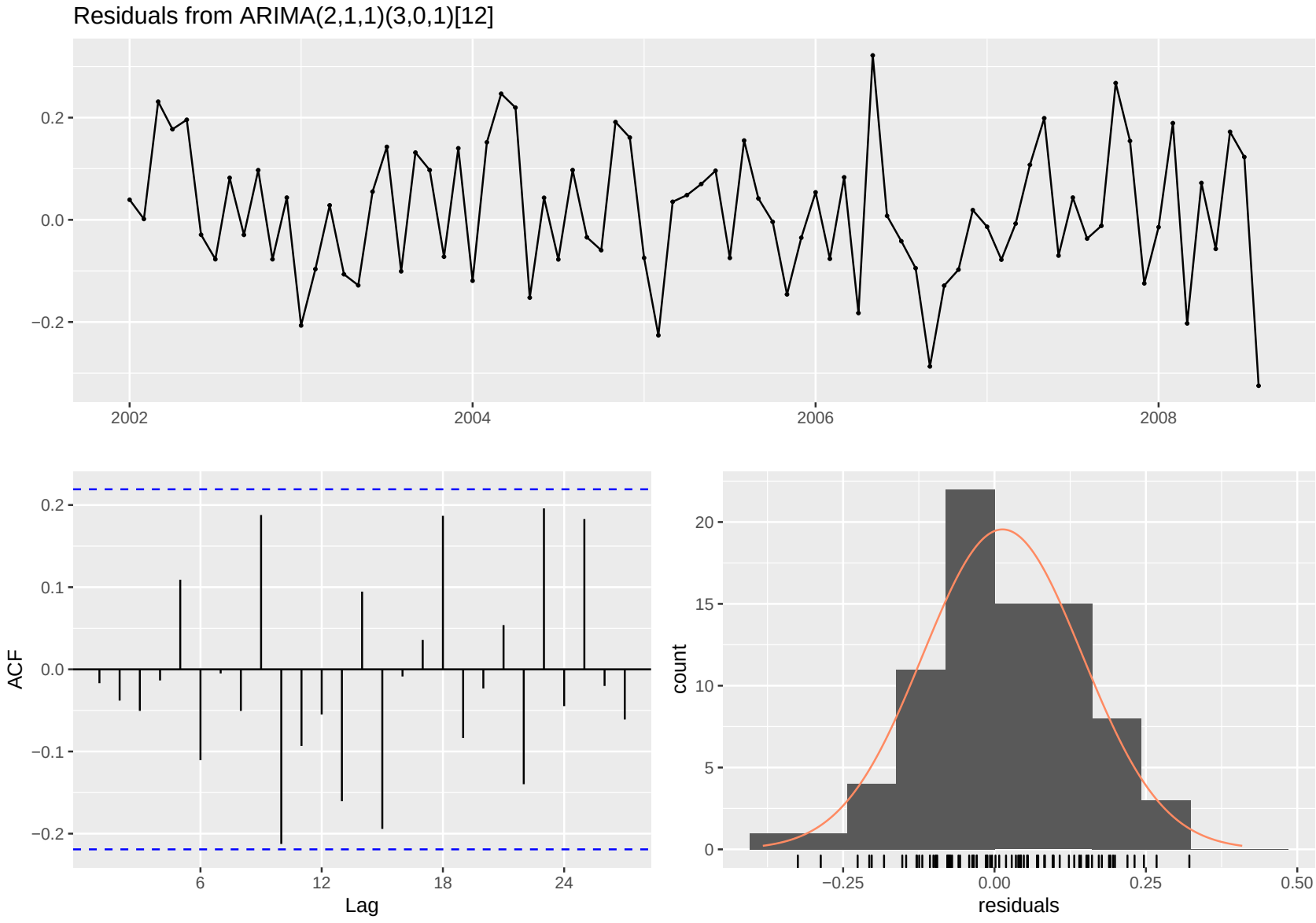




- Both non-seasonal and seasonal ACFs decrease slowly, indicating an auto regressive model.
- For the non seasonal PACFs, we have the first two significant lags and for the seasonal we have the first three significant lags.
- Going to test the following models:
  - SARIMA(2, 1, 1)(3, 0, 1)
  - SARIMA(2, 1, 1)(2, 0, 1)
  - SARIMA(2, 1, 1)(2, 1, 1)

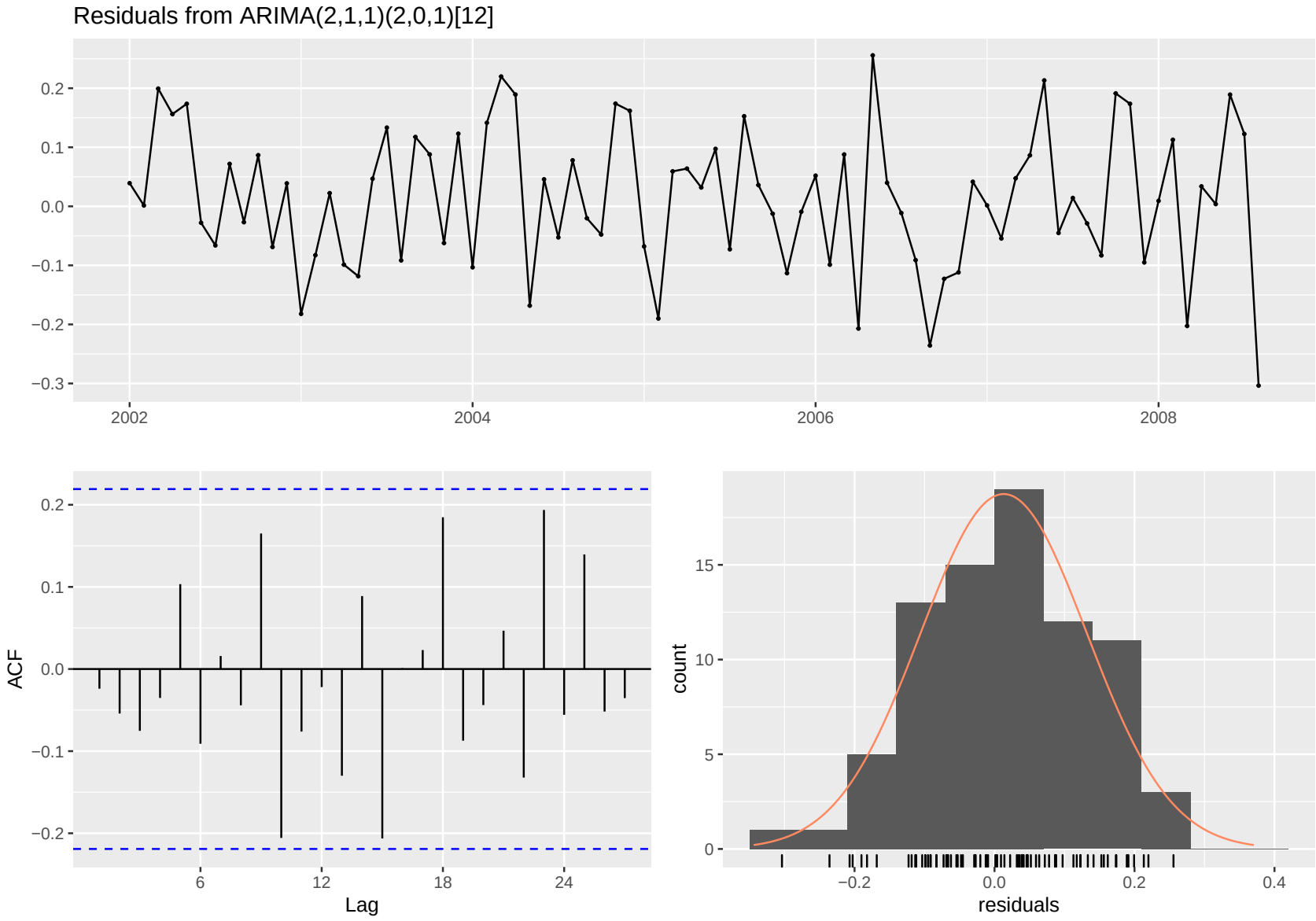
SARIMA(2, 1, 1)(3,0 ,1)

```
## Series: FRED_pre
## ARIMA(2,1,1)(3,0,1)[12]
## Box Cox transformation: lambda= 0.1370143
##
## Coefficients:

## Warning in sqrt(diag(x$var.coef)): NaNs produzidos

##           ar1      ar2     ma1     sar1    sar2    sar3    sma1
##       -1.0841  -0.6725  0.3762  -0.0028  0.6105  0.2732  0.8088
## s.e.   0.1669   0.0988  0.2214      NaN     NaN  0.1557     NaN
##
## sigma^2 estimated as 0.0193:  log likelihood=34.32
## AIC=-52.63   AICc=-50.57   BIC=-33.67
##
## Training set error measures:
##                     ME     RMSE      MAE       MPE     MAPE      MASE
## Training set 1886.588 20274.27 16172.38 0.1815222 1.621103 0.2541459
##                      ACF1
## Training set -0.04002457
```


Residuals from ARIMA(2,1,1)(3,0,1)[12]
0.2
0.0
−0.2
2002
2004
2006
2008
ACF
0.2
0.1
0.0
−0.1
−0.2
6
12
18
24
Lag
count
20
15
10
5
0
−0.25
0.00
0.25
0.50
residuals

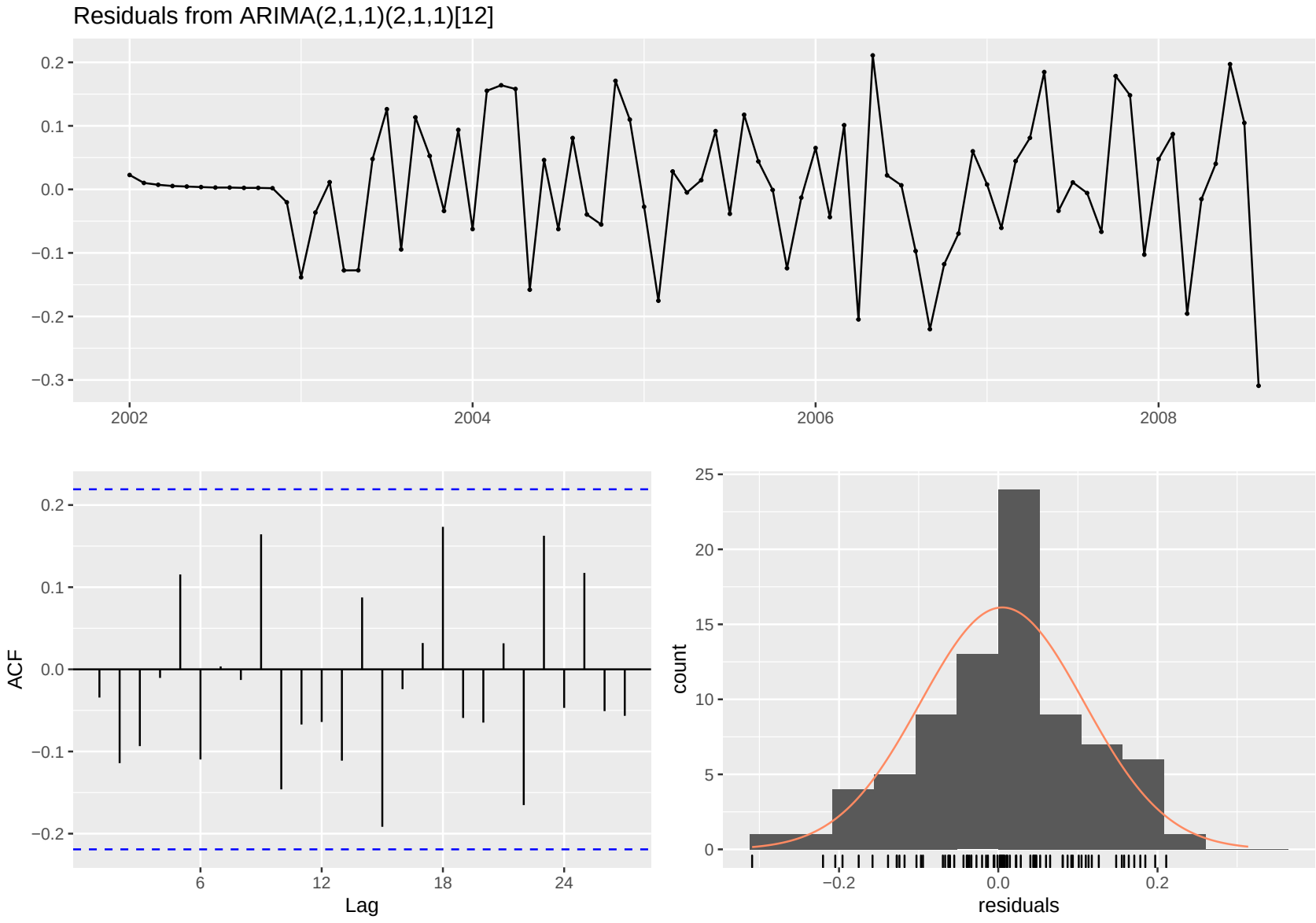
SARIMA(2, 1, 1)(2,0 ,1)

```
## Series: FRED_pre
## ARIMA(2,1,1)(2,0,1)[12]
## Box Cox transformation: lambda= 0.1370143
##
## Coefficients:
##           ar1      ar2     ma1    sar1     sar2     sma1
##       -1.0998  -0.6944  0.3887  1.3894  -0.3895  -0.9628
## s.e.   0.1522   0.1011  0.2013  0.1652   0.1651   0.1334
##
## sigma^2 estimated as 0.01554:  log likelihood=36.49
## AIC=-58.99   AICc=-57.41   BIC=-42.4
##
## Training set error measures:
##                     ME  RMSE      MAE      MPE     MAPE      MASE         ACF1
## Training set 1927.329 18390 14675.48 0.182487 1.468923 0.2306223 -0.03922664
```

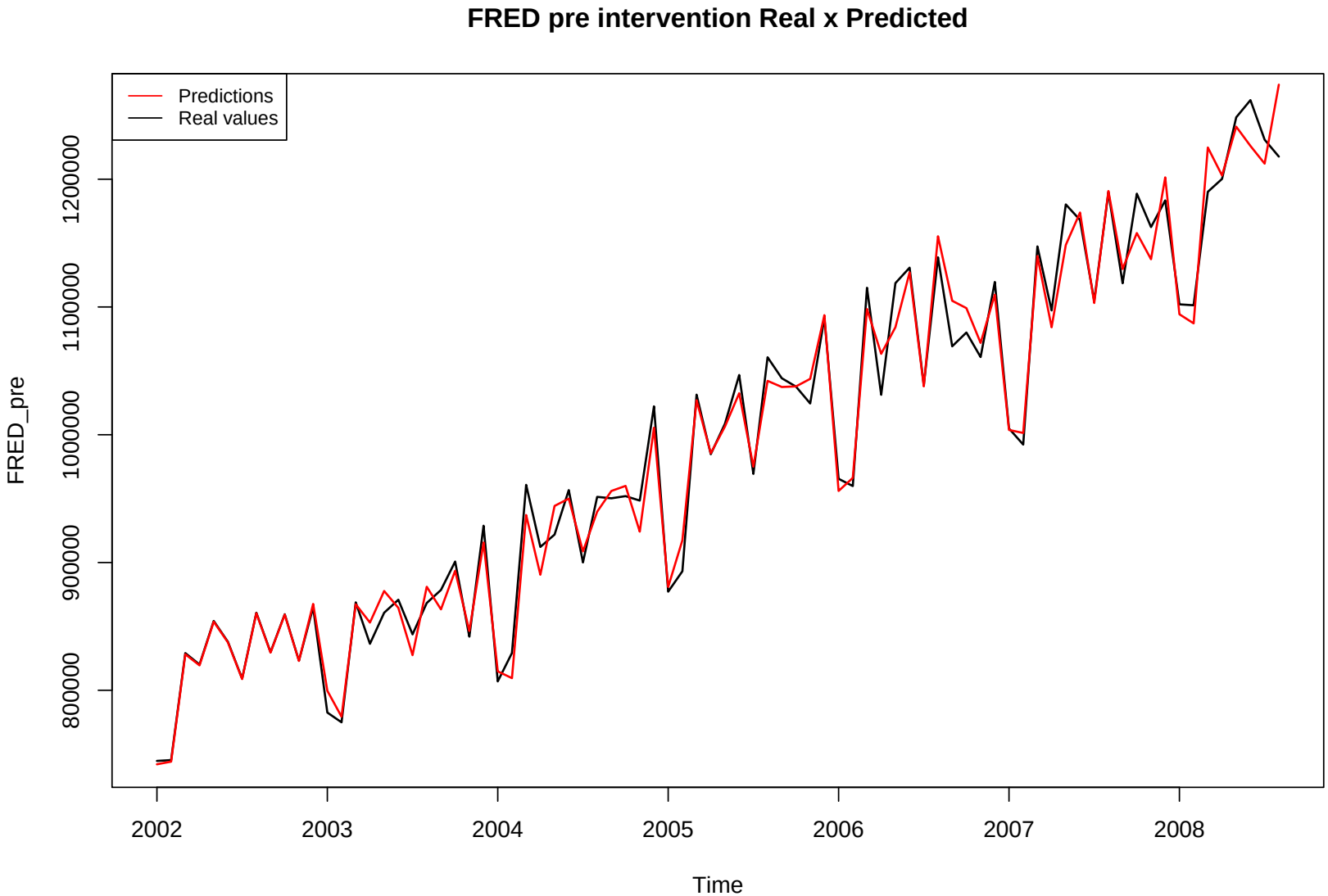

Residuals from ARIMA(2,1,1)(2,0,1)[12]
0.2
0.1
0.0
−0.1
−0.2
−0.3
2002
2004
2006
2008
ACF
Lag
6
12
18
24
count
residuals
15
10
5
0
−0.2
0.0
0.2
0.4

SARIMA(2, 1, 1)(2,1 ,1)

```
## Series: FRED_pre
## ARIMA(2,1,1)(2,1,1)[12]
## Box Cox transformation: lambda= 0.1370143
##
## Coefficients:
##           ar1      ar2     ma1    sar1     sar2     sma1
##       -1.0979  -0.7046  0.4394  0.3592  -0.2377  -0.9998
## s.e.   0.1599   0.0989  0.2389  0.1512   0.1785   0.4891
##
## sigma^2 estimated as 0.01376:  log likelihood=41.06
## AIC=-68.13   AICc=-66.23   BIC=-52.69
##
## Training set error measures:
##                    ME     RMSE      MAE       MPE     MAPE      MASE         ACF1
## Training set 824.437 16261.08 11894.78 0.0635975 1.159562 0.1869242 -0.03438936
```

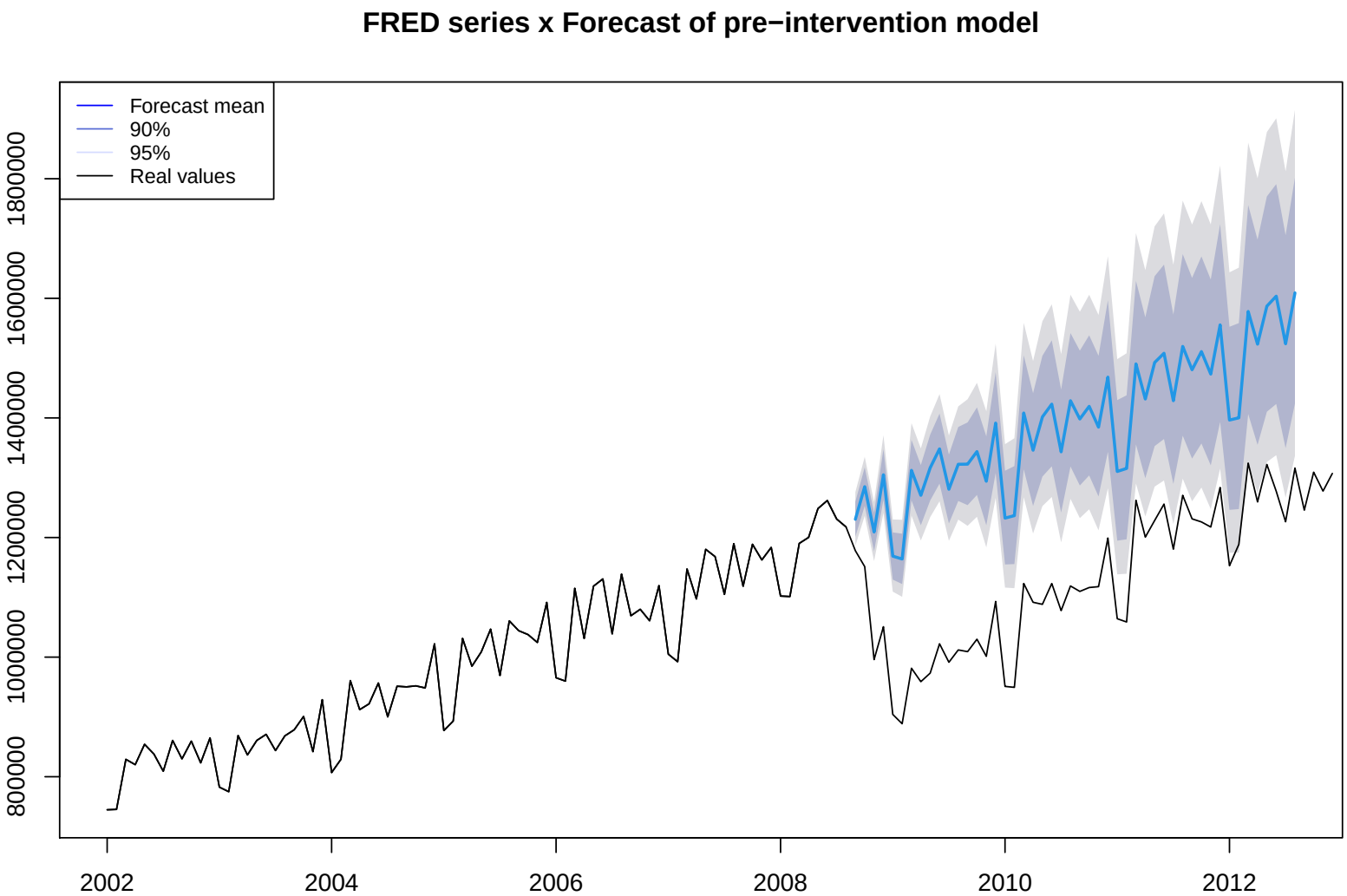

Residuals from ARIMA(2,1,1)(2,1,1)[12]
0.2
0.1
0.0
−0.1
−0.2
−0.3
2002
2004
2006
2008
ACF
Lag
6
12
18
24
count
residuals
25
20
15
10
5
0
−0.2
0.0
0.2

- The model with lowest AIC is the SARIMA(2, 1, 1)(2, 1, 1).



Forecast of pre intervention

- We now look on how our model says that the series should be without the intervention, predicting the next six years after july 2008.

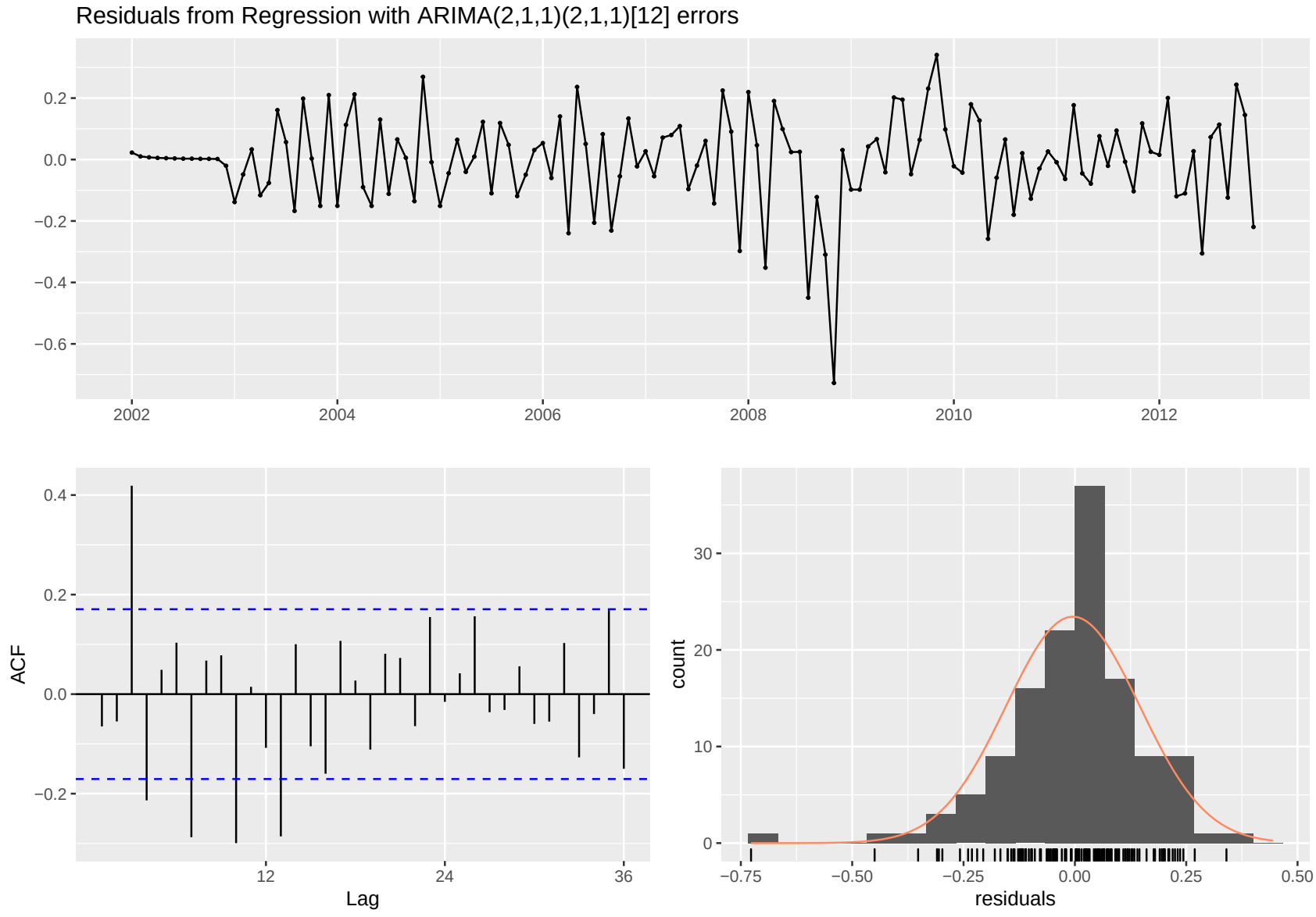


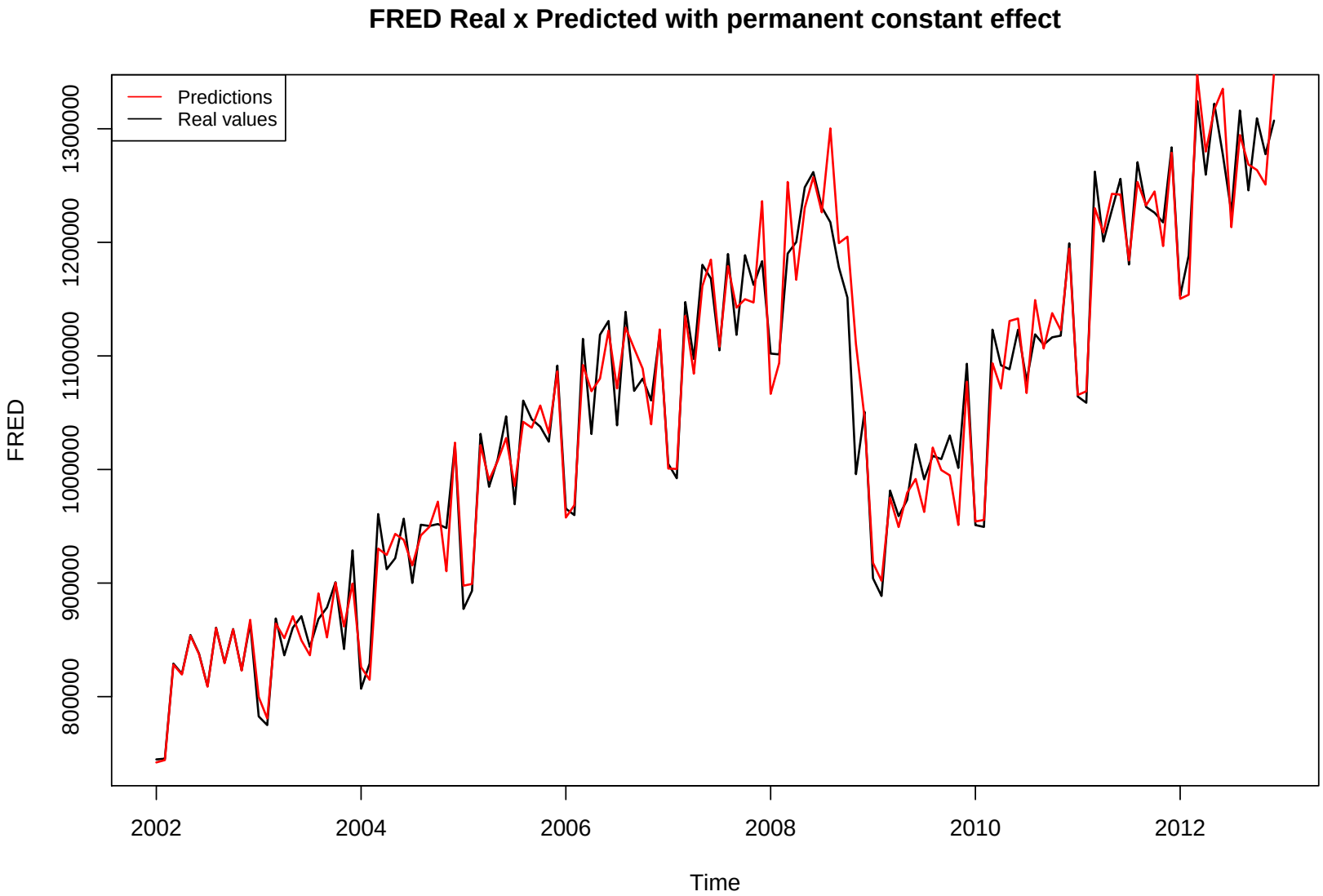
Intervention modeling

Permanent constant effect

- For the first intervention model we will define $h_t = I(t > jul2008)\delta_0$, so there will be a permanent effect equal to δ_0 after july of 2008.
- The estimated δ_0 value is 0.1105 with s.e. 0.1492, so the interval includes 0.

```
## Series: FRED
## Regression with ARIMA(2,1,1)(2,1,1)[12] errors
## Box Cox transformation: lambda= 0.1370143
##
## Coefficients:
##          ar1     ar2      ma1    sar1     sar2  sma1    xreg
##       0.4543  0.2465  -0.5897  0.2995  -0.3439  -1.0  0.1105
## s.e.  0.1585  0.0904   0.1395  0.0969   0.0946   0.1  0.1492
##
## sigma^2 estimated as 0.0263:  log likelihood=35.34
## AIC=-54.69   AICc=-53.38   BIC=-32.46
##
## Training set error measures:
##                      ME  RMSE      MAE         MPE     MAPE      MASE       ACF1
## Training set -845.9794 24414 17468.18 -0.09032853 1.629346 0.2050465 -0.0618996
```


Residuals from Regression with ARIMA(2,1,1)(2,1,1)[12] errors
0.2
0.0
−0.2
−0.4
−0.6
2002
2004
2006
2008
2010
2012
0.4
0.2
0.0
−0.2
ACF
12
24
36
Lag
30
20
10
0
count
−0.75
−0.50
−0.25
0.00
0.25
0.50
residuals

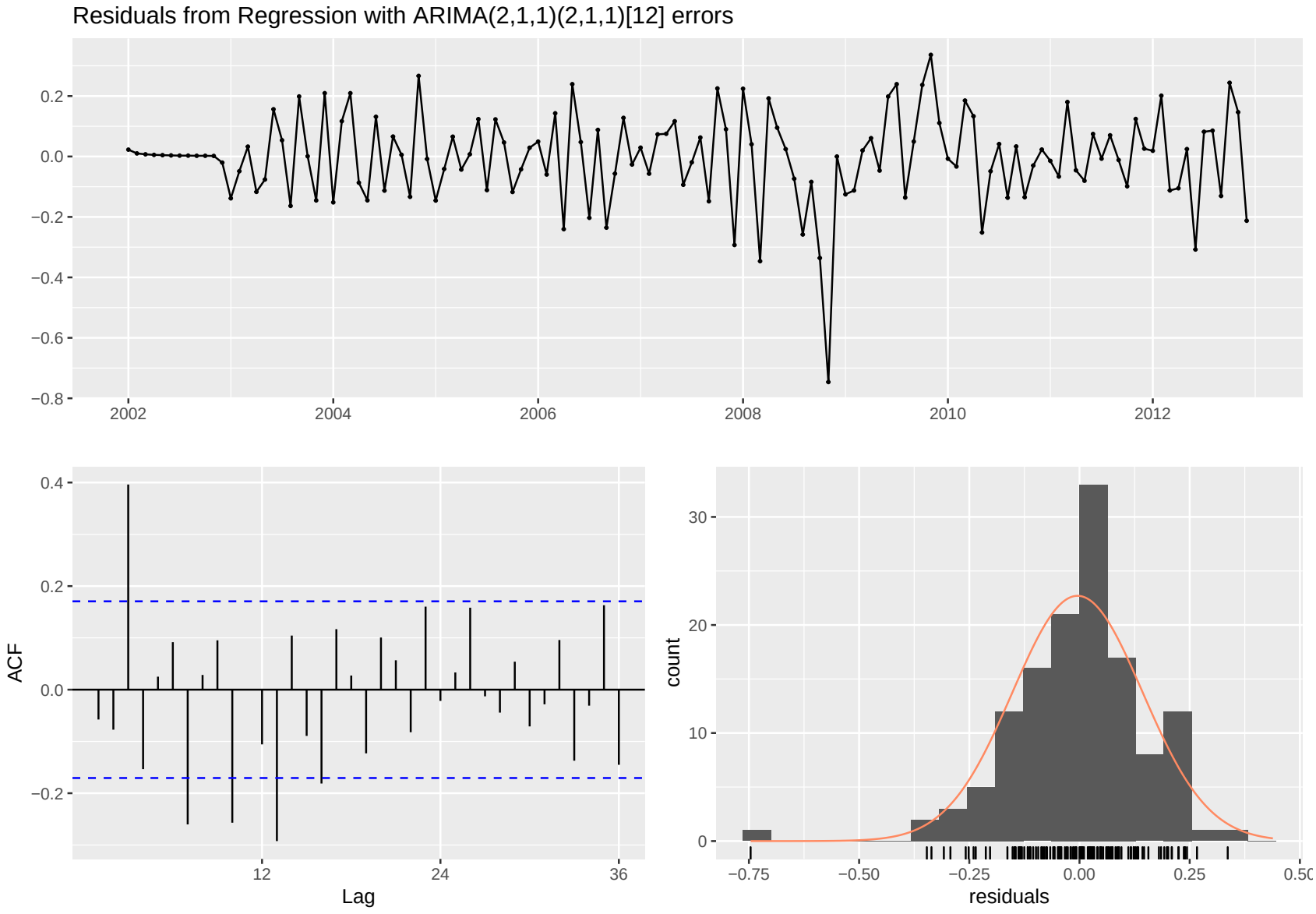

FRED Real x Predicted with permanent constant effect
Predictions
Real values
FRED
800000
900000
1000000
1100000
1200000
1300000
2002
2004
2006
2008
2010
2012
Time

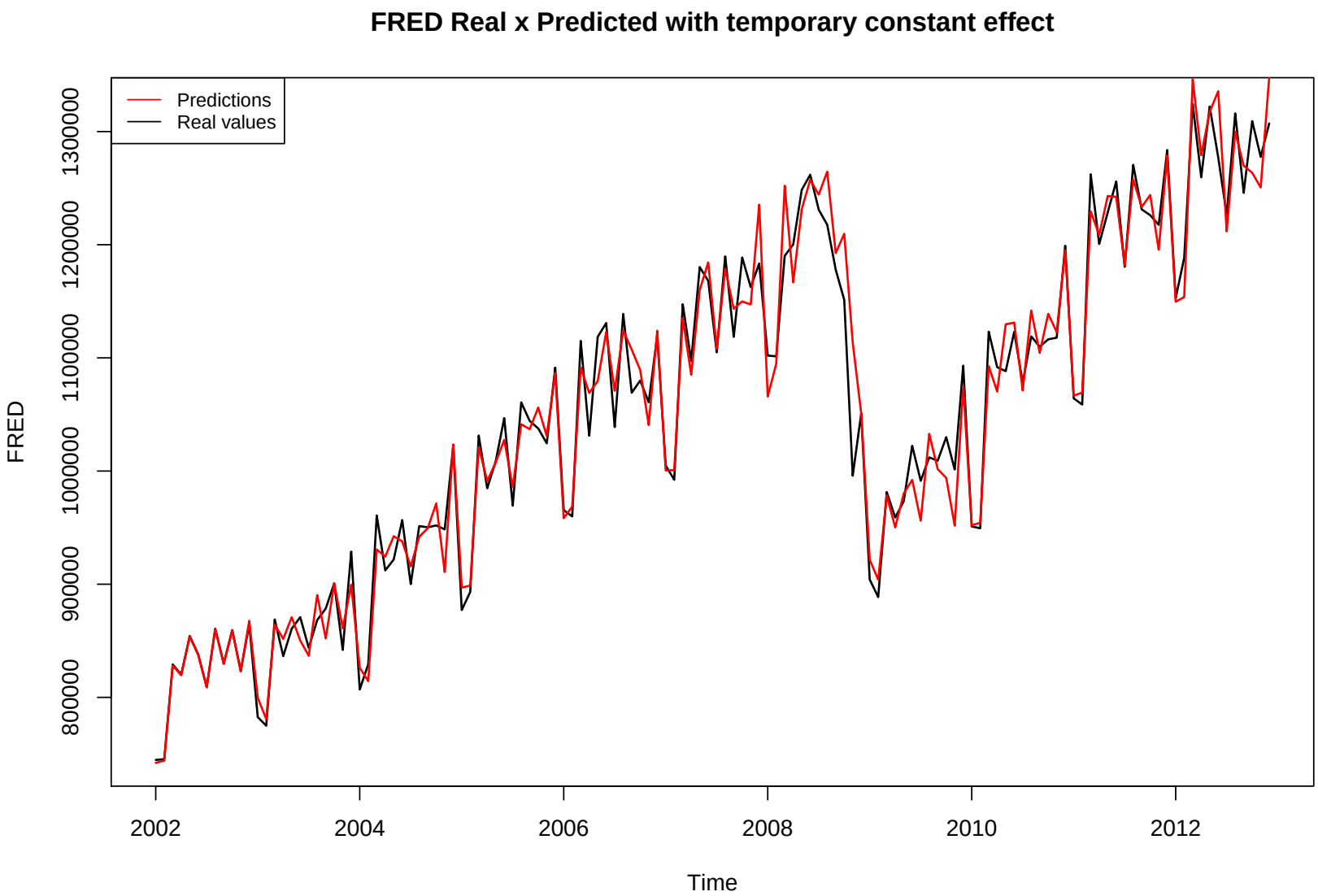
Temporary constant effect

- Now for the intervention model we will define $h_t = I(t = jul2008)\delta_0$, so there will be a temporary effect equal to δ_0 on july of 2008.
- The estimated δ_0 value is 0.2209 with s.e. 0.1134, so the interval doesn't include 0.

```
## Series: FRED
## Regression with ARIMA(2,1,1)(2,1,1)[12] errors
## Box Cox transformation: lambda= 0.1370143
##
## Coefficients:
##          ar1     ar2      ma1    sar1     sar2     sma1    xreg
##       0.4749  0.2230  -0.6151  0.2804  -0.3408  -1.0000  0.2209
## s.e.  0.1695  0.0918   0.1521  0.0989   0.0955   0.0992  0.1134
##
## sigma^2 estimated as 0.02554:  log likelihood=37
## AIC=-58   AICc=-56.69   BIC=-35.77
##
## Training set error measures:
##                      ME  RMSE      MAE         MPE     MAPE      MASE
## Training set -805.7189 23866 17205.32 -0.08929967 1.609214 0.2019609
##                      ACF1
## Training set -0.05589136
```

```
##
## z test of coefficients:
##
##       Estimate Std. Error  z value  Pr(>|z|)
## ar1   0.474910   0.169522   2.8015 0.0050873 **
## ar2   0.223007   0.091760   2.4303 0.0150849 *
## ma1  -0.615083   0.152137  -4.0430 5.278e-05 ***
## sar1  0.280412   0.098862   2.8364 0.0045626 **
## sar2 -0.340759   0.095457  -3.5698 0.0003573 ***
## sma1 -0.999978   0.099198 -10.0806 < 2.2e-16 ***
## xreg  0.220942   0.113434   1.9478 0.0514443 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
```

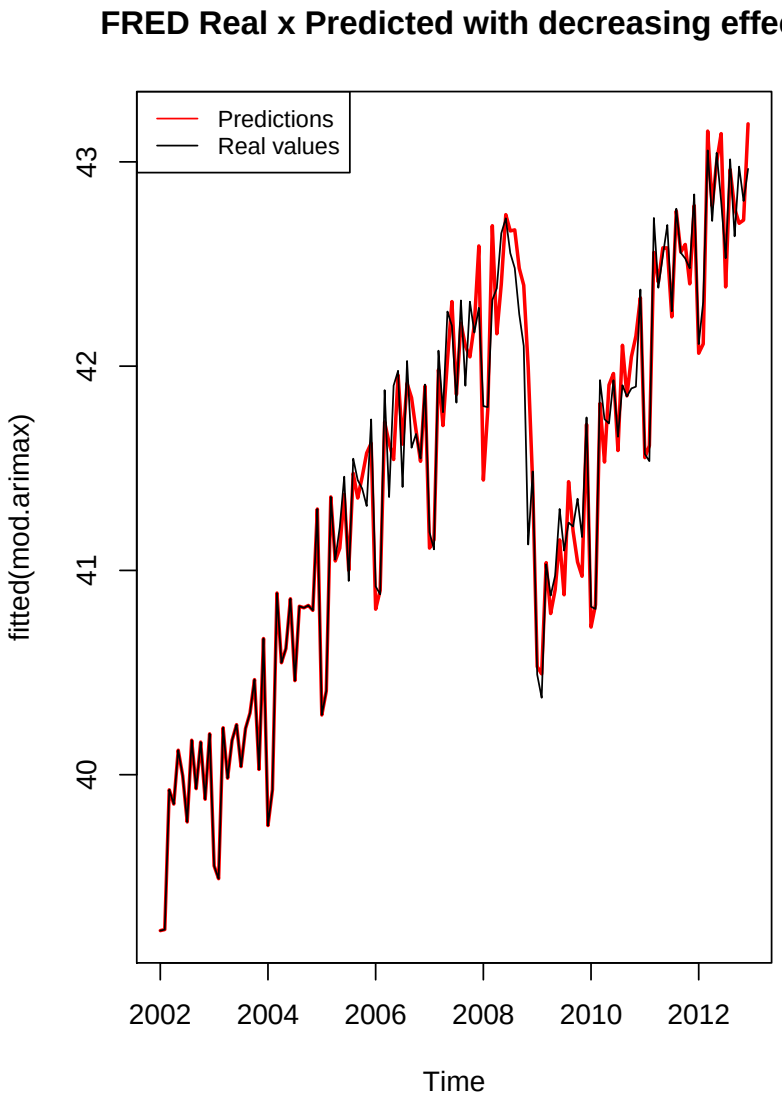

Residuals from Regression with ARIMA(2,1,1)(2,1,1)[12] errors
0.2
0.0
−0.2
−0.4
−0.6
−0.8
2002
2004
2006
2008
2010
2012
ACF
0.4
0.2
0.0
−0.2
12
24
36
Lag
count
30
20
10
0
−0.75
−0.50
−0.25
0.00
0.25
residuals

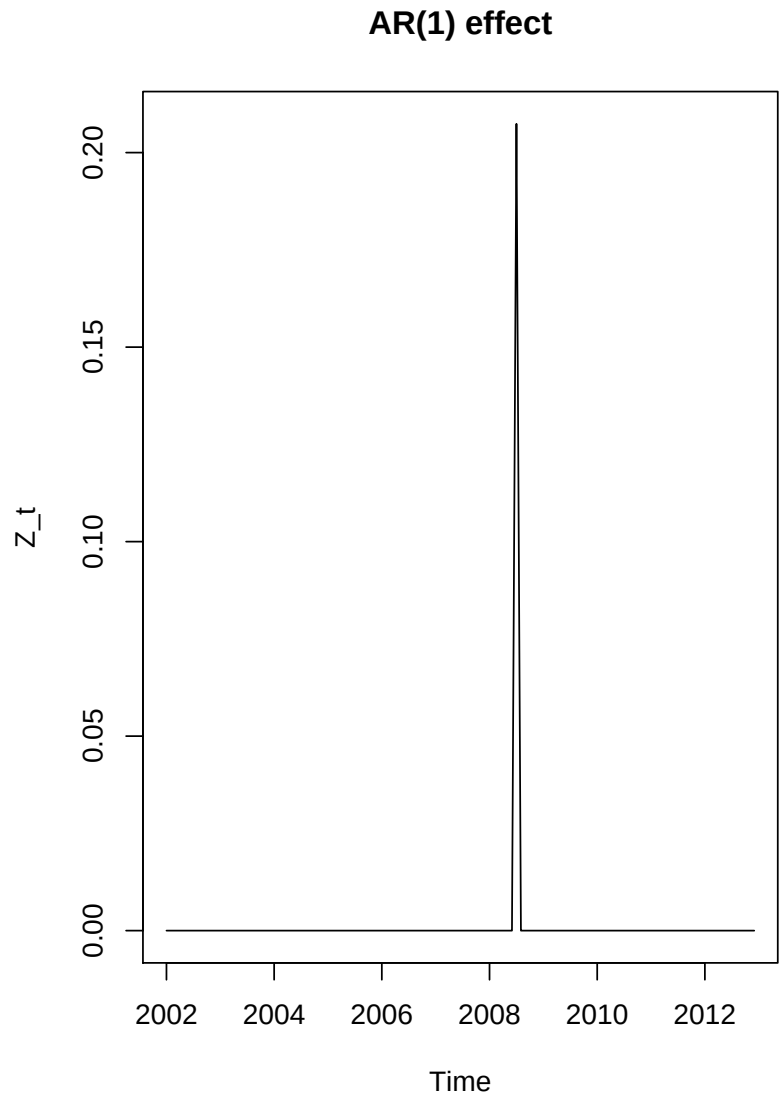

FRED Real x Predicted with temporary constant effect
Predictions
Real values
FRED
800000
900000
1000000
1100000
1200000
1300000
2002
2004
2006
2008
2010
2012
Time

Changing intervention effect

- Now with we model the transfer function as one $AR(1)$, it is, $h_t = \frac{I(t=jul2008)\delta_0}{1-\omega_0 B}$ we can have a more complex effect of intervention, without it being a constant change in the mean.
- With the estimated values for δ_0 and ω_0 we can compute the transfer curve.

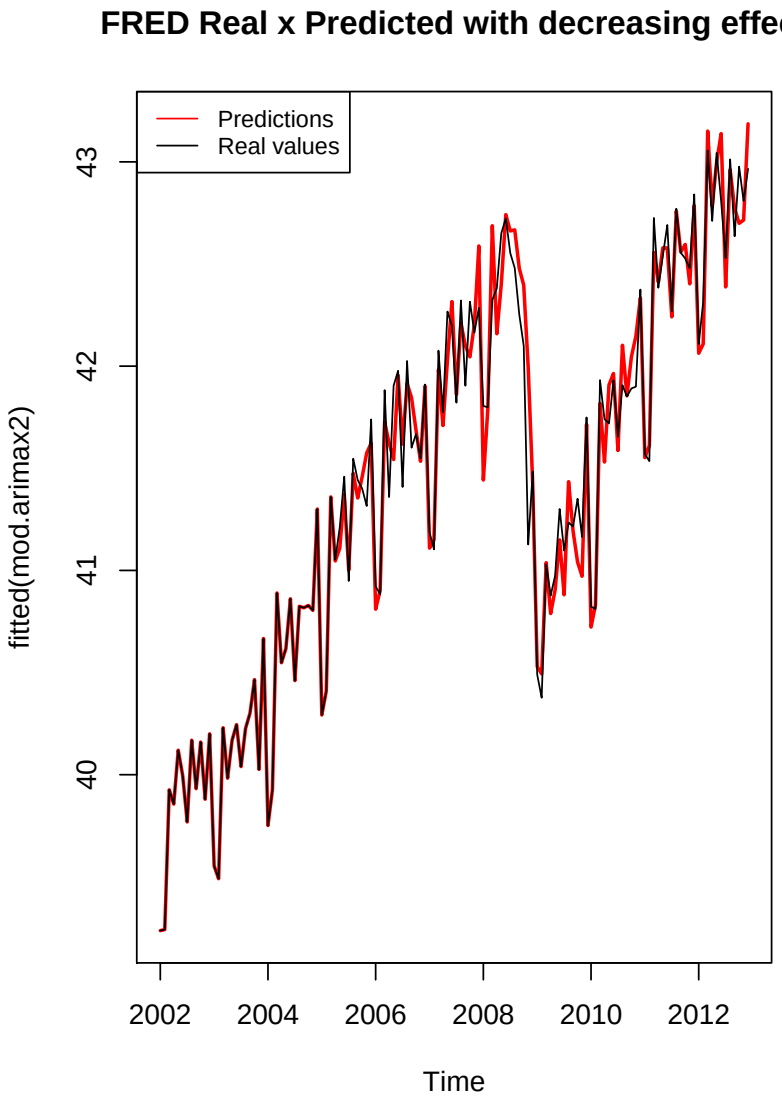
```
##
## Call:
## arimax(x = BoxCox(FRED, lambda), order = c(2, 1, 1), seasonal = list(order = c(2,
##     1, 1), frequency = 12), include.mean = FALSE, method = "CSS", xtransf = temporary_const_eff,
##     transfer = list(c(1, 0)))
##
## Coefficients:
##          ar1     ar2      ma1    sar1     sar2     sma1   T1-AR1  T1-MA0
##       0.6262  0.1537  -0.5044  0.1578  -0.5649  -0.8734  -0.2868  0.2074
## s.e.  0.3524  0.0957   0.2007  0.0557   0.1168   0.0582   0.2576  0.0922
##
## sigma^2 estimated as 0.03239:  part log likelihood = 35.23
##
## Training set error measures:
##                        ME      RMSE        MAE          MPE      MAPE      MASE
## Training set -0.001950638 0.1510535 0.09214914 -0.004856425 0.2199494 0.2738843
##                      ACF1
## Training set -0.03711821
```

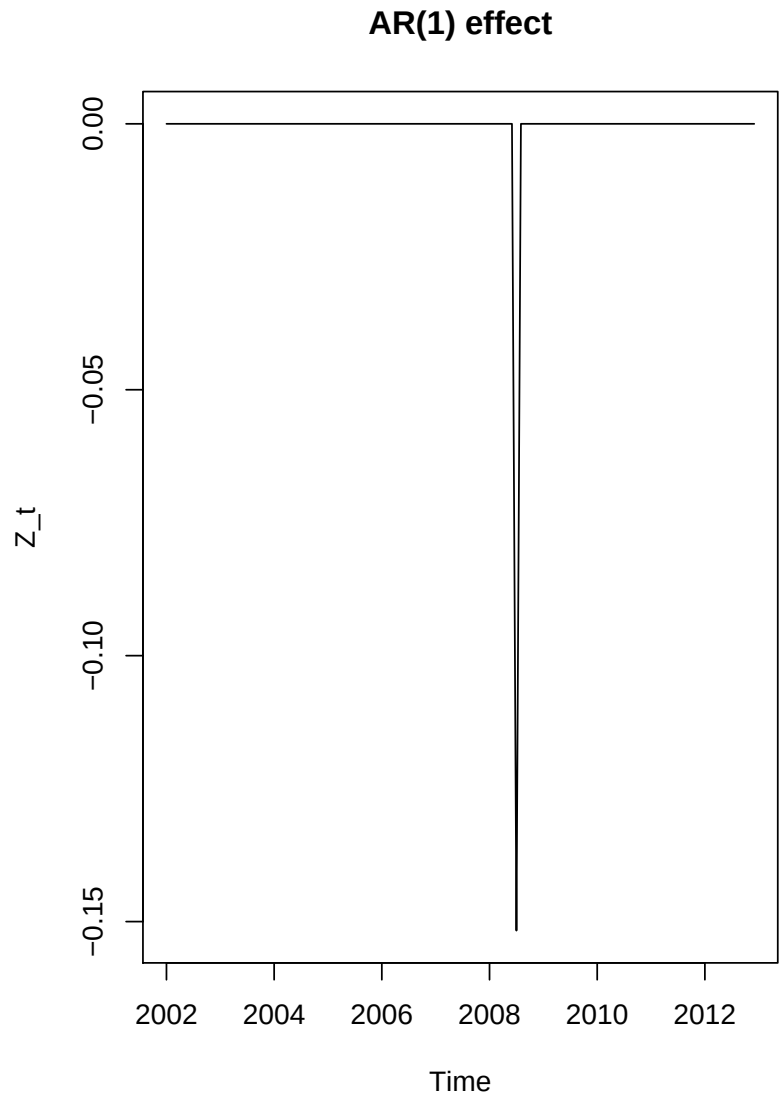

FRED Real x Predicted with decreasing effect
Predictions
Real values
fitted(mod.arimax)
40
41
42
43
2002
2004
2006
2008
2010
2012
Time


AR(1) effect
Z_t
0.00
0.05
0.10
0.15
0.20
2002
2004
2006
2008
2010
2012
Time

- The curve show a increase and after a decrease, while we should expect a decrease. From the results of TSA, we may think that the effect of the intervention only show in the following month, august, so we my try to model with $I(t = aug2008)$.

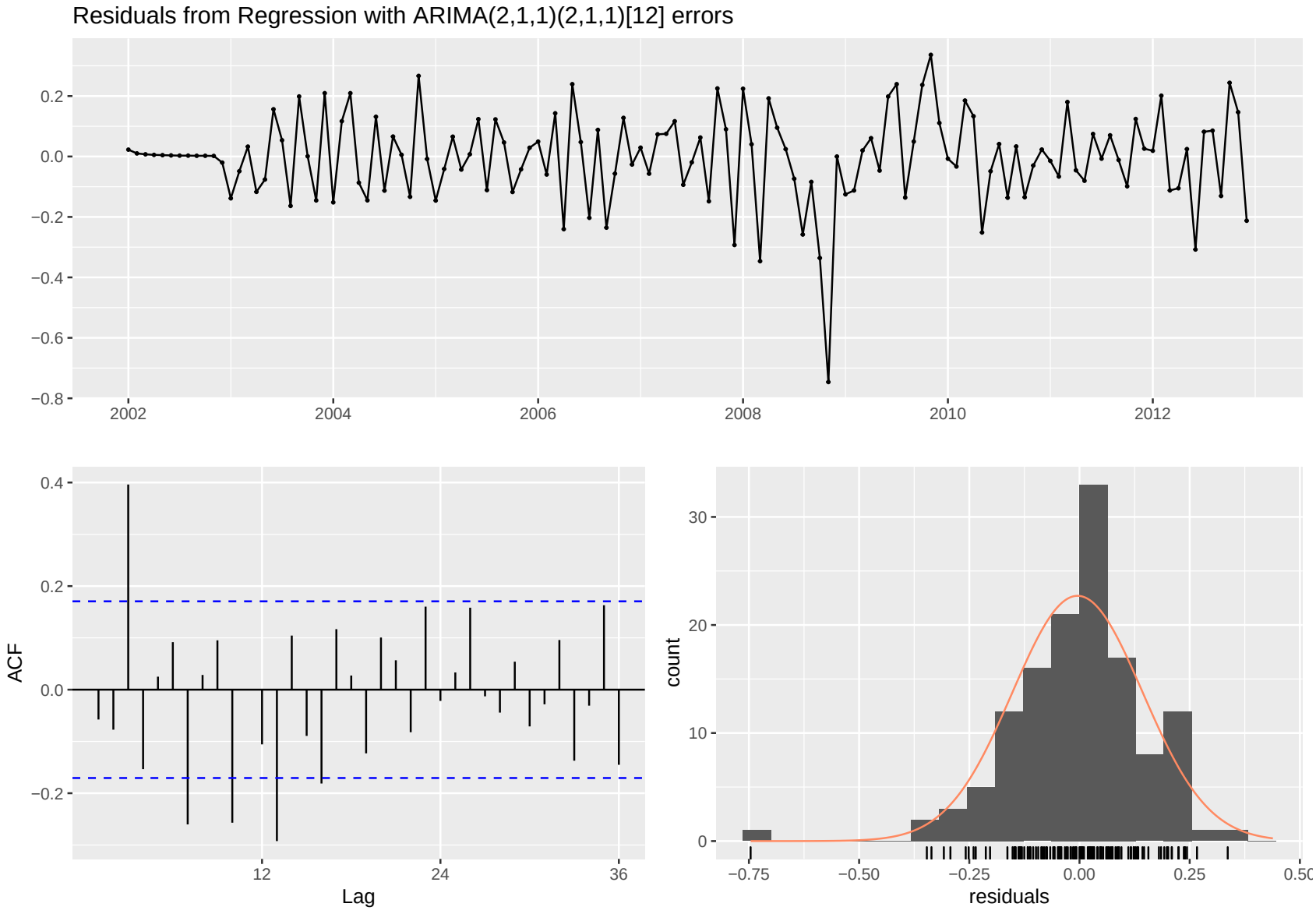
```
##
## Call:
## arimax(x = BoxCox(FRED, lambda), order = c(2, 1, 1), seasonal = list(order = c(2,
##     1, 1), frequency = 12), include.mean = FALSE, method = "CSS", xtransf = 1 *
##     (seq_along(fred_FIT$DATE) == 79), transfer = list(c(1, 0)))
##
## Coefficients:
##          ar1     ar2      ma1    sar1     sar2     sma1   T1-AR1  T1-MA0
##       0.6262  0.1537  -0.5044  0.1578  -0.5649  -0.8734  -0.2868  0.2074
## s.e.  0.3524  0.0957   0.2007  0.0557   0.1168   0.0582   0.2576  0.0922
##
## sigma^2 estimated as 0.03239:  part log likelihood = 35.23
##
## Training set error measures:
##                        ME      RMSE        MAE          MPE      MAPE      MASE
## Training set -0.001950638 0.1510535 0.09214914 -0.004856425 0.2199494 0.2738843
##                      ACF1
## Training set -0.03711821
```

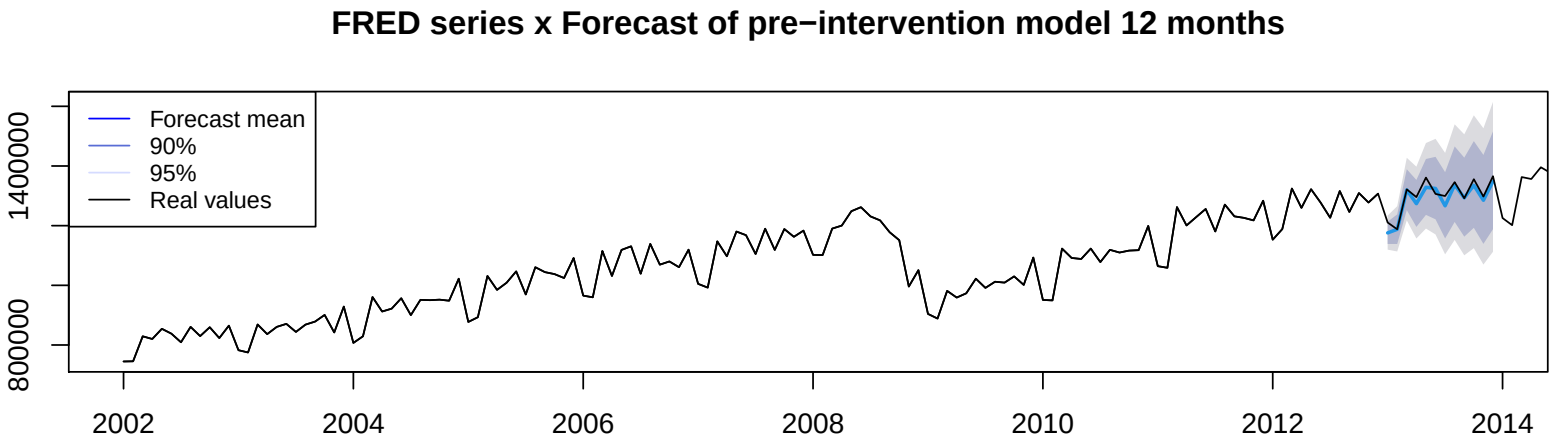

FRED Real x Predicted with decreasing effect
Predictions
Real values
fitted(mod.arimax2)
40
41
42
43
2002
2004
2006
2008
2010
2012
Time

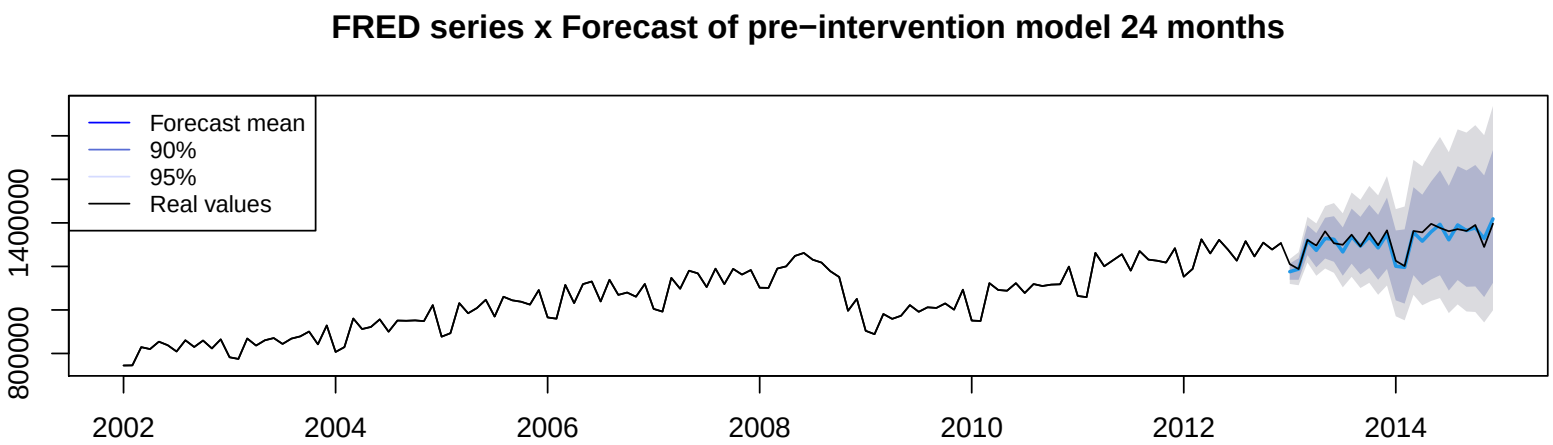

AR(1) effect
Z_t
0.00
−0.05
−0.10
−0.15
2002
2004
2006
2008
2010
2012
Time

- With the computed transfer curve, we use it in a Arima model as regressor.

```
## Series: FRED
## Regression with ARIMA(2,1,1)(2,1,1)[12] errors
## Box Cox transformation: lambda= 0.1370143
##
## Coefficients:
##          ar1     ar2      ma1    sar1     sar2     sma1    xreg
##       0.4749  0.2230  -0.6151  0.2804  -0.3408  -1.0000  1.0653
## s.e.  0.1695  0.0918   0.1521  0.0989   0.0955   0.0992  0.5469
##
## sigma^2 estimated as 0.02554:  log likelihood=37
## AIC=-58   AICc=-56.69   BIC=-35.77
##
## Training set error measures:
##                      ME  RMSE      MAE         MPE     MAPE      MASE
## Training set -805.7189 23866 17205.32 -0.08929967 1.609214 0.2019609
##                      ACF1
## Training set -0.05589136
```


Residuals from Regression with ARIMA(2,1,1)(2,1,1)[12] errors
0.2
0.0
−0.2
−0.4
−0.6
−0.8
2002
2004
2006
2008
2010
2012
ACF
0.4
0.2
0.0
−0.2
12
24
36
Lag
count
30
20
10
0
−0.75
−0.50
−0.25
0.00
0.25
residuals

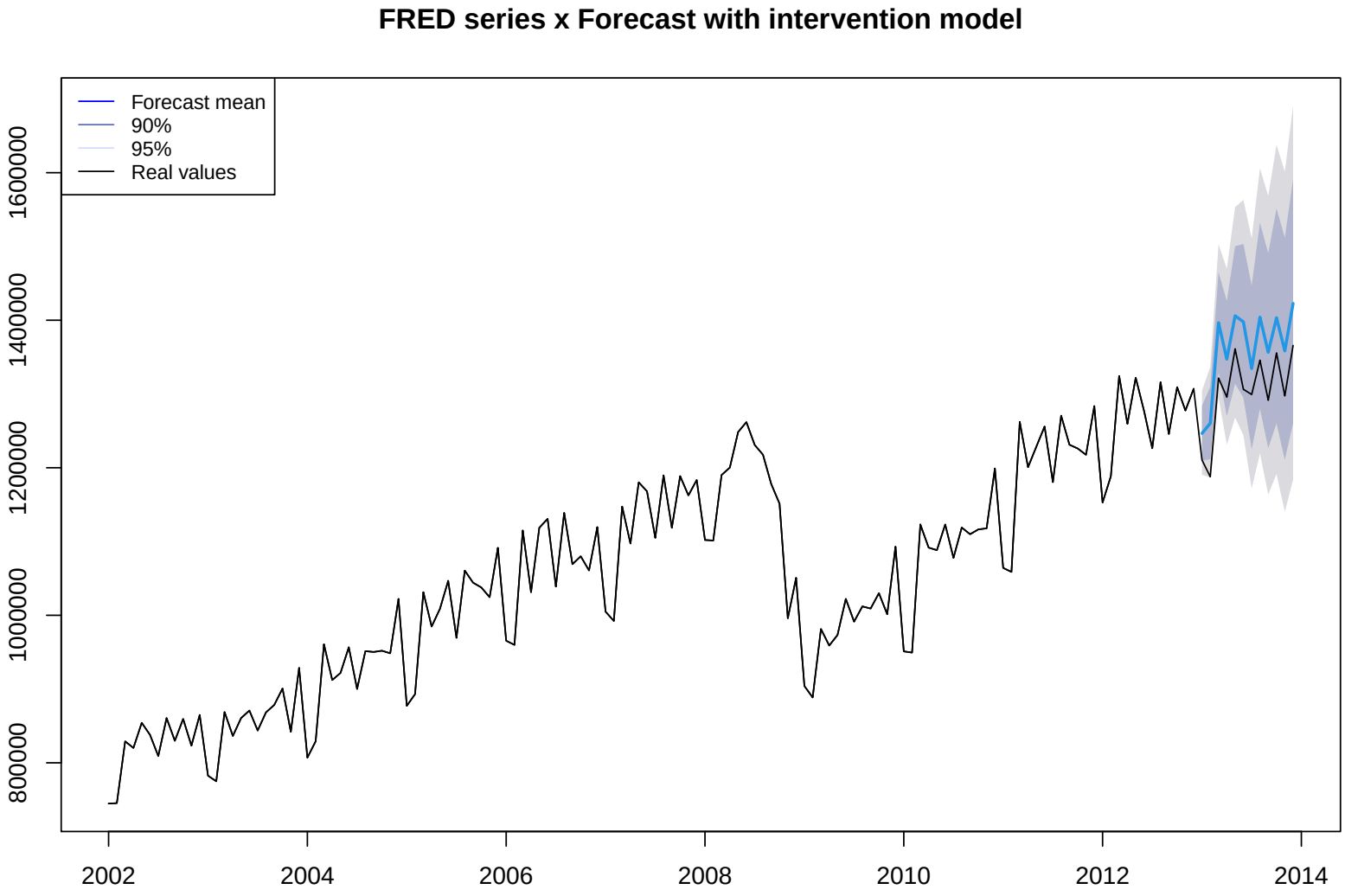

FRED series x Forecast of pre−intervention model 12 months
Forecast mean
90%
95%
Real values
1400000
800000
2002
2004
2006
2008
2010
2012
2014


FRED series x Forecast of pre−intervention model 24 months
Forecast mean
90%
95%
Real values
1400000
800000
2002
2004
2006
2008
2010
2012
2014

Predictive model

```
## Series: FRED_train
## Regression with ARIMA(2,1,1)(2,1,1)[12] errors
## Box Cox transformation: lambda= 0.1370143
##
## Coefficients:
##          ar1     ar2      ma1    sar1     sar2     sma1    xreg
##       0.4704  0.2517  -0.6299  0.3276  -0.3571  -1.0000  0.3937
## s.e.  0.1516  0.0908   0.1328  0.0983   0.0977   0.1024  0.0936
##
## sigma^2 estimated as 0.02298:  log likelihood=43.44
## AIC=-70.88   AICc=-69.57   BIC=-48.65
##
## Training set error measures:
##                      ME     RMSE      MAE        MPE     MAPE      MASE
## Training set -867.4581 22991.08 17055.82 -0.0850767 1.586538 0.2002061
##                      ACF1
## Training set -0.05841938
```

Predict 12 months



Predict 24 months