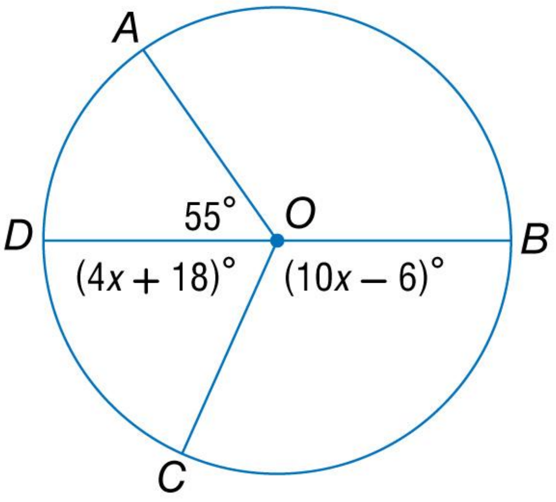
Lesson Menu

5-Minute Check

Over Lesson 9–2

1 In $\odot O$, $\overline{BD}$ is a diameter and $m\angle AOD = 55$. Find $m\angle COB$.



A. 105

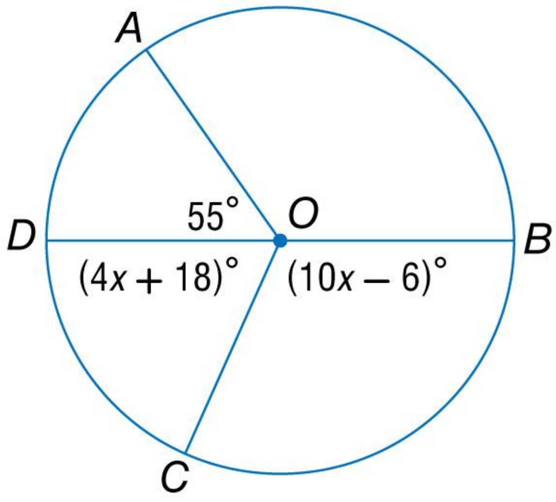
B. 114

C. 118

D. 124

5-Minute Check

2 In $\odot O$, $\overline{BD}$ is a diameter and $m\angle AOD = 55$. Find $m\angle DOC$.



A. 35

B. 55

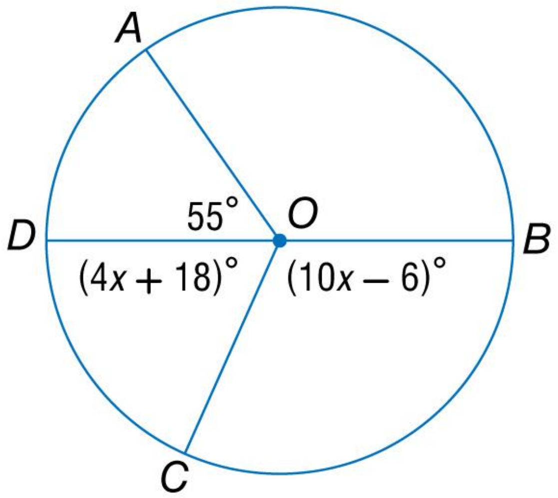
C. 66

D. 72

5-Minute Check

Over Lesson 9–2

3 In $\odot O$, $\overline{BD}$ is a diameter and $m\angle AOD = 55$. Find $m\angle AOB$.



A. 125

B. 130

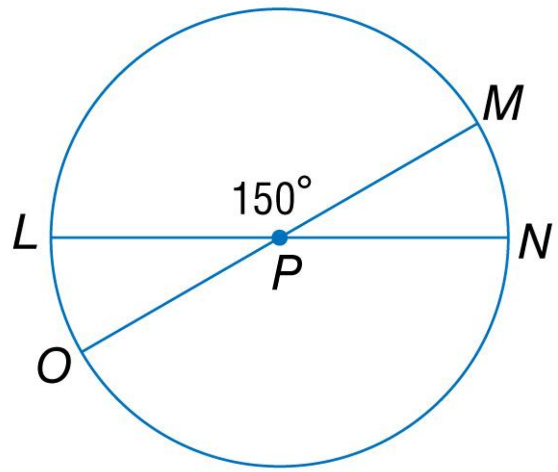
C. 135

D. 140

5-Minute Check

Over Lesson 9–2

4 Refer to $\odot P$. Find $m\overset{\frown}{LM}$.



A. 160

B. 150

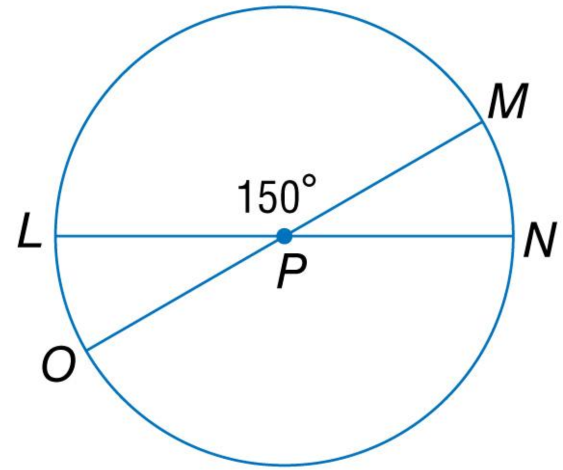
C. 140

D. 130

5-Minute Check

Over Lesson 9–2

5 Refer to $\odot P$. Find $m\widehat{MOL}$.



A. 180

B. 190

C. 200

D. 210

5-Minute Check

Over Lesson 9–2

6 Dianne runs around a circular track that has a radius of 55 feet. After running three quarters of the distance around the track, how far has she run?

A. 129.6 ft

B. 165 ft

C. 259.2 ft

D. 345.6 ft

MP

Mathematical Practices

4 Model with mathematics.

3 Construct viable arguments and critique the reasoning of others.

Content Standards

G.C.2 Identify and describe relationships among inscribed angles, radii, and chords.

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

Then

You used the relationships between arcs and angles to find measures.

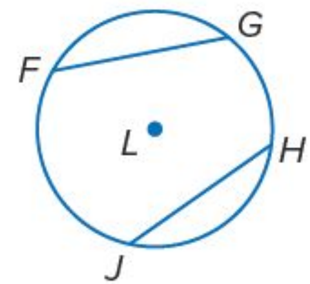
Now

- Recognize and use relationships between arcs and chords.
- Recognize and use relationships between arcs, chords, and diameters.

Theorem 9.2

Words In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

Example $\overset{\frown}{FG} \cong \overset{\frown}{HJ}$ if and only if $\overline{FG} \cong \overline{HJ}$.

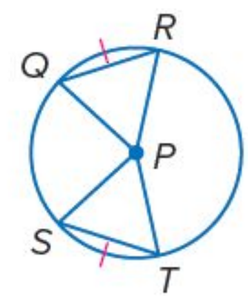


Proof Theorem 9.2 (part 1)

Given: $\odot P$; $\overset{\frown}{QR} \cong \overset{\frown}{ST}$

Prove: $\overline{QR} \cong \overline{ST}$

Proof:



Statements	Reasons
1. $\odot P$, $\overset{\frown}{QR} \cong \overset{\frown}{ST}$	**1.** Given
2. $\angle QPR \cong \angle SPT$	**2.** If arcs are $\cong$, their corresponding central $\measuredangle$s are $\cong$.
3. $\overline{QP} \cong \overline{PR} \cong \overline{SP} \cong \overline{PT}$	**3.** All radii of a circle are $\cong$.
4. $\triangle PQR \cong \triangle PST$	**4.** SAS
5. $\overline{QR} \cong \overline{ST}$	**5.** CPCTC

Real-World Example 1 Use Congruent Chords to Find Arc Measure

JEWELRY A circular piece of jade is hung from a chain by two wires wrapped around the stone. $\overline{JM} \cong \overline{KL}$ and $m\overset{\frown}{KL} = 90$. Find $m\overset{\frown}{JM}$.

Real-World Example 1 Use Congruent Chords to Find Arc Measure

$\overline{JM}$ and $\overline{KL}$ are congruent chords, so the corresponding arcs $\overset{\frown}{JM}$ and $\overset{\frown}{KL}$ are congruent.

Answer : $m\overset{\frown}{KL} = m\overset{\frown}{JM} = 90$

Real-World Example 1

Guided Practice

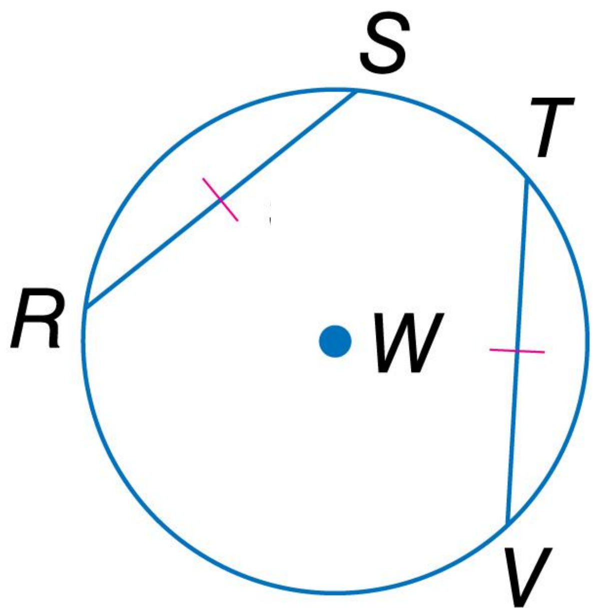
$\odot W$ has congruent chords $\overline{RS}$ and $\overline{TV}$. If $m\overset{\frown}{RS} = 85$, find $m\overset{\frown}{TV}$.

A. 42.5

B. 85

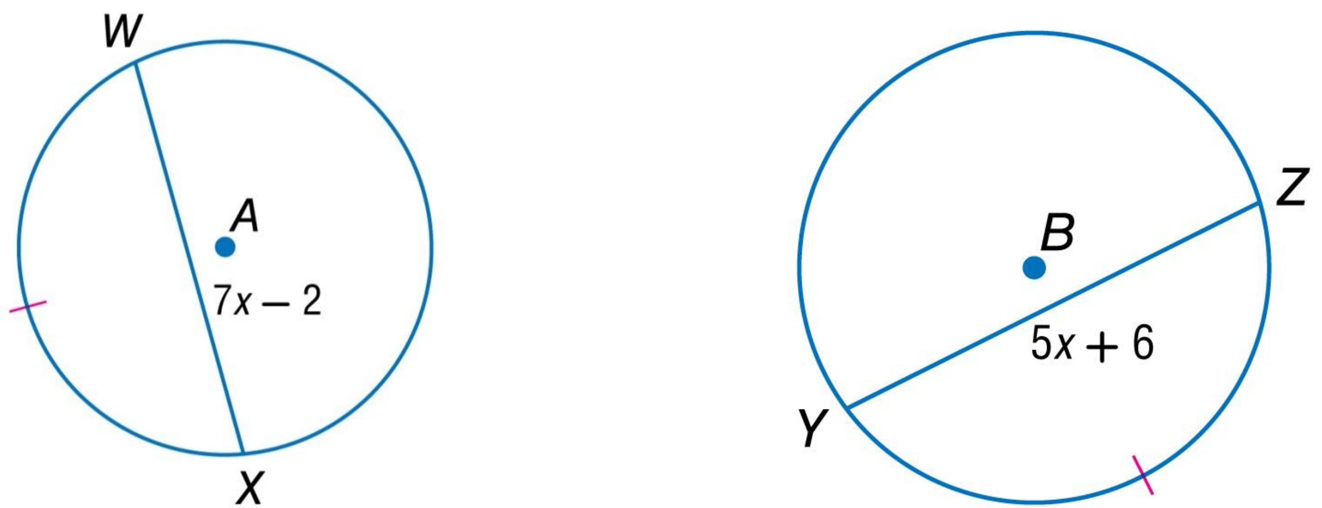
C. 127.5

D. 170



Example 2 Use Congruent Arcs to Find Chord Lengths

ALGEBRA In the figure, $\odot A \cong \odot B$ and $\widehat{WX} \cong \widehat{YZ}$. Find WX.



$\widehat{WX}$ and $\widehat{YZ}$ are congruent arcs in congruent circles, so the corresponding chords $\overline{WX}$ and $\overline{YZ}$ are congruent.

Example 2 Use Congruent Arcs to Find Chord Lengths

$WX = YZ$ Definition of congruent segments

$7x - 2 = 5x + 6$ Substitution

$7x = 5x + 8$ Add 2 to each side.

$2x = 8$ Subtract $5x$ from each side.

$x = 4$ Divide each side by 2.

So, $WX = 7x - 2 = 7(4) - 2$ or 26.

Answer: $WX = 26$

Example 2

Guided Practice

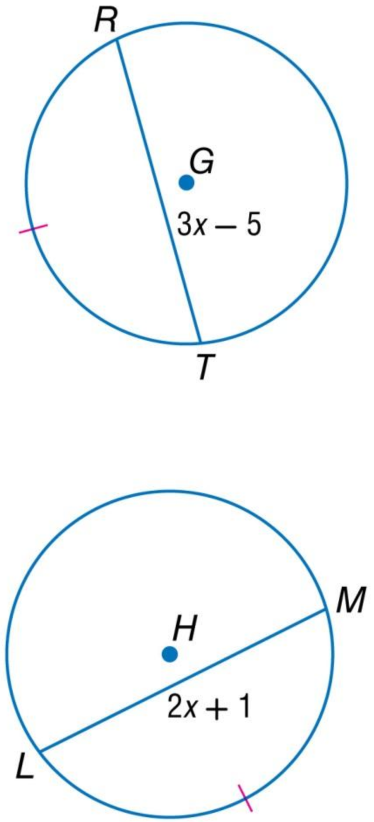
ALGEBRA In the figure, $\odot G \cong \odot H$ and $\widehat{RT} \cong \widehat{LM}$. Find LM.

A. 6

B. 8

C. 9

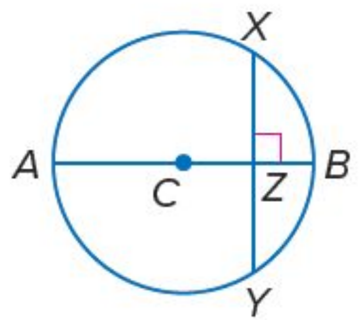
D. 13



Theorems

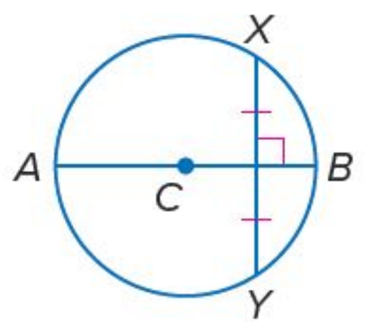
9.3 If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc.

Example If diameter $\overline{AB}$ is perpendicular to chord $\overline{XY}$, then $\overline{XZ} \cong \overline{ZY}$ and $\widehat{XB} \cong \widehat{BY}$.



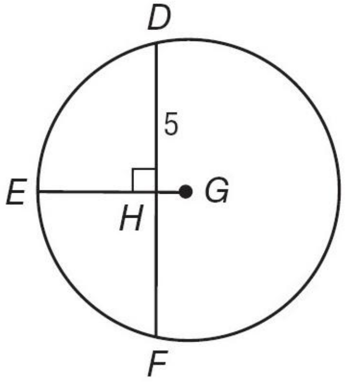
9.4 The perpendicular bisector of a chord is a diameter (or radius) of the circle.

Example If $\overline{AB}$ is a perpendicular bisector of chord $\overline{XY}$, then $\overline{AB}$ is a diameter of $\odot C$.



Example 3 Use a Radius Perpendicular to a Chord

In $\odot G$, $m\overset{\frown}{DEF} = 150$. Find $m\overset{\frown}{DE}$.



Radius $\overline{EG}$ is perpendicular to chord $\overline{DF}$. So by Theorem 10.3, $\overline{EG}$ bisects $\overset{\frown}{DEF}$. Therefore, $m\overset{\frown}{DE} = m\overset{\frown}{EF}$. By substitution, $m\overset{\frown}{DE} = \frac{150}{2}$ or 75.

Answer: $m\overset{\frown}{DE} = 75$

Example 3

Guided Practice

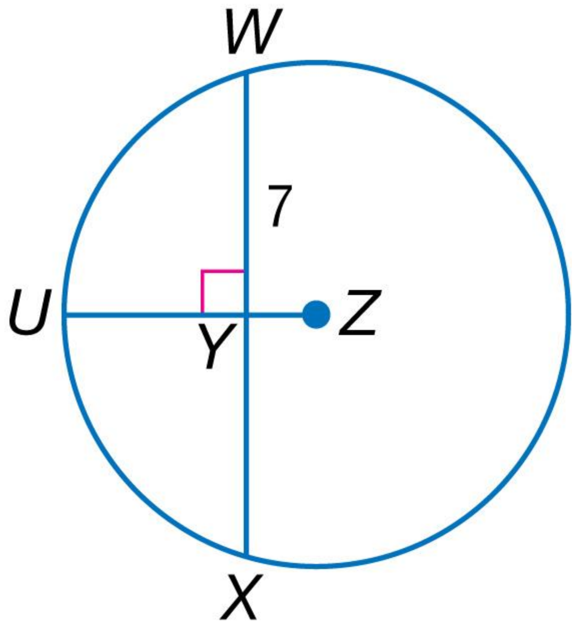
In $\odot Z$, $m\widehat{WUX} = 160$. Find $m\widehat{UX}$.

A. 14

B. 80

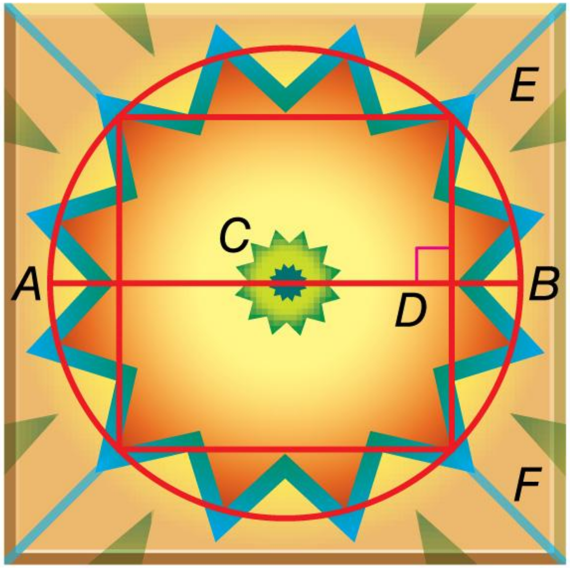
C. 160

D. 180



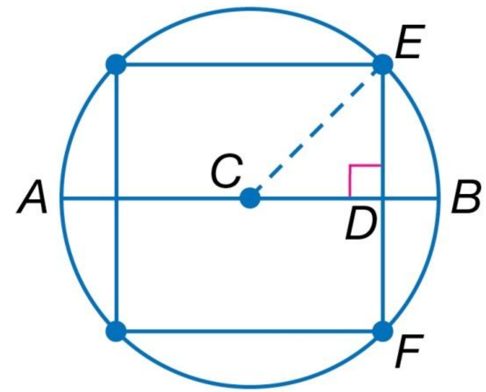
Real-World Example 4 Use a Diameter Perpendicular to a Chord

CERAMIC TILE In the ceramic stepping stone below, diameter $\overline{AB}$ is 18 inches long and chord $\overline{EF}$ is 8 inches long. Find CD.



Real-World Example 4 Use a Diameter Perpendicular to a Chord

Step 1 Draw radius $\overline{CE}$.



This forms right ΔCDE.

Real-World Example 4 Use a Diameter Perpendicular to a Chord

Step 2 Find *CE* and *DE*.

Since *AB* = 18 inches, *CB* = 9 inches. All radii of a circle are congruent, so *CE* = 9 inches.

Since diameter $\overline{AB}$ is perpendicular to $\overline{EF}$, $\overline{AB}$ bisects chord $\overline{EF}$ by Theorem 10.3. So, $DE = \frac{1}{2}(8)$ or 4 inches.

Real-World Example 4 Use a Diameter Perpendicular to a Chord

Step 3 Use the Pythagorean Theorem to find *CD*.

$CD^2 + DE^2 = CE^2$ Pythagorean Theorem

$CD^2 + 4^2 = 9^2$ Substitution

$CD^2 + 16 = 81$ Simplify.

$CD^2 = 65$ Subtract 16 from each side.

$CD = \sqrt{65}$ Take the positive square root.

Answer: $CD = \sqrt{65}$ or about 8.06 inches.

Real-World Example 4

Guided Practice

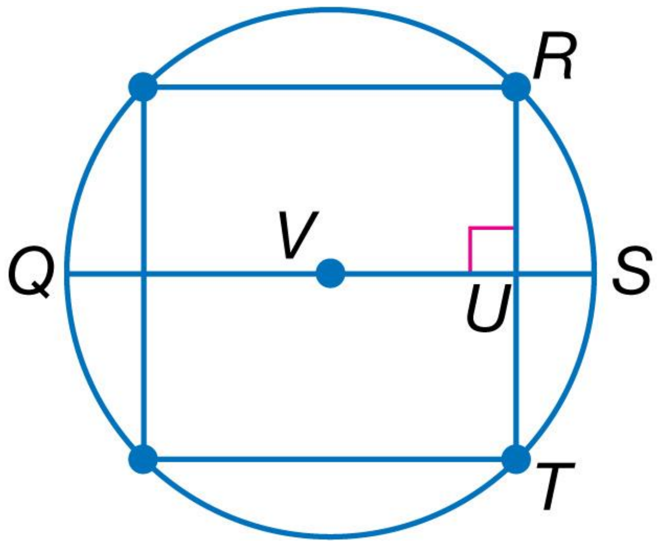
In the circle below, diameter $\overline{QS}$ is 14 inches long and chord $\overline{RT}$ is 10 inches long. Find VU.

A. 3.87

B. 4.25

C. 4.90

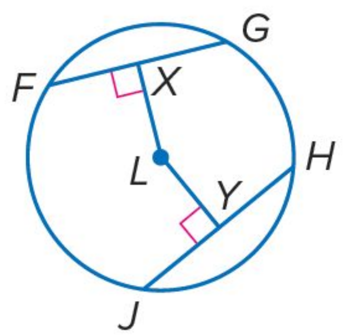
D. 5.32



Theorem 9.5

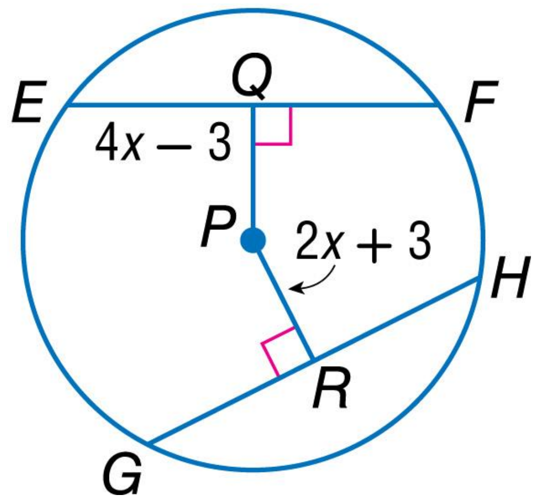
Words In the same circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

Example $\overline{FG} \cong \overline{JH}$ if and only if $LX = LY$.



Example 5 — Chords Equidistant from Center

ALGEBRA In $\odot P$, $EF = GH = 24$. Find PQ.



Since chords $\overline{EF}$ and $\overline{GH}$ are congruent, they are equidistant from P. So, $PQ = PR$.

Example 5 — Chords Equidistant from Center

$PQ = PR$

$4x - 3 = 2x + 3$ Substitution

$x = 3$ Simplify.

So, $PQ = 4(3) - 3$ or 9

Answer: $PQ = 9$

Example 5 — Guided Practice

ALGEBRA In $\odot R$, $MN = PO = 29$. Find RS.

A. 7

B. 10

C. 13

D. 15