Lesson Menu

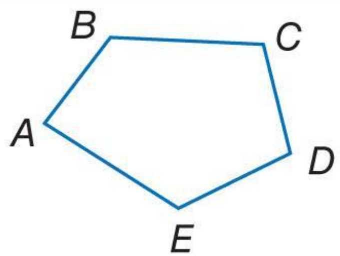
ANGLES OF POLYGONS

Theorem 6.1 Polygon Interior Angles Sum

The sum of the interior angle measures of an n-sided convex polygon is $(n - 2) \cdot 180$.

Example $m\angle A + m\angle B + m\angle C + m\angle D + m\angle E = (5 - 2) \cdot 180$

$$= 540$$

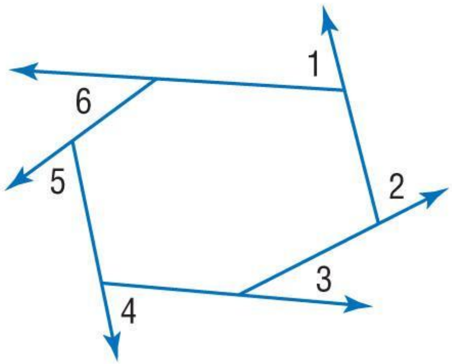


Theorem 6.2 Polygon Exterior Angles Sum

The sum of the exterior angle measures of a convex polygon, one angle at each vertex, is 360.

Example

$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 = 360$



Lesson Menu

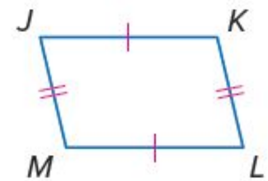
PARALLELOGRAMS

Theorems Properties of Parallelograms

6.3 If a quadrilateral is a parallelogram, then its opposite sides are congruent.

Abbreviation *Opp. sides of a ▱ are ≅.*

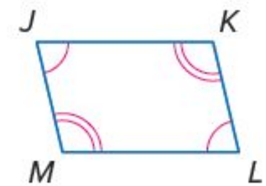
Example If $JKLM$ is a parallelogram, then $\overline{JK} \cong \overline{ML}$ and $\overline{JM} \cong \overline{KL}$.



6.4 If a quadrilateral is a parallelogram, then its opposite angles are congruent.

Abbreviation *Opp. ∠s of a ▱ are ≅.*

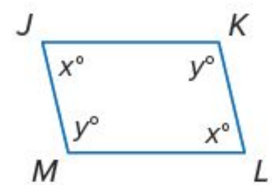
Example If $JKLM$ is a parallelogram, then $\angle J \cong \angle L$ and $\angle K \cong \angle M$.



6.5 If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

Abbreviation *Cons. ∠s in a ▱ are supplementary.*

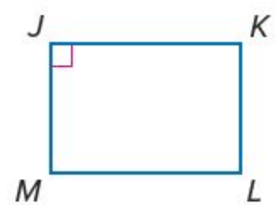
Example If $JKLM$ is a parallelogram, then $x + y = 180$.



6.6 If a parallelogram has one right angle, then it has four right angles.

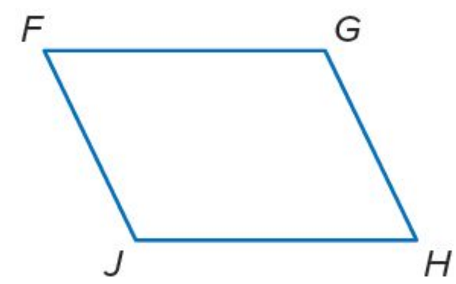
Abbreviation *If a ▱ has 1 rt. ∠, it has 4 rt. ∠s.*

Example In ▱$JKLM$, if $\angle J$ is a right angle, then $\angle K$, $\angle L$, and $\angle M$ are also right angles.



Proof Theorem 6.4

Write a two-column proof of Theorem 6.4.



Given: $\square FGHJ$

Prove: $\angle F \cong \angle H$, $\angle J \cong \angle G$

Proof:

Statements	Reasons
1. $\square FGHJ$	**1.** Given
2. $\overline{FG} \parallel \overline{JH}$; $\overline{FJ} \parallel \overline{GH}$	**2.** Definition of parallelogram
3. $\angle F$ and $\angle J$ are supplementary. $\angle J$ and $\angle H$ are supplementary. $\angle H$ and $\angle G$ are supplementary.	**3.** If parallel lines are cut by a transversal, consecutive interior angles are supplementary.
4. $\angle F \cong \angle H$, $\angle J \cong \angle G$	**4.** Supplements of the same angles are congruent.

Concept Summary

Prove that a Quadrilateral Is a Parallelogram

- Show that both pairs of opposite sides are parallel. (Definition)
- Show that both pairs of opposite sides are congruent. (Theorem 6.9)
- Show that both pairs of opposite angles are congruent. (Theorem 6.10)
- Show that the diagonals bisect each other. (Theorem 6.11)
- Show that a pair of opposite sides is both parallel and congruent. (Theorem 6.12)

Lesson Menu

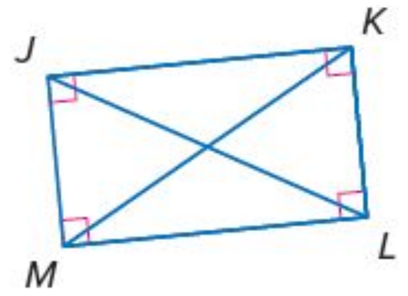
RECTANGLES

Theorem 6.13 Diagonals of a Rectangle

If a parallelogram is a rectangle, then its diagonals are congruent.

Abbreviation *If a ▱ is a rectangle, diag. are ≅.*

Example If ▱ *JKLM* is a rectangle, then $\overline{JL} \cong \overline{MK}$.

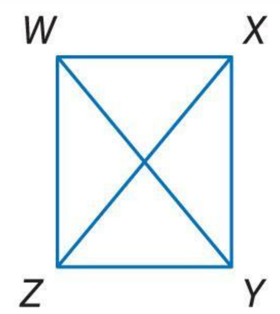


Theorem 6.14 Diagonals of a Rectangle

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Abbreviation *If diag. of a ▱ are ≅, then ▱ is a rectangle.*

Example If $\overline{WY} \cong \overline{XZ}$ in ▱$WXYZ$, then ▱$WXYZ$ is a rectangle.

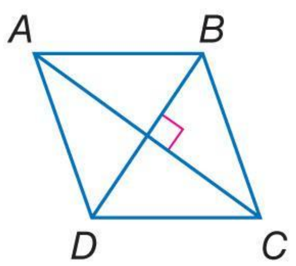


RHOMBI AND SQUARES

Theorems Diagonals of a Rhombus

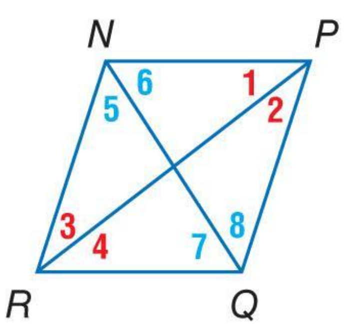
6.15 If a parallelogram is a rhombus, then its diagonals are perpendicular.

Example If $\square ABCD$ is a rhombus, then $\overline{AC} \perp \overline{BD}$.



6.16 If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.

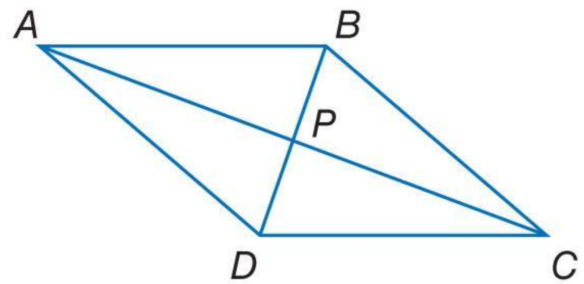
Example If $\square NPQR$ is a rhombus, then $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$, $\angle 5 \cong \angle 6$, and $\angle 7 \cong \angle 8$.



Proof Theorem 6.15

Given: $ABCD$ is a rhombus.

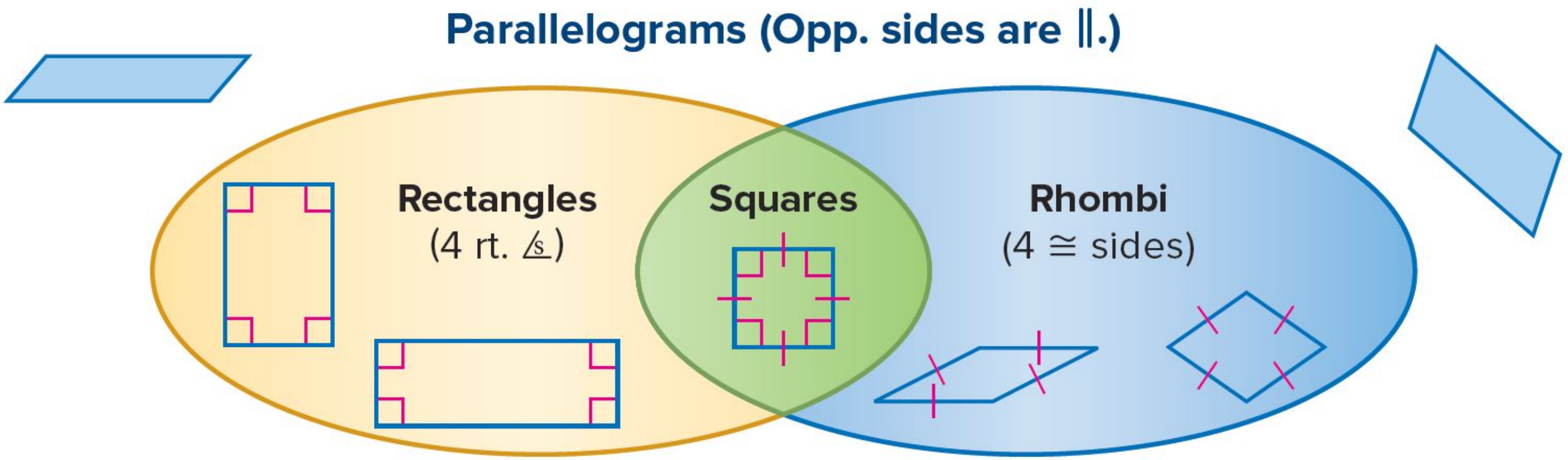
Prove: $\overline{AC} \perp \overline{BD}$



Paragraph Proof:

Since $ABCD$ is a rhombus, by definition $\overline{AB} \cong \overline{BC}$. A rhombus is a parallelogram and the diagonals of a parallelogram bisect each other, so $\overline{BD}$ bisects $\overline{AC}$ at P. Thus, $\overline{AP} \cong \overline{PC}$. $\overline{BP} \cong \overline{BP}$ by the Reflexive Property. So, $\triangle APB \cong \triangle CPB$ by SSS. $\angle APB \cong \angle CPB$ by CPCTC. $\angle APB$ and $\angle CPB$ also form a linear pair. Two congruent angles that form a linear pair are right angles. $\angle APB$ is a right angle, so $\overline{AC} \perp \overline{BD}$ by the definition of perpendicular lines.

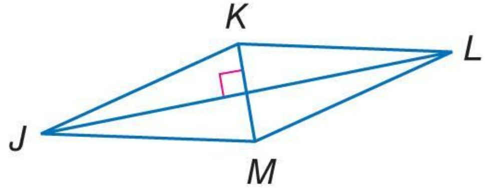
Concept Summary Parallelograms



Theorems Conditions for Rhombi and Squares

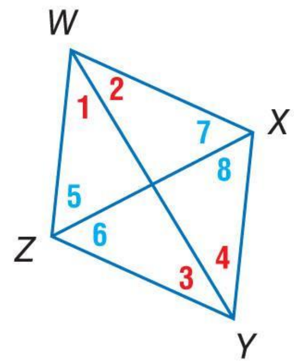
6.17 If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (Converse of Theorem. 6.15)

Example If $\overline{JL} \perp \overline{KM}$, then $\square JKLM$ is a rhombus.



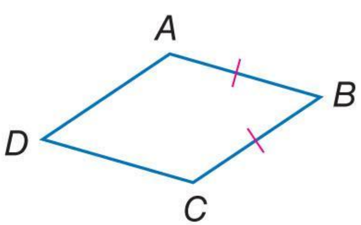
6.18 If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus. (Converse of Theorem. 6.16)

Example If $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$, or $\angle 5 \cong \angle 6$ and $\angle 7 \cong \angle 8$, then $\square WXYZ$ is a rhombus.



6.19 If one pair of consecutive sides of a parallelogram are congruent, the parallelogram is a rhombus.

Example If $\overline{AB} \cong \overline{BC}$, then $\square ABCD$ is a rhombus.



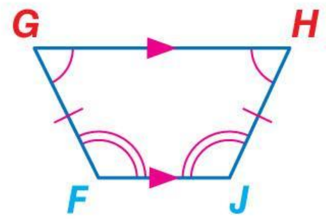
6.20 If a quadrilateral is both a rectangle and a rhombus, then it is a square.

TRAPEZOIDS AND KITES

Theorems Isosceles Trapezoids

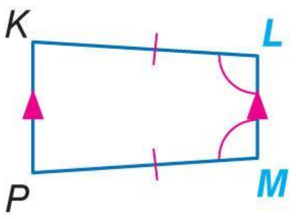
6.21 If a trapezoid is isosceles, then each pair of base angles is congruent.

Example If trapezoid *FGHJ* is isosceles, then $\angle G \cong \angle H$ and $\angle F \cong \angle J$.



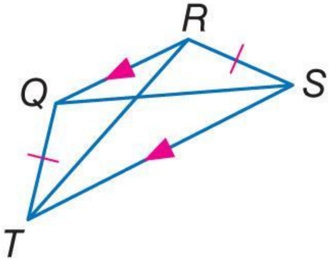
6.22 If a trapezoid has one pair of congruent base angles, then it is an isosceles trapezoid.

Example If $\angle L \cong \angle M$, then trapezoid *KLMP* is isosceles.



6.23 A trapezoid is isosceles if and only if its diagonals are congruent.

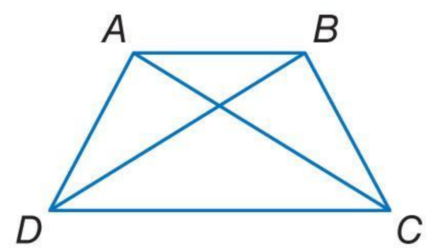
Example If trapezoid *QRST* is isosceles, then $\overline{QS} \cong \overline{RT}$. Likewise, if $\overline{QS} \cong \overline{RT}$, then trapezoid *QRST* is isosceles.



Proof Part of Theorem 6.23

Given: *ABCD* is an isosceles trapezoid.

Prove: $\overline{AC} \cong \overline{BD}$



ABCD is an isosceles trapezoid.
Given

$\overline{DC} \cong \overline{CD}$
Reflexive Property

$\overline{AD} \cong \overline{BC}$
Def. Isos. Trapezoid

$\angle ADC \cong \angle BCD$
Base $\measuredangle$s of trapezoid are $\cong$.

$\triangle ADC \cong \triangle BCD$
SAS

$\overline{AC} \cong \overline{BD}$
CPCTC

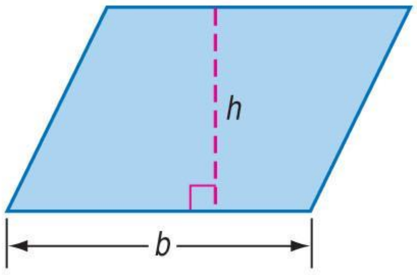
AREAS OF PARALLELOGRAMS AND TRIANGLES

Postulate 10.1 Area Addition Postulate
The area of a region is the sum of the areas of its nonoverlapping parts.

KeyConcept Area of a Parallelogram

Words The area A of a parallelogram is the product of a base b and its corresponding height h.

Symbols $A = bh$

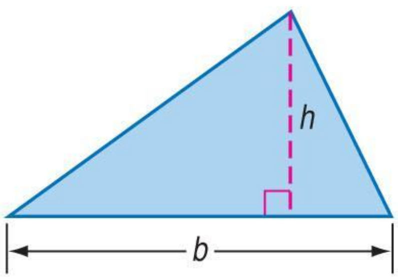


Postulate 10.2 Area Congruence Postulate
If two figures are congruent, then they have the same area.

KeyConcept Area of a Triangle

Words The area A of a triangle is one half the product of a base b and its corresponding height h.

Symbols $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

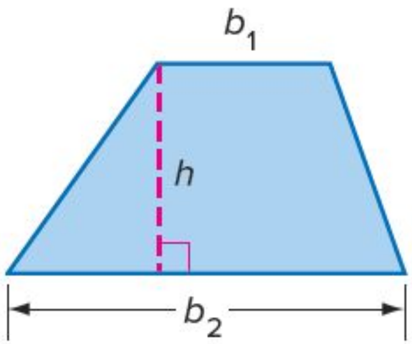


AREAS OF TRAPEZOIDS AND KITES

Key Concept Area of a Trapezoid

Words The area A of a trapezoid is one half the product of the height h and the sum of its bases, b_1 and b_2.

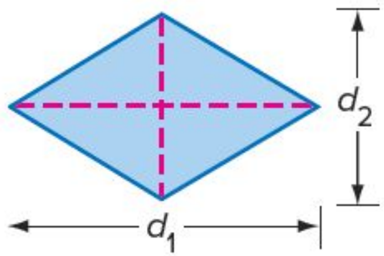
Symbols $A = \frac{1}{2}h(b_1 + b_2)$

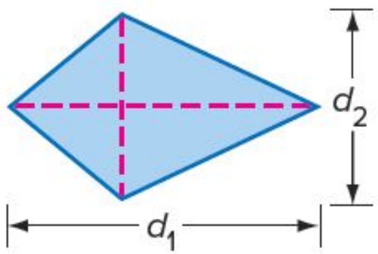


Key Concept Area of a Rhombus or Kite

Words The area A of a rhombus or kite is one half the product of the lengths of its diagonals, d_1 and d_2.

Symbols $A = \frac{1}{2}d_1d_2$



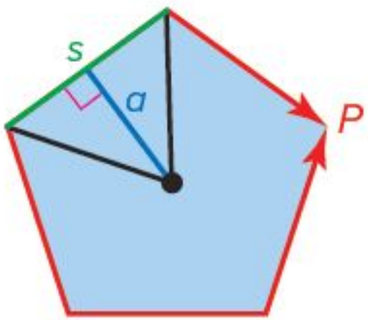


AREAS OF REGULAR POLYGONS AND COMPOSITE FIGURES

Key Concept Area of a Regular Polygon

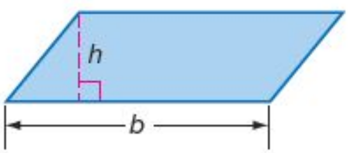
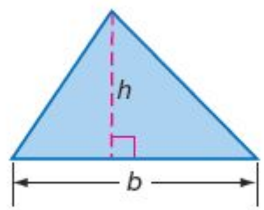
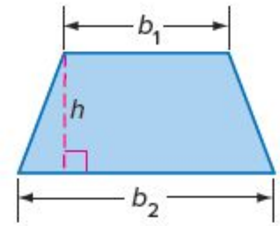
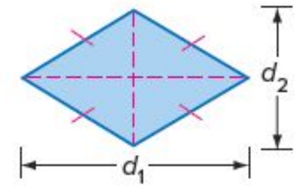
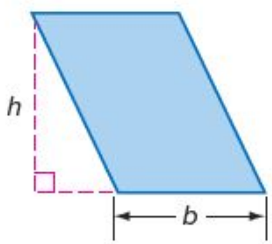
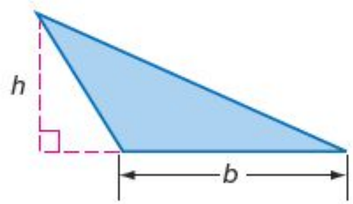
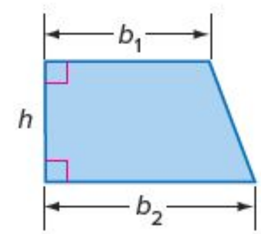
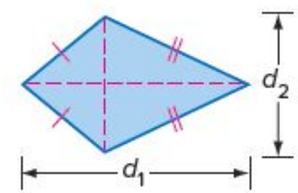
Words The area A of a regular n-gon with side length s is one half the product of the apothem a and perimeter P.

Symbols $A = \frac{1}{2}a(ns)$ or $A = \frac{1}{2}aP$.



SUMMARY

Concept Summary Areas of Polygons

Parallelogram	Triangles	Trapezoids	Rhombi and Kites
			
			
$A = bh$	$A = \frac{1}{2}bh$	$A = \frac{1}{2}h(b_1 + b_2)$	$A = \frac{1}{2}d_1d_2$