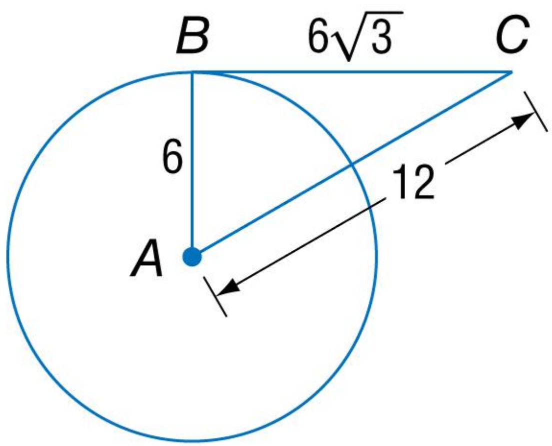
# Lesson Menu

## 5-Minute Check

Over Lesson 9–5

**1** Determine whether $\overline{BC}$ is tangent to the given circle.




**A.** yes

**B.** no

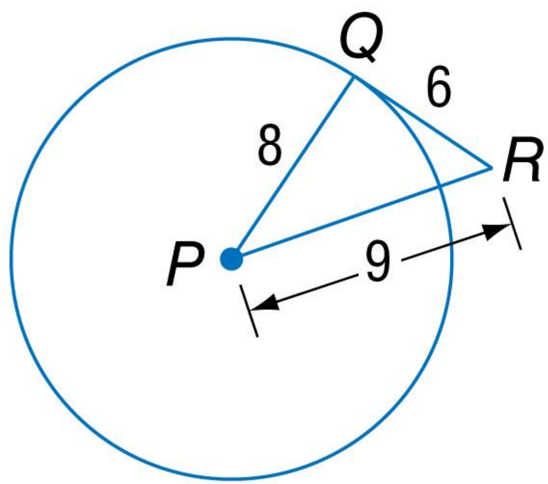
## 5-Minute Check

Over Lesson 9–5

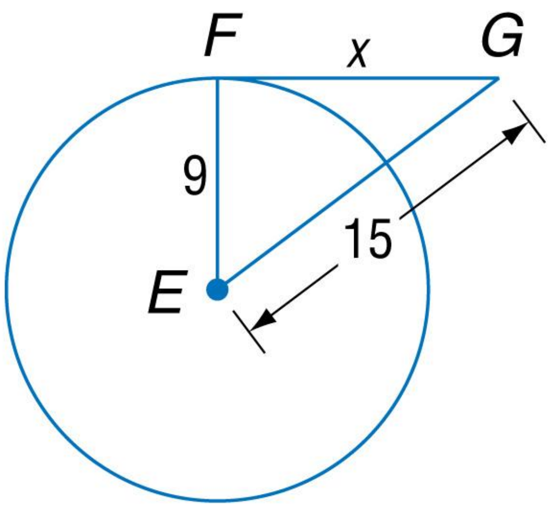
**2** Determine whether $\overline{QR}$ is tangent to the given circle.

A. yes

B. no



**3** Find *x*. Assume that segments that appear to be tangent are tangent.




A. 10

B. 11

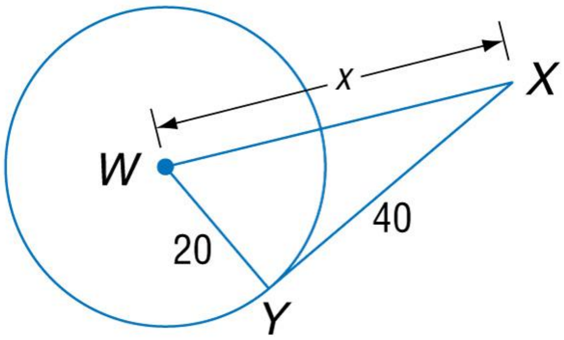
C. 12

D. 13

## 5-Minute Check

Over Lesson 9–5

**4** Find *x*. Assume that segments that appear to be tangent are tangent.




A. $17\sqrt{2}$

B. $18\sqrt{3}$

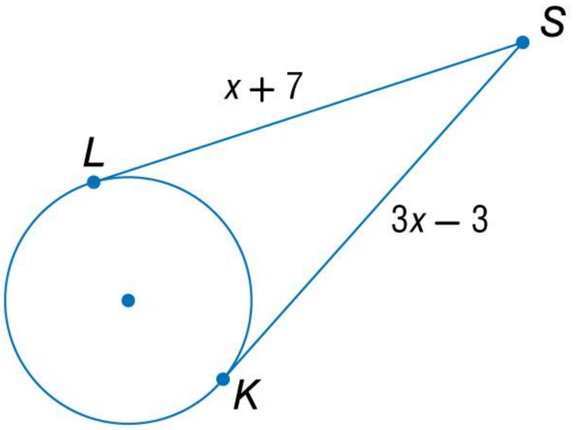
C. 20

D. $20\sqrt{5}$

## 5-Minute Check

Over Lesson 9–5

**5** $\overline{SL}$ and $\overline{SK}$ are tangent to the circle. Find $x$.




**A.** 1

**B.** $\frac{5}{2}$

**C.** 5

**D.** 44

## Mathematical Practices

G.C.2 Identify and describe relationships among inscribed angles, radii, and chords.

## Then

You found measures of segments formed by tangents to a circle.

## Now

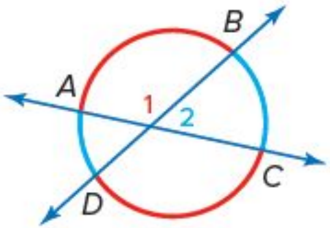
- Find measures of angles formed by lines intersecting on or inside a circle.
- Find measures of angles formed by lines intersecting outside the circle.

## New Vocabulary

- secant

## Theorem 9-12

**Words** If two secants or chords intersect in the interior of a circle, then the measure of an angle formed is one half the *sum* of the measure of the arcs intercepted by the angle and its vertical angle.



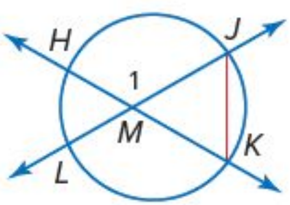

**Example** $m\angle 1 = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$ and $m\angle 2 = \frac{1}{2}(m\widehat{DA} + m\widehat{BC})$

## Proof

Given: $\overleftrightarrow{HK}$ and $\overleftrightarrow{JL}$ intersect at $M$.

Prove: $m\angle 1 = \frac{1}{2}(m\widehat{JH} + m\widehat{LK})$

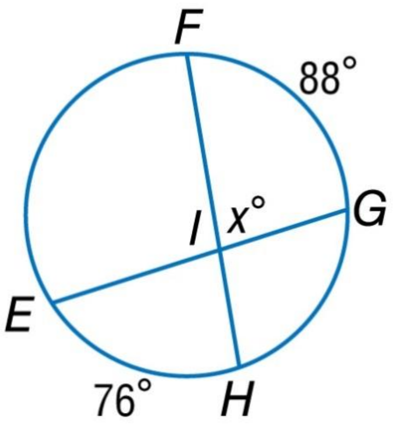
Proof:




| Statements | Reasons |
|---|---|
| 1. $\overleftrightarrow{HK}$ and $\overleftrightarrow{JL}$ intersect at $M$. | 1. Given |
| 2. $m\angle 1 = m\angle MJK + m\angle MKJ$ | 2. Exterior Angle Theorem |
| 3. $m\angle MJK = \frac{1}{2}m\widehat{LK}$, $m\angle MKJ = \frac{1}{2}m\widehat{JH}$ | 3. The measure of an inscribed $\angle$ equals half the measure of the intercepted arc. |
| 4. $m\angle 1 = \frac{1}{2}m\widehat{LK} + \frac{1}{2}m\widehat{JH}$ | 4. Substitution |
| 5. $m\angle 1 = \frac{1}{2}(m\widehat{JH} + m\widehat{LK})$ | 5. Distributive Property |

## Example 1 — Use Intersecting Chords or Secants

**A. Find *x*.**




$$m\angle FIG = \frac{1}{2}\left(m\widehat{FG} + m\widehat{EH}\right)$$ Theorem 10.12

$$m\angle FIG = \frac{1}{2}(88 + 76)$$ Substitution

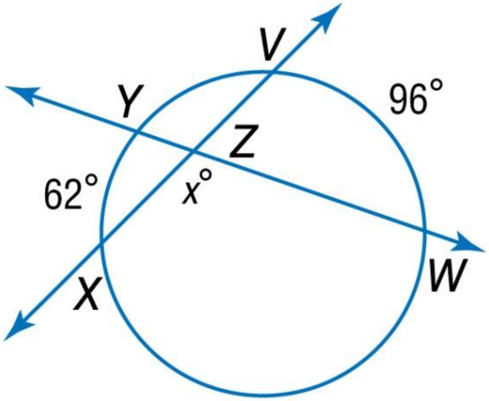
$$m\angle FIG = \frac{1}{2}(164) \text{ or } 82$$ Simplify.

**Answer:** $x = 82$

## Example 1

**Use Intersecting Chords or Secants**

**B. Find *x*.**




**Step 1** Find $m\angle VZW$.

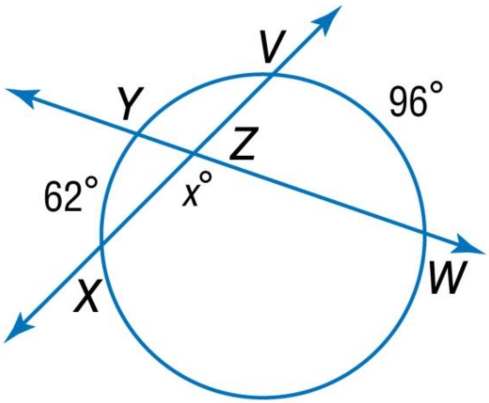
$$m\angle VZW = \frac{1}{2}\left(m\overset{\frown}{VW} + m\overset{\frown}{XY}\right)$$ Theorem 10.12

$$m\angle VZW = \frac{1}{2}(96 + 62)$$ Substitution

$$m\angle VZW = \frac{1}{2}(158) \text{ or } 79$$ Simplify.

## Example 1 — Use Intersecting Chords or Secants

**Step 2** Find $m\angle WZX$.




$m\angle WZX = 180 - m\angle VZW$ Definition of supplementary angles

$x = 180 - 79$ Substitution

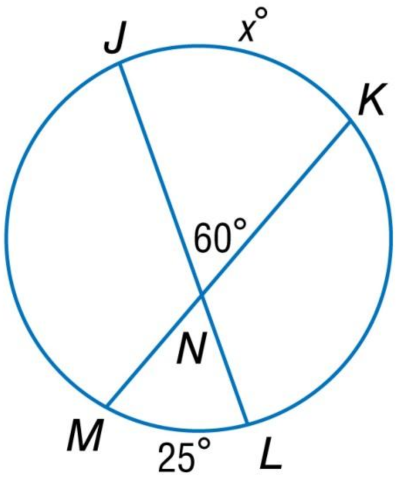
$x = 101$ Simplify.

**Answer:** $x = 101$

## Example 1

**Use Intersecting Chords or Secants**

**C. Find *x*.**




$$m\angle JNK = \frac{1}{2}\left(m\overset{\frown}{JK} + m\overset{\frown}{LM}\right)$$ Theorem 10.12

$$60 = \frac{1}{2}(x + 25)$$ Substitution

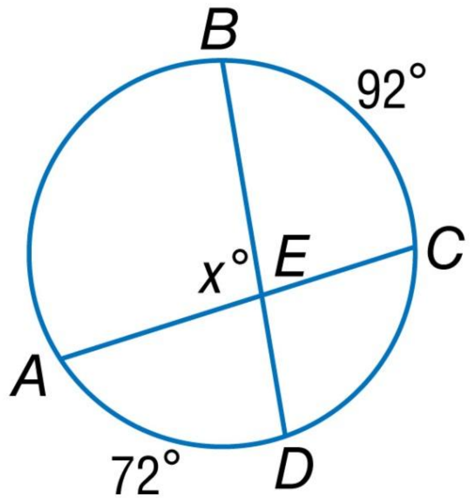
$$120 = x + 25$$ Multiply each side by 2.

$$95 = x$$ Subtract 25 from each side.

**Answer:** $x = 95$

## Example 1 Guided Practice

**A. Find *x*.**




**A.** 92

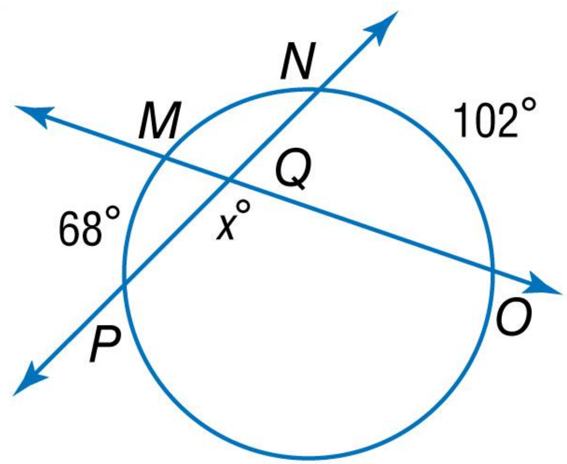
**B.** 95

**C.** 98

**D.** 104

## Example 1 Guided Practice

**B. Find *x*.**




A. 92

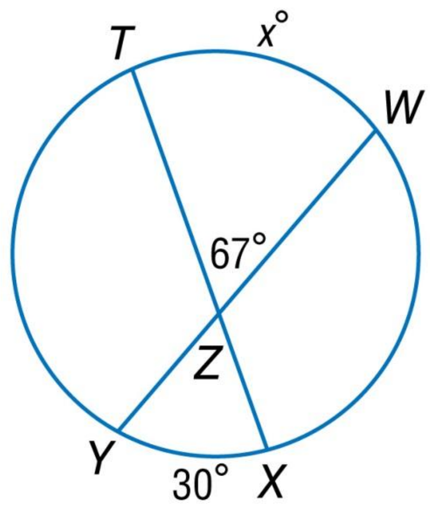
B. 95

C. 97

D. 102

## Example 1 — Guided Practice

**C. Find *x*.**




A. 96

B. 99

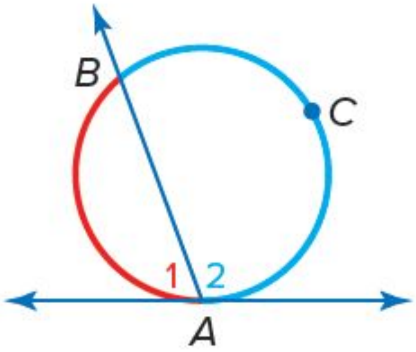
C. 101

D. 104

## Theorem 9-13

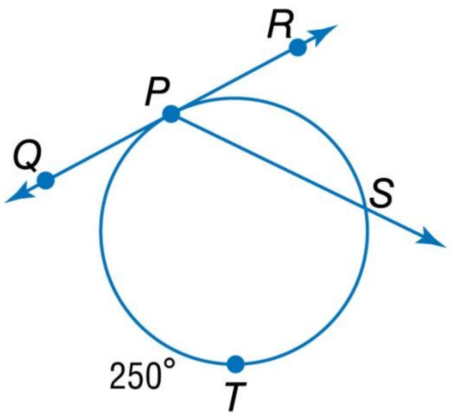
**Words** If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one half the measure of its intercepted arc.

**Example** $m\angle 1 = \frac{1}{2}m\widehat{AB}$ and $m\angle 2 = \frac{1}{2}m\widehat{ACB}$



## Example 2 Use Intersecting Secants and Tangents

**A. Find $m\angle QPS$.**



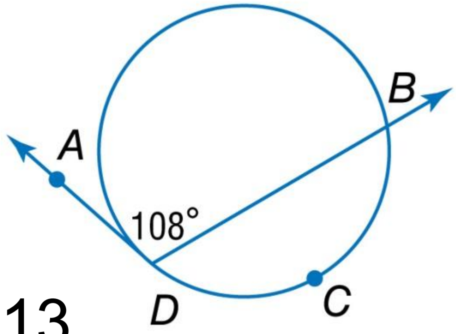

$$m\angle QPS = \frac{1}{2} m\overparen{PTS}$$ Theorem 10.13

$$= \frac{1}{2}(250) \text{ or } 125$$ Substitute and simplify.

**Answer:** $m\angle QPS = 125$

## Example 2 Use Intersecting Secants and Tangents

**B. Find $m\overparen{BCD}$.**




$$m\angle ADB = \frac{1}{2} m\overparen{DB}$$ Theorem 10.13

$$108 = \frac{1}{2} m\overparen{DB}$$ Substitution

$$216 = m\overparen{DB}$$ Multiply each side by 2.

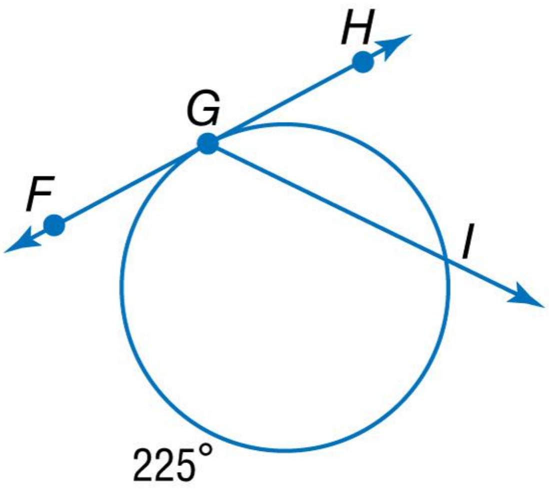
$$m\overparen{BCD} = 360 - m\overparen{DB} = 360 - 216 \text{ or } 144$$

**Answer:** $m\overparen{BCD} = 144$

## Example 2

Guided Practice

**A.** Find $m\angle FGI$.




A. 98

B. 108

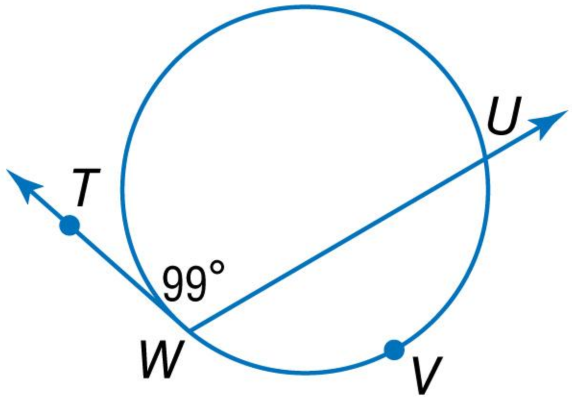
C. 112.5

D. 118.5

## Example 2

Guided Practice

**B. Find $m\overset{\frown}{UVW}$.**




A. 99

B. 148.5

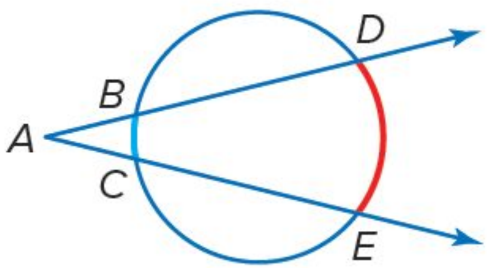
C. 162

D. 198

## Theorem 9.14

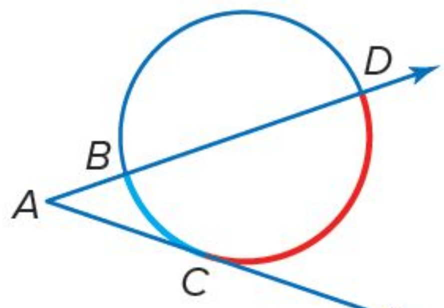
**Words** If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs.

**Examples**



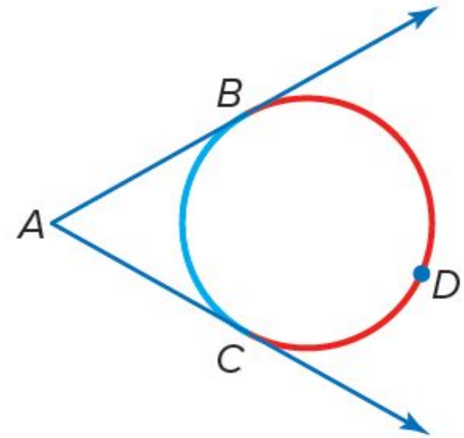

**Two Secants**

$m\angle A = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$




**Secant-Tangent**

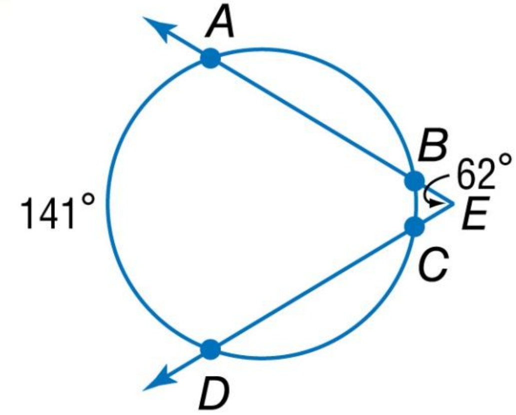
$m\angle A = \frac{1}{2}(m\widehat{DC} - m\widehat{BC})$




**Two Tangents**

$m\angle A = \frac{1}{2}(m\widehat{BDC} - m\widehat{BC})$

## Example 3 Use Tangents and Secants That Intersect Outside a Circle



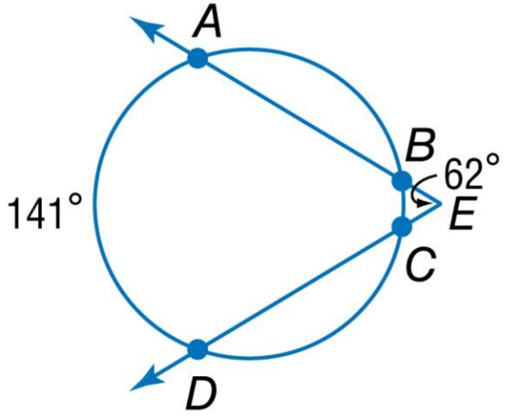

**A. Find $m\overset{\frown}{BC}$.**

$$m\angle AED = \frac{1}{2}\left(m\overset{\frown}{AD} - m\overset{\frown}{BC}\right)$$ Theorem 10.14

$$62 = \frac{1}{2}\left(141 - m\overset{\frown}{BC}\right)$$ Substitution

$$124 = \left(141 - m\overset{\frown}{BC}\right)$$ Multiply each side by 2.

## Example 3 Use Tangents and Secants That Intersect Outside a Circle

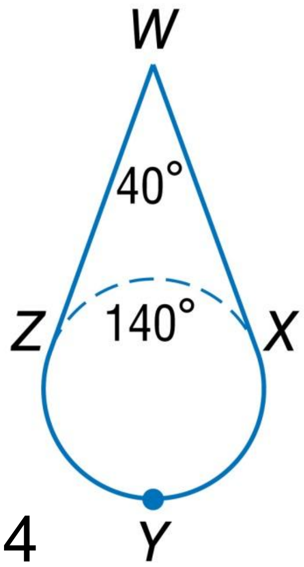



$-17 = -m\overset{\frown}{BC}$ Subtract 141 from each side.

$17 = m\overset{\frown}{BC}$ Multiply each side by –1.

**Answer:** $m\overset{\frown}{BC} = 17$

## Example 3 Use Tangents and Secants That Intersect Outside a Circle



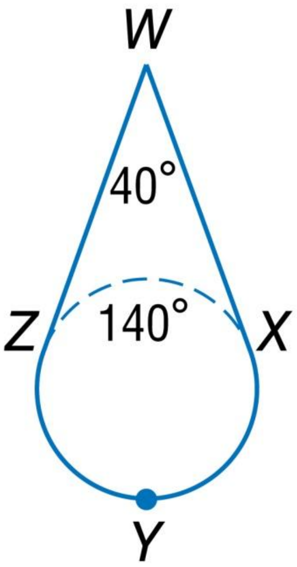

**B. Find $m\overset{\frown}{XYZ}$.**

$$m\angle W = \frac{1}{2}\left(m\overset{\frown}{XYZ} - m\overset{\frown}{ZX}\right)$$ Theorem 10.14

$$40 = \frac{1}{2}\left(m\overset{\frown}{XYZ} - 140\right)$$ Substitution

$$80 = \left(m\overset{\frown}{XYZ} - 140\right)$$ Multiply each side by 2.

## Example 3 Use Tangents and Secants That Intersect Outside a Circle

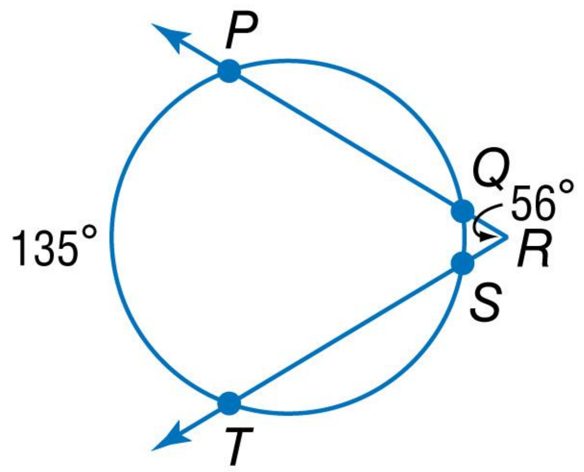



$220 = m\overparen{XYZ}$ Add 140 to each side.

**Answer :** $m\overparen{XYZ} = 220$

## Example 3 Guided Practice

**A. Find $m\overset{\frown}{QS}$.**




A. 23

B. 26

C. 29

D. 32

## Example 3

**Guided Practice**

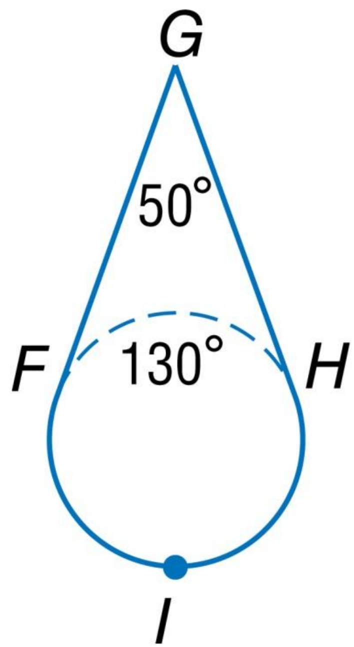
**B. Find $m\overset{\frown}{FIH}$.**

**A.** 194

**B.** 202

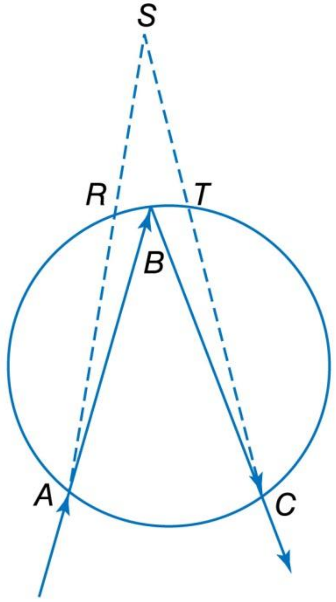
**C.** 210

**D.** 230



## Real-World Example 4 Apply Properties of Intersecting Secants

**PHYSICS** **The diagram shows the path of a light ray as it hits a cut diamond. The ray is bent, or *refracted*, at points $A$, $B$, and $C$. If $m\overset{\frown}{AC} = 96°$ and $m\angle S = 35°$, what is $m\overset{\frown}{RBT}$?**




$$m\angle S = \frac{1}{2}\left(m\overset{\frown}{AC} - m\overset{\frown}{RBT}\right)$$ Theorem 10.14

$$35 = \frac{1}{2}\left(96 - m\overset{\frown}{RBT}\right)$$ Substitution

## Real-World Example 4 Apply Properties of Intersecting Secants

$$70 = \left(96 - m\overset{\frown}{RBT}\right)$$ Multiply each side by 2.

$$-26 = -m\overset{\frown}{RBT}$$ Subtract 96 from each side.

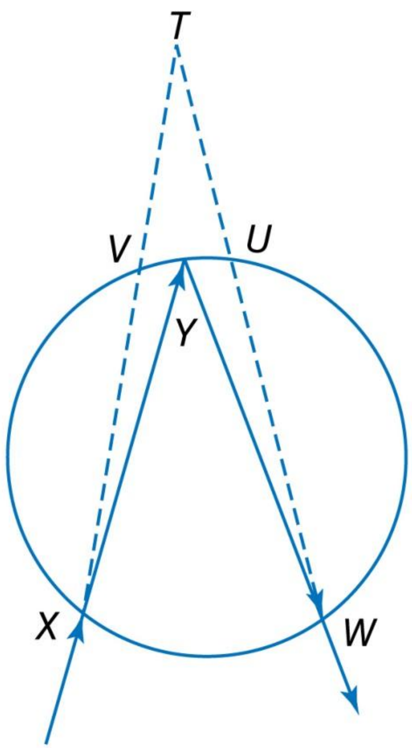
$$26 = m\overset{\frown}{RBT}$$ Multiply each side by –1.

**Answer :** $m\overset{\frown}{RBT} = 26$

## Real-World Example 4

**Guided Practice**

**PHYSICS** The diagram shows the path of a light ray as it hits a cut crystal. The ray is bent, or *refracted*, at points $X$, $Y$, and $W$. If $m\overset{\frown}{XW} = 100°$ and $m\angle T = 30°$, what is $m\overset{\frown}{VYU}$?



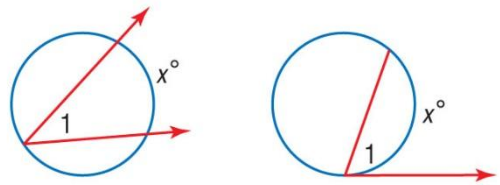
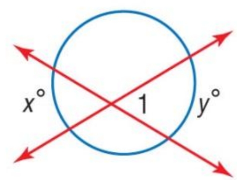
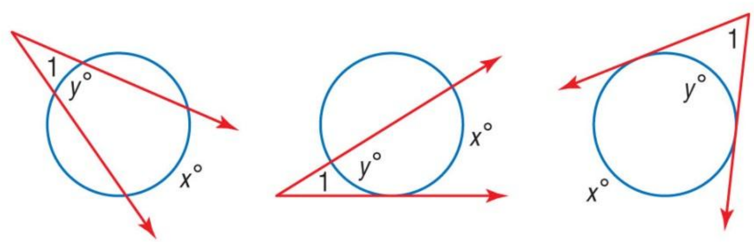

A. 25

B. 35

C. 40

D. 45

## KeyConcept Circle and Angle Relationships

| Vertex of Angle | Model(s) | Angle Measure |
|---|---|---|
| on the circle |   | one half the measure of the intercepted arc $m\angle 1 = \frac{1}{2}x$ |
| inside the circle |   | one half the measure of the sum of the intercepted arc $m\angle 1 = \frac{1}{2}(x + y)$ |
| outside the circle |   | one half the measure of the difference of the intercepted arcs $m\angle 1 = \frac{1}{2}(x - y)$ |