SLOPES & EQUATION OF A LINE

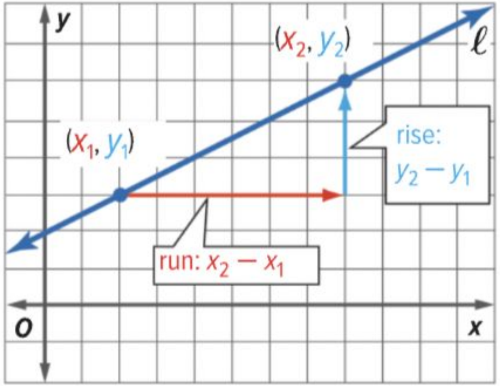
By Ms. Gabbie Dayrit

Key Concept Slope of a Line

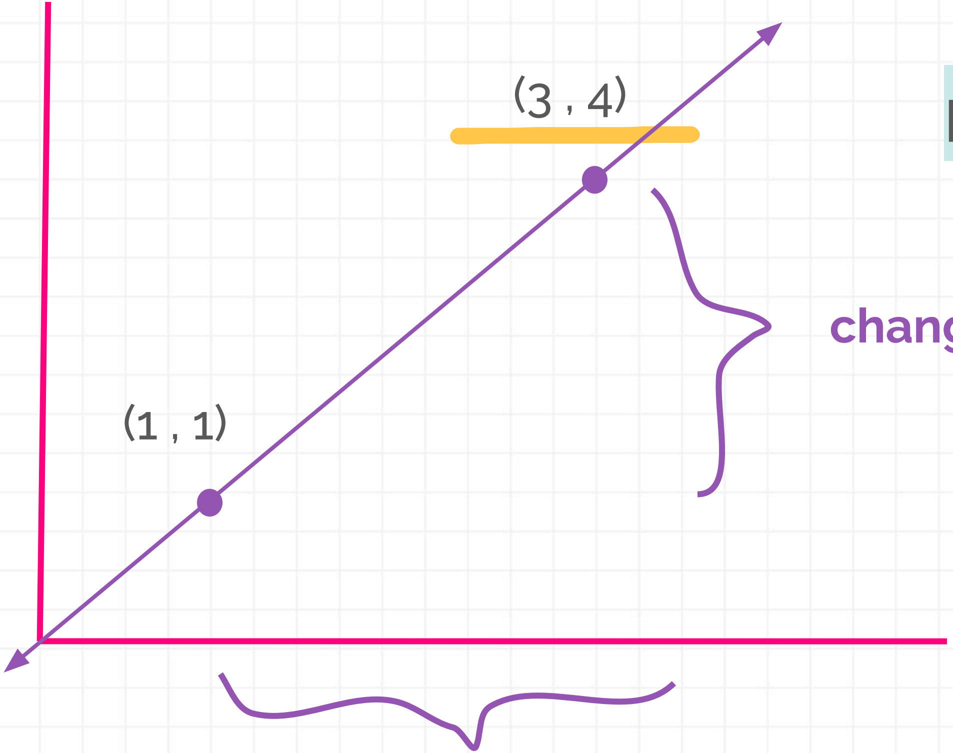
In a coordinate plane, the **slope** of a line is the ratio of the change along the *y*-axis to the change along the *x*-axis between any two points on the line.

The slope m of a line containing two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2.$$



$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$



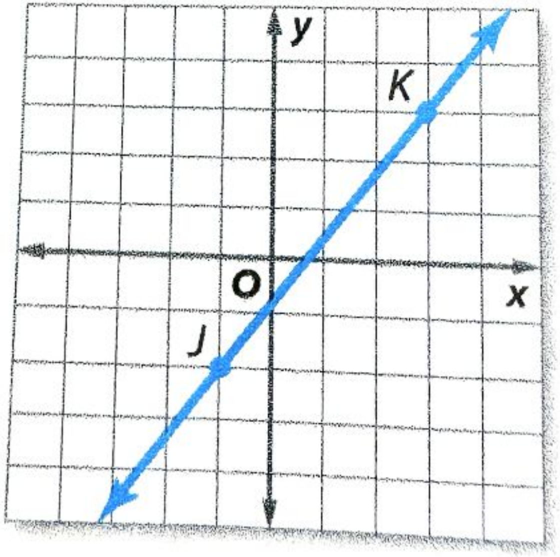
m = 3 / 2

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

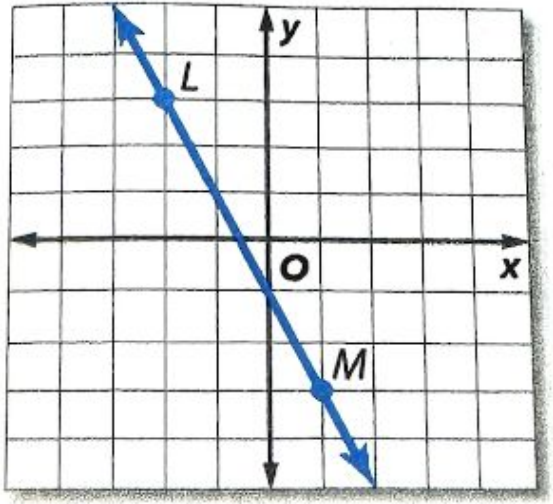
Let's do it!

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

a.



b.

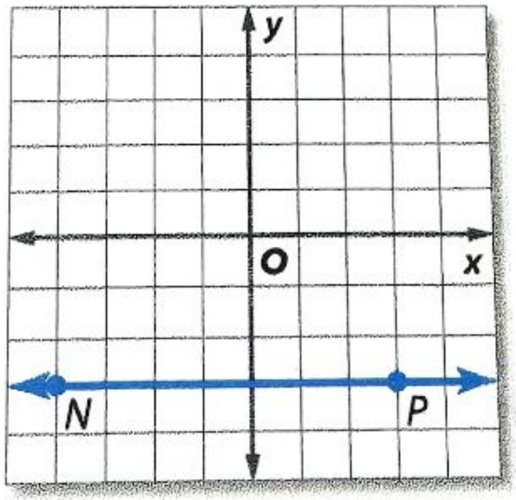


Let's do it!

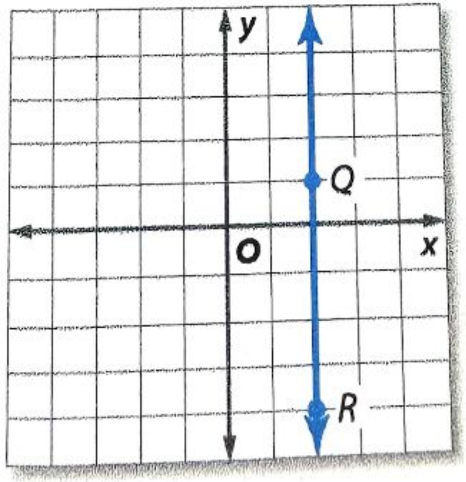
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

All vertical lines have undefined slopes!

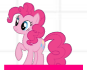
c.



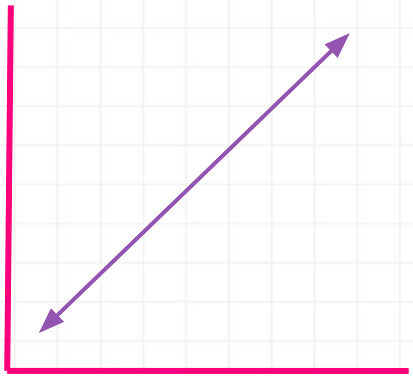
d.



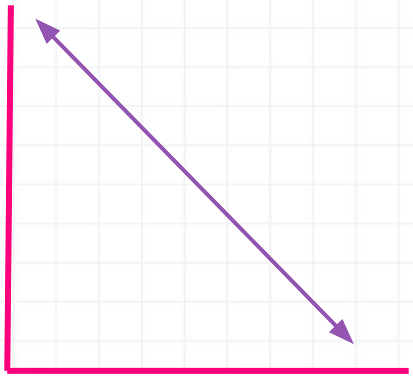
Why? Anything divided by 0 is undefined!



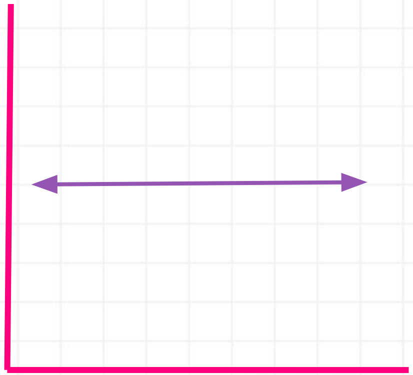
Types of slopes:



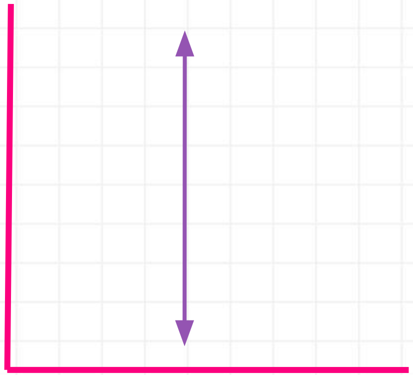
Positive



Negative



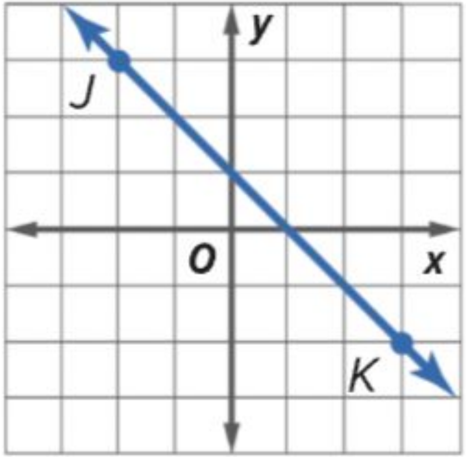
Zero



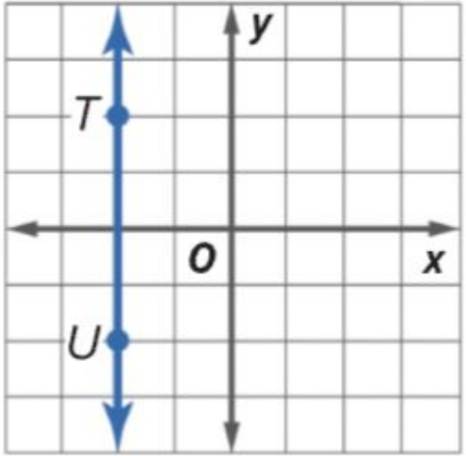
Undefined

Find the slope of each line.

1.

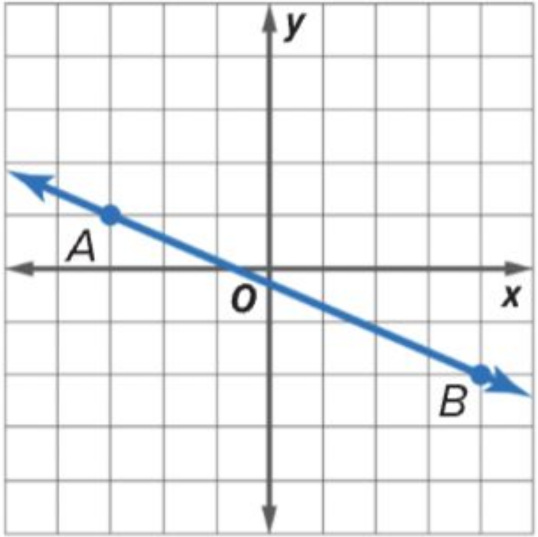


2.

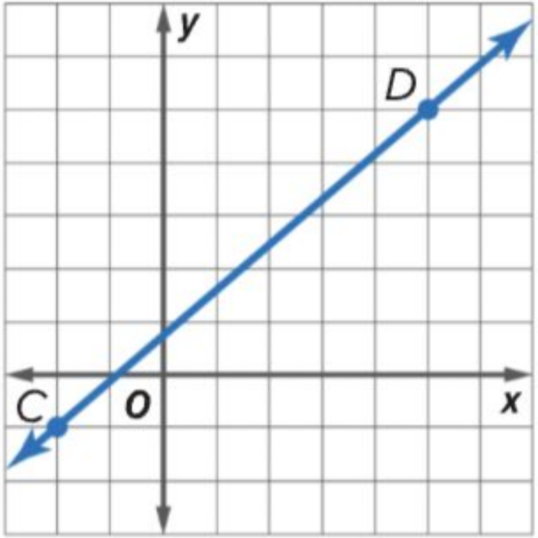


Find the slope of each line.

13



14.



Key Concept Nonvertical Line Equations

The **slope-intercept form** of a linear equation is $y = mx + b$, where m is the slope of the line and b is the y-intercept.

slope

$$y = mx + b \qquad y = 3x + 8$$

y-intercept

The **point-slope form** of a linear equation is $y - y_1 = m(x - x_1)$, where (x_1, y_1) is any point on the line and m is the slope of the line.

point of line (3, 5)

$$y - 5 = -2(x - 3)$$

slope

All lines are read via slope-intercept form!

$$y = mx + b$$

slope

y-intercept (where the line intersects the y-axis)

← →

Use the point-slope formula to transform the equation to slope-intercept form!

make

$$y - y_1 = m(x - x_1)$$

look like:

$$y = mx + b$$

through algebra!

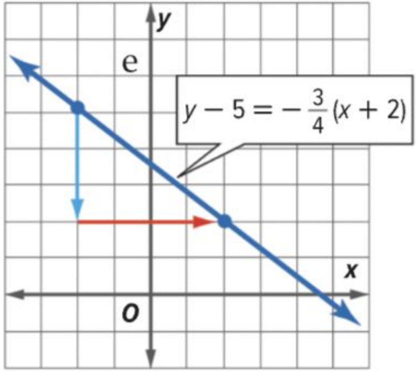
← →

Write an equation in point-slope form of the line with slope $-\frac{3}{4}$ that contains $(-2, 5)$. Then graph the line.

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -\frac{3}{4}[x - (-2)]$$

$$y - 5 = -\frac{3}{4}(x + 2)$$



Write a slope-intercept equation of the following:

4. $m = 5, (3, -2)$

5. $m = \frac{1}{4}, (-2, -3)$

6. $m = -4.25, (-4, 6)$

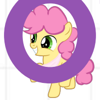
Write a slope-intercept equation of the following:

27 $m = 2, (3, 11)$

28. $m = -\frac{4}{5}, (-3, -6)$

29. $m = -2.4, (14, -12)$

KAHOOT!



ADDITIONAL PRACTICE

MB: Page 182 - 24, 25, 26, 36

EB: Page 183 - 51

Parallel & Perpendicular Lines

By Ms. Gabbie Dayrit

Theorems Parallel and Perpendicular Lines

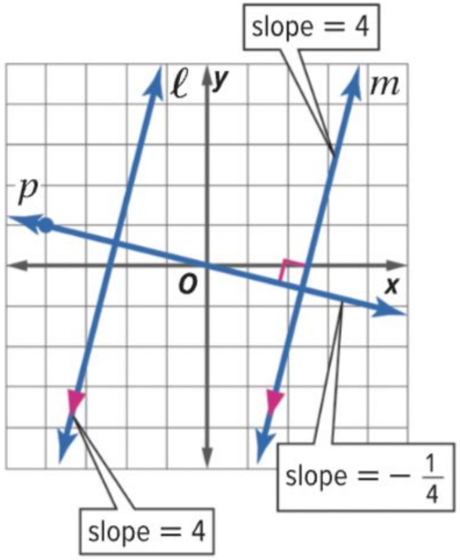
2.18 Slopes of Parallel Lines Two distinct nonvertical lines have the same slope if and only if they are parallel. All vertical lines are parallel.

Example Parallel lines ℓ and m have the same slope, 4.

2.19 Slopes of Perpendicular Lines Two nonvertical lines are perpendicular if and only if the product of their slopes is −1. Vertical and horizontal lines are perpendicular.

Example line $m \perp$ line p

product of slopes $= 4 \cdot -\frac{1}{4}$ or -1



Think of it this way...

PARALLEL = SAME SLOPE

4 = 4

PERPENDICULAR = NEGATIVE RECIPROCAL!

4 & -1/4

Think of it this way...

When two lines are perpendicular, their intersection forms a 90-degree angle!

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel, perpendicular,* or *neither* for $A(1, 1)$, $B(-1, -5)$, $C(3, 2)$, and $D(6, 1)$. Graph each line to verify your answer.

Step 1 Find the slope of each line.

slope of $\overleftrightarrow{AB} = \frac{-5-1}{-1-1} = \frac{-6}{-2}$ or 3

slope of $\overleftrightarrow{CD} = \frac{1-2}{6-3}$ or $\frac{-1}{3}$

Step 2 Determine the relationship, if any, between the lines.

The two lines do not have the same slope, so they are *not* parallel. To determine if the lines are perpendicular, find the product of their slopes.

$3\left(-\frac{1}{3}\right) = -1$ Product of slopes for $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$

Because the product of their slopes is -1, $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{CD}$.

Check your understanding!

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel, perpendicular,* or *neither.* Graph each line to verify your answer.

3A. $A(14, 13)$, $B(-11, 0)$, $C(-3, 7)$, $D(-4, -5)$

3B. $A(3, 6)$, $B(-9, 2)$, $C(5, 4)$, $D(2, 3)$

7. $W(2, 4)$, $X(4, 5)$, $Y(4, 1)$, $Z(8, -7)$

8. $W(1, 3)$, $X(-2, -5)$, $Y(-6, -2)$, $Z(8, 3)$

9. $W(-7, 6)$, $X(-6, 9)$, $Y(6, 3)$, $Z(3, -6)$

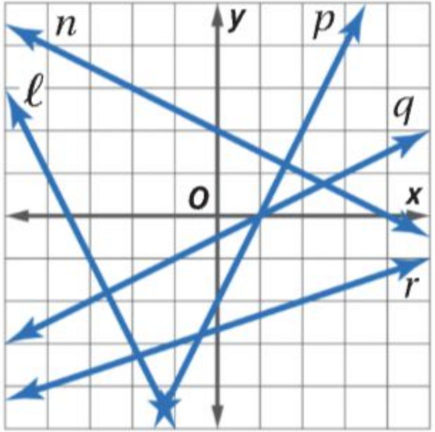
10. $W(1, -3)$, $X(0, 2)$, $Y(-2, 0)$, $Z(8, 2)$

Check your understanding!

Name the line(s) on the graph shown that match each description.

37. parallel to $y = 2x - 3$

38. perpendicular to $y = \frac{1}{2}x + 7$



Check your understanding!

Determine whether the lines are *parallel*, *perpendicular*, or *neither*.

40. $y = 2x + 4$, $y = 2x - 10$

41. $y = -\frac{1}{2}x - 12$, $y = 2x + 7$

←   →

Word Problems

11. **QUILTS** Tamiko uses a coordinate plane to help her cut out patches for a quilt. Her first cut is along the line $y = -2x + 6$. Her second cut will be perpendicular to the first and will pass through $(3, 2)$. Write an equation in slope-intercept form for the second cut.

Challenge!

45 Write an equation for a line containing $(-8, 12)$ that is perpendicular to the line containing the points $(3, 2)$ and $(-7, 2)$.

Challenge!

49. Without graphing, what type of geometric shape is enclosed by the lines $x - 2y = -6$, $2x + y = 13$, $x - 2y = 4$, and $2x + y = -2$? Explain.

Challenge!

CHALLENGE Find the value of n so that the line perpendicular to the line with the equation $-2y + 4 = 6x + 8$ passes through the points at $(n, -4)$ and $(2, -8)$.

Challenge!

54. MP **CRITIQUE ARGUMENTS** Mark and Josefina wrote an equation of a line with slope -5 that passes through the point $(-2, 4)$. Is either of them correct? Explain your reasoning.

Mark

$$y - 4 = -5(x - (-2))$$
$$y - 4 = -5(x + 2)$$
$$y - 4 = -5x - 10$$
$$y = -5x - 6$$

Josefina

$$y - 4 = -5(x - (-2))$$
$$y - 4 = -5(x + 2)$$

ADDITIONAL PRACTICE

MB:

Page 182 - 30-35, 40-43

Page 183 - 44, 46

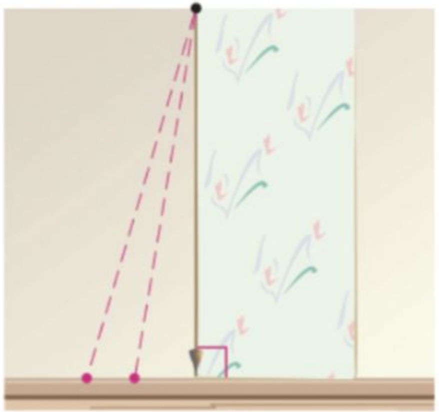
EB:
Page 183 - 53

Parallel/Perpendicular Lines & Distance

By Ms. Gabbie Dayrit

1 Distance from a Point to a Line

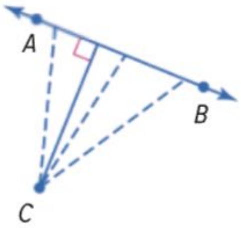
The plumb bob also indicates the shortest distance between the point at which it is attached on the ceiling and a level floor below. This perpendicular distance between a point and a line is the shortest in all cases.



Key Concept Distance Between a Point and a Line

Words The distance between a line and a point not on the line is the length of the segment perpendicular to the line from the point.

Model



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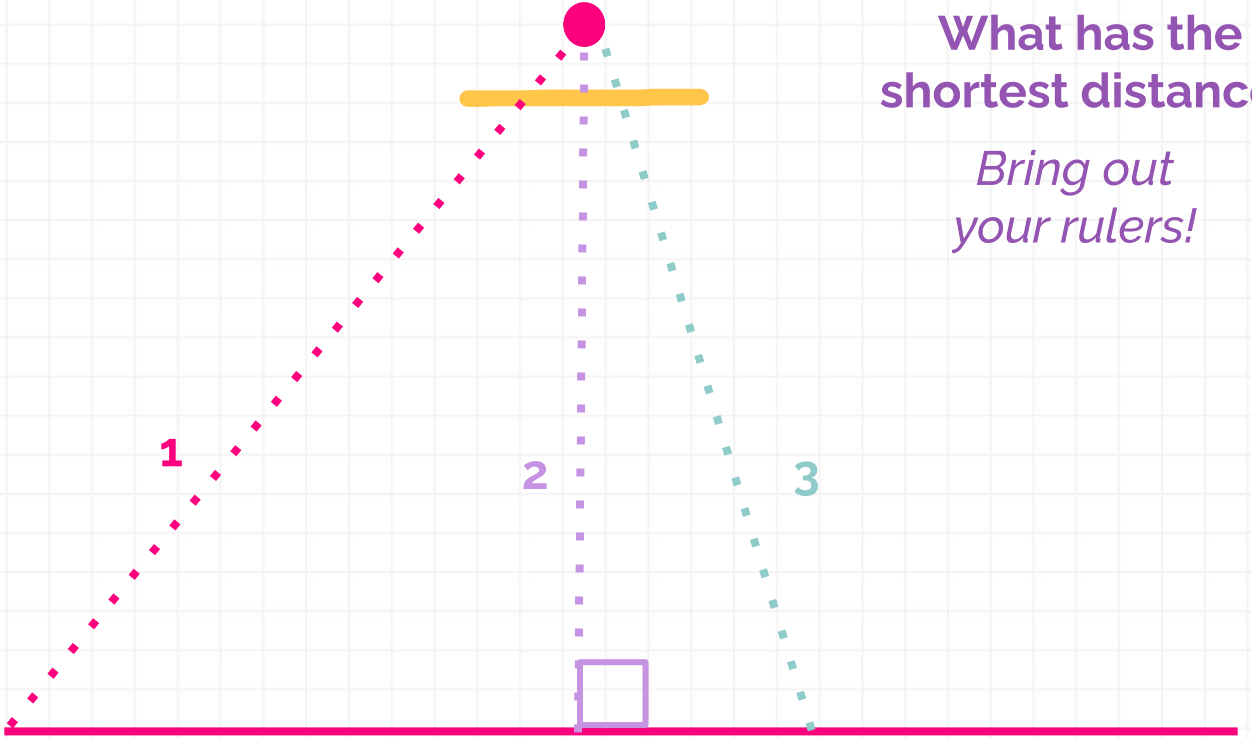
What has the shortest distance?

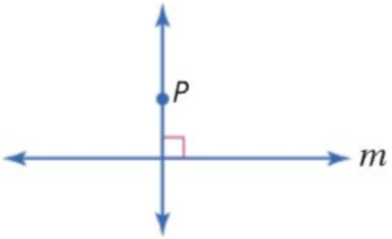
Bring out your rulers!



Postulate 2.15 Perpendicular Postulate

Words If given a line and a point not on the line, then there exists exactly one line through the point that is perpendicular to the given line.

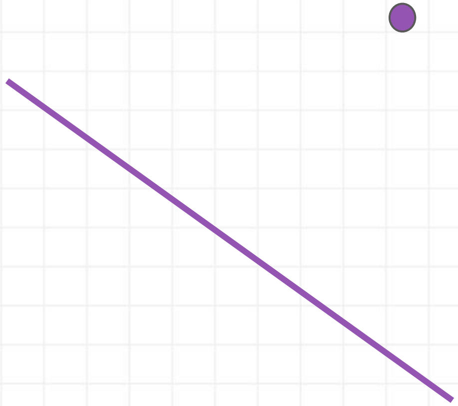
Model



MEANING! There can *only be one line* that connects the point P to line m, and be perpendicular to it.

To find the distance between a line and a point not on the line...

1. Get the equation of the line in slope-intercept form
2. Write an equation for the line passing through the point, perpendicular to the original line
3. Find the coordinates of intersection
4. Apply the distance formula between the point of interaction and point not on the line



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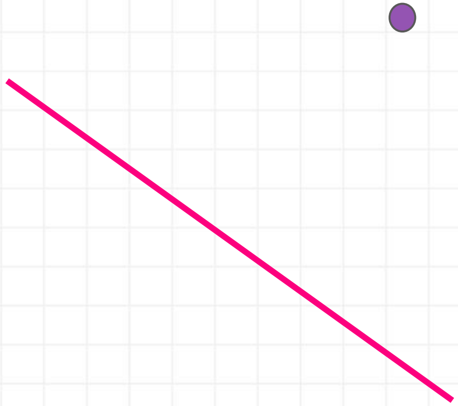
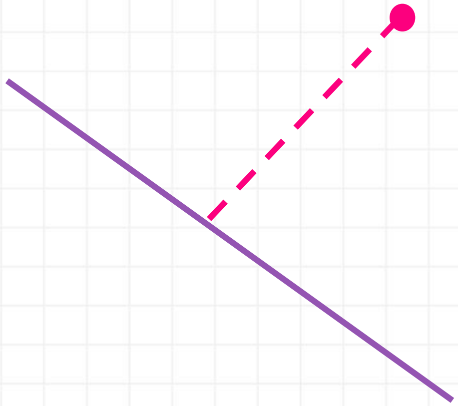


To find the distance between a line and a point not on the line…

1. Get the equation of the line in slope-intercept form
2. **Write an equation for the line passing through the point, perpendicular to the original line**
3. Find the coordinates of intersection
4. Apply the distance formula between the point of interaction and point not on the line



To find the distance between a line and a point not on the line...

1. Get the equation of the line in slope-intercept form
2. Write an equation for the line passing through the point, perpendicular to the original line
3. **Find the coordinates of intersection**
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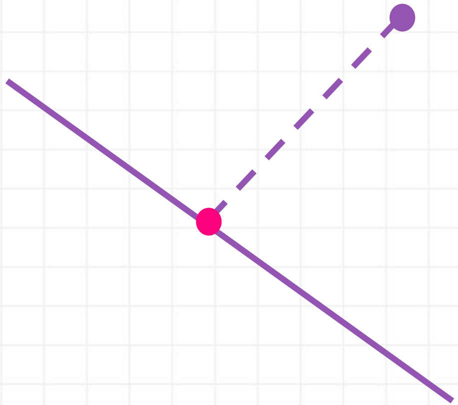
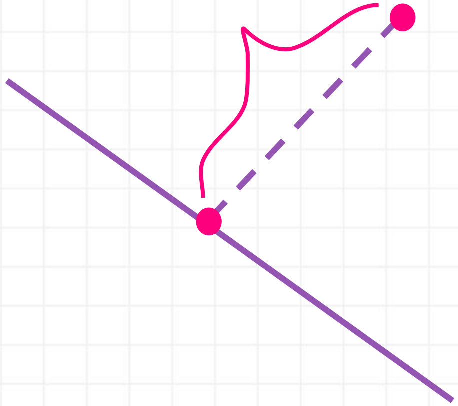
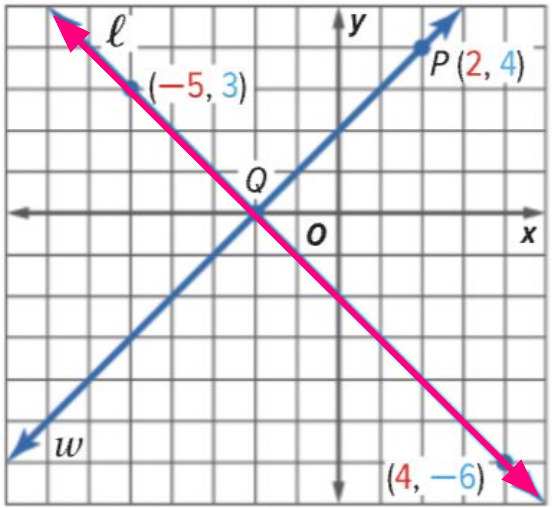


To find the distance between a line and a point not on the line...

1. Get the equation of the line in slope-intercept form
2. Write an equation for the line passing through the point, perpendicular to the original line
3. Find the coordinates of intersection
4. **Apply the distance formula between the point of interaction and point not on the line**



COORDINATE GEOMETRY Line ℓ contains points at $(-5, 3)$ and $(4, -6)$. Find the distance between line ℓ and point $P(2, 4)$.



y = -x -2

STEP 1: Get the equation of the line in slope-intercept form

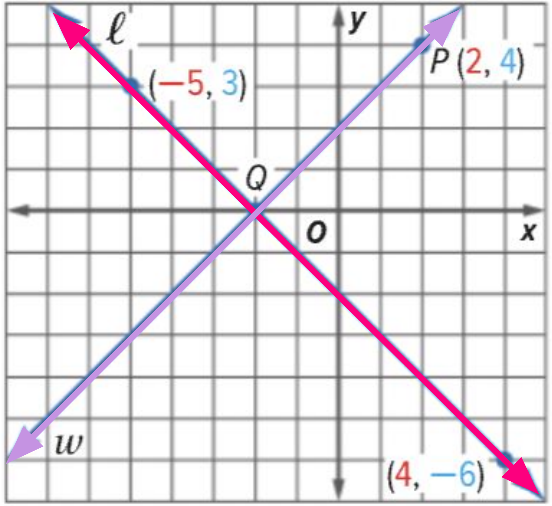
$m = [-6 - 3] / [4 - (-5)] = -1$

$y - 3 = -1\,[x - (-5)]$

$y - 3 = -1\,(x + 5)$

$y = -x - 2$

COORDINATE GEOMETRY Line ℓ contains points at $(-5, 3)$ and $(4, -6)$. Find the distance between line ℓ and point $P(2, 4)$.



$y = x + 2$

$y = -x - 2$

STEP 2: Write an equation for the line passing through the point, perpendicular to the original line

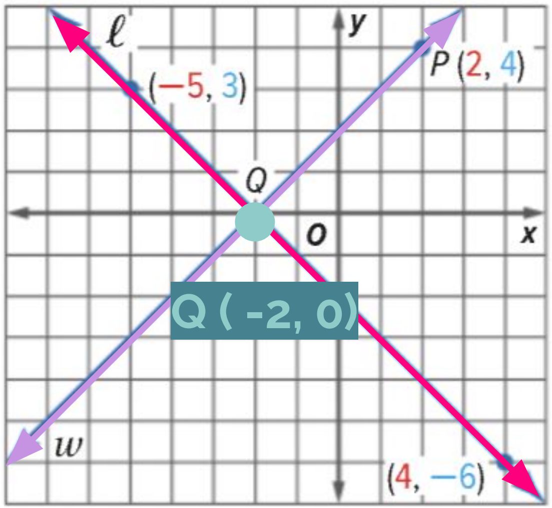
m = 1, since perpendicular ; P (2,4)

$y - 4 = 1(x-2)$

$y - 4 = x - 2$

$y = x + 2$

COORDINATE GEOMETRY Line ℓ contains points at $(-5, 3)$ and $(4, -6)$. Find the distance between line ℓ and point $P(2, 4)$.



y = x + 2

y = -x -2

STEP 3: Get the intersection between the lines (point Q!)

We use systems here because *we want the point to satisfy both lines!*

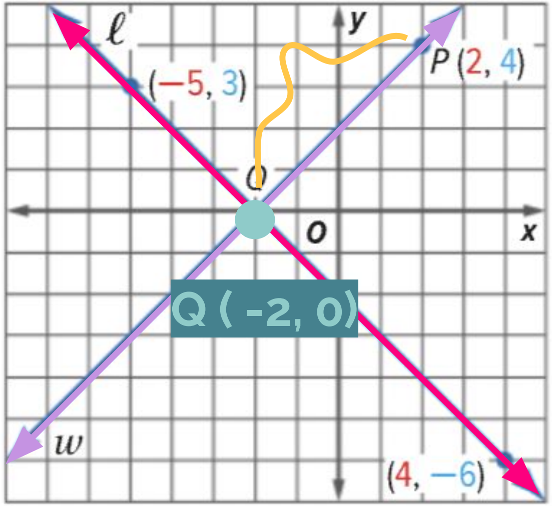
$$\begin{array}{r} y = -x - 2 \\ + \quad y = x + 2 \\ \hline y = 0 \end{array}$$

$0 = x + 2$

$x = -2$

Q (-2, 0)

COORDINATE GEOMETRY Line ℓ contains points at $(-5, 3)$ and $(4, -6)$. Find the distance between line ℓ and point $P(2, 4)$.



y = x + 2

y = -x -2

STEP 4: Get the distance between points Q & P using the distance formula

Step 4 Use the Distance Formula to determine the distance between $P(2, 4)$ and $Q(-2, 0)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance formula}$$

$$= \sqrt{(-2 - 2)^2 + (0 - 4)^2} \quad x_2 = -2, x_1 = 2, y_2 = 0, y_1 = 4$$

$$= \sqrt{32} \quad \text{Simplify.}$$

The distance between the point and the line is $\boxed{\sqrt{32} \text{ or about } 5.66 \text{ units.}}$

←

→

Your Turn!

Construct a line perpendicular to line l, passing through point P. Then, find the distance between point p and line l.

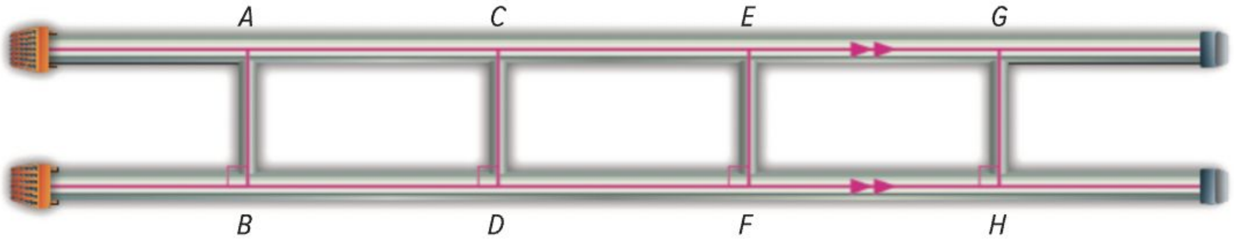
4. Line ℓ contains points $(4, 3)$ and $(-2, 0)$. Point P has coordinates $(3, 10)$.

5. Line ℓ contains points $(-6, 1)$ and $(9, -4)$. Point P has coordinates $(4, 1)$.

6. Line ℓ contains points $(4, 18)$ and $(-2, 9)$. Point P has coordinates $(-9, 5)$.

2 Distance Between Parallel Lines

By definition, parallel lines do not intersect. An alternate definition states that two lines in a plane are parallel if they are everywhere **equidistant**. Equidistant means that the distance between two lines measured along a perpendicular line to the lines is always the same.



$AB = CD = EF = GH$

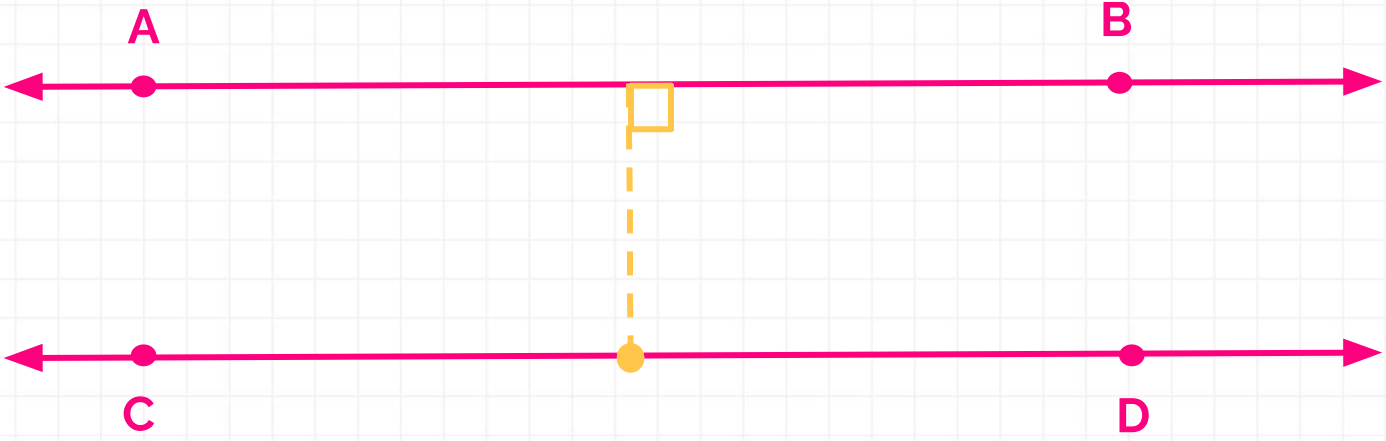
This leads to the definition of the distance between two parallel lines.

Key Concept Distance Between Parallel Lines

The distance between two parallel lines is the perpendicular distance between one of the lines and any point on the other line.

Meaning, at any point between the two parallel lines, the distance is exactly the same!

Meaning, at any point between the two parallel lines, the distance is exactly the same!



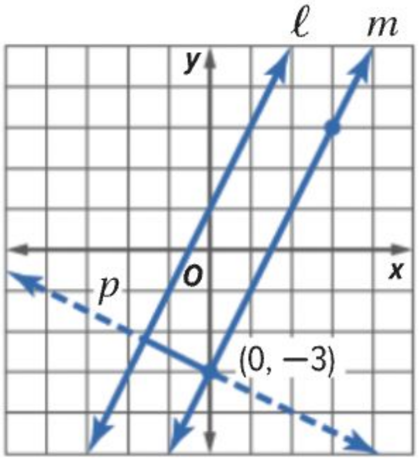
We treat this as getting the perpendicular distance between a line and a point Q not on the line!

Example 3 Distance Between Parallel Lines

Find the distance between the parallel lines ℓ and m with equations $y = 2x + 1$ and $y = 2x - 3$, respectively.

You will need to solve a system of equations to find the endpoints of a segment that is perpendicular to both ℓ and m. From their equations, we know that the slope of line ℓ and line m is 2.

Sketch line p through the y-intercept of line m, $(0, -3)$, perpendicular to lines m and ℓ.



Step 1 Write an equation of line p. The slope of p is the opposite reciprocal of 2, or $-\frac{1}{2}$. Use the y-intercept of line m, $(0, -3)$, as one of the endpoints of the perpendicular segment.

$(y - y_1) = m(x - x_1)$	Point-slope form
$[y - (-3)] = -\frac{1}{2}(x - 0)$	$x_1 = 0$, $y_1 = 3$, and $m = -\frac{1}{2}$
$y + 3 = -\frac{1}{2}x$	Simplify.
$y = -\frac{1}{2}x - 3$	Subtract 3 from each side.

Step 2 Use a system of equations to determine the point of intersection of lines ℓ and p.

ℓ: $y = 2x + 1$

p: $y = -\frac{1}{2}x - 3$

$2x + 1 = -\frac{1}{2}x - 3$	Substitute $2x + 1$ for y in the second equation.
$2x + \frac{1}{2}x = -3 - 1$	Group like terms on each side.
$\frac{5}{2}x = -4$	Simplify on each side.
$x = -\frac{8}{5}$	Multiply each side by $\frac{2}{5}$.
$y = -\frac{1}{2}\left(-\frac{8}{5}\right) - 3$	Substitute $-\frac{8}{5}$ for x in the equation for p.
$= -\frac{11}{5}$	Simplify.

The point of intersection is $\left(-\frac{8}{5}, -\frac{11}{5}\right)$ or $(-1.6, -2.2)$.

Step 3 Use the Distance Formula to determine the distance between $(0, -3)$ and $(-1.6, -2.2)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ Distance Formula

$$= \sqrt{(-1.6 - 0)^2 + [-2.2 - (-3)]^2}$$ $x_2 = -1.6$, $x_1 = 0$, $y_2 = -2.2$, and $y_1 = -3$

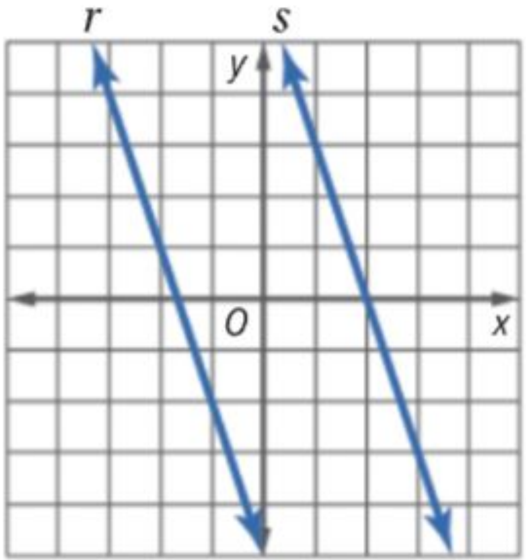
$$\approx 1.8$$ Simplify using a calculator.

The distance between the lines is about 1.8 units.

Your Turn!

3A. Find the distance between the parallel lines r and s whose equations are $y = -3x - 5$ and $y = -3x + 6$, respectively.

3B. Find the distance between parallel lines a and b with equations $x + 3y = 6$ and $x + 3y = -14$, respectively.



Your Turn!

Find the distance between each pair of parallel lines with the given equations.

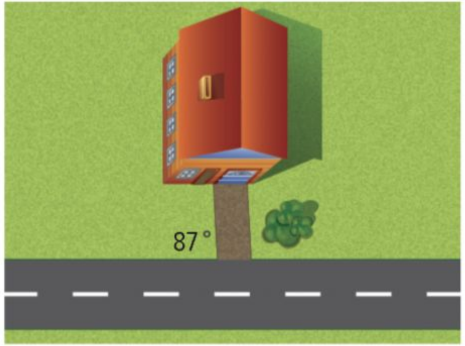
7 $y = -2x + 4$
$y = -2x + 14$

8. $y = 7$
$y = -3$

← →

Think, think, think!

13. **DRIVEWAYS** In the diagram at the right, is the driveway shown the shortest possible one from the house to the road? Explain why or why not.



Challenge!

43. CHALLENGE Suppose a line perpendicular to a pair of parallel lines intersects the lines at the points $(a, 4)$ and $(0, 6)$. If the distance between the parallel lines is $\sqrt{5}$, find the value of a and the equations of the parallel lines.

REVIEW: SAMPLE EB QUESTION

When John bought his new computer, he purchased an online computer help service. The help service has a yearly fee of $25.50 and a $10.50 charge for each help session a person uses.

If John can only spend $170 for the computer help this year, what is the maximum number of help sessions he can use this year?

REVIEW: SAMPLE EB QUESTION

Linda rented a cotton candy machine for $75.00 to use at a community fair. In addition to the machine rental cost, the cotton candy supplies cost her $0.15 per serving sold.

Linda sold the cotton candy for $3.00 per serving. How much profit did Linda make if she sold 150 servings of cotton candy? Box your final answer.