

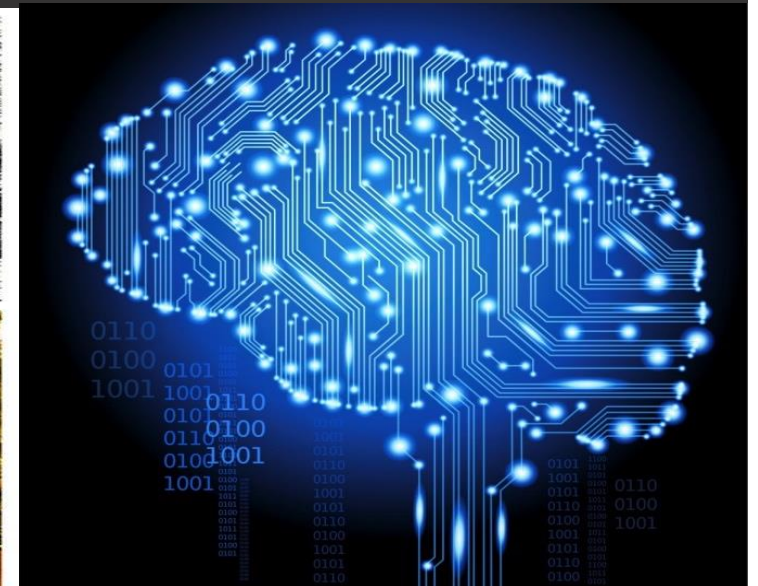
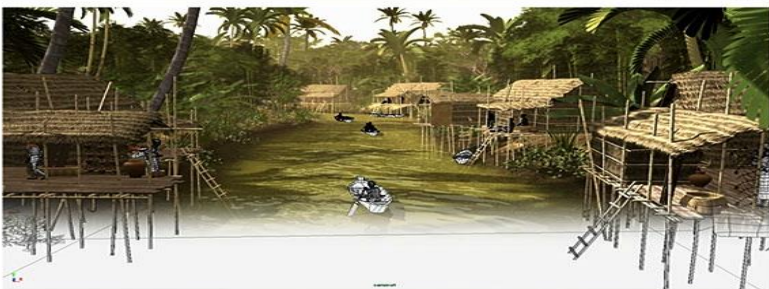
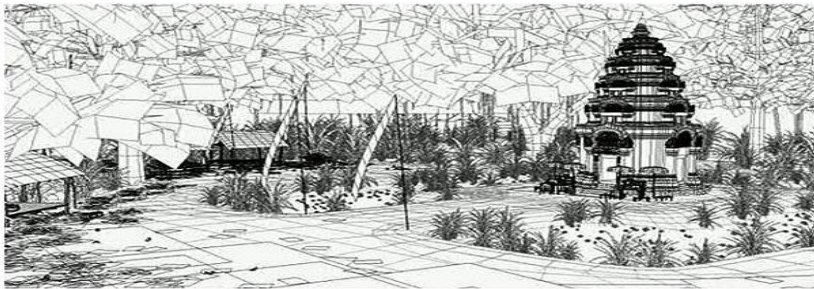


MONASH University

Information Technology

Linked Stacks & Queues

Prepared by Maria Garcia de la Banda
Updated by Brendon Taylor



Objectives for this lesson

- **To understand the use of linked data structures in implementing**
 - Stacks
 - Queues
- **To be able to:**
 - Implement, use and modify linked stacks and linked queues
 - Decide when it is appropriate to use them (rather than arrays)

Linked Stacks

```
from abc import ABC, abstractmethod
from typing import TypeVar, Generic
T = TypeVar('T')
```

```
class Stack(ABC, Generic[T]):
    def __init__(self) -> None:
        self.length = 0

    @abstractmethod
    def push(self, item: T) -> None:
        pass

    @abstractmethod
    def pop(self) -> None:
        pass

    @abstractmethod
    def peek(self) -> T:
        pass

    def __len__(self) -> int:
        return self.length

    def clear(self):
        self.length = 0
```

Remember:

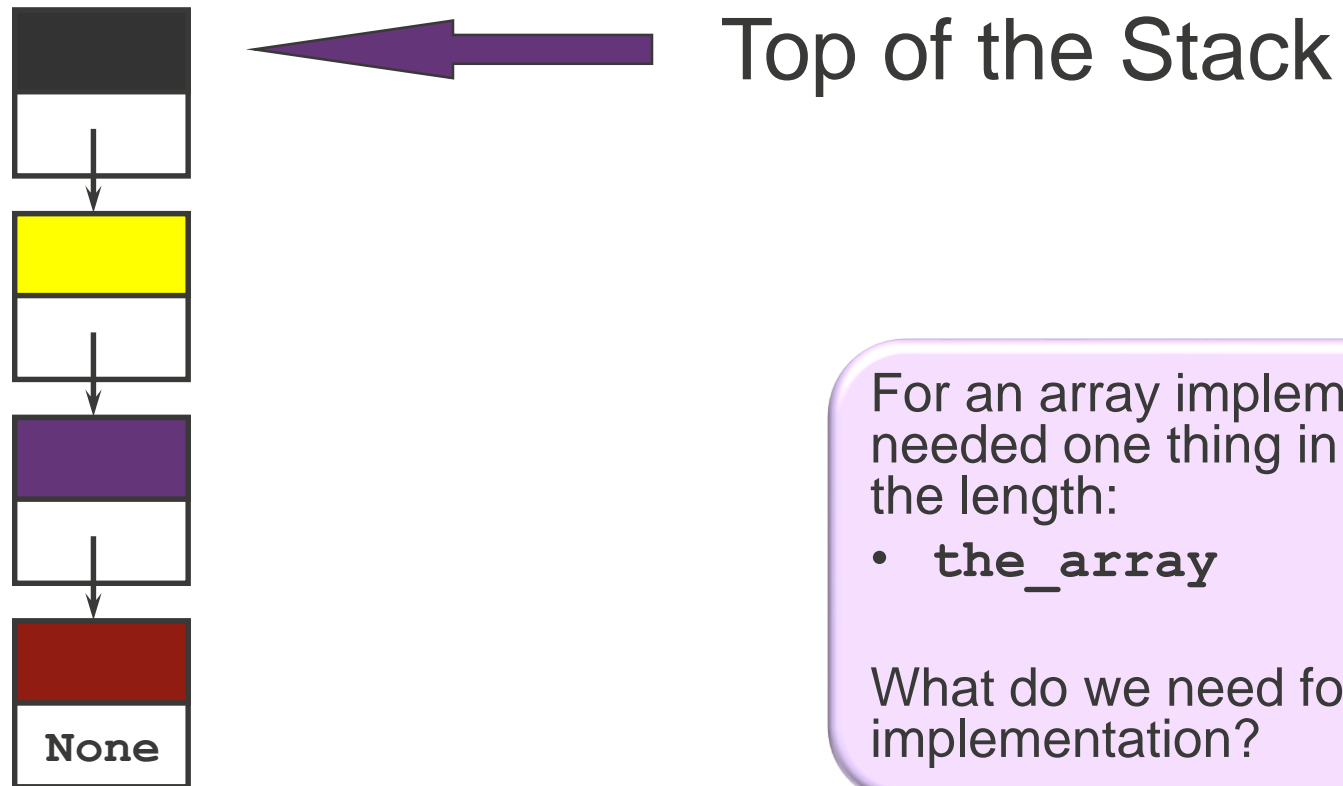
Abstract base Stack class

```
def is_empty(self) -> bool:
    return len(self) == 0
```

```
@abstractmethod
def is_full(self) -> bool:
    pass
```

```
@abstractmethod
def __toString__(self) -> str:
    pass
```

Linked Stack implementation



For an array implementation we needed one thing in addition to the length:

- `the_array`

What do we need for a linked implementation?

Nodes!

Class for a Linked Stack

```
from typing import TypeVar
from abstract_stack import Stack
from node import Node
T = TypeVar('T')
```

No need for **size** when initialising the object

```
class LinkStack(Stack[T]):
    def __init__(self):
        Stack.__init__(self)
        self.top = None
```

```
    def is_full(self):
        return False
```

Big O?

O(1)

```
    def clear(self):
        Stack.clear()
        self.top = None
```

Did not do that for LinkLists,
but it is good to free memory



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Push method for Linked Stacks

Push: algorithm

- In the **array** implementation:

- If the array is full: raise exception (or resize, if we wanted to do that)
- Else
 - Add the item in the position marked by top (was the same as the length of the list)
 - Increase top

- In a **linked data structure**:

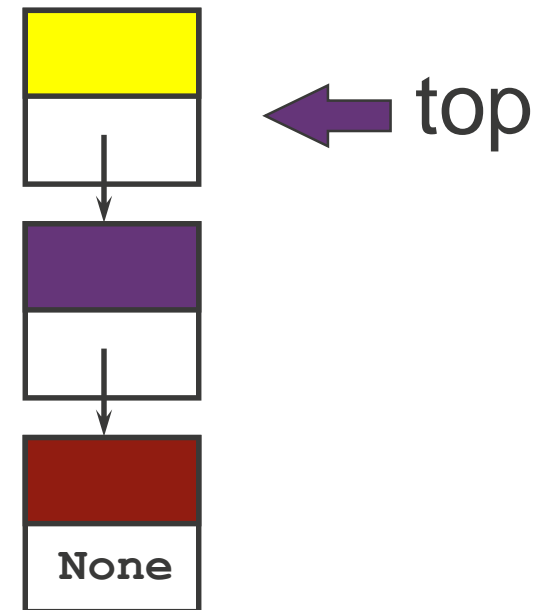
- Create a new node that contains the item
- We link it to the current top
- Make the new node the new top

- **No need for `is_full` check**

- **If no more memory can be allocated:**

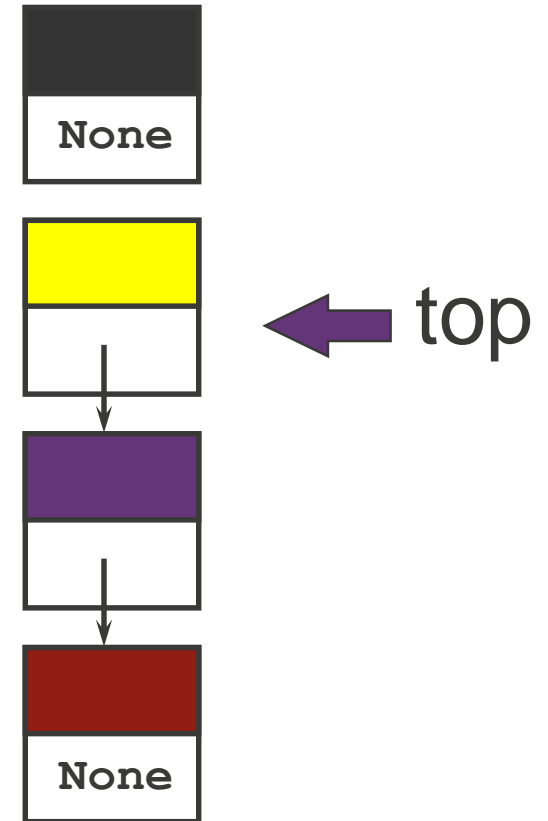
- The system will raise an **exception**

Push: algorithm

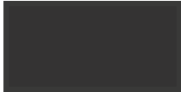


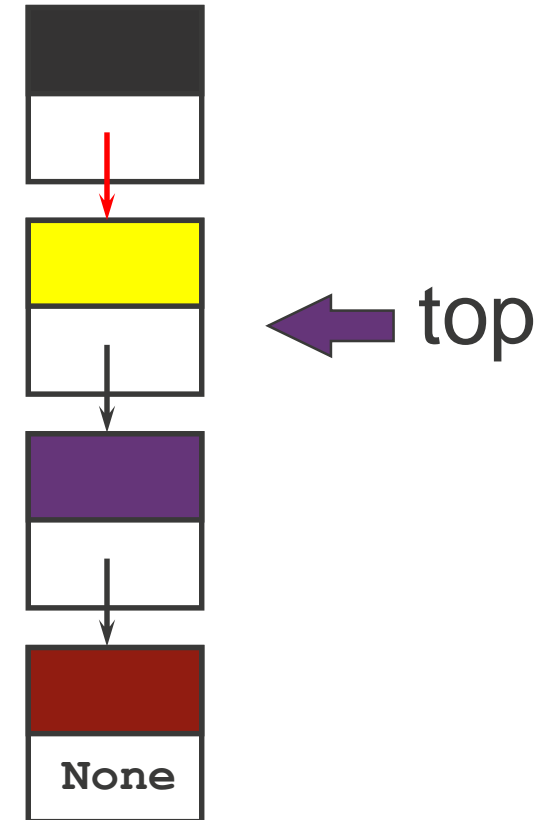
Push: algorithm

- Create a new node for item

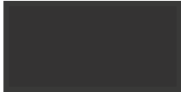


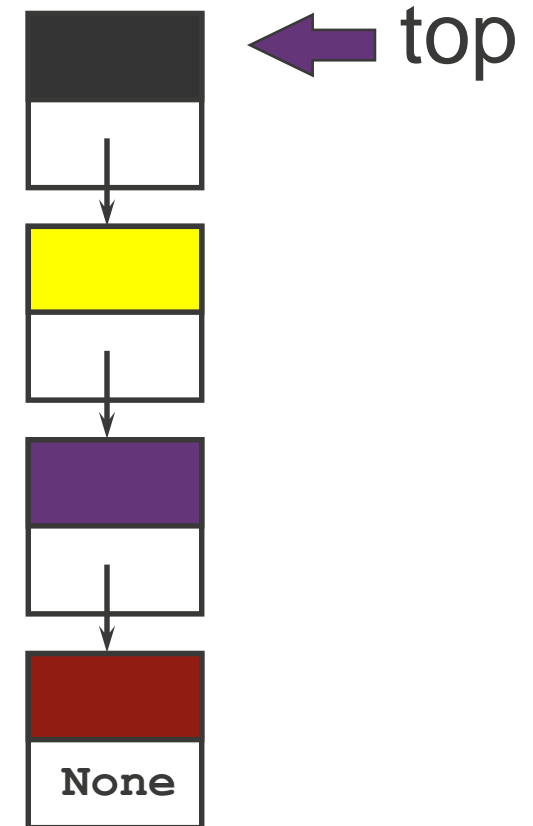
Push: algorithm

- Create a new node for item 
- Link it to the current top node



Push: algorithm

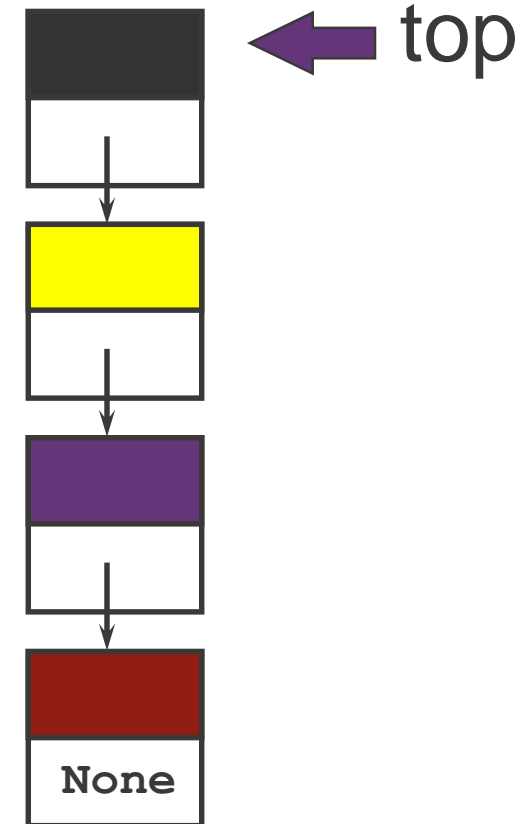
- Create a new node for item 
- Link it to the current top node
- **Make the new node the new top**



Push: algorithm

- Create a new node for item
- Link it to the current top node
- Make the new node the new top

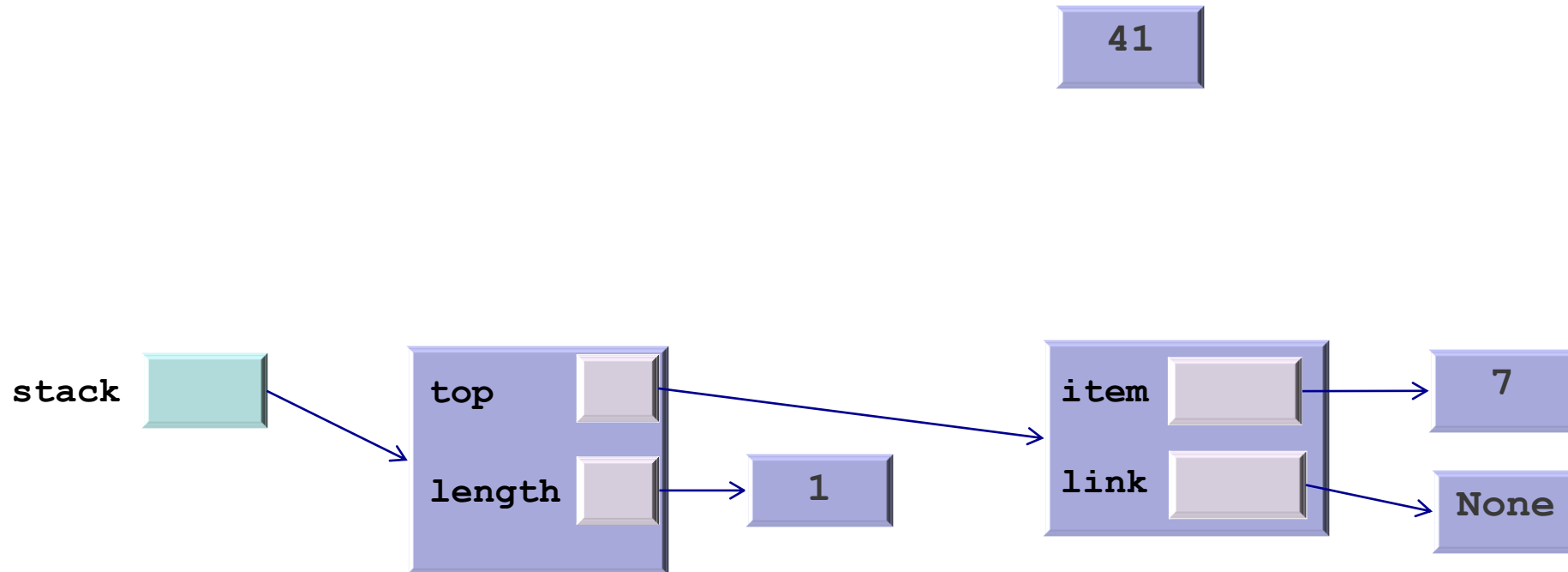
```
def push(self, item: T):  
    new_node = Node(item)  
    new_node.link = self.top  
    self.top = new_node  
    self.length += 1
```



Consider a **stack**
with a node whose
item is **7**

Lets see the memory
diagram for
stack.push(41)

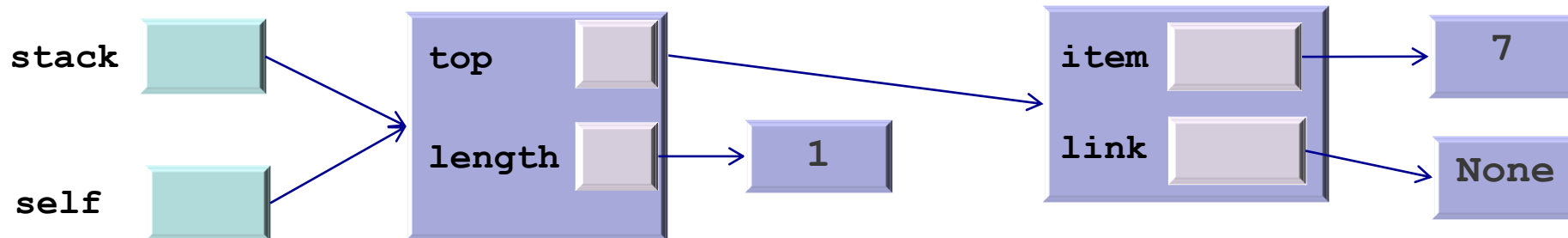
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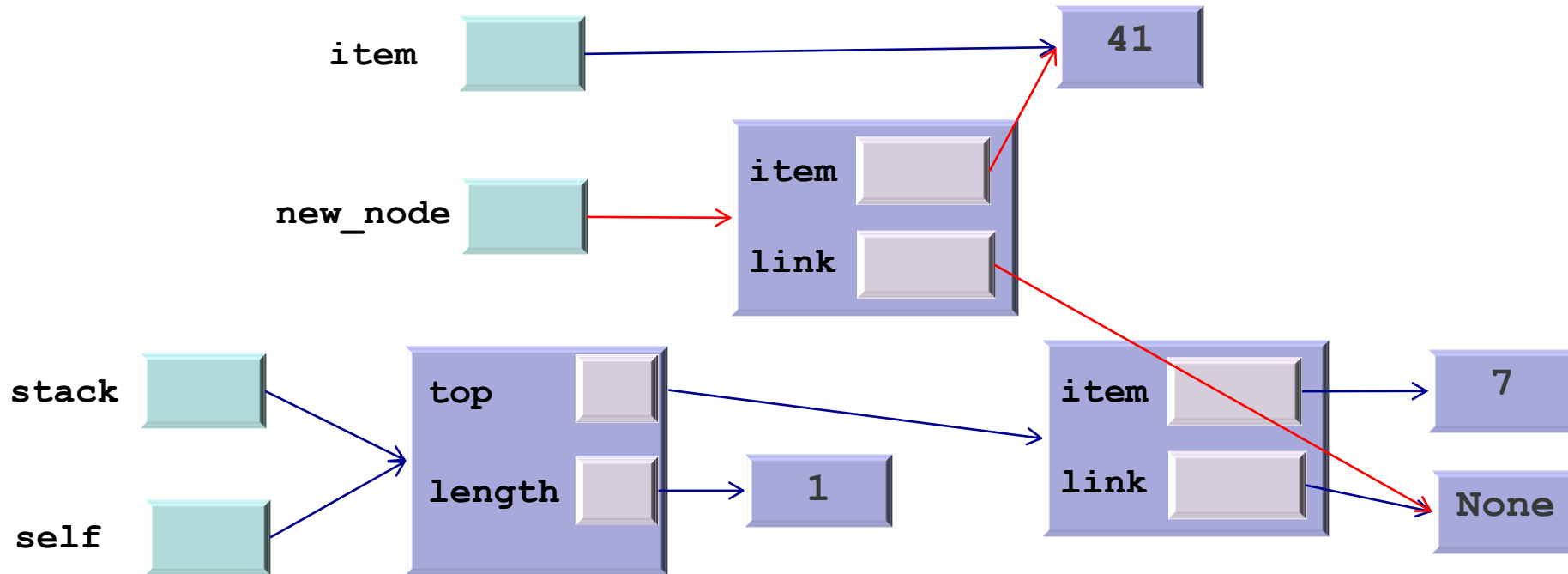
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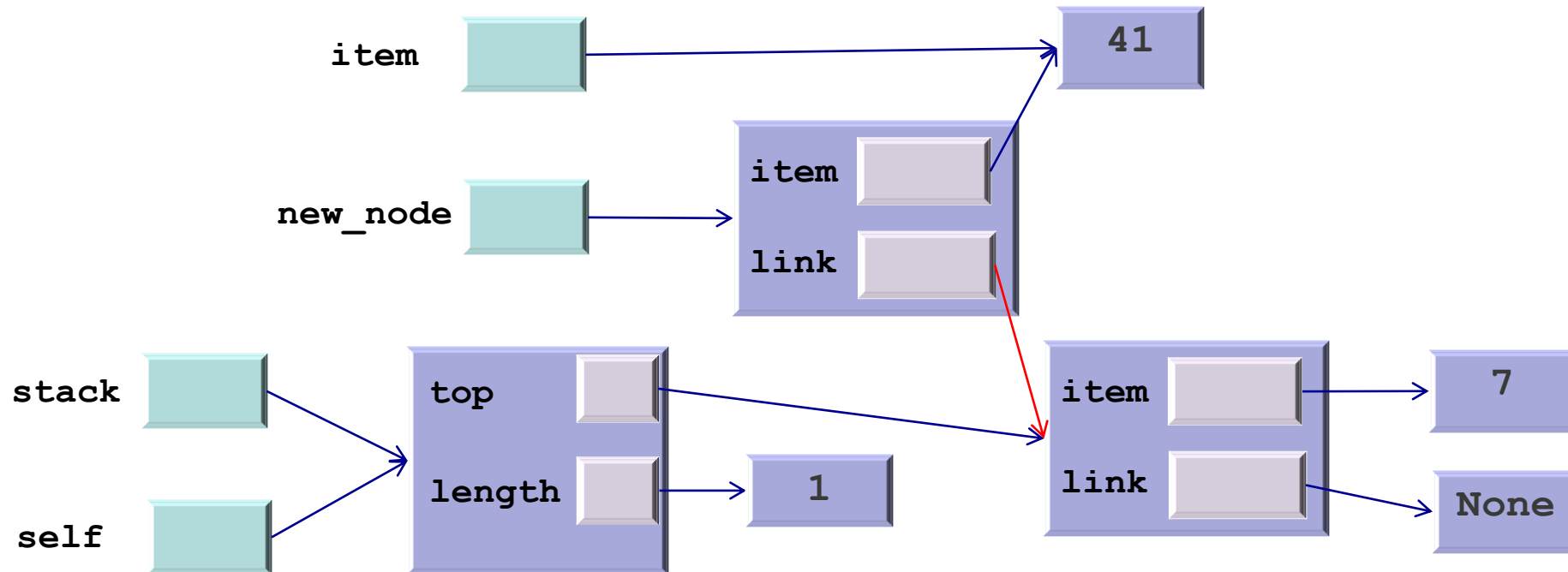
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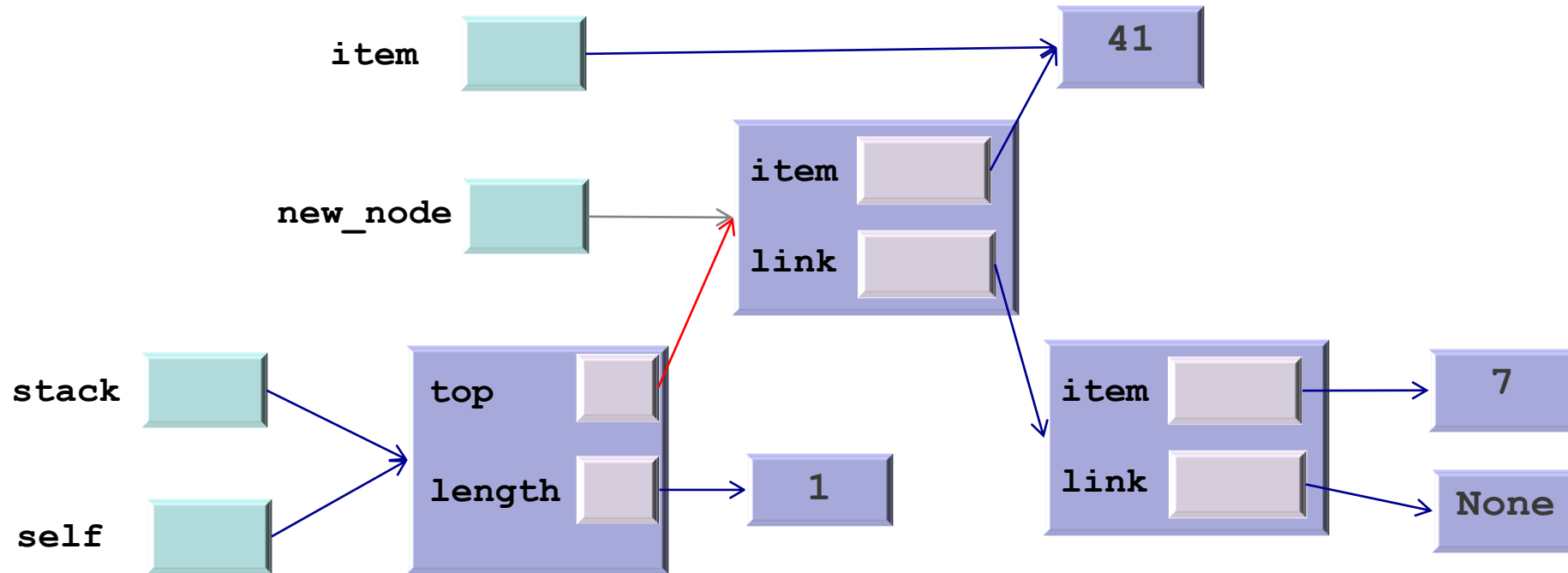
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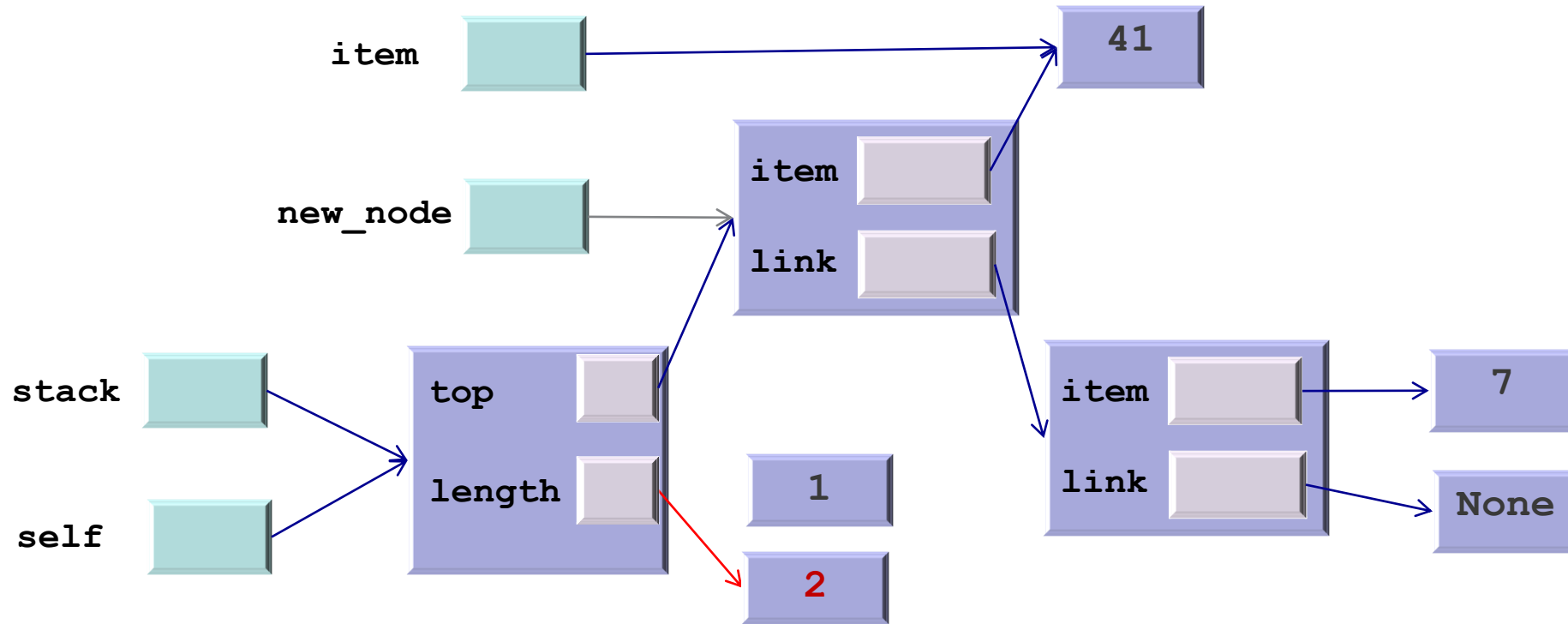
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Pop method for Linked Stacks

Pop algorithm

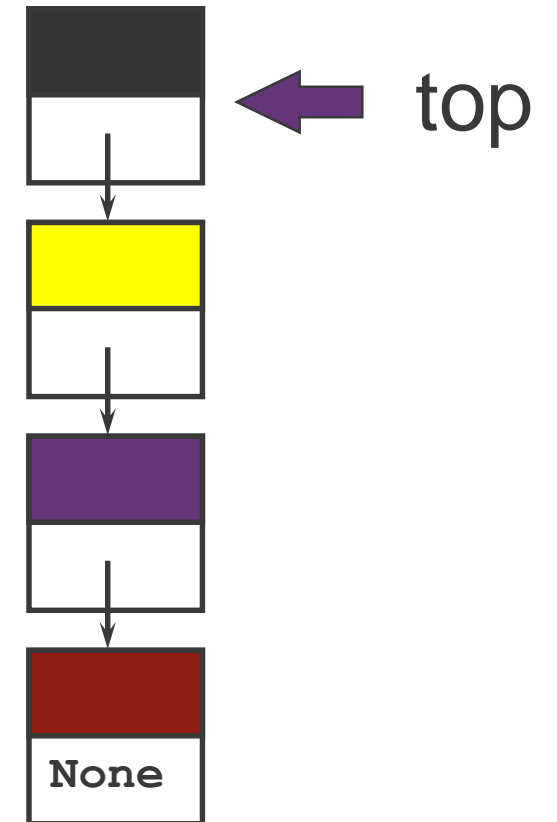
- **Array implementation:**

- If it is empty: raise exception
- Else:
 - Remember the top item
 - Decrease top
 - Return the item

- **Linked nodes:**

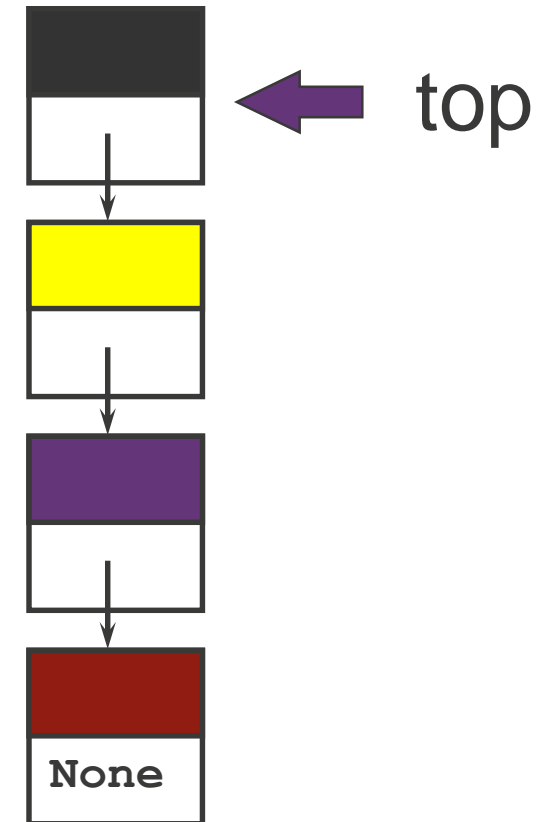
- Almost identical
- We simply move top along, rather than increase it

Pop: algorithm



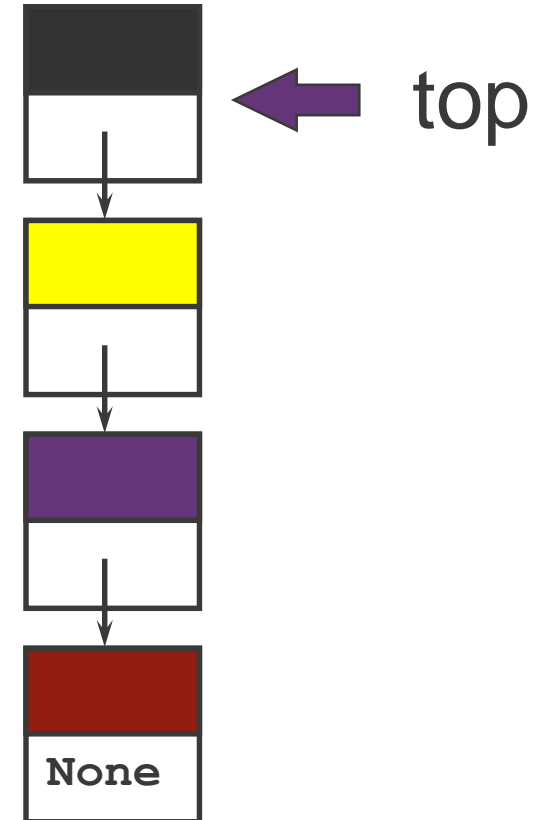
Pop: algorithm

- Check if the stack is empty



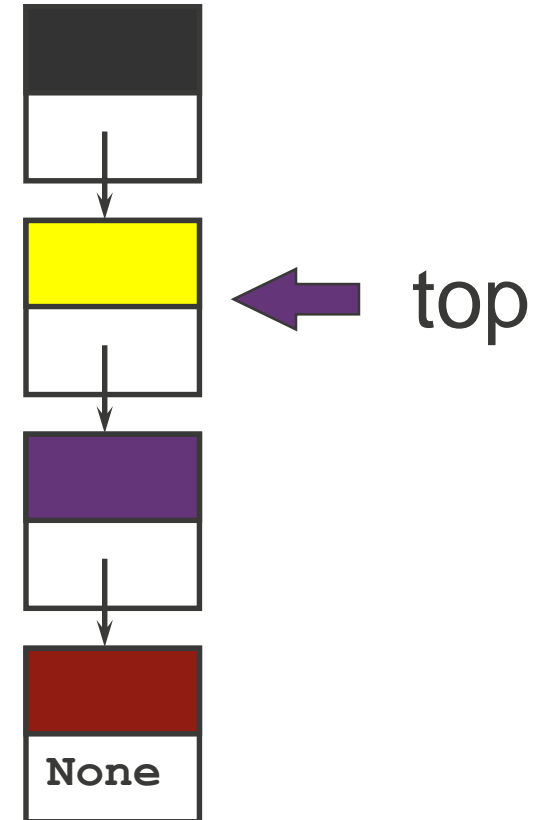
Pop: algorithm

- Check if the stack is empty
- Remember the item in the top node



Pop: algorithm

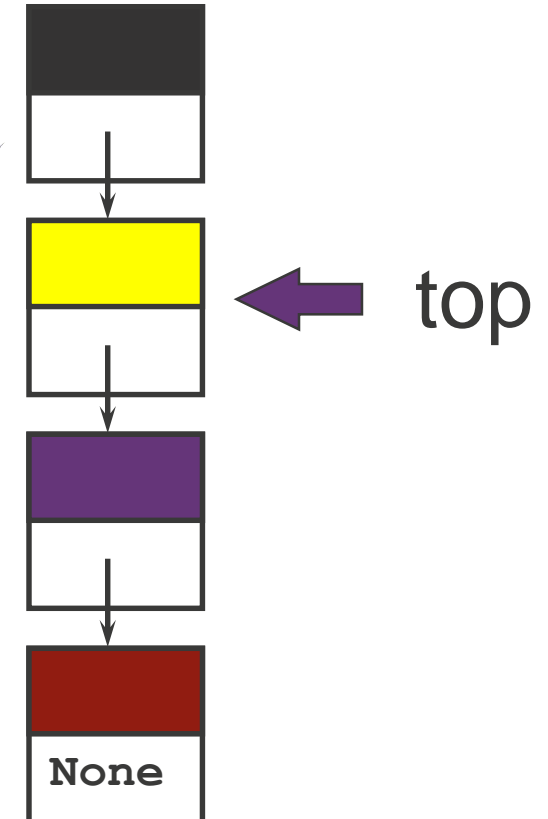
- Check if the stack is empty
- Remember the item in the top node
- **Make the next node the new top**



Pop: algorithm

- Check if the stack is empty
- Remember the item in the top node
- Make the next node the new top
- **Return the item**

As usual, no need to do anything about this. Python will automatically free the memory



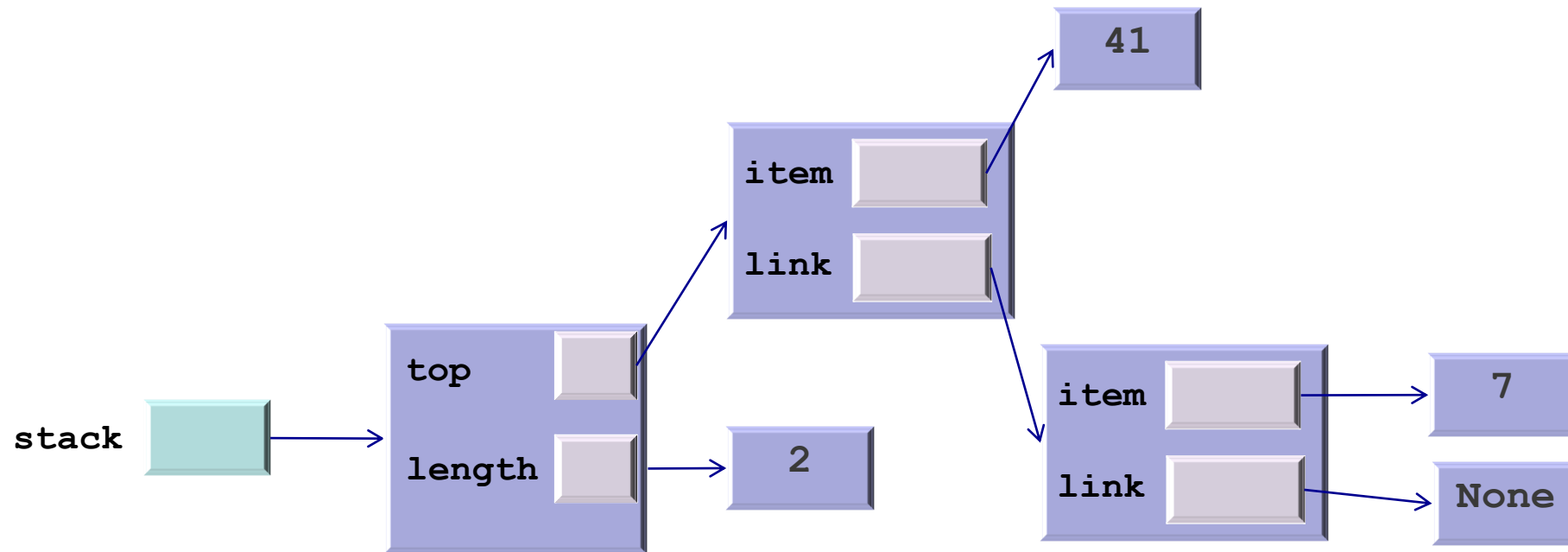
Pop: algorithm and method

```
def pop(self) -> T:
    if not self.is_empty():
        item = self.top.item
        self.top = self.top.link
        self.length -= 1
        return item
    else:
        raise ValueError("Stack is empty")
```

Complexity? $O(1)$

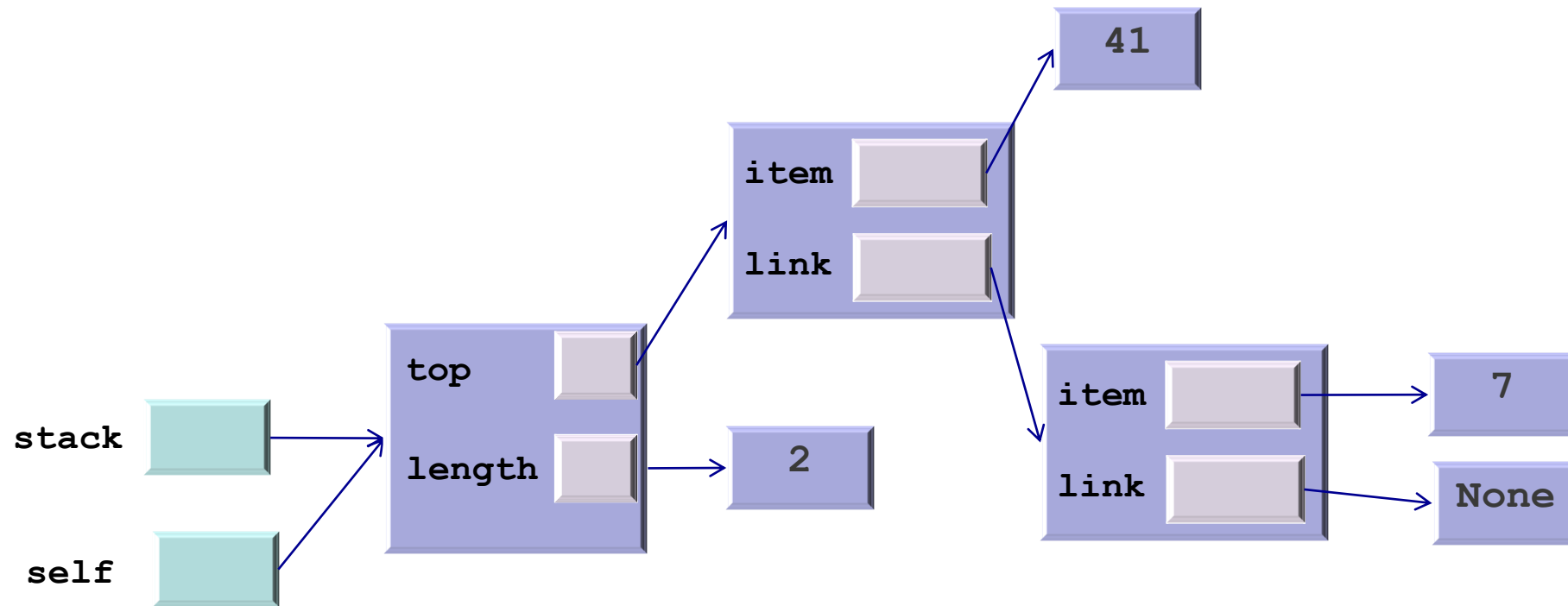
Consider a **stack**
with two nodes whose
items are **41** and **7**
Let's see the memory
diagram for
stack.pop()

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def pop(self) -> T:
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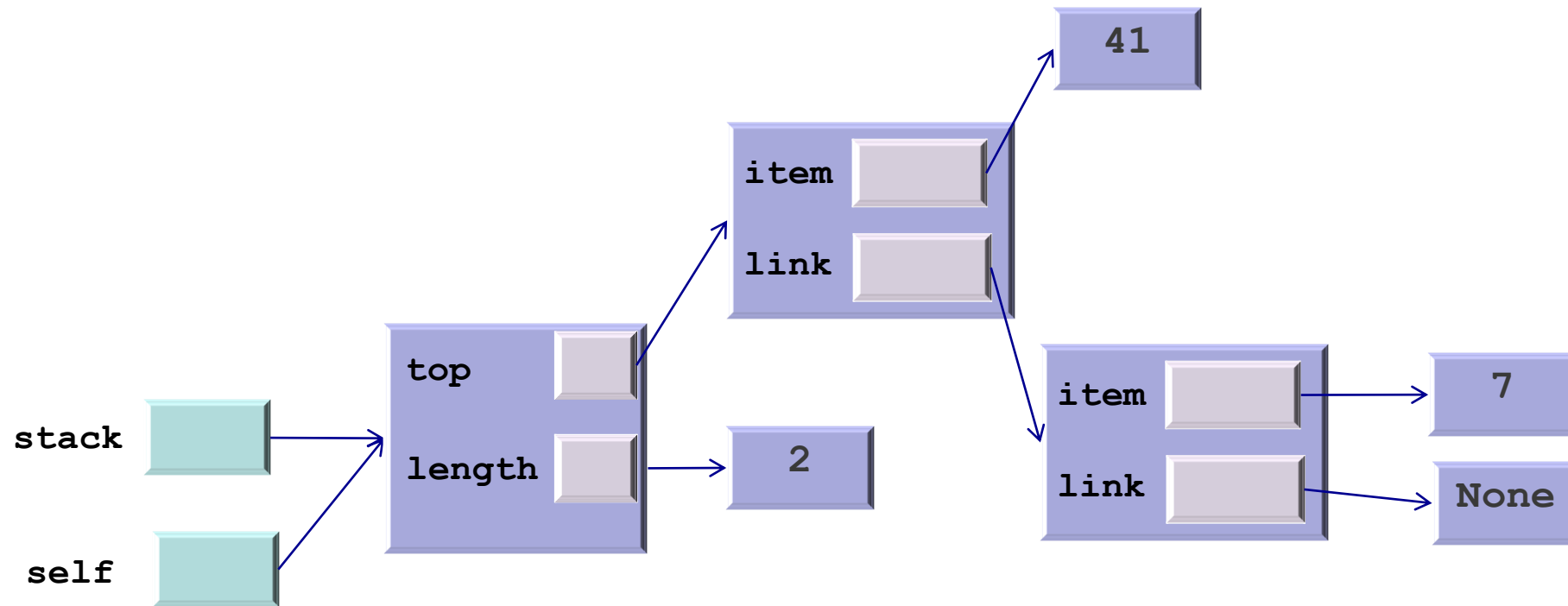
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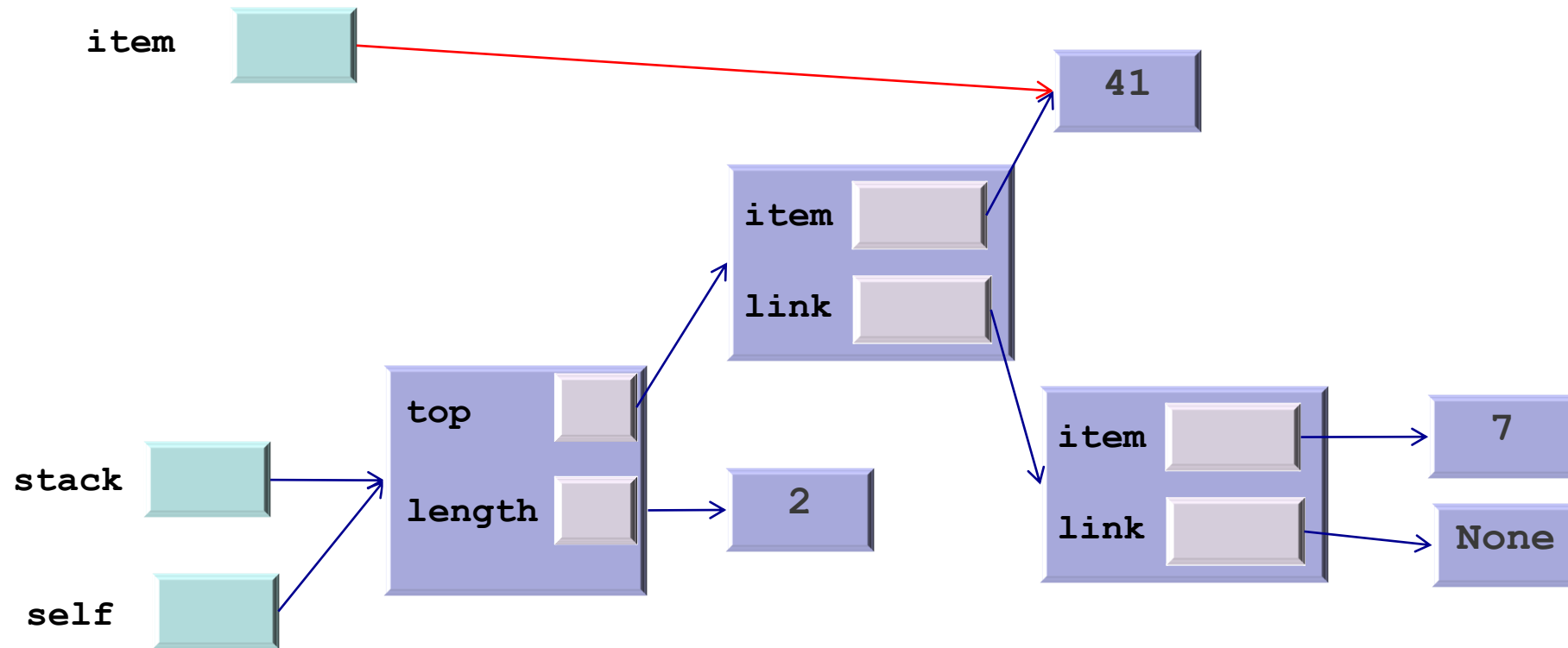
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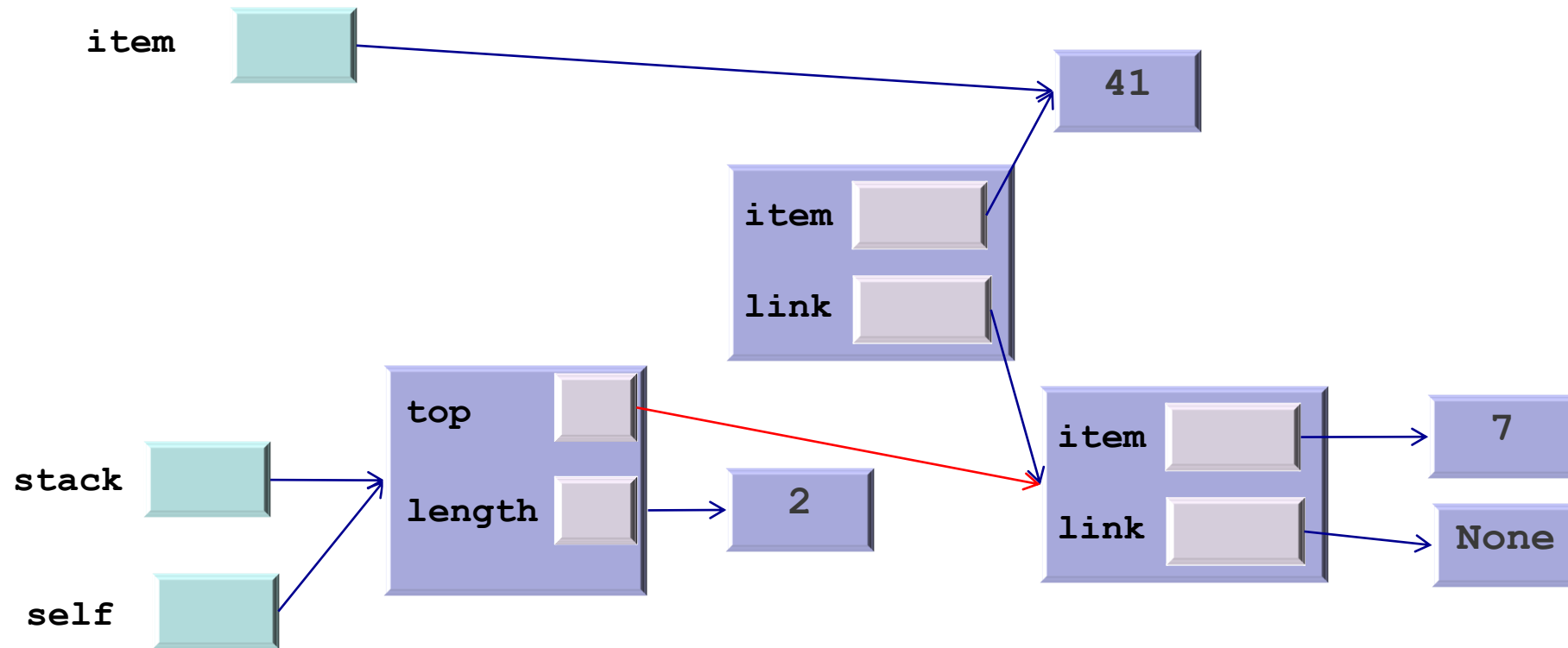
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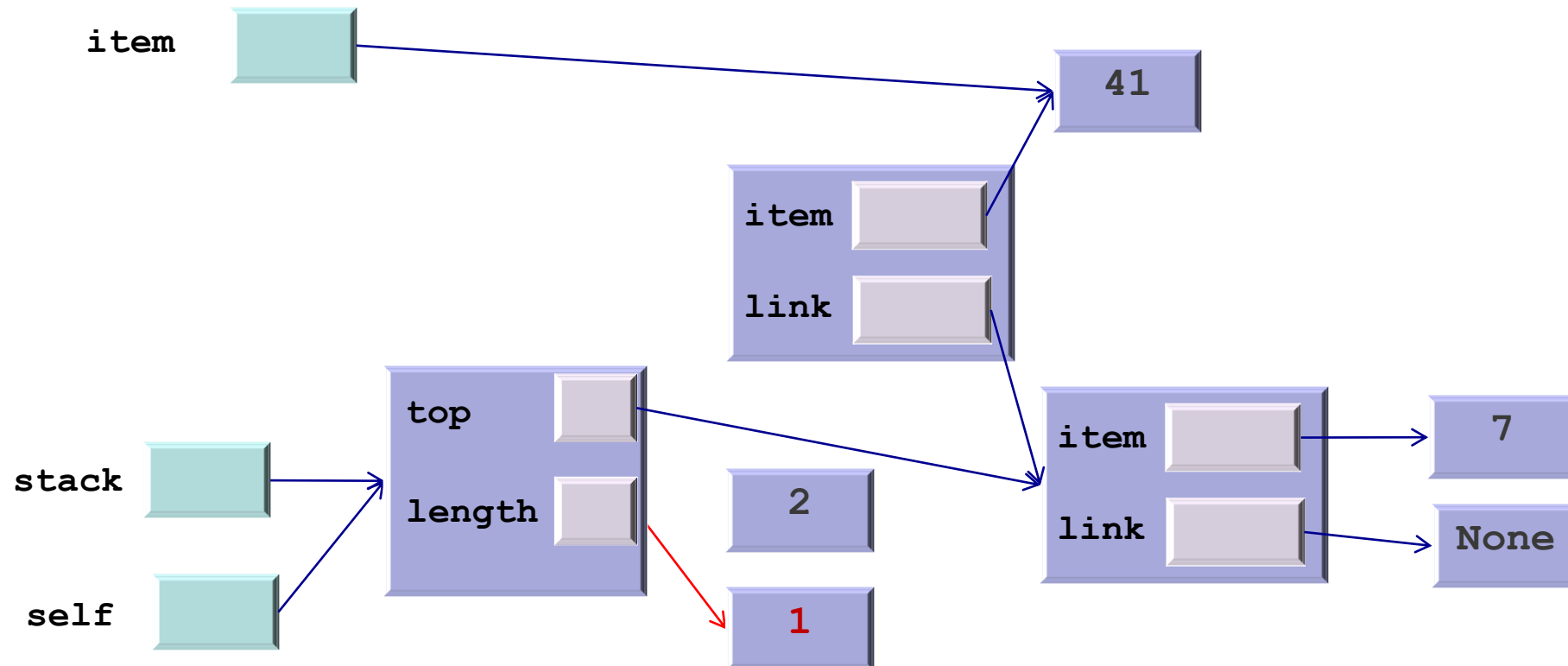
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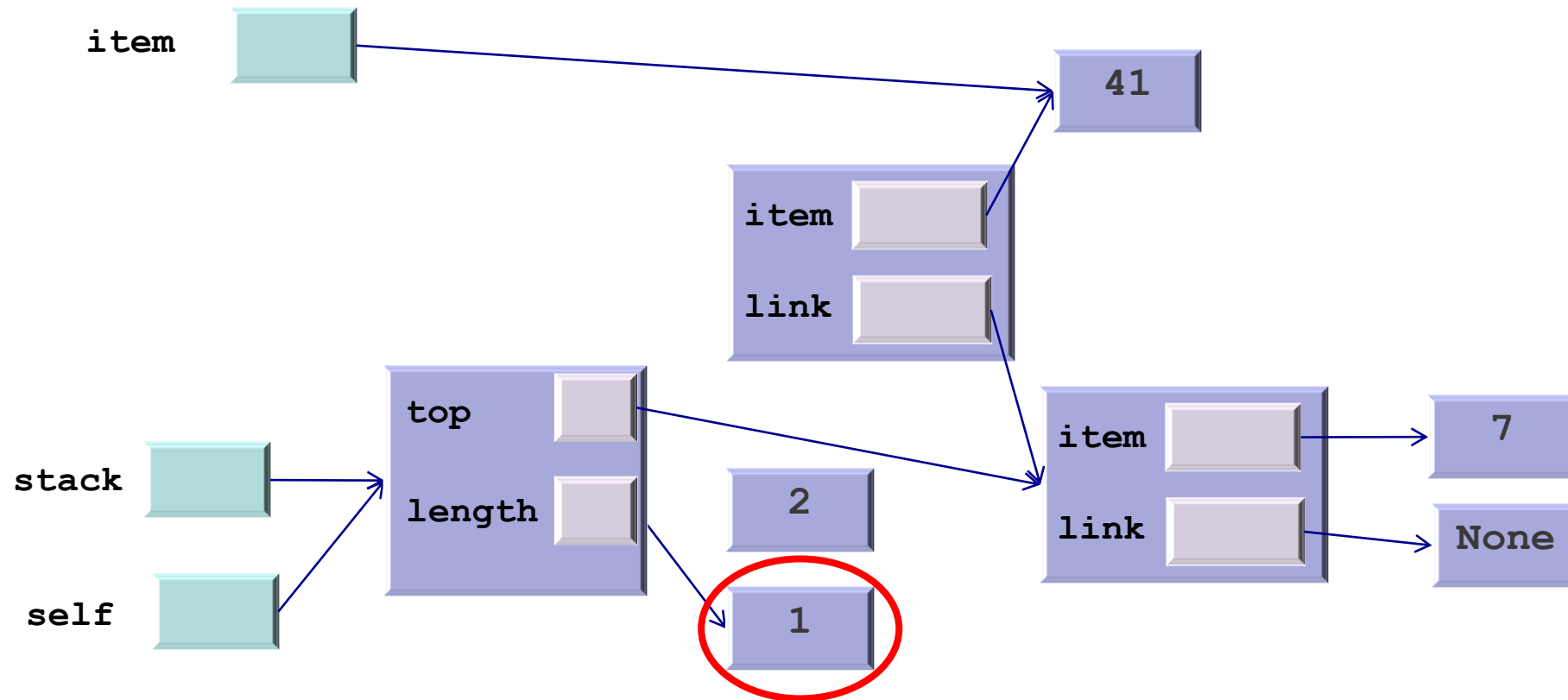
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Consider a **stack**
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    if not self.is_empty():
        item = self.top.item
        self.top = self.top.link
        self.length -= 1
        return item
    else:
        raise ValueError("Stack is empty")
```



Example: modify for using linked stacks

```
def reverse(string: str) -> str:
    my_stack = ArrayStack(len(string))

    for char in string:
        my_stack.push(char)

    output = ""

    while not my_stack.is_empty():
        char = my_stack.pop()
        output += char

    return output
```

What needs to change?

Only the class name for instantiating the object

That is the point of ADTs!

Advantages/Disadvantages for Stacks

▪ Main advantages:

- Good to **resize**:
 - Push: never full so no need to copy, just add element at top
 - Pop: uses less memory when elements are popped
- Needs **less space** than the array, **if the array is relatively empty** (less than half)

▪ Main disadvantage:

- Needs **more space** (for the links) than the array, if the array is relatively full

▪ Other disadvantages:

- A bit **slower**
 - Still constant time but a bigger constant (create nodes, etc)

Note: Lack of random access is not a problem for a stack: its operations do not need this!

Linked Queues

```
from abc import ABC, abstractmethod
from typing import TypeVar, Generic
T = TypeVar('T')
```

```
class Queue(ABC, Generic[T]):
    def __init__(self) -> None:
        self.length = 0

    @abstractmethod
    def append(self, item: T) -> None:
        pass

    @abstractmethod
    def serve(self) -> None:
        pass

    def __len__(self) -> int:
        return self.length

    def clear(self):
        self.length = 0
```

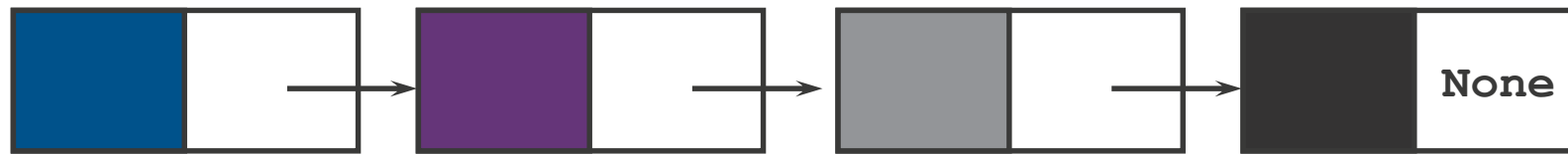
Remember:

Abstract base Queue class

```
def is_empty(self) -> bool:
    return len(self) == 0
```

```
@abstractmethod
def is_full(self) -> bool:
    pass
```

Linked Queue



↑
front

Careful: rear now
marks the last node

↑
rear

No need for circularity
What do we need in
the class?

Class for Linked Queue

```
from typing import TypeVar
from abstract_queue import Queue
from node import Node
T = TypeVar('T')
```

```
class LinkQueue(Queue[T]):
    def __init__(self):
        Queue.__init__(self)
        self.front = None
        self.rear = None
```

The code must ensure that when **front** is **None**, **rear** is also **None**

```
def is_empty(self) -> bool:
    return self.front is None
```

```
def is_full(self) -> bool:
    return False
```

```
def clear(self) -> None:
    Queue.clear()
    self.front = None
    self.rear = None
```


Linked Queues

Append

Append: algorithm

- **Linear array implementation:**

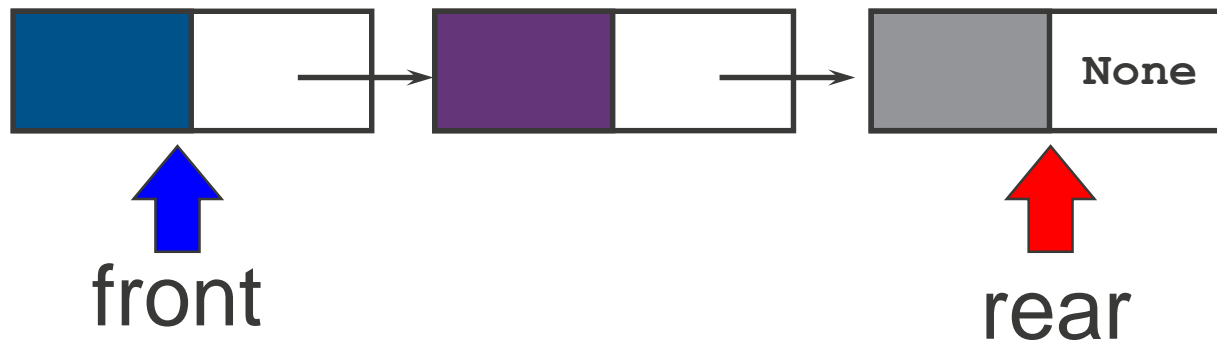
- If it is full: raise exception
- Else:
 - Increase rear
 - Add item at position marked by rear

- **In a linked list:**

- Create a new node that contains item and points to **None**
- Link the current rear to it
- Make the new node the new rear

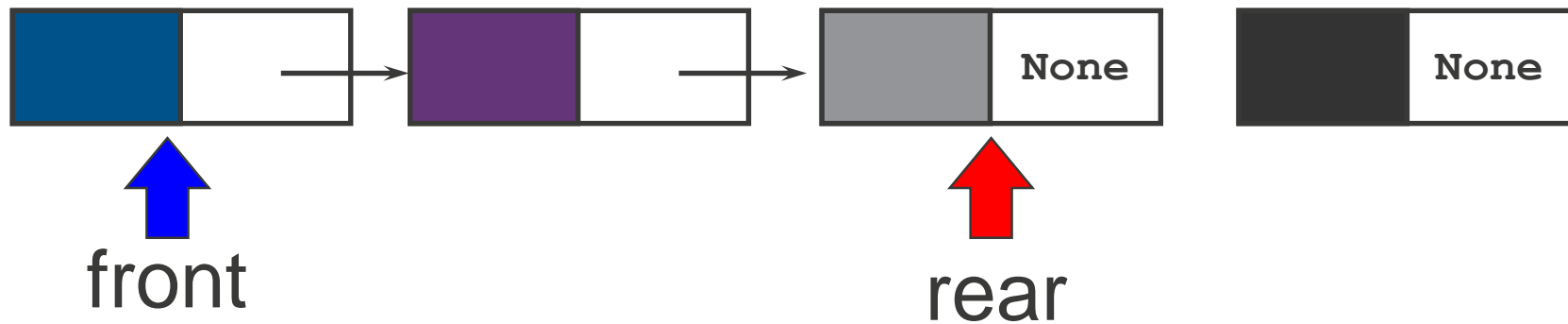
- **Again, no need for `is_full` check**

Append: algorithm



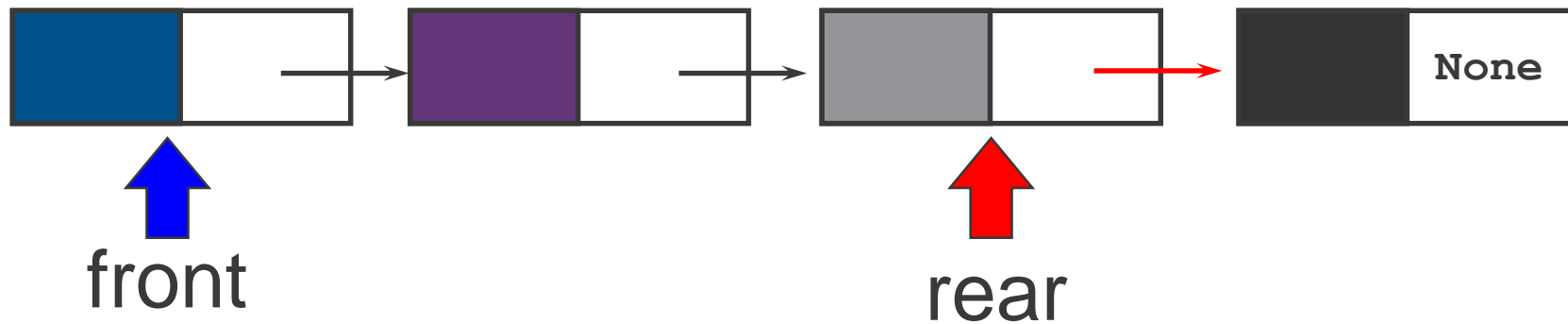
Append: algorithm

- Create a new node for item



Append: algorithm

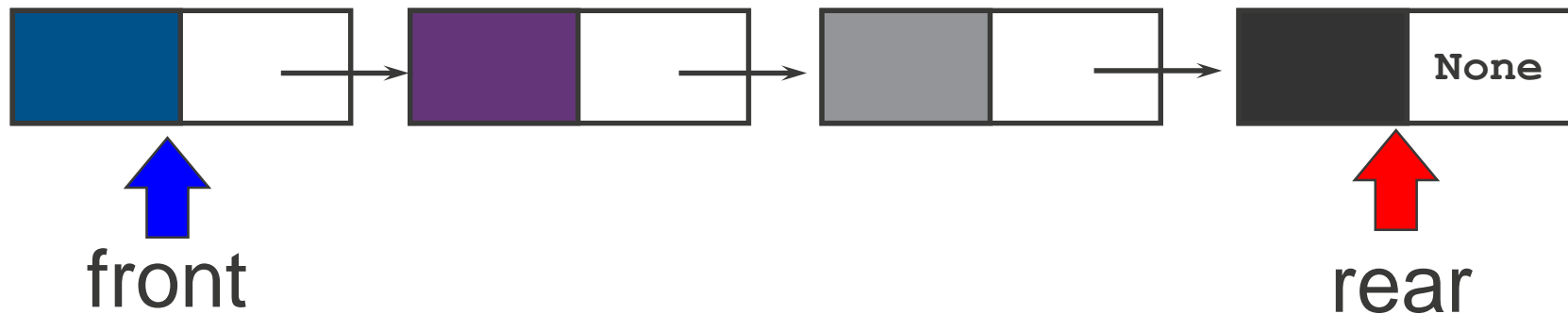
- Create a new node for item 
- Make a link from the current rear to the new node



Append: algorithm

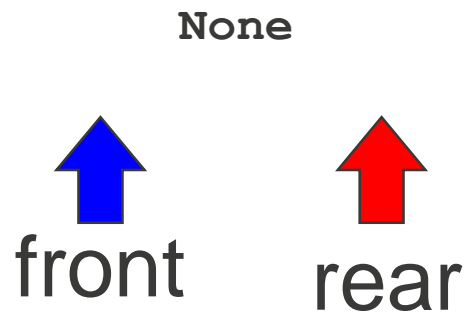
- Create a new node for item 
- Make a link from the current rear to the new node
- **The new node becomes the new rear**

Does this general algorithm always work?



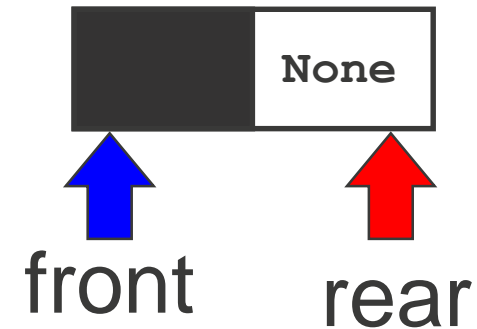
Append: algorithm

- No, if the queue is empty, we must modify front too
- How?
 - Create a new node for item



Append: algorithm

- No, if the queue is empty, we must modify front too
- How?
 - Create a new node for item 
- The new node become the new front and rear

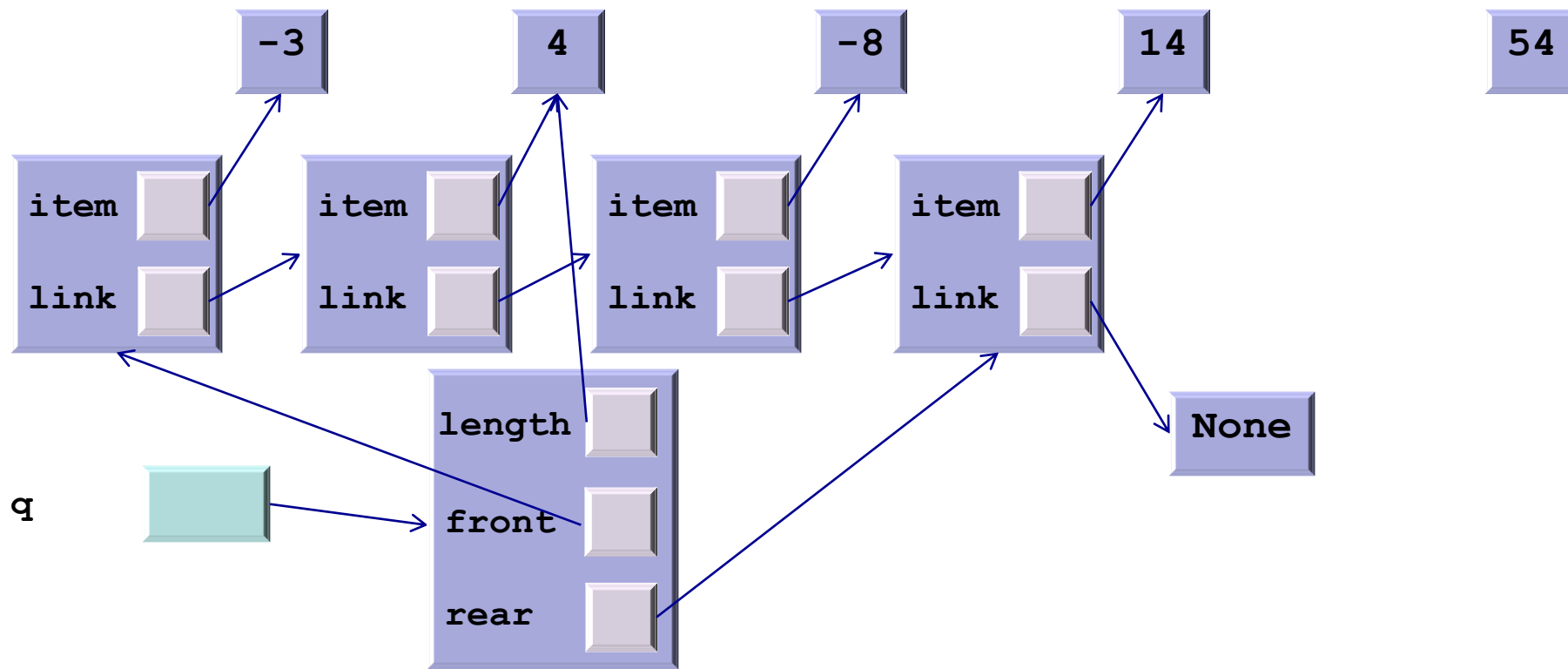


Append method

...

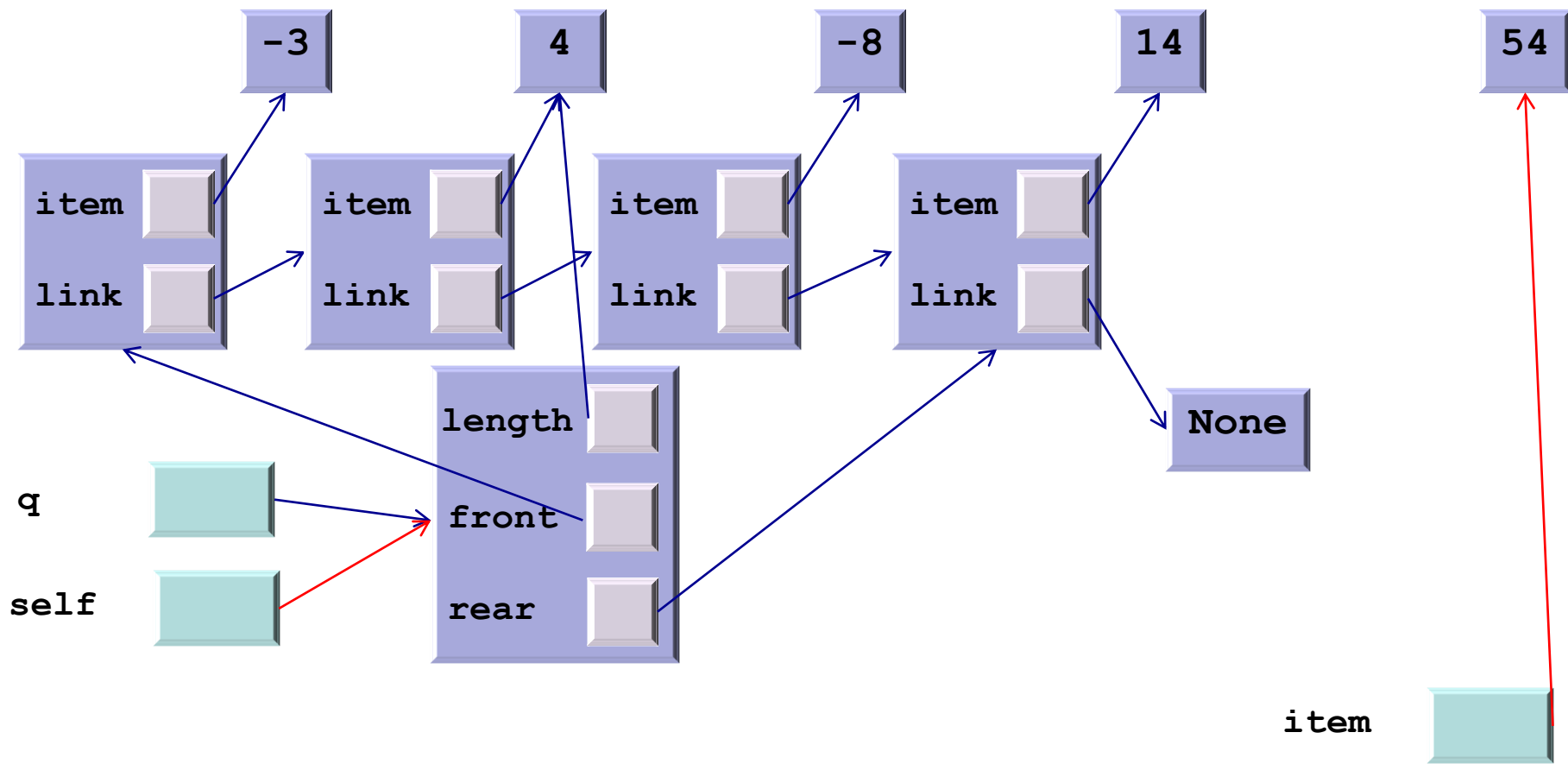
```
def append(self, item: T) -> None:
    new_node = Node(item) # create new node
    if self.is_empty():
        self.front = new_node # move head
    else:
        self.rear.link = new_node #link it in
    self.rear = new_node # move rear to new node
    self.length += 1
```

Complexity? $O(1)$



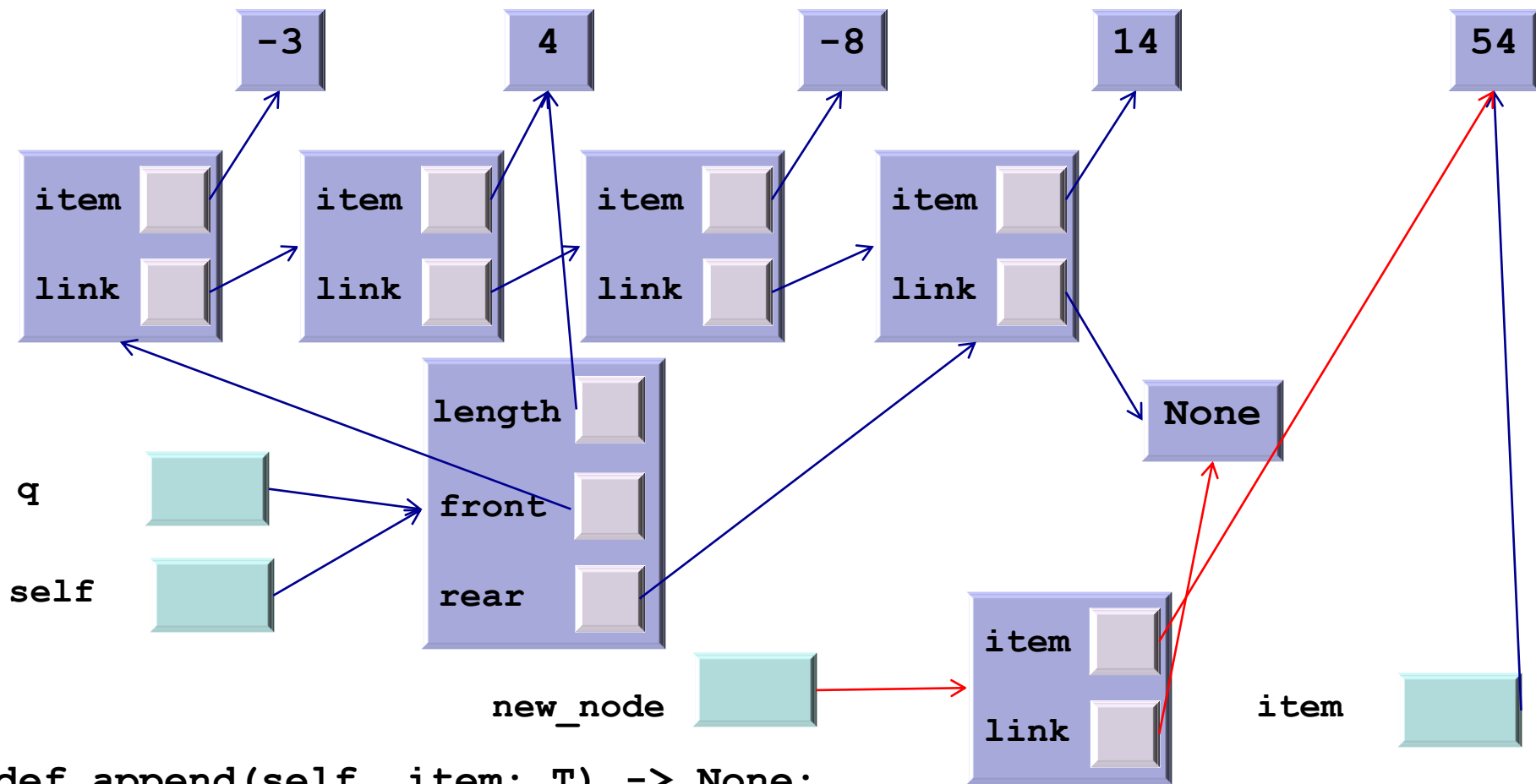
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    if self.is_empty():
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    self.length += 1
```

q.append(54)



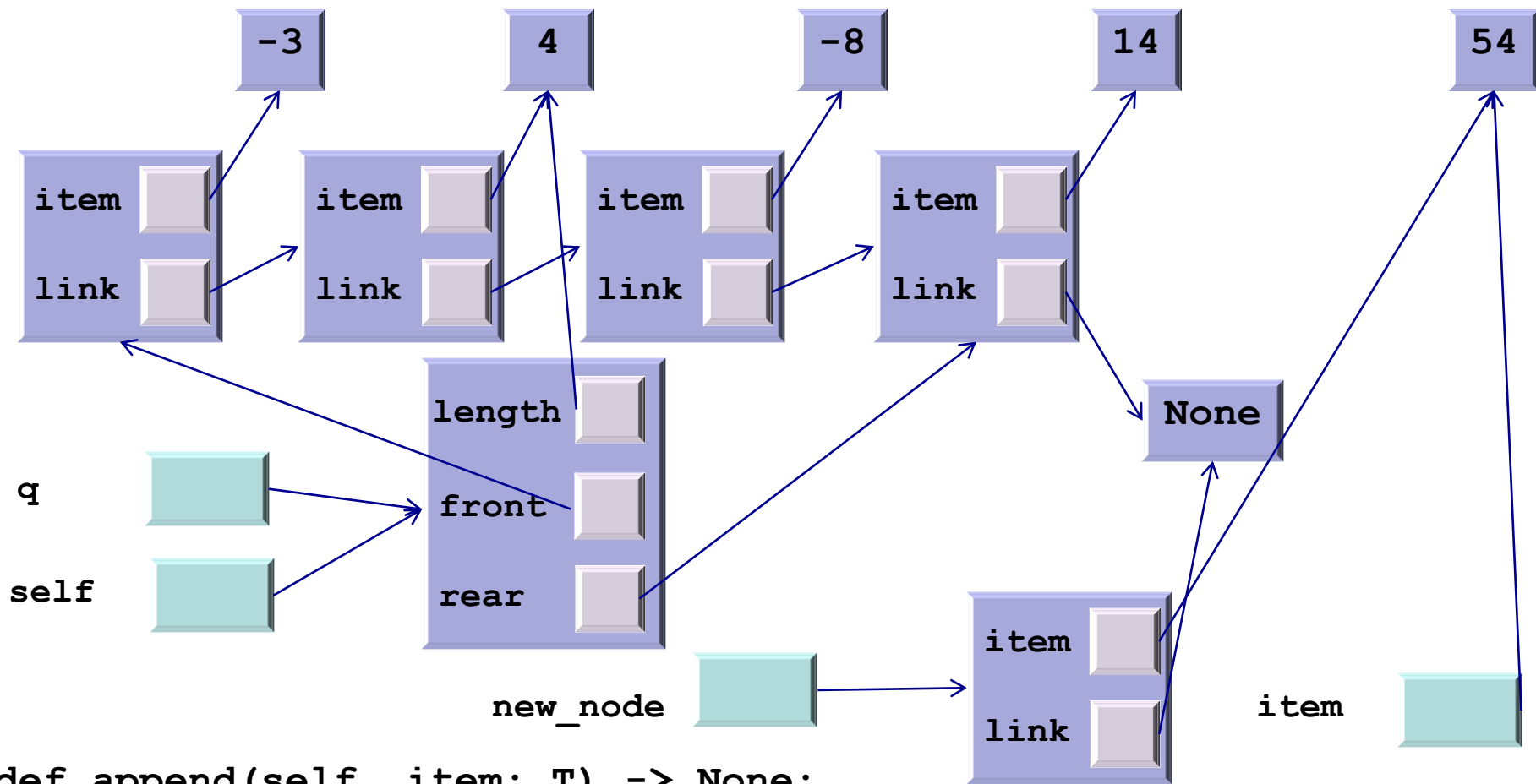
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`q.append(54)`



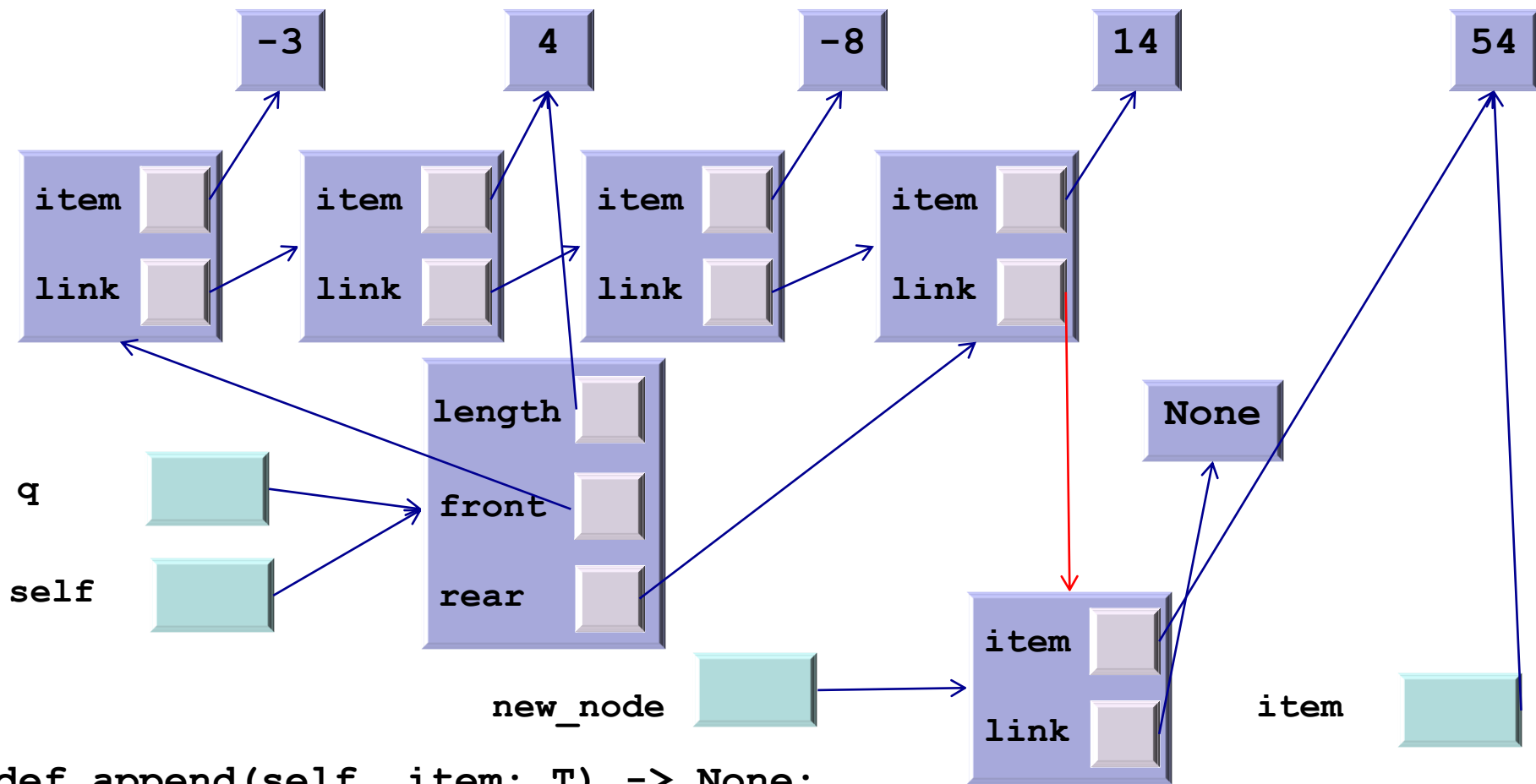
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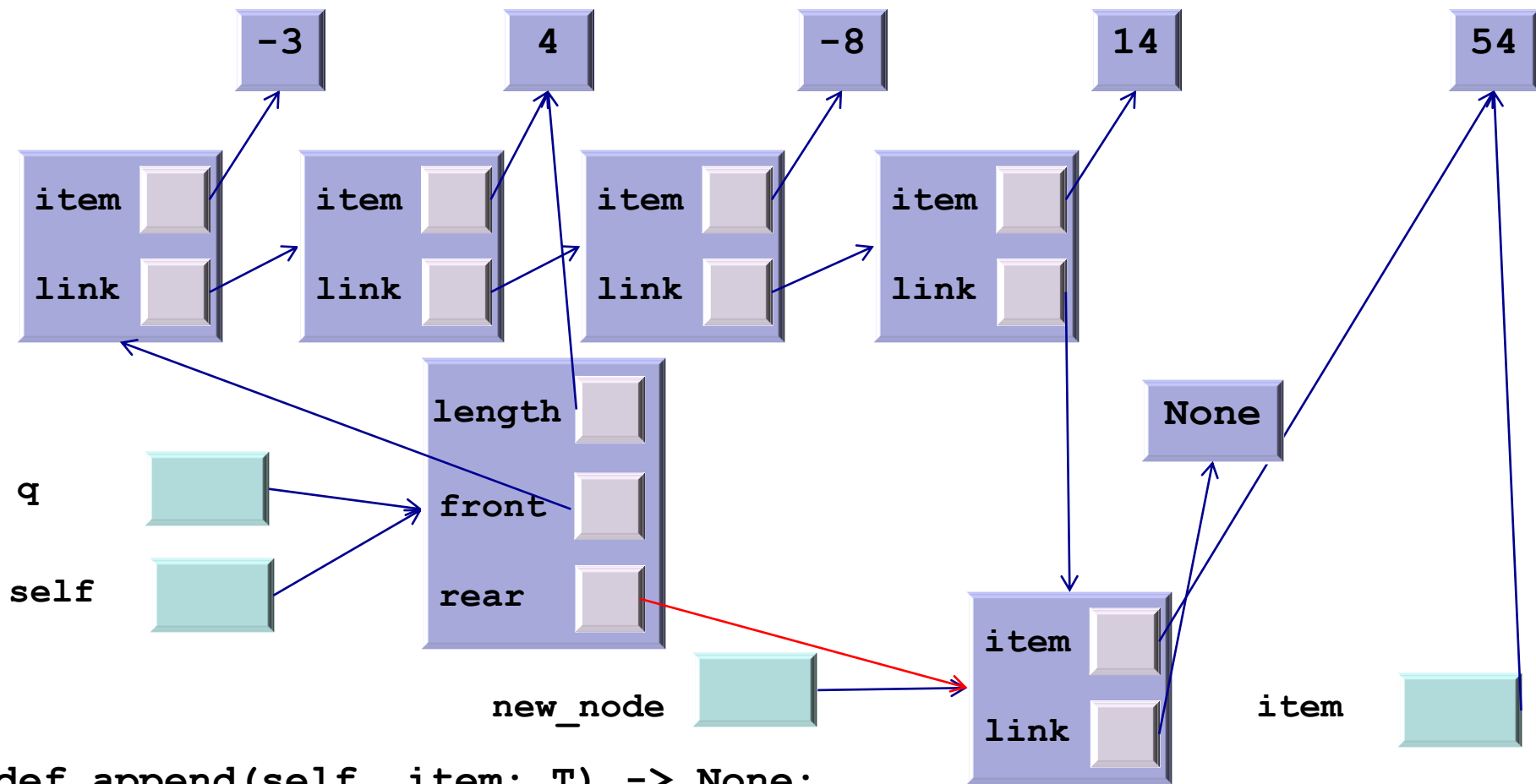
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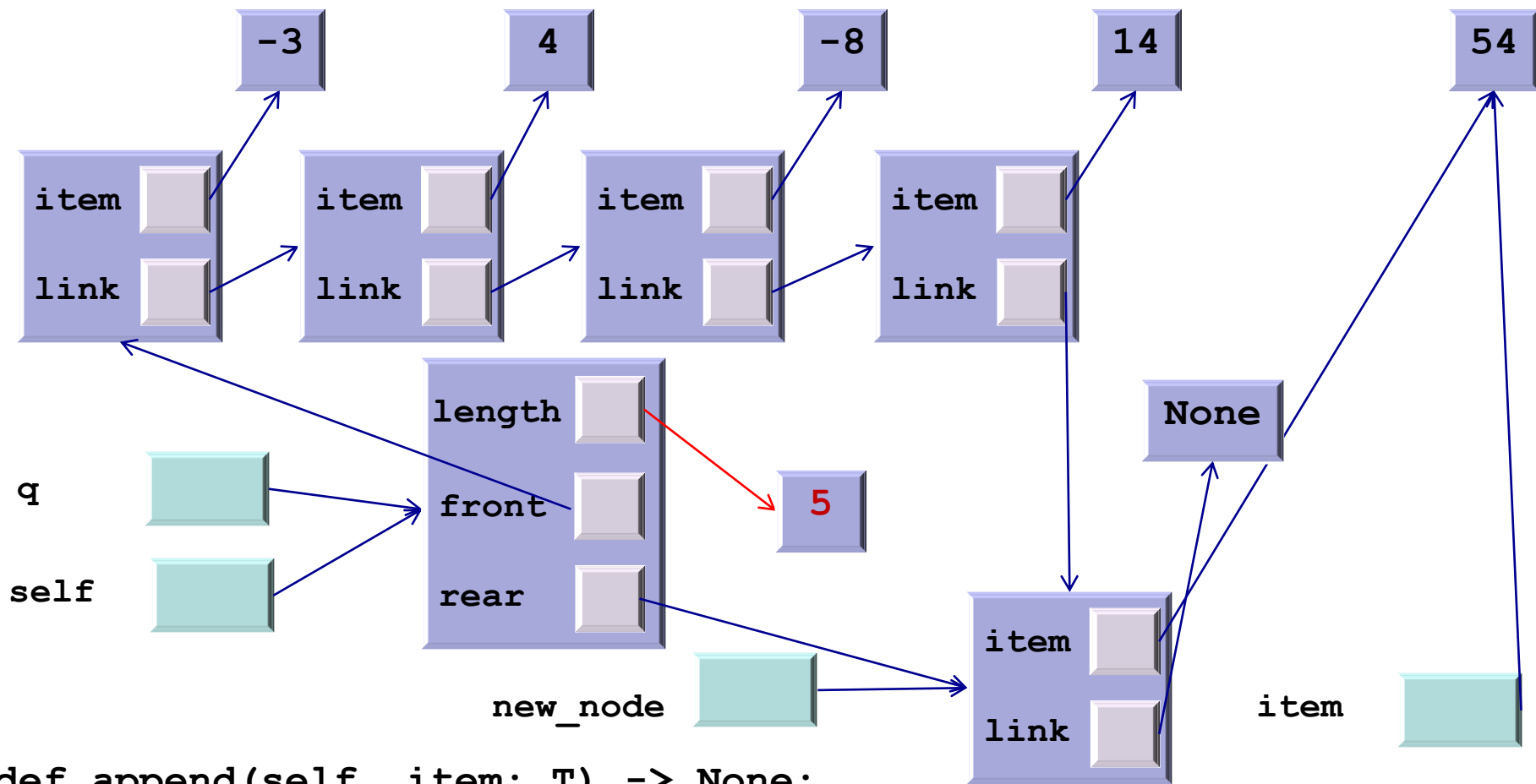
```
def append(self, item: T) -> None:
    new_node = Node(item)
    if self.is_empty():
        self.front = new_node
    else:
        self.rear.link = new_node
    self.rear = new_node
    self.length += 1
```

q.append(54)



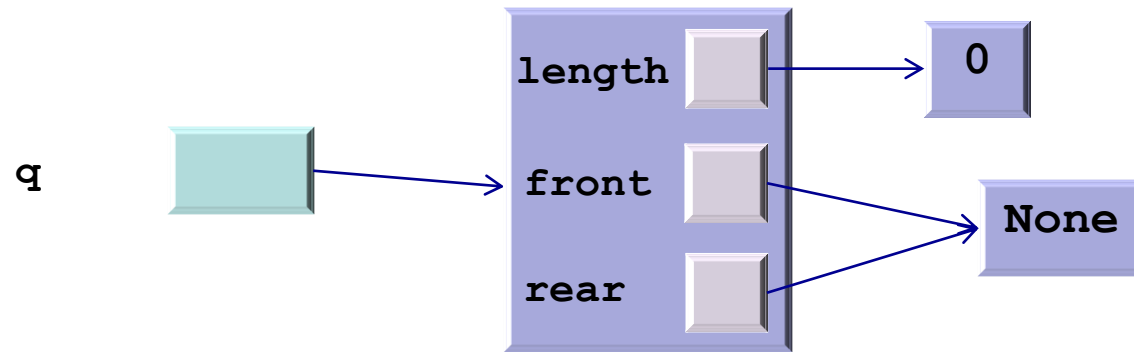
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def append(self, item: T) -> None:
    new_node = Node(item)
    if self.is_empty():
        self.front = new_node
    else:
        self.rear.link = new_node
    self.rear = new_node
    self.length += 1
```

q.append(54)



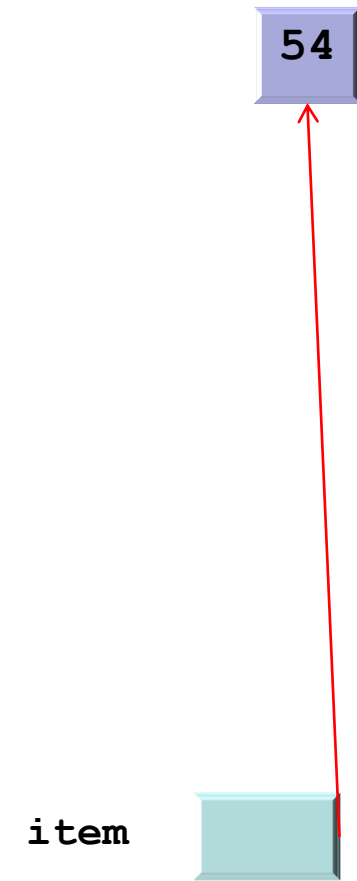
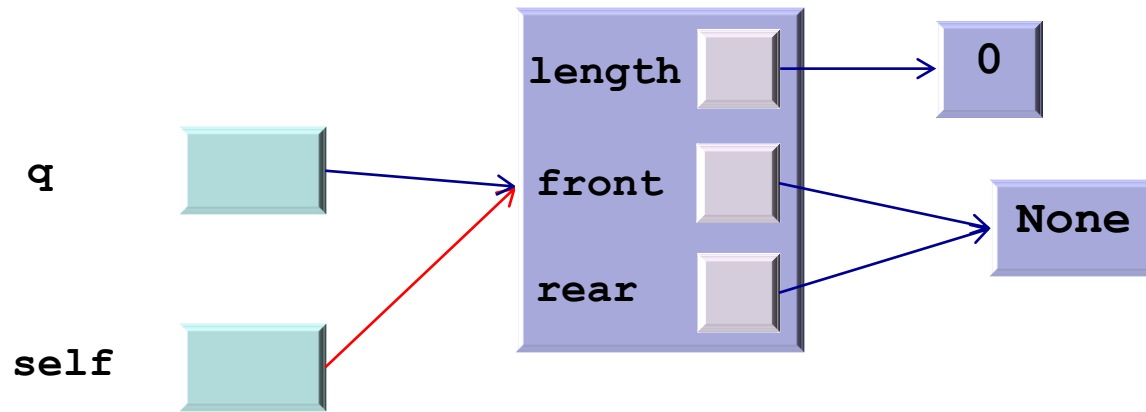
```
def append(self, item: T) -> None:
    new_node = Node(item)
    if self.is_empty():
        self.front = new_node
    else:
        self.rear.link = new_node
    self.rear = new_node
    self.length += 1
```

q.append(54)



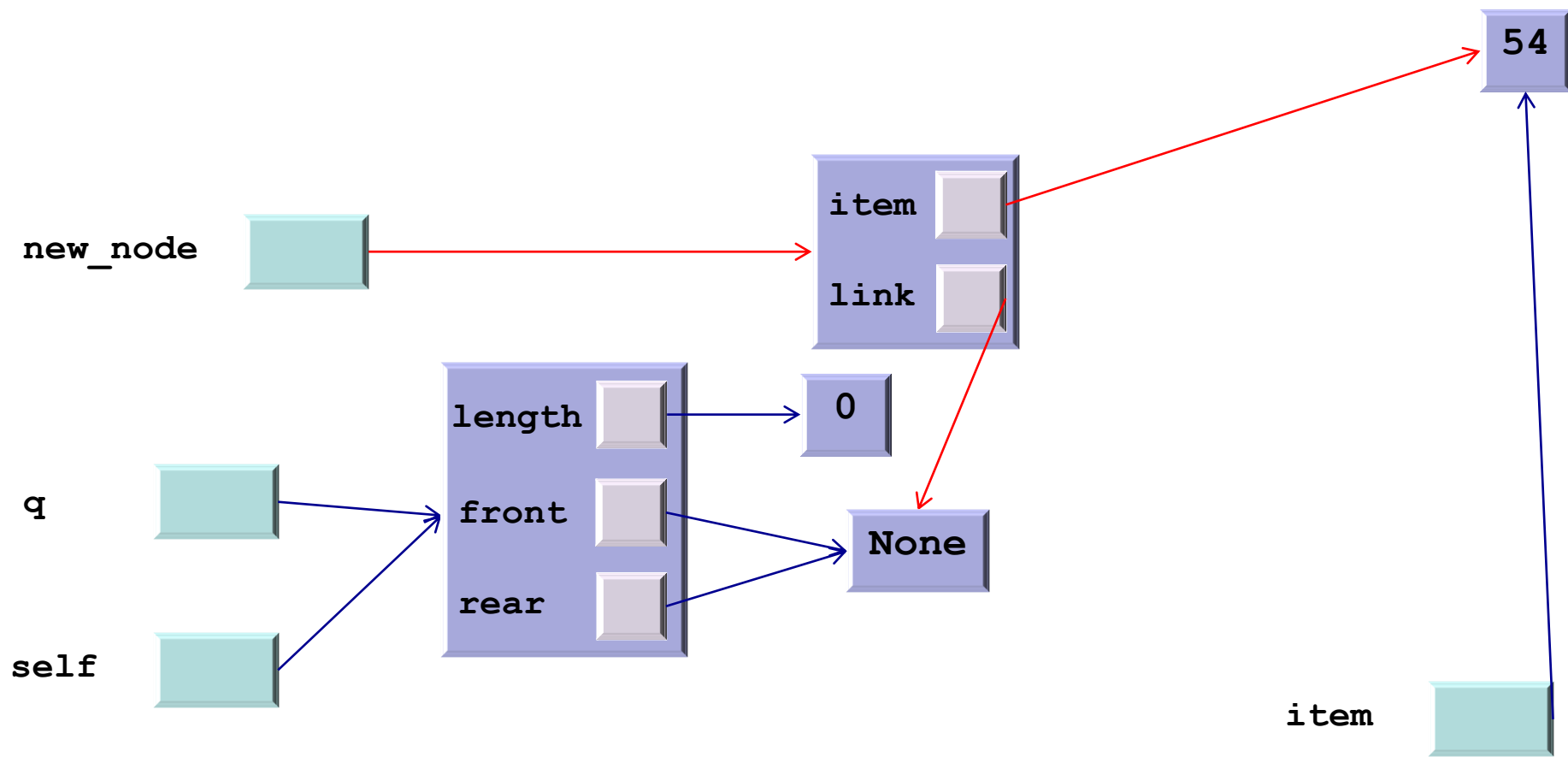
```
def append(self, item: T) -> None:
    new_node = Node(item)
    if self.is_empty():
        self.front = new_node
    else:
        self.rear.link = new_node
    self.rear = new_node
    self.length += 1
```

q.append(54)



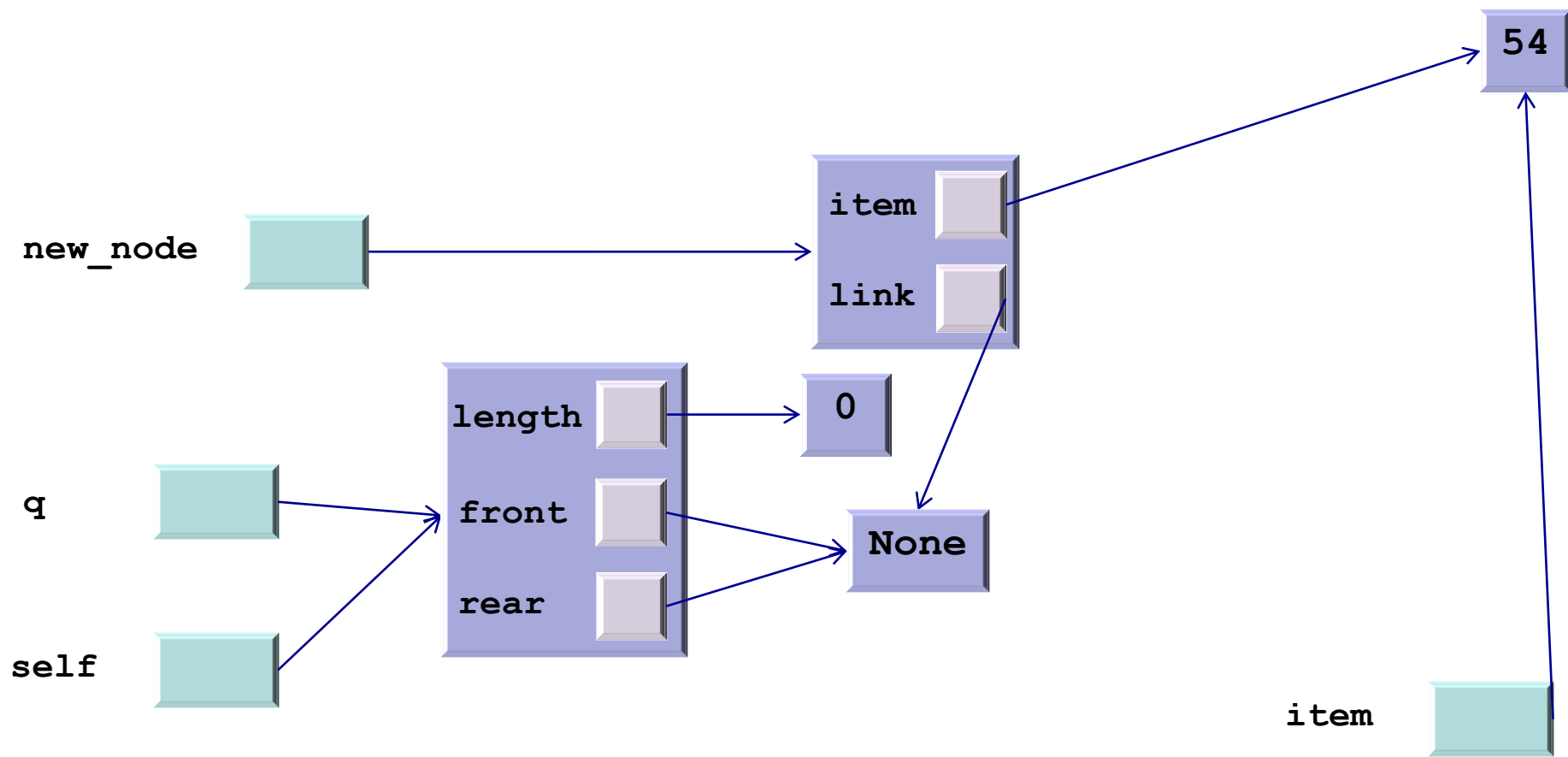
```
def append(self, item: T) -> None:
    new_node = Node(item)
    if self.is_empty():
        self.front = new_node
    else:
        self.rear.link = new_node
    self.rear = new_node
    self.length += 1
```

q.append(54)



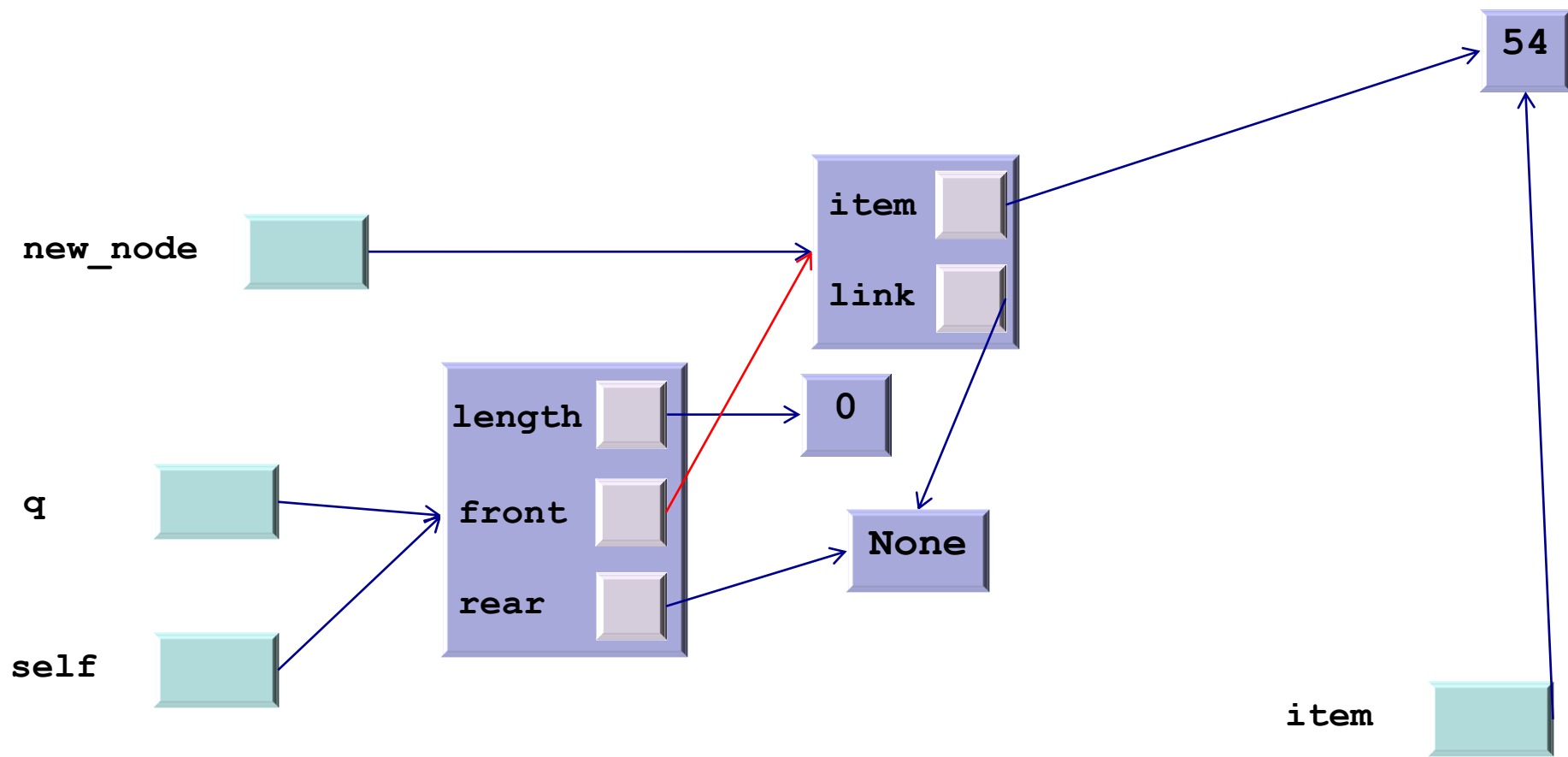
```
def append(self, item: T) -> None:
    new_node = Node(item)
    if self.is_empty():
        self.front = new_node
    else:
        self.rear.link = new_node
    self.rear = new_node
    self.length += 1
```

q.append(54)



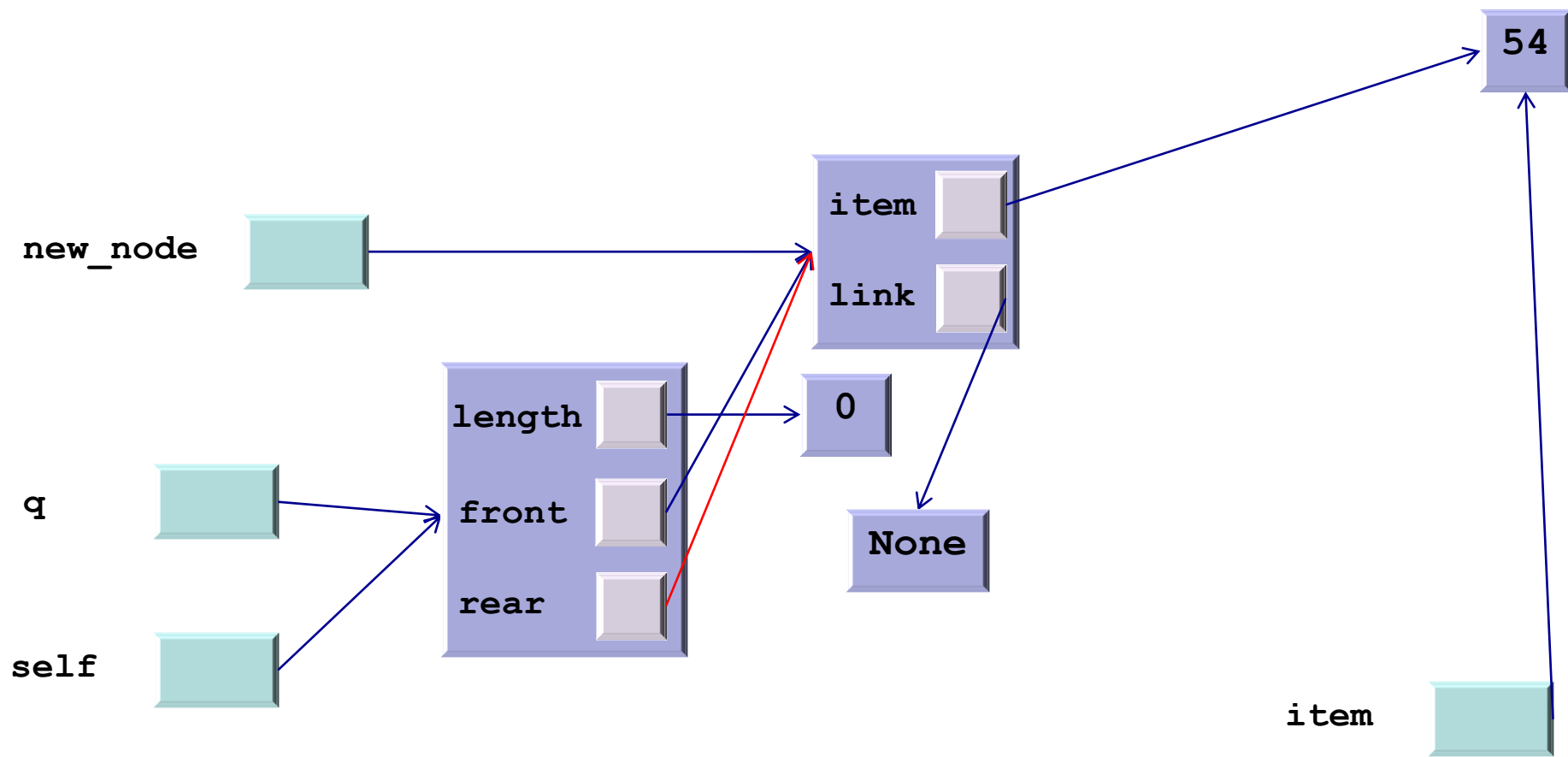
```
def append(self, item: T) -> None:
    new_node = Node(item)
    if self.is_empty():
        self.front = new_node
    else:
        self.rear.link = new_node
    self.rear = new_node
    self.length += 1
```

`q.append(54)`



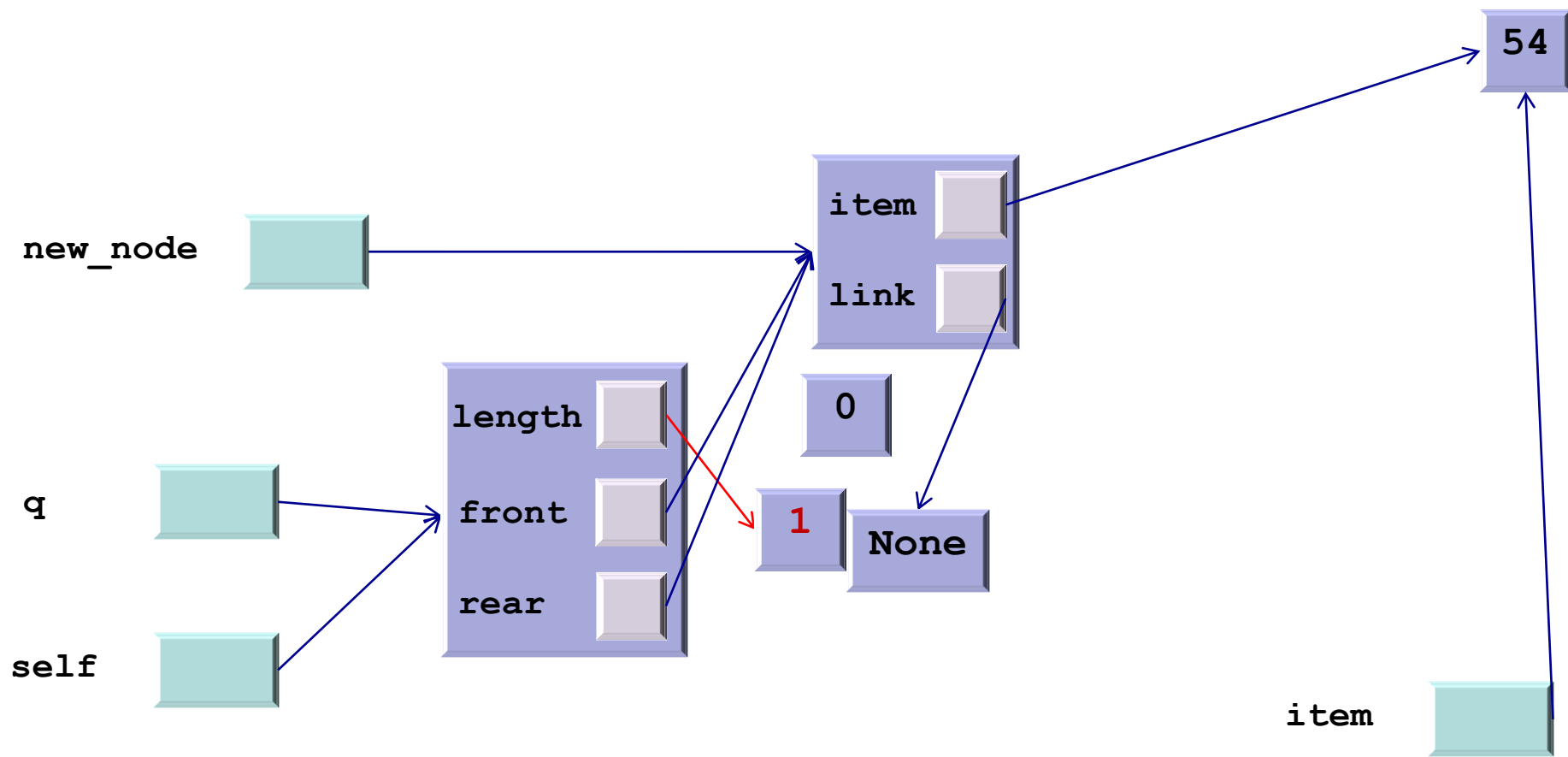
```
def append(self, item: T) -> None:
    new_node = Node(item)
    if self.is_empty():
        self.front = new_node
    else:
        self.rear.link = new_node
    self.rear = new_node
    self.length += 1
```

`q.append(54)`



```
def append(self, item: T) -> None:
    new_node = Node(item)
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        self.front = new_node
    else:
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    self.rear = new_node
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```

`q.append(54)`



```
def append(self, item: T) -> None:
    new_node = Node(item)
    if self.is_empty():
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    else:
        self.rear.link = new_node
    self.rear = new_node
    self.length += 1
```

`q.append(54)`

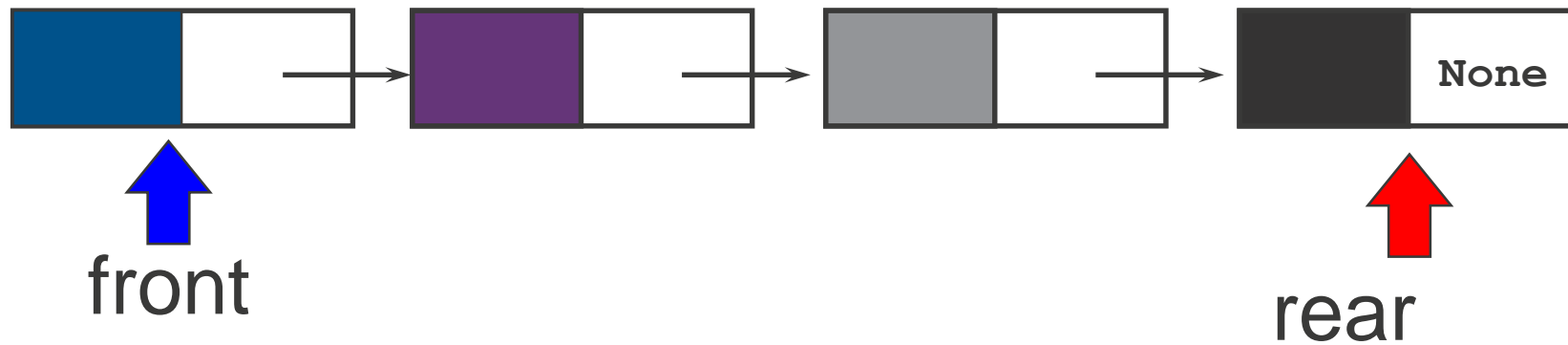
Linked Queues Serve

Serve: algorithm

- **Linear array implementation:**
 - If it is empty: raise exception
 - Else:
 - Remember item to return
 - Increase front
 - Return item
- **In a linked list**
 - Almost identical
 - We simply move front along rather than increase it

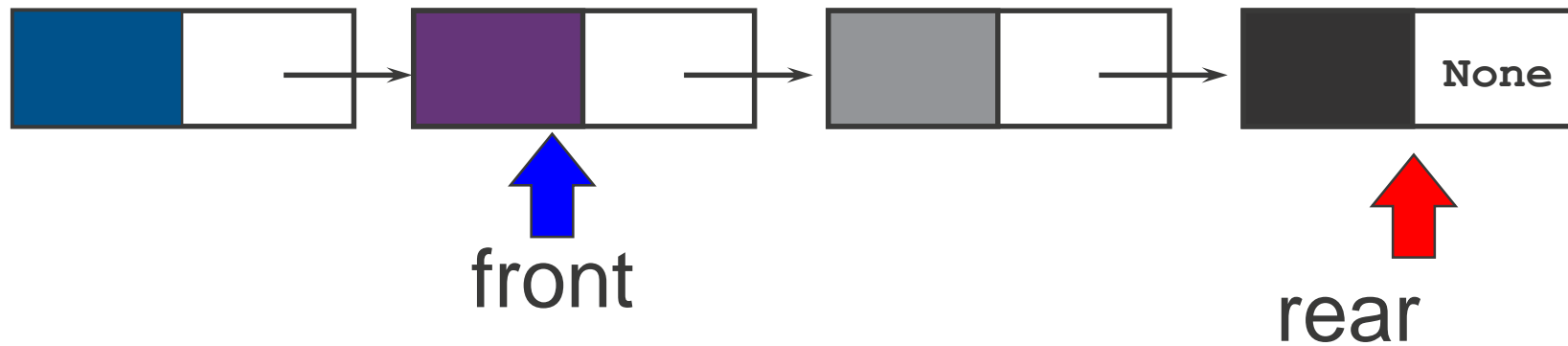
Serve: algorithm

- Remember the item in the front node



Serve: algorithm

- Remember the item in the front node
- Make the next node the new front

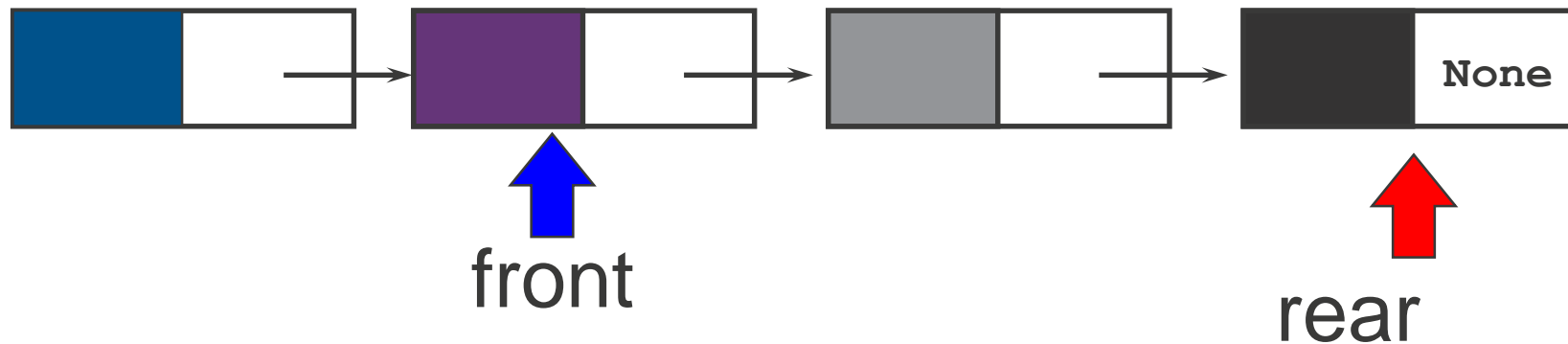


Serve: algorithm

- Remember the item in the front node
- Make the next node the new front
- **Return the item**

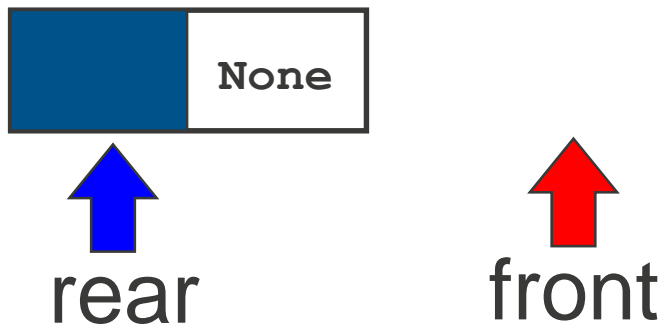


Does this general algorithm always work?



Serve: algorithm

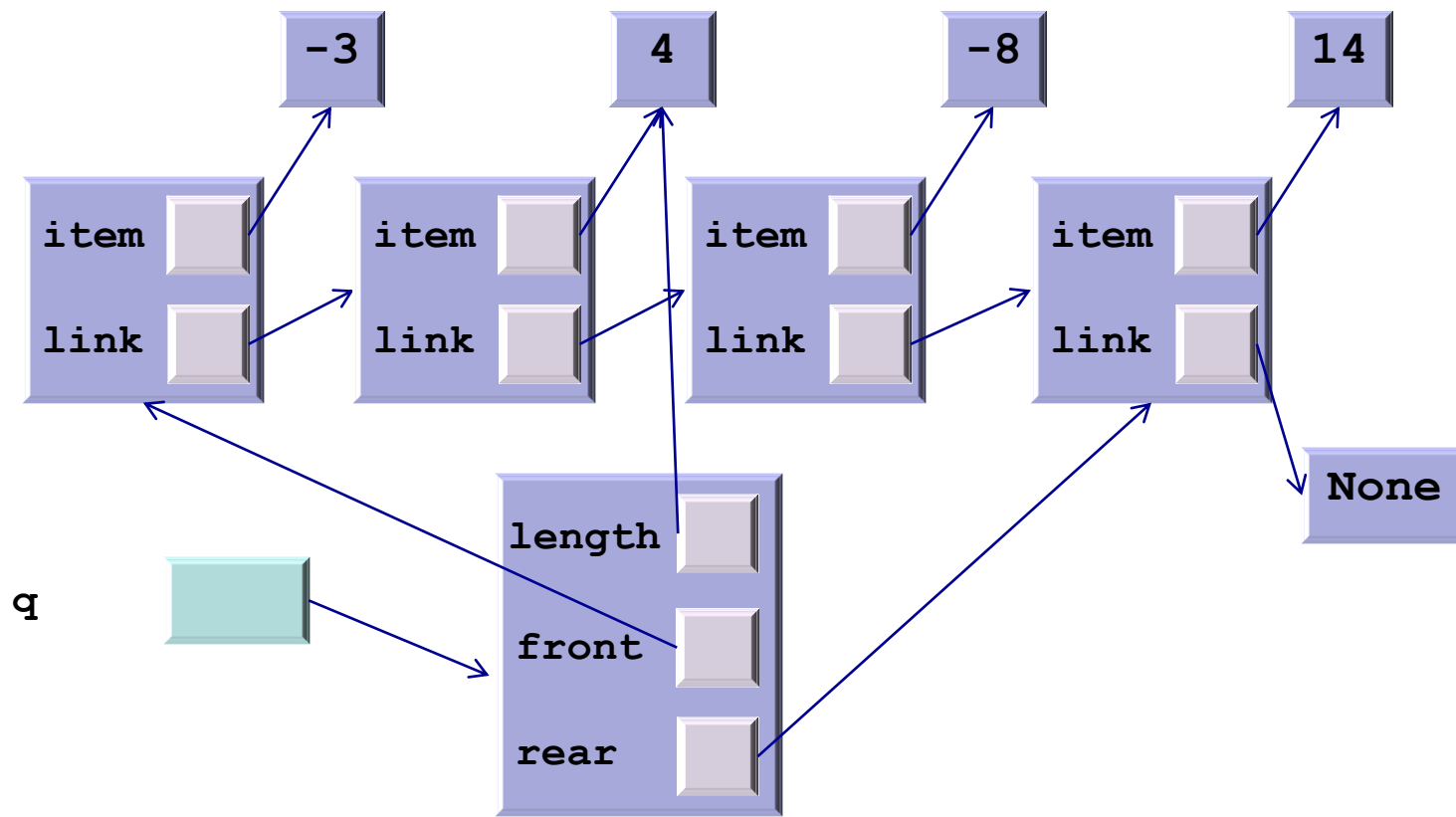
- If the Queue becomes empty, we end up in a possibly dangerous configuration
- Better to set rear to `None`



Serve method

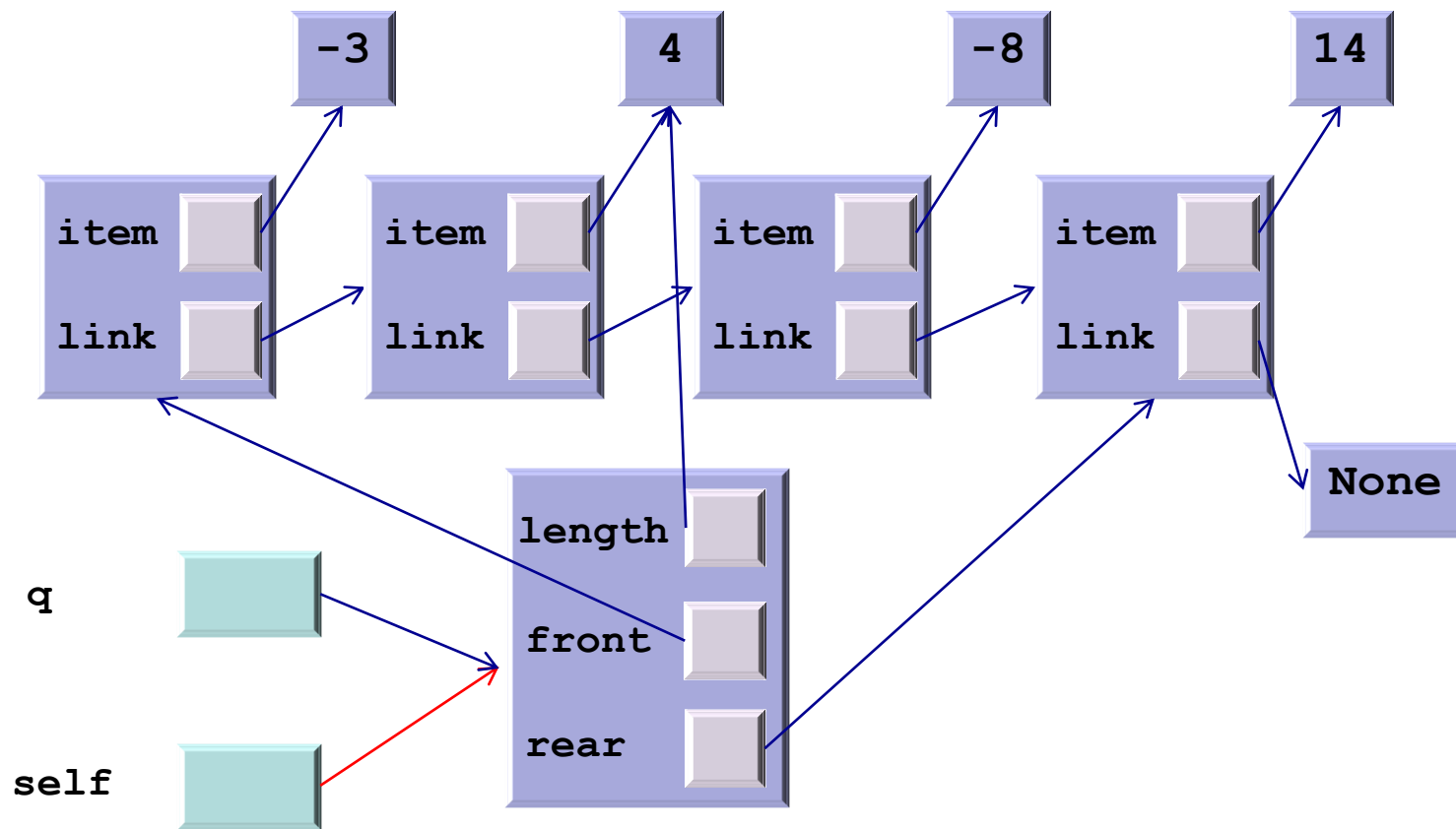
```
def serve(self) -> T:
    if not self.is_empty():
        item = self.front.item # store the item to serve
        self.front = self.front.link # move front
        self.length -= 1
        if self.is_empty(): # if now empty
            self.rear = None # move rear
        return item
    else:
        raise ValueError("Queue is empty")
```

Complexity? $O(1)$



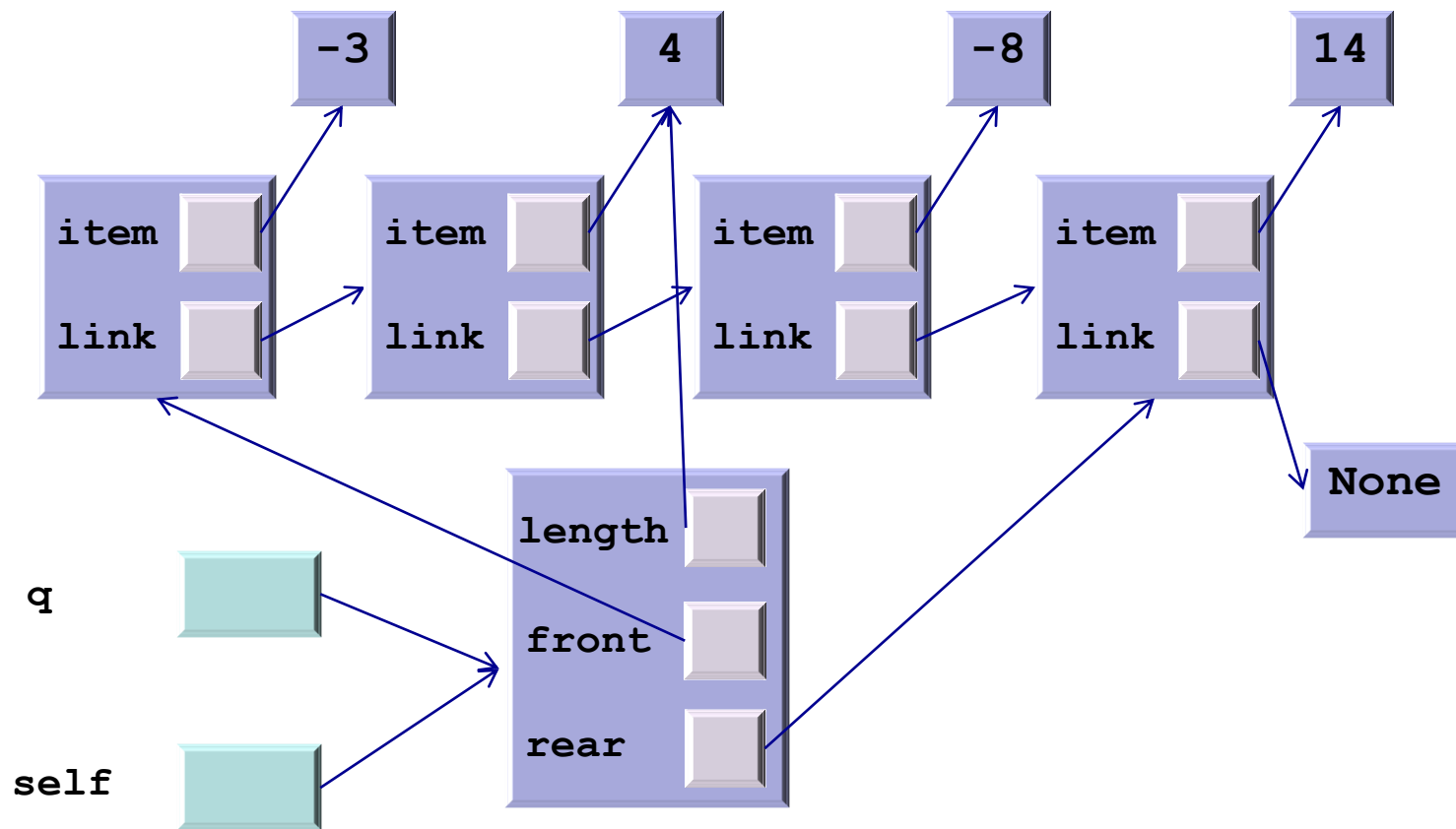
`q.serve()`

```
def serve(self) -> T:
    if not self.is_empty():
        item = self.front.item # store the item to serve
        self.front = self.front.link # move front
        self.length -= 1
        if self.is_empty(): # if now empty
            self.rear = None # move rear
        return item
    else:
        raise ValueError("Queue is empty")
```



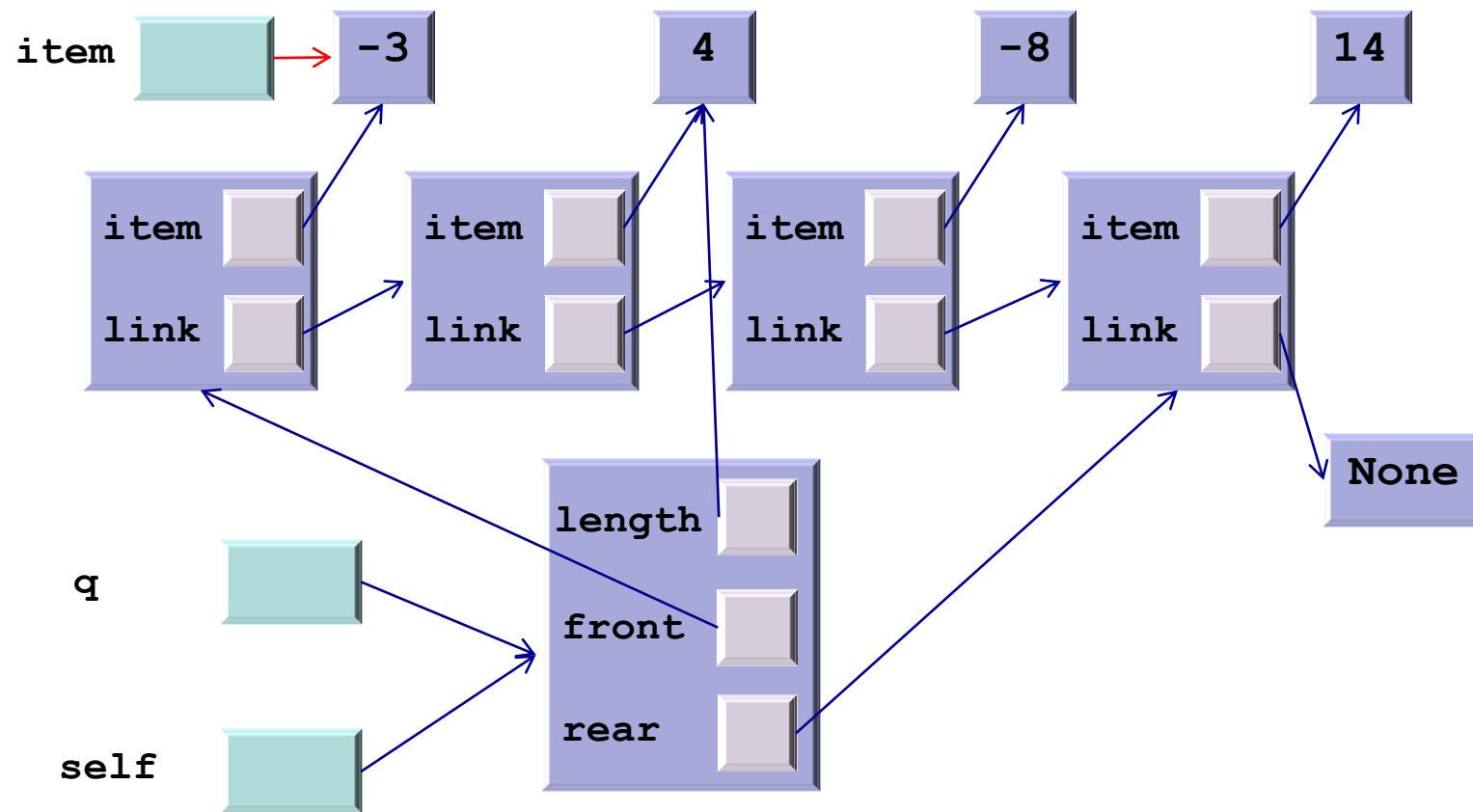
q. serve()

```
def serve(self) -> T:
    if not self.is_empty():
        item = self.front.item # store the item to serve
        self.front = self.front.link # move front
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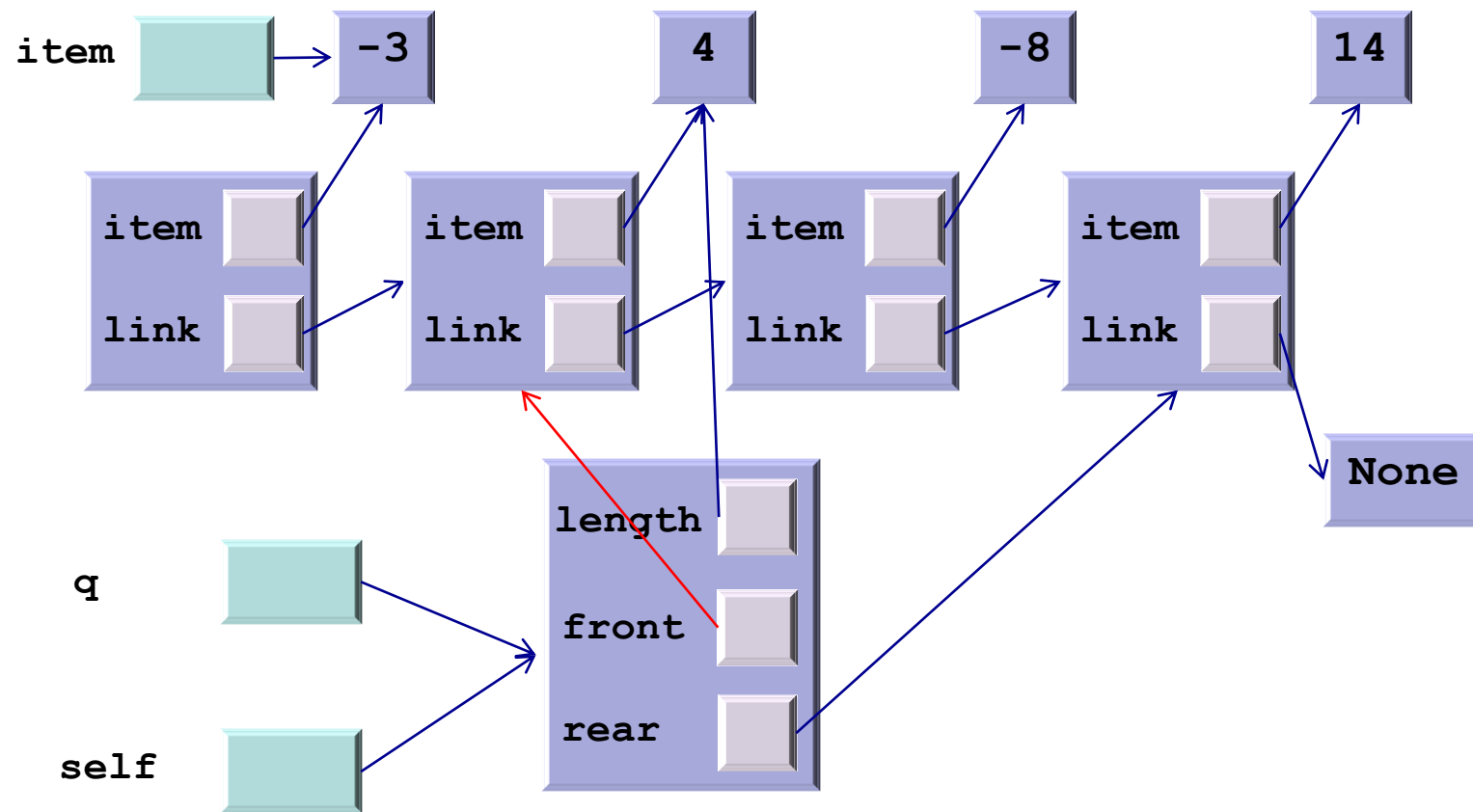
q. serve()

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    if not self.is_empty():
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        self.length -= 1
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            self.rear = None # move rear
        return item
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        raise ValueError("Queue is empty")
```



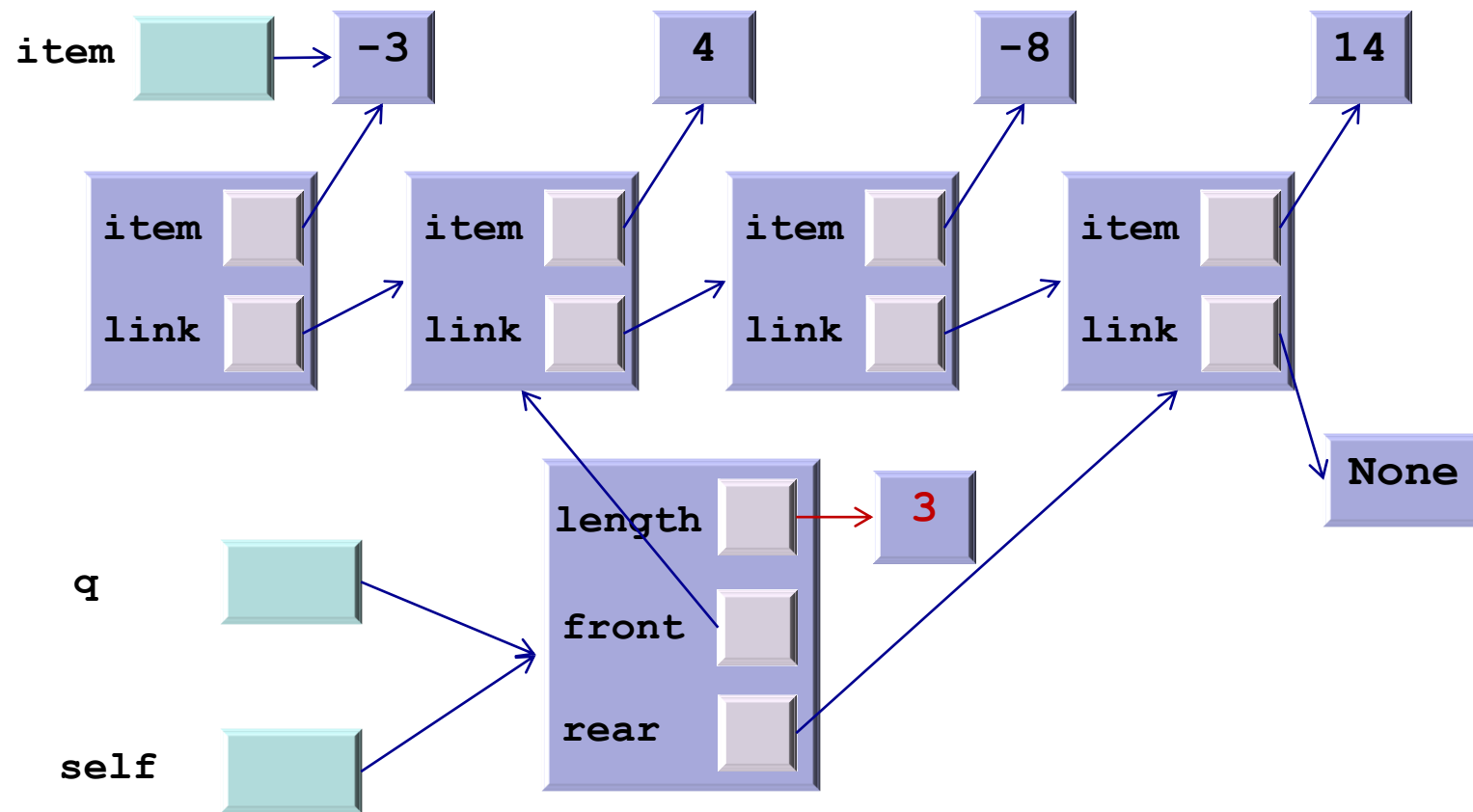
q.serve()

```
def serve(self) -> T:
    if not self.is_empty():
        item = self.front.item # store the item to serve
        self.front = self.front.link # move front
        self.length -= 1
        if self.is_empty(): # if now empty
            self.rear = None # move rear
        return item
    else:
        raise ValueError("Queue is empty")
```



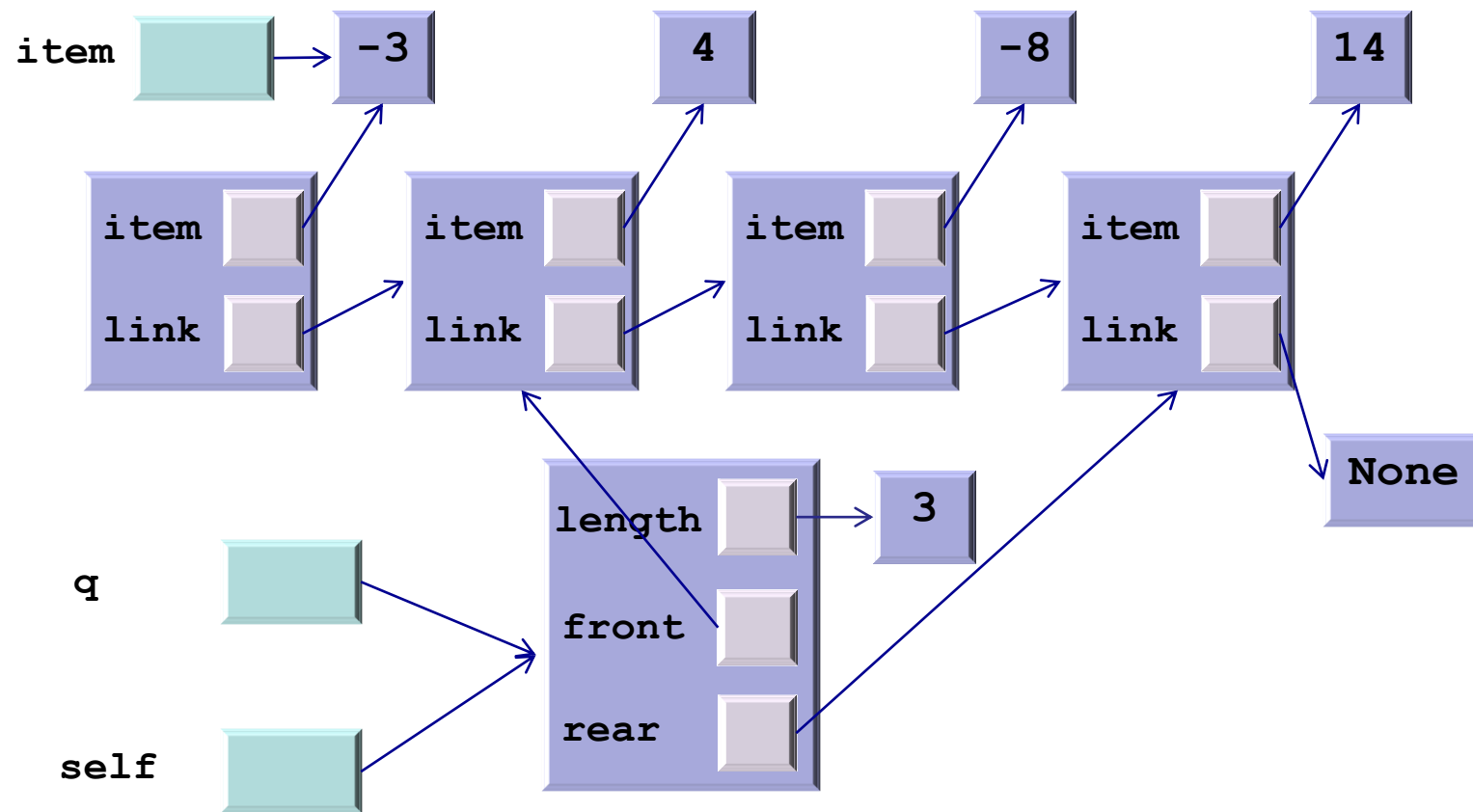
q.serve()

```
def serve(self) -> T:
    if not self.is_empty():
        item = self.front.item # store the item to serve
        self.front = self.front.link # move front
        self.length -= 1
        if self.is_empty(): # if now empty
            self.rear = None # move rear
        return item
    else:
        raise ValueError("Queue is empty")
```



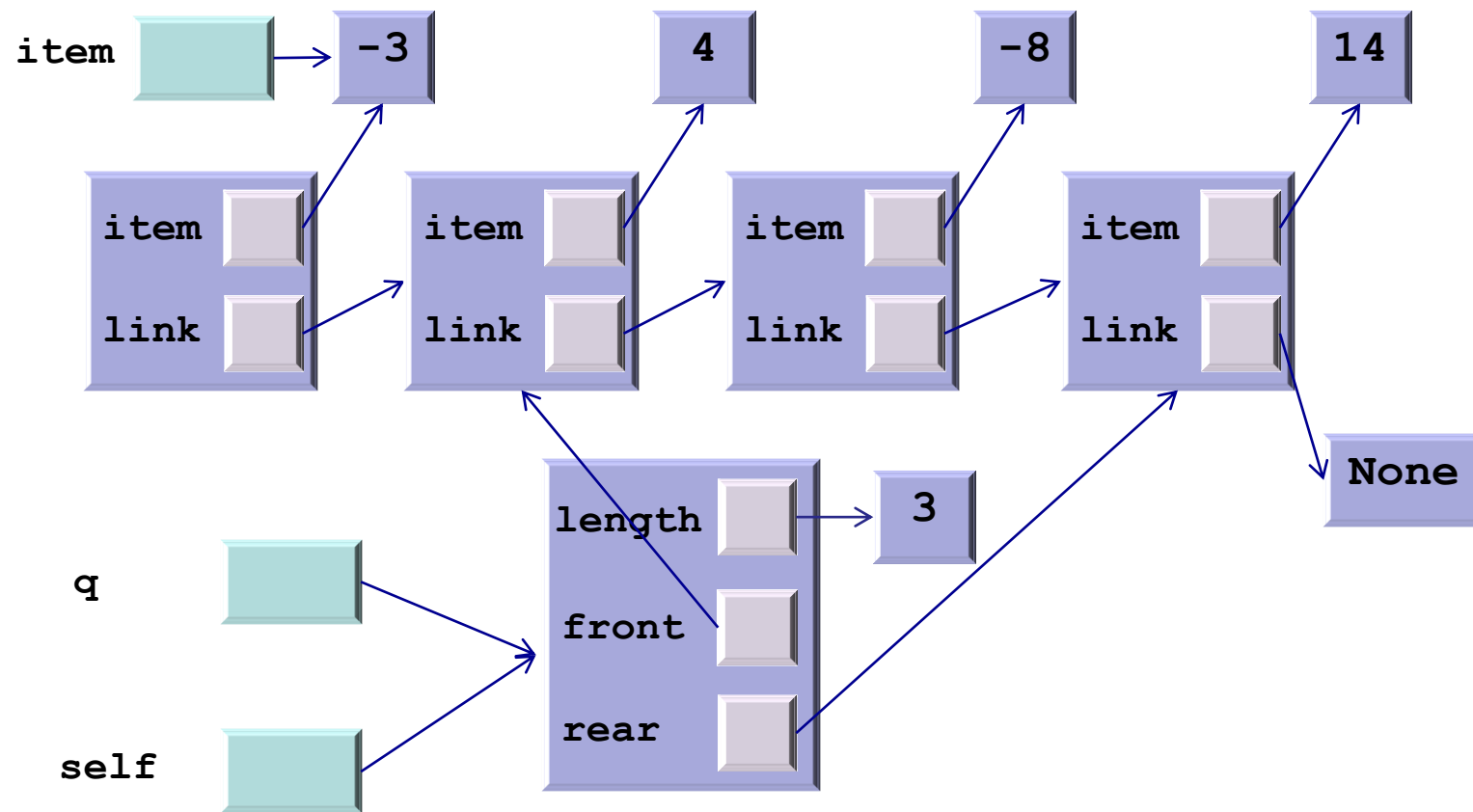
`q.serve()`

```
def serve(self) -> T:
    if not self.is_empty():
        item = self.front.item # store the item to serve
        self.front = self.front.link # move front
        self.length -= 1
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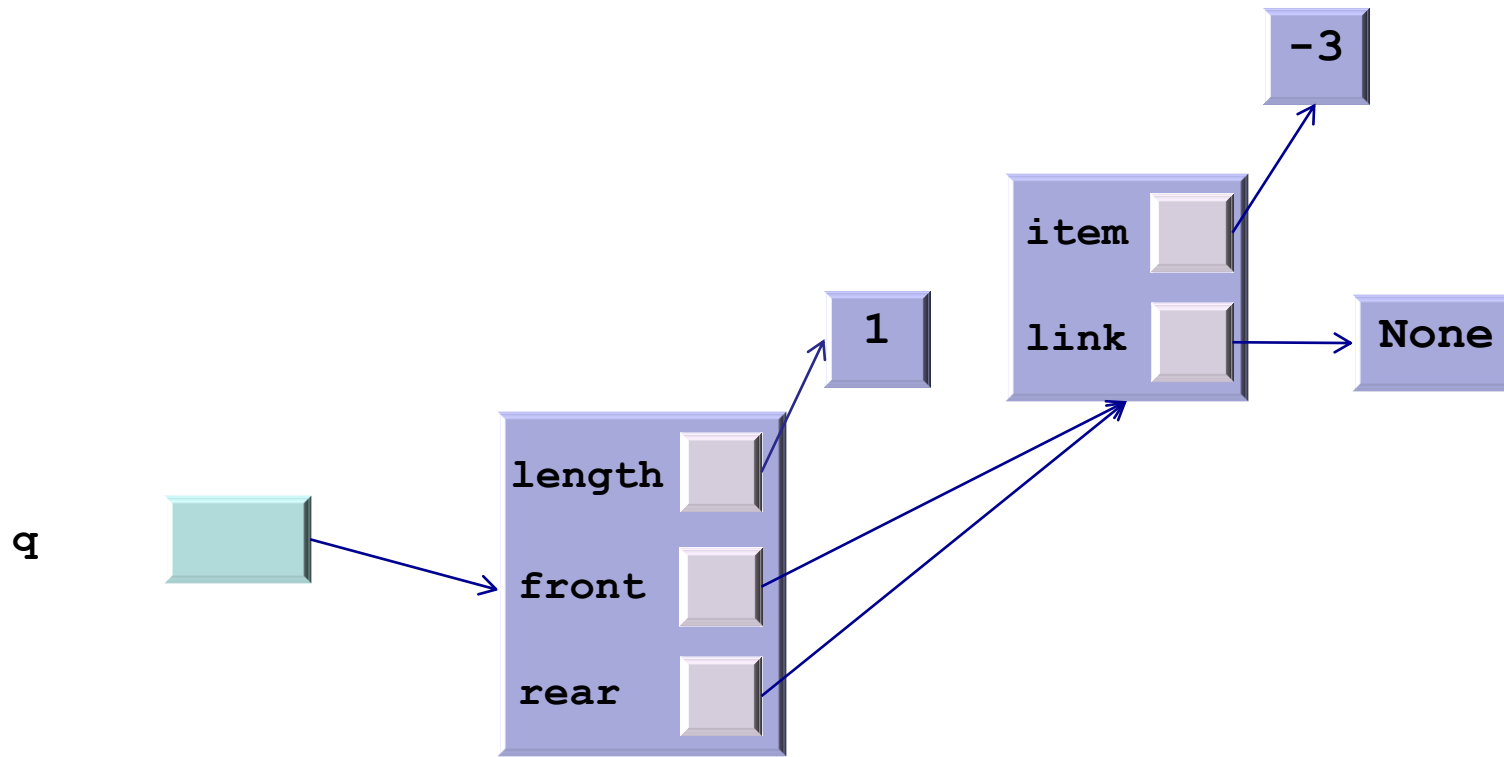
q.serve()

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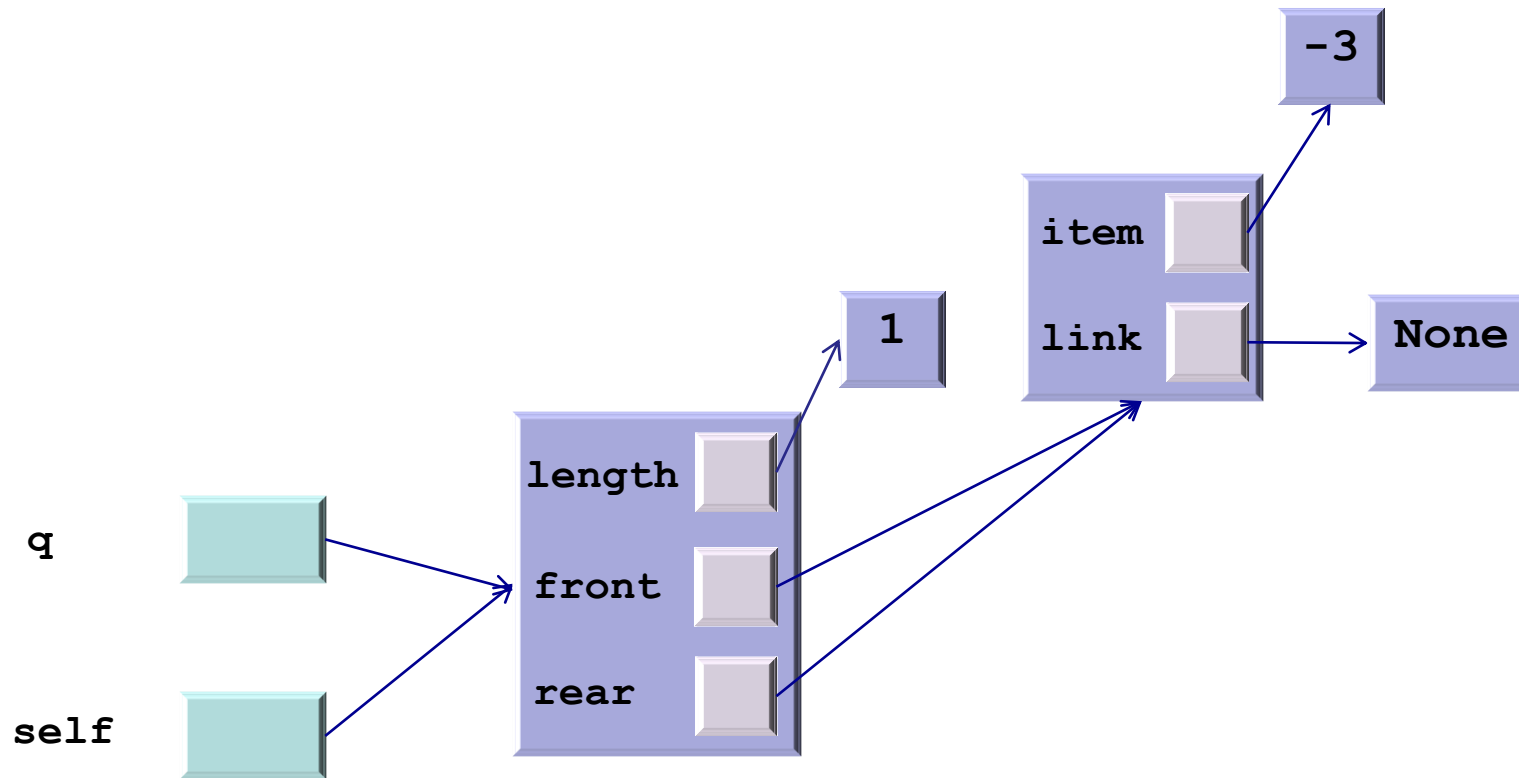
q.serve()

```
def serve(self) -> T:
    if not self.is_empty():
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        self.length -= 1
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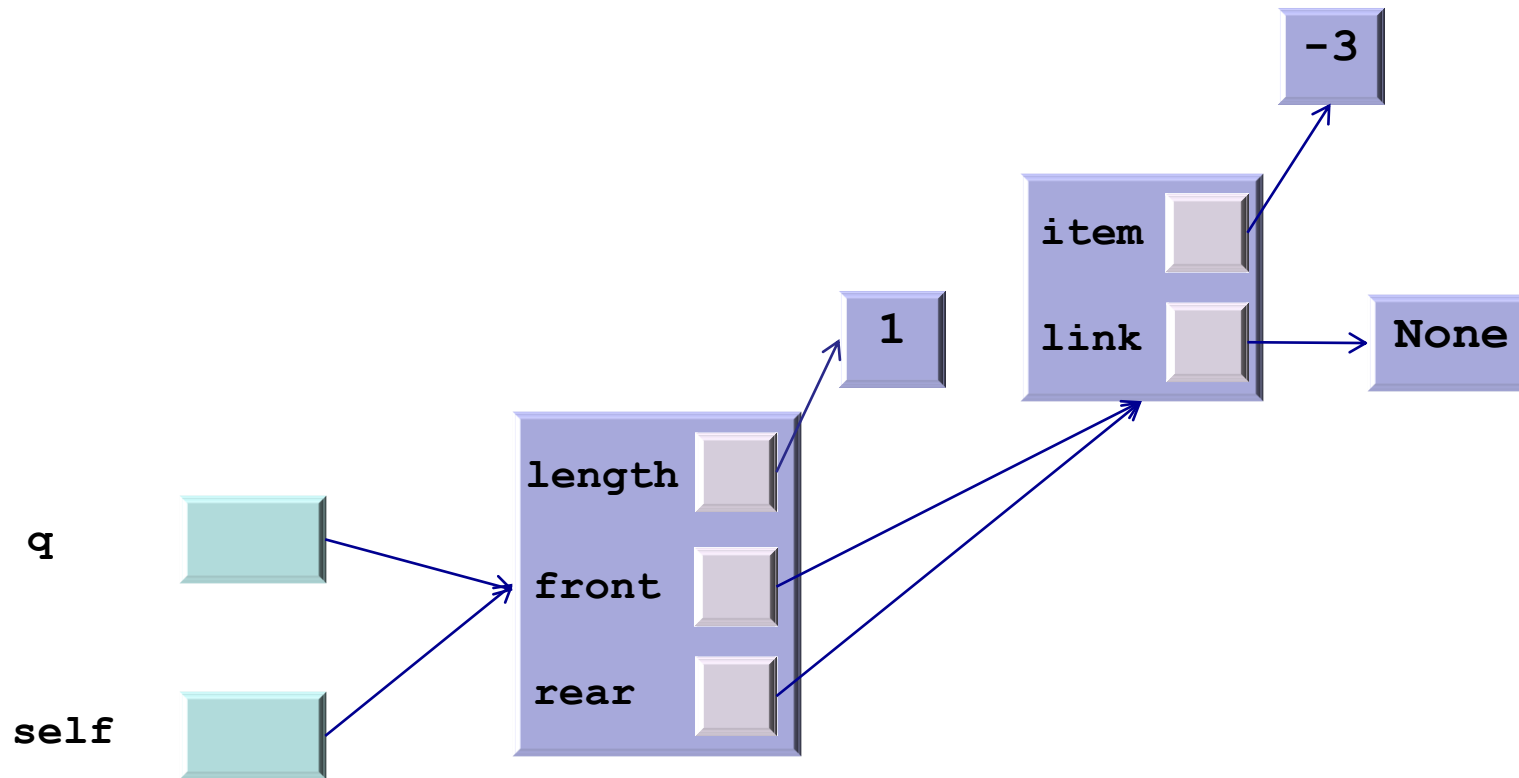
`q.serve()`

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        return item
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```



q. serve()

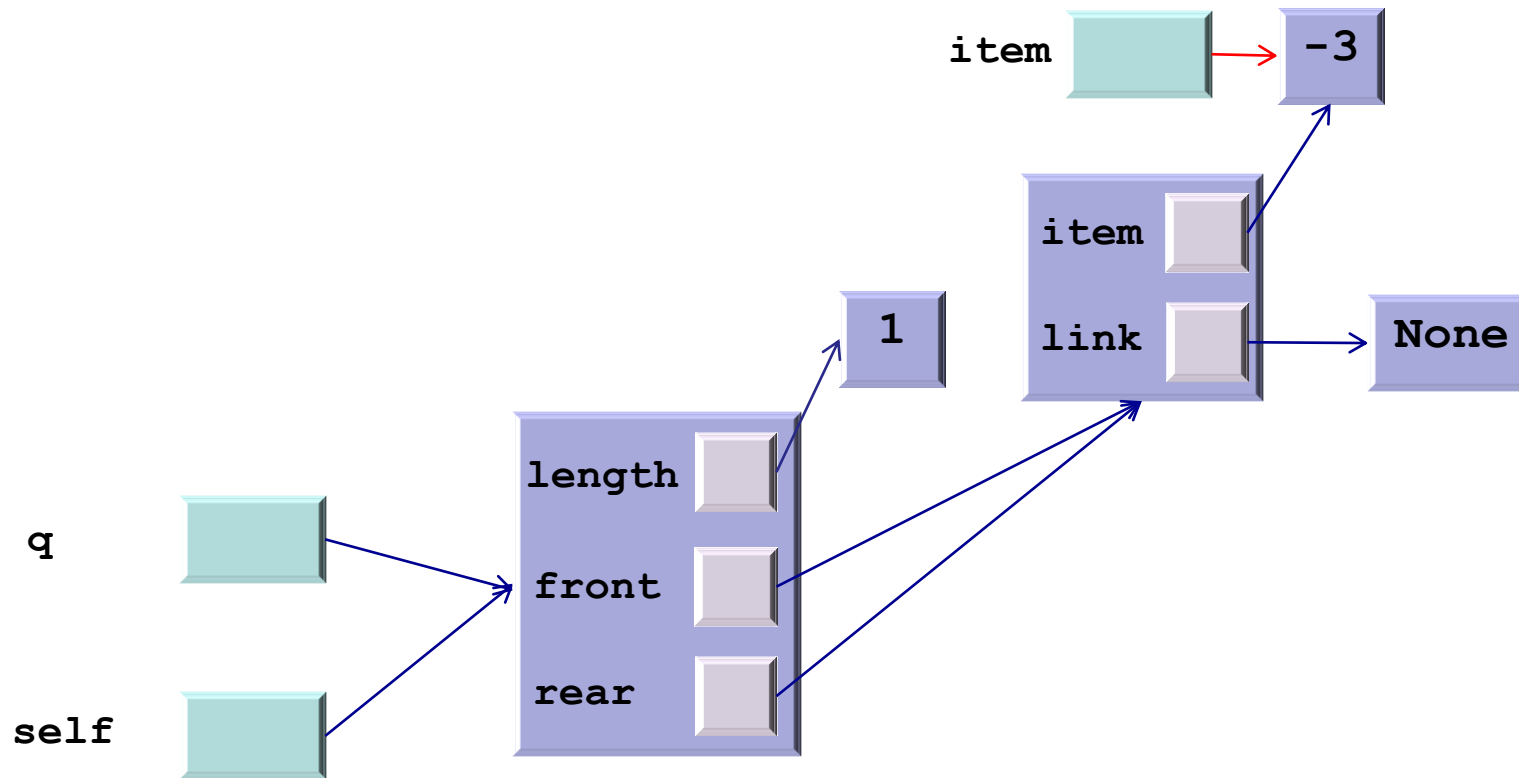
```
def serve(self) -> T:
    if not self.is_empty():
        item = self.front.item # store the item to serve
        self.front = self.front.link # move front
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q.serve()

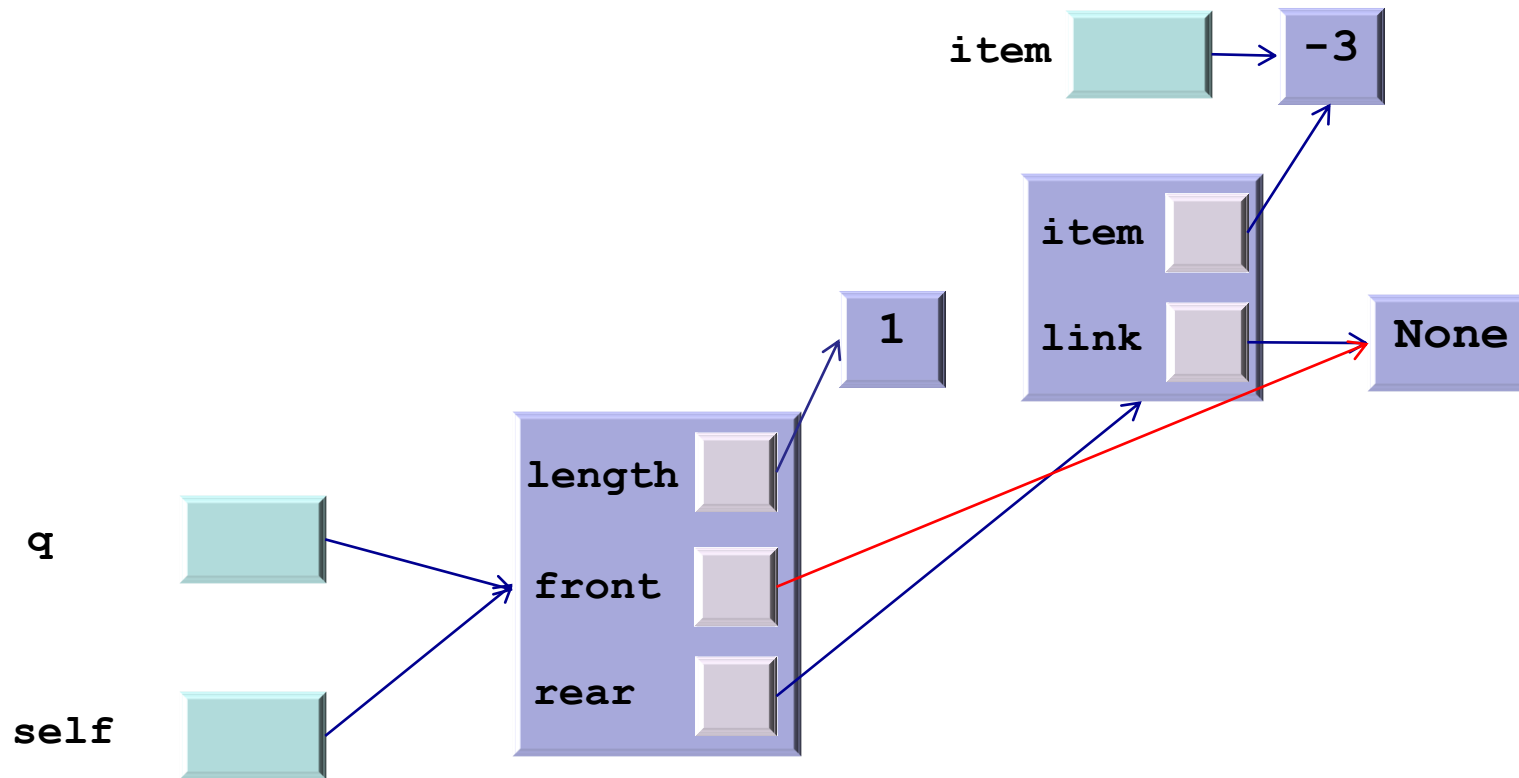
```

def serve(self) -> T:
    if not self.is_empty():
        item = self.front.item # store the item to serve
        self.front = self.front.link # move front
        self.length -= 1
        if self.is_empty(): # if now empty
            self.rear = None # move rear
        return item
    else:
        raise ValueError("Queue is empty")
  
```



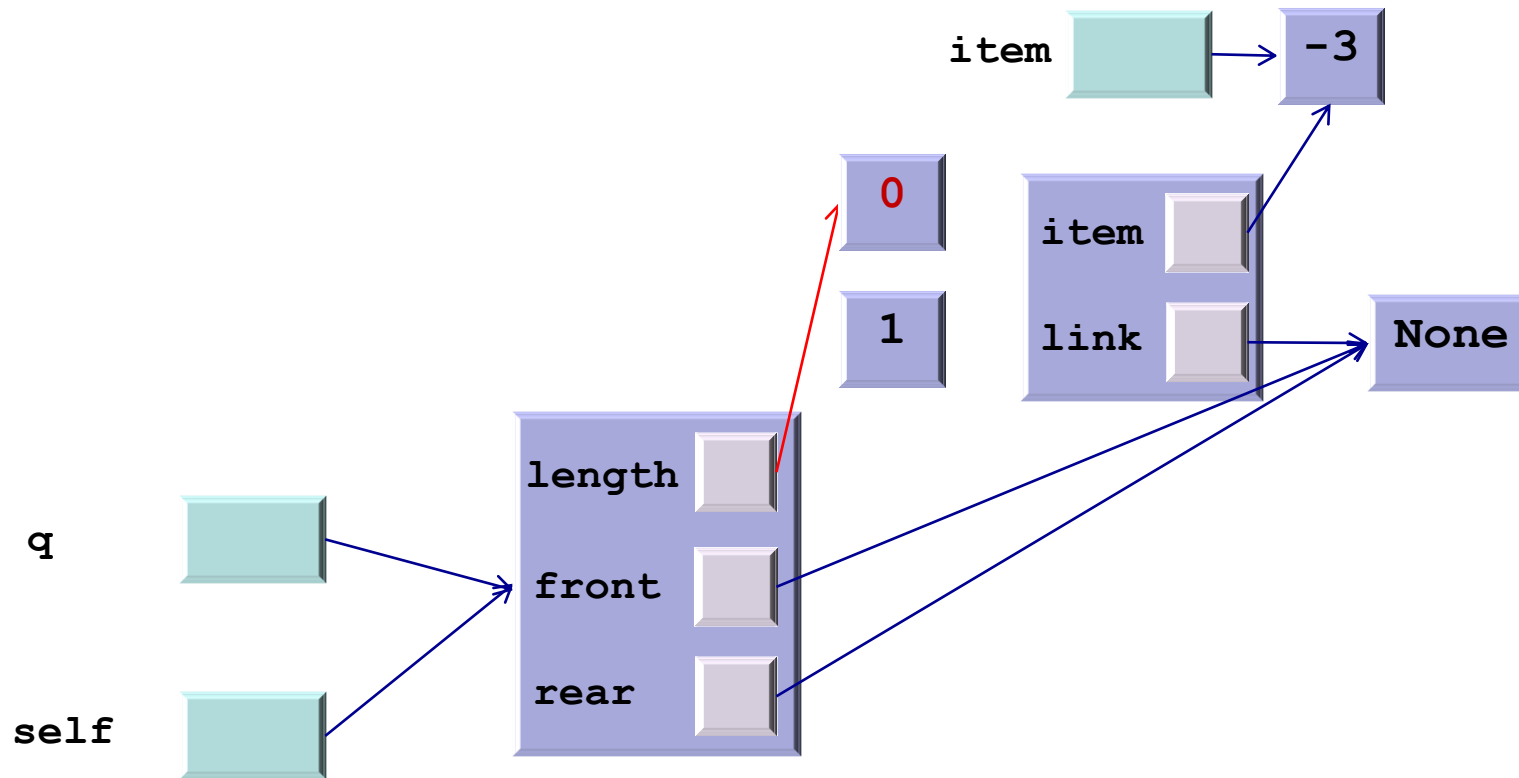
`q.serve()`

```
def serve(self) -> T:
    if not self.is_empty():
        item = self.front.item # store the item to serve
        self.front = self.front.link # move front
        self.length -= 1
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```



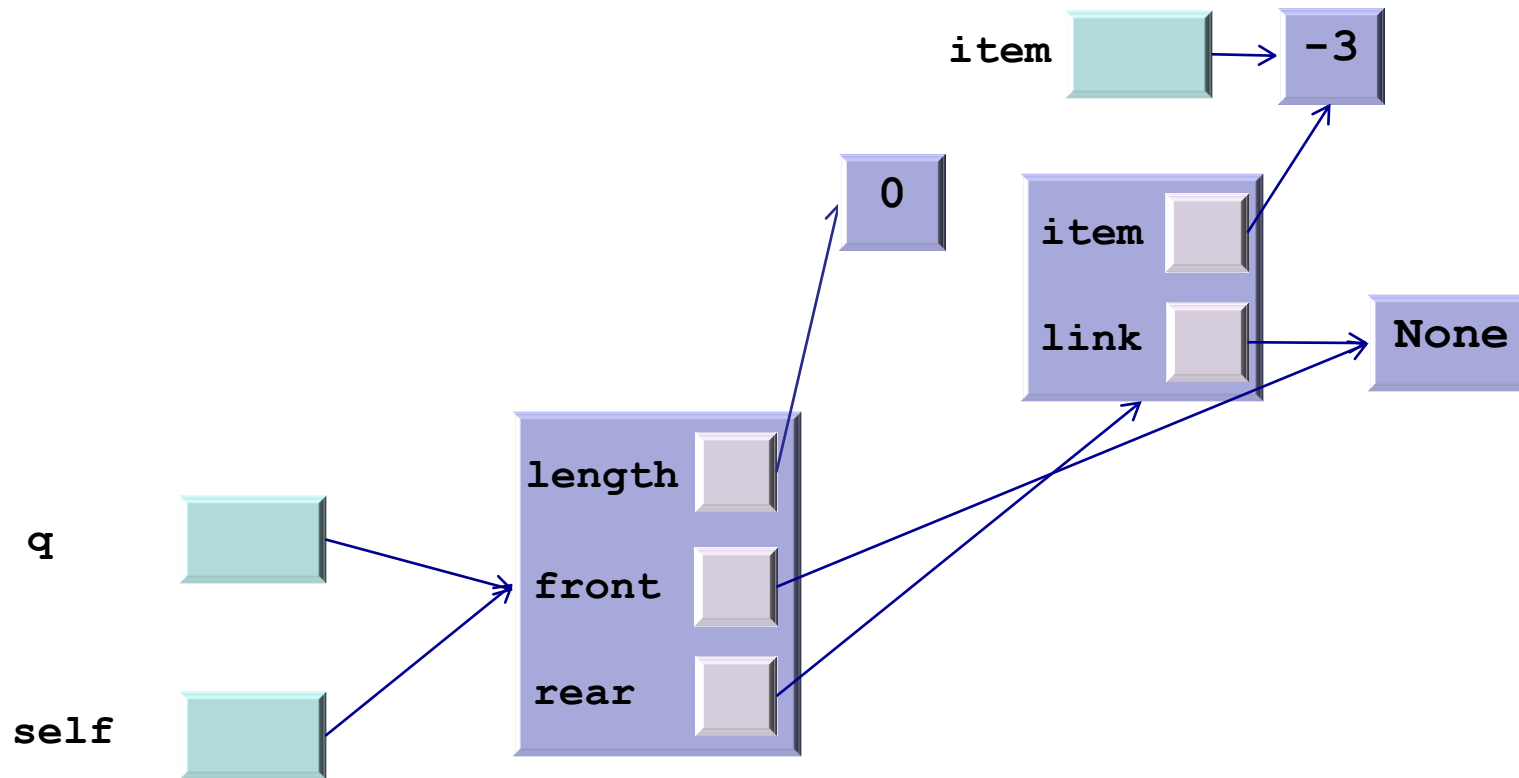
q. serve()

```
def serve(self) -> T:
    if not self.is_empty():
        item = self.front.item # store the item to serve
        self.front = self.front.link # move front
        self.length -= 1
        if self.is_empty(): # if now empty
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q. serve()

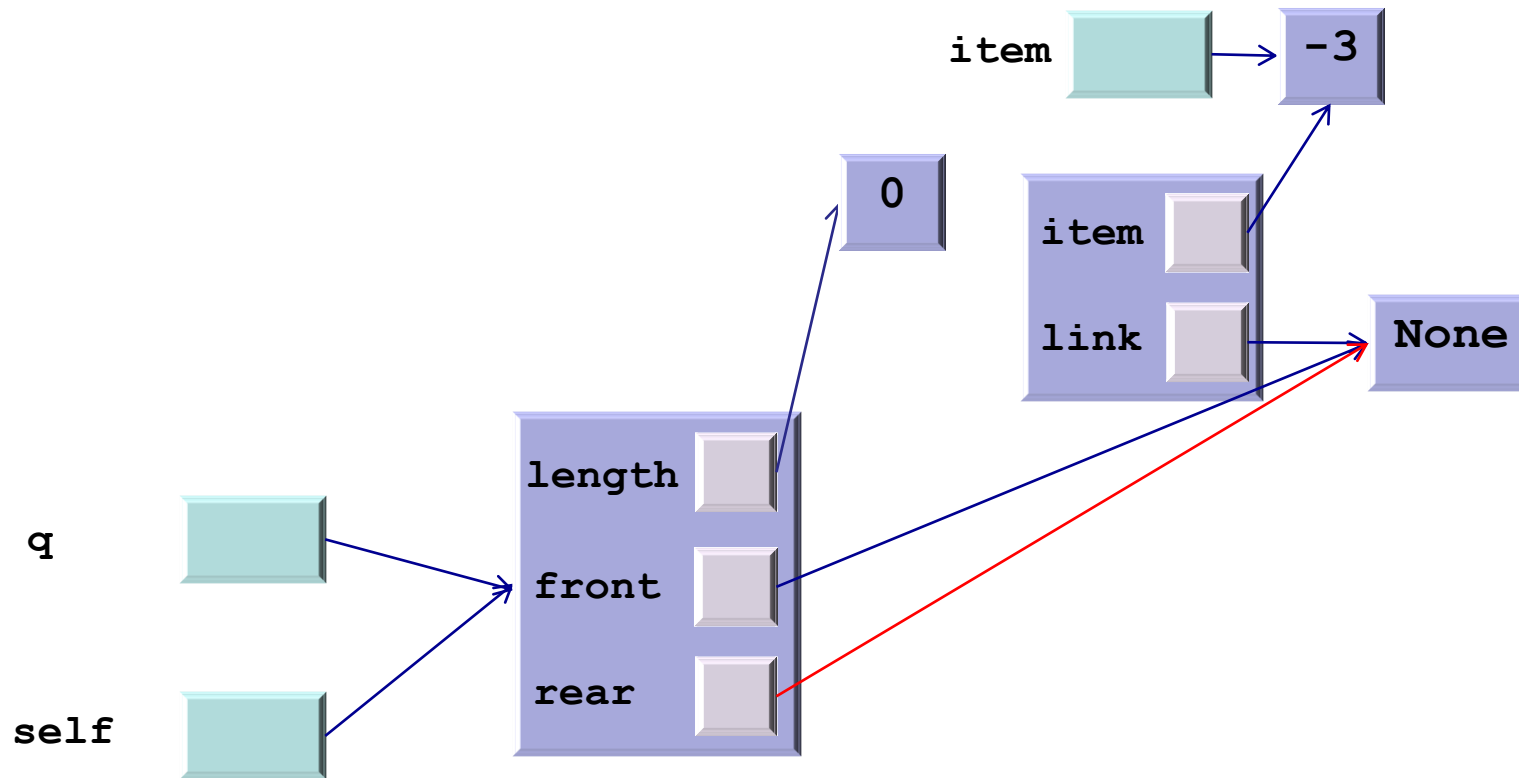
```
def serve(self) -> T:
    if not self.is_empty():
        item = self.front.item # store the item to serve
        self.front = self.front.link # move front
        self.length -= 1
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q. serve()

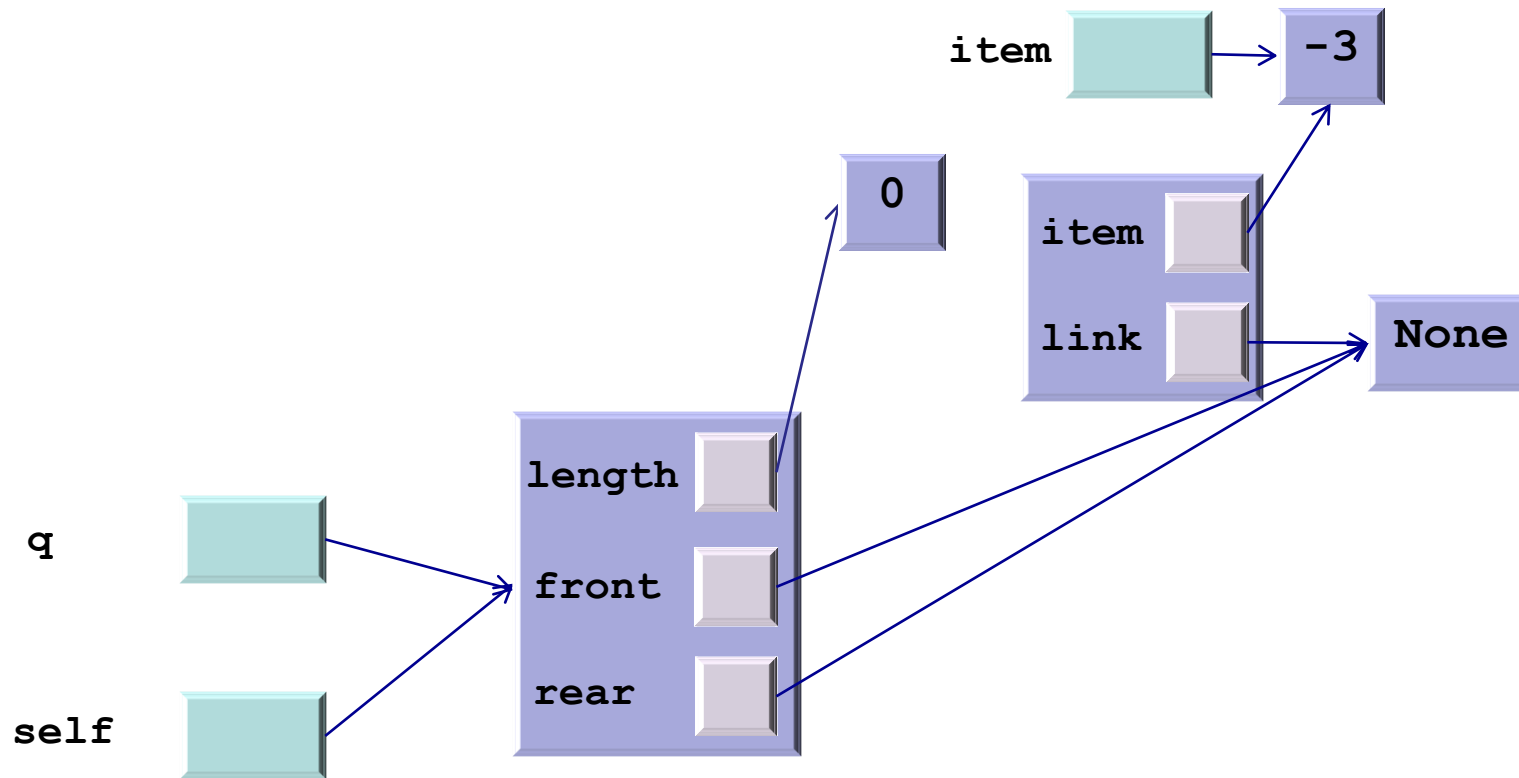
```

def serve(self) -> T:
    if not self.is_empty():
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        self.front = self.front.link # move front
        self.length -= 1
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q. serve()

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def serve(self) -> T:
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q. serve()

```
def serve(self) -> T:
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        self.length -= 1
        if self.is_empty(): # if now empty
            self.rear = None # move rear
        return item
    else:
        raise ValueError("Queue is empty")
```



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Using LinkQueue

Exercise: add to **LinkQueue** a method that halves the queue by deleting the nodes at index 1, 3, 5, etc

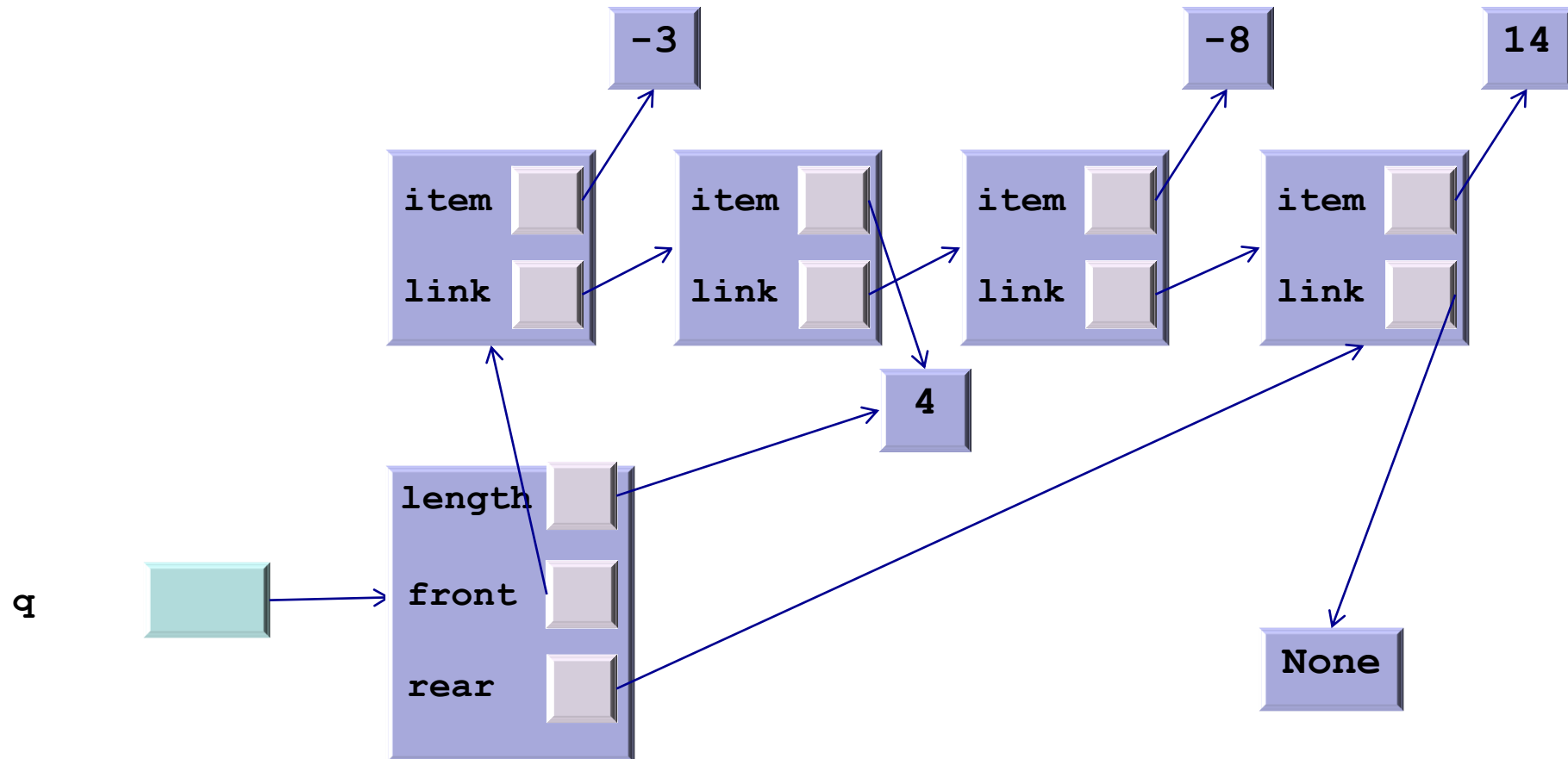
```
def halve(self) -> None:
    current = self.front
    # while at least two elements not traversed in the queue
    while current is not None and current.link is not None:
        if current.link is self.rear: # if even node is last
            self.rear = current # move rear up
        current.link = current.link.link # bypass odd node
        current = current.link # keep on traversing - next two
    length -= 1
```

```

def halve(self) -> None:
    current = self.front
    while current is not None and current.link is not None:
        if current.link is self.rear:
            self.rear = current
        current.link = current.link.link
        current = current.link
    length -= 1

```

q.halve()



```
def halve(self) -> None:
```

```
    current = self.front
```

```
    while current is not None and current.link is not None:
```

```
        if current.link is self.rear:
```

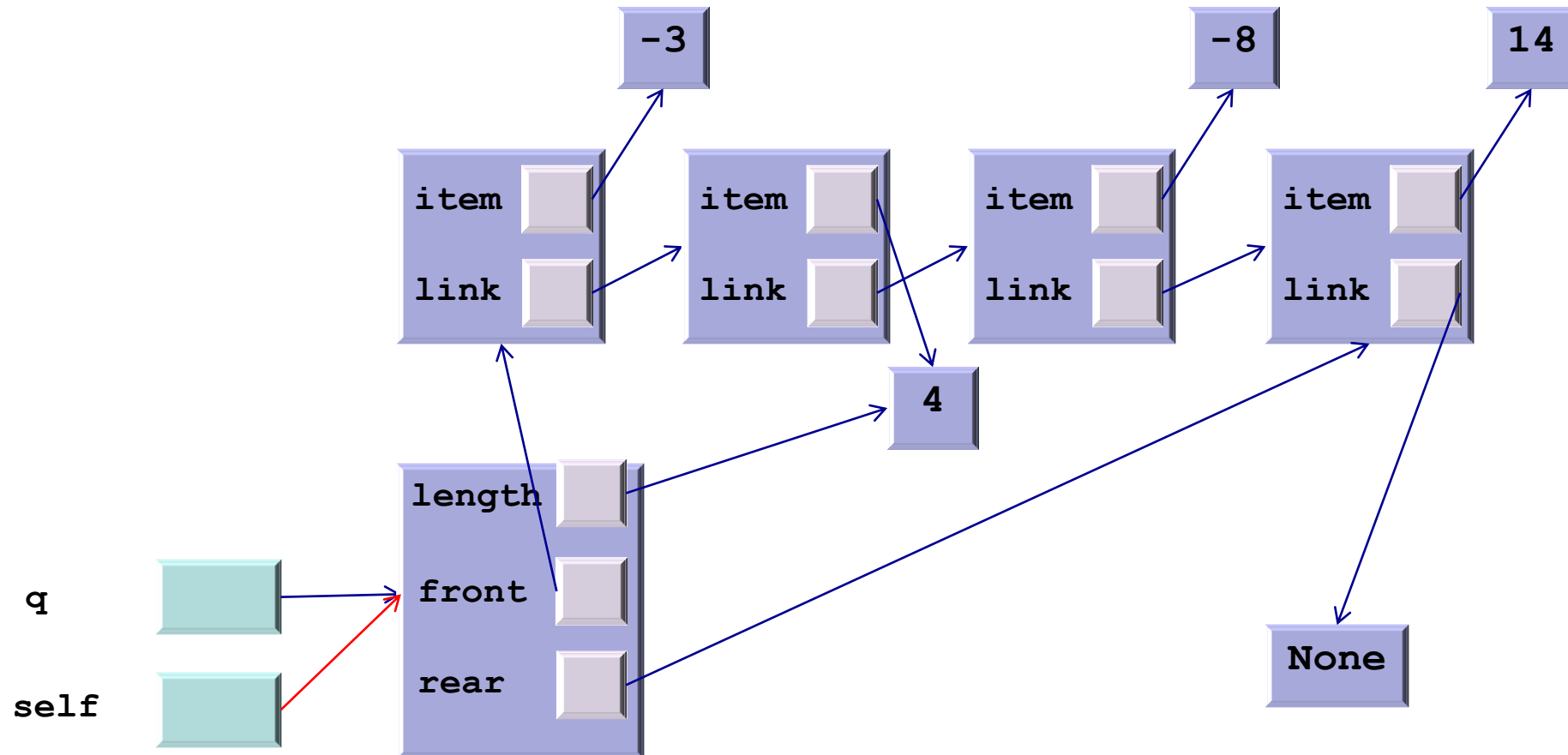
```
            self.rear = current
```

```
            current.link = current.link.link
```

```
            current = current.link
```

```
            length -= 1
```

q.halve()

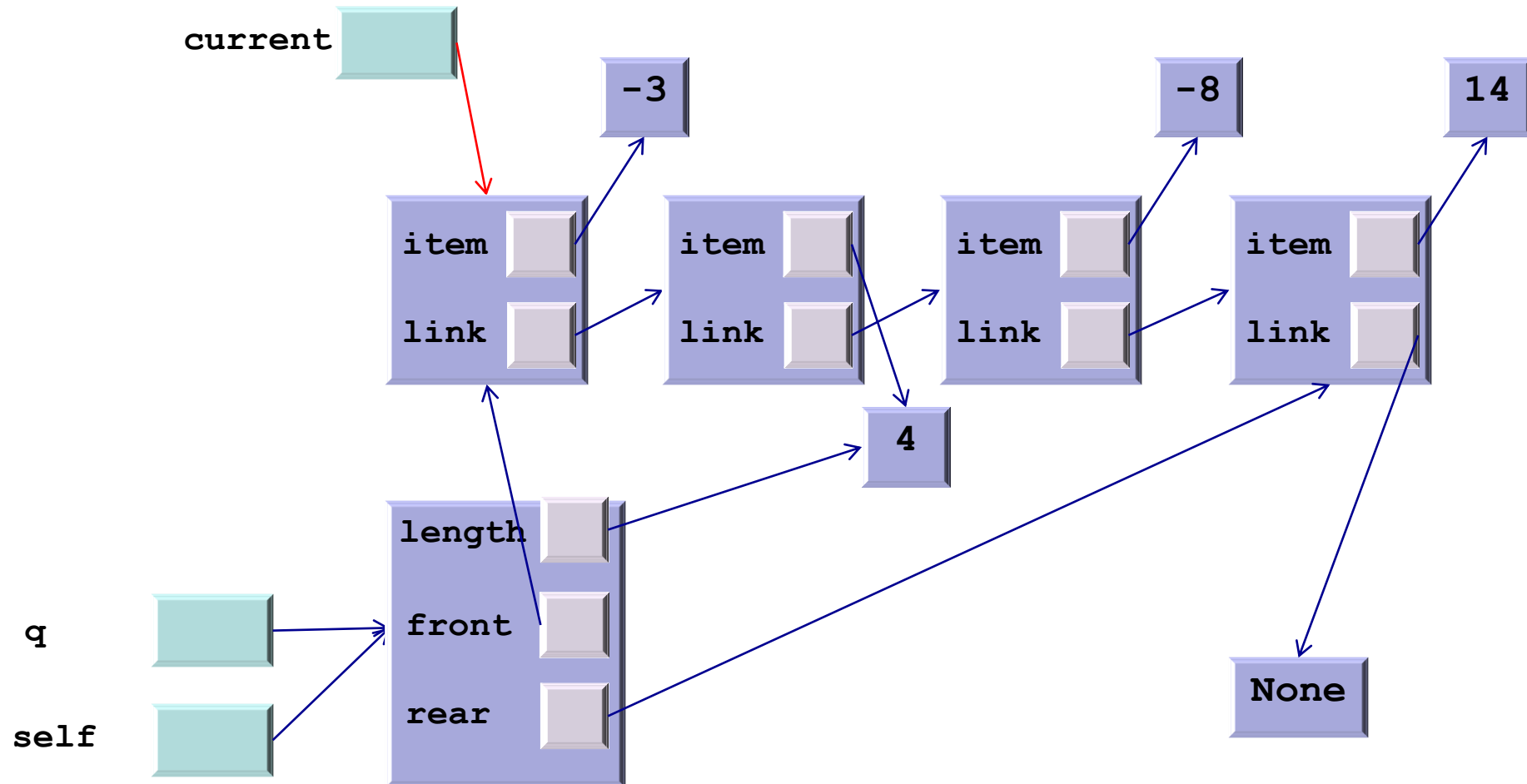


```

def halve(self) -> None:
    current = self.front
    while current is not None and current.link is not None:
        if current.link is self.rear:
            self.rear = current
        current.link = current.link.link
        current = current.link
        length -= 1

```

q.halve()

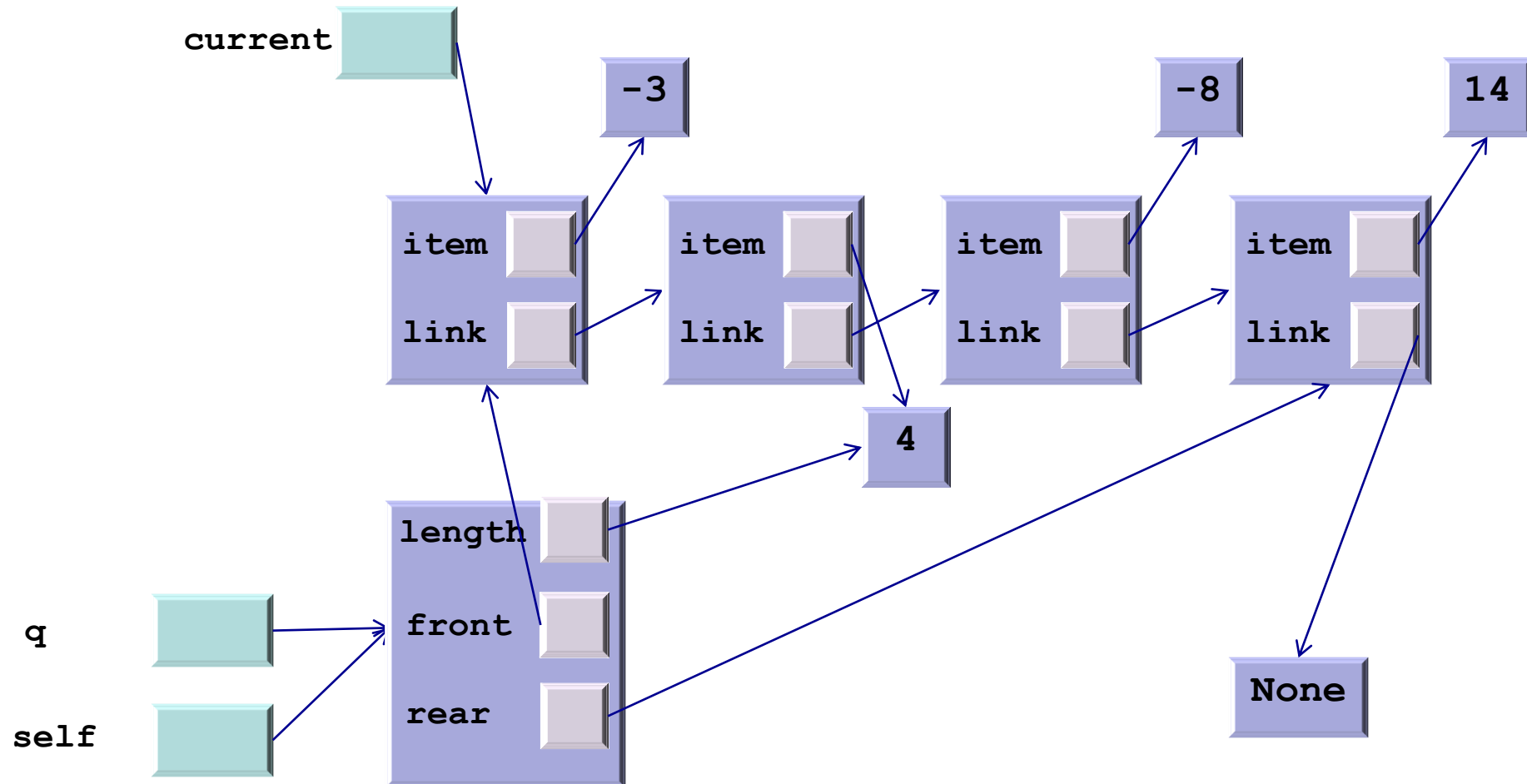


```

def halve(self) -> None:
    current = self.front
    while current is not None and current.link is not None:
        if current.link is self.rear:
            self.rear = current
        current.link = current.link.link
        current = current.link
    length -= 1

```

q.halve()

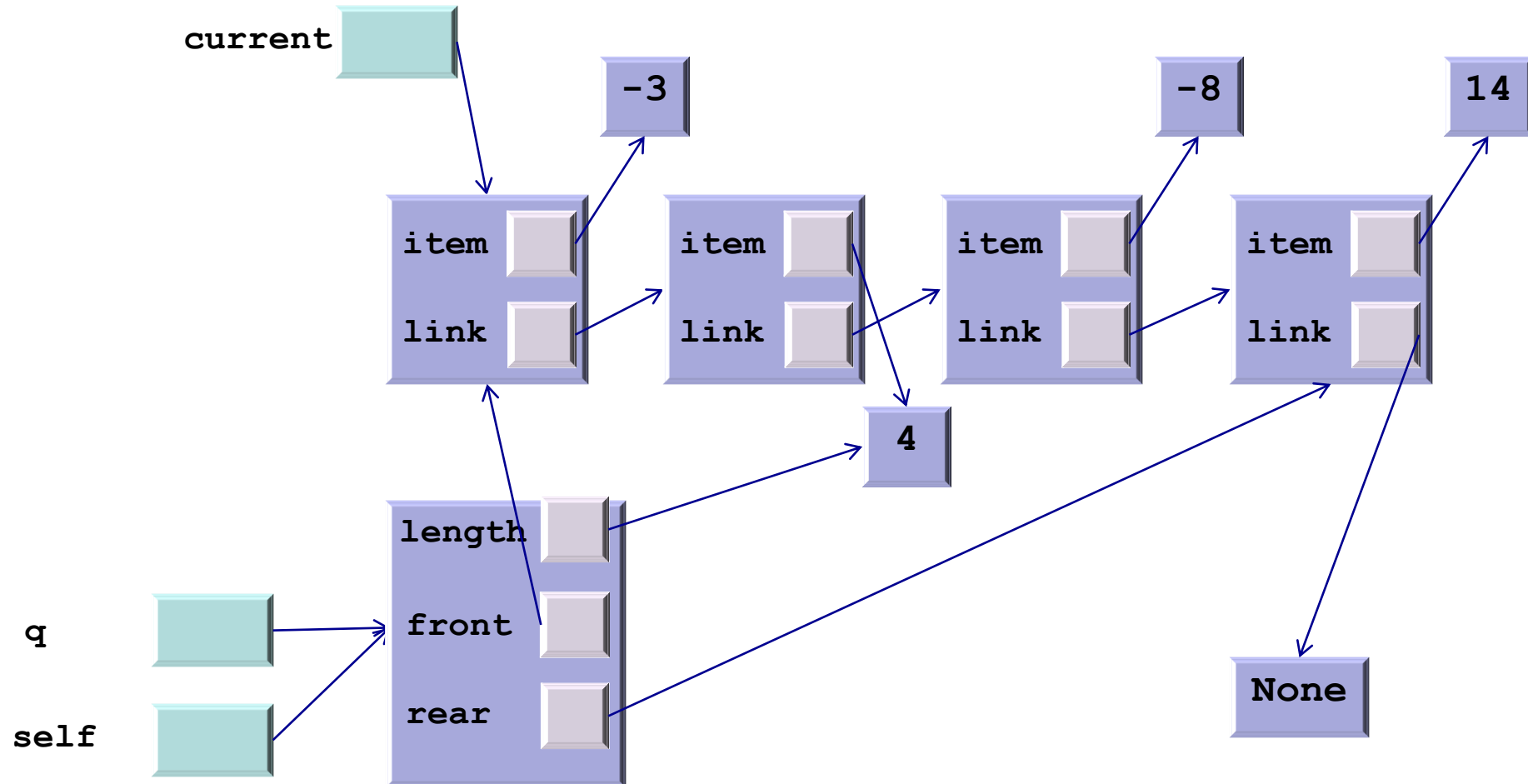


```

def halve(self) -> None:
    current = self.front
    while current is not None and current.link is not None:
        if current.link is self.rear:
            self.rear = current
        current.link = current.link.link
        current = current.link
    length -= 1

```

q.halve()

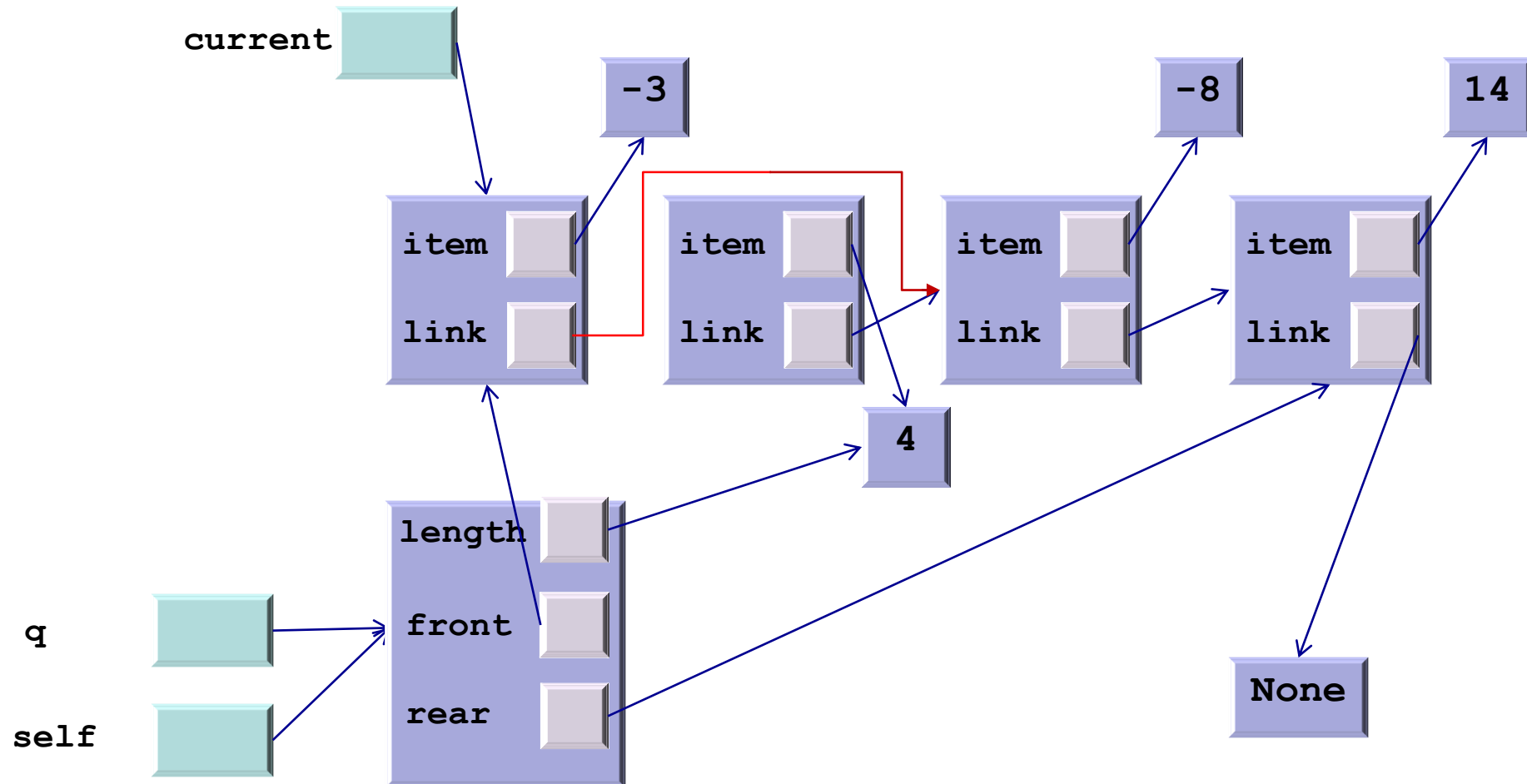


```

def halve(self) -> None:
    current = self.front
    while current is not None and current.link is not None:
        if current.link is self.rear:
            self.rear = current
            current.link = current.link.link
            current = current.link
        length -= 1

```

q.halve()

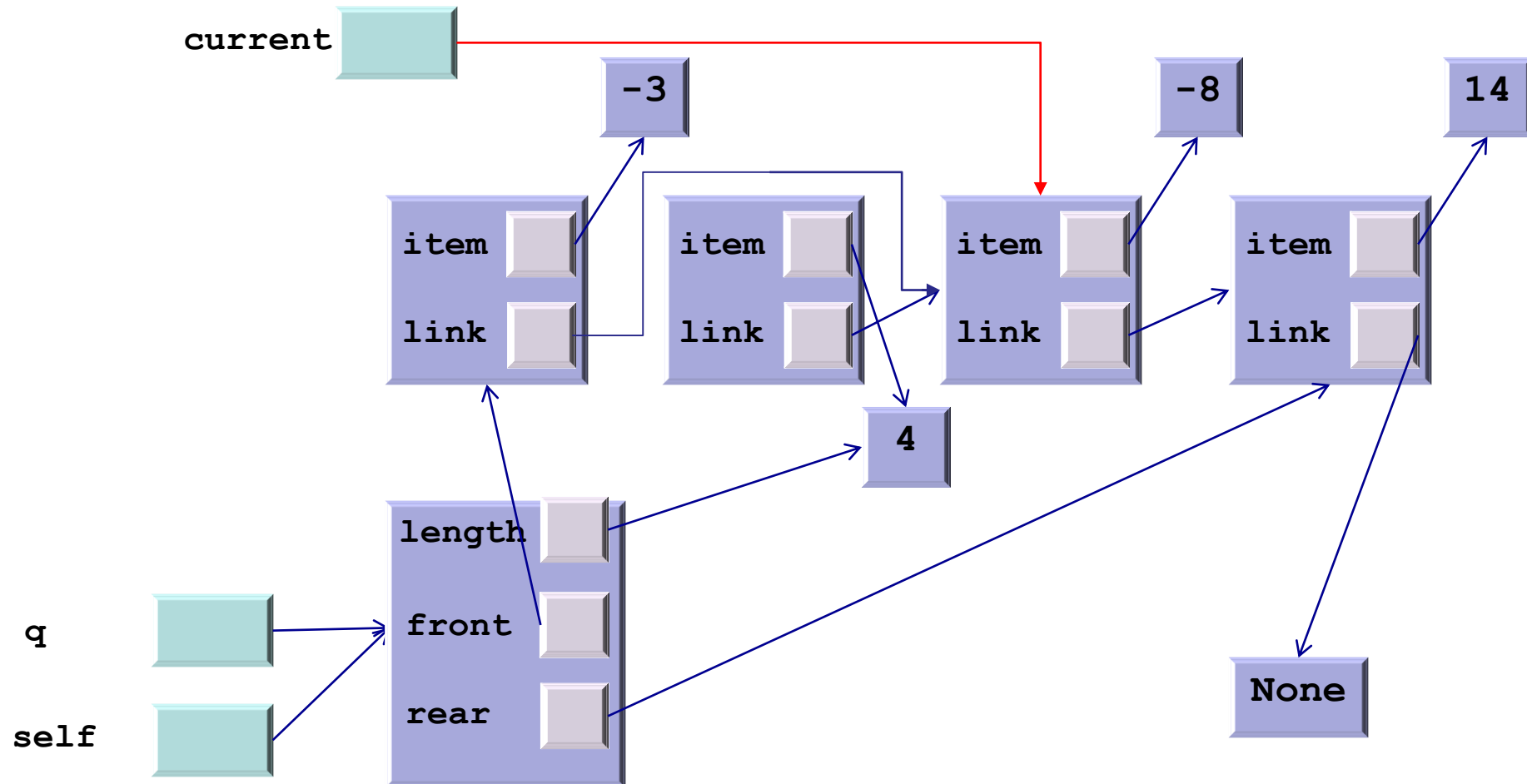


```

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    current = self.front
    while current is not None and current.link is not None:
        if current.link is self.rear:
            self.rear = current
        current.link = current.link.link
        current = current.link
        length -= 1

```

q.halve()

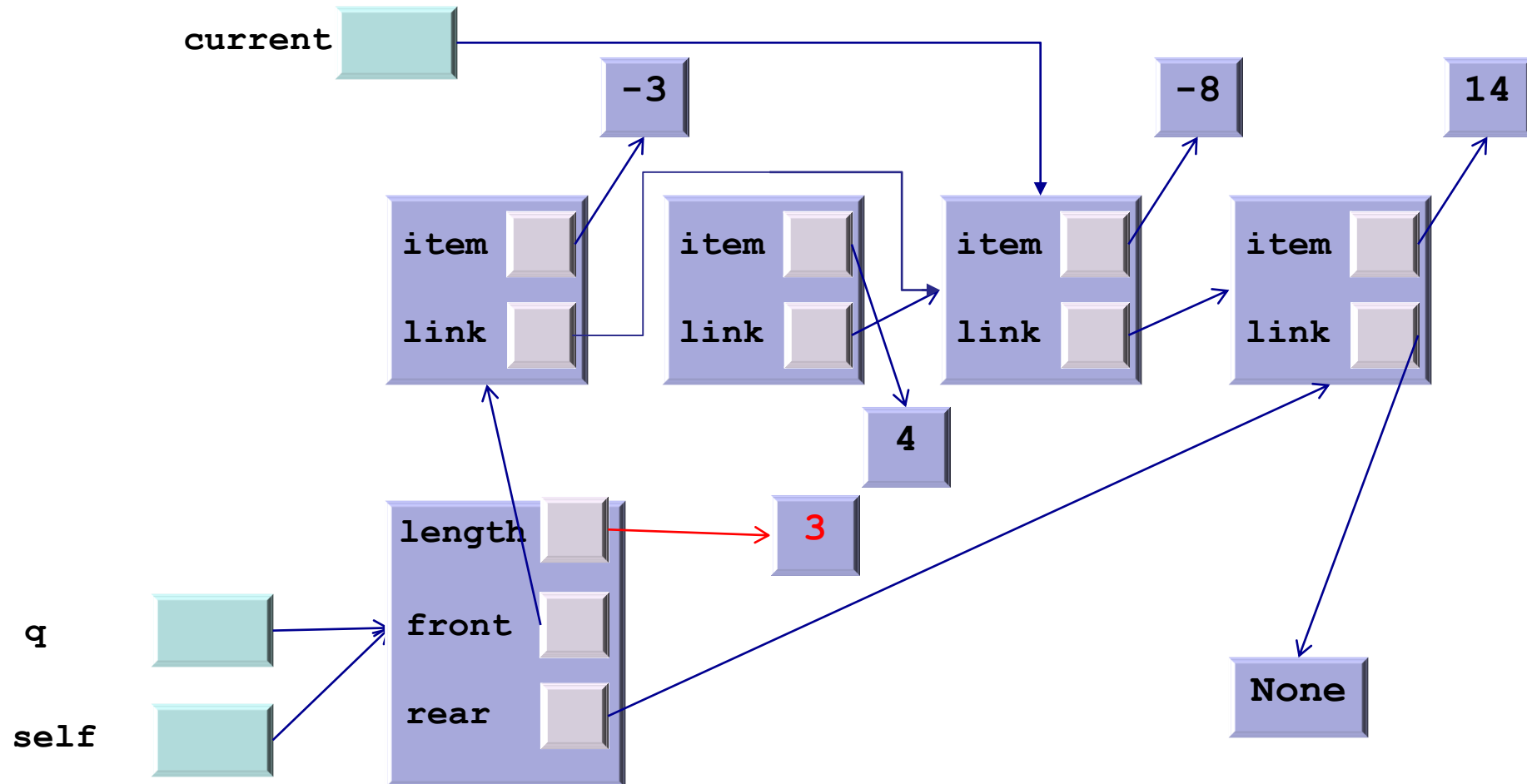



```

def halve(self) -> None:
    current = self.front
    while current is not None and current.link is not None:
        if current.link is self.rear:
            self.rear = current
        current.link = current.link.link
        current = current.link
    length -= 1

```

q.halve()

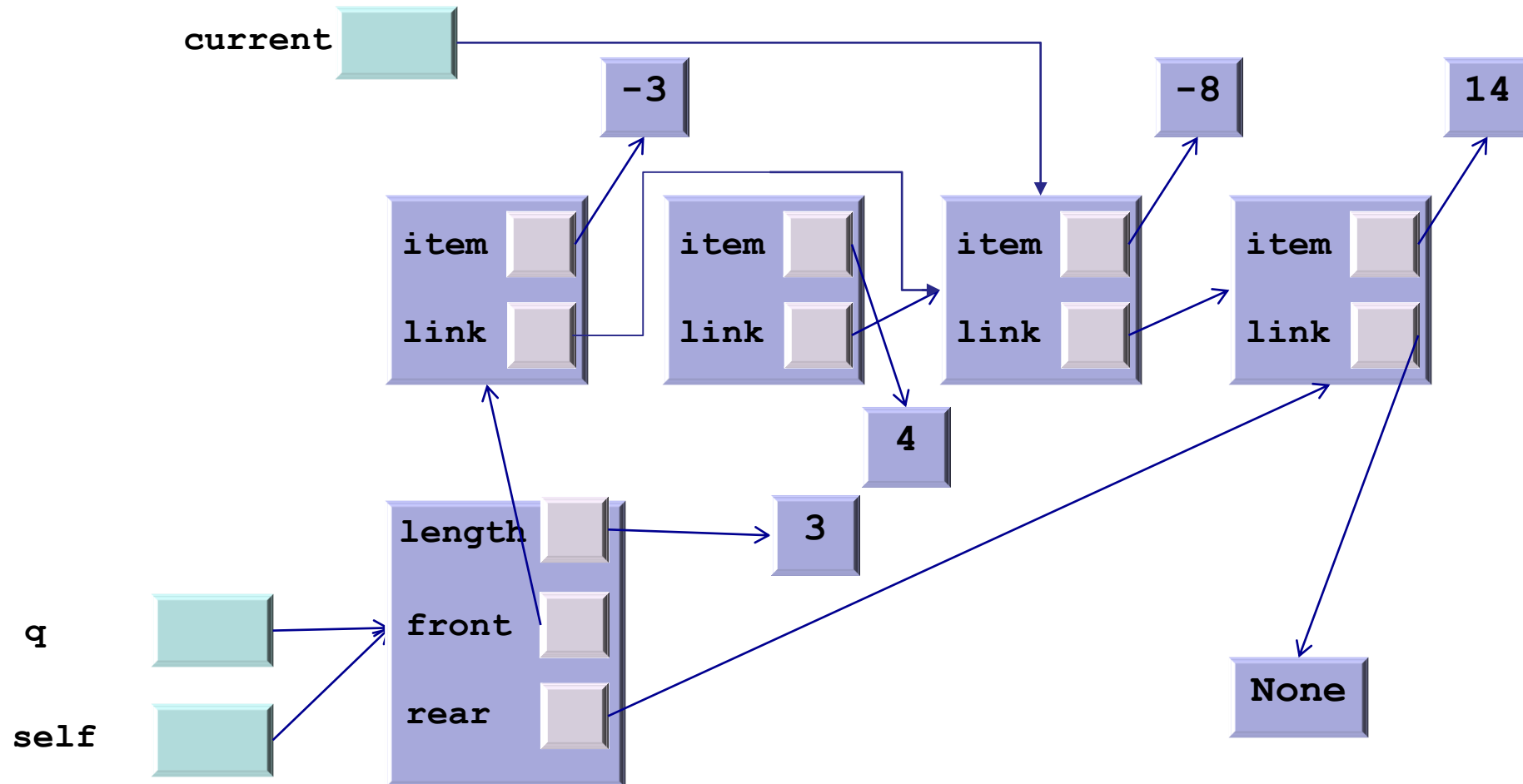


```

def halve(self) -> None:
    current = self.front
    while current is not None and current.link is not None:
        if current.link is self.rear:
            self.rear = current
        current.link = current.link.link
        current = current.link
        length -= 1

```

q.halve()

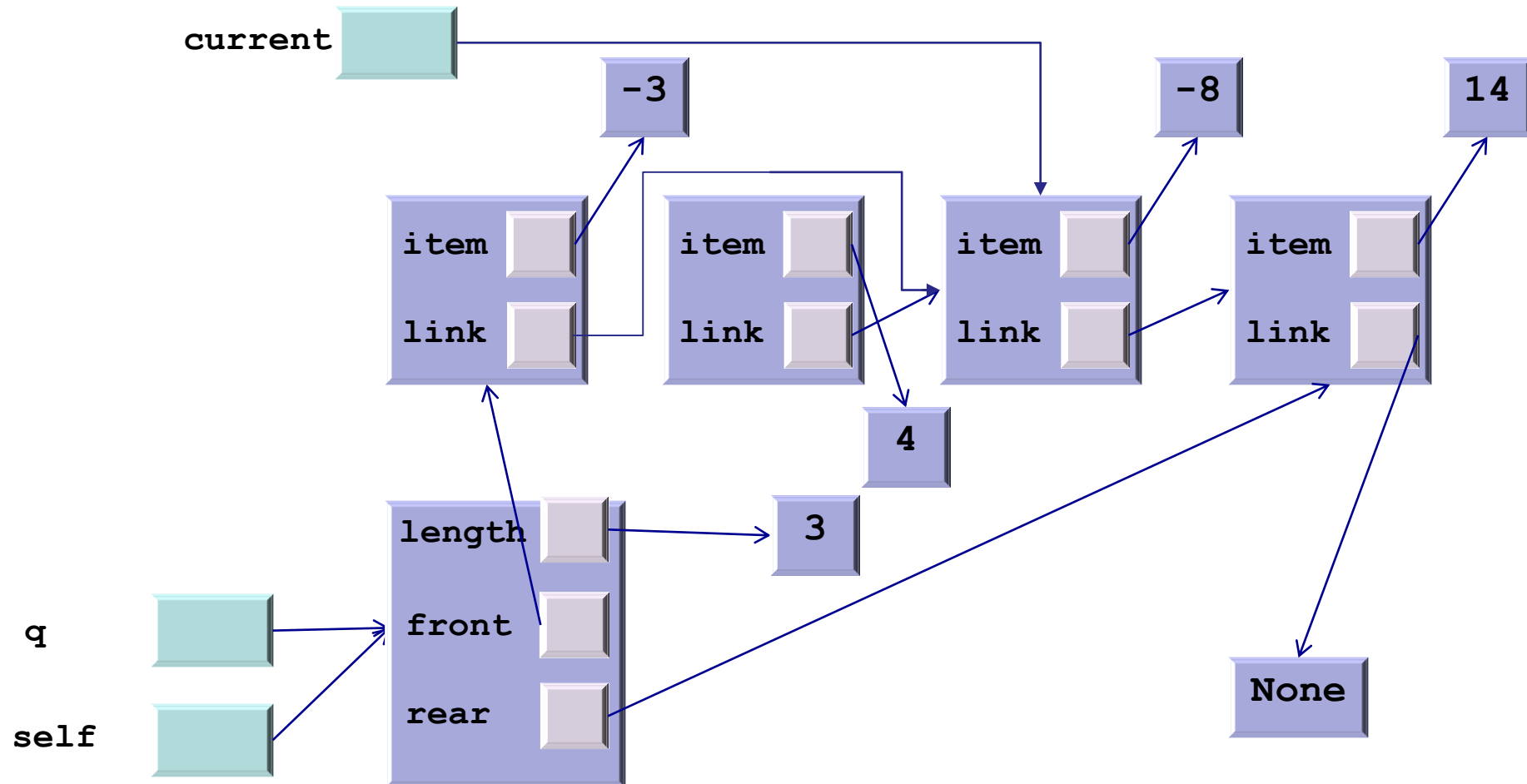


```

def halve(self) -> None:
    current = self.front
    while current is not None and current.link is not None:
        if current.link is self.rear:
            self.rear = current
        current.link = current.link.link
        current = current.link
        length -= 1

```

q.halve()

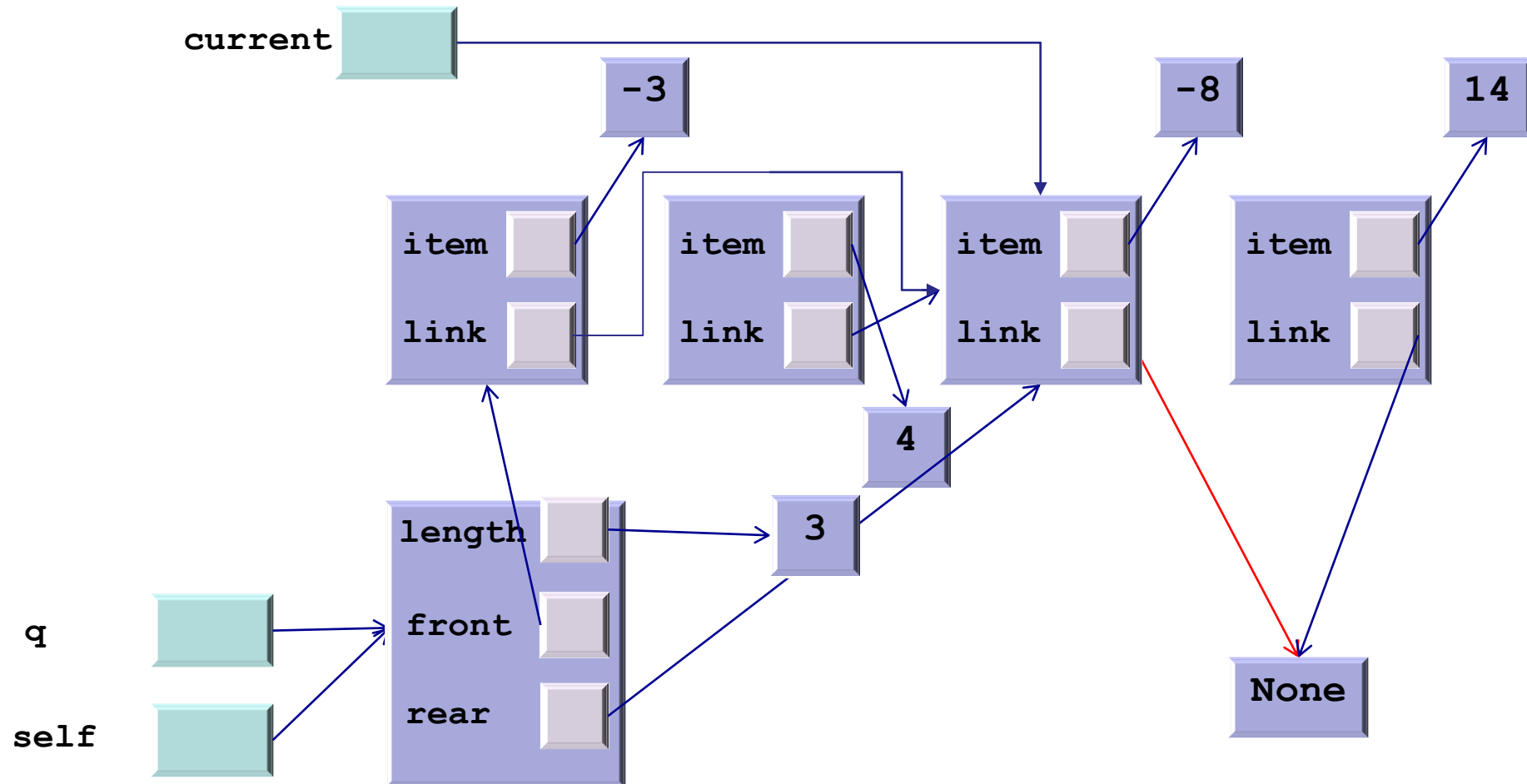


```

def halve(self) -> None:
    current = self.front
    while current is not None and current.link is not None:
        if current.link is self.rear:
            self.rear = current
        current.link = current.link.link
        current = current.link
    length -= 1

```

q.halve()

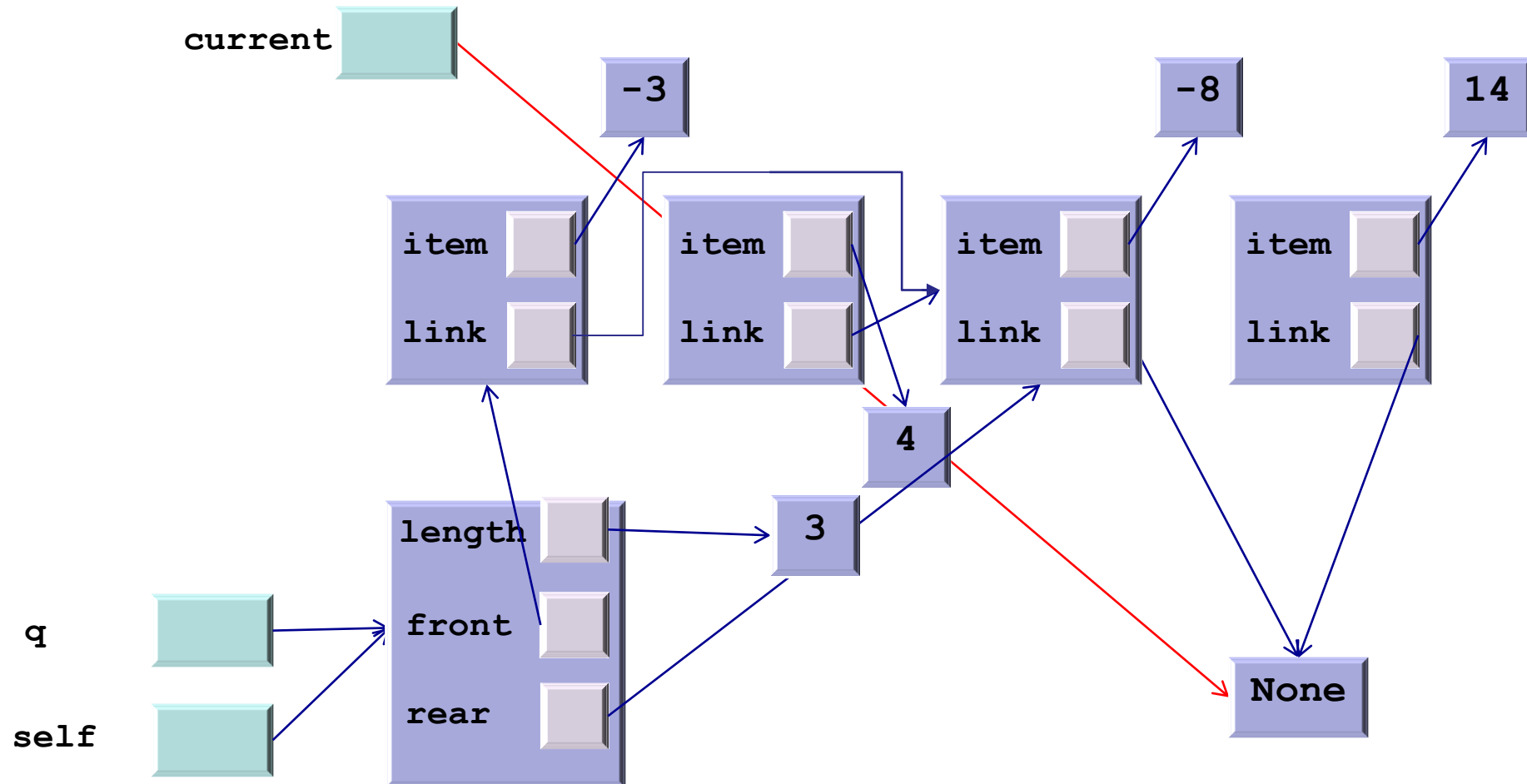


```

def halve(self) -> None:
    current = self.front
    while current is not None and current.link is not None:
        if current.link is self.rear:
            self.rear = current
        current.link = current.link.link
        current = current.link
        length -= 1

```

q.halve()

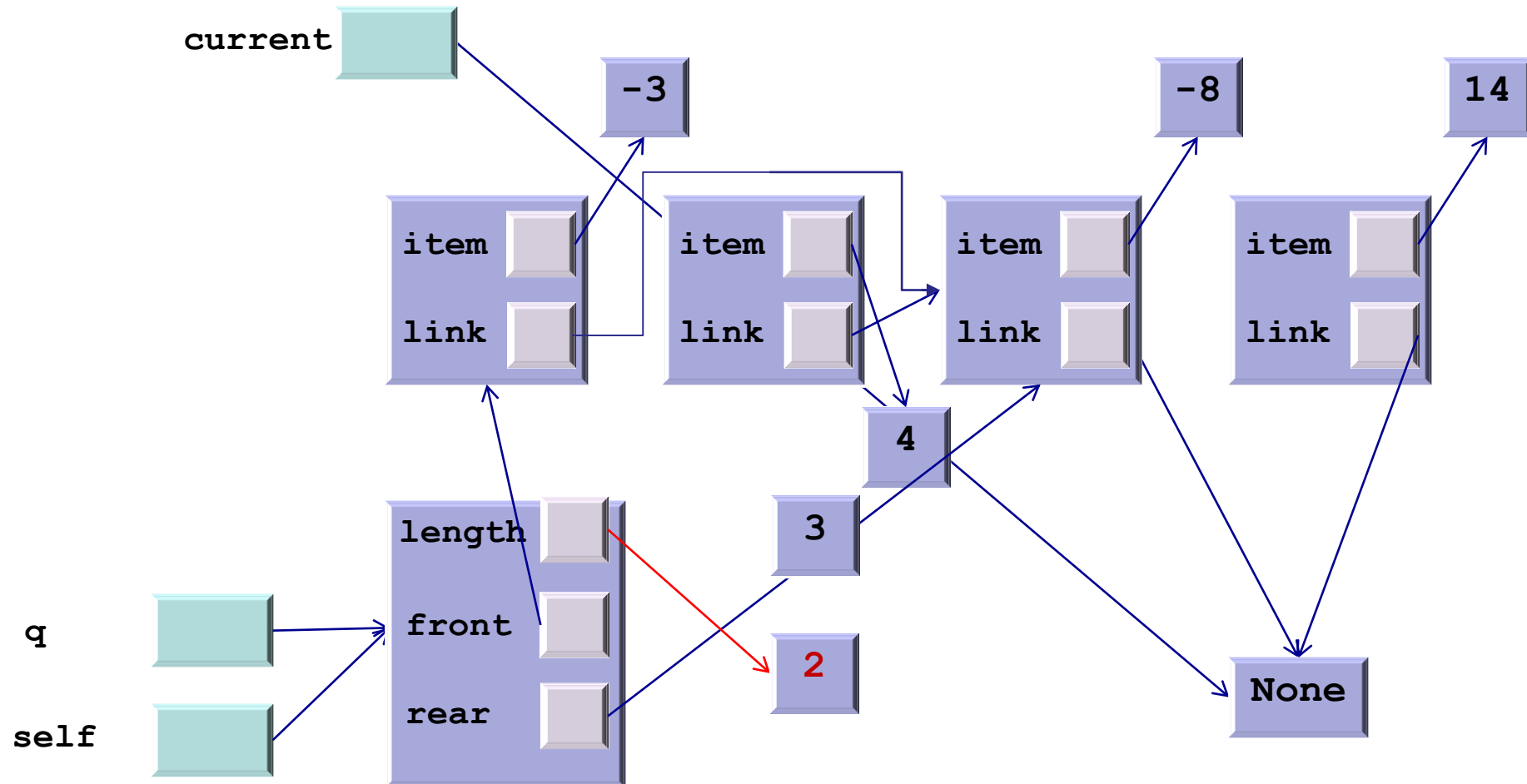


```

def halve(self) -> None:
    current = self.front
    while current is not None and current.link is not None:
        if current.link is self.rear:
            self.rear = current
        current.link = current.link.link
        current = current.link
    length -= 1

```

q.halve()



Remember: Abstract Data Types

- Any data type provides:
 - Storage for a collection of data items
 - A set of operations to interact with the data
- An abstract one does not provide any info on how:
 - The storage is organised
 - The operations are implemented
- Users can only interact with the data through the provided operations

*Separates the WHAT from the HOW
and ignores the HOW*

Abstract Data Types (cont)

- **Example: a Stack ADT has operations:**
 - push, pop, is_empty, reset, etc
 - Implementation? Could be array, could be linked, or something else, a user does not know
- **As a user I just need to know its operations**
- **Do not confuse Data Type with Data Structure**
 - Data Structure: particular way in which the data is organised (structured) in memory
 - The way a given Data Type is implemented

Abstract Data Types: pros and cons

- **Main advantage: maintenance**

- Changing the implementation of the ADT does not mean changing the user's code

- **Main disadvantage: efficiency**

- Having access to the implementation (ADT as an inner class) might allow good programmers to improve time/space performance

Abstract Data Types: advice

- **Always design your data types abstractly**
 - Use the class methods if you can (even as god!)
- **Late modifications to its implementation will not affect the rest of your code**
- **Readability** is also improved: use meaningful names for operations
- **Correctness** easier to verify: after proper testing to all methods

Summary

- **We now understand how to use linked data structures in implementing**
 - Stacks
 - Queues
- **We are able to:**
 - Implement, use and modify linked stacks and linked queues
 - Decide when it is appropriate to use them (rather than arrays)