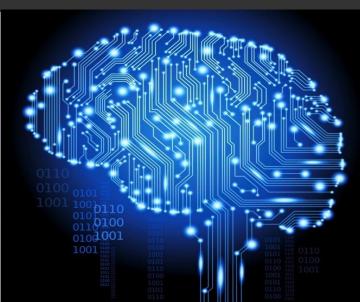


Information Technology

Conflict Resolution

Prepared by Maria Garcia de la Banda Updated by Brendon Taylor





Objectives for these two lectures

- To understand the main method of conflict resolution:
 - Open addressing:
 - Linear Probing
- To understand its advantages and disadvantages
- To be able to implement it





Conflict Resolution Add

Hash Table operations: Add

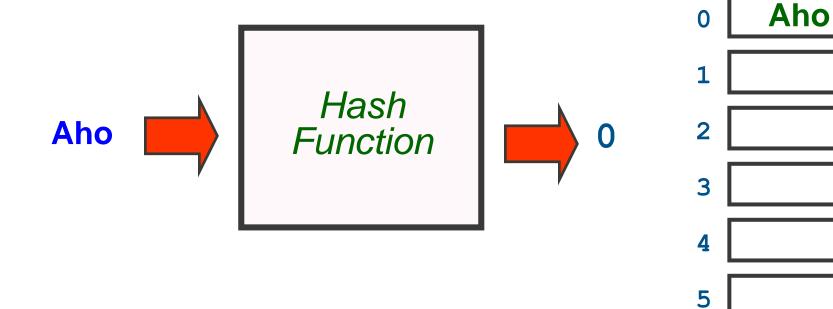
- Apply the hash function to get a position N
- Try to add key at position N
- Deal with collision if any



Aho, Kruse, Standish, Horowitz, Langsam, Sedgewick, Knuth

hash table

6

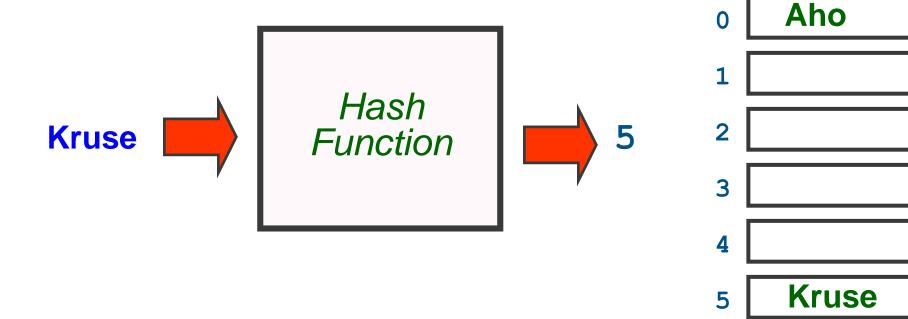




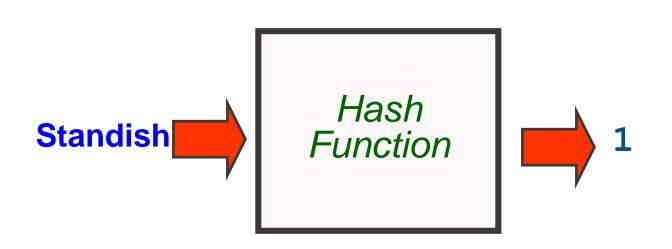
Aho, Kruse, Standish, Horowitz, Langsam, Sedgewick, Knuth

hash table

6



Aho, Kruse, Standish, Horowitz, Langsam, Sedgewick, Knuth

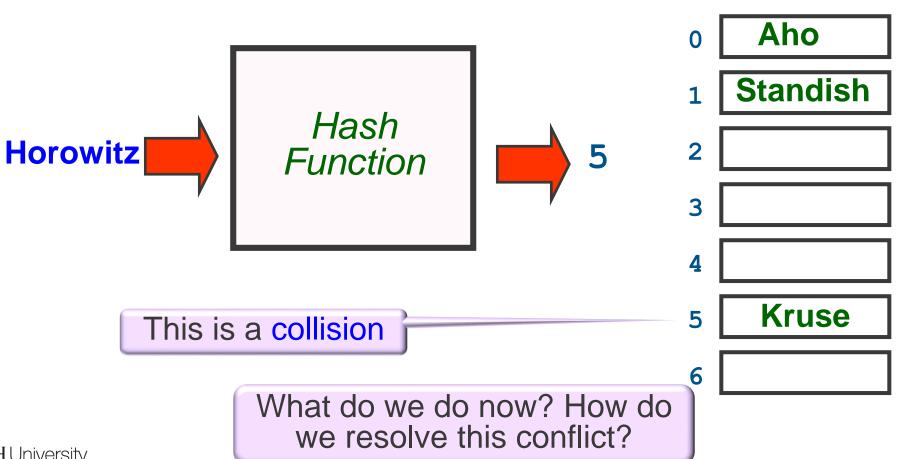


hash table

- 0 Aho
- 1 Standish
- 2
- 3
- 4
- 5 Kruse
- 6

Aho, Kruse, Standish, Horowitz, Langsam, Sedgewick, Knuth

hash table



Conflict resolution: two main approaches

Separate chaining:

- Each array position contains a linked list of items
- Upon collision, either update (same key) or add the element to the linked list

Open addressing:

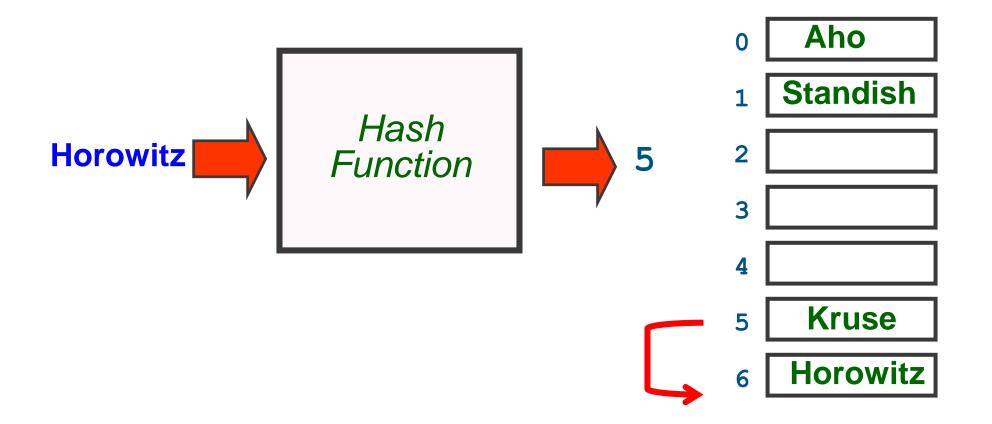
- Each array position contains a single item
- Upon collision, either update (same key) or use an empty space to store the new item (which empty space depends on the technique)
- As we will see:
 - Requires an array of at least double the size of the number of elements
 - Thus, we must be able to estimate in advance the number of elements (or risk a dynamic resize)

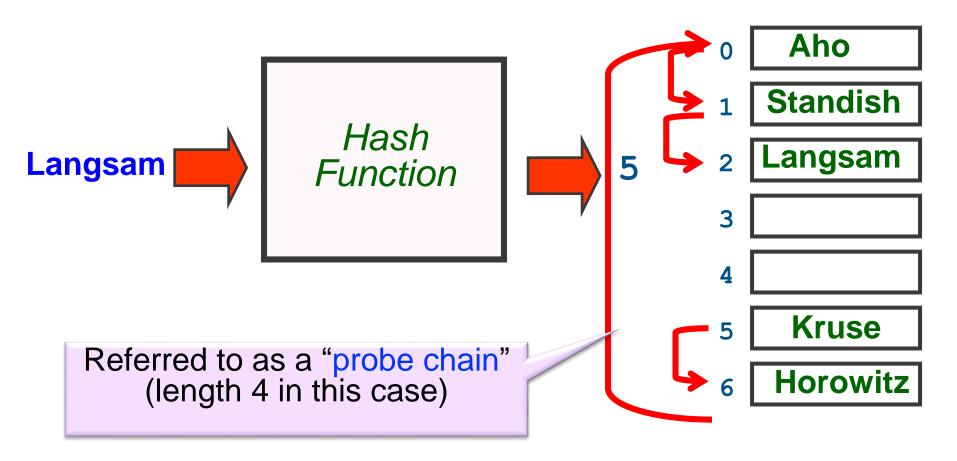


Open Addressing: Linear Probing

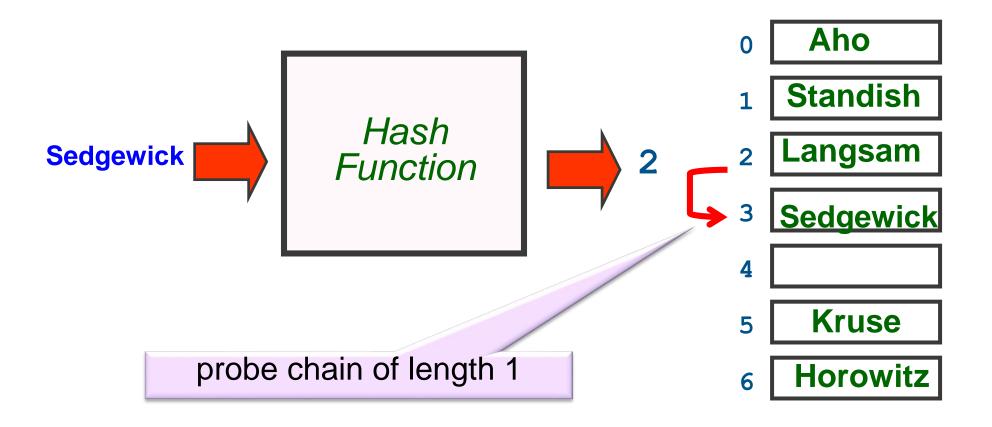
- Add item with hash value N:
 - If array[N] is empty: put item there
 - If there is already an item there with:
 - A different key:
 - search for the first empty space in the array from N+1
 - add the item there (if any)
 - Same key: update the data associated to the key
- Basically: linear search from N until an empty slot is found
- But careful, you must deal with:
 - Full table (to avoid going into an infinite loop)
 - Restarting from position 0 if the end of table is reached



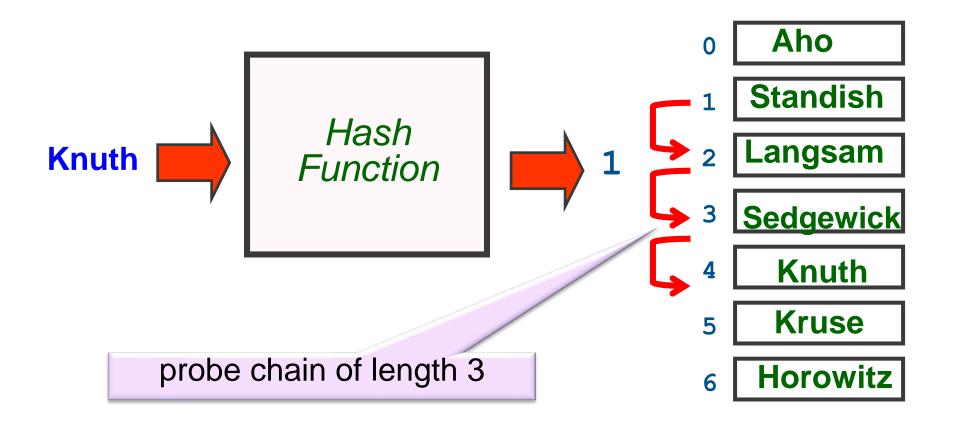




Let's keep on going



Let's keep on going



```
from typing import TypeVar, Generic
T = TypeVar('T')
                                         Default size (a prime)
class LinearProbeTable(Generic[T]):
    def __init__ (self, size: int = 7919) -> None:
         self.count = 0
                                                     How many elements?
         self.table = ArrayR(size)
                                                    The array to store them
    def len (self) -> int:
         return self.count
    def hash(self, key: str) -> int:
                                                Universal hashing
        value = 0
         a = 31415
                             h = ((\dots (a_0x + a_1)x + \dots + a_{n-3})x + a_{n-2})x + a_{n-1})x + a_n
        b = 27183
         for char in key:
            value = (ord(char) + a*value) % len(self.table)
            a = a * b % (len(self.table)-1)
         return value
                             Base changes for each position pseudo randomly
```

Reminder: Adding in Linear Probing

Add item with hash value N:

But what is an item?

Up to now, we were storing only the key (the item was the key)

- If array[N] is empty: put item there
- If there is already an item there with:
 - A different key:
 - search for the first empty space in the array from N+1
 - add the item there (if any)
 - Same key: update the data associated to the key
- Basically: linear search from N until an empty slot is found
- But careful, you must deal with:
 - Full table (to avoid going into an infinite loop)
 - Restarting from position 0 if the end of table is reached



We were storing the key only

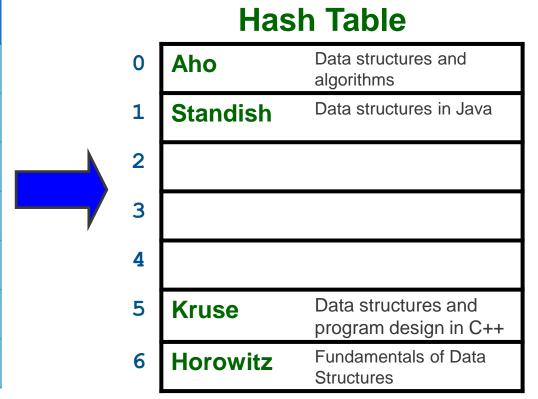
Key	Hash	Hash Table		
Aho	0	0	Aho	
Kruse	5	1	Standish	
Standish	1	2		
Horowitz	5	3		
Langsam	5	4		
Sedgewick	2	5	Kruse	
Knuth	1	6	Horowitz	

In practice we want to store also data associated to each key



We also need to store the data

Key	Hash	Data		
Aho	0	Data structures and algorithms		
Kruse	5	Data structures and program design in C++		
Standish	1	Data structures in Java		
Horowitz	5	Fundamentals of Data Structures		
Langsam	5	Data structures using C and C++		
Sedgewick	2	Algorithms in C++		
Knuth	1	The art of computer programming		

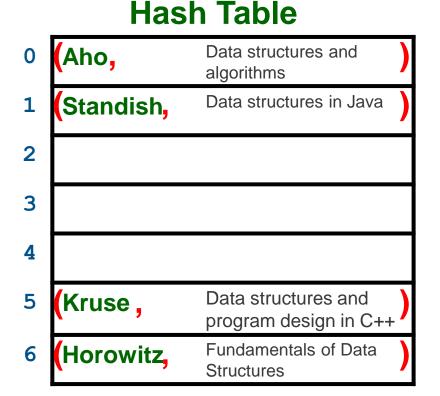




We also need to store the data (cont)

- How do we store both key and data in the hash table?
- We need an object that stores:
 - Key
 - Data
- Have we seen anything already?
- Tuples! We could use: (key, data)
- We can then access them as usual

```
>>> my_tuple = ("Aho","Data structures")
>>> my_tuple[0]
'Aho'
>>> my_tuple[1]
'Data structures'
>>>
```



Reminder: Adding in Linear Probing

Add item with hash value N:

But what is an item?

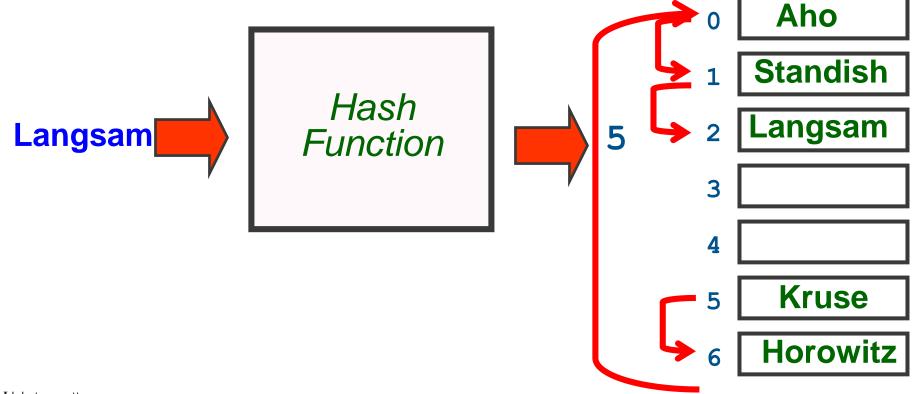
Now we know: an item is a tuple (key,data)

- If array[N] is empty: put item there
- If there is already an item there with:
 - A different key:
 - search for the first empty space in the array from N+1
 - add the item there (if any)
 - Same key: update the data associated to the key
- Basically: linear search from N until an empty slot is found
- But careful, you must deal with:
 - Full table (to avoid going into an infinite loop)
 - Restarting from position 0 if the end of table is reached

Adding algorithm for Linear Probing

- The algorithm for add(key,data) for linear probing is as follows
- Get the position N using the hash function N = hash(key)
- If array[N] is empty, just put the item (key,data) there
- Else, if there is already an item there:
 - If the item has the same key: update the data
 - If it has a different key, keep looking in next cell (wrapping around)
 - What if we never find it and there is no empty spot?
 - Then we need to rehash: create a bigger array and reinsert all items
- We must traverse the table before we can rehash:
 - Even if it is known to be full, in case the key is already in (we are doing an update)
 - Also, as we will see later, rehash in Linear Probing should happen much earlier than when the table is full...







Allows our container class to provide the [] notation

```
def setitem (self, key: str, data: T) -> None:
                                                                Traverse each
   position = self.hash(key) # get the position using hash
                                                                item in our hash
                                                                  table from
                                                                   position
   for in range(len(self.table)): # start traversing
       if self.table[position] is None: # found empty slot
           self.table[position] = (key, data)
           self.count += 1
                                                                Item already
           return
                                                               exists, overwrite
                                                                  the data
       elif self.table[position][0] == key:# found key
           self.table[position] = (key, data)
           return
                                            Linear Probing
       else: # not found, try next
           position = (position+1) % len(self.table)
   self.rehash() # move everything to a new, larger table
   self. setitem (key, data) #try again
```

Your turn...

Write __str__ for a Linear Probe hash table, e.g.:

(Aho, Data structures and algorithms) (Standish, Data Structures in Java)

```
def __str__(self) -> str:
    result = ""
    for item in self.array:
        if item is not None:
            (key, value) = item
            result += "(" + str(key) + "," + str(value) + ")\n"
    return result
```

Hash Table

0 Aho
1 Standish
2 Langsam
3
4
5 Kruse
6 Horowitz

But we are traversing the Hash Table! Didn't we say not to do that?

No! We said not to traverse IN A PARTICULAR ORDER



Conflict Resolution Search

Searching in Linear Probing

- Search for an item with hash value N:
 - Perform a linear search from array[N] until either the item or an empty space is found (if so, raise a KeyError(key) exception)
- But careful, you must deal again with:
 - Full table (to avoid going into an infinite loop)
 - Restarting from position 0 if the end of table is reached



Searching algorithm for Linear Probing

- The algorithm for search (key, data) is as follows
- Get the position N using the hash function N = hash(key)
- If array[N] is empty, raise a KeyError (key) exception
- Else, if there is already an item there:
 - If the item has the same key: return the associated data
 - If it has a different key, keep looking
 - What if we never find the key and there is no empty spot?
 - Then we raise a **KeyError** (**key**) exception
- We used __setitem__ for adding
- We will use __getitem for searching



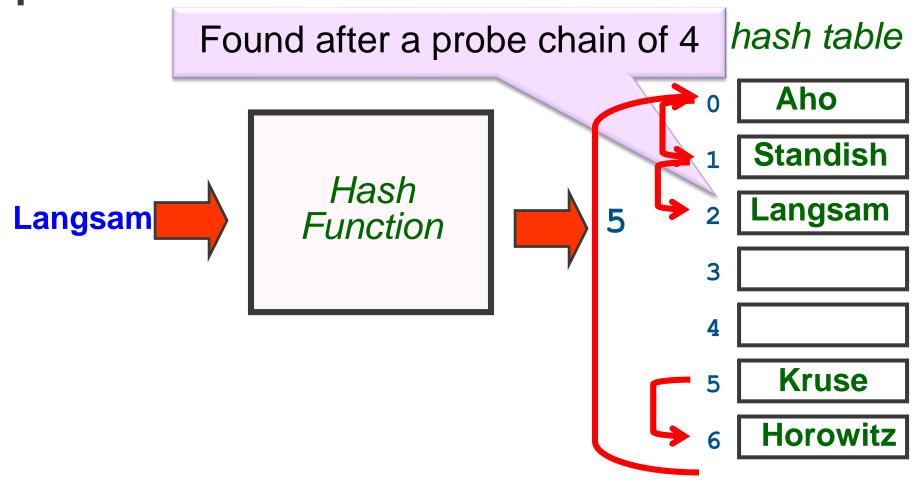
```
def getitem (self, key: str) -> T:
                                                                             Traverse each
                position = self.hash(key) # get the position using hash
                                                                            item in our hash
                                                                              table from
                                                                               position
                for in range(len(self.table)): # start traversing
                    if self.table[position] is None: # found empty slot
Stop if we find an
                        raise KeyError(key) # so the key is not in
  empty slot
                    elif self.table[position][0] == key:# found key
                                                                            Linear Probing
                        return self.table[position][1] #return data
                    else: # there is something but not the key, try next
                        position = (position+1) % len(self.table)
                # At this point, I have gone through the table and not found
                raise KeyError(key)
```



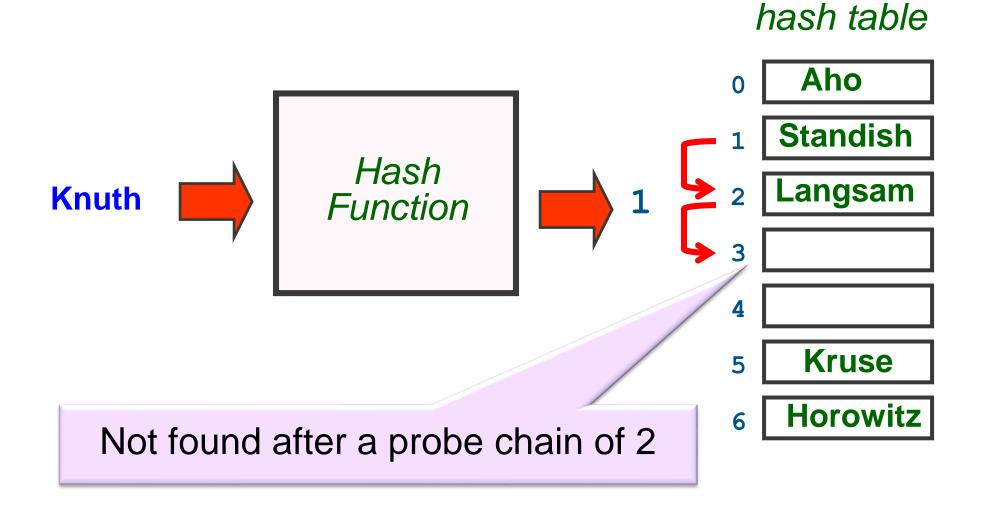
```
def linear probe(self, key: str, is search: bool) -> int:
                position = self.hash(key)
                                                            Traverse each item in our
                                                             hash table from position
                for in range(len(self.table)):
                     if self.table[position] is None: # found empty slot
                         if is search: # if searching
If we're searching for an
                             raise KeyError(key) # key is not in
item (eg. __getitem_
                         else:
                             return position # if adding, return position
                     elif self.table[position][0] == key: # found key
                                                                             Linear Probing
                         return position
                     else: # there is something but not the key, try next
                         position = (position + 1) % len(self.table)
                raise KeyError(key)
```

```
Will raise a KeyError if not
def getitem (self, key: str) -> T:
                                                             found
      position = self. linear probe(key, True)
       return self.table[position][1]
def setitem (self, key: str, data: T) -> None:
       try:
           position = self. linear probe(key, False)
       except KeyError:
                             Full Hash Table, need to resize
           self. rehash()
           self. setitem (key, data) # try again
       else:
           if self.table[position] is None: # if it's a new item
               self.count += 1
           self.table[position] = (key, data)
```

Example: search



Example: search





Conflict Resolution Delete

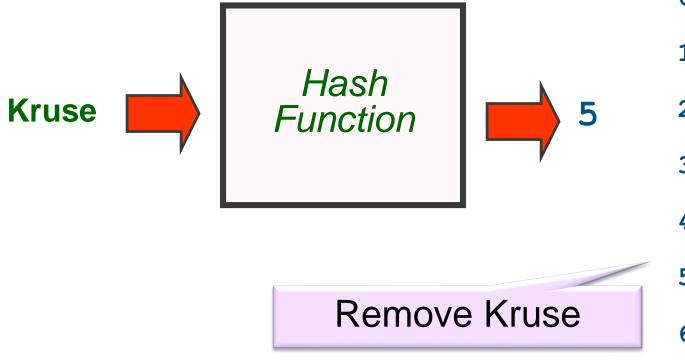
Deleting in Linear Probing

- What about delete?
 - Use the search function to find the item
 - If found at N, then what?
- Should we simply delete it and leave it empty?
 - No, as empty spots have meaning in linear probing...

Invariant in Linear Probing: if an item with hash(key)=N is in the table, it will always appear between N and the first empty position (wrapping around)

Example: bad delete I

Assume we delete Kruse and just leave it empty



hash table

- 0 Aho
- 1 Standish
- 2 Langsam
- 3
- 4
- 5 Kruse
- 6 | Horowitz

Example: bad delete I

hash table Assume we delete Kruse and just leave it empty Aho • If we now search for Horowitz (key 5)? It will say it does not find it (raise KeyError) **Standish** def linear probe(self, key: str, is search: bool) -> int: Langsam position = self.hash(key) for in range(len(self.table)): if self.table[position] is None: if is search: 4 raise KeyError(key) else: return position elif self.table[position][0] == key: **Horowitz** return position else: position = (position + 1) % len(self.table) raise KeyError(key)

Deleting in Linear Probing

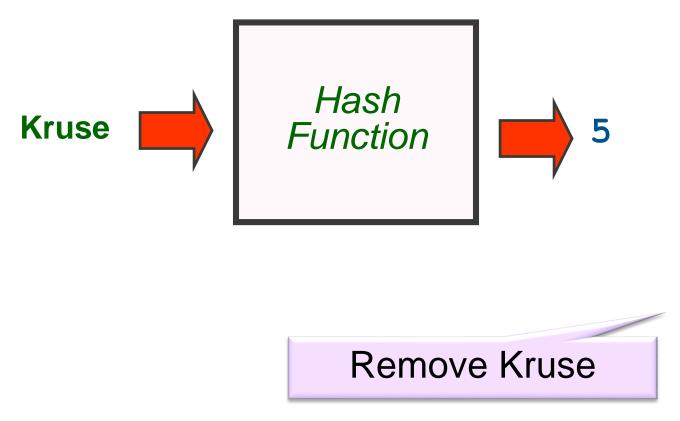
- What about delete?
 - Use the search function to find the item
 - If found at N, then what?
- Should we simply delete it and leave it empty?
 - No, as empty spots have meaning in linear probing...
- Should we shuffle everything from N+1 upwards?
 - From N+1 to what?
 - To the first empty position
 - Is shuffling a good idea though?
- first empty position (wrapping around)

Invariant in Linear Probing: if an item with hash(key)=N is in the table, it will always appear between N and the

No, we might move items that were in the correct positions!



Assume we delete Kruse and shuffle



hash table

- 0 Aho
- 1 Standish
- 2 | Langsam
- 3
- 4
- 5 Kruse
- 6 | Horowitz

Assume we delete Kruse and shuffle

hash table

Aho

1 Standish

2 | Langsam

3

4

5 Horowitz

6

Shuffle up until first empty position



Assume we delete Kruse and shuffle

2 Langsam345 Horowitz

Aho

hash table

Standish

Shuffle up until first empty position



Assume we delete Kruse and shuffle

2 Langsam345 Horowitz

Aho

hash table

Standish

Shuffle up until first empty position



- Assume we delete Kruse and shuffle
- If we now search for Horowitz (key 5)?
 - Will find it without problem
- And if we search for Aho (key 0)?
 - Will not find it
- Shuffling can incorrectly move elements

hash table

o | Standish

1 | Langsam

2

3

4

5 Horowitz

6 Aho

Deleting in Linear Probing

- What about delete?
 - Use the search function to find the item
 - If found at N, then what?
- Should we simply delete it and leave it empty?
 - No, as empty spots have meaning...
- Should we shuffle everything from N+1 upwards?
 - From N+1 to what?
 - To the first empty position
 - Is shuffling a good idea though?

Invariant in Linear Probing: if an item with hash(key)=N is in the table, it will always appear between N and the first empty position (wrapping around)

- No, we might move items that were in the correct positions!
- One possibility:
 - If found at N, reinsert every item from N+1 to the first empty position
 Time consuming! (though should not be many)

```
pos
                                                                     hash table
                                             Hash
                                                                        Aho
                           Kruse
                                           Function
                                                                      Standish
 def delitem (self, key: str) -> None:
                                                                       Langsam
     pos = self. linear probe(key, False)
     self.table[pos] = None
                                                                   3
     self.count -= 1
     pos = (pos + 1) % len(self.table)
     while self.table[pos] is not None:
                                                                        Kruse
                                             Need to delete it
         item = self.table[pos]
         self.table[pos] = None
                                                                       Horowitz
         self.count -= 1
         self[str(item[0])] = item[1]
         pos = (pos + 1) % len(self.table)
```



```
def __delitem__ (self, key: str) -> None:
    pos = self.__linear_probe(key, False)
    self.table[pos] = None
    self.count -= 1

pos = (pos + 1) % len(self.table)
    while self.table[pos] is not None:
        item = self.table[pos]
        self.table[pos] = None
        self.count -= 1
        self[str(item[0])] = item[1]
        pos = (pos + 1) % len(self.table)
```

aeiete

Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

hash table

0	Aho
1	Standish
2	Langsam
3	
4	
5	
6	Horowitz

Reinsert Horowitz



```
def __delitem__ (self, key: str) -> None:
    pos = self.__linear_probe(key, False)
    self.table[pos] = None
    self.count -= 1

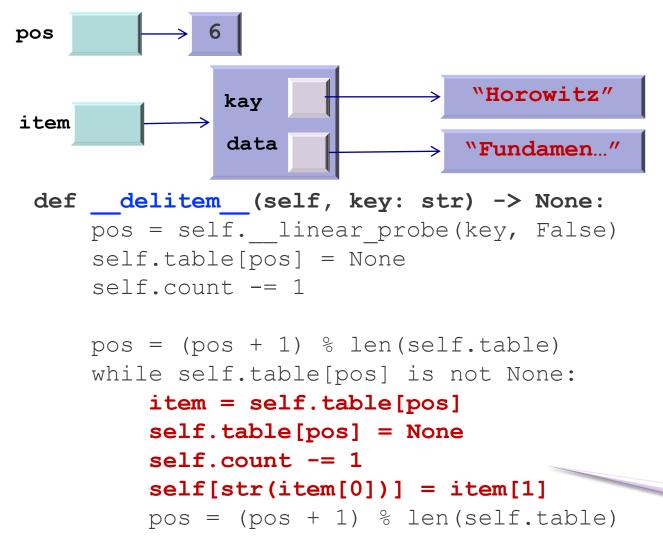
pos = (pos + 1) % len(self.table)
    while self.table[pos] is not None:
        item = self.table[pos]
        self.table[pos] = None
        self.table[pos] = None
        self.count -= 1
        self[str(item[0])] = item[1]
        pos = (pos + 1) % len(self.table)
```

hash table

Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

0	Aho
1	Standish
2	Langsam
3	
4	
5	
6	Horowitz

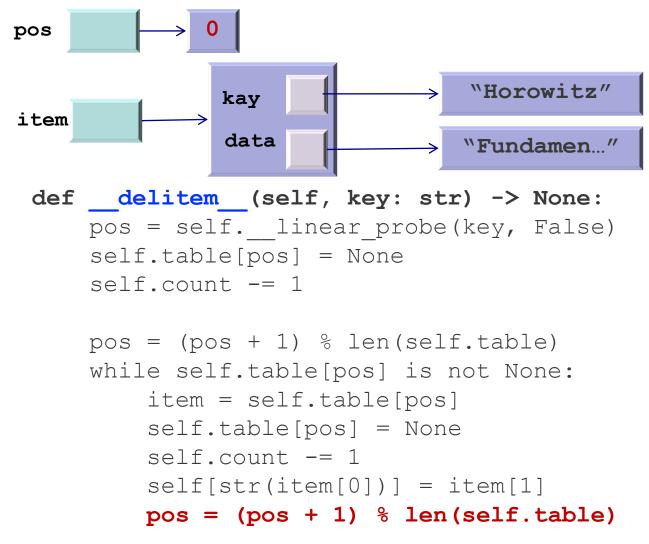
Reinsert Horowitz



hash table

Key	Hash	0	Aho
Aho	0	1	Standish
Kruse	5	2	Langsam
Standish	1	2	Langsam
Horowitz	5	3	
Langsam	5	4	
Sedgewick	2	5	Horowitz
Knuth	1	6	

Reuse our setitem



Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

hash table

0 Aho

└ Standish

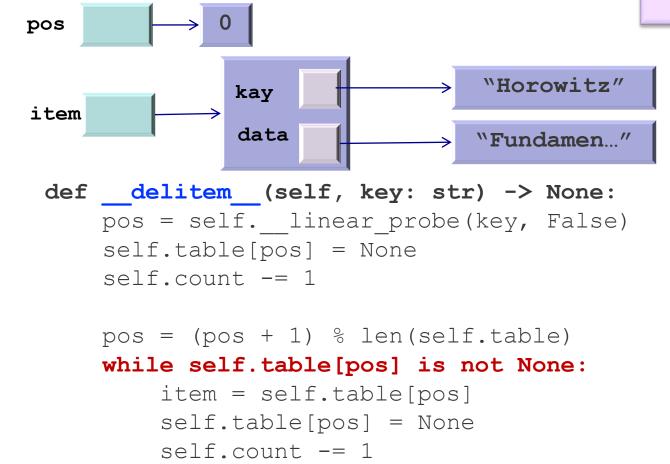
₂ | Langsam

3

4

5 | Horowitz

6



self[str(item[0])] = item[1]

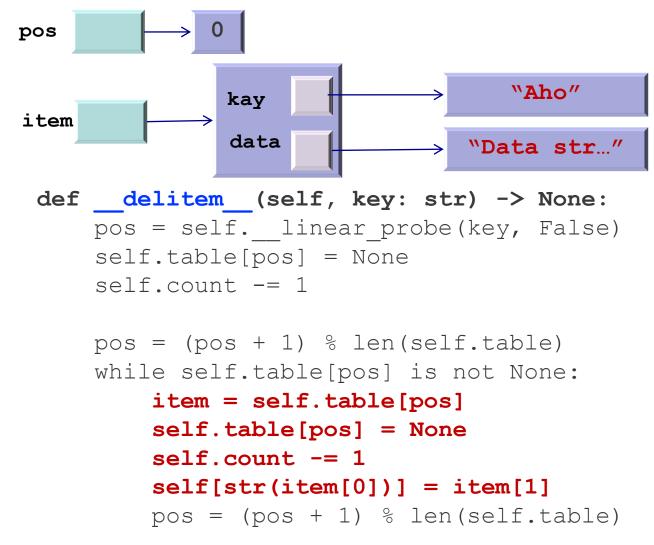
pos = (pos + 1) % len(self.table)

Reinsert Aho

Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

hash table

Aho	0	Hash	∍y
Standish	1	0	าด
_		5	use
Langsam	2	1	andish
	3	5	orowitz
	4	5	ngsam
Horowitz	5	2	edgewick
	6	1	nuth



Stays the same

Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

hash table

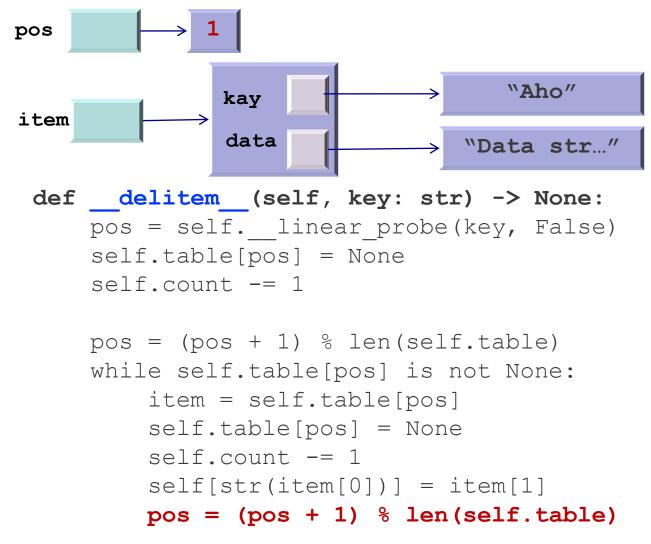
0	Aho



2	Langsam
---	---------

3





Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

hash table

0 Aho

L | Standish

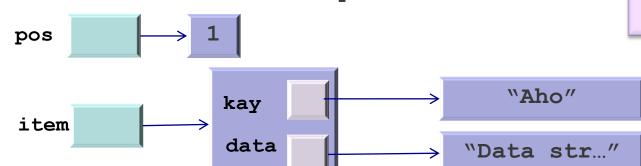
₂ | Langsam

3

4

5 | Horowitz

6



```
def __delitem__ (self, key: str) -> None:
    pos = self.__linear_probe(key, False)
    self.table[pos] = None
    self.count -= 1
```

```
pos = (pos + 1) % len(self.table)
```

while self.table[pos] is not None:

```
item = self.table[pos]
self.table[pos] = None
self.count -= 1
self[str(item[0])] = item[1]
pos = (pos + 1) % len(self.table)
```

Reinsert Standish

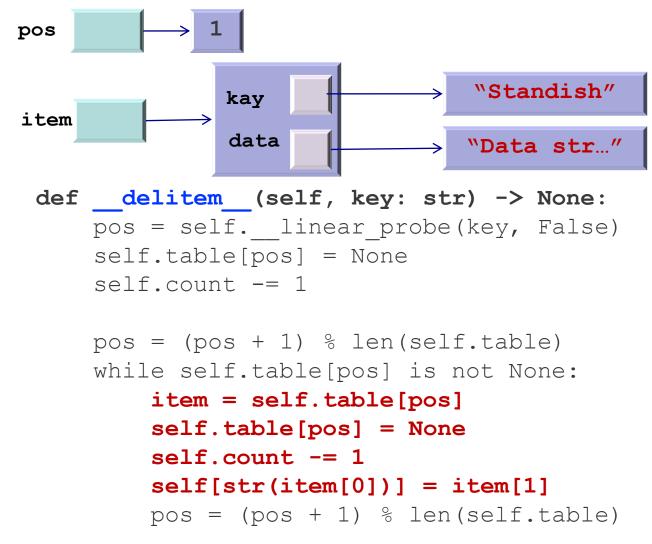
Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

•		4		
na	ch	ta	n	
IICA	OLI		IJ	

0	Ano		
1	Standish		







Stays the same

Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

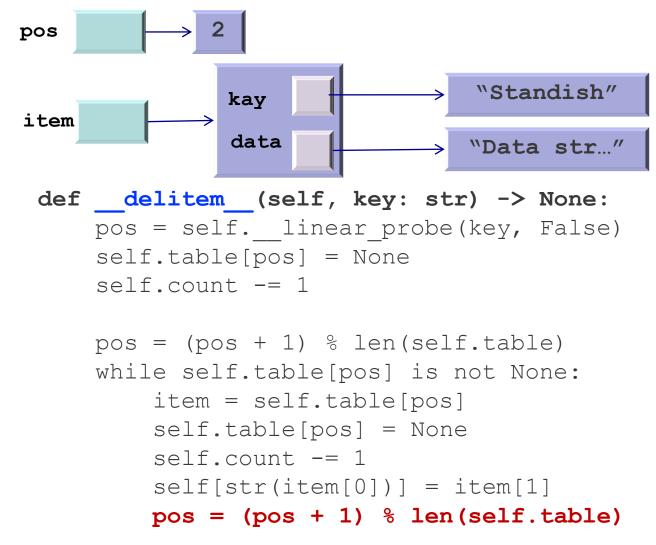
hash table

0	Aho



3





Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

hash table

0 Aho

L | Standish

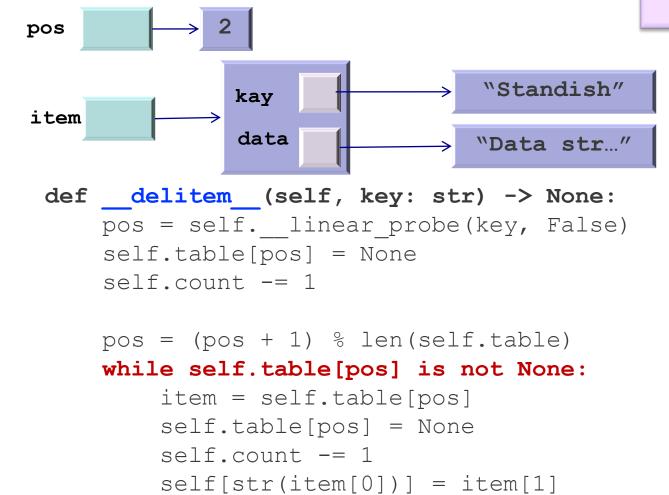
₂ | Langsam

3

4

5 | Horowitz

6



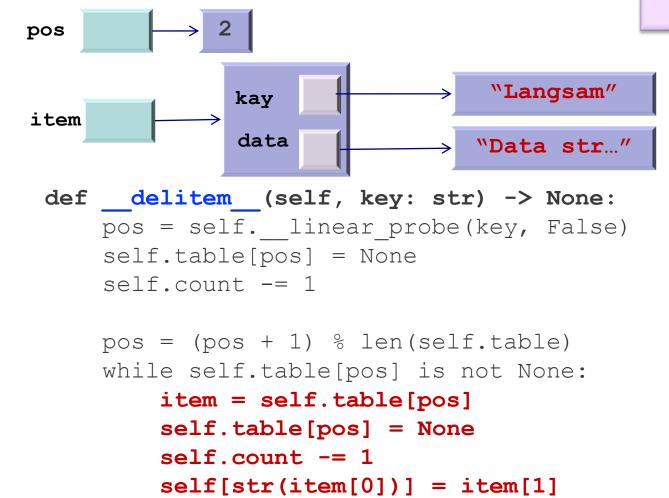
pos = (pos + 1) % len(self.table)

Reinsert Langsam

Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

hash table

		\	
ey	Hash	0	Aho
ho	0	1	Standish
ruse	5		Langsam
tandish	1	2	Langsam
orowitz	5	3	
angsam	5	4	
edgewick	2	5	Horowitz
outh.	1		



pos = (pos + 1) % len(self.table)

Reinsert Langsam

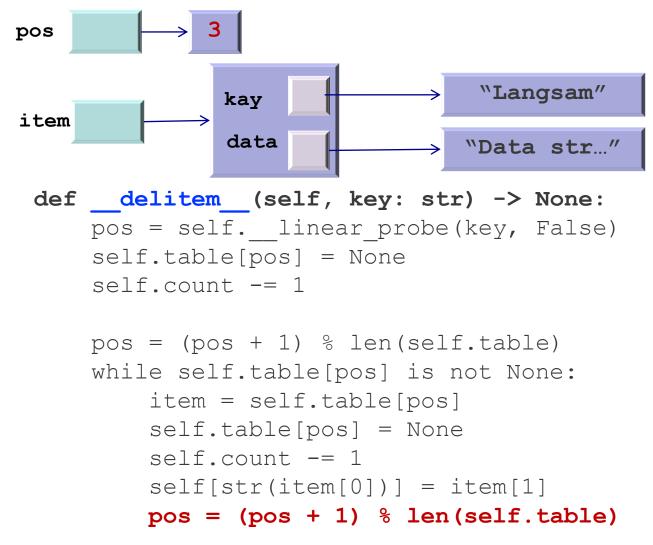
Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

hash table

	Пазп	\ 0	Ano
)	0	1	Standish
se	5		
ndish	1	Z	

4

5 Horowitz



Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

hash table

Aho

L | Standish

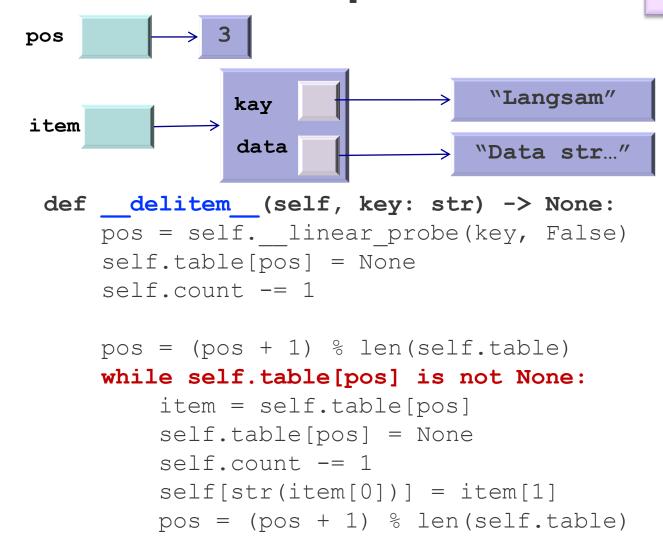
2

3

4

5 | Horowitz

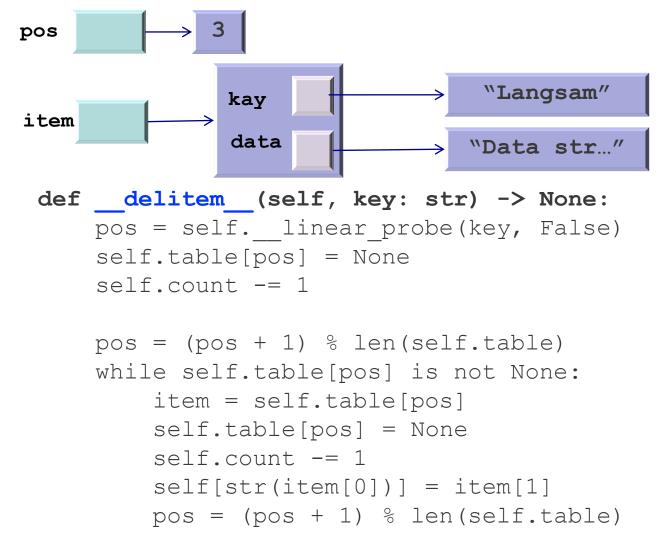
Reached empty, finish



Key	Hash	1
Aho	0	
Kruse	5	
Standish	1	
Horowitz	5	
Langsam	5	
Sedgewick	2	
Knuth	1	

\ '	Tasir table	
0	Aho	
1	Standish	
2		
3		
4		
5	Horowitz	

hash table



Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

hash table

O Aho

└ Standish

2

3

4

5 | Horowitz



Conflict Resolution Open Addressing

Open Addressing: Linear Probing

• Another possibility for delete:

- Use a special symbol (a sentinel) to denote delete
- Modify add and search to take that symbol into account
 - For search: treat the sentinel as you would treat a cell with a key different to the one you are searching for (keep on looking)
 - If found, you could move it to the first deleted cell you found
 - What else would need to be done?
 - For add: treat it as empty and add it in the first deleted cell
 - But this only works for new keys (those not already in the table)
 - If you want updates… think about it!



Open Addressing: Linear Probing

- Load factor: total number of items/TABLESIZE
- Cluster: sequence of full hash table slots (i.e., without an empty slot)
- Cluster can form even when the load is small
- Once a cluster forms, it tends to grow larger
 - Items that hash to a value within the cluster, get added at the end making it bigger
 - This might involve more than one hash value



Example of cluster

- All 4 elements are part of a cluster
- Two of them have the same hash value:
 - Kruse and Horowtiz (5)
- The other two have different hash values (0 and 1)
- From then on, any element mapped to 0,1,5 or 6 will be part of the cluster. And adding elements mapped to 0,1,2,4,5, or 6 will make it grow.

hash table

Aho

1 | Standish

2

3

4

5 Kruse

6 | Horowitz



Linear Probing: Problems

- Tendency for clustering to occur as the load is > 0.5
- Low speed on clustering:
 - Adding a key with hash value N can drastically increase the search time for keys with values other than N
 - Deletion can also be time consuming, as the entire cluster needs to be rehashed
 - This means we start to under-deliver on the O(1) promise
- If implemented in arrays table may become full fairly quickly, resizing is time and resource consuming

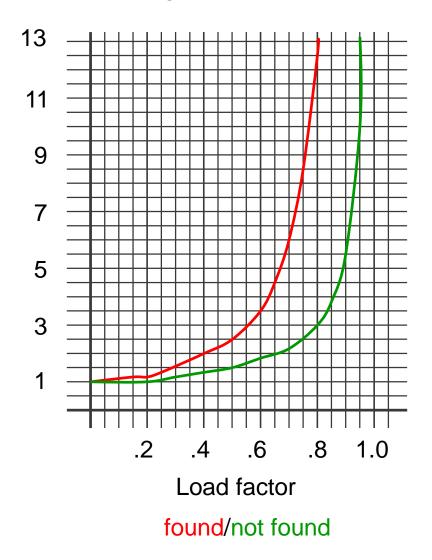
What can we do?

Idea: reduce clustering by taking bigger and bigger steps (rather than always using +S)

Open Addressing

- You must keep the load under 2/3
 - Otherwise the probe length (i.e., number of items visited before the element is found/not found) is too high
- Even better: under 1/2

Length of probe chain





Conclusion

- Hash Tables are one of the most used data types, as they have expected
 O(1) complexity for adding, deleting and searching, if built properly
- You have a very good chance of using them in your professional career
- They are very simple conceptually
- But they are also very "empirical":
 - A significant amount of experimental evaluation is usually needed to fine tune the hash function and the TABLESIZE
- A good choice of hash function, collision handling and load factor are crucial to maintaining an efficient hash table (i.e., keep the O(1) promise)



Summary

- Open Addressing
 - Linear Probing
- Advantages/Disadvantages
 - Length of probe
 - Memory usage
 - Resizing