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Course: ECE 16:332:573 DSA

Assignment-2

Q1. **Implement Shellsort which reverts to insertion sort. (Use the increment sequence 7, 3, 1). Create a plot for the total number of comparisons made in the sorting the data for both cases (insertion sort phase and shell sort phase). Explain why Shellshort is more effective than Insertion sort in this case. Also, discuss results for the relative (physical wall clock) time taken when using (i) Shellsort that reverts to insertions sort, (ii) Shellsort all the way.**

Answer:

Problem Scenario:

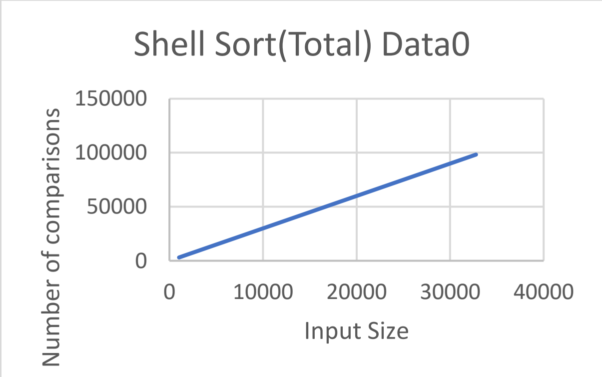
Develop an implementation for shell sort that reverts to insertion sort. i.e. when the gap is one call Insertion sort.

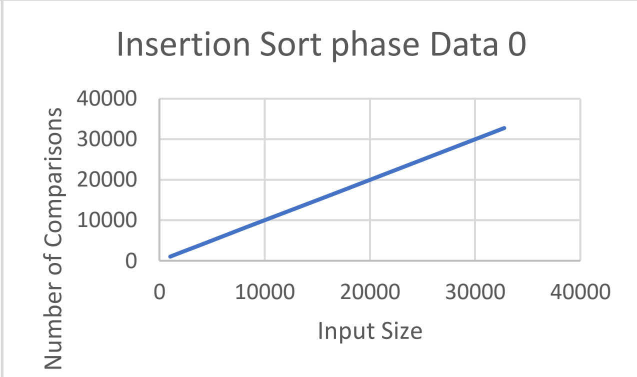
For the above implementation, the below observations are obtained for Data0 and Data1.

Data0:

The number of comparisons made for the shell sorting(total) and Insertion phase for data0 are as below :

Size	Shell Sort(Total)	Insertion Sort phase
1024	3061	1023
2048	6133	2047
4096	12277	4095
8192	24565	8191
16384	49141	16383
32768	98293	32767





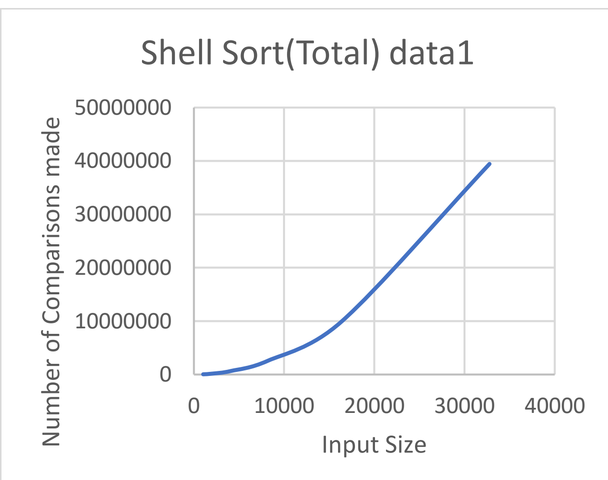
The above graphs indicate that for a sorted data the number of comparisons made for Shell sort and Insertion sort will be Linear.

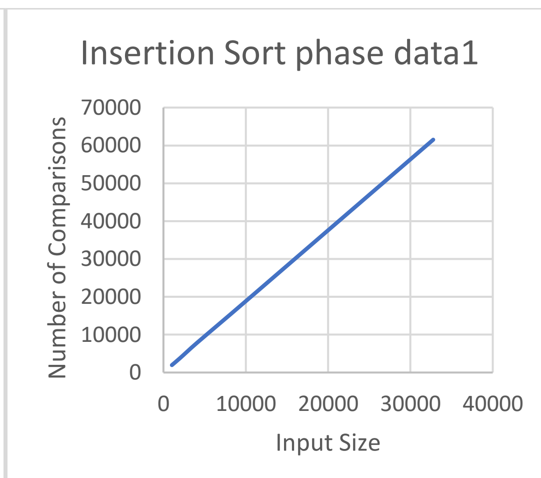
Data1:

Now let us have the data that is not sorted and see how many comparisons are required for sorting the data for both the Shel sort and Insertion phase of the Shell sort.

The below table shows the number of comparisons made in the sorting (both total and Insertion phase)

Size	Shell Sort(Total)	Insertion Sort phase
1024	46768	1961
2048	169081	3873
4096	660673	7887
8192	2576322	15473
16384	9950984	30805
32768	39442505	61508





From the table and the graphs we can see that the comparisons follow quadratic trend for the entire sorting but it is almost linear for Insertion sort phase i.e. when gap is ‘1’.

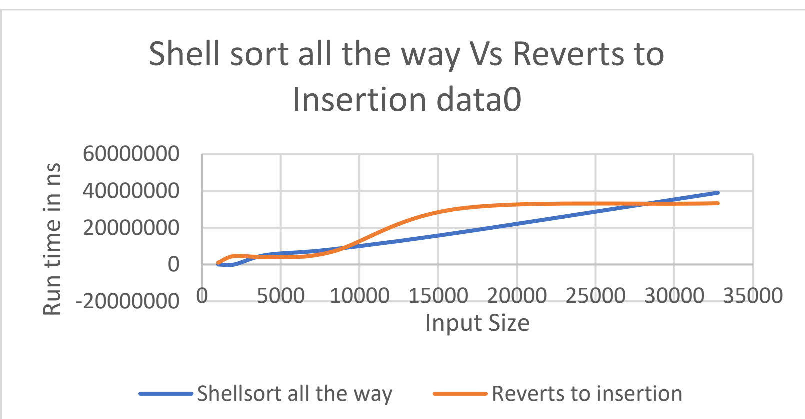
The average complexity of Insertion sort is O(N^2), which is very slow for arrays with large size as the number of Inversion might be a high value. This was changed to linear with the help of shell sorting which has a complexity of less than O(N^2) in case of array size 32768.

Why Shell Sort is effective than Insertion Sort:
The Shell sort tries to sort the array elements which are known distance apart. This distance known as gap is reduced to a value using a sequence, once the array elements with this gap are sorted. The decrease in this gap is done until the gap is '1' i.e. it reaches Insertion Sorting. The main purpose of this h-sorting or the gap is to reduce the number of inversions. Insertion Sort is the best sorting Algorithm when the number of Inversions is small.

Comparison of Physical Wall clock for Shell sort that reverts to Insertion and Shell sort all the way:
I have run the two implementations shell sort that reverts to insertion and shell sort all the way on data0 and data1.

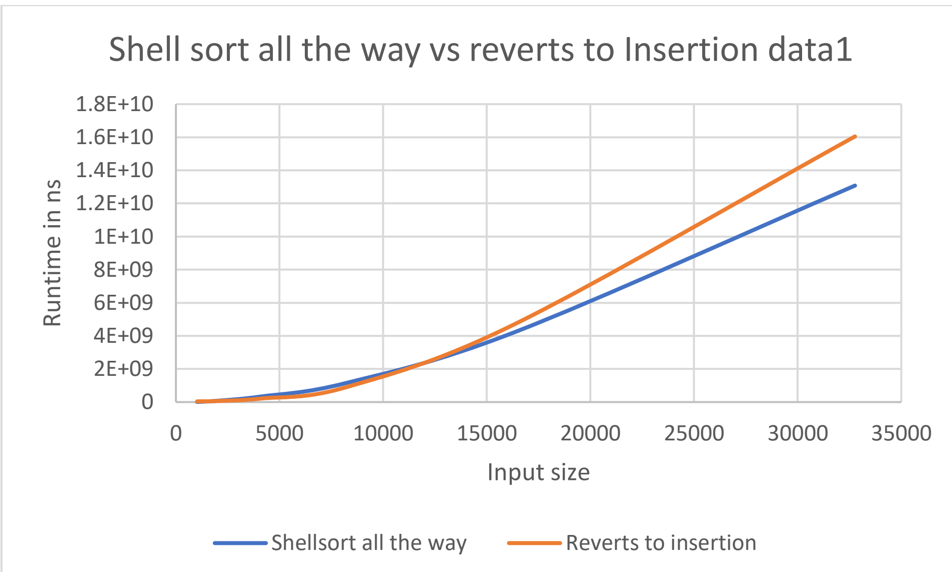
Data0:

Size	Shellsort all the way	Reverts to insertion
1024	0	996900
2048	0	4592400
4096	5169500	4151400
8192	8144800	6762700
16384	17396900	30366000
32768	38927800	33214500



Data1:

Size	Shellsort all the way	Reverts to insertion
1024	7460500	41686400
2048	82984900	67412800
4096	333650200	213652000
8192	1144792600	890690700
16384	4243010500	4731353600
32768	13079526700	16048733700



From the above two tables and plots, we can observe that Shell sort that reverts to Insertion sort takes more time compared to sell sort all the way.

Conclusion:
Shell sorting is effective than insertion sorting as it reduces the number of inversions by sub-sorting the array, this leads to an almost Linear Insertion Sort for h=1 i.e gap=1 as seen in the above graphs.

Q2. **The Kendall Tau distance is a variant of the "number of inversions". It is defined as the number of pairs that are in different order in two permutations. Write an efficient program that computes the Kendall Tau distance in less than quadratic time on average. Plot your results and discuss.**

Answer:

The number of inversions i.e the Kendall Tau Distance can be determined by the number of swaps a sorting algorithm uses. The number of swaps or exchange of the array elements directly gives the number of inversions present in an array.

Any sorting Algorithm that has a worst complexity of NlgN gives the number of swaps made in less than quadratic time(N^2). One such Algorithm is Merge Sorting.

By using the sorted data, we can verify if the merge sort provides values that are valid for our Analysis.

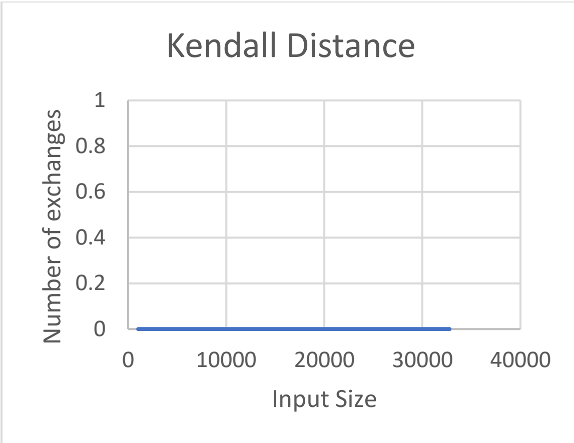
Data0:

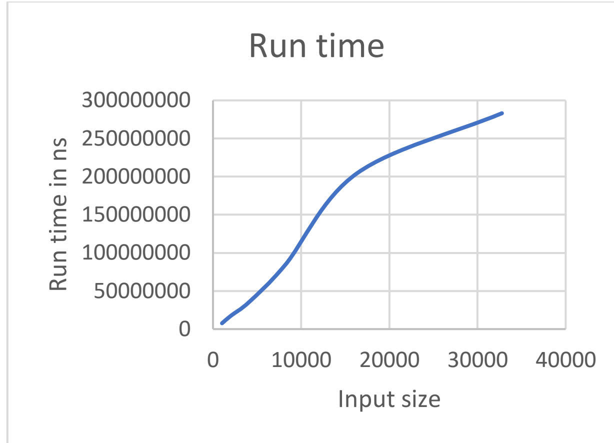
Number of pairs of Inversions for data0:

Size	Kendall Distance
1024	0
2048	0
4096	0
8192	0
16384	0
32768	0

Run time of the Algorithm for data0

Size	Run time
1024	7648400
2048	17631200
4096	35451400
8192	84180800
16384	204765800
32768	283124600



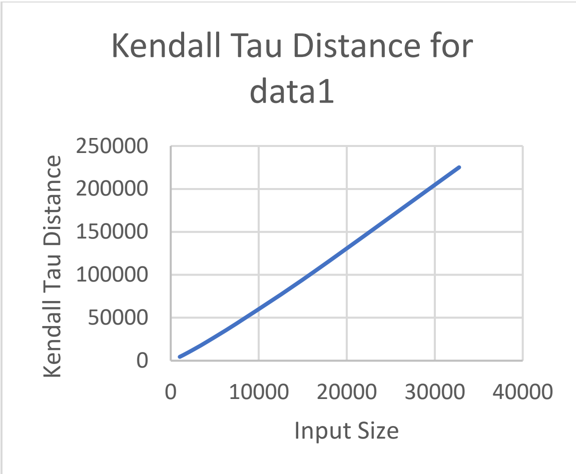


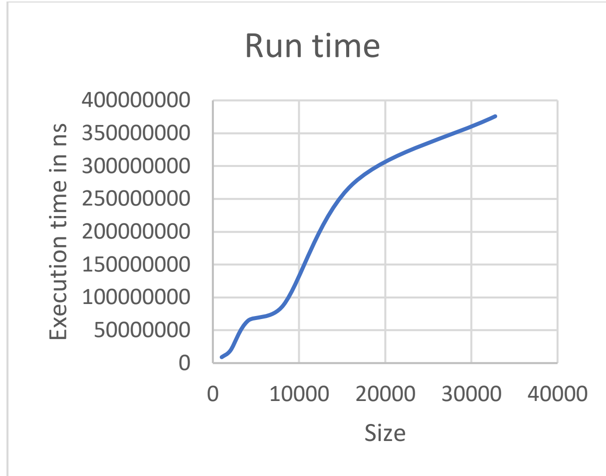
Here we can see that the runtime of the algorithm to check if there are any inversions is less than a quadratic curve.
With same algorithm, lets find out the number of inversions in the arrays of data set data1 i.e data which is not sorted
Number of Pairs of Inversions for data1

size	Kendall Distance
1024	4491
2048	9959
4096	21925
8192	48024
16384	104287
32768	225166

The runtime of the Algorithm for data1

size	Run time
1024	9018300
2048	19358700
4096	64731900
8192	88348700
16384	274464400
32768	375824100





For a data that is not sorted, the above tables and plots indicate that the average complexity of Merge sort is less than quadratic time, O(N^2), and hence It is one of the best sorting algorithm to find the Kendall Tau Distance in less than quadratic time on average.

Q3 **Implement the two versions of MergeSort that we discussed in class. Create a table or a plot for the total number of comparisons to sort the data (using data set here) for both cases. Discuss (i) relative number of operations, (ii) relative (physical wall clock) time taken.**

Answer:

The two versions of Merge sort are

(i) Top-Down approach which uses the recursive calls of the sort function

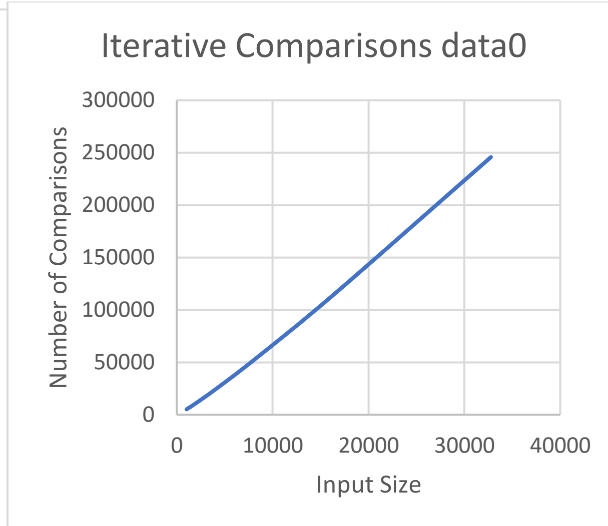
(ii) Bottom-Up approach which is an iterative implementation of merge sort.

We first use both the implementations on data0 which is already sorted.

The number of comparisons made for both the implementations on same data is as in the table below:

The below table shows the comparisons for data0:

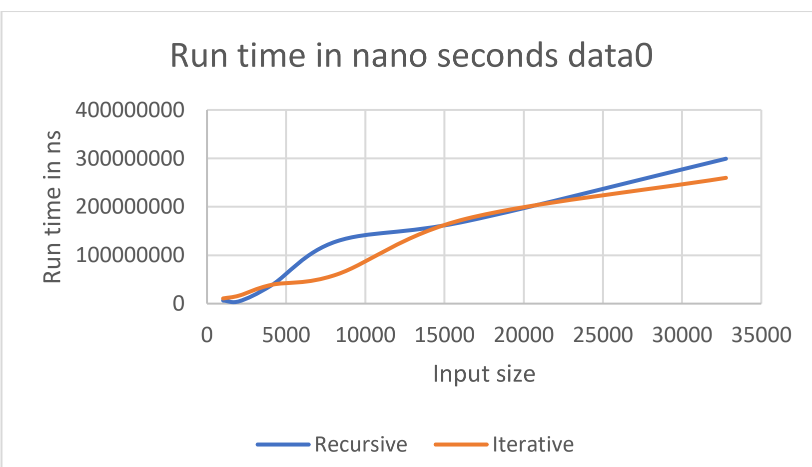
Size	Recursive	Iterative
1024	5120	5120
2048	11264	11264
4096	24576	24576
8192	53248	53248
16384	114688	114688
32768	245760	245760



It can be observed that both the implementations make the same number of comparisons.

The run time of each implementation for data0 is as below:

size	Recursive	Iterative
1024	6399200	10793700
2048	4988600	16721400
4096	38327700	38955100
8192	128713100	60122200
16384	170249000	175382600
32768	299261100	259784800



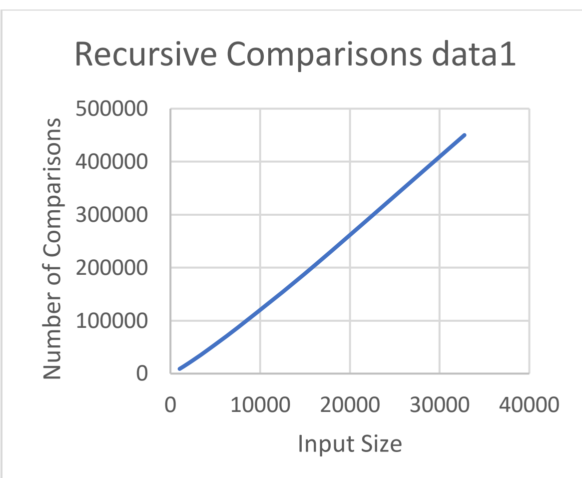
It can be observed that even though both the implementations take almost same amount of execution time, Iterative implementation has a relatively low run time.

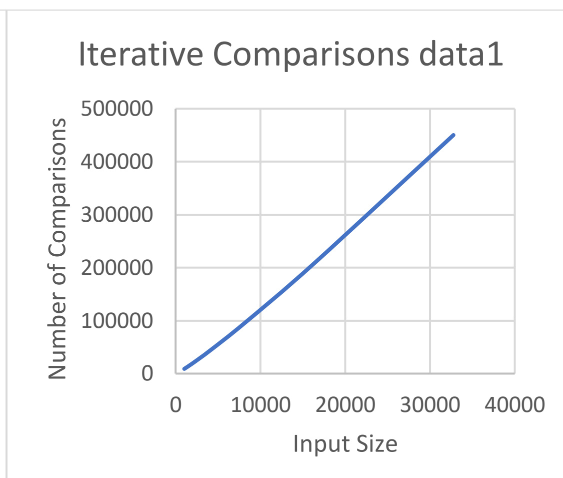
Now we implement both the implementations on data1.
Number of Comparisons made

size	Recursive	Iterative
1024	8954	8954
2048	19934	19934
4096	43944	43944
8192	96074	96074
16384	208695	208695
32768	450132	450132

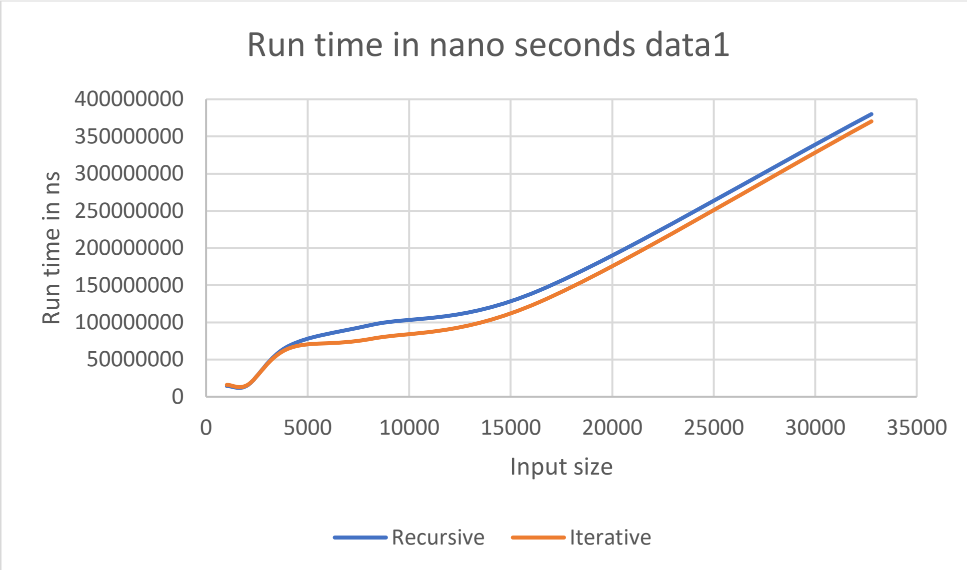
The run time of each implementation is as below:

size	Recursive	Iterative
1024	14424900	15993900
2048	15511600	16039900
4096	68658900	65167700
8192	96882700	78172800
16384	142641900	126951500
32768	380046100	370339400





The above tables and plots indicate that both the implementations have the same number of comparisons made for a given size.



The runtime of both the algorithms follow the same trend as the size increases but the iterative implementation has relatively less run time.

Hence, it can be concluded that Iterative implementation of merge sort is faster than the recursive implementation as there is no additional calls to the function in iterative implementation.

Q4 **Create a data set of 8192 entries which has in the following order: 1024 repeats of 1, 2048 repeats of 11, 4096 repeats of 111 and 1024 repeats of 1111. Write a sort algorithm that you think will sort this set "most" effectively. Explain why you think so.**

Answer:

The data created is already sorted and no further sorting is required.

However, It will be a healthy practice to check if the data is sorted and try to sort if not. The best approach for this will be the bubble sort algorithm with a check for already sorted array. The algorithm only makes N comparison for an array of size N for the best case i.e. sorted data and N^2 comparisons for worst case.

The main reason of selecting bubble sort with a check for this problem is it only takes N comparisons for a sorted data i.e. Linear comparisons.

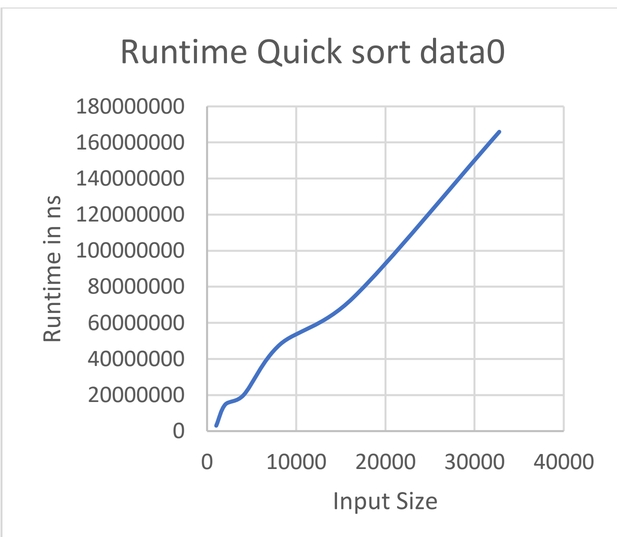
Q5. **Implement Quicksort using median-of-three to determine the partition element. Compare the performance of Quicksort with the Mergesort implementation and dataset from Q3. Is there any noticable difference when you use N=7 as the cut-off to insertion sort. Experiment if there is any value of "cut-off to insertion" at which the performance inverts.**

Answers:

The run times of the Quick sort for both the data sets are as below.

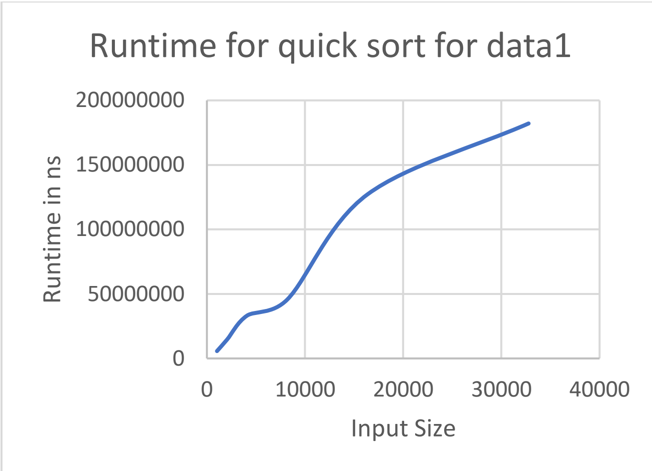
Data0:

Size	Runtime
1024	2990400
2048	14780900
4096	19995400
8192	48081600
16384	73948200
32768	165932400



Data1:

size	Runtime
1024	5682500
2048	14561900
4096	33201100
8192	45684600
16384	127311500
32768	182118500



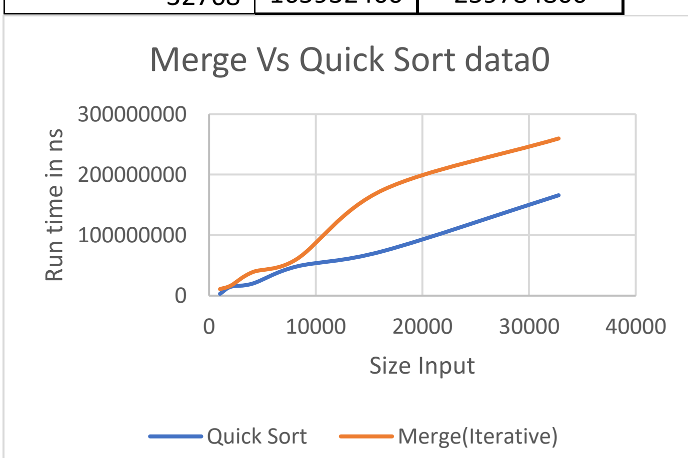
Merge vs Quick Sort:

Let us compare the runtimes in nano seconds for both the sorts on both the data sets. I have taken the runtime of iterative merge sort from the question 3.

Data0:

For Data 0, we can observe that Quick sort has a relatively less runtime compared to Merge sort.

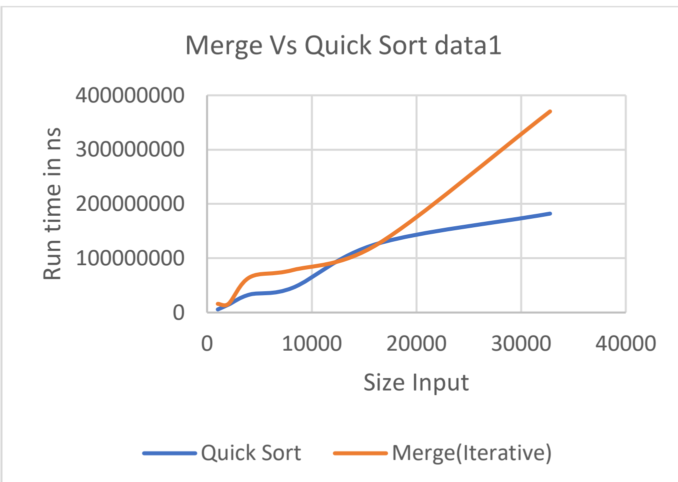
Size	Quick Sort	Merge
1024	2990400	10793700
2048	14780900	16721400
4096	19995400	38955100
8192	48081600	60122200
16384	73948200	175382600
32768	165932400	259784800



Data1:

For Data 1, we can observe that Quick sort has a relatively less runtime compared to Merge sort.

Size	Quick Sort	Merge
1024	5682500	15993900
2048	14561900	16039900
4096	33201100	65167700
8192	45684600	78172800
16384	127311500	126951500
32768	182118500	370339400

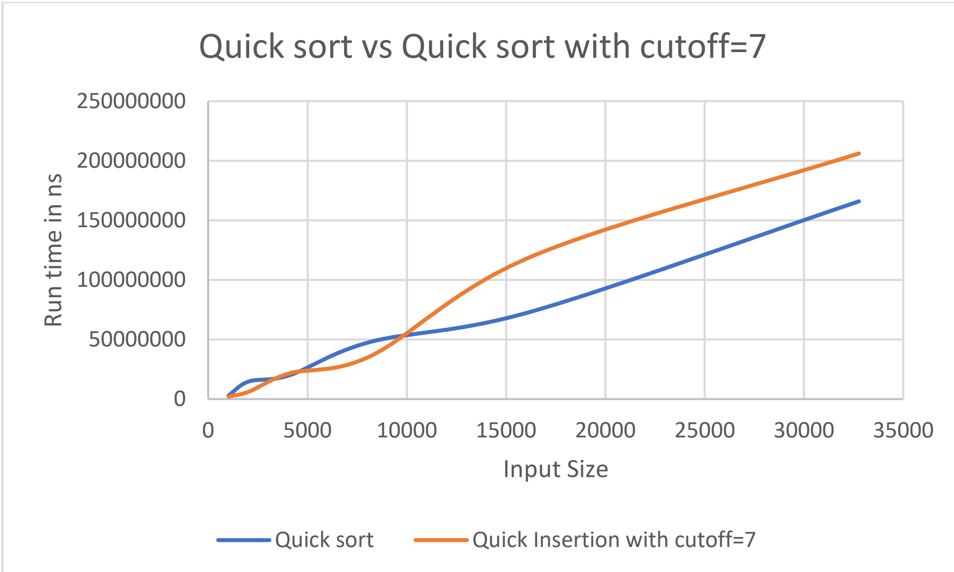


From the above observations above, We can observe that Quick sort takes comparatively less time than the Merge sort (Here I have taken the case of bottom up approach, as it has less run time than the recursive merge).

When N=7 is used as a cutoff to Insertion sort:

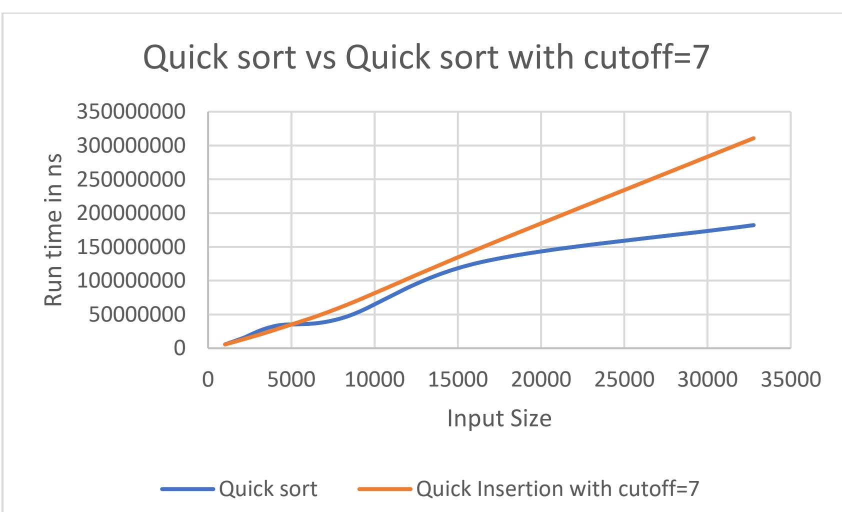
Data0:

Size	Quick sort	Quick Insertion with cutoff=7
1024	2990400	2140800
2048	14780900	6212300
4096	19995400	21637700
8192	48081600	36225700
16384	73948200	120353200
32768	165932400	206092900



Data1:

size	Quick sort	Quick Insertion with cutoff=7
1024	5682500	5642400
2048	14561900	12698700
4096	33201100	27875500
8192	45684600	62812200
16384	127311500	148559900
32768	182118500	310754100



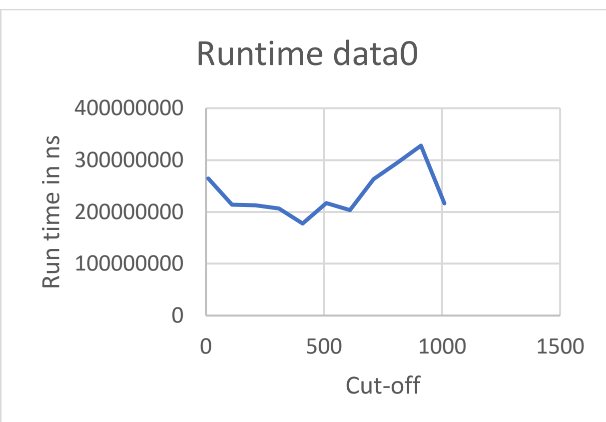
From the above two graphs we can observe that there is a noticeable difference when we use N=7 as cutoff to Insertion sort.

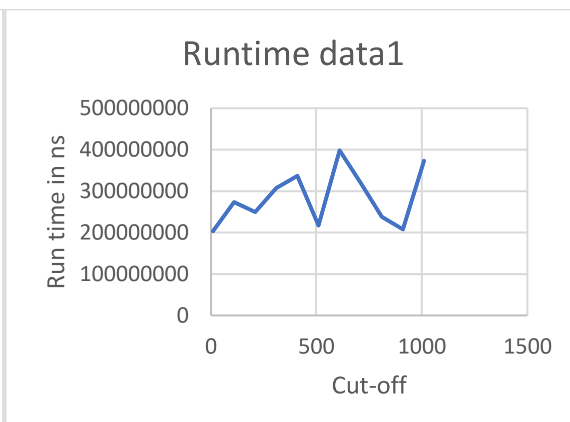
We can observe that the runtime is almost linear from 16384 for both the data.

Experiment if a cutoff value causes inversion of performance:

In order to find out a value of cut-off which inverts the performance, I have ran the function with cut-off ranging from 10 to 1010 by incrementing 100 each time. The data set 32768 from data0 and data1 was used for the purpose.

Cutoff	Data0	Data1
10	264645500	203358300
110	214008900	273494900
210	212692300	249629900
310	206382900	307378900
410	177504400	336424900
510	216884600	216756000
610	203680300	398202000
710	263414100	319883500
810	295053200	238059400
910	327787800	207799100
1010	216300400	372774100





From the above graphs, we can observe that there is no value of cut-off to insertion which inverts the performance.

Q6.

Extra Points: View the following Data Set here. The column on the left is the original input of strings to be sorted or shuffled; the column on the extreme right are the string in sorted order; the other columns are the contents at some intermediate step during one of the 8 algorithms listed below. Match up each algorithm under the corresponding column. Use each algorithm exactly once: (1) Knuth shuffle (2) Selection sort(3) Insertion sort (4) Mergesort(top-down)(5) Mergesort (bottom-up) (6) Quicksort (standard, no shuffle) (7) Quicksort (3-way, no shuffle) (8) Heapsort.

Answer:

Original	**5**	**6**	**1**	**4**	**3**	**8**	**2**	**7**	**Sorted**
navy	coal	corn	blue	blue	blue	wine	bark	mist	bark
plum	jade	mist	gray	coal	coal	teal	blue	coal	blue
coal	navy	coal	rose	gray	corn	silk	cafe	jade	cafe
jade	plum	jade	mint	jade	gray	plum	coal	blue	coal
blue	blue	blue	lime	lime	jade	sage	corn	cafe	corn
pink	gray	cafe	navy	mint	lime	pink	dusk	herb	dusk
rose	pink	herb	jade	navy	mint	rose	gray	gray	gray
gray	rose	gray	teal	pink	navy	jade	herb	leaf	herb
teal	lime	leaf	coal	plum	pink	navy	jade	dusk	jade
ruby	mint	dusk	ruby	rose	plum	ruby	leaf	mint	leaf
mint	ruby	mint	plum	ruby	rose	pine	lime	lime	lime
lime	teal	lime	pink	teal	ruby	palm	mint	bark	mint
silk	bark	bark	silk	bark	silk	coal	silk	corn	mist
corn	corn	navy	corn	corn	teal	corn	plum	navy	navy
bark	silk	silk	bark	dusk	bark	bark	navy	wine	palm
wine	wine	wine	wine	leaf	wine	gray	wine	silk	pine
dusk	dusk	ruby	dusk	silk	dusk	dusk	pink	ruby	pink
leaf	herb	teal	leaf	wine	leaf	leaf	ruby	teal	plum
herb	leaf	rose	herb	cafe	herb	herb	rose	sage	rose
sage	sage	sage	sage	herb	sage	blue	sage	rose	ruby
cafe	cafe	pink	cafe	mist	cafe	cafe	teal	pink	sage
mist	mist	plum	mist	palm	mist	mist	mist	pine	silk
pine	palm	pine	pine	pine	pine	mint	pine	palm	teal
palm	pine	palm	palm	sage	palm	lime	palm	plum	wine
||	||	||	||	||	||	||	||	||	
Original	merge-TD	quick	K. shuffle	Merge	Insertion	heap	selection	quick 3-way	