Project Euler #226: A Scoop of Blancmange

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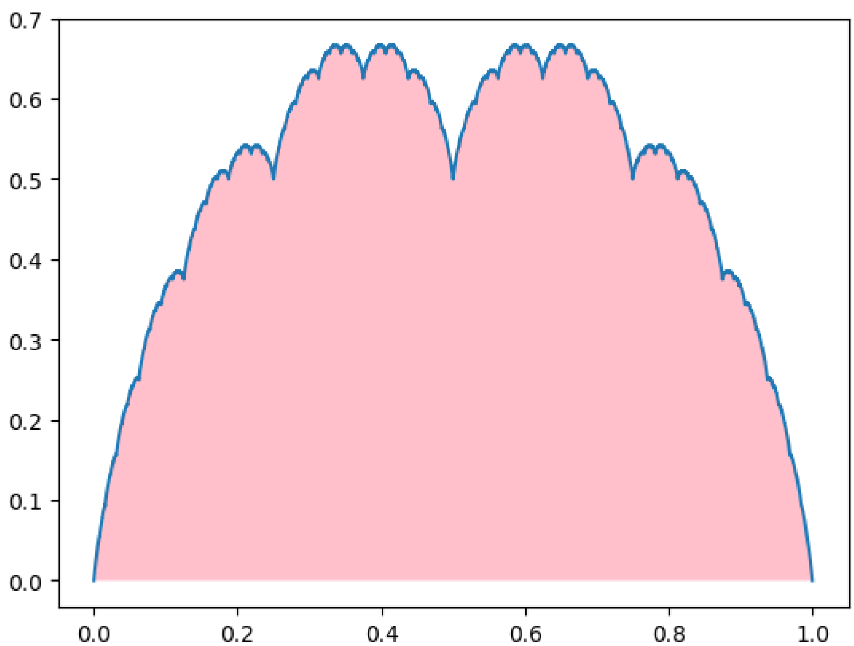
This problem is a programming version of Problem 226 from projecteuler.net

For any real number x, define $d(x)$ as the distance from x to its nearest integer.
Let $r, s \geq 2$ be positive integers and consider the function $f_{r,s}$ defined on the real inteval $[0, 1]$ by:

$$f_{r,s}(x) = \sum_{n \geq 0} \frac{d(r^n x)}{s^n}$$

For example, when $r = s = 2$ we get the blancmange function shown bellow



Given a polynomial $P = \sum_{i=0}^{m} a_i X^i$, where a_i are integers. Let

$$I = \int_0^1 f_{r,s}(x) P(x) dx$$

It can be proved that I is a rational number, therefore we can write it as $I = \frac{p}{q}$ where p and q are integers.
In addition, the constraints on the inputs guarantee that q is not divisible by the prime number 1004535809.
In this case, find $p \cdot q^{-1}$ modulo 1004535809 (q^{-1} is the the inverse of q modulo 1004535809).

Input Format

The first line of each test file contains three space-separated integers r, s and m.
The next line contains $m + 1$ space-separated integers $a_0, \ldots, a_m$.

Constraints

- $2 \le r, s \le 10^9$.
- $0 \le m \le 2 \cdot 10^5$.
- $s \cdot r^i - 1$ is not divisible by 1004535809 for all $0 \le i \le m+1$.
- $0 \le a_i \le 10^9$.
- $a_m > 0$.

Output Format

Print your answer in one line.

Sample Input 0

```
2 2 0
1
```

Sample Output 0

```
502267905
```

Explanation 0

The graph of $f_{2,2}$ is shown in the statement.
$I = \int_0^1 f_{2,2}(x)dx = \frac{1}{2}$, hence $I = 1 \cdot 2^{-1} = 502267905 \mod 1004535809$.

Sample Input 1

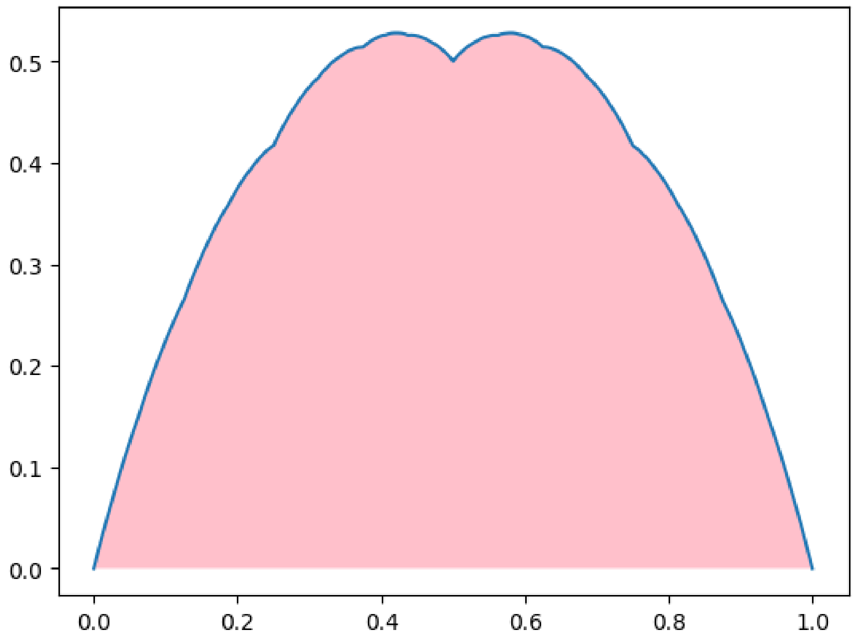
```
2 3 0
1
```

Sample Output 1

```
627834881
```

Explanation 1

Below is the graph of $f_{2,3}$



$I = \int_0^1 f_{2,3}(x)dx = \frac{3}{8}$, hence $I = 3 \cdot 8^{-1} = 3 \cdot 878968833 = 627834881 \mod 1004535809$.

Sample Input 2

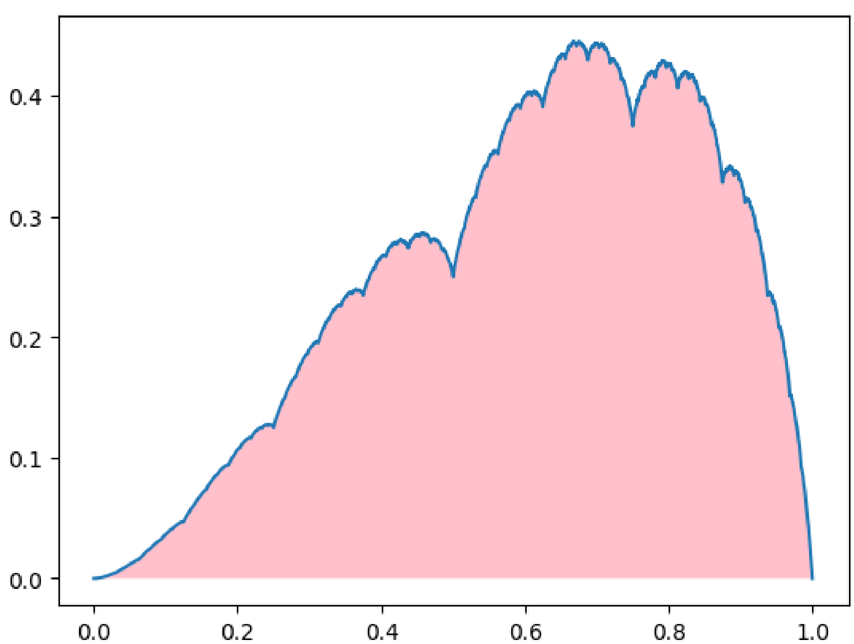
```
2 2 1
0 1
```

Sample Output 2

```
753401857
```

Explanation 2

The following is the graph of $x \to x f_{2,2}(x)$



$I = \int_0^1 x f_{2,2}(x) dx = \frac{1}{4}$, hence $I = 1 \cdot 4^{-1} = 753401857 \mod 1004535809$.

Sample Input 3

```
42 57 5
490 480 625 34 405 968
```

Sample Output 3

```
617014829
```