Project Euler #209: Circular Logic

This problem is a programming version of Problem 209 from projecteuler.net

A k-input binary truth table is a map from k input bits (binary digits, 0 [$false$] or 1 [$true$]) to 1 output bit. For example, the 2-input binary truth tables for the logical AND and XOR functions are:

x	y	x AND y	x XOR y
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

How many n-input binary truth tables, τ, satisfy the formula $\tau(F_1(a_1, a_2, \ldots, a_n), F_2(a_1, a_2, \ldots, a_n), \ldots, F_n(a_1, a_2, \ldots, a_n))\ AND$ $\tau(G_1(a_1, a_2, \ldots, a_n), G_2(a_1, a_2, \ldots, a_n), \ldots, G_n(a_1, a_2, \ldots, a_n)) = 0$ for all n-bit inputs $(a_1, a_2, \ldots, a_n)$?

Input Format

The first line of each test file contains a single integer n. n lines follow with descriptions of the functions F_i on each line. n lines follow then with descriptions of the functions G_i on each line.

Every description follow the grammar described below:

$$Formula \rightarrow Summand | Summand + Formula$$
$$Summand \rightarrow 0|1|Product$$
$$Product \rightarrow Letter|Letter\&Product$$
$$Letter \rightarrow aIndex$$
$$Index \rightarrow 1..n$$

where $\&$ means logical AND, $+$ means logical XOR, $aIndex$ result into $a_1 \ldots a_n$.

For example, one of the possible function descriptions could look as follows:

```
a1&a2+a1+1
```

One should interprete this as the function $(a_1\ AND\ a_2)\ XOR\ a_1\ XOR\ 1$

Constraints

- $1 \leq n \leq 6$
- Every description of a function has length < 600. Moreover, every possible summand occurs in each description not more than once.

Output Format

Print exactly one number, which is the answer to the problem.

Sample Input 0

```
1
1
a1
a1+1
```

Sample Output 0

```
3
```

Explanation 0

Let's look at all possible τ:

- $\tau(x) = 0$. Then it doesn't depend on $a1$ and the statement is always true
- $\tau(x) = 1$. It also doesn't depend on $a1$ but now the statement is always false
- $\tau(x) = x$ and $\tau(x) = x\ XOR\ 1$ both lead us to the statement $a1\ AND\ (a1\ XOR\ 1) = 0$ which is always true.

That said, our answer is 3.

Sample Input 1

```
1
1
a1
0
```

Sample Output 1

```
2
```

Explanation 1

Using the same logic as in previous sample, we can deduce that $\tau(x) = 0$ is good and $\tau(x) = 1$ is bad. Let's take a look into $\tau(x) = x$ and $\tau(x) = x\ XOR\ 1$:

- $\tau(x) = x$. After substitution we get $a1\ AND\ 0 = 0$ which is always true.
- $\tau(x) = x\ XOR\ 1$. Now we get $(a1\ XOR\ 1)\ AND\ 1 = 0$. It is wrong for $a1 = 0$.

That leaves us with only two good τ.

Sample Input 2

```
2
2
a1&a2
a1&a2+1
a1
a1&a2+1
2
1
a1
a1&a2+a2+a1+1
a2+a1
```

Sample Output 2

```
4
5
```

Sample Input 3

```
2
3
```

```
a2&a3+a1&a3+a1
a1&a2&a3+a2&a3+a2
a2&a3+a3+a2
a1&a2&a3+a1&a3
a2&a3+a1&a3+a1&a2+a2
a2&a3+a1&a3+a3+a1&a2+a1
3
a1&a2&a3+a2&a3+a3+a2+a1
a1&a2&a3+a2&a3+a1&a2+1
a1&a2&a3+a2&a3+a1&a3+a1&a2+a1
a1&a2&a3+a2&a3+a3+a1&a2+a2+a1+1
a1&a2&a3+a2&a3+a3+a1&a2+a2+a1
a1&a2&a3+a1&a3+a1&a2+a2+a1+1
```

Sample Output 3

```
48
80
```