Project Euler #191: Prize Strings

This problem is a programming version of Problem 191 from projecteuler.net

A particular school offers cash rewards to children based on their score history.

During an $l-\textbf{day}$ period, a string (scores) is formed, for each child, in the following way:

$s_1|s_2|\ldots|s_l$

where $0 \leq s_i \leq c-1$ is the score of the child at the i^{th} day.

If they get 1 for m consecutive days $\textbf{or}$ 0 on more than n occasion(s) then they forfeit their prize.

For a given l, n, m and c, let's call strings for which the child gets his prize $\textbf{prize strings}$, and denote $f(l,n,m,c)$ the number of prize strings for these parameters.

For example, with $l=4$ ($4-\textbf{day}$ period), $n=1$, $m=3$ and $c=3$, it can be verified that $f(4,1,3,3)=43$ and here are the different prize strings in this case:

$2|2|2|2$ $2|2|2|1$ $2|2|2|0$ $2|2|1|2$ $2|2|1|1$ $2|2|1|0$ $2|2|0|2$ $2|2|0|1$ $2|1|2|2$ $2|1|2|1$

$2|1|2|0$ $2|1|1|2$ $2|1|1|0$ $2|1|0|2$ $2|1|0|1$ $2|0|2|2$ $2|0|2|1$ $2|0|1|2$ $2|0|1|1$ $1|2|2|2$

$1|2|2|1$ $1|2|2|0$ $1|2|1|2$ $1|2|1|1$ $1|2|1|0$ $1|2|0|2$ $1|2|0|1$ $1|1|2|2$ $1|1|2|1$ $1|1|2|0$

$1|1|0|2$ $1|1|0|1$ $1|0|2|2$ $1|0|2|1$ $1|0|1|2$ $1|0|1|1$ $0|2|2|2$ $0|2|2|1$ $0|2|1|2$ $0|2|1|1$

$0|1|2|2$ $0|1|2|1$ $0|1|1|2$

You are given L, N, m and c, what is $\sum_{l=1}^{L}\sum_{n=1}^{N} f(l,n,m,c) \bmod 7+10^9$.

Input Format

The only line of each test case contains exactly four integers separated by single spaces: L, N, m and c

Constraints

- List Item $1 \leq N \times L \leq 10^7$
- List Item $1 \leq m \leq L$
- List Item $2 \leq c \leq 10^{18}$

Output Format

Print the answer modulo $7+10^9$

Sample Input

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4 1 3 3
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Sample Output

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73
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Explanation

$f(4,1,3,3) = 43$, $f(3,1,3,3) = 19$, $f(2,1,3,3) = 8$ and $f(1,1,3,3) = 3$. Hence the sum is 73.