

# Project Euler #198: Ambiguous Numbers

This problem is a programming version of [Problem 198](#) from [projecteuler.net](#)

A best approximation to a real number  $x$  for the denominator bound  $d$  is a rational number  $r/s$  (in reduced form) with  $s \leq d$ , so that any rational number  $p/q$  which is closer to  $x$  than  $r/s$  has  $q > d$ .

Usually the best approximation to a real number is uniquely determined for all denominator bounds. However, there are some exceptions, e.g.  $9/40$  has the two best approximations  $1/4$  and  $1/5$  for the denominator bound  $6$ . We shall call a real number  $x$  *ambiguous*, if there is at least one denominator bound for which  $x$  possesses two best approximations. Clearly, an ambiguous number is necessarily rational.

How many ambiguous numbers  $x = p/q$ ,  $a/b < x < c/d$ , are there whose denominator  $q$  does not exceed  $N$ ?

## Input Format

The only line of each test case contains exactly five space-separated integers:  $a$ ,  $b$ ,  $c$ ,  $d$  and  $N$ .

## Constraints

- $a/b, c/d \leq 10^3$
- $0 \leq a, c \leq N$
- $0 < b, d \leq N$
- $1 < N \leq 2 \times 10^9$

## Output Format

On a single line print the answer modulo  $10^9 + 7$ .

## Sample Input 0

```
1 4 1 2 25
```

## Sample Output 0

```
3
```

## Explanation 0

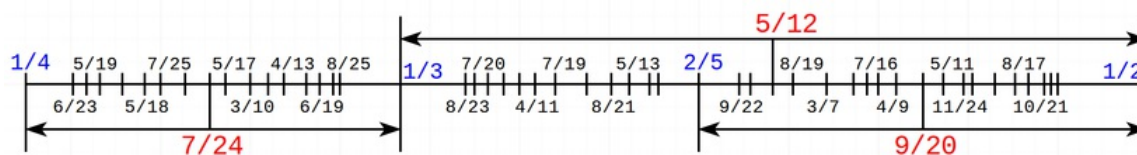
There are 49 rational numbers between  $1/4$  and  $1/2$  with the denominator no greater than 25:

$6/23, 5/19, 4/15, 3/11, 5/18, 7/25, 2/7, 7/24, 5/17, 3/10,$   
 $7/23, 4/13, 5/16, 6/19, 7/22, 8/25, 1/3, 8/23, 7/20, 6/17,$   
 $5/14, 9/25, 4/11, 7/19, 3/8, 8/21, 5/13, 7/18, 9/23, 2/5,$   
 $9/22, 7/17, 5/12, 8/19, 3/7, 10/23, 7/16, 11/25, 4/9, 9/20,$   
 $5/11, 11/24, 6/13, 7/15, 8/17, 9/19, 10/21, 11/23$  and  $12/25$ .

Only three of them are *ambiguous* numbers:  $7/24$ ,  $5/12$  and  $9/20$

- $1/3$  and  $1/4$  are the two best approximations of  $7/24$  for the denominator bound 5;

- $\frac{1}{2}$  and  $\frac{1}{3}$  are the two best approximations of  $\frac{5}{12}$  for the denominator bound 4;
- $\frac{1}{2}$  and  $\frac{2}{5}$  are the two best approximations of  $\frac{9}{20}$  for the denominator bound 6.



Therefore the answer is **3**.