

Duration: 3 hours

Maximum Marks: 100

Instructions:

1. All questions are compulsory.
2. Do not write anything other than your Reg. No. on the question paper.
3. Programmable calculator is not permitted.

Part -A

(10x1=10)

- A1 [CO1] The asymptotes parallel to x-axis are obtained by equating to zero the coefficient of the highest power of y in the equation of the curve, provided it is not merely a constant. (True/False)
- A2 [CO1] If $\phi_1(m)=0$ but $\phi_2(m)\neq 0$, then the number of asymptotes are ∞ (a) 0 (b) 1 (c) $n-1$ (d) n
- A3 [CO2] If $u = e^{2xy}$, then $\frac{\partial^2 u}{\partial y \partial x} = \dots$ (a) $xe^{xy} + e^{xy}$ (b) $xe^{xy} + x^2 e^{xy}$ (c) $xe^{xy} + y^2 e^{xy}$ (d) none of these
- A4 [CO2] $\frac{x^2 - y^2}{\sqrt{x^2 + y^2}}$ is a homogeneous function of degree _____. (a) 0 (b) 1 (c) 2 (d) none of these
- A5 [CO3] The value of the integral $\int_1^a \int_1^b \frac{dx dy}{xy} = \dots$
- A6 [CO3] $I(n+1) = \dots$ (a) $I(n)$ (b) $n! I(n)$ (c) $(n-1) I(n-1)$ (d) none of these
- A7 [CO4] If $\phi = x^2 + xy + z^2$ then $\nabla \phi$ at the point (1,2,3) is $\frac{1}{\sqrt{14}} \sqrt{14} \hat{i} + \frac{2}{\sqrt{14}} \sqrt{14} \hat{j} + \frac{6}{\sqrt{14}} \sqrt{14} \hat{k}$ (a) $\sqrt{14} \hat{i} + 2\sqrt{14} \hat{j} + 6\sqrt{14} \hat{k}$ (b) $\sqrt{14} \hat{i} + \sqrt{14} \hat{j} + \sqrt{14} \hat{k}$ (c) $\sqrt{14} \hat{i} + 2\sqrt{14} \hat{j} + 3\sqrt{14} \hat{k}$ (d) none of these
- A8 [CO4] If $\vec{F} = (2x - 5y)\hat{i} + (x + \lambda y)\hat{j} + (3x - z)\hat{k}$ is solenoidal, the value of λ is (a) 1 (b) -1 (c) 0 (d) none of these
- A9 [CO5] Any integral which is evaluated along a curve is called a ____ integral. (a) surface (b) line (c) volume (d) none of these
- A10 [CO5] Which of the following theorem convert line integral to surface integral?

- (a) Stoke's theorem only (b) Green's theorem only
- (c) Stoke's and Green's theorem (d) Gauss divergence and Stoke's theorem

Part -B

(5x2=10)

- B1 [CO1] Find the radius of curvature of the curve $s = 4a \sin(\psi/3)$
- B2 [CO2] If $u = \tan^{-1}(\frac{x^2 + y^2}{x - y})$ the obtain the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$
- B3 [CO3] Evaluate $\int_0^1 x^2 (1 - x^2)^3 dx$
- B4 [CO4] If $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ then evaluate $\nabla \cdot (\frac{\vec{F}}{r^3})$
- B5 [CO5] State Gauss Divergence Theorem. (5x6=30)

Part -C

(5x6=30)

- C1 [CO1] For the curve $y = \frac{ax}{a+x^2}$, prove that $(\frac{z}{y})^2 + (\frac{y}{x})^2 = (\frac{2a}{a})^{2/3}$.
- C2 [CO2] If $u = f(y - z, z - x, x - y)$, prove that $\frac{\partial u}{\partial z} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial x} = 0$
- C3 [CO3] Prove that $B(m, n) = B(m+1, n) + B(m, n+1)$
- C4 [CO4] Find a unit vector normal to the surface $x^2 + y^2 + 3xyz = 3$ at the point (1,2,-1).
- C5 [CO5] Evaluate by Stoke's theorem $\int_C (e^x dx + 2y dy - dz)$ where C is the curve $x^2 + y^2 = 4$ and $z = 2$

Part -D

(5x10=50)

- D1 [CO1] Find the asymptotes of the curve: $y^3 - x^2 y - 2xy^2 + 2x^3 - 7xy + 3y^2 + 2x^2 + 2x + 2y + 1 = 0$
- D2 [CO2] Obtain the maximum and minimum value of the given function $f(x, y) = \sin x + \sin y + \sin(x + y)$
- D3 [CO3] Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} y^2 \sqrt{(x^2 + y^2)} dx dy$ by changing into polar coordinates.
- D4 [CO4] Find the values of a, b and c such that $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational vector field. Also find its scalar potential.
- D5 [CO5] A vector field \vec{F} is given by $\vec{F} = \sin y \hat{i} + x(1 + \cos y)\hat{j}$. Evaluate the line integral $\oint_C \vec{F} \cdot d\vec{r}$ using Green's theorem, where C is the circular path given by $x^2 + y^2 = a^2$

JECRC UNIVERSITY, JAIPUR
B. Tech. I SEMESTER (Common for all Branches)
Special In-Sem Examination-2021-22
Subject: Engineering Mathematics-I
Paper code: DMA001A

Maximum Marks: 50

Duration: 1.30 Hours

Instructions:

1. All questions are compulsory.
 2. Write the question number clearly while answering. Draw figures whenever necessary.
 3. Write legibly on both the sides of answer-book.
- CO2 Understand the functions of more than one independent variable and calculate partial derivatives along with their applications. Also obtain an idea for finding the extreme values of functions of more than one variable.
- CO3 Will able to integrate a continuous function of two or three variables over a bounded region and able to trace the curves.
- CO4 Understand the representation of vector and its properties.

SECTION A

(10×1 = 10 marks)

- A1.[CO2] What is the condition for a function $f(x, y)$ to be minimum?
- A2.[CO2] The minimum value of $f(x, y) = 2x^2 + 2y^2 + 6x + 6$ is
 (a) -3/2 (b) 5/2 (c) 3/2 (d) No minima
- A3.[CO3] If the revolution is about x-axis, then volume $V = \int_a^b \pi y^2 dx$. (True/False)

A4.[CO3] The value of $\int_0^{\infty} \sqrt{x} e^{-x} dx = \dots\dots\dots$

A5.[CO3] The value of $B(m+1, n) + B(m, n+1) = \dots\dots\dots$

- (a) $B(m, n)$ (b) $B(m-n)$ (c) $B(m+n)$ (d) none of these

A6.[CO3] The value of $B\left(\frac{5}{2}, \frac{1}{2}\right)$ is.....

- (a) $3\pi/2$ (b) $3\pi/8$ (c) $3\pi/4$ (d) none of these

A7.[CO3] Evaluate the value of $\Gamma\left(-\frac{1}{2}\right) = \dots\dots\dots$

A8.[CO4] $\vec{a} \times (\vec{b} \times \vec{c}) = \dots\dots\dots$

A9.[CO4] A vector \vec{A} is called the solenoidal vector if

A10.[CO4] If $\vec{a} = 2x\hat{i} + y\hat{j} + 2z\hat{k}$, $\vec{b} = 2\hat{i} + 2xy\hat{j} + 2yz\hat{k}$, and $\vec{c} = \hat{i} + \hat{j} + \hat{k}$ then $[\vec{a}\vec{b}\vec{c}] = \dots$

SECTION B

(4×2 = 8 marks)

B1.[CO2] Find the stationary points of the function $f(x, y) = x^2 + y^2 + 2/x + 2/y$.

B2.[CO3] Find the value of the integral $\int_0^1 \int_0^1 e^{y/x} dx dy$.

B3.[CO3] Using beta or gamma function Evaluate, $\int_0^{\infty} x^4 e^{-x^2} dx$.

B4.[CO3] Change the order of integration of the integral $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} f(x, y) dx dy$.

SECTION C

(2×6 = 12 marks)

C1.[CO2] Find the dimensions of the rectangular box, without top, of maximum capacity whose surface area is 108 cm^2 .

C2.[CO4] Prove that $\nabla^2 f(r) = f''(r) + 2f'(r)/r$.

SECTION D

(2×10 = 20 marks)

D1.[CO3] Evaluate the integral by changing to polar coordinates $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2 \sqrt{x^2+y^2} dx dy$.

D2.[CO4] A fluid motion is given by $\vec{q} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$, is this motion irrotational? If so, find the velocity potential.

JECRC UNIVERSITY, JAIPUR
I In-Sem Examination, November 2021
B. Tech- I Semester
Branch: Common to all branches
Subject: Engineering Mathematics-I
Code: DMA001A

Time: 1.5 hrs.

Instructions:

Maximum Marks: 50

1. Attempt all the questions.
2. Illustrate your answers with suitable examples and diagrams, wherever necessary.
3. Write relevant question number before writing the answer.

Part -A

(10x1=10)

- A1 [CO1] The asymptote of the curve $x^2y - 3x^2 - 5xy + 6y + 2 = 0$ which is parallel to the x -axis is $y = 3$.
- A2 [CO1] The equation of the asymptote is $y = mx + c$, where c is calculated by the formula:
 (a) $\frac{\phi_{n-1}(m)}{\phi_n'(m)}$ (b) $-\frac{\phi_{n-1}(m)}{\phi_n'(m)}$ (c) $\frac{\phi_{n-1}'(m)}{\phi_n(m)}$ (d) $-\frac{\phi_{n-1}'(m)}{\phi_n(m)}$
- A3 [CO1] The curve $x^3 + y^3 = 3axy$ is symmetric about the line $y = -x$. (True/False)
- A4 [CO1] The curve $x^2(y^2 + x^2) = a^2(x^2 - y^2)$ has no asymptotes. (True/False)
- A5 [CO1] A curve of degree n has maximum n asymptotes.
- A6 [CO2] If $u = \log(y \sin x + x \sin y)$, then $\frac{\partial u}{\partial x} =$ _____.
- A7 [CO2] If $z = x^2 - y^2 + 3xy$, then the value of $\frac{\partial^2 z}{\partial x \partial y}$ is
 (a) 0 (b) 1 (c) 2 (d) 3
- A8 [CO2] Define homogeneous function in two variables.
- A9 [CO2] If $u = x^4 + y^4 + 3x^2y^2$, then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} =$ _____
 (a) 0 (b) $4u$ (c) $\frac{4}{u}$ (d) $3u$
- A10 [CO1] The formula to find the radius of curvature of the curve $y = f(x)$ at any point (x, y) is _____.

Part -B

(4x2=8)

- B1 [CO1] Find the asymptotes parallel to the axes to the curve $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.
- B2 [CO1] Find the radius of curvature at origin of the curve $y = x^3 + 5x^2 + 6x$.
- B3 [CO2] If $z = x^5y^4$, where $x = t^3, y = t^2$, find $\frac{dz}{dt}$.
- B4 [CO2] Find $\frac{dy}{dx}$ if $x^y = y^x$.

Part -C

(2x6=12)

- C1 [CO1] Find the asymptotes of the curve: $x^3 - 5x^2y + 8xy^2 - 4y^3 + x^2 - 3xy + 2y^2 - 1 = 0$.
- C2 [CO1] In the curve $y = ae^{\frac{x}{a}}$, prove that $\rho = a \sec^2 \theta \operatorname{cosec} \theta$, where $\theta = \tan^{-1}\left(\frac{y}{a}\right)$.

Part -D

(2x10=20)

- D1 [CO1] Trace the curve $y^2(a-x) = x^2(a+x)$
- D2 [CO2] If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$.