Exercise 2.1

Bayes rule: $P(H \mid E_1 \cap E_2) = \frac{P(E_1 \cap E_2 \mid H) \cdot P(H)}{P(E_1 \cap E_2)}$

a.) i) insufficient, b/c $P(E_1 \cap E_2 \mid H) \neq P(E_1 \mid H) \cdot P(E_2 \mid H)$ in general.

ii) sufficient

iii) insufficient

b.) conditional independence implies $P(H \mid E_1 \cap E_2) = \frac{P(E_1 \mid H) \cdot P(E_2 \mid H) \cdot P(H)}{P(E_1 \cap E_2)}$

i.) sufficient

ii.) sufficient

iii) use $P(E_1 \cap E_2) = \sum_{k=1}^{K} p(H_k) \cdot p(E_1 \cap E_2 \mid H_k)$

$= \sum_{k=1}^{K} p(H_k) \cdot p(E_1 \mid H_k) \cdot p(E_2 \mid H_k)$ $\Rightarrow$ sufficient.

Exercise 2.3

a.) Independence $\Rightarrow$ factorization

if $p(x,y \mid z) = p(x \mid z) \cdot p(y \mid z)$ then simply set $g(x,z) = p(x,z)$ and $h(y,z) = p(y,z)$ then we get the factorization.

b.) factorization $\Rightarrow$ independence. This is tricky!

if $p(x,y \mid z) = g(x,z) \cdot h(y,z)$ then

$$1 = \sum_{x,y} p(x,y \mid z) = \sum_{x,y} g(x,z)\, h(y,z) = \sum_x g(x,z) \cdot \sum_y h(y,z) \qquad (*)$$

and

$$p(x \mid z) = \sum_y p(x,y \mid z) = \sum_y g(x,z)\, h(y,z) = g(x,z) \cdot \left(\sum_y h(y,z)\right)$$

$$p(y \mid z) = \sum_x p(x,y \mid z) = \sum_x g(x,z) \cdot h(y,z) = \left(\sum_x g(x,z)\right) \cdot h(y,z)$$

multiply the last two equations

$$p(x \mid z) \cdot p(y \mid z) = g(x,z) \cdot h(y,z) \cdot \underbrace{\sum_y h(y,z) \cdot \sum_x g(x,z)}_{=1 \text{ by } (*)} = p(x,y \mid z)$$

by assumption

so independent.

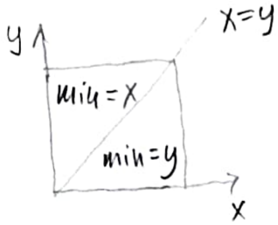
Exercise 2.5 $X \in [0,1] \quad p(x) = 1$ (pdf's)

$Y \in [0,1] \quad p(y) = 1$

also $p(x,y) = p(x) \cdot p(y) = 1$ (independent)

$z = \min\{x, y\}$

First solution, using 2-dimensional integration



$$\mathbb{E}(Z) = \int_0^1 \int_0^1 \min\{x,y\}\, dy\, dx$$

$$= \int_0^1 \int_0^x y\, dy\, dx + \int_0^1 \int_x^1 x\, dy\, dx$$

$$= \int_0^1 \frac{1}{2} y^2 \Big|_0^x dx + \int_0^1 x(1-x)\, dx$$

$$= \frac{1}{2} \int_0^1 x^2\, dx + \int_0^1 x - x^2\, dx = \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} - \frac{1}{3} = \frac{1+3-2}{6} = \frac{2}{6} = \frac{1}{3}$$

Second solution, using cpdfs

$$P(Z > z) = P(X > z \wedge Y > z) = P(X > z) \cdot P(Y > z) = (1-z)^2$$

$$\Rightarrow p(z) = \frac{d}{dz} P(Z > z) = \frac{d}{dz} (1-z)^2 = +2(1-z) = 2 - 2z$$

$$\Rightarrow \mathbb{E}(Z) = \int_0^1 z \cdot p(z)\, dz = \int_0^1 2z - 2z^2\, dz = 1 - \frac{2}{3} = \frac{1}{3}$$

Exercise 2.11

a.) Event space $X = \{2B, B+G, 2G\}$

$p \rightarrow \quad \frac{1}{4} \quad \frac{1}{2} \quad \frac{1}{4}$

any boys: $Y = \{2B, B+G\} \Rightarrow p(Y) = \frac{3}{4}$

$p(H \cap Y) = p(\{B+G\}) = \frac{1}{2}$

one child is a girl: $H = \{B+G, 2G\} \Rightarrow p(H) = \frac{3}{4}$

$\Rightarrow p(H|Y) = p(H \cap Y)/p(Y) = \frac{1/2}{3/4} = \frac{2}{3}$

b.) Event space $X = \{BB, BG, GB, GG\}$ first indicates the child that ran by

$Y = \{BB, BG\} \quad p(Y) = \frac{1}{2}$

$p(H \cap Y) = p(\{BG\}) = \frac{1}{4}$

$H = \{BG\} \quad p(H) = \frac{1}{4}$

$\Rightarrow p(H|Y) = p(H \cap Y)/p(Y) = \frac{1/4}{1/2} = \frac{1}{2}$

Exercise 2.9

This should be the result of the 2nd part of the programming assignment

prob. of negative if tested negative 0.99999.9..

" " positive " " positive 0.009804..