A Qualitative Study on Online Platform Product Ranking

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Abstract

When a person uses an online platform to purchase any object/item that platform fetches ‘n’ number of relevant items on ‘k’ number of pages. Usually these n,k are very huge numbers as online marketing is huge. Well observed phenomenon that consumer having no knowledge on what the sellers are offering, many one goes to purchase the items of first few pages only. That means high ranked items/products are almost capturing the most of market while remaining low rank positioned market losing it’s balance with competitors. From consumer side also he/she is only choosing the product which is more visible at high rank where remaining items are remained invisible, which affect the consumer welfare. This term paper "Product Ranking on Online Platform" aims is to study and try to emphasize work on this two objectives i)Maximizing market welfare ii) Maximizing consumer welfare with improved novel methods proposed by Mahsa Derakhshan[1] with her team. These methods deals with consumer search model and ranking schemes supporting above consumer search method. This paper proposed a novel approach on two-stage consumer search model and two other novel algorithms on online ranking which improves market share maximization and consumer welfare respectively.

1 Introduction

Digital advances caused rapid phase changes in almost everything in human life, Digital market is one of it. Online platform has grown very much rapidly such that now a days an average product offered online by various sellers. More numbers of sellers in one digital platform makes so many choices for the consumer. This may be a good for consumer because of having lot’s of supply. But a simple study shows that when a online platform offered ‘n’ number of various versions of same product in ‘k’ number of pages user mostly going only first few pages. Millward Brown reports that 70% of Amazon users do not go beyond the first page of search results [2]. This behavior of consumer making ranking of product is prominent factor to consider. Product ranking is making products on top positions imposing externality in other products [1]. This will cause imbalance market shares who provide products but positioned low. Maintaining market welfare is mandatory factor for balance of supply - demand in an online platform.

Usually consumers had different levels of patience and time in order to go through deeper pages, this behaviour can be captured by imposing **search cost** for each customer. This cost captures the type of different buyers and how much deep they search. Each consumer search keywords,type of product depends to his/her perspective. Empirical studies shows that probability of purchase on product decreases as positioning it in to deeper page. That decrease rate also very different user to user. This puts a question that does consumer really extracting the type of product he needed

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or he just getting satisfied with top ranked product because of the reason that consumer can't go through all product descriptions.This concludes that from consumer side he/she is unable to go through all choices to make choices according to their taste. This is an important factor to consider because a market can only gain trust and stability by consumers if he/she getting satisfaction on their product. This leads to another question that if markets are aiming for consumer satisfaction does online platforms could able to make their profits. Without profits there is no stability to market.

In Product ranking as consumer choice has become increasingly important. Empirical evidence shows that, when two products say Product1, Product2 and Product1 has more popularity, if you put Product1 at higher position probability of purchasing Product2 decreases 51% and if Product2 positioned in top purchase probability for Product1 will drop to 35%. Similarly another study says that when products positioned randomly or by popularity either way probability of click decreases as going deeper pages, and **placing popular products in top decreases overall market purchase probability** which suggest that there is huge impact of ranking on this online markets welfare. From above knowledge we can highlight two following objectives that online market-customer facing, which we solve with efficient methods.

- Maximizing Market Share
- Maximizing Consumer Welfare

This term paper following chapters as followed. In chapter 2 proposed consumer search model which is optimal, In chapter 3 we discuss a ranking problem we needed to solve if we followed method proposed in chapter 2. Next discussed some conclusions and Extension study of this paper in chapter 5.

2 Optimal Consumer Search Model

Consumer choice on product captures a key features about his behaviour. Usually consumer choice always towards maximization of his utility. While he is trying to maximize his utility due to above mentioned reasons consumer not really getting his optimal product which he needed. So modelling a search model in which he can follow so that he could maximize his welfare is key thing.

This chapter proposes novel a consumer search model **Two-stage screening process** for a product. At first consumer selects some number of products(**consideration set**) then he do deep evaluation of those considered products to finalise one product. Formally At first stage he forms a consideration set based on extracting product preference weights through screening, these product preference weights are decided by platform and product features. At stage two, again screens a product from consideration set based on his own taste. This model utilities for consumer behaviour are modelled as **i)Intrinsic Utility ii)Idiosyncratic Utility**.

Every product has some preference weights and based on these preference weights consumer goes through screening process. Intrinsic utility extract this screening behaviour by imposing positioned based search cost to consumer. Any consumer after screening some products into consideration set, he proceeds to select one product based on his own interest in it. This behaviour was captured by Idiosyncratic utility of user. More elaborately a patient person search for deeper pages so that he could get good product which means he has very less penalty/cost to his Intrinsic Utility, so considers huge no of products into consideration set, where as an impatient person not prefers to go to deeper pages which implies he has high penalty/cost to his Intrinsic utility, while he screens he indeed screens very less number of products. And Idiosyncratic utility can be defined as difference between Utility of purchased product and search cost for that product to be selected.

2.1 Related Work

A single-stage model such as work by Weitzman[12] assumes that consumer learns both preference weights and idiosyncratic utility at screening process. And above process adopted by Usru[6] and proposed a model which resembles like, in platform consumer first examines product fully and then go to next product. In practice such model also not captures consumer behaviour clearly. This two-stage model we can relate closely to consider-then-choose model proposed in marketing-and-operations literature. Some papers like Auoad[9,10,11],Rusmevichientong[13] is considers consideration set formed exogenously, some other work[14] which considers evaluation cost after it included in consideration set.Here this two-stage approach different than other above is consideration set formed by sequential screening process with which for this screening itself consumer has to pay position based cost.

2.2 Intuition

Following Two-stage screening process intuitively mimics the actual person how he will choose a product. An actual person in online first he will add all reliable products into one list and then he choose best suitable one among all. This paper proposed method highly matches this method among other literatures proposed.

2.3 Preliminaries

Consider n products in On-line platform $i \in [n] = \{1, 2, 3, ..., n\}$ ordered by index $j \in [n]$ and assume **Position with lower index have higher visibility**. Let denote U_i is Intrinsic utility random variable and Z_i is i product idiosyncratic utility random variable. Assume Z_i drawn Gumbel[15] distribution. At first stage consumer seeks preference weights and extracted through sequential screening. Previous work[6] considers consumer know these weights but in our model he doesn't know weights which he need to extract at first stage. These preference weights captures features of product and can be defined as exponent of intrinsic utility. $W_i = e^{u_i}$. Assume each consumer has type set $k \in [K]$ characterised by set of **search costs** $\{s_1^k, s_2^k, .., s_n^k\}$ where s_j^k is search cost imposed to consumer of type k when he screens out product positoined in j and one general assumption that we make when consumer goes deeper(j increases) search cost to screen a product increases $\{s_{j+1}^k \geq s_j^k\}$. This modelling mimics that type k person with how much patience he can screen a product. Simple words this nature of search cost defines every person has to stop screening process at some point of time according to his Intrinsic utility otherwise he is imposing higher penalties as going to deeper pages.

2.4 Two-Stage Optimal Search policy

By defining permutation $\pi : [n] \longrightarrow [n]$ where product i placed in position j represented as $i = \pi(j)$ Let C be a consideration set and when consumer screens a product he has to pay position search cost as well as idiosyncratic cost which essentially he has to pay at second stage evaluation, so here screening process incur two costs to products which he screens. Every consumer try to maximise his utility so consumer welfare can be defined as

$$Wel_\pi^k(C, W) = \mathbb{E}\left[\max_{i \in C \cup \{\phi\}} \{\log(w_i) + Z_i\}\right] - \sum_{i \in C} s_{\pi^{-1}(i)}^k \qquad (1)$$

where above consumer welfare is difference between Expected maximum utility of he could get and search cost to screen positions, consideration set C is union with choice ϕ because leaving platform without screening anything also a possibility for consumer. For above welfare $Wel(C, W)$ consumer adds a product weight w_i to W if he decides that paying search cost for it is worthy, and if not he don't consider it. This behaviour can be defined with $V(C, W)$ expected welfare of consumer of type k if he follows above two stage approach. This sequential screening process is a dynamic problem(DP) and $V(C, W)$ is recursive equation as follows

$$V(C, W)_\pi^k = max\left\{Wel_\pi^k(C, W), \max_{j \in [n] \setminus \pi^{-1}(C)} \mathbb{E}\left[V(C \cup \{\pi(j)\}), W \cup \{W_{\pi(j)}\}\right]\right\} \qquad (2)$$

From above we can observe consumer welfare updates for every screening step he does. If he decided to choose product welfare updates to eq.(2), if not search cost added to eq(1). At final when he stops screening we will get $W(C, W)$ welfare for consumer. When consumer decides to screen next position term $\max_{j \in [n] \setminus \pi^{-1}(C)} \mathbb{E}\left[V(C \cup \{\pi(j)\}), W \cup \{W_{\pi(j)}\}\right]$ captures this decision. But this is DP problem which we needed to essentially solve. This paper proposed an optimal search policy which essentially a solution to this above DP problem.

Consumer can not randomly go through products and evaluates its for weights instead from a key result of following lemma we can modify our welfare eq.(1) as following

Lemma: *The expected utility of consumer who screens products in set C is given by*

$$\mathbb{E}\left[\max_{i\in C\cup\{0\}}\{\log(w_i)+Z_i\}\right]=log(1+w(C))+\gamma \qquad (3)$$

where $w(C)=\sum_{i\in C} w_i$ *and* γ *is Eular-Mascheroni constant*

proof to above lemma provided in [1].

from above lemma we can infer that, consumer only needed product weights and search cost that he has to screen each position. Based on that we define new measure r_j^k which is called reservation prices. This r_j^k satisfies below equation for type k consumer when he screened $j-1$ products and when he screens j positioned product.

$$\mathbb{E}\left[\log(1+r_j^k+W)\right]-log(1+r_j^k)=s_j^k \qquad (4)$$

for more elaboration Let consumer screened out product i positioned on $j=\pi(i)$ and observed weight w_j and $C=\{\phi,\pi(j)\}$. Now if consumer consider next product positioned on $j+1$ and he screens it out, his expected welfare would be $-s_{j+1}^k+\mathbb{E}\left[\log(1+w_j+W)\right]+\gamma$ and $C=\{\phi,\pi(j),\pi(j+1)\}$ or if he does not chose to screen it, welfare would be $log(1+w_j)+\gamma$ and $C=\{\phi,\pi(j)\}$ now based on this if you solved (4) for reservation price r_{j+1}^k this price resembles consumer decision as he would select product if $w_j<r_{j+1}^k$ and not screens the product if $w_j>r_{j+1}^k$ so these reservation prices giving a selection criteria so consumer follow it once he calculated it.

Above analogy is just for 2 products, when there is n products he sequentially screens through products using these reservation price, this price is obtained from eq.(4) using screened weights $w(C)$ till now, search costs $(s_j^k)_{j\in\{0,1,..,n\}}$. As initially mentioned $\{s_{j+1}^k\geq s_j^k\}$ makes property on reservation prices $\{r_{j+1}^k\leq r_j^k\}$ this property making sequential process to an index-based search which results a search policy that consumer starts search by index and when he encountered weights in consideration set exceeds reservation price he stops screening. Following inequality is stopping condition.

$$\sum_{j'\in|j|} w_{\pi(j')}\geq r_{j+1}^k \qquad (5)$$

suppose above condition is satisfied at position j then consideration set would be $C_\pi^k=\{\pi(1),\pi(2),..,\pi(j)\}$ and if initial start itself if $r_1^k\leq 0$ then consideration set would be $C=\{\phi\}$ means consumer left without buying. Proof to this above index-based policy indeed maximizes welfare from eq.(2) is provided as Theorem 1 in [1].

Algorithm

initialise $C_\pi^k=\phi$;$j=1$

while $\left(\sum_{j'\in|j|} w_{\pi(j')}\leq r_{j+1}^k\right)$

  Add product $\pi(j+1)$ **into** W;

  j $\leftarrow$ **j+1**;

Stop

And above such search model is optimum when search cost increases with index increased. Now if consumer follow above optimal index-based search policy he screens products obviously from highest visibility position why because highest visible position lead to lowers search cost. So still there is huge impact of product ranking in this search model including consumer type. Although this model is efficient for consumer welfare, we needed supportive algorithms to rank products in platform to achieve our objectives.

3 Optimal Product ranking to index-based search policy

With huge discussion on how much product ranking is important, we came across an optimal and consumer welfare oriented search policy. But as we addressed before maximizing market welfare and consumer welfare are two eyes of this online market. Product ranking which is feasible to increase purchase probability is main concern now. This following section discuss the some of work on product ranking and proposes optimal product ranking approximation algorithms to above proposed search policy.

3.1 Related Work

Mahajan and Van Ryzin[15] observed that for gumbel distribution effect idiosyncratic utility probability of purchasing a product. And Varian and Edelman [16] discussed effect of positioning along with Click-through-rate of consumer, Similarly Athey and Ellison[17] observed externality of higher positioned products with lower positioned products. All this work are proved effectively allocating products but not as effective as concerning the objective of maximizing consumer welfare and market shares. In above literature work done Chu et al [18] is an elegant method that sequential search process calls surplus-ordered-ranking but it was not able generalise well to our objectives.

3.2 Intuition

This ranking algorithm also mimics realtime analogy. Suppose consider case to a fruit shop when buyer comes to buy some fruits if seller kept his fruits in order with freshness and size, consumer screening will very short he will take out best among he could afford but seller is left out with least ones which gives him loss. Instead when seller put his best fresh and big fruits with smaller ones as stacks(buckets) consumer buy best out those stacks, but in this case seller gets his maximum market share. Similarly if online platform also follow proposed PTAS algorithm which resembles above procedure it is easy to get good shares.

3.3 Preliminaries

As ranking/ordering to above consumer search policy is crucial let's start with defining probability of consumer belong to type k given $[K]$ by θ^k which gives distribution as $(\theta^1, \theta^2, ..\theta^K)$. This distribution is what online platform need to know in-order to rank the product. We start with an assumption that platform already know preference weights of products but what they don't know idiosyncratic cost of the consumer type k. The assumption that platform already know preference weights is valid since with lot of computational data and historical records a platform can really model such product weights distribution. So for our methodology we assume product preference weights w_i are drawn from distribution $f_w : [w_{min}, w_{max}] \to \mathbb{R}^+$ and w_{min}, w_{max} are minimum and maximum weights of n products.

In ranking of products for n products there are $n!$ ways of permutations. Defining there possibilities as set Π and an ordering that consumer follow $\pi \in \Pi$. From this basic definition we can define for type k consumer with ordering π consideration set would be $\{C_\pi^k\}_{\pi\in\Pi, k\in[K]}$ and preference weights $w_i \in W$ range can be defined with $\rho = \frac{w_{max}}{w_{min}}$ is measure to complexity of ranking algorithm. To be precise the following algorithm which is going to be proposed is works only if when $\rho = O(1)$.

3.4 Optimal Ranking

Optimal ranking for above consumer search policy discussed separately with respect to our objectives. We will first show that market share maximization problem M_s and consumer welfare maximization problem Wel are NP-complete problems. This NP-complete problems hard to solve so we develop approximation algorithms. We will discuss two of these approximation algorithms, first algorithm will take π ranking/ordering with decreasing order of preference weights which we call it as **w-ordered** algorithm, with this we will propose new algorithm called **PTAS** (polynomial time approximation scheme). Brief introduction about these two algorithms as follows

W-ordered Algorithm : This algorithm defined simply ordering products based on decreasing order of preference weights. So as platform has preference weights this algorithm has no need of any knowledge about type of consumer and his search costs. Even though it is matching with consumer search policy it is not optimum algorithm when product has high preference weight. High preference weight products decreases consideration set size with this algorithm we need to balance this problem.

PTAS Algorithm : In contrast to above algorithm this PTAS follows two step process, First it will put each weight into some predefined no.of buckets by rounding weights , Second reformulate NP-complete problem into Dynamic programming problem.(DP-problem). This reformulation yield solution to our NP-complete problems and gives best optimum ranking using approximation factors. And this proposed algorithm finds this solution with time complexity of polynomial in n and linear with K.

3.4.1 Optimal Ranking for maximizing market Share

We defined needed notions of ranking π and consumer type distribution $\{\theta^k\}_{k\in[K]}$ in preliminaries. Let C_π^k be consideration set of consumer type k when products are positioned with permutation π. Now denoting weights $\{w_i\}_{i\in C_\pi^k}$ and their summation as $w(C_\pi^k)$ platform market share M_s can be defined as fraction of consumers who purchase product when ordering followed permutation π. The initial assumption was when consumer buying a product in consideration set it depends on idiosyncratic utility Z_i of consumer. With the assumption that this z_i has Gumbel distribution we can say Probability of buying product from consideration set

$$P_{buying}\left[i \in C_\pi^k\right] = \frac{\sum_{i\in C_\pi^k} w_i}{1+\sum_{i\in C_\pi^k} w_i} = \frac{w(C_\pi^k)}{1+w(C_\pi^k)} \qquad (6)$$

above eq.(6) comes from property of Gumbel distribution. This implies market share maximization is a optimization problem of finding permutation π which maximises above eq.(6) probability for all type $k \in [K]$ consumers.

$$M_s = \max_{\pi\in\Pi} \sum_{k\in[K]} \theta^k \frac{w(C_\pi^k)}{1+w(C_\pi^k)} \qquad (7)$$

Now defining our market share maximization problem we can observe this problem is NP-complete. So we apply our proposed approximation algorithms which essentially solve eq.(7). Lets apply above mentioned algorithms

W-ordered Algorithm

In this algorithm we don't need to find permutation since this algorithm ranks products in decreasing order of preference weights. By following this algorithm we will yield multiplicative approximation factor as $\frac{1}{2}$ and additive approximation factor as 0.1716 to our NP-complete problem by following below equation.

$$M_{s(w-ord)} \geq max\left\{\frac{M_s}{2}, M_s - 0.1716\right\} \approx max\left\{\frac{M_s}{2}, M_s - 0.1716\right\} \qquad (8)$$

where $M_{s(w-ord)}$ is market share for w-ordered algorithm and M_s is maximum possible market share if ordering followed optimal ordering. For above approximation factors with a simple example[1] we can prove for high weight products positioning at first causes worst market share.

When there is high weight product we need to make modifications to arrangement to get high market share. That means we needed to put medium weight products in high position go get high market share, conversely we need to put high weight product at high position to shorten screening process of consumer this two problems intuitively viewed as to make consumer purchase a product platform has to put as many as possible products in his consideration set in contrast consumer needed less search efforts to put and get good product.

PTAS Algorithm

In this algorithm when $\rho = O(1)$ it indeed produces optimal ranking which maximises market share. This algorithm guarantees atleast $(1-\epsilon)M_s$ market share for $\epsilon \in \left(\frac{log(\rho)}{n}, 1\right)$. Just as introduced, this algorithm follows two step process first it creates bucket of weights by rounding product weights. After these weights are bucketed, we will choose bucket which maximizes share and then product from it. More elaborately explained after defined this procedure clearly.

Rounding Weights : Let there are $[B]$ buckets and bucket $b \in [B]$ we put product with weight w_i in bucket b if

$$\underline{w}(1+\epsilon)^{b-1} \leq w_i \leq \underline{w}(1+\epsilon)^b \qquad (9)$$

where $\underline{w} = min_{i\in[n]} w_i$, denoting $\bar{w} = max_{i\in[n]} w_i$ we can generalise $B \leq O(\frac{log(\rho)}{\epsilon})$ and $\rho = \frac{w_{max}}{w_{min}} \geq \frac{\bar{w}}{\underline{w}}$. For each bucket b has unique weight $Rw_b = \underline{w}(1+\epsilon)^b$ so all products which fall into this bucket has this value Rw_b.

In market share maximization problem in eq.(6) $w(C^k_\pi)$ is sum of original weights w_i, denoting rounded weights as $\tilde{w_i}$ we can say $w(\tilde{C^k_\pi}) = \sum_{i\in C^k_\pi} \tilde{w_i}$. now with all requirements we had, and small mathematical fact for $\delta > 0$ and $x \geq 0$, $\frac{\delta x}{1+\delta x} \leq \delta \frac{x}{1+x}$. we can say below.

$$\frac{\tilde{w}(C^k_\pi)}{1+\tilde{w}(C^k_\pi)} \leq (1+\epsilon)\frac{w(C^k_\pi)}{1+w(C^k_\pi)} \qquad (10)$$

eq.(10) proves that rounded weights always captures all consideration set products into account, but not proves that it not captures additional products hence there could be chances that it screens additional products which has high weight due to round off it considers it. This problem can be mitigated by limiting search space of permutation π to **Low-weight-priority class(LWP)**.

A permutation is belongs to this LWS class if products in each bucket has arranged increasing order according to original weight(Low weight product take first place and selected first when bucket is selected). One question arise is does this above class contain our optimal permutation that we needed to find, Yes indeed it consist. We simply argue this result by eq.(10) consideration set formed by this class permutation is always greater than optimal permutation consideration set.

DP-based algorithm : This algorithm solves our eq.(7). If $j-1$ products are already placed from different buckets,then we needed to know for j position which bucket had to select, From LWS property we just place low weight product in that position. Knowing bucket index for position j becomes DP-based problem which results approximated solution to eq.(7).

We have property of $s^k_j \geq s^{k+1}_j$ gives us when consumer type k not stop screening process at position j all consumers of type$k' > k$ also not stops screening. Assume bucket b selected for j position and $w_{b(min)}$ be minimum weight product in b at that instant. Now let k' be minimum consumer type who stops screening after consumer k stops screening. we can define this as

$$k' = min\left\{k \in [k, K] : w_{b(min)} + \sum_{l=1}^{j-1} w_l \leq r^k_{j+1}\right\} \qquad (11)$$

from above we can see at position j, type k consumer stop screening and next consumer who stops screening would be k' so between these consumers types no other stops screening process.Market profit they get in this transition j to $j+1$ would be

$$\frac{Rw_{\mathbb{B}}+Rw_b}{1+Rw_{\mathbb{B}}+Rw_b}\sum_{\tilde{k}=k}^{k'-1}\theta^{\tilde{k}} \qquad (12)$$

Where $Rw_{\mathbb{B}}$ is sum of rounded weights till $j-1$ positions and Rw_b is rounded weight of bucket b in position j . Total Market share can be written as recursive equation with profit they get in each transition as followed

$$W_j(S) = \max_{b\in[B]}\left\{W_{j+1}(S') + \frac{Rw_{\mathbb{B}}+Rw_b}{1+Rw_{\mathbb{B}}+Rw_b}\sum_{\tilde{k}=k}^{k'-1}\theta^{\tilde{k}}\right\} \qquad (13)$$

this formulation we can solve optimal b which gives maximum welfare. From that we choose next consumer type for filling $j+1$ position and repeat this process till we fill up n products. By the end of algorithm we get positioning of products according to market welfare maximization.

Algorithm

initialise Buckets, Round products weights and put sorted weights (LWS) order in buckets

while j not equals n

Find $b^* = arg \max_{b \in [B]} \left\{ W_{j+1}(S') + \frac{Rw_{\mathbb{B}} + Rw_b}{1 + Rw_{\mathbb{B}} + Rw_b} \sum_{\tilde{k}=k}^{k'-1} \theta^{\tilde{k}} \right\}$

Fill position j **with** $\min\{w_i \in b^*\}$ **product**

k $\leftarrow$ **k'**;

Update $Rw_{\mathbb{B}}, Rw_b$;

j $\leftarrow$ **j+1**;

Stop

3.4.2 Optimal Ranking for maximizing Consumer welfare

Unlike market share, consumer welfare increases with screening best product with less effort. With above mathematical definitions we can easily define welfare maximization problem. For type k consumer having product ranking as permutation π Consumer welfare can be defined as optimization problem as follows.

$$Wel = \max_{\pi \in \Pi} \sum_{k \in [K]} \theta^k Wel_\pi^k (C_\pi^k, \{w_i\})_{i \in C_\pi^k} \qquad (14)$$

Above problem also poses as NP-Complete. Solving this problem also done with PTAS algorithms with slight modification.

W-ordered Algorithm : We will get multiplicative factor of 1 and additive factor of 0.6931 to this setting. Following ideas just as market share maximization although this w-ordered algorithm appealing we have high weight product here too. So we apply PTAS algorithm.

PTAS-Algorithm : We imply PTAS idea here along with w-ordered algorithm. As consumer screens products this algorithm guarantees $Wel - \epsilon$ market share when $\epsilon \in \left(\frac{\rho}{\epsilon}, 1\right)$ this algorithm makes permutation to first $M = \frac{\rho}{\epsilon}$ positions following PTAS and then next positions are just filled with decreasing order of product weights(This is like a mixed version of two algorithms) but it guarantees for a consumer who is less patient he get his maximum share because first M positions are obtained from PTAS optimal ranking scheme which maximises consumer welfare problem eq.(14) above and decreasing weights order ensures more patient persons welfare maximum.

Algorithm

initialise $M = \frac{\rho}{\epsilon} \quad \rho = \frac{w_{max}}{w_{min}} \quad j = 0$

while$j \leq M$

obtain Ordered set S **and places** i^{th} **element of ordered set in** i^{th} **position.**

j $\leftarrow$ **j+1**;

Stop

while$M < j \leq n$

put i^{th} **highest weight product in** $n - M$ **product in** i^{th} **position.**

j $\leftarrow$ **j+1**;

Stop

4 Conclusion

This above procedure to solve our two specified objectives indeed solved effectively and PTAS has given results as Very good approximation algorithms to proposed Consumer Optimal index based search policy. The team has done practical implementation of this algorithm to bench mark this algorithm with existing algorithm and showed 5% increase comparably with existing ones. And algorithm time complexity is indeed $O(K^2 n^{log(\rho)/\epsilon+1})$ for market share maximization and $O(Kn^{1+\rho/\epsilon})$

5 External Study, Observations and Extension ideas

- With Future study that they mentioned that ρ need not be the constant we can extend this work to variable ρ which implies good additive approximation to this algorithm.
- We can make this algorithm more robust to preference weights. As the part of literature survey to this paper I got interest in work done by Negin Golrezaei[19] which is on Robust algorithm to fake users. This idea is of fake users can affect the preference weights which cause fail to this algorithm.
- This algorithm was proposed in such away that it works predefined preference weights but in online platform preference weights and consumer type sets are continuously altered. To be precise this proposed methodology work like offline algorithm. We can this of possibilities that making this algorithm online.
- This proposed algorithm fully depends on the assumption that preference weights are genuine and obtained previously by platform as extension of this we can try decrease the dependency of this preference weights.
- We can improve/implement this algorithm to Auctions which essentially we will have multiple consumers and multiple providers on items if possible we can implement to this multiple agent auctions and observe results.
- From the insight of the work done by Wenjia Ba, Haim Mendelson [21] we have considered in our algorithm that consumer patients level based dependency, but for both patient and impatient consumers we do descending order of valuations. They have showed that if you put ascending order to patient consumer and descending order of valuations gives maximum market share.
- We can put up another stage in consumer search policy so that consumer screens product in 3 stages this results definitely more guaranteed consumer welfare but as an impatient person it leads to very complex problem.
- Our work is taking multiple consumers type as probability distribution and finding One optimal ranking which shown for all consumers. This property can be extended dynamic product ranking according to his past/present consideration set.
- As proposed we can try to emphasize this optimum algorithm to dynamic page ranking instead Static one. Inspired from the work done by Kris Ferreira,Shreyas Sekar [20].
- The work proposed by Aouad and segav[9,10,11] considers that consumer each consumer views random number of pages and forms consideration set. But with slight modification we can think of to our optimal search policy we can add this random glance by consumer as one stage in screening process.
- Consumer consideration set made and then from consideration set consumer again selected one product. But consumer may screen for multiple products which essentially leads to question does this algorithm extendible to multinomial logit algorithm.
- We have not considered the fact that this algorithm taking product Intrinsic utility that not consumer dependent but real case along with idiosyncratic utility we can include the case of where product intrinsic utility is also varies with consumer type.
- From above point we can add extra observation that we are assuming platform has no information about consumer but we can add some common and diverse properties from consumer details which are obtained from previous history of platform user details.

- We are observing consumer search strategy and making search model along with it we can observe additional properties such as how he decided to purchase particular product and what sequence he is following to buy particular product to get more reasonable market share maximization.

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