Set theory

Set →

A Set is well-defined collection of distinct objects.

Ex (i) Rivers in India
(ii) Students who speak either Hindi or English
(iii) the vowels in English alphabet.
(iv) Countries in the world.

Elements of a Set → the objects in a set are called its elements or members

* Generally Capital letters A, B, C, - - etc. are used to denote sets and its elements by lower case letters a, b, c, — etc.
* the symbol $\in$ (epsilon) is used to indicate, belongs to
* the symbol $\notin$ (epsilon not) is used to indicate, not belongs to

Ex (i) $x \in A$ (ii) $y \notin A$

Standard Sets →

$N = \{1, 2, 3, 4, \ldots\}$, the set of natural no's

$I = \{\ldots, -3, -2, -1, 0, 1, 2, 3 \ldots\}$, the set of integers.

$Q = \{x : x = \frac{p}{q} ; p, q \in I \text{ \& } q \neq 0\}$, the set of rational no.

R = the set of real no's.

$C = \{x : x = a + ib ; a, b \in R, i = \sqrt{-1}\}$, the set of complex no's.

Representation of a Set → there are two ways of representing a set

(i) Roster or Tabular form
(ii) Rule Method or Set builder form

Roster or Tabular form → In this from all the elements of the set are listed, the elements being separated by commas and are enclosed within braces.

ex (i) $A = \{a, e, i, o, u\}$, the set of vowels in the english alphabets.

Rule or Set builder form → In this method, a set is define by specifying a property that elements of the set have in common.

ex (i) $A = \{1, 2, 3, 4, 5, 6\}$

$A = \{x : x \in N, 1 \leq x \leq 6\}$

(ii) $B = \{1, 4, 9, 16, 25, 36\}$

$B = \{x : x = n^2, n \in N \cdot n \leq 6\}$

(iii) I or $Z = \{x : x$ is an integer$\}$

Finite and Infinite set → A set with finite no's of elements in it, is called a finite set. those sets which are not finite are called infinite sets.

ex (i) the set of students in a class (finite set)

(ii) the set of Natural no's (infinite set)

Null or Empty Set → A set which contains no element is called null or empty set. denoted by ϕ or $\{\}$

ex (i) $A = \{x : 1 < x < 2$, x is an integer$\}$

Singleton Set → A set which has only one element is called a Singleton set.

ex (i) $A = \{x\}$ is a Singleton set.

Sub set → If A and B are sets such that every element of A is also an element of B, then A is said to be a subset of B. denoted by $A \subseteq B$

i.e., $A \subseteq B$, if $x \in A$ and $x \in B$

* Every set A is a subset of itself i.e., $A \subseteq A$
* the null set ϕ is subset of every non-empty set
* If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.
* Number of subsets of a set $= 2^n$, n is no. of elements in a set.

Super Set → If A is a subset of B, then B is called the superset of A. denoted by $B \supseteq A$ & read as "B is a super set of A"

Proper Subset → Any subset A is said to be proper subset of another set B if A is a subset of B, but there is at least one element of B which does not belongs to A

i.e., $A \subseteq B$ but $A \neq B$ It is denoted by $A \subset B$ read as "A is proper subset of B"

Equal Set → Two sets A and B are said to be equal if and only if every element of A is an element of B and consequently every element of B is an element of A.

i.e., $A \subseteq B$ and $B \subseteq A \Rightarrow A = B$.

Universal Set → All the sets under investigation are likely to be considered as subsets of particular set. this set is called the Universal set. denoted by U

Operations on Sets →

(i) **Union** → the union of two sets A and B denoted by $A \cup B$.

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

(ii) **Intersection** → the intersection of two sets A and B denoted by $A \cap B$

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

(iii) **Complements** → let U be the universal set and A, B are two subsets of U. Complement of A defined as

$$A' = \{x : x \in U \text{ and } x \notin A\}$$

* let $x \in A' \Rightarrow x \notin A$

* $A - B = \{x : x \in A \text{ and } x \notin B\}$ (Difference)

* let $x \in A \cup B \Rightarrow x \in A$ or $x \in B$

* let $x \notin A \cup B \Rightarrow x \notin A$ and $x \notin B$

(iv) **Symmetric Difference** → the symmetric difference of two sets A and B, denoted by $A \Delta B$ or $A \oplus B$.

$$A \Delta B = (A - B) \cup (B - A)$$

or

$$A \Delta B = \{x : x \text{ belongs to exactly one of A and B}\}$$

Disjoint set → two sets A and B are said to be disjoint set if $A \cap B = \phi$

Cardinal Number of a Set → the number

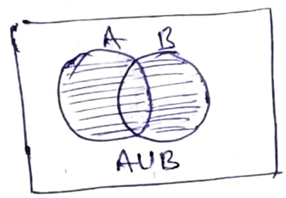
Operations on Sets →

(1) **Union of Sets** → Let A and B are two non-empty sets the union of A and B is denoted by $A \cup B$ and defined as

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

Properties of unions of Sets →

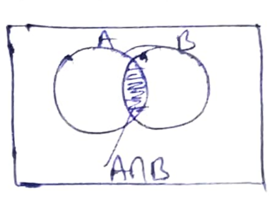
(i) The union of sets is commutative $A \cup B = B \cup A$
(ii) The union of sets is Associative $(A \cup B) \cup C = A \cup (B \cup C)$
(iii) The union of sets is idempotent $A \cup A = A$
(iv) If A is any set, then $A \cup \phi = A$
(v) If A is any subset of the universal set U, then
$A \cup U = U$



(2) **Intersection of Sets** → Let A and B are two non-empty sets. The intersection of A and B is denoted by $A \cap B$ defined as $A \cap B = \{x : x \in A \text{ and } x \in B\}$

Properties of Intersection →

(i) The intersection of set is Commutative $A \cap B = B \cap A$
(ii) " " is Associative $(A \cap B) \cap C = A \cap (B \cap C)$
(iii) " " set A is idempotent $A \cap A = A$
(iv) If A is any set, then $A \cap \phi = \phi$
(v) If A is any set & U is the universal set then
$A \cap U = A$



Difference of Two Sets → If A and B are any two sets then the difference of A and B denoted by A−B. defined as $A-B=\{x : x\in A \text{ and } x\notin B\}$

Properties

(i) $A-A=\phi$ (ii) $A-\phi=A$ (v) $(A-B)\cup A=A$

(iii) $A-B\subseteq A$ (iv) $(A-B)\cap B=\phi$

Complement of a Set → Let A be any set. the Complement of A is the set of elements that belongs to the Universal set but do not belongs to A. denoted by A^c or $\overline{A}$.

$$A^c=U-A=\{x : x\in U \text{ and } x\notin A\}$$

Properties

(i) $A\cup\overline{A}=U$ (ii) $A\cap\overline{A}=\phi$ (iii) $\overline{(\overline{A})}=A$ (iv) $\overline{U}=\phi$

(v) $\overline{(A\cup B)}=\overline{A}\cap\overline{B}$ (vi) $\overline{(A\cap B)}=\overline{A}\cup\overline{B}$

Symmetric difference of sets → Let A and B be two non-empty sets. then the symmetric difference of A and B, denoted by $A\oplus B$ or $A\Delta B$. & defined as

$$A\Delta B=(A-B)\cup(B-A)$$
or
$$A\Delta B=(A\cup B)-(A\cap B)$$

Properties

(i) Commutative $A\oplus B=B\oplus A$

(ii) Associative $(A\oplus B)\oplus C=A\oplus(B\oplus C)$

(iii) $A\oplus A=\phi$

(iv) $A\oplus B=\phi \iff A=B$