2022-Jul-28 Shift-2

AI24BTECH11003 - Badde Vijaya Sreyas

1) Let $S = \left\{x \in [-6, 3] - \{-2, 2\} : \frac{|x+3|-1}{|x|-2} \geq 0\right\}$ and $T = \left\{x \in Z : x^2 - 7|x| + 9 \leq 0\right\}$. Then the number of elements in $S \cap T$ is

a) 7
b) 5
c) 4
d) 3

2) Let α, β be roots of the equation $x^2 - \sqrt{2}x + \sqrt{6} = 0$ and $\frac{1}{\alpha^2} + 1, \frac{1}{\beta^2} + 1$ be the roots of the equation $x^2 + ax + b = 0$. Then the roots of the equation $x^2 - (a + b - 2)x + (a + b + 2) = 0$ are:

a) non-real complex numbers
b) real and both negative
c) real and both positive
d) real and exactly one of them is positive

3) Let A and B be any two 3×3 symmetric and skew-symmetric matrices respectively. Then which of the following is NOT true?

a) $A^4 - B^4$ is a symmetric matrix
b) AB−BA is a symmetric matrix
c) $B^5 - A^5$ is a skew-symmetric matrix
d) AB+BA is a skew-symmetric matrix

4) Let $f(x) = ax^2 + bx + c$ be such that $f(1) = 3, f(-2) = \lambda$ and $f(3) = 4$. If $f(0) + f(1) + f(-2) + f(3) = 14$, then λ is equal to

a) -4
b) $\frac{13}{2}$
c) $\frac{23}{2}$
d) 4

5) The function $f : R \to R$ defined by $f(x) = \lim_{n\to\infty} \frac{\cos(2\pi x) - x^{2n}\sin(x-1)}{1 + x^{2n+1} - x^{2n}}$ is continuous for all x in

a) $R - \{-1\}$
b) $R - \{-1, 1\}$
c) $R - \{1\}$
d) $R - \{0\}$

6) The function $f(x) = xe^{x(1-x)}, x \in R$, is

a) increasing in $\left(-\frac{1}{2}, 1\right)$
b) decreasing in $\left(\frac{1}{2}, 2\right)$
c) increasing in $\left(-1, -\frac{1}{2}\right)$
d) decreasing in $\left(-\frac{1}{2}, \frac{1}{2}\right)$

7) The sum of the absolute maximum and absolute minimum values of the function $f(x) = \arctan(\sin x - \cos x)$ in the interval $[0, \pi]$ is

a) 0
b) $\arctan\left(\frac{1}{\sqrt{2}}\right) - \frac{\pi}{4}$
c) $\arccos\left(\frac{1}{\sqrt{3}}\right) - \frac{\pi}{4}$
d) $\frac{\pi}{4}$

8) Let $x(t) = 2\sqrt{2}\cos t\sqrt{\sin 2t}$ and $y(t) = 2\sqrt{2}\sin t\sqrt{\sin 2t}, t \in \left(0, \frac{\pi}{2}\right)$. Then $\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$ at $t = \frac{\pi}{4}$ is equal to

a) $\frac{-2\sqrt{2}}{3}$ b) $\frac{2}{3}$ c) $\frac{1}{3}$ d) $\frac{-2}{3}$

9) Let $I_n(x) = \int_0^x \frac{1}{(t^2+5)^n} dt, n = 1, 2, 3 \cdots$ Then

a) $50I_6 - 9I_5 = xI_5'$ b) $50I_6 - 11I_5 = xI_5'$ c) $50I_6 - 9I_5 = I_5'$ d) $50I_6 - 11I_5 = I_5'$

10) The area enclosed by the curves $y = \log_e\left(x + e^2\right), x = \log_e\left(\frac{2}{y}\right)$ and $x = \log_e 2$, above the line $y = 1$ is

a) $2 + e - \log_e 2$ b) $1 + e - \log_e 2$ c) $e - \log_e 2$ d) $1 + \log_e 2$

11) Let $y = y(x)$ be the solution curve of the differential equation $\frac{dy}{dx} + \frac{1}{x^2-1}y = \left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}, x > 1$ passing through the point $\left(2, \sqrt{\frac{1}{3}}\right)$. Then $\sqrt{7}y(8)$ is equal to

a) $11 + 6\log_e 3$ b) 19 c) $12 - 2\log_e 3$ d) $19 - 6\log_e 3$

12) The differential equation of the family of circles passing through the points $(0, 2)$ and $(0, -2)$ is

a) $2xy\frac{dy}{dx} + \left(x^2 - y^2 + 4\right) = 0$ c) $2xy\frac{dy}{dx} + \left(y^2 - x^2 + 4\right) = 0$
b) $2xy\frac{dy}{dx} + \left(x^2 + y^- + 4\right) = 0$ d) $2xy\frac{dy}{dx} - \left(x^2 - y^2 + 4\right) = 0$

13) Let the tangents at two points A and B on the circle $x^2 + y^2 - 4x + 3 = 0$ meet at the origin $O(0, 0)$. Then the area of the triangle OAB is

a) $\frac{3\sqrt{3}}{2}$ b) $\frac{3\sqrt{3}}{4}$ c) $\frac{3}{2\sqrt{3}}$ d) $\frac{3}{4\sqrt{3}}$

14) Let the hyperbola $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ pass through the point $\left(2\sqrt{2}, -2\sqrt{2}\right)$. A parabola is drawn whose focus is same as the focus of H with positive abscissa and the directrix of the parabola passes through the other focus of H. If the length of the latus rectum of the parabola is e times the length of the latus rectum of H, where e is the eccentricity of H, then which of the following points lies on the parabola?

a) $\left(2\sqrt{3}, 3\sqrt{2}\right)$ b) $\left(3\sqrt{3}, -6\sqrt{2}\right)$ c) $\left(\sqrt{3}, -\sqrt{6}\right)$ d) $\left(3\sqrt{6}, 6\sqrt{2}\right)$

15) Let the lines $\frac{x-1}{\lambda} = \frac{y-2}{1} = \frac{z-3}{2}$ and $\frac{x+26}{-2} = \frac{y+18}{3} = \frac{z+28}{\lambda}$ be coplanar and P be the plane containing these two lines. Then which of the following points does NOT lie on P?

a) $(0, -2, -2)$ b) $(-5, 0, -1)$ c) $(3, -1, 0)$ d) $(0, 4, 5)$