2022-Jul-27 Shift-1

AI24BTECH11003 - Badde Vijaya Sreyas

16) $(p \cap r) \iff (p \cap (\sim q))$ is equivalent to $(\sim p)$ when r is

a) p b) $\sim p$ c) q d) $\sim q$

17) If the plane P passes through the intersection of two mutually perpendicular planes $2x + ky - 5z = 1$ and $3kx - ky + z = 5, k < 3$ and intercepts a unit length on the positive x-axis, then the intercept made by the plane P on the y-axis is

a) $\frac{1}{11}$ b) $\frac{5}{11}$ c) 6 d) 7

18) Let A(1, 1), B(4, 3), C(−2, −5) be vertices of a triangle ABC, P be a point on the side BC, and Δ_1 and Δ_2 be the areas of the triangles APB and ABC respectively. If $\Delta_1 : \Delta_2 = 4 : 7$, then the area enclosed by the lines AP, AC and the x-axis is

a) $\frac{1}{4}$ b) $\frac{3}{4}$ c) $\frac{1}{2}$ d) 1

19) If the circle $x^2 + y^2 - 2gx + 6y - 19c = 0,\ g, c \in R$ passes through the point $(6, 1)$ and its centre lies on the line $x - 2cy = 8$, then the length of intercept made by the circle n x-axis is

a) $\sqrt{11}$ b) 4 c) 3 d) $2\sqrt{23}$

20) Let a function $f : R \to R$ be defined as: $f(x) = \begin{cases} \int_0^x (5 - |t - 3|)\, dt & x > 4 \\ x^2 + bx & x \le 4 \end{cases}$ where $b \in R$. If f is continuous at $x = 4$, then which of the following statements is NOT true?

a) f is not differentiable at $x = 4$
b) $f'(3) + f'(5) = \frac{35}{4}$
c) f is increasing in $\left(-\infty, \frac{1}{8}\right) \cup (8, \infty)$
d) f has a local minima at $x = \frac{1}{8}$

21) For $k \in R$, let the solutions of the equation $\cos(\arcsin(x \cot(\arctan(\cos(\arcsin x))))) = k, 0 < |x| < \frac{1}{\sqrt{2}}$ be α and β, where the inverse trigonometric functions take only principal values. If the solutions of the equation $x^2 - bx - 5 = 0$ are $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ and $\frac{\alpha}{\beta}$, then $\frac{b}{k^2}$ is equal to

22) The mean and variance of 10 observations were calculated as 15 and 15 respectively by a student who took by mistake 25 instead of 15 for one observation. Then the correct standard deviation is

23) Let the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z-3}{-4}$ intersect the plane containing the lines $\frac{x-4}{1} = \frac{y+1}{-2} = \frac{z}{1}$ and $4ax - y + 5z - 7a = 0 = 2x - 5y - z - 3, a \in R$ at the point $P(\alpha, \beta, \gamma)$. Then, the value of $\alpha + \beta + \gamma$ equals

24) An ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the vertices of the hyperbola $H : \frac{x^2}{49} - \frac{y^2}{64} = -1$. Let the major and minor axes of the ellipse E coincide with the transverse and conjugate axes of the hyperbola H. Let the product of the eccentricities of E and H be $\frac{1}{2}$. If l is the length of the latus rectum of the ellipse E, then the value of 113l is equal to:

25) Let $y = y(x)$ be the solution curve of the differential equation $\sin\left(2x^2\right)\log_e\left(\tan x^2\right)dy + \left(4xy - 4\sqrt{2}x\sin\left(x^2 - \frac{\pi}{4}\right)\right)dx = 0, 0 < x < \sqrt{\frac{\pi}{2}}$, which passes through the point $\left(\sqrt{\frac{\pi}{6}}, 1\right)$. Then $\left|y\left(\sqrt{\frac{\pi}{3}}\right)\right|$ is equal to

26) Let M and N be the number of points on the curve $y^5 - 9xy + 2x = 0$, where the tangents on the curve are parallel to x-axis and y-axis, respectively. Then the value of $M + N$ equals

27) Let $f(x) = 2x^2 - x - 1$ and $S = \{n \in Z : |f(n)| \leq 800\}$. Then, the value of $\sum\limits_{n \in S} f(n)$ is equal to

28) Let S be the set containing all 3×3 matrices with entries from $\{-1, 0, 1\}$. The total number of matrices $A \in S$ such that the sum of all the diagonal elements of $A^\top A$ is 6 is

29) If the length of the latus rectum of the ellipse $x^2 + 4y^2x + 8y - \lambda = 0$ is 4, and l is the length of its major axis, then $\lambda + l$ is equal to

30) Let $S = \left\{z \in C : z^2 + \bar{z} = 0\right\}$. Then $\sum\limits_{z \in S} (Re(z) + Im(z))$ is equal to