9-9.3-15

AI24BTECH11003 - Badde Vijaya Sreyas

Question:
Find the equation of the tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x-2y+5 = 0$. Also write the equation of the normal to the curve at the point of contact.
Solution:

Information	**Equation**
Parabola	$4y = 3x^2$
Line	$2y = 3x + 12$

TABLE 0: Information

From (**??**) the curve can be rewritten as:

$$\mathbf{g}\left(\mathbf{x}\right) = \mathbf{x}^\top \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} \frac{-3}{2} \\ 0 \end{pmatrix} \mathbf{x} + 2 \tag{0.1}$$

Now, since we know $\mathbf{m}$ already, we can use (**??**) to find a point on the tangent, $\mathbf{q} = \begin{pmatrix} x \\ y \end{pmatrix}$.

$$\mathbf{m}^\top \left(\mathbf{V}\mathbf{q} + \mathbf{u}\right) = 0 \tag{0.2}$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix}^\top \left(\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{q} + \begin{pmatrix} -\frac{3}{2} \\ 0 \end{pmatrix} \right) = 0 \tag{0.3}$$

$$y = \frac{3}{4} \tag{0.4}$$

Now with this value of y, and the equation of the curve,

$$\mathbf{q} = \begin{pmatrix} \frac{41}{48} \\ \frac{3}{4} \end{pmatrix} \tag{0.5}$$

$\therefore$ From this point on the line, and the value of $\mathbf{m}$, the tangent can be written as:

$$\mathbf{X} = \begin{pmatrix} \frac{41}{48} \\ \frac{3}{4} \end{pmatrix} + \kappa \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{0.6}$$

or simplified:

$$\mathbf{X} = \frac{1}{24} \begin{pmatrix} 0 \\ -23 \end{pmatrix} + \kappa \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{0.7}$$

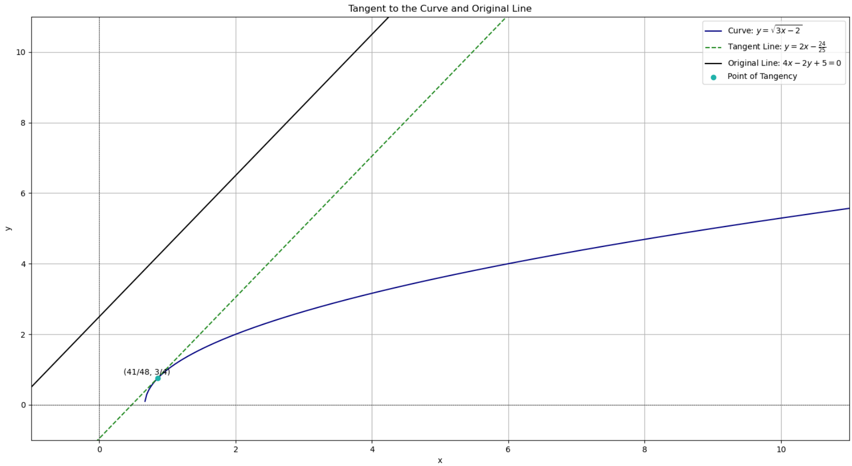

Fig. 0.1: Lines and Curve