AI24BTECH11003 - Badde Vijaya Sreyas

Question:

Find the equation of the tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line 4x-2y+5=0. Also write the equation of the normal to the curve at the point of contact. **Solution:**

Information	Value
Curve	$y = \sqrt{3x - 2}$
Given Line	4x - 2y + 5 = 0
Tangent	$\mathbf{X} = \frac{1}{24} \begin{pmatrix} 0 \\ -23 \end{pmatrix} + \kappa \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

TABLE 0: Information

From (??) the curve can be rewritten as:

$$\mathbf{g}(\mathbf{x}) = \mathbf{x}^{\mathsf{T}} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} \frac{-3}{2} \\ 0 \end{pmatrix} \mathbf{x} + 2 \tag{0.1}$$

Now, since we know **m** already, we can use (??) to find a point on the tangent, $\mathbf{q} = \begin{pmatrix} x \\ y \end{pmatrix}$.

$$\mathbf{m}^{\mathsf{T}} \left(\mathbf{V} \mathbf{q} + \mathbf{u} \right) = 0 \tag{0.2}$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix}^{\mathsf{T}} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{q} + \begin{pmatrix} -\frac{3}{2} \\ 0 \end{pmatrix} = 0 \tag{0.3}$$

$$y = \frac{3}{4} \tag{0.4}$$

Now with this value of y, and the equation of the curve,

$$\mathbf{q} = \begin{pmatrix} \frac{41}{48} \\ \frac{3}{4} \end{pmatrix} \tag{0.5}$$

... From this point on the line, and the value of **m**, the tangent can be written as:

$$\mathbf{X} = \begin{pmatrix} \frac{41}{48} \\ \frac{3}{4} \end{pmatrix} + \kappa \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{0.6}$$

or simplified:

$$\mathbf{X} = \frac{1}{24} \begin{pmatrix} 0 \\ -23 \end{pmatrix} + \kappa \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{0.7}$$

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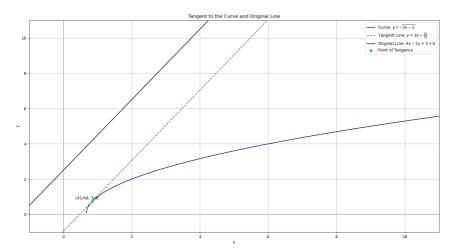


Fig. 0.1: Lines and Curve