2024-Apr-4 Shift-1

AI24BTECH11003 - Badde Vijaya Sreyas

1) Let $f(x) = \begin{cases} -2 & -2 \le x \le 0 \\ x-2 & 0 < x \le 2 \end{cases}$ and $h(x) = f(|x|) + |f(x)|$. Then $\int_{-2}^{2} h(x)\,dx$ is equal to

a) 1 b) 2 c) 6 d) 4

2) Let $f : R \to R$ be a function given by $f(x) = \begin{cases} \frac{1-\cos 2x}{x^2} & x < 0 \\ \alpha & x = 0 \\ \frac{\beta\sqrt{1-\cos x}}{x} & x > 0 \end{cases}$ where $\alpha, \beta \in R$. If f is continuous at $x = 0$, then $\alpha^2 + \beta^2$ is equal to

a) 6 b) 48 c) 12 d) 3

3) A square is inscribed in the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. One side of this square is parallel to $y = x + 3$. If (x_i, y_i) are the vertices of the square, then $\sum \left(x_i^2 + y_i^2\right)$ is equal to:

a) 148 b) 156 c) 160 d) 152

4) The vertices of a triangle are $A(-1, 3), B(-2, 2)$ and $C(3, -1)$. A new triangle is formed by shifting the sides of the triangle by one unit inwards. Then the equation of the side of the new triangle nearest to origin is:

a) $x + y - \left(2 - \sqrt{2}\right) = 0$
b) $x - y - \left(2 + \sqrt{2}\right) = 0$
c) $x + y + \left(2 + \sqrt{2}\right) = 0$
d) $-x + y - \left(2 - \sqrt{2}\right) = 0$

5) The sum of all rational terms in the expansion of $\left(2^{\frac{1}{5}} + 5^{\frac{1}{3}}\right)^{15}$ is equal to:

a) 3133 b) 633 c) 931 d) 6131

6) Let the sum of the maximum and the minimum values of the function $f(x) = \frac{2x^2-3x+8}{2x^2+3x+8}$ be $\frac{m}{n}$, where $\gcd(m, n) = 1$. Then $m + n$ is equal to:

a) 217 b) 201 c) 182 d) 195

7) Let the point on the line passing through the points $P(1, -2, 3)$ and $Q(5, -4, 7)$, farther from the origin and at a distance of 9 units from the point P, be (α, β, γ). Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to

a) 165 b) 150 c) 160 d) 155

8) There are 5 points P_1, P_2, P_3, P_4, P_5 on the side AB, excluding points A and B, of a triangle ABC. Similarly, there are 6 points $P_6, P_7, \cdots, P_{11}$ on side BC and 7 points $P_{12}, P_{13}, \cdots, P_{18}$ on the side CA of the triangle. The number of triangles, that can be formed using the points $P_1, P_2, \cdots, P_{18}$ as vertices, is:

a) 771 b) 776 c) 751 d) 796

9) If the domain of the function $\arcsin\left(\frac{3x-22}{2x-19}\right) + \log_e\left(\frac{2x^2-8x+5}{x^2-3x-10}\right)$ is $(\alpha, \beta]$, then $3\alpha + 10\beta$ is equal to

a) 100 b) 95 c) 98 d) 97

10) Let $\alpha \in (0, \infty)$ and $A = \begin{pmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$. If $\det(adj(2A - A^\top)) \cdot adj(A - 2A^\top) = 2^8$, then $(\det(A))^2$ is equal to

a) 16 b) 1 c) 49 d) 36

11) Let $\alpha, \beta \in R$. Let the mean and variance of 6 observations $-3, 4, 7, -6\alpha, \beta$ be 2 and 23, respectively. The mean deviation about the mean of these 6 observations is:

a) $\frac{14}{3}$ b) $\frac{11}{3}$ c) $\frac{13}{3}$ d) $\frac{16}{3}$

12) Let a unit vector which makes an angle of $60°$ with $2\hat{i} + 2\hat{j} - \hat{k}$ and an angle of $45°$ with $\hat{i} - \hat{k}$ be $\vec{C}$. Then $\vec{C} + \left(-\frac{1}{2}\hat{i} + \frac{1}{3\sqrt{2}}\hat{j} - \frac{\sqrt{2}}{3}\hat{k}\right)$ is:

a) $-\frac{\sqrt{2}}{3}\hat{i} + \frac{\sqrt{2}}{3}\hat{j} + \left(\frac{1}{2} + \frac{2\sqrt{2}}{3}\right)\hat{k}$
b) $\frac{\sqrt{2}}{3}\hat{i} - \frac{1}{2}\hat{k}$
c) $\frac{\sqrt{2}}{3}\hat{i} + \frac{1}{3\sqrt{2}}\hat{j} - \frac{1}{2}\hat{k}$
d) $\left(\frac{1}{\sqrt{3}} + \frac{1}{2}\right)\hat{i} + \left(\frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{2}}\right)\hat{j} + \left(\frac{1}{\sqrt{3}} + \frac{\sqrt{2}}{3}\right)\hat{k}$

13) If 2 and 6 are the roots of the equation $ax^2 + bx + 1 = 0$, then the quadratic equation, whose roots are $\frac{1}{2a+b}$ and $\frac{1}{6a+b}$, is:

a) $x^2 + 8x + 12 = 0$ b) $x^2 + 10x + 16 = 0$ c) $2x^2 + 11x + 12 = 0$ d) $4x^2 + 14x + 12 = 0$

14) If the system of equations $x + \left(\sqrt{2}\sin\alpha\right)y + \left(\sqrt{2}\cos\alpha\right)z = 0, x + (\cos\alpha)y + (\sin\alpha)z = 0, x + (\sin\alpha)y - (\cos\alpha)z = 0$ has a non-trivial solution, then $\alpha \in \left(0, \frac{\pi}{2}\right)$ is equal to:

a) $\frac{11\pi}{24}$ b) $\frac{7\pi}{24}$ c) $\frac{3\pi}{4}$ d) $\frac{5\pi}{24}$

15) Three urns A, B, and C contain 7 red, 5 black; 5 red, 7 black and 6 red, 6 black balls, respectively. One of the urns is selected at random and a ball is drawn from it. If the ball drawn is black, then the probability that it is drawn from urn A is:

a) $\frac{5}{18}$ b) $\frac{7}{18}$ c) $\frac{5}{16}$ d) $\frac{4}{17}$