2021-Jul-20 Shift-2

AI24BTECH11003 - Badde Vijaya Sreyas

1) For the natural numbers m, n, if $(1-y)^m (1+y)^n = 1 + a_1 y + a_2 y^2 + \cdots + a_{m+n} y^{m+n}$ and $a_1 = a_2 = 10$, then the value of $(m+n)$ is equal to

a) 88 b) 664 c) 100 d) 80

2) The value of $\tan\left(2\arctan\left(\frac{3}{5}\right) + \arcsin\left(\frac{5}{13}\right)\right)$ is equal to

a) $\frac{-181}{69}$ b) $\frac{220}{21}$ c) $\frac{-291}{76}$ d) $\frac{151}{63}$

3) Let r_1 and r_2 be the radii of the largest and smallest circles, respectively, which pass through the point $(-4, 1)$ and having their centres on the circumference of the circle $x^2 + y^2 + 2x + 4y - 4 = 0$. If $\frac{r_1}{r_2} = a + b\sqrt{2}$, then $a + b$ is equal to:

a) 3 b) 11 c) 5 d) 7

4) Consider the following three statements:
(A): If $2 + 4 = 7$, then $3 + 4 = 8$
(B): If $3 + 5 = 8$, then the earth is flat
(C): If (A) and (B) are true, then $5 + 6 = 17$
Then which of the following statements is correct?

a) (A) is false but (B) and (C) are true
b) (A) and (C) are true while (B) is false
c) (A) is true while (B) and (C) are false
d) (A) and (B) are false while (C) is true

5) The lines $x = ay - 1 = z - 2$ and $x = 3y - 2 = bz - 2$, $(ab \neq 0)$ are coplanar, if:

a) $b = 1, a \in R - \{0\}$ b) $a = 1, b \in R - \{0\}$ c) $a = 2, b = 2$ d) $a = 2, b = 3$

6) If $[x]$ denotes the greatest integer less than or equal to x, then the value of the integral $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} [[x] - \sin x]\, dx$ is equal to:

a) $-\pi$ b) π c) $-$ d) 1

7) If the real part of the complex number $(1 - \cos\theta + 2i\sin\theta)^{-1}$ is $\frac{1}{5}$ for $\theta \in (0, \pi)$, then the value of the integral $\int_0^{\theta} \sin x\, dx$ is equal to:

a) 1 b) 2 c) -1 d) 0

8) Let $f : R - \left\{\frac{\alpha}{6}\right\} \to R$ be defined by $f(x) = \frac{5x+3}{6x-\alpha}$. Then the value of α for which $(f \circ f)(x) = x$, for all $x \in R - \{\{\alpha\}6\}$, is :

a) No such α exists b) 5 c) 8 d) 6

9) If $f : R \to R$ is given by $f(x) = x + 1$, then the value of $\lim\limits_{x\to\infty} \frac{1}{n}\left[f(0) + f\left(\frac{5}{n}\right) + f\left(\frac{0}{n}\right) + \cdots + f\left(\frac{5(n-1)}{n}\right)\right]$ is:

a) $\frac{3}{2}$ b) $\frac{5}{2}$ c) $\frac{1}{2}$ d) $\frac{7}{2}$

10) Let A, B and C be three events such that the probability that exactly one of A and B occurs is $(1 - k)$, the probability that exactly one of B and C occurs is $(1 - 2k)$, the probability that exactly one of C and A occurs is $(1 - k)$ and the probability of all A, B and C occur simultaneously is k^2, where $0 < k < 1$. Then the probability that at least one of A, B and C occur is:

a) greater than $\frac{1}{8}$ but less than $\frac{1}{4}$
b) greater than $\frac{1}{2}$
c) greater than $\frac{1}{4}$ but less than $\frac{1}{2}$
d) exactly equal to $\frac{1}{2}$

11) The sum of all the local minimum values of the twice differentiable function $f : R \to R$ defined by $f(x) = x^3 - 3x^2 - \frac{3f''(x)}{2} + f''(1)$ is:

a) -22 b) 5 c) -27 d) 0

12) Let in a right angled triangle, the smallest angle be θ.If a triangle formed by taking the reciprocal of it's sides is also a right angled triangle, then $\sin\theta$ is equal to:

a) $\frac{\sqrt{5}+1}{4}$ b) $\frac{\sqrt{5}-1}{2}$ c) $\frac{\sqrt{2}-1}{2}$ d) $\frac{\sqrt{5}-1}{4}$

13) Let $y = y(x)$ satisfies the equation $\frac{dy}{dx} - |A| = 0$, for all $x > 0$, where $A = \begin{pmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x} \end{pmatrix}$.

If $y(\pi) = \pi + 2$, then the value of $y\left(\frac{\pi}{2}\right)$ is:

a) $\frac{\pi}{2} + \frac{4}{\pi}$ b) $\frac{\pi}{2} - \frac{1}{\pi}$ c) $\frac{3\pi}{2} - \frac{1}{\pi}$ d) $\frac{\pi}{2} - \frac{4}{\pi}$

14) Consider the line L given by the equation $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z-2}{1}$. Let Q be the mirror the image of the point $(2, 3, -1)$ with respect to L. Let a plane P be such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is on the plane P?

a) $(-1, 1, 2)$ b) $(1, 1, 1)$ c) $(1, 1, 2)$ d) $(1, 2, 2)$

15) If the mean and variance of six observations 7, 10, 11, 15, a, b are 10 and $\frac{20}{3}$, respectively, then the value of $|a - b|$ is equal to:

a) 9 b) 11 c) 7 d) 1