Assignment - 1

AI24BTECH11003 - B. Vijaya Sreyas

17.Indefinite Integrals - Section B

5. The value of $\sqrt{2}\int\frac{\sin x dx}{\sin\left(x-\frac{\pi}{4}\right)}$ [2008]
(a)$x+log|\cos\left(x-\frac{\pi}{4}\right)|+c$
(b)$x-log|\sin\left(x-\frac{\pi}{4}\right)|+c$
(c)$x+log|\sin\left(x-\frac{\pi}{4}\right)|+c$
(d)$x-log|\cos\left(x-\frac{\pi}{4}\right)|+c$
6. If the $\int\frac{5\tan x}{\tan x-2}dx=x+aln|\sin x-2\cos x|+k$, then a is equal to [2018]

(a) -1 (b) 2 (c) 1 (d) 2

7. If $\int f(x)\,dx=\psi(x)$, then $\int x^5 f\left(x^3\right)dx$ is equal to: [JEE M 2013]
(a) $\frac{1}{3}[x^3\psi\left(x^3\right)-\int x^2\psi(x^3)dx]+C$
(b) $\frac{1}{3}x^3\psi\left(x^3\right)-3\int x^3\psi(x^3)dx+C$
(c) $\frac{1}{3}x^3\psi\left(x^3\right)-\int x^2\psi(x^3)dx+C$
(d) $\frac{1}{3}[x^3\psi\left(x^3\right)-\int x^3\psi(x^3)dx]+C$
8. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to [JEE M 2014]

(a) $(x+1)\,e^{x+\frac{1}{x}}+c$ (c) $(x-1)\,e^{x+\frac{1}{x}}+c$
(b) $-xe^{x+\frac{1}{x}}+c$ (d) $xe^{x+\frac{1}{x}}+c$

9. The integral $\int\frac{dx}{x^2(x^4+1)^{3/4}}$ equals: [JEE M 2015]

(a) $-\left(x^4+1\right)^{\frac{1}{4}}+c$ (c) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$
(b) $-\left(\frac{x^4+1}{x^4}\right)+c$ (d) $\left(x^4+1\right)^{\frac{1}{4}}+c$

10. The integral $\int\frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}dx$ is equal to [JEE M 2016]

(a) $\frac{x^5}{2\left(x^5-x^3+1\right)^2}+C$ (c) $\frac{-x^5}{\left(x^5+x^3+1\right)^2}+C$
(b) $\frac{-x^{10}}{2\left(x^5+x^3+1\right)^2}+C$ (d) $\frac{x^{10}}{2\left(x^5+x^3+1\right)}+C$

where C is an arbitrary constant
11. Let $I_n=\int\tan^x dx$, $(n>1)$. $I_4+I_6=a\tan^5 x+bx^5+C$, where C is constant of integration, then the ordered pair (a,b) is equal to : [JEE M 2017]

(a) $\left(-\frac{1}{5},0\right)$ (b) $\left(-\frac{1}{5},1\right)$ (c) $\left(\frac{1}{5},0\right)$ (d) $\left(\frac{1}{5},-1\right)$

12. The integral $\int\frac{sin^2xcos^2x}{\left(sin^5x+cos^3xsin^2x+sin^3xcos^2x+cos^5x\right)^2}$dx is equal to [JEE M 2018]

(a) $\frac{-1}{3\left(1+\tan^3 x\right)}+C$ (c) $\frac{-1}{1+\cot^3 x}+C$
(b) $\frac{1}{1+\cot^3 x}+C$ (d) $\frac{1}{3\left(1+\tan^3 x\right)}+C$

13. For $x^2\neq n\pi+1$, n∈N (the set of natural numbers), the integral $\int x\sqrt{\frac{2\sin\left(x^2-1\right)-\sin 2\left(x^2-1\right)}{2\sin\left(x^2-1\right)+\sin 2\left(x^2-1\right)}}$dx is equal to: [JEE M 2019 - 9 Jan(M)]

(a) $\log_e|\frac{1}{2}\sec^2\left(x^2-1\right)|+c$ (c) $\frac{1}{2}\log_e|\sec^2\left(\frac{x^2-1}{2}\right)|+c$
(b) $\frac{1}{2}\log_e|\sec^2\left(\frac{x^2-1}{2}\right)|+c$ (d) $\log_2|\sec\left(\frac{x^2-1}{2}\right)|+c$

(where c is a constant of integration)
14. The integral $\int\sec^{2/3}x\,\text{cosec}^{4/3}x dx$ is equal to [JEE M 2019 - 9 April (M)]

(a) -3tan$^{-1/3}$x+C (c) -3cot$^{-1/3}$x+C
(b) $-\frac{3}{4}\tan^{-4/3}$x+C (d) 3tan$^{-1/3}$+C

(Here, C is a constant of integration)

18. Definite Integrals - Section B

31. The area of the region bounded by the parabola $(y-2)^2=x-1$, the tangent of the parabola at the point $(2,3)$ and the x-axis is: [2009]

(a) 6 (b) 9 (c) 12 (d) 3

32. $\int_0^{\pi}[\cot x]dx$, where [.] denotes the greatest integer function, is equal to [2009]

(a) 1 (b) -1 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

33. The area bounded between the curves $y=\cos x$ and $y=\sin x$ between the ordinates $x=0$ and $x=\frac{3\pi}{2}$ is [2010]

(a) $4\sqrt{2}+2$ (c) $4\sqrt{2}+1$
(b) $4\sqrt{2}-1$ (d) $4\sqrt{2}-2$

34. Let $p(x)$ be a function defined on $\mathbf{R}$ such that $p'(x)=p'(1-x)$, for all $x\in[0,1], p(0)=1$ and $p(1)=41$. Then $\int_0^1 p(x)\,dx$ equals [2010]

(a) 21 (b) 41 (c) 42 (d) $\sqrt{41}$

35. The value of $\int_0^1\frac{8log(1+x)}{1+x^2}dx$ is [2011]

(a) $\frac{\pi}{8}\log 2$ (c) $\log 2$
(b) $\frac{\pi}{2}\log 2$ (d) $\pi \log 2$

36. The area of the region enclosed by the curves $y = x, x = e, y = \frac{1}{x}$ and the positive x axis is [2011]

(a) 1 square unit (c) $\frac{5}{2}$ square units
(b) $\frac{3}{2}$ square units (d) $\frac{1}{2}$ square unit

37. The area between the parabolas:$x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is: [2012]

(a) $20\sqrt{2}$ (b) $\frac{10\sqrt{2}}{3}$ (c) $\frac{20\sqrt{2}}{3}$ (d) $10\sqrt{2}$

38. If $g(x) = \int_0^x \cos 4t dt$, then $g(x+\pi)$ equals [2012]

(a) $\frac{g(x)}{g(\pi)}$ (c) $g(x) - g(\pi)$
(b) $g(x) + g(\pi)$ (d) $g(x).g(\pi)$

39. **Statement-1 :** The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\pi/6$
Statement-2 : $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx.$ [JEE M 2013]
(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation fo Statement-1
(c) Statement-1 is true; Statement-2 is false
(d) Statement-1 is false; Statement-2 is true

40. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is : [JEE M 2013]

(a) 9 (b) 36 (c) 18 (d) $\frac{27}{4}$

41. The integral $\int_0^{\pi} \sqrt{1 + 4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}}dx$ equals: [JEE M 2014]

(a) $4\sqrt{3} - 4$ (c) $\pi - 4$
(b) $4\sqrt{3} - 4 - \frac{\pi}{3}$ (d) $\frac{2\pi}{3} - 4 - 4\sqrt{3}$

42. The area of the region described by $A = \{(x,y) : x^2+y^2 \le 1$ and $y^2 \le 1 - x\}$ is: [JEE M 2014]

(a) $\frac{\pi}{2} - \frac{2}{3}$ (b) $\frac{\pi}{2} + \frac{2}{3}$ (c) $\frac{\pi}{2} + \frac{4}{3}$ (d) $\frac{\pi}{2} - \frac{4}{3}$

43. The area (in sq. units) of the region described by $\{(x,y) : y^2 \le 2x$ and $y \ge 4x - 1\}$ is [JEE M 2015]

(a) $\frac{15}{64}$ (b) $\frac{9}{32}$ (c) $\frac{7}{32}$ (d) $\frac{5}{64}$

44. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36-12x+x^2)}dx$ is equal to: [JEE M 2015]

(a) 1 (b) 6 (c) 2 (d) 4

45. The area (in sq. units) of the region $\{(x,y) : y^2 \ge 2x$ and $x^2 + y^2 \le 4x, x \ge 0, y \ge 0\}$ is [JEE M 2016]

(a) $\pi - \frac{4\sqrt{2}}{3}$ (c) $\pi - \frac{4}{3}$
(b) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$ (d) $\pi - \frac{8}{3}$