

9-9.3-15

AI24BTECH11003 - Badde Vijaya Sreyas

Question:

Find the equation of the tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x-2y+5=0$. Also write the equation of the normal to the curve at the point of contact.

Solution:

Information	Value
Curve	$y = \sqrt{3x-2}$
Given Line	$4x-2y+5=0$
Tangent	$\mathbf{X}_T = \frac{1}{48} \begin{pmatrix} 41 \\ 36 \end{pmatrix} + \kappa \begin{pmatrix} 1 \\ 2 \end{pmatrix}$
Normal	$\mathbf{X}_N = \frac{1}{48} \begin{pmatrix} 41 \\ 36 \end{pmatrix} + \gamma \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

TABLE 0: Information

From (??) the curve can be rewritten as:

$$\mathbf{g}(\mathbf{x}) = \mathbf{x}^\top \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -\frac{3}{2} \\ 0 \end{pmatrix} \mathbf{x} + 2 \quad (0.1)$$

Now, since we know \mathbf{m} already, we can use (??) to find a point on the tangent, $\mathbf{q} = \begin{pmatrix} x \\ y \end{pmatrix}$.

$$\mathbf{m}^\top (\mathbf{V}\mathbf{q} + \mathbf{u}) = 0 \quad (0.2)$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix}^\top \left(\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{q} + \begin{pmatrix} -\frac{3}{2} \\ 0 \end{pmatrix} \right) = 0 \quad (0.3)$$

$$y = \frac{3}{4} \quad (0.4)$$

Now with this value of y , and the equation of the curve,

$$\mathbf{q} = \begin{pmatrix} \frac{41}{48} \\ \frac{3}{4} \end{pmatrix} \quad (0.5)$$

\therefore From this point on the line, and the value of \mathbf{m} , the tangent can be written as:

$$\mathbf{X}_T = \begin{pmatrix} \frac{41}{48} \\ \frac{3}{4} \end{pmatrix} + \kappa \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (0.6)$$

From (??), the normal to the curve at the point of contact can be written as:

$$\mathbf{X}_N = \begin{pmatrix} \frac{41}{48} \\ \frac{3}{4} \end{pmatrix} + \gamma \begin{pmatrix} -2 \\ 1 \end{pmatrix} \quad (0.7)$$

which can also be written as

$$\mathbf{X}_T = \frac{1}{48} \begin{pmatrix} 41 \\ 36 \end{pmatrix} + \kappa \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (0.8)$$

$$\mathbf{X}_N = \frac{1}{48} \begin{pmatrix} 41 \\ 36 \end{pmatrix} + \gamma \begin{pmatrix} -2 \\ 1 \end{pmatrix} \quad (0.9)$$

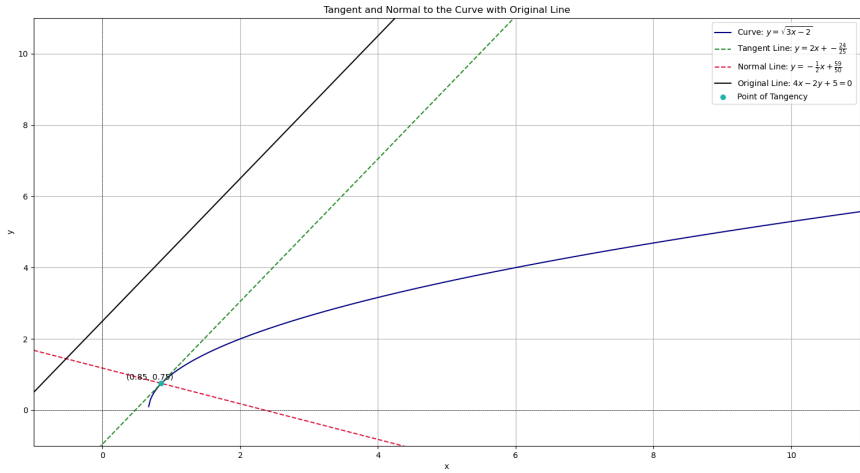


Fig. 0.1: Lines and Curve