2022-Jun-28 Shift-2

AI24BTECH11003 - Badde Vijaya Sreyas

1) Let $R_1 = \{(a,b) \in N \times N : |a-b| \leq 13\}$ and $R_2 = \{(a,b) \in N \times N : |a-b| \neq 13\}$. Then on N:

   a) Both R_1 and R_2 are equivalence relations
   b) Neither R_1 nor R_2 is an equivalence relation
   c) R_1 is an equivalence relation but R_2 is not
   d) R_2 is an equivalence relation but R_1 is not

2) Let $f(x)$ be a quadratic polynomial such that $f(-2) + f(3) = 0$. If one of the roots of $f(x) = 0$ is -1, then the sum of the roots of $f(x) = 0$ is equal to:

   a) $\frac{11}{3}$
   b) $\frac{7}{3}$
   c) $\frac{13}{3}$
   d) $\frac{14}{3}$

3) The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives at least four and at most 7 candies, C_3 receives at least 2 and at most 6 candies, is equal to:

   a) 205
   b) 615
   c) 510
   d) 430

4) The term independent of x in the expression of $\left(1 - x^2 + 3x^3\right)\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^1 1, x \neq 0$ is

   a) $\frac{7}{40}$
   b) $\frac{33}{200}$
   c) $\frac{39}{200}$
   d) $\frac{11}{50}$

5) If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is $1 : 7$ and $a + n = 33$, then the value of n is

   a) 21
   b) 22
   c) 23
   d) 24

6) Let $f, g : R \to R$ be functions defined by $f(x) = \begin{cases} [x] & x < 0 \\ |1-x| & x \geq 0 \end{cases}$ and $g(x) = \begin{cases} e^x - x & x < 0 \\ (x-1)^2 - 1 & x \geq 0 \end{cases}$ where $[x]$ denotes the greatest integer less than or equal to x. Then, the function $f \circ g$ is discontinuous at exactly:

   a) one point
   b) two points
   c) three points
   d) four points

7) Let $f : R \to R$ be a differentiable function such that $f\left(\frac{\pi}{4}\right) = \sqrt{2}, f\left(\frac{\pi}{2}\right) = 0$ and $f'\left(\frac{\pi}{2}\right) = 1$ and let $g(x) = \int_x^{\frac{\pi}{4}} \left(f'(t)\sec t + \tan t \sec t f(t)\right) dt$ for $x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then $\lim_{x \to \left(\frac{\pi}{2}\right)^-} g(x)$ is equal to

a) 2 b) 3 c) 4 d) -3

8) Let $f: R \to R$ be a continuous function satisfying $f(x) + f(x+k) = n$, for all $x \in R$ where $k > 0$ and n is a positive integer. If $I_1 = \int_0^{4nk} f(x)\,dx$ and $I_2 = \int_{-k}^{3k} f(x)\,dx$, then

a) $I_1 + 2I_2 = 4nk$ b) $I_1 + 2I_2 = 2nk$ c) $I_1 + nI_2 = 4n^2k$ d) $I_1 + nI_2 = 6n^2k$

9) The area of the bounded region enclosed by the curve $y = 3 - \left|x - \frac{1}{2}\right| - |x+1|$ and the x-axis is

a) $\frac{9}{4}$ b) $\frac{45}{16}$ c) $\frac{27}{8}$ d) $\frac{63}{16}$

10) Let $x = x(y)$ be the solution of the differential equation $2ye^{\frac{x}{y^2}}dx + \left(y^2 - 4xe^{\frac{x}{y^2}}\right)dy = 0$ such that $x(1) = 0$. Then, $x(e)$ is equal to

a) $e\log_e(2)$ b) $e^2\log_e(2)$ c) $-e\log_e(2)$ d) $-e^2\log_e(2)$

11) Let the slope of the tangent to a curve $y = f(x)$ at (x, y) be given by $s\tan(\cos x - y)$. If the curve passes through the point $\left(\frac{\pi}{4}, 0\right)$, then the value of $\int_0^{\frac{\pi}{2}} y dx$ is equal to

a) $\left(2 - \sqrt{2}\right) + \frac{\pi}{\sqrt{2}}$ b) $2 - \frac{\pi}{\sqrt{2}}$ c) $\left(2 + \sqrt{2}\right) + \frac{\pi}{\sqrt{2}}$ d) $2 + \frac{\pi}{\sqrt{2}}$

12) Let a triangle be bounded by the lines $L_1 : 2x + 5y = 10; L_2 : -4x + 3y = 12$ and the line L_3, which passes through the point P$(2, 3)$, intersect L_2 at A and L_1 at B. If the point P divides the line-segment AB, internally in the ratio $1 : 3$, then the area of the triangle is equal to

a) $\frac{110}{13}$ b) $\frac{110}{13}$ c) $\frac{110}{13}$ d) $\frac{110}{13}$

13) Let $a > 0, b > 0$. Let e and l respectively be the eccentricity and length of latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Let e' and l' respectively the eccentricity and length of latus rectum of its conjugate hyperbola. If $e^2 = \frac{11}{14}l$ and $(e')^2 = \frac{11}{8}l'$, then the value of $77a + 44b$ is equal to

a) 100 b) 110 c) 120 d) 130

14) Let $\overrightarrow{a} = \alpha\hat{i} + 2\hat{j} - \hat{k}$ and $\overrightarrow{b} = -2\hat{i} + \alpha\hat{j} + \hat{k}$, where $\alpha \in R$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ is $\sqrt{15\left(\alpha^2 + 4\right)}$, then the value of $2\left|\overrightarrow{a}\right|^2 + \left(\overrightarrow{a} \cdot \overrightarrow{b}\right)\left|\overrightarrow{b}\right|^2$ is equal to

a) 10 b) 7 c) 9 d) 14

15) If vertex of a parabola is $(2, -1)$ and the equation of its directrix is $4x - 3y = 21$, then the length of its latus rectum is

a) 2 b) 8 c) 12 d) 16