2021-Aug-31 Shift-1

AI24BTECH11003 - Badde Vijaya Sreyas

1) If $\alpha+\beta+\gamma=2\pi$, then the system of equations
$x+(\cos\gamma)\,y+(\cos\beta)\,z=0$
$(\cos\gamma)\,x+y+(\cos\alpha)\,z=0$
$(\cos\beta)\,x+(\cos\alpha)\,y+z=0$
has:

a) no solution
b) infinitely many solutions
c) exactly two solutions
d) a unique solution

2) Let $\vec{a},\vec{b},\vec{c}$ be three vectors mutually perpendicular to each other and have the same magnitude. If a vector $\vec{r}$ satisfies $\vec{a}\times\left\{\left(\vec{r}-\vec{b}\right)\times\vec{a}\right\}+\vec{b}\times\left\{\left(\vec{r}-\vec{c}\right)\times\vec{b}\right\}+\vec{c}\times\left\{\left(\vec{r}-\vec{a}\right)\times\vec{c}\right\}=\vec{0}$, then $\vec{r}$ is equal to:

a) $\frac{1}{3}\left(\vec{a}+\vec{b}+\vec{c}\right)$
b) $\frac{1}{3}\left(2\vec{a}+\vec{b}+\vec{c}\right)$
c) $\frac{1}{2}\left(\vec{a}+\vec{b}+\vec{c}\right)$
d) $\frac{1}{2}\left(\vec{a}+\vec{b}+2\vec{c}\right)$

3) The domain of the function $f(x)=\arcsin\left(\frac{3x^2+x-1}{(x-1)^2}\right)+\arccos\left(\frac{x-1}{x+1}\right)$ is:

a) $\left[0,\frac{1}{4}\right]$
b) $[-2,0]\cup\left[\frac{1}{4},\frac{1}{2}\right]$
c) $\left[\frac{1}{4},\frac{1}{2}\right]\cup\{0\}$
d) $\left[0,\frac{1}{2}\right]$

4) Let $S=\{1,2,3,4,5,6\}$. Then the probability that a randomly chosen onto function g from S to S satisfies $g(3)=2g(1)$ is:

a) $\frac{1}{10}$
b) $\frac{1}{15}$
c) $\frac{1}{5}$
d) $\frac{1}{30}$

5) Let $f:N\to N$ be a function such that $f(m+n)=f(m)+f(n)$ for every $m,n\in N$. If $f(6)=18$, then $f(2)\cdot f(3)$ is equal to:

a) 6
b) 54
c) 18
d) 36

6) The distance of the point $(-1,2,-2)$ from the line of intersection of the planes $2x+3y+2z=0$ and $x-2y+z=0$ is:

a) $\frac{1}{\sqrt{2}}$
b) $\frac{5}{2}$
c) $\frac{\sqrt{42}}{2}$
d) $\frac{\sqrt{34}}{2}$

7) Negation of the statement $(p\vee q)\implies(q\vee r)$ is:

a) $p\wedge \sim q\wedge \sim r$ b) $\sim p \wedge q\wedge \sim r$ c) $\sim p \wedge q \wedge r$ d) $p \wedge q \wedge r$

8) If $\alpha = \underset{x\to\frac{\pi}{4}}{lim} \frac{\tan^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$ and $\beta = \underset{x\to 0}{lim} (\cos x)^{\cot x}$ are the roots of the equation $ax^2+bx-4 = 0$, then the ordered pair (a, b) is:

a) $(1, -3)$ b) $(-1, 3)$ c) $(-1, -3)$ d) $(1, 3)$

9) The locus of midpoints of the line segments joining $(-3, -5)$ and the points on the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is:

a) $9x^2 + 4y^2 + 18x + 8y + 145 = 0$
c) $36x^2 + 16y^2 + 108x + 80y + 145 = 0$
b) $36x^2 + 16y^2 + 90x + 56y + 145 = 0$
d) $36x^2 + 16y^2 + 72x + 32y + 145 = 0$

10) If $\frac{dy}{dx} = \frac{2^x y+2^y\cdot 2^x}{2^x+2^{x+y}\log_e 2}, y(0) = 0$, then for $y = 1$, the value of x lies in the interval:

a) $(1, 2)$ b) $\left(\frac{1}{2}, 1\right]$ c) $(2, 3)$ d) $\left(0, \frac{1}{2}\right]$

11) An angle of intersection of the curves, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $x^2 + y^2 = ab, a > b$, is:

a) $\arctan\left(\frac{a+b}{\sqrt{ab}}\right)$ b) $\arctan\left(\frac{a-b}{2\sqrt{ab}}\right)$ c) $\arctan\left(\frac{a-b}{\sqrt{ab}}\right)$ d) $\arctan\left(2\sqrt{ab}\right)$

12) If $y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right], x > 0, \phi > 0$, and $y(1) = -1$, then $\phi\frac{y^2}{x^4}$ is equal to:

a) $4\phi(2)$ b) $4\phi(1)$ c) $2\phi(1)$ d) $\phi(1)$

13) The sum of the roots of the equation $x + 1 - 2\log_2(2 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$, is:

a) $\log_2 14$ b) $\log_2 11$ c) $\log_2 12$ d) $\log_2 13$

14) If z is a complex number such that $\frac{z-i}{z-1}$ is purely imaginary, then the minimum value of $|z - (3 + 3i)|$ is:

a) $2\sqrt{2} - 1$ b) $3\sqrt{2}$ c) $6\sqrt{2}$ d) $2\sqrt{2}$

15) Let $a_1, a_2, a_3, \cdots$ be an A.P. If $\frac{a_1+a_2+\cdots+a_{10}}{a_1+a_2+\cdots+a_p} = \frac{100}{p^2}, p \neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to:

a) $\frac{19}{21}$ b) $\frac{100}{121}$ c) $\frac{21}{19}$ d) $\frac{121}{100}$