

Assignment - 1

AI24BTECH11003 - B. Vijaya Sreyas

17. Indefinite Integrals - Section B

5) The value of $\sqrt{2} \int \frac{\sin x dx}{\sin(x - \frac{\pi}{4})}$

(2008)

(a) $x + \log \left| \cos \left(x - \frac{\pi}{4} \right) \right| + c$

(b) $x - \log \left| \sin \left(x - \frac{\pi}{4} \right) \right| + c$

(c) $x + \log \left| \sin \left(x - \frac{\pi}{4} \right) \right| + c$

(d) $x - \log \left| \cos \left(x - \frac{\pi}{4} \right) \right| + c$

6) If the $\int \frac{5 \tan x}{\tan x - 2} dx = x + a \ln |\sin x - 2 \cos x| + k$, then a is equal to

(2018)

1) -1

2) 2

3) 1

4) 2

7) If $\int f(x) dx = \psi(x)$, then $\int x^5 f(x^3) dx$ is equal to:

(JEE M 2013)

(a) $\frac{1}{3} [x^3 \psi(x^3) - \int x^2 \psi(x^3) dx] + C$

(b) $\frac{1}{3} x^3 \psi(x^3) - 3 \int x^3 \psi(x^3) dx + C$

(c) $\frac{1}{3} x^3 \psi(x^3) - \int x^2 \psi(x^3) dx + C$

(d) $\frac{1}{3} [x^3 \psi(x^3) - \int x^3 \psi(x^3) dx] + C$

8) The integral $\int \left(1 + x - \frac{1}{x}\right) e^{x + \frac{1}{x}} dx$ is equal to

(JEE M 2014)

1) $(x+1) e^{x + \frac{1}{x}} + c$

2) $-x e^{x + \frac{1}{x}} + c$

3) $(x-1) e^{x + \frac{1}{x}} + c$

4) $x e^{x + \frac{1}{x}} + c$

9) The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals:

(JEE M 2015)

1) $-\left(x^4 + 1\right)^{\frac{1}{4}} + c$

2) $-\left(\frac{x^4+1}{x^4}\right) + c$

3) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$

4) $\left(x^4 + 1\right)^{\frac{1}{4}} + c$

10) The integral $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx$ is equal to

(JEE M 2016)

1) $\frac{x^5}{2(x^5 - x^3 + 1)^2} + C$

2) $\frac{-x^{10}}{2(x^5 + x^3 + 1)^2} + C$

3) $\frac{-x^5}{(x^5 + x^3 + 1)^2} + C$

4) $\frac{x^{10}}{2(x^5 + x^3 + 1)} + C$

where C is an arbitrary constant

11) Let $I_n = \int \tan^x x dx$, ($n > 1$). $I_4 + I_6 = a \tan^5 x + b x^5 + C$, where C is constant of integration, then the ordered pair (a, b) is equal to :

(JEE M 2017)

1) $\left(-\frac{1}{5}, 0\right)$ 2) $\left(-\frac{1}{5}, 1\right)$ 3) $\left(\frac{1}{5}, 0\right)$ 4) $\left(\frac{1}{5}, -1\right)$

12) The integral $\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$ is equal to

(JEE M 2018)

1) $\frac{-1}{3(1+\tan^3 x)} + C$

2) $\frac{1}{1+\cot^3 x} + C$

3) $\frac{-1}{1+\cot^3 x} + C$

4) $\frac{1}{3(1+\tan^3 x)} + C$

13) For $x^2 \neq n\pi + 1$, $n \in \mathbb{N}$ (the set of natural numbers), the integral $\int x \sqrt{\frac{2 \sin(x^2-1) - \sin 2(x^2-1)}{2 \sin(x^2-1) + \sin 2(x^2-1)}} dx$ is equal to:

(JEE M 2019 - 9 Jan(M))

1) $\log_e \left| \frac{1}{2} \sec^2(x^2 - 1) \right| + c$ 3) $\frac{1}{2} \log_e \left| \sec^2 \left(\frac{x^2-1}{2} \right) \right| + c$

2) $\frac{1}{2} \log_e \left| \sec^2 \left(\frac{x^2-1}{2} \right) \right| + c$ 4) $\log_2 \left| \sec \left(\frac{x^2-1}{2} \right) \right| + c$

(where c is a constant of integration)

14) The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x dx$ is equal to

(JEE M 2019 - 9 April (M))

1) $-3 \tan^{-1/3} x + C$

2) $-\frac{3}{4} \tan^{-4/3} x + C$

3) $-3 \cot^{-1/3} x + C$

4) $3 \tan^{-1/3} x + C$

(Here, C is a constant of integration)

18. Definite Integrals - Section B

31) The area of the region bounded by the parabola $(y-2)^2 = x-1$, the tangent of the parabola at the point $(2, 3)$ and the x -axis is:

(2009)

1) 6

2) 9

3) 12

4) 3

32) $\int_0^\pi [\cot x] dx$, where $[.]$ denotes the greatest integer function, is equal to

(2009)

- 1) 1 2) -1 3) $-\frac{\pi}{2}$ 4) $\frac{\pi}{2}$ 1) 9 2) 36 3) 18 4) $\frac{27}{4}$

33) The area bounded between the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is

(2010)

- 1) $4\sqrt{2} + 2$ 3) $4\sqrt{2} + 1$
2) $4\sqrt{2} - 1$ 4) $4\sqrt{2} - 2$

34) Let $p(x)$ be a function defined on \mathbf{R} such that $p'(x) = p'(1-x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x) dx$ equals

(2010)

- 1) 21 2) 41 3) 42 4) $\sqrt{41}$

35) The value of $\int_0^1 \frac{8 \log(1+x)}{1+x^2} dx$ is

(2011)

- 1) $\frac{\pi}{8} \log 2$ 3) $\log 2$
2) $\frac{\pi}{2} \log 2$ 4) $\pi \log 2$

36) The area of the region enclosed by the curves $y = x$, $x = e$, $y = \frac{1}{x}$ and the positive x axis is

(2011)

- 1) 1 square unit 3) $\frac{5}{2}$ square units
2) $\frac{3}{2}$ square units 4) $\frac{1}{2}$ square unit

37) The area between the parabolas: $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is:

(2012)

- 1) $20\sqrt{2}$ 2) $\frac{10\sqrt{2}}{3}$ 3) $\frac{20\sqrt{2}}{3}$ 4) $10\sqrt{2}$

38) If $g(x) = \int_0^x \cos 4t dt$, then $g(x + \pi)$ equals

(2012)

- (a) $\frac{g(x)}{g(\pi)}$ (c) $g(x) - g(\pi)$
(b) $g(x) + g(\pi)$ (d) $g(x) \cdot g(\pi)$

39) **Statement-1** : The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$ is equal to $\pi/6$

Statement-2 : $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$.

(JEE M 2013)

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(c) Statement-1 is true; Statement-2 is false

(d) Statement-1 is false; Statement-2 is true

40) The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x -axis, and lying in the first quadrant is :

(JEE M 2013)

41) The integral $\int_0^{\pi} \sqrt{1 + 4 \sin^2 \frac{x}{2}} - 4 \sin \frac{x}{2} dx$ equals:

(JEE M 2014)

- 1) (a) $4\sqrt{3} - 4$ 3) (c) $\pi - 4$
2) (b) $4\sqrt{3} - 4 - \frac{\pi}{3}$ 4) (d) $\frac{2\pi}{3} - 4 - 4\sqrt{3}$

42) The area of the region described by $A = \{(x, y) : x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\}$ is:

(JEE M 2014)

- 1) $\frac{\pi}{2} - \frac{2}{3}$ 2) $\frac{\pi}{2} + \frac{2}{3}$ 3) $\frac{\pi}{2} + \frac{4}{3}$ 4) $\frac{\pi}{2} - \frac{4}{3}$

43) The area (in sq. units) of the region described by $\{(x, y) : y^2 \leq 2x \text{ and } y \geq 4x - 1\}$ is

(JEE M 2015)

- 1) $\frac{15}{64}$ 2) $\frac{9}{32}$ 3) $\frac{7}{32}$ 4) $\frac{5}{64}$

44) The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$ is equal to:

(JEE M 2015)

- 1) 1 2) 6 3) 2 4) 4

45) The area (in sq. units) of the region $\{(x, y) : y^2 \geq 2x \text{ and } x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is

(JEE M 2016)

- 1) $\pi - \frac{4\sqrt{2}}{3}$ 3) $\pi - \frac{4}{3}$
2) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$ 4) $\pi - \frac{8\sqrt{2}}{3}$