2023-Jan-29 Shift-2

AI24BTECH11003 - Badde Vijaya Sreyas

16) If the tangent at a point P on the parabola $y^2 = 3x$ is parallel to the line $x + 2y = 1$ and the tangents at the points Q and R on the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ are perpendicular to the line $x - y = 2$, then the area of the triangle PQR is:

a) $\frac{9}{\sqrt{5}}$ b) $5\sqrt{3}$ c) $\frac{3}{2}\sqrt{5}$ d) $3\sqrt{5}$

17) Let $y = y(x)$ be the solution of the differential equation $x \log_e x \frac{dy}{dx} + y = x^2 \log_e x, (x > 1)$. If $y(2) = 2$, then $y(e)$ is equal to

a) $\frac{4+e^2}{4}$ b) $\frac{1+e^2}{4}$ c) $\frac{2+e^2}{2}$ d) $\frac{1+e^2}{2}$

18) The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is

a) 472 b) 432 c) 507 d) 400

19) Let R be a relation defined on N as $a\ R\ b$ is $2a + 3b$ is a multiple of 5, $a, b \in N$. Then R is

a) not reflexive
b) transitive but not symmetric
c) symmetric but not transitive
d) an equivalence relation

20) Consider a function $f : N \to R$, satisfying $f(1) + 2f(2) + 3f(3) + \cdots + xf(x) = x(x+1) f(x); x > 2$ with $f(1) = 1$. Then $\frac{1}{(2022)} + \frac{1}{f(2028)}$ is equal to

a) 8200 b) 8000 c) 8400 d) 8100

21) The total number of 4-digit numbers whose greatest common divisor with 54 is 2, is
22) A triangle is formed by the tangents at the point $(2, 2)$ on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$, and the line $x + y + 2 = 0$. If r is the radius of its circumcircle, then r^2 is equal to
23) A circle with centre $(2, 3)$ and radius 4 intersects the line $x + y = 3$ at the points P and Q. If the tangents at P and Q intersect at the point $S(\alpha, \beta)$, then $4\alpha - 7\beta$ is equal to
24) Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + +(n-1), b_n = b_{n-1} + a_{n-1} \forall n \geq 2$. If $S = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $T = \sum_{n=1}^{8} \frac{n}{2^{n-1}}$, then $2^7 (2S - T)$ is equal to
25) If the equation of the normal to the curve $y = \frac{x-a}{(x+b)(x-2)}$ at the point $(1, -3)$ is $x - 4y = 13$, then the value of $a + b$ is equal to

26) If A be the symmetric matrix such that $|A| = 2$ and $\begin{pmatrix} 2 & 1 \\ 2 & \frac{3}{2} \end{pmatrix} A = \begin{pmatrix} 1 & 2 \\ \alpha & \beta \end{pmatrix}$. If the sum of the diagonal elements of A is s, then $\frac{\beta s}{\alpha^2}$ is equal to

27) Let $\{a_k\}$ and $\{b_k\}, k \in N$, be two G,P,s with common ratio r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k, k \in N$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$ then $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to

28) Let $X = \{11, 12, 13, \cdots, 40, 41\}$ and $Y = \{61, 62, 63, \cdots, 90, 91\}$ be the two sets of observations. If $\overline{x}$ and $\overline{y}$ are their respective means and σ^2 is the variance of all the observations in $X \cup Y$, then $\left|\overline{x} + \overline{y} - \sigma^2\right|$ is equal to

29) Let $\alpha = 8 - 14i$, $A = \left\{z \in C : \frac{\alpha z \overline{\alpha z}}{z^2 - (\overline{z})^2 - 112i} = 1\right\}$ and $B = \{z \in C : |z + 3i| = 4\}$. Then $\sum_{z \in A \cap B} (Re\ z - Im\ z)$ is equal to

30) Let $\alpha_1, \alpha_2, \cdots, \alpha_7$ be the roots of the equation $x^7 + 3x^5 - 13x^3 - 15x = 0$ and $|\alpha_1| \geq |\alpha_2| \geq \cdots \geq |\alpha_7|$. Then $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5 + \alpha_6$ is equal to