Assignment - 1

AI24BTECH11003 - B. Vijaya Sreyas

17.Indefinite Integrals - Section B

5) The value of $\sqrt{2}\int \frac{\sin x dx}{\sin\left(x-\frac{\pi}{4}\right)}$ (2008)

(a)$x + log|\cos\left(x - \frac{\pi}{4}\right)| + c$
(b)$x - log|\sin\left(x - \frac{\pi}{4}\right)| + c$
(c)$x + log|\sin\left(x - \frac{\pi}{4}\right)| + c$
(d)$x - log|\cos\left(x - \frac{\pi}{4}\right)| + c$

6) If the $\int \frac{5\tan x}{\tan x-2}dx = x + a\ln|\sin x - 2\cos x| + k$, then a is equal to (2018)

(a) -1 (b) 2 (c) 1 (d) 2

7) If $\int f(x)\,dx = \psi(x)$, then $\int x^5 f\left(x^3\right)dx$ is equal to:

(JEE M 2013)

(a) $\frac{1}{3}\left[x^3\psi\left(x^3\right) - \int x^2\psi(x^3)dx\right] + C$
(b) $\frac{1}{3}x^3\psi\left(x^3\right) - 3\int x^3\psi(x^3)dx + C$
(c) $\frac{1}{3}x^3\psi\left(x^3\right) - \int x^2\psi(x^3)dx + C$
(d) $\frac{1}{3}\left[x^3\psi\left(x^3\right) - \int x^3\psi(x^3)dx\right] + C$

8) The integral $\int\left(1 + x - \frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to (JEE M 2014)

(a) $(x+1)\,e^{x+\frac{1}{x}} + c$ (b) $-xe^{x+\frac{1}{x}} + c$ (c) $(x-1)\,e^{x+\frac{1}{x}} + c$ (d) $xe^{x+\frac{1}{x}} + c$

9) The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals: (JEE M 2015)

(a) $-\left(x^4+1\right)^{\frac{1}{4}} + c$ (b) $-\left(\frac{x^4+1}{x^4}\right) + c$ (c) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$ (d) $\left(x^4+1\right)^{\frac{1}{4}} + c$

10) The integral $\int \frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}dx$ is equal to (JEE M 2016)

(a) $\frac{x^5}{2\left(x^5-x^3+1\right)^2} + C$ (b) $\frac{-x^{10}}{2\left(x^5+x^3+1\right)^2} + C$ (c) $\frac{-x^5}{\left(x^5+x^3+1\right)^2} + C$ (d) $\frac{x^{10}}{2\left(x^5+x^3+1\right)} + C$

where C is an arbitrary constant

11) Let $I_n = \int\tan^x dx$, $(n > 1)$. $I_4 + I_6 = a\tan^5 x + bx^5 + C$, where C is constant of integration, then the ordered pair (a, b) is equal to : (JEE M 2017)

(a) $\left(-\frac{1}{5}, 0\right)$ (b) $\left(-\frac{1}{5}, 1\right)$ (c) $\left(\frac{1}{5}, 0\right)$ (d) $\left(\frac{1}{5}, -1\right)$

12) The integral $\int \frac{sin^2xcos^2x}{\left(sin^5x+cos^3xsin^2x+sin^3xcos^2x+cos^5x\right)^2}dx$ is equal to (JEE M 2018)

(a) $\frac{-1}{3\left(1+\tan^3 x\right)} + C$ (b) $\frac{1}{1+\cot^3 x} + C$ (c) $\frac{-1}{1+\cot^3 x} + C$ (d) $\frac{1}{3\left(1+\tan^3 x\right)} + C$

13) For $x^2 \neq n\pi + 1$, n∈N (the set of natural numbers), the integral $\int x\sqrt{\frac{2\sin\left(x^2-1\right)-\sin 2\left(x^2-1\right)}{2\sin\left(x^2-1\right)+\sin 2\left(x^2-1\right)}}dx$ is equal to:

(JEE M 2019 - 9 Jan(M))

(a) $\log_e|\frac{1}{2}\sec^2\left(x^2-1\right)|$ + c
(b) $\frac{1}{2}\log_e|\sec^2\left(\frac{x^2-1}{2}\right)|$ + c
(c) $\frac{1}{2}\log_e|\sec^2\left(\frac{x^2-1}{2}\right)|$ + c
(d) $\log_2|\sec\left(\frac{x^2-1}{2}\right)|$ + c

(where c is a constant of integration)

14) The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x dx$ is equal to (JEE M 2019 - 9 April (M))

(a) -3$\tan^{-1/3}$x+C (b) -$\frac{3}{4}\tan^{-4/3}$x+C (c) -3$\cot^{-1/3}$x+C (d) 3$\tan^{-1/3}$+C

(Here, C is a constant of integration)

18. Definite Integrals - Section B

31) The area of the region bounded by the parabola $(y-2)^2 = x-1$, the tangent of the parabola at the point $(2, 3)$ and the x-axis is: (2009)

(a) 6 (b) 9 (c) 12 (d) 3

32) $\int_0^{\pi}[\cot x]\,dx$, where [.] denotes the greatest integer function, is equal to (2009)

(a) 1 (b) -1 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

33) The area bounded between the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is

(2010)

(a) $4\sqrt{2}+2$ (c) $4\sqrt{2}+1$
(b) $4\sqrt{2}-1$ (d) $4\sqrt{2}-2$

34) Let $p(x)$ be a function defined on $\mathbf{R}$ such that $p'(x) = p'(1-x)$, for all $x \in [0,1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)\,dx$ equals (2010)

(a) 21 (b) 41 (c) 42 (d) $\sqrt{41}$

35) The value of $\int_0^1 \frac{8log(1+x)}{1+x^2}dx$ is (2011)

(a) $\frac{\pi}{8}$log2 (c) log 2
(b) $\frac{\pi}{2}$log2 (d) π log2

36) The area of the region enclosed by the curves $y = x, x = e, y = \frac{1}{x}$ and the positive x axis is (2011)

(a) 1 square unit (c) $\frac{5}{2}$ square units
(b) $\frac{3}{2}$ square units (d) $\frac{1}{2}$ square unit

37) The area between the parabolas:$x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is: (2012)

(a) $20\sqrt{2}$ (b) $\frac{10\sqrt{2}}{3}$ (c) $\frac{20\sqrt{2}}{3}$ (d) $10\sqrt{2}$

38) If $g(x) = \int_0^x \cos 4t dt$, then $g(x+\pi)$ equals (2012)

(a) $\frac{g(x)}{g(\pi)}$ (c) $g(x) - g(\pi)$
(b) $g(x) + g(\pi)$ (d) $g(x).g(\pi)$

39) **Statement-1 :** The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\pi/6$

Statement-2 : $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx.$ (JEE M 2013)

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(c) Statement-1 is true; Statement-2 is false

(d) Statement-1 is false; Statement-2 is true

40) The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is : (JEE M 2013)

(a) 9 (b) 36 (c) 18 (d) $\frac{27}{4}$

41) The integral $\int_0^\pi \sqrt{1 + 4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}}dx$ equals:

(JEE M 2014)

(a) (a) $4\sqrt{3}-4$ (c) (c) $\pi - 4$
(b) (b) $4\sqrt{3}-4-\frac{\pi}{3}$ (d) (d) $\frac{2\pi}{3}-4-4\sqrt{3}$

42) The area of the region described by $A = \{(x,y) : x^2 + y^2 \le 1 \text{ and } y^2 \le 1 - x\}$ is: (JEE M 2014)

(a) $\frac{\pi}{2}-\frac{2}{3}$ (b) $\frac{\pi}{2}+\frac{2}{3}$ (c) $\frac{\pi}{2}+\frac{4}{3}$ (d) $\frac{\pi}{2}-\frac{4}{3}$

43) The area (in sq. units) of the region described by $\{(x,y) : y^2 \le 2x \text{ and } y \ge 4x - 1\}$ is (JEE M 2015)

(a) $\frac{15}{64}$ (b) $\frac{9}{32}$ (c) $\frac{7}{32}$ (d) $\frac{5}{64}$

44) The integral $\int_2^4 \frac{logx^2}{logx^2+log\left(36-12x+x^2\right)}dx$ is equal to:

(JEE M 2015)

(a) 1 (b) 6 (c) 2 (d) 4

45) The area (in sq. units) of the region $\{(x,y) : y^2 \ge 2x \text{ and } x^2 + y^2 \le 4x, x \ge 0, y \ge 0\}$ is (JEE M 2016)

(a) $\pi - \frac{4\sqrt{2}}{3}$ (c) $\pi - \frac{4}{3}$
(b) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$ (d) $\pi - \frac{8}{3}$