Assignment - 1

AI24BTECH11003 - B. Vijaya Sreyas

I. 17.Indefinite Integrals - Section B

5. The value of $\sqrt{2}\int \frac{sinxdx}{sin(x-\frac{\pi}{4})}$ [2008]
(a)$x + log|cos(x-\frac{\pi}{4})| + c$
(b)$x - log|sin(x-\frac{\pi}{4})| + c$
(c)$x + log|sin(x-\frac{\pi}{4})| + c$
(d)$x - log|cos(x-\frac{\pi}{4})| + c$

6. If the $\int \frac{5tanx}{tanx-2}dx = x + aln|sinx - 2cosx| + k$, then a is equal to [2012]

(a) -1 (b) 2 (c) 1 (d) 2

7. If $\int f(x)dx = \psi(x)$, then $\int x^5 f(x^3)dx$ is equal to: [JEE M 2013]
(a) $\frac{1}{3}[x^3\psi(x^3) - \int x^2\psi(x^3)dx] + C$
(b) $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)dx + C$
(c) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + C$
(d) $\frac{1}{3}[x^3\psi(x^3) - \int x^3\psi(x^3)dx] + C$

8. The integral $\int(1 + x - \frac{1}{x})e^{x+\frac{1}{x}}dx$ is equal to [JEE M 2014]

(a) $(x+1)e^{x+\frac{1}{x}} + c$ (c) $(x-1)e^{x+\frac{1}{x}} + c$
(b) $-xe^{x+\frac{1}{x}} + c$ (d) $xe^{x+\frac{1}{x}} + c$

9. The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals: [JEE M 2015]

(a) $-(x^4+1)^{\frac{1}{4}} + c$ (c) $(\frac{x^4+1}{x^4})^{\frac{1}{4}} + c$
(b) $-(\frac{x^4+1}{x^4}) + c$ (d) $(x^4+1)^{\frac{1}{4}} + c$

10. The integral $\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}dx$ is equal to [JEE M 2016]

(a) $\frac{x^5}{2(x^5-x^3+1)^2} + C$ (c) $\frac{-x^5}{(x^5+x^3+1)^2} + C$
(b) $\frac{-x^{10}}{2(x^5+x^3+10^2} + C$ (d) $\frac{x^{10}}{2(x^5+x^3+1)} + C$

where C is an arbitrary constant

11. Let $I_n = \int \tan^x dx$, (n>1). $I_4+I_6 = a\tan^5 x + bx^5 + C$, where C is constant of integration, then the ordered pair (a, b) is equal to : [JEE M 2017]

(a) $(-\frac{1}{5}, 0)$ (b) $(-\frac{1}{5}, 1)$ (c) $(\frac{1}{5}, 0)$ (d) $(\frac{1}{5}, -1)$

12. The integral $\int \frac{sin^2xcos^2x}{(sin^5x+cos^3xsin^2x+sin^3xcos^2x+cos^5x)^2}dx$ is equal to [JEE M 2018]

(a) $\frac{-1}{3(1+tan^3x)} + C$ (c) $\frac{-1}{1+cot^3x} + C$
(b) $\frac{1}{1+cot^3x} + C$ (d) $\frac{1}{3(1+tan^3x)} + C$

13. For $x^2 \neq n\pi + 1$, n∈N (the set of natural numbers), the integral $\int x\sqrt{\frac{2sin(x^2-1)-sin2(x^2-1)}{2sin(x^2-1)+sin2(x^2-1)}}dx$ is equal to: [JEE M 2019 - 9 Jan(M)]

(a) $\log_e|\frac{1}{2}sec^2(x^2-1)| + c$ (c) $\frac{1}{2}\log_e|sec^2(\frac{x^2-1}{2})| + c$
(b) $\frac{1}{2}\log_e|sec^2(\frac{x^2-1}{2})| + c$ (d) $\log_2|sec(\frac{x^2-1}{2})| + c$

(where c is a constant of integration)

14. The integral $\int sec^{2/3}xcosec^{4/3}xdx$ is equal to [JEE M 2019 - 9 April (M)]

(a) -3tan$^{-1/3}$x+C (c) -3cot$^{-1/3}$x+C
(b) -$\frac{3}{4}$tan$^{-4/3}$x+C (d) 3tan$^{-1/3}$+C

(Here, C is a constant of integration)

II. 18. Definite Integrals - Section B

31. The area of the region bounded by the parabola $(y-2)^2 = x-1$, the tangent of the parabola at the point (2, 3) and the x-axis is: [2009]

(a) 6 (b) 9 (c) 12 (d) 3

32. $\int_0^\pi [cotx]dx$, where [.] denotes the greatest integer function, is equal to [2009]

(a) 1 (b) -1 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

33. The area bounded between the curves $y = cosx$ and $y = sinx$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is [2010]

(a) $4\sqrt{2} + 2$ (c) $4\sqrt{2} + 1$
(b) $4\sqrt{2} - 1$ (d) $4\sqrt{2} - 2$

34. Let $p(x)$ be a function defined on **R** such that $p'(x) = p'(1-x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)dx$ equals [2010]

(a) 21 (b) 41 (c) 42 (d) $\sqrt{41}$

35. The value of $\int_0^1 \frac{8log(1+x)}{1+x^2}dx$ is [2011]

(a) $\frac{\pi}{8}$log2 (c) log 2
(b) $\frac{\pi}{2}$log2 (d) π log2

36. The area of the region enclosed by the curves $y = x, x = e, y = \frac{1}{x}$ and the positive x axis is [2011]

(a) 1 square unit (c) $\frac{5}{2}$ square units
(b) $\frac{3}{2}$ square units (d) $\frac{1}{2}$ square unit

37. The area between the parabolas:$x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is: [2012]

(a) $20\sqrt{2}$ (b) $\frac{10\sqrt{2}}{3}$ (c) $\frac{20\sqrt{2}}{3}$ (d) $10\sqrt{2}$

38. If $g(x) = \int_0^x cos4tdt$, then $g(x+\pi)$ equals [2012]

(a) $\frac{g(x)}{g(\pi)}$ (c) $g(x) - g(\pi)$
(b) $g(x) + g(\pi)$ (d) $g(x).g(\pi)$

39. **Statement-1 :** The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{tanx}}$ is equal to $\pi/6$
Statement-2 : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$. [JEE M 2013]
(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation fo Statement-1
(c) Statement-1 is true; Statement-2 is false
(d) Statement-1 is false; Statement-2 is true

40. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is : [JEE M 2013]

(a) 9 (b) 36 (c) 18 (d) $\frac{27}{4}$

41. The integral $\int_0^{\pi} \sqrt{1 + 4sin^2\frac{x}{2} - 4sin\frac{x}{2}}dx$ equals: [JEE M 2014]

(a) $4\sqrt{3} - 4$ (c) $\pi - 4$
(b) $4\sqrt{3} - 4 - \frac{\pi}{3}$ (d) $\frac{2\pi}{3} - 4 - 4\sqrt{3}$

42. The area of the region described by $A = \{(x, y) : x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\}$ is: [JEE M 2014]

(a) $\frac{\pi}{2} - \frac{2}{3}$ (b) $\frac{\pi}{2} + \frac{2}{3}$ (c) $\frac{\pi}{2} + \frac{4}{3}$ (d) $\frac{\pi}{2} - \frac{4}{3}$

43. The area (in sq. units) of the region described by $\{(x, y) : y^2 \leq 2x \text{ and } y \geq 4x - 1\}$ is [JEE M 2015]

(a) $\frac{15}{64}$ (b) $\frac{9}{32}$ (c) $\frac{7}{32}$ (d) $\frac{5}{64}$

44. The integral $\int_2^4 \frac{logx^2}{logx^2 + log(36-12x+x^2)}dx$ is equal to: [JEE M 2015]

(a) 1 (b) 6 (c) 2 (d) 4

45. The area (in sq. units) of the region $\{(x, y) : y^2 \geq 2x \text{ and } x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is [JEE M 2016]

(a) $\pi - \frac{4\sqrt{2}}{3}$ (c) $\pi - \frac{4}{3}$
(b) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$ (d) $\pi - \frac{8}{3}$