

1-1.5-18

AI24BTECH11003 - Vijaya Sreyas

Question:

Find the coordinated of a point **A** where AB is the diameter of a circle whose center is $(2, -3)$ and **B** is the point $(1, 4)$. (10, 2019)

Solution:

Say center of the circle is **O**, and **A** be located at $\begin{pmatrix} x \\ y \end{pmatrix}$.

Point	Position	Description
A	$\begin{pmatrix} x \\ y \end{pmatrix}$	Unknown end of the diameter
B	$\begin{pmatrix} 1 \\ 4 \end{pmatrix}$	Known end of the diameter
O	$\begin{pmatrix} 2 \\ -3 \end{pmatrix}$	Center of the circle

TABLE 0: Points Involved

We know that the midpoint of any diameter of a circle is its center, i.e., in this case, midpoint of AB is $O = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

Applying section formula, we get

$$\mathbf{O} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad (0.1)$$

$$2\mathbf{O} = \mathbf{A} + \mathbf{B} \quad (0.2)$$

$$2 \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad (0.3)$$

$$\begin{pmatrix} 4 \\ -6 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad (0.4)$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad (0.5)$$

$$= \begin{pmatrix} 3 \\ -10 \end{pmatrix} \quad (0.6)$$

\therefore The point **A** is $\begin{pmatrix} 3 \\ -10 \end{pmatrix}$.

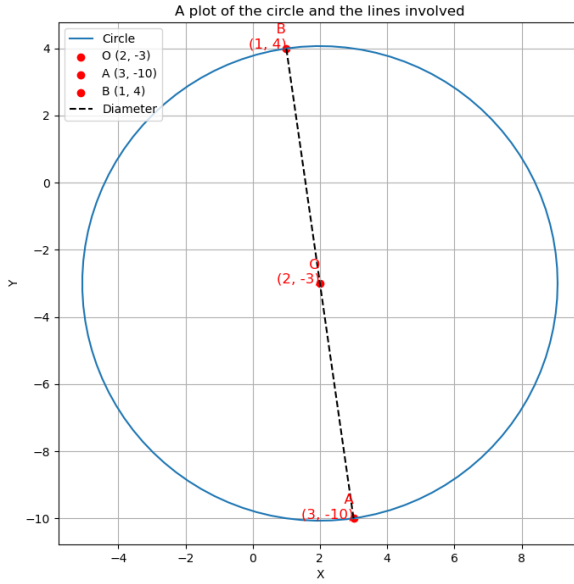


Fig. 0.1: Plot of the points and circle involved

Code for plotting points and circle

codes/code.py