Chevireddi, Vijay Dev Reddy
UID: 119491485
Homework 5

UR10e ROBOTIC MANIPULATOR:

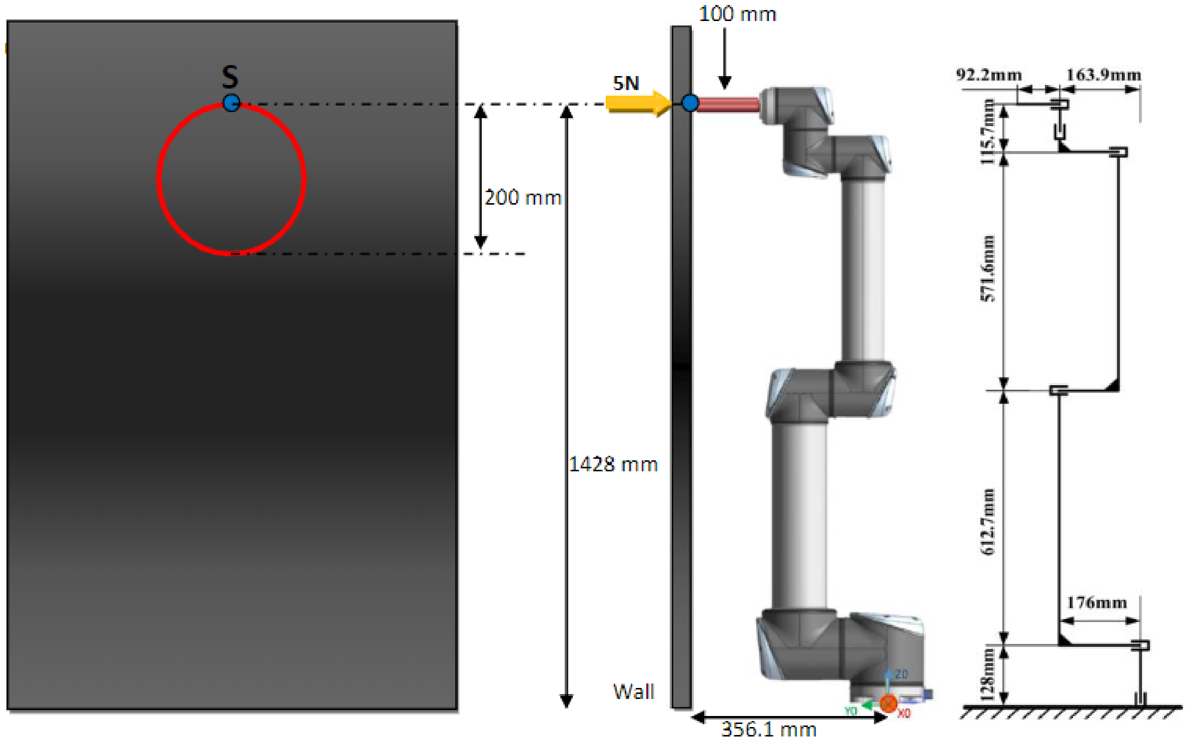

Figure 1 UR10 MANIPULATOR

UPDATED CIRCULAR TRAJECTORY EQUATIONS

```
x_dot = -sp.N(100*sp.pi/100*sp.sin(sp.N((sp.pi*i)/100)+sp.pi/2))
y_dot = 0.0
z_dot = sp.N(100*sp.pi/100*sp.cos(sp.N((sp.pi*i)/100)+sp.pi/2))
```

THE UPDATED CIRCULAR TRAJECTORY EQUATIONS:

$$\begin{bmatrix} -3.14159265358979\cdot\cos(0.0314159265358979\cdot\theta) \\ 0 \\ -3.14159265358979\cdot\sin(0.0314159265358979\cdot\theta) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

These is the path the end effector must follow to trace a circle in 200 seconds.

EQUATION OF MOTION

Using Euler-Lagrange equations for dynamics the equations of motion for a manipulator can be proved as:

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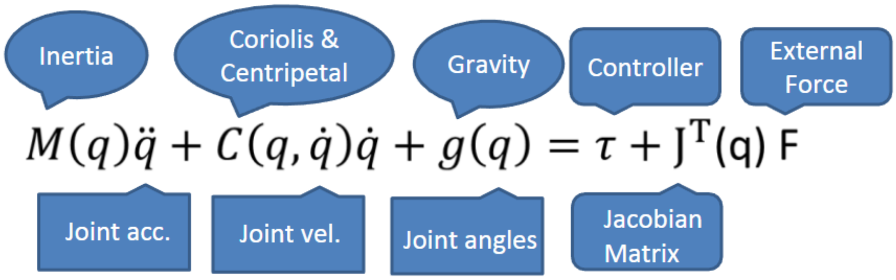

As the robot is assumed to be quasi-static the joint acceleration ($\ddot{q}$) and joint velocity ($\dot{q}$) can be taken as zero.

Now, the above equation can be written as:

$$\tau = g(q) - J^T(q)F$$

Where,

$\boldsymbol{\tau}$ is a 6×1 vector that represents joint torques that we are calculating.

$\boldsymbol{g(q)}$ is gravity matrix that is given by $g(q) = \frac{\partial P}{\partial q_k}$($P$=> Total Potential Energy, and q_k is the joint variable.

$\boldsymbol{F}$ is the external force acting on the manipulator.

$g(q)$ GRAVITY MATRIX

$g(q) = \frac{\partial P}{\partial q_k}$

P , total potential energy of the given UR10 manipulator is as follows:

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```
l1 = 128
a2 = 612.7
a3 = 571.6
l4 = 163.9
l5 = 115.7
l6 = 192.2

# Mass of the LINKS
m1 = 7.1
m2 = 12.7
m3 = 4.27
m4 = 2
m5 = 2
m6 = 0.365



# Height of Centre of Mass of all the LINKS
h1 = l1/2
h2 = l1 + (a2*ssin(theta2-sp.pi/2))/2
h3 = l1 + (a2*ssin(theta2-sp.pi/2)) + (a3*ssin(theta2-sp.pi/2+theta3))/2
h4 = l1 + (a2*ssin(theta2-sp.pi/2)) + (a3*ssin(theta2-sp.pi/2+theta3))
h5 = l1 + (a2*ssin(theta2-sp.pi/2)) + (a3*ssin(theta2-sp.pi/2+theta3)) + (l5*ssin(theta2-sp.pi/2+theta3+theta4+sp.pi/2))/2
h6 = l1 + (a2*ssin(theta2-sp.pi/2)) + (a3*ssin(theta2-sp.pi/2+theta3)) + (l5*ssin(theta2-sp.pi/2+theta3+theta4+sp.pi/2))


#Total Potential Energy
p = 9.8*((m1*h1)+(m2*h2)+(m3*h3)+(m4*h4)+(m5*h5)+(m6*h6))
```

LENGTHS-

I used the lengths of the links as shown above using the *figure 1*.

MASSES-

I used the mass of links as shown above these were provided in the official universal robots website (https://www.universal-robots.com/articles/ur/application-installation/dh-parameters-for-calculations-of-kinematics-and-dynamics/) as such:

UR10							
Kinematics	theta [rad]	a [m]	d [m]	alpha [rad]	**Dynamics**	Mass [kg]	Center of Mass [m]
Joint 1	0	0	0.1273	π/2	Link 1	7.1	[0.021, 0.000, 0.027]
Joint 2	0	-0.612	0	0	Link 2	12.7	[0.38, 0.000, 0.158]
Joint 3	0	-0.5723	0	0	Link 3	4.27	[0.24, 0.000, 0.068]
Joint 4	0	0	0.163941	π/2	Link 4	2	[0.000, 0.007, 0.018]
Joint 5	0	0	0.1157	-π/2	Link 5	2	[0.000, 0.007, 0.018]
Joint 6	0	0	0.0922	0	Link 6	0.365	[0, 0, -0.026]

The total potential energy equation was found to be:

```
TOTAL POTENTIAL ENERGY EQUATION:
P =  1547.7189*sin(theta2 + theta3 + theta4) - 89976.8331*cos(theta2) - 36410.92*cos(theta2 + theta3) + 31215.744
```

$g(q)$ the gravity matrix can be found in this way:

$$g(q) = \left[\frac{\partial P}{\partial q_1}\ \frac{\partial P}{\partial q_2}\ \frac{\partial P}{\partial q_3}\ \frac{\partial P}{\partial q_4}\ \frac{\partial P}{\partial q_5}\ \frac{\partial P}{\partial q_6}\right]^T$$

The generic gravity matrix for the UR10 robot was found to be:

THE GRAVITY MATRIX:

$$\begin{bmatrix} 0 \\ 89976.8331\cdot\sin(\theta_2) + 36410.92\cdot\sin(\theta_2 + \theta_3) + 1547.7189\cdot\cos(\theta_2 + \theta_3 + \theta_4) \\ 36410.92\cdot\sin(\theta_2 + \theta_3) + 1547.7189\cdot\cos(\theta_2 + \theta_3 + \theta_4) \\ 1547.7189\cdot\cos(\theta_2 + \theta_3 + \theta_4) \\ 0 \\ 0 \end{bmatrix}$$

F External Force

An external force of 5N is being applied on the end effector in the negative y direction while the manipulator is tracing a circle. Therefore, the force matrix can be taken as:

External Force Matrix:

$$\begin{bmatrix} 0 \\ -5 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

JOINT TORQUES

Now, using this equation, $\tau = g(q) - J^T(q)F$ we can find out τ which is a 6×1 vector with its entries as joint torques.

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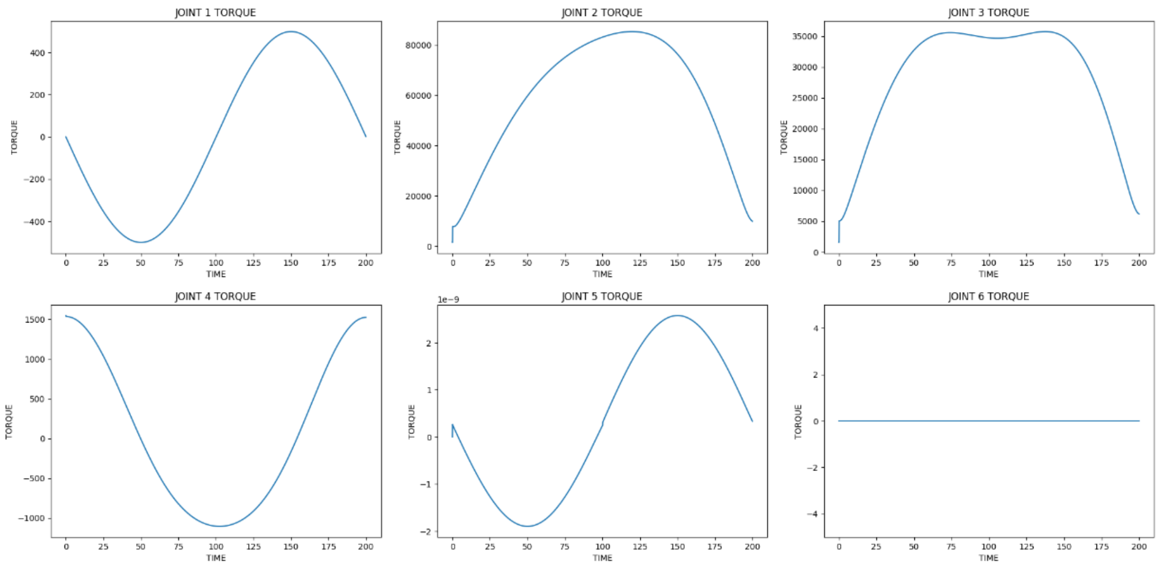
The joint torques that are required to draw the circle in 200 seconds were plotted.



Circle Obtained: