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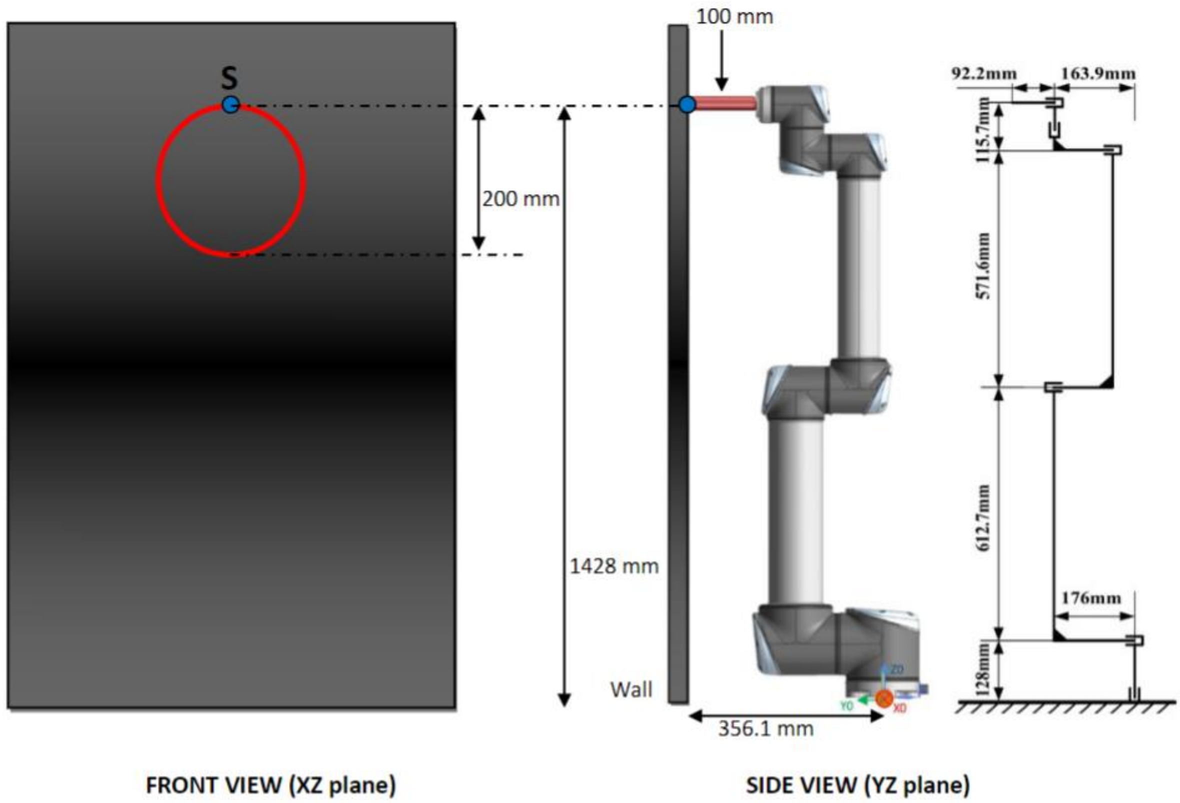
Homework 4

UR10e ROBOTIC MANIPULATOR:



From Figure 1 we can see that the length of the pen is 100mm. This 100mm needs to be added in the DH table for the last link in the "d" column.

DH TABLE:

	a_i	α_i	d_i	θ_i
$0-1$	0	-90	128	θ_1
$1-2$	612.7	180	0	$\theta_2 - 90$
$2-3$	571.6	180	0	θ_3
$3-4$	0	90	163.9	$\theta_4 + 90$
$4-5$	0	-90	115.7	θ_5
$5-6$	0	0	$92.2 +$ 100(length of pen)	θ_6

JACOBIAN MATRIX CALCULATION:

- Jacobian Matrix was found using the first method.
- Steps for the first method are:

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Step 1: Calculate ${}^{0}{}_{i}T$
Step 2: Calculate O_i
Step 3: Calculate Z_i
Step 4: Calculate J_i
Step 5: Write J

- The J_i for revolute joint and prismatic joint varies as follows.

where if joint *(i)* is revolute

$$J_i = \begin{bmatrix} Z_{i-1} \times (O_n - O_{i-1}) \\ Z_{i-1} \end{bmatrix}$$

and if joint *(i)* is prismatic

$$J_i = \begin{bmatrix} Z_{i-1} \\ 0 \end{bmatrix}$$

Where Z_i is the first three elements in the 3rd column of the T_0^i matrix, and O_i is the first three elements in the 4th column of the T_0^i matrix.

- The J_i vectors of size $6 \times 1$for all the joints are calculated and put together in this way, $J = [J_1\, J_2 \ldots \ldots J_n]$ to get the Jacobian Matrix (J).
- Generic Origin Vectors were found to be:

```
****************************************************************************************************
THE ORIGIN VECTORS ARE:
O0:
⎡0⎤
⎢ ⎥
⎢0⎥
⎢ ⎥
⎣0⎦
O1:
⎡ 0 ⎤
⎢   ⎥
⎢ 0 ⎥
⎢   ⎥
⎣128⎦
O2:
⎡612.7⋅sin(θ₂(t))⋅cos(θ₁(t))⎤
⎢                           ⎥
⎢612.7⋅sin(θ₁(t))⋅sin(θ₂(t))⎥
⎢                           ⎥
⎣  612.7⋅cos(θ₂(t)) + 128   ⎦
O3:
⎡571.6⋅sin(θ₂(t))⋅cos(θ₁(t))⋅cos(θ₃(t)) + 612.7⋅sin(θ₂(t))⋅cos(θ₁(t)) - 571.6⋅sin(θ₃(t))⋅cos(θ₁(t))⋅cos(θ₂(t))⎤
⎢                                                                                                              ⎥
⎢571.6⋅sin(θ₁(t))⋅sin(θ₂(t))⋅cos(θ₃(t)) + 612.7⋅sin(θ₁(t))⋅sin(θ₂(t)) - 571.6⋅sin(θ₁(t))⋅sin(θ₃(t))⋅cos(θ₂(t))⎥
⎢                                                                                                              ⎥
⎣             571.6⋅sin(θ₂(t))⋅sin(θ₃(t)) + 571.6⋅cos(θ₂(t))⋅cos(θ₃(t)) + 612.7⋅cos(θ₂(t)) + 128              ⎦
O4:
⎡-163.9⋅sin(θ₁(t)) + 571.6⋅sin(θ₂(t))⋅cos(θ₁(t))⋅cos(θ₃(t)) + 612.7⋅sin(θ₂(t))⋅cos(θ₁(t)) - 571.6⋅sin(θ₃(t))⋅cos(θ₁(t))⋅cos(θ₂
⎢
⎢571.6⋅sin(θ₁(t))⋅sin(θ₂(t))⋅cos(θ₃(t)) + 612.7⋅sin(θ₁(t))⋅sin(θ₂(t)) - 571.6⋅sin(θ₁(t))⋅sin(θ₃(t))⋅cos(θ₂(t)) + 163.9⋅cos(θ₁(
⎢
⎣                        571.6⋅sin(θ₂(t))⋅sin(θ₃(t)) + 571.6⋅cos(θ₂(t))⋅cos(θ₃(t)) + 612.7⋅cos(θ₂(t)) + 128

(t))⎤
    ⎥
t)) ⎥
    ⎥
    ⎦
```

```
O5:
⎡115.7·(sin(θ₂(t))·sin(θ₃(t))·cos(θ₁(t)) + cos(θ₁(t))·cos(θ₂(t))·cos(θ₃(t)))·sin(θ₄(t)) + 115.7·(sin(θ₂(t))·cos(θ₁(t))·cos(θ₃(
⎢
⎢115.7·(sin(θ₁(t))·sin(θ₂(t))·sin(θ₃(t)) + sin(θ₁(t))·cos(θ₂(t))·cos(θ₃(t)))·sin(θ₄(t)) + 115.7·(sin(θ₁(t))·sin(θ₂(t))·cos(θ₃(
⎢
⎣                                             115.7·(sin(θ₂(t))·sin(θ₃(t)) + cos(θ₂(t))·cos(θ₃(t)))·cos(θ₄(t)) + 115.7·(-sin(θ₂

t)) - sin(θ₃(t))·cos(θ₁(t))·cos(θ₂(t)))·cos(θ₄(t)) - 163.9·sin(θ₁(t)) + 571.6·sin(θ₂(t))·cos(θ₁(t))·cos(θ₃(t)) + 612.7·sin(θ₂(

t)) - sin(θ₁(t))·sin(θ₃(t))·cos(θ₂(t)))·cos(θ₄(t)) + 571.6·sin(θ₁(t))·sin(θ₂(t))·cos(θ₃(t)) + 612.7·sin(θ₁(t))·sin(θ₂(t)) - 57

(t))·cos(θ₃(t)) + sin(θ₃(t))·cos(θ₂(t)))·sin(θ₄(t)) + 571.6·sin(θ₂(t))·sin(θ₃(t)) + 571.6·cos(θ₂(t))·cos(θ₃(t)) + 612.7·cos(θ₂

t))·cos(θ₁(t)) - 571.6·sin(θ₃(t))·cos(θ₁(t))·cos(θ₂(t))⎤
                                                       ⎥
1.6·sin(θ₁(t))·sin(θ₃(t))·cos(θ₂(t)) + 163.9·cos(θ₁(t))⎥
                                                       ⎥
(t)) + 128                                             ⎦
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- Generic Z-axis unit vectors were found to be:

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****************************************************************************************************
THE Z-AXES UNIT VECTORS OF LOCAL FRAMES WITH RESPECT TO BASE FRAME:
z0:
⎡0⎤
⎢ ⎥
⎢0⎥
⎢ ⎥
⎣1⎦
z1:
⎡-sin(θ₁(t))⎤
⎢           ⎥
⎢cos(θ₁(t)) ⎥
⎢           ⎥
⎣     0     ⎦
z2:
⎡sin(θ₁(t)) ⎤
⎢           ⎥
⎢-cos(θ₁(t))⎥
⎢           ⎥
⎣     0     ⎦
z3:
⎡-sin(θ₁(t))⎤
⎢           ⎥
⎢cos(θ₁(t)) ⎥
⎢           ⎥
⎣     0     ⎦
z4:
⎡(sin(θ₂(t))·sin(θ₃(t))·cos(θ₁(t)) + cos(θ₁(t))·cos(θ₂(t))·cos(θ₃(t)))·sin(θ₄(t)) + (sin(θ₂(t))·cos(θ₁(t))·cos(θ₃(t)) - sin(θ₃
⎢
⎢(sin(θ₁(t))·sin(θ₂(t))·sin(θ₃(t)) + sin(θ₁(t))·cos(θ₂(t))·cos(θ₃(t)))·sin(θ₄(t)) + (sin(θ₁(t))·sin(θ₂(t))·cos(θ₃(t)) - sin(θ₁
⎢
⎣                     (sin(θ₂(t))·sin(θ₃(t)) + cos(θ₂(t))·cos(θ₃(t)))·cos(θ₄(t)) + (-sin(θ₂(t))·cos(θ₃(t)) + sin(θ₃(t))·cos(θ₂

(t))·cos(θ₁(t))·cos(θ₂(t)))·cos(θ₄(t))⎤
                                      ⎥
(t))·sin(θ₃(t))·cos(θ₂(t)))·cos(θ₄(t))⎥
                                      ⎥
(t)))·sin(θ₄(t))                      ⎦
z5:
⎡-((sin(θ₂(t))·sin(θ₃(t))·cos(θ₁(t)) + cos(θ₁(t))·cos(θ₂(t))·cos(θ₃(t)))·cos(θ₄(t)) - (sin(θ₂(t))·cos(θ₁(t))·cos(θ₃(t)) - sin(
⎢
⎢-((sin(θ₁(t))·sin(θ₂(t))·sin(θ₃(t)) + sin(θ₁(t))·cos(θ₂(t))·cos(θ₃(t)))·cos(θ₄(t)) - (sin(θ₁(t))·sin(θ₂(t))·cos(θ₃(t)) - sin(
⎢
⎣                                 -(-(sin(θ₂(t))·sin(θ₃(t)) + cos(θ₂(t))·cos(θ₃(t)))·sin(θ₄(t)) + (-sin(θ₂(t))·cos(θ₃(t)) + si

θ₃(t))·cos(θ₁(t))·cos(θ₂(t)))·sin(θ₄(t)))·sin(θ₅(t)) - sin(θ₁(t))·cos(θ₅(t))⎤
                                                                            ⎥
θ₁(t))·sin(θ₃(t))·cos(θ₂(t)))·sin(θ₄(t)))·sin(θ₅(t)) + cos(θ₁(t))·cos(θ₅(t))⎥
                                                                            ⎥
n(θ₃(t))·cos(θ₂(t)))·cos(θ₄(t)))·sin(θ₅(t))                                 ⎦
z6:
⎡-((sin(θ₂(t))·sin(θ₃(t))·cos(θ₁(t)) + cos(θ₁(t))·cos(θ₂(t))·cos(θ₃(t)))·cos(θ₄(t)) - (sin(θ₂(t))·cos(θ₁(t))·cos(θ₃(t)) - sin(
⎢
⎢-((sin(θ₁(t))·sin(θ₂(t))·sin(θ₃(t)) + sin(θ₁(t))·cos(θ₂(t))·cos(θ₃(t)))·cos(θ₄(t)) - (sin(θ₁(t))·sin(θ₂(t))·cos(θ₃(t)) - sin(
⎢
⎣                                 -(-(sin(θ₂(t))·sin(θ₃(t)) + cos(θ₂(t))·cos(θ₃(t)))·sin(θ₄(t)) + (-sin(θ₂(t))·cos(θ₃(t)) + si

θ₃(t))·cos(θ₁(t))·cos(θ₂(t)))·sin(θ₄(t)))·sin(θ₅(t)) - sin(θ₁(t))·cos(θ₅(t))⎤
                                                                            ⎥
θ₁(t))·sin(θ₃(t))·cos(θ₂(t)))·sin(θ₄(t)))·sin(θ₅(t)) + cos(θ₁(t))·cos(θ₅(t))⎥
                                                                            ⎥
n(θ₃(t))·cos(θ₂(t)))·cos(θ₄(t)))·sin(θ₅(t))                                 ⎦
****************************************************************************************************
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- Generic Jacobian Matrix was also found; however, it is too long to include in this report. It can be seen by executing the code I provided.

JACOBIAN MATRIX at t =0

- We know that at t = 0 (initial condition) all the values of theta are 0(i.e, $q = [\ 0\ 0\ 0\ 0\ 0\ 0]^T$). For this initial condition we get the manipulator at erect home position as seen in figure 1.
- At this initial condition values for theta and the values shown in the DH table we can build the intermediate transformation matrices ($T_1^0, T_2^1, T_3^2, T_4^3, T_5^4, T_6^5$).
- Using these intermediate transformation matrices, we can build these matrices $T_2^0, T_3^0, T_4^0, T_5^0, T_6^0$ by respective matrix multiplication respectively as follows:

$$\left[\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & -1 & 0 & 128\\ 0 & 0 & 0 & 1\end{bmatrix}, \begin{bmatrix}0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 1 & 0 & 0 & 740.7\\ 0 & 0 & 0 & 1\end{bmatrix}, \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 1 & 0 & 0 & 1312.3\\ 0 & 0 & 0 & 1\end{bmatrix}, \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 163.9\\ 0 & 0 & 1 & 1312.3\\ 0 & 0 & 0 & 1\end{bmatrix}, \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 163.9\\ 0 & -1 & 0 & 1428.0\\ 0 & 0 & 0 & 1\end{bmatrix}, \begin{bmatrix}-1 & 0 & 0 & 0\\ 0 & 0 & 1 & 356.1\\ 0 & 1 & 0 & 1428.0\\ 0 & 0 & 0 & 1\end{bmatrix}\right]$$

- From the above matrices we built we can derive the Z-axes unit vectors of all the frames with respect to the base frame and origins of all the frames as follows:

  We know that for revolute joint the J_{i-1} can be calculated as follows,

$$J_i = \begin{bmatrix} Z_{i-1} \times (O_n - O_{i-1}) \\ Z_{i-1} \end{bmatrix}$$

  Where Z_i is the first three elements in the 3rd column of the T_0^i matrix, and O_i is the first three elements in the 4th column of the T_0^i matrix.

  Using this concept, the origin vectors $O_0, O_1, O_2, O_3, O_4, O_5, O_6$ were respectively derived as

$$\left[\begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\128\end{bmatrix}, \begin{bmatrix}0\\0\\740.7\end{bmatrix}, \begin{bmatrix}0\\0\\1312.3\end{bmatrix}, \begin{bmatrix}0\\163.9\\1312.3\end{bmatrix}, \begin{bmatrix}0\\163.9\\1428.0\end{bmatrix}, \begin{bmatrix}0\\356.1\\1428.0\end{bmatrix}\right]$$

  And Z unit vectors were rescpectively derived as

$$\left[\begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\-1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right]$$

The Jacobian Matrix was found as

Jacobian Matrix:

$$\begin{bmatrix}-356.1 & 1300.0 & -687.3 & 115.7 & -192.2 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & -1 & 1 & 0 & 1\\ 1 & 0 & 0 & 0 & 1 & 0\end{bmatrix}$$

- These steps need to be repeated for other instances of time to calculate Jacobian matrices for other instances.

CIRCLE TRAJECTORY

We know that polar coordinates of the circle that needs to be traced by the manipulator are:

$$x = r \times \cos(\theta + 90),$$

$$z = r \times \sin(\theta + 90)\,, and$$

$$y = 0.$$

A phase difference of 90° was added to make the manipulator exactly trace the in the way it's asked (from the erect home position of the manipulator).

Differentiation of these equations gives us end-effector velocities:

$$\dot{x} = -r \times \sin(\theta + 90) \times \dot{\theta}$$

$$\dot{z} = r \times \cos(\theta + 90) \times \dot{\theta}$$

$$\dot{y} = 0$$

We know the relationship between JOINT VELOCITY and END-EFFECTOR VALUES as

$$\dot{q} = J^{-1} \times \epsilon$$

Where, ϵ is vector that represents end-effector velocities.

From Numerical Integration concept $q = q + \dot{q}\delta t$.

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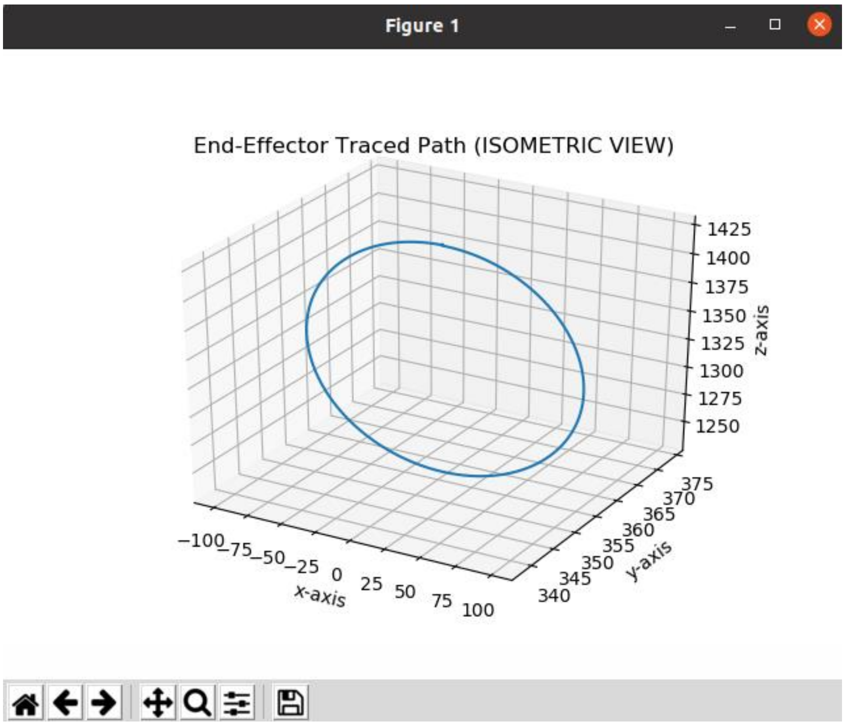
Now as time passes, we keep getting updates q values. And, for each q we need to find its corresponding Jacobian Matrix, and correspondingly the new (x,y,z) position of the end effector.

All these (x,y,z) positions were plotted and resulting graph was obtained:

ISOMETRIC VIEW:



FRONT VIEW: