6. A uniform beam with both ends fixed is subjected to an arbitrary transverse dynamic load $p(x,t)$ as shown in Figure Q6. The beam which is laid on an elastic foundation has a constant EI, and mass per unit length, m. The transverse displacement of the beam is denoted by $v(x,t)$. The elastic reaction of the foundation is $\bar{k} \cdot v(x,t)$ per unit length, in which $\bar{k}$ is called equivalent spring constant of the foundation.

(a) Describe the boundary conditions of the beam.

(4 marks)

(b) Derive the governing dynamic equation for the transverse displacements of the beam.

(14 marks)

(c) Derive the characteristic equation to determine the natural circular frequencies (do not need to solve it) and the corresponding expression for the normal modes of the beam if $\bar{k}=0$.

(15 marks)

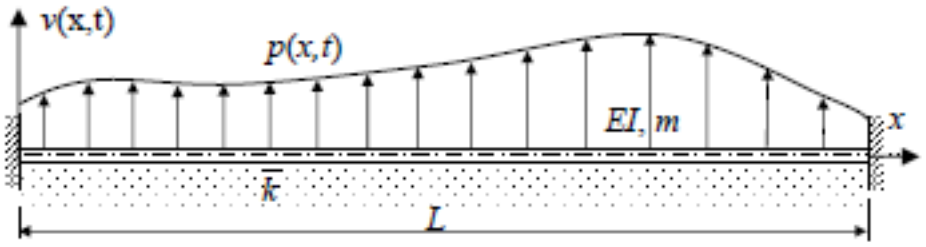

Figure Q6 Beam on an elastic foundation subjected to a dynamic loading