Financial Statistics coursework.

Please address all four questions.

You are invited to develop your own MatLab code but if you prefer you can use any software package available at the School.

Question 1 (CAPM)

Let r_{it} be the rate of return from holding the ith stock, let r_{mt} be the (equity) market index return, let r_{ft} be the risk-free rate.

$$r_{it} - r_{ft} = \alpha + \beta(r_{Mt} - r_{ft}) + u_t, \tag{1}$$

and

$$r_{it} - r_{ft} = \alpha + \beta_1[D_t(r_{Mt} - r_{ft})] + \beta_2[(1 - D_t)(r_{Mt} - r_{ft})] + u_t, \tag{2}$$

where $D_t = 1$ if $(r_{Mt} - r_{ft}) > 0$ and $D_t = 0$ otherwise.

- Write a programme which estimates the two above linear regression model using OLS over the whole sample period.
- With respect to the null hypothesis

  $$H_0: \ \ \beta_1 = \beta_2$$

  write down a code to perform the Wald test and apply the test to the data.

  Second, can we evaluate the Wald test by using the residuals from both equations (1) and (2)? Show the results and discuss its analogies with the conventional Wald test.

Question 2 (CIR model for the term structure of interest rate)

The discrete time version of the CIR model for the term structure postulates that the short-term interest rate r_t satisfies the following dynamic equation:

$$r_t = \mu(1 - \phi) + \phi r_{t-1} + r_{t-1}^{\frac{1}{2}} u_t,$$

with $u_t \sim NID(0, \sigma^2)$.

Write the code to estimate this model using MLE, deriving also the asymptotic covariance matrix.

Does the model resemble any linear time series model? How does its fit goes as compared with the standard AR(1) model

$$r_t = \mu(1-\phi) + \phi r_{t-1} + u_t,$$

with $u_t \sim NID(0, \sigma^2)$? You can simply adapt the code now to estimate this latter model and compare the results.

Perform the analysis using both the US 1-month interest rate r_t^{US} and the UK 1-month interest rate r_t^{UK}.

Question 3 (Backtesting)

Given the time series of the portfolio loss r_t (minus the portfolio return) and of its VaR_t at 1%, evaluate the time series of exceptions:

$$x_t = \begin{cases} 1 & \text{if } r_t > VaR_t \\ 0 & \text{if } r_t \leq VaR_t \end{cases}$$

Bearing in mind that x_t is distributed like a Bernoulli with parameter α, implying that the likelihood of the sample $(x_1, ..., x_T)$ is

$$L(\alpha) = \alpha^{\sum_{t=1}^{T} x_t}(1-\alpha)^{T-\sum_{t=1}^{T} x_t}.$$

Write the code to evaluate the LR (likelihood ratio) test, the Wald test and the LM (Lagrange multiplier) test for the null hypothesis:

$$H_0: \quad \alpha = 1\%.$$

Perform the tests using the data and comment on the results.

Do present the results using both time series of portfolio losses.

Question 4 (Unit root and co-integration)

Are the short-term (1-month) interest rate for US r_t^{US} and UK r_t^{UK} bond integrated, that is do they display a unit root? What does this imply in terms of the sample autocorrelation function? (Hint: bear in mind that in order to test for a unit root in the residual you will need to consider the Augmented-Dickey-Fuller test with an intercept term.)

More importantly, if the assumption of integration is not rejected, are them also cointegrated?

Write down a code to test both hypothesis.

It turns out that once the cointegration vector, say $(1 - \hat{\beta})$ is estimated, we can treat the Error Correction Mechanism term $ECM_{t-1} = (r_{t-1}^{US} - \hat{\beta} r_{t-1}^{UK})$ as a regressor and we can estimate the bivariate VAR:

$$\begin{aligned}
\Delta r_t^{US} &= \alpha_1 ECM_{t-1} + u_{1t}, \\
\Delta r_t^{UK} &= \alpha_2 ECM_{t-1} + u_{2t},
\end{aligned}$$

equation by equation by OLS. Do perform both aspects of this empirical analysis.