FMTH0301/Rev.5.3

Course Plan

Semester: FIRST SEMESTER Year: 2024-25

Course Title: **Introduction to Calculus**	Course Code: **24EMAB101**
Total Contact Hours: 74	Duration of ESA: 3 Hrs
ESA Marks: 50	ISA Marks: 50
Lesson Plan Author: Dr. Sumedha S.S, Roopa S.A, Dr. Shaila V. C, Dr. G. N. Bhadri, Dr. N.S. Kabbur	Date:18-09-2024
Checked By: Dr. Uma I. Neeli	Date: 19-09-2024

Prerequisites:

This subject requires the student to know about pre-university mathematics.

Course Outcomes (COs):

At the end of the course the student should be able to:

1. Construct and analyze the behavior of functions and use the properties of functions to solve application problems
2. Analyze the behavior of functions using limits, continuity, and derivatives, and apply these concepts to solve problems involving rates of change and asymptotic behavior.
3. Apply calculus concepts, including derivatives, extrema, concavity, and numerical methods, to analyze and solve real-world problems involving changing quantities, optimization, and limit evaluation
4. Analyze integration under the curve using Riemannian sum and use different integration techniques to evaluate integrals and employ numerical methods to approximate definite integrals.
5. Apply integral calculus techniques to find areas, arc lengths, volumes, and surface areas, as well as determine the average value of functions and the center of mass of objects in a plane.

Course Articulation Matrix: Mapping of Course Outcomes (COs) with Program Outcomes (POs)

Course Title: Introduction to Calculus	**Semester: First**
Course Code: 24EMAB101	**Year: 2024-25**

Course Outcomes (COs) / Program Outcomes (POs)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1.Construct and analyze the behavior of functions and use the properties of functions to solve application problems	H													
2.Analyze the behavior of functions using limits, continuity, and derivatives, and apply these concepts to solve problems involving rates of change and asymptotic behavior.	H													
3.Apply calculus concepts, including derivatives, extrema, concavity, and numerical methods, to analyze and solve real-world problems involving changing quantities, optimization, and limit evaluation	H													
4. Analyze integration under the curve using Riemannian sum and use different integration techniques to evaluate integrals and employ numerical methods to approximate definite integrals.	H													
5.Apply integral calculus techniques to find areas, arc lengths, volumes, and surface areas, as well as determine the average value of functions and the center of mass of objects in a plane.	H													

Degree of compliance **L**: Low **M**: Medium **H**: High

Competency addressed in the Course and corresponding Performance Indicators

Competency	**Performance Indicators**
1.1 - Demonstrate the competence in mathematical modeling.	1.1.1 – Apply Mathematical Techniques to solve problems.

Eg: 1.2.3: Represents program outcome ‘1’, competency ‘2’ and performance indicator ‘3’.

Course Content

<table>
<tr><td colspan="4">Program: UG</td></tr>
<tr><td colspan="2">Course Title: Introduction to Calculus</td><td colspan="2">Course Code: 24EMAB101</td></tr>
<tr><td>L-T-P: 4-0-1</td><td>Credits: 05</td><td colspan="2">Contact Hours: 74</td></tr>
<tr><td>CIE Marks: 50</td><td>SEE Marks: 50</td><td colspan="2">Total Marks: 100</td></tr>
<tr><td>Teaching Hours: 50</td><td>Laboratory Hrs:24</td><td colspan="2">Exam Duration: 3 hrs</td></tr>
<tr><td colspan="3">Content</td><td>Hrs</td></tr>
<tr><td colspan="4">Unit I</td></tr>
<tr><td colspan="3">1.Functions and Graphs
Review of Functions-Four ways to represent a function, Domain and Range. Basic Classes of Functions-Linear, polynomials and transcendental. Inverse of functions. Transformations-New functions from old functions. Compositions of Functions. Symmetry of functions-even and odd.</td><td>8 hrs</td></tr>
<tr><td colspan="3">2.Limits and derivatives
Tangent and velocity problems, Limit and continuity of function-laws of limits-examples. Infinite limits. Limits at infinity-horizontal and vertical asymptotes. Derivatives and rates of change, Derivative as a function, Rules of differentiation-product and quotient rule, Chain rule. Implicit differentiation, Derivatives of exponential, logarithmic and trigonometric functions.</td><td>12hrs</td></tr>
<tr><td colspan="4">Unit-II</td></tr>
<tr><td colspan="3">3.Applications of differentiation
Maximum and minimum values-local and global. Derivative and shape of the graph-first derivative test and second derivative test. Optimization problems. Indeterminate forms- L' Hospital rule. Roots of functions-Bisection method and Newton Raphson method.</td><td>10hrs</td></tr>
<tr><td colspan="3">4.Integrals
Approximating the areas, Definite integrals –integration formulas-properties, Fundamental theorem of calculus. Techniques of integration. Integrals involving exponential and logarithmic functions.</td><td>10hrs</td></tr>
<tr><td colspan="4">Unit III</td></tr>
<tr><td colspan="3">5.Applications of integration
Area between curves, Arc length, Area of surface of Revolution, Volume of surface of revolution, Average value of a function, Moments and center of mass.</td><td>10hrs</td></tr>
<tr><td colspan="4">Text Books
1. Calculus Volume-1, Edwin “Jed” Herman, Gilbert Strang, Openstax.
2. Early Transcendentals Calculus- James Stewart, Thomson Books, 7e 2010</td></tr>
<tr><td colspan="4">Reference Books:
1. Hughues- Hallett Gleason, Calculus Single and Multivariable, 4ed, Wiley India, 2009.
2. Thomas Calculus, George B Thomas, Pearson India, 12ed, 2010</td></tr>
</table>

In-Semester Assessment Scheme

Assessment	Conducted for marks	Weightage in Marks
ISA-1 (Theory)	40	40
ISA-2 (Theory)	40	
Laboratory Assessment	80	10
Total		**50**

End-Semester Assessment Scheme

Assessment	Conducted for marks	Weightage in Marks
Theory	100	40
Lab	20	10
	Total	**50**

Experiment wise plan

List of experiments planned to meet the requirements of the course

Expt. No.	Experiment	No. of Lab. Session/s per batch (estimate)	Marks/ Experiment	PI Code, BL, CO	Correlation of Experiment with the theory
1.	Introduction to GeoGebra, Sketch a graph, domain and range, Operation on functions, composition	1	--	1.1.1, L2, CO1	Functions and Graphs
2.	Inverse, Function from dataset, Transformation and Linear functions	1	10	1.1.1, L3, CO1	Functions and Graphs
3.	Exponential and Trigonometric	1	10	1.1.1, L3, CO1	Functions and Graphs
4.	Limit for piece wise/ continuous/rational function, continuity, asymptotes, slope of tangent line, derivative curve	1	10	1.1.1, L2, CO2	Limits and derivatives
5.	Introduction to MatLab	1	--	--	--
6.	Increasing or decreasing, local maxima and minima	1	10	1.1., L3, CO3	Application of Derivatives
7.	Optimization problems	1	10	1.1.1, L3, CO3	Application of Derivatives
8.	Roots of a function	1	10	1.1.1, L3, CO3	Application of Derivatives
9.	Integral by using Riemann sum	1	10	1.1.1, L3, CO4	Integrals
10.	Trapezoidal and Simpson's rule	1	10	1.1.1, L3, CO4	Integrals
11.	Arc length, SA and volume	1	--	1.1.1, L3, CO5	Integrals

Rubrics for Laboratory Experiments

Parameter	Excellent	Good	Needs improvement
Understanding the problem, Identify the mathematical concept (3 Marks)	**(3 M)**	**(2 M)**	**(1 M)**
	Student is able to identify all variables and constrains. Student is able to identify all suitable mathematical concepts	Student is able to identify most of the variables and constrains. Student is able to identify most of the suitable mathematical concepts	Student is able to identify only few variables, constrains and mathematical concepts
Program Execution using GeoGebra / MatLab (5 Marks)	**(5 M)**	**(4-3 M)**	**(2-1M)**
	Student is able to write logically correct program and plot the graph with proper labels	Student is able to write program that works up to 75%	Student is able to write program that works up to 50%
Interpretation of Results (2 Marks)	**(2 M)**	**(1 M)**	**(0 M)**
	Student is able to interpret all the results	Student is able to interpret few results	Student is unable to interpret the results

Course Unitization for ISA and ESA

Topics / Chapters	**Teaching hours**	**No. of Questions in ISA - I**	**No. of Questions in ISA - II**	**No. of Questions in Practical**	**No. of Questions in ESA Practical**	**No. of Questions in ESA**
UNIT-I						
1.Functions and Graphs	08	04	--	02	01	04
2.Limits and Derivatives	12	05	--	01		05
UNIT-II						
3.Applications of Differentiation	10	--	05	03	01	05
4. Integrals	10	--	04	02		04
UNIT-III						
5.Applications of Integration	10	--	--	--	01	02

Note

1. Each Question carries 20 marks and may consists of sub-questions.
2. Mixing of sub-questions from different chapters within a unit (only for Unit I and Unit II) is allowed in ISA - I, ISA -II and ESA
3. Answer 5 full questions of 20 marks each (two full questions from Unit I, II and one full questions from Unit III) out of 8 questions in ESA.

Course Assessment Plan

Course Title: Introduction to Calculus						Code: 24EMAB101
Course outcomes (COs)	Weightage in assessment	Assessment Methods				
		ISA-1	ISA-2	ISA (L)	ESA (T)	ESA(L)
1.Construct and analyze the behavior of functions and use the properties of functions to solve application problems	20%	✓		✓	✓	✓
2.Analyze the behavior of functions using limits, continuity, and derivatives, and apply these concepts to solve problems involving rates of change and asymptotic behavior.	20%	✓		✓	✓	✓
3.Apply calculus concepts, including derivatives, extrema, concavity, and numerical methods, to analyze and solve real-world problems involving changing quantities, optimization, and limit evaluation	20%		✓	✓	✓	✓
4. Analyze integration under the curve using Riemannian sum and use different integration techniques to evaluate integrals and employ numerical methods to approximate definite integrals.	20%		✓	✓	✓	✓
5.Apply integral calculus techniques to find areas, arc lengths, volumes, and surface areas, as well as determine the average value of functions and the center of mass of objects in a plane.	20%			✓	✓	✓
Weightage		15%	15%	10%	10%	50%

Date: 19-09-2024 **Head of Department**

Chapter-wise Plan

Course Code and Title: **24EMAB101 / Introduction to Calculus**	
Chapter Number and Title: **1. Functions and Graphs**	Planned Hours: **8 hrs.**

Learning Outcomes:

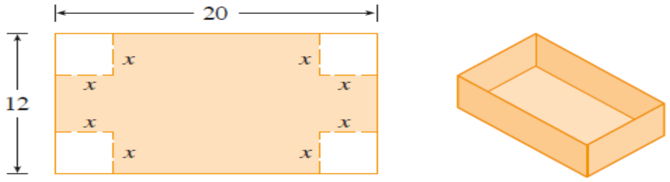
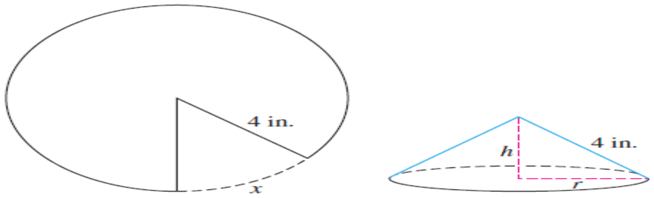
At the end of the topic the student should be able to:

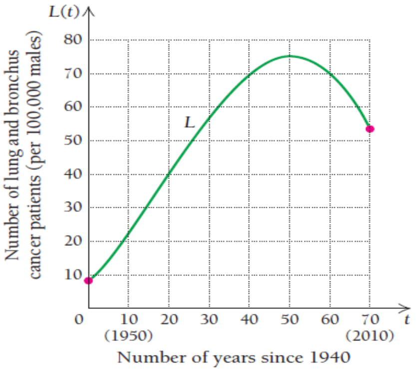
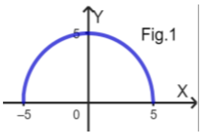
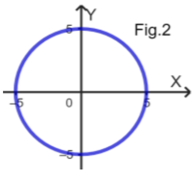
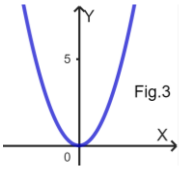
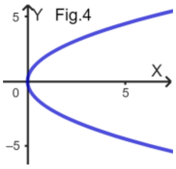
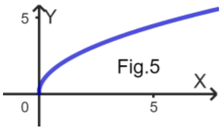
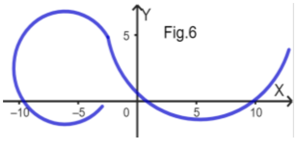
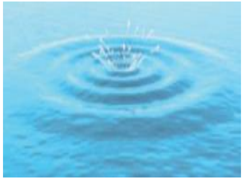
Topic Learning Outcomes	CO's	BL	CA Code
1. Identify and construct a function.	CO1	L3	1.1
2. Determine the domain and range of a function.	CO1	L2	1.1
3. Sketch the graph of a function.	CO1	L2	1.1
4. Make new functions from two or more given functions.	CO1	L2	1.1
5. Identify and apply different types of transformation to a given function.	CO1	L2	1.1
6. Recognize a function from a table of values.	CO1	L3	1.1
7. Apply the concept of basic functions to solve application problems.	CO1	L3	1.1

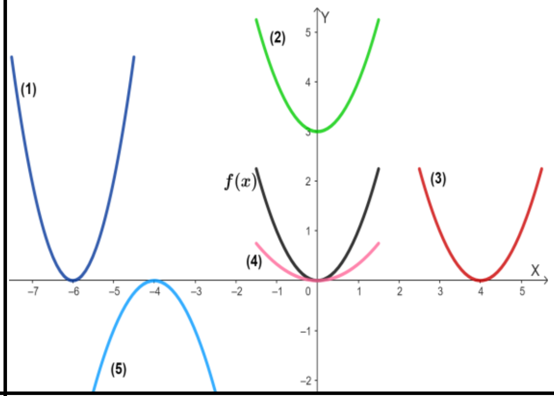
Lesson Schedule Class No. - Portion covered per hour
1. Review of Functions-Four ways to represent a function
2. Domain and Range with example
3. Compositions of Functions. Symmetry of functions, Inverse of function.
4. Transformations-New functions from old functions
5. Basic Classes of Functions-Linear function
6. Polynomials- Quadratic and Cubic functions
7. Exponential and Logarithmic functions
8. Trigonometric functions

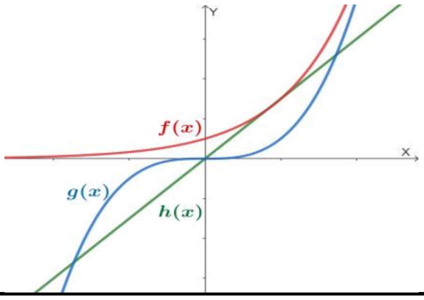
Review Questions

Sl.No	Questions	TLO	BL	PI Code
1.	A car starts with an initial velocity of 10 m/s and accelerates at a constant rate of 2 m/s². Find the velocity of a car at any given instant of time Identify the parameters and express the velocity of a car as a function of time t. Also, represent this function in four different ways.	TLO1	L3	1.1.1
2.	Identify the parameter that the area of a circle depends on. Write a formula for the area A of a circle based on that parameter. Then, use the formula to find the area of a circle with a diameter of 4 units.	TLO1	L3	1.1.1
3.	A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side x at each corner and then folding up the sides as in	TLO1	L3	1.1.1

	the figure. Express the volume V of the box as a function of x and hence find its domain.  			
4.	A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 30 ft, express the area A of the window as a function of the width x of the window and find its domain.  	TLO1	L3	1.1.1
5.	Begin with a circular piece of paper with a 4-in. radius as shown in (a). Cut out a sector with an arc length of *x.* Join the two edges of the remaining portion to form a cone with radius *r* and height *h*, as shown in (b).  (a) (b) **(a)** Explain why the circumference of the base of the cone is $8\pi - x$ **(b)** Express the radius *r* as a function of *x.* **(c)** Express the height *h* as a function of *x.* **(d)** Express the volume *V* of the cone as a function of *x.*	TLO1	L3	1.1.1
6.	A tank initially holds 50 gallons of water. As the water drains through a leak at the bottom, the tank eventually becomes empty. The drain is faster when it is nearly full because the pressure on the leak is greater. According to Torricelli's Law, the volume of water remaining in the tank after t minutes is given by the function: $V(t) = 50\left(1 - \frac{t}{20}\right)^2$ **(a)** Identify the domain and range of the function V(t). **(b)** Find *V*(0) and *V*(20) . **(c)** Make a table of values of *V(t)* for *t* = 0, 5, 10, 15, 20. **(d)** Find the net change in the volume *V* as *t* changes from 0 min to 20 min. 	TLO2	L2	1.1.1

7.	The following graph approximates the incidence of lung and bronchus cancer *L*, per 100,000 males, as a function of *t*, the number of years since 1940. The equation for this graph is the function given by  $L(t) = -0.00054t^3 + 0.02917t^2 + 1.2329t + 8$ a) Use the graph to estimate the domain of *L(t)*. b) Use the graph to estimate the range of *L(t)*. c) Use the graph to approximate the t-value where the cancer rate is 40 per 100,000 and verify it analytically d) Use the formula to approximate the lung and bronchus cancer rate in 2010 and verify it graphically.	TLO2	L2	1.1.1
8.	Determine whether the given graphs represent a function or not  Fig.1  Fig.2  Fig.3  Fig.4  Fig.5  Fig.6 	TLO1	L2	1.1.1
9.	Find the functions (i) $f \circ g$ (ii) $g \circ f$ (iii) $f \circ f$ a) $f(x) = x^2 - 1,$ $g(x) = 2x + 1$ b) $f(x) = x - 2,$ $g(x) = x^2 + 3x + 4$	TLO4	L2	1.1.1
10.	A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 60 cm/s.  (a) Express the radius r of this circle as a function of the time t (in seconds) (b) If A is the area of this circle as a function of the radius, find $A \circ r$ and interpret it.	TLO4	L2	1.1.1
11.	Sketch the graph of $\mathrm{f(x)} = \sqrt{-1 - \mathrm{x}}$ and its inverse on the same coordinate axes.	TLO5	L2	1.1.1
12.	Explain how each graph is obtained from the graph of y = f(x). $(i) y = f(x) + 8$ $(ii) y = f(x + 8)$ $(iii) y = 8f(x)$ $(iv) y = f(8x)$ $(v) y = -f(x) - 1$ $(vi) y = 8f\left(\frac{1}{8}x\right)$	TLO5	L2	1.1.1

13.	The graph of $y = f(x)$ is given. Match each equation with its graph and give reasons for your choice. 1 $Y = f(x-4)$ 2 $Y = -f(x+4)$ 3 $Y = \left(\frac{1}{3}\right) f(x)$ 4 $Y = f(x) + 3$ 5 $Y = 2f(x+6)$  	TLO5	L2	1.1.1
14.	Graph the following function, by identifying its parent function and then applying the appropriate transformations.Also find domain and range $i)\ y = -x^3$ $ii)\ y = 1 - x^2$ $iii)\ y = (x+1)^2$ $iv)\ y = \|4 - x^2\|$ $v)\ y = \sqrt(4 - x)$ $vi)\ y = x^2 - 4x + 3$ $vii)\ y = \|x^2 - 2x\|$ $viii) y = \sqrt{4 - 2x}$	TLO5	L2	1.1.1
15.	a) The data in the table relate study time and test scores. b) Create a linear function to the data. c)Make a scatterplot of the data and super impose the linear function on the scatterplot. Study Time (In Hrs) — Test Grade (In percent) 7 — 83 8 — 85 9 — 88 10 — 91 11 — ? d)Use the linear model to predict the test score received when one has studied for 11 hr.	TLO6	L3	1.1.1
16.	The manager of a furniture factory finds that it costs Rs2200 to manufacture 100 chairs in one day and Rs4800 to produce 300 chairs in one day. (a) Express the cost as a function of the number of chairs produced, assuming that it is linear. Then sketch the graph. (b) What is the slope of the graph and what does it represent? (c) What is the y-intercept of the graph and what does it represent? (d) Estimate the cost to produce 400 chairs.	TLO7	L3	1.1.1
17.	A small-appliance manufacturer finds that it costs Rs9000 to produce 1000 toaster ovens a week and Rs12,000 to produce 1500 toaster ovens a week. (i) Express the cost as a function of the number of toaster ovens produced, assuming that it is linear. Then sketch the graph. (ii)What is the slope of the graph and what does it represent? (iii) What is the y-intercept of the graph and what does it represent?	TLO7	L3	1.1.1
18.	A company wants to predict how much money it will make based on the price it charges for an item. They came up with the following quadratic function to model the revenue $R(p)$ (in thousands of rupees) based on the price per item p:	TLO7	L3	1.1.1

<table>
<tr><td></td><td>$R(p) = -1.04p^2 + 26p$ for $0 \le p \le 25.$
a. Predict the revenue if the item is sold at $p = 5$ and $p = 17$.
b. Find the prices where this function gives a revenue of zero and explain what these prices mean.
c. Draw a graph of *R(p)*.</td><td></td><td></td><td></td></tr>
<tr><td>19.</td><td>Match each equation with its graph and give reasons for your answer.
(i) $y = 3x$ (ii) $y = 3^x$ (iii) $y = x^3$
 </td><td>TLO1</td><td>L2</td><td>1.1.1</td></tr>
<tr><td>20.</td><td>Starting with the graph of $y = ln\ x$, find the equation of the graph that results from (i) Shifting 3 units upward (ii)Shifting 3 units to the left (iii) Reflecting about the x-axis (iv) Reflecting about the y-axis (v) Reflecting about the line y = x (vi) Shifting 3 units to the left and then Reflecting about the line y = x</td><td>TLO5</td><td>L2</td><td>1.1.1</td></tr>
<tr><td>21.</td><td>Graph the function by applying the appropriate transformations. Also find domain and range
i) $y = 4^{-x}$ ii)$y = \ln(x-1)$</td><td>TLO5</td><td>L2</td><td>1.1.1</td></tr>
<tr><td>22.</td><td>Use the data in Table and an exponential model to predict the population in the year 2010<table><tr><td>Year</td><td>1998</td><td>1999</td><td>2000</td><td>2001</td><td>2002</td><td>2003</td></tr><tr><td>Population (millions)</td><td>276.1</td><td>279.3</td><td>282.4</td><td>285.3</td><td>288.2</td><td>291.0</td></tr></table></td><td>TLO6</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>23.</td><td>If a bacteria population starts with 100 bacteria and doubles every three hours, then the number of bacteria after t hours is
$n = f(t) = 100 \cdot 2^{t/3}$
(a) Find the inverse of this function and explain its meaning.
(b) When will the population reach 50,000?</td><td>TLO7</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>24.</td><td>The half life of phosphorous-32 is about 14 days. There are 6.6 grams present initially.
(a) Express the amount of phosphorous-32 remaining as a function of time t.
(b) When will there be 1 gram remaining?</td><td>TLO7</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>25.</td><td>For the following functions determine (a) amplitude and period, (b) the domain, (c) the range, and (d) draw the graph of the function.
$i)y = 2\sin(4t+\pi)+3$ (ii) $y = 2\sin\left(2t+\frac{\pi}{3}\right)$
(iii) $y = 1 + 2\sin(t)$ (iv) $y = 3\cos(2t)$
(v)$y = -2\sin\left(\frac{t}{2}\right)$ (vi) $y = 1 - 2\sin 3x$</td><td>TLO5</td><td>L2</td><td>1.1.1</td></tr>
</table>

26.	Suppose that $T = 50 + 10\, sin\left[\frac{\pi}{12}(t-8)\right]$ is a mathematical model of the temperature (in degrees Fahrenheit) at t hours after midnight on a certain day of the week. (a) Determine the amplitude and period (b) Find the temperature 7 hours after midnight (c) At what time does $T = 60^0$? (d) Sketch the graph of T over $0 \leq t \leq 24$.	TLO7	L3	1.1.1
27.	On Feb 10, 2023, high tide in Goa was at midnight. The water level at high tide was 9.9 feet; at low tide, it was 0.1 feet. Assuming the next high tide is at exactly 12 noon and that the height of the water is given by a sine or cosine curve, find a formula for the water level in Goa as a function of time.	TLO7	L3	1.1.1

Course Code and Title: **24EMAB101 / Introduction to Calculus**	
Chapter Number and Title: **2. Limits and derivatives**	Planned Hours: **12 hrs**

Learning Outcomes:

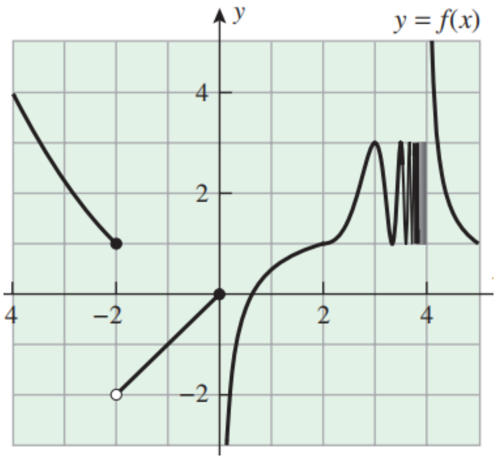
At the end of the topic the student should be able to:

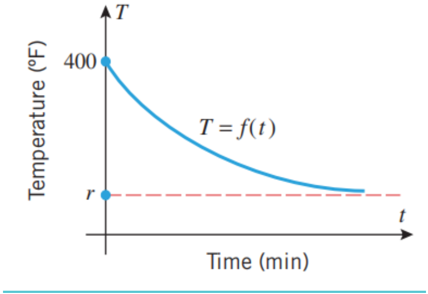
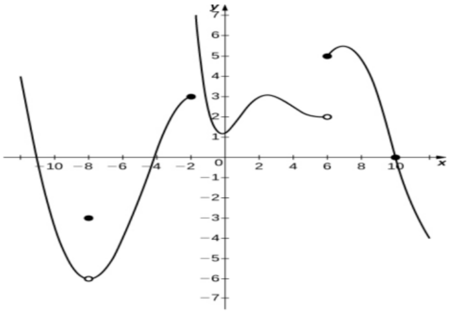
Topic Learning Outcomes	CO's	BL	CA Code
1. Define and interpret the limit of a function analytically and graphically.	CO2	L3	1.1
2. Define and identify vertical and horizontal asymptotes .	CO2	L2	1.1
3. Use the laws of limit an evaluate the limit of a function.	CO2	L2	1.1
4. Define continuity at a point and on an interval.	CO2	L2	1.1
5. Describe three kinds of discontinuities.	CO2	L2	1.1
6. Define derivatives of a function and calculate the derivative of a function at a point	CO2	L3	1.1
7. Explain average rate of change and instantaneous rate of change	CO2	L3	1.1

Lesson Schedule Class No. - Portion covered per hour
1. Limit of a function
2. Limit at infinity
3. Infinite limits
4. Examples
5. Continuity and discontinuity
6. Examples
7. Defining derivative
8. The derivative as a function

9. Examples
10. Differentiation Rules
11. Examples
12. Examples

Review Questions

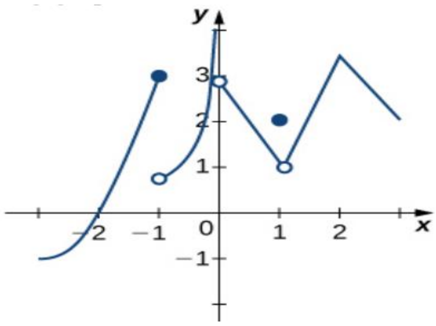
<table>
<tr><th>S.No</th><th>Questions</th><th>TLO</th><th>BL</th><th>PI Code</th></tr>
<tr><td>1.</td><td>Define limit of a function and Explain why graphically
(i) $\lim\limits_{x->0} \frac{1}{x^2}$ (ii) $\lim\limits_{x\to 0} \frac{|x|}{x}$ does not exists.</td><td>TLO1</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>2.</td><td>For the function f graphed in given figure, find the following limits

(a) $\lim\limits_{x\to -2^-} f(x)$
(b) $\lim\limits_{x\to -2^+} f(x)$
(c) $\lim\limits_{x\to 0^-} f(x)$
(d) $\lim\limits_{x\to 0^+} f(x)$
(e) $\lim\limits_{x\to 4^-} f(x)$
(f) $\lim\limits_{x\to 4^+} f(x)$</td><td>TLO1</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>3.</td><td>Determine $\lim\limits_{x\to c^+} f(x)$ and $\lim\limits_{x\to c^-} f(x)$ .Does limit exists? If so, what is it? If not, explain.
i) $c = 2, f(x) = \begin{cases} 3 - x, x < 2 \\ \frac{x}{2} + 1, x > 2 \end{cases}$ ii) $c = 2, f(x) = \begin{cases} 3 - x\,,\ x < 2 \\ 2, \quad x = 2 \\ \frac{x}{2}, \quad x > 2 \end{cases}$</td><td>TLO1</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>4.</td><td>A track coach uses a camera with a fast shutter to estimate the position of a runner with respect to time. A table of the values of position of the athlete versus time is given here, where x is the position in meters of the runner and t is time in seconds.
What is $\lim\limits_{x\to 2} x(t)$?
What does it mean physically?
<table>
<tr><th>t (sec)</th><th>x (m)</th></tr>
<tr><td>1.75</td><td>4.5</td></tr>
<tr><td>1.95</td><td>6.1</td></tr>
<tr><td>1.99</td><td>6.42</td></tr>
<tr><td>2.01</td><td>6.58</td></tr>
<tr><td>2.05</td><td>6.9</td></tr>
<tr><td>2.25</td><td>8.5</td></tr>
</table></td><td>TLO1</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>5.</td><td>Evaluate i) $\lim\limits_{x\to 1} \frac{x^3+3x^2+5}{4-7x}$ ii) $\lim\limits_{x\to -2} \sqrt{x^2 - 6x + 3}$
$iii)$ $\lim\limits_{x\to 0} \frac{1}{1+sinx}$ iv) $\lim\limits_{x\to 2} e^{2x-x^2}$</td><td>TLO1</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>6.</td><td>Let T=f(t) denote the temperature of a baked potato t minutes after it has been removed from a hot oven. The accompanying figure shows</td><td>TLO1</td><td>L3</td><td>1.1.1</td></tr>
</table>

	the temperature versus time curve for the potato, where ‘r’ is the temperature of the room. (a) What is the physical significance of $\lim_{x\to 0^+} f(t)$? (b) What is the physical significance of $\lim_{x\to +\infty} f(t)$? 			
7.	Find the horizontal and vertical asymptotes of each curve. a) $y = \frac{x^2+1}{2x^2-3x-2}$ b) $y = \frac{x^3-x}{x^2-6x+5}$ c) $y = \frac{2x}{x-3}$	TLO1	L3	1.1.1
8.	Consider the graph of the function $y = f(x)$ shown here. Which of the statements about $y = f(x)$ are true and which are false? Explain.  (i) $\lim_{x\to 10} f(x) = 0$ (ii) $\lim_{x\to -2^+} f(x) = 3$ (iii) $\lim_{x\to -8^-} f(x) = f(-8)$ (iv) $\lim_{x\to 6^-} f(x) = 5$	TLO1	L3	1.1.1
9.	Define continuity. For what value of the constant c is the function $\mathrm{f(x)} = \begin{cases} \mathrm{x}^2 - \mathrm{c}^2, \mathrm{x} < 4 \\ \mathrm{cx} + 20, \mathrm{x} \geq 4 \end{cases}$ continuous on $(-\infty, \infty)$	TLO2	L3	1.1.1
10.	Let $f(x) = \begin{cases} 3x, & x > 1 \\ x^3, & x < 1 \end{cases}$ i) Sketch the graph of f. ii) Is it possible to find a value k such that $f(1) = k$, which makes $f(x)$ continuous for all real numbers? Briefly explain.	TLO2	L3	1.1
11.	A lab technician controls the temperature T inside a kiln. From an initial temperature of 0 degrees Celsius (°C), he allows the kiln to increase by 2°C per minute for the next 60 min. After the 60th minute, he allows the kiln to cool at the rate of 3°C per minute. The temperature function T is defined by $T(t) = \begin{cases} 2t, & t\leq 60 \\ k-3t, & t>60 \end{cases}$ a) Find k such that T is continuous at t=60min b) Explain why T must be continuous at t = 60 min	TLO2	L3	1.1.1

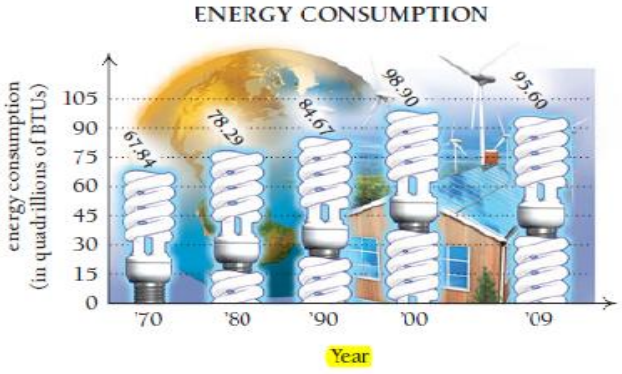
<table>
<tr><td>12.</td><td>Rick's Rocks sells decorative landscape rocks in bulk quantities. For quantities up to and including 500 lb, Rick charges 2.50 per pound. For quantities above 500 lb, he charges 2 per pound. The price function can be stated as a piecewise function:

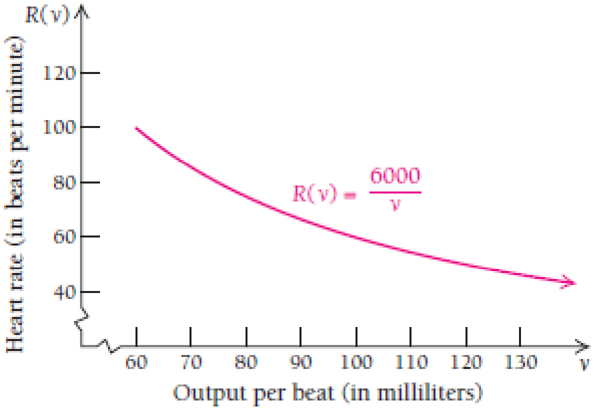
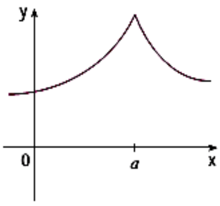
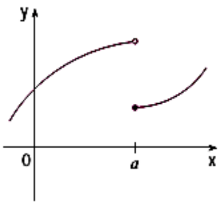
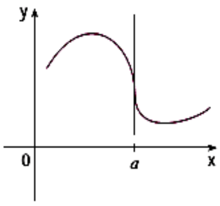
$$p(x)=\begin{cases}2.5x, & 0<x\leq 500\\ 2x, & x>500\end{cases}$$

Where p is the price in rupees and x is the quantity in pounds. Is the price function continuous at $x=500$ why or why not?</td><td>TLO2</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>13.</td><td>Consider the graph of the function $y=f(x)$ shown in the following graph.



i) Find all values of x for which the function is discontinuous.
ii) For each value in part a., state why the formal definition of Continuity does not apply.
iii) Classify each discontinuity as either jump, removable, or infinite.</td><td>TLO2</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>14.</td><td>Use the following graph to find the average rate of change in energy consumption from 1970 to 1980, from 1980 to 1990, and from 2000 to 2009


</td><td>TLO3</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>15.</td><td>Consider an athlete running a 40-m dash. The position of the athlete is given by $d(t)=\frac{t^3}{6}+4t$, Where d is the position in meters and t is the time elapsed, measured in seconds
(i) Compute the average velocity of the runner over the given time intervals.
a. [1.95, 2.05] b. [1.995, 2.005] c. [1.9995, 2.0005] d. [2, 2.00001].
(ii) Compute the instantaneous velocity of the runner at $t=2$ sec.</td><td>TLO3</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>16.</td><td>The table shows the estimated percentage of the population of Europe that use cell phones. (Midyear estimates are given.)
<table>
<tr><td>Year</td><td>1998</td><td>1999</td><td>2000</td><td>2001</td><td>2002</td><td>2003</td></tr>
<tr><td>P</td><td>28</td><td>39</td><td>55</td><td>68</td><td>77</td><td>83</td></tr>
</table></td><td>TLO3</td><td>L3</td><td>1.1.1</td></tr>
</table>

	(a) Find the average rate of cell phone growth (i) from 2000 to 2002 (ii) from 2000 to 2001 (iii) from 1999 to 2000 (b) Estimate the instantaneous rate of growth in 2000 by taking the average of two average rates of change. What are its units? (c) Estimate the instantaneous rate of growth in 2000 .			
17.	A manufacturer produces bolts of a fabric with a fixed width. The cost of producing x yards of this fabric is dollars. (a) What is the meaning of the derivative? What are its units? (b) In practical terms, what does it mean to say that? (c) Which do you think is greater, or? What about?	TLO3	L3	1.1.1
18.	The equation $R(v)=\frac{6000}{v}$ can be used to determine the heart rate, of a person whose heart pumps 6000 milliliters (mL) of blood per minute and milliliters of blood per beat.  a) Find the rate of change of heart rate with respect to the output per beat. b) Find the heart rate at per beat. c) Find the rate of change at per beat	TLO3	L3	1.1.1
19.	In following graphs of f, explain why f is not differentiable at $x=a$   	TLO3	L3	1.1.1
20.	Differentiate the following i) $y=(2x+1)^2$ ii) $y=(x-4)^8(2x+3)$ iii) $y=\frac{1}{(3x+8)^2}$ iv) $y=\frac{4x^2}{(7-5x)^3}$ v) $y=\sqrt[3]{2x-1}+(4-x)^2$	TLO3	L3	1.1.1
21.	Find $\frac{dy}{dt}$, if $y=\frac{1}{u^2+u}$ $and\ u=5+3t.$	TLO3	L3	1.1.1
22.	Find $\frac{dy}{dx}$, if $y=\sqrt[3]{u}\ and\ u=\frac{x^3-x}{1+x}.$	TLO3	L3	1.1.1

Course Code and Title: **24EMAB101 / Introduction to Calculus**	
Chapter Number and Title: **3. Applications of differentiation**	Planned Hours: **10hrs**

***Learning Outcomes*:**

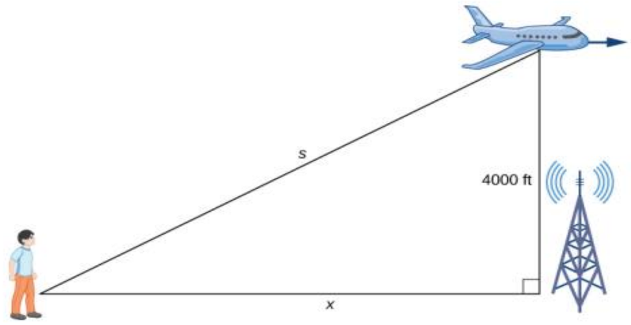
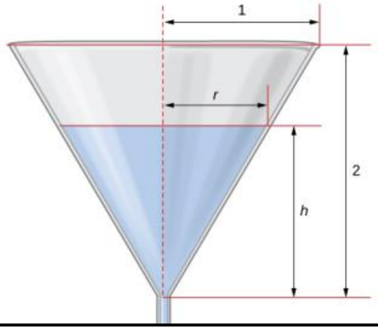
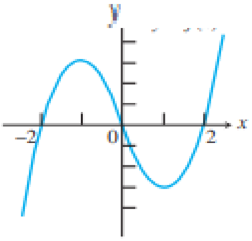
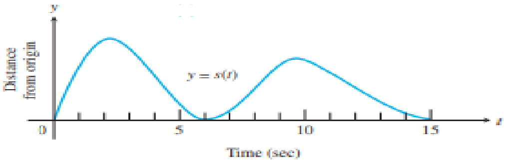
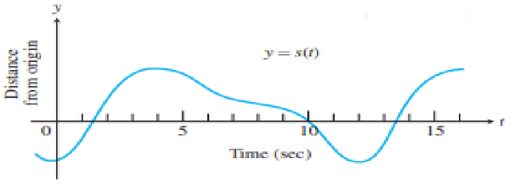
At the end of the topic the student should be able to:

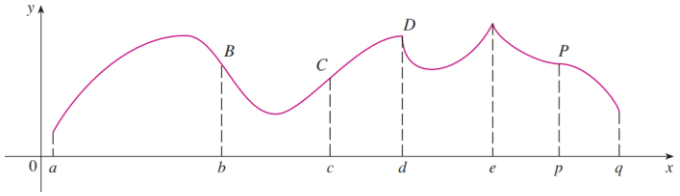
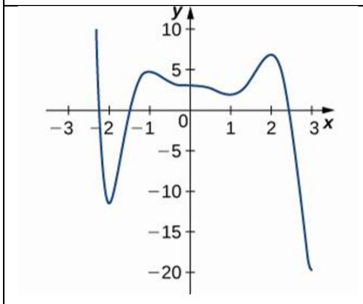
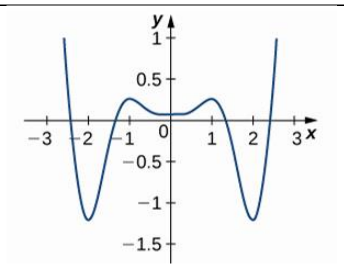
Topic Learning Outcomes	CO's	BL	CA Code
1. Express changing quantities in terms of derivatives	CO3	L3	1.1
2. Define relative extrema of a function and apply first and second derivative tests to find relative extrema and intervals for increase and decrease for a given function	CO3	L3	1.1
3. Define and find extrema of a function on an closed interval	CO3	L3	1.1
4. Find points of inflection and intervals of concavity for a given function.	CO3	L3	1.1
5. Identify the indeterminate forms; use the algebra of limits and L-Hospital's rule to determine limits of indeterminate forms.	CO3	L3	1.1
6. Apply Bisection and NR -method to solve application problems	CO3	L3	1.1

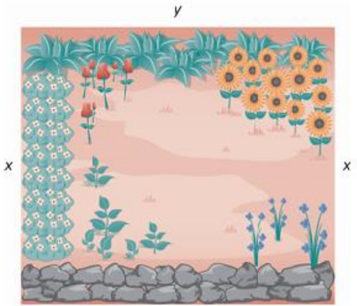
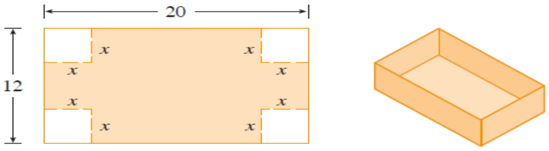
Lesson Schedule Class No. - Portion covered per hour
1) Related rates
2) Definitions of Local and Global maximum and minimum values
3) First derivative test
4) Second derivative test
5) Global maximum and minimum-Closed interval test
6) Optimization
7) Indeterminate forms- L' Hospital rule
8) Examples
9) Roots of functions- Bisection method
10) Roots of functions- Newton Raphson method

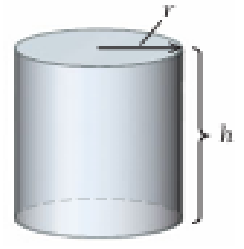
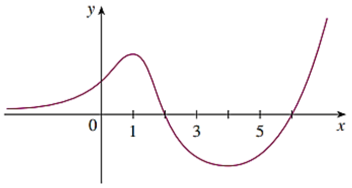
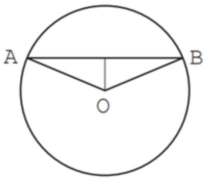
Review Questions

S.No	Questions	TLO	BL	PI Code
1.	An Airplane Flying at a Constant Elevation An airplane is flying overhead at a constant elevation of 4000 ft. A man is viewing the plane from a position 3000 ft from the base of a radio tower. The airplane is flying horizontally away from the man. If the plane is flying at the rate of 600 ft/sec, at what rate is the distance between the man and the plane increasing when the plane passes over the radio tower?	TLO1	L3	1.1.1

				
2.	Water is draining from the bottom of a cone-shaped funnel at the rate of 0.03 ft^3 /sec. The height of the funnel is 2 ft and the radius at the top of the funnel is 1 ft. At what rate is the height of the water in the funnel changing when the height of the water is $\frac{1}{2}$ ft? 	TLO1	L3	1.1.1
3.	The base of a triangle is shrinking at a rate of 1 cm/min and the height of the triangle is increasing at a rate of 5 cm/ min. Find the rate at which the area of the triangle changes when the height is 22 cm and the base is 10 cm	TLO1	L3	1.1.1
4.	Use the graph of the function f to estimate where (a) f and (b) f' are 0, positive, and negative 	TLO2	L3	1.1.1
5.	The graph of the position function $y = s(t)$ of a particle moving along a line is given. At approximately what times is the particle's (a) velocity equal to zero? (b) acceleration equal to zero?  	TLO2	L3	1.1.1

6.	Write the nature of the second derivative $f''(x)$ between ab, bc, cd, de, ep, pq.  	TLO2	L3	1.1.1
7.	For the following graphs, determine where the local and absolute maxima and minima occurs. Assume the graph represents the entirety of each function. a.  b.  	TLO2	L3	1.1.1
8.	For the following exercises, find the critical points in the domains of the following functions. i) $y = 4x^3 - 3x$ ii) $y = 4x - x^2$. iii)y = ln(x − 2) iv) y = tan(x)	TLO2	L3	1.1.1
9.	For the following functions, determine (a) intervals where f is increasing or decreasing, (b) local minima and maxima of f (c) intervals where f is concave up and concave down, and (d) the inflection points of f. (i) $f(x) = x^2 -$ 6x (ii) $f(x) = x^3 - 6x^2$ (iii) $f(x) = x + x^2 - x^3$ (iv) $f(x) = x^4 + x^3$ (v) $f(x) = (x-2)^2(x-4)^2$	TLO2 ,4	L3	1.1.1
10.	Ex:For the given function $f(x) = cos^2(x) - 2\,sin(x),\ \ 0 \le x \le 2\pi$ (a)Find the intervals on which f is increasing or decreasing (b)Find the local maximum and minimum values of f (c)Find the intervals of concavity and the inflection points.	TLO2 ,4	L3	1.1.1
11.	A cricketer hits a ball into the air and its position is observed as $h(t) = -6.2t2 + 80t$ ft. A fielder catches the ball. (i) How long after does the downfall happen? (ii) What is the height at which the ball starts falling? (iii) How much time (sec) did the ball take to reach the ground?	TLO2	L3	1.1.1
12.	Consider the production of gold for 40 years from 1948–1988. The production of gold is modeled by $G(t) = \frac{25t}{t^2+16}$, Where t (0 ≤ t ≤ 40) is the number of years since the mining began and G is the ounces of gold produced (in millions). (i) Find when the maximum (local) gold production occurred, and the amount of gold produced during that maximum. (ii) Find when the minimum (local) gold production occurred. What was the amount of gold produced during this minimum?	TLO2	L3	1.1.1

13.	Using second derivative test classify the critical points as local maxima or local minima (i)$\mathrm{f(x)} = 3\mathrm{x}^5 - 20x^3$ (ii) $\mathrm{f(x)} = \frac{\mathrm{x}}{\mathrm{x}^2+4}$	TLO2	L3	1.1.1
14.	Determine by inspection whether $p(x) = 3x^4 + 4x^3$ has any absolute extrema. If so, find them and state where they occur.	TLO3	L3	1.1.1
15.	Find the absolute maximum and minimum values of the function $f(x) = 2x^3 - 15x^2 + 36x$ on the interval [1, 5], and determine where these values occur.	TLO3	L3	1.1.1
16.	A rectangular garden is to be constructed using a rock wall as one side of the garden and wire fencing for the other three sides. Given 100 ft of wire fencing, determine the dimensions that would create a garden of maximum area. What is the maximum area?  	TLO2 ,3	L3	1.1.1
17.	A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. What size square should be cut out of each corner to get a box with the maximum volume?  	TLO2 ,3	L3	1.1.1
18.	A rectangular box with a square base, an open top, and a volume of 216 in^3 is to be constructed. What should the dimensions of the box be to minimize the surface area of the box? What is the minimum surface area?	TLO2 ,3	L3	1.1.1
19.	A hobby store has 20ft of fencing to fence off a rectangular area for an electric train in one corner of its display room. The two sides up against the wall require no fence. What dimensions of the rectangle will maximize the area? What is the maximum area?  	TLO2 ,3	L3	1.1.1

20.	A manufacturer of food-storage containers makes a cylindrical can with a volume of 500 milliliters What dimensions (height and radius) will minimize the material needed to produce each can, that is, minimize the surface area? 	TLO2	L3	1.1.1
21.	Identify the following indeterminate forms and evaluate using L-Hospital's rule (i)$\lim\limits_{x\to 0}\frac{2\sin x-\sin 2x}{x^3}$ (ii) $\lim\limits_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$ (iii)$\lim\limits_{x\to\infty}\left(\frac{2x-3}{2x+5}\right)^{2x+1}$ (iv)$\lim\limits_{x\to 0}(\tan x)^{\tan 2x}$ (v) $\lim\limits_{x\to 0}(\cot x)^{1/\log x}$ (vi) $\lim\limits_{x\to 0}\left[\frac{\sin x}{x}\right]^{1/x^2}$ (vii) $\lim\limits_{x\to 0}(sinx)^{\sqrt{x}}$ (viii) $\lim\limits_{x\to 0}(1+\sin 4x)^{cotx}$	TLO5	L3	1.1.1
22.	Find a root to $x^2 - x - 3 = 0$ accurate to three decimal places by Bisection method	TLO6	L3	1.1.1
23.	Find a root to $sinx + 3x = 0$ accurate to four decimal places by Bisection method	TLO6	L3	1.1.1
24.	Find the real root of the following equations by using (a) Bisection method (b) Newton-Raphson method (i) $xe^x = 1$ (ii) $x\sin x = \cos x$ (iii) $x^3 - x - 1 = 0$	TLO6	L3	1.1.1
25.	For each initial approximation, determine graphically what happens if Newton's method is used for the function whose graph is shown. (a) x_1=0 (b) x_1=1 (c) x_1=3 (d) x_1=4 (e) x_1=5 	TLO6	L3	1.1.1
26.	Use Newton's method with initial approximation to the root in [1, 2] of the equation $x^4 - x - 1=0$.	TLO6	L3	1.1.1
27.	Use Newton's method with initial approximation to the root in [–2, –1] of the equation $x^3 + x + 3=0$.	TLO6	L3	1.1.1
28.	The circle below has a radius of 1, and the longer circular arc joining A and B is twice as long as the chord AB. Use Newton-Raphson method to find the length of the chord AB, correct to 4 decimal places. 	TLO6	L3	1.1.1

Course Code and Title: **24EMAB101 / Introduction to Calculus**	
Chapter Number and Title: **4. Integrals**	Planned Hours: 10 **hrs**

Learning Outcomes: -

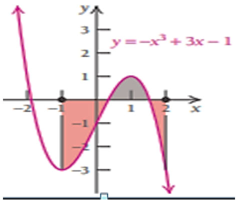
At the end of the topic the student should be able to:

Topic Learning Outcomes	CO's	BL	CA Code
1. Apply the sum of rectangular areas to approximate the area under the curve	CO4	L3	1.1
2. Apply the basic integration formulas, properties and techniques of integration	CO4	L3	1.1
3. Apply the Fundamental theorem of calculus	CO4	L3	1.1
4. Approximate the value of a definite integral using trapezoidal and Simpson's rule and estimate the errors in approximations.	CO4	L3	1.1

Lesson Schedule Class No. - Portion covered per hour
1. Introduction
2. Approximating the areas
3. Definite integrals
4. Examples
5. Integration formulas-properties
6. Fundamental theorem of calculus
7. Techniques of integration
8. Integrals involving exponential and logarithmic functions.
9. Numerical Integration
10. Examples

Review Questions

S.No.	Questions	TLOs	BL	PI Code
1.	Estimate the area under the graph of $f(x) = \sqrt{x}$ from x=0 to x=4 using four approximating rectangles.	TLO1	L3	1.1.1
2.	Consider the graph of $f(x) = 600x - x^2$ over the interval $[0,600]$. (a) Approximate the area by dividing the interval into 6 subintervals. (b) Approximate the area by dividing the interval in 12 subintervals.	TLO1	L3	1.1.1
3.	The rates of change in population for two cities are as follows: Alphaville: $P'(t) = 45.$ Betaburgh: $Q'(t) = 105e^{0.03t}$	TLO1	L3	1.1.1

	where t is the number of years since 1990, and both P' and Q' are measured in people per year. In 1990, Alphaville had a population of 5000, and Betaburgh had a population of 3500. a) Determine the population models for both cities. b) What were the populations of Alphaville and Betaburgh, to the nearest hundred, in 2000? c) Sketch the graph of each city's population model and estimate the year in which the two cities have the same population.			
4.	Consider $\int_{-1}^{2}(-x^3+3x+1)dx$. Predict the sign of the result by examining the graph and then evaluate the integral.  	TLO1	L3	1.1.1
5.	The velocity of a moving object is given by the function $v(x) = 3x$, where x is in hours and v is in miles per hour. Use geometry to find the area under the graph, which is the distance the object has traveled: a) during the first 3 hr $(0 \le x \le 3)$; b) between the third hour and the fifth hour $(3 \le x \le 5)$	TLO1	L3	1.1.1
6.	Use 5 subinervals to approximate the area under the graph of $f(x) = 0.1x^3 - 2.3x^2 + 12x + 25$ over the interval [1, 16].	TLO1	L3	1.1.1
7.	Find the area under the graph $y = x^2 + 1$ over the interval [-1,2].	TLO1	L3	1.1.1
8.	Evaluate each of the following: $(a)\int_{-1}^{2}(x^2-x)dx, \quad (b)\int_{0}^{3} e^x dx, (c)\int_{0}^{e}\left(1+2x-\frac{1}{x}\right)dx$	TLO2	L3	1.1.1
9.	***Total Profit from Marginal Profit***: Northeast Airlines determines that the marginal profit resulting from the sale of x seats on a jet traveling from Atlanta to Kansas City, in hundreds of dollars, is given by $P'(x) = \sqrt{x} - 6$. Find the total profit wen 60 seats are sold.	TLO2	L3	1.1.1
10.	Find the area under the graph of *y=f(x)* from -4 to 5, where $f(x) = \begin{cases} 9, & x < 3 \\ x^2, & x \ge 3. \end{cases}$	TLO2	L3	1.1.1
11.	Evaluate each definite integral: $(a)\int_{-3}^{4}\lvert x\rvert dx,\ (b)\int_{0}^{3}\lvert 1-x^2\rvert dx,$	TLO2	L3	1.1.1
12.	Evaluate the following integrals by interpreting each in terms of areas, $(\mathrm{a})\int_{0}^{1}\sqrt{1-x^2}\ \mathrm{dx}, \qquad (\mathrm{b})\int_{0}^{3}(x-1)\ \mathrm{dx}$	TLO2	L3	1.1.1
13.	Use the properties of definite integral to express the definite integral of $f(x) = -3x^3 + 2x + 2$ over the interval [-2, 1] as the sum of three definite integrals.	TLO2	L3	1.1.1

<table>
<tr><td>14.</td><td>Find the average value of the function $f(x) = 8 - 2x$ over the interval [0, 4] and find c such that $f(c)$ equals the average value of the function over [0, 4].</td><td>TLO3</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>15.</td><td>Given $\int_0^3 (2x^2 - 1)dx = 15$, find c such that f(c) equals the average value of $f(x) = 2x^2 - 1$ over [0, 3].</td><td>TLO3</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>16.</td><td>Integrate the function $\int_{-2}^{2} (3x^8 - 2)dx$ and verify that the integration formula for even functions holds.</td><td>TLO2</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>17.</td><td>Use basic integration formulas to compute the following antiderivatives or integrals
(i) $\int (x^2 + x^{-2})\, dx$ (ii) $\int (u + 4)(2u + 1)\, du$
(iii) $\int_0^1 (5x - 5^x)dx$ (iv) $\int_0^{\pi} (\sin x - \cos x)dx$ (v) $\int_0^{\frac{\pi}{4}} \frac{1+\cos^2\theta}{\cos^2\theta}\, d\theta$</td><td>TLO2</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>18.</td><td>Write an integral that expresses the increase in the perimeter P(s) of a square when its side length s increases from 2 units to 4 units and evaluate the integral.</td><td>TLO2</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>19.</td><td>Use substitution method to evaluate the following integrals
(i) $\int x^2\sqrt{x^3 + 1}\, dx$ (ii) $\int \cos^3\theta \sin\theta\, d\theta$ (iii) $\int \frac{dx}{5-3x}$
(iv) $\int e^x \sin(e^x)\, dx$ (v) $\int_0^1 xe^{-x^2}\, dx$ (vi) $\int_0^2 \frac{\cos(\ln(2x))}{x}\, dx$
(vii) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos x}\, dx$</td><td>TLO2</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>20.</td><td>Suppose a population of fruit flies increases at a rate of $g(t) = 2e^{0.02t}$, in flies per day. If the initial population of fruit flies is 100 flies, how many flies are in the population after 10 days?</td><td>TLO2</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>21.</td><td>Evaluate the following integrals using integration by parts
(i) $\int \ln x\, dx$ (ii) $\int x^2 \ln x\, dx$ (iii) $\int z^3 e^z\, dz)$ (iv) $\int_0^{2\pi} t^2 \sin 2t\, dt$</td><td>TLO2</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>22.</td><td>When a particle is located at a distance x meters from the origin, a force of $x^2 + 2x$ N acts on it. How much work is done in moving it from x=1 to x=3?</td><td>TLO2</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>23.</td><td>Write an integral to express the area under the graph of $y = e^t$ between t=0 and t=ln x, and evaluate the integral.</td><td>TLO2</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>24.</td><td>Use Trapezoidal rule to approximate the given integral with the specified value of n. $\int_0^{\frac{1}{2}} \mathrm{sintdt},\ \mathrm{n} = 8.$</td><td>TLO4</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>25.</td><td>Using Simpson $\frac{1}{3}$rd rule evaluate $\int_0^1 \frac{\mathrm{dx}}{1+\mathrm{x}^2}$ by dividing the interval into 6-equal sub-intervals and hence find the value of π correct to4-decimal places.</td><td>TLO4</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>26.</td><td>A river is 80 meters wide. The depth ‘y’ of the river at a distance ‘x’ from one bank is given in the following table:
<table>
<tr><td>x</td><td>0</td><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td></tr>
<tr><td>y</td><td>0</td><td>4</td><td>7</td><td>9</td><td>12</td><td>15</td><td>14</td><td>8</td><td>3</td></tr>
</table>
Find approximately the area of cross section of the river using Simpson’s 1/3 rule.</td><td>TLO4</td><td>L3</td><td>1.1.1</td></tr>
</table>

<table>
<tr><td>27.</td><td>The velocity v of a particle at a distances s from a point on its path is given by the table
<table>
<tr><td>s(ft)</td><td>0</td><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td></tr>
<tr><td>v(ft/sec)</td><td>47</td><td>58</td><td>64</td><td>65</td><td>61</td><td>52</td><td>38</td></tr>
</table>
Estimate the time taken to travel 60ft. by using Simpson's 1/3rd rule.</td><td>TLO4</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>28.</td><td>The table shows values of a force function $f(x)$, where x is measured in meters and $f(x)$ in Newton's. Use Simpson's rule to estimate the work done by the force in moving an object a distance of 18 m.
<table>
<tr><td>x</td><td>0</td><td>3</td><td>6</td><td>9</td><td>12</td><td>15</td><td>18</td></tr>
<tr><td>$f(x)$</td><td>9.8</td><td>9.1</td><td>8.5</td><td>8.0</td><td>7.7</td><td>7.5</td><td>7.4</td></tr>
</table></td><td>TLO4</td><td>L3</td><td>1.1.1</td></tr>
</table>

Course Code and Title: **24EMAB101 / Introduction to Calculus**	
Chapter Number and Title: **5. Applications of integration**	Planned Hours: 10**hrs**

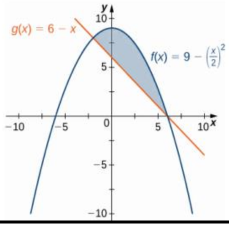
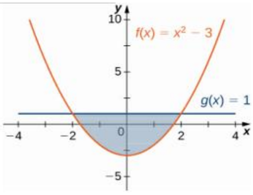
Learning Outcomes:

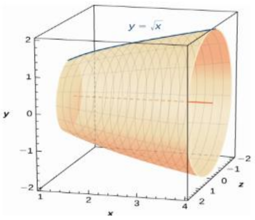
At the end of the topic the student should be able to:

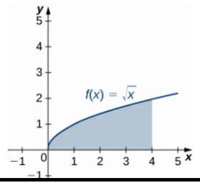
Topic Learning Outcomes	CO's	BL	CA Code
1.Determine the area bounded by a curve and the area of a region between two curves	CO5	L3	1.1
2.Find the arc length of a curve, volume of revolution and surface area of revolution	CO5	L3	1.1
3. Find the Average value of a function using integration	CO5	L3	1.1
4.Find the center of mass of objects in a plane.	CO5	L3	1.1

Lesson Schedule Class No. - Portion covered per hour
1. Applications to find Area
2. Examples Continued
3. Applications to find arc length
4. Examples Continued
5. Applications to find surface area of revolution
6. Applications to find Volume of revolution
7. Examples Continued
8. Average value of a function
9. Moments and center of mass
10.Examples Continued

Review Questions

S.No.	Questions	TLOs	BL	PI Code
1.	A vehicle is moving with the velocity as a function of time t is v(t)=3t+5. What is the distance traveled by the vehicle from t=0 to t=6 ?	TLO1	L3	1.1.1
2.	**The tortoise versus the hare:** The speed of the hare is given by the sinusoidal function H(t) = (1/2) − (1/2) cos(2πt) , whereas the speed of the tortoise is T(t) = t, where t is time measured in hours and speed is measured in kilometers per hour. If the race is over in 1 hour, who won the race and by how much? 	TLO1	L3	1.1.1
3.	If R is the region bounded above by the graph of the function $f(x) = 9 - \left(\frac{x}{2}\right)^2$ and below by the graph of the function $g(x) = 6 - x$, find the area of region R. 	TLO1	L3	1.1.1
4.	If R is the region bounded by the graph of the functions $y = x^2 - 3$ and g(x) = 1 , find the area of region R. 	TLO1	L3	1.1.1
5.	If birth rate of a population of a city is modelled as $\mathrm{f}(t) = 2200 + 52.3t$ people per year and the death rate of a same city is modelled as $\mathrm{g}(t) = 1460 + 28.8t$ people per year. Find the area between these curves for $0 \leq t \leq 10$. What does this area represent?	TLO1	L3	1.1.1
6.	A factory selling cell phones has a marginal cost function $C(x) = 0.01x^2 - 3x + 229$, where x represents the number of cell phones, and a marginal revenue function given by $R(x) = 429 - 2x$. Find the area between the graphs of these curves and x = 0. What does this area represent?	TLO1	L3	1.1.1
7.	An amusement park has a marginal cost function $C(x) = 0.01x^2 - 2x + 105$, where x represents the number of tickets sold, and a marginal revenue function given by $R(x) = 105 - 0.1x$. Find the total profit generated when selling 150 tickets.	TLO1	L3	1.1.1
8.	Find the arc length of the following curves: (i)$y = lnx,\ 1 \leq x \leq \sqrt{3}$ (ii)$y = 1 + 6x^{\frac{3}{2}}$,$0 \leq x \leq 1$	TLO2	L3	1.1.1

9.	A hawk flying at 15m/s at an altitude of 180 m accidentally drops its prey. The parabolic trajectory of the falling prey is described by the equation $y = 180 - \frac{x^2}{45}$ until it hits the ground, where is its height above the ground and is the horizontal distance traveled in meters. Calculate the distance traveled by the prey from the time it is dropped until the time it hits the ground. 	TLO2	L3	1.1.1
10.	A steady wind blows a kite due west. The kite's height above ground from horizontal position x= 0 to x=80 ft is given by $y = 150 - \frac{1}{40}(x-50)^2$. Find the distance traveled by the kite. 	TLO2	L3	1.1.1
11.	A lampshade is constructed by rotating $y = \frac{1}{x}$ around the y-axis from $y = 1$ to $y = 2$. Determine how much material required to construct this lampshade. 	TLO2	L3	1.1.1
12.	Find the area of the surface obtained by rotating the curve $y = x^3$ between x=0 and x=2 about the x-axis.	TLO2	L3	1.1.1
13.	Find the surface area of the volume generated when the curve $y = \sqrt{x}$ revolves around the x-axis from (1, 1) to (4, 2), as seen here. 	TLO2	L3	1.1.1
14.	Find the volume of the reel shaped solid generated by revolution about y-axis of the part of the parabola $y^2 = 4ax$ cut off by its latus rectum.	TLO2	L3	1.1.1
15.	The cost per unit c of producing CD players over two year period is modeled by $c = 0.005t^2 + 0.01t + 13.15,\ 0 \le t \le 24$, where t is the time in months. Approximate the average cost per unit over the two year period.	TLO3	L3	1.1.1
16.	The concentration y of a drug (in milligrams per milliliter) in a patient's blood stream after t hours can be modeled by $y =$	TLO3	L3	1.1.1

	$500e^{-0.4t}$. Determine the average concentration of the drug during the first 5 hours after the drug is administered.			
17.	Suppose four point masses are placed on a number line as follows: m1=30kg at x1=-2m, m2=5kg at x2=3m, m3=10kg at x3=6m and m4=15kg at x4=-3m.Find the moment of the system with respect to the origin and find the center of mass of the system.	TLO4	L3	1.1.1
18.	Suppose four point masses are placed on a number line as follows: m1=12kg at x1=-4m,m2=12kg at x2=4m, m3=30kg at x3=2m and m4=6kg at x4=-6m.Find the moment of the system with respect to the origin and find the center of mass of the system.	TLO4	L3	1.1.1
19.	Suppose three point masses are placed in the xy-plane as follows: m1=2kg at (-1,3),m2=6kg at (1,1) , m3=4kg at (2,-2).Find the center of mass of the system.	TLO4	L3	1.1.1
20.	Suppose three point masses are placed in the xy-plane as follows: m1=5kg at (-2,-3),m2=3kg at (2,3) , m3=2kg at (-3,-2).Find the center of mass of the system.	TLO4	L3	1.1.1
21.	Let R be the region bounded above by the graph of the function $f(x) = \sqrt{x}$ and below by the x-axis over the interval [0,4]. Find the center of mass of the region. 	TLO4	L3	1.1.1
22.	Find the center of mass of a semicircular plate of radius r.	TLO4	L3	1.1.1