Convolutional Neural Networks (CNNs) - Detailed Explanation with Examples

1. Introduction to Convolutional Neural Networks (CNNs)

Convolutional Neural Networks (CNNs) are a specialized type of **Deep Neural Networks** designed for processing **grid-like data** such as images and videos. CNNs are widely used in **computer vision** tasks like **image classification, object detection, and facial recognition**.

Why Use CNNs Instead of Fully Connected Networks?

Traditional **fully connected neural networks** (MLPs) have too many parameters when dealing with high-dimensional inputs like images. CNNs reduce the number of parameters using **convolutional layers**, making them more **efficient and effective for image processing**.

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2. Architecture of CNNs

A typical CNN consists of the following key layers:

1. **Convolutional Layer**
2. **Activation Function (ReLU)**
3. **Pooling Layer (Max Pooling or Average Pooling)**
4. **Fully Connected Layer (Dense Layer)**
5. **Softmax Layer (for classification tasks)**

Overview of CNN Layers:

Input Image → Convolution → Activation (ReLU) → Pooling → Fully Connected Layer → Output

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3. CNN Layers Explained in Detail

3.1 Convolutional Layer

- **Purpose:** Extracts important **features** from the input image.
- **How it Works:**
    - A small **filter (kernel)** slides over the input image.
    - Each filter extracts specific features like **edges, corners, and textures**.
    - The output of this process is called a **feature map**.

Example of a 3×3 Filter Convolution Operation:

Input Image:

[[3, 1, 2, 4],

[0, 1, 3, 2],

[1, 2, 0, 1],

[2, 3, 1, 0]]

3×3 Filter:

[[1, 0, -1],

[1, 0, -1],

[1, 0, -1]]

Feature Map (After Applying Filter):

[[5, 4],

[2, -1]]

- Filters detect patterns such as **horizontal edges, vertical edges, textures**, etc.

Mathematical Representation

Each pixel value in the output feature map is computed as:

FeatureMap(i,j)=∑(Image×Kernel)+BiasFeatureMap(i, j) = \sum (Image \times Kernel) + BiasFeatureMap(i,j)=∑(Image×Kernel)+Bias

Where:

- **Image** = Input image matrix
- **Kernel** = Convolution filter
- **Bias** = Learnable parameter
- ∑ = Summation over the kernel window

3.2 Activation Function (ReLU)

- **Purpose:** Introduces **non-linearity** into the model.
- **Formula:** f(x)=max(0,x)f(x) = max(0, x)f(x)=max(0,x)

- **Effect:** Converts negative values to zero, keeping only significant positive activations.

Example:

Feature Map (Before ReLU):

[[-2, 4],

[3, -1]]

Feature Map (After ReLU):

[[0, 4],

[3, 0]]

3.3 Pooling Layer

- **Purpose:** Reduces the **spatial dimensions** of the feature map while preserving important information.
- **Types:**
    - **Max Pooling:** Keeps the maximum value in a region.
    - **Average Pooling:** Computes the average value in a region.

Example of 2×2 Max Pooling:

Feature Map:

[[1, 3, 2, 4],

[5, 6, 1, 2],

[2, 4, 3, 1],

[1, 5, 2, 6]]

After 2×2 Max Pooling:

[[6, 4],

[5, 6]]

Pooling helps to:

- Reduce **computational complexity**.
- Make the model **invariant to small translations**.

3.4 Fully Connected Layer (Dense Layer)

- **Purpose:** The extracted features are **flattened** and passed to a fully connected neural network.
- **Example:** If the previous layer outputs a **7×7×64 feature map**, it is reshaped into a **1D vector** and passed to a dense layer.

3.5 Output Layer (Softmax for Classification)

- **Purpose:** Converts the final output into **class probabilities**.
- **Formula:** P(yi)=ezi∑jezjP(y_i) = \frac{e^{z_i}}{\sum_{j} e^{z_j}}P(yi)=∑jezjezi
- **Example:** If we classify images into three categories: **Dog, Cat, Horse**, the output might be:

Dog: 0.7

Cat: 0.2

Horse: 0.1

The highest probability (**0.7 for Dog**) is chosen as the final class.

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4. Example: CNN for Image Classification

Let's say we want to classify an image as either **"Dog" or "Cat"**.

Step 1: Input Image

- Size: **128×128×3** (RGB Image)

Step 2: CNN Layers

1. **Convolutional Layer:**
    - 32 filters (3×3) → Feature Map (128×128×32)
2. **ReLU Activation**
3. **Max Pooling:**
    - Reduces size to (64×64×32)
4. **Convolutional Layer:**
    - 64 filters (3×3) → Feature Map (64×64×64)
5. **ReLU Activation**

6. **Max Pooling:**
    - Reduces size to (32×32×64)
7. **Flatten Layer:** Converts (32×32×64) to 1D vector.
8. **Fully Connected Layer:**
    - 128 neurons
9. **Output Layer (Softmax):**
    - Outputs **"Dog" or "Cat"**.

Step 3: Prediction

- Model outputs a probability for each class.
- The class with the highest probability is chosen.

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5. Advantages of CNNs

✅ **Automatic Feature Extraction** – No need for manual feature engineering.
✅ **Parameter Efficiency** – Uses shared weights, reducing parameters.
✅ **Handles Large Images Well** – Works effectively on high-dimensional data.
✅ **Great for Transfer Learning** – Pre-trained CNNs (e.g., ResNet, VGG) can be used.

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6. Applications of CNNs

1. Image Classification

- CNNs classify images into categories like **cats, dogs, cars, planes** (e.g., **ImageNet** dataset).

2. Object Detection

- Used in **autonomous vehicles** to detect **pedestrians, traffic signs**.

3. Facial Recognition

- Used in **Face ID (Apple), security systems**.

4. Medical Diagnosis

- Detects diseases in **X-ray, MRI, CT scans**.

5. Natural Language Processing (NLP)

- CNNs extract features from text data for **sentiment analysis, chatbots**.

Fully Connected Neural Networks (FCNNs) - Explanation

A **Fully Connected Neural Network (FCNN)** is a type of **Artificial Neural Network (ANN)** where **each neuron in one layer is connected to every neuron in the next layer**. These networks are also called **Dense Neural Networks (DNNs)** because all connections are present between layers.

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1. Structure of a Fully Connected Neural Network

A fully connected neural network consists of three main types of layers:

1.1 Input Layer

- The first layer where data enters the network.
- Example: If an image has **28×28 pixels (grayscale)**, it has **784 input neurons**.

1.2 Hidden Layers

- Layers between the input and output layers.
- Each neuron in a hidden layer applies a mathematical transformation using **weights, biases, and an activation function**.
- More hidden layers create a **Deep Neural Network (DNN)**.

1.3 Output Layer

- The final layer that gives the network's prediction.
- Example:
    - **Binary classification:** 1 neuron with **sigmoid activation** (output: 0 or 1).
    - **Multi-class classification:** n neurons with **softmax activation** (output: class probabilities).

Example of a 3-Layer Fully Connected Neural Network

Input Layer (784 neurons) → Hidden Layer 1 (128 neurons) → Hidden Layer 2 (64 neurons) → Output Layer (10 neurons for digit classification)

Each neuron in one layer is connected to **all** neurons in the next layer.

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2. Mathematical Representation of FCNN

Each neuron computes:

$$y=f(W\cdot x+b)$$

Where:

- x= Input features
- W= Weights (learnable parameters)
- b= Bias term
- f = Activation function
- y = Output of the neuron

Example Calculation

If:

$x=[0.5,0.3,0.7]$

Then:

$y=(0.5\times0.2)+(0.3\times0.8)+(0.7\times-0.5)+0.1=0.$

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3. Activation Functions in FCNN

Activation functions introduce **non-linearity**, helping networks learn complex patterns.

- **ReLU (Rectified Linear Unit)**: $f(x)=\max(0,x)$(Common in hidden layers)
- **Sigmoid**: $f(x)=11+e-x$ (For binary classification)
- **Softmax**: Converts outputs into probabilities (For multi-class classification)

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4. Advantages of Fully Connected Neural Networks

✅ **Simple and Versatile** – Can handle a variety of problems.
✅ **Powerful Function Approximation** – Can model any function given enough layers and neurons.
✅ **Works with Structured Data** – Well-suited for classification and regression tasks.

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5. Limitations of FCNNs

❌ **High Computational Cost** – Too many parameters lead to **overfitting** and slow training.
❌ **Not Efficient for Image Data** – CNNs perform better for images due to local feature extraction.
❌ **Lack of Spatial Awareness** – Doesn't preserve spatial relationships in data.

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6. When to Use FCNNs?

- Tabular data (structured data in databases).
- Text classification (when combined with word embeddings).
- Small-scale image datasets (though CNNs are better).

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7. Difference Between FCNN and CNN

Feature	Fully Connected Neural Network (FCNN)	Convolutional Neural Network (CNN)
Connections	All neurons are connected	Uses local connections
Parameters	High number of weights	Fewer parameters (shared filters)
Best for	Structured/tabular data	Image/video processing
Performance	Prone to overfitting	Generalizes better

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8. Example: Building a Fully Connected Neural Network in Python (TensorFlow/Keras)

```
import tensorflow as tf
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense
# Define model
model = Sequential([
    Dense(128, activation='relu', input_shape=(784,)),  # Input layer + Hidden layer
    Dense(64, activation='relu'),  # Second hidden layer
    Dense(10, activation='softmax')  # Output layer (10 classes)
])
# Compile model
model.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])

# Summary
model.summary()
```

Fully Connected Neural Networks (MLPs) – Explanation

A **Fully Connected Neural Network (FCNN)**, also known as a **Multilayer Perceptron (MLP)**, is a fundamental type of **Artificial Neural Network (ANN)** where **each neuron in one layer is connected to every neuron in the next layer**. These networks are widely used in **classification, regression, and feature extraction** tasks.

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1. Structure of an MLP (Fully Connected Network)

An MLP consists of three types of layers:

1.1 Input Layer

- This is the first layer where data enters the network.
- Each neuron represents a feature of the input data.
- Example: An image with **28×28 pixels (grayscale)** would have **784 input neurons**.

1.2 Hidden Layers

- Layers between the input and output layers.
- Each neuron applies a transformation using **weights, biases, and an activation function**.
- The more hidden layers, the deeper the network.

1.3 Output Layer

- The final layer that produces predictions.
- Example:
    - **Binary classification:** 1 neuron with **sigmoid activation** (output: 0 or 1).
    - **Multi-class classification:** n neurons with **softmax activation** (output: class probabilities).

Example of a 3-Layer MLP

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Input Layer (784 neurons) → Hidden Layer 1 (128 neurons) → Hidden Layer 2 (64 neurons) → Output Layer (10 neurons)

Each neuron in one layer is connected to **all** neurons in the next layer.

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2. Working of MLP

Each neuron computes:

$y=f(W\cdot x+b)$

Where:

- x= Input features
- W = Weights (learnable parameters)
- b = Bias term
- f = Activation function
- y = Output of the neuron

Example Calculation

If:

$x=[0.5,0.3,0.7]$

Then:

$y=(0.5\times0.2)+(0.3\times0.8)+(0.7\times-0.5)+0.1=0.$

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3. Activation Functions in MLP

Activation functions introduce **non-linearity**, allowing MLPs to learn complex patterns.

- **ReLU (Rectified Linear Unit)**: $f(x)=\max(0,x)$ (Common in hidden layers).
- **Sigmoid**: $f(x)=11+e-x$ (Used in binary classification).
- **Softmax**: Converts outputs into probabilities (Used in multi-class classification).

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4. Advantages of MLPs

✅ **Simple and Versatile** – Can handle a variety of problems.
✅ **Powerful Function Approximation** – Can model any function given enough layers and neurons.
✅ **Works with Structured Data** – Well-suited for classification and regression tasks.

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- Small-scale image datasets (though CNNs are better).

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7. Difference Between MLP and CNN

Feature	MLP (Fully Connected Network)	CNN (Convolutional Neural Network)
Connections	All neurons are connected	Uses local connections
Parameters	High number of weights	Fewer parameters (shared filters)
Best for	Structured/tabular data	Image/video processing
Performance	Prone to overfitting	Generalizes better

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8. Example: Building an MLP in Python (TensorFlow/Keras)

```
import tensorflow as tf
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense
# Define model
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    Dense(128, activation='relu', input_shape=(784,)),  # Input layer + Hidden layer
    Dense(64, activation='relu'),  # Second hidden layer
    Dense(10, activation='softmax')  # Output layer (10 classes)
])
```

```
# Compile model
model.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])

# Summary
model.summary()
```

Difference Between MLP and CNN with Examples

Feature	MLP (Fully Connected Network)	CNN (Convolutional Neural Network)
Connections	All neurons in one layer are connected to every neuron in the next layer.	Uses local connections (each neuron connects only to a small region of the previous layer).
Parameters	High number of trainable weights because every neuron is connected to all neurons in the next layer.	Fewer parameters due to weight sharing (filters are applied across the image, reducing total weights).
Best for	Works well with structured/tabular data like numerical datasets (e.g., financial data, customer analytics).	Best for image, video, and spatial data where patterns and local features are important.
Performance	Prone to overfitting due to too many parameters, making it less efficient for large datasets.	Generalizes better by detecting spatial hierarchies and reducing overfitting.

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1. Connections (Dense vs. Local)

MLP (Fully Connected Network)

- Every neuron in one layer is connected to **all** neurons in the next layer.
- No spatial relationships are preserved.

🔹 **Example: Handwritten Digit Recognition (MNIST)**

- An image of 28×28 pixels is **flattened** into 784 input neurons.
- Each neuron has a weight connecting it to **every** neuron in the next layer.

🔹 **Why is this a problem?**

- The spatial structure of the image (rows and columns) is lost.

- Leads to excessive computation and inefficient learning.

CNN (Convolutional Neural Network)

- Uses **local connections** by applying filters (kernels) to small regions of the input.
- Spatial features (edges, textures, shapes) are **preserved**.

🔹 **Example: Handwritten Digit Recognition (MNIST)**

- Instead of flattening the image, **small filters (e.g., 3×3, 5×5)** scan portions of the image.
- Detects features like **edges, curves, and digits**.

✅ **Advantage:** CNN captures meaningful patterns (e.g., "0" has round edges, "7" has sharp edges) while reducing unnecessary connections.

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2. Parameters (High vs. Low)

MLP (Fully Connected Network)

- If an MLP has:
    - **Input layer**: 784 neurons (28×28 pixels)
    - **First hidden layer**: 128 neurons
    - **Second hidden layer**: 64 neurons
    - **Output layer**: 10 neurons (for 10-digit classification)

🔹 **Total weights (parameters):**

(784×128)+(128×64)+(64×10)=100,352(784 \times 128) + (128 \times 64) + (64 \times 10) = 100,352(784×128)+(128×64)+(64×10)=100,352

🔹 **Problem:** Too many trainable parameters → **overfitting & slow training**.

CNN (Convolutional Neural Network)

- Instead of connecting every pixel to every neuron, CNNs use small **filters (kernels)**.
- Example: A **3×3 filter** slides over the image detecting patterns.

🔹 **Advantage:**

- CNN uses **shared weights**, meaning fewer parameters.
- Extracts useful information with far fewer parameters than an MLP.

✅ **Example:**

- A single **3×3 filter** in CNN has **only 9 weights**, but can scan an entire image.

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3. Best for (Tabular Data vs. Image Processing)

MLP: Best for Structured Data

- 🔹 **Example 1: Predicting House Prices**
  - Features: **Square footage, number of bedrooms, location, price history**.
  - MLP learns from structured numerical inputs.
- 🔹 **Example 2: Fraud Detection in Banking**
  - Features: **Transaction amount, location, account history, frequency of transactions**.
  - A fully connected network works well because relationships between numerical values matter.

CNN: Best for Image and Spatial Data

- 🔹 **Example 1: Image Classification (Cats vs. Dogs)**
  - The model needs to identify features like **ears, tails, whiskers, and fur patterns**.
  - CNN's **filters** learn patterns efficiently.
- 🔹 **Example 2: Self-Driving Cars**
  - CNNs detect objects like **traffic signs, pedestrians, and vehicles** from camera feeds.
  - **Preserving spatial features is crucial** for making correct driving decisions.

✅ **Advantage:** CNN understands **visual structure** better than MLP.

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4. Performance (Overfitting vs. Generalization)

MLP (Fully Connected Network)

- **Prone to overfitting** because:
  - It memorizes data instead of generalizing.
  - Too many parameters → Needs **more data** to avoid overfitting.
  - Poor performance on new/unseen data.
- 🔹 **Example: Overfitting in MLP**
  - If an MLP is trained on 1000 handwritten digit samples, it may memorize noise rather than actual features.

- When tested on **new** images, its accuracy drops.

CNN (Convolutional Neural Network)

- **Better generalization** because:
    - Uses **shared filters**, reducing the risk of memorization.
    - Captures important patterns **without learning noise**.

🔹 **Example: CNN Generalization**

- If a CNN is trained on 1000 handwritten digit samples, it learns **shapes and edges** (e.g., how "8" has loops).
- When tested on **new** images, it still identifies digits correctly.

✅ **Advantage:** CNN generalizes better on new data because it learns meaningful patterns, **not just memorized examples**.

Convolutional Neural Networks (CNNs) and the Role of Filters (Kernels)

CNNs use **small filters (kernels)** to extract important features from images while preserving spatial relationships. This approach makes CNNs highly effective for image recognition, object detection, and other computer vision tasks.

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1. What Are Filters (Kernels)?

A **filter (kernel)** is a small matrix that slides over an image and performs a mathematical operation called **convolution**. The filter extracts meaningful features like **edges, textures, and patterns** at different layers.

Key Properties of Filters:

- **Size:** Usually small (e.g., 3×3, 5×5).
- **Shared weights:** The same filter is applied across the entire image.
- **Detects patterns:** Captures edges, corners, and textures.

Example: 3×3 Edge Detection Filter

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

This filter detects edges by highlighting regions with high contrast.

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2. How Do Filters Work in CNNs? (Convolution Operation)

Step 1: Apply Filter on a Small Patch

- Each filter **slides (strides)** over the input image.
- At each position, element-wise multiplication is performed between the filter and the image patch.
- The results are summed to produce a **single pixel** in the output.

Step 2: Move the Filter Across the Image

- The filter moves **left to right**, **top to bottom**.
- Each step extracts a feature.

Example: Applying a 3×3 Filter to a 5×5 Image

Original Image (5×5)

$$\begin{bmatrix} 2 & 3 & 4 & 1 & 2 \\ 1 & 5 & 7 & 3 & 1 \\ 3 & 2 & 8 & 4 & 5 \\ 6 & 1 & 2 & 7 & 3 \\ 4 & 3 & 6 & 5 & 2 \end{bmatrix}$$

3×3 Filter (Edge Detector)

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

Convolution Process

1. Place the 3×3 filter on the top-left corner of the image.
2. Multiply each element in the filter with the corresponding element in the image.
3. Sum the results.
4. Store the result in the output feature map.
5. Move the filter by **one step (stride)** and repeat.

This process produces a **smaller feature map** that highlights edges.

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3. Why Use Small Filters?

CNNs use small filters like **3×3 or 5×5** instead of fully connected layers for several reasons:

1. Capturing Local Features

- Small filters detect **basic features** like edges, corners, and textures.
- Deeper layers combine these features to recognize **complex patterns** (e.g., eyes, noses, objects).

2. Reducing Parameters

- A fully connected layer in an image of **256×256 pixels** would need **millions of parameters**.
- A **3×3 filter** has only **9 weights** but is applied across the image, drastically reducing the number of parameters.

3. Translational Invariance

- Since the **same filter** is used across different regions, CNNs learn **general features** that work for different parts of the image.

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4. Multiple Filters in CNNs

CNNs use **multiple filters** in each layer to extract different features:

Example:

- **Edge detection filter**: Captures outlines.
- **Blur filter**: Smooths the image.
- **Texture filter**: Identifies patterns.
- **Shape filter**: Recognizes curves or lines.

Each filter creates a **separate feature map**, which is stacked together to create a **rich representation** of the input image.

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5. Pooling Layer After Filters

After convolution, CNNs apply **pooling layers** to:

- Reduce the size of feature maps.
- Extract the most important information.

Types of Pooling:

1. **Max Pooling (Most Common)**
    - Takes the **maximum value** in a region.
    - Example: [1342]\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}[1432] → **Max = 4**
2. **Average Pooling**
    - Takes the **average value** in a region.

Max pooling ensures CNNs focus on the **strongest features**.

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6. Deep CNNs: Stacking Multiple Filters

Modern CNN architectures stack **multiple layers of filters**:

- **Layer 1 (3×3 filters)** → Detects edges.
- **Layer 2 (3×3 filters)** → Detects corners.
- **Layer 3 (3×3 filters)** → Detects textures.
- **Layer 4 (Fully Connected Layer)** → Classifies the image.

Each deeper layer learns more **abstract** and **high-level** features.

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7. Example: CNN for Digit Recognition (MNIST Dataset)

```
import tensorflow as tf
from tensorflow import keras
from tensorflow.keras import layers
# Load MNIST dataset
(x_train, y_train), (x_test, y_test) = keras.datasets.mnist.load_data()
# Normalize data
x_train, x_test = x_train / 255.0, x_test / 255.0
# CNN Model
model = keras.Sequential([
    layers.Conv2D(32, (3,3), activation='relu', input_shape=(28, 28, 1)),  # 32 filters of 3x3
    layers.MaxPooling2D((2,2)),  # Reduce size
```

```
    layers.Conv2D(64, (3,3), activation='relu'),  # 64 filters of 3x3
    layers.MaxPooling2D((2,2)),
    layers.Flatten(),
    layers.Dense(128, activation='relu'),
    layers.Dense(10, activation='softmax')  # 10 classes (digits 0-9)
])

# Compile model
model.compile(optimizer='adam', loss='sparse_categorical_crossentropy', metrics=['accuracy'])

# Train model
model.fit(x_train, y_train, epochs=5, validation_data=(x_test, y_test))
```

Explanation:

1. **First convolutional layer (3×3 filters, 32 filters)**
    - Detects edges and simple patterns.
2. **Second convolutional layer (3×3 filters, 64 filters)**
    - Detects textures and shapes.
3. **Max pooling layers**
    - Reduces the size of feature maps.
4. **Fully connected layer**
    - Makes predictions.

This CNN learns to classify digits (0-9) efficiently.
