Quantum Computing – Notes Ver 1.1

Prepared By: Vijayananda Mohire

Sources: Various open courses, MOOC trainings and self study; no intention for any copyright infringements

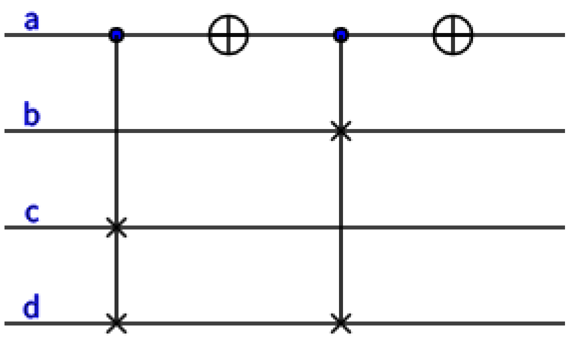
Question 1

Design a reversible circuit, using NOT, CNOT, Toffoli, and Fredkin gates, which acts on the four inputs a,b,c,d, to perform the operation $\mathrm{swap}_{243}(a,b,c,d)$ which swaps b and d if $a=0$, and swaps c and d if $a=1$. Bit a should be left unchanged

Answer 1

High level function with the circuit

```
fredkin(a,c,d)
not(a)
fredkin(a,b,d)
not(a)
```



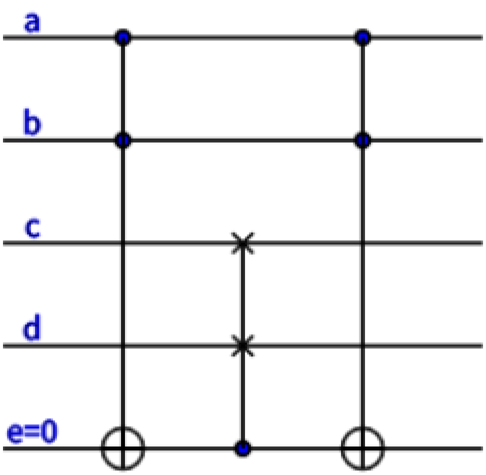
Question 2

Design a reversible circuit, using NOT, CNOT, Toffoli, and Fredkin gates, which acts on the four inputs a,b,c,d, to swap c and d only when both $a=1$ and $b=1$. You may use a fifth bit e, given as initialized to $e=0$, in your circuit; this bit must also end as $e=0$. C

Answer 2

High level function with the circuit

```
toffoli(a,b,e)
fredkin(e,c,d)
toffoli(a,b,e)
```



Question 3

Sample RandomNumber using Q#

Answer 3

```
open Microsoft.Quantum.Arrays;
open Microsoft.Quantum.Measurement;

operation SampleRandomNumber(nQubits : Int) : Result[] {
    // We prepare a register of qubits in a uniform
    // superposition state, such that when we measure,
    // all bitstrings occur with equal probability.
    use register = Qubit[nQubits] {
        // Set qubits in superposition.
        ApplyToEachA(H, register);

        // Measure all qubits and return.
        return ForEach(MResetZ, register);
    }
}
```

Question 4

Run a basic quantum circuit expressed using the [Qiskit library]() to an IonQ target via the Azure Quantum service.

Answer 4

First, import the required packages for this sample:

```
from qiskit import QuantumCircuit
from qiskit.visualization import plot_histogram
from qiskit.tools.monitor import job_monitor
from azure.quantum.qiskit import AzureQuantumProvider

#Connect to backend Azure quantum service, using below function
from azure.quantum.qiskit import AzureQuantumProvider

provider = AzureQuantumProvider ( resource_id = " ", location = " " )

# Create a Quantum Circuit acting on the q register
circuit = QuantumCircuit(3, 3)
circuit.name = "Qiskit Sample - 3-qubit GHZ circuit"
circuit.h(0)
circuit.cx(0, 1)
circuit.cx(1, 2)
circuit.measure([0,1,2], [0, 1, 2])
# Print out the circuit
circuit.draw()
```

```
     ┌───┐          ┌─┐
q_0: ┤ H ├──■───────┤M├──────
     └───┘┌─┴─┐     └╥┘┌─┐
q_1: ─────┤ X ├──■───╫─┤M├───
          └───┘┌─┴─┐ ║ └╥┘┌─┐
q_2: ──────────┤ X ├─╫──╫─┤M├
               └───┘ ║  ║ └╥┘
c: 3/════════════════╩══╩══╩═
                     0  1  2
```

```
#Create a Backend object to connect to the IonQ Simulator back-end:
simulator_backend = provider.get_backend("ionq.simulator")

job = simulator_backend.run(circuit, shots=100)
job_id = job.id()
print("Job id", job_id)

#Create a job monitor object
job_monitor(job)

#To wait until the job is completed and return the results, run:
result = job.result()
```

```
qiskit.result.result.Result
```

```
print(result)

connect to real hardware (Quantum Processing Unit or QPU)
qpu_backend = provider.get_backend("ionq.qpu")

# Submit the circuit to run on Azure Quantum
qpu_job = qpu_backend.run(circuit, shots=1024)
job_id = qpu_job.id()
print("Job id", job_id)

# Monitor job progress and wait until complete:
job_monitor(qpu_job)

# Get the job results (this method also waits for the Job to complete):
result = qpu_job.result()
print(result)
counts = {format(n, "03b"): 0 for n in range(8)}
counts.update(result.get_counts(circuit))
print(counts)
plot_histogram(counts)
```

Question 5

Develop Google AI sample Cirq circuit

Answer 5

```
import cirq
qubits = [cirq.GridQubit(x, y) for x in range(3) for y in range(3)]
print(qubits[0])

# This is an Pauli X gate. It is an object instance.
x_gate = cirq.X
# Applying it to the qubit at location (0, 0) (defined above)
# turns it into an operation.
x_op = x_gate(qubits[0])
print(x_op)

cz = cirq.CZ(qubits[0], qubits[1])
x = cirq.X(qubits[2])
moment = cirq.Moment([x, cz])
x2 = cirq.X(qubits[2])
cz12 = cirq.CZ(qubits[1], qubits[2])
moment0 = cirq.Moment([cz01, x2])
```

```
moment1 = cirq.Moment([cz12])
circuit = cirq.Circuit((moment0, moment1))
print(circuit)
```

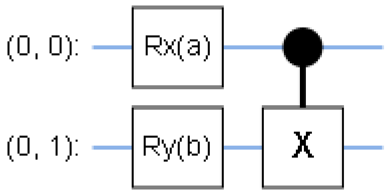
```
(0, 0): ───@───────
           │
(0, 1): ───@───@───
               │
(0, 2): ───X───@───
```

Question 6

Design a simple Tensorflow based quantum Colab sample

Answer 6

```
!pip install tensorflow==2.4.1
!pip install tensorflow-quantum

import tensorflow as tf
import tensorflow_quantum as tfq
import cirq
import sympy
import numpy as np
# visualization tools
%matplotlib inline
import matplotlib.pyplot as plt
from cirq.contrib.svg import SVGCircuit
a, b = sympy.symbols('a b')
# Create two qubits
q0, q1 = cirq.GridQubit.rect(1, 2)

# Create a circuit on these qubits using the parameters you created above.
circuit = cirq.Circuit(
    cirq.rx(a).on(q0),
    cirq.ry(b).on(q1), cirq.CNOT(control=q0, target=q1))
SVGCircuit(circuit)
```



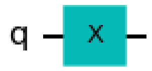

```
# Calculate a state vector with a=0.5 and b=-0.5.
resolver = cirq.ParamResolver({a: 0.5, b: -0.5})
output_state_vector = cirq.Simulator().simulate(circuit, resolver).final_state_vector
output_state_vector
```

Question 7

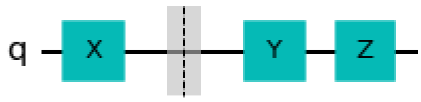
Design a simple qubit based quantum circuit using IBM Qiskit

Answer 7

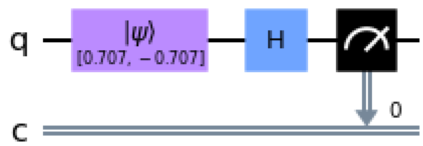
```
import numpy as np
# Importing standard Qiskit libraries
from qiskit import QuantumCircuit, transpile, Aer, IBMQ, assemble
from qiskit.tools.jupyter import *
from qiskit.visualization import *
from ibm_quantum_widgets import *
from math import pi, sqrt
# Loading your IBM Quantum account(s)
provider = IBMQ.load_account()
sim = Aer.get_backend('aer_simulator')

# Let's do an X-gate on a |0> qubit
qc = QuantumCircuit(1)
qc.x(0)
qc.draw()
```




```
qc.y(0) # Do Y-gate on qubit 0
qc.z(0) # Do Z-gate on qubit 0
qc.draw()
```




```
# Create the X-measurement function:
def x_measurement(qc, qubit, cbit):
"""Measure 'qubit' in the X-basis, and store the result in 'cbit'"""
qc.h(qubit)
qc.measure(qubit, cbit)
return qc
initial_state = [1/sqrt(2), -1/sqrt(2)]
# Initialize our qubit and measure it
qc = QuantumCircuit(1,1)
qc.initialize(initial_state, 0)
x_measurement(qc, 0, 0) # measure qubit 0 to classical bit 0
qc.draw()
```



Question 8

How to find if matrix is Unitary

Answer 8

Consider a 2*2 Matrix A with different values. We take 2 examples as shown below to prove how these are valid or not for quantum representation

A = $\begin{matrix} 0 & 1 \\ i & 0 \end{matrix}$ and A^T = $\begin{matrix} 0 & i \\ 1 & 0 \end{matrix}$

Next, $A*A^T$ = $\begin{matrix} i & 0 \\ 0 & i \end{matrix}$ = i * $\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$ which is an Identity matrix I

So this matrix is **Unitary** and valid for quantum representations

Next example,
A = $\begin{matrix} 1 & -1 \\ 0 & 1 \end{matrix}$ and A^T = $\begin{matrix} 1 & 0 \\ -1 & 1 \end{matrix}$

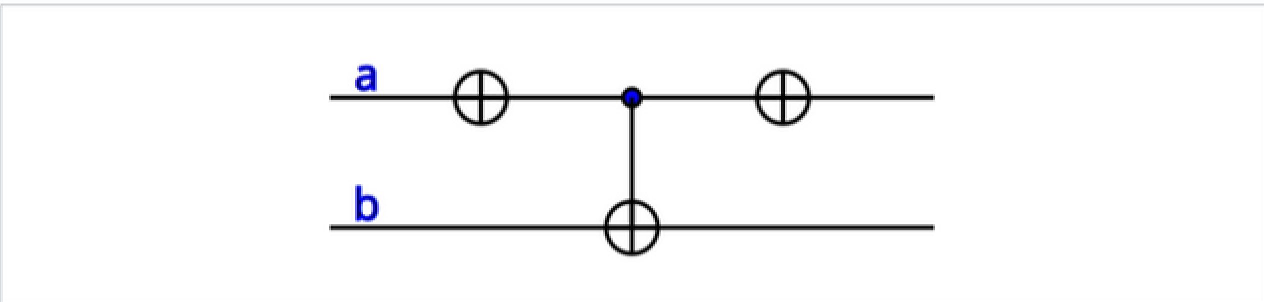
Next, $A*A^T$ = $\begin{matrix} 2 & 0 \\ -1 & 0 \end{matrix}$ = which is NOT an Identity matrix, as 2 is not correct

So this matrix is **NOT Unitary** and NOT valid for quantum representations

Question 9

Generate the Unitary matrix for the given quantum circuit

Consider the following quantum circuit *C*, composed of two NOT gates and one CNOT gate:



Answer 9

First let me get the matrices for NOT and CNOT gates

NOT = $\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$ and for CNOT $\left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}\right)$

Gate Matrices have to be multiplied. However, when matrix is generated for single qubit ,tensor product with identity is required.

So getting the I for the NOT gates

$\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$ tensor product $\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$ = $\left(\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\right)$ this is the Identity I

Now multiply these as per circuit order

I * CNOT Matrix *I

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

The multiplication can be made easier using online tool like

https://www.dcode.fr/matrix-multiplication

This is based on theory, however this needs to be done using simulator like Qiskit based Composer and get the Unitary matrix

Question 10: Derive Pauli's X gate

Answer 10: There are 3 Pauli's gates namely X, Y and Z that represent the various gate operations on the Bloch sphere.

Pauli's X gate offer a NOT type of operation and is represented by bra-ket and matrix notations. Below is an example of deriving the X gate

Please note bra is represented by < 0 | and ket by |0 >. Arranging the matrices in proper shape is the key in getting the proper results. There is also a conjugate transpose required, meaning the cols matrix is transformed to row matrix and these are then multiplied

I have used a different method to represent the state vector rows and columns; however this is not the best one. You can use KET based COLS first and BRA based ROWS, and then do the operation. Pauli X is a NOT gate, so the 0->1 and 1>0 are reflected in the matrices. Please get these things clear first

Pauli's X = NOT = $|0\rangle\langle 1| + |1\rangle\langle 0| = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ ⇒ value 1
$(0 \to 1, 1 \to 0)$

Representing Bra-ket in matrix form

$$\underbrace{|0\rangle\langle 1|}_{\mathrm{I}} + \underbrace{|1\rangle\langle 0|}_{\mathrm{II}} = \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \end{bmatrix}}_{\mathrm{I}} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \end{bmatrix}}_{\mathrm{II}}$$

Ket (cols) Bra (rows) ~~(cols)~~

These rows are conjugate transpose.

Now, multiply the 2 matrices of I

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 1\times 0 & 1\times 1 \\ 0\times 0 & 0\times 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$ ⇒ computed ~~Required~~ Value of I (a)

Ilerly, multiply the 2 matrices of II

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 0\times 1 & 0\times 0 \\ 1\times 1 & 1\times 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$ ⇒ computed val of II. (b)

Now adding I & II (I + II)

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0+0 & 1+0 \\ 0+1 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$ ⇒ This is the Value 1

Hence the above method to solve the Pauli's X is correct.

Question 11: Derive Pauli's Y gate

Answer 11: In a similar way the Pauli's X is derived, Pauli's Y is derived

Pauli y :- $\sigma_y = \underbrace{i|1\rangle\langle 0|}_{\mathrm{I}} - \underbrace{i|0\rangle\langle 1|}_{\mathrm{II}}$

Should result in $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

(I) → matrix form.

$$i|1\rangle\langle 0| = i\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \end{bmatrix} = i\begin{bmatrix} 0\times 1 & 0\times 0 \\ 1\times 1 & 1\times 0 \end{bmatrix} =$$

$$i\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \rightarrow \mathrm{I}^{st} \text{ term}$$

(II) → matrix form.

$$i|0\rangle\langle 1| = i\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \end{bmatrix} = i\begin{bmatrix} 1\times 0 & 1\times 1 \\ 0\times 0 & 0\times 1 \end{bmatrix}$$

$$= i\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

(I) minus (II)

$$i\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} - i\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} 0 & 0 \\ i & 0 \end{bmatrix} - \begin{bmatrix} 0 & i \\ 0 & 0 \end{bmatrix}$$

Adding

$$\equiv \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \Rightarrow \text{Desired result.}$$

Question 12: Derive Pauli's Z gate

Answer 11: Pauli's Z does not change any value, only flips the sign

Paulis: ✓

$X = \sigma_x = NOT = |0\rangle\langle 1| + |1\rangle\langle 0| = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$Z = \sigma_z = \text{Signflip} = |0\rangle\langle 0| - |1\rangle\langle 1| = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$Y = \sigma_y = -i|0\rangle\langle 1| + i|1\rangle\langle 0| = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

---

uv^T outer products (Tensor product) $\quad |0\rangle\langle 1| \rightarrow$ kets first (col)
bra ~~rows~~ next (row)

Inner products $\quad \langle 0|1\rangle \longrightarrow$ bras ~~rows~~ first (row).
(Dot product) o/p's scalar. $\quad$ kets next (col).
$u^T v$

Kronecker product (i/p pair of matrices, o/p → block matrix).
— Similar to outer product.

(orthonormal decomposition)

Z is an ~~orthog~~ diagonal representation, so using braket is not correct ans.

$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = |0\rangle\langle 0| - |1\rangle\langle 1|$

Question 13: Show an example of inner product

Answer 13: Inner product of 2 matrices is the dot product and results in a scalar.

Inner & outer products: (for Column vectors ONLY)

Inner

$u = \begin{bmatrix}-1\\3\end{bmatrix}$ $v = \begin{bmatrix}2\\5\end{bmatrix}$ $u^Tv = \underbrace{\begin{bmatrix}-1 & 3\end{bmatrix}\begin{bmatrix}2\\5\end{bmatrix}}_{\text{bra-ket}} = -1\cdot 2 + 3\cdot 5 = -2+15 = 13$

Scalar

outer

$uv^T = \begin{bmatrix}-1\\3\end{bmatrix}\begin{bmatrix}2 & 5\end{bmatrix} = \begin{bmatrix}-2 & -5\\6 & 15\end{bmatrix} \rightarrow$ vector matrix (Tensor)

ket-bra

Inner is bra-ket, outer is ket-bra

Ex. pauli X → Inner & outer products

Inner

say, $u = \begin{bmatrix}-1\\3\end{bmatrix}$ and

$X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

$u^TX = \begin{bmatrix}-1 & 3\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} = \begin{bmatrix}0 & -1\\3 & 0\end{bmatrix}$

Inner products are Scalar

$= 0+3-1+0 = 2$ is inner product with pauliX.

Question 14: Show an example of outer product

Answer 14: Outer product of 2 matrices is the tensor product and results in a vector matrix.

say, $u = \begin{bmatrix}-1\\3\end{bmatrix}$

Inner

$X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

$u^TX = \begin{bmatrix}-1 & 3\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} = \begin{bmatrix}0 & -1\\3 & 0\end{bmatrix}$

Inner products are Scalar

$= 0+3-1+0 = 2$ is inner product with pauliX.

outer

$u = \begin{bmatrix}-1\\3\end{bmatrix}$

$x = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} \Rightarrow$ transpose

$ux^T = \begin{bmatrix}-1\\3\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} = \begin{bmatrix}0 & -1\\3 & 0\end{bmatrix} =$ outer products are tensor product

Question 15: Show an example of outer product using Pauli X & Y with an example of Trace

Answer 15: Using Pauli's X & Y matrices

Tensor product of Pauli X & Y is (outer product).

$$X \otimes Y = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} x_{11} \cdot \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} & x_{21} \cdot \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \\ x_{12} \cdot \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} & x_{22} \cdot \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \cdot \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} & 1 \cdot \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \\ 1 \cdot \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} & 0 \cdot \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix} \checkmark$$

$\Rightarrow$ Kronecker is also same steps.

Trace of a matrix is $tr(A^T) \rightarrow$ Sum of the main diagonal entries

Ex:

$$A = \begin{bmatrix} -1 & 2 & 4 \\ 3 & 7 & 0 \\ 5 & 8 & -6 \end{bmatrix} \quad A^T = \begin{bmatrix} -1 & 3 & 5 \\ 2 & 7 & 8 \\ 4 & 0 & -6 \end{bmatrix} \quad tr(A^T) = -1 + 7 - 6 = 0$$

rows $\rightarrow$ cols for AT

Question 16: Show how Bell State is derived

Answer 16: Bell state preparation uses 3 steps:

1. State initialization
2. Use Hadamard and Identity gate for superposition and getting the Kronecker matrix
3. Use a CNOT to multiply with the Kronecker matrix

Details in the following notes below

Bell state:

CNOT $(H \otimes I)$ $(|0\rangle \otimes |0\rangle)$

$\leftarrow$ step 3 $\quad \leftarrow$ step 2 $\quad \leftarrow$ step 1

Step 1 :- $|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ → first qbit & second qbit initi to zero

→ state matrix.

Step 2:

$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ & $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Kronecker / outer product = $H \otimes I$ =

$H \otimes I = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & 1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ 1 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & -1 \begin{bmatrix} 1 & 0 \\ 0 & +1 \end{bmatrix} \end{bmatrix}$

$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$

~~Step3~~ Now multiply step 1 & 2.

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&-1&0\\0&1&0&-1\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix} = \begin{bmatrix}1+0+0+0\\0+0+0+0\\1+0-0+0\\0+0+0-0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\\0\end{bmatrix}$$

This result needs to be applied CNOT. in step3.

Step3.

$$\text{CNOT matrix} = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$$

So,

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\begin{bmatrix}1\\0\\1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1+0+0+0\\0+0+0+0\\0+0+0+0\\0+0+1+0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$$

Desired Bell state for first qubit state

Question 17: State the types of quantum states

Answer 17: Quantum qubit can have 6 possible states, 2 each for the X, Y and Z directions of the Bloch sphere ket state rep (general rep)

for first qubit state $|00\rangle$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow$ superposition state

$\Downarrow$

4 states

$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \rightarrow$ plus state. $= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \Rightarrow$ minus state

$|i\rangle = \frac{1}{\sqrt{2}}|0\rangle + i\frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \rightarrow$ i state.

$|-i\rangle = \frac{1}{\sqrt{2}}|0\rangle - i\frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle) \rightarrow$ −i state

These are total 6 Quantum states for a given qubit. $\underbrace{|0\rangle, |1\rangle}_{\text{for z axis}}, \underbrace{|+\rangle, |-\rangle}_{\text{for x axis}}, \underbrace{|i\rangle \text{ and } |-i\rangle}_{\text{for y axis.}}$

Another way to represent these are shown below, |0>, |1>,| +>, | ->,| I > and | –I >

$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \alpha\begin{pmatrix}1\\ 0\end{pmatrix} + \beta\begin{pmatrix}0\\ 1\end{pmatrix} = \alpha|0\rangle + \beta|1\rangle = |\Psi\rangle \text{ "ket"}$$

$$\langle\Psi| \text{ "bra"}$$

$$\langle\Psi| = (|\Psi\rangle)^{\dagger} = (|\Psi\rangle)^{*T}$$

$$\langle\Psi| = \begin{pmatrix}\alpha^*\\ \beta^*\end{pmatrix}^{T} = (\alpha^* \;\; \beta^*) = \alpha^*(1 \;\; 0) + \beta^*(0 \;\; 1) = \alpha^*\langle 0| + \beta^*\langle 1|$$

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad \text{"superposition state"}$$

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \quad \text{"plus state"}$$

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \quad \text{"minus state"}$$

$$|i\rangle = \frac{1}{\sqrt{2}}|0\rangle + i\frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \quad \text{"i state"}$$

$$|-i\rangle = \frac{1}{\sqrt{2}}|0\rangle - i\frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle) \quad \text{"} -i \text{ state"}$$

Image source: https://andisama.medium.com/qubit-an-intuition-1-first-baby-steps-in-exploring-the-quantum-world-16f693e456d8

Question 18: Define the notations for the different types of quantum states like plus, minus etc

Answer 18: Quantum qubit state notations are mainly represented in matrix and bra-ket forms with transformation from one notation to another as required to solve a problem .Below are matrix notations for 0,1, + and – states. These can be re-written from matrix to state, like col matrix [1 0] can be written as ket notation | 0> as per the need of the problem to be solved

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow |+\rangle$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = |-\rangle$$

Question 19: Apply an H gate on the |+> and show the results

Answer 19: First we get the matrix notation for H and |+> states, then we multiply them, details shown below

Apply a H gate on the $|+\rangle$ state.

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$ ⇒ see prev. pg

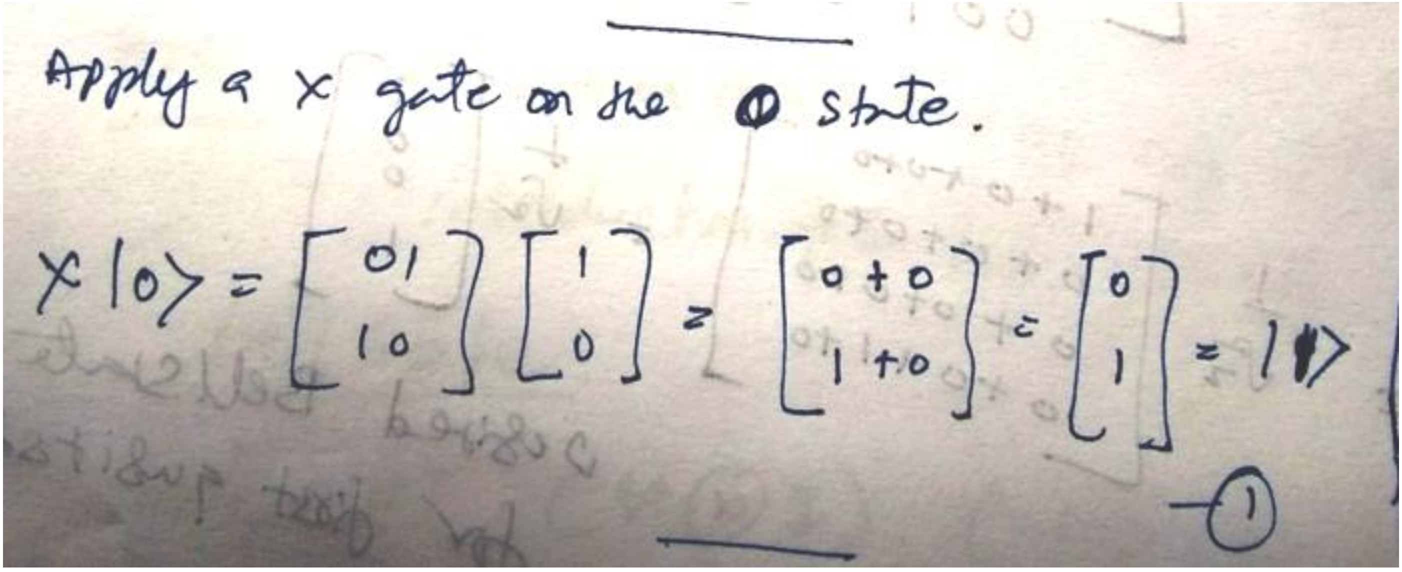
$$H|+\rangle = \frac{1}{2}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Rows of first matrix multiplied by cols of sec. matrix.

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ state matrix represents $|0\rangle$ state ket.

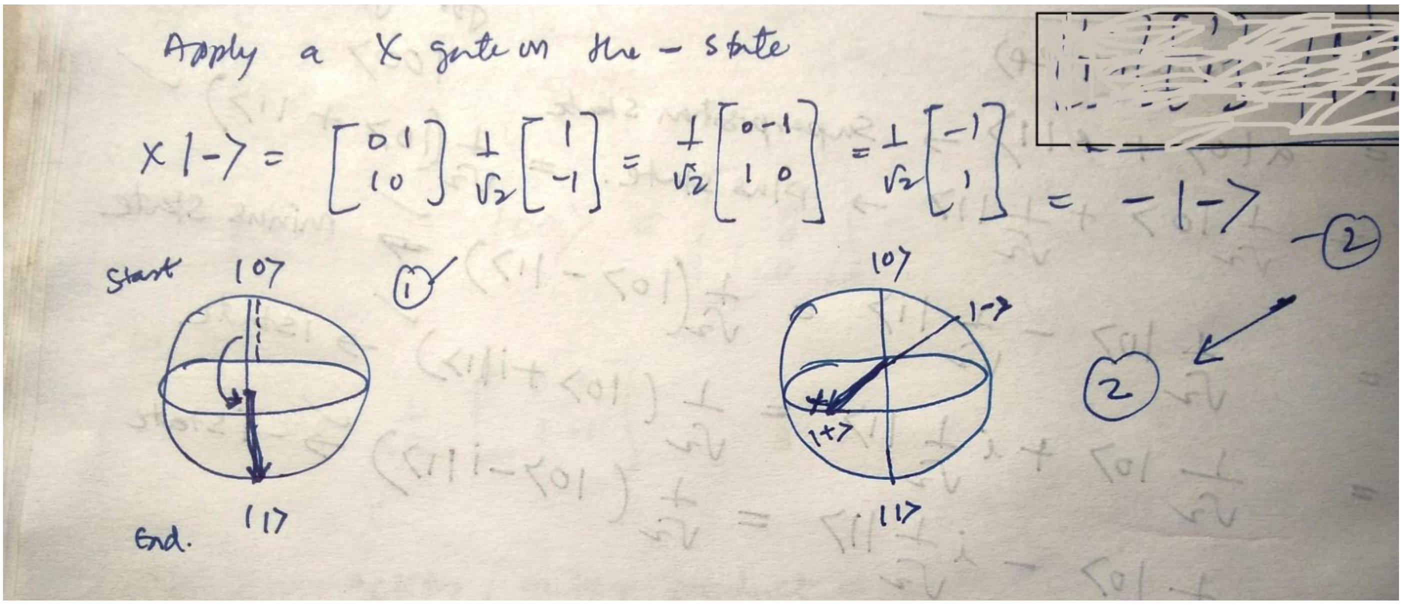
Question 20: Apply an X gate on the |0> and show the results

Answer 20: First we get the matrix notation for X and |0> states, then we multiply them, as shown below



Question 21: Apply an X gate on the |-> and show the results

Answer 21: First we get the matrix notation for X and |-> states, then we multiply them, details shown below ,results show on Bloch sphere for Question 19 and 20



Question 22: Test the below matrices for the validity of being the bitflip X gate

Answer 22: First we get the matrix notation of the X gate and test it against each given matrix that should result in the NOT operation

Which of these are valid bit flip X gate

A) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ ×

Test:

A) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+0 \\ 0+0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ → which is required to flip the $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ state ×

B) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+0 \\ 0+0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ ↗ same issue ×

C) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+0 \\ 0-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ → Does not flip ×

D) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+0 \\ 1+0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ⇒ ✓ as $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is now $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Question 23: Given H acting on |0> produces |+> & H|1> = |->, which is the correct H operator

Answer 23: First we get the matrix for H and test each given matrix that produces the required results

Given H acting on $|0\rangle$ produces $|+\rangle$ & $H|1\rangle = |-\rangle$, which is correct H operator.

A) $\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ B) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ c) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ d) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$

Test:

A) $\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0+0 \\ 1+0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ⇒ this is NOT $|+\rangle$ ⇒ $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ×

B) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1-0 \\ -1 & 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ ↗ same ×

c) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1+0 \\ 1-0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ↗ correct as this is $|+\rangle$

✓

chk for $|-\rangle$

$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0+1 \\ 0-1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ ⇒ correct as this is $|-\rangle$

References:

1. MIT OpenCourseWare , https://ocw.mit.edu/
2. IBM Quantum Lab, https://quantum-computing.ibm.com/lab
3. Azure Quantum, https://azure.microsoft.com/en-in/services/quantum/
4. QuTech Academy, https://www.qutube.nl/
5. Andi Sama Blog, https://andisama.medium.com/qubit-an-intuition-1-first-baby-steps-in-exploring-the-quantum-world-16f693e456d8

Disclaimer: I have no intention for any copyright infringement, nor I promise that the results are true and right. Please use your caution to self-check the results against the quantum postulates. I am reachable at vijaymohire@gmail.com for any clarifications