

Prepared By: Vijayananda Mohire

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Previous questions at: <https://www.slideshare.net/Vijaymohire/quantum-computing-notes-ver-12>

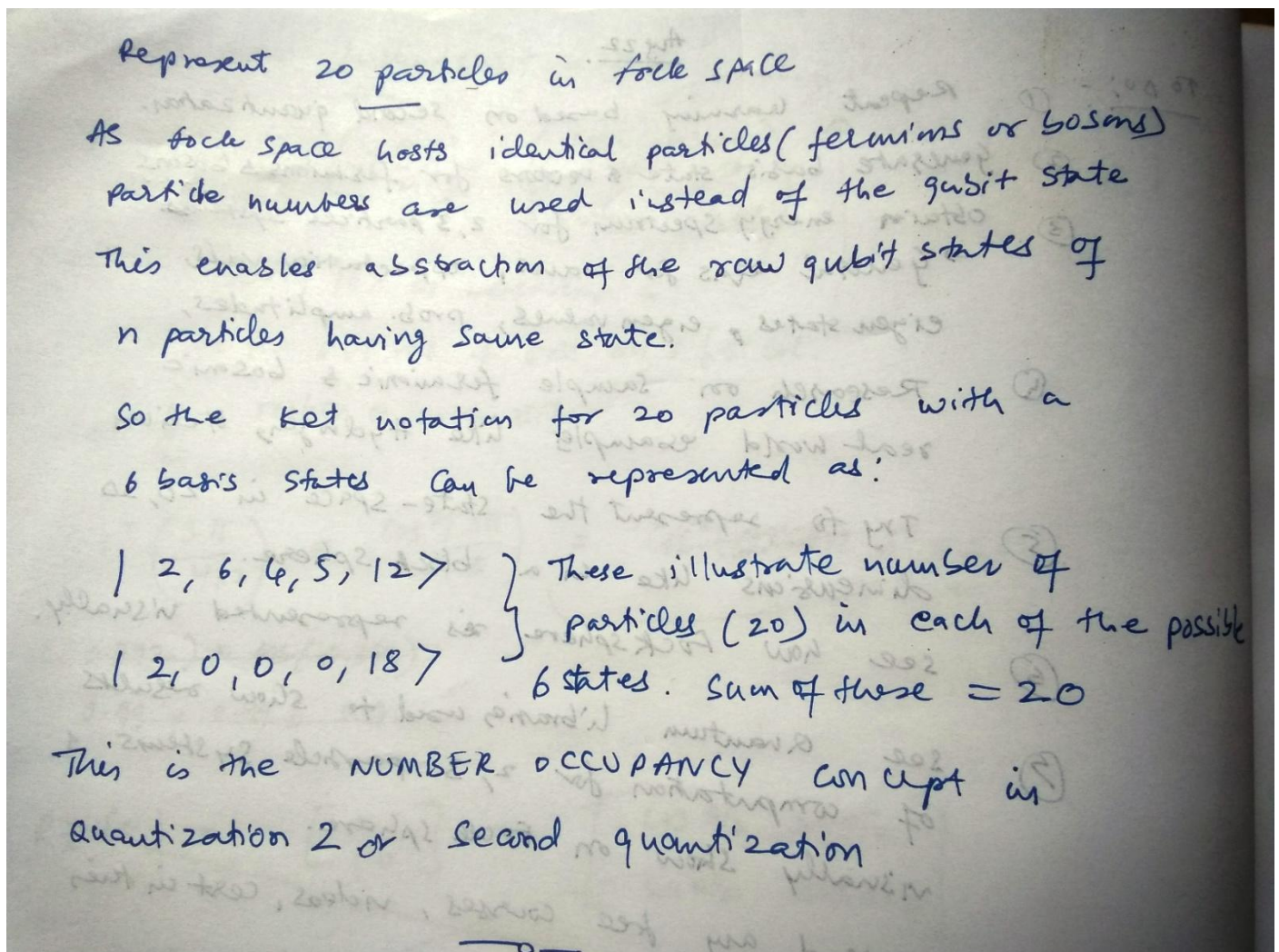
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### Question 30

Describe how you would represent a large set of particles in a Fock space rather than the Hilbert space

### Answer 30

Fock space is a newer way (Second Quantization) to represent multi-particles in an easier way unlike in the Hilbert space. Below is the broad way in simple terms



### Question 31

Describe in simple words how Fock space uses Hilbert space

### Answer 31

Fock space offers newer way of abstracting the state-space representations as previously done in the First Quantization. This helps in easier and shorter way in showing the quantum states.

Hilbert space  $H$  & Fock space  $F$

$H$  space is good for state-space representation of single particles in a Bloch sphere.

The  $F$  space is  $F(H)$  is another Hilbert space and used to represent state of an  $N$  number of identical particles.

The state of the pair of particles belong to the tensor product of the state-spaces of the particles separately.

The particles are identical & subject to symmetry.

Bosons have symmetrical states under exchange

Fermions state change sign under exchange

Fock space takes the sequence of  $n$ -fold tensor products of  $H$ , imposes symmetry and adds them.

### Question 32

Describe the Fock space mathematically

### Answer 32



The Fock space is the (Hilbert) direct sum of tensor products of copies of a single-particle Hilbert space  $H$ .

$$F_v(H) = \bigoplus_{n=0}^{\infty} S_v H^{\otimes n} = C \oplus H \oplus (S_v(H \otimes H)) \oplus (S_v(H \otimes H \otimes H)) \dots$$

$C \Rightarrow$  complex scalars of states  $\psi$  with no particles

$H \Rightarrow$  states of one particle,  $S_v(H \otimes H) \dots$

A general state for  $F_v(H)$  is

$$|\psi\rangle_v = |\psi_0\rangle_v \oplus |\psi_1\rangle_v \oplus \dots = a|0\rangle \oplus \sum_i a_i |\psi_i\rangle \oplus \sum_{ij} a_{ij} |\psi_i, \psi_j\rangle_v \oplus \dots$$

Where

$|0\rangle$  is a vector of length 1 called vacuum state and  $a \in C$  is a complex coefficient

$|\psi_i\rangle \in H$  is a state in the single particle Hilbert space and

$a_i \in C$  is a complex coeff.

$|\psi_i, \psi_j\rangle_v = a_{ij} |\psi_i\rangle \otimes |\psi_j\rangle + a_{ji} |\psi_j\rangle \otimes |\psi_i\rangle \in S_v(H \otimes H)$  and

$a_{ij} = v a_{ji} \in C$  is a complex coeff.

### Question 33

Describe in general how two states are represented in Fock space and their inner product

### Answer 33

### Fock space contd...

For two general states:

$$|\psi\rangle_v = |\psi_0\rangle_v \oplus |\psi_1\rangle_v \oplus \dots = a|0\rangle \oplus \sum_i a_i |\psi_i\rangle \oplus \sum_{ij} a_{ij} |\psi_i, \psi_j\rangle_v \oplus \dots$$

and

$$|\phi\rangle_v = |\phi_0\rangle_v \oplus |\phi_1\rangle_v \oplus \dots = b|0\rangle \oplus \sum_i b_i |\phi_i\rangle \oplus \sum_{ij} b_{ij} |\phi_i, \phi_j\rangle_v \oplus \dots$$

The inner product in  $F_v(H)$  is

$$\langle \psi | \phi \rangle_v = \sum_n \langle \psi_n | \phi_n \rangle_v = a^* b + \sum_i a_i^* b_i \langle \psi_i | \phi_i \rangle +$$

$$\sum_{ijkl} a_{ij}^* b_{kl} \langle \psi_i | \phi_k \rangle \langle \psi_j | \phi_l \rangle_v + \dots$$

### Question 34

Describe the general format of the product state in Fock space and the key operators used

### Answer 34

product state in fock space is

$$|\psi\rangle_v = |\phi_1, \phi_2, \dots, \phi_n\rangle_v = |\phi_1\rangle \otimes |\phi_2\rangle \otimes \dots \otimes |\phi_n\rangle$$

$n \rightarrow$  particles, one particle in state  $\phi_1$ , another  $\phi_2$

A state that cannot be written as a convex sum of product states is in an entangled state.

$a^\dagger(\phi)$  is the creation operator

$a(\phi)$  is the annihilator operator

} serve as generator for more general operators like number of particles operator

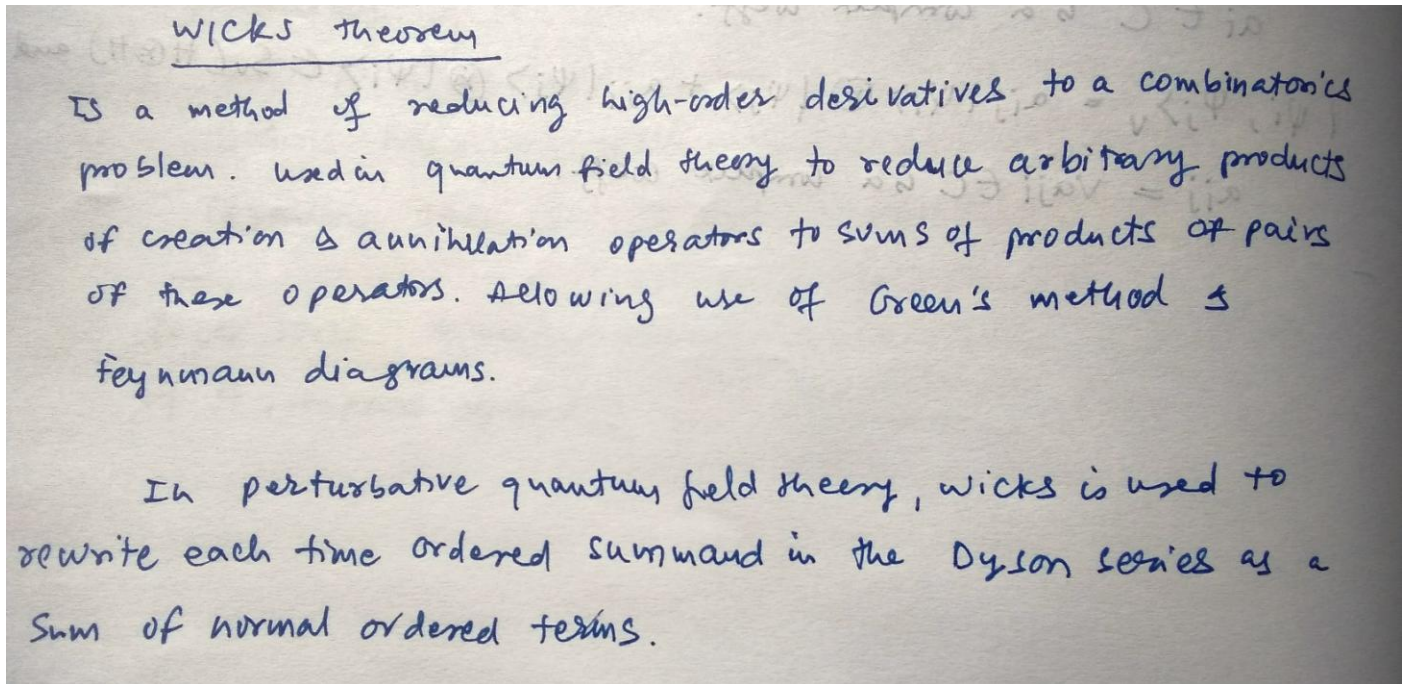
$$a^\dagger(\phi_i) a(\phi_i) \Rightarrow \text{number operator}$$



### Question 35

Define in simple words what **Wicks method** is and where it is used

### Answer 35



### References:

1. MIT OpenCourseWare , <https://ocw.mit.edu/>
2. IBM Quantum Lab, <https://quantum-computing.ibm.com/lab>
3. Azure Quantum, <https://azure.microsoft.com/en-in/services/quantum/>
4. QuTech Academy, <https://www.qutube.nl/>
5. Wikipedia and the web

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