OVERVIEW

Localized heating of a white dwarf has the potential to ignite the star. Namely, if a region of size λ_T or greater is raised to a critical temperature T_f, this would initiate runaway thermonuclear fusion and cause the white dwarf to explode in a supernovae. According to [?], $\lambda_T \sim 10^{-5}$ cm for carbon-oxygen white dwarfs at the highest densities $\rho \sim 5 \times 10^9 \frac{\text{gm}}{\text{cm}^3}$. This is then analytically scaled in [?] for varying densities. Get values from Surjeet

Consider an ultra-heavy dark matter (DM) state transit through the white dwarf. Assume that the DM interacts with white dwarf constituents (ions or electrons) in a general manner as shown in Figure ??, releasing n_i particles of species i each with kinetic energy ϵ. The cross section for this interaction is denoted as $\sigma_{i,\epsilon}$.

Of course, this released energy must be transferred to the stellar medium in order for the white dwarf to be heated. For a given particle type, each value of ϵ is characterized by a distance R_ϵ from the point of release over which it is deposited. This "range", and therefore the nominal size of the resulting hot region, is set by the various ways in which the emitted particle interacts with the stellar constituents and is able to dumps its energy. In particular, we define R_ϵ as the distance over which a particle i and any secondaries transfer $\mathcal{O}(1)$ of the initial energy ϵ to electrons or ions in the white dwarf. To demonstrate the significance of this parameter, suppose that the DM simply scattered off nuclei elastically with no particles released. In this case, R_ϵ effectively vanishes as ϵ is transferred directly to the kinetic energy of nuclei. However, in the other extreme limit, suppose the interaction released energy into neutrinos. In this case, R_ϵ is of astronomical length scales and there would be no chance of thermalizing any local region in the star.

Consider the two possibilities relevant for ignition. If $R_\epsilon > \lambda_T$, the DM must deposit a minimum energy $E_{min} \sim R_\epsilon^3 n T_f$ in order to heat up the entire region of size R_ϵ to the critical temperature T_f, where n is the number density of nuclei in the white dwarf. On the other hand, if $R_\epsilon < \lambda_T$ then the minimum energy required is independent of R and given by $E_{min} \sim \lambda_T^3 n T_f$. Setting $T_f \sim$ MeV, $\lambda_T \sim 10^{-5}$ cm, and $n \sim 10^{32}$ cm^{-3}, we find that an energy $E_{min} \sim 10^{14}$ GeV transferred to the white dwarf within a localized region smaller than λ_T will eventually trigger runaway fusion. This "explosion" energy must be compared to the total energy released during the DM transit over a distance $\min\{\lambda_T, R_\epsilon\}$. As a result, we find a lower bound on the interaction cross section $\sigma_{i,\epsilon}$ sufficient to trigger runaway fusion:

$$n_i \sigma_{\epsilon,i} \gtrsim \begin{cases} \lambda_T^2 \left(\dfrac{T_f}{\epsilon}\right) & R_\epsilon < \lambda_T \\ \lambda_T^2 \left(\frac{R_\epsilon}{\lambda_T}\right)^2 \left(\frac{T_f}{\epsilon}\right) & R_\epsilon > \lambda_T \end{cases} . \quad (1)$$

Note that in deriving this explosiveness bound, we have assumed the DM transit time is less than the corresponding diffusion time. After a time Δt the DM has traversed $v_{esc}\Delta t$, where the velocity is set by the escape velocity of a white dwarf. Therefore, this amounts the following condition:

$$\begin{array}{ll} \tau_d^{\lambda_T, T_f} \gtrsim \frac{\lambda_T}{v_{esc}} & R_\epsilon < \lambda_T \\ \tau_d^{R_\epsilon, T_f} \gtrsim \frac{R_\epsilon}{v_{esc}} & R_\epsilon > \lambda_T \end{array} , \quad (2)$$

where $\tau_d^{\lambda_T, T_f}$ is the time for a region of size λ_T and temperature T_f to diffuse $\mathcal{O}(1)$ of its heat. Within the validity of the heat equation this is simply given by $\tau_d^{\lambda_T, T_f} \sim \frac{\lambda_T^2}{\alpha}$, where α is the (temperature-dependent) diffusivity. Show condition is true for all densities. We also assume that the time to transfer energy ϵ out to its characteristic range R_ϵ is less than the diffusion time scale $\tau_d^{\lambda_T, T_f}$.

DETERMINATION OF R_ϵ

In this section, we enumerate the different ways in which standard model particles can lose energy to the white dwarf. For the purpose of depositing sufficient energy to trigger supernovae, we focus on high-energy particles $\epsilon \gtrsim 100$ MeV interacting via the strong and electromagnetic forces.

The interior of a (carbon-oxygen) white dwarf is a complex environment. Famously, the star is supported against collapse by the degeneracy pressure of a degenerate electron gas at a characteristic Fermi energy $E_F \sim n_e^{1/3} \sim \mathcal{O}(\text{MeV})$. In addition, the nuclei are at an ambient temperature $T \sim$ keV and form a strongly-coupled plasma with a plasma parameter $\Gamma \sim \frac{Ze^2}{n^{1/3}T} \gg 1$. Ions are like a solid or liquid, perhaps already crystallized at sufficiently high densities.

Electromagnetic Interactions

For charged particles, Coulomb scattering is a useful mechanism for energy transfer. Generically, an incident (spin-0) particle of mass m_i, charge e, and velocity β scattering off a target M_t of charge Ze is described by the "Rutherford" differential cross section Originally derived by Bhabha?

$$\frac{d\sigma(E', \beta)}{dE'} = \frac{2\pi\alpha^2 Z^2}{M_t \beta^2} \frac{1}{E'^2} \left(1 - \frac{\beta^2 E'}{E_{max}}\right), \quad (3)$$

where we have assumed a sufficiently fast incident particle so that interactions are governed by single collisions with energy transfer E' [?]. E_{max} denotes the maximum energy transfer possible satisfying kinematic constraints target at rest and zero relative angle between incoming incident and outgoing target momenta:

$$E_{max} = \frac{2M_t\beta^2\gamma^2}{1+2\gamma(M_t/m_i)+(M_t/m_i)^2}. \tag{4}$$

For sufficiently heavy incident particles, the differential cross section depends only on the velocity of the incident particle. Note that higher-spin particles receive additional corrections to the cross section, but for small energy transfers these corrections are negligible. It is straightforward to understand the parametric dependences of (3): there is increased likelihood to scatter for slowly moving incident particles undergoing "soft-scatters" against lighter targets. Therefore, one would expect that soft scattering dominates the energy loss and that collisions with nuclei of mass M are suppressed by a factor $\mathcal{O}\left(\frac{M}{Zm_e}\right)$ as compared to collisions with electrons. This is certainly true for incident charged particles in ordinary matter. However, both of these naive expectations turn out to be false when considering scattering off a degenerate medium.

To understand the effect of degeneracy, we first consider the energy loss from scattering a high-energy charged particles off non-degenerate targets in the white dwarf (i.e. nuclei). In this case, the stopping power due to collisions with a number density n is given by:

$$\frac{dE}{dx} = -\int dE'\left(\frac{d\sigma}{dE'}\right)nE' \tag{5}$$

$$= -\frac{2\pi nZ^2\alpha^2}{M_t\beta^2}\left(\log\left(\frac{E_{upper}}{E_{lower}}\right)-\beta^2\right). \tag{6}$$

This integration must be performed over all E' within the regime of validity for (3), fixing the lower and upper bounds of the "Coulomb Logarithm". Quantum mechanical uncertainty sets a limit to the accuracy that can be achieved in "aiming" an incident particle at a target. In terms of impact parameter b for the collision, this translates to a bound $b > \frac{1}{\min\{m_i,M_t\}\beta\gamma}$ or, in terms of energy transfer, $E' < E_q = \frac{2(\min\{m_i,M_t\})^2Z^2\alpha^2\gamma^2}{M_t}$. In addition, the expression for the differential cross section (3) no longer holds when E' becomes larger than the mass of the target. For our calculations, we take the maximum energy that an incident particle is able to transfer to be $E_{upper} = \min\{E_q, E_{max}, M_t\}$. On the other hand, the maximum impact parameter b_{max} is determined by charge screening. In a white dwarf, this is simply the screening induced by a degenerate electron gas similar to a solid and is given by the Thomas-Fermi length $l_{sc} = \left(\frac{6\pi Ze^2n_e}{E_F}\right)^{1/2} \sim \frac{1}{m_e}$. This is the equivalent of the Debye length for a degenerate gas at Fermi energy E_F. This corresponds to a lower bound $E_{lower} = \frac{2m_e^2Z^2\alpha^2}{M_t\beta^2}$. Note that when the incident particle reaches velocity $\beta\gamma \approx \frac{m_e}{\min\{m_i,M_t\}}$, the minimum possible energy transfer due to Thomas-Fermi screening will in fact exceed the maximum. At this point, the Coulomb Logarithm of (5) becomes modified to avoid non-negative values of dE/dx and there is negligible stopping power due to collisions.

However when considering collisions with the degenerate electrons, an incident particle transferring energy E' can only scatter those electrons within E' of the Fermi surface. We define a modified density of electrons $n_e(E')$ as:

$$n_e(E') = \begin{cases} \int_{E_F-E'}^{E_F} dE\ g(E) & E' \leq E_F \\ n_e & E_F \leq E' \end{cases}, \tag{7}$$

where $g(E)$ is the density of states per unit volume for a three-dimensional free electron gas. This can also be expressed as a suppression of the differential cross section of order $\mathcal{O}(E'/E_F)$ whenever energy less than E_F is transferred. Therefore, unlike in the non-degenerate case, the energy loss due to soft-scatters are in fact subdominant to the contributions from rare, hard-scatters. The stopping power is also highly sensitive to the upper and lower bounds of integration. We have calculated the stopping power of various high-energy particles (i.e. muons and electrons) solely due to Coulomb collisions, differentiating between target nuclei and degenerate electrons. This is shown in Figure figure. It is found that for a given incident particle, the stopping power is dominated by collisions with nuclei at low-energies although it is dominated by collisions with degenerate electrons at high-energies. In addition, scattering off degenerate electrons becomes completely screened at a higher-energy as compared to scattering off nuclei.

In order to determine the effective R_ϵ, we

$$L = \int dE\left(\frac{dE}{dx}\right)^{-1}, \tag{8}$$

integrated over the incident particle kinetic energy.

Note that the stopping power due to radiative effects i.e. bremsstrahlung only becomes comparable to Coulomb scattering off nuclei to at much higher energies - LPM suppression.

Hadronic Interactions

Charged Hadrons

Neutral Hadrons

The main difference between charged and neutral particles is that the latter do not have appreciable electromagnetic interactions. These couplings are typically suppressed by higher dimension operators.

Q-BALLS

In various supersymmetric extensions of the standard model (SM), non-topological solitons called Q-balls can be produced in the early universe. If these Q-balls were stable, they would comprise a component of the dark matter today. Q-balls can be classified into two groups: supersymmetric electrically charged solitons (SECS) and supersymmetric electrically neutral solitons (SENS). When a neutral baryonic Q-ball interacts with a nucleon, it absorbs its baryonic charge as a minimum-energy configuration and induces the dissociation of the nucleon into free quarks. In this process (known as the "KKST" process), $\sim$ GeV of energy is released through the emission of 2-3 pions. The KKST process provides a useful way to detect such Q-balls. The cross section for interaction is approximately the geometric cross section

$$\sigma_Q \simeq \pi R_Q^2. \tag{9}$$

In gauge-mediated models with flat scalar potentials, the Q-ball mass and radius are given by

$$M_Q \sim m_F Q^{3/4}, \quad R_Q \sim m_F^{-1} Q^{1/4}, \tag{10}$$

where m_F is related to the scale of supersymmetry breaking (messenger scale). The condition $M_Q/Q < m_p$ ensures that the Q-ball is stable against decay to nucleons.

Note that a sufficiently massive Q-ball will become a black hole if the Q-ball radius is less than the Schwarzschild radius $R_Q \lesssim R_s \sim GM_Q$. In the model described above, this translates into the condition

$$m_F \left(\frac{M_{\rm pl}}{m_F}\right)^3 \lesssim m_Q, \quad \left(\frac{M_{\rm pl}}{m_F}\right)^4 \lesssim Q. \tag{11}$$

For Q-ball masses of this order, gravitational interactions become relevant while the KKST interaction ceases to exist.

Q-ball Explosiveness

Note that a given DM transit will be most explosive for higher mass white dwarfs. This can be seen explicitly in the density dependence of λ_T. We consider densities in the range $\rho \sim 10^6 - 10^9 \, \frac{\rm gm}{\rm cm^3}$. For carbon-oxygen white dwarfs, this translates to $n \sim 10^{29} - 10^{32}$ cm^{-3} and $n_e \sim 10^{30} - 10^{33}$ cm^{-3} for number densities of nuclei and electrons, respectively. Someone check these density numbers Over this range of densities, the trigger size approximately varies between $\lambda_T \sim 10^{-5}$ cm $- 10^{-2}$ cm.

We assume that for each Q-ball collision, there is equal probability to produce π^0, π^+ and π^- under the constraint of charge conservation. Since $\sim$ 10 GeV is released in $\mathcal{O}(10)$ pions per nuclei dissociation, pions are emitted with velocity $\gamma \approx 5$. The mean distance travelled by a relativistic particle before decaying is $d = \gamma v \tau$. For neutral pions $d_{\pi^0} \sim 10^{-5}$ cm while for charged pions (which decay via weak interactions and have characteristically longer lifetimes), $d_{\pi^\pm} \sim 10$ m. Note that d_{π^0} and $d_{\pi^\pm}$ do not depend on the ambient white dwarf density.

Because of the effect of degeneracy on electromagnetic energy loss, nuclear interactions play a key role in determining the range of the KKST process in the white dwarf. The cross section for any nuclear interaction is approximately set by the nuclear length scale $\sim$ fm^2. Numerous experiments have studied the effects of $50 - 500$ MeV pions incident upon complex nuclei targets such as carbon. It is found that there is approximately equal cross section of order $\mathcal{O}(100\,\text{mb})$ for a (neutral or charged) pion to either scatter elastically, scatter inelastically, or become absorbed with no final state pion. Elastic scattering is not a dominant source of energy loss due to large nuclei masses. Of these possibilities, pion absorption is the most relevant for energy loss. During this process, an incident pion is absorbed in the nucleus and transfers energy greater than the typical binding energy per nucleon $\sim$ 10 MeV. This leads to the emission of $\sim 2-4$ protons, neutrons, or deuterons with an $\mathcal{O}(1)$ fraction of the total initial energy split among the final states. Furthermore, at high energies these emitted nucleons have considerable $\sim$ 100 mb nonelastic nuclear cross sections which result in multiple final state hadrons including protons, neutrons, pions, etc. The details of these interactions are beyond the scope of this work and typically involve complicated nuclear dynamics. spallation - nuclear evaporation Regardless, the process qualitatively resembles a "hadronic shower" in which an initial high energy nucleon eventually produces many lower energy hadronic final states.

At a nuclear density $n_C \sim 10^{32}$ cm^{-3}, the mean free path for nuclear interactions of cross section $\sim$ 100 mb is given by $l_n \sim 10^{-7}$ cm. Therefore, the range of KKST process is determined as follows. In terms of l_n, the initial pions and resulting high-energy "shower" traverse a distance $\sim$ few $\times\, l_n$ until final states of $\sim$ MeV energy are produced. At sufficiently low energies the range of charged hadrons due to electromagnetic scattering off nuclei (8) becomes comparable to l_n, and at this point these particles immediately stop. For protons in the specified density, this critical energy occurs at $\mathcal{O}(10\text{ MeV})$. As for long-lived neutral hadrons i.e. neutrons, the nonelastic cross section effectively vanishes $\sim$ MeV. At this point, the dominant stopping mechanism is nuclear elastic scattering. For an $\sim$ MeV neutron interacting with C and O nuclei, it is found that $\mathcal{O}(100)$ elastic collisions are needed to sufficiently slow down to thermal velocity. Therefore, neutrons have to traverse an additional distance $\sim 10 \times l_n$ in the form of a random walk before stopping. In summary, an $\mathcal{O}(1)$ fraction of the energy deposited in the KKST process gets transferred within a range $R \sim 10^{-7} - 10^{-6}$ cm.

Q-ball Constraints