I. Q-BALL PROPERTIES

In various supersymmetric extensions of the standard model (SM), non-topological solitons called Q-balls can be produced in the early universe. If these Q-balls were stable, they would comprise a component of the dark matter today. Q-balls can be classified into two groups: supersymmetric electrically charged solitons (SECS) and supersymmetric electrically neutral solitons (SENS). When a neutral baryonic Q-ball interacts with a nucleon, it absorbs its baryonic charge as a minimum-energy configuration and induces the dissociation of the nucleon into free quarks. In this process (known as the "KKST" process), $\sim$ GeV of energy is released through the emission of 2-3 pions. The KKST process provides a useful way to detect such Q-balls. The cross section for interaction is approximately the geometric cross section

$$\sigma_Q \simeq \pi R_Q^2. \tag{1}$$

In gauge-mediated models with flat scalar potentials, the Q-ball mass and radius are given by

$$M_Q \sim m_F Q^{3/4}, \quad R_Q \sim m_F^{-1} Q^{1/4}, \tag{2}$$

where m_F is related to the scale of supersymmetry breaking (messenger scale) and is at least of $\mathcal{O}(\text{TeV})$. The condition $M_Q/Q < m_p$ ensures that the Q-ball is stable against decay to nucleons.

Note that a sufficiently massive Q-ball will become a black hole if the Q-ball radius is less than the Schwarzschild radius $R_Q \lesssim R_s \sim G M_Q$. In the model described above, this translates into a condition on the Q-ball interaction cross section

$$\sigma_Q \lesssim \frac{M_{\rm pl}^2}{m_F^4}. \tag{3}$$

For cross sections of this order, gravitational interactions become relevant. In fact, values of σ_Q greater than this bound have no meaning since black holes do not interact via the KKST process.

II. Q-BALL EXPLOSIVENESS

Localized heating of a white dwarf has the potential to ignite the star. Namely, if a region of size λ_T or greater is raised to a critical temperature T_f, this would initiate runaway thermonuclear fusion and cause the white dwarf to explode in a supernovae. According to [**?**], $\lambda_T \sim 10^{-5}$ cm for white dwarf densities $\sim 5 \times 10^9 \ \frac{\text{gm}}{\text{cm}^3}$. This is then analytically scaled in [**?**] for varying densities.

Consider a Q-ball (or similar dark matter candidate) transit through the white dwarf. The energy released per distance travelled is $n_C \sigma_Q \epsilon$, where ϵ is the typical energy released per nuclei collision (denoted as C for simplicity). Of course, this released energy must be transferred to the stellar medium in order for the white dwarf to be heated. In particular, any ϵ is characterized by a region of size R from the point of release over which it is deposited. This range, and therefore the size of the heated region, is set by the various standard model processes by which the energy is released and subsequently interacts with stellar constituents. To demonstrate the significance of R, suppose each Q-ball collision resulted in a simple elastic scattering process. In this case, R effectively vanishes as ϵ is transferred directly to the kinetic energy of nuclei. However, in the other extreme limit, suppose ϵ were released purely into neutrinos. In this case, R is of astronomical length scales and the released energy would leave the white dwarf before having a chance to thermalize any region in the star.

Therefore, it is necessary to determine the relevant R for a Q-ball transit, during which $\epsilon \sim 10$ GeV of nuclear energy is released per collision in the form of energetic pions. This is done in Section **??**. For now, consider the two possibilities relevant for ignition: if $R > \lambda_T$, then the Q-ball must deposit a minimum energy $E_{min} \sim R^3 n_C T_f$ in order to heat up the entire region of size R to the critical temperature T_f. On the other hand, if $R < \lambda_T$ then the minimum energy required is independent of R and given simply by $E_{min} \sim \lambda_T^3 n_C T_f$. Setting $T_f \sim$ MeV as in [**?**], we see that an energy $E_{min} \sim 10^{14}$ GeV $\left(\frac{\lambda_T}{10^{-5}\text{cm}}\right)^3 \left(\frac{n_C}{10^{32}\text{cm}^3}\right)$ transferred to the white dwarf within a localized region smaller than λ_T will eventually trigger runaway fusion.

After a time Δt the Q-ball will have traversed a distance $v_Q \Delta t$, where the Q-ball velocity is set by the escape velocity of the white dwarf $v_Q \approx 2 \times 10^{-2}$.

Of course, as we are only interested in sufficiently heating regions of size λ_T, we can set $\Delta t = \frac{\lambda_T}{v_Q}$. If this time scale is shorter than the typical diffusion time scale for a trigger-sized region of thermal diffusivity α, $\tau_d \sim \frac{\lambda_T^2}{\alpha}$, then we may effectively treat the energy deposited in this region as a cylinder.

We first find the σ_Q necessary to deposit a total energy inside the white dwarf equivalent to heating a cylinder of radius λ_T to a temperature $\sim$ MeV. If the white dwarf is comprised of primarily carbon nuclei, $\mathcal{O}(10 \text{ GeV})$ is released per collision. Assuming the Q-ball traverses the full length of the white dwarf with minimal change to its baryon number and mean free path $l \sim \frac{1}{n_C \sigma_Q}$, we have $10^{-4}\ \lambda_{\min}^2 \lesssim \sigma_Q$. This is effectively the minimum σ_Q needed to release a total amount of energy capable of triggering runaway fusion.

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Rather, the pions emitted in the KKST process will travel a distance in the stellar medium before depositing their full kinetic energy. It is therefore necessary to compute the typical pion range in the degenerate medium. We can assume that there is an equal probability to produce π^0, π^+ and π^- in each collision under the constraint of charge conservation. The mean distance travelled by a relativistic particle before decaying is $d = \gamma v \tau$. Since energy equal to the nucleon mass ($\sim$ GeV) is released in 2-3 pions, each pion travels with velocity roughly $\gamma \approx 5$. For neutral pions we find that $d_{\pi^0} \sim \mathcal{O}(10^{-5}$ cm) while for charged pions, which decay via the weak interactions and have characteristically longer lifetimes, $d_{\pi^\pm} \sim \mathcal{O}(10$ m).

Q-balls (or any other dark matter trigger) will be most explosive for higher density white dwarfs. We consider densities in the upper edge of the range $\rho \sim 10^6 - 10^9 \ \frac{\text{gm}}{\text{cm}^3}$. For carbon-oxygen white dwarfs, this translates to $n_C \sim 10^{29} - 10^{32}$ cm^{-3} and $n \sim 10^{30} - 10^{33}$ cm^{-3} for number densities of carbon nuclei and electrons, respectively. Someone check these density numbers.

First we estimate the cross section for (neutral and charged) pions colliding with carbon nuclei. This is approximately set by the hadronic length scale $\sim$ fm and is confirmed by experiments to be $\sigma \sim 100$ mb. Therefore, the mean free path for pion-nuclei interactions is estimated to be $l_{\pi c} \sim \mathcal{O}(10^{-8} - 10^{-5}$ cm).

Now we calculate the charged pion range. To leading order, the stopping of charged pions in the white dwarf will be through electromagnetic scattering off the degenerate electron gas. In other words, as pertaining to energy loss, collisions with nuclei have a negligible effect compared with collisions of electrons. For particles of charge Ze scattering off free electrons, this is described by the "Bhabha" differential cross section (for a spin-0 particle)

$$\frac{d\sigma_R(E', \beta)}{dE'} = \frac{2\pi\alpha^2 Z^2}{m_e \beta^2} \frac{1}{E'^2} \left(1 - \frac{\beta^2 E'}{E_{max}}\right), \tag{4}$$

where we have assumed a sufficiently fast charged particle so that interactions are governed by single-electron collisions with energy transfer E'. This is the equation quoted in the PDG and the textbook by Rossi. For the case of E' much less than E_{max}, this reduces to the "Rutherford" cross section:

$$\frac{d\sigma_R(E', \beta)}{dE'} = \frac{2\pi\alpha^2 Z^2}{m_e \beta^2} \frac{1}{E'^2}. \tag{5}$$

It is straightforward to understand the parametric dependences of this formula: there is increased likelihood to deposit energy for slowly moving particles undergoing "soft-scatters".

It is important to understand the range of validity of this formula. According to the classical derivation of 5, the energy transfer is given in terms of an impact parameter b (distance of closest approach):

$$E' = \frac{2m_e Z^2 r_e^2}{\beta^2 b^2}, \qquad r_e = \frac{\alpha}{m_e}. \tag{6}$$

The first breakdown occurs when $E' > m_e$, or in terms of impact parameter, $b < b_{rel} = \frac{Z}{\beta} r_e$. In other words, 5 ceases to make sense when the impact parameter to deposit energy E' is less than the classical electron radius. This seems reasonable. The second breakdown follows from the fact that 5 was derived neglecting the deflection of the incident particle and the motion of the electron during the collision. These conditions also amount to a lower limit of the impact parameter of order $b > Z r_e \sqrt{1-\beta^2}$. However, for our processes this constraint is less restrictive than the previous one. Another breakdown comes from quantum mechanical arguments: the uncertainty principle sets a limit to the accuracy that can be achieved in "aiming" the pion at a target electron. According to Rossi, this translates to a bound on the impact parameter $b > b_q = \frac{1}{\beta\gamma m_e}$ or a bound on the energy transfer $E' < 2m_e\gamma^2\alpha^2$.

Since the pion is only emitted with $\sim$ GeV energy, it appears that $b_q > b_{rel}$. Therefore, it makes sense to set an upper limit for the possible energy transfers described by this formula $E' < 2m_e\gamma^2\alpha^2$.

On the other hand, E_{max} denotes the maximum energy transfer possible solely due to kinematic constraints (conservation of energy and momentum), which is the case for an electron initially at rest and a "head-on" collision (zero angle between incoming pion and outgoing electron momenta). By relativistic kinematics, E_{max} is given by

$$E_{max} = 2m_e \frac{p^2}{m_e^2 + M^2 + 2m_e(p^2+m^2)^{1/2}}, \tag{7}$$

where M and p the mass and momentum of the incident particle. For the pions in consideration we have $m_e \ll m_\pi$ and $2\gamma m_e \ll m_\pi$. Therefore $E_{max} \approx 2m_e\beta^2\gamma^2$ only depends on the velocity of the incoming pion. Of course, the typical temperature of the white dwarf interior is $\sim 10^7$ K $\sim$ keV, so no electron is actually at rest. In fact, the fastest electrons have momentum of order the Fermi momentum $p_F \sim E_F \sim n^{1/3} \approx$ MeV (the approximation of an extreme relativistic Fermi gas is valid since $m_e \lesssim p_F$). Implicit in the derivation of 5 is the assumption of small momentum transfers and therefore a target electron at rest. Numerically, we find that $E_{max} \sim 10$ MeV when the pion kinetic energy is ~ 500 MeV.

We now compute the rate of energy loss of the charged pion along its track:

$$\frac{dE}{dx} = \int dE' \left(\frac{d\sigma_R}{dE'}\right) n(E')E'. \tag{8}$$

Typically the number density of electrons is not a function of energy but for a degenerate electron gas, the differential cross section is suppressed by a Pauli-blocking factor of order $\mathcal{O}(E'/E_F)$. This can instead be expressed as a modified density of electrons $n(E')$ where for a given E', the pion can only scatter those electrons of energy within E' of the Fermi surface. Assuming a perfect Fermi gas, we define $n(E')$ as:

$$n(E') = \begin{cases} \int_{E_F-E'}^{E_F} dE \; g(E) & E' \le E_F \\ n & E_F \le E' \end{cases} . \tag{9}$$

Here $g(E) = \frac{E^2}{\pi^2}$ is the density of states per unit volume for a three-dimensional relativistic free electron gas. With the correct form of the stopping power, the range of the pion is simply

$$R_\pi = \int_0^{T_\pi} dE \left(\frac{dE}{dx}\right)^{-1}, \tag{10}$$

where we have chosen to integrate over E, the pion kinetic energy and T_π denotes the initial kinetic energy ~ 500 MeV.

The question then becomes the limits of integration for 8.

Take 1:

$$\frac{dE}{dx} = \int_0^{Min[E_F,E_{max}]} dE' \left(\frac{d\sigma_R}{dE'}\right) n(E')E' + \int_{E_F}^{Max[E_F,E_{max}]} dE' \left(\frac{d\sigma_R}{dE'}\right) nE' \tag{11}$$

Doing so yields ranges $R_\pi \sim 1 \times 10^{-6} - 4 \times 10^{-4}$ cm. This seems magical, given the trigger sizes and their scaling with density. However, both E_F and the initial value of E_{max} are larger than the constraint $E' < 2m_e\gamma^2\alpha^2 \sim \mathcal{O}(\text{keV})$. Therefore, is this formula valid?

Take 2:

$$\frac{dE}{dx} = \int_0^{Min[2m_e\gamma^2\alpha^2,E_{max}]} dE' \left(\frac{d\sigma_R}{dE'}\right) n(E')E'. \tag{12}$$

This yields $R_\pi \sim 10^{-2} - 1$ cm, so considerable difference.