White Dwarfs as Dark Matter Detectors

I.   INTRODUCTION

[sketch of intro text]

Heavy DM requires (multiple) astrophysical probes. We are presenting one astrophysical probe of heavy DM: WD supernovae.

White dwarfs are degenerate, so they do not regulate their temperature via expansion - this means they will explode if locally heated. This explosion can be used to detect DM species that provides sufficient localalized heating.

Weakness of WD detection: other ignition sources exist (i.e., detector backgrounds). What constraints can we place in light of this?

Focus of this work: constrain local interactions of DM that release energy in the form of SM particles. Contrast to long-range gravitational heating, mention other possibilities.

Conventional paper outline.

[end sketch]

II.   OVERVIEW

White dwarfs and stuff...

III.   EXPLOSION CONDITIONS

A.   Explosion Condition for Localized Heating

An energy deposition event in the white dwarf can be roughly characterized by two numbers: a heating length L and temperature T. In any such event the energy deposit will eventually take the form of a local peak in the star's temperature profile. We take T to be the characteristic temperature of this peak and L its characteristic length scale. This peak will evolve through various processes, and we take the above quantities to describe the peak as it initially appears.

For a specific deposition mechanism, L is determined by the efficiency with which that mechanism couples to the stellar medium. For example, if one was able to elastic scatter kinetic energy direction into a number of neighboring ions, those ions would thermalize over their collisional time scale to give an L of order the ion mean free path. This is the minimal L that one could achieve. In the other extreme, one might imagine a process that produces a large number of electrons with energy just above the Fermi energy. These electrons have Pauli-suppressed interactions with the

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medium and will travel a long distance before their energy is scattered and thermalized, resulting in a large L - possibly of order the stellar radius. We are concerned here with similar, but much more efficient, inelastic processes wherein a DM interaction heats the star via production of a large number of high-energy standard model particles. Understanding the heating lengths of these processes is critical to assessing their explosive potential - these are presented in section V and computed in detail in appendix B. In our processes, typical L will range from 10^{-7}cm to 0.1cm.

We need a condition that determines if a given heating event L, T will result in the destruction of the star as a supernova. It will be more useful to phase this in terms of the excess energy E within the temperature peak instead of the temperature itself. These are related by the number of particles within the peak's volume: $E \sim nTL^3$, where n is the number density of the stellar medium. The explosion condition is then:

$$E \gtrsim nT_B \mathrm{Max} \left[L, \lambda_T \right]^3 \tag{1}$$

where λ_T and T_B are two characteristic scales. The *fusion temperature* T_B is the onset temperature of carbon-carbon fusion. This is set by the energy required for carbon ions in the star to overcome their mutual Coulomb barriers, and is thus a constant with value $\sim \mathrm{MeV}$. The *trigger size* λ_T is a length scale set by the efficiency of diffusive cooling in the stellar medium, and is thus sensitive to the medium's density, which can vary considerable from star to star. The trigger size as a function of white dwarf mass is computed in [2] and reproduced in figure 1. For white dwarf masses of $0.4M_\odot$ to $1.4M_\odot$, the trigger sizes range roughly from 10^{-5} cm to 0.1 cm. The physics of condition 1 and the scales T_B and λ_T are reviewed in more detail in appendix A.

The moral of condition 1 is that to destroy a white dwarf one needs to deposit both enough energy and enough energy density. There is an absolute minimum explosion energy:

$$E_{\mathrm{min}} \sim nT_B \lambda_T \sim 10^{\mathrm{something}} - 10^{14} \mathrm{GeV}$$

(the range here is due to the range in white dwarf densities). Yet this energy is sufficient only if it thermalizes below the trigger size. If your energy is deposited on a length scale larger than λ_T, the explosion energy required is parametrically larger. As we are concerned with processes that have a fixed deliverable energy (the energy of the incoming DM), the most explosive processes will be those that result in the most localized heating.
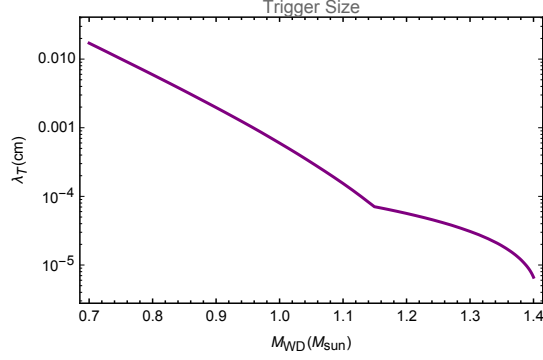
FIG. 1: Trigger sizes for carbon-oxygen white dwarfs, calculated in [2].

B. Localize Heating via Dark Matter Interactions

1. Parameterizing the Dark Matter Interaction

2. Explosion Conditions For Dark Matter Interaction Parameters

¡¡¡¡¡¡¡ Updated upstream Consider an ultra-heavy dark matter (DM) state transit through the WD. Assume that the DM interacts with WD constituents (ions or electrons) in a general manner as shown in Figure III B 2, releasing n_i particles of a single species i each with kinetic energy ϵ. The cross section for this interaction is denoted as $\sigma_{i,\epsilon}$. Of course, this released energy must be transferred to the stellar medium in order for the WD to be heated. For a given particle type, each value of ϵ is characterized by a distance R_ϵ from the point of release over which it is deposited. This "range", and therefore the nominal size of the resulting hot region, is set by the various ways in which the emitted particle interacts with the stellar constituents and is able to dumps its energy. In particular, we define R_ϵ as the distance over which a particle i and any secondaries transfer $\mathcal{O}(1)$ of the initial energy ϵ to electrons or ions in the WD. To demonstrate the significance of this parameter, suppose that the DM simply scattered off nuclei elastically with no particles released. In this case, R_ϵ effectively vanishes as ϵ is transferred directly to the kinetic energy of nuclei. However, in the other extreme limit, suppose the interaction released energy into neutrinos. In this case, R_ϵ is of astronomical length scales and there would be no chance of thermalizing any local region in the star. ======= Consider an ultra-heavy dark matter (DM) state transit through the WD. Assume that the DM interacts with WD constituents (ions or electrons) in a general manner as shown in Figure III B 2, releasing n_i particles of species i each with kinetic energy ϵ. The cross section for this interaction is denoted as $\sigma_{i,\epsilon}$. Of course, this released energy must be transferred to the stellar medium in order for the WD to be heated. For a given particle type, each value
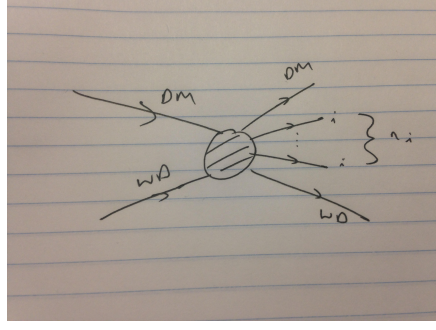
FIG. 2: General interaction between ultra-heavy DM and WD constituents, producing n_i additional particles of species i.

of ϵ is characterized by a distance R_ϵ from the point of release over which it is deposited. This thermalization distance is the nominal size of any resulting hot region and is set by the various ways in which emitted particles interact with stellar constituents. In particular, we define R_ϵ as the distance over which a particle i and any secondaries transfer $\mathcal{O}(1)$ of the initial energy ϵ to electrons or ions in the WD. To demonstrate the significance of this parameter, suppose that the DM simply scattered off nuclei elastically with no particles released. In this case, R_ϵ effectively vanishes as ϵ is transferred directly to the kinetic energy of nuclei. However, in the other extreme limit, suppose the interaction released energy into neutrinos. In this case, R_ϵ is of astronomical length scales and there would be no chance of thermalizing any local region in the star. ¿¿¿¿¿¿¿ Stashed changes

¡¡¡¡¡¡¡ Updated upstream Consider the two possibilities relevant for ignition. If $R_\epsilon > \lambda_T$, the DM must deposit a minimum energy $E_{\mathrm{boom}} \sim R_\epsilon^3 n T_f$ in order to heat up the entire region of size R_ϵ to the critical temperature T_f, where n is the number density of nuclei in the WD. On the other hand, if $R_\epsilon < \lambda_T$ then the minimum energy required is independent of R and given by $E_{\mathrm{boom}} \sim \lambda_T^3 n T_f$. Setting $T_f \sim \mathrm{MeV}$, $\lambda_T \sim 10^{-5}$ cm, and $n \sim 10^{32}$ cm^{-3}, we find that an energy $E_{\mathrm{boom}} \sim 10^{14}$ GeV transferred to the WD within a localized region smaller than λ_T will eventually trigger runaway fusion. This "explosion" energy must be less than the total energy released during the DM transit over a distance $\min\{\lambda_T, R_\epsilon\}$. Doing so, we find a lower bound on the interaction cross section sufficient to trigger runaway fusion: ======= Consider the two possibilities relevant for ignition. If $R_\epsilon > \lambda_T$, the DM must deposit a minimum energy $E_{\mathrm{boom}} \sim R_\epsilon^3 n T_f$ in order to heat up the entire region of size R_ϵ to the critical temperature T_f, where n is the number density of nuclei in the WD. On the other hand, if $R_\epsilon < \lambda_T$ then the minimum energy required is independent of R_ϵ and given by $E_{\mathrm{boom}} \sim \lambda_T^3 n T_f$. Setting $T_f \sim \mathrm{MeV}$, $\lambda_T \sim 10^{-5}$ cm, and $n \sim 10^{32}$ cm^{-3}, we find that

an energy $E_{\text{boom}} \sim 10^{14}$ GeV transferred to the WD within a localized region smaller than λ_T^3 will eventually trigger runaway fusion. This "explosion" energy must be less than the total energy released during the DM transit. Doing so, we find a lower bound on the interaction cross section sufficient to trigger runaway fusion: ¿¿¿¿¿¿¿ Stashed changes

$$n_i \sigma_{\epsilon,i} \gtrsim \begin{cases} \lambda_T^2 \left(\dfrac{T_f}{\epsilon} \right) & R_\epsilon < \lambda_T \\ \lambda_T^2 \left(\dfrac{R_\epsilon}{\lambda_T} \right)^2 \left(\dfrac{T_f}{\epsilon} \right) & R_\epsilon > \lambda_T \end{cases}. \tag{2}$$

Note that in deriving this explosiveness bound, we have assumed the DM transit time is less than the corresponding diffusion time. After a time Δt the DM has traversed $v_{\text{esc}} \Delta t$, where the velocity is set by the escape velocity of a WD. Therefore, this amounts to the following condition:

$$\tau_d(\lambda_T, T_f) \gtrsim \frac{\lambda_T}{v_{\text{esc}}}, \qquad R_\epsilon < \lambda_T$$

$$\tau_d(R_\epsilon, T_f) \gtrsim \frac{R_\epsilon}{v_{\text{esc}}}, \qquad R_\epsilon > \lambda_T \tag{3}$$

where τ_d is the characteristic time for a region of given size and temperature to diffuse $\mathcal{O}(1)$ of its heat. An estimate from the heat equation gives $\tau_d(\lambda_T, T_f) \sim \frac{\lambda_T^2}{\alpha}$, where α is the (temperature-dependent) diffusivity. Show condition is true for all densities. We also assume that the time to transfer energy ϵ out to its characteristic range R_ϵ is less than the diffusion time scale $\tau_d(\lambda_T, T_f)$.

IV. GENERAL CONSTRAINTS

A. Annihilation Events: Rate and Constraints

B. Transit Events: Rate and Constraints

V. HEATING LENGTHS

¡¡¡¡¡¡¡ Updated upstream In this section, we enumerate the different ways in which standard model particles can lose energy to the WD. (Unless otherwise noted, we will assume a carbon-oxygen white dwarf.) The interior of a WD is a complex environment. Famously, the star is supported against collapse by electron degeneracy pressure with a characteristic Fermi energy $E_F \sim n_e^{1/3} \sim \mathcal{O}(\text{MeV})$ at all densities?. In addition, the nuclei are at an ambient temperature $T \sim$ keV and form a strongly-coupled plasma $\frac{Ze^2}{n^{1/3}T} \gg 1$. In what follows, we calculate the WD medium sensitivity to deposited energy and the deposit ranges R_ϵ for the different possible DM interactions. For the purpose of depositing sufficient energy to trigger supernovae, we focus

on high-energy particles $\epsilon \gg$ MeV which interact via the strong and electromagnetic forces (i.e. electrons, muons, photons, pions, and neutral hadrons). In this way, the WD may be thought of as a "particle detector" with electromagnetic and hadronic "calorimeter" components. ======= In this section, we enumerate the different ways in which standard model particles can lose energy to the WD. The interior of a (carbon-oxygen) WD is a complex environment. Famously, the star is supported against collapse by the degeneracy pressure of an electron gas at a characteristic Fermi energy $E_F \sim n_e^{1/3} \sim \mathcal{O}(\text{MeV})$. In addition, the nuclei are at an ambient temperature $T \sim$ keV and form a strongly-coupled plasma $\frac{Ze^2}{n^{1/3}T} \gg 1$. In what follows, we calculate the WD medium sensitivity to deposited energy and, in turn, R_ϵ for the different possible DM interactions. For the purpose of depositing sufficient energy to trigger supernovae, we focus on high-energy particles $\epsilon \gg$ MeV which interact via the strong and electromagnetic forces (i.e. electrons, muons, photons, pions, and neutral hadrons). In this sense, the WD may be thought of as a "particle detector" with electromagnetic and hadronic "calorimeter" components. ¿¿¿¿¿¿¿ Stashed changes

A.   Electromagnetic Interactions

For charged particles, Coulomb scattering is a useful mechanism for energy transfer. Generically, an incident (spin-0) particle of mass m_i, charge e, and velocity β scattering off a target M_t of charge Ze is described by the "Rutherford" differential cross section Originally derived by Bhabha?

$$\frac{d\sigma}{dE'} = \frac{2\pi\alpha^2 Z^2}{M_t\beta^2}\frac{1}{E'^2}\left(1 - \frac{\beta^2 E'}{E_{\text{kin}}}\right), \tag{4}$$

where we have assumed a sufficiently fast incident particle so that interactions are governed by single collisions with energy transfer E' [3]. E_{kin} denotes the maximum energy transfer possible satisfying kinematic constraints target at rest and zero relative angle between incoming incident and outgoing target momenta:

$$E_{\text{kin}} = \frac{2M_t\beta^2\gamma^2}{1 + 2\gamma(M_t/m_i) + (M_t/m_i)^2}, \tag{5}$$

where $\gamma = (1-\beta)^{-1/2}$.

For sufficiently heavy incident particles, the differential cross section depends only on the velocity of the incident particle. Note that higher-spin particles receive additional corrections to the cross section, but for small energy transfers these corrections are negligible. It is straightforward to understand the parametric dependences of (4): there is increased likelihood to scatter for slowly moving incident particles undergoing "soft-scatters" against lighter targets. Therefore, one would

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expect that soft scattering dominates the energy loss and that collisions with nuclei of mass M are suppressed by a factor $\mathcal{O}\left(\frac{Zm_e}{M}\right)$ as compared to collisions with electrons. This is certainly true for incident charged particles in non-degenerate matter. However, both of these naive expectations turn out to be false when considering scattering off a degenerate species.

To understand the effect of degeneracy, we first consider the energy loss from scattering a high-energy charged particles off non-degenerate targets in the WD, for instance, the carbon nuclei. In this case, the stopping power due to collisions with a number density n is given by:

$$\frac{dE}{dx} = -\int dE' \left(\frac{d\sigma}{dE'}\right) nE' \tag{6}$$

$$\sim -\frac{2\pi n Z^2 \alpha^2}{M_t \beta^2} \log\left(\frac{E_{\max}}{E_{\min}}\right). \tag{7}$$

This integration must be performed over all E' within the regime of validity for (4), fixing the lower and upper bounds of the "Coulomb Logarithm". Quantum mechanical uncertainty sets a limit to the accuracy that can be achieved in "aiming" an incident particle at a target. In terms of impact parameter b for the collision, this translates to a bound $b \gtrsim (\min\{m_i, M_t\}\beta\gamma)^{-1}$ corresponding to the larger de Broglie wavelength. In terms of energy transfer, this becomes

$$E' \lesssim E_q = \frac{2 \min\{m_i, M_t\}^2 Z^2 \alpha^2 \gamma^2}{M_t}. \tag{8}$$

In addition, the expression for the differential cross section (4) no longer holds when E' becomes larger than the mass of the target [4]. For our calculations, we take the maximum energy that an incident particle is able to transfer to be

$$E_{\max} = \min\{E_q, E_{\mathrm{kin}}, M_t\}. \tag{9}$$

On the other hand, the maximum impact parameter is determined by charge screening. In a WD, this is simply the screening due to a degenerate electron gas similar to a solid and is given by the Thomas-Fermi length [5]

$$l_{\mathrm{sc}} = \left(\frac{6\pi Z e^2 n_e}{E_F}\right)^{-1/2} \sim \frac{1}{m_e}. \tag{10}$$

This is the equivalent of the Debye screening length for a degenerate gas at Fermi energy E_F. This corresponds to a lower bound

$$E_{\mathrm{sc}} = \frac{2 m_e^2 Z^2 \alpha^2}{M_t \beta^2}. \tag{11}$$

Note that when the incident particle reaches velocity $\beta\gamma \approx \frac{m_e}{\min\{m_i, M_t\}}$, the minimum possible energy transfer due to Thomas-Fermi screening will in fact exceed the maximum. At this point,

the form of equation (6) becomes modified to avoid unphysical non-negative values of dE/dx and there is negligible stopping power due to collisions. However, the lattice structure of ions in the WD introduces further complications [5]. The typical lattice binding energy between ions is given by the electric potential

$$E_B \sim \frac{Z^2 e^2}{n^{-1/3}}. \tag{12}$$

For energy transfers greater than E_B, any lattice effects can be safely ignored. On the other hand, momentum transfers from collisions below this threshold may lead to suppressed energy loss due to collective effects. For simplicity we set the lower bound on nuclei scattering (not applicable for electron targets) to be

$$E_{\min} = \max\{E_B, E_{\text{sc}}\} \tag{13}$$

so that we only account for energy transfers greater than the ionic lattice binding energy.

When considering collisions with the degenerate electrons, an incident particle transferring energy E' can only scatter those electrons within E' of the Fermi surface. We define a modified density of electrons $n_e(E')$ as:

$$n_e(E') = \begin{cases} \displaystyle\int_{E_F - E'}^{E_F} dE \; g(E) & E' \leq E_F \\ n_e & E_F \leq E' \end{cases}, \tag{14}$$

where $g(E)$ is the density of states per unit volume for a three-dimensional free electron gas. This can also be expressed as a suppression of the differential cross section of order $\mathcal{O}(E'/E_F)$ whenever energy less than E_F is transferred. Therefore, unlike in the non-degenerate case, the energy loss due to soft-scatters are in fact subdominant to the contributions from rare hard-scatters. The stopping power is also highly sensitive to $E_{\max}$ and $E_{\min}$, governed by a power-law dependence rather than the logarithmic sensitivity in the case of a non-degenerate target.

We have calculated the stopping power of high-energy particles solely due to Coulomb collisions, differentiating between nuclei and degenerate electron targets. This is shown in Figure figure for an electron density $n_e \sim 10^{33}$ cm^{-3}. It is found that for heavy incident particles, the stopping power is dominated by collisions with nuclei at low-energies although it is dominated by collisions with degenerate electrons at high-energies. In addition, scattering off degenerate electrons becomes completely screened at a comparatively higher energies.

Electron Energy Loss

Muon Energy Loss
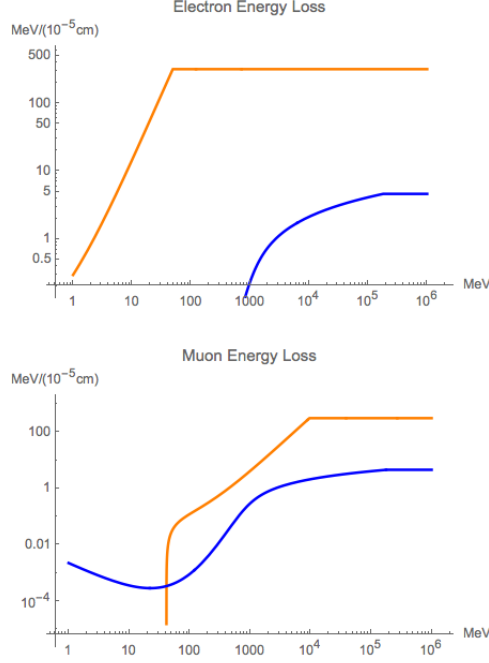
B. Strong Interactions

Strong interactions are a significant channel for energy transfer in the WD. For sufficiently energetic particles greater than the nuclear binding energy $E_{nuc} \sim \mathcal{O}(10 \text{ MeV})$, nuclear interactions (absorption) result in an $\mathcal{O}(1)$ number of energetic secondary hadrons (protons, neutrons, pions, etc.) emitted roughly in the direction of the primary particle. These secondary particles approximately split the initial total energy of the absorbed primary particle and can collide with other nuclei in the WD. In addition, the nucleus will generally be left in an excited state and relax through the emission of low-energy $\mathcal{O}(10 \text{ MeV})$ nucleons ("nuclear evaporation") and photons [4]. The cross section for any such process is approximately set by the nuclear length scale $\sim$ fm. A hadronic shower is a result of all such reactions caused by primary and secondary particles.

To estimate the typical shower length, we assume a primary particle of energy ϵ interacting with nuclei to produce N secondary particles each of energy ϵ/N. The shower will end once final-

Proton Energy Loss

R_ϵ for Hadronic Shower (n+p)
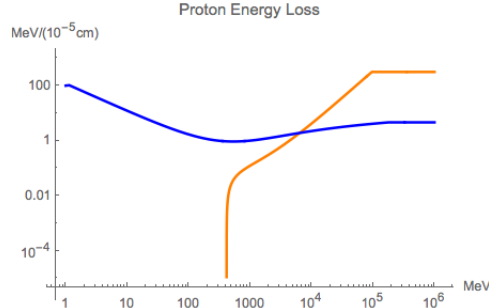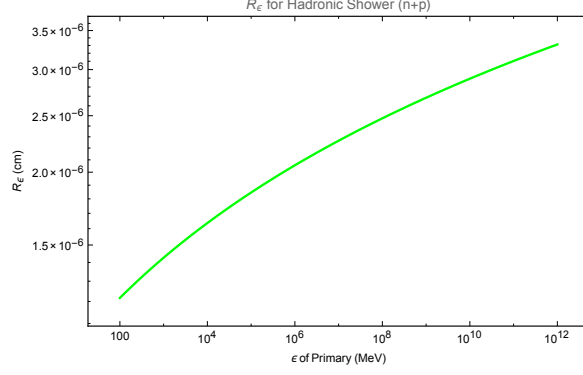
state particles reach energies of order E_{nuc}. We ignore the effects of the evaporative stage in each reaction as this will only introduce minor corrections at sufficiently large ϵ. We find a hadronic shower length $L = l_{\mathrm{nuc}} \left(\frac{\log{(\epsilon/E_{nuc})}}{\log N} \right)$, where l_{nuc} is the mean free path for nuclear collisions. For a number density $n \sim 10^{32}$ cm^{-3} and hadronic cross section ~ 0.1 barn match nuclear data for carbon nonelastic scattering this results in a shower length $L \approx 10\ l_{\mathrm{nuc}} \approx 10^{-6}$ cm. Note that this only has a mild, logarithmic sensitivity to the initial energy $\epsilon \gg 10$ MeV and number of secondaries $N \sim \mathcal{O}(1)$.

The end products of a hadronic shower will generally be hadrons of kinetic energy $1 - 10$ MeV which are incapable of inducing further nuclear disintegration. Charged hadrons have appreciable electromagnetic interactions with nuclei, while collisions with degenerate electrons are completely screened at these energies. Integrating (6), we find that in a density of $n \sim 10^{32}$ cm^{-3} protons traverse $\approx 10^{-6}$ cm slowing down from 5 MeV to 1 MeV. The corresponding distance is significantly larger for charged pions $\sim 10^{-2}$ cm On the other hand, electromagnetic couplings are highly suppressed for final-state neutral hadrons. The dominant source of energy loss for neutrons in this range is elastic scattering off nuclei neutron capture cross section in C+O is negligible. Each scatter transfers a fraction $\left(1 - \left(\frac{m}{m+M} \right)^2 \right) \approx 0.1$ of the neutron energy to nuclei (averaging over scattering angles), where m and M are the masses of the neutron and nuclei, respectively. At kinetic energies below $\sim$ MeV, the typical energy transfer in an elastic collision is of order the binding energy in the Coulomb lattice E_B. We find that $\mathcal{O}(10)$ elastic scatters are needed to slow neutrons from 5 MeV to 1 MeV. For a number density $n \sim 10^{32}$ cm^{-3} and elastic cross section $\sim$ barn set to match nuclear data for carbon nonelastic scattering, we find that neutrons traverse a total distance (in the form of a random-walk) $\sim 10^{-8}$ cm during this energy deposition.

VI. EXAMPLE CONSTRAINT: Q-BALL DARK MATTER

In various supersymmetric extensions of the standard model (SM), non-topological solitons called Q balls can be produced in the early universe [6, 7]. If these Q balls were stable, they would comprise a component of the dark matter today. Q balls can be classified into two groups: supersymmetric electrically charged solitons (SECS) and supersymmetric electrically neutral solitons (SENS). When a neutral baryonic Q ball interacts with a nucleon, it absorbs its baryonic charge as a minimum-energy configuration and induces the dissociation of the nucleon into free quarks. In this process (known as the "KKST" process), $\sim$ GeV of energy is released through the emission of 2-3 pions [8]. The KKST process provides a useful way to detect such Q balls. The cross section for interaction is approximately the geometric cross section

$$\sigma_Q \simeq \pi R_Q^2. \tag{15}$$

In gauge-mediated models with flat scalar potentials, the Q ball mass and radius are given by

$$M_Q \sim m_F Q^{3/4}, \quad R_Q \sim m_F^{-1} Q^{1/4}, \tag{16}$$

where m_F is related to the scale of supersymmetry breaking (messenger scale). The condition $M_Q/Q < m_p$ ensures that the Q ball is stable against decay to nucleons [8].

Note that a sufficiently massive Q ball will become a black hole if the Q ball radius is less than the Schwarzschild radius $R_Q \lesssim R_s \sim GM_Q$. In the model described above, this translates into the condition

$$m_F \left(\frac{M_{\mathrm{pl}}}{m_F}\right)^3 \lesssim m_Q, \quad \left(\frac{M_{\mathrm{pl}}}{m_F}\right)^4 \lesssim Q. \tag{17}$$

For Q ball masses of this order, gravitational interactions become relevant while the KKST interaction ceases to exist.

A. Q ball Explosiveness

We assume that for each Q ball collision, there is equal probability to produce π^0, π^+ and π^- under the constraint of charge conservation. In the notation of Section overview, $n_\pi \sim \mathcal{O}(10)$ pions are released with $\epsilon_\pi \sim$ GeV. The mean distance travelled by a relativistic particle before decaying is $d = \gamma v \tau$. $\gamma \approx 5$, so for neutral pions $d_{\pi^0} \sim 10^{-5}$ cm while for charged pions, $d_{\pi^\pm} \sim 10$ m.

Numerous experiments have studied the effects of $50 - 500$ MeV pions incident upon complex nuclei targets such as carbon. It is found that there is roughly equal cross section of order ~ 0.1 barn

Q ball Explosiveness
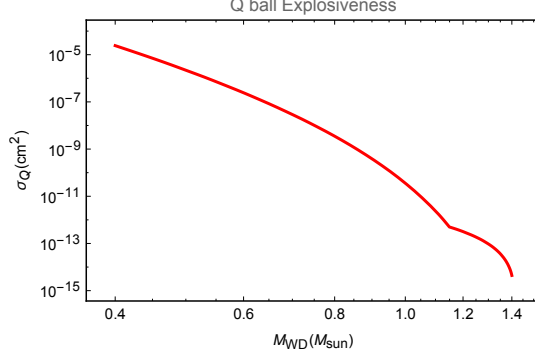
for a (neutral or charged) pion to either scatter elastically, scatter inelastically, or become absorbed with no final state pion [9]. Of these possibilities, pion absorption is the most relevant for energy loss as this will induce a hadronic shower. During this process, $\sim$ 2-3 hadrons (i.e. protons, neutrons, alpha particles) are emitted. Point of this paragraph: argue that KKST range lies within trigger size

Therefore, Q balls with sufficiently large cross-section $\sigma_Q \gtrsim 10^{-4}\lambda_T^2$ will result in explosions. This is plotted in Figure figure. For the model described in Section previous section, the Q ball cross section is related to its mass $m_Q = \sigma_Q^{3/2}m_F^4$ and $Q = \sigma_Q^2 m_F^4$. Therefore, find that

$$m_Q \gtrsim 10^8 \text{ g} \left(\frac{m_F}{\text{TeV}}\right)^4, \quad Q \gtrsim 10^{38}\left(\frac{m_F}{\text{TeV}}\right)^4 \tag{18}$$

is capable of triggering runaway fusion in a heavy $\sim 1.25 M_\odot$ WD for which $\lambda_T \sim 10^5$ cm.

B. Q ball Constraints

The transit rate for an ultra-heavy DM state through a WD is given by

$$\Gamma_{\text{DM}} = n_Q \sigma_g v \sim \frac{\rho_{\text{DM}}}{m_{\text{DM}}}\pi R_s^2 \left(\frac{v_{\text{esc}}^2}{v}\right), \tag{19}$$

where $v \sim 10^{-3}$ is the virial velocity of DM and ρ_{DM} is the energy density of DM in the region of interest. σ_g denotes the capture cross-section including the gravitational Sommerfeld enhancement.

The existence of heavy white dwarfs can rule out Q ball DM. Considering a $1.25 M_\odot$ WD in the local dark matter halo $\rho_{\text{DM}} \sim 0.3$ GeV/cm^3, we find that a Q ball of mass $m_Q \lesssim 10^{20}$ g will transit with a rate $\Gamma_Q \gtrsim$ Gyr^{-1}. If we instead consider (recently discovered) heavy white dwarfs in the galactic center $\rho_{\text{DM}} \sim 10^3$ GeV/cm^3, Q balls of mass $m_Q \lesssim 10^{24}$ g will transit within a Gyr.

VII. CONCLUSIONS

A. RUNAWAY FUSION AND DIFFUSIVE COOLING: T_B AND λ_T

Runaway fusion is characterized by the following situation: a region of the star is hot enough to undergo fusion, which releases additional energy that heats the neighboring regions until they can support fusion as well. In this manner the fusing region grows to encompass the star. Now, this process can be stopped - the fusion region is also cooling by a variety of mechanisms, and if any of these processes drain energy from the region faster than it is liberated by fusion, it will quench the nascent fusion chain-reaction.

A local deposition of energy can destroy a white dwarf by initiating a runaway fusion chain reaction. To do this, the energy deposition must heat a region of linear size L inside the star to a temperature T such that L is larger than the *trigger size* λ_T and T is larger than the *fusion temperature* T_F. The fusion temperature is set by the energy required for two ions to overcome their mutual Coulomb barrier, so it is common to all stars of the same composition. For a carbon-oxygen white dwarf, $T_F \sim$ MeV [2]. The trigger size is determined by the efficiency of cooling in the star. The rate at which the heated region L cools will scale as L^2, whereas the energy liberated by fusion within L scales as L^3. The equality of these two rates defines the critical length scale λ_T, below which fusion is quenched. The trigger size therefore depends on the thermal conductivity of the stellar gas and will be sensitive to its number density, varying from star to star. The trigger size as a function of white dwarf mass is computed in [2] and reproduced in figure 1. For white dwarf masses of $0.4 M_\odot$ to $1.4 M_\odot$, the trigger sizes range roughly from 10^{-5} cm to 0.1 cm.

The two conditions above determine if a region of size L and temperate T *immediately* initiate runaway fusion. But what we are truly concerned with is whether a given energy deposit *eventually* initiates runaway fusion. This is an important distinction only if the initial deposit has $T > T_F$ but $L < \lambda_T$. Then, the initial thermal evolution will be dominated by diffusive cooling and the temperature will diffuse outward, becoming a heated region of larger L and smaller T. Eventually, the temperature profile will diffuse out to a length scale λ_T, at which point runaway fusion begins if the now cooler temperature is still above T_F. The condition for eventually ignition after heating

a region of size L to temperature T is thus:

$$T > \begin{cases} T_F & \text{if } L > \lambda_T \\ T_F \left(\frac{\lambda_T}{L} \right)^3 & \text{if } L < \lambda_T \end{cases}$$

$$\Rightarrow T > T_F \text{Max} \left[1, \frac{\lambda_T}{L} \right]^3$$

It will be more useful in what follows to phrase this explosion condition in terms of the total energy deposited E instead of the resulting temperature, which differs by a factor of the total number of particles in the heated region, $\sim nL^3$ for a stellar number density n:

$$E \gtrsim nT_F \text{Max} \left[L, \lambda_T \right]^3$$

B. DETAIL STUDY OF HEATING LENGTHS

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