Thus for a heating event characterized by its L_0, $\mathcal{E}_0$, and $T_0 \gtrsim T_f$, there is a *boom condition*:

$$\mathcal{E}_0 \gtrsim \mathcal{E}_{\text{boom}} \cdot \max\left\{1, \frac{L_0}{\lambda_T}\right\}^3. \tag{1}$$

A. Transits

a. Boom Condition. The energy deposited during a continuous heating event such as a DM transit is best described in terms of a linear energy transfer $(dE/dx)_{\text{LET}}$, the kinetic energy of SM particles produced per distance traveled by the DM. If these products have a heating length L_0 then the relevant energy deposit must at minimum be taken as the energy transferred over the transit distance L_0. Of course, we can always choose to consider energy deposits over a longer segment of the DM trajectory. Importantly, as per the general condition (1) such a deposition is *less* explosive unless L_0 is smaller than the trigger size λ_T. Thus, we consider the energy deposited in a transit over the larger of these two length scales. Assuming the energy of the DM is roughly constant over this heating event, the boom condition for transit heating is:

$$\left(\frac{dE}{dx}\right)_{\text{LET}} \gtrsim \frac{\mathcal{E}_{\text{boom}}}{\lambda_T} \cdot \text{Max}\left\{\frac{L_0}{\lambda_T}, 1\right\}^2. \tag{2}$$

The above argument sums the individual energy deposits along the DM trajectory as though they are all deposited simultaneously. This is possible if the DM moves sufficiently quickly so that this energy does not diffuse out of the region of interest before the DM has traversed the region. We therefore require that the diffusion time $\tau_{\text{diff}} \approx 10^{-12}$ s across a heated region at temperature T_f be larger than the DM crossing-time:

$$\tau_{\text{diff}} \sim \frac{L^2}{\alpha(T_f)} \gg \frac{L}{v_{\text{esc}}}, \tag{3}$$

where $\alpha(T)$ is the temperature-dependent diffusivity, and the DM transits at the stellar escape velocity $v_{\text{esc}} \approx 10^{-2}$. This condition is more stringent for smaller regions, so we focus on the smallest region of interest, $L = \lambda_T$. (3) is then equivalent to demanding that the escape speed is greater than the conductive speed of the fusion wave front, $v_{\text{cond}} \sim \alpha(T_f)/\lambda_T$. Numerical calculations of v_{cond} are tabulated in [?], and indeed condition (3) is satisfied for all WD densities.

b. Event Rate: Wind Scenario. The rate of transit events is given by the flux of DM passing through a WD

$$\Gamma_{\text{transit}} \sim \frac{\rho_\chi}{m_\chi} R_{\text{WD}}^2 \left(\frac{v_{\text{esc}}}{v_{\text{halo}}}\right)^2 v_{\text{halo}}, \tag{4}$$

where m_χ is the DM mass, ρ_χ is the local DM density near the WD, and $R_{\text{WD}} \approx 4000$ km is the WD radius. Here $v_{\text{halo}} \sim 10^{-3}$ is galactic virial velocity, and the transit rate contains an $\mathcal{O}(100)$ enhancement due to gravitational focusing.

c. WD Shielding. Runaway fusion only occurs in the degenerate WD interior where thermal expansion is suppressed as a cooling mechanism. The outer layers of the WD, however, are composed of a non-degenerate gas and it is therefore essential that a DM candidate penetrate this layer in order to ignite a SN. We parameterize this by a DM stopping power $(dE/dx)_{\text{SP}}$, the kinetic energy lost by the DM per distance traveled in the non-degenerate layer, and demand that

$$\left(\frac{dE}{dx}\right)_{\text{SP}} \ll \frac{m_\chi v_{\text{esc}}^2}{R_{\text{envelope}}}, \tag{5}$$

where $R_{\text{envelope}} \approx 50$ km is the width of a WD envelope [?]. Note that the DM stopping power in the non-degenerate layer $(dE/dx)_{\text{SP}}$ and the linear energy transfer in the degenerate interior $(dE/dx)_{\text{LET}}$ are possibly controlled by different physics and may have very different numerical values. In addition, a transit heating event satisfying condition (5) will have negligible energy loss over the parametrically smaller trigger size or heating length L_0, validating the boom condition (2).

B. Collisions and Decays

a. Boom Condition. For a point-like DM-DM collision or DM decay event releasing particles of heating length L_0, ignition will occur if the total energy in SM products satisfies condition (1). Such an event will likely result in

both SM and dark sector products, so we parameterize the resulting energy in SM particles as a fraction $f_{\rm SM}$ of the DM mass. For non-relativistic DM, the DM mass is the dominant source of energy and therefore $f_{\rm SM} \lesssim 1$ regardless of the interaction details, although we may well suspect that $f_{\rm SM} \ll 1$ for realistic models. With this parameterization, a single DM-DM collision or DM decay has a boom condition:

$$m_\chi f_{\rm SM} \gtrsim \mathcal{E}_{\rm boom} \cdot \max\left\{\frac{L_0}{\lambda_T}, 1\right\}^3. \tag{6}$$

We are thus sensitive to DM masses $m_\chi \gtrsim 10^{16}$ GeV.

    b. Event Rate: DM Wind. DM with negligible energy loss in the WD medium will traverse the star in $\sim R_{\rm WD}/v_{\rm esc} \approx 0.1$ s and have a number density within the WD enhanced relative to the average galactic density by a factor $(v_{\rm esc}/v_{\rm halo}) \sim 10$. In the wind scenario, the DM-DM collision rate inside the WD parameterized by a cross-section $\sigma_{\chi\chi}$ is:

$$\Gamma_{\rm collision} \sim \left(\frac{\rho_\chi}{m_\chi}\right)^2 \sigma_{\chi\chi} \left(\frac{v_{\rm esc}}{v_{\rm halo}}\right)^3 v_{\rm halo} R_{\rm WD}^3. \tag{7}$$

Similarly the net DM decay rate inside the WD parameterized by a lifetime τ_χ is:

$$\Gamma_{\rm decay} \sim \frac{1}{\tau_\chi} \frac{\rho_\chi}{m_\chi} \left(\frac{v_{\rm esc}}{v_{\rm halo}}\right) R_{\rm WD}^3. \tag{8}$$

    c. Event Rate: DM Capture. For the DM to be captured in a WD, it must have sufficiently strong interactions with stellar constituents:

$$\left(\frac{dE}{dx}\right)_{\rm SP} \gtrsim \frac{m_\chi v^2}{R_{\rm WD}}, \tag{9}$$

where v is the DM galactic virial velocity relative to the WD. For $v \ll v_{\rm esc}$, we demand that the DM lose a fraction $(v/v_{\rm esc})^2$ of its energy to eventually become stopped in the star. Of course, the most probable relative velocity will be $v_{\rm halo}$.

    The physics of DM capture can be made more precise for a specific interaction: the simplest scenario is a spin-independent, elastic scatter off ions characterized by cross section $\sigma_{\chi A}$. For $m_\chi \gg m_{\rm ion}$, the typical momentum transfer in an elastic scatter is $q \sim \mu_A v_{\rm esc} \sim 100$ MeV, where μ_A is the reduced mass of the DM-ion system. The typical energy transfer in a nuclear elastic scatter is simply $q^2/m_{\rm ion} \sim m_{\rm ion} v_{\rm esc}^2 \approx$ MeV. If the DM has an initial relative velocity with the WD $v \approx v_{\rm halo}$, then DM masses $m_\chi \gtrsim 10^3$ GeV will require multiple scatters to lose the necessary fraction of its energy. The stopping power of the DM with this interaction is

$$\left(\frac{dE}{dx}\right)_{\rm SP} \sim n_{\rm ion} \sigma_{\chi A} m_{\rm ion} v^2. \tag{10}$$

Demanding (9) yields a condition on the DM elastic nuclear cross section sufficient for capture:

$$\sigma_{\chi A} \gtrsim \left(\frac{m_\chi}{m_{\rm ion}}\right) \left(\frac{v}{v_{\rm esc}}\right)^2 \frac{1}{n_{\rm ion}} \frac{1}{R_{\rm WD}}. \tag{11}$$

Evidently, the assumption that the scatters responsible for slowing the DM are not sufficient to blow up the WD (2) is a valid one if the cross section satisfies

$$\sigma_{\chi A} < \left(\frac{\mathcal{E}_{\rm boom}}{\lambda_T}\right) \left(\frac{1}{m_{\rm ion} v_{\rm esc}^2}\right) \left(\frac{1}{n_{\rm ion}}\right) \sim 10^{-8} \text{ cm}^2, \tag{12}$$

evaluated at a WD density $n_{\rm ion} \sim 10^{32} {\rm cm}^{-3}$. Since the momentum transfer q is of order the inverse nuclear size, it is reasonable to expect the DM coherently scatters off all nucleons. Therefore, the average per-nucleon cross section (spin-independent)

$$\sigma_{\chi n} = A^2 \left(\frac{\mu_A}{\mu_n}\right)^2 F^2(q) \sigma_{\chi A} \sim A^4 F^2(q) \sigma_{\chi A}, \tag{13}$$

where $F^2(q) \approx 0.2$ is the Helm form factor, calculated based on the analytic expression in [?]. Therefore, the cross section needed for capture (11) can be compared to limits from direct detection experiments. Currently, the most stringent bound on DM-nuclear elastic scatters comes from XENON 1T:

$$\sigma_{\chi n} < 10^{-45} \text{ cm}^2 \left(\frac{m_\chi}{10^3 \text{ GeV}} \right). \tag{14}$$

First we review the evolution of DM within the star once it has been captured. The DM eventually thermalizes at a velocity

$$v_{\text{th}} \sim \sqrt{\frac{T}{m_\chi}} \approx 10^{-12} \left(\frac{10^{16} \text{ GeV}}{m_\chi} \right)^{1/2}, \tag{15}$$

where $T \sim$ keV is the WD temperature. At this point the DM will accumulate at the thermal radius where its kinetic energy balances against the gravitational potential energy of the enclosed WD mass:

$$R_{\text{th}} \sim \left(\frac{T}{G m_\chi \rho_{\text{WD}}} \right)^{1/2} \tag{16}$$

$$\approx 0.1 \text{ cm} \left(\frac{10^{16} \text{ GeV}}{m_\chi} \right)^{1/2} \left(\frac{10^{31} \text{ cm}^{-3}}{n_{\text{ion}}} \right)^{1/2},$$

where we have assumed a constant WD density $\rho_{\text{WD}} \sim n_{\text{ion}} m_{\text{ion}}$ within R_{th}. DM will collect at this radius until its total mass exceeds the WD mass within R_{th},

$$M_{\text{crit}} \sim \rho_{\text{WD}} R_{\text{th}}^3 \tag{17}$$

$$\approx 10^{29} \text{ GeV} \left(\frac{10^{16} \text{ GeV}}{m_\chi} \right)^{3/2} \left(\frac{10^{31} \text{ cm}^{-3}}{n_{\text{ion}}} \right)^{1/2}.$$

The DM cloud will then begin self-gravitational collapse. The exact nature of this collapse is model-dependent, eventually being arrested by DM-DM interactions or the formation of a black hole. For composite DM, it is reasonable to suspect that the collapse stabilizes into a core of radius R_{sta} larger than the Schwarzschild radius due to some unknown physics.

There are several timescales relevant to the DM capture scenario. Initially there is a timescale for DM to slow down to the thermal velocity v_{th} and settle to the thermal radius. If the DM thermalizes rapidly in the star e.g. via inelastic scatters, this time is at most

$$t_{\text{drift}} \lesssim \frac{R_{\text{WD}}}{v_{\text{th}}} \approx 50 \text{ yr} \left(\frac{m_\chi}{10^{16} \text{ GeV}} \right)^{1/2}. \tag{18}$$

However, the settle time can be explicitly calculated for a given interaction. Consider the case of DM-nuclear elastic scatters.

Once formed, the DM cloud at R_{th} will continue to collect DM until it reaches a critical core mass (17) in a time

$$t_{\text{collect}} \sim \left(\frac{M_{\text{crit}}}{m_\chi} \right) \frac{1}{\Gamma_{\text{transit}}} \tag{19}$$

$$\approx 10 \text{ s} \left(\frac{10^{16} \text{ GeV}}{m_\chi} \right)^{3/2} \left(\frac{0.4 \text{ GeV/cm}^3}{\rho_\chi} \right),$$

evaluated at $n_{\text{ion}} \sim 10^{31} \text{ cm}^{-3}$. Note that t_{collect} has a non-trivial dependence on WD density: this is manifest in the values for v_{esc} and R_{WD}. Subsequently, the actual timescale for the collapse is independent of DM mass:

$$t_{\text{collapse}} \sim \frac{R_{\text{th}}}{v_{\text{th}}} \approx 3 \text{ s} \left(\frac{10^{31} \text{ cm}^{-3}}{n_{\text{ion}}} \right)^{1/2}. \tag{20}$$

This expression is valid for the two cases that the DM core either initially remains thermalized while self-gravitating or collapses at free-fall velocity.

We now turn towards the rate of explosive collisions and decays for captured DM. Of course, if the slowing of DM itself is sufficient to ignite a SN, then the boom condition is instead given by the transit condition (2). Assuming this is not the case, we calculate the rate of DM events during the various stages of the capture described above.

For decay heating, capture gives an enhancement due to the increased number of DM particles within the WD. This can be very large if the DM core admits decays, however it is still significantly enhanced over the wind scenario even for inert cores (as in the case that the DM forms a black hole). We have an enhancement of the net decay rate (8) by a factor

$$\frac{v_{\text{esc}}}{v_{\text{th}}} \approx 10^{10} \left(\frac{m_\chi}{10^{16} \text{ GeV}} \right)^{1/2} \tag{21}$$

due to the increased time spent by the DM in the WD medium before joining the inert core.

In the case of DM-DM collision heating, it is possible that the collapse of the core will induce an ignition event due to the enhancement of DM number density during the collapse. This would set the lifetime of WDs to t_{collect}. During the collapse, the rate of collisions taking place at a radius r within the enclosed volume is given by

$$\Gamma_{\text{collision}}(r) \sim \left(\frac{M_{\text{core}}}{m_\chi} \right)^2 \frac{1}{r^3} \sigma_{\chi\chi} v(r), \tag{22}$$

where $v(r)$ is the velocity of DM - this could be either free-fall velocity or v_{th} if the DM remains thermalized. Integrating to the stable radius R_{sta}, we find the total number of collisions during the collapse is

$$N_{\text{col}} \sim \left(\frac{M_{\text{core}}}{m_\chi} \right)^2 \frac{\sigma_{\chi\chi}}{R_{\text{sta}}^2}. \tag{23}$$

Assuming the collapse proceeds until the DM core becomes a black hole, the number of collisions is

$$N_{\text{col}} \sim \frac{\sigma_{\chi\chi}}{G^2 m_\chi^2}. \tag{24}$$

If the collapse itself is not explosive, there is still an enhanced collision rate relative to the wind scenario due to DM colliding while in-falling to the core. Again we look at the conservative situation of an inert core - the rate is obviously much greater if the core is stabilized in a fluid state which admits DM-DM collisions. The rate of in-falling collisions is enhanced over the wind collision rate (7) by a factor

$$\frac{R_{\text{WD}}}{R_{\text{sta}}} \times \frac{v_{\text{esc}}}{v_{\text{th}}}, \tag{25}$$

which again depends on the physics of R_{sta}.

d. Event Rate: DM Capture and Multiple Collisions Up until now, we only considered the case that a single DM-DM collision releases sufficient energy (6) in order to trigger runaway fusion. However, the possibility of a DM core collapse in the star provides up an interesting alternative: