A. Transits

a. Ignition Condition. Runaway fusion only occurs in the degenerate WD interior where thermal expansion is suppressed as a cooling mechanism. The outer layers of the WD, however, are composed of a non-degenerate gas and it is therefore essential that a DM candidate penetrate this layer in order to ignite a SN. We parameterize this by a DM stopping power $(dE/dx)_{\text{SP}}$, the kinetic energy lost by the DM per distance traveled in the non-degenerate layer, and demand that

$$\left(\frac{dE}{dx}\right)_{\text{SP}} \ll \frac{m_\chi v_{\text{esc}}^2}{R_{\text{envelope}}}, \tag{1}$$

where $R_{\text{envelope}} \approx 50$ km is the width of a WD envelope [?].

The energy deposited during a continuous heating event such as a DM transit is best described in terms of a linear energy transfer $(dE/dx)_{\text{LET}}$, the kinetic energy of SM particles produced per distance traveled by the DM. If these products have a heating length L_0 then the relevant energy deposit must at minimum be taken as the energy transferred over the transit distance L_0. Of course, we can always choose to consider energy deposits over a longer segment of the DM trajectory. Importantly, as per the general condition (**??**) such a deposition is *less* explosive unless L_0 is smaller than the trigger size λ_T. Thus, we consider the energy deposited in a transit over the larger of these two length scales. Assuming the energy of the DM is roughly constant over this heating event, the ignition condition for transit heating is:

$$\left(\frac{dE}{dx}\right)_{\text{LET}} \gtrsim \frac{\mathcal{E}_{\text{boom}}}{\lambda_T} \cdot \text{Max}\left\{\frac{L_0}{\lambda_T}, 1\right\}^2. \tag{2}$$

Note that the DM stopping power in the non-degenerate layer $(dE/dx)_{\text{SP}}$ and the linear energy transfer in the degenerate interior $(dE/dx)_{\text{LET}}$ are possibly controlled by different physics and may have very different numerical values. In addition, a transit heating event satisfying condition (1) will have negligible energy loss over the parametrically smaller trigger size or heating length L_0, validating (2).

The above argument sums the individual energy deposits along the DM trajectory as though they are all deposited simultaneously. This is possible if the DM moves sufficiently quickly so that this energy does not diffuse out of the region of interest before the DM has traversed the region. We therefore require that the diffusion time $\tau_{\text{diff}} \approx 10^{-12}$ s across a heated region at temperature T_f be larger than the DM crossing-time:

$$\tau_{\text{diff}} \sim \frac{L^2}{\alpha(T_f)} \gg \frac{L}{v_{\text{esc}}}, \tag{3}$$

where $\alpha(T)$ is the temperature-dependent diffusivity, and the DM transits at the stellar escape velocity $v_{\text{esc}} \sim 10^{-2}$. This condition is more stringent for smaller regions, so we focus on the smallest region of interest, $L = \lambda_T$. (3) is then equivalent to demanding that the escape speed is greater than the conductive speed of the fusion wave front, $v_{\text{cond}} \sim \alpha(T_f)/\lambda_T$. Numerical calculations of v_{cond} are tabulated in [?], and indeed condition (3) is satisfied for all WD densities.

b. Event Rate: Wind Scenario. The rate of transit events is given by the flux of DM passing through a WD

$$\Gamma_{\text{trans}} \sim \frac{\rho_\chi}{m_\chi} R_{\text{WD}}^2 \left(\frac{v_{\text{esc}}}{v_{\text{halo}}}\right)^2 v_{\text{halo}}, \tag{4}$$

where m_χ is the DM mass, ρ_χ is the DM density in the region of the WD, and R_{WD} is the WD radius. Here $v_{\text{halo}} \sim 10^{-3}$ is the galactic virial velocity, and the transit rate contains an $(v_{\text{esc}}/v_{\text{halo}})^2 \sim 100$ enhancement due to gravitational focusing.

B. Collisions and Decays

a. Ignition Condition. For a point-like DM-DM collision or DM decay event releasing particles of heating length L_0, ignition will occur if the total energy in SM products satisfies condition (**??**). Such an event will likely result in both SM and dark sector products, so we parameterize the resulting energy in SM particles as a fraction f_{SM} of the DM mass. For non-relativistic DM, the DM mass is the dominant source of energy and therefore $f_{\text{SM}} \lesssim 1$ regardless of the interaction details. With this parameterization, a single DM-DM collision or DM decay has an ignition condition:

$$m_\chi f_{\text{SM}} \gtrsim \mathcal{E}_{\text{boom}} \cdot \max\left\{\frac{L_0}{\lambda_T}, 1\right\}^3. \tag{5}$$

We are thus sensitive to DM masses $m_\chi \gtrsim 10^{16}$ GeV.

However, there is the possibility if DM is captured in the WD that allows collisions of lower mass DM to ignite the star. Multiple DM-DM collisions in a sufficiently small region can occur rapidly enough to be counted as a single heating event. This is similar in nature to a transit heating event, where multiple scatters across a transit length λ_T can release an energy $\mathcal{E}_{\rm boom}$ and satisfy (2) even if any individual scatter is not explosive by itself. If a single DM-DM collision is unable to ignite the star, the sum total of the energy released in many collisions can still result in a SN if

$$m_\chi f_{\rm SM} \gtrsim \frac{\mathcal{E}_{\rm boom}}{N_{\rm mult}} \cdot \max\left\{\frac{L_0}{\lambda_T}, 1\right\}^3, \qquad N_{\rm mult} \gtrsim 1, \tag{6}$$

We define $N_{\rm mult}$ as the number of collisions happening within a region of size $\max\{\lambda_T, L_0\}^3$ (or smaller) during a diffusion time $\tau_{\rm diff}$. This necessarily depends on the DM-DM collision cross section, the DM-SM scattering cross section, and the evolution of the captured DM in the star. These are discussed in detail below.

b. Event Rate: DM Wind. For the remainder of this section, all numerical quantities are evaluated assuming a WD lifetime $\tau_{\rm WD} \sim 5$ Gyr and central WD density $n_{\rm ion} \sim 10^{31}$ cm^{-3}. At this density, the relevant WD parameters are approximately:

$$M_{\rm WD} \approx 1.25\ M_\odot, \qquad R_{\rm WD} \approx 4000\ \text{km}, \qquad v_{\rm esc} \approx 2 \times 10^{-2}. \tag{7}$$

We also assume a typical WD temperature $T \sim$ keV.

DM that is not captured traverses the WD in $R_{\rm WD}/v_{\rm esc} \approx 0.1$ s, and the rate of DM-DM collisions within the WD parameterized by cross-section $\sigma_{\chi\chi}$ is:

$$\Gamma_{\rm ann} \sim \left(\frac{\rho_\chi}{m_\chi}\right)^2 \sigma_{\chi\chi} \left(\frac{v_{\rm esc}}{v_{\rm halo}}\right)^3 v_{\rm halo} R_{\rm WD}^3. \tag{8}$$

Similarly the net DM decay rate inside the WD parameterized by a lifetime τ_χ is:

$$\Gamma_{\rm decay} \sim \frac{1}{\tau_\chi}\frac{\rho_\chi}{m_\chi}\left(\frac{v_{\rm esc}}{v_{\rm halo}}\right) R_{\rm WD}^3. \tag{9}$$

c. Event Rate: DM Capture. For the DM to be captured in a WD, it must lose energy $\sim m_\chi v^2$, where v is the relative DM velocity (in the rest frame of the WD) asymptotically far away. Properly, this DM velocity is described by a (boosted) Maxwell distribution peaked at the galactic virial velocity $v_{\rm halo} \sim 10^{-3}$. Since typically $v \ll v_{\rm esc}$, the DM has initial velocity $v_{\rm esc}$ in the star and must lose a fraction $(v/v_{\rm esc})^2$ of its energy to become captured.

The physics of DM capture can be made more precise for a specific interaction. Consider a spin-independent, elastic scattering off ions characterized by cross section $\sigma_{\chi A}$. Assuming $m_{\rm ion} \ll m_\chi$, the typical momentum transfer in an elastic scatter is $q \sim \mu_A v_{\rm esc} \approx 200$ MeV, where μ_A is the reduced mass of the DM-nuclei system. This corresponds to an energy transfer $q^2/m_{\rm ion} \sim m_{\rm ion} v_{\rm esc}^2 \approx 10$ MeV. The average number of DM scatters during a full transit of the WD is simply a ratio of the mean free path to the size of the WD

$$N_{\rm scat} \sim n_{\rm ion} \sigma_{\chi A} R_{\rm WD}. \tag{10}$$

If $N_{\rm scat} < 1$, then $N_{\rm scat}$ is the probability for a *single* scatter to occur during the transit. Thus, DM with initial velocites less than

$$v_{\rm cap}^2 \sim v_{\rm esc}^2 \left(\frac{m_{\rm ion}}{m_\chi}\right) \max\{N_{\rm scat}, 1\}. \tag{11}$$

will be captured in the WD. The full calculation of the rate of DM capture [**?**] yields

$$\Gamma_{\rm cap} \sim \Gamma_{\rm trans} \cdot \min\{N_{\rm scat}, 1\} \left(\frac{v_{\rm cap}}{v_{\rm halo}}\right)^2. \tag{12}$$

This is assuming that $v_{\rm cap} < v_{\rm halo}$, otherwise the capture rate is simply $\Gamma_{\rm cap} \sim \Gamma_{\rm trans}$. Evidently, the assumption that the scatters responsible for slowing the DM are not sufficient to blow up the WD (2) is a valid one for cross sections

$$\sigma_{\chi A} < \left(\frac{\mathcal{E}_{\rm boom}}{\lambda_T}\right)\left(\frac{1}{m_{\rm ion} v_{\rm esc}^2}\right)\left(\frac{1}{n_{\rm ion}}\right) \approx 10^{-8}\ \text{cm}^2, \tag{13}$$

Since the momentum transfer q is roughly of order the inverse nuclear size, it is reasonable to expect the DM coherently scatters off all nucleons in the nucleus. Indeed, the average per-nucleon cross section (spin-independent) is

$$\sigma_{\chi A} = A^2 \left(\frac{\mu_A}{\mu_n}\right)^2 F^2(q)\sigma_{\chi n}, \tag{14}$$

where $F^2(q) \approx 0.1$ is the Helm form factor [?]. We can compare the cross section sufficient for capture (11) to the limits from direct detection experiments. Currently, the bound on spin-independent DM nuclear elastic scatters from XENON 1T is

$$\sigma_{\chi n} < 10^{-45}\ \mathrm{cm}^2 \left(\frac{m_\chi}{10^3\ \mathrm{GeV}}\right). \tag{15}$$

We now review the evolution of DM within the star once it has been captured. The DM slowly thermalizes to an average velocity

$$v_{\mathrm{th}} \sim \sqrt{\frac{T}{m_\chi}} \approx 10^{-12} \left(\frac{10^{16}\ \mathrm{GeV}}{m_\chi}\right)^{1/2}. \tag{16}$$

and settles at its thermal radius

$$R_{\mathrm{th}} \sim \left(\frac{T}{G m_\chi \rho_{\mathrm{WD}}}\right)^{1/2} \approx 0.1\ \mathrm{cm} \left(\frac{10^{16}\ \mathrm{GeV}}{m_\chi}\right)^{1/2} \tag{17}$$

where its kinetic energy balances against the gravitational potential energy of the enclosed WD mass. Here we assume for simplicity a constant WD density $\rho_{\mathrm{WD}} \sim n_{\mathrm{ion}} m_{\mathrm{ion}}$ within R_{th}. Of course, the timescale for captured DM to settle depends on the nature of the DM-SM interaction. This has been explicitly calculated in the case that the DM loses energy via elastic nuclear scatters, see [?]. First, the DM passes through the WD many times before the size of its orbit becomes fully contained within the star. This occurs after a time

$$t_1 \sim \left(\frac{m_\chi}{m_{\mathrm{ion}}}\right)^{3/2} \frac{R_{\mathrm{WD}}}{v_{\mathrm{esc}}} \frac{1}{N_{\mathrm{scat}}} \frac{1}{\max\{N_{\mathrm{scat}}, 1\}^{1/2}} \approx 2 \times 10^3\ \mathrm{yr} \left(\frac{m_\chi}{10^{10}\ \mathrm{GeV}}\right)^{3/2} \left(\frac{10^{-38}\ \mathrm{cm}^2}{\sigma_{\chi A}}\right)^{3/2}. \tag{18}$$

Note that this stage is relevant only if the energy loss after a single transit is does not exceed $\sim m_\chi v_{\mathrm{esc}}^2$:

$$\left(\frac{m_\chi}{m_{\mathrm{ion}}}\right) \max\{N_{\mathrm{scat}}, 1\} < 1. \tag{19}$$

This is the case for any cross sections which satisfy the XENON bound (15). Subsequently, the DM completes many orbits within the star until dissipation from elastic scatters reduces the orbital size to the thermal radius. This occurs after a characteristic time

$$t_2 \sim \left(\frac{m_\chi}{m_{\mathrm{ion}}}\right) \frac{1}{n_{\mathrm{ion}}\sigma_{\chi A}} \frac{1}{v_{\mathrm{ion}}} \approx 30\ \mathrm{yr} \left(\frac{m_\chi}{10^{10}\ \mathrm{GeV}}\right) \left(\frac{10^{-38}\ \mathrm{cm}^2}{\sigma_{\chi A}}\right). \tag{20}$$

where $v_{\mathrm{ion}} \sim \sqrt{\frac{T}{m_{\mathrm{ion}}}}$ is the thermal velocity of ions. For our purposes, we simply require that the settling time, which is an initial off-set time before steady collection, is shorter than the age of the WD

$$t_1 + t_2 < \tau_{\mathrm{WD}}. \tag{21}$$

The settling DM constitutes a number density of DM throughout the WD volume as well as outside the star. We can compare the total rate of annihilations of infalling DM to the rate of DM capture. This annihilation rate is dominated by the DM density inside the star with orbits of order the thermal radius

$$\Gamma_{\mathrm{infall}} \sim \frac{\Gamma_{\mathrm{cap}}^2 \sigma_{\chi\chi}}{R_{\mathrm{th}} v_{\mathrm{th}}}. \tag{22}$$

Thus, depletion of the infalling DM can be ignored as long as

$$\Gamma_{\mathrm{infall}} < \Gamma_{\mathrm{cap}}. \tag{23}$$

For the rest of this section we will evaluate all numerical quantities assuming efficient capture of the DM, i.e. $\Gamma_{\rm cap} \sim \Gamma_{\rm trans}$. As such, condition (23) is independent of m_χ and yields an upper bound on the cross section $\sigma_{\chi\chi} < 10^{-13}$ cm^2.

After a settling time has passed, DM will be steadily accumulating at the thermal radius $R_{\rm th}$. If (23) is satisfied, the accumulation rate is roughly the same as the capture rate. However, this density of accumulating DM is also depleting due to annihilations. Eventually, these two rates become comparable and there is an equilibrium number of DM particles

$$N_{\rm eq} \sim \left(\frac{\Gamma_{\rm cap} R_{\rm th}^3}{\sigma_{\chi\chi} v_{\rm th}}\right)^{1/2} \approx 10^{19} \left(\frac{10^{16}~{\rm GeV}}{m_\chi}\right) \left(\frac{10^{-30}~{\rm cm}^2}{\sigma_{\chi\chi}}\right)^{1/2} \left(\frac{\rho_\chi}{0.4~{\rm GeV/cm}^3}\right)^{1/2}. \tag{24}$$

Of course, there is no guarantee that this equilibrium is achieved within the age of the WD. In that case, annihilations can be ignored and the total number of DM particles accumulated is simply

$$N_{\rm life} \sim \Gamma_{\rm cap} \tau_{\rm WD} \approx 10^{29} \left(\frac{10^{16}~{\rm GeV}}{m_\chi}\right) \left(\frac{\rho_\chi}{0.4~{\rm GeV/cm}^3}\right) \tag{25}$$

As expected, the total *mass* of DM that the WD can possibly accumulate $N_{\rm life} m_\chi \sim 10^{45}$ GeV is independent of m_χ. However, if the collected mass of DM at the thermal radius ever exceeds the WD mass within this volume, then there is the possibility of self-gravitational collapse of the DM. The critical number of DM particles needed for collapse is given by

$$N_{\rm crit} \sim \frac{\rho_{\rm WD} R_{\rm th}^3}{m_\chi} \approx 10^{12} \left(\frac{10^{16}~{\rm GeV}}{m_\chi}\right)^{5/2}. \tag{26}$$

This can only be achieved if the time to collect a critical mass of DM is shorter than the time for annihilations to deplete this mass sufficiently *and* shorter than the WD lifetime. Thus the condition for a collapse is:

$$N_{\rm crit} < N_{\rm eq}, \qquad N_{\rm crit} < N_{\rm life}. \tag{27}$$

Importantly, we see that DM masses less than $\sim 10^6$ GeV do not have enough time within the age of the WD to collect a number $N_{\rm crit}$ and begin a collapse. At a given radius r, the time it takes for the DM to free-fall an $\mathcal{O}(1)$ fraction of this distance is roughly

$$t_{\rm ff} \sim \frac{r}{v_{\rm ff}}, \qquad v_{\rm ff} \sim \sqrt{\frac{G N m_\chi}{r}}. \tag{28}$$

Thus, the timescale for self-gravitational collapse at the thermal radius is independent of DM mass:

$$t_{\rm col} \sim \frac{R_{\rm th}}{v_{\rm th}} \sim \sqrt{\frac{1}{G\rho_{\rm WD}}} \approx 0.1~{\rm s}. \tag{29}$$

Of course, it is possible that the DM initially remains thermalized while collapsing due to sufficiently strong DM-SM interactions. In the case of elastic nuclear scatters the DM loses a fraction $\sim m_{\rm ion}/m_\chi$ of its energy per collision, so the DM is free-falling at the thermal radius as long as

$$\sigma_{\chi A} \lesssim \frac{1}{n_{\rm ion} R_{\rm th}} \left(\frac{m_\chi}{m_{\rm ion}}\right) \approx 10^{-30}~{\rm cm}^2 \left(\frac{m_\chi}{10^6~{\rm GeV}}\right)^{3/2}. \tag{30}$$

It is straightforward to see that this is the case for any cross sections which satisfy the XENON bound (15). Annihilations in the collapsing DM density become significant when the free-fall time is of order the time for a single DM to annihilate. This occurs at the characteristic radius

$$R_{\chi\chi} \sim \sqrt{N_{\rm crit} \sigma_{\chi\chi}} \approx 10^{-9}~{\rm cm} \left(\frac{10^{16}~{\rm GeV}}{m_\chi}\right)^{5/4} \left(\frac{\sigma_{\chi\chi}}{10^{-30}~{\rm cm}^2}\right)^{1/2}. \tag{31}$$

As a check of consistency, such a collapse of the accumulated DM in the WD is only sensible if

$$R_{\chi\chi} < R_{\rm th}, \tag{32}$$

which is trivially satisfied if both (23) and (27) are true. The number (and mass) of collapsing DM is depleting by an $\mathcal{O}(1)$ fraction at a distance $R_{\chi\chi}$, while below this radius the number is determined by:

$$\frac{dN(r)}{dr} \sim \frac{N(r)^2}{r^3}\sigma_{\chi\chi}. \tag{33}$$

Of course the enclosed WD mass is also dropping by $M_{\rm WD}(r) \propto r^3$ during the collapse, so if $N(r)$ depletes as a stronger function of radius then the collapse will halt below $R_{\chi\chi}$. Note that the number of DM particles that are initially collapsing can be greater than $N_{\rm crit}$ if the capture rate is sufficiently large. This is only feasible if captured DM is able to pass through the thermal radius even before sufficiently slowing down to the thermal velocity, as is the case for elastic scatters. Once the accumulated DM reaches $N_{\rm crit}$, evolution of the DM profile can either be collapse or further collection. The later occurs if the collapse time $t_{\rm col}$ is greater than time needed collect a critical number of DM particles $\sim N_{\rm crit}/\Gamma_{\rm cap}$. For efficient capture $\Gamma_{\rm cap} \sim \Gamma_{\rm trans}$ and a local DM density $\rho_\chi \sim 0.4$ GeV/cm^3, this takes place if the DM mass is greater than $\sim 10^{17}$ GeV. In this scenario, the DM cloud at $R_{\rm th}$ will continue to collect more DM until a saturated number $N_{\rm sat} > N_{\rm crit}$ at which the timescale for free-fall matches the timescale for collection.

$$\frac{R_{\rm th}}{\sqrt{\frac{GN_{\rm sat}m_\chi}{R_{\rm th}}}} \sim \frac{N_{\rm sat}}{\Gamma_{\rm cap}} \Rightarrow N_{\rm sat} \sim N_{\rm crit}^{\frac{1}{3}}(\Gamma_{\rm cap}t_{\rm col})^{\frac{2}{3}}. \tag{34}$$

Therefore, the initial number of collapsing DM particles is simply $\sim \max\{N_{\rm crit}, N_{\rm sat}\}$.

There are two potential evolutions of the captured DM: either the DM collapses or it does not. In the later case, either the DM has reached its equilibrium number at the thermal radius or is still continuing to accumulate, not yet having the critical mass necessary for collapse within its lifetime:

$$\min\{N_{\rm eq}, N_{\rm life}\} < N_{\rm crit}. \tag{35}$$

First we see if this scenario allows for any meaningful constraints. The number of collisions that can be counted as a single heating event is roughly

$$N_{\rm multi} \sim \left(\frac{\min\{N_{\rm eq}, N_{\rm life}\}}{R_{\rm th}^3}\right)^2 \sigma_{\chi\chi} v_{\rm th} \max\{\lambda_T, L_0\}^3 \tau_{\rm diff}. \tag{36}$$

Even in the "best-case" scenario of efficient capture and $L_0 \sim \lambda_T$, we find there is no parameter space $\{m_\chi, \sigma_{\chi\chi}\}$ where both (35) and (6)—with $N_{\rm multi}$ given by (36)—are simultaneously satisfied.

We instead turn our attention to collapsing DM, characterized by (27). Of course, the number of collisions $N_{\rm multi}$ that can be counted as a single heating event depends on where we examine the collapse. In general, this is given as an integral of the annihilation rate

$$N_{\rm multi} \sim \int \left(\frac{N}{r^3}\right)^2 \sigma_{\chi\chi} \min\{L_{\rm heat}, r\}^3 dr, \qquad L_{\rm heat} \equiv \max\{\lambda_T, L_0\} \tag{37}$$

integrating over the distance fell within a fixed time interval $\tau_{\rm diff}$. The expectation is that there exists an optimal value of the lower radius at which $N_{\rm multi}$ is maximized. We denote this as R_*. However, even without knowing the details of this optimum choice, we can calculate (37) by considering the following limits. If the free-fall time (28) at a distance of order R_* is much larger than the diffusion time, the annihilation rate can be approximated as constant over a time $\tau_{\rm diff}$. If this free-fall time is instead much smaller than the diffusion time, the annihilation rate is a rapidly increasing function over the interval $\tau_{\rm diff}$. Therefore, (37) is approximated by the peak value of the annihilation rate (which is maximized at R_*) multiplied by the time spent at this peak (which is the time to free-fall $\sim R_*$). Considering both these possibilities, the maximum value of (37) is of the form:

$$N_{\rm multi} \sim \left(\frac{N}{R_*^3}\right)^2 \sigma_{\chi\chi} v_{\rm ff} \min\{L_{\rm heat}, R_*\}^3 \min\left\{\tau_{\rm diff}, \frac{R_*}{v_{\rm ff}}\right\}, \tag{38}$$

The questions is: what is R_*? Ultimately, the answer depends on the parameters m_χ and $\sigma_{\chi\chi}$. Suppose $\sigma_{\chi\chi}$ is independent of velocity or position. In this case, the scaling is such that $N_{\rm multi}$ is maximized at the smaller of the two scales $R_* \sim \min\{R_{\chi\chi}, L_{\rm heat}\}$. However, there may be some stabilizing pressure which prevents the DM from collapsing below a certain radius. This is reasonable to expect in the case of composite DM, although such a stable radius would depend on unknown physics. Famously, gravity itself provides such a "pressure", arresting collapses below the Schwarzschild radius by the formation of a black hole:

$$R_{\rm BH} \sim GN_{\rm crit}m_\chi \approx 5 \times 10^{-24}\ {\rm cm}\left(\frac{10^{16}\ {\rm GeV}}{m_\chi}\right)^{3/2}. \tag{39}$$

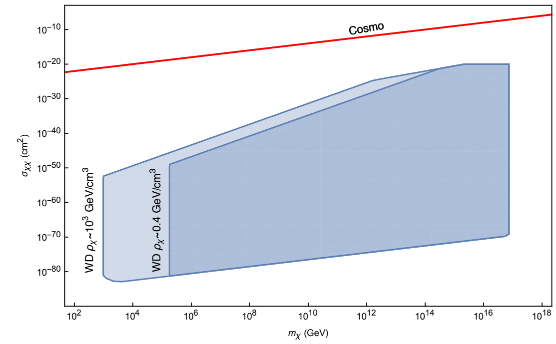

FIG. 1: Constraints on DM-DM annihilation cross-section into SM particles which deposit their energy compactly within a trigger size λ_T during self-gravitational collapse in a WD. Bounds come from observation of a single 1.25 $M_\odot$ WD assuming efficient capture of the DM ($\Gamma_{\rm cap} \sim \Gamma_{\rm trans}$). We also assume the DM collapse is stabilized by formation of a BH.

Of course, this choice of radius will necessarily change for a specific model that relates $\sigma_{\chi\chi}$ to velocity in some way. For instance if $\sigma_{\chi\chi} \propto 1/v$ then the optimum radius is instead just $R_* \sim R_{\chi\chi}$. For the sake of simplicity, we choose to examine the collapse at a radius

$$R_* = \max\{R_{\chi\chi}, R_{\rm BH}\}. \tag{40}$$

Given this choice, one can check that $N_{\rm multi}$ is at most $N_{\rm crit}$, the total number of DM particles collapsing, as expected.

C. Constraints

We now turn towards constraints on DM interactions in the capture scenario. In Figure 2, we show the constraints on DM-DM annihilation cross section $\sigma_{\chi\chi}$ into SM particles during self-gravitational collapse in a WD, i.e. multiple collisions. We have assumed that the DM is efficiently captured, $\Gamma_{\rm cap} \sim \Gamma_{\rm trans}$. The bounds in Figure 2 are valid for any SM annihilation products which deposit their energy compactly upon release within a trigger size λ_T. The plotted region is the set of all paramteres which satisfy (27) and (6)—-with $N_{\rm multi}$ given by (38)—assuming that the DM collapse is stabilized by formation of a BH. One can check that the additional conditions (23) and (32) are also satisfied for such parameters. We can qualitatively understand the range of excluded cross sections as follows: If $\sigma_{\chi\chi}$ is too large, then the DM will effectively deplete before reaching the critical number needed for collapse. However, $\sigma_{\chi\chi}$ cannot be arbitrarily small while still satisfying (6).

Of course, the constraints on multiple DM collisions in a collapse can be made concrete for a specified DM interaction. As in Section **??**, we consider the case where the DM is captured in a WD via elastic scatters. Suppose the DM scatters off nuclear targets through Z boson exchange, with a per-nucleon cross section

$$\sigma_{\chi n} \sim \frac{G_F^2 \mu_{\chi n}^2}{2\pi} Y^2 \left[\frac{(A-Z)-(1-4\sin^2\theta_W)Z}{A}\right]^2 \approx 10^{-39}\ {\rm cm}^2, \tag{41}$$

where G_F is Fermi's constant and Y is the hyper-charge of the DM. In order to satisfy the XENON bound (15), such a DM must be sufficiently heavy $m_\chi \gtrsim 10^9$ GeV. In addition, suppose such a DM has an annihilation cross section into electroweak gauge bosons. This is of course a generic class of DM models, although there are many specific manifestations of these interactions, e.g. heavy sneutrino DM. A particular example considered by keisuke and friends is a type of heavy WIMPzilla DM they call "GUTzilla". The naive estimate for such an annihilation cross section is of order

$$\sigma_{\chi\chi} v \sim \frac{1}{8\pi}\frac{g_W^2}{m_\chi^2}. \tag{42}$$

However, the cross section can be larger, e.g. due to a Sommerfeld enhancement. There is also the possibility of an upper limit on the annihilation cross section (the so-called unitarity limit) if the DM is "point-like" in nature:

$$\sigma_{\chi\chi} v \lesssim \frac{4\pi}{m_\chi^2}\frac{1}{v}. \tag{43}$$

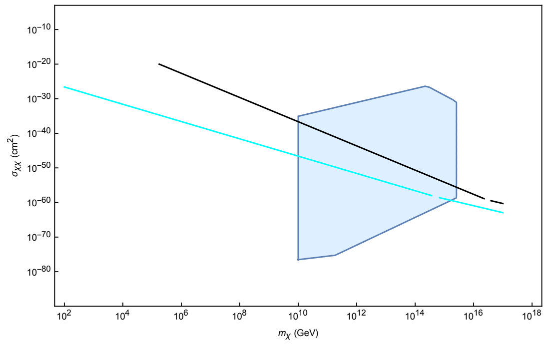

FIG. 2: Constraints on a generic class of DM models which elastically scatter off nuclei through Z boson-exchange and annihilate into electroweak gauge bosons. Bounds come from observation of a single 1.25 $M_\odot$ WD. We assume the DM collapse is stabilized by formation of a BH. Also shown are the naive order of magnitude (blue) estimate and unitarity limit (black) for this annihilation cross section.

W and Z bosons decay predominantly to quarks with a decay length of order

$$\delta_W \sim \frac{8\pi}{g_W^2 m_W}\left(\frac{m_\chi}{m_W}\right) \sim 10^{-7}\ \text{cm}\left(\frac{m_\chi}{10^9\ \text{GeV}}\right). \tag{44}$$

Since hadrons stop efficiently within the trigger size at all energies and WD densities elaborate, the heating length of any one annihilation is set by the decay length of the gauge boson. Evidently, DM masses $m_\chi > 10^{11}$ will have a heating length larger than the trigger size of a 1.25 $M_\odot$ WD. In this case we take $L_{\text{heat}} = L_0 \approx \delta_W$ and, as expected, the ignition condition on the DM becomes more stringent as more collisions are needed to deposit sufficient energy in the larger volume L_0^3 to trigger a SN.