An Efficient Method for Computer-Aided Noise Analysis of Linear Amplifier Networks

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An Efficient Method for Computer Aided Noise Analysis of Linear Amplifier Networks

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Abstract—**A method for computer aided noise analysis is presented which is based on a description of noise by means of correlation matrices. The method is a two-port analysis and it is, therefore, applicable to circuits which are composed of simple two-ports with known noise performance. The correlation matrix concept holds two main advantages over other methods of noise analysis. Partially correlated noise sources can be treated without any loss of efficiency and information concerning minimum noise figure and noise matching conditions is obtained.**

I. Introduction

In recent years, several methods for computer aided noise analysis of linear networks have been reported [1]–[4]. The basic concept which is common to all these methods consists in replacing noise sources by nonrandom sinusoidal sources of the same available power and then applying a straightforward ac analysis for the noise power calculations. The advantages of this concept are obvious: noise power and noise figure calculations can be carried out with any existing ac analysis program. However, the efficiency of the method is restricted to applications where noise sources occurring in the circuit are either uncorrelated or fully correlated. If partially correlated sources are involved serious difficulties will arise, since the whole information about correlation gets lost when noise sources are replaced by nonrandom sources. In principle, the difficulties can be overcome by either two methods, first by transforming the noise network to its canonical form where only uncorrelated sources are contained and second by establishing a set of noise sources which are either uncorrelated or fully correlated and then representing each noise source occurring in the network by a linear combination of this set. Both methods are tedious and time consuming and make the analysis concept unefficient for circuits containing partially correlated sources (e.g., circuits containing any type of transistors). As another weakness of this analysis concept, no information is provided about minimum noise figure and noise matching conditions. In the opinion of the authors, this is also an important disadvantage. There are many applications where this information is absolutely required.

It is the purpose of this paper to present quite another approach to computer aided noise analysis which does not show the disadvantages of the above mentioned methods. It is based on the circuit theory of linear noisy networks. In this theory, noise in linear circuits is described in terms of correlation matrices rather than by voltages and currents. The properties of such a description are usually presented in rather general form and so the theory is not directly applicable to computer aided noise analysis. This may be one reason why the correlation matrix approach has not been used so far.

The noise analysis will be a two-port analysis. The philosophy behind such a method is as follows. The two-port which is to be analyzed is viewed as an interconnection of basic two-ports whose noise behavior is known. Starting from these basic two-ports, the analysis proceeds by interconnecting simpler two-ports to more complicated two-ports until finally the noise performance of the original two-port is obtained. During the whole analysis only two-ports are involved. Therefore, the following theoretical considerations concerning the correlation matrices are applied to two-ports only.

Obviously, the two-port analysis concept is applicable only to a certain class of networks. However, this restriction is not too severe since most practical networks, particularly in the high-frequency and microwave region, belong to this class. On the other hand, the procedure is very simple, easy to program and represents a basis for investigations towards a more general approach.

II. Correlation Matrix Representation of Noisy Two-Ports

The circuit theory of linear noisy networks shows that any noisy two-port can be replaced by a noise equivalent circuit which consists of the original two-port (now assumed to be noiseless) and two additional noise sources [5]. In general, six different forms of noise equivalent circuits exist depending upon the type of the additional noise sources and their arrangement relative to the noiseless two-port. Each form is called a representation. For common applications only three of these representations

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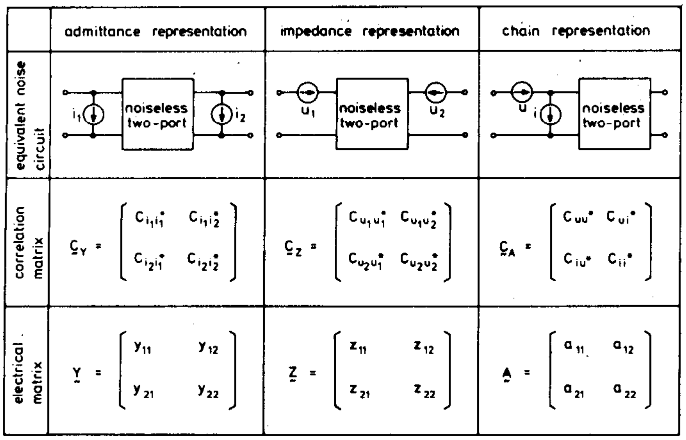

Fig. 1. Correlation matrices of various representations.

TABLE I
TRANSFORMATION MATRICES

		original representation		
		admittance	impedance	chain
resulting representation	admittance	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} -y_{11} & 1 \\ -y_{21} & 0 \end{bmatrix}$
	impedance	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & -z_{11} \\ 0 & -z_{21} \end{bmatrix}$
	chain	$\begin{bmatrix} 0 & a_{12} \\ 1 & a_{22} \end{bmatrix}$	$\begin{bmatrix} 1 & -a_{11} \\ 0 & -a_{21} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

are required. They are shown in Fig. 1. The additional noise sources are indicated by circles. The admittance representation uses two current noise sources i_1 and i_2, the impedance representation two voltage noise sources u_1 and u_2 and the chain representation a voltage noise source u and a current noise source i.

A physically significant description of these sources is given by their self- and cross-power spectral densities which are defined as the Fourier transform of their auto- and cross-correlation functions.[1] Arranging these spectral densities in matrix form leads to the so-called correlation matrices [6]. The correlation matrices belonging to admittance, impedance, and chain representation are shown in Fig. 1. The elements of matrices are denoted by $C_{s_1 s_2^*}$ where the subscript indicates that the spectral density refers to the noise sources s_1 and s_2. The matrices themselves are denoted by $\boldsymbol{C}$ and by a subscript which specifies the representation. The noiseless part of the noise equivalent two-port is described by electrical matrices. These matrices are the conventional two-port matrices. They are also shown in Fig. 1.

Noise sources are usually characterized by their mean fluctuations in bandwidth Δf centered on frequency f. For two noise sources s_1 and s_2, the mean fluctuations are $\langle s_1 s_1^* \rangle$, $\langle s_1 s_2^* \rangle$, $\langle s_2 s_1^* \rangle$, and $\langle s_2 s_2^* \rangle$ ($\langle s_i s_j^* \rangle$ denotes the mean fluctuation of a product containing the signal s_i and the complex conjugate of the signal s_j). Mean fluctuations are closely related to power spectral densities. This relation has the form

$$\langle s_i s_j^* \rangle = 2\Delta f C_{s_i s_j^*}, \qquad i,j = 1,2. \tag{1}$$

The factor 2 occurs because the frequency range has been taken from $-\infty$ to $+\infty$. The correlation matrix $\boldsymbol{C}$ belonging to the noise sources s_1 and s_2 can then be written as

$$\boldsymbol{C} = \frac{1}{2\Delta f} \begin{bmatrix} \langle s_1 s_1^* \rangle & \langle s_1 s_2^* \rangle \\ \langle s_2 s_1^* \rangle & \langle s_2 s_2^* \rangle \end{bmatrix}. \tag{2}$$

[1] The noise sources are assumed to be stationary random processes.

III. CHANGES OF REPRESENTATION, INTERCONNECTION OF NOISY TWO-PORTS

If two or more representations exist (and they generally do) these representations can be transformed into each other by simple transformation operations. The derivation of these formulas is straightforward. As the system is linear the noise signals of the new representation (denoted by the vector $\boldsymbol{x}'(t)$) can be expressed in terms of the noise signals of the old representation (denoted by the vector $\boldsymbol{x}(t)$) by the convolution integral

$$\boldsymbol{x}'(t) = \int_{-\infty}^{+\infty} \boldsymbol{H}(s)\boldsymbol{x}(t-s)\,ds \tag{3}$$

where the transformation is characterized by the weighting matrix $\boldsymbol{H}(s)$ [7]. Using this relation the auto- and cross-correlation functions are calculated. Fourier transforming them leads to the transformation formula

$$\boldsymbol{C}' = \boldsymbol{T}\boldsymbol{C}\boldsymbol{T}^+ \tag{4}$$

where $\boldsymbol{C}$ and $\boldsymbol{C}'$ denote the correlation matrix of the original and resulting representation, respectively. The transformation matrix $\boldsymbol{T}$ is the Fourier transform of $\boldsymbol{H}(s)$. The plus sign (+) is used to denote Hermitian conjugation. Obviously, the $\boldsymbol{T}$ matrix depends only upon the system but not upon the noise parameters of the two-port.

A simple procedure for determining the transformation matrix is as follows. The noise sources of both the original and the resulting equivalent circuits are replaced by nonrandom sinusoidal sources. Now a frequency domain analysis can be applied to establish relations between the Fourier amplitudes of the original and resulting circuit. By expressing these relations in matrix form the desired transformation matrix is obtained.

A set of matrices covering all possible transformations between impedance, admittance, and chain representation is presented in Table I.

Interconnection of noisy two-ports is also formulated by means of operations with correlation matrices. Formulas corresponding to the various types of interconnections can be obtained by the general concept demonstrated above.

In deriving these formulas it is assumed that there is no correlation between the noise sources of different two-ports. This is by no means a restriction to the applicability as far as the basic two-ports correspond to individual devices. In this case, clearly no noise correlation exists. Problems may arise, however, in applications for device modeling. Then the correlation matrix concept has to be extended to a more general form.

For applications in noise analysis interconnections of two two-ports either in parallel, in series or in cascade are of particular interest. For these interconnections the resulting correlation matrix is related to the correlation matrices of the original two-ports by

$$C_Y = C_{Y1} + C_{Y2} \qquad \text{(parallel)} \qquad (5)$$

$$C_Z = C_{Z1} + C_{Z2} \qquad \text{(series)} \qquad (6)$$

$$C_A = A_1 C_{A2} A_1^+ + C_{A1} \qquad \text{(cascade)} \qquad (7)$$

where the subscripts 1 and 2 refer to the two-ports to be connected. As shown by these equations interconnection in parallel and in series corresponds to addition of the correlation matrices in admittance and impedance representation, respectively. For cascading (in an order indicated by the subscripts) a more complicated relation is obtained which additionally contains the electrical matrix A_1 of the first two-port.

IV. Correlation Matrix of the Basic Two-Ports

The two-port analysis starts from basic two-ports whose correlation matrices have to be known. These matrices are obtained from either theoretical considerations or noise measurements. An important example belonging to the former case is the thermal noise of two-ports consisting only of passive elements. On thermodynamic grounds, the correlation matrices in impedance and admittance representation of such a two-port are

$$C_Z = 2kT\,\mathrm{Re}\,\{Z\} \qquad (8)$$

$$C_Y = 2kT\,\mathrm{Re}\,\{Y\}. \qquad (9)$$

They are completely determined by the temperature T and the real part of their electrical matrices in impedance and admittance representation, respectively. Theoretical estimations of the correlation matrix are also obtained if noise equivalent circuits of the elements are used. This is demonstrated by a simplified transistor model as shown in Fig. 2. The equivalent circuit is considered as a cascade of two two-ports. The correlation matrix of the first two-port is obtained from (9). The correlation matrix of the second two-port is according to (2)

$$C_Y = \frac{1}{2\Delta f}\begin{bmatrix} \langle i_b i_b^* \rangle & \langle i_b i_c^* \rangle \\ \langle i_c i_b^* \rangle & \langle i_c i_c^* \rangle \end{bmatrix}. \qquad (10)$$

The quantities $\langle i_b i_b^* \rangle$, $\langle i_b i_c^* \rangle$, $\langle i_c i_b^* \rangle$, and $\langle i_c i_c^* \rangle$ can be taken from the literature, e.g., [1], [2].

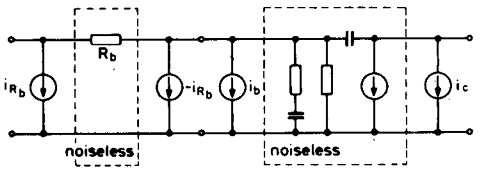

Fig. 2. Simplified noise equivalent circuit for transistor.

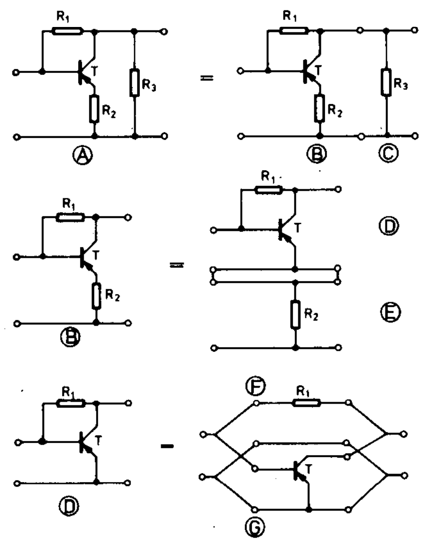

Fig. 3. Principles of noise analysis demonstrated for amplifier.

In cases where the correlation matrix cannot be derived from theory measurements of the noise performance provide the required information. Such measurements are usually done by determining the equivalent noise resistance R_n, the optimal source admittance Y_{opt} and the minimum noise figure $NF_{\min}$. With these quantities estimated the chain representation of the correlation matrix is obtained as

$$C_A = 2kT\begin{bmatrix} R_n & \dfrac{NF_{\min}-1}{2} - R_n Y_{\mathrm{opt}} \\ \dfrac{NF_{\min}-1}{2} - R_n Y_{\mathrm{opt}}^* & R_n |Y_{\mathrm{opt}}|^2 \end{bmatrix} \qquad (11)$$

where T is the absolute temperature.

V. The Concept of Noise Analysis

The principles of the analysis procedure are explained with reference to Fig. 3 where the amplifier A is to be analyzed. In a first step, a decomposition of the two-port A into basic two-ports C, E, F, and G is carried out. The basic two-ports have to be specified by their electrical and correlation matrices. The electrical matrices are obtained by any usual procedure, either by calculation or by measurement. The correlation matrices are determined using the results of Section IV. Once all matrices are known the basic two-ports are successively interconnected in a manner that finally the two-port A is obtained. For example, two-port D results from interconnecting F and E in parallel. The matrices characterizing two-port D are determined by the following two-step procedure: 1) the matrices of E and F are transformed into admittance representation which is the appropriate representation for parallel interconnection; 2) adding the electrical matrices of D and E yields the electrical matrix of D, adding the correlation matrices the correlation matrix of D (cf. (5)).

It is demonstrated by this example that the correlation matrix of a two-port can be calculated from the matrices of the basic two-ports by a consequent application of the interconnection rules given in Section III. Once the correlation matrix has been determined in chain representation the noise parameters can be computed. Denoting this correlation matrix by $\boldsymbol{C}_A$ and its elements by (C_{uu^*}, C_{ui^*}, $C_{ui}{}^*$, C_{ii^*}) the noise figure relating to a source impedance Z_S is given by

$$NF = 1 + \frac{z^+ C_A z}{2kT \,\mathrm{Re}\,\{Z_S\}} \tag{12}$$

where

$$z = \begin{bmatrix} 1 \\ Z_S^* \end{bmatrix}. \tag{13}$$

This result follows immediately from the definition of the noise figure. Equation (11) is used to express the optimal source admittance and the minimum noise figure as function of the correlation matrix. The following relations are obtained:

$$Y_{\mathrm{opt}} = \sqrt{\frac{C_{ii^*}}{C_{uu^*}} - (\mathrm{Im}\,\{C_{ui^*}/C_{uu^*}\})^2} \; - j\,\mathrm{Im}\,\{C_{ui^*}/C_{uu^*}\} \tag{14}$$

$$NF_{\min} = 1 + (C_{ui^*} + C_{uu^*}\,Y_{\mathrm{opt}})/kT. \tag{15}$$

Minimum noise figure and optimal source admittance can be directly determined by this noise analysis method.

Basic to this concept is that the analysis of any network consists in a consequent application of only a few operations (change of representation, interconnection, noise figure calculation). This makes it well suited for a computer aided analysis where these operations are left to the computer. The sequence of operations is controlled by a supervisor which can be either the designer itself or the computer. Obviously, there is a large variety of actual implementations. In principle, any of the existing implementations of ac two-port analysis can be extended to include noise analysis.

VI. Conclusions

The noise analysis concept presented in this paper is an application of the circuit theory of linear noisy networks. The correlation matrices represent a systematic description of noise in linear networks which includes both the uncorrelated and the (partially or fully) correlated case. In contrast to other noise analysis methods, partially correlated noise sources can be treated with the same efficiency.

The noise analysis method is based on the two-port analysis concept and is, therefore, suited for a wide class of networks which covers most applications occurring in practice. It can easily be adapted to already existing two-port analysis programs for ac analysis.

The set of noise parameters which can be calculated by the method includes the noise figure, the minimum noise figure, and the optimal source admittance. All noise parameters are obtained by one single analysis.

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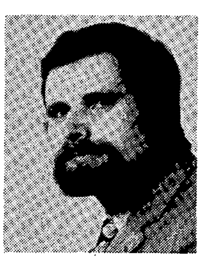

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