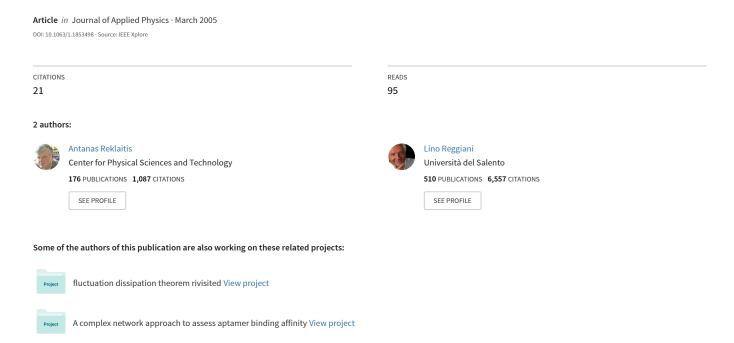
Monte Carlo investigation of current voltage and avalanche noise in GaN double-drift impact diodes



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By Monte Carlo simulations, we investigate the current voltage characteristics and the current noise in GaN homojunction double-drift impact avalanche diodes. We have found that a suppression of avalanche noise from the standard excess noise factor starts when the dielectric relaxation time becomes comparable or less than the carrier transit time. The suppression reaches values down to three orders of magnitude when the current approaches the electrical breakdown regime. The negative feedback between fluctuations of the space charge and of the number of electron-hole pairs generated under avalanche conditions is found to be responsible for this giant suppression of noise. © 2005 American Institute of Physics. [DOI: 10.1063/1.1853498]

I. INTRODUCTION

Among semiconductor microwave emitters, impact avalanche transit time (IMPATT) diodes are generally recognized as exhibiting the best characteristics. ^{1,2} Recently, wide band gap semiconductors, primarily GaN, have emerged as promising materials for optoelectronics in general and microwave generation in particular. Indeed, a large breakdown voltage results in a large ac voltage swing, greater carrier injection into the drift region³ and, hence, high power performances.

In previous work,⁴ through extended Monte Carlo simulations we have modelled a near-terahertz IMPATT diode made of wurtzite GaN with an active region of nanometric length. The knowledge of the noise characteristics is thus a mandatory issue to assess the quality of this device. To this purpose, it is commonly accepted that avalanche noise is the main noise source in IMPATT diodes. This source originates from the random nature of impact ionization processes and it was quantified by Tager⁵ and McIntyre.⁶ However, in these seminal works the effects of long range Coulomb interaction were not detailed from a microscopic point of view. Furthermore, a direct application of McIntyre theory⁶ to the nanometric length of the structure considered here is questionable because charge transport takes a local character and exhibits hot carrier effects.

The aim of this article is to go beyond the McIntyre approach⁶ and provide a detailed investigation of avalanche noise in GaN based IMPATT diodes. To this purpose, we investigate the I-V and the noise characteristics of an avalanche double-drift diode starting from a microscopic model of the steady current and using a self-consistent Monte Carlo (MC) simulation.⁷ By comparing the results coming from simulations performed with a static and a dynamic Poisson

The article is organized as follows. The theoretical approach is briefly described in Sec. II. Section III presents the I-V and avalanche noise characteristics of GaN double-drift IMPATT diodes. Major conclusions are summarized in Sec. IV.

II. THEORY

Avalanche noise is known to be the main source of noise in avalanche devices (IMPATT diodes, avalanche photodiodes, etc.). This kind of noise was investigated by McIntyre^{6,8} who found the following expression for the low-frequency current noise spectral density S_0 :

$$S_0 = 2eI_t M^2 F(M), \tag{1}$$

where I_t is the primary leakage current (Zener tunneling and/or thermal) before multiplication, M is the average value of the multiplication factor and F(M) is the excess noise factor given by

$$F(M) = kM + (2 - 1/M)(1 - k), \tag{2}$$

where k is the ratio between the ionization coefficient of electrons, α , and that of holes, β . If the multiplication is initiated by electrons only, then $k = \alpha/\beta$ and if it is initiated by holes, then $k = \beta/\alpha$.

Equation (2) predicts the reduction of avalanche noise at decreasing values of k. Therefore, semiconductor materials and structures with disparate ionization coefficients (i.e., $\alpha \gg \beta$ or $\beta \gg \alpha$) have attracted particular attention. The phenomenological theory of McIntyre was generalized in a number of successive works. In particular, Refs. 15 and 18–20 implemented the Monte Carlo procedure to evaluate the current multiplication factor M and the excess noise factor F(M).

Here, we use an alternative approach to investigate the noise performance of avalanche devices. The fluctuations of

solver we will determine unambiguously the role played by space charge fluctuations on avalanche noise.

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the steady current in an IMPATT diode are evaluated microscopically from direct Monte Carlo simulations of the full diode under consideration. Thus, the resulting current fluctuations take into account all the noise sources simultaneously. To this purpose, following Ref. 21, the autocorrelation function of current fluctuations is calculated by considering the different contributions in which it can be decomposed to analyze the contributions of the different noise sources.

In accordance with the Shockley–Ramo theorem, ^{22,23} the instantaneous steady current is given by ²⁴

$$I(t) = \frac{1}{L} \sum_{i=1}^{N(t)} q_i v_{zi}(t),$$
 (3)

where q_i is the carrier charge (we take $q_i = e$ for electrons and $q_i = -e$ for holes), L is the device length, $v_{zi}(t)$ is the instantaneous velocity in the z direction of the ith carrier, and N(t) is the instantaneous total number of carriers (electrons and holes) in the structure. The instantaneous current can be rewritten as

$$I(t) = \frac{eN(t)v_d(t)}{I},\tag{4}$$

where $v_d(t)$ is the average instantaneous drift velocity of the electrons and holes ensemble in the structure

$$v_d(t) = \frac{1}{eN(t)} \sum_{i=1}^{N(t)} q_i v_{zi}(t).$$
 (5)

By linearizing Eq. (5), the instantaneous current fluctuation, $\delta I(t)$, is found to be

$$\delta I(t) = \frac{e}{I} \left[\bar{N} \delta v_d(t) + \bar{v}_d \delta N(t) \right], \tag{6}$$

where the bar denotes time average. From Eq. (6), the autocorrelation function of current fluctuations and its decomposition is given by

$$C_I(\tau) = \overline{\delta I(t)\,\delta I(t+\tau)} = C_v + C_N + C_{vN},\tag{7}$$

where

$$C_v(\tau) = \frac{e^2 \overline{N^2}}{L^2} \overline{\delta v_d(t) \, \delta v_d(t+\tau)},\tag{8}$$

$$C_N(\tau) = \frac{e^2 \overline{v_d^2}}{L^2} \overline{\delta N(t) \, \delta N(t+\tau)},\tag{9}$$

$$C_{vN}(\tau) = \frac{e^2 \overline{Nv_d}}{L^2} \left[\overline{\delta v_d(t) \delta N(t+\tau)} + \delta \overline{N(t) \delta v_d(t+\tau)} \right], \quad (10)$$

are the terms corresponding to the different contributions that compose the autocorrelation function. Accordingly, the terms C_v , C_N , and C_{vN} are associated, respectively, with fluctuations in carrier drift velocity, carrier number, and the cross-correlation between these quantities. In accordance with the Wiener–Khinchine theorem, 25 the spectral density of current fluctuations is obtained from the autocorrelation function through Fourier transform as

$$S_I(f) = 2 \int_{-\infty}^{\infty} C_I(\tau) \exp(2\pi j f \tau) d\tau = S_v + S_N + S_{vN}, \qquad (11)$$

where j is the imaginary unit and S_v , S_N , and S_{vN} are, respectively, the spectral densities of velocity fluctuations, number fluctuations, and cross-correlation between fluctuations of these quantities.

In Monte Carlo simulations, instead of the total number of carriers, we calculate the total carrier concentration n(t) by using the following relation:

$$N(t) = LSn(t), (12)$$

where S is cross-sectional area of the structure. The correlation functions obtained from the Monte Carlo simulations, $C_{\rm MC}$, are related with the real correlation functions, C, as

$$C = \frac{e}{\sigma^{S}} C_{MC}, \tag{13}$$

where σ is the surface charge density of the Monte Carlo particle ensemble. By inserting Eqs. (12) and (13) into Eqs. (8)–(10), we obtain the real autocorrelation functions, $c(\tau)$, which are normalized to the device cross-sectional area. The result is

$$c_v(\tau) = \frac{C_v(\tau)}{S} = \frac{e^3 \overline{n^2}}{\sigma} \overline{\delta v_d(t) \delta v_d(t+\tau)}, \tag{14}$$

$$c_N(\tau) = \frac{C_N(\tau)}{S} = \frac{e^3 \overline{v_d}^2}{\sigma} \frac{\partial \overline{n(t)} \, \delta n(t+\tau)}{\partial n(t+\tau)},\tag{15}$$

$$c_{vN}(\tau) = \frac{C_{vN}(\tau)}{S}$$

$$= \frac{e^3 \overline{nv}_d}{\sigma} \left[\overline{\delta n(t) \delta v_d(t+\tau)} + \overline{\delta v_d(t) \delta n(t+\tau)} \right], \quad (16)$$

where the fluctuations of: the total current, the total carrier density, and the drift velocity are obtained separately from direct Monte Carlo simulations, while the cross correlation between fluctuations in velocity and number is calculated by difference using Eq. (7), which gives an exact decomposition of the total noise. ²⁶ The corresponding real spectral densities normalized to the device surface, s(f), are obtained as

$$s_v(f) = 2 \int_{-\infty}^{\infty} c_v(\tau) \exp(2\pi j f t) d\tau, \tag{17}$$

$$s_n(f) = 2 \int_{-\infty}^{\infty} c_N(\tau) \exp(2\pi j f \tau) d\tau, \tag{18}$$

$$s_{vN}(f) = 2 \int_{-\pi}^{\infty} c_{vN}(\tau) \exp(2\pi j f \tau) d\tau.$$
 (19)

The fluctuations of the potential profile can play a significant role in the noise spectrum, especially in the presence of number fluctuations. We recall, that for correlated current pulses the fluctuation of the potential profile may cause an enhancement²⁷ or a suppression²⁸ of the noise with respect to the full shot noise value corresponding to uncorrelated cur-

TABLE I. Structure of the diode.

Layer	Thickness (nm)	Doping concentration (cm ⁻³)
p ⁺ contact layer		5×10^{19}
P	30	3×10^{17}
P	30	7×10^{17}
p^+	10	4×10^{19}
n^{-}	10	1×10^{15}
n^+	10	4×10^{19}
N	60	3×10^{17}
N	60	1×10^{17}
n+ contact layer		1×10^{19}

rent pulses. Accordingly, the influence of these fluctuations on the noise performance of IMPATT devices is studied by comparing the noise properties obtained with a static (i.e., nonfluctuating) and a dynamic (i.e., fluctuating) potential profile. In the simulations, the static profile corresponds to the self-consistent potential averaged over a sufficiently long time of simulation.

III. RESULTS AND DISCUSSION

In this section we present the results of the I-V and noise characteristics of an homojunction wurtzite GaN double-drift IMPATT diode when avalanche phenomena are of importance. We consider a lattice temperature of 600 K, which is found appropriate for a realistic operating condition of the diode. The structure of the diode under test and the microscopic model used for the simulations are the same of those reported in Ref. 4 where we investigated the small and large signal response of the diode in the context of a near terahertz generator. For the sake of completeness, Table I summarizes the parameters of the structure, while the microscopic parameters of the model are those of Ref. 4. In brief, simulations make use of a three-valley, spherical, and nonparabolic conduction band model. The minimum of the conduction band is located at the center of the Brillouin zone (Γ_1 point). In wurtzite GaN, the higher valleys of the conduction band are at Γ point (Γ_2) and U point (located two thirds of the way between L and M symmetry points). For the valence band we consider heavy, light, and split-off nonparabolic subbands. The electron scattering mechanisms include: (i) ionized impurity, (ii) phonon from polar optical, deformation acoustic, and intervalley transfer and, (iii) impact ionization. The hole scattering mechanisms include: (i) phonons from deformation optical, deformation acoustic, and polar optical for intrasubband and intersubband mechanisms, (ii) ionized impurity and, (iii) interband impact ionization. Interband impact ionization takes into account the carrier multiplication initiated by electrons and holes.

If not stated otherwise, the uncertainty of Monte Carlo simulations is estimated to be within 1% and 10% for the I-V and noise characteristics, respectively.

A. Current voltage characteristics

The current voltage characteristic of the diode is reported in Fig. 1. Here, the continuous curve shows the results

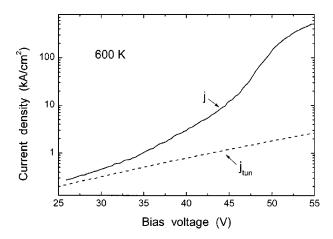


FIG. 1. Steady-state I-V characteristic of the homojunction IMPATT diode in Table I at T=600 K. Solid curve shows the I-V characteristic calculated for the full model of the device. The dashed curve is obtained when carrier multiplication by impact ionization is neglected, and the current is induced by band to band Zener tunneling process.

when carrier multiplication through band-band impact ionization is taken into account. For the purpose of comparison, the dashed curve reports the results when impact ionization is neglected and electron-hole pairs, constituting the leakage current, are created by Zener tunneling only. The leakage current exhibits an exponential increase with applied voltage as predicted by the Kane theory,²⁹ while the presence of avalanche originates a superexponential increase of the current. We individuate an avalanche regime that starts at low voltages around 20 V and then transforms into a breakdown regime at voltages above about 50 V. The breakdown voltage estimated by the simulations is found to be around 55 V.

From the results reported in Fig. 1, the current multiplication factor M is determined as

$$M = \frac{I}{I_t},\tag{20}$$

where I is the current calculated taking into account carrier multiplication, and I_t is the Zener tunneling current. The resulting current multiplication factor is reported in Fig. 2.

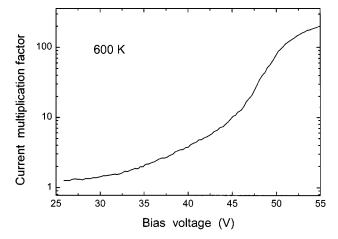


FIG. 2. Current multiplication factor vs bias voltage. The multiplication factor is obtained from the results presented in Fig. 1 in accordance with Eq.

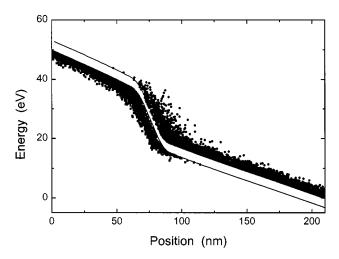


FIG. 3. Potential profile (solid line) and distribution of carrier kinetic energy (points) for an applied voltage of 50 V in the diode under test.

Figure 3 shows the potential profile of the diode in the avalanche regime. We notice, that electron and hole drift velocities are near to their saturation values $(1.1 \times 10^7 \text{ and } 0.6 \times 10^7 \text{ cm/s})$ for electrons and holes, respectively) in the different diode regions, apart from some overshoot effects in the highest field regions.

B. Current noise characteristics

Figure 4 reports the time series of the stochastic current density as obtained from direct simulations. The device is biased in the regime of avalanche breakdown at 50 and 52.7 V, respectively. The corresponding average current densities are close to 100 and 300 kA/cm², respectively. From these simulations we evaluate the correlation functions and the corresponding spectral densities. These are shown in Figs. 5 and 6 for average current values of 100 and 300 kA/cm², respectively. The initial spike in carrier current [see Figs. 5(a) and 6(a)] is explained in terms of thermal fluctuations of hot carrier velocity only. The duration of this spike is very short (about 0.1 ps) due to the strength of polar phonon scattering in GaN. Indeed, the optical phonon scat-

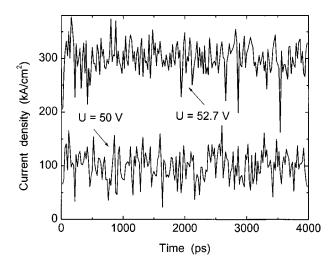


FIG. 4. Current density vs time in the diode under test biased at U=50 V and U=52.7 V, respectively. The results of Monte Carlo simulations are obtained at T=600 K.

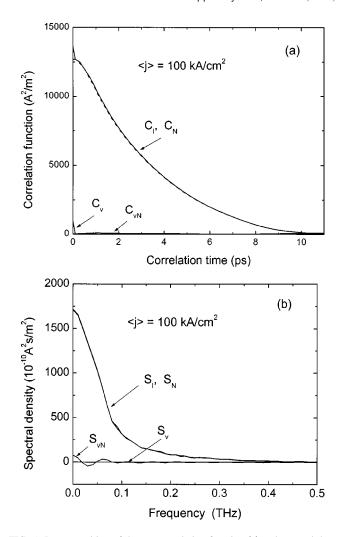


FIG. 5. Decomposition of the autocorrelation function (a) and spectral density (b) of current fluctuations in the diode under test for an average dc density $\langle j \rangle = 100 \text{ kA/cm}^2$ at U = 50 V and T = 600 K.

tering rate in GaN exceeds that in GaAs for about one order of magnitude. At longer times, the correlation functions exhibit a nearly exponential decay with a correlation time of 3.0 and 1.5 ps, respectively, for the cases of low and high current. As a consequence, the spectral densities follow a nearly Lorenzian behavior [see Figs. 5(b) and 6(b)]. The noise contributions that have been registered in the device are: (i) shot noise associated with the discreteness of the charge and due to fluctuations in carrier number, (ii) thermal noise due to fluctuations in carrier velocity and, (iii) their crosscorrelation. As expected, being under carrier multiplication, shot noise is the dominant noise source. From Figs. 5 and 6 it is clear that the contributions due to thermal noise, as well as cross-correlation noise, are negligible when compared with that due to shot noise.

In order to evaluate the intrinsic thermal noise, we have performed Monte Carlo simulations of the device by neglecting carrier multiplication. In these simulations, impact ionization is considered only as a scattering mechanism for electrons and holes on the same ground of phonon and impurity scatterings. The diode is biased at 50 V and the carrier concentrations of injecting contacts adjacent to the p- and n-drift regions are taken of $p_c = 1.3 \times 10^{17}$ cm⁻³ and $n_c = 4.0$

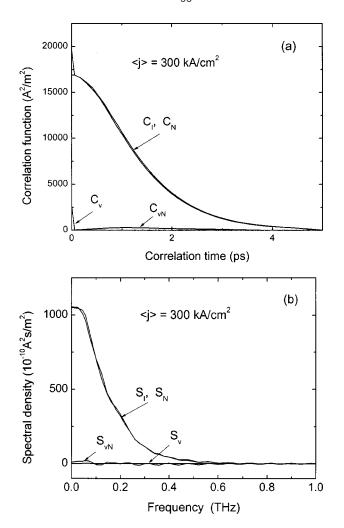


FIG. 6. Decomposition of the autocorrelation function (a) and spectral density (b) of current fluctuations in the diode under test for an average dc density $\langle j \rangle = 300 \text{ kA/cm}^2$ at U = 52.7 V and T = 600 K.

× 10¹⁶ cm⁻³, respectively. When carrier multiplication is neglected, such contact concentrations lead to an average current density of 100 kA/cm² at 50 V, i.e., of the same value as for the avalanche case. Figure 7 reports the results of the simulation which show a fast decay of the correlation function typical of the hot-carrier regime when relaxation processes are controlled by momentum (the initial positive decay) and energy (the final negative decay) relaxation times²¹ [see Fig. 7(a)]. For completeness, the corresponding spectral density is reported in Fig. 7(b). The comparison between the results of Figs. 7 and those reported in Figs. 5 and 6 confirms that thermal noise can be neglected in the noise performance of IMPATT devices. Indeed, the values of the thermal noise spectrum remains below that of the total noise by several orders of magnitude.

To investigate the role played by the fluctuating self-consistent potential, we have carried out Monte Carlo simulations for the same cases presented in Figs. 5 and 6, but with the static potential profile corresponding to stationary conditions. The results of the simulations are reported in Figs. 8 and 9. Here, the correlation functions [see Figs. 8(a) and 9(a)] still exhibit a nearly exponential decay. However, the

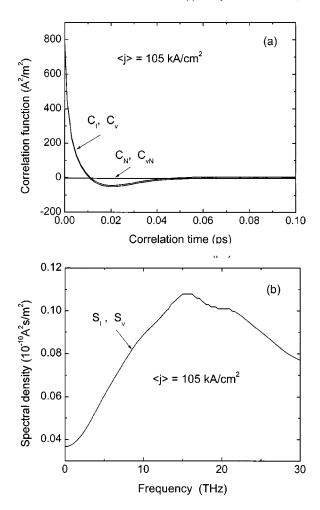


FIG. 7. Correlation functions and spectral densities of current fluctuations in the diode under test obtained when carrier multiplication is neglected. The diode is biased at 50 V, the average dc density is $\langle j \rangle = 105 \text{ kA/cm}^2$ and T = 600 K.

values of both the amplitudes and the correlation times far exceed those of the corresponding dynamic simulations (see Figs. 5 and 6).

Figure 10 reports a systematic comparison between the results of the static and dynamic simulations in the whole region of avalanche regime. The amplitude of the correlation functions as function of the steady current is shown in Fig. 10(a). Here, we see that potential fluctuations start suppressing the amplitude of correlation functions for currents above about 30 kA/cm². Analogously, from Fig. 10(b) we see that also the correlation time of dynamic simulations remains shorter than that of static simulations for current values above about 30 kA/cm². Indeed, we have found that above about 30 kA/cm² the (differential) dielectric relaxation time of the diode, determined as

$$\tau_d = \frac{dV}{dj} \frac{\varepsilon \varepsilon_0}{L} \tag{21}$$

with ε_0 as the static dielectric constant of GaN and ε as the vacuum permittivity, becomes comparable with or shorter than the correlation time of the dynamic case. This is shown in Fig. 10(b) where the dielectric relaxation time is reported together with the correlation times. From Figs. 1, 2, and 10(b) we also note that the value of the static correlation time

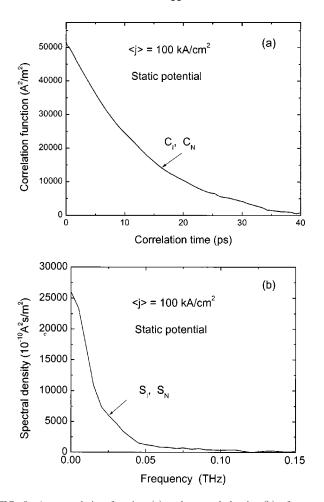


FIG. 8. Autocorrelation function (a) and spectral density (b) of current fluctuations in the diode under test for an average dc density $\langle j \rangle$ = 100 kA/cm². Calculations are performed for a static, i.e., nonfluctuating potential distribution, at U=50 V and T=600 K.

scales with the value of the multiplication factor. This scaling is an evidence of the so called multiplication time effect already predicted by McIntyre⁶ and which implies that in the whole avalanche region the ratio τ_d/M remains practically constant and, in this case, equal to 0.27 ± 0.10 ps. This value represents the typical average time between successive ionization processes. We conclude, that the effect of potential fluctuations on the avalanche noise becomes of importance when the dielectric relaxation time becomes shorter than the dynamic correlation time. Under such a condition, when taken into account, the net effects of potential fluctuations are: (i) to suppress the amplitude of the correlation function of current fluctuations and (ii) to shorten the correlation time.

The overall picture of the avalanche noise in the IM-PATT diode is reported in Fig. 11. Here, Monte Carlo simulations for the static and dynamic cases are shown together with the results predicted by the McIntyre theory⁶ for the simple case when the electron and hole impact coefficients are taken to be equal, so that it is

$$S_0 = 2eI_{\text{tun}}M^3. \tag{22}$$

From Fig. 11 we note, that the results of the static Monte Carlo simulation obtained for nonfluctuating potential profiles are in good qualitative agreement with the theoretical predictions given by Eq. (22). The discrepancies should be

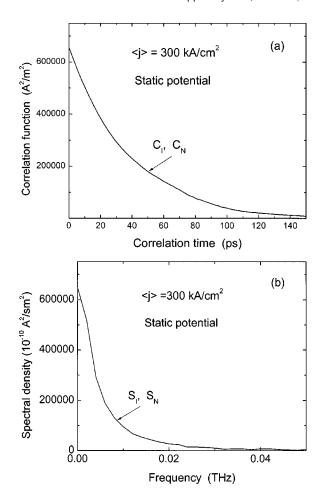


FIG. 9. Autocorrelation function (a) and spectral density (b) of current fluctuations in the diode under test for an average dc density $\langle j \rangle$ = 300 kA/cm². Calculations are performed for a static potential distribution at U=52.7 V and T=600 K.

attributed to the simplifying assumption of taking the ionization coefficients of electrons and holes equal, while according to the results of Ref. 4 this assumption is not correct for wurtzite GaN in the field range below 3 MV/cm. By contrast, when potential fluctuations are taken into account, the current noise level is reduced by 2–3 orders of magnitude at high current densities.

The mechanism of shot noise suppression due to fluctuations in the self-consistent potential and the associated electric field is as follows. Electrons and holes generated by the impact ionization drift in opposite directions and the corresponding space charge reduces the built-in electric field in the avalanche zone. Now, in case the number of generated electron-hole pairs exhibits a positive fluctuation, the electric field in the avalanche zone is reduced and the successive carrier generation rate is suppressed. Hence, a positive fluctuation in the created number of pairs (and, consequently, in current) is damped. In the opposite case that the number of generated pairs exhibits a negative fluctuation, the electric field in the avalanche zone is enhanced and the successive carrier generation rate is also enhanced. Again, a negative fluctuation in the created number of pairs is damped. This mechanism of suppression of avalanche shot noise was considered by Tager,⁵ however, the expression he provided cannot be quantified to the present case.

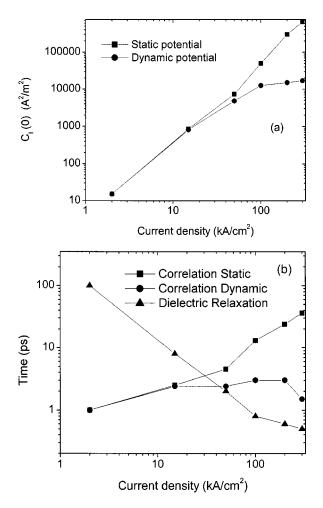


FIG. 10. (a) Amplitude of the current correlation functions obtained from the simulations under static (full square) and dynamic (full circles) conditions vs the steady current for the IMPATT diode under test. (b) Different times scales relevant to current fluctuations vs the steady current for the IMPATT diode under test. Full squares refer to the static correlation time, full circle to the dynamic correlation time and full triangles to the differential dielectric relaxation time of the diode. Curves are guide to the eyes.

The low-frequency spectral densities of avalanche noise presented in Fig. 11 shows that the noise suppression due to long range Coulomb interaction is very efficient in the prebreakdown regime. Indeed, at high current densities, j =300 kA/cm², a giant suppression is obtained. The level of avalanche noise is suppressed down to a factor of 1000, and is well below the noise level predicted by the McIntyre theory.6 (We note, that in his paper McIntyre was already arguing on the reduction of the current noise appearing in the external circuit because of the shunting effect of the diode differential impedance.⁶ However, he did not present quantitative results.) It should be remarked, that this mechanism could be applied for an essential reduction of noise in the gained current of proper designed avalanche photodiodes. Indeed, the noise suppression pertaining to this mechanism can be much more efficient than that due to the introduction of multiple quantum wells. 14,30

IV. CONCLUSIONS

Transport and noise characteristics of a GaN homojunction IMPATT diodes have been theoretically investigated by

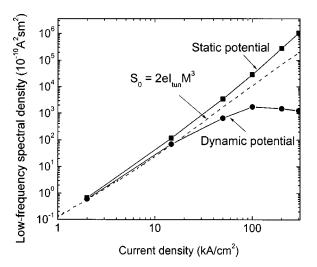


FIG. 11. Low-frequency value of spectral density of current fluctuations in the IMPATT diode under test vs the dc density flowing through the device. Open (solid) circles refer to the case in which the instantaneous fluctuations of the self-consistent potential distribution are considered (neglected) in the simulation. Dashed curve shows the theoretical $S_0 = 2eI_{tun}M^3$ with the multiplication factor M obtained from Fig. 2. Continuous curves are guides to the eye.

performing microscopic Monte Carlo simulations. To this purpose, by complementing a previous investigation⁴ we have considered a double drift nanometric structure at 600 K when the primary leakage current is controlled by Zener tunneling. By comparing the results of simulations performed under dynamic and static conditions of the potential profile we have determined unambiguously the role played by space charge fluctuations on avalanche noise. Giant suppression of avalanche noise is found at increasing current density because of the negative feedback between fluctuations in the number of electron-hole pair generated by impact ionization and fluctuations of the self-consistent potential. This suppression mechanism occurs when the dielectric relaxation time becomes comparable with or shorter than the carrier transit time between contacts. Simulations prove that suppression acts on both, the variance of current fluctuations and the correlation time, thus shedding light in the physical understanding of multiplication noise since the seminal works of Tager⁵ and McIntyre. ⁶ This suppression mechanism, which is reminiscent of that occurring in vacuum diodes before current saturation conditions, ³¹ can be of valuable help to design low noise photodetectors.

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