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Microelectronics Reliability 43 (2003) 585–599

MICROELECTRONICS  
RELIABILITY

www.elsevier.com/locate/microrel

## Introductory Invited Paper

# Low-frequency noise study in electron devices: review and update

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Received 20 November 2002

### Abstract

Low-frequency noise or flicker noise has been found in many systems and has become a hot research topic for more than eight decades. It was believed that there exists a common origin of this kind of noise for different systems. The common origin theories were shook as more experiments on electron devices were conducted. For electronic system, it is easier to produce samples with different noise behaviors via different fabrication processes, measurement conditions such as temperature, stressing, biasing etc. More and more studies suggest that if there is a common regime for the low-frequency noise, it must be mathematical rather than physical ones. These mathematical processes give rise to  $1/f$  spectrum could be due to the distribution of time constant in spatial or energy-wise and the non-linear transformation of Gaussian signal. This paper presents a historical review on the development of low-frequency noise study in electron devices and the recent progresses in the understanding and modeling are updated.

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### 1. Introduction

The Noise is so great, one cannot hear God  
thunder.

R.C. Trench

If a constant voltage is applied to a semiconductor sample or device, to a resistor or vacuum tube, the current will exhibit fluctuations. The frequency spectrum is in general constant at high frequencies, superimposing with shot-noise components. However, at low frequency  $f$ , ( $f < 10$  Hz) the noise is found to be proportional to  $1/f^\gamma$  ( $\gamma$  in the range 0.8–1.4). This kind of noise is frequently called as low-frequency noise, flicker noise or  $1/f$  noise. Appearing in all kinds of electronic devices and many other non-physical systems, the  $1/f$  spectrum has captured the attention of researchers from various disciplines [1,2], especially in the field of electronics and physics, for several decades. However, as more experi-

mental results were collected, the more inexplicable its nature turned out to be.

Although the low-frequency fluctuation phenomena are also observed in other non-electronic systems, such as thermovoltage of thermocell, earth rotation frequency, loudness of speech, nerve membrane potential, highway traffic current, etc. [1,2], none of them is as crucial as the noise in electronic systems because the noise directly deteriorates the device and system performances. In addition, as it is easy to build an electronic system whose sensitivity is limited not by incidental imperfections in its construction but solely by the more fundamental limitations imposed by the atomic structure and statistical behavior of matter, noise in electronic system has turned out to be a basic research interest for many disciplines. Particularly, to the physicist the noise in an electronic system represents a practical manifesting of the phenomena described by statistical mechanics, and an understanding of its practical consequences helps to illuminate and clarify some concepts of the physical theory; to the electronics engineer, noise is a constraint of the real systems, but a better understanding of its physical origins helps the

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engineers to minimize its effects by informed and careful design. Indeed low-frequency noise is one of the main problems in designing a low-noise amplifier in low-frequency applications. It becomes more crucial in modern integrated circuit engineering, as the device size continues to shrink and more MOS devices are used in analog circuits. Better understanding of the physical regime of this noise will be helpful for deciding whether the performance of the electronic systems can be improved by lowering this type of noise.

Flicker noise, however, has a reputation for difficulty and obscurity. Despite that lots of efforts have been dedicated to it up to now, its exact physical origins are still unclear in most systems and the disputes on the origin or  $1/f$  noise is still unresolved so far. Even for electrical noise, several models of different origins have been proposed to interpret the observed  $1/f$  noise in different electronic devices or experiments. Hooge et al. [3] interpreted the  $1/f$  spectrum in terms of mobility fluctuations and gave an empirical formula while McWhorter [4] ascribed the  $1/f$  noise to the number fluctuations of carrier and looked upon this kind of noise as a surface effect. Handel [2], on the other hand, introduced a quantum model and treated  $1/f$  noise as an infrared phenomenon. Nevertheless, all these models can only be applied to a very specific situation. Since the  $1/f$  noise widely exists in various physical systems [1,2], it is possible that the observed flicker noise may be of different physical origins. Even for metal film conductor, the most fundamental structure, whether the real mechanism of low-frequency fluctuation is a bulk effect or a surface effect is still unclear [5]. On the other hand, for noise in metal-oxide-semiconductor (MOS) devices which may be the most pragmatic topic in noise study as the low-frequency noise becomes a limiting factor of the device performance, a consistent picture of the noise generation mechanism has not yet emerged. Generally the measured noise power in MOS field-effect transistors (MOSFETs) has a more complicated dependence on the gate bias and oxide than the existing noise theory predicts [6–8]. Fortunately, since flicker noise in MOSFETs is usually regarded as the superposition of Lorentzian components due to capture and emission of a channel carrier by the trap, developing a more accurate model with sound physical justifications is now possible. This is because of the availability of sub-micrometer-sized MOSFETs which provide an opportunity to study the noise generated by individual oxide traps [9].

Aside from the aspect of real origin, the actual observed spectra are ordinarily of the form  $S(f) \propto f^{-\gamma}$  ( $0.8 < \gamma < 1.4$ ) over a very large-frequency range, and the temperature dependence of the noise is always more complicated than theories. These observations have not been interpreted satisfactorily. In brief, the low-frequency noise is enigmatic in its nature, and there are lots of problems not yet resolved even though they may be

critically important for some practical applications. The impetus therefore is great and the characterization and modeling on various electron devices are still hot [10–18].

## 2. Review of early noise models [1,2]

In this section, a brief review of the early works on the source of noise will be given. The progress of noise study in the early stage seems very interesting and outstanding. The first attention paid to the electrical noise is Einstein [19], who predicted that Brownian motion of charge carriers would lead to a fluctuation in the potential across the ends of any resistance in thermal equilibrium. This kind of fluctuation was called “wärmeeffekt” or “thermal noise”, the effect was then observed by Johnson [20], and its power spectrum was calculated by Nyquist [21]. From classical thermodynamics Nyquist showed that for a resistor  $R$  at temperature  $T$ , the noise power is given by

$$S = 4k_B T R B_w \quad (1)$$

where  $B_w$  is the bandwidth of the measurement system.

Thermal noise is the main type of noise and exists in almost all kinds of electronic systems. It is now always treated as “white noise”. Strictly speaking, it is true only within the frequency limits  $[-f_c, +f_c]$  where  $f_c$  can be approximated by  $2.6 \times 10^{10} T$ , where  $T$  is the absolute temperature [21] (see Fig. 1). Thermal noise is of so fundamental origin that it is frequently used as a yardstick with which to compare other sorts of noise. Thermal noise is in a sense inherently white noise in that the fundamental relationship between the noise source, the resistance or conductance, and the amount of noise per unit bandwidth is independent of frequency.

Not until the World War I did engineers begin to realize that the essential deterioration of vacuum-tube amplifier was not due to the thermal noise but the shot noise. In attempting to design a high-gain vacuum-tube amplifier, there is a limit to the number of stages which could be cascaded in the quest because of the large background noise. Schottky [22] first gave explanations of these effects and formulated the random component using the Campbell's theorem in the plate current of a vacuum tube. For a mean current,  $I_{DC}$  pass through the tube, Schottky showed that the noise power should be

$$S = 2qI_{DC}B_w \quad (2)$$

In his classic paper Schottky formulated the above equation based on the fact that the plate current is not composed of a continuum but rather a sequence of discrete increments of charge carried by each electron arriving at the plate at random times. The average rate of charge arrival constitutes the DC component of the

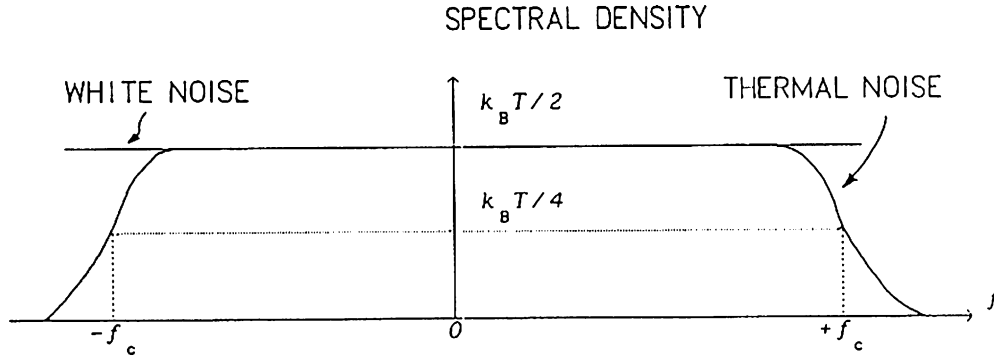


Fig. 1. Frequency spectra difference between white noise and thermal noise.

plate current,  $I_{DC}$  on which is a DC superimposed fluctuation component as each discrete charge arrives. He referred to this phenomenon as “schroteffekt” or “shot-effect” as we call it today. It is again a fundamental noise, namely it arises from the fundamental physical process and cannot be eliminated. Unlike the thermal noise which can be reduced by lowering the operating temperature (see Eq. (1)), the frequently used technique is to reduce the bandwidth over which the noise appears. Note that reducing the operation current may not be a realistic technique to alleviate the shot effect as it will also reduce the signal simultaneously, i.e. it cannot improve the signal to noise ratio.

### 2.1. Low-frequency noise models

The low-frequency or flicker noise was found nearly at the same time. With some cathodes Johnson [23] revealed that there exists another source of noise at low frequencies superimposed on the pure shot noise and yield a fluctuation in current much greater than the shot current itself (i.e.  $2qI_{DC}B_W$ ). Fig. 2 illustrates this observation. In the linear portion of the curve shows the pure shot noise obtained from tubes having filaments of tungsten and thoriated tungsten. In the barium oxide tube, the noise increased more rapidly and reached a maximum value approximately 10 times the amplitude of the pure shot noise. The noise magnitude decreases as the frequency increases. Johnson attributed this excess noise to the fluctuations in the work-function of the cathode surface due to particle migration. The interpretation was discussed at length by Schottky, who called it “Fackelneffekt”.

After that, the flicker noise in the electronic device receives increasing attention and becomes an active research topic in electronics and physics. In 1934 Bernamont [24] reported the investigation of noise in current carrying non-metallic resistors, he found that the spectral density of the noise was almost inversely proportional to the frequency at low frequencies resembling to the flicker

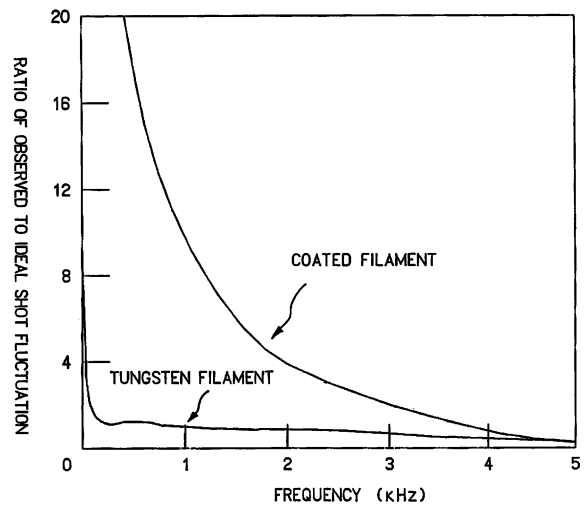


Fig. 2. First observation of flicker noise (after Johnson [20]).

effect in vacuum tubes discovered earlier by Johnson. For higher frequencies, thermal noise usually dominates the flicker one. A change from a  $1/f$  spectrum into an  $f^{-2}$  spectrum was also observed in some samples. This change in the spectral slope matches well with some theoretical models. Montgomery [25] reported a detailed investigation on germanium single crystals as soon as it was available shortly after World War II. Noise spectrum of a  $1/f$  nature was again reported. Moreover, the evidence of noise associated with the surface of germanium was found. Another pronounced progress was that a model for the  $1/f$  dependence in germanium was consequently established by Herzog and van der Ziel [26]. The spectrum showed a characteristic time constant of  $\sim 1 \mu s$  which later on was identified as the minority carrier lifetime. The noise was thus attributed to the random excitation and capture of free carriers. During their stay in the conduction band (or valence band), the carriers give rise to a small fluctuation in current. Later,

McWhorter [4] developed a more sophisticated modeling which the noise was attributed to the trapping and detrapping of surface states. McWhorter's model is now a basis of several models will be quoted in next section. Unlike the thermal or shot noise,  $1/f$  noise was defined in terms of the nature of the noise power spectrum, without reference to a specific physical mechanism. It is, to a certain extent, a reflection of our understanding about the noise. Even though several mechanisms of  $1/f$  noise had been proposed, no conclusive theory appears so far. Among these models, those of Hooge, McWhorter, Voss and Clarke and Handel were the most influenced ones, even though these models may not be the best ones and could only be applied in a very specific situation.

### 2.1.1. Hooge's phenomenological equation [3–13]

Hooge carried out a number of experiments in metal film and found that the noise in metal film conductors can be characterized by

$$\frac{S_V(f)}{V^2} = \frac{\alpha_H}{N_C f} \quad (3)$$

where  $\alpha \approx 2 \times 10^{-3}$  is a dimensionless constant.  $N_C$  is the number of charge carriers in the conductor.

Since Eq. (3) is independent on temperature and material parameters, it is as Hooge declared a universal equation. Soon it was found that this model is very restrictive. McWhorter [4] found that the noise was strongly dependent on the semiconductor surface state. Voss and Clarke [28] found that the magnitude of the noise in semimetal bismuth is outside the range of validity for Eq. (3) could predict. Later, the Hooge's relation was modified in order to correlate more experiments on other materials. The modified version is

$$\frac{S_V(f)}{V^2} = \frac{\mu}{\mu_{ph}} \frac{\alpha_H}{N_C f} \quad (4)$$

where  $\mu$  is the total mobility and  $\mu_{ph}$  is the mobility of electron due to electron phonon scattering. This modification implies that only phonon scattering will result in  $1/f$  fluctuation, which makes the original correlation with the results of metal films incorrect.

Nevertheless, it is unlikely that such a universal equation should exist. There is neither theoretical nor experimental reason. The common acceptable point is that the flicker noise from different sources may have different origins and properties. Similarities of spectra in different systems may be due to the similar mathematical processes. In fact, it is often found exceptions in characterizing or tabulating the properties of flicker noise [29].

In addition, the spectral slope is directly proportional to  $1/f$  for all frequencies in Eq. (4), it is unrealized because it is not even and real function of frequency, i.e.

the noise is a non-stationary process, contradicting with experiments. Meanwhile, the noise power will be infinite unless roll off at low frequency exists. Hence, the Hooge's relation is at most an approximation of some real physical mechanism at some frequencies, temperatures, materials, etc.

### 2.1.2. McWhorter's number fluctuations model [4]

The McWhorter's model has remained the most acceptable basis for  $1/f$  noise in MOSFET so far. In his model, McWhorter considered the carriers number fluctuation due to trapping of charge carriers in traps located at a distance from the semiconductor–oxide interface as the noise source; namely, the noise is a surface effect. The  $1/f$  spectrum comes from the superimposing shot noise spectra of the type  $\tau/[1 + (\tau\omega)^2]$  over a wide range of  $\tau$  whose distribution function is proportional to  $1/\tau$ . In the original version of McWhorter's model, the  $1/\tau$  distribution of  $\tau$  is ascribed to the non-uniform traps distribution or non-linear effect or surface barrier at the semiconductor–oxide interface, having several modifications in the field-effect devices applications. With the assumption given above, the noise spectrum can be approximated by

$$S(\omega) \propto [\tan^{-1} \omega\tau_2 - \tan^{-1} \omega\tau_1]/\omega \quad (5)$$

$$S(\omega) \propto \begin{cases} \text{const.}, & \text{for } \omega \ll 1/\tau_1 \\ 1/\omega, & \text{for } 1/\tau_1 < \omega < 1/\tau_2 \\ \omega^{-2}, & \text{for } \omega \gg 1/\tau_2 \end{cases} \quad (6)$$

In Eqs. (5) and (6),  $\tau_1$  and  $\tau_2$  are the time constants governed by the smallest and largest distances of tunneling respectively.

It is clear that the McWhorter's model has different noise spectrum compared with Hooge's one, as given in Eq. (6) or as depicted in Fig. 3. At extremely low frequency, the noise level is equal to a constant and no

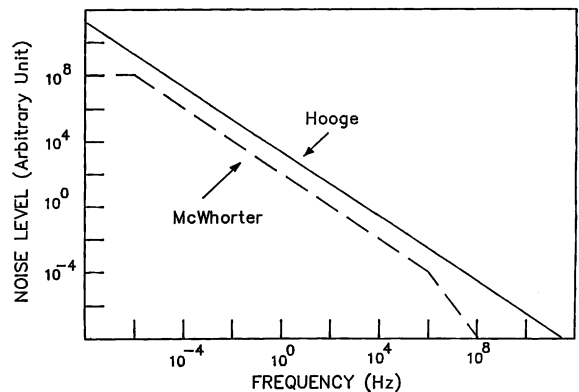


Fig. 3. Frequency spectra difference between the Hooge's and McWhorter's theories.

longer depends on the frequency. In other words, it does not have the problems found in Hooge's relation, such as non-stationary. In addition, the roll-off at high frequency to an  $f^2$  is also in agreement with many experiments. Therefore, the spectrum in the form of Eq. (6) is much preferred.

### 2.1.3. Voss and Clark's temperature fluctuations model [28]

Due to the spontaneous enthalpy fluctuations, the resistance and the voltage will fluctuate,

$$\langle \Delta V^2 \rangle = V^2 \beta^2 k_B T^2 C_V^{-1} \quad (7)$$

where  $\beta$  and  $C_V$  are respectively temperature coefficient of resistance and the specific heat of material.

As the fluctuation quantity goes through the sample, the local temperature at  $x$  for a given instant  $t$  can be given by Langevin diffusion equation

$$\frac{\partial T}{\partial t} = D \nabla^2 T + C_V^{-1} \nabla \cdot \mathbf{F} \quad (8)$$

where  $D$  is the thermal diffusivity and  $\mathbf{F}$  is an uncorrelated random driving term.

The noise spectrum is

$$S_T(\omega) = \int \langle T(t)T(t+\tau) \rangle \exp(-j\omega\tau) d\tau \quad (9)$$

Solving Eq. (8) for  $T$  and making use of Eq. (9), Voss and Clarke found that for a sample with dimensions  $l_1 \times l_2 \times l_3$  where  $l_1 \gg l_2 \gg l_3$ , four frequency regions can be identified

$$S_T(\omega) \propto \begin{cases} \omega^{-3/2}, & \text{for } \omega \gg \frac{D}{2l_3^2} \\ \omega^{-1/2}, & \text{for } \frac{D}{2l_3^2} \gg \omega \gg \frac{D}{2l_2^2} \\ (\text{const.} - \ln \omega), & \text{for } \frac{D}{2l_2^2} \gg \omega \gg \frac{D}{2l_1^2} \\ \text{const.}, & \text{for } \omega \gg \frac{D}{2l_1^2} \end{cases} \quad (10)$$

Particularly, if the  $\nabla \cdot \mathbf{F}$  term is simply given by an energy fluctuation in the system. The temperature fluctuations can be given by

$$\frac{S_V(f)}{V^2} = \frac{\beta^2 k_B T^2}{C_V [3 + 2 \ln(l_1/l_2)] f} \quad (11)$$

for

$$\frac{D}{2l_2^2} \gg \omega \gg \frac{D}{2l_1^2}$$

The temperature-fluctuation model comes off all right in Josephson junctions and tin films near the superconducting transition. However, it cannot be applied to the metal films as a much stronger temperature dependence

of the noise was found. In addition, the temperature fluctuation model also failed to predict the  $1/f$  region, as there is no roll off of  $1/f$  spectrum at frequencies  $\omega < (D/2l_1^2)$ .

### 2.1.4. Handel's quantum mechanic model [2]

Handel splits the electron wave function into a large unperturbed part and a small part perturbed by the Bremsstrahlung emission. This two parts beat with each other and result in  $1/f$  spectrum. Handel thus finds the following

$$S_I(f) = \frac{4\alpha}{3\pi} \frac{\Delta v^2}{c^2} \frac{qI}{f\tau} \quad (12)$$

where  $c$  is the velocity of light,  $\Delta v$  the vectorial change in velocity along the electron path,  $\tau$  the electron transit time, and  $\alpha$  the fine structure constant.

As shown in Eq. (12), Handel's theory has a spectral density which agrees with Hooge's phenomenological formula in the frequency dependence, it therefore still has the problem found in Hooge's model. Meanwhile, the magnitude of the noise is also in the same order of Hooge's one, this agreement is most probably an accident. This is because Handel's model is a zero-temperature theory for a beam of electrons which can emit low-energy photons. However, most electrons in a metal cannot emit low-energy photons because all the nearby states are occupied. In addition, Handel's theory does not consider the equilibrium distribution of charged carriers in any way, and hence the magnitude prediction cannot be relevant for experiments in either metals or semiconductors.

### 2.1.5. Noise model in MOSFET [30]

Although noise due to bulk mobility fluctuation as proposed by Hooge is occasionally incorporated into the model, the main trend of noise theories in MOSFET is developed using the McWhorter's approach with the assumption that the noise is resulting from carrier density fluctuations. The variations in the different versions are mainly in the trap distribution. This section outlines the typical formulation processes for the MOSFET flicker noise model.

The noise spectrum due to a trap center  $n_T$  at energy  $E$  in a unit volume  $\Delta x \Delta y \Delta z$  can be given by

$$S_{\Delta n_i}(\omega) = 4n_T(E) \Delta E \Delta x \Delta y \Delta z f_i (1 - f_i) \frac{\tau}{1 + \omega^2 \tau^2} \quad (13)$$

where  $f_i = \{1 + \exp[(E - E_F)/k_B T]\}^{-1}$  is the trapping probability. The relaxation time constant  $\tau$  or the trapping process for a trap located at a distance  $z$  from the silicon-oxide interface is

$$\tau = \tau_0 \exp(az) \quad (14)$$

where  $\tau_0$  is a proportional constant.

Since  $f_i(1 - f_i)$  in Eq. (13) behaves like a delta function around the Fermi level, the major contribution to the integral will be due to the traps around  $E_F$  i.e. the integration of Eq. (13) over the energy space can be effectively obtained using the value at Fermi level,  $E_F$ . Integrating over  $z$  (from 0 to an upper limit of  $z_1$ ), we have

$$S_{\Delta N_i}(\omega) = \frac{k_B T n_T(E_F) \Delta x \Delta y z_1}{f \ln(\tau_1/\tau_0)}, \quad \text{for } \frac{1}{\tau_1} < \omega < \frac{1}{\tau_0} \quad (15)$$

where  $\tau_1$  is the relaxation time constant or trapping process for a trap at  $z_1$ .

Since the normalized noise spectrum due to drain current fluctuation is equal to that due to carrier number fluctuation, we have

$$S_{I_D}/I_D^2 = S_N/N^2 \quad (16)$$

At weak inversion, the total number of carrier in the channel region is

$$N = C_{ox}(LW)(V_G - V_T)/q \quad (17)$$

where  $C_{ox}$ ,  $V_G$  and  $V_T$  are the oxide capacitance per unit area, gate voltage, and the threshold voltage of the MOS transistor, respectively.

Integrating Eq. (15) over the whole channel region,  $L \times W$ , we have

$$\frac{S_{I_D}(f)}{I_D^2} = \frac{k_B T q^2 N_T(E_F)}{C_{ox}^2 L W (V_G - V_T)^2 \ln(\tau_1/\tau_0)} \frac{1}{f}, \quad \text{for } \frac{1}{\tau_1} < \omega < \frac{1}{\tau_0} \quad (18)$$

Most of the flicker noise models in MOSFET have relationship similar to that given in Eq. (18).

It is noted that although many investigations have been made on the  $1/f$  noise in MOS transistors, inconsistencies still exist among the experimental results from different workers and between theory and experiment. Here we summarize some important theoretical and experimental results of the low-frequency noise in MOS transistors [9].

- (1) *Device size dependence*: Almost all experiments [31–34] and models showed that the noise level is inversely proportional to the channel area, except for a relative shorter channel (2–5  $\mu\text{m}$ ) device in which the noise increases with the third power of the channel length approximately [35].
- (2) *Traps density*: There is general agreement that the noise spectral density is proportional to the traps density at the silicon-oxide interface, both from the experimental [32,34,36,37] and the theoretical [35,38,39] results.
- (3) *Drain bias*: Noise in general depends weakly on the drain bias for a small  $V_D$  or at the linear region [39].
- (4) *Oxide capacitance dependence*: Two different classes of results were reported on the dependence of the

noise power on oxide capacitance. Hsu [40] and Christensson and Lundstrom [41] found that the drain noise power is proportional to  $C_{ox}^{-2}$  whereas the equivalent gate noise power measured by another groups of workers is found to increase directly with  $1/C_{ox}$  [31,32,38,42].

- (5) *Gate bias dependence*: The normalized drain noise spectrum is almost independent of  $V_G$  at weak inversion of the conduction channel but varies closely with  $V_G^{-2}$  in strong inversion [35,39]. The gate noise,  $S_{V_G}(f)$ , instead, is found to be proportional to  $V_G$ . Some researchers incorporate these two dependencies into a single model by considering the mobility fluctuation (making use of the Hooge relation [27]).
- (6) *Frequency index*: It was found that the frequency index can either monotonically increase or decrease with  $V_G$  depending on the nature of the type of devices used in the investigation [43,44], and in some cases it even fluctuates with the gate bias [7,45–47].
- (7) *Temperature dependence*: Almost all models give very weak temperature dependence. However, experimental results show that the temperature dependence is strong and very complicated [7,48]. Several peaks in the noise power versus temperature plots were found in these experiments. The number of peaks and their locations vary from sample to sample. The noise power can vary over two orders of magnitude even in a narrow range of temperature (100–300 K). Moreover, these experiments also demonstrate that the frequency index is also a function of temperature.

### 3. Non-linear transformation induced $1/f$ spectrum

In modeling the flicker noise, there are two approaches to account for this observation. One is the phenomenological expression as given by Hooge [27], the noise spectrum is purely  $1/f$ , and is satisfactory for modeling the low-frequency noise in metal films and semiconductors in most situation although it disagrees with the strong temperature dependence of the noise in some noble metal films [5]. On the other hand, several models emerge by using the superposition of Lorentzian spectra with widely distributed relaxation time constants, albeit dissimilar in details. For instance, the distribution of time constant is due to different tunneling distances or charged carrier into the oxide layer in MOS structures in McWhorter's model [4] whereas in Jäntschi's model, it is due to the different distances of random walks at the surface or interface. All these mechanisms are likely to happen. However, since the assumption of the existence of a distribution or relaxation times has less enough experimental confirmation, it is, therefore, difficult to examine the validity of the existing models. This section will present an attempt to find another

possible regime resulting the  $f^{-\gamma}$  spectrum. The regime which does not need a distribution of relaxation times is the non-linear transform of a single or multiple relaxation process. Since the non-linear transform is a ubiquitous phenomenon in physical world, this approach is expected to sound well.

For most physical processes, there exists a centered random signal, probably a weakly stationary Gaussian signal. Considering this kind of random signal  $x$  with variance  $\sigma_x^2$ , the autocorrelation function is

$$R_x(\alpha) = \sigma_x^2 \exp(-|\alpha|/\tau) \quad (19)$$

According to the Wiener–Khinchine theorem [49], the power spectrum density of a signal is the Fourier transform of its autocorrelation function, i.e.

$$S_x(\omega) = \frac{2\tau\sigma_x^2}{1 + (\omega\tau)^2} \quad (20)$$

The spectrum indicated in Eq. (20) will be nearly a constant at low frequency and proportional to  $\omega^{-2}$  at frequency higher than  $1/\tau$ . This spectrum has been observed in many processes such as in Langevin equation, random telegraph signal, Markov sequence and other diffusion and Umklapp processes in a lot of natural and electrical systems. The most frequent explanation of the  $1/f$  spectrum is assuming a wide distribution of relaxation time constant  $\tau$  whose probability distribution function (p.d.f.) is inversely proportional to its magnitude.

This is the common treatment of McWhorter [4], Jäntch [50], Fu and Sah [45], and many other models for the  $1/f$  fluctuations. A distinct approach was proposed that the noise spectrum is due to transformation or a Gaussian random signal [29]. This signal may be modulated by a physical system and yield another random signal  $\eta$ , let  $\eta(t) = f(x(t))$ , the autocorrelation function of  $\eta(t)$  is given by

$$\begin{aligned} R_\eta(\alpha) &= E[\eta(t)\eta^*(t-\alpha)] \\ &= \int_0^\infty \int_0^\infty f(x_1)f^*(x_2)p_x(x_1, x_2, \alpha) dx_1 dx_2 \\ &= \int_0^\infty \int_0^\infty F(v_1)F^*(v_2)\Phi(v_1, v_2, \alpha) dv_1 dv_2 \end{aligned} \quad (21)$$

where  $F(v)$  is Fourier transform of  $f(x)$  and  $\Phi(v_1, v_2, \alpha)$  is characteristic function of  $p_x(x_1, x_2, \alpha)$  which is given by [51]

$$\Phi(v_1, v_2, \alpha) = \exp\{-2\pi^2\sigma^2(v_1^2 + 2v_1v_2R_x(\alpha)/\sigma^2 + v_2^2)\} \quad (22)$$

Substituting Eq. (22) into Eq. (21) and expanding the term containing  $R_x(\alpha)$ , we have

$$R_\eta(\alpha) = \sum_{n=0}^N \frac{a_n}{n!} R_x^n(\alpha) \quad (23)$$

where

$$a_n = \frac{1}{\sqrt{2\pi}\sigma^2} \int_0^\infty x^n f^{(n)}(x) \exp\left(-\frac{x^2}{2\sigma^2}\right) dx$$

With expansion of  $f(x)$  in terms of Taylor series at  $x = 0$ , we have,

$$a_n = \sum_{m=0}^\infty \frac{(2\sigma^2)^{\frac{m+n}{2}}}{2\sqrt{\pi}m!} \Gamma\left(\frac{m+n+1}{2}\right) f^{(m+n)}(0) \quad (24)$$

and

$$S_\eta(f) = a_0\delta(f) + \sum_{n=1}^\infty \frac{a_n}{n!} \cdot S_x^{*n}(f) \quad (25)$$

where  $S_x^{*n}(f)$  is  $n$ -fold convolution of the spectrum  $S_x$ . If  $S_x$  is given by Eq. (20), we have

$$S_x(\omega) = \Im\{R^n(\alpha)\} = \frac{\tau\sigma_x^{2n}/n}{1 + (\omega\tau/n)^2} \quad (26)$$

By substituting, the final expression for the spectrum of the transformed process can be written as

$$S_\eta(f) = a_0\delta(f) + \sum_{n=1}^\infty \frac{a_n}{n!} \cdot \frac{\tau\sigma_x^{2n}/n}{1 + (\omega\tau/n)^2} \quad (27)$$

In Eq. (27), it was noted that the frequency band of the  $1/f$  fluctuations can be broad enough even with a single relaxation time constant  $\tau$ . The lower limit of the noise is given by  $\tau^{-1}$  and the upper one may extend to an extremely higher limit depending on the coefficients  $a_n/n!$ . In addition, if there are several relaxation processes with different relaxation time constants, the resultant spectrum will be a summation of Eq. (27) for the different relaxation time constants.

### 3.1. Symmetrical clipper

Consider a random signal  $x(t)$  passing through a symmetrical clipper and the output signal  $\eta(t)$  is given by

$$\eta(t) = \begin{cases} 1, & \text{for } x(t) \geq 0 \\ -1, & \text{for } x(t) < 0 \end{cases} \quad (28)$$

The autocorrelation function can be shown to be

$$R_\eta(\alpha) = \frac{2}{\pi} \sin^{-1} R_x(\alpha) = \frac{2}{\pi} \sum_{n=1}^\infty T(n) R_x^n(\alpha) / \sigma^{2n} \quad (29)$$

where

$$T(n) = \frac{(2n-3)!!}{(2(n-1))!!(2n-1)}$$

Taking Fourier transform of Eq. (29) and making use of Eq. (26), we obtain

$$S_{\eta}(f) = \frac{2}{\pi} \sum_{n=1}^{\infty} T(n) \frac{\tau/(2n-1)}{1 + (2\pi\tau f/(2n-1))^2} \quad (30)$$

### 3.2. Junction noise model

Another example is the voltage fluctuation  $x(t)$  applied to a junction of a semiconductor device. Supposing  $x(t)$  is a stationary normal process with zero mean and its spectrum is given by Eq. (20), the noise current is given by

$$\delta I(t) = I \exp(-\beta x(t)), \quad (31)$$

where  $\beta = q/k_B T$  is the thermal voltage.

Using Eq. (27), we have the current noise spectrum below,

$$S_{\delta I}(f) = I^2 \exp\left(\frac{(\sigma\beta)^2}{2}\right) \times \left\{ \delta(f) + \sum_{n=1}^{\infty} \frac{(\sigma\beta)^2}{n!} \frac{2\tau/n}{1 + (2\pi\tau f/n)^2} \right\} \quad (32)$$

If  $\sigma\beta = 10$ , the span of  $f^{-\gamma}$  ( $\gamma = 1 \pm 0.3$ ) spectrum is about three decades. This result is approximately in the same range of some or the experiments on the low-frequency noise of diodes and bipolar transistors [52].

In the above examples, it is noted that the  $1/f$  spectrum is very narrow in the frequency domain, normally does not exceed three decades unless a very special transform is found. A broader frequency range for that spectrum can be readily obtained by assuming more than one relaxation time constant in the systems but need not be as wide as the McWhorter one. Fig. 4 shows the Lorentzian spectra for different relaxation time

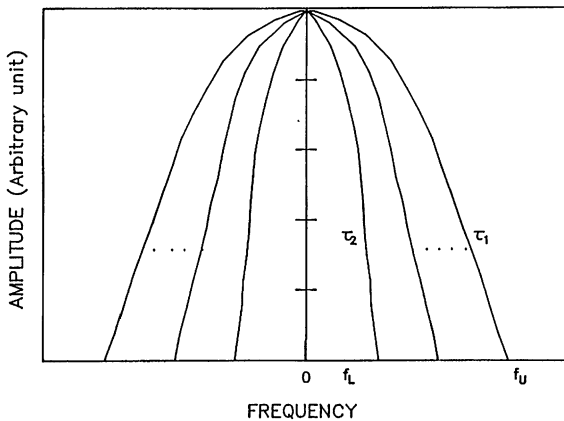


Fig. 4. Plot of noise spectra versus frequency with relaxation time constant as a parameter.

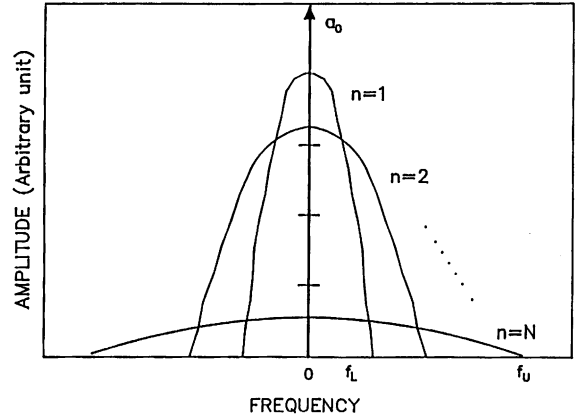


Fig. 5. Theoretical plot of components of noise spectrum as a function of frequency for a non-linear transformed signal.

constants. In general, the larger the range of the time constant, the broader the spectrum is. The resultant spectrum is given by the superposition of these spectra between  $\tau_2$  and  $\tau_1$ , and the upper frequency limit or  $1/f$  spectrum is given by  $f_U = (2\pi\tau_2)^{-1}$  whereas the lower  $f_L = (2\pi\tau_1)^{-1}$ . In contrast to Fig. 4, Fig. 5 illustrates some spectrum components of a non-linear transform of a Gaussian process with a spectrum given by Eq. (20). The frequency band of the  $n$ th component will be  $n$  times wider than the first component. The slope of the resultant spectrum will be close to  $-1$  over a few decades of frequency, depending on the transform and on the coefficient or the spectral components. The spectra in Figs. 4 and Fig. 5 are dissimilar in that the former assumes a wider range or relaxation time constant distribution ( $10^{-7}$ – $10^5$  s) in order to correlate with the experimental facts. On the other hand, the spectrum from a non-linear transform, similar to the McWhorter one, would have both upper and lower limits. Although the limit of  $1/f$  spectrum have extended down to  $10^{-6}$  Hz or even lower by some researchers [53], a pure  $1/f$  spectrum of infinite band limits cannot exist unless the spectrum is time variant, or the process is non-stationary.

### 4. Trapping–detrapping MOSFET noise model [9]

The study of low-frequency noise in electronic devices, especially the MOS transistor, has received increasing attention, not only due to its importance in the scaled-down structures in integrated circuits and the wide-spread applications of MOS devices in analog electronics, but also due to the fact that it provides a possible test vehicle to examine the model of  $1/f$  or flicker noise which is an universal phenomenon existing in most physical systems [54–56]. Several models have been proposed to account for the  $1/f$  noise in MOSFET



[29,30], but these models seem to have many limitations and need some modifications. Nevertheless, either these models or other experiments reported that the interface traps should contribute to the low-frequency noise generation in MOSFET. And new insight was shed into the trapping–detrapping physics of electrons at the silicon-oxide interface [57–62]. An electrical signal with two discrete levels is now frequently reported when monitoring the noise in the time domain for a small MOS device. Ralls et al. [57] observed a discrete change (random telegraph signal) in the channel conductance of a submicron device as a result of the capture and emission of electrons at the interface traps in the gate oxide. Welland and Koch [58] even obtained trap profiles on the silicon surface in great details using the scanning tunneling microscopy in an extremely small MOS device. They found that the trapping or detrapping time constant is energy-activated and could be either enhanced or retracted by applied gate voltage. The traditional McWhorter's approach [4] in model formulation may have problems on this account. In fact, the past investigations of the low-frequency noise in MOS transistors had already encountered several discrepancies that will be summarized in Section 2. A new low-frequency noise model for MOS transistors based on the results given by Ralls et al. [57], Welland and Koch [58], and Restle [60] were formulated [9].

Several researchers have found a signal with two discrete levels in MOS transistors and attribute this observation to the trapping and detrapping of electrons in the silicon oxide. These results show that the trapping and detrapping time-constants ( $\tau_t$  and  $\tau_d$ ) are different from each other [57], and depend exponentially on temperature and gate voltage [57,58]. More specifically, both  $\tau_t$  and  $\tau_d$  are thermally activated and there is a slight difference in their activation energies. With the gate dependence of these two constants is opposite in sign and different in value, i.e. it decreases with gate bias for trapping but increases for detrapping (for p-type silicon). For an oxide trap located at a distance  $z$  beyond the silicon-oxide interface, the time constants can be approximated by [57,58]

$$\begin{aligned}\tau_t &= \tau_{ot} \exp\left(E\left(1 + \lambda_t V_{GT} \frac{z}{d_{ox}}\right)/k_B T\right) \\ \tau_d &= \tau_{od} \exp\left(E\left(1 + \lambda_d V_{GT} \frac{z}{d_{ox}}\right)/k_B T\right)\end{aligned}\quad (33)$$

where  $V_{GT} = V_G - V_T$ ,  $d_{ox}$  is the oxide thickness,  $\tau_{ot}$ ,  $\tau_{od}$ ,  $\lambda_t$ , and  $\lambda_d$  are the trapping and detrapping parameters.

The rate of a trapping–detrapping process relaxing to an equilibrium state can be defined in terms of its characteristic time constant ( $\tau_c$ )

$$\tau_c = \frac{\tau_t \tau_d}{\tau_t + \tau_d} \approx \tau_{oc} \exp\left(\frac{E}{k_B T}(1 + \zeta V_{GT})\right) \quad (34)$$

$$\text{where } \tau_{oc} = \frac{\tau_{ot} \tau_{od}}{\tau_{ot} + \tau_{od}}.$$

The noise spectrum generated by a single electron trap with different trapping and detrapping time constants can be given by [43]

$$\Delta S_N(f) = \frac{\tau_t \tau_d}{(\tau_t + \tau_d)^2} \frac{4\tau_c}{1 + (\omega \tau_c)^2} \quad (35)$$

For the case of multiple traps

$$S_N(f) = \int_0^\infty \frac{\tau_c D(E) dE}{1 + (\omega \tau_c)^2} \quad (36)$$

In Eq. (36),  $D(E)$  is the energy distribution function of traps which can be approximated by [9]

$$D(E) = N_t \exp(-\xi E) \quad (37)$$

where the sign of  $\xi$  is negative for p-channel and positive for n-channel MOS transistors.

For the sake of simplicity, we assume that the quantities of  $\lambda_t V_{GT} z/d_{ox}$  and  $\lambda_d V_{GT} z/d_{ox}$  in Eq. (33) are less than unity so that the first term in Eq. (35) can be reduced to a constant equal to 1/4. (These assumptions are rational but are not critical. And they do not affect the major conclusions even if they are not valid.) In addition, let

$$\zeta = (\lambda_t + \lambda_d)z/d_{ox} \quad (38)$$

then the noise spectrum induced by multiple traps located at a distance  $z$  can be approximated by

$$\frac{S_{I_D}(\omega)}{I_D^2} = \frac{q^2}{C_{ox}^2 V_{GT}^2} \frac{\pi k_B T N_t \tau_{oc}^{1-\gamma}}{2 \sin\left(\frac{\pi\gamma}{2}\right) L W (1 + \zeta V_{GT}) \omega^\gamma} \quad (39)$$

If  $\zeta V_{GT}$  is much smaller than unity, the term  $1/(1 + \zeta V_{GT})$  can be expanded into a series. Neglecting the terms containing  $\zeta V_{GT}$  with order higher than two, we have

$$\frac{S_{I_D}(\omega)}{I_D^2} \approx \frac{\pi k_B T N_t \tau_{oc}^{1-\gamma}}{2 \sin\left(\frac{\pi\gamma}{2}\right) L W \omega^\gamma} \left\{ \frac{q^2}{C_{ox}^2 V_{GT}^2} - \frac{(\lambda_t + \lambda_d) q^2 z}{\epsilon_{ox} C_{ox} V_{GT}^2} \right\} \quad (40)$$

For some experiments, the noise is measured in terms of an equivalent input noise,  $S_{V_G}(\omega)$

$$S_{V_G}(\omega) \approx \frac{\pi k_B T N_t \tau_{oc}^{1-\gamma}}{2 \sin\left(\frac{\pi\gamma}{2}\right) L W \omega^\gamma} \left\{ \frac{q^2}{C_{ox}^2} - \frac{(\lambda_t + \lambda_d) q^2 z}{\epsilon_{ox} C_{ox}} V_{GT} \right\} \quad (41)$$

As the theories developed by others [4,30–35], the low-frequency noise generation was due to the occupation fluctuations of electron traps at the oxide-silicon interface or in the bulk or the gate oxide, and the almost unity frequency indices of the noise spectra from several experiments arise mainly from a wide distribution of

activation energies for the trapping–detrapping process. An unusual treatment in deriving Eq. (41) is that we make use of the experimental fact: the gate-bias and temperature dependencies or the trapping and detrapping time constants are not the same. This yields the phenomenon of different gate dependent noise amplitude and frequency index in the noise spectrum.

This model had been verified using the results summarized in Section 2. Firstly, it was noted that the low-frequency noise level in this model is inversely proportional to the channel area, directly proportional to the traps density, and depends weakly on the drain voltage. For MOS transistors biased in the linear region (see Eqs. (39), (40) or (41)). These match well with the experimental observations from several sources as mentioned in Section 2. For the oxide capacitance dependence, two different behaviors of dependence, i.e.  $C_{\text{ox}}^{-1}$  and  $C_{\text{ox}}^{-2}$  exist simultaneously in this model. Noted that since the magnitude of the second term inside the bracket in Eq. (40) is almost three orders less than that of the first term, it has negligible contribution to the total noise in general. The equivalent gate noise power is then independent on gate-voltage and simply scaled by  $C_{\text{ox}}^{-2}$ . However, if we calculate the slope or gate-noise versus gate-voltage plot, i.e.  $dS_{V_G}(f)/dV_G$  the first term drops out and the second term becomes dominant, then  $1/C_{\text{ox}}$  dependence as reported by Mikoshiba [33] will be found.

On the other hand, although two dissimilar gate-bias dependencies were found in a single MOS transistor, it does not imply that there are at least two noise sources dominant in this device. In the model given above, these dependencies are interpreted in term of a single noise source, the trapping–detrapping process. Although incorporating the mobility fluctuation model (use of the Hooge relation) into number fluctuation model could also obtain this type of dependence, Hooge formula turns out to be an oversimplified model and even has problems in modeling the low-frequency noise in metal film for which it was first established.

In addition to the gate dependent noise amplitude, the frequency index of the noise spectrum is again governed by the applied gate-voltage. Eqs. (39)–(41) show that the frequency index which is generally close to unity, is a function of gate-voltage ( $V_G$ ) and the ad hoc sample features (i.e. the parameters  $\xi$ ,  $\lambda_t$ ,  $\lambda_d$ ,  $z$  and  $d_{\text{ox}}$ ). For instance, the frequency index will decrease with  $V_G$  for a p-MOSFET if the gate sensitivity of the relaxation constants is given by Ralls et al. [54]. This agrees with some experimental results [43,44]. Inconsistencies are found in the findings of some reports [45–47] unless a change or the sign of the quantity  $\xi$  is allowed. In these experiments, the frequency index shows fluctuation with the gate bias. The results of our study showed that the frequency index is process-sensitive and highly non-linearly depends on the applied gate voltage [6,9].

Also note that the temperature dependencies of noise amplitude and frequency index in the newly proposed model were governed by some similar factors. Although almost all models are very weakly temperature dependent, most frequently, scaled by  $k_B T$ , several experiments suggested that the noise should be scaled by the temperature with a power law. Christensson and Lundstron [41] found that the noise level increases with temperature for a p-channel MOS transistor but decreases for an n-channel device in temperature range from 50 to 400 K. Fig. 6 shows that the time sequence of the drain current noise at different temperatures. Since the frequency index generally can either rise or fall for a larger range of temperature, all models have difficulties in this account. However, Eq. (4)) has potential to correlate the complicated dependencies of the frequency index and even the noise amplitude. Because there are several types of traps, e.g. neutral traps, positive traps, negative traps, etc. in the oxide, each type of traps has its own activation energy and gate dependence, and distributes over the gate insulator in different locations, a change in sign of the second term in the frequency index

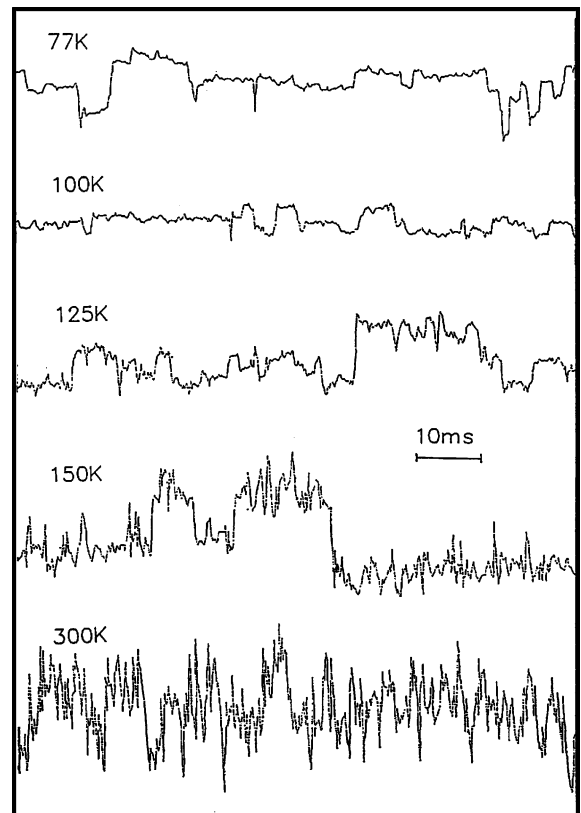


Fig. 6. Random signal in drain current for a transistor with an aspect ratio of  $W/L = 60/0.6$  (in  $\mu\text{m}$ ) biased at  $V_G = 2$  V and  $I_D = 0.14$  mA at different temperatures.

expression  $\{\gamma = 1 - \xi k_B T / (1 + \zeta V_{GT})\}$  for different temperature is possible even though the gate bias remains unchanged.

Finally, several refinements to the model might be made. First, since the gate potential also governs the position of the Fermi level, the trap energy distribution will vary with the bias and thus leading to the additional gate-potential dependencies for noise power and frequency index. With this connection, a modification of the traps distribution shall be reasonable, and thus further investigation on exploring the trap details in gate oxide is indispensable. Second, it should be noted that even though Ralls et al. [57] had not found any evidence of tunneling of traps, it seems that the tunneling of electron into the bulk of silicon dioxide cannot be neglected [61]. The low-frequency noise resulting from this tunneling for a silicon-gate metal-oxide-silicon capacitors was measured by Neri et al. [61], and since the slopes of the noise spectra are sensitive to traps location in the oxide, it is interesting to consider the spatial trap distributions for all  $z$  in the oxide. Third, for the widespread observations of the frequency index varying with both temperature and gate bias, our model can only correlate part of the experimental results, unless the sign of  $1 + \zeta V_{GT}$  can be changed. Though there is no evidence that this change is appropriate in the experiment given by Ralls et al. [57], the argument is that the experiment of Ralls et al. was carried out in a very small device, and the temperature range (4–111 K) is not the same as the investigation of low-frequency noise in other experiments. Further experimental confirmations are needed for better modeling of the low-frequency noise in a MOSFET.

## 5. Surface noise model in metal films

Compared with the same study in MOSFET, the exploration of the noise origin in metal film seems lacking experimental supports as no attempt was made to study the physical structure of metal film. A surface-related experiment was carried out to verify this hypothesis [5]. Unlike the noise model used in MOS devices which ascribes the number fluctuation to the quantum-mechanical tunneling of carriers into the gate insulator, the number fluctuations model in metal films are attributed to the tunneling or thermionic emission of electrons over empty regions between hillocks on the surface of thin film conductor [5,29].

Figs. 7 and 8 depict the relations of the normalized noise amplitude ( $= \text{noise level} \times f^{\gamma} / V^2$ , where  $V$  is the voltage across the conductor under test) with length and thickness respectively, for copper-film conductors. It is noted that the noise amplitude of copper film on silicon wafer is almost inversely proportional to the conductor length. These results, which resemble the experiments

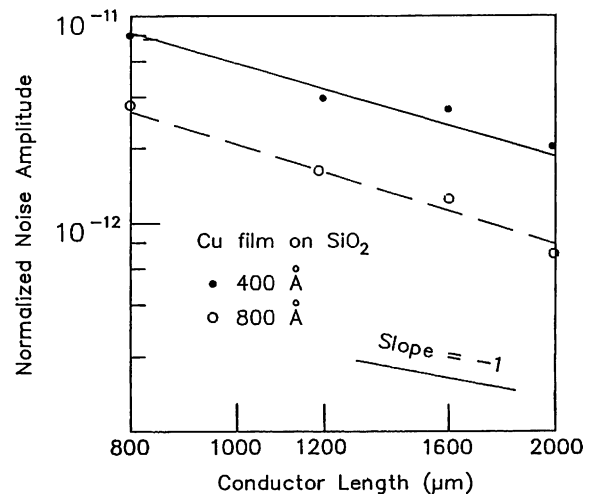


Fig. 7. Normalized noise amplitude vs lengths of copper-film conductors on oxidized-silicon wafer.

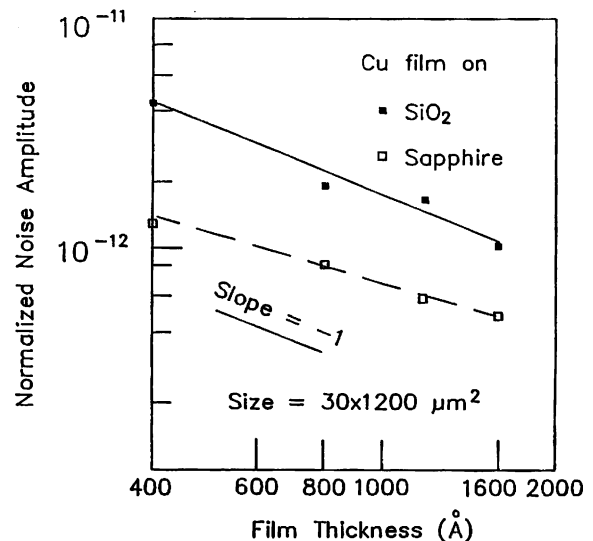


Fig. 8. Plot of normalized noise amplitude as a function of copper films thickness for conductors with a size of  $30 \times 1200 \mu\text{m}^2$ .

reported by Hooge and his co-workers [3,27], were used to be considered one of the main evidence of the bulk origin  $1/f$  noise in metal films, because the  $1/f$  noise is inversely proportional to the total number of free carriers [4]. However, this speculation may not be true because the surface noise can also possess this relationship. As the film becomes thicker, the portion of current conduction through the surface layer is reduced. As a result, the noise level is lower for thicker films, and the relationship is not necessarily in exactly inverse proportion. In fact, as shown in Fig. 8, the

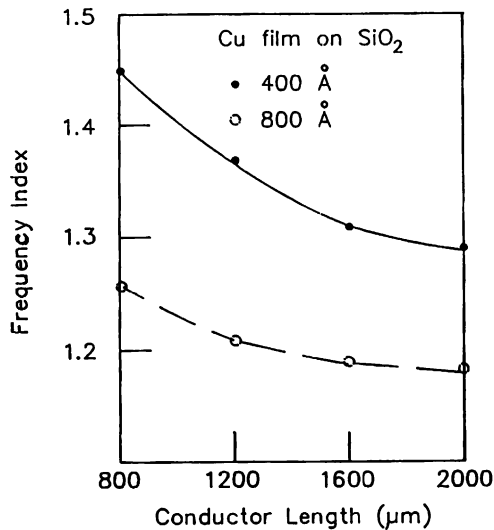


Fig. 9. Plot of frequency index of noise spectrum for copper-film conductor on oxidized silicon wafer as a function of conductor lengths.

thickness-dependence obviously deviates from the inverse proportion law for sapphire substrate.

As shown in Figs. 9 and 10, the frequency index decreases with either the length or thickness of the conductor. However the decrease with the thickness is much faster than the length. The frequency indices were found to range from a value of about 1.45 to a value slightly smaller than 1.1 in this investigation. In addition, for same thickness, copper films on sapphire substrate has a smaller frequency index than that on silicon. These ob-

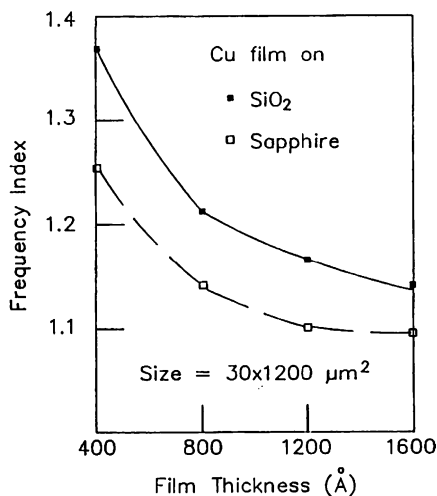


Fig. 10. Plot of frequency index as a function of copper films thickness.

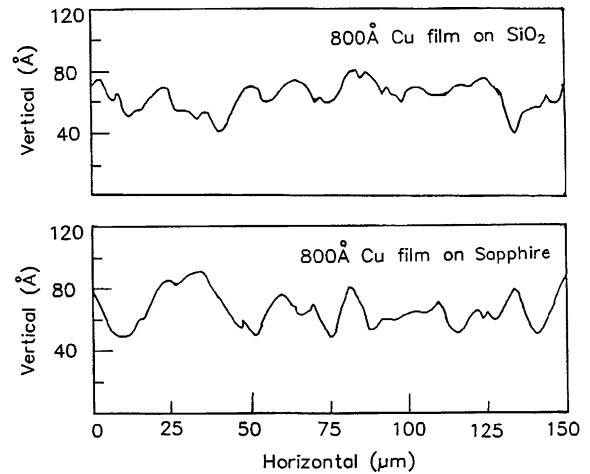


Fig. 11. Surface profiles of sputtered copper films on oxidized silicon and sapphire.

servations have never been reported previously and obviously could not be explained by Hooge's model.

Although sputtered samples are believed to have a greater uniform coverage of the surface, the surface under the detection of a stylus instrument reveals that the surface is rough and has a lot of hillocks. The typical profile of metal films on oxidized-silicon and sapphire substrates are shown in Fig. 11. The roughness or irregularities of surface for sputtered film depends strongly on substrate. For example, the surface roughness of film on oxidized-silicon wafer is 5 Å whereas it is 10 Å for the sample on sapphire substrate. The detailed difference between the samples on these two kinds of substrates is that the former has smaller-sized hillocks whereas the latter has larger ones.

Based on these experimental data, a surface noise model for metal films was developed [5]. The basic mechanism of the surface noise in metal films is due to the charge-carrier number fluctuation which results from the electron tunneling through or emitting over the empty regions between any two adjacent hillocks on the surface. The theory of electron tunneling through a small dielectric gap was well developed, and the time required to transmit an electron from one hillock to the other with a separation of  $w$  can be expressed as

$$\tau = \tau_0 \exp(\phi w) \quad (42)$$

where  $\phi$  is a parameter governed by the potential barrier between the metal and the empty region.

Assuming that the distribution function of  $w$  can be approximated by

$$D(w) = \exp(-\xi w) \quad (43)$$

which means that the larger of  $w$ , the smaller of the occurrence probability. This is a good approximation for the practical cases.

Taking summation of the Lorentian spectra for all  $w$ , (the formulation process is similar to the McWhorter approach [4]) we have

$$\frac{S(f)}{I^2} = \frac{\tau_0^{\xi/\phi} (C/N)^\beta}{\phi (2\pi f)^\gamma} \int \frac{x^{\gamma-1}}{1+x^2} dx \quad (44)$$

where  $\gamma = 2 - \xi/2\phi$  is defined as the frequency index of the noise spectrum;  $C$ ,  $\beta$  are some process-dependent parameters and  $N$  is the total number of electron inside the conductor. The factor  $(C/N)^\beta$  represents the portion of current transport in the surface layer. For some range of  $w$ , Eq. (44) can be approximated by

$$\frac{S_I(f)}{I^2} = \frac{\alpha}{N^\beta f^\gamma} \quad (45)$$

where  $\alpha$  is a parameter governed by the material and surface structure of the metal film. Equation (45) is different from the Hooge's formula [3] as in Eq. (3).

As the measured noise level is inversely proportional to the total number of electrons inside the conductor, the  $1/f$  noise mechanism is always regarded as a bulk effect; however, this observation could not reject the possibility that the surface noise can also possess this relationship. As shown in Eq. (44), just as the case of the thin film resistivity, when the conductor becomes thicker, the portion of current conduction  $((C/N)^\beta)$  through the surface layer is reduced; namely, the noise level is lower for a thicker film. In addition, the relationship is not necessarily exactly in inverse proportion. Indeed the present experiments showed that the thickness-dependence could deviate from inverse-proportion law significantly for sapphire substrates.

On the other hand, the frequency index was found to change from a value of about 1.45 to a value slightly smaller than 1.1 for different samples. Additionally, for the same thickness, copper films on sapphire substrate have a smaller frequency index than that on silicon. These observations obviously deviate from Hooge's formula. Meanwhile, the frequency index was found to decrease with either the length or thickness of the conductor, but the decrease with the thickness is more significant than with the length. The variation of the frequency index in our model can be explained by the different surface structures. In this model, the frequency index,  $\gamma$ , is given by  $2 - \xi/2\phi$ , and is strongly governed by the distribution of the empty spaces between the adjacent hillocks. Particularly, if there is only one value of  $w$ , the frequency index should be equal to 2, whereas for a uniform distribution of  $w$  over a very large range,  $\gamma$  approximates to unity. More generally, any form of  $w$  distribution other than the above two cases could occur depending strongly on type of substrate and method of preparation, and  $\gamma$  may be any value between 1 and 2. Since the surface of sample on oxidized-silicon has smaller-sized hillocks whereas the latter has larger hill-

ocks, it is reasonable to expect that the copper films on sapphire substrate have a smaller frequency index than that on silicon. The conductor-size dependence could also be explained accordingly.

The temperature dependence of the low-frequency noise may now become an important factor in clarifying the proposed noise models since most of the models are very weakly temperature dependent or even independent. Although  $1/f$  noise was believed to be insignificantly temperature-dependent, recent experiments had shown that noise in various metal films are strongly temperature-dependent [63–69]. In some experiments, the noise could even vary with the temperature over three orders of magnitude between 150 and 490 K for copper films [64,65]. Moreover, the dependence is quite arbitrary in these experiments. Eberhard and Horn [64,65] found that the noise magnitude first increases and then decreases with temperature from 100 to 600 K for copper, silver and nickel films, with the peak locations different for all these three metal films. Fleetwood et al. [67,68] showed that the shapes of the noise-temperature plots are dissimilar to each other for tin, indium, platinum and bismuth films and even for different methods of deposition. However, unlike the MOS devices, it is not easy to establish a temperature dependence model even though the metal film is simple in its physical structure. Because of the thermal expansion, a change in the dimensions and distances between hillocks, which in turn may have a pronounced effect on the probability of tunneling, could be expected. In addition, the energy provided to the electrons in the metal film could also vary for different temperatures. Furthermore, changes or defect density or activation energies could occur due to the high-temperature ( $>400$  K) thermal treatment where the morphology of the metal film could even be modified. These changes can cause further modification in the scattering process and the relaxation constant ( $\tau$ ). Governed by so many factors, the temperature dependence of low-frequency noise in metal film is therefore very complicated. Hence, even though the study of  $1/f$  noise in metal films is now dominated by investigating the temperature dependence [64–69] and the newly proposed model could be used to interpret the various experimental results by considering the factors mentioned earlier. However, no quantitative correlation was attained owing to the difficulties in monitoring these factors.

Besides, defects may be important for a bulk sample. The role of defects in determining the noise power and temperature dependence had been investigated by Pelz and Clarke [69]. with electron bombardment and subsequent annealing, Pelz and Clarke found that there exists a direct connection between the noise level and defects in polycrystalline copper films. They found that the noise could increase with electron-beam irradiation and decrease with thermal annealing. Additionally,

different types of temperature dependence for different annealing temperatures after the electron irradiation were also observed in the investigation. Thus, the model needs to be refined further.

Above all, in conducting any investigation on the  $1/f$  noise in metal films, one should bear in mind that the disagreement of the electrical properties in metal film for different experiments cannot be avoided as it is difficult to monitor the film parameters during preparation and to completely reproduce the samples. It is, therefore, strongly suggested that the future noise studies in metal film should also report some other characteristics, such as surface morphology of the samples.

## 6. Conclusions

Low-frequency noise or flicker noise has been found in many systems and has become a hot research topic for many decades. More and more studies in electronic system suggest that if there is a common regime for the low-frequency noise, it must be mathematical rather than physical ones [29,70,71]. These mathematical processes give rise to  $1/f$  spectrum could be due to the distribution of time constant in spatial or energy-wise and the non-linear transformation of Gaussian signal. This paper presents a historical review on the development of low-frequency noise study in electron devices and the recent progresses in the understanding and modeling are updated. Several key issues were noted:

- (1) In developing the noise models, the models should be stationary from random process point of view. Various mathematical processes exist and are valid for generating the flicker noise spectrum [29,70,71]. It is noted that the low-frequency noise spectrum could originate from non-linear transformation of single relaxation process [29]. Results show that processes with only one relaxation time constant might also give a  $1/f^\gamma$  ( $\gamma = 1 \pm 0.3$ ) spectrum with a wide frequency band by an appropriate transformation or modulation of an weakly stationary Gaussian noise. That was different from the classical treatment of the noise spectrum where a wide range of relaxation time constant ( $10^{-7}$ – $10^5$  s) is always assumed in order to correlate with the experiments. Other mathematical models are also possible [70,71].
- (2) The models should be based on some fundamental physics and correlate well with the related properties of the materials or devices. For MOS transistor, a better low-frequency noise model had been derived based on the studies of the trapping and detrapping processes at the silicon-oxide interface [9]. For metal film, correlation between the low-frequency noise with the surface characteristics of metal films was attempted [5].

- (3) In verifying or validating the model, the temperature dependence and the variation of the frequency index had been treated as the major test engines [5–7, 46,47].
- (4) In applications of the low-frequency noise measurement as diagnostic tools, both the physics and the mathematics are important [11,37,54,59].

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