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1. We know D_n is distributive iff

$$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$$

where $x, y, z \in (D_n, \vee, \wedge)$

Let $a \in (D_n, \vee, \wedge)$

then $a = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}$

where p_i is prime divisor of n

then

$$LHS = x \wedge (y \vee z) = \left(p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k}\right) \wedge \left(\left(p_1^{y_1} \cdots p_k^{y_k}\right) \vee \cdots\right)$$

$$= \prod p_i^{\min(x_i, \max(y_i, z_i))} \quad \text{— (1)}$$

$[\because \wedge$ is gcd & $\vee$ is LCM$]$

Similarly

$$RHS = (x \wedge y) \vee (x \wedge z) = \prod p_i^{\max(\min(x_i, y_i), \min(x_i, z_i))} \quad \text{— (2)}$$

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From ① & ②, we can conclude that

LHS = RHS if

$$\min(x_i, \max(y_i, z_i)) = \max(\min(x_i, y_i), \min(x_i, z_i))$$

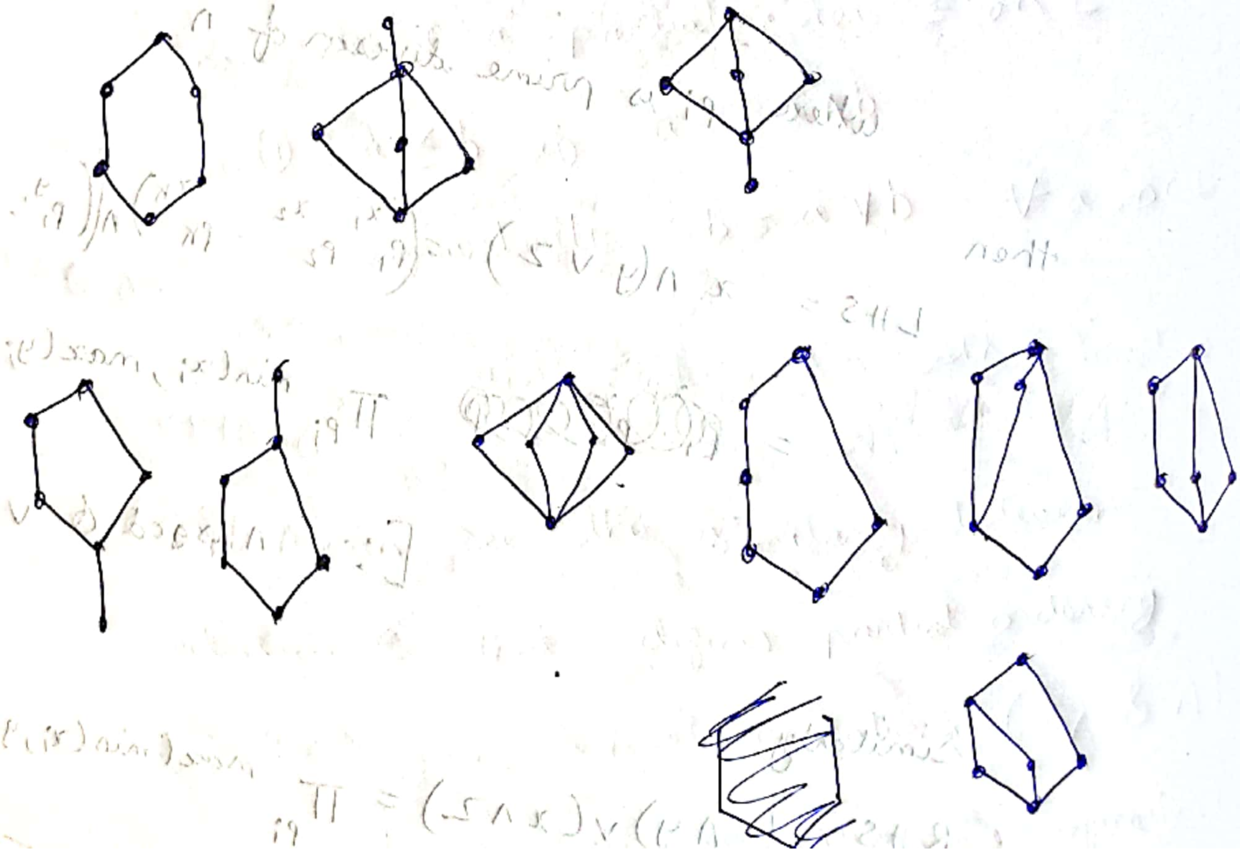
and we already know $L = (W, \wedge, \vee)$ is distributive lattice where

$$x \wedge y = \min\{x, y\}$$

$$x \vee y = \max\{x, y\}$$

$\therefore (D_n, \vee, \wedge)$ is distributive lattice

Q4 There are 10 elements



Q2 given:

$(L_1, \wedge_1, \vee_1)$ and $(L_2, \wedge_2, \vee_2)$ modular lattice

then show that

$(L, \wedge, \vee)$ is modular lattice

where $L = L_1 \times L_2$

Let (x_1, y_1), (x_2, y_2) and $(x_3, y_3) \in L$

then $(L, \wedge, \vee)$ is modular if we can

show

$(x_1, y_1) \leq (x_3, y_3)$ implies

$$(x_1, y_1) \vee ((x_2, y_2) \wedge (x_3, y_3))$$

$$= ((x_1, y_1) \vee (x_2, y_2)) \wedge (x_3, y_3))$$

where $a \leq b \Leftrightarrow a \wedge b = a$ $\Leftrightarrow a \vee b = b$

$$LHS = (x_1, y_1) \vee ((x_2, y_2) \wedge (x_3, y_3))$$

$$= (x_1, y_1) \vee (x_2 \wedge_1 x_3, y_2 \wedge_2 y_3)$$

$$= (x_1 \vee_1 (x_2 \wedge_1 x_3), y_1 \vee_2 (y_2 \wedge_2 y_3))$$

$$= ((x_1 \vee_1 x_2) \wedge_1 x_3, (y_1 \vee_2 y_2) \wedge_2 y_3)$$

$$= ((x_1, y_1) \vee (x_2, y_2)) \wedge (x_3, y_3)) = R.HS$$

Hence Proved.

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Q3 Lattice as ~~algebra~~ algebra and lattice as order are essentially the same concepts having different approach and perspective

For example

(1) In ordered theory, lattice has two operations $\vee$ and $\wedge$.

We can easily verify commutative, associative and absorption laws for them.

$\therefore$ They make $(L, \vee, \wedge)$ into a lattice in algebraic sense.

(2) Also, given a lattice $(L, \vee, \wedge)$, we can define a partial order $\leq$ on L

(1) $a \leq b$ if $a = a \wedge b$

equivalently $b = a \vee b$ $\forall a, b \in L$

and duality for other direction

One can see the similarly between relation $\leq$ that defines partial ordering and the join & meet operations ($\vee$ & $\wedge$)

Hence both definations are ~~equivalent~~ equivalent

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