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① A lattice $(L, \vee, \wedge)$ is modular iff

$\forall x, y, z \in L$

$$x \leq z \Rightarrow x \vee (y \wedge z) = (x \vee y) \wedge z \quad -①$$

So, to show that modular identity is self dual, we need to prove that

$$z \leq x \Rightarrow x \wedge (y \vee z) = (x \wedge y) \vee z \quad -②$$

Proof:-

$$x \leq z \Rightarrow x \vee (y \wedge z) = (x \vee y) \wedge z$$

So, we can also say that

if $z \leq x \Rightarrow z \vee (y \wedge x) = (z \vee y) \wedge x$

Rearranging by using commutative property

$$(x \wedge y) \vee z = x \wedge (y \vee z) \quad -③$$

Hence proved (② & ③)

② We know all non-modular Lattice L contain the lattice N_5 as sub lattice

$\rightarrow$ length $= 3$

$\therefore$ length two lattice can't have N_5 ($\because 2 < 3$)

$\therefore$ All length two lattice are modular

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Q3 There does not exist a modular lattice of seven elements in which complemented elements do not form a sublattice

Q4 To prove that a finite meet-semilattice with universal upper bound I is lattice, we need to ~~show~~ define a join operation

i.e ~~$a, b \in L$~~ Let L be meet semilattice

~~for ea~~ $\forall a, b \in L$

$$a \vee b = c$$

where $c \in L$ and $a \leq c$ & $b \leq c$

Since upper bound always exists, we can define

$$a \vee b = glb(\{c, \; a \leq c \; \& \; b \leq c\})$$

$\Downarrow$

This set is non-empty since universal upper bound is I and finite

i.e $glb(\{c, \; a \leq c \; \& \; b \leq c\})$ is finite meet of set of elements

$\therefore$ Join exists for any two elements $\in L$

Hence L is a lattice

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