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Q1 a) Let $E = x \wedge (y \wedge z)$

$= glb(x, y \wedge z) = glb(x, glb(y, z))$

$= glb(x, y, z)$ — ①

Similarly $(x \wedge y) \wedge z = glb(x \wedge y, z)$

$= glb(glb(x, y), z)$

$= glb(x, y, z)$ — ②

From ① & ②

$x \wedge (y \wedge z) = (x \wedge y) \wedge z$

$\Rightarrow x \vee (y \vee z) = (x \vee y) \vee z$ (Duality)

b) Let a be glb of $x, (x \vee y)$

i.e $a = glb(x, x \vee y)$

$\therefore a \leq x$

We can also say that x is lower bound of x and $(x \vee y)$

$\therefore x \leq a$ ($\because a$ is glb)

$\Rightarrow x = a$ (Antisymmetric)

$\Rightarrow x \wedge (x \vee y) = x$

$\Rightarrow x \vee (x \wedge y) = x$ (Duality)

Q2 i) y is lower bound of $y \vee z$

$\therefore x \wedge (y \vee z) \geq x \wedge y$ —①

Similarly $x \wedge (y \vee z) \geq x \wedge z$ —②

$\therefore x \wedge (y \vee z)$ is upper bound (From ①&②)

$\therefore x \wedge (y \vee z) \geq \text{lub}(x \wedge y, x \wedge z)$

$\Rightarrow x \wedge (y \vee z) \geq (x \wedge y) \vee (x \wedge z)$ —③

ii) Applying duality on ③

$x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$

Hence proved.

Q3 To prove:-

$(a \vee b) \wedge (c \vee d) \geq (a \wedge c) \vee (b \vee d)$

Proof: $a \leq a \vee b$

$c \leq c \vee d$

$\Rightarrow (a \wedge c) \leq (a \vee b) \wedge (c \vee d)$ —①

Similarly $b \leq a \vee b$

$d \leq c \vee d$

$(b \wedge d) \leq (a \vee b) \wedge (c \vee d)$ —②

From ① & ②, we can say

$(a \vee b) \wedge (c \vee d)$ is upper bound of $a \wedge c$ and $b \wedge d$

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$(a \vee b) \wedge (c \vee d)$ is greater than lub of $(a \wedge c)$ & $(b \wedge d)$

$\Rightarrow (a \vee b) \wedge (c \vee d) \geq (a \wedge c) \vee (b \wedge d)$

Hence proved.

(4) Let C be the set of all convex subsets of V

let $L = (X, \leq)$ where $\leq$ is the subset operation

Join operation

We define it to be convex hull of the union of two convex set

Let S, T be two subset of vector space V.

then

$conv(S) \vee conv(T) = conv(S \cup T) = conv(conv(S) \cup conv(T))$

Meet operation

The intersection of any collection of convex set is convex.

So the convex subsets of any vector space form a complete lattice