Name - T. Vijay

Roll no - CS17BTECH11040

1. In this question, we pad the image such that the convolved image size is same as the input image.

In other words, if $F * I \rightarrow I'$

then $\text{shape}(I) = \text{shape}(I')$

a) $I = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ $\quad F = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

padded $I = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix}$, $F' = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$

We know $F * I = F' \oplus I$ where $F'$ is flipped filter and $\oplus$ is correlation

$F * I = \begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & 4 \end{bmatrix}$

b) $F = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix}$

$\Rightarrow F_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad F_2 = \begin{bmatrix} 1 & -1 \end{bmatrix}$

Calculating $F_1 * I$

$$\text{padded } I = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix} \qquad F_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$I' = F_1 * I = \begin{bmatrix} 2 & 0 & 1 \\ 3 & -1 & 3 \end{bmatrix}$$

Calculating $F_2 * I'$

$$\text{padded } I' = \begin{bmatrix} 0 & 2 & 0 & 1 \\ 0 & 3 & -1 & 3 \end{bmatrix} \qquad F_2 = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$F_2 * I' = F_2 * (F_1 * I) = \begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & 4 \end{bmatrix}$$

Q1 c) We know,

$$(F * I)[i, j] = \sum_{k,l} I[i-k, j-l] F[k, l] \quad -①$$

$$\therefore (F_1 * I)[i, j] = \sum_{k,l} I[i-k, j-l] F_1[k, l]$$

Since $F_1$ is column vector, we can change it to

$$I' = (F_1 * I)[i, j] = \sum_{k} I[i-k, j] F_1[k] \quad -②$$

Now, we convolve with $F_2$

$$F_2 * I'[i, j] = (F_2 * (F_1 * I))[i, j]$$

$$= \sum_{k,l} (F_1 * I)[i-k, j-l] F_2[k, l] \quad (\text{using } ①)$$

$\because F_2$ is row vector, we can change it to

$$= \sum_{l} (F_1 * I)[i, j-l] F_2[l]$$

$$= \sum_{k,l} I[i-k, j-l] F_1[k] F_2[l] \quad (\text{using } ②)$$

$$= \sum_{k,l} I[i-k, j-l] F[k, l]$$

$$= (F * I)[i, j] \quad (\text{using } ①)$$

$$\therefore F_2 * (F_1 * I) = F * I$$

Hence proved

Q1 d) In part ⓐ

1. For producing each output pixel, we perform $(2\times2)$ multiplications, i.e 4

No. of pixels in Image = $2\times3$
= 6

$\therefore$ Total = $4\times6 = 24$

In part ⓑ

No. of multiplications per 1D conv = 2
(For $F_1$)

No. of operations = Size of output image
= $2\times3 = 6$

$\therefore$ Total multiplications = $2\times6 = 12$ —①

Similarly for $F_2$
Total multiplications for $F_2$ = 12 —②

$\therefore$ Total = 12+12 [Adding ① & ②]
= 24

e) i) Shape of Image = $(M_1, N_1)$ = Shape of convolved Image

Shape of filter = $(M_2, N_2)$

Total multiplication = (Operations per Image pixel) $\times$ (No. of pixels)

$= (M_2 \times N_2) \times (M_1 \times N_1)$

$= M_1 N_1 M_2 N_2$

ii) Let $F = (F_1)(F_2)$

Shape of $F_1 = M_2$

Shape of $F_2 = N_2$

No. of operations for each element of output image for $F_1 = M_2$

Size of output image $= M_1 N_1$

$\therefore$ No. of multiplication $= M_1 N_1 M_2$

Similarly for $F_2$,

No. of multiplications $= M_1 N_1 N_2$

$\therefore$ Total $= M_1 N_1 (M_2 + N_2)$

iii) 2D - convolution $= O(M_1 N_1 M_2 N_2)$

1D - convolution $= O(M_1 N_1 (M_2 + N_2))$

and we know $O(M_2 N_2) \geq O(M_2 + N_2)$

Hence 1D convolution is more efficient

Q2 a) Let the gradient at point $(x, y)$ be $(D_x, D_y)$

Since we rotate the system by angle $\theta$

the new ~~gradient~~ gradient is $(D'_x, D'_y)$

So $$D'_x = (D_x \cos\theta - D_y \sin\theta)$$

$$D'_y = (D_x \sin\theta + D_y \cos\theta)$$

$\therefore$ Magnitude of new gradient

$$= \sqrt{(D'_x)^2 + (D'_y)^2}$$

$$= \sqrt{(D_x^2 \cos^2\theta + D_y^2 \sin^2\theta - 2 D_x D_y \sin\theta \cos\theta) + (D_x^2 \sin^2\theta + D_y^2 \cos^2\theta + 2 D_x D_y \sin\theta \cos\theta)}$$

$$= \sqrt{D_x^2 (\cos^2\theta + \sin^2\theta) + D_y^2 (\cos^2\theta + \sin^2\theta)}$$

$$= \sqrt{D_x^2 + D_y^2}$$

= Original Magnitude

Since canny edge detector depends only on magnitude, and magnitude does not change in the new system, we can say that edge will be detected.

b) (1) Handling gaps - Decrease LOW threshold

(2) Handling spurious edge - Increase HIGH threshold

Reasoning

(1) Gaps occur because the LOW threshold is too high. So decreasing it will allow some additional positions to be marked as edge

(2) Too many ~~edges~~ positions are classified as edge because HIGH threshold is too ~~to~~ low. So increasing it will decrease spurious edge.