Name - T. Vijay

Roll no - CS17BTECH11040

Q1 a) $A_L = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$\therefore (a_{11} \wedge a_{12}) \vee (a_{21} \wedge a_{22}) =$

$= \vee(\wedge(\text{Rows}))$

and $(a_{11} \vee a_{21}) \wedge (a_{12} \vee (a_{22}))$

$= \wedge(\vee(\text{columns}))$

$\therefore \vee(\wedge(\text{Rows})) \leq \wedge(\vee(\text{columns}))$

b) The above eqn can be generalized as follows for $A_L(n \times m)$ matrix

$$\bigvee_{i=1}^{n} \left(\bigwedge_{j=1}^{m} a_{ij} \right) \leq \bigvee_{i=1}^{m} \left(\bigwedge_{j=1}^{n} a_{ji} \right)$$

c) $RHS = \bigwedge_{i=1}^{m} \bigvee_{j=1}^{n} a_{ji} \geq \bigwedge_{i=1}^{m} a_{ki}$

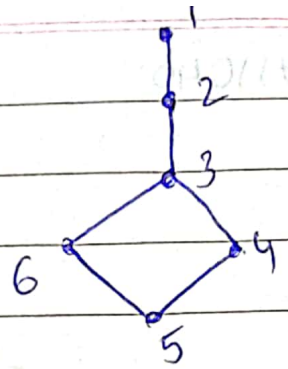
where $1 \leq k \leq n$

$\therefore$ RHS is upper bound of $\bigwedge_{i=1}^{m} a_{ki}$

$\therefore \forall k$ it must be greater than LUB

Hence $\bigvee_{i=1}^{n} \bigwedge_{j=1}^{m} a_{ij} \leq \bigwedge_{i=1}^{m} \bigvee_{j=1}^{n} a_{ji}$

Q2 i)



Maximal ideal:

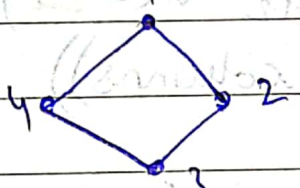
{2, 3, 4, 5, 6}

Prime

{5, 6} {4, 5} {3, 4, 5, 6}

{2, 3, 4, 5, 6}

ii)



Maximal ideal

{3, 4}. {3, 2}

Hence there exists a lattice with two maximal ideals.