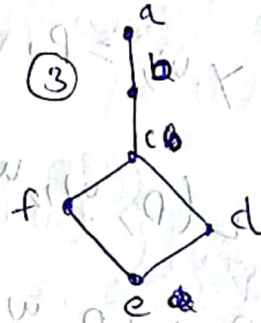
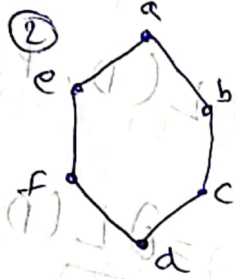


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Q2



→ Three 6 element Hasse diagram

a)

①

$\{a, b, c, d, e, f\}$

$\{b, c, d, e, f\}$

$\{c, d, e, f\}$

$\{d, e, f\}$

$\{e, f\}$

$\{f\}$

② $\{a, b, c, d, e, f\}$

$\{e, f, d\}$ $\{b, c, d\}$

$\{f, d\}$ $\{c, d\}$ $\{d\}$

③ $\{a, b, c, d, e, f\}$

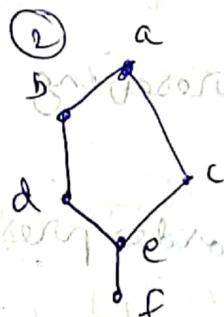
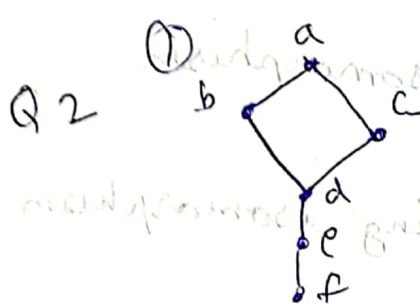
$\{b, c, d, f, e\}$

$\{c, d, f, e\}$ $\{f, e\}$ $\{d, e\}$ $\{e\}$

Q3 Any set of rational number that is bigger than
for some $a \in \mathbb{Q}$, i.e.

(motivated) $\{q \mid q \in \mathbb{Q} \text{ and } q \geq a\}$
where $a \in \mathbb{Q}$

$L \geq x$



- ⑥ ① $\{a, b, c, d, e, f\}$ $\{a, b, c, d, e\}$
 $\{a, b, c, d\}$ $\{a, b\}$ $\{a, c\}$ $\{a\}$
- ② $\{a, b, c, d, e, f\}$ $\{a, b, c, d, e\}$
 $\{a, b, d\}$ $\{a, c\}$ $\{a, b\}$ $\{a\}$

Q1

Let f is a lattice isomorphism (wrt join & meet)

then let $x, y \in L$

$$f(x \vee y) = f(x) \vee f(y)$$

$$f(x \wedge y) = f(x) \wedge f(y)$$

So if $x \leq y$
then $f(x) \leq f(y)$ } - ①

as $y = x \vee y$ and $f(y) = f(x) \vee f(y)$

Now if $f(x) \leq f(y)$

$$f(y) = f(y) \vee f(x)$$

$$= f(y \vee x)$$

$$y = y \vee x$$

$$x \leq y$$

(bijection)

\therefore From ① & ② and duality

f is order preserving isomorphism

Now let f is order preserving isomorphism

Let $x, y \in L$

$$\text{then } x \leq y \Rightarrow f(x) \leq f(y)$$

$$\text{and } f(x) \leq f(y) \Rightarrow x \leq y$$

- ④ Since L is finite, then $\text{Id}(L)$ is finite
 \therefore Ideals are principle and have a generator

Isomorphic mapping

Let $x \in L$

then $f(x) = \downarrow x$

f is isomorphic because

one-one mapping exists

i.e. $f(x) = f(y)$

$$\downarrow x = \downarrow y$$

then $\text{Join}(\downarrow x) = \text{Join}(\downarrow y)$

$$\Rightarrow x = y$$

Onto

$$x \leq y \Rightarrow f(x) \leq f(y)$$

$$\text{and } \cancel{f(x) \leq f(y)} \Rightarrow x \leq y$$