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Q1 i) No. of elements in $P = (D_*, 1)$

where $* = (x_1^{p_1})(x_2^{p_2})(x_3^{p_3}) \cdots (x_n^{p_n})$

(Prime factorization)

$\therefore$ No. of factors $= (p_1+1)(p_2+1)(p_3+1) \cdots (p_n+1)$

No. of elements is $P(A) = 2^{|A|}$ where $|A|$ = size of A

So we want

$$(p_1+1)(p_2+1)\cdots(p_n+1) = 2^{|A|}$$

Let $p_1 = p_2 = \cdots = p_n = 1$

So by observation, we can say that $P \cong Q$

if $*$ is made up of $|A|$ distinct prime numbers

ii)

Now let $* = x_1 x_2 x_3 \cdots x_n$

where $\forall i$ x_i is prime number (Distinct)

and $n = |A|$, where A is non-empty set

$\{a_1, a_2, a_3, \cdots a_n\}$

Isomorphic mapping

Let $a \in D_*$ i.e, $a = x_{i_1} x_{i_2} x_{i_3} \cdots x_{i_k}$

then $f(x) = \{a_{i_1}, a_{i_2}, a_{i_3} \cdots a_{i_k}\}$

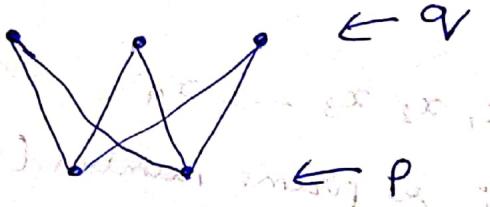
$$f^{-1}(\{a_{i_1}, a_{i_2} \cdots a_{i_k}\}) = a$$

$\therefore$ 1 will map to null set

and $*$ will map to A

Hence we have a isomorphic mapping

2. i) Form a complete bipartitate graph with q (maximal elements) at top and p (minimal elements) at bottom



ii) Complete bipartitate graph with $q+m$ (maximal elements) at top and p (minimal elements) at bottom

Q3 Given: $x, y \in S$ and x, y are maximum elements of S

$\therefore$ $s \leq x \quad \forall s \in S$ — ①

and $s \leq y \quad \forall s \in S$ — ②

$\therefore x \leq y$ and $y \leq x$ (From ① & ②)

$\therefore x = y$ (Anti-symmetric)

Similarly if x, y are minimum elements of S

$x \leq s$ and $y \leq s \quad \forall s \in S$

$\Rightarrow x \leq y$ and $y \leq x$

$\Rightarrow x = y$ (Anti-symmetric)

Q4 There are 25 lattices with size ≤ 6

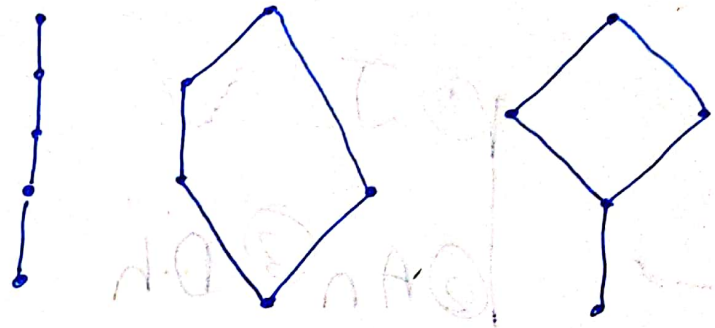
Size = 1

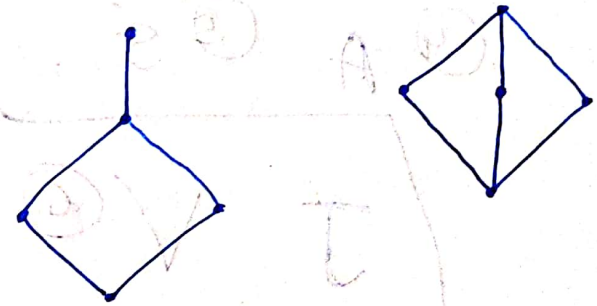
Size = 2

Size = 3

Size = 4

Size = 5





Size = 6