VIJAY SHAHWAL

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Q1.

Y (Rev)	X (Year)	X^2	XY
100	2007	4028049	200700
95	2009	4036081	190855
85	2010	4040100	170850
90	2011	4044121	180990
92	2012	4048144	185104
80	2013	4052169	161040
89	2014	4056196	179246
84	2015	4060225	169260
93	2016	4064256	187488
96	2017	4068289	193632
97	2018	4072324	195746

Since we know

$$Y = h_w(X) = w_0 + w_1 X \quad -(1)$$

$$w_0 = \frac{BC - AD}{mC - A^2}$$

$$w_1 = \frac{AB - mD}{A^2 - mC}$$

where $A = \sum_{i=1}^{m} x_i$, $B = \sum_{i=1}^{m} y_i$, $C = \sum_{i=1}^{m} x_i^2$,

$$D = \sum_{i=1}^{m} x_i y_i$$

$A = 2007 + 2009 + 2010 + 2011 + 2012 + 2013 + 2014 + 2015 + 2016 + 2017 + 2018$

$= 22,142$

$B = 100 + 95 + 85 + 90 + 92 + 80 + 89 + 84 + 93 + 96 + 97 = 1001$

$C = 2007^2 + 2009^2 + 2010^2 + 2011^2 + 2012^2 + 2013^2 + 2014^2 + 2015^2 + 2016^2 + 2017^2 + 2018^2 = 44,569,954$

$D = 200700 + 170850 + 180990 + 185104 + 161040 + 179246 + 169260 + 187488 + 193632 + 195746 = 2,014,911$

$$w_0 = \frac{1001 * 44569954 - 22142 * 2014911}{11 * 44569954 - 490268164}$$

$$= \frac{44614523954 - 44614159362}{490269494 - 490268164}$$

$$= \frac{364592}{1330} = 274.129$$

$$w_1 = \frac{22142 \times 1001 - 11 \times 2014911}{490268164 - 490269494}$$

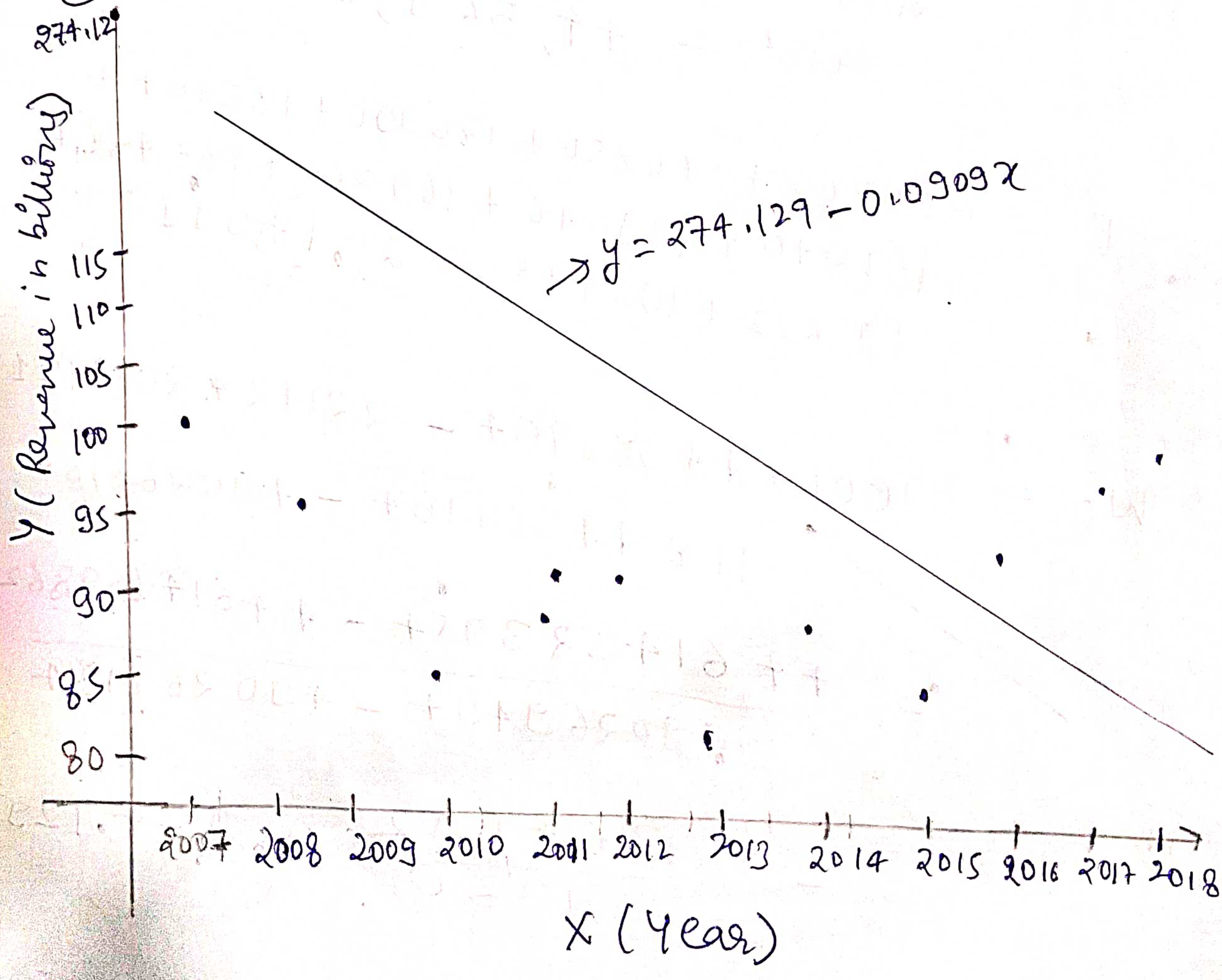
$$= \frac{22164142 - 22164021}{-1330}$$

$$= -\frac{121}{1330} = -0.0909$$

Putting w_0, w_1 in eq'n (1)

$$y = h_w(x) = 274.129 - 0.09\cancel{0}9x$$

(a) Draw line fitting points



(b) Expected Revenue in 2022

$$Y = h_w(2022) = 274.129 - 0.09 \times 2022$$
$$= 274.129 - 183.798$$
$$= 90.33$$
$$\simeq 90 \text{ billion}$$

(c) $\text{Error} = J = \frac{1}{2m} \sum (h_w(x) - y)^2$

$$= \frac{1}{2 \times 11} \sum (w_0 + w_1 x_i - y)^2$$

$$\Rightarrow \frac{1}{22} \left[(-8.3073)^2 + (-3.4891)^2 + (1.3291)^2 + (-0.7618)^2 + (6.42)^2 + (11.1473)^2 + (2.0564)^2 + (6.9655)^2 + (-5.2163)^2 + (-6.3072)^2 + (-2.1254)^2 \right.$$

$$\Rightarrow \frac{1}{22} [373.26544] = 16.966$$

Q2.

ML(X)	HUR(Y)
82	80
85	88
93	96
65	72
87	91
71	80
98	95
68	72
84	89
87	84

$$\bar{X} = \frac{82+85+93+65+87+71+98+68+84+87}{10}$$

$$= \frac{820}{10} = 82$$

$$\bar{Y} = (80+88+96+72+91+80+95+72+89+84) * \frac{1}{10}$$

$$= \frac{847}{10} = 84.7$$

(ML) X	(HUR) Y	$X-\bar{X}$	$Y-\bar{Y}$	$(X-\bar{X})^2$	$(Y-\bar{Y})^2$	$(X-\bar{X})(Y-\bar{Y})$
82	80	0	-4.7	0	22.09	0
85	88	3	3.3	9	10.89	9.9
93	96	11	11.3	121	127.69	124.3
65	72	-17	-12.7	289	161.29	215.9
87	91	5	6.3	25	39.69	31.5
71	80	-11	-4.7	121	22.09	51.7
98	95	16	10.3	256	106.09	164.8
68	72	14	-12.7	196	161.29	-173.6
84	89	2	4.3	4	18.49	8.6
87	84	5	-0.7	25	0.49	-3.5
			Σ =	1046	670.1	429.6

(a) $Y = w_0 + w_1 X$

$$w_0 = \bar{Y} - w_1 \bar{X}$$

$$w_1 = \frac{\Sigma (X-\bar{X})(Y-\bar{Y})}{\Sigma (X-\bar{X})^2} =$$

$$w_1 = \frac{429.6}{1046} = 0.41$$

$$w_0 = 84.7 - 0.41 * 82 = 51.08$$

$$\boxed{Y = 51.08 + 0.41X}$$

(b) $X = w_0 + w_1 Y$

$$w_0 = \bar{X} - w_1 \bar{Y}$$

$$w_1 = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\Sigma (y-\bar{y})^2} = \frac{429.6}{670.1} = 0.641$$

$$w_0 = 82 - 0.641 * 84.7 = 27.70$$

$$\boxed{X = 27.70 + 0.641Y}$$

(c) $Y = 51.08 + 0.41 * 96$

$$\boxed{Y = 90.44}$$

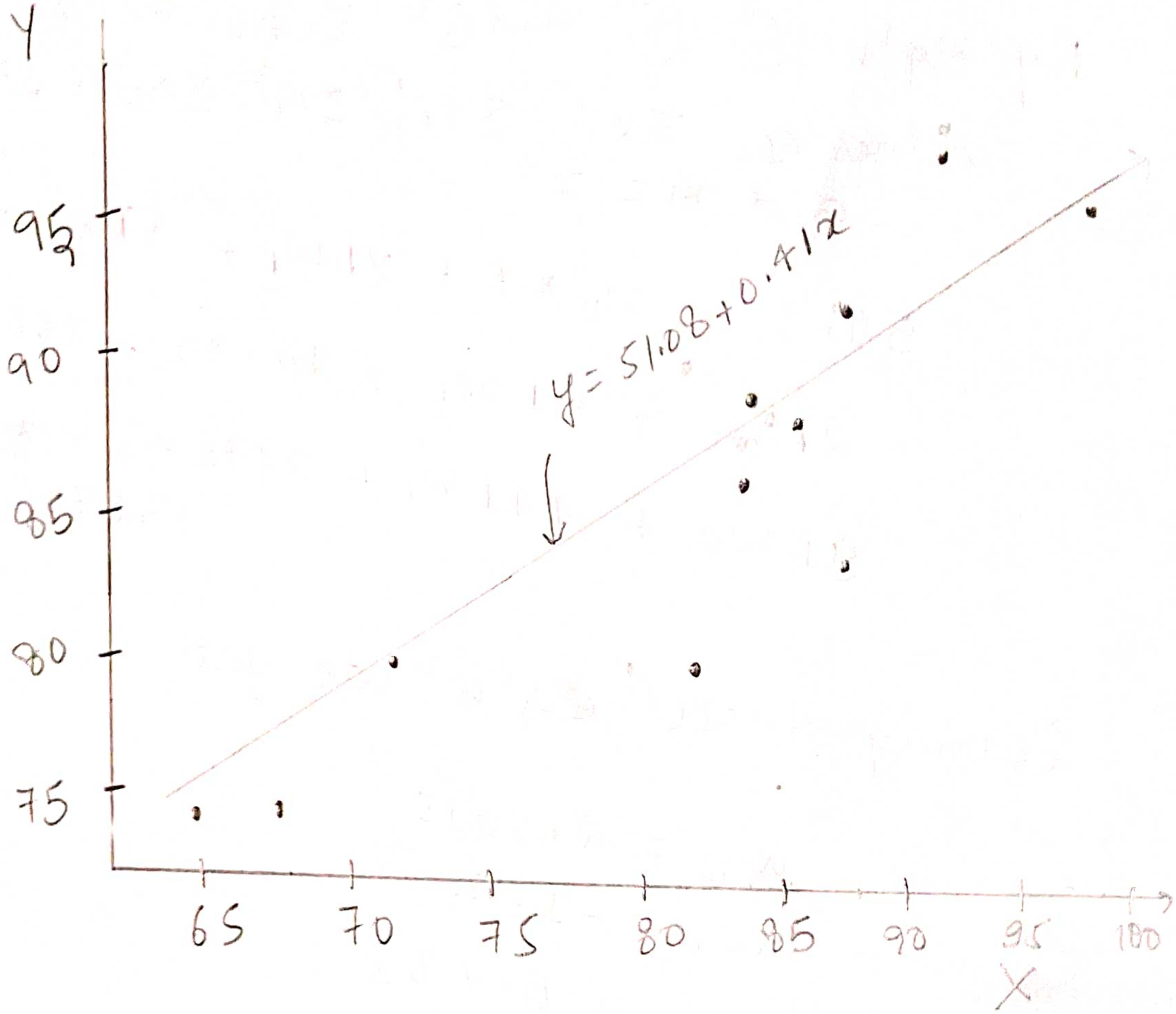
Marks in HUR is 90.44

(d) $X = 27.70 + 0.641 * 95$

$$= 88.595$$

Marks in ML is 88.595

(a) $Y = 51.08 + 0.41x$



Error: $J = \frac{1}{2m} \sum (h_w(x) - y)^2$

$= \frac{1}{2*10} \Big[(51.08 + 0.41*82 - 80)^2 +$

$(51.08 + 0.41*93 - 96)^2 +$

$(51.08 + 0.41*85 - 88)^2 + (51.08 + 0.41*65 - 72)^2 +$

$(51.08 + 0.41*65 - 72)^2 + (51.08 + 0.41*71 - 80)^2 +$

$(51.08 + 0.41*87 - 91)^2 + (51.08 + 0.41*68 - 72)^2 +$

$(51.08 + 0.41*98 - 95)^2 + (51.08 + 0.41*87 - 84)^2 \Big]$

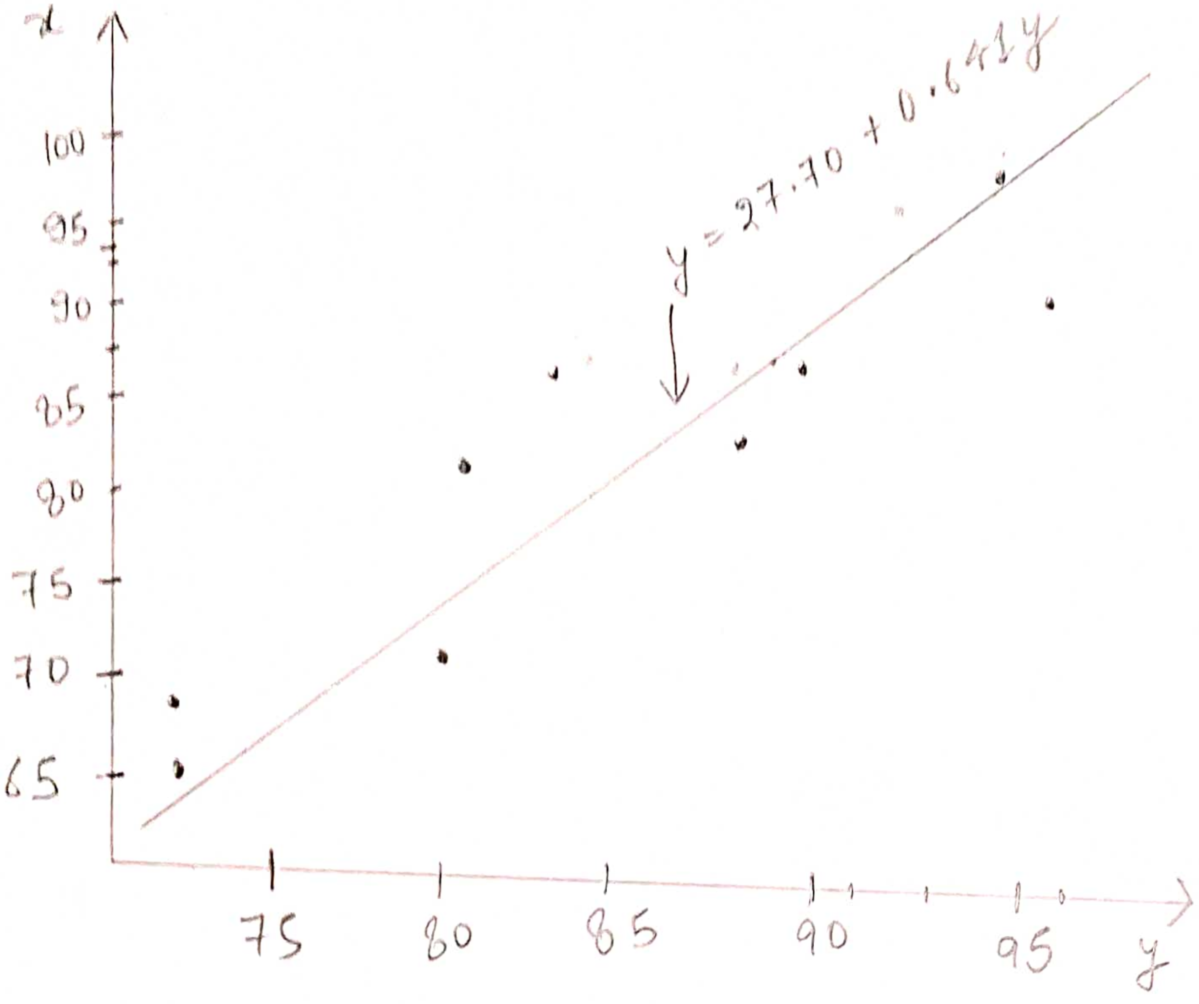
$(51.08 + 0.41 \times 84 - 89)^2 + (51.08 + 0.41$

$(51.08 + 0.41$

$= 10.2756$

(b)

$x = 27.70 + 0.611\,y$



Error $= \frac{1}{20}\big[(27.70 + 0.641 * 80 - 82)^2 + (27.7 + 0.641 * 88 - 85)^2 + (27.7 + 0.641 * 96 - 93)^2$
$+ (27.7 + 0.641 * 72 - 65)^2 + (27.7 + 0.641 * 91 - 87)^2 +$
$(27.7 + 0.641 * 80 - 71)^2 + (27.7 + 0.641 * 95 - 98)^2 +$
$(27.7 + 0.641 * 89 - 84)^2 +$
$(27.7 + 0.641 * 72 - 68)^2 + (27.7 + 0.641 * 84 - 87)^2\big] = 16.0044$

Thus in 1st figure line gives more accurate result as the error corresponding to it is less.

V (volume)	Pressure (P)
54.3	61.2
61.8	49.5
72.4	37.5
88.7	28.4
118.6	19.2
194	10.1

(a) $PV^n = C$, $m = 6$

Applying log both side

$$\log P + n \log V = \log C$$

$$\log P - n \log \frac{1}{V} = \log C$$

$$\boxed{\log P = n \log \frac{1}{V} + \log C}$$

Hypothesis : $Y = w_0 + w_1 X$

let $Y = \log P$ & $X = \log\left(\frac{1}{V}\right)$

$$A = \sum_{i=1}^{m} x_i = \sum_{i=1}^{m} \log\left(\frac{1}{V}\right)$$

$$= \log\left(\frac{1}{54.3}\right) + \log\left(\frac{1}{61.8}\right) + \log\left(\frac{1}{72.4}\right)$$
$$+ \log\left(\frac{1}{88.7}\right) + \log\left(\frac{1}{118.6}\right) + \log\frac{1}{194}$$

$$A = -26.92$$

$$B = \sum_{i=1}^{m} Y_i = \log 61.2 + \log 49.5 + \log 37.5 + \log 28.4 + \log 19.2 + \log 10.1$$

$$B = 20.954$$

$$C = \sum_{i=1}^{m} x_i^2 = \left(\log\left(\frac{1}{54.3}\right)\right)^2 + \left(\log\left(\frac{1}{61.8}\right)\right)^2 + \left(\log\frac{1}{72.4}\right)^2 + \left(\log\left(\frac{1}{88.7}\right)\right)^2$$
$$+ \left(\log\left(\frac{1}{118.6}\right)\right)^2 + \left(\log\frac{1}{194}\right)^2$$

$C = 121.975$

$$D = \sum_{i=1}^{m} \log\left(\frac{1}{v}\right) \log P = -\sum_{i=1}^{m} \log v \log P$$

$$\Rightarrow -[\log(61.2)\log(54.3) + \log(49.5)\log(61.8) + \log(37.5)\log(72.4) + \log(28.4)\log(88.7) + \log(19.2)\log(118.6) + \log(10.1)\log(194)]$$

$$= -89.349$$

$$w_0 = \frac{BC - AD}{mC - A^2}, \quad w_1 = \frac{AB - mD}{A^2 - mC}$$

$$w_0 = \frac{20.254 * 121.975 + (-26.92) * 89.349}{6 * 121.975 - 725.198}$$

$$= \frac{2470.481 - 2405.275}{731.85 - 725.198}$$

$$\Rightarrow \frac{64.408}{6.656}$$

$$= 9.675$$

$$w_1 = \frac{20.254 * (-26.92) - 6 * (-89.849)}{725.198 - 731.85}$$

$$= \frac{-9.343}{-6.656} = 1.4037$$

(a) find n and C

$w_0 = \log C \Rightarrow C = e^{w_0}$

$C = e^{9.675}$

$C = 15914.72$

$w_1 = 1.40 = n$

(b) As we have calculate $n = 1.4$

and $C = 15914.72$

$$\boxed{PV^{1.4} = 15914.72}$$

(c) $PV^{1.4} = 15914.72$

$$P = \frac{15914.72}{(100)^{1.4}} = 25.2231$$

The above hypothesis has three unknown w_0, w_1, w_2 so we will have three eq'n

$$\sum_{i=1}^{m} y_i = w_0 m + w_1 \sum_{i=1}^{m} x_i + w_2 \sum_{i=1}^{m} x_i^2 \quad \text{--- (1)}$$

$$\sum_{i=1}^{m} x_i y_i = w_0 \sum_{i=1}^{m} x_i + w_1 \sum_{i=1}^{m} x_i^2 + w_2 \sum_{i=1}^{m} x_i^3 \quad \text{--- (2)}$$

$$\sum_{i=1}^{m} x_i^2 y_i = w_0 \sum_{i=1}^{m} x_i^2 + w_1 \sum_{i=1}^{m} x_i^3 + w_2 \sum_{i=1}^{m} x_i^4 \quad \text{--- (3)}$$

X	Y	X^2	X^3	X^4	XY	X^2Y
0	2.4	0	0	0	0	0
1	2.1	1	1	1	2.1	2.1
2	3.2	4	8	16	6.4	12.8
3	5.6	9	27	81	16.8	50.4
4	9.3	16	64	256	37.2	148.8
5	14.6	25	125	625	73	36.5x10 = 365
6	21.9	36	216	1296	131.4	788.4

	Σx_i	Σy_i	Σx_i^2	Σx_i^3	Σx_i^4	$\Sigma x_i y_i$	$\Sigma x_i^2 y_i$
Σ :	21	59.1	91	441	2275	266.9	1367.5

By eq'n ①, ② and ③ Substituting values of Σx_i, Σx_i^2, Σx_i^3, $\Sigma x_i^2 y_i$ etc. and $m = 7$

$$59.1 = w_0 \times 7 + 21 w_1 + 91 w_2 \quad -①$$

$$21 w_0 + 91 w_1 + 441 w_2 = 266.9 \quad -②$$

$$91 w_0 + 441 w_1 + 2275 w_2 = 1367.5 \quad -③$$

Solving these eq'n we get

$$w_0 = 2.5095$$
$$w_1 = -1.2$$
$$w_2 = 0.733$$

$$\boxed{Y = 2.5095 - 1.2x + 0.733x^2}$$