**Understanding Principle Component Analysis(PCA) step by step.**

**Introduction**

**Principal component analysis** (**PCA**) is a statistical procedure that is used to reduce the dimensionality.

**Steps Involved in the PCA**

***Step 1:*** Standardize the dataset.

***Step 2:***Calculate the covariance matrix for the features in the dataset.

***Step 3:***Calculate the eigenvalues and eigenvectors for the covariance matrix.

***Step 4:***Sort eigenvalues and their corresponding eigenvectors.

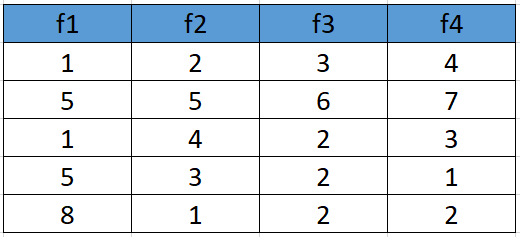
***Step 5:***Pick k eigenvalues and form a matrix of eigenvectors.

**Step 6:** Transform the original matrix.

Let's go to each step one by one.

**1. Standardize the Dataset**

Assume we have the below dataset which has 4 features and a total of 5 training examples.



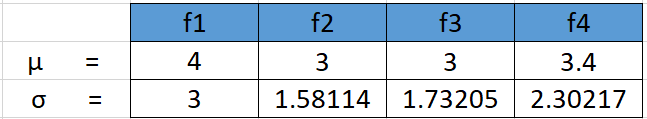
Dataset matrix

First, we need to standardize the dataset and for that, we need to calculate the mean and standard deviation for each feature.

A picture containing text, watch

Description automatically generated

Standardization formula



Mean and standard deviation before standardization

After applying the formula for each feature in the dataset is transformed as below:

Table

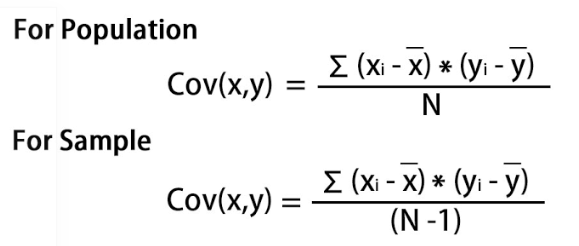
Description automatically generated

Standardized Dataset

**2. Calculate the covariance matrix for the whole dataset**

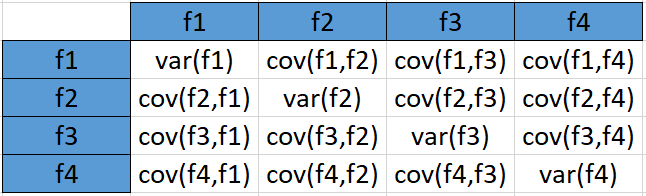
Covariance indicates **the relationship of two variables whenever one variable changes**. If an increase in one variable results in an increase in the other variable, both variables are said to have a positive covariance.

The formula to calculate the covariance matrix:



Covariance Formula

The covariance matrix for the given dataset will be calculated as below

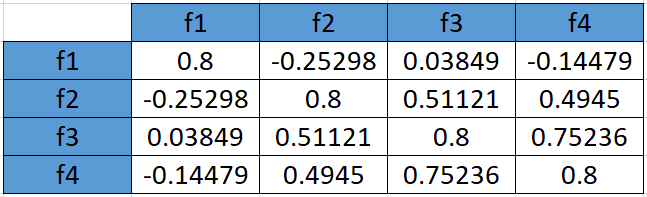


Since we have standardized the dataset, so the **mean for each feature is 0** and the standard deviation is 1.

var(f1) = ((-1.0-0)² + (0.33-0)² + (-1.0-0)² +(0.33–0)² +(1.33–0)²)/5  
**var (f1) = 0.8**

cov(f1,f2) =  
((-1.0–0)\*(-0.632456-0) +  
(0.33–0)\*(1.264911-0) +  
(-1.0–0)\* (0.632456-0)+  
(0.33–0)\*(0.000000 -0)+  
(1.33–0)\*(-1.264911–0))/5  
**cov(f1,f2 = -0.25298**

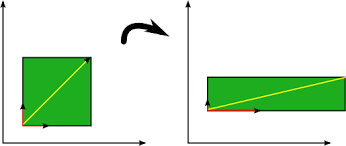
In the similar way we can calculate the other covariances and which will result in the below covariance matrix

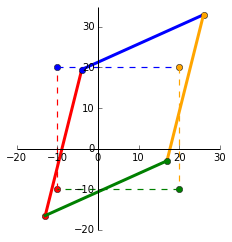


covariance matrix (population formula)

**3. Calculate eigenvalues and eigen vectors.**

An **eigenvector** is a nonzero vector that changes at most by a scalar factor when that linear transformation is applied to it. The corresponding **eigenvalue** is the factor by which the eigenvector is scaled.





Eigenvectors **make understanding linear transformations easy**. They are the "axes" (directions) along which a linear transformation acts simply by "stretching/compressing" and/or "flipping"; eigenvalues give you the factors by which this compression occurs

Let A be a square matrix (in our case the covariance matrix), ν a vector and λ a scalar that satisfies Aν = λν, then λ is called eigenvalue associated with eigenvector ν of A.  
Rearranging the above equation,

Aν-λν =0 ; (A-λI)ν = 0

Since we have already know ν is a non- zero vector, only way this equation can be equal to zero, if

det(A-λI) = 0

Table

Description automatically generated

A-λI = 0

Solving the above equation = 0

***λ = 2.51579324 , 1.0652885 , 0.39388704 , 0.02503121***

**Eigenvectors:**

Solving the (A-λI)ν = 0 equation for ν vector with different λ values:

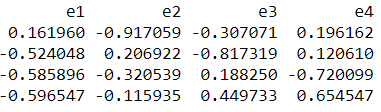
Text

Description automatically generated

For λ = *2.51579324, solving the above equation using Cramer's rule, the values for v vector are*

*v1 = 0.16195986  
v2 = -0.52404813  
v3 = -0.58589647  
v4 = -0.59654663*

Going by the same approach, we can calculate the eigen vectors for the other eigen values. We can from a matrix using the eigen vectors.



eigenvectors(4 \* 4 matrix)

***4.*Sort eigenvalues and their corresponding eigenvectors.**

Since eigenvalues are already sorted in this case so no need to sort them again.

***5.*Pick k eigenvalues and form a matrix of eigenvectors**

If we choose the top 2 eigenvectors, the matrix will look like this:

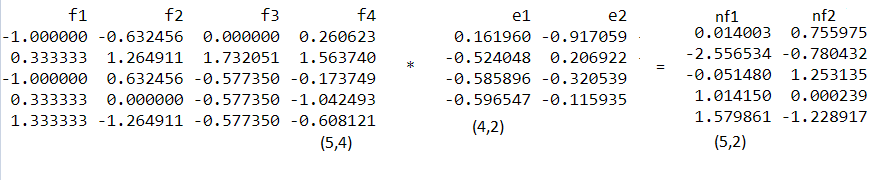
Text

Description automatically generated

Top 2 eigenvectors(4\*2 matrix)

**6. Transform the original matrix.**

Feature matrix \* top k eigenvectors = Transformed Data



Data Transformation