Space Flight Mechanics
Assignment 2

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The report consists of two parts. The former deals with the estimation of life time of spacecraft with varying ballistic coefficient and orbit radius. The latter deals with the design of Hohmann and Bielliptic transfers.

Nomenclature

a = orbit altitude
B = ballistic coefficient
ρ = density
μ = Gravitational constant of earth
t = time

I. Part I

THIS section deals with the estimation of lifetime numerically using MATLAB. The following equation was used for the same.

$$\frac{da}{dt} = \frac{-\sqrt{\mu a}\rho}{B} \tag{1}$$

The life of the satellite was considered to be over when it reaches an altitude of 100 km. Based on this, we have obtained Fig.1, Fig.2, and Fig.3.

A. Discussion

We observe that as the orbit is placed lower, the life time is lesser. The life increases drastically as the orbit is placed higher as the density decreases as altitude increases.

As the ballistic coefficient is lower, lower the lifetime. Higher the ballistic coefficient, the life time increases linearly.

Code

```
global B;
global atm_alt;
global atm_dens;
global mu;
global j;
a = 200 : 1 : 1000;
B = (50*10^6 : 25*10^6 : 300*10^6)';
atm_alt = table2array(data(:,1));
atm_dens = table2array(data(:,2));
mu = 398600;
```

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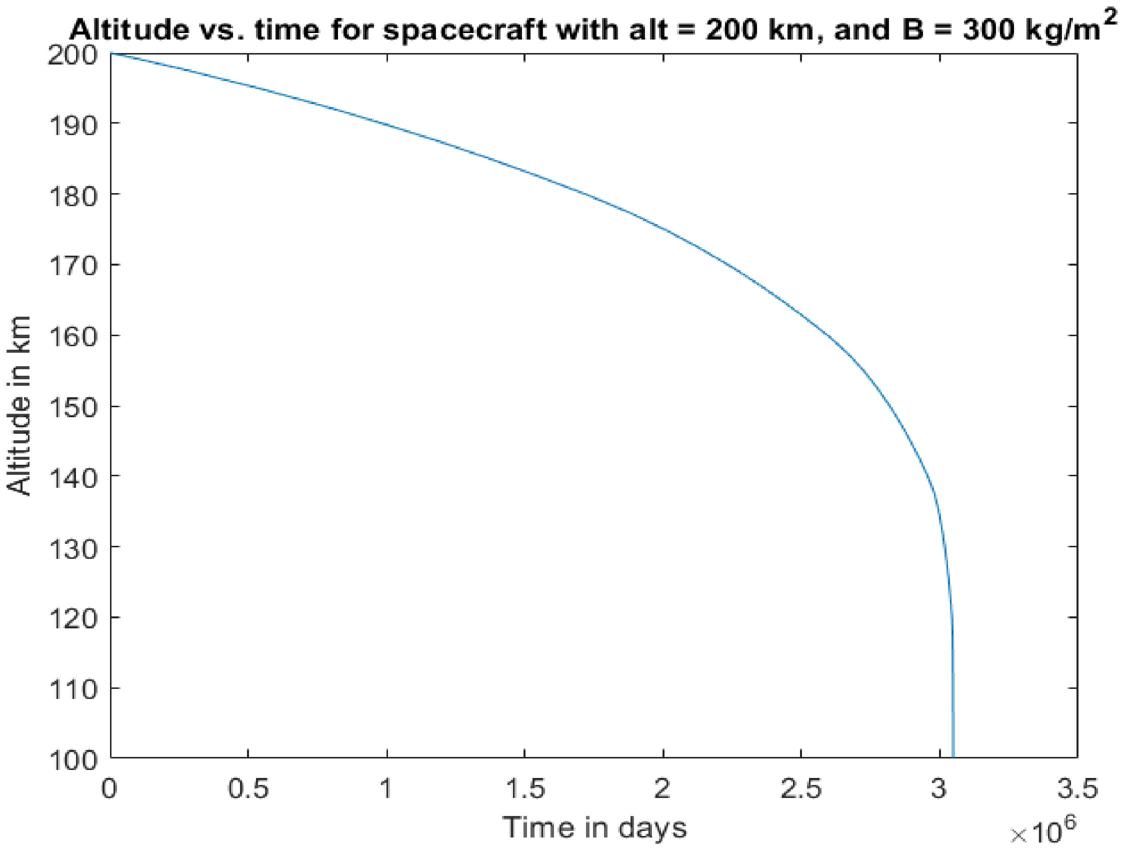

Fig. 1 The altitude of a satellite initially at 200 km altitude with a ballistic coefficient of $300kg/m^2$

```
for j = 1 : length(B)
    t0 = 0;
    [y,time] = ode45(@(y,t) f(y,t),[200 100],t0);
    t(j,1) = max(time);
    for i = 2 : length(a)
        i
        [yt,tim] = ode45(@(y,t) f(y,t),[a(i) 100],t0);
        t(j,i) = max(tim);
    end
end
t = t/(24*3600);

figure
plot(a,t(1,:),a,t(3,:),a,t(5,:),a,t(7,:),a,t(9,:),a,t(11,:));
xlabel('Orbit size in kms');
ylabel('Life time in days');
legend('B = 50 kg/m^2','B = 100 kg/m^2','B = 150 kg/m^2','B = 200 kg/m^2','B =
    250 kg/m^2','B = 300 kg/m^2');

figure
plot(time,y)
xlabel('Time in days');
ylabel('Altitude in km');
```

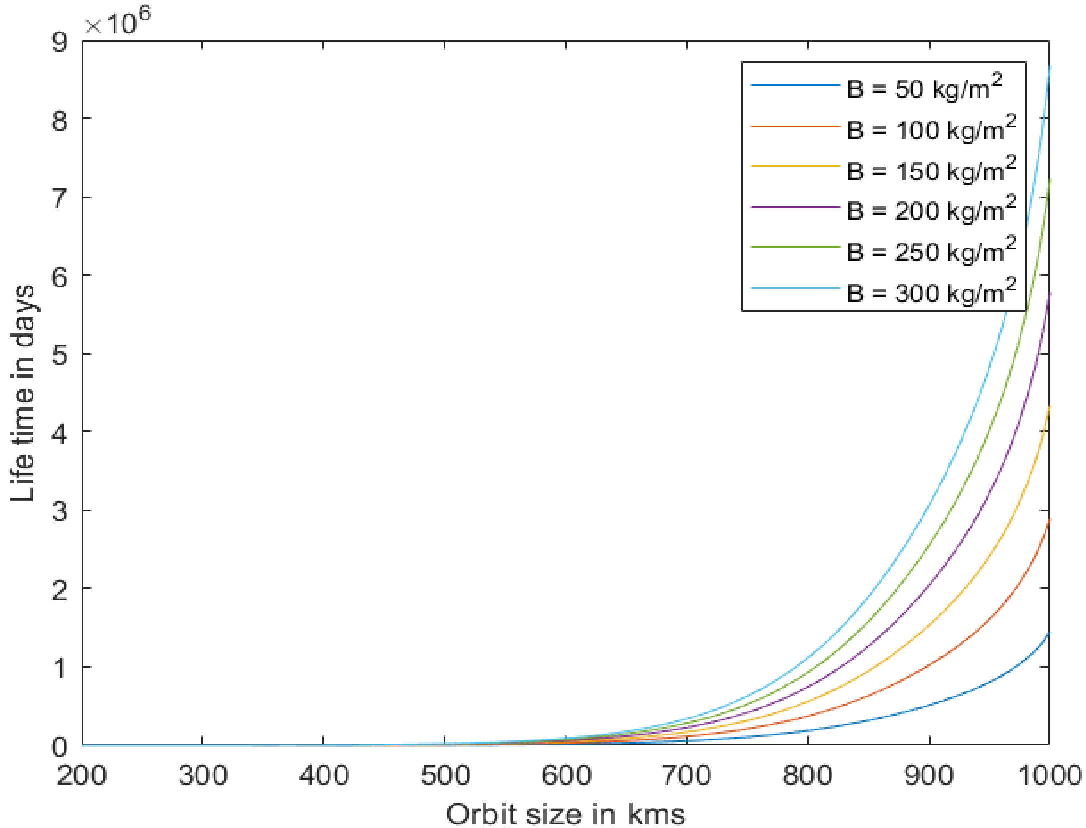

Fig. 2 Variation of the lifetime of a satellite w.r.t initial altitude

```
title('Altitude vs. time for spacecraft with alt = 200 km, and B = 300 kg/m^2');

figure
plot(B*10^-6,t(:,1),B*10^-6,t(:,201),B*10^-6,t(:,401),B*10^-6,t(:,601),B*10^-6,t(:,651),B*10^-6,t(:,701),B*10^-6,t(:,751),B*10^-6,t(:,801),0,0);
xlabel('Ballistic coefficient in kg/m^2');
ylabel('Life time in days');
legend('a = 200 km','a = 400 km','a = 600 km','a = 800 km','a = 850 km','a = 900 km','a = 950 km','a = 1000 km');

function tdot = f(y,t)
    global B
    global atm_alt
    global atm_dens
    global mu
    global j
    if y<=900
    density = interp1((atm_alt),(atm_dens),y,'linear');
    else
    density = interp1((atm_alt),(atm_dens),y,'linear','extrap');
    end
    tdot = -B(j)/(sqrt(mu*y)*density) + 0*t;
```

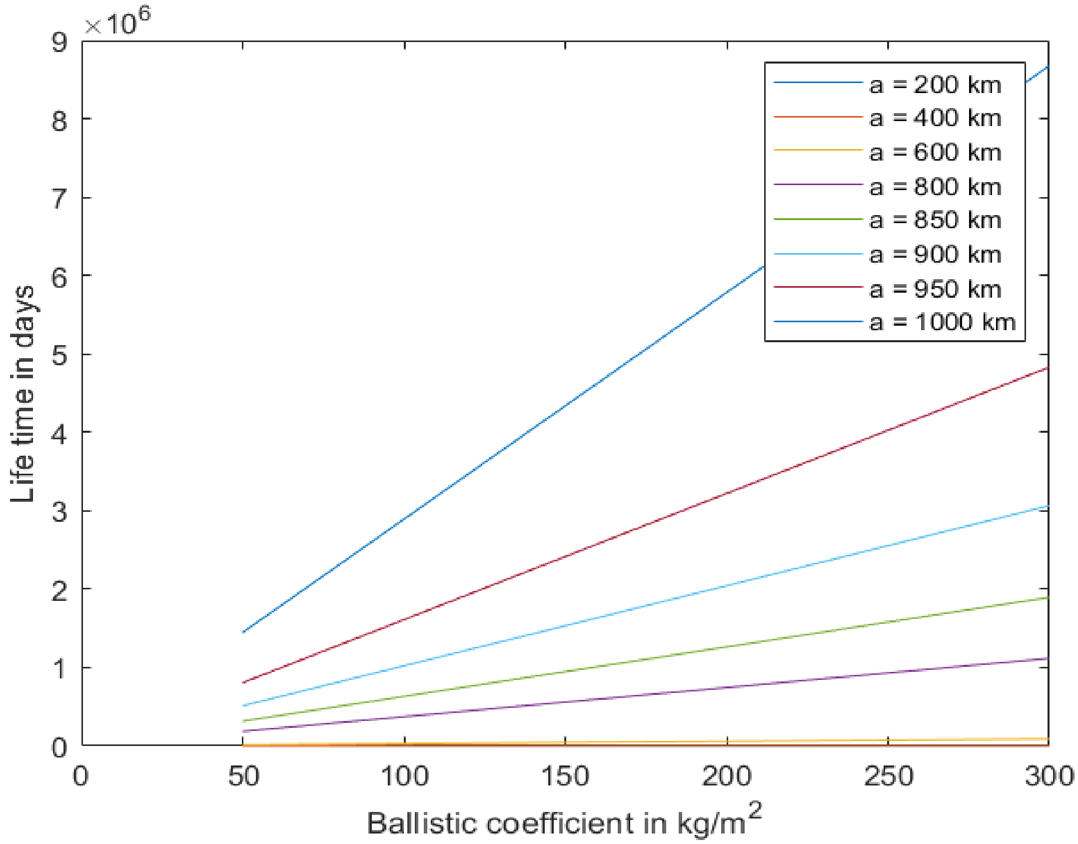

Fig. 3 Variation of the lifetime of a satellite w.r.t ballistic coefficient

```
end
```

II. Part II

A. Hohmann Transfers

For Hohmann transfers, we calculate the velocity impulses required for transfer from the initial circular orbit to the elliptical transfer orbit and from the transfer orbit to the final circular orbit.

$$v_I = \sqrt{\frac{\mu}{r_I}}$$

$$v_{T_p} = \sqrt{\frac{2\mu}{r_I} - \frac{\mu}{a_T}}$$

$\Delta v_1 = v_{T_p} - v_I$ in the direction of velocity

$$v_{T_a} = \sqrt{\frac{2\mu}{r_F} - \frac{\mu}{a_T}}$$

Orbit altitude (km)	First velocity impulse (km/s)	Second velocity impulse (km/s)
125000	2.764422	1.173813
200000	2.863586	1.024609
300000	2.921610	0.893258

Table 1 Velocity impulses for Hohmann Transfers

$$v_F = \sqrt{\frac{\mu}{r_F}}$$

$\Delta v_2 = v_F - v_{T_a}$ in the direction of velocity

In our case, we have $r_I = 6378 + 1000$ km. We compare the velocity impulses required when the final orbit altitudes are 125000, 200000, and 300000 kms respectively.

Following are the results:

As expected, the first impulse velocity increases as the final orbit altitude increases. However, surprisingly, we note that the second impulse velocity required has a maximum as orbit altitude increases, similar to total velocity impulse required.

Code

```
global total_del_v_h
global time_h
mu = 398600;
R_E = 6378;
h_I = 1000;
r_I = h_I+R_E;
r_F = [1000 : 1000 : 300000]+R_E;
v_r_I = sqrt(mu*(1/r_I));
a_T = (r_I+r_F)/2;
v_r_T_p = sqrt(mu*((2/r_I)-(1./a_T)));
del_v_1 = v_r_T_p-v_r_I;
v_r_T_a = sqrt(mu*((2./r_F)-(1./a_T)));
v_r_F = sqrt(mu*(1./r_F));
del_v_2 = v_r_F-v_r_T_a;
time_h = pi*sqrt((a_T.^3)/mu);
disp("Hohmann Transfer to 125000 km orbit:");
if del_v_1(125) >= 0
    fprintf('The first velocity impulse is %f km/s in the direction of
        velocity.\n',del_v_1(125));
else
    fprintf('The first velocity impulse is %f km/s opposite to the direction
        of velocity.\n',abs(del_v_1(125)));
end
if del_v_2(125) >= 0
    fprintf('The second velocity impulse is %f km/s in the direction of
        velocity.\n',del_v_2(125));
```

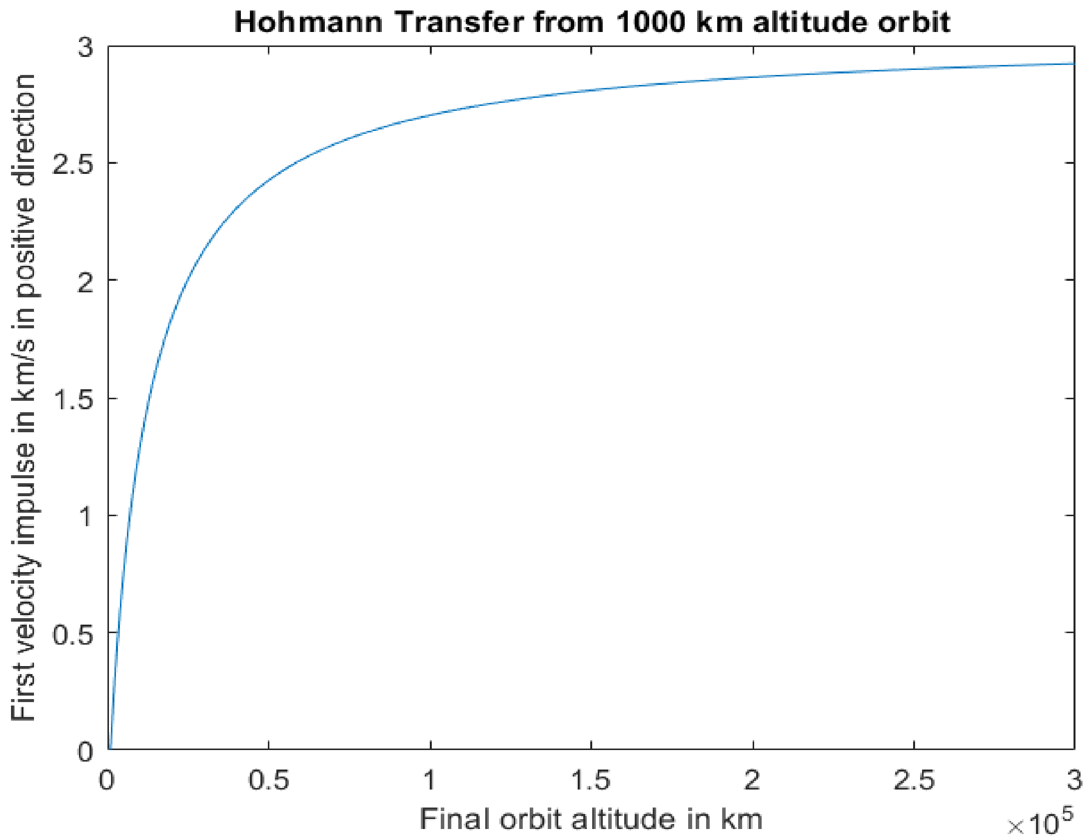

Fig. 4 First velocity impulse for Hohmann transfer

```
else
    fprintf('The second velocity impulse is %f km/s opposite to the direction of velocity.\n\n\n',abs(del_v_2(125)));
end
disp("Hohmann Transfer to 200000 km orbit:");
if del_v_1(200) >= 0
    fprintf('The first velocity impulse is %f km/s in the direction of velocity.\n',del_v_1(200));
else
    fprintf('The first velocity impulse is %f km/s opposite to the direction of velocity.\n',abs(del_v_1(200)));
end
if del_v_2(200) >= 0
    fprintf('The second velocity impulse is %f km/s in the direction of velocity.\n',del_v_2(200));
else
    fprintf('The second velocity impulse is %f km/s opposite to the direction of velocity.\n\n\n',abs(del_v_2(200)));
end
disp("Hohmann Transfer to 300000 km orbit:");
if del_v_1(300) >= 0
    fprintf('The first velocity impulse is %f km/s in the direction of velocity.\n',del_v_1(300));
```

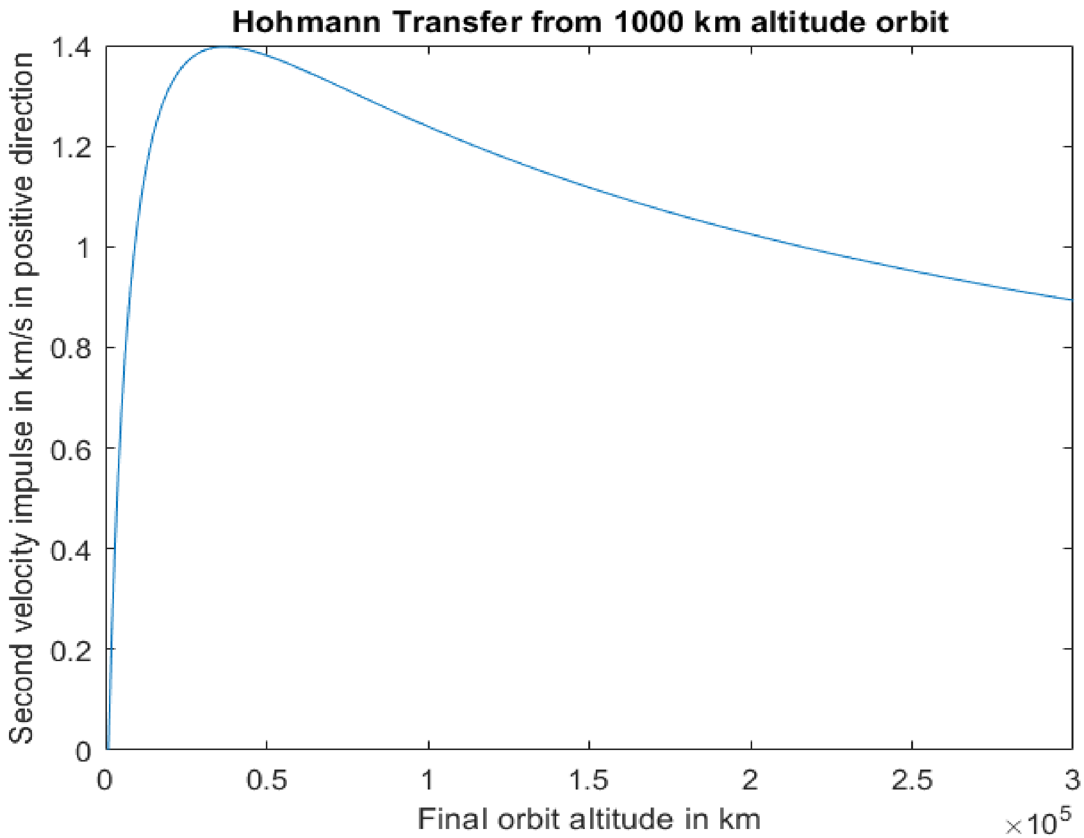

Fig. 5 Second velocity impulse for Hohmann transfer

```
else
    fprintf('The first velocity impulse is %f km/s opposite to the direction of velocity.\n',abs(del_v_1(300)));
end
if del_v_2(300) >= 0
    fprintf('The second velocity impulse is %f km/s in the direction of velocity.\n',del_v_2(300));
else
    fprintf('The second velocity impulse is %f km/s opposite to the direction of velocity.\n\n\n',abs(del_v_2(300)));
end
total_del_v_h = abs(del_v_1)+abs(del_v_2);
figure
plot(r_F-R_E,del_v_1);
xlabel("Final orbit altitude in km");
ylabel("First velocity impulse in km/s in positive direction");
title("Hohmann Transfer from 1000 km altitude orbit");
figure
plot(r_F-R_E,del_v_2);
xlabel("Final orbit altitude in km");
ylabel("Second velocity impulse in km/s in positive direction");
title("Hohmann Transfer from 1000 km altitude orbit");
figure
```

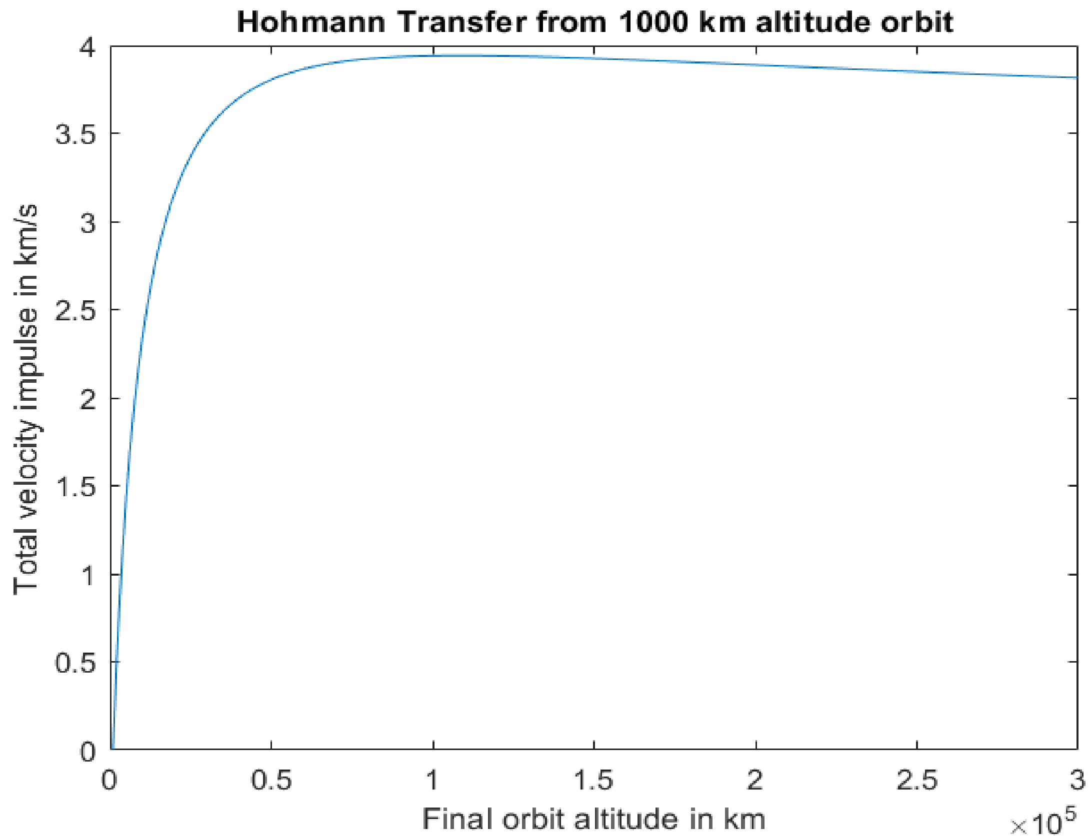

Fig. 6 Total velocity impulse for Hohmann transfer

Orbit altitude (km)	First velocity impulse (km/s)	Second velocity impulse (km/s)	Third velocity impulse (km/s)
125000	2.919043	0.647057	0.312417
200000	2.919043	0.788125	0.123027
300000	2.919043	0.906169	0.006014

Table 2 Velocity impulses for Bielliptic Transfers with intermediate orbit radius of 300000 km

```
plot(r_F-R_E,total_del_v_h);
xlabel("Final orbit altitude in km");
ylabel("Total velocity impulse in km/s");
title("Hohmann Transfer from 1000 km altitude orbit");
figure
plot(r_F-R_E,time_h);
xlabel("Final orbit altitude in km");
ylabel("Total time for transfer in seconds");
title("Hohmann Transfer from 1000 km altitude orbit");
```

B. Bielliptic transfers

For bielliptic transfer, a new intermediate transfer orbit is used. The same method as above applies for the calculation of velocity impulses required. A negative velocity impulse implies thrust is required in the opposite direction to velocity. For the total velocity impulse required, all the absolute magnitudes of the velocity impluses are added.

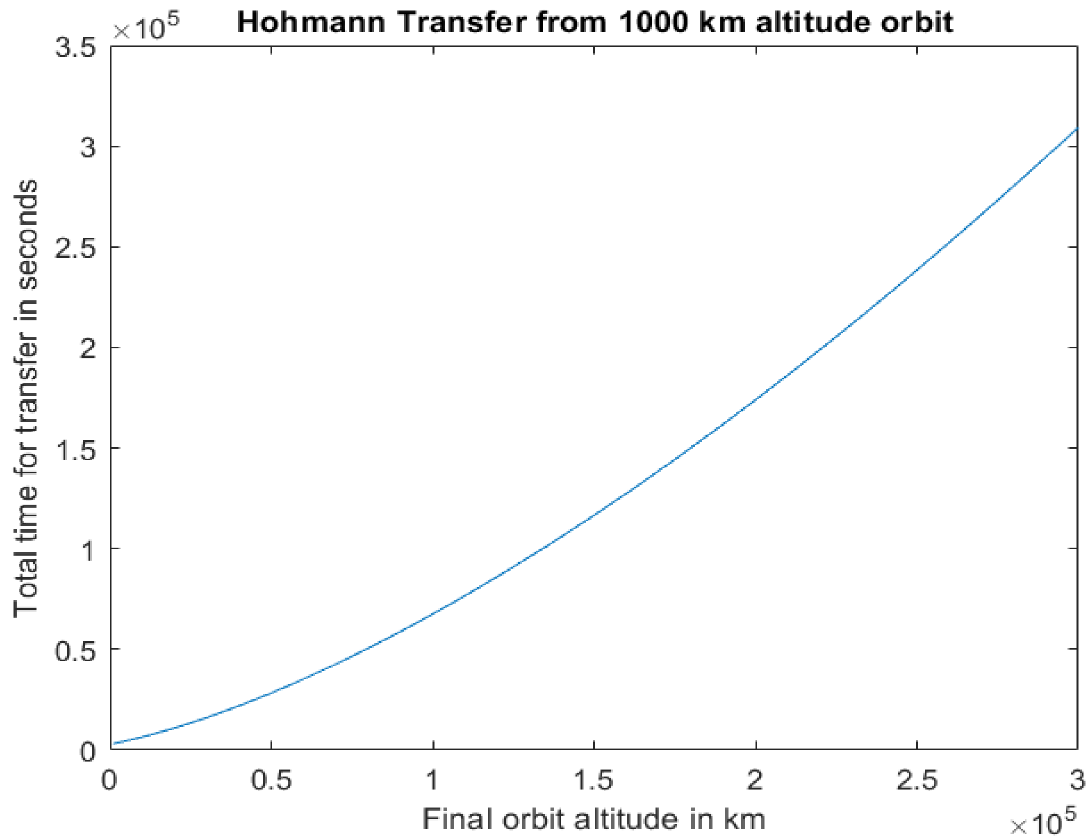

Fig. 7 Total time for Hohmann transfer

The following are the results:

Code

```
global time_h
mu = 398600;
R_E = 6378;
h_I = 1000;
r_I = h_I+R_E;
r_F = [1000 : 1000 : 300000]+R_E;
v_r_I = sqrt(mu*(1/r_I));
r_T = [150000 : 1000 : 500000]';
a_T_1 = (r_I+r_T)/2;
a_T_2 = (r_F+r_T)/2;
time_bi = pi*sqrt((a_T_2.^3)/mu)+pi*sqrt((a_T_1.^3)/mu);
time_bi = time_bi/(24*3600);
time_h = time_h/(24*3600);
v_r_T_1_p = sqrt(mu*((2/r_I)-(1./a_T_1)));
del_v_1 = v_r_T_1_p-v_r_I;
v_r_T_1_a = sqrt(mu*((2./r_T)-(1./a_T_1)));
v_r_T_2_a = sqrt(mu*((2./r_T)-(1./a_T_2)));
del_v_2 = v_r_T_2_a-v_r_T_1_a;
```

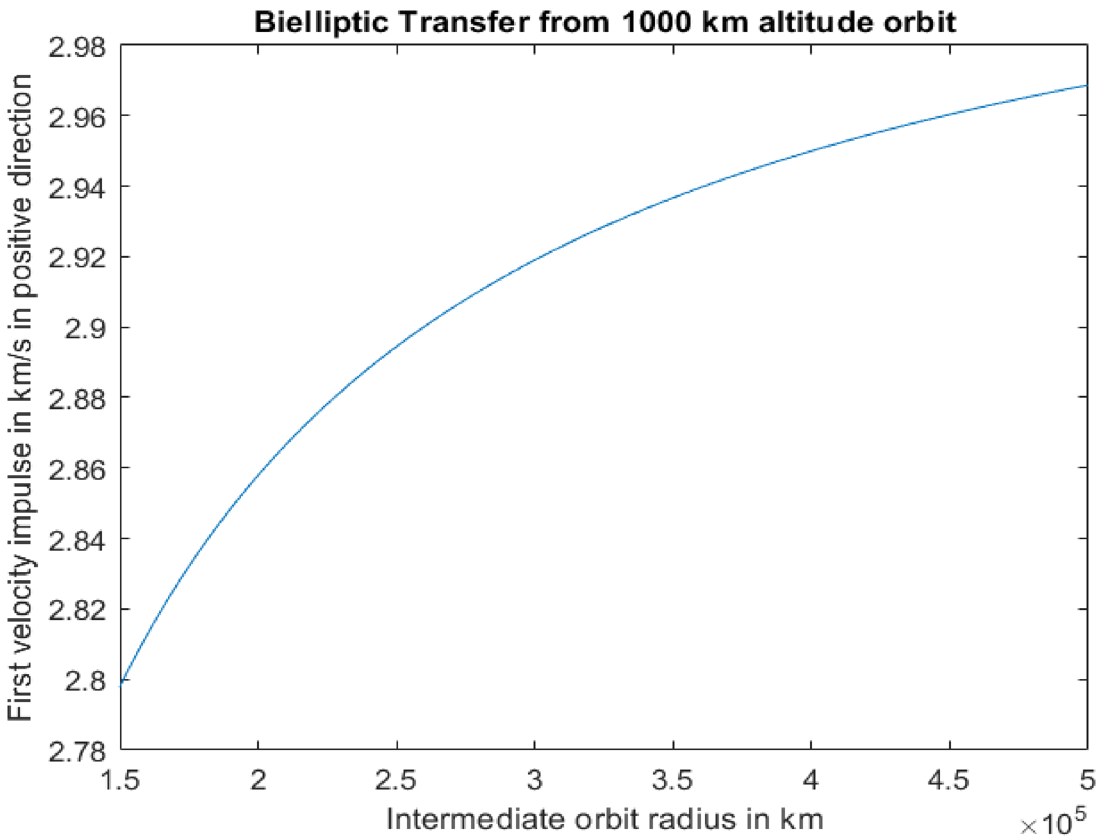

Fig. 8 First velocity impulse for Bielliptic transfer

```
v_r_T_2_p = sqrt(mu*((2./r_F)-(1./a_T_2)));
v_r_F = sqrt(mu*(1./r_F));
del_v_3 = v_r_F-v_r_T_2_p;
disp("Bielliptic Transfer to 125000 km orbit:");
if del_v_1(151,1) >= 0
    fprintf('The first velocity impulse is %f km/s in the direction of velocity.\n',del_v_1(151,1));
else
    fprintf('The first velocity impulse is %f km/s opposite to the direction of velocity.\n',abs(del_v_1(151)));
end
if del_v_2(151,125) >= 0
    fprintf('The second velocity impulse is %f km/s in the direction of velocity.\n',del_v_2(151,125));
else
    fprintf('The second velocity impulse is %f km/s opposite to the direction of velocity.\n\n\n',abs(del_v_2(151,125)));
end
if del_v_3(151,125) >= 0
    fprintf('The third velocity impulse is %f km/s in the direction of velocity.\n',del_v_3(151,125));
else
    fprintf('The third velocity impulse is %f km/s opposite to the direction
```

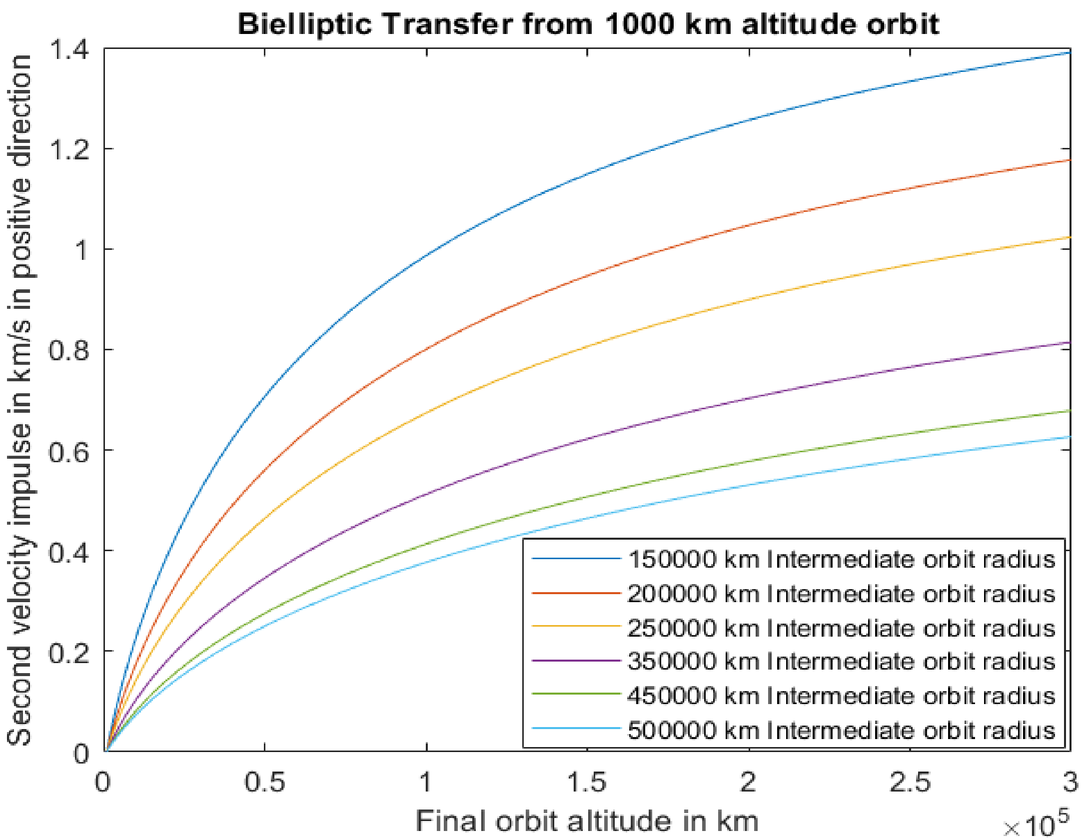

Fig. 9   Second velocity impulse for Bielliptic transfer

```
        of velocity.\n\n\n',abs(del_v_3(151,125)));
end
disp("Bielliptic Transfer to 200000 km orbit:");
if del_v_1(151) >= 0
    fprintf('The first velocity impulse is %f km/s in the direction of
        velocity.\n',del_v_1(151));
else
    fprintf('The first velocity impulse is %f km/s opposite to the direction
        of velocity.\n',abs(del_v_1(151)));
end
if del_v_2(151,200) >= 0
    fprintf('The second velocity impulse is %f km/s in the direction of
        velocity.\n',del_v_2(151,200));
else
    fprintf('The second velocity impulse is %f km/s opposite to the direction
        of velocity.\n\n\n',abs(del_v_2(151,200)));
end
if del_v_3(151,200) >= 0
    fprintf('The third velocity impulse is %f km/s in the direction of
        velocity.\n',del_v_3(151,200));
else
    fprintf('The third velocity impulse is %f km/s opposite to the direction
        of velocity.\n\n\n',abs(del_v_3(151,200)));
```

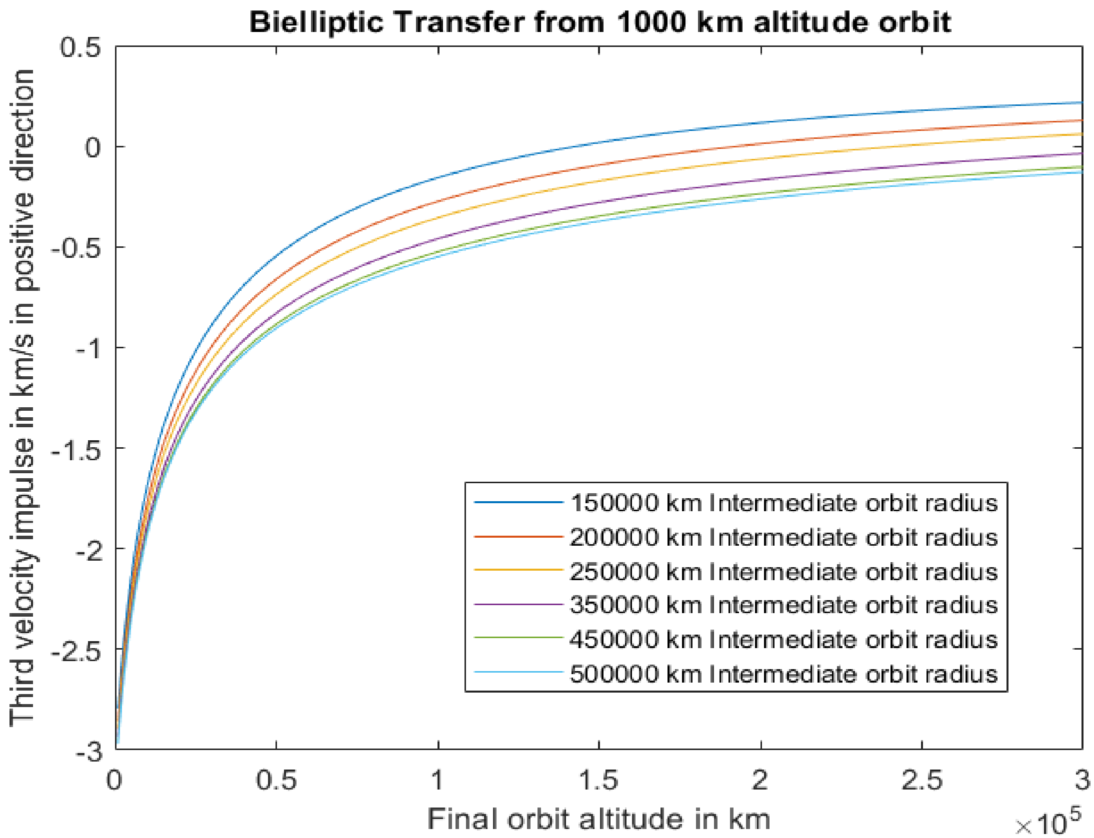

Fig. 10 Third velocity impulse for Bielliptic transfer

```
end
disp("Bielliptic Transfer to 300000 km orbit:");
if del_v_1(151) >= 0
    fprintf('The first velocity impulse is %f km/s in the direction of
        velocity.\n',del_v_1(151));
else
    fprintf('The first velocity impulse is %f km/s opposite to the direction
        of velocity.\n',abs(del_v_1(151)));
end
if del_v_2(151,300) >= 0
    fprintf('The second velocity impulse is %f km/s in the direction of
        velocity.\n',del_v_2(151,300));
else
    fprintf('The second velocity impulse is %f km/s opposite to the direction
        of velocity.\n\n\n',abs(del_v_2(151,300)));
end
if del_v_3(151,300) >= 0
    fprintf('The third velocity impulse is %f km/s in the direction of
        velocity.\n',del_v_3(151,300));
else
    fprintf('The third velocity impulse is %f km/s opposite to the direction
        of velocity.\n\n\n',abs(del_v_3(151,300)));
end
```

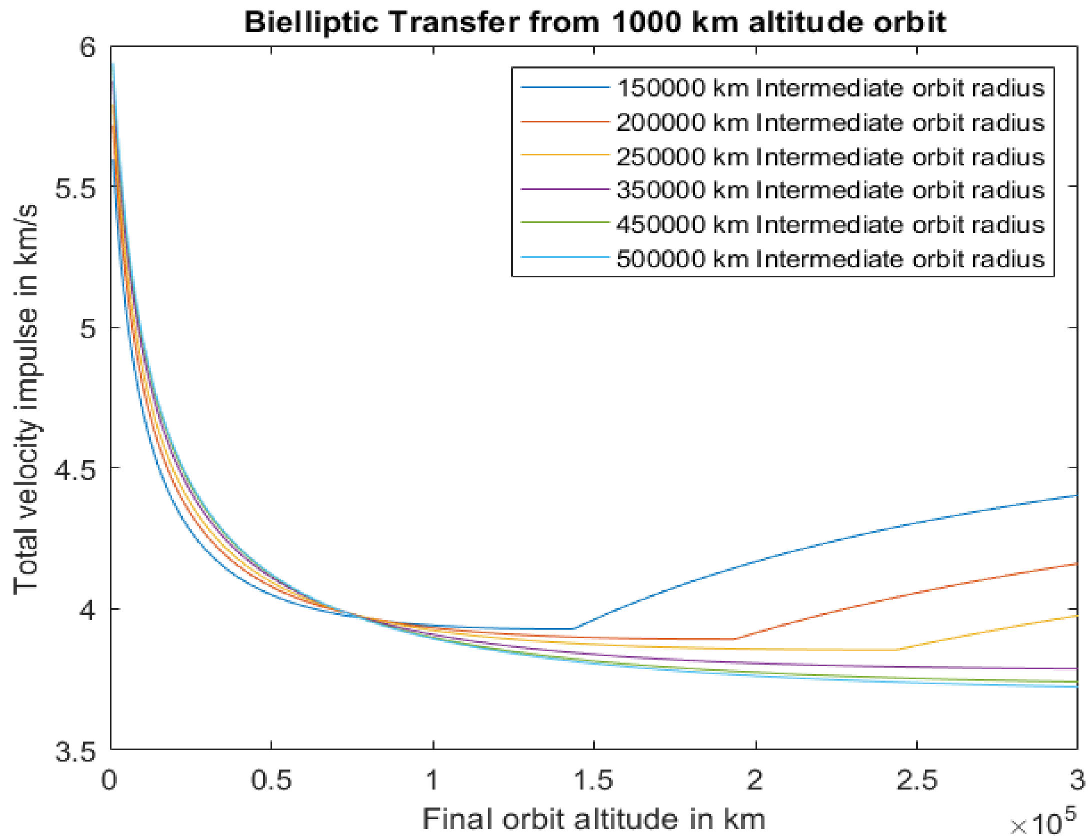

Fig. 11 Total velocity impulse for Bielliptic transfer

```
total_del_v = abs(del_v_1)+abs(del_v_2)+abs(del_v_3);
figure
plot(r_T,del_v_1);
xlabel("Intermediate orbit radius in km");
ylabel("First velocity impulse in km/s in positive direction");
title("Bielliptic Transfer from 1000 km altitude orbit");
figure
plot(r_F-R_E,del_v_2(1,:),r_F-R_E,del_v_2(51,:),r_F-R_E,del_v_2(101,:),r_F-R_E
    ,del_v_2(201,:),r_F-R_E,del_v_2(301,:),r_F-R_E,del_v_2(351,:));
xlabel("Final orbit altitude in km");
ylabel("Second velocity impulse in km/s in positive direction");
title("Bielliptic Transfer from 1000 km altitude orbit");
legend("150000 km Intermediate orbit radius","200000 km Intermediate orbit
    radius","250000 km Intermediate orbit radius","350000 km Intermediate
    orbit radius","450000 km Intermediate orbit radius","500000 km
    Intermediate orbit radius");
figure
plot(r_F-R_E,del_v_3(1,:),r_F-R_E,del_v_3(51,:),r_F-R_E,del_v_3(101,:),r_F-R_E
    ,del_v_3(201,:),r_F-R_E,del_v_3(301,:),r_F-R_E,del_v_3(351,:));
xlabel("Final orbit altitude in km");
ylabel("Third velocity impulse in km/s in positive direction");
title("Bielliptic Transfer from 1000 km altitude orbit");
legend("150000 km Intermediate orbit radius","200000 km Intermediate orbit
```

```
radius","250000 km Intermediate orbit radius","350000 km Intermediate orbit radius","450000 km Intermediate orbit radius","500000 km Intermediate orbit radius");
figure
plot(r_F-R_E,total_del_v(1,:),r_F-R_E,total_del_v(51,:),r_F-R_E,total_del_v(101,:),r_F-R_E,total_del_v(201,:),r_F-R_E,total_del_v(301,:),r_F-R_E,total_del_v(351,:),r_F-R_E,total_del_v_h);
xlabel("Final orbit altitude in km");
ylabel("Total velocity impulse in km/s");
title("Bielliptic Transfer from 1000 km altitude orbit");
legend("150000 km Intermediate orbit radius","200000 km Intermediate orbit radius","250000 km Intermediate orbit radius","350000 km Intermediate orbit radius","450000 km Intermediate orbit radius","500000 km Intermediate orbit radius", "Hohmann Transfer");
plot(r_F-R_E,time_bi(1,:),r_F-R_E,time_bi(51,:),r_F-R_E,time_bi(101,:),r_F-R_E,time_bi(201,:),r_F-R_E,time_bi(301,:),r_F-R_E,time_bi(351,:),r_F-R_E,time_h);
xlabel("Final orbit altitude in km");
ylabel("Total transfer time in days");
title("Bielliptic Transfer from 1000 km altitude orbit");
legend("150000 km Intermediate orbit radius","200000 km Intermediate orbit radius","250000 km Intermediate orbit radius","350000 km Intermediate orbit radius","450000 km Intermediate orbit radius","500000 km Intermediate orbit radius", "Hohmann Transfer");
```

III. Comparison

We see that the energy required for bielliptic transfer is sometimes lesser than the energy required for Hohmann transfers. For small orbit raising however, it is seen that Hohmann transfer is very much better than the bielliptic transfer. As bielliptic transfer involves three velocity impulses, it also takes more time. Bigger the intermediate orbit radius, lower the energy required for transfers from small initial orbits to bigger final orbits.

IV. Conclusion

All the required results and plots have been obtained successfully using MATLAB, and the codes have been attached then and there for further modification and development.

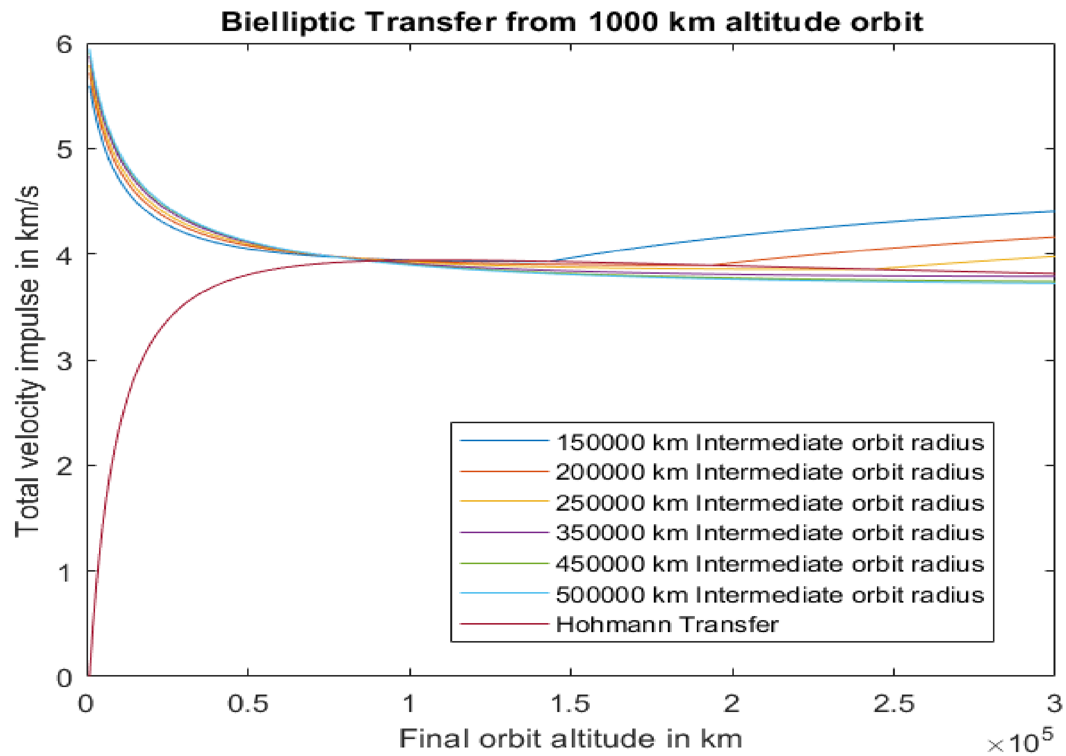

Fig. 12 Total velocity impulse for transfer

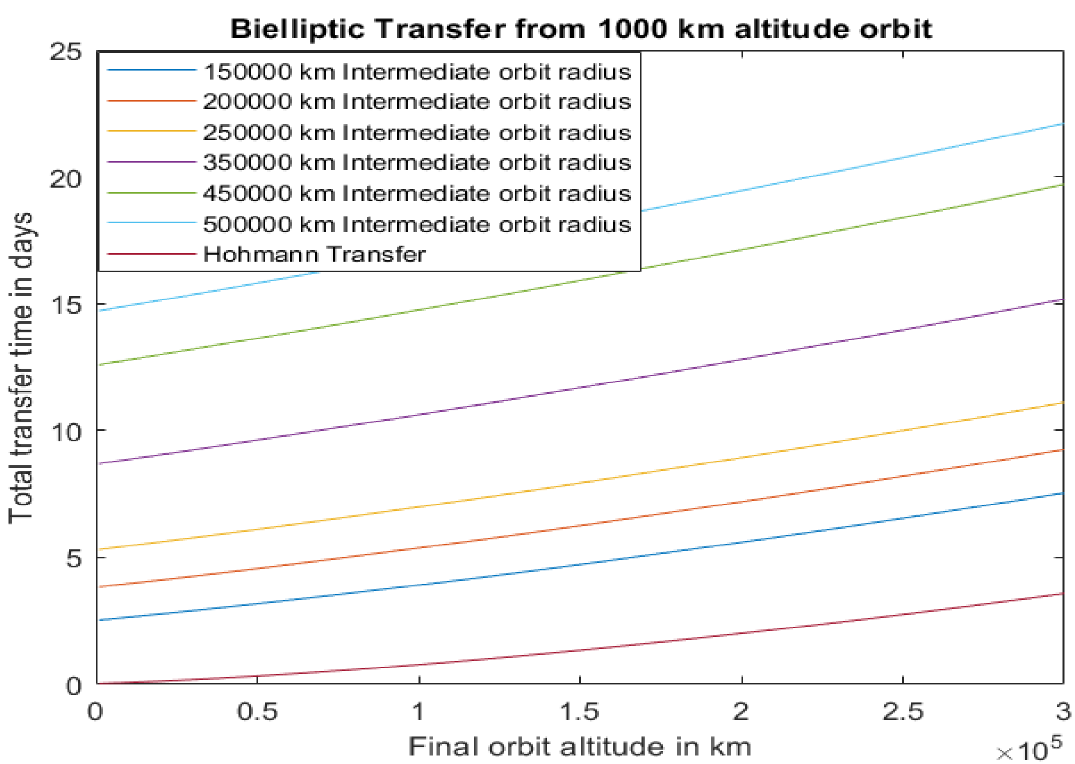

Fig. 13 Total time for transfer