Simultaneous color constancy

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Observers matched patches (simulated Munsell papers) in two simultaneously presented computer-controlled displays, a standard array presented under 6500-K illumination and a test array under 4000 or 10,000 K. Adaptation to the test illuminants was limited. The adjusted patch was surrounded by a single color (annulus display) or by many colors (Mondrian display). Observers either matched hue and saturation or made surface-color (paper) matches in which the subject was asked to make the test patch look as if it were cut from the same piece of paper as the standard patch. For two of the three subjects, the paper matches were approximately color constant. The hue–saturation matches showed little color constancy. Moreover, the illumination difference between the two displays was always visible. Our data show that simultaneous mechanisms alone (e.g., simultaneous color contrast) alter hues and saturations too little to produce hue constancy.

INTRODUCTION

There has recently been considerable interest in color constancy, which, in general terms, refers to a process whereby perceived object colors remain invariant under changes of illuminant color. This renewed interest in human color constancy is due in part to the potential value of low-level (knowledge- and task-independent) constancy mechanisms in the complex artificial-intelligence task of object recognition.[1]

Much of the recent work has been theoretical, involving studies of the mathematical constraints under which color constancy might be possible and analyses of the quantitative properties of previously proposed mechanisms.[2,3] In this paper we are concerned with the empirical evidence regarding human color constancy. Although it is generally agreed that humans have extensive color-constancy capabilities, there are remarkably few systematic studies of color constancy in the literature.

Proposed color-constancy mechanisms can be divided into two classes, which we will refer to as adaptation mechanisms and simultaneous mechanisms. Adaptation mechanisms are defined primarily in terms of temporal interactions whereby the sensitivities of the chromatic channels of the visual system change over time in response to the changed illuminant. In their simplest forms, adaptation mechanisms are spatially local, requiring no spatial interactions. Von Kries receptor adaptation[4] is an example of this type of mechanism.

Simultaneous mechanisms, on the other hand, are defined primarily in terms of spatial interactions among responses of chromatic channels to light at various locations in the retinal image. In their simplest forms, simultaneous mechanisms depend only secondarily on temporal parameters. Examples of this type of mechanism are simultaneous color contrast[4,5] and normalization[6] within neural edge-ratio computations.[7–9]

There are extensive data showing that adaptation, alone or in combination with simultaneous mechanisms, can produce large hue and saturation shifts.[4] There are also data showing hue and saturation shifts under conditions in which reduced possibility of adaptation isolates simultaneous mechanisms.[10] The design of most experiments of both types, however, makes it difficult or impossible to determine their meaning with respect to color constancy and constancy mechanisms.

For example, McCann *et al.*[9] studied apparent colors of complex arrays of colored papers under several illuminants. Quantitatively, their data met the requirements of color constancy well in the yellow–blue direction and somewhat poorer in the red–green direction.[3] Their constancy study is one of the best, with careful design, procedures, and analysis. With regard to discriminating among constancy models, however, there are several major problems. First, both adaptation and simultaneous mechanisms potentially contributed significantly to the observed constancy. Their subjects viewed only the test illuminant with the right eye and only the standard illuminant with the left eye.

A second problem for constancy interpretation is that the observer's task may have permitted intermixture of two types of perceptual judgment. It is not clear whether subjects were matching hue and saturation or were selecting papers with the same apparent surface color. As the experiments reported below show, this distinction has a major effect on matching data.

A third major problem is that the papers in the test array were surrounded by different reflectances from those surrounding the match papers, so illuminant color was not the only important difference between test and matching arrays.

We report here experiments on the role of simultaneous mechanisms in color constancy. The experiments are part of a program in which we attempt to discriminate among various classes of constancy mechanisms by controlling four kinds of variables: We make a clear distinction between hue and saturation judgments and judgments of surface color; we limit the subject's state of chromatic adaption; we use identical reflectances in test and matching arrays; and we control the spatial complexity of the patterns.

In our experiments the subjects viewed simulations of colored-paper arrays on a computer-controlled video monitor. In these simulations a standard patch of colored paper was displayed in an array under one illuminant and a test

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patch in an array under a different illuminant. The subject was asked either to adjust the test patch to match its hue and saturation to those of the standard patch (hue–saturation match) or to adjust the test patch to look as if it were "cut from the same piece of paper" as the standard, i.e., to match its surface color (paper match).

The two tasks were performed with spatial paper arrays of three different complexities. We used center-surround arrays (two surfaces) and complex arrays (32 surfaces) surrounded by darkness, i.e., with the edges of the simulated illuminant falling exactly on the edges of the paper arrays. It seemed possible that important information about the illuminants might be obtained if the illuminants were to spill over the edges of the papers onto the surround. We therefore also had our subjects make hue matches in a simulation in which the illuminants of the standard and test arrays slightly overlapped onto the simulated black paper of the surround.

METHODS

Observers

The two authors, LA and AR, and a naive observer, JS, were the observers. All observers were color normal (100-hue test) and wore their eyeglasses.

Equipment

Color displays were presented on a carefully calibrated (Appendix A) Tektronix 690SR high-resolution color monitor under the control of an Adage 3000 image processor and a VAX 11/750 minicomputer. Chromaticities were based on measurements of the spectral radiances of the phosphors (3-nm resolution) at approximately the luminance levels used in the experimental work. Chromaticity and luminance did not vary appreciably over the effective viewing area. Nonlinearity of the red, green, and blue channels of the image-processor–display system was measured. A linear relationship between digital data and luminance was obtained in each color channel through 10-bit lookup tables based on the nonlinearity measurements.

Subjects controlled the chromaticity of the test patch with a high-resolution graphics tablet. The computer interpreted the hand-held cursor's position on the tablet as a position within an RGB color triangle and set the test patch to the corresponding chromaticity. Vertical cursor movement controlled yellow–blue content, and horizontal movement controlled red–green content. Between trials the computer randomly offset the horizontal and vertical position of the color mapping on the bit pad within a range of ±10% to prevent position cues from influencing the matches. The spatial resolution of the bit pad exceeded the 10-bit resolution of the image-processor digital-to-analog convertors.

Stimuli

All the stimuli were simulations of arrays of Munsell matte colored papers in a dark surround. Chromaticities of 32 papers under three daylight illuminants (4000-, 6500-, and 10,000-K correlated color temperatures) were based on published tables of Kelly *et al.*[11] The 4000- and 10,000-K illuminants represent extremes of daylight, and 6500 K represents average sunlight.[12]

The logic of the simulation is based on superposition of lights: Conceptually, the 4000- and 6500-K correlated-color-temperature white lights were obtained by mixing appropriate amounts of illuminants A and C and the 1000-K light by mixing sources S and D. The light reflected from each of the 32 colored papers under, for example, the 4000-K light is just the sum of the reflected lights from the A and C components. In practice, tristimulus values for the simulated paper–illuminant combinations were obtained by linear interpolation between the tabled tristimulus values for the selected Munsell papers under standard illuminants A, C, D, and S. The proper ratios for the mixtures of the sources were obtained by using correlated-color-temperature lines from LeGrand.[12] The chromaticities will hereafter be referred to by the names of the simulated papers (Munsell notation) and illuminants.

The papers were chosen from those in the tables to provide as large and uniform coverage of the Munsell set (at value 5/) as possible, given the limitations imposed by our requirement that our monitor produce all the selected papers under all illuminants. We restricted our papers to Munsell value 5/ in an attempt to isolate color constancy from lightness constancy. Blue and purple-blue papers had high-excitation purity (re D6500) under the 10,000-K illuminant, medium-excitation purity under 6500 K, and low-excitation purity under 4000 K. The reverse was true for red and yellow-red papers.

In the annulus condition, two 1-deg-square patches were presented 7.1 deg apart (center to center) on the screen. Each patch was surrounded by a gray, square annulus (Munsell N 5/) whose inner border coincided with the edge of the patch and whose outer borders subtended 3 deg.

In the Mondrian condition (Fig. 1a), two 5-deg-square multicolored patchwork arrays were presented 7.1 deg apart (center to center). The two arrays had identical spatial arrangements of 32 irregular rectilinear patches (Fig. 1b).

The left-hand-side display (standard) was given a simulated illumination of 6500 K on all trials, the right-hand-side display (test) either 4000 or 10,000 K. Patches in the standard all had about the same luminance (20 cd/m^2). At exactly equal luminance, some color combinations did not create adequate borders. Therefore, in the Mondrians, luminances of individual patches were varied by ±10% to ensure that all borders between patches appeared about equally salient (Fig. 1c).

For the test Mondrians there were two luminance arrangements. In the full-simulation condition, the luminance of each patch was that required for simulation of the interaction of the illuminant with the reflectance. For example, the R 5/8 paper was most luminous under the 4000-K illuminant, less under the 6500-K illuminant, and least under the 10,000-K illuminant. The required luminances were obtained from the published tables[11] by the same interpolation as for chromaticities. As a consequence, some of the borders between test Mondrian patches were more clearly defined than others. In this condition the subject adjusted the chromaticity of the test patch with the luminance fixed by the computer at that required by the simulation, i.e., the interpolated value for the test-array illuminant and the reflectance of the patch being matched.

In the equal-luminance condition the patches of the test Mondrian had approximately equal luminances, the same luminances as their standard Mondrian counterparts (Fig. 1c), and all borders were approximately equally defined. In

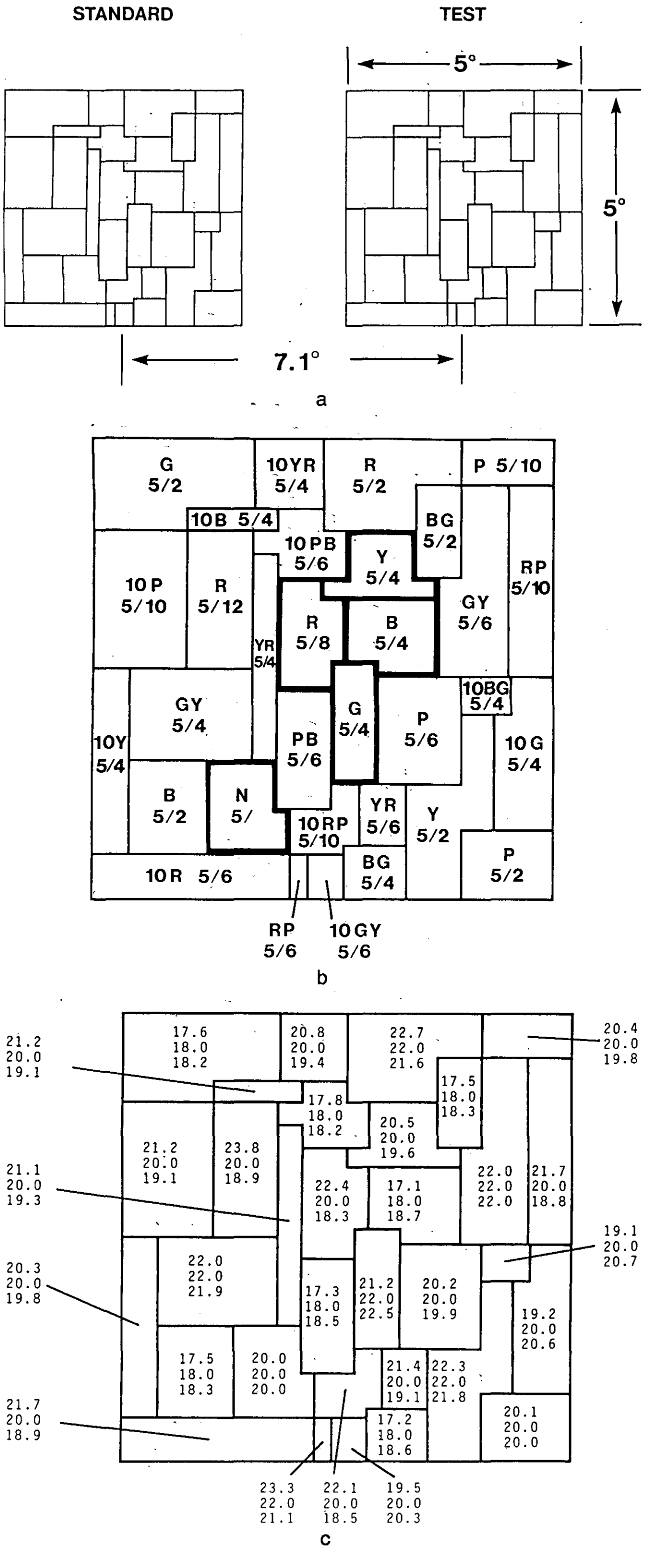

Fig. 1. a, The Mondrian display. One patch in the test display (on the right) was varied to match the equivalent patch in the standard display on the left. Both displays were viewed simultaneously; observers move their eyes frequently between them. The simulated illuminant in the standard was 6500 K; in the test, it was either 4000 or 1000 K. b, Munsell designations of the patches in the standard display. Darker contours surround potential test patches and are marked to aid the reader; no outlines were present in the display. c, Luminances (in candellas per square meter) for each patch in the full-simulation condition. Upper numbers, the 4000-K test Mondrian; middle, the 6500-K standard Mondrian; lower, the 10,000-K test Mondrian.

this condition the test patch being adjusted had the same luminance as the corresponding standard patch. Setting the luminances equal is equivalent to building the test Mondrian with different papers from those in the standard Mondrian; that is, with papers of the same relative spectral reflectances but with higher or lower absolute reflectance.

In the overlapped illuminant condition the test and standard Mondrians were surrounded with 0.5-deg-wide annular zones simulating the test and standard illuminants, respectively, falling on Munsell N 1/, a black paper.

In all cases the simulations were convincing, clearly appearing to be matte reflective surfaces evenly illuminated by an external source.

Procedure

Observers initially adapted to a 6500-K, 20-cd/m^2 white for 3 min. They then viewed the two continuously presented displays and matched a designated standard patch in the left display with the corresponding test patch in the right display, using the tablet to vary the chromaticity of the test patch. Observers were requested to spend about the same amount of time looking at each display and to alternate between the displays approximately once per second. Trials for the five different test patches, red (Munsell R 5/8), yellow (Y 5/4), green (G 5/4), blue (B 5/4), and gray (N 5/), occurred in pseudorandom order, once under 4000 K and once under 10,000 K in each of the five blocks of trials.

Task

In the hue-match condition, observers were instructed to match the hue and saturation of the test patch to those of the standard patch by varying the chromaticity of the (fixed-luminance) test patch. They were instructed to disregard, as much as possible, other areas of the screen.

In the paper-match condition, observers were told that the two displays were identical paper arrays illuminated by different sources. They were instructed to make the test patch "look as if it were cut from the same piece of paper" as the corresponding patch in the standard. It was pointed out that other patches in the Mondrians had closely related colors (there were no identical patches) and that the relations among such groups might be useful. Hence they could, for example, match the standard red at 6500 K with the corresponding test patch at 10,000 K by comparing the standard with other reds (or oranges or violets) under the 6500-K standard and by then transposing the relevant relations to the 10,000-K display.

RESULTS

Results for the annulus condition are shown in CIE (x, y) coordinates in Fig. 2. Solid lines with no symbols are the triangular color space of our monitor and the spectrum locus. The stimuli are plotted in the same fashion in each panel, with the (x, y) coordinates of the 6500-K standard patches that were matched represented by circles. Coordinates of patches under 4000 and 10,000 K are indicated by squares and triangles, respectively. The mean chromaticities set by the observers are indicated by filled symbols with error bars of ±1 standard error (shown only when larger than the symbol). The open symbols indicate the chromaticities that the standard paper would have under the test illuminants. In this representation of the data, perfect color con-

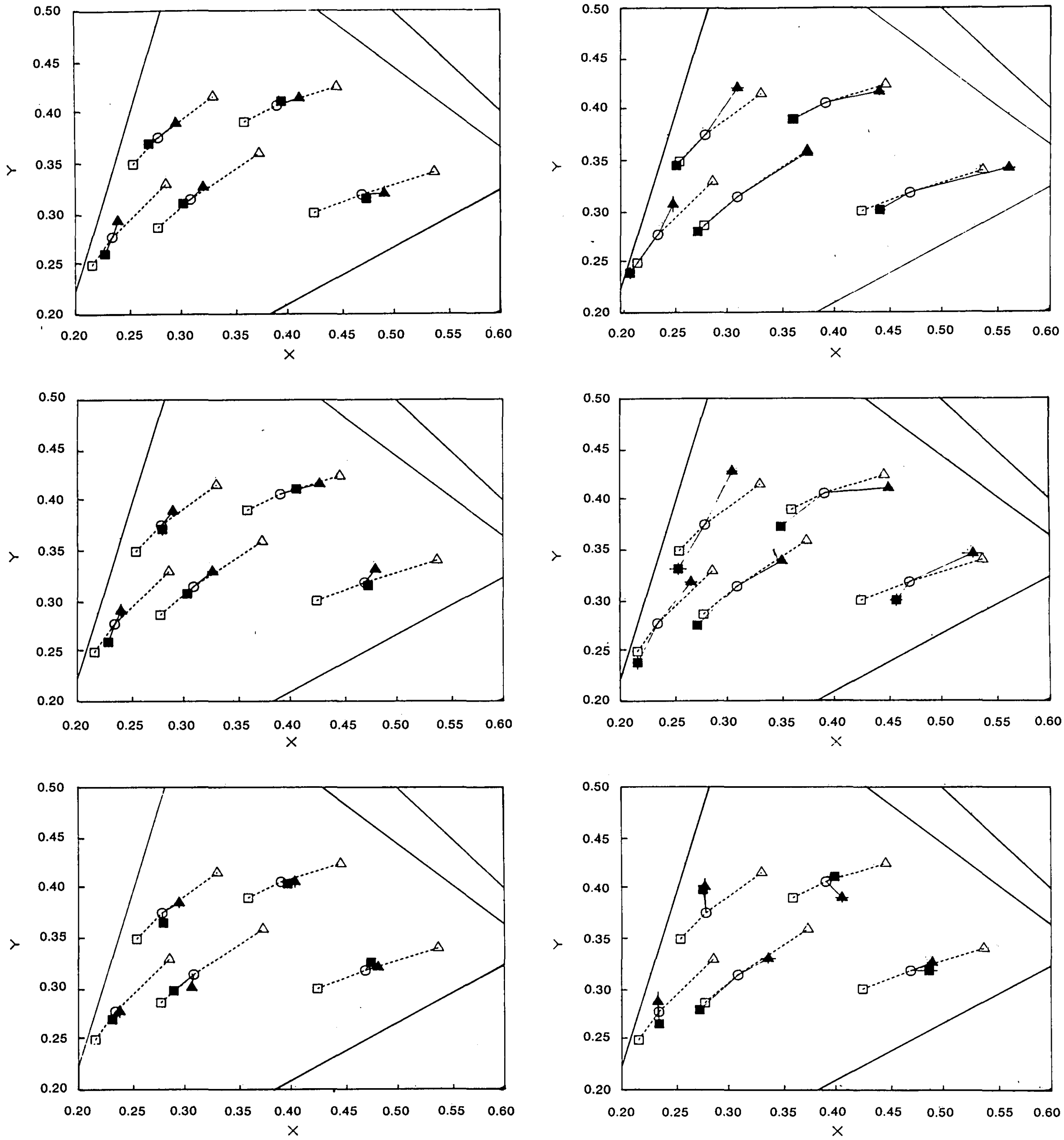

Fig. 2. Annular stimuli. Each plot shows the CIE (x, y) coordinates of the standard stimulus patches (open symbols) and the mean test-patch matches (filled symbols) under simulated illuminants of 6500 (circles), 4000 (triangles), and 10,000 K (squares). Bars represent ±1 standard error of the mean match. Where no bars appear, the bars are smaller than the plotted symbol. Solid lines with no symbols are the monitor color triangle and spectrum locus (outside). Top, middle, and bottom panels, observers LA, AR, and JS, respectively; right-hand panel, paper matches, left-hand panel, hue and saturation matches. If color constancy held perfectly, open and filled symbols would become superimposed. Color constancy is adequate with paper matches but poor with the direct hue and saturation matches.

stancy would occur if the subject set the test-patch chromaticity to the chromaticity that the standard patch would have under the test illuminant, i.e., if open and filled symbols coincided.

Results for the hue match are shown on the left-hand side of Fig. 2 and those for the paper match on the right. The paper matches show quite good color constancy for most of the test patches, although the shift is typically slightly erroneous in magnitude or direction. The hue-match data show much less constancy by comparison; the data depart much less from the 6500-deg chromaticity being matched, though the small shifts obtained are typically in the general direction of constancy.

Similar results are shown for the Mondrian patterns in the full-simulation (both luminance and chromaticity determined by illuminant) condition in Fig. 3 and the equal-luminance condition (constant luminance, chromaticity determined by illuminant) in Fig. 4. As with the annuli, the

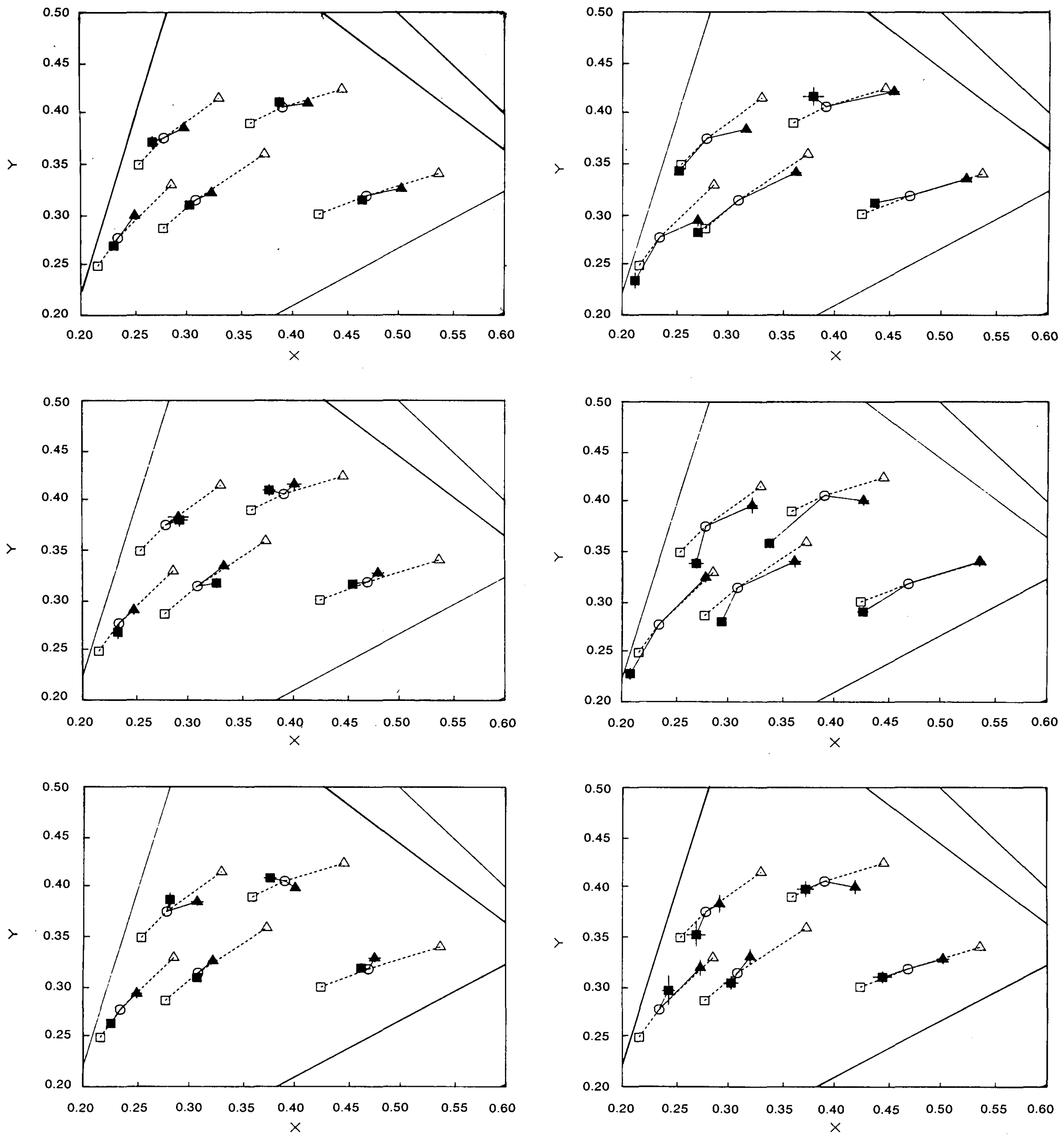

Fig. 3. Mondrian stimuli, equal-luminance condition. Symbols are as in Fig. 2.

hue matches shifted substantially less than required by color constancy.

In Fig. 5, results are plotted for hue matches for full-simulation Mondrians surrounded by the annulus of illuminant spillover. Even though the illuminants were clearly isolated on neutral surrounds, color matches were no closer to color constancy than in the other conditions. (No paper matches were made in this condition.)

In summary, we found little simultaneous color constancy for hue and saturation for any of our spatial configurations when we asked for direct hue and saturation matches. The 4000- and 10,000-K test illuminants were always easily discriminated from the 6500-K standard display and from each other. All the test pattern's hues shifted systematically from those of their standard pattern counterparts, i.e., there was a substantial preservation of illuminant information.

This absence of major hue and saturation shifts indicates that our instructions that the subject not fixate too long on either Mondrian and the random interleaving of the test illuminants were successful in avoiding large temporal adaptation effects.

In the paper-match condition two of the three subjects had sufficient knowledge of the color that the paper should take on in the test illuminant to produce approximately color-constant settings, with standard errors approximating those in the hue-matching task. However, the paper match-

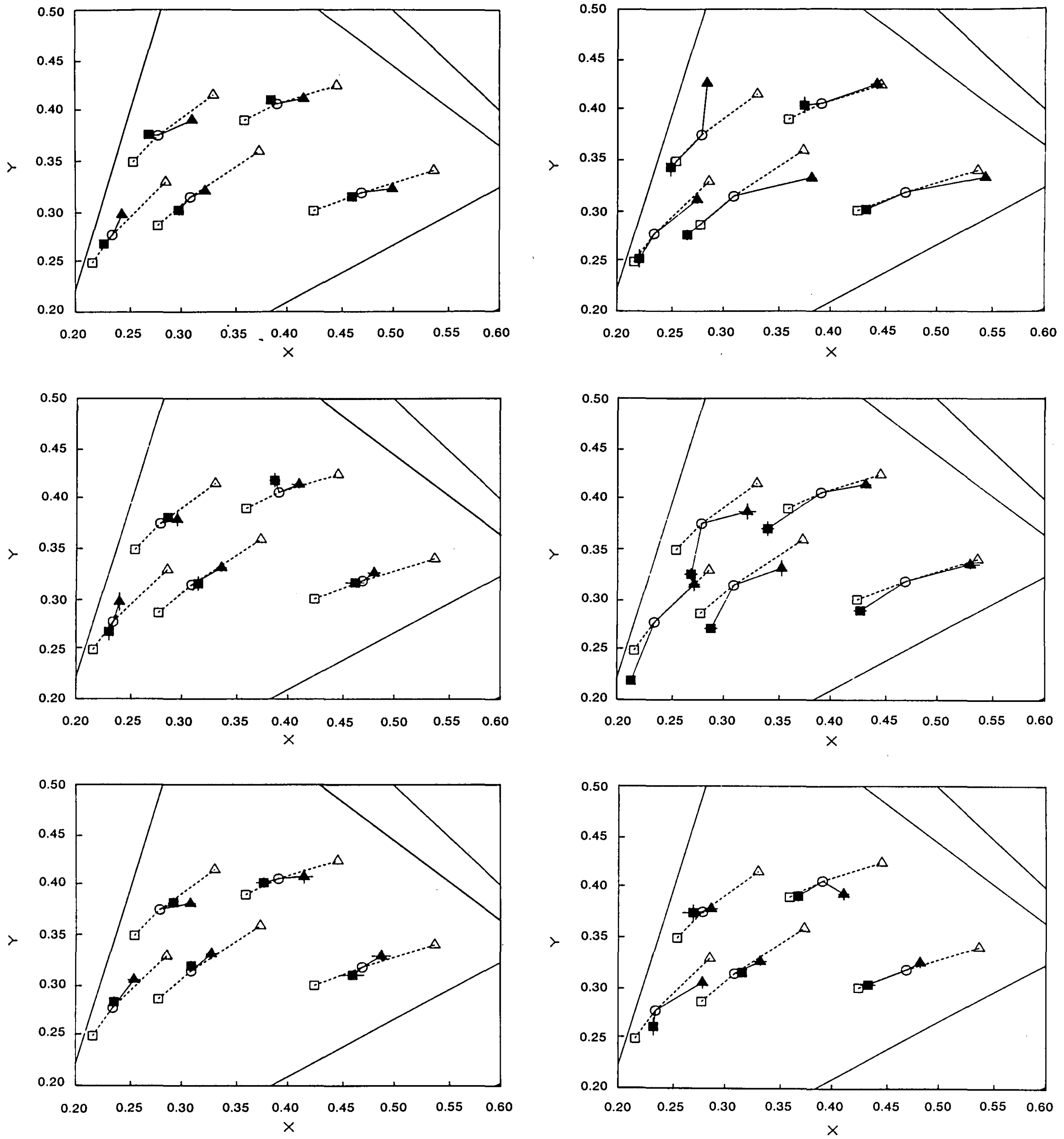

Fig. 4. Mondrian stimuli, full-simulation condition. Symbols are as in Fig. 2.

es were slower, were reported to be more difficult, and were more variable between subjects than were the hue and saturation matches.

Pattern complexity had little effect on the amount of simultaneous color constancy. The hue matches in the Mondrian condition showed no more deviation from the 6500-K chromaticity point than in the annulus condition. In fact, the directions of the deviations seem to be more nearly in the constancy direction for the simpler pattern, perhaps because of the less-complicated local contrast relations. In the paper-match condition, the single gray paper of the annulus gave sufficient illuminant information for the subjects to know fairly accurately the appearance that the test patch should take on.

The luminance differences in the full-simulation condition seem to have produced slightly larger deviations from the 6500-K chromaticities, but they are still far short of constancy. The constancy of the paper matches is as good in the equal-luminance condition as in the full-simulation condition.

The addition of the large, gray-paper annulus in the illuminant spillover condition had no effect on the hue matches.

In informal observations, we established that the illuminant differences were just as detectable and hues in the two Mondrians just as different when the Mondrians were surrounded by a 20-cd/m^2 6500-K reference white, when the room lights were turned on, or when the Mondrians were viewed from close to the display.

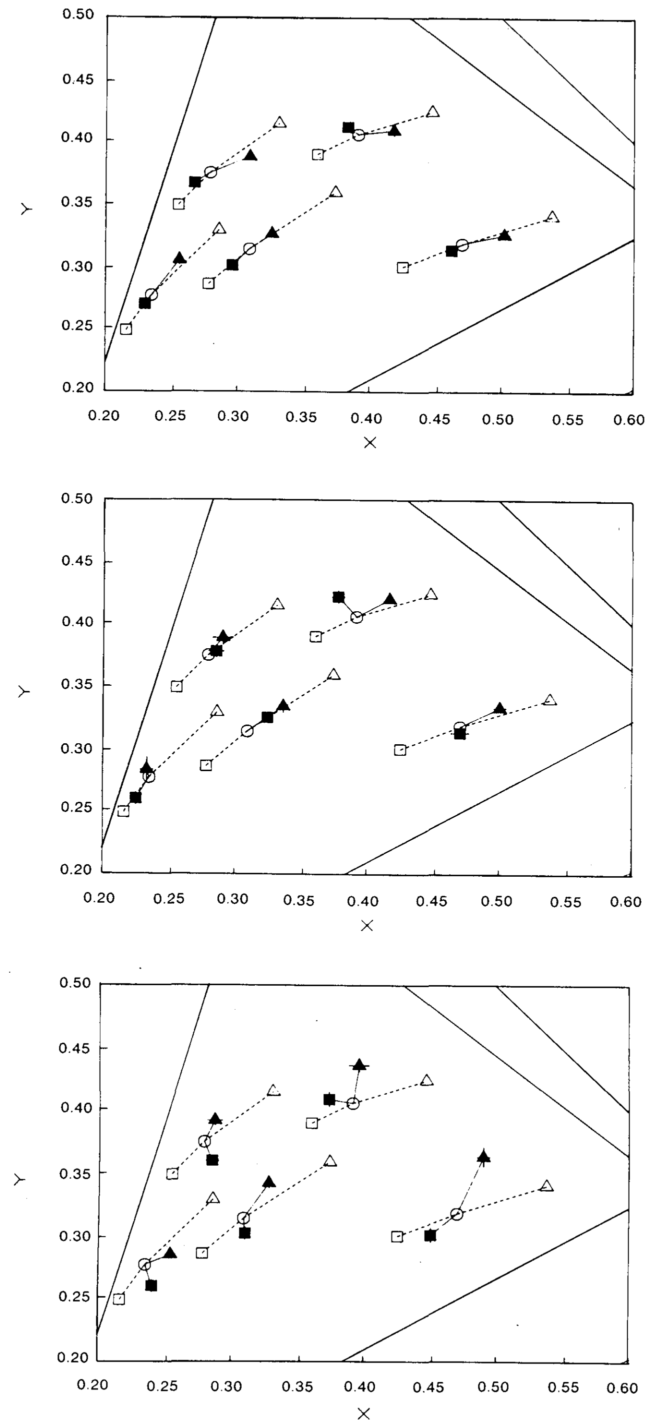

Fig. 5. Mondrian stimuli, full-simulation condition with a simulated overspill of the illuminant [a 0.5-deg annular surround that simulated the illuminant falling on a black Munsell (N 1/) paper]. Only hue matches were made. Results were not altered by the overspill. Symbols are as in Fig. 2.

DISCUSSION

Constancy of surface color may be perceptually represented in at least two different ways.[13] Constancy of perceived surface color might occur because hues and saturations are invariant under illuminant change (spatial or temporal), with no perceptual representation of the illuminant difference in terms of local color qualities. In this case the invariance is produced by a sensory mechanism (e.g., adaptation or contrast) that adjusts the response to cancel the proximal stimulus differences produced by the illuminant change. For example, a paper that looks unique yellow under direct sunlight might continue to look unique yellow in the greenish reflected illuminant under a tree.

Alternatively, hues and saturations might change when the illuminant changes but be perceived to result from constant surface colors and varying illumination. The paper that looks unique yellow under direct sunlight might look greenish yellow under the tree and yet might be clearly identifiable as a yellow paper. That is, perfect constancy could still obtain if the viewer, by a perceptual computation, were able to see the paper as an object of the same surface color under illumination perceived to be greener than the direct sunlight. The difference between this type of mechanism and the sensory mechanisms is not in the computation required but in the perceptual representation of the information computed.

Color constancy was weak for our hue matches (direct sensory representation), although two of the three observers could, if required, approximate the latter type of color constancy (the paper matches). This result held true over variations in relative patch luminances (full-simulation and equal-luminance conditions) over display complexity (annuli versus Mondrians), and of surround luminance (black, dimly illuminated gray annuli or, in informal observations, a mean-luminance uniform gray field). Under our viewing conditions, the illumination difference was always clearly perceptible.

We conclude that mechanisms such as simultaneous color contrast, which inflexibly discard, through a normalizing process, information about abrupt spatial illumination changes, make little contribution to color constancy within a single scene. Humans can, however, compute approximate chromatic reflectance information in such scenes while retaining some illumination information.

Lightness Constancy

We restricted our papers to Munsell value 5/ in an attempt to isolate color constancy from lightness constancy. Arend and Goldstein[14] have recently investigated lightness constancy in achromatic patterns (annuli and Mondrians) analogous to our chromatic patterns. As in our chromatic experiments, subjects matched separately apparent surface color (lightness, analog of our paper matching) and local sensory quality (brightness, analogue of our hue and saturation matching). The lightness matches showed nearly perfect constancy, better constancy than the paper data in our chromatic experiments.

In the annulus condition, brightness data were also analogous to our hue data in the sense that they showed little constancy, departing only slightly from stimulus (luminance) matches.

In the Mondrian condition, on the other hand, there was a major difference between the achromatic and chromatic data, suggesting a fundamental difference between simultaneous lightness and color mechanisms. In contrast to our hue-match data, in the Mondrian condition even the brightness matches showed a strong tendency toward constancy. In other words, Arend and Goldstein found evidence for significant simultaneous brightness constancy as well as almost perfect simultaneous lightness constancy.

Simultaneous Color Contrast

Simultaneous color contrast, when invoked as a possible color-constancy mechanism, predicts a reduction of hue and saturation differences between identical surface colors under different illuminants in the same scene. Simultaneous contrast between adjacent stimulus regions reduces the contribution to perceived color of stimulus components common to the regions, relative to the contribution of components unique to one or another region.

In our data, both for conventional disk–annulus patterns and Mondrians, hue and saturation shifts attributable to simultaneous contrast were often in the constancy direction but were much smaller than required for hue constancy. Kinney[10] reported much larger shifts of matching chromaticity for her "maximum induction" condition (high inducing purity, large luminance contrast, large inducing area). While her paradigm differed in important ways from ours, for parameters roughly comparable to ours (low inducing purity, low luminance contrast) she also found very little induction.

Kinney observed larger changes of match chromaticity when the inducing field luminance was twice the test-field luminance, suggesting that we might see larger hue shifts when we introduce large lightness differences in our chromatic patterns. However, both for Kinney's induction effects and for these hypothetical Mondrian effects, the responsible mechanisms could not properly be called chromatic mechanisms, since they have a strong interactive response to lightness variation.

Retinex

Land and his co-workers[7] have suggested as a color-constancy mechanism a process that calculates within each of three wavebands the ratios among the responses to the various regions of the stimulus pattern. When there are no shallow spatial gradients in the stimulus, the process is equivalent to normalizing the local responses to the geometric mean of the responses to all stimulus regions.[15] The normalization process is directly represented as hue and saturation changes and is simultaneous, predicting the same shifts in briefly flashed patterns as in continuously presented patterns. Thus the small chromaticity shifts in our hue-match condition argue against such a mechanism contributing substantially to constancy under our viewing conditions.

Land based his model in part on demonstrations that produced substantial color shifts under brief-flash conditions in which little adaptation was possible.[16] There are a number of experimental differences that may account for the apparent discrepancy between those effects and our failure to find large hue shifts. One possible explanation is our small luminance contrast. As noted above, there is good reason to believe that we might obtain larger hue shifts with large luminance contrast. If so, however, the strong dependence of chromatic change on luminance variation will require major theoretical adjustments.

Finally, several authors[1,7–9] have argued that illumination changes over natural scenes tend to be gradual and that the visual system's relative insensitivity to shallow spatial gradients may contribute to constancy. Our experiments do not rule out the possibility that shallow chromatic illumination gradients may be unperceived or underweighted perceptually, producing partial or full simultaneous constancy of hue and saturation within such scenes.

APPENDIX A

Calibration of the display system consisted of measurement and calculation of the CIE coordinates of the monitor phosphors, measurement and correction of the nonlinear relationship between digital data levels and color-gun luminances, and measurement of spatial homogeneity.

Phosphor CIE Coordinates

The relative spectral power distributions of the red, green, and blue phosphors of the monitor were determined using a Princeton Applied Research optical multichannel analyzer spectroradiometer. The radiometer had 3-nm resolution and was calibrated with a National Bureau of Standards traceable incandescent source (Optronic Laboratories). The blue and green guns have unimodal spectra, while the red consists of several narrow bands (Fig. 6). The 1931 CIE chromaticity coordinates of the phosphors (Fig. 6), r = (0.637, 0.360), g = (0.271, 0.593), b = (0.157, 0.043), were calculated by integrating over wavelength the weighted relative spectral power distributions.

Monitor Linearization

The relationship between the digital data values applied to the image processor's 10-bit digital-to-analog convertors and the monitor luminances was measured separately for the three color guns by using a United Detector Technology photometer with photopic filter. The curves for the three guns all closely approximated the same power function,

$$L/L_{\max} = (D/D_{\max}) ** 2.58,$$

where L and $L_{\max}$ are the gun luminance and maximum luminance and D and $D_{\max}$ are the digital data and the maximum data, respectively. The relationship between image data and luminance could therefore be made linear by loading each channel's intervening 10-bit lookup table with the inverting power function.

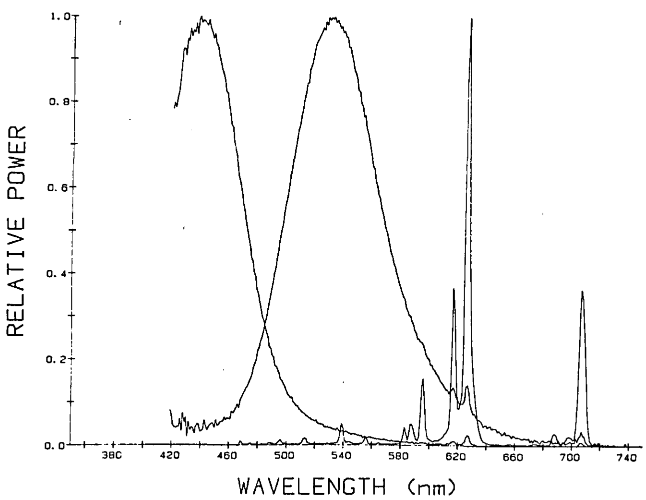

Fig. 6. The relative spectral power of the R, G, and B phosphors of the monitor measured at every 3 nm and used in calculating the chromaticity coordinates of the patches.

Spatial Inhomogeneity

The spatial nonuniformity of the monitor was measured for each color channel by scanning the face at close spatial intervals with a narrow-acceptance-angle photometer head.

For all three guns the luminance profile was approximately the same, thereby giving the same relationship between image data and chromaticity at all locations. The luminance was approximately constant (5%) within a central disk-shaped area subtending 10-deg visual angle at the viewing distance of the present experiment. The luminance fell off gradually outside the disk to 79% of the center luminance at the worst corner of the display. Most of the two Mondrians (and all the test and comparison patches) fell within the uniform disk.

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REFERENCES

1. B. K. P. Horn, "Determining lightness from an image," Comput. Graphics Image Process. **3**, 277–299 (1974); D. Marr, *Vision* (Freeman, San Francisco, 1982).
2. G. Buchsbaum, "A spatial processor model for object color perception," J. Franklin Inst. **310**, 1–26 (1980); L. T. Maloney, *Computational Approaches to Color Constancy* (Stanford U. Press, Stanford, Calif., 1985); L. T. Maloney and B. A. Wandell, "Color constancy: a method for recovering surface spectral reflectance," J. Opt. Soc. Am. A **3**, 29–33 (1986); M. Brill and G. West, "Contributions to the theory of invariance of color under the conditions of varying illumination," J. Math. Biol. **11**, 337–350 (1981).
3. J. A. Worthey, "Limitations of color constancy," J. Opt. Soc. Am. A **2**, 1014–1026 (1985).
4. J. von Kries, "Die Gesichtsempfindungen," Handbuch Physiol. Menchen **3**, 211 (1904); H. von Helmholtz, *Handbuch der Physiologischen Optik*, 2nd ed. (Voss, Hamburg, 1896); H. E. Ives, "The relation between the color of the illuminant and the color of the illuminated object," Trans. Illum. Eng. Soc. **7**, 62–72 (1912); H. Helson, "Fundamental problems in color vision. I. The principle governing changes in hue, saturation and lightness of nonselective samples in chromatic illumination," J. Exp. Psychol. **23**, 439–436 (1938); H. Helson, D. B. Judd, and M. Wilson, "Color rendition with fluorescent sources of illumination," Illum. Eng. **51**, 329–346 (1956); review by G. Wyszecki and W. S. Stiles, *Color Science* (Wiley, New York, 1967).
5. H. Wallach and A. Galloway, "Constancy of colored objects in colored illumination," J. Exp. Psychol. **36**, 119–126 (1946).
6. T. N. Cornsweet, *Visual Perception* (Academic, New York, 1970), pp. 382–383.
7. E. H. Land, "The retinex," Am. Sci. **52**, 247 (1964); E. H. Land, "The retinex theory of color vision," Sci. Am. **237**, 108–128 (1977); E. H. Land, "Recent advances in retinex theory and some implications for cortical computations: Color vision and the natural image," Proc. Natl. Acad. Sci. USA Phys. **80**, 5163–5169 (1983); E. H. Land and J. J. McCann, "Lightness and retinex theory," J. Opt. Soc. Am. **61**, 1–11 (1971).
8. J. J. McCann and K. Houston, "Calculating color sensations from arrays of physical stimuli," IEEE Trans. Systems Man Cybern. **SMC-13**, (1983).
9. J. J. McCann, S. P. McKee, and T. H. Taylor, "Quantitative studies in retinex theory," Vision Res. **16**, 445–458 (1976).
10. J. A. S. Kinney, "Factors affecting induced color," Vision Res. **2**, 503–525 (1962).
11. K. L. Kelly, K. S. Gibson, and D. Nickerson, "Tristimulus specification of the Munsell Book of Color from spectrophotometric measurements," J. Opt. Soc. Am. **33**, 355–376 (1943).
12. Y. Le Grand, *Light, Colour and Vision* (Chapman, London, 1968).
13. R. M. Evans, *Light, Colour and Vision* (Chapman and Hall, London, 1968).
14. L. E. Arend and R. Goldstein, "Lightness and brightness in complex achromatic arrays," Invest. Ophthalmol. Vis. Sci. **27**, 292 (1986).
15. D. Brainard and B. Wandell, *An Analysis of the Retinex Theory of Color Vision* (Stanford U. Press, Stanford, Calif., 1985).
16. E. Land and N. Daw, "Colors seen in a flash of light," Proc. Natl. Acad. Sci (USA) **48**, 1000–1008 (1962).