A Spectral Estimation Theory for Color Appearance Matching

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Abstract

There are several reports describing color vision in subjects who are dichromatic in one eye and trichromatic in the other. Formulae fit to the between-eye color appearance matches in such unilateral dichromats have been used to predict color appearance for dichromats [1][2]. In this paper, we describe a general principle, spectral estimation theory, that guides how to predict the mapping from two cone class absorptions in a dichromatic eye to three cone class absorptions in a trichromatic eye. The theory predicts matches by first estimating the smoothest, non-negative, spectral power distribution that is consistent with the measured cone absorption rates (e.g., of a dichromat). We then use this spectral estimate to calculate the absorption rate for the missing cone type (the standard color observer). In addition to predicting color appearance of dichromats, the theory predicts color appearance matches between color anomalous subjects and the standard color observer. Finally, the theory offers guidance about the possible effects on color appearance of gene therapy treatment for colorblindness.

Introduction

Many students of color vision have asked how the color experienced by one person may appear to another person. A fascinating series of papers describe one attempt to answer this question by measuring between-eye color matches measured in unilateral dichromats: these people are dichromatic in one eye and trichromatic in the other. The between-eye color matching experiments were first reported by Judd et al. [3] and subsequently by Graham et al. [4] and Alpern et al. [5]. Judd et al. [3] reported that equal energy light and two narrow band lights at 475 nm and 575 nm look the same when presented to both eyes of a unilateral protanope and a unilateral deuteranope. Alpern et al. [5] conducted a similar experiment with a unilateral tritanope and reported three eigen-colors: equal energy light, narrow-band light at 485 and 660. [5] reported We refer to lights whose color appearance match between the two eyes of a unilateral dichromat as eigen-colors. In 1995, Vienot et al. [1] used the data from these unilateral dichromats to calculate the equivalent color appearance in a standard color observer. Vienot et al. [1] and later Brettel et al. [2] noted that eigen-colors define an empirical map between two cone coordinates in a dichromatic eye and three cone coordinates in the trichromatic eye. They fit a piecewise linear relationship from the measured to missing cone types using the eigen-colors.

$$\beta_D L_{eigen} = L_{eigen}$$

Here, β_D is the transformation matrix and L_{eigen} denotes the LMS values for eigen-colors for a particular type of dichromat. In general, three eigen-colors can be used to solve for a single linear mapping: equal energy light and two narrow band eigen-colors. However, in some cases a single linear transformation produces physically unrealizable results. To solve this problem, Brettel et al. [2] used two different transforms $\beta_D^{(1,2)}$ to solved for different eigen-colors. That is

$$\beta_D^{(1)} L_{eigen}^{(1)} = L_{eigen}^{(1)}$$

$$\beta_D^{(2)} L_{eigen}^{(2)} = L_{eigen}^{(2)}$$

Here, $L_{eigen}^{(i)}$ is a matrix composed of equal energy light and the i^{th} narrowband eigen-color. For a pair of dichromatic cone responses, L_D, one determines which linear transform, $\beta_D^{(i)}$ applies then calculates the standard observer cone responses using $L_S = \beta_D^{(i)} L_D$.

Brettel's calculation has proved to be a very useful phenomenological model, but it does not provide a general principle that allows us to extend the calculation to other observers and conditions. In this paper, we provide a general principle that predicts the between-eye color matches. The principle supposes that both dichromats and trichromats solve a spectral estimation problem. Given the available cone absorptions, every type of subject implicitly calculates the smoothest, non-negative spectral radiance distribution that is consistent with their cone absorptions. Two people experience the same color appearance when the estimated spectral radiances match. We show that this spectral estimation calculation conforms closely to Brettel's numerical summary of the unilateral dichromatic measurements. The theory adds value because it shows how to generalize beyond the dichromatic appearance data to many other applications, such as color appearance of anomalous subjects, the effects of biological differences such as lens-aging, macular pigment density, and the potential consequences of gene therapy for color blindness.

Spectral Estimation Theory

We propose that all individuals, whether dichromat or trichromat, use the same computational strategy to estimate the spectral radiance that gives rise to the cone responses (i.e. different retinae, same neural computation). For standard color observers (trichromats), the spectral radiance is estimated from the responses in three different channels - L, M and S cones. For dichromats, spectral radiance is inferred from two channels, MS (protanope), LS (deuteranope), or LM (tritanope).

As Helmholtz wrote, *"The general rule determining the ideas of vision that are formed whenever an impression is made on the eye, is that such objects are always imagined as being present in the field of vision as would have to be there in order to produce the same impression on the nervous mechanism"* [6].

In this case, as in many others, the estimate is highly underdetermined and an infinite number of different spectral lights can create the same pattern of cone responses (metamers). To choose one possible spectrum additional constraints are required. A first plausible constraint is to select the smoothest spectral radiance function among all the metamers. A second criterion is physical realizability: constrain the spectral radiance function to be nonnegative. We can state this as a convex optimization problem

$$minimize \frac{1}{2}||\nabla w||_2^2, s.t. Sw = c, Sw \geq 0$$

Here, the rows of S contain the cone fundamentals for that subject, including transmittance of human lens and macular pigments [7]; c is the cone absorption values and w is the estimated smooth spectra. For trichromats, c is consists of three cone absorption values while for dichromats c contains two values.

We can also write the convex problem in pure matrix form as

$$minimize \frac{1}{2}||Zw||_2^2, s.t. S_e w = c, S_m w \geq 0$$

in which, $Z \in R^{n-1 \times n}$ is the differentiation matrix.

Linear Approximation for Protanopes and Deuteranopes

We evaluate the accuracy of the spectral estimation theory in the results section. Here, we first discuss some numerical properties of the theory.

Estimates for protanope and deuteranope rarely violate the non-negativity constraints. Removing the non-negativity constraints results in a closed-form linear solution. Specifically, if we remove the non-negativity constraints

$$minimize \frac{1}{2}||Zw||_2^2, s.t. S_e w = c$$

Expressing the joint error with a Lagrangian multiplier, λ, the solution will have the property that the derivative of $\frac{1}{2}||Zw||_2^2 + \lambda(Sw - c)$ with respect to w should be zero, i.e.

$$Z^T Zw + \lambda S = 0$$

Expressing this together with the constraints equations in matrix form, we have

$$\begin{pmatrix} Z^T Z & S^T \\ S & 0 \end{pmatrix} \begin{pmatrix} w \\ \lambda^T \end{pmatrix} = \begin{pmatrix} 0 \\ c \end{pmatrix}$$

Where λ is a row vector of length equal to the number of cone types, and 0 is a vector of zeros of the same length as λ. Taking the inverse, we have

$$\begin{pmatrix} w \\ \lambda^T \end{pmatrix} = \begin{pmatrix} Z^T Z & S^T \\ S & 0 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ c \end{pmatrix}$$

Thus, the estimated spectra w is in the first n rows of the solution. Plugging in all the constants, we find two simple linear transformations that calculate the missing cone type for protanopes and deuteranopes

$$\hat{L} = 1.3924M - 0.2652S(Protanope)$$

$$\hat{M} = 0.6918L + 0.2439S(Deuteranope)$$

The solution for the tritanopes is not linear, but it does satisfy the property of homogeneity. Suppose a tritanopes (L,M) cone responses are matched by a normal with (L,M,S), so that $(L,M) \Rightarrow S$.Then scaling the (L,M) cones will also scale the S cone: $\alpha(L,M) \Rightarrow \alpha S$. This makes it possible to code the nonlinear relationship between the (L,M) cones and the interpolated S value very efficiently using a one-dimensional lookup table in which the ratio of the L and M cones is an input that is assigned to an S value. The final value of S is multiplied by a scalar, say the level of the L cone input, to determine the final value.

Results

Comparison with reported data

Eigen-color for dichromats

To evaluate the spectral estimation theory, we predict the results of unilateral dichromatic cross-eye matching experiment and compare them against the reported measurement data. First we compare the eigen-colors, which appear to be the same to both eyes (dichromatic and trichromatic) in a unilateral dichromat.

In [3], the authors reported eigen-colors for a unilateral protanope and a unilateral deuteranope that are equal energy lights and two narrow-band spectral lights. In [5], a similar experiment was conducted with a unilateral tritanope and eigen-colors were documented among narrow-band lights.

Spectral estimation theory proposes that the eigen-colors are cases in which the dichromatic and trichromatic cone absorptions result in the same estimated scene radiance. The theory predicts the the eigen-colors. The comparison of predicted and measured eigen-colors among all narrow-band lights are shown in Table 1 below.

The predicted eigen-colors differ by only a few nanometers from the measurements, and the values are probably within the population or measurement variation. Furthemore, the equal energy light has zero gradient and is smoothest in wavelength, so that the theory predicts that the equal energy light is an eigen-color for all types of dichromats as reported in the literature [3][5].

Table 1. Comparison of predicted and measured eigen-colors among narrow-band lights (nm)

	Measurements	Theory
Protan	475	470.11
	575	573.61
Deuteran	475	473.25
	575	575.66
Tritan	485	485.68
	660	589-700

Comparison with Brettel's Method

We compared spectral estimation theory and Brettels algorithm by calculating their predictions for the color match in the trichromatic eye when presented with a spectral light in the dichromatic eye. The two calculations are very similar for protanopes and deuteranopes (Figure 1ab). The spectral estimation theory prediction is linear and thus the matches plot along a line in chromaticity space. The lines showing the chromaticity of the predicted match in the trichromatic eye give a series of monochromatic lights in the dichromatic eye that match the predictions of the Brettel calculation closely.

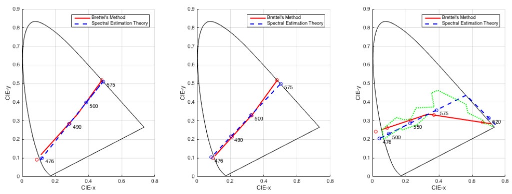

Figure 1. *Comparison between Brettel et al. algorithm and the proposed Spectral Estimation Theory. Lines show the predictions for unilateral color matches for monochrome lights between 400 and 700 nm by Brettel (red line) and the proposed spectral estimation theory (blue line) for protanope (left panel), deuteranope (middle panel) and tritanope (right panel).*

There are some differences in the predictions for the tritanopic subject (right panel in Figure 1). The two methods agree on their predictions for short wavelength (bluish) spectral lights, but the predictions differ for long wavelength spectral lights (reddish). Neither method perfectly predicts the measurements from the one subject reported by Alpern et al. [5]. Both methods miss the data by the similar amount, about 5-10 ΔE_{ab} units.

Simulating color appearance of color anomalous subjects

The added value of spectral estimation theory is that it provides a computational explanation of the phenomenological model proposed by Vienot et al. [1] and Brettel al. [2]. This enables us to extend the calculation to make predictions in new contexts. Here we apply the idea to anomalous subjects and to analyze the perceptual consequences of gene therapy, in which new types of photoreceptors are created within the retina.

The spectral estimation theory defines a principle for predicting the color appearance seen by color anomalous subjects. We model different color anomalous observers by shifting the wavelength sensitivity of a cone type along the wavenumber axis [8]. For example, the sensitivity of M cones in the eyes of deuteranomalous observers are mutated and shifted towards L sensitivity curve.

In applying spectral estimation theory, we assume that during brain development the neural circuitry learns the wavelength sensitivity of the cone types in color anomalous observers [9]. We estimate the spectral radiance from the absorptions in the anomalous observer and predict the cone absorptions in the standard color observer. These are the predicted color matches between the anomalous and standard observer. We note that a different model might assume that the retinal and brain circuitry is fixed and does not respond during development to the specific properties of the cone mosaic.

Furthermore, we can simulate the color matches for color anomalous subjects with different amounts of wavenumber shifts. The top row of Figure 2 shows simulated sensitivities of different observers. The bottom row shows the predicted color appearance. The leftmost column contains images for normal trichromatic observers and the rightmost ones are for a deuteranope. The predicted color appearance only changes subtly until the peak wavelength of the M-cone type is within 10 nm of the L-cone type.

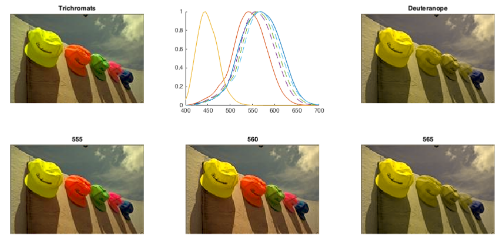

Figure 2. *Color Appearance Simulation for Deuteranomalous Trichromats. The peak wavelength of the shifted M cone type is shown at the bottom. The peak wavelength of the standard observers M-cone is 540 nm and of the L-cone is 570 nm, so that the upper left panel is a standard color observer and the upper right panel is a deuteranope.*

When peak M-cone sensitivity changes by less than 20 nm towards the L-cone, the color appearance is relatively constant. This indicates that population variation of cone sensitivities in this range will not have big effect on color appearance. When the M-cone peak sensitivity approaches within 10 nm of the L-cone peak sensitivity, the color appearance changes dramatically.

Simulating color appearance following gene therapy

Spectral estimation also permits us to analyze the potential color appearance effects arising from new gene therapies designed to transform subjects from dichromats to trichromats [10]. The proposal is to introduce an adenosine associated virus into the eye of a dichromat that will cause some of the, say, M-cones to produce L-cone pigment. The transform produces a new cone mosaic comprising three cone types: $(L, L+M, S)$.

Two possible color appearance outcomes of gene therapy have been discussed [11][12]. The predictions differ depending on the assumptions one makes about the ability of neural circuitry to account for the spectral properties of the new cone type. If the circuitry learns the new cone type accurately the predicted color appearance will be that of an anomalous trichromatic observer (Figure 3 d). If the retinal and brain circuitry does not adjust in response to the new cone type, the subject will still be dichromatic but appearance will change mainly by the presence of an overlay of small spots (Figure 3 e). The neuroscience literature on cortical plasticity in the early visual pathways mainly suggests that there is only limited flexibility in adults [13]. Hence, the extent to which the adult neural circuitry can learn and adapt to the effects of gene therapy remains an open and controversial question [11][12].

Summary

We propose the spectral estimation theory as a means to predict color appearance matches between subjects with different sensors. The theory predicts critical features of the color matches in unilateral dichromats (eigen-colors) and is consistent with the phenomenological model based on these data that are used to predict color appearance experienced by dichromatic subjects [1][2]. The theory also provides a rationale for comparing color appear-

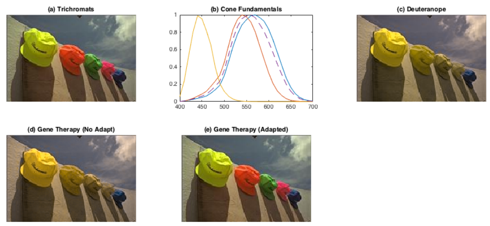

Figure 3. *Color appearance simulation of the consequences of gene therapy for a deuteranope. (a) Appearance for a trichromatic subject. (b) Model of the spectral sensitivities of the gene therapy subject. (c) Appearance for the deuteranope. (d) Appearance if the neural circuitry learns to account for the new cone type. (e) Appearance of the neural circuitry fails to account for the new cone type.*

ance between anomalous subjects and gene therapy patients with the standard color observer. Finally, the theory can be used to develop a color metric for dichromatic and color anomalous observers [14]. Exploration of spectral estimation theory predictions clarifies that a key issue in gene therapy is to what degree the neural circuitry of the retina and brain adjusts its properties to account for the spectral sensitivities of the cones. The predicted color appearance can depend significantly on the relative plasticity or stability of the neural processing.

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