Appendix: Luminance constancy under fixed geometry

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1 Illumination Spectra

Let us denote the i'th sample in the Granada dataset as $I_i^G(\lambda)$, where $\{i \in [1, M]\}$ and M is the total number of spectra in the dataset. Since the measured spectra vary over several orders of magnitude in overall intensity, we normalize each spectrum by dividing it by its mean power

$$I_i^s(\lambda) = \frac{I_i^G(\lambda)}{\int d\lambda I_i^G(\lambda)}. \tag{1}$$

For simplicity of notation, we denote wavelength λ as a continuous variable; in the actual calculations wavelength is discretely sampled and integrals are approximated by sums.

The Granada dataset was measured at 5 nm sampling intervals between 300 and 1100 nm. We subsampled the spectra to the 400-700 nm interval, 10 nm spacing representation used for rendering, and performed our calculations at this sampling.

The rescaled spectra $I_i^s(\lambda)$ were mean centered for PCA by subtracting out the sample mean ($\bar{I}^s(\lambda)$), over all rescaled spectra in the dataset,

$$\bar{I}^s(\lambda) = \frac{1}{M} \sum_i I_i^s(\lambda). \tag{2}$$

The mean centered dataset, $I_i^{MC}(\lambda) = I_s(\lambda) - \bar{I}_s(\lambda)$ was decomposed as:

$$I_i^{MC}(\lambda) = \sum_j w_{ij} \hat{\mathbf{e}}_j^{PCA}, \tag{3}$$

where the $\hat{\mathbf{e}}_j^{PCA}$ are the PCA basis vectors obtained using singular value decomposition (SVD) applied to $I_i^{MC}(\lambda)$, and w_{ij} are the weights obtained by projecting each of $I_i^{MC}(\lambda)$ onto the basis vectors $\hat{\mathbf{e}}_j^{PCA}$ $\left(w_{ij} = I_i^{\mathrm{MC}}(\lambda) \cdot \hat{\mathbf{e}}_j^{\mathrm{PCA}}\right)$. We used the basis vectors corresponding to the largest six SVD eigenvalues, so that $\{j \in [1, 6]\}$. For the rescaled Granada dataset, these six eigenvalues account for more than 90% of the variance.

These steps can be summarized as follows:

$$I_i^G(\lambda) \rightarrow I_i^s(\lambda) = \frac{I_i^G(\lambda)}{\int d\lambda I_i^G(\lambda)} \rightarrow I_i^{MC}(\lambda) = I_s(\lambda) - \bar{I}_s(\lambda) \rightarrow I_i^{MC}(\lambda) \approx \sum_{j=1}^{6} w_{ij} \hat{\mathbf{e}}_j^{PCA}. \tag{4}$$

To get a random illuminant spectrum $\tilde{I}_i(\lambda)$, we sample random weights $\tilde{w}_{ij}$ from a multivariate Gaussian distribution with mean $\bar{w}_j = \frac{1}{M} \sum_i w_{ij}$, and co-variance Σ given as:

$$\Sigma_{jj'} = \frac{1}{M} \sum_i \left(w_{ij} - \bar{w}_j\right) \left(w_{ij'} - \bar{w}_{j'}\right). \tag{5}$$

The corresponding spectrum is generated from these weights as $\left(\sum_j \tilde{w}_{ij}\hat{\mathbf{e}}_j^{PCA} + \bar{I}_s(\lambda)\right)$. This spectrum can sometimes have values that are less than zero. In such cases, the weights are discarded and a new draw obtained, until the condition $\left(\sum_j \tilde{w}_{ij}\hat{\mathbf{e}}_j^{PCA} + \bar{I}_s(\lambda)\right) > 0$, is satisfied for all λ. The random illuminant spectrum is

$$\tilde{I}_i(\lambda) = \left(\sum_j \tilde{w}_{ij}\hat{\mathbf{e}}_j^{PCA} + \bar{I}_s(\lambda)\right). \tag{6}$$

2 Surface Reflectance Spectra

Let us denote the i'th sample in the reflectance spectrum dataset as $R_i(\lambda)$, where $\{i \in [1, M]\}$ and M is the total number of spectra in the dataset. We first calculated the mean reflectance spectrum, $\bar{R}(\lambda)$, by taking the sample mean over all spectra in the dataset

$$\bar{R}(\lambda) = \frac{1}{M}\sum_{i=1}^{M} R_i(\lambda). \tag{7}$$

The dataset is then mean centered by subtracting the mean spectrum, $R_i^{\mathrm{MC}}(\lambda) = R_i(\lambda) - \bar{R}(\lambda)$ and decomposed using PCA as:

$$R_i^{\mathrm{MC}}(\lambda) = \sum_j w_{ij}\ \hat{\mathbf{e}}_j^{\mathrm{PCA}}, \tag{8}$$

where $\hat{\mathbf{e}}_j^{\mathrm{PCA}}$ are PCA basis vectors obtained using SVD applied to $R_i^{\mathrm{MC}}(\lambda)$ and w_{ij} are the weights obtained by projecting each of $R_i^{\mathrm{MC}}(\lambda)$ onto the basis vectors $\hat{\mathbf{e}}_j^{\mathrm{PCA}}$ $\left(w_{ij} = R_i^{\mathrm{MC}}(\lambda)\cdot\hat{\mathbf{e}}_j^{\mathrm{PCA}}\right)$. We used the basis vectors corresponding to the largest six SVD eigenvalues, which account for more than 90% of the variance in the combined Munsell and Vrhel datasets.

To get a random reflectance spectrum, we generate samples of weights $(\tilde{w}_{ij})$ drawn from the multivariate Gaussian distribution with mean $\bar{w}_j = \frac{1}{M}\sum_i w_{ij}$, and co-variance $\Sigma_{jj'} = \frac{1}{M}\sum_i (w_{ij} - \bar{w}_j)(w_{ij'} - \bar{w}_{j'})$. If the randomly sampled weights satisfy the condition $\left(0 < \sum_j \tilde{w}_{ij}\hat{\mathbf{e}}_j^{\mathrm{PCA}} + \bar{R}(\lambda) < 1\right)$ at every λ, the randomly generated reflectance spectrum is given as:

$$\tilde{R}_i(\lambda) = \sum_j \tilde{w}_{ij}\hat{\mathbf{e}}_j^{\mathrm{PCA}} + \bar{R}(\lambda). \tag{9}$$

Otherwise the draw is discarded and a new set of weights is drawn.

For generating the target object reflectance at a particular LRV (Y_{T}), the values in a generated spectrum were scaled by

$$\frac{Y_{\mathrm{T}}}{\int d\lambda \tilde{R}(\lambda) D_{65}(\lambda)\bar{y}(\lambda)} \tag{10}$$

with $\bar{y}(\lambda)$ being the CIE photopic luminosity (or luminous efficiency) function, which describes the average spectral sensitivity of human visual perception of luminance. The target reflectance is then given by:

$$\tilde{R}_i^{\mathrm{T}}(\lambda) = \tilde{R}_i(\lambda)\cdot\left(\frac{Y_{\mathrm{T}}}{\int d\lambda \tilde{R}(\lambda) D_{65}(\lambda)\bar{y}(\lambda)}\right). \tag{11}$$