Definition of "albedo" from Wikipedia: "Albedo (/lbido/) (Latin: albedo, meaning "whiteness") is the measure of the diffuse reflection of solar radiation out of the total solar radiation received by an astronomical body (e.g. a planet like Earth). It is dimensionless and measured on a scale from 0 (corresponding to a black body that absorbs all incident radiation) to 1 (corresponding to a body that reflects all incident radiation)." Not quite the same as LRV. Note that Attwell and Badderly (2007) use albedo to refer to ratio of measured luminance from a surface and luminance from a referene card, but across varying illumination. That's not quite LRV either.

\*\*\*\*\*\*

Reference to previous work might be extended, especially concerning research in perception. In fact, although in my understanding, the presented work is about lightness constancy, there is no definition of lightness and it is not clear what are the factors involved in lightness constancy. For a definition of lightness and brightness I recommend referring to "Lightness Perception and Lightness Illusions"- Adelson, 2000. For the factors contributing to lightness constancy I suggest "Seeing black and white" - Gilchrist, 2006.

We modified the end of the first paragraph to add a definition of lightness constancy and to include some key references:

"The ability of a visual system to compute a representation of object color that is stable against variation in object-extrinsic factors is called color constancy. A well-studied special case of color constancy is when the stimuli are restricted to be achromatic. This special case is called lightness constancy (Gilchrist, 2006). Although human lightness and color constancy are not perfect, they are often very good (Foster, 2011; Brainard & Radonjic, 2014; Adelson, 2000; Kingdom, 2011)."

Adelson, E.H. 2000 Lightness perception and lightness illusions. In *The New Cognitive Neurosciences, 2nd ed.* (ed. M. Gazzaniga), pp. 339-351. Cambridge, MA, MIT Press.

Gilchrist, A.L. 2006 *Seeing Black and White*. Oxford, Oxford University Press.

Kingdom, F.A.A. 2011 Lightness, brightness and transparency: A quarter century of new ideas, captivating demonstrations and unrelenting controversy. *Vision Research* **51**, 652-673.

Also, fix citations to Radonjic -> Radonjić

Also, there is a certain body of work on which scenes aspects potential cues for lightness (e.g "Cues to an Equivalent Lighting Model" Boyaci, Doerschner & Maloney, 2006; "Illumination estimation in three-dimensional scenes with and without specular cues" - Snyder, Doerschner & Maloney). Specular reflections are one of those cues. However, there are human and simulation studies reporting that specular highlight are discounted in lightness judgments and that specular reflections potentially impair lightness discrimination (e.g. "Lightness constancy in the presence of specular highlights" - Todd, Normal & Mingolla, 2004; "Lightness perception for matte and glossy complex shapes", Toscani, Valsecchi & Gegenfurtner, 2017; "The effect of gloss on perceived lightness" - Beck, 1964 ).

We agree that providing a bit more in the way of pointers into the relevant literature will be helpful, although reviewing this literature is beyond the scope of the current paper. We have now edited the following paragraph in the discussion, and added citations along the lines suggested above. Changes to this paragraph also address the three points made by the reviewer that follow this one.

"In the work presented here, we studied computational luminance constancy in virtual scenes with naturalistic spectral variation in light sources and in surface reflectance functions, with only matte surfaces in the scenes. It is natural to by studying with spectral variation, because this variation is at the heart of what makes luminance constancy a rich computational problem. In natural scenes, however, there are other sources of variation that add additional richness. These include variation in non-spectral properties of objects and lighting in the scene, including object texture, material, and shape as well as lighting geometry. The methods we developed here may be generalized to study the effects of variation in these factors. That is, one could incorporate these other sources of variation into the generation of the scenes and again learn estimators from the corresponding labeled images. A challenge for this approach will be to thoughtfully control the increase in problem complexity, both to keep compute time feasible and to ensure that it is possible to extract meaningful insight from the results. Extending the work to include variation of material may provide insights not only about luminance constancy but also for computations that relate to material perception (see Fleming, 2017); extending the work to include variation in object shape and lighting geometry may clarify the role of object boundaries versus object interiors for providing information that supports perception of object color and lightness (see Land & McCann, 1971; Rudd, 2016). We also note that there is a literature on how increasing stimulus complexity along the various lines listed above affects human color and lightness perception (e.g. Beck, 1964; Yang & Maloney, 2001; Yang & Shevell, 2002; Todd et al., 2004; Snyder et al., 2005; Boyaci et al., 2006; Xiao & Brainard, 2008; Kingdom, 2011; Xiao et al., 2012; Anderson, 2015; Toscani et al., 2017), as well as the computational problem of color and lightness constancy (e.g. Lee, 1986; D'Zmura & Lennie, 1986; Funt & Drew, 1988; Tominaga & Wandell, 1989; Barron & Malik, 2012; Barron, 2015; Finlayson, 2018)."

References added to paragraph above. Asterisk below indicates already in our bibliography.

Fleming, R.W. 2017 Material perception. *Annual Review of Vision Science* **3**, 365-388.

Beck, J. 1964 The effect of surface gloss on perceived lightness. *American Journal of Psychology* **77**, 54-63. – Highlights affect perceived lightness

Todd, J.T., Norman, J.F. & Mingolla, E. 2004 Lightness constancy in the presence of specular highlights. *Psychological Science* **15**, 33-39. – Specular highlights discounted in lightness perception.

Boyaci, H., Doerschner, K. & Maloney, L.T. 2006 Cues to an equivalent lighting model. *J Vis* **6**, 106-118. – Review of that lab's work. Not quite what we want here.

Snyder, J.L., Doerschner, K. & Maloney, L.T. 2005 Illumination estimation in three-dimensional scenes with and without specular cues. *J Vis* **5**, 863-877. – Specularities improve constancy.

Toscani, M., Valsecchi, M. & Gegenfurtner, K.R. 2017 Lightness perception for matte and glossy complex shapes. *Vision Res* **131**, 82-95. – Highlights affect perceived lightness of glossy surfaces.

Yang, J.N. & Maloney, L.T. 2001 Illuminant cues in surface color perception: tests of three candidate cues. *Vision Research* **41**, 2581-2600. – Hightlights can improve constancy.

Yang, J.N. & Shevell, S.K. 2002 Stereo disparity improves color constancy. *Vision Research* **42**, 1979-1989. – Constancy improves with stereo when there are highlights.

Lee, H.C. 1986 Method for computing the scene-illuminant chromaticity from specular highlights. *Journal of The Optical Society of America A* **3**, 1694-1699. – How specularities could improve color constancy.

Xiao, B. & Brainard, D.H. 2008 Surface gloss and color perception of 3D objects. *Visual Neuroscience* **25**, 371-385. – Object color appearance is somewhat stabilized against desaturation from specular highlights

Xiao, B., Hurst, B., MacIntyre, L. & Brainard, D.H. 2012 The color constancy of three-dimensional objects. *Journal of Vision* **12**, 1-15. – No improvement of constancy with addition of highlights.

D'Zmura, M. & Lennie, P. 1986 Mechanisms of color constancy. *Journal of the Optical Society of America A* **3**, 1662-1672. – How specularities could improve color constancy.

Tominaga, S. & Wandell, B.A. 1989 The standard surface reflectance model and illuminant estimation. *Journal of the Optical Society of America A* **6**, 576-584. – How specularities could improve color constancy.

\*Barron, J. T. (2015). Convolutional color constancy. Proceedings of the IEEE International Conference on Computer Vision, 379-387.

\*Barron, J. T., & Malik, J. (2012). Color constancy, intrinsic images, and shape estimation. Proceedings of the European Conference on Computer Vision (ECCV) , 57-70.

\*Funt, B. V., & Drew, M. S. (1988). Color constancy computation in near-mondrian scenes using a finite dimensional linear model. Computer Society Conference on Computer Vision and Pattern Recognition, 1988 , 544-549.

Finlayson, G. D. (2018). Colour and illumination in computer vision. *Interface Focus*, *8*(4), 20180008.

Anderson, B.L. 2015 The perceptual representation of transparency, lightness, and gloss. In *Handbook of Perceptual Organization* (ed. J. Wagemans). Oxford, Oxford University Press. – Review.

Kingdom, F.A.A. 2011 Lightness, brightness and transparency: A quarter century of new ideas, captivating demonstrations and unrelenting controversy. *Vision Research* **51**, 652-673.

Land, E.H. & McCann, J.J. 1971 Lightness and retinex theory. *Journal of the Optical Society of America* **61**, 1-11.

Rudd, M.E. 2016 Retinex-like computations in human lightness perception and their possible realization in visual cortex. In *Electronic Imaging 2016* (pp. 1-8. San Francisco, CA, Society for Imaging Science and Technology.

I think that the approach presented in the manuscript might help investigating the role of specular highlights for an ideal observers. In fact, with a fixed geometry of the scene and the illumination (as it was in the reported simulations) the distribution of the weights in the receptive fields is informative about the role of the elements in the scene. Given the interest that specular reflection received by color and lightness constancy investigations, I would add this in the "Future Directions" section.

Good point. We have adopted this suggestion in the revised paragraph above.

Also, I suggest stating that the rendered scenes were matte in the "Images of Virtual vs. Real Scenes" section, as a limitation of the simulation given that specular reflections might interact with lightness constancy, as discussed in the literature.

This restriction is now noted explicitly as described above, albeit in the Future Directions rather than Virtual vs. Real Scenes section.

Classical ("Lightness and retinex theory", Land & McCann, 1971) but also recent theories of lightness constancy ("A cortical edge-integration model of object-based lightness computation that explains effects of spatial context and individual differences" - Rudd, 2014) propose that visual system spatially integrates the luminance steps corresponding to reflectance edges (as given object boundaries). By looking at the shape of the receptive field in condition 3, it seems that rather large positive weights are flanked by negative weights corresponding to borders between the objects in the background, suggesting edge related computations. I suppose one of the potentiality of the approach is to reveal such local computations, thus if the authors find my speculation sensible, I would add it in the discussion, showing how the proposed approach has the power to reveal lightness constancy computations as proposed in the literature.

This is an interesting connection, which we now make, again in the same revised future directions paragraph quoted above. (We cited a 2016 paper by Rudd, though, rather than the 2014 paper suggested by the reviewer.)

The idea of generating large datasets of rendered surfaces in order to investigate classification of an ideal observers (ROC and linear classification) on their material properties is not new ("Optimal sampling of visual information for lightness judgments" - Toscani, Valsecchi & Gegenfurtner 2013; "Lightness perception for matte and glossy complex shapes" - Toscani, Valsecchi & Gegenfurtner 2017; "Statistical correlates of perceived gloss in natural images" -Wiebel, Toscani & Gegenfurtner, 2015).

However, to my knowledge this is the first time that reflectance spectra are taken into account, as opposed to grayscale images, as the toolbox presented in the paper allows. I would stress the novelty respect to previous work.

Thanks for pointing us at the work above, which indeed we should have cited. We have now remedied this and a few other related omissions through modifications to the following paragraphs in the discussion.

DB to work on these paragraphs still.

Unfortunately, such datasets are not readily available for studying correlates of object surface reflectance. To obtain ground truth information about the reflectance of objects in a natural image, it is necessary to make an independent measurement of the of the reflectance function of each object (or of the illumination impinging on each object). Although this can be done at a small number of image locations by inserting discretely spaced reflectance standards into the scene, interpolating to locations between such standards requires strong assumptions that are often unjustified in real scenes.

Here we used labeled images rendered from virtual scene descriptions. There are many advantages to using rendered images (Butler, Wulff, Stanley, & Black , 2012 ). One advantage is that they allow us to work with large number of labeled images where object reflectance is precisely known at each pixel. A second advantage is that we can control the variation in distinct scene factors that might affect the difficulty of the estimation problem. This flexibility allows the study of individual scene factors as well as their combined effects. Here, we exploited this flexibility to quantify how variation in the relative reflectance spectrum of the target object, the spectrum of the illumination, and the reflectance spectrum of the background objects limit LRV estimation. We also exploited our use of rendered images to explore how the presence or absence of secondary reflections from background objects affected estimation of target object LRV. This type of question cannot be addressed using real images. The basic approach we use here can be extended to include parametric control over the amount of variation of different factors. For example, we could systematically vary the variances of the distribution over the weights that control the relative spectrum of the illumination.

Toscani, M., Valsecchi, M. & Gegenfurtner, K.R. 2013 Optimal sampling of visual information for lightness judgments. *Proceedings of the National Academy of Sciences* **110**, 11163-11168. – "Simulations with rendered physical lighting showthat higher values in an object’s luminance distribution are particularly informative about reflectance. This sampling strategy is an efficient and simple heuristic for the visual system to achieve accurate and invariant judgments of lightness."

Toscani, M., Valsecchi, M. & Gegenfurtner, K.R. 2017 Lightness perception for matte and glossy complex shapes. *Vision Res* **131**, 82-95. – Uses graphics to generate a bunch of images and uses psychophysics to "study how different statistics (i.e. percentiles) based on the luminance distributions of matte and glossy objects predict the overall surface albedo. We found that the brightest parts of matte surfaces are good predictors of the surface albedo." Also has some psychophysics.

Wiebel, C.B., Toscani, M. & Gegenfurtner, K.R. 2015 Statistical correlates of perceived gloss in natural images. *Vision Res* **115**, 175-187. Uses rendered images and natural images to study how simple characterizations of the distribution of luminances reflected from an object predict material (glossiness).

Prokott, K.E. & Fleming, R.W. 2017 Applying machine learning to gloss perception. European Conference on Visual Perception (Berlin). – CNN learning gloss from computer generated dataset.

Flachot, A. & Gegenfurtner, K.R. 2018 Processing of chromatic information in a deep convolutional neural network. *J Opt Soc Am A Opt Image Sci Vis* **35**, B334-B346. – This doesn't use graphics, but does analyze representation of color within an extant CNN, and might be cited by use somewhere.

The distribution of surface albedos in natural environments is approximated by a specific beta distribution ("The distribution of reflectances within the visual environment", Attewell & Baddeley, 2006) and the discernible colors present in nature only cover a specific portion of theoretical solid of visible colors ("The number of discernible colors in natural scenes" Linhares, Pinto & Nascimento, 2008). For the simulation presented in the manuscript, reflectance spectra are sampled from a statistical model approximating a largely variable set of colors, as the Munsell chips is supposed to represent the space of visible colors rather than resembling the occurrence of colors in the word. I suppose this gives an upper limit to the limitation in performance due to the increasing complexity with conditions, and results might change considering the natural distribution of reflectance spectra.

There are databases providing a large collection of reflectance spectra or reflected spectra from isolated surfaces under a known illuminant, although they do not span color spaces as well as the munsell system. In fact, they focus on leaves fruits and vegetables ("Fruits, foliage and the evolution of primate colour vision" - Regan, Julliot, Simmen, Vienot, Charles-Dominique & Mollon, 2011; "Hyperspectral database of fruits and vegetables" - Ennis, Schiller, Toscani & Gegenfurtner, 2018).

We have added discussion of these and related papers into the section of the paper that relates to our approximation to naturally occurring surface reflectances.

DB to work on this paragraph.

To increase the representativeness of our rendered images, we used datasets of natural surface reflectance spectra and natural daylight illumination spectra. Although we believe these datasets provide reasonable approximations of the statistical variation in reflectance and illumination spectra, they can be extended and improved. For example, the surface reflectance datasets capture regularities in naturally occurring reflectance functions, but the surfaces in the datasets were not sampled to reflect the relative frequency of different reflectance spectra in natural viewing.

Attewell, D. & Baddeley, R.J. 2007 The distribution of reflectances within the visual environment. *Vision Res* **47**, 548-554. – Only measurements of albedo were made here, although they were done in a manner that makes them more or less consistent with our measure of LRV. (Luminance reflected from objects divided by luminance from a white card, but illumination not controlled.) Argues that beta better than Gaussian. Not clear how to generalize this to full spectra, but worth citing.

Ennis, R., Schiller, F., Toscani, M. & Gegenfurtner, K.R. 2018 Hyperspectral database of fruits and vegetables. *Journal of the Optical Society of America A* **35**, B256-B266. – These data are available and could be used in future work. Hyperspectral mages of 57 fruits and vegetable under a calibrated illuminant.

Regan, B.C., Julliot, C., Simmen, B., Vienot, F., Charles-Dominique, P. & Mollon, J.D. 2001 Fruits, foliage and the evolution of primate color vision. *Philosophical Transactions: Biological Sciences* **356**, 229-283. – Measurements of fruit and foliage spectra relevant to new world monkeys.

Sumner, P. & Mollon, J.D. 2000 Catarrhine photopigments are optimized for detecting targets against a foliage background. *Journal of Experimental Biology* **203**, 1963-1986. – Similarly, measurements of fruit and foliage.

Linhares, J.M.M., Pinto, P.D. & Nascimento, S.C.M. 2008 The number of discernible colors in natural scenes. *Journal of the Optical Society of America A* **25**, 2918-2924. – This analysis is in terms of light reflected to the eye, not surface reflectance.

[1] Cheng, D., Prasad, D.K. & Brown, M.S. 2014 Illuminant estimation for color constancy: why spatial-domain methods work and the role of the color distribution. *Journal of the Optical Society of America A* **31**, 1049-1058. NUS database of camera images – Says there is a database but I get a permissions error at the published URL (http://www.comp.nus.edu.sg/~whitebal/illuminant/illuminant.html)

<http://colorconstancy.com>. Lists various databases.

* [Nascimento, Ferreira, and Foster, 2002](https://personalpages.manchester.ac.uk/staff/david.foster/Research/My_PDFs/Nascimento_etal_JOSAA_02.pdf) – hyperspectral images with calibration in some places.
* Barnard et al. – Illuminant and reflectance data for synthesized images. There are also calibrated images. We should cite this one. Kobus Barnard, Lindsay Martin, Brian Funt, and Adam Coath, [A Data Set for Colour Research](http://kobus.ca/research/publications/data_for_colour_research/index.html), Color Research and Application, Volume 27, Number 3, pp. 147-151, 2002. Contains a reflectance database: "The surface reflectance data ([reflect\_db.reflect](http://www.cs.sfu.ca/~colour/data/colour_constancy_synthetic_test_data/reflect_db.reflect.gz)) is a set of 1995 spectra compiled from several sources. These surfaces include the 24 Macbeth color checker patches, 1269 Munsell chips, 120 Dupont paint chips [1], 170 natural objects [1], the 350 surfaces in Krinov data set [2], and 57 additional surfaces measured by ourselves."
* Ciurea, F. and Funt, B. "A Large Image Database for Color Constancy Research," Proceedings of the Imaging Science and Technology Eleventh Color Imaging Conference, pp. 160-164, Scottsdale, Nov. 2003.
* Data used in this paper is available here. [Bayesian Color Constancy Revisited](http://www.kyb.mpg.de/publications/attachments/CVPR2008-Gehler_%5B0%5D.pdf) - [Peter V. Gehler](http://www.kyb.mpg.de/~pgehler), [Carsten Rother](http://research.microsoft.com/~carrot/), [Andrew Blake](http://research.microsoft.com/~ablake), [Toby Sharp](http://research.microsoft.com/~tsharp/) and [Tom Minka](http://research.microsoft.com/~minka), [CVPR 2008](http://vision.eecs.ucf.edu/), [Bibtex Entry](http://files.is.tue.mpg.de/pgehler/projects/color/bibtex.bib). The dataset these guys used was reprocessed by Funt's group; Lilong Shi and Brian Funt, "Re-processed Version of the Gehler Color Constancy Dataset of 568 Images," accessed from <http://www.cs.sfu.ca/~colour/data/>
* The Parraga group's papers, with an image dataset online 1) [Vazquez-Corral, J., Párraga, C. A., Vanrell, M., & Baldrich, R. (2009). Color Constancy Algorithms: Psychophysical Evaluation on a New Dataset. Journal of Imaging Science and Technology, 53(3), 0311051-0311059.](http://www.imaging.org/store/epub.cfm?abstrid=42927) 2) [Parraga, C. A., Baldrich, R. & Vanrell, M. (2010).  Accurate Mapping of Natural Scenes Radiance to Cone Activation Space: A New Image Dataset. CGIV 2010/MCS'10 - 5th European Conference on Colour in Graphics, Imaging, and Vision - 12th International Symposium on Multispectral Colour Science, Society for Imaging Science and Technology.](http://www.cvc.uab.es/color_calibration/to_share/LMScharacterization_CGIV2010.pdf)3) [Párraga, C. A., Vazquez-Corral, J., & Vanrell, M. (2009). A new cone activation-based natural images dataset. Perception, 36(Suppl), 180.](http://www.cvc.uab.es/color_calibration/to_share/ECVP09_Parraga_Vazquez_Vanrell.pdf)
* Dataset of HDR images with MCC. http://www.cs.sfu.ca/~colour/data/funt\_hdr/
* This has some data links with it: @InProceedings{vdm\_iccv2013, author = {V. Prinet and D. Lischinski and M. Werman}, title = {Illuminant Chromaticity from Image Sequences}, journal = {International Conference on Computer Vision (ICCV)}, year = {2013}, month = {December}}
* An image database for color constancy. <http://eidomatica.di.unimi.it/index.php/research/idb/yaccd2>
* There are data that go with this. {S. Beigpour, C. Riess}, J. van de Weijer, E. Angelopoulou: "Multi-Illuminant Estimation with Conditional Random Fields", *submitted*.
* Data and links to papers here. https://ipg.fer.hr/ipg/resources/color\_constancy

I found only one typo at the end of page 2: "(?, ?; Brainard and Freeman, 1997)", probably due to the reference manager.

Fixed. [Carefully check submission for any such typos, search on "?" in PDF, etc.]

I would find interesting to have the shape of the receptive fields reported also for the analysis about the scenes in condition 2.

We have added these to the appendix.