3) (5 pts) ANL (Recurrence Relations)

Using the iteration technique, just solve for the **next two** iterations of the following recurrence relation:

$$T(n) = 3T(n-1) + n^2, for\ integers\ n > 0$$
$$T(0) = 1$$

Your answers should be of the form

$T(n) = aT(n-2) + bn^2 - cn + d$ and
$T(n) = eT(n-3) + fn^2 - gn + h$, where $a, b, c, d, e, f, g,$ and h are positive integers.

$$T(n) = 3(3T(n-2) + (n-1)^2) + n^2$$

$$T(n) = 9T(n-2) + 3n^2 - 6n + 3 + n^2$$

$$T(n) = 9T(n-2) + 4n^2 - 6n + 3$$

It follows that a = 9, b = 4, c = -6, and d = 3

Now, plug in one more iteration

$$T(n) = 9(3T(n-3) + (n-2)^2) + 4n^2 - 6n + 3$$

$$T(n) = 27T(n-3) + 9n^2 - 36n + 36 + 4n^2 - 6n + 3$$

$$T(n) = 27T(n-3) + 13n^2 - 42n + 39$$

It follows that e = 27, f = 13, g = -42 and d = 39

Grading: 2 pts first iteration, (give 1 pt if some terms correct)
3 pts second iteration (give 1 pt if 1 or 2 terms correct,
2 pts if 3 terms correct)

Take off 1 pt if they leave 1ˢᵗ unsimplified: $9T(n-2) + 3(n-1)^2 + n^2$
Take off 1 pt if they leave 2ⁿᵈ unsimplified: $27T(n-3) + 9(n-2)^2 + 3(n-1)^2 + n^2$