

## 3) (10 pts) ANL (Summations)

Recall that  $\sum_{i=0}^{n-1} 2^i = 2^n - 1$ .

(a) (8 pts) Using this result, determine a closed-form solution in terms of  $n$ , for the summation below.

(b) (2 pts) Determine the numeric value of the summation for  $n = 9$ .

$$\sum_{i=0}^n \left( \sum_{j=0}^{i-1} 2^j \right)$$

(a)

$$\begin{aligned} \sum_{i=0}^n \left( \sum_{j=0}^{i-1} 2^j \right) &= \sum_{i=0}^n (2^i - 1) \\ &= \sum_{i=0}^n 2^i - \sum_{i=0}^n 1 \\ &= 2^{n+1} - 1 - (n + 1) \\ &= 2^{n+1} - n - 2 \end{aligned}$$

(b) Plugging in  $n = 9$  into the closed-form solution obtained in part (a), we get:

$$\underline{2^{9+1} - 9 - 2 = 1024 - 11 = \mathbf{1013}}$$

**Grading: Part A -2 pts for inner sum, 2 pts split sum, 1 pt left sum, 2 pts right sum, 1 pt simplifying difference, Part B - 2 pts correct answer, 1 pt plug in correct but made an arithmetic error, 0 otherwise**