2) (10 pts) ANL (Algorithm Analysis)

(a) (5 pts) An algorithm for searching for a housing contract in a database of n records takes $O(lg\ n)$ time. When $n = 2^{20}$, one million searches can be performed in one fifth of a second. If we increase the database to size $n = 2^{25}$, how long will 500,000 searches take?

Let T(n) represent the time one search takes. Thus, $T(n) = clgn$, for some constant c. Using the given information, we have:

$$10^6 T(n) = .2sec = 10^6 clg(2^{20})$$
$$.2sec = 10^6(20)c$$
$$c = 10^{-8}sec$$

We are being asked to find $500000T(2^{25})$:

$$500000T(2^{25}) = 5(10^5)(10^{-8}sec)\lg(2^{25})$$
$$= 5(10^5)(10^{-8}sec)25$$
$$= 125(10^{-3}sec)$$
$$= 125ms, or\ .125\ sec$$

Grading: 1 pt setting up valid equation, 2 pts solving for constant, 2 pts for plugging into second part and getting the answer - can be represented in any unit of time, though sec and ms are probably going to be the most common.

(b) (5 pts) A shortest distance algorithm on an $n \times m$ street grid runs in $O(nm)$ time. If the algorithm takes 2 seconds to run on a 4000 × 3000 sized grid, how long will it take on a grid of size 2000 × 18000 sized grid?

Let T(n, m) represent the time algorithm takes . Thus, $T(n, m) = cnm$, for some constant c. Using the given information, we have:

$$T(4000{,}3000) = 2sec = c(4000)(3000)$$
$$c = \frac{1}{6}10^{-6}sec$$

Now we solve for T(2000, 18000):

$$T(2000{,}18000) = \left(\frac{1}{6}10^{-6}\text{sec}\right)(2000)(18000) = \frac{36}{6}sec = 6\ sec$$

Grading: 1 pt setting up valid equation, 2 pts solving for constant, 2 pts for plugging into second part and getting the answer - can be represented in any unit of time, though sec is probably going to be the most common.