3) (10 pts) ANL (Summations)

Recall that $\sum_{i=0}^{n-1} 2^i = 2^n - 1$.

(a) (8 pts) Using this result, determine a closed-form solution in terms of *n*, for the summation below.

(b) (2 pts) Determine the numeric value of the summation for n = 9.

$$\sum_{i=0}^{n}\left(\sum_{j=0}^{i-1} 2^j\right)$$

(a)

$$\sum_{i=0}^{n}\left(\sum_{j=0}^{i-1} 2^j\right) = \sum_{i=0}^{n}(2^i - 1)$$

$$= \sum_{i=0}^{n} 2^i - \sum_{i=0}^{n} 1$$

$$= 2^{n+1} - 1 - (n+1)$$

$$= 2^{n+1} - n - 2$$

(b) Plugging in n = 9 into the closed-form solution obtained in part (a), we get:

$$\underline{2^{9+1} - 9 - 2 = 1024 - 11 = \mathbf{1013}}$$

Grading: Part A -2 pts for inner sum, 2 pts split sum, 1 pt left sum, 2 pts right sum, 1 pt simplifying difference, Part B - 2 pts correct answer, 1 pt plug in correct but made an arithmetic error, 0 otherwise