Summer 2021 Algorithms and Analysis Tools Exam, Part A

3) (10 pts) ANL (Summations)

What is the closed form of the following summation? Your solution should be a function in terms of n. For full credit work must be shown.

$$\sum_{i=0}^{n} \sum_{j=0}^{i} 2^j$$

$$\sum_{i=0}^{n} \sum_{j=0}^{i} 2^{j} = \sum_{i=0}^{n} (2^{i+1} - 1)$$

$$= \sum_{i=0}^{n} 2^{i+1} - \sum_{i=0}^{n} 1$$
$$= \sum_{i=0}^{n} (2)2^{i} - (n+1)$$

$$=2\sum_{i=0}^{n}2^{i}-(n+1)$$

$$= 2(2^{n+1} - 1) - (n+1)$$

$$= 2^{n+2} - 2 - n - 1$$

$$=2^{n+2}-n-3$$

Grading: 3 pts inner sum

1 pt split

1 pt second sum

4 pts first sum (lots of ways to break this down)

1 pt simplification at the end