3) (10 pts) ANL (Summations)

With proof, find the ordered pair of values (a, b) which satisfy the equation below?

$$\sum_{k=1}^{2n}(ak+b) = 7n^2 + 3n$$

Simplify the left hand side in terms of a and b to get to this point:

$$\sum_{k=1}^{2n}(ak+b) = 7n^2 + 3n$$

$$\frac{a(2n)(2n+1)}{2} + b(2n) = 7n^2 + 3n$$

$$an(2n+1) + 2bn = 7n^2 + 3n$$

$$2an^2 + (2b+a)n = 7n^2 + 3n$$

In order for this equation to always be true, we have to equate coefficients, giving us the two following simultaneous equations:

$$2a = 7 \qquad\qquad 2b + a = 3$$

Solving the first equation, we find that $a = \frac{7}{2}$. Plugging this into the second equation, we have

$$2b + \frac{7}{2} = 3$$

$$2b = -\frac{1}{2}$$

$$b = -\frac{1}{4}$$

Thus, the desired ordered pair (a, b) is $(\frac{7}{2}, -\frac{1}{4})$.

Grading: 2 pts sum of ak, 1 pt sum of b, 2 pts simplifying expression, 2 pts equating coefficients, 1 pt solving for a, 2 pts solving for b.