3) (5 pts) ALG (AVL Trees)

Suppose we randomly shuffle the six words in the list below and insert them into an AVL tree. (In other words, we insert them in random order – not necessarily the order given – with each of those words ending up in the AVL tree exactly once.)

Fill in the blank next to each word to indicate whether it could **ever** end up at the root of the resulting AVL tree ("yes") or not ("no"). (If you answer "no" for a given word, you are saying it could **never** end up at the root of the resulting AVL tree.)

You may assume the AVL tree is ordered alphabetically. So, all the words in the left subtree of "apple" would have to come before "apple" in alphabetical order, and all the words in its right subtree would have to come after "apple" in alphabetical order.

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__no___ apple

__yes__ mango

__no___ papaya

__no___ banana

__no__ mulberry

__yes_ blueberry


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Fill in each blank with "yes" or "no" to indicate whether the word could
serve as the root of an AVL tree that results from shuffling these six words
and inserting them into an AVL tree in random order.
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See notes on following page for guidance on how to solve this problem.

Grading for Question #3:

Subtract 1 point for each incorrect answer (including any spaces that were left blank), for a minimum score of 0 points.

(So, if they got 5 blanks right and 1 blank wrong, that's -1, for a total of 4/5 on this question. If they got 1 blank right and 5 blanks wrong, that's -5, for a final score of 0/5. If they got all 6 blanks wrong, that's also 0/5.)

Notes for Solving Question #3:

Probably the first thing to do is to list the words in alpha order:

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apple
banana
blueberry
mango
mulberry
papaya
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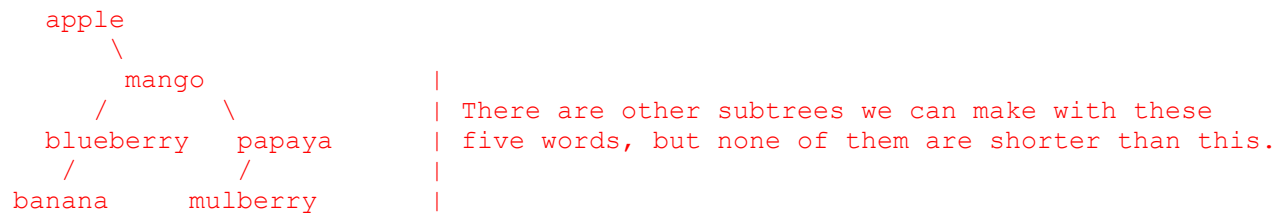
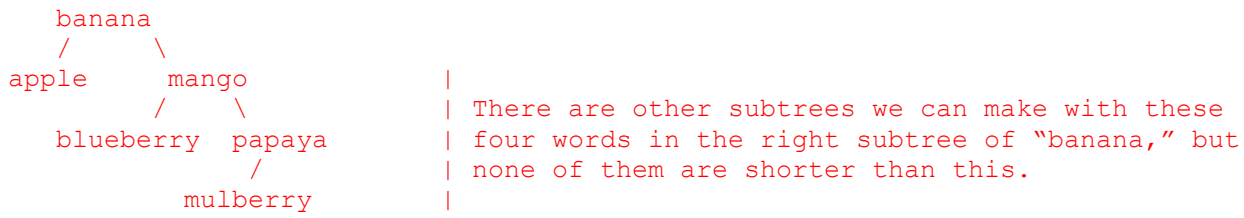
If we try to construct an AVL tree with "apple" at the root, all other words go into its right subtree. The smallest height that right subtree can have is 2, meaning we have an unbalanced root. For example:



Therefore, "apple" cannot be at the root of the AVL tree no matter what order we use to insert these words. Rotations would displace "apple" from the root. A symmetric argument can be used to show that "papaya" cannot ever be the root, since every other word would have to go into the left subtree of "papaya."

Similarly, if we try to place "banana" at the root, its left subtree must contain "apple," and its right subtree must contain all other words. There would be 4 words in its right subtree, with a minimum height of 2. For example:



That means we necessarily have a bad balance factor at "banana" if it's the root, and so it cannot serve as the root of an AVL tree with these words. A symmetric argument can be used to demonstrate that "mulberry" cannot be at the root, since "papaya" would go to the right of "mulberry" and all remaining words would go in its left subtree.

That leaves just "mango" and "blueberry" for us to consider. Either of them can serve as the root. E.g.: