3) (10 pts) ANL (Summations and Recurrence Relations)

Find the Big-Oh solution to the following recurrence relation using the iteration technique. Please show all of your work, including 3 iterations, followed by guessing the general form of an iteration and completing the solution. Full credit will only be given if all of the work is accurate (and not just for arriving at the correct answer.)

$$T(n) = 2T\left(\frac{n}{2}\right) + n^2, T(1) = 1$$

First, iterate three times:

$$\begin{aligned}
T(n) &= 2T\left(\frac{n}{2}\right) + n^2 \\
&= 2\left[2T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^2\right] + n^2 \\
&= 2\left[2T\left(\frac{n}{4}\right) + \frac{n^2}{4}\right] + n^2 \\
&= 4T\left(\frac{n}{4}\right) + \frac{n^2}{2} + n^2 \\
&= 4T\left(\frac{n}{4}\right) + \frac{3n^2}{2} \\
&= 4[2T\left(\frac{n}{8}\right) + (\frac{n}{4})^2] + \frac{3n^2}{2} \\
&= 4[2T\left(\frac{n}{8}\right) + \frac{n^2}{16}] + \frac{3n^2}{2} \\
&= 8T\left(\frac{n}{8}\right) + \frac{n^2}{4} + \frac{3n^2}{2} \\
&= 8T\left(\frac{n}{8}\right) + \frac{7n^2}{4}
\end{aligned}$$

In general, after k iterations we will have $T(n) = 2^k T\left(\frac{n}{2^k}\right) + \frac{(2^k-1)n^2}{2^{k-1}}$. We want to plug in the value of k for which $\frac{n}{2^k} = 1$,which is when $n = 2^k$. Note that for this value of k, $2^{k-1} = n/2$, since $2 \times 2^{k-1} = 2^k$:

$$T(n) = nT(1) + \frac{(n-1)n^2}{\frac{n}{2}} = n(1) + 2n(n-1) = 2n^2 - n = O(n^2)$$

Grading: 2 pts for second iteration ($4T(n/4)+3n^2/2$), 2 pts for third iteration ($8T(n/8)+7n^2/4$), 2 pts for general form, 2 pts for value to plug into general form, 2 pts for final solution.