3) (10 pts) ANL (Summations and Recurrence Relations)

(a) (5 pts) Determine the following sum in terms of n: $\sum_{i=1}^{2n-1}(3i-2)$.

$$\sum_{i=1}^{2n-1}(3i-2) = 3(\sum_{i}^{2n-1} i) - 2\sum_{i=1}^{2n-1} 1$$
$$= \frac{3(2n-1)(2n)}{2} - 2(2n-1)$$
$$= (2n-1)(3n-2)$$
$$= 6n^2 - 7n + 2$$

(**Grading:** 1 pt split, 2 pts formula for i, 1 pt const formula, 1 pt final answer - can leave in either factored for polynomial form.)

(b) (5 pts) Let $T(n) = 3T\left(\frac{n}{2}\right) + n^2$. In using the iteration technique (3 steps) to solve the recurrence, we arrive at an equation of the form: $T(n) = AT\left(\frac{n}{8}\right) + Bn^2$. Find A and B.

$$T(n) = 3T\left(\frac{n}{2}\right) + n^2$$
$$= 3(3T\left(\frac{n}{4}\right) + (\frac{n}{2})^2) + n^2$$
$$= 3(3T\left(\frac{n}{4}\right) + \frac{n^2}{4}) + n^2$$
$$= 9T\left(\frac{n}{4}\right) + \frac{3n^2}{4} + n^2$$
$$= 9(3T\left(\frac{n}{8}\right) + (\frac{n}{4})^2) + \frac{3n^2}{4} + n^2$$
$$= 9(3T\left(\frac{n}{8}\right) + \frac{n^2}{16}) + \frac{3n^2}{4} + n^2$$
$$= 27T\left(\frac{n}{8}\right) + \frac{9n^2}{16} + \frac{3n^2}{4} + n^2$$
$$= 27T\left(\frac{n}{8}\right) + \frac{37n^2}{16}$$

In follows that A = 27 and B = $\frac{37}{16}$.

(**Grading:** 2 pts to get to second iteration, 3 pts to get to third iteration)