2) (10 pts) ANL (Algorithm Analysis)

(a) (5 pts) A matrix factorization algorithm that is run on a input matrix of size *n* x *n*, runs in O(n^3) time. If the algorithm takes 54 seconds to run for an input of size 3000 x 3000, how long will it take to run on an input of size 1000 x 1000?

Let T(n) be the run-time of the algorithm on a matrix input of size n x n. We have:

$$T(3000) = c3000^3 = 54\ sec$$
$$c = \frac{54}{27 \times 10^9} sec = \frac{2}{10^9} sec$$

We desire to find T(1000):

$$T(1000) = c(1000^3) = \frac{2\ sec}{10^9} \times 10^9 = 2\ sec$$

Grading: 2 pts solving for c, 2 pts plugging in c, 1 pt for simplifying to 2 sec. Ratio method is valid as well, map points accordingly.

(b) (5 pts) A string algorithm with inputs of lengths *n* and *m* runs in O(n^2m) time. If the algorithm takes 2 seconds to run on an input with *n* = 1000 and *m* = 500, how long will the algorithm take to execute on an input with *n* = 250 and *m* = 1000?

Let T(n, m) be the run-time of the algorithm on strings inputs with lengths n and m. We have:

$$T(1000, 500) = c(1000^2)(500) = 2\ sec$$
$$c = \frac{2\ sec}{5 \times 10^8}$$

We desire to find T(250, 1000):

$$T(250, 1000) = c(250^2)(1000) = \frac{2\ sec}{5 \times 10^8} \times 625 \times 10^5 = \frac{250}{1000} sec = .25\ sec$$

Grading: 2 pts solving for c, 3 pts obtaining final answer. Ratio method is valid as well, map points accordingly.