

3) (10 pts) ANL (Recurrence Relations)

Use the iteration technique to solve the following recurrence relation in terms of n:

$$T(n) = 3T(n-1) + 1, \text{ for all integers } n > 1$$

$$T(1) = 1$$

Please give an exact closed-form answer in terms of n, instead of a Big-Oh answer.

(Note: A useful summation formula to solve this question is $\sum_{i=0}^n x^i = \frac{x^{n+1}-1}{x-1}$.)

$$\begin{aligned} T(n) &= 3T(n-1) + 1 \\ &= 3(3T(n-2) + 1) + 1 \\ &= 9T(n-2) + 3 + 1 \\ &= 9(3T(n-3) + 1) + 3 + 1 \\ &= 27T(n-3) + 9 + 3 + 1 \end{aligned}$$

After k steps, we have: $= 3^k T(n-k) + \sum_{i=0}^{k-1} 3^i$

Let k = n-1, then we have that $T(n) = 3^{n-1} T(n - (n-1)) + \sum_{i=0}^{n-2} 3^i$

$$\begin{aligned} &= 3^{n-1} T(1) + \sum_{i=0}^{n-2} 3^i \\ &= 3^{n-1} + \sum_{i=0}^{n-2} 3^i \\ &= \sum_{i=0}^{n-1} 3^i \\ &= \frac{3^n - 1}{3 - 1} = \frac{3^n - 1}{2} \end{aligned}$$

Grading: 2 pts for iteration with T(n-2), 2 pts for iteration with T(n-3), 2 pts for general guess after k steps. 1 pt for plugging in k = n-1, 3 pts for simplifying that to the final answer.