3) (10 pts) ANL (Recurrence Relations)

Using the iteration technique, determine a closed-form solution for the following recurrence relation in terms of n. Note: Your answer should be **EXACT** and not a Big-Oh bound.

$$T(0) = 1$$
$$T(n) = 4T(n-1) + 2^n, for\ integers\ n > 0$$

Using the iteration technique for three iterations, we get:

$T(n) = 4T(n-1) + 2^n$ **Grading: 1 pt**
$T(n) = 4(4T(n-2) + 2^{n-1}) + 2^n$
$T(n) = 16T(n-2) + (2^2)2^{n-1} + 2^n$
$T(n) = 16T(n-2) + [2^{n+1} + 2^n]$ **Grading: 2 pts**
$T(n) = 16(4T(n-3) + 2^{n-2}) + [2^{n+1} + 2^n]$
$T(n) = 64T(n-3) + (2^4)2^{n-2} + [2^{n+1} + 2^n]$
$T(n) = 64T(n-3) + [2^{n+2} + 2^{n+1} + 2^n]$ **Grading: 2 pts**

From here we see that after k iterations, the recurrence is:

$T(n) = 4^k T(n-k) + \sum_{i=0}^{k-1} 2^{n+i}$ **Grading: 2 pts**

Since we know T(0), let's plug in k = n into the formula above to yield:

$T(n) = 4^n T(n-n) + \sum_{i=0}^{n-1} 2^{n+i}$ **Grading: 1 pt**
$T(n) = 4^n T(0) + \sum_{i=0}^{n-1} 2^n 2^i$
$T(n) = 2^{2n} + 2^n \sum_{i=0}^{n-1} 2^i$
$T(n) = 2^n(2^n) + 2^n \sum_{i=0}^{n-1} 2^i$
$T(n) = 2^n\left[2^n + \sum_{i=0}^{n-1} 2^i\right]$
$T(n) = 2^n\left[\sum_{i=0}^{n} 2^i\right]$
$T(n) = 2^n(\frac{2^{n+1}-1}{2-1})$
$T(n) = 2^n(2^{n+1} - 1)$ **Grading: 2 pts**

Grading Note: Please accept any reasonable final form (there are quite a few that are reasonably simplified). For the last two points, award 1 pt if there is some algebra but not quite enough to get to a reasonable final form.