3) (10 pts) ANL (Recurrence Relations)

Use the iteration technique to solve the following recurrence relation in terms of n:

$$T(n) = 2T(n-1) + 2^n, for\ all\ integers\ n > 0$$
$$T(0) = 1$$

Please give an **exact closed-form answer in terms of n,** instead of a Big-Oh answer.

$$T(n) = 2T(n-1) + 2^n$$

$$= 2(2T(n-2) + 2^{n-1}) + 2^n$$

$$= 4T(n-2) + 2^n + 2^n$$

$$= 4T(n-2) + 2(2^n)$$

$$= 4(2T(n-3) + 2^{n-2}) + 2(2^n)$$

$$= 8T(n-3) + 2^n + 2(2^n)$$

$$= 8T(n-3) + 3(2^n)$$

After k steps, we have: $= 2^k T(n-k) + k(2^n)$

Let k = n, then we have that $T(n) = 2^n T(n-n) + n(2^n)$

$$= 2^n T(0) + n(2^n)$$

$$= 2^n + n(2^n)$$

$$= (n+1)(2^n)$$

Grading: 2 pts for iteration with T(n-2), 2 pts for iteration with T(n-3), 2 pts for general guess after k steps. 1 pt for plugging in k = n (or k = n-1), 3 pts for simplifying that to the final answer.