**3)** (10 pts) ANL (Summations)

Recall that $\sum_{i=0}^{n-1} 2^i = 2^n - 1$.

Use the iteration technique to find a Big-Oh bound for the recurrence relation below. Note: you may find the following mathematical results helpful: $2^{log_3 n} = n^{log_3 2}$, and $\sum_{i=0}^{\infty}(\frac{2}{3})^i = 3$. You may use these without proof in your work below.

$$T(n) = 2T\left(\frac{n}{3}\right) + O(n), for\ n > 1$$
$$T(1) = O(1)$$

$$T(n) = 2T\left(\frac{n}{3}\right) + cn$$
$$T(n) = 2(2T\left(\frac{n}{9}\right) + c\left(\frac{n}{3}\right)) + cn$$
$$T(n) = 4T\left(\frac{n}{9}\right) + c(\left(\frac{2n}{3}\right) + n)$$
$$T(n) = 4(2T\left(\frac{n}{27}\right) + c\left(\frac{n}{9}\right)) + c(\left(\frac{2n}{3}\right) + n)$$
$$T(n) = 8T\left(\frac{n}{27}\right) + c(\left(\frac{4n}{9}\right) + \left(\frac{2n}{3}\right) + n)$$

Now that we've done three iterations, we can guess the form of the recurrence after k iterations:

$$T(n) = 2^k T\left(\frac{n}{3^k}\right) + cn(\sum_{i=0}^{k-1}(\frac{2}{3})^i)$$

We want to plug in a value of k to this formula such that $\frac{n}{3^k} = 1$, which occurs when $n = 3^k$. By definition of log, we have that $k = log_3 n$. We will bound the summation by taking it to infinity instead of k-1:

$$T(n) \leq 2^{log_3 n} T(1) + cn(\sum_{i=0}^{\infty}(\frac{2}{3})^i)$$

Now, we can use both given hints to arrive at:

$$T(n) \leq n^{log_3 2} + 3cn = O(n)$$

Note that $\log_3 3 = 1$, so if follows that $\log_3 2 < 1$. Thus, the dominant term is 3cn, which is O(n).

**Grading: Part A - 1 pt for restating original recurrence, 1 pt for getting to second iteration, 2 pts for getting to third iteration, 2 pts for the correct guess of the general form after k iterations, 1 pt for getting the appropriate value of k to plug in, 2 pts to properly simplify both terms, 1 pt to decide which of the two terms is dominant and give the final answer.**