3) (10 pts) ANL (Summations)

What is the closed form of the following summation? Your solution should be a function in terms of n. For full credit work must be shown.

$$\sum_{i=0}^{n}\sum_{j=0}^{i} 2^j$$

$$\sum_{i=0}^{n}\sum_{j=0}^{i} 2^j = \sum_{i=0}^{n}(2^{i+1}-1)$$

$$= \sum_{i=0}^{n} 2^{i+1} - \sum_{i=0}^{n} 1$$

$$= \sum_{i=0}^{n} (2)2^i - (n+1)$$

$$= 2\sum_{i=0}^{n} 2^i - (n+1)$$

$$= 2(2^{n+1}-1) - (n+1)$$

$$= 2^{n+2} - 2 - n - 1$$

$$= \mathbf{2^{n+2} - n - 3}$$

Grading: 3 pts inner sum
1 pt split
1 pt second sum
4 pts first sum (lots of ways to break this down)
1 pt simplification at the end