3) (10 pts) ANL (Summations and Recurrence Relations)

Determine the following summation in terms of n (assume n is a positive integer 2 or greater), expressing your answer in the form $an^3 + bn^2 + cn$, where a, b and c are rational numbers. (Hint: Try rewriting the summation into an equivalent form that generates less algebra when solving.)

$$\sum_{i=n^2-3}^{n^2+n-4} (i+4)$$

To simplify the algebra, do an index shift. Notice that the terms getting added are actually $n^2 + 1$, $n^2 + 2$, …, $n^2 + n$:

$$\sum_{i=n^2-3}^{n^2+n-4} (i+4) = \sum_{i=1}^{n} (n^2 + i)$$

$$= \left(\sum_{i=1}^{n} n^2\right) + \left(\sum_{i=1}^{n} i\right)$$

$$= n(n^2) + \frac{n(n+1)}{2}$$

$$= n^3 + \frac{1}{2}n^2 + \frac{1}{2}n$$

Grading: 4 pts for a correct index shift (give partial as necessary), 1 pt for splitting the sum correctly, 2 pts for the sum of the constant, 3 pts for the sum of i.

If they don't do the index shift, then they are likely to be subtracting two sums. 3 pts for each of the two sums, 4 pts for the algebra of subtracting those sums.

If they try something else, try your best to map points to one of these two schemes.