3) (10 pts) ANL (Recurrence Relations)

Use the iteration technique to solve the following recurrence relation in terms of n:

$$T(n) = 3T(n-1) + 1, for\ all\ integers\ n > 1$$
$$T(1) = 1$$

Please give an exact closed-form answer in terms of n, instead of a Big-Oh answer.

(Note: A useful summation formula to solve this question is $\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$.)

$$T(n) = 3T(n-1) + 1$$

$$= 3(3T(n-2) + 1) + 1$$

$$= 9T(n-2) + 3 + 1$$

$$= 9(3T(n-3) + 1) + 3 + 1$$

$$= 27T(n-3) + 9 + 3 + 1$$

After k steps, we have: $= 3^k T(n-k) + \sum_{i=0}^{k-1} 3^i$

Let k = n-1, then we have that $T(n) = 3^{n-1} T\big(n-(n-1)\big) + \sum_{i=0}^{n-2} 3^i$

$$= 3^{n-1}T(1) + \sum_{i=0}^{n-2} 3^i$$

$$= 3^{n-1} + \sum_{i=0}^{n-2} 3^i$$

$$= \sum_{i=0}^{n-1} 3^i$$

$$= \frac{3^n - 1}{3-1} = \frac{3^n - 1}{2}$$

Grading: 2 pts for iteration with T(n-2), 2 pts for iteration with T(n-3), 2 pts for general guess after k steps. 1 pt for plugging in k = n-1, 3 pts for simplifying that to the final answer.