

3) (5 pts) ANL (Recurrence Relations)

Using the iteration technique, just solve for the **next two** iterations of the following recurrence relation:

$$T(n) = 3T(n-1) + n^2, \text{ for integers } n > 0$$
$$T(0) = 1$$

Your answers should be of the form

$$T(n) = aT(n-2) + bn^2 - cn + d \text{ and}$$

$$T(n) = eT(n-3) + fn^2 - gn + h, \text{ where } a, b, c, d, e, f, g, \text{ and } h \text{ are positive integers.}$$

$$T(n) = 3(3T(n-2) + (n-1)^2) + n^2$$

$$T(n) = 9T(n-2) + 3n^2 - 6n + 3 + n^2$$

$$T(n) = 9T(n-2) + 4n^2 - 6n + 3$$

It follows that  $a = 9$ ,  $b = 4$ ,  $c = -6$ , and  $d = 3$

Now, plug in one more iteration

$$T(n) = 9(3T(n-3) + (n-2)^2) + 4n^2 - 6n + 3$$

$$T(n) = 27T(n-3) + 9n^2 - 36n + 36 + 4n^2 - 6n + 3$$

$$T(n) = 27T(n-3) + 13n^2 - 42n + 39$$

It follows that  $e = 27$ ,  $f = 13$ ,  $g = -42$  and  $d = 39$

**Grading: 2 pts first iteration, (give 1 pt if some terms correct)**

**3 pts second iteration (give 1 pt if 1 or 2 terms correct,**

**2 pts if 3 terms correct)**

**Take off 1 pt if they leave 1<sup>st</sup> unsimplified:  $9T(n-2) + 3(n-1)^2 + n^2$**

**Take off 1 pt if they leave 2<sup>nd</sup> unsimplified:  $27T(n-3) + 9(n-2)^2 + 3(n-1)^2 + n^2$**