2) (10 pts) ANL (Algorithm Analysis)

A program takes $O(n^3)$ time to process data about a map with **n** points of interest. For n = 100, the program completes in 25 milliseconds. The time limit for running the program has been set at a maximum of 12.8 seconds. What is the largest value of **n** for which the program is expected to complete within the time limit?

Let the run time of the program on a map with n points be $T(n) = cn^3$, for some constant c. Using the given information, we have:

$$T(100) = c(100)^3 = 25ms$$

$$c = \frac{25}{10^6} ms$$

Now, we want to find a value of n, such that $T(n) \leq 12.8$ seconds. Since we want to maximize n, set both sides equal to each other, converting 12.8 seconds to 12800 ms.

$$T(n) = \frac{25ms}{10^6} n^3 = 12800ms$$

$$n^3 = \frac{128 \times 10^2 \times 10^6}{5^2}$$

$$n^3 = \frac{2^7 \times 10^8}{5^2} = \frac{2^7 \times 2^8 \times 5^8}{5^2} = 2^{15}5^6$$

Taking the cube root of both sides, we get:

$$n^3 = 2^5 \times 5^2 = 32 \times 25 = 8 \times (4 \times 25) = \mathbf{800}$$

Grading: 1 pt setting up equation for c, 2 pts solving for c (without simplification)
2 pts setting up equation with n, 1 pt converting 12.8 to 12,800 or equivalent conversion
3 pts for algebra to get to answer
1 pt for final answer
If they get to cube root of 512,000,000, then give 8/10 (so -2 total of the last 4 pts)

Note: Please give full credit to 799, just in case someone used a strictly less than sign.