3) (10 pts) ANL (Summations)

Using the fact that if $x \neq 1$, then $\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$, for positive integers n, determine the following summation, in terms of n (assume n is a positive integer):

$$\sum_{i=2n+1}^{3n} 4^i$$

First, notice that we can factor out 4^{2n+1} from each term of our summation. Next, we can re-index the summation by noticing that inside the new sum, the terms are $4^0 + 4^1 + \ldots + 4^{n-1}$. Formally, we set j = i - (2n+1).

$$\sum_{i=2n+1}^{3n} 4^i = 4^{2n+1} \sum_{i=2n+1}^{3n} 4^{i-(2n+1)}$$

$$= 4^{2n+1} \sum_{j=0}^{n-1} 4^j$$

$$= 4^{2n+1}\left(\frac{4^n - 1}{4 - 1}\right)$$

$$= \frac{4^{3n+1} - 4^{2n+1}}{3}$$

Another way to solve the sum is to take the sum from i=1 to 3n, and subtract from it the same sum form i=1 to 2n. If we proceed in this way, we'll get $\frac{4^{3n+1}-1}{4-1} - \frac{4^{2n+1}-1}{4-1}$, after evaluating both sums. Notice that both terms equal to one-third (first -, second +) cancel out and that we arrive at the same answer as above.

Grading Method #1: 3 pts factor out, 3 pts rewrite sum, 3 pts apply formula, 1 pt final answer (note, final answer can be factored form instead.)

Grading Method #2: 2 pts split sum, 3 pts apply formula first sum, 3 pts apply formula second sum, 2 pts algebra to get to final answer