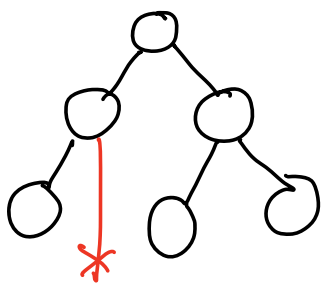


Heaps

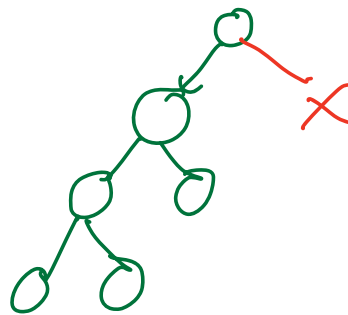
Complete Binary Tree (Proc - req)

A BT is a CBT if ;

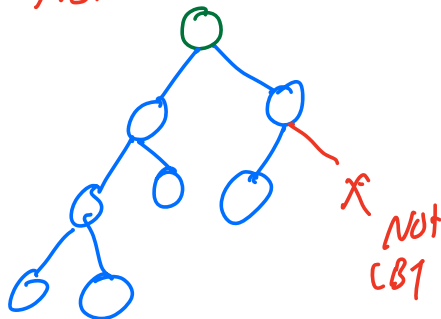
- 1) A BT is said to be CBT if
 - i) All nodes have to be filled level by level from left \rightarrow right
 - ii) At all levels it should be completely filled except last level (we can either fill it or not)



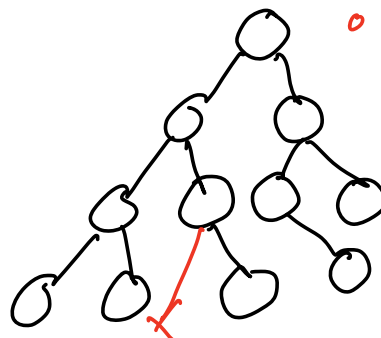
Not CBT



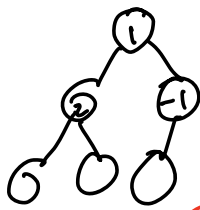
Not a CBT



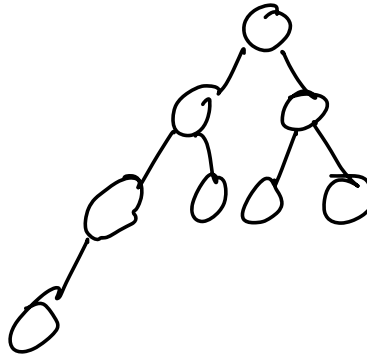
Not CBT



Not a CBT



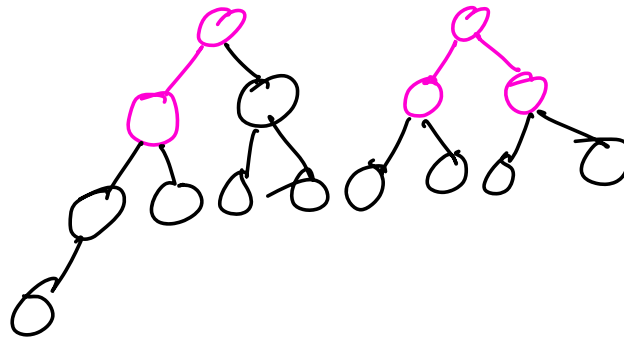
CBT ✓



CBT ✓

Balance Binary tree: ~~CBT~~ CBT
for every Node.

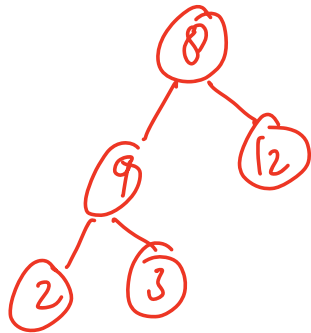
Height of CBT	Min Nodes	Max Nodes
1	2 : 2^1	3 : $2^2 - 1$
2	4 : 2^2	7 : $2^3 - 1$
3	8 : 2^3	15 : $2^4 - 1$
h	2^h	$2^{h+1} - 1$



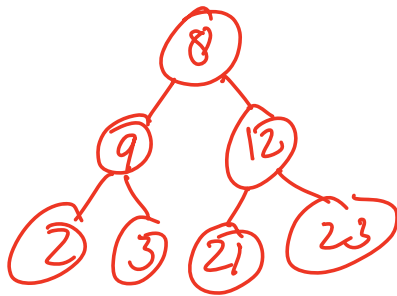
$$\begin{aligned}
 H \rightarrow 2^h = N &\Rightarrow h = O(\log_2 N) \\
 H \rightarrow 2^{h+1} - 1 = N &\Rightarrow h = O(\log_2 N)
 \end{aligned}$$

Height of N nodes CBT is $O(\log_2 N)$

Implementation of CBT



8, 9, 12, 2, 3, 21, 23, 26



Whenever a new node is created
insert in a queue, &
delete from front of queue
only if both left & right
child are filled

TC: $O(N)$

SC: $O(N)$

2 Problems:

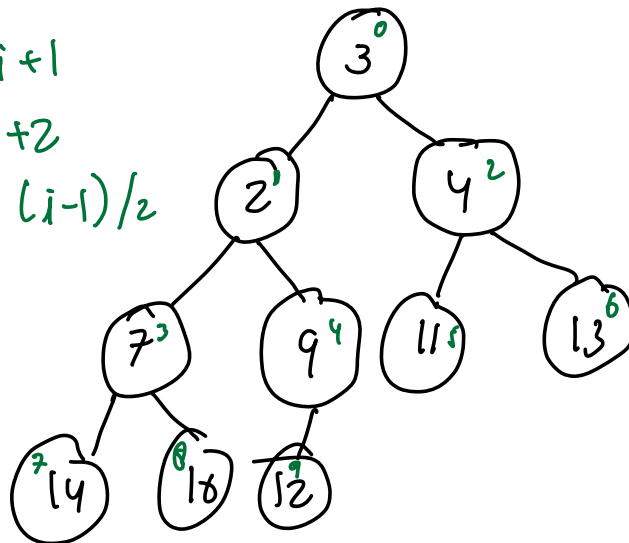
→ Extra space

→ Node → parent is tough

Arrays

list C++
 $0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9$
 $3, 2, 4, 7, 9, 11, 13, 14, 15, 12$

$l: 2i+1$
 $r: 2i+2$
 $\text{parent}: (i-1)/2$

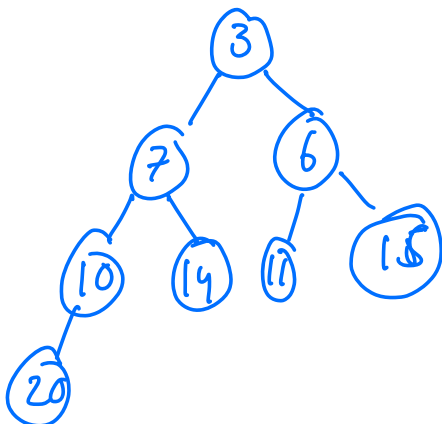


node	l	right
0	1	2
1	3	4
2	5	6

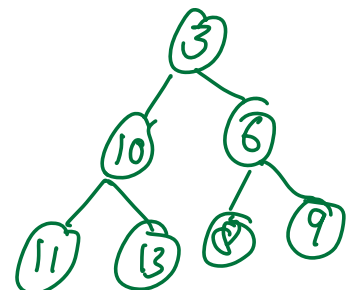
$i/2$

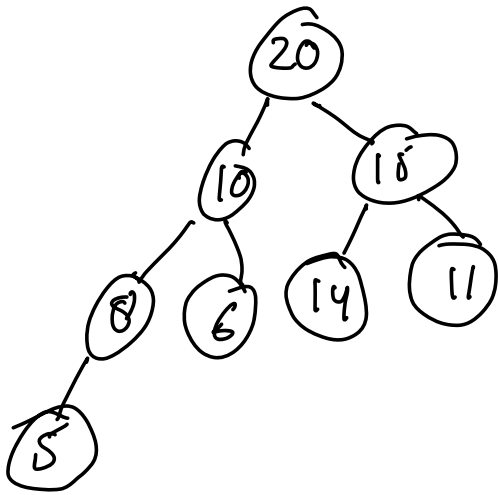
Heap:

- 1) CBT
- 2) (i) for any node $>$ both child : Maxheap
- (ii) for any node $<$ both child : Minheap



: min heap



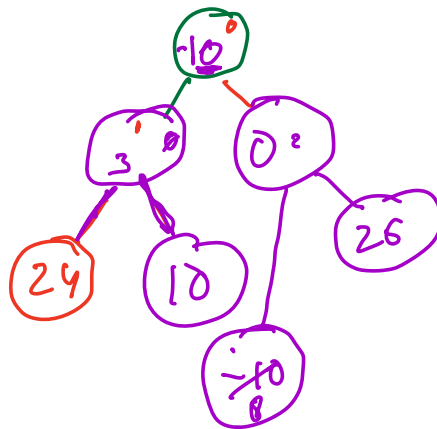


: maxHeap

Min heap:

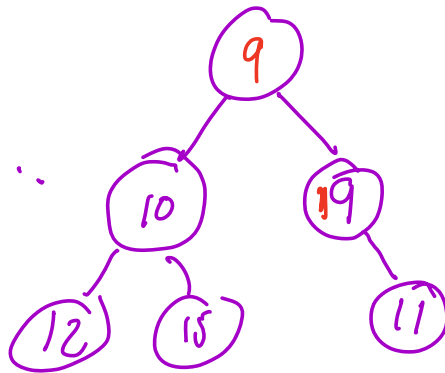
10 , 3 , 8 , 24 , 0 , -10 , 26

⁰
~~10~~
 3
 8
 -10
¹
~~3~~
 10
 3
²
~~8~~
 0
³
 24
⁴
~~0~~
 10
⁵
~~-10~~
 8
⁶
 26
 Min heap



⁰10, ¹3, ²8, ³24, ⁴0, ⁵-10, ⁶26
₃₀ ₁₀ ₀ ₁₀

min Heap



Break: 8:52

10, 3, 8, 24, 0, -10, 26
 arr → 10, 3

Pseudo Code

```

void Insert ( List<int> arr, int ele )
{

```

```

    arr.add ( ele )

```

```

    int ind = arr.length - 1;

```

```

    int par = (ind - 1) / 2;

```

```

    while ( ind != 0 & arr[par] > arr[ind] )
    {

```

```

        swap ( arr[ind], arr[par] )

```

```

        ind = par;

```

```

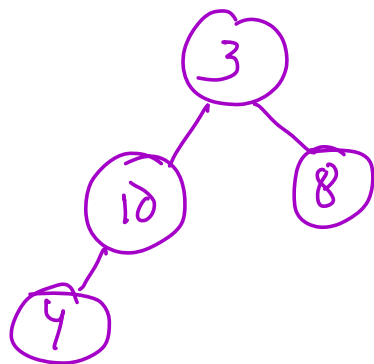
        par =  $\frac{(ind - 1)}{2}$ ;
    }

```

} }

10

arr \rightarrow $\begin{matrix} 0 & 1 & 2 & 3 \\ 3 & 4 & 8 & 10 \\ 1 & 3 & & \end{matrix}$



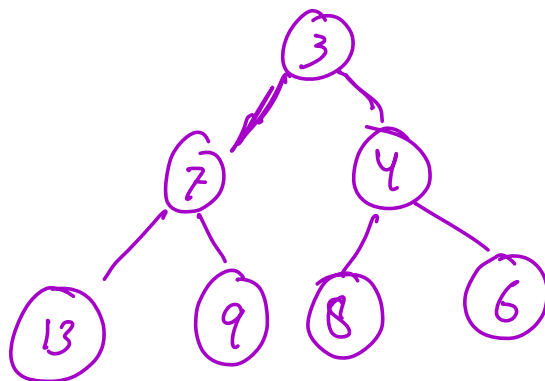
insert $\rightarrow O(\log N)$

getMin $\rightarrow O(1)$

search $\rightarrow O(N)$

delete Min / Max $\rightarrow O(\log N)$

delete Random $\rightarrow O(N)$



Delete

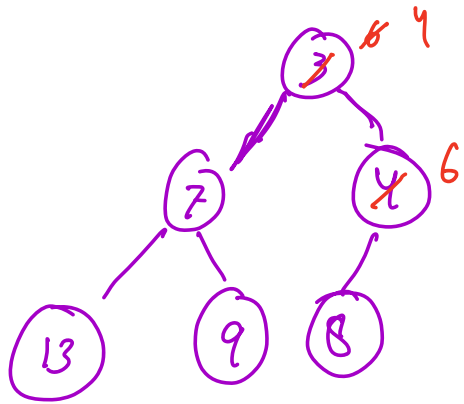
$O(\log N)$

3, 7, 4, 13, 9, 8, 6

- ① Swap root & last ele
- ② Delete last element
- ③ Propagate down

$N \rightarrow n \log N$
 \Downarrow

Not optimal
 insert Heap $\rightarrow O(N)$



4, 6, 9
 3, 7, 4, 13, 9, 8,

