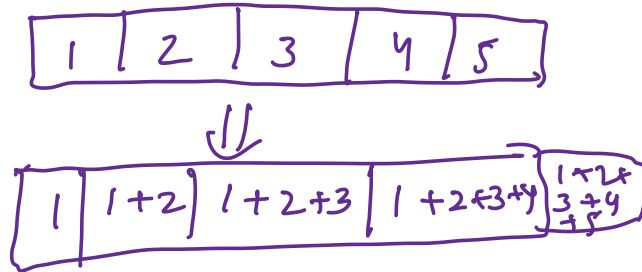


Arrays 2

Q \Rightarrow Prefix Sum



a_0	b_0	c_0
a_1	b_1	c_1
a_2	b_2	c_2

\Rightarrow

a_0	$a_0 + b_0$	$a_0 + b_0 + c_0$
$a_0 + a_1$	$a_0 + b_0 + a_1 + b_1$	$a_0 + b_0 + c_0 + a_1 + b_1 + c_1$
$a_0 + a_1 + a_2$	$a_0 + b_0 + a_1 + b_1 + a_2 + b_2$	$a_0 + b_0 + c_0 + a_1 + b_1 + c_1 + a_2 + b_2 + c_2$

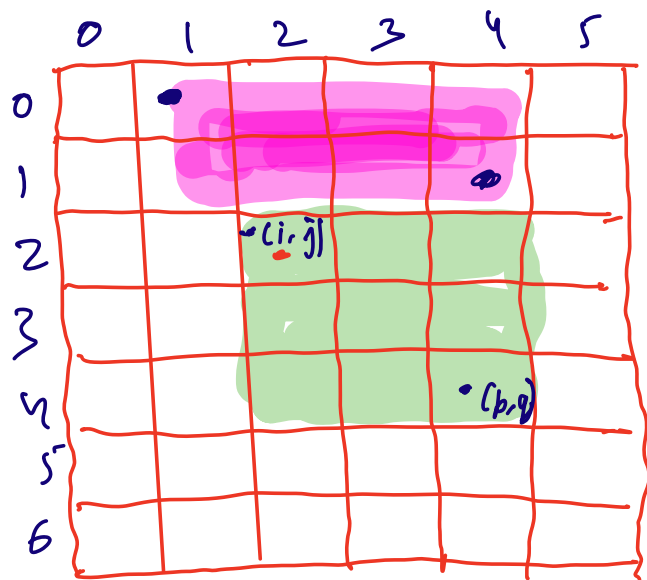
\Downarrow Row PS

	0	1	2
0	a_0	$a_0 + b_0$	$a_0 + b_0 + c_0$
1	a_1	$a_1 + b_1$	$a_1 + b_1 + c_1$
2	a_2	$a_2 + b_2$	$a_2 + b_2 + c_2$

PF (1, 2)

Col PS

Q \Rightarrow Given a matrix of size $N \times M$ & Q queries



TL
(i, j)

BR
(p, q)

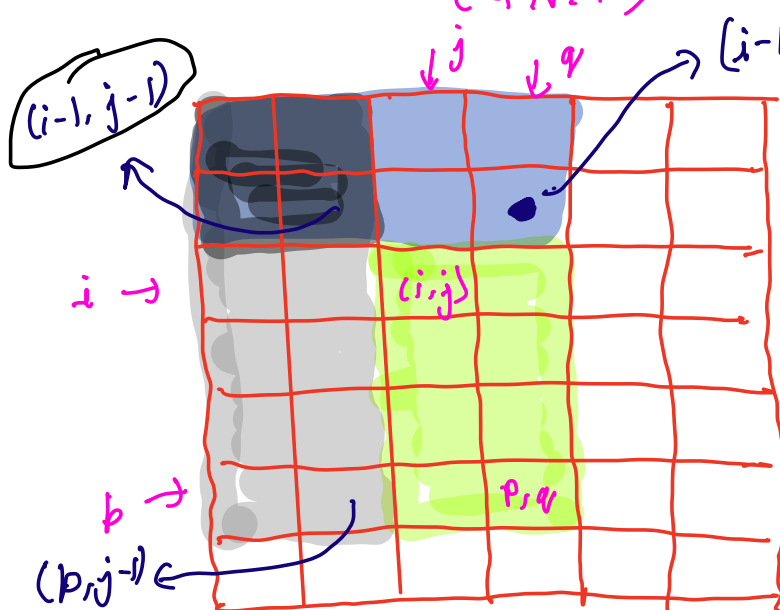
(0, 1)

(1, 4)

Q \rightarrow for every query $\} \Rightarrow Q$
sum = 0

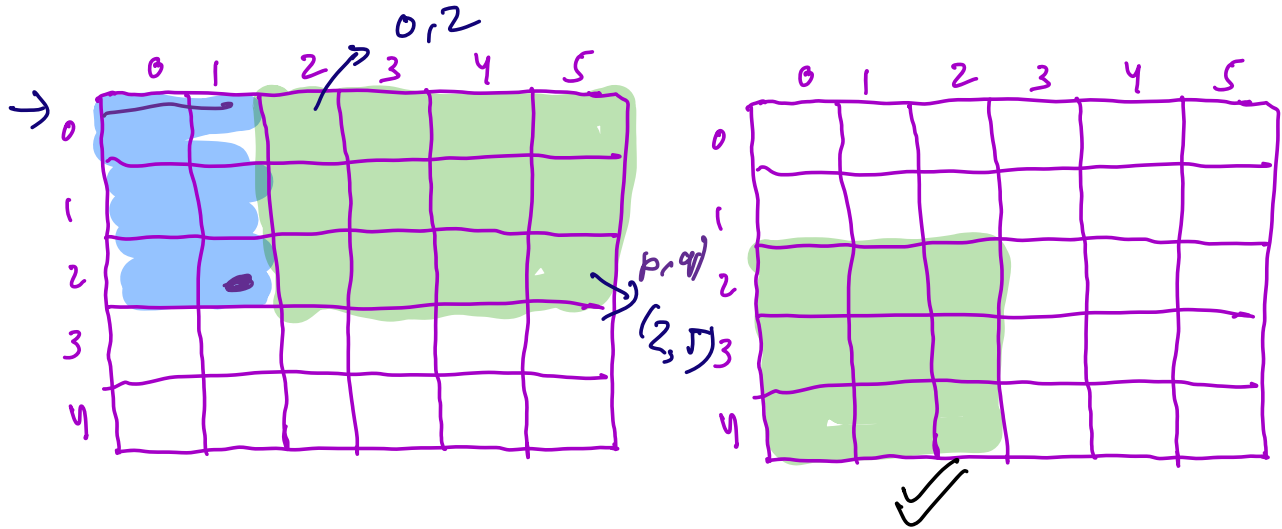
N \rightarrow for (row = i; row <= p; row++)
M \rightarrow for (col = j; col <= q; col++)
sum += A[row][col];

TC: $O(QNM)$



PS (P, q)

Sum from (i, j) to $(p, q) \Rightarrow PS[p][q] - PS[i-1, q] - PS[p][j-1] + PS[i-1][j-1]$

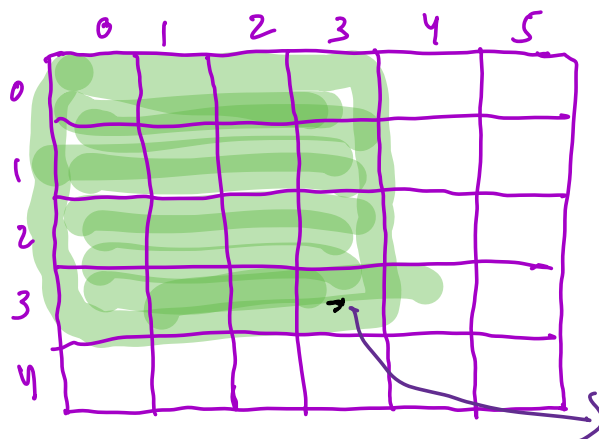


if $(i == 0)$

$$sum = PS[p, q] - PS[p][j-1]$$

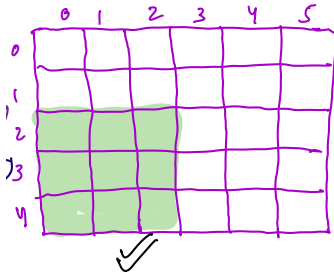
if $(j == 0)$

$$sum = PS[p, q] - PS[i-1][q]$$



$$PS[p][q]$$

Code:



→ (a_1, b_1) (a_2, b_2)

→ $sum = PF[a_2][b_2]$

if $(b_1 > 0)$

{
 $sum = sum - PF[a_2][b_1 - 1];$
 }

if $(a_1 > 0)$

$sum = sum - PF[a_1 - 1][b_2];$

if $(a_1 > 0 \& \& b_1 > 0)$

$sum = sum + PF[a_1 - 1][b_1 - 1]$

PFS $\Rightarrow O(NM)$

TC: $O(Q + NM)$

Google

FBQ \Rightarrow

Given a Matrix of size $N * M$
 calculate sum of all submatrix sum

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 \end{bmatrix}$

$\begin{bmatrix} 1 \end{bmatrix}$

$\begin{bmatrix} 1 \end{bmatrix}$

$\begin{bmatrix} 1 \end{bmatrix}$

$+ \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow 16$$

$$\begin{bmatrix} 4 & 9 & 6 \\ 5 & -1 & 2 \end{bmatrix}$$

2 ✖

$$[4] \rightarrow 4$$

$$[4 \ 9] \rightarrow 13$$

$$\begin{bmatrix} 4 \\ 5 \end{bmatrix} \rightarrow 9$$

$$\begin{bmatrix} 4 & 9 \\ 5 & -1 \end{bmatrix} \rightarrow 17$$

$$[9] \rightarrow 9$$

$$[9 \ 6] \rightarrow 15$$

$$\begin{bmatrix} 9 \\ -1 \end{bmatrix} \rightarrow 8$$

$$\begin{bmatrix} 9 & 6 \\ -1 & 2 \end{bmatrix} \rightarrow 16$$

$$[6] \rightarrow 6$$

$$[5 \ -1] \rightarrow 4$$

$$[5] \rightarrow 5$$

$$[-1 \ 2] \rightarrow 1$$

$$[-1] \rightarrow -1$$

$$[4 \ 9 \ 6] \rightarrow 19$$

$$\begin{bmatrix} 6 \\ 2 \end{bmatrix} \rightarrow 8$$

$$\begin{bmatrix} 4 & 9 & 6 \\ 5 & -1 & 2 \end{bmatrix} \rightarrow 25$$

$$[2] \rightarrow 2$$

$$[5 \ -1 \ 2] \rightarrow 6$$

$$\text{Sum} = 166$$

$$4 \times 6 + 9 \times 8 + 6 \times 6 + 5 \times 6 + (-1) \times 8 + 2 \times 6 = 166$$

	0	1	2	3	4	5
0	✓	✓	✓	✓		
1	✓	✓	✓	✓		
2	✓	✓	✓	✓	✓	✓
3				✓	✓	✓
4				✓	✓	✓

TL

BR

$$(0,0) \rightarrow 9$$

$$(0,1) \rightarrow 9$$

$$(0,3) \rightarrow 9$$

$$100 * \text{Add}[2][3]$$

$$12 * 9 = 108$$

	0		j		M-1	
0	✓	✓	✓	✓		
	✓	✓	✓	✓		
	✓	✓	✓	✓		
i	✓	✓	✓	✓	✓	✓
				✓	✓	✓
				✓	✓	✓
N-1				✓	✓	✓

BR:

$$\text{rows} \rightarrow [i, N-1]$$

$$N-1 - i + 1$$

$$N-i$$

$$\text{cols} \rightarrow [j, M-1]$$

$$M-1 - j + 1$$

$$M-j$$

$$\text{cells} : (N-i) * (M-j)$$

TL: rows $\rightarrow [0 \dots i]$ $i-0+1$

cols $\rightarrow [0 \dots j]$ $j-0+1$

$$\text{cells} = (i+1) * (j+1)$$

\Rightarrow Total submatrix in which i, j shall be present

$= \text{TL} * \text{TR}$

cont : $(\text{TL} * \text{BR}) * \text{mat}[i][j]$

Break: 9:40

ans = 0

```
for (i = 0; i < N; i++)
```

```
{
    for (j = 0; j < M; j++)
```

```
    → TL = (i+1) * (j+1)
```

```
    BR = (N-i) * (M-j)
```

```
    total = TL * BR
```

```
    ans += (total * mat[i][j]);
```

```
}
```

TC: $O(N * M)$

SC: $O(1)$

⇒ Given n array elements, calculate
max subarray sum

ex: { 3, 2, -6, 8, 2, 9, 4 }

arr[7] = { -3, 2, 4, -1, 3, -4, 3 }

Approach 1 :

all sub array
& get the sum
 $O(N^3)$

App 2 : Prefix sum / Contribution Tech
 $O(N^2)$

App 3 :
c1 : ≥ 0

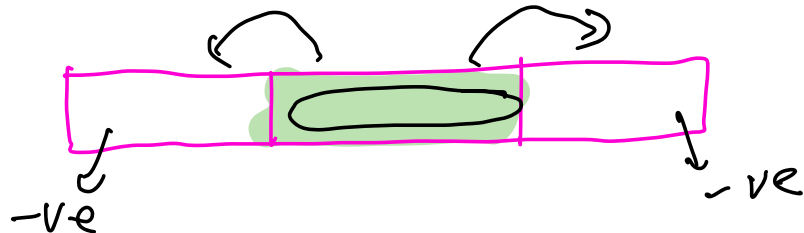
3	2	1	6
---	---	---	---

 = 12

case 2 : < 0

-8	-4	-2	-10
----	----	----	-----

case 3 :



case 4 :



~ [~ ~ ~] +ve
 ~~~~~  
 max

~ [ ~ ~ ~ ] ~~~~~  
 ~~~~~  
 max

6 -1 5 10
 5
 -ve +ve

~ [~ ~ ~] ~~~~~
 ~~~~~  
 max

6 -7 10  
 -ve +ve  
 10  
 +ve  
 9

1, 2, 3, -1, 10 ✓  
 ~~~~~  
 6 (5) 15

1, 2, 3, -7, 10 ✓
 ~~~~~  
 6 -7 10 ✓  
 ~~~~~  
 10 → 10
 ~~~~~  
 9

|          |   |    |    |    |    |     |                 |    |    |    |    |    |
|----------|---|----|----|----|----|-----|-----------------|----|----|----|----|----|
|          | 5 | 6  | 7  | -3 | 2  | -10 | -12             | 0  | 12 | 21 | -4 | 7  |
| curr-sum | 5 | 11 | 18 | 15 | 17 | 7   | -5 <sup>0</sup> | 8  | 20 | 41 | 37 | 44 |
| max-sum  | 5 | 11 | 18 | 18 | 18 | 18  | 18              | 18 | 20 | 41 | 41 | 44 |

|          |                             |    |                             |                             |    |    |    |    |    |
|----------|-----------------------------|----|-----------------------------|-----------------------------|----|----|----|----|----|
|          | -20                         | 10 | -20                         | -12                         | 6  | 5  | -3 | 0  | -2 |
| curr-sum | <del>-20</del> <sup>0</sup> | 10 | <del>-10</del> <sup>0</sup> | <del>-12</del> <sup>0</sup> | 16 | 11 | 8  | 16 | 14 |
| max-sum  | <u>-20</u>                  | 10 | 10                          | 10                          | 16 | 16 | 16 | 16 | 16 |

maxSum = A[0]

for (i = 0; i < N; i++)

sum += A[i];

maxSum = max(sum, maxSum);

if (sum < 0)

sum = 0;

}

return maxSum;

}

TC:  $O(N)$

SC:  $O(1)$

$$\{-20, -163\}$$

$$\downarrow$$

$$-20$$

$$\begin{array}{r} -20, -163 \\ -20^0, -163 \\ -20, -20 \end{array}$$

shivank agrawal\_1 @scudexon

$$\{ \underbrace{2, 4}_{6}, \underbrace{-7}_{-1}, \underbrace{10}_{9} \}$$

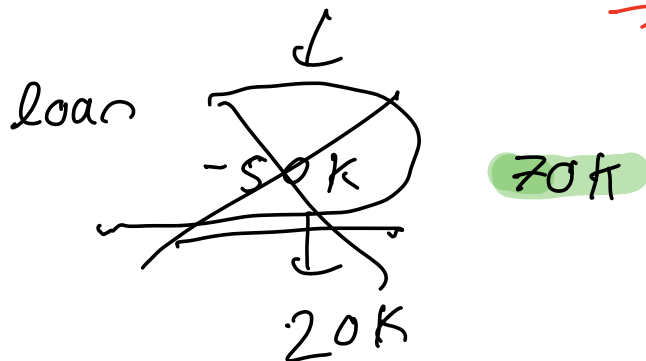
$$\quad \quad \quad 0 \quad \quad \quad 10$$

$$\left\{ \begin{array}{l} -20, 10, -20, -12, 6, 5, -3, 0, -2 \\ \text{cum-sum} \quad -20^0, 10, -10^0, -12^0, 6, 11, 0, 16, 14 \\ \text{max-sum} \quad -20, 10, 10, 10, 10, 11, 11, 16, 16 \end{array} \right.$$

$$\text{cum-sum} \quad -20^0 \quad (-10)$$

$$\text{max} \rightarrow \min \left\{ \begin{array}{l} -20 \\ -10 \end{array} \right.$$

$$\begin{array}{c} \boxed{\{-20, 10\}} \Rightarrow 10 \\ \downarrow \quad \downarrow \\ \{-20\} \quad \{10\} \quad \{-20, 10\} \\ \downarrow \quad \downarrow \quad \downarrow \\ \text{max} \left( \begin{array}{cc} -20 & 10 \\ -10 \end{array} \right) \\ \Rightarrow 10 \end{array}$$



$$\text{maxsum} = A[0]$$

$$\text{sum} = 0 \quad i = 0 \rightarrow n$$

$$j = i \rightarrow n$$

$$\text{sum} = \text{sum} + A[j]$$

$$\text{maxsum} = \underline{\hspace{2cm}}$$

max

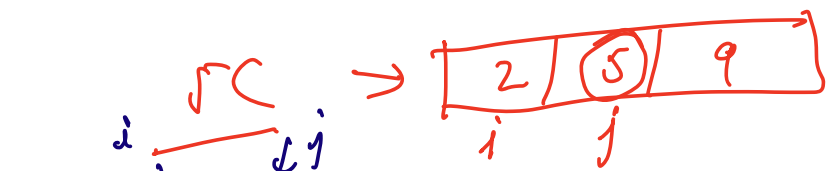
$$i = 0 \rightarrow N$$

$$j = i \rightarrow N$$

$$\text{sum} = 0;$$

prejst  $\rightarrow$   $\left\{ \begin{array}{l} k = i \rightarrow j \\ \text{sum} = A[k] \end{array} \right\} \rightarrow O(1) \text{ using PS}$

$$\{ \overset{i}{2}, \overset{j}{3}, 4 \}$$



$$PS \Rightarrow \{2, 5, 9\}$$

$$K=3 \rightarrow 4$$

$$PF[1] \rightarrow PF[2] \rightarrow$$

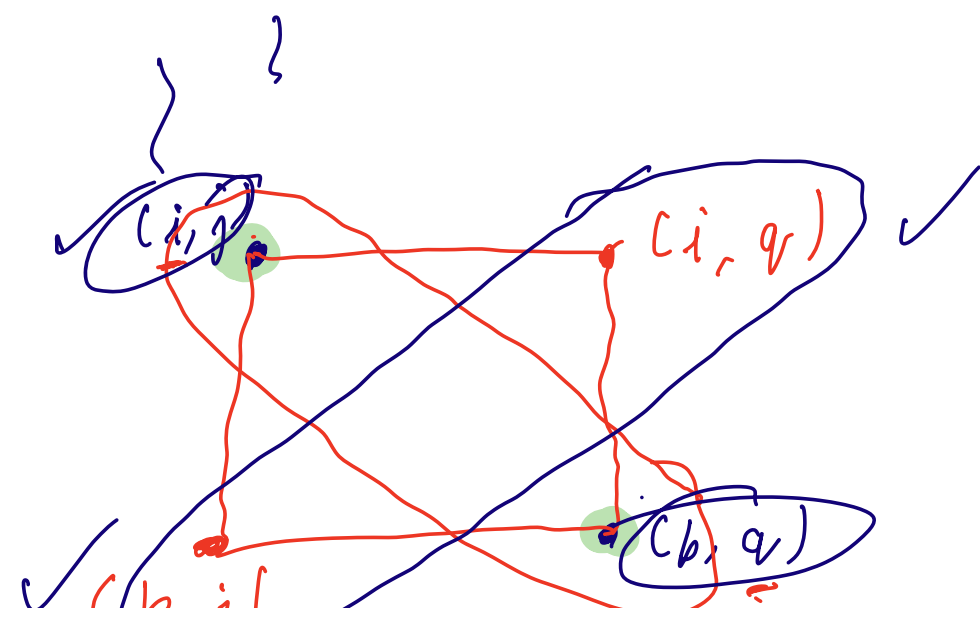
$$\rightarrow \text{for } i=0, i < N;$$

$$\rightarrow \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \\ \hline \end{array}$$

$$\text{for } j=0, \dots$$

$$1 \quad \textcircled{j+2} \quad \underline{j+2}$$

$$PF[i] = arr[i][j] + \text{PF}[i][j-1]$$



$\underbrace{(x, y)}$

—

•  $(i, q)$

•  $(b, j)$

